Radiometry
Part 2: Continuation

Computer Graphics, CSCD18

Fall 2007

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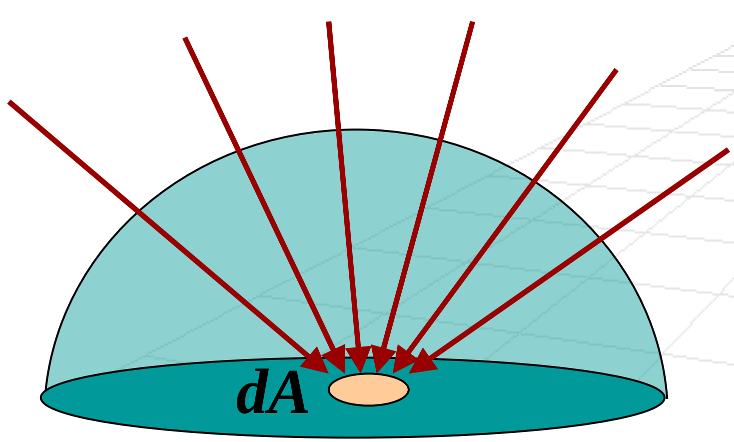
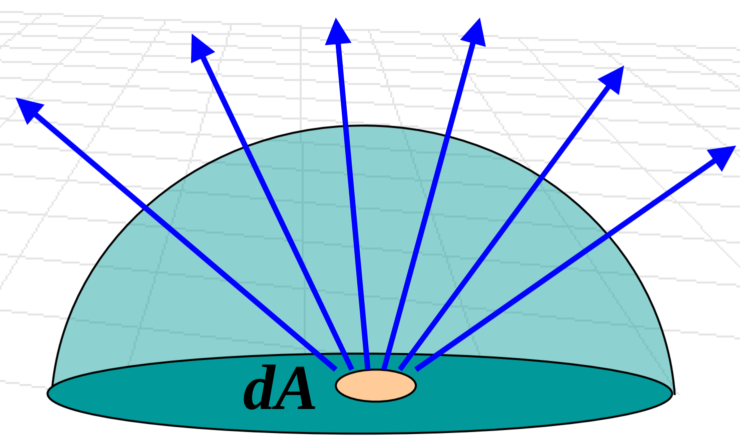
Light

- Light is manifested as photons
  - Number of photons at a point is zero
  - Hence, we going to talk about flux density (*i.e.* number of photons per unit area)
- **Irradiance** – amount of the light falling on the surface patch (measured in Watts/meters^2)
- **Radiance** – amount of light leaving the point per area (measured in Watts/(sr * meters^2))



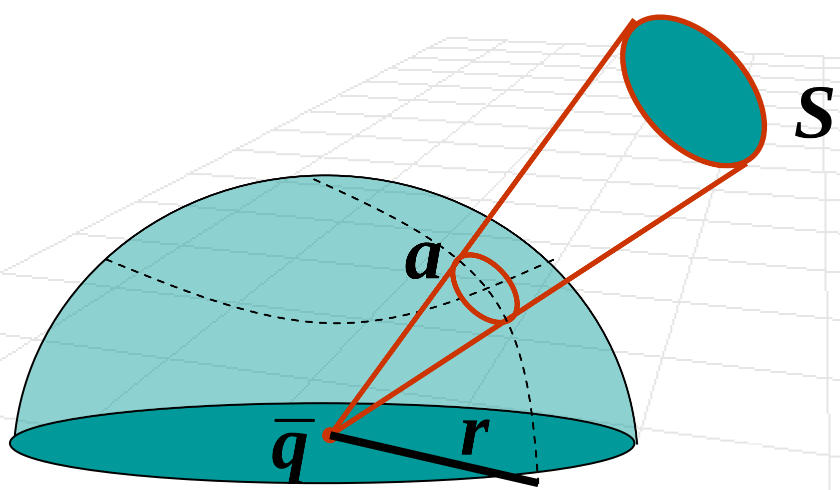


Solid Angle

- **Solid Angle** - measured as the area $\boldsymbol{a}$ of a patch of a sphere, divided by the squared radius $\boldsymbol{r}$ of the sphere

$$\omega = \frac{\boldsymbol{a}}{\boldsymbol{r}^2}$$

- **Intuition:** imagine you are at point $\boldsymbol{q}$ and you look out in all possible directions, solid angle measures the amount of your view that a patch of the surface $\boldsymbol{S}$ is taken up



Irradiance

- What is irradiance at surface patch $\boldsymbol{S}$ at point $\overline{\boldsymbol{p}}$ due to point light source at $\overline{\boldsymbol{e}}$ in direction $\vec{\boldsymbol{d}}$, with radiance $\boldsymbol{I}$?
- First compute the solid angle of $\boldsymbol{S}$ with respect to $\boldsymbol{e}$

$$d\omega = \frac{dA_s}{\left\|\overline{p}-\overline{e}\right\|^2}\boxed{(\vec{n}\cdot\vec{d})}$$

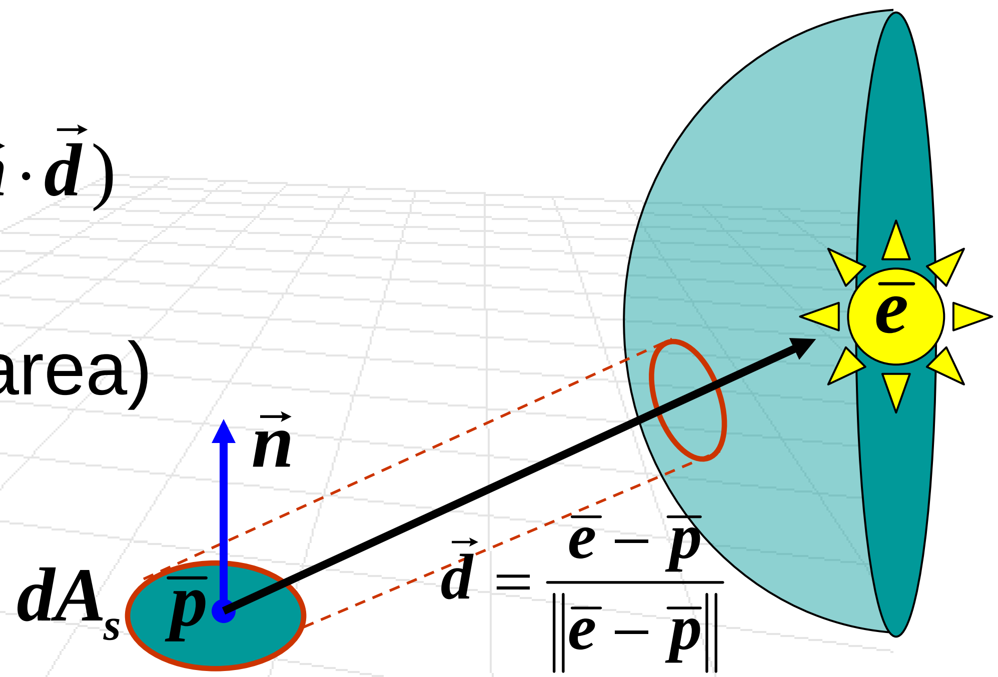
foreshortening

- Light reaching $\boldsymbol{S}$

$$S = I\,d\omega = I\,\frac{dA_s}{\left\|\overline{p}-\overline{e}\right\|^2}(\vec{n}\cdot\vec{d})$$

- Irradiance (divide by area)

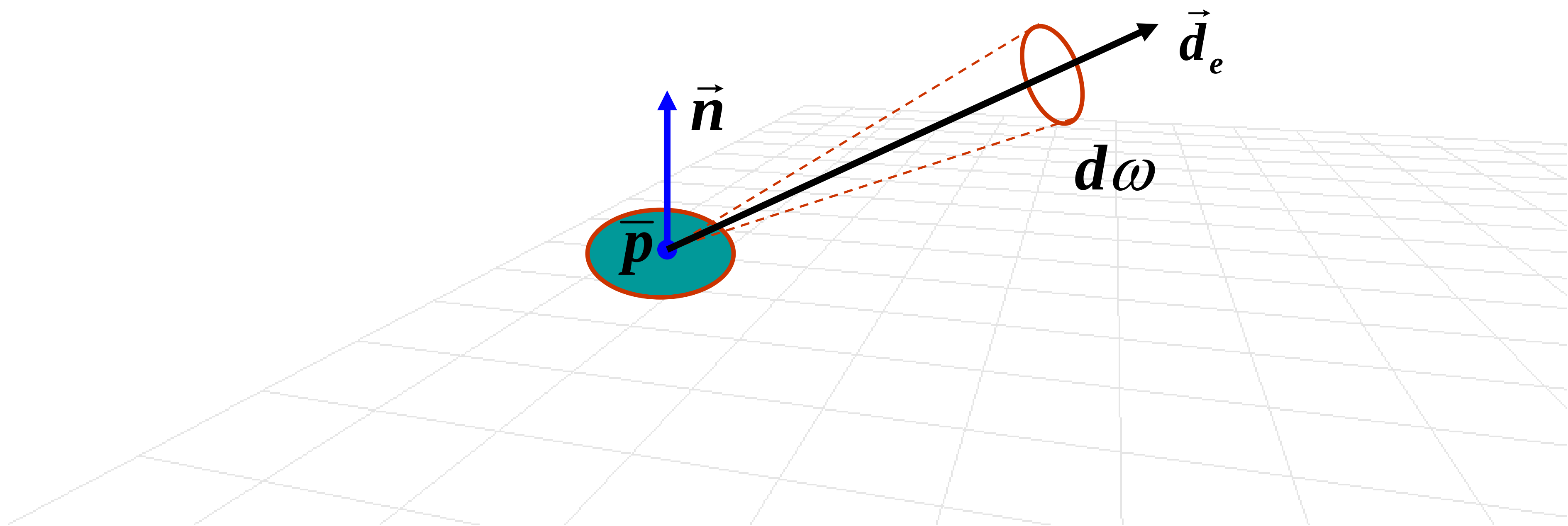
$$H(\overline{p}) = \frac{I\,d\omega}{dA_s} = \frac{I(\vec{n}\cdot\vec{d})}{\left\|\overline{p}-\overline{e}\right\|^2}$$



Radiance

- Light emitted in direction $\boldsymbol{d_e}$ through small surface patch $\boldsymbol{S}$ at point $\bar{\boldsymbol{p}}$, is called radiance $\boldsymbol{L}(\bar{\boldsymbol{p}}, \vec{\boldsymbol{d}})$

- We need to integrate this quantity over all possible directions to obtain the radiosity (or radiant exitance)



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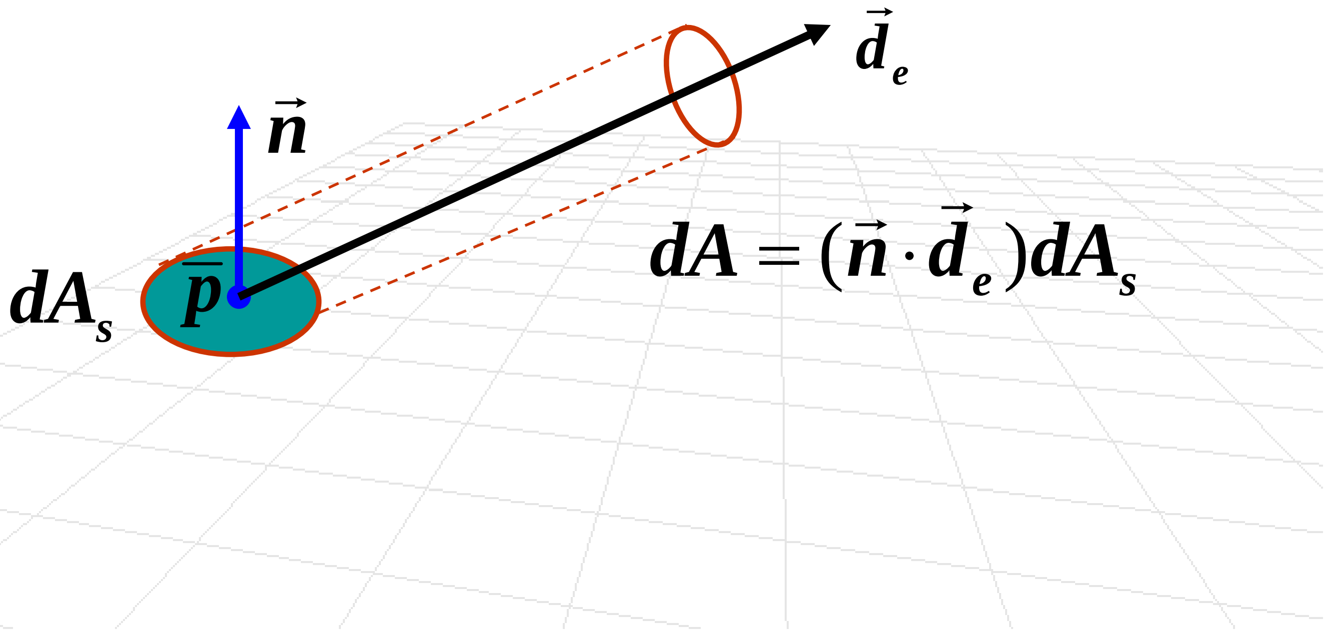

$$dA = (\vec{n} \cdot \vec{d}_e)dA_s$$

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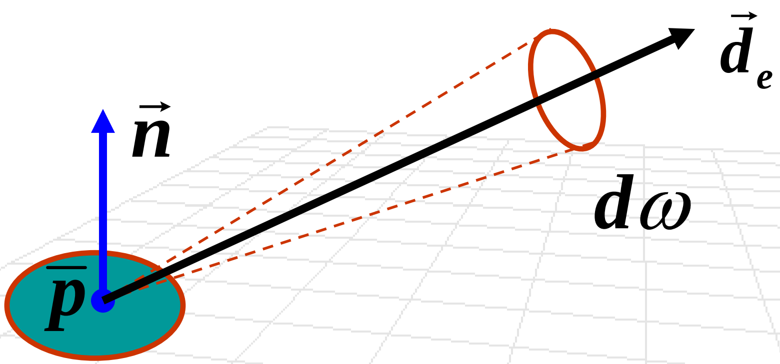
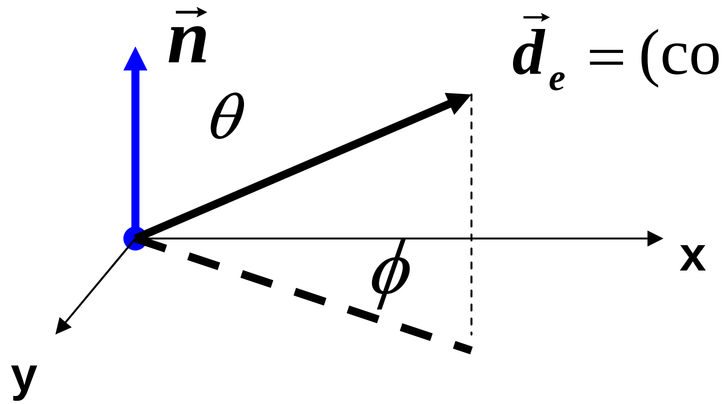


- Taking this into account we get

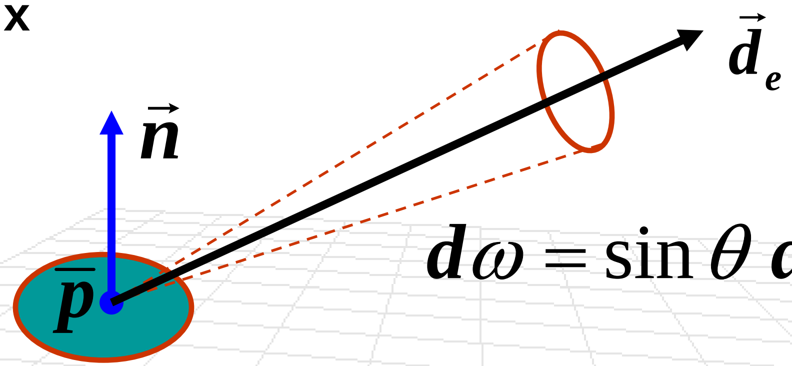
$$E(\bar{p}) = \int_{\vec{d}_e \in \Omega_e} L(\bar{p}, \vec{d}_e)(\vec{n} \cdot \vec{d}_e)\, d\omega$$

Radiance

- In spherical coordinates, we can express this as a double integral (assuming infinitesimally small patch)



$$\vec{d}_e = (\cos\phi \sin\theta, \sin\phi \sin\theta, \cos\theta)$$



$$d\omega = \sin\theta \, d\theta \, d\phi$$

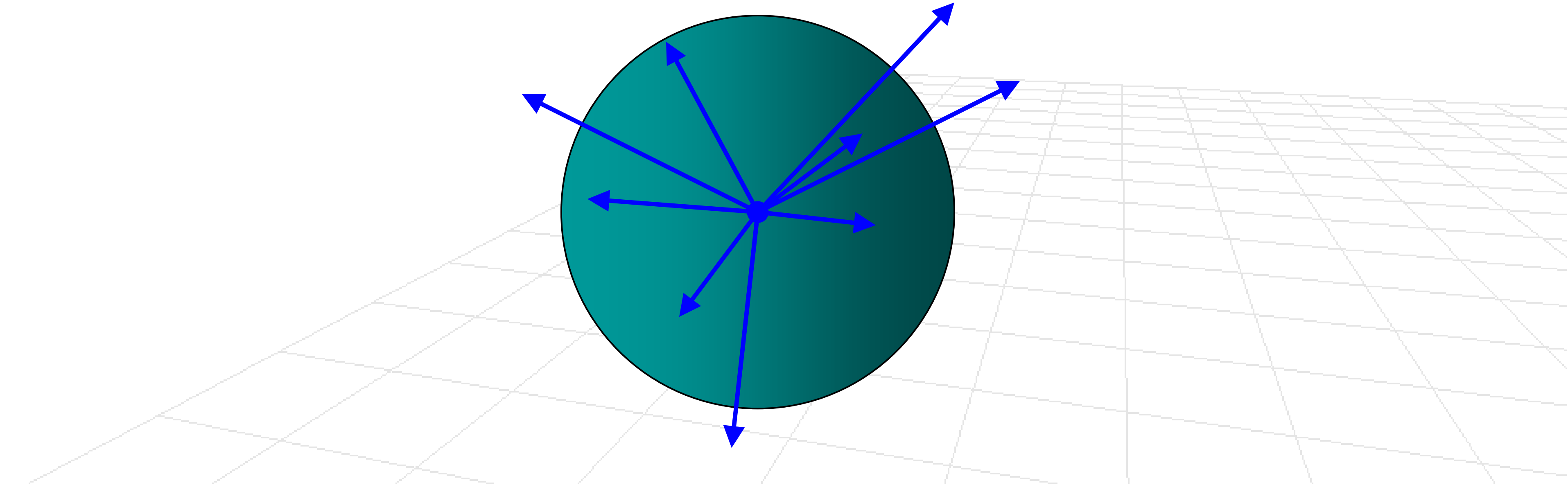
$$E(\bar{p}) = \int_{\vec{d}_e \in \Omega_e} L(\bar{p}, \vec{d}_e)(\vec{n} \cdot \vec{d}_e) \, d\omega$$

$$E(\bar{p}) = \int_{\phi} \int_{\theta} L(\bar{p}, \vec{d}_e)(\vec{n} \cdot \vec{d}_e) \sin\theta \, d\theta \, d\phi$$

Irradiance from Radiance

- We can get irradiance by integrating radiance over the entire sphere
- **Intuition:** Light that is hitting the surface is equal to the light emitted by everything else in the direction of the point

$$\boldsymbol{H}(\overline{\boldsymbol{p}}) = \int_{\phi}\int_{\theta} \boldsymbol{L}(\overline{\boldsymbol{p}}, -\vec{\boldsymbol{d}}_i)(\vec{\boldsymbol{n}} \cdot \vec{\boldsymbol{d}}_i) \sin\theta \, \boldsymbol{d}\theta \, \boldsymbol{d}\phi$$



Radiance vs. Irradiance

- Radiance
  - Describes light emitted from a surface (per area)
  - Function of direction
  - Units: $W \cdot sr^{-1} \cdot m^{-2}$

- Irradiance
  - Describes light incident on a surface
  - Not a directional quantity
  - Units: $W \cdot m^{-2}$

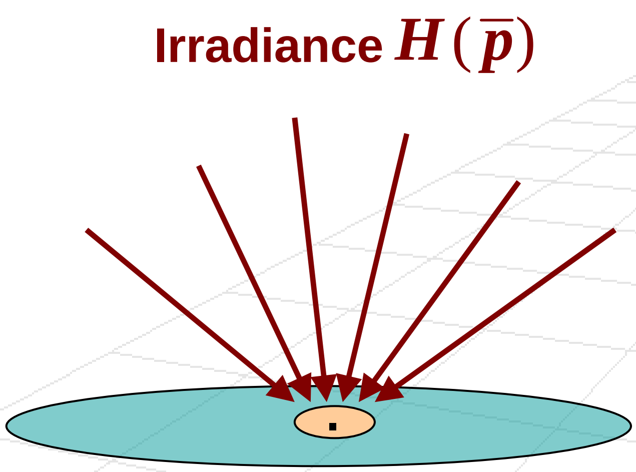
- From the radiance emitted from one surface we can compute the incidence irradiance at a nearby surface

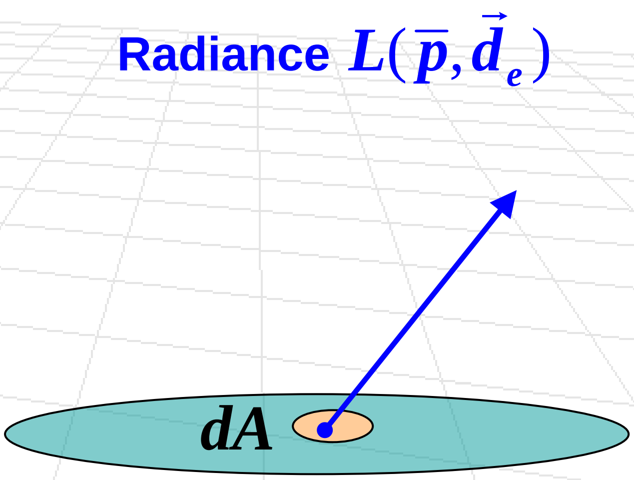
Bidirectional Reflectance Distribution Function (BRDF)

- **BRDF:** Ratio of emittant to incident light (i.e. radiance to irradiance)

$$\rho(\vec{d}_e, \vec{d}_i) = \frac{L(\bar{p}, \vec{d}_e)}{H(\bar{p})}$$

- Models reflectance of simple materials
- Often BRDF must be empirically determined (measured in a laboratory)

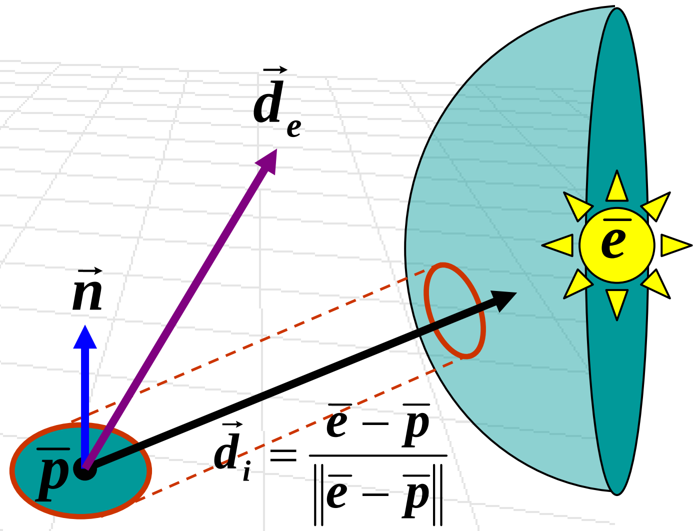




Point Light Sources

- Let's compute surface radiance for a point light source with radiant intensity ***I***
  - I = flux for a solid angle dw
- We already know (from earlier slides) that for a point light source irradiance is given by: $H(\bar{p}) = \dfrac{I(\vec{n}\cdot\vec{d}_i)}{\|\bar{p}-\bar{e}\|^2}$
- We can then get surface radiance by rearranging terms in the definition of BRDF

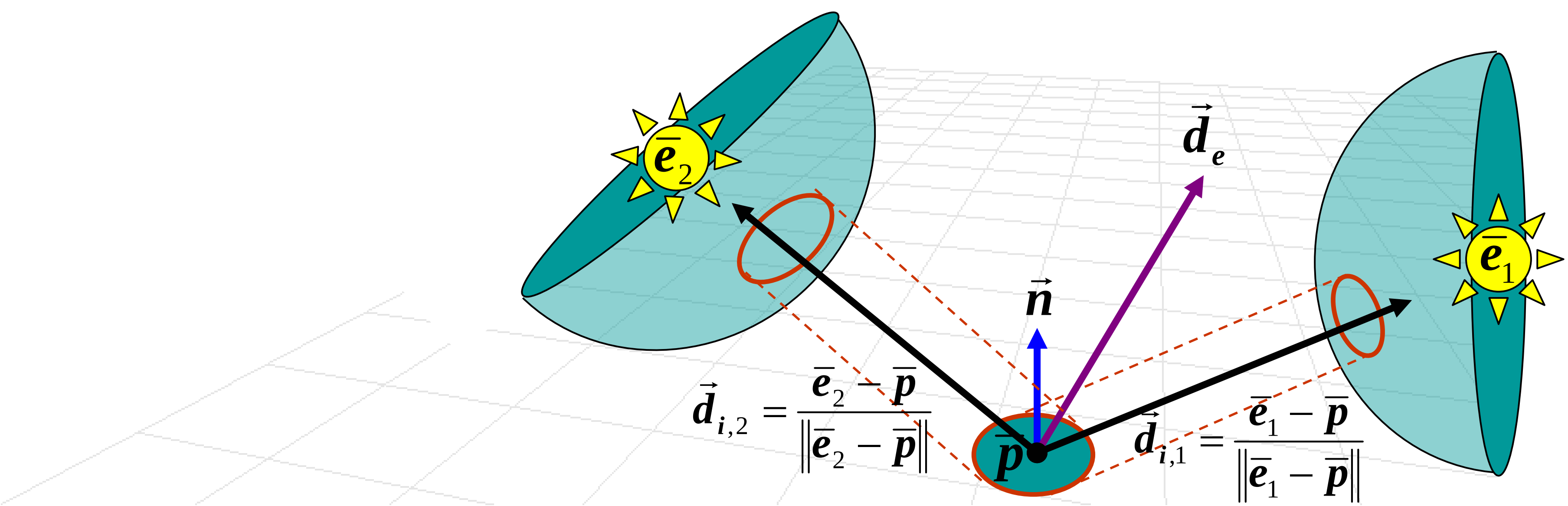
$$
\begin{aligned}
L(\bar{p},\vec{d}_e) &= \rho(\vec{d}_e,\vec{d}_i)H(\bar{p}) \\
&= \rho(\vec{d}_e,\vec{d}_i)\frac{I(\vec{n}\cdot\vec{d}_i)}{\|\bar{p}-\bar{e}\|^2}
\end{aligned}
$$



Multiple Point Light Sources

- Simple to handle, since light is additive

$$L(\overline{\boldsymbol{p}},\vec{\boldsymbol{d}}_e) = \sum_{j=1}^{J} \rho(\vec{\boldsymbol{d}}_e,\vec{\boldsymbol{d}}_{i,j}) \frac{I(\vec{\boldsymbol{n}} \cdot \vec{\boldsymbol{d}}_{i,j})}{\left\| \overline{\boldsymbol{p}} - \overline{\boldsymbol{e}}_j \right\|^2}$$



Extended Light Sources

- We can use radiance to compute required irradiance at a point by integrating over the incident directions

- Remember

$$\boldsymbol{H}(\overline{\boldsymbol{p}}) = \int_{\phi}\int_{\theta} \boldsymbol{L}(\overline{\boldsymbol{p}}, -\vec{\boldsymbol{d}}_i)(\vec{\boldsymbol{n}} \cdot \vec{\boldsymbol{d}}_i) \sin\theta \, \boldsymbol{d}\theta \, \boldsymbol{d}\phi$$

hence

$$\boldsymbol{L}(\overline{\boldsymbol{p}}, \vec{\boldsymbol{d}}_e) = \int_{\phi}\int_{\theta} \rho(\vec{\boldsymbol{d}}_e, \vec{\boldsymbol{d}}_i) \boldsymbol{L}(\overline{\boldsymbol{p}}, -\vec{\boldsymbol{d}}_i)(\vec{\boldsymbol{n}} \cdot \vec{\boldsymbol{d}}_i) \sin\theta \, \boldsymbol{d}\theta \, \boldsymbol{d}\phi$$

Idealizing Lighting and Reflectance

- We will consider a few special cases of the general BRDF models that facilitate lighting

- How do we do Phong lighting in terms of BRDFs?

Diffuse Reflection

- The only factor that determines appearance (radiance) of a **Lambertian** surface is irradiance (incident light)
- In other words, BRDF is constant and independent of incident and emittent direction. i.e. $\rho(\vec{\boldsymbol{d}}_e, \vec{\boldsymbol{d}}_i) = \rho_0$
- The radiance

$$L_d(\overline{\boldsymbol{p}}, \vec{\boldsymbol{d}}_e) = \rho_0 \int_{\vec{\boldsymbol{d}}_i \in \Omega_i} L(\overline{\boldsymbol{p}}, -\vec{\boldsymbol{d}}_i)(\vec{\boldsymbol{n}} \cdot \vec{\boldsymbol{d}}_i) \boldsymbol{d}\omega_i$$

$$L_d(\overline{\boldsymbol{p}}, \vec{\boldsymbol{d}}_e) = \rho_0 \int_{\vec{\boldsymbol{d}}_i \in \Omega_i} L(\overline{\boldsymbol{p}}, -\vec{\boldsymbol{d}}_i) \cos\theta_i \boldsymbol{d}\omega_i$$

- Since total irradiance must equal radiant exitance (conservation of energy), we can show that $\rho_0 = \dfrac{1}{\pi}$

Diffuse Reflection

- Despise simple BRDF, it's still hard to compute radiance because of the integral

$$L_d(\bar{p},\vec{d}_e) = \rho_0 \int_{\vec{d}_i \in \Omega_i} L(\bar{p},-\vec{d}_i)(\vec{n}\cdot\vec{d}_i)d\omega_i$$

- Assuming point light source helps
  - Lets assume single point light source with intensity I
  - Then irradiance is as before $H(\bar{p}) = \frac{I(\vec{n}\cdot\vec{d}_i)}{\|\bar{p}-\bar{e}\|^2}$

$$L_d(\bar{p},\vec{d}_e) = \rho_0 \frac{I(\vec{n}\cdot\vec{d}_i)}{\|\bar{p}-\bar{e}\|^2}$$

- Assuming that light is far away removes the denominator $L_d(\bar{p},\vec{d}_e) = \rho_0 I(\vec{n}\cdot\vec{d}_i)$

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Remember the Phong model?

- Remember Phong lighting equation?

$$L(\overline{\boldsymbol{p}},\vec{\boldsymbol{c}}) = \boxed{\boldsymbol{r}_d \boldsymbol{I}_d \max(0,\vec{\boldsymbol{d}}_i \cdot \vec{\boldsymbol{n}})} + \boldsymbol{r}_a \boldsymbol{I}_a + \boldsymbol{r}_s \boldsymbol{I}_s \max(0,\vec{\boldsymbol{r}} \cdot \vec{\boldsymbol{c}})^{\alpha}$$

$$\boldsymbol{r}_d = \rho_0 \overset{?}{\leq} \frac{1}{\pi}$$

- Assuming that light is far away removes the denominator

$$L_d(\overline{\boldsymbol{p}},\vec{\boldsymbol{d}}_e) = \rho_0 \boldsymbol{I}(\vec{\boldsymbol{n}} \cdot \vec{\boldsymbol{d}}_i)$$

Ambient Illumination

- **Remember:** we need ambient illumination, because diffuse lighting looks artificial (parts of the object are black)

- Ambient illumination is equivalent to uniform illumination and constant BRDF (as in the diffuse case)

$$L_a(\overline{p},\vec{d}_e) = \rho_a \int_{\vec{d}_i \in \Omega_i} L(\overline{p},-\vec{d}_i)(\vec{n}\cdot\vec{d}_i)d\omega_i$$

- It's easy to see that the integral in the above equation is simply a constant

$$L_a(\overline{p},\vec{d}_e) = \rho_a I_a$$

Remember the Phong model?

- Remember Phong lighting equation?

$$L(\bar{\boldsymbol{p}},\vec{\boldsymbol{c}}) = \boldsymbol{r}_d \boldsymbol{I}_d \max(0,\vec{\boldsymbol{d}}_i \cdot \vec{\boldsymbol{n}}) + \boxed{\boldsymbol{r}_a \boldsymbol{I}_a} + \boldsymbol{r}_s \boldsymbol{I}_s \max(0,\vec{\boldsymbol{r}} \cdot \vec{\boldsymbol{c}})^{\alpha}$$

$$\boldsymbol{r}_a = \rho_a$$

- It's easy to see that the integral in the above equation is simply a constant

$$L_a(\bar{\boldsymbol{p}},\vec{\boldsymbol{d}}_e) = \rho_a \boldsymbol{I}_a$$

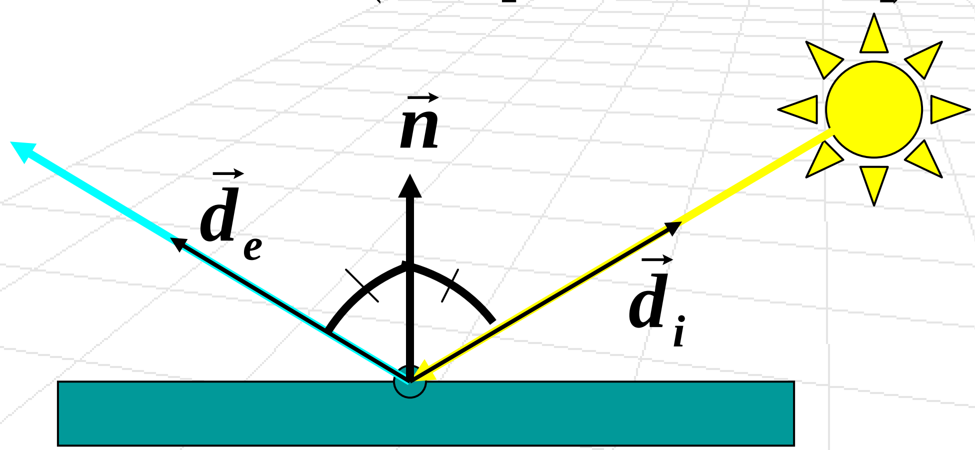
Specular Reflection

- For specular (mirror) surfaces each incident direction is reflected toward unique emittant direction
- The emittant direction can be derived as before in the Phong model

$$\vec{d}_e = 2(\vec{n} \cdot \vec{d}_i)\vec{n} - \vec{d}_i$$

- Since all of the light is reflected into a single direction, the corresponding BRDF can be formulated as follows:

$$\rho(\vec{d}_e, \vec{d}_i) \propto \delta\left(\vec{d}_e - \left[2(\vec{n} \cdot \vec{d}_i)\vec{n} - \vec{d}_i\right]\right)$$



Specular Reflection

- If we assume that light emitted is the same amount of light incident (conservation of energy), we can derive the proportionality constant

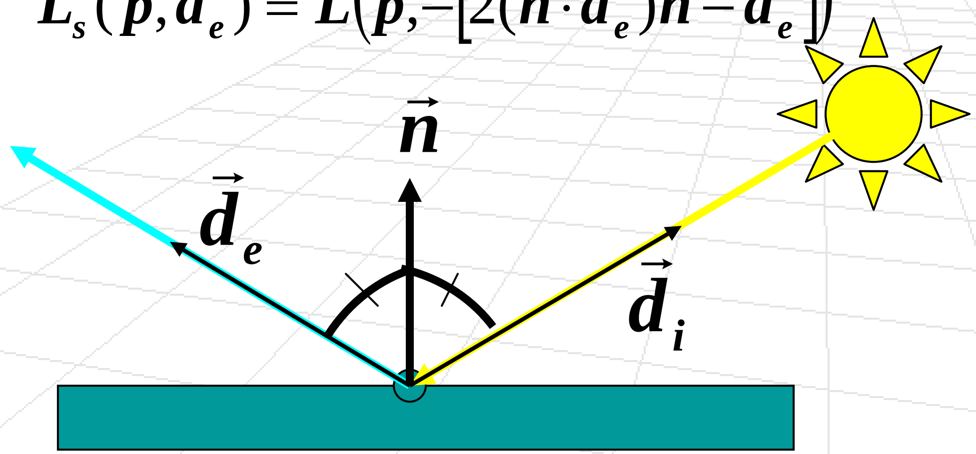
$$\rho(\vec{d}_e, \vec{d}_i) = \frac{1}{\vec{n} \cdot \vec{d}_i} \left(\vec{d}_e - \left[2(\vec{n} \cdot \vec{d}_i)\vec{n} - \vec{d}_i \right] \right)$$

- Specular radiance can then be computed as for other components

$$L_s(\bar{p}, \vec{d}_e) = \int_{\vec{d}_i \in \Omega_i} \rho(\vec{d}_e, \vec{d}_i) L(\bar{p}, -\vec{d}_i)(\vec{n} \cdot \vec{d}_i) d\omega_i$$

which simplifies in this case to:

$$L_s(\bar{p}, \vec{d}_e) = L\left(\bar{p}, -\left[2(\vec{n} \cdot \vec{d}_e)\vec{n} - \vec{d}_e \right] \right)$$



Off-axis Secularity

- If we have more complex surfaces (not just mirrors) we will have off-axis secularities

- In that case the BRDF will not be a simple delta function and we need to go back to the full integral formulation for the radiance

- Phong model makes the point light source assumption that is far away, this leads to the approximation we already encountered

$$L(\bar{p},\vec{c}) = r_d I_d \max(0,\vec{d}_i \cdot \vec{n}) + r_a I_a + \boxed{r_s I_s \max(0,\vec{r} \cdot \vec{c})^{\alpha}}$$

How will all of this help in Ray Tracing?

- We will consider a more accurate (and much more expensive) approximation to the radiance at the "hit point" based on the integral of the BRDF and incident irradiance

- What do we integrate over?
  - We integrate over area of a pixel