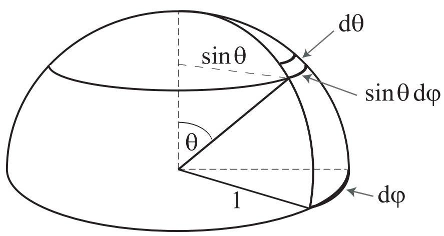
1 Summary of Radiometry and Reflection

1. **Area differential.** Differential element for a surface. Surface area can be computed by integrating: $A = \int dA$. See notes and homework for examples.

2. **Spherical coordinates.**

$$\vec{d} = (\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta)^T \tag{1}$$



3. **Solid angle.** Solid angle is measured as the area a of a patch on a sphere, divided by the squared radius of the sphere (Figure 1); i.e.,

$$\omega = \frac{a}{r^2} \tag{2}$$

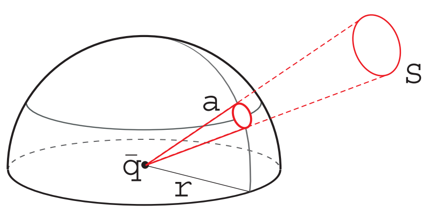


Figure 1: The solid angle of a patch S is given by the area a of its projection onto a sphere of radius r, divided by the squared radius, r^2.

4. **Differential Solid Angle.**

$$d\omega = \sin\theta\, d\theta\, d\phi \tag{3}$$

5. **Foreshortening.**

   Foreshortening is the reduction in the (projected) area of a surface patch as seen from a particular point or viewer. When the surface normal points directly at the viewer its effective size (solid angle) is maximal. As the surface normal rotates away from the viewer it appears smaller (Figure 2). Eventually when the normal is pointing perpendicular to the viewing direction you see the patch "edge on"; so its projection is just a line (with zero area).

Figure 2: Foreshortening in 2D. *Left:* For a patch with area A, seen from a point $\bar{q}$, the patch's foreshortened area is approximately $A\cos\theta$. This is an approximation, since the distance r varies over the patch. The angle θ is the angle between the patch normal and the direction to $\bar{q}$. *Right:* For an infinitesimal patch with area dA, the foreshortened area is exactly $dA\cos\theta$.

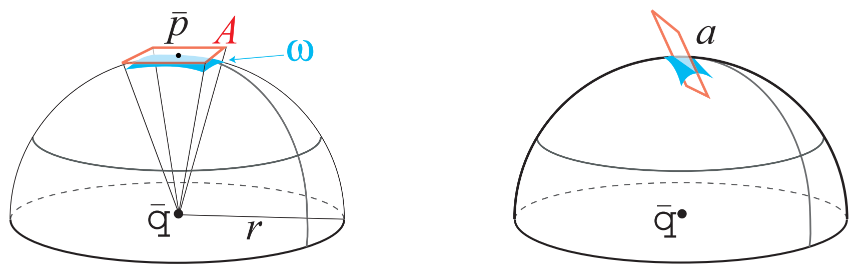


Figure 3: Solid angle of a patch. *Left:* A patch with normal pointing at $\bar{l}$. *Right:* A patch with arbitrary orientation.

6. **Solid angle of an infinitesimal patch.**

$$d\omega = \frac{dA\cos\theta}{r^2}\,, \tag{4}$$

This formula takes into account distance and foreshortening.

7. **Energy:** $Q(t)$, measured in Joules

8. **Flux (power)**, measured in Joules/second

$$\Phi(t) = \frac{dQ(t)}{dt} \tag{5}$$

Hence,

$$Q(t) = \int_0^t \Phi(\tau)\,d\tau \tag{6}$$

9. **Irradiance**

$$H(\bar{p}) = \frac{d\Phi}{dA} \tag{7}$$

at a point $\bar{p}$, where dA refers to differential surface area. Irradiance is power per unit surface area ($\mathrm{W}\cdot\mathrm{m}^{-2}$). Irradiance measures the amount of light incidence on a surface.

10. **Point light source** with I watts per steradian into all directions:

$$d\Phi = I d\omega \tag{8}$$

Irradiance at a patch is then:

$$H \;=\; \frac{d\Phi}{dA} \;=\; \frac{I d\omega}{dA} \;=\; \frac{I dA \cos\theta}{dA r^2} \;=\; \frac{I\cos\theta}{r^2} \tag{9}$$

where $\bar{p}$ is the position of S, $r = ||\bar{l} - \bar{p}||$, and θ is the angle between the surface normal and the vector $\bar{l} - \bar{p}$.

11. **Radiance** Radiance is a measure of the rate at which light energy is emitted from a surface in a particular direction. It is a function of position and direction, and it is often denoted by L (or $L(\bar{p}, \vec{d})$). Formally, it is defined as power per steradian per surface area ($\mathrm{W} \cdot \mathrm{sr}^{-1} \cdot \mathrm{m}^{-2}$), where the surface area is defined with respect to a surface patch at $\bar{p}$ that is perpendicular to the direction $\vec{d}$.

    Note that radiance and irradiance are different concepts; see lecture notes ("Radiance vs. Irradiance.")

12. **Bidirectional Reflectance Distribution Function (BRDF)** The BRDF describes how a surface reflects light, and represents the "material" of a surface. The BRDF is a function of emittant and incident directions $\vec{d}_e$ and $\vec{d}_i$. It is defined to be the ratio of radiance to irradiance:

$$\rho(\vec{d}_e, \vec{d}_i) = \frac{L}{H} \tag{10}$$

    We use the BRDF to compute outgoing radiance from a point, given all incoming radiance:

$$L(\vec{d}_e) = \int_{\vec{d}_i \in \Omega_i} \rho(\vec{d}_e,\, \vec{d}_i)\, L(\bar{p}, -\vec{d}_i)\, \cos\theta_i \, d\omega_i \tag{11}$$