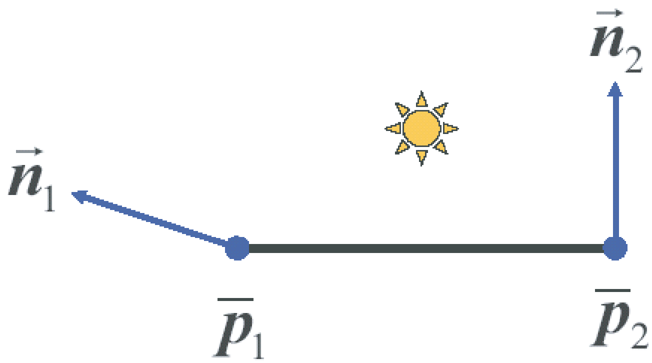
CSCD18 Computer Graphics, Fall 2007
Assignment 3

Part B, Written: Due at the drop-box by 11:59pm on Monday, November 26 [50 marks]

1. *[17 marks]* Suppose that we are doing lighting and shading in 2D (for simplicity) in the scene illustrated bellow. The scene consists of a single line segment, illustrated by the dark black line, with normals at the two vertices and a single light source.



   (a) *[12 marks]* Describe how the line segment will look shaded by each one of the following models: Flat, Phong, and Gouraud. In each of these three cases tell where the brightest point will be and why. For simplicity assume that the properties of the line are such that it does not reflect any ambient or specular light (*i.e.* only consider diffuse component). 1-2 sentences for each one of the models will suffice.

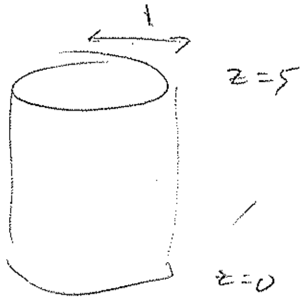
   Solution:

   - **Flat -** In flat shading the entire segment will be lit with the same color, so there will not be a single brightest point. The radiance (or color) of the segment will be a function of the average of the two normals, $\vec{n}_1$ and $\vec{n}_2$, which will correspond to the normal at the center of the segment.
   - **Gouraud -** In gouraud shading segment will have the radiance interpolated from one vertex to the other linearly, hence the brightest point must occur at one of the two vertices. Since the dot product of the normal $\vec{n}_1$ at $\bar{p}_1$ with the vector from $\bar{p}_1$ to the light source is < 0, the contribution of the diffuse component will be 0 as well, and $\bar{p}_1$ will be assigned 0 radiance (black color). The dot product of the normal $\vec{n}_2$ at $\bar{p}_2$ with the vector from $\bar{p}_2$ to the light source is > 0, and hence at $\bar{p}_2$ we will have non-zero radiance. Consequently because of this $\bar{p}_2$ will be the brightest point.
   - **Phong -** In Phong shading the normals themselves are interpolated, and hence the brightest point can occur not just at the vertex. In particular the brightest point, in this case, will occur somewhere between $\bar{p}_1$ and $\bar{p}_2$ (closer to $\bar{p}_2$), where the dot product, between the interpolated normal and the vector from the point at which the interpolated normal is computed to the lightsource, is maximized.

(b) *[5 marks]* Now consider the same scenario, but with different material properties for the line, where specular component is also present. For simplicity assume that we have a very narrow specularity. Which of the three models will be effected by this change? Why?

Solution: Both flat shading and the Gouraud shading will be unaffected (to the large extent) by this change in the material properties. The Phong shading, however, will be effected and will result in a very differently shaded light segment. In particular, the brightest point on the segment will now correspond to the point of secularity, which will be a function of the eye position.

2. *[20 marks]* Derive an intersection algorithm for ray with a cylindrical *vase*. Assume that the cylinder (in the form of which the *vase* is made) has an equation of $x^2 + y^2 = 1$ for $0 \leq z \leq 5$. Since the surface is a *vase*, it has a single cap at the bottom (at $z = 0$) and an open top. First, derive the equation for the *hit point(s)*. Second, enumerate all the special cases, and describe what needs to be done for each one (*i.e.* is the *hit point* visible, which of the *hit points* needs to be rendered, *etc.*). Note that we have done this in class for a sphere. (Hint: the basic algorithm for doing this was already discussed in class and is outlined in the lecture notes; all you need to do is fill in the missing details).



intersect w/ tube $x^2+y^2=1$

$$r(\lambda) = [p_x, p_y, p_z] + \lambda [d_x, d_y, d_z]$$

$$(p_x+\lambda d_x)^2 + (p_y+\lambda d_y)^2 = 1$$

$$p_x^2 + 2\lambda d_x p_x \;\lambda^2 d_x^2 + p_y^2 + 2\lambda d_y p_y + \lambda^2 d_y^2 = 1$$

$$\underbrace{p_x^2+p_y^2-1}_{c} + \lambda \underbrace{2(d_x p_x + d_y p_y)}_{b} + \lambda^2 \underbrace{(d_x^2+d_y^2)}_{a} = 0$$

$$\lambda = \frac{-2(d_x p_x + d_y p_y) \pm \sqrt{4(d_x p_x + d_y p_y)^2 - 4(d_x^2+d_y^2)(p_x^2+p_y^2-1)}}{2(d_x^2+d_y^2)}$$

$$\text{if} = \frac{(d_x p_x + d_y p_y) \pm \sqrt{(d_x p_x + d_y p_y)^2 - (d_x^2+d_y^2)(p_x^2+p_y^2-1)}}{(d_x^2+d_y^2)}$$

if term under sqrt is negative : no intersection
positive : two intersections
zero : grazes.

take candidate λs found and verify z component is within 0..5.

i.e. take z component of $r(\lambda_i)$, λ_i candidate found, check $0 \le r_z(\lambda_i) \le 5$.

next, intersect w/ plane. i.e. $z=0$

$r_z(\lambda)=0$

$p_z+\lambda d_z=0$
$$\lambda = \frac{-p_z}{d_z}$$

so
$$x(\lambda) = p_x - p_z \frac{d_x}{d_z}$$
$$y(\lambda) = p_y - p_z \frac{d_y}{d_z}$$

and ensure it lies within the circular profile of the tube: i.e. $x^2+y^2 \leq 1$.

verify $\left(p_x - p_z \frac{d_x}{d_z}\right)^2 + \left(p_y - p_z \frac{d_y}{d_z}\right)^2 \leq 1$.
discard this λ if it fails.

Finally, collect all remaining lambdas and pick the smallest one to find the first hit point.

If none left, ray misses the cylinder.

3. *[13 marks]* Recall that the radiant intensity I of a point light source is defined as the flux per solid angle: $I = d\phi/d\omega$. Solid angle is measured in steradians; radiant intensity is measured in Watts per steradian.

   (a) *[4 marks]* Consider a spotlight modeled as a point light source that radiates light over a restricted solid angle of c steradians. Derive the total power output of this spotlight, as measured in Watts. What is the power if $I = 6W/sr$ and $c = 1sr$?

   Solution:

$$\Phi = \int d\Phi = \int I d\omega = I \int d\omega = I\omega = Ic \tag{1}$$

   If $I = 6W/sr$ and $c = 1sr$, then $\Phi = 6W$.

   (b) *[4 marks]* Suppose the spotlight is placed at the center of a sphere with radius r. Derive the surface area of the region of the sphere illuminated by the spotlight.

   Solution: $\omega = A/r^2$, hence, $A = \omega r^2 = cr^2$

   (c) *[5 marks]* What is the irradiance $H(\bar{p})$ at a surface point $\bar{p}$ inside the region of the sphere illuminated by the spotlight? You may assume that the inward-facing surface normal $\vec{n}$ at this point is given.

   Solution:

$$H = \frac{d\phi}{dA} = \frac{I d\omega}{dA} = \frac{I d\omega}{dA} = \frac{I dA \cos\theta}{dA r^2} = \frac{I \cos\theta}{r^2} = \frac{I}{r^2} \tag{2}$$

   where $\cos\theta = \vec{n} \cdot (\bar{l} - \bar{p})/r$, $r = ||\bar{l} - \bar{p}|| = 1$, and $\bar{l}$ is the location of the center of the sphere.