CSCD18 Assignment #1 Solutions v1.1

(1) $640 \times 480 \times 12$ bits
$= 3686400$ bits
$= 460800$ bytes
$= 450$ KB

$1280 \times 1024 \times 12$ bits
$= 15728640$ bits
$= 1966080$ bytes
$= 1920$ KB

$2560 \times 2048 \times 12$ bits
$= 62914560$ bits
$= 7864320$ bytes
$= 7680$ KB

$\times 24$ bits
$= 921600$ bytes
$= 900$ KB

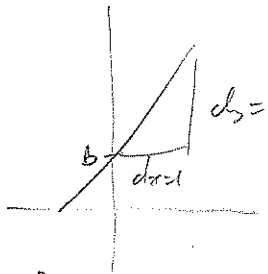
$\times 24$ bits
$= 3932160$ bytes
$= 3840$ KB

$\times 24$ bits
$= 15728640$ bytes
$= 15360$ KB

(3) Reflection Matrix (dx, dy)

$$= \frac{1}{dx^2+dy^2}\begin{bmatrix} dx^2-dy^2 & 2dxdy \\ 2dxdy & dy^2-dx^2 \end{bmatrix}$$

for $y = mx + b$



to generate the transform,

$$\vec{x}' = \text{translate}(x=0, y=b)\ \text{reflect}(dx=1, dy=m)\ \text{translate}(x=0, y=-b)\ \vec{x}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -b \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \frac{1-m^2}{1+m^2} & \frac{2m}{1+m^2} & 0 \\ \frac{2m}{1+m^2} & \frac{m^2-1}{1+m^2} & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -b \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -b \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \frac{1-m^2}{1+m^2} & \frac{2m}{1+m^2} & \frac{-2mb}{1+m^2} \\ \frac{2m}{1+m^2} & \frac{m^2-1}{1+m^2} & b\frac{1-m^2}{1+m^2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1-m^2}{1+m^2} & \frac{2m}{1+m^2} & \frac{-2mb}{1+m^2} \\ \frac{2m}{1+m^2} & \frac{m^2-1}{1+m^2} & b\frac{1-m^2}{1+m^2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1-m^2}{1+m^2} & \frac{2m}{1+m^2} & \frac{-2mb}{1+m^2} \\ \frac{2m}{1+m^2} & \frac{m^2-1}{1+m^2} & b\frac{1-m^2}{1+m^2}+b \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1-m^2}{1+m^2} & \frac{2m}{1+m^2} & \frac{-2mb}{1+m^2} \\ \frac{2m}{1+m^2} & \frac{m^2-1}{1+m^2} & \frac{2b}{1+m^2} \\ 0 & 0 & 1 \end{bmatrix}$$

(4) $\text{reflect}(dx, dy) = \frac{1}{dx^2+dy^2} \begin{bmatrix} dx^2 - dy^2 & 2dxdy \\ 2dxdy & dy^2 - dx^2 \end{bmatrix}$

want to show: $\text{reflect}(dx_1, dy_1)\ \text{reflect}(dx_2, dy_2) = \text{rotate}(\theta)$

for some θ.

without loss of generality, let $dx_1 = \cos\theta \quad dx_2 = \cos\phi$
$dy_1 = \sin\theta \quad dy_2 = \sin\phi$

$\text{reflect}(\cos\theta, \sin\theta) = \frac{1}{1} \begin{bmatrix} \cos^2\theta - \sin^2\theta & 2\cos\theta\sin\theta \\ 2\cos\theta\sin\theta & \sin^2\theta - \cos^2\theta \end{bmatrix}$

recall $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\text{reflect}(\cos\theta, \sin\theta) = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$

reflect $(\cos\theta, \sin\theta)$ reflect $(\cos\phi, \sin\phi)$

$$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix} \begin{bmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta \cos 2\phi + \sin 2\theta \sin 2\phi & \cos 2\theta \sin 2\phi - \sin 2\theta \cos 2\phi \\ \cos 2\phi \sin 2\theta - \cos 2\theta \sin 2\phi & \sin 2\theta \sin 2\phi + \cos 2\theta \cos 2\phi \end{bmatrix}$$

$$= \begin{bmatrix} \cos(2\theta - 2\phi) & \sin(2\theta - 2\phi) \\ \sin(2\phi - 2\theta) & \cos(2\theta - 2\phi) \end{bmatrix}$$

because of evenness of cos & oddness of sin,

$$= \begin{bmatrix} \cos(2\phi - 2\theta) & -\sin(2\phi - 2\theta) \\ \sin(2\phi - 2\theta) & \cos(2\phi - 2\theta) \end{bmatrix} = \text{rotate}\,(2\phi - 2\theta)$$

(5) $R(\theta) R(\phi) \stackrel{?}{=} R(\theta + \phi)$

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta \cos\phi - \sin\theta \sin\phi & -\cos\theta \sin\phi - \sin\theta \cos\phi \\ \sin\theta \cos\phi + \cos\theta \sin\phi & -\sin\phi \sin\theta + \cos\theta \cos\phi \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta + \phi) & -(\sin(\theta + \phi)) \\ \sin(\theta + \phi) & \cos(\theta + \phi) \end{bmatrix} = R(\theta + \phi)$$

(6) a) $P = [3,2]$

origin $= [3,4]$

$i = [-1,-1]$

$j = [-1,1]$

(lower left corner of diagram is $(0,0)$)

$$\begin{bmatrix} | & | \\ \vec{i} & \vec{j} \\ | & | \end{bmatrix} \vec{x} = \vec{P} - \vec{o}$$

$$\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} \vec{x} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} - \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} -1 & -1 & 0 \\ -1 & 1 & -2 \end{array}\right] \overset{R_1+R_2}{\sim} \left[\begin{array}{cc|c} -2 & 0 & -2 \\ -1 & 1 & -2 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 1 \\ -1 & 1 & -2 \end{array}\right]$$

$$\underset{R_2+R_1}{\sim} \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -1 \end{array}\right] \qquad [3,2] \equiv [1,-1]_{C5a}$$

b) origin $= [5,5]$

$i = [1,0]$

$j = [0,-1]$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \vec{x} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} - \begin{bmatrix} 5 \\ 5 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & -1 & -3 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & 3 \end{array}\right]$$

$$[3,2] \equiv [-2,3]_{C5b}$$

c) origin $= [1,1]$

$i = [1,0]$

$j = [0,1]$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \vec{x} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$[3,2] \equiv [2,1]_{C5c}$$

d) origin $= [5,1]$
$i = [-2,-1]$
$j = [1,0]$

$$\begin{bmatrix} -2 & 1 \\ -1 & 0 \end{bmatrix} \vec{x} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} - \begin{bmatrix} 5 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

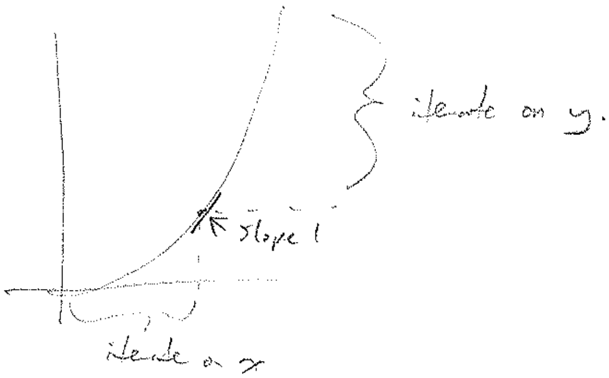
$$\left[\begin{array}{cc|c} -2 & 1 & -2 \\ -1 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cc|c} -2 & 1 & -2 \\ 1 & 0 & -1 \end{array}\right]$$

$$\underset{R_1+2R_2}{\sim} \left[\begin{array}{cc|c} 0 & 1 & -4 \\ 1 & 0 & -1 \end{array}\right]$$

$$[3,2] = [-1,-4]_{CS_d}$$

(2) Assume a, b integer.

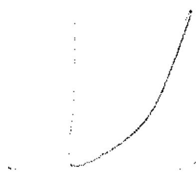
initially assume $a > 0$.
x starts at 0 and increases.

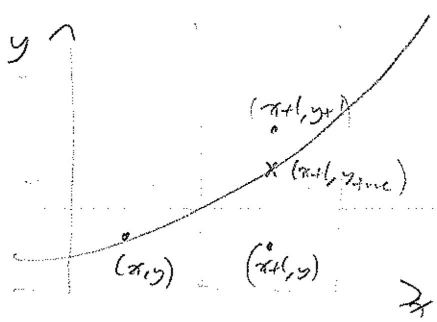


$$y = ax^2 - b$$
$$y' = 2ax = 1$$
$$x = \frac{1}{2a}$$

```
while x < 1/(2a)
    x = x+1
    y = y or y+1
    draw (x,y)
end
while y < ymax
    y = y+1
    x = x or x+1
    draw (x,y)
end
```

essential framework for drawing this section





what is $y_{true} - y_{pixel}$?

$$\underbrace{ax^2 - b - y}$$

all integer, can use as decision variable.

let $d = y_{true} - y_{pixel}$

when $d > \frac{1}{2}$, $y = y+1$

$d = ax^2 - b - y$

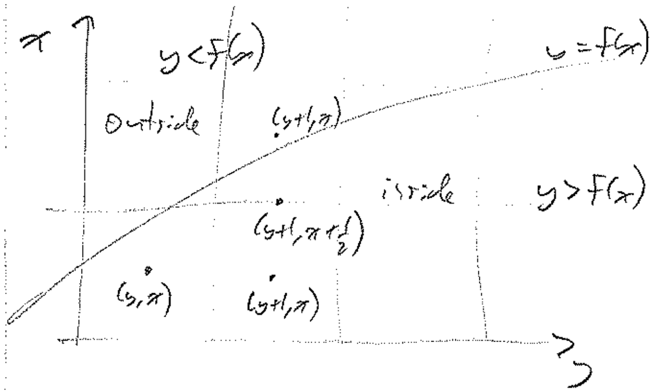
assume drew pixel x, y last.
if we assume $x+1, y$ is the next one, what is d?

$$
\begin{aligned}
d &= a(x+1)^2 - b - y \\
&= ax^2 + 2ax + a - b - y \\
&= \underbrace{ax^2 - b - y} + 2ax + a \\
&= d_{old} + 2ax + a.
\end{aligned}
$$

so algo is so far

$x = 0$
$y = -b$ $\quad (a \cdot 0^2 - b = -b)$
$d = 0$

```
draw (x,y)
while x < 1/(2a)
    d = d + 2ax + a
    x = x+1
    if d > 1/2
        d = d-1
        y = y+1
    end
    draw (x,y)
end.
```



can't use $x_{true} - x_{pixel}$ because $y = ax^2 - b$

$x = \sqrt{\frac{y+b}{a}}$

square root is unpleasant

instead, we inside/outside.

let $d = ax^2 - b - y$.

so $d > 0$, then outside

$d < 0$, then inside

assume we drew (x, y) last.

what is d for $(x+\frac{1}{2}, y+1)$?

where does the midpoint lie?

assume d stores the calculation for $(x+\frac{1}{2}, y)$.

$$d = a(x+\tfrac{1}{2})^2 - b - y.$$

d_{new} to calc is

$$d_{new} = a(x+\tfrac{1}{2})^2 - b - (y+1)$$
$$= d_{old} - 1.$$

if $d_{new} < 0$, then increment x (draw at $(x+1, y+1)$)

corresponding d is

$$d_{new} = a(x+\tfrac{1}{2}+1)^2 - b - (y+1)$$

$$
\begin{aligned}
d_{new} &= a\left(x+\tfrac{1}{2}+1\right)^2 - b - y - 1 \\
&= a\left(x+\tfrac{1}{2}\right)^2 + 2a\left(x+\tfrac{1}{2}\right) + a - b - y - 1 \\
&= a\left(x+\tfrac{1}{2}\right)^2 - b - y + 2ax + a + a - 1 \\
&= d_{old} + 2a(x+1) - 1.
\end{aligned}
$$

so adding this code is given.

$d = a(x+\frac{1}{2})^2 - b - y$ (need to reset d to new defs)

$y_{max} = a x_{max}^2 - b.$

```
while y < ymax
    y = y+1
    d = d-1
    if d < 0
        x = x+1
        d = d + 2ax        (x has been incremented previously)
    end
    draw (x,y)
end.
```

now, if $a < 0$, how do we deal with that?
imagine the reflection of the parabola in the x axis.
the upright parabola mirrors the upside down one.
the b shift can be ignored as it is integral;
it doesn't introduce any floating points which will interfere
with the d defns.
so.

```
x=0
y=-b
d=0
draw (x,y)
while  x < 1/(2|a|)                  <- integer optimize
                                        diffent code in increments of 2|a|.
    d=d+2|a|x+|a|
    x=x+1
    if d>1/2                            <- multiplys d by 2 to avoid half here.
        d=d-1
        y=y+1sgn(a)
    end
    draw(x,y)        < draw (-x,y)
                       draw (x,y)       mirror
end

d=a(x+1/2)^2-b-y                        <- multiplys d by a to avoid 1/a factors.
ymax= a xmax^2 -b

while  y sgn(a) < ymax sgn(a)
    y=y+1sgn(a)
    d=d-1sgn(a)
    if  d sgn(a) < 0
        x=x+1
        d=d+2ax
    end
    draw(x,y)        < draw (-x,y)
                       draw (x,y)       mirror
end.
```