Course Updates

- Assignment 2
  - **Programming** is due on Friday
  - Example surface of revolution file is now available on the web
- Assignment 3
  - Out on Friday
  - **Due date:** November 23rd
- Assignment 4
  - **Due date:** December 3rd
  - **Demo Day:** December 3rd

Ray Tracing
Part 2: Introduction

Computer Graphics, CSCD18

Fall 2007

Instructor: Leonid Sigal

Finding Intersections (for spheres)

- Let's consider simple case, unit length sphere at a given point $\bar{p}$, $\|\bar{p}\|^2=1$, by first substituting:

$$(\bar{a}+\lambda^* \vec{d})\cdot(\bar{a}+\lambda^* \vec{d})-1=0$$

then expanding:

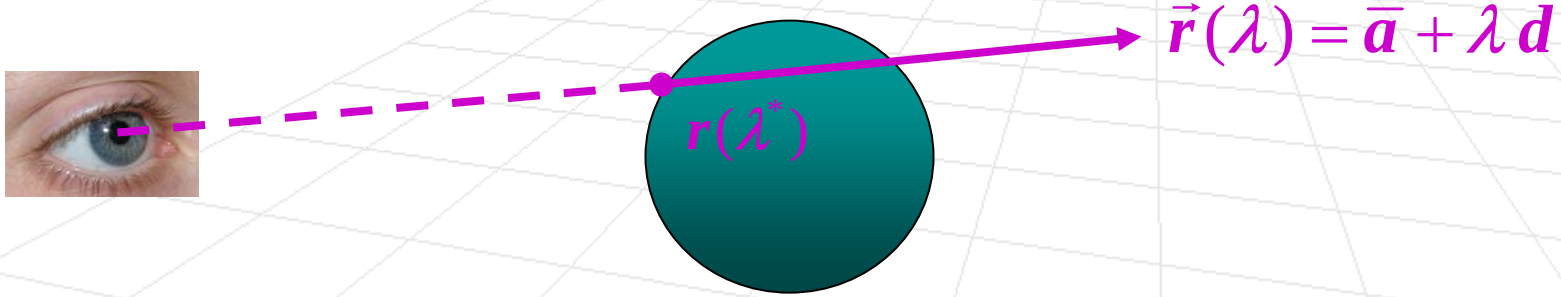
$$A(\lambda^*)^2+2B\lambda^*+C=0 \quad \text{, where}$$

$$A=\vec{d}\cdot\vec{d} \qquad C=\bar{a}\cdot\bar{a}-1$$

$$B=\bar{a}\cdot\vec{d} \qquad D=B^2-AC$$

Producing solution:

$$\lambda^*=\frac{-2B\pm\sqrt{4B^2-4AC}}{2A}=-\frac{B}{A}\pm\frac{\sqrt{D}}{A}$$



Finding Intersections (for spheres)

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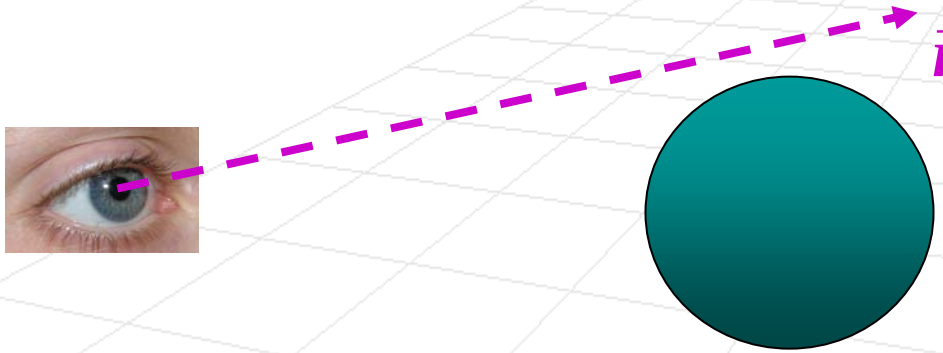
- Cases
  - D < 0 no intersection
  - D = 0 1 hit (ray grazes the sphere)
  - D > 0 2 hits with at λ_1^*, λ_2^*

$\lambda_1^* < 0, \lambda_2^* < 0$ - Both hits are behind viewplane (not visible)

$\lambda_1^* > 0, \lambda_2^* < 0$ - Eye inside sphere, first hit is valid

$\lambda_1^* > \lambda_2^* > 0$ - Both hits are valid (in front og viewplane), second closer



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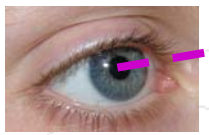

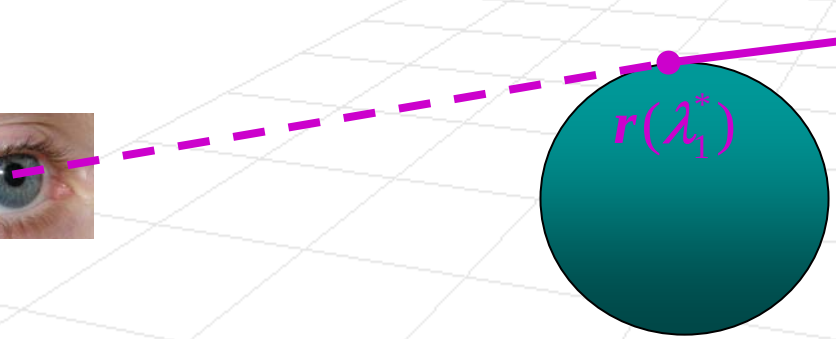

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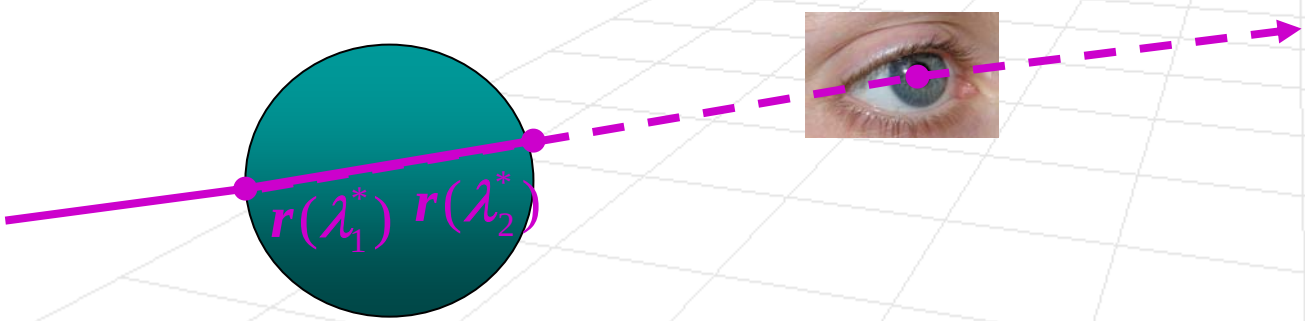

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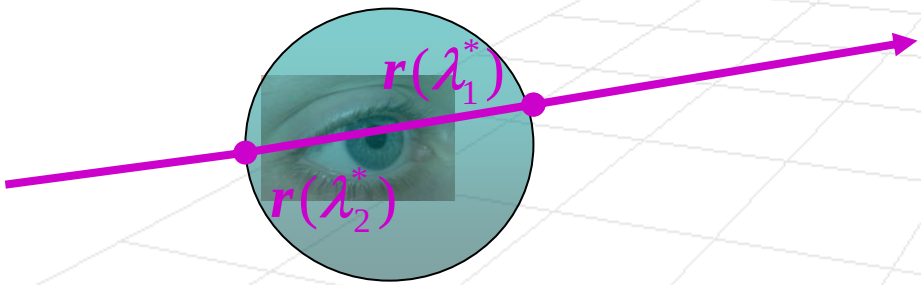

$$\vec{\boldsymbol{r}}(\lambda) = \bar{\boldsymbol{a}} + \lambda \vec{\boldsymbol{d}}$$

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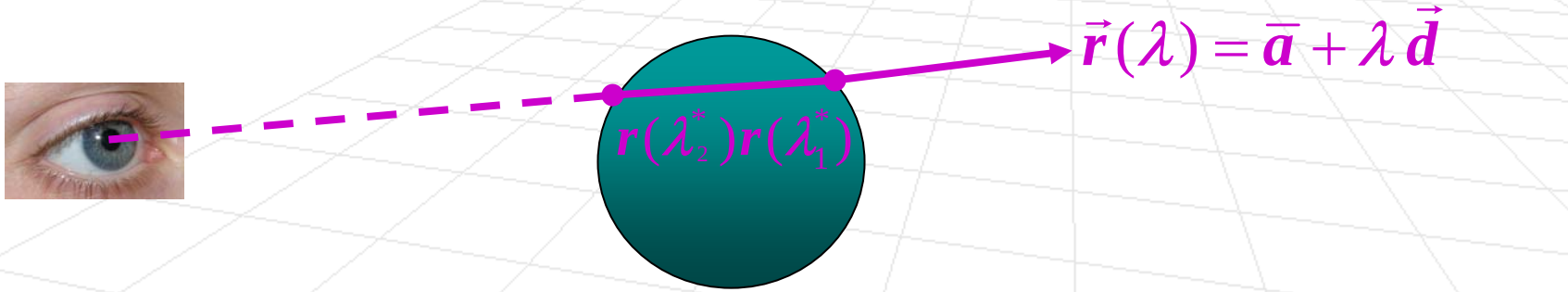


What if we want to intersect with a general sphere?

- **Property:** Given an intersection method for an object, it is easy to intersect rays with affinely deformed versions of the object

- Assuming we have an invertible affine transformation $\boldsymbol{f(\bar{x}) = A\bar{x}+\vec{t}}$ that is applied to the object, it is relatively easy to show that intersection of the deformed object is the same as the intersection of the original object with inversely transformed ray:

$$\vec{r}'(\lambda) = \bar{a}' + \lambda \vec{d}'$$

$$\bar{a}' = A^{-1}(\bar{a} - \vec{t})$$

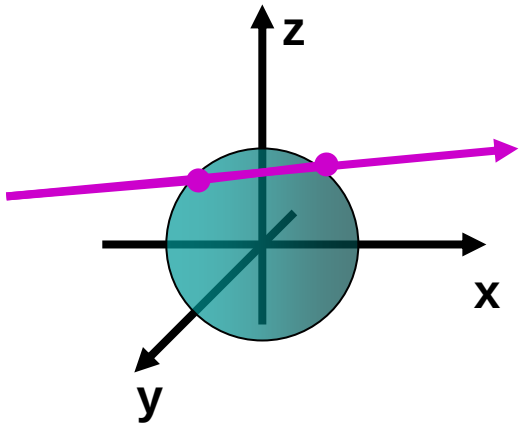
$$\vec{d}' = A^{-1}\vec{d}$$

Full proof is in the lecture notes

Short Proof

Given intersection method for

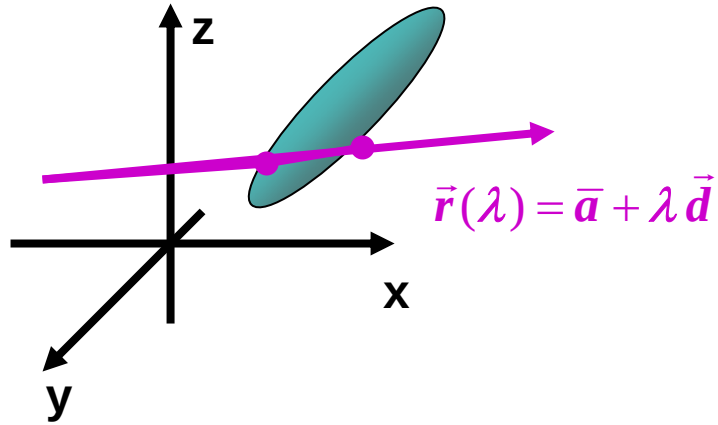
$$f(\bar{x})$$



$$f(\bar{x}) = \bar{x} \cdot \bar{x} - 1 = 0$$

Find intersection with affine version

$$F(\bar{y}) = f\left(A^{-1}(\bar{y} - \vec{t})\right)$$



$$F(\bar{y}) = A^{-1}(\bar{y} - \vec{t}) \cdot A^{-1}(\bar{y} - \vec{t}) - 1 = 0$$

We substitute equation of the ray into implicit function of the affinely transformed surface

$$\begin{aligned} F(\bar{r}(\lambda)) &= f\left(A^{-1}\left(\bar{r}(\lambda) - \vec{t}\right)\right) \\ &= f\left(A^{-1}\left(\bar{a} + \lambda \vec{d} - \vec{t}\right)\right) \\ &= f\left(A^{-1}\left(\bar{a} - \vec{t}\right) + \lambda A^{-1}\vec{d}\right) \\ &= f\left(\bar{r}'(\lambda)\right) \end{aligned}$$

where

$$\begin{aligned} \bar{r}'(\lambda) &= \bar{a}' + \lambda \vec{d}' \\ \bar{a}' &= A^{-1}\left(\bar{a} + \vec{t}\right) \\ \vec{d}' &= A^{-1}\vec{d} \end{aligned}$$

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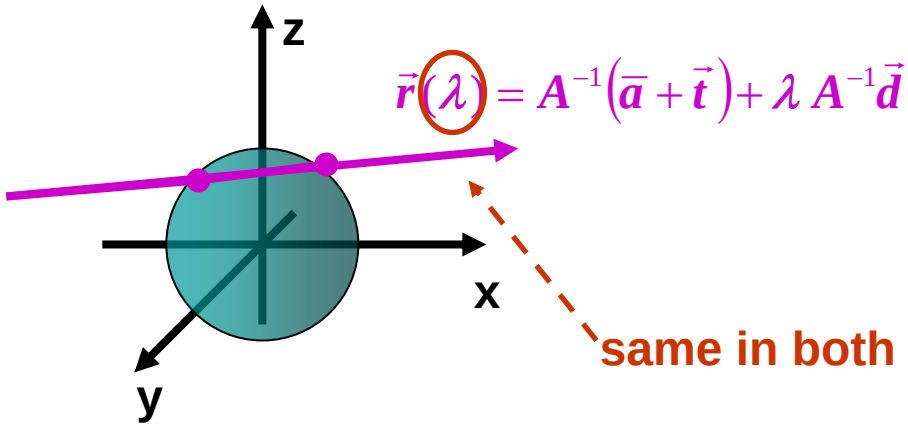

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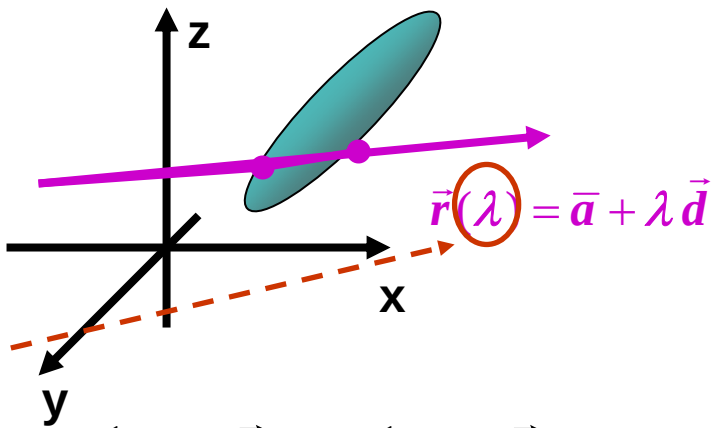

same in both

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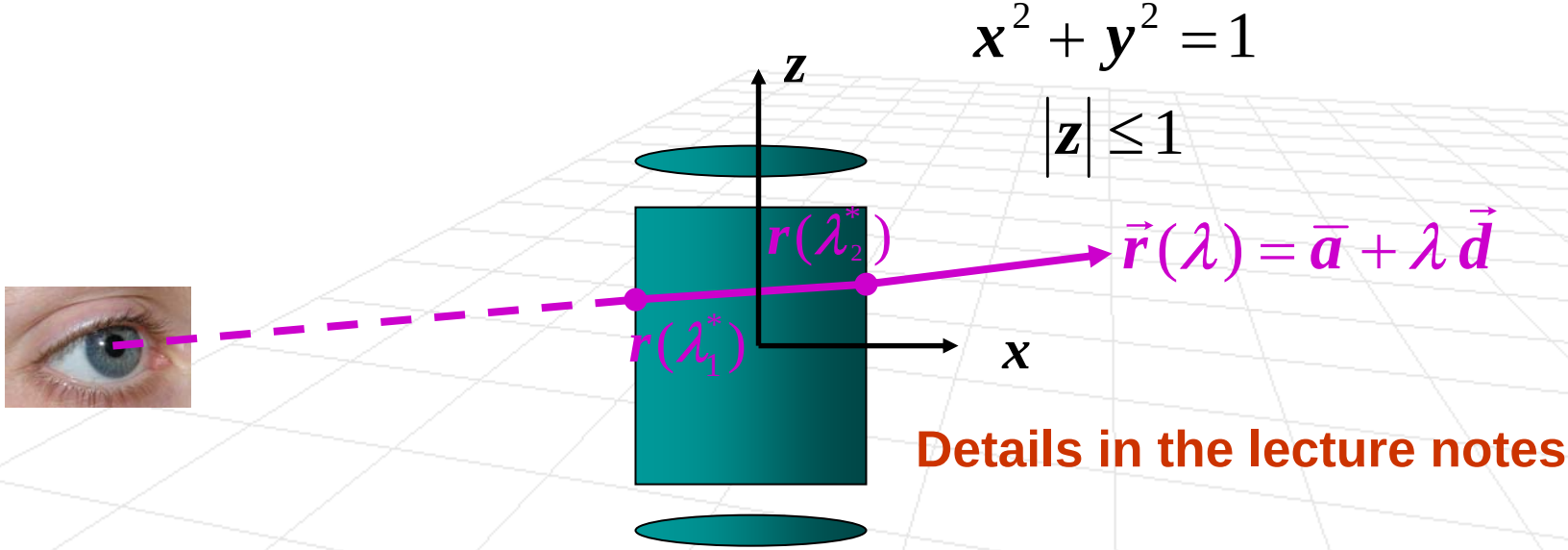
Intersection with Cylinders and Cones

- Find intersection with “quadratic walls”, ignoring constraints on z
- Test the resulting z component of $\bar{p}(\lambda^*)$ against the constraints on z
- Intersect the ray with plains constraining end caps
- Test to ensure they satisfy interior circular constraint
- If there are multiple intersections take the intersection with smallest positive λ^*



Details in the lecture notes

Intersection with Cylinders and Cones

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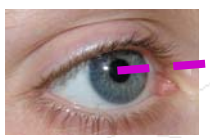
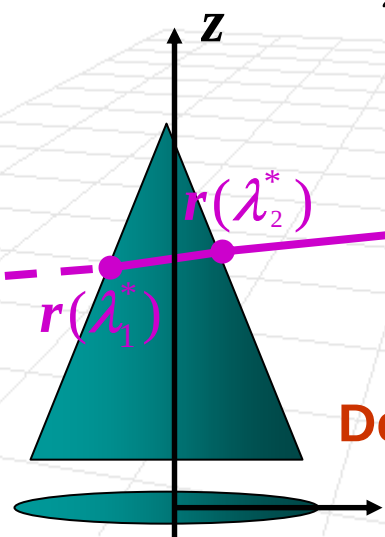
$$x^2 + y^2 - \frac{1}{4}\left(1 - z^2\right) = 0$$

$$0 \le z \le 1$$





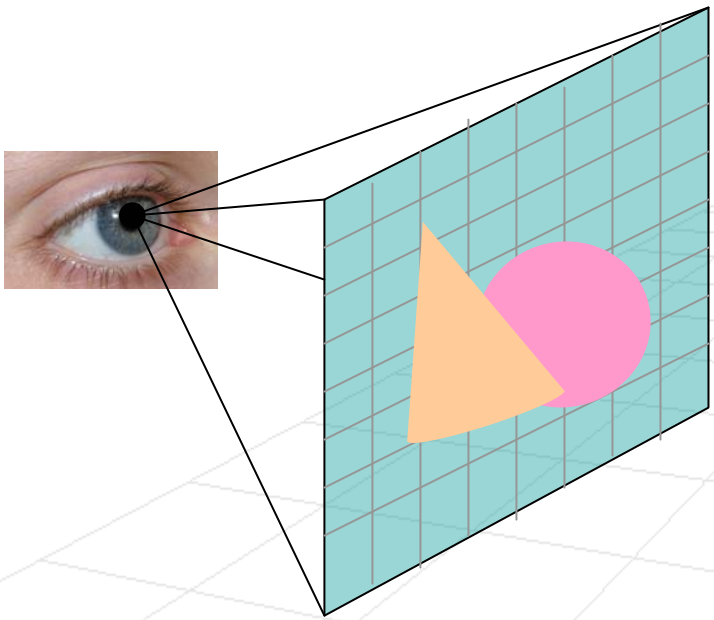
$$\vec{r}(\lambda) = \bar{a} + \lambda \vec{d}$$

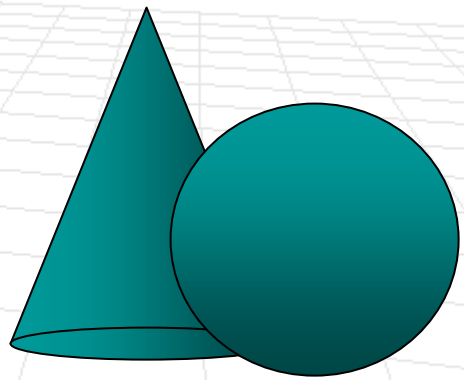
Details in the lecture notes

Scene Signature

- Simple way to test geometry and intersection methods (great tool for debugging)

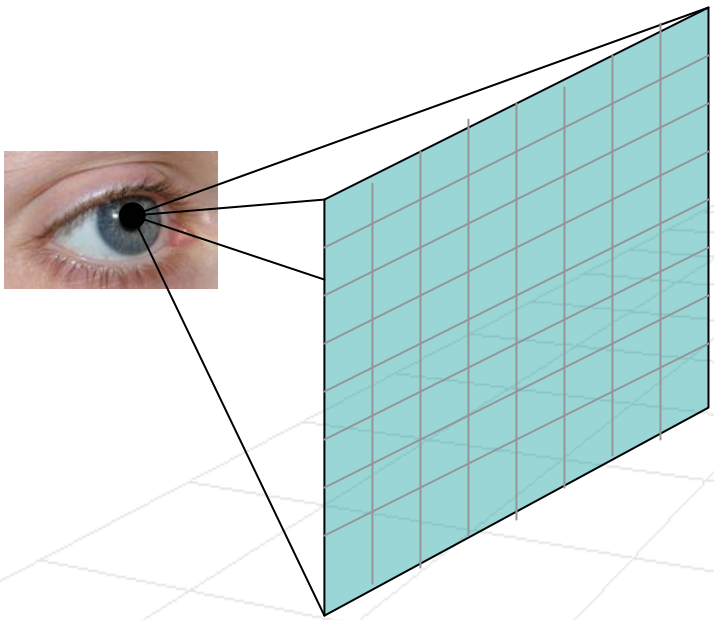
- **Idea:** Create image in which pixel (i,j) has intensity k if object k is the first object hit by the ray through (i,j)
  - Each object gets one unique color

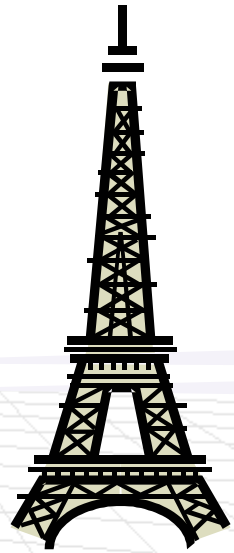




Efficiency

- Intersection tests are expensive, especially for complex geometries.
- Data structures are typically used to avoid testing intersections with objects that are not hit.

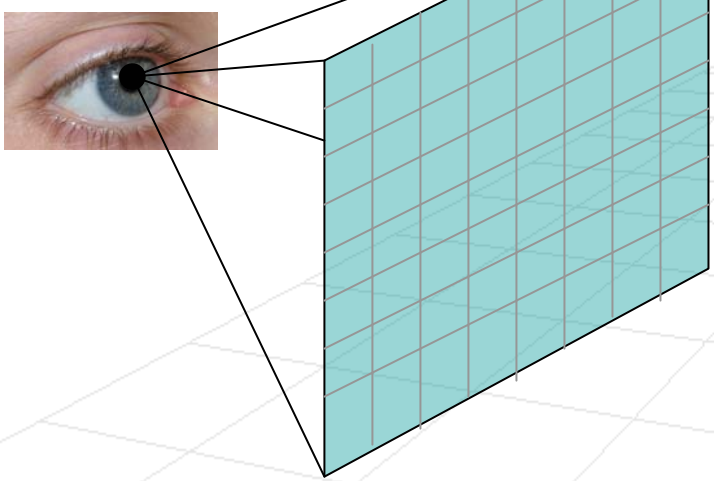


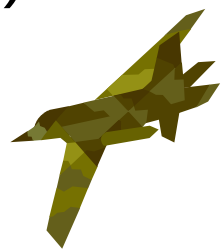


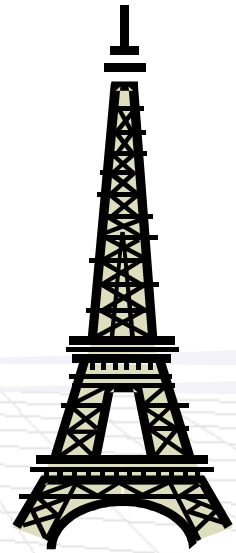
Efficiency

- **Idea:** Bound 3D objects (meshes) with single simple bounding volume (e.g. sphere or a cube).
  - Only test intersections with the object if there exists positive intersection with bounding volume

(This can also be done hierarchically)







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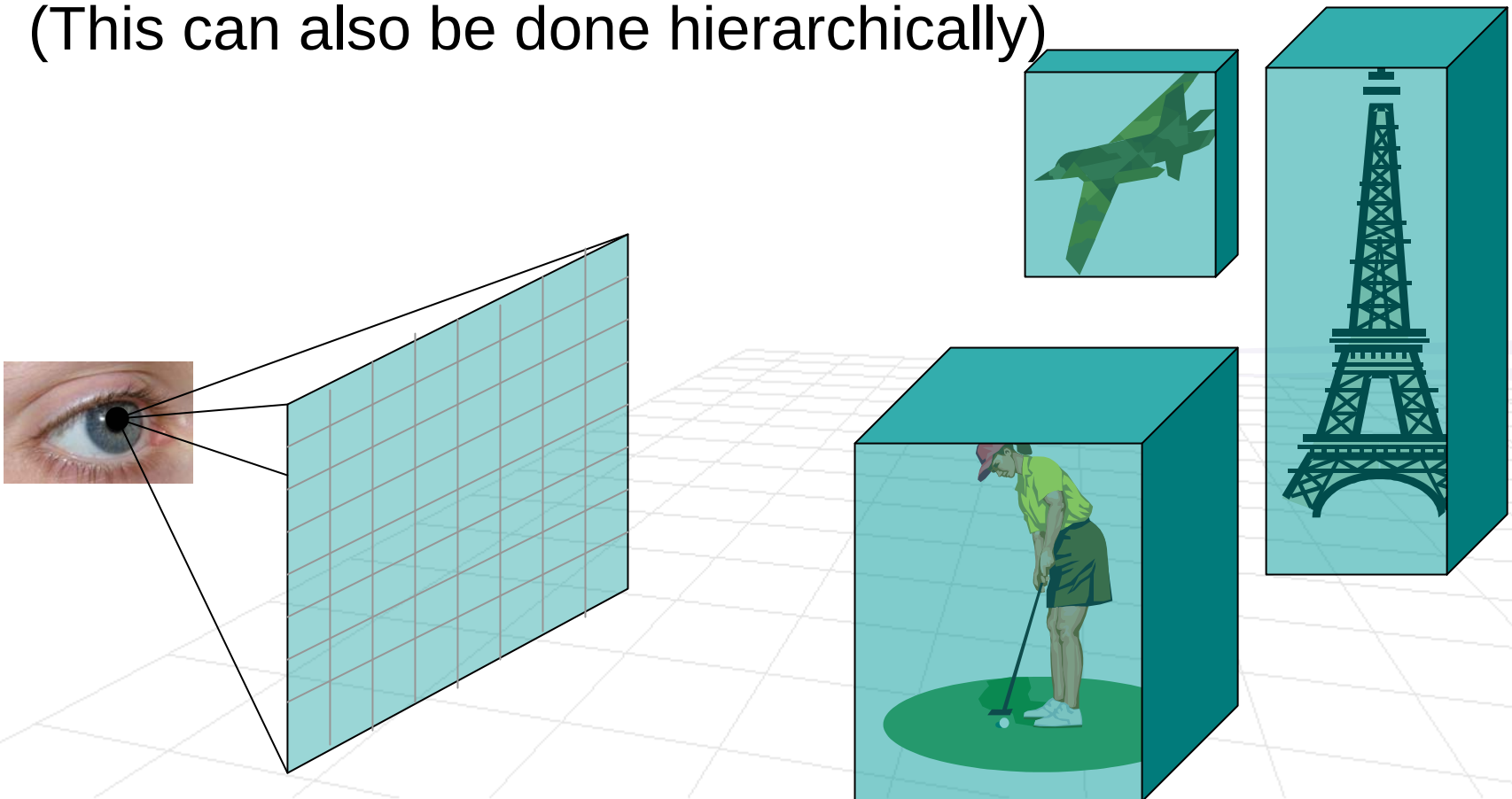
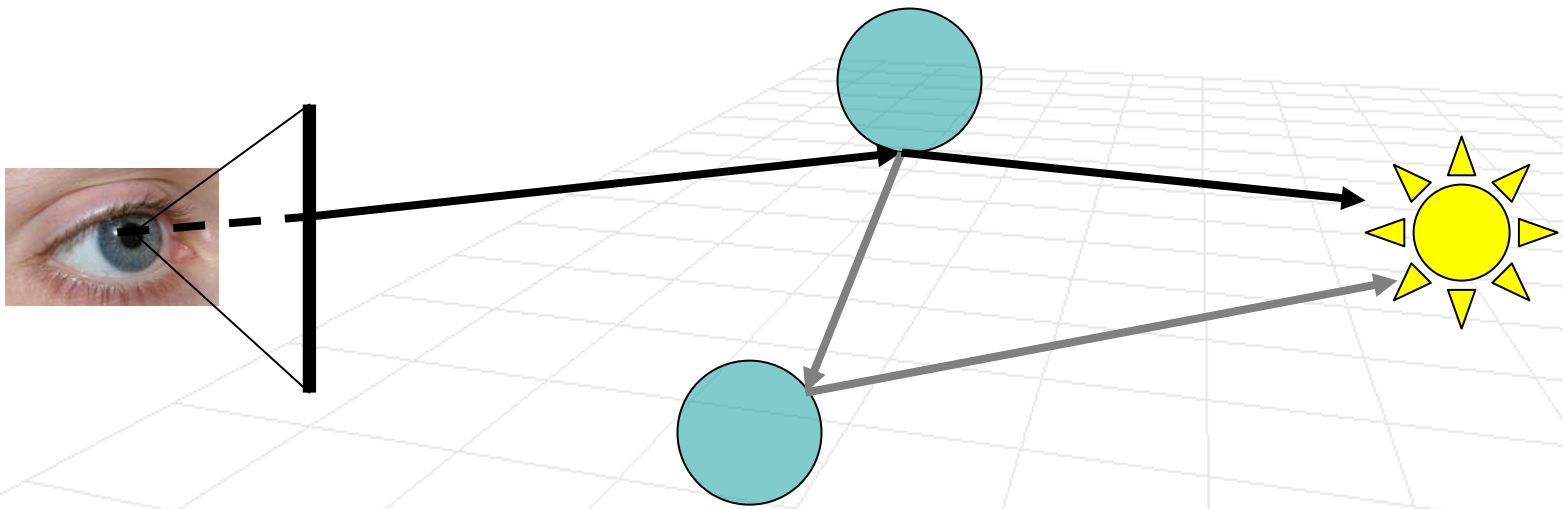


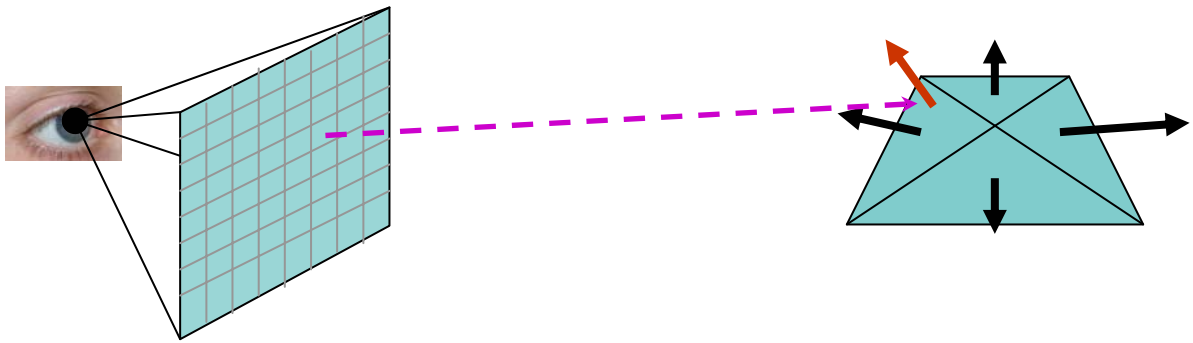
Computational Issues to Consider

- Form rays (a.k.a. ray casting)
- Find intersection of rays with objects
- Find closest object intersection (there could be multiple object intersections for any given ray)
- Find normal at the closest intersection point (a.k.a hit point)
- Evaluate reflectance model at the hit point



Finding surface normal at the hit point

- For mesh surface one might smoothly interpolate normal at the hit point from the nearby face normals (same as for the vertex before)



- Given parametric shape, we can compute normal $\vec{n}_j$ for the hit point $\bar{p}_j$ explicitly
  - Implicit form $\vec{n}_j(\bar{p}_j) = \nabla f(\bar{p}_j)$
  - Explicit form $\vec{n}_j(\bar{p}_j) = \left.\frac{\partial s(\alpha,\beta)}{\partial \alpha}\right|_{\alpha_0,\beta_0} \times \left.\frac{\partial s(\alpha,\beta)}{\partial \beta}\right|_{\alpha_0,\beta_0}$

Normals for affinely-deformed surfaces

- Let $f(\bar{p}) = 0$ be an implicit surface, and let $Q(\bar{p}) = A\bar{p} + \vec{t}$ be an affine transformation.
- The new affinely-deformed surface can then be written as follows:

$$F(\bar{q}) = f\left(A^{-1}(\bar{p} - \vec{t})\right) = 0$$

- The normal of F at a point $\bar{q}$ is given by

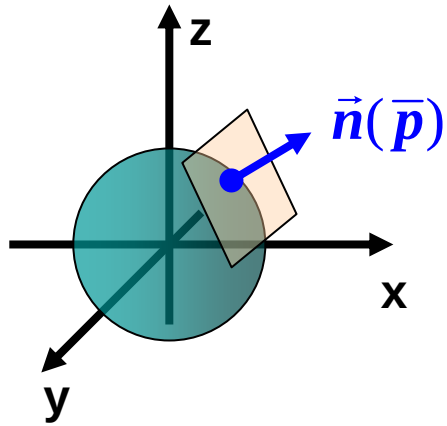
$$\frac{\left(A^{-1}\right)^T \vec{n}}{\left\|\left(A^{-1}\right)^T \vec{n}\right\|}$$

Full proof in given in the lecture notes

Short Proof

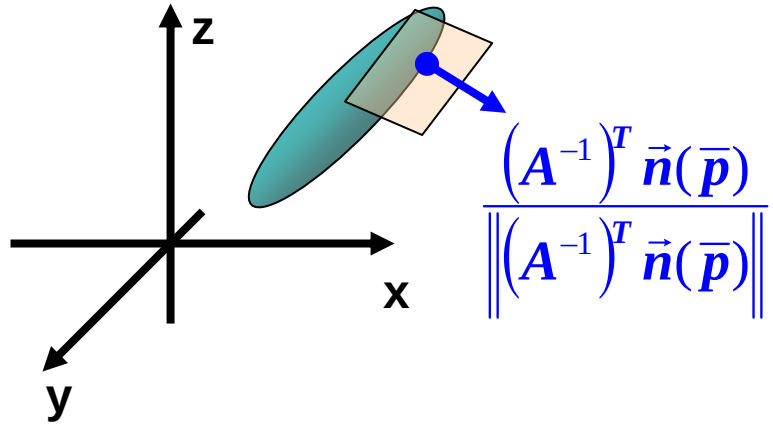
Given a normal

$$\vec{n}(\bar{p}) = \nabla f(\bar{p}) / \left\| \nabla f(\bar{p}) \right\|$$



Find a normal for affine version

$$F(\bar{q}) = f\left(A^{-1}(\bar{q} - \vec{t})\right)$$



Assume that the tangent plane at hit point, $\bar{r}(\lambda^*)$, on the generic surface is given by $\left(\bar{p} - \bar{r}(\lambda^*)\right) \cdot \vec{n} = 0$ or $\bar{p}^T \vec{n} = D$, where $D = \bar{r}(\lambda^*) \cdot \vec{n}$

Under affine transformation, planarity is preserved so tangent plane on deformed surface is given by

$$\left(A^{-1}(\bar{q} - \vec{t})\right)^T \vec{n} = D$$

$$\left(A^{-1}\bar{q} - A^{-1}\vec{t}\right)^T \vec{n} = D$$

$$\left(A^{-1}\bar{q}\right)^T \vec{n} = D + \left(A^{-1}\vec{t}\right)^T$$

$$\bar{q}^T \left(A^{-1}\right)^T \vec{n} = D + \left(A^{-1}\vec{t}\right)^T$$

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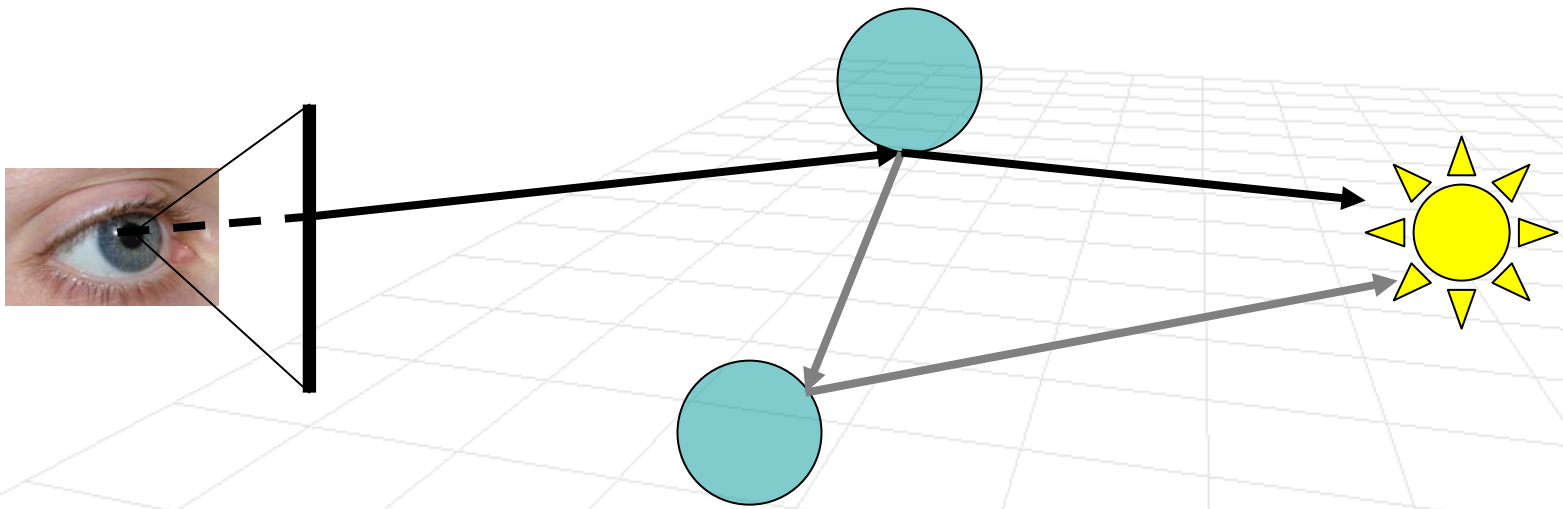

Conventional (Whitted) Ray Tracing

- **Local** model (e.g. Phong) to account for diffuse and specular highlights due to the direct lighting
- Use ambient term to approximate **global** diffuse lighting
- Cast rays to estimate ideal “mirror” reflections of other objects
  - Feasible since for perfect specular reflection there is a unique direction of preference
  - In general, however, we need a Bidirectional Reflectance Distribution Function (BRDF)
- So radiance at a hit point becomes

$$E_j = \underbrace{r_d I_d \max(0, \vec{s}_j \cdot \vec{n}_j) + r_a I_a + r_s I_s \max(0, \vec{r}_j \cdot \vec{c}_j)^{\alpha}}_{\text{Local Phong Model}} + \underbrace{r_g I_{spec}}_{\text{Global Specular Model}}$$

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r_g can depend on distance

$$E_j = r_d I_d \max(0, \vec{s}_j \cdot \vec{n}_j) + r_a I_a + r_s I_s \max(0, \vec{r}_j \cdot \vec{c}_j)^{\alpha} + r_g I_{spec}$$

Local Phong Model

Global Specular Model

Texture

- Texture can be used to modulate diffuse and ambient reflection coefficients, as with Gouraud or Phong shading
- All we need, is a way of mapping a point on the surface (hit point) to a point in the texture space
  - e.g. given a hit point of parametric surface, we can convert the 3D point coordinates to surface parameters, and use them to get texture coordinates (as with standard texture mapping)
- Unlike with Gouraud or Phong shading models we don't need to interpolate texture coordinates over polygons
- Anti-aliasing and super-sampling we will cover later (next week)