

THE UNIVERSITY OF BRITISH COLUMBIA

Topics in AI (CPSC 532S): **Multimodal Learning with Vision, Language and Sound**

Lecture 5: Convolutional Neural Networks (Part 2)





Assignment 2 is out

Last time: Convolutional Layer

32 x 32 x 3 **image**





activation map

Last time: Convolutional Neural Network (ConvNet)







Convolutional Neural Networks



VGG-16 Network



CNNs: Reminder Fully Connected Layers

Input

3072

(32 x 32 x 3 image -> stretches to 3072 x 1)





Convolutional Neural Networks



VGG-16 Network

CNNs: Reminder Fully Connected Layers



102,760,448 parameters!

* adopted from Fei-Dei Li, Justin Johnson, Serena Yeung, cs231n Stanford

Activation

4,096



Convolutional Neural Networks



VGG-16 Network

Pooling Layer



Let us assume the filter is an "eye" detector

How can we make detection spatially invariant (insensitive to position of the eye in the image)

* slide from Marc'Aurelio Renzato

Pooling Layer



Let us assume the filter is an "eye" detector

How can we make detection spatially invariant (insensitive to position of the eye in the image)

> By "pooling" (e.g., taking a max) response over a spatial locations we gain robustness to position variations



* slide from Marc'Aurelio Renzato

Pooling Layer

- Makes representation smaller, more manageable and spatially invariant
- Operates over each activation map independently



e manageable and spatially invariant independently



Max **Pooling**

activation map





max pool with 2 x 2 filter and stride of 2

6 8 3 4

Average **Pooling**

activation map





avg pool with 2 x 2 filter and stride of 2

3.25 5.25 2 2

Pooling Layer Receptive Field

If convolutional filters have size KxK and stride 1, and pooling layer has pools of size PxP, then each unit in the pooling layer depends upon a patch (at the input of the preceding conv. layer) of size: **(P+K-1)x(P+K-1)**



* slide from Marc'Aurelio Renzato

Pooling Layer Receptive Field

If convolutional filters have size KxK and stride 1, and pooling layer has pools of size PxP, then each unit in the pooling layer depends upon a patch (at the input of the preceding conv. layer) of size: (P+K-1)x(P+K-1)



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Pooling Layer Summary

Accepts a volume of size: $W_i \times H_i \times D_i$ Requires hyperparameters: - Spatial extent of filters: K- Stride of application: FProduces a volume of size: $W_o \times H_o \times D_o$ $W_o = (W_i - F)/S + 1$ $H_o = (H_i - F)/S + 1$

Number of total learnable parameters: 0

$D_o = D_i$

Convolutional Neural Networks



VGG-16 Network

Improving Single Model

Regularization

- L2, L1
- Dropout / Inverted Dropout
- Data augmentation

L2 Regularization: Learn a more (dense) distributed representation

$R(\mathbf{W}) = ||\mathbf{W}|$

L1 Regularization: Learn a sparse representation

 $R(\mathbf{W}) = ||\mathbf{W}|$



Dropout

$$||_{2} = \sum_{i} \sum_{j} \mathbf{W}_{i,j}^{2}$$

n (few non-zero wight elements)

$$\|_1 = \sum_i \sum_j |\mathbf{W}_{i,j}|$$





Transform image

Horizontal flips

Random crops & scales

Color Jitter

Horizontal flips

Random crops & scales





Color Jitter

Horizontal flips

Training: sample random crops and scales e.g., ResNet:

- 1. Pick random L in range [256, 480]
- 2. Resize training image, short size = L
- 3. Sample random 224x224 patch

Testing: average a fix set of crops e.g., ResNet:

Resize image to 5 scales (224, 256, 384, 480, 640) 2. For each image use 10 224x224 crops: 4 corners + center, + flips

Random crops & scales

Color Jitter



Horizontal flips

Random perturbations in contrast and brightness



Random crops & scales

Color Jitter



Regularization: Stochastic Depth

Effectively "dropout" but for layers

some layer (for each batch)



Huang et al., ECCV 2016]



Common "Wisdom": You need a lot of data to train a CNN



This strategy is PERVASIVE.



Solution: Transfer learning — taking a model trained on the task that has lots of data and adopting it to the task that may not

Train on **ImageNet**

FC-1000
FC-4096
FC-4096
MaxPool
Conv-512
Conv-512
MaxPool
Conv-512
Conv-512
MaxPool
Conv-256
Conv-256
MaxPool
Conv-128
Conv-128
MaxPool
Conv-64
Conv-64
Image

- Why on **ImageNet**?
 - Convenience, lots of data
 - We know how to train these well



[Yosinski et al., NIPS 2014] Donahue et al., ICML 2014 Razavian et al., CVPR Workshop 2014

However, for some tasks we would need to start with something else (e.g., videos for optical flow)

Train on **ImageNet**

FC-1000
FC-4096
FC-4096
MaxPool
Conv-512
Conv-512
MaxPool
Conv-512
Conv-512
MaxPool
Conv-256
Conv-256
MaxPool
Conv-128
Conv-128
MaxPool
Conv-64
Conv-64
Image

Re-initialize and train



[Yosinski et al., NIPS 2014] Donahue et al., ICML 2014 Razavian et al., CVPR Workshop 2014

Small dataset with C classes

Lower levels of the CNN are at task independent anyways

Freeze these layers

Train on **ImageNet**

Small dataset with C classes

FC-1000
FC-4096
FC-4096
MaxPool
Conv-512
Conv-512
MaxPool
Conv-512
Conv-512
MaxPool
Conv-256
Conv-256
MaxPool
Conv-128
Conv-128
MaxPool
Conv-64
Conv-64
Image

Re-initialize and train



[Yosinski et al., NIPS 2014] [Donahue et al., ICML 2014] [Razavian et al., CVPR Workshop 2014]

Larger dataset

* adopted from Fei-Dei Li, Justin Johnson, Serena Yeung, cs231n Stanford

Freeze these layers



Model **Ensemble**

Training: Train multiple independent models **Test:** Average their results

~ 2% improved performance in practice

Alternative: Multiple snapshots of the single model during training!

- **Improvement:** Instead of using the actual parameter vector, keep a moving average of the parameter vector and use that at test time (Polyak averaging)

CPU vs. GPU (Why do we need Azure?)



Data from https://github.com/jcjohnson/cnn-benchmarks

Frameworks: Super quick overview

1. Easily **build computational graphs**

2. Easily **compute gradients** in computational graphs

3. Run it all efficiently on a GPU (weap cuDNN, cuBLAS, etc.)



Frameworks: Super quick overview

Core DNN Frameworks

Caffe (UC Berkeley) Caffe 2 (Facebook)

(Baidu)

Torch (NYU/Facebook)

PyTorch (Facebook)

Theano (U Montreal) **TensorFlow** (Google)

Puddle

CNTK (Microsoft)

MXNet (Amazon)

Wrapper Libraries

Keras TFLearn TensorLayer tf.layers **TF-Slim** tf.contrib.learn Pretty Tensor

Frameworks: PyTorch vs. TensorFlow

Dynamic vs. Static computational graphs
Frameworks: PyTorch vs. TensorFlow

Dynamic vs. **Static** computational graphs

With static graphs, framework can optimize the graph for you before it runs!



Frameworks: PyTorch vs. TensorFlow

Dynamic vs. Static computational graphs

Graph building and execution is intertwined. Graph can be different for every sample.



PyTorch: Three levels of abstraction

Tensor: Imperative ndarray, but runs on GPU

Variable: Node in a computational graph; stores data and gradients

Module: A neural network layer; may store state or learnable weights



Categorization



Horse Multi-class: Church Toothbrush Person IM GENET

Multi-label: Horse

Church Toothbrush Person



Categorization

Detection





Multi-**class:** Horse Church Toothbrush Person **IM** GENET

Multi-label: Horse

Church Toothbrush Person

Horse (x, y, w, h) Horse (x, y, w, h) Person (x, y, w, h) Person (x, y, w, h)





Categorization

Detection





Multi-**class:** Horse Church Toothbrush Person **IM** GENET

Multi-label: Horse

Church Toothbrush Person

Horse (x, y, w, h) Horse (x, y, w, h) Person (x, y, w, h) Person (x, y, w, h)





Segmentation

Horse Person



Categorization

Detection





Multi-class: Horse Church Toothbrush Person **IM** GENET

Multi-label: Horse

Church Toothbrush Person

Horse (x, y, w, h) Horse (x, y, w, h) Person (x, y, w, h) Person (x, y, w, h)





Segmentation

Horse Person



Instance Segmentation

Horse1 Horse₂ Person1 Person2



Object Classification



Problem: For each image predict which category it belongs to out of a fixed set

Object Classification

	Category	Predictio
	Dog	No
	Cat	No
	Couch	No
	Flowers	No
	Leopard	Yes

Problem: For each image predict which category it belongs to out of a fixed set

Object Classification

Problem: For each image predict which category it belongs to out of a fixed set

 \mathbf{x}^t

CNN Architectures: LeNet-5

Architecture: $CONV \longrightarrow POOL \longrightarrow CONV \longrightarrow POOL \longrightarrow FC \longrightarrow FC$ Conv filters: 5x5, Stride: 1 **Pooling:** 2x2, Stride: 2

[LeCun et al., 1998]

ImageNet Dataset

Over **14 million** (high resolution) web **images** Roughly labeled with **22K synset** categories Labeled on Amazon Mechanical Turk (AMT)

Popular Synsets

Animal

fish bird mammal invertebrate

Plant

tree flower vegetable

Activity

sport

Material

fabric

Instrumentation

utensil appliance tool musical instrument

Scene

room geological formation

Food

beverage

ImageNet Competition (ILSVRC)

Annual competition of image classification at scale Focuses on a subset of **1K synset** categories **Scoring:** need to predict true label within top K (K=5)

AlexNet

Architecture: CONV1 MAX POOL1 NORM1 CONV2 MAX POOL2 NORM2 CONV3 CONV4 CONV5 Max POOL3 FC6 FC7 FC8

Output: 55 x 55 x 96

[Krizhevsky et al., 2012]

Input: 227 x 227 x 3 images

CONV1: 96 11 x 11 filters applied at stride 4

Parameters: 35K

MAX POOL1: 96 11 x 11 filters applied at stride 4 **Output:** 27 x 27 x 96 **Parameters:** 0

Local Contrast Normalization Layer

ensures response is the same in both case (details omitted, no longer popular)

* images from Marc'Aurelio Renzato

AlexNet

Full (simplified) AlexNet architecture: [227x227x3] INPUT [55x55x96] CONV1: 96 11x11 filters at stride 4, pad 0 [27x27x96] MAX POOL1: 3x3 filters at stride 2 [27x27x96] NORM1: Normalization layer [27x27x256] CONV2: 256 5x5 filters at stride 1, pad 2 [13x13x256] MAX POOL2: 3x3 filters at stride 2 [13x13x256] NORM2: Normalization layer [13x13x384] CONV3: 384 3x3 filters at stride 1, pad 1 [13x13x384] CONV4: 384 3x3 filters at stride 1, pad 1 [13x13x256] CONV5: 256 3x3 filters at stride 1, pad 1 [6x6x256] MAX POOL3: 3x3 filters at stride 2 [4096] FC6: 4096 neurons [4096] FC7: 4096 neurons [1000] FC8: 1000 neurons (class scores)

Krizhevsky et al., 2012

Details / Comments

- First use of ReLU
- Used contrast normalization layers
- Heavy data augmentation
- Dropout of 0.5
- Batch size of 128
- SGD Momentum (0.9)
- Learning rate (1e-2) reduced by 10 manually when validation accuracy plateaus
- L2 weight decay
- -7 CNN ensample: 18.2% -> 15.4%

ILSVRC winner 2012

ZF Net

AlexNet with small modifications:

- CONV1 (11 x 11 stride 4) to (7 x 7 stride 2)
- CONV3 # of filters 384 -> 512
- CONV4 # of filters 384 -> 1024
- CONV5 # of filters 256 -> 512

[Zeiler and Fergus, 2013]

ILSVRC winner 2012

VGG Net

Trend:

- -smaller filters (3 x 3)
- -deeper network (16 or 19 vs. 8 in Alex

Why?

- receptive field of a 3 layer ConvNet with filter size is the same as 1 layer ConvNet with filter 7x7 (at stride

- deeper = more non-linearity (so richer filters)
- fewer parameters

[Simonyan and Zisserman, 2014]

			Solumax
			FC 1000
		Softmax	FC 4096
		FC 1000	FC 4096
		FC 4096	Pool
		FC 4096	3x3 conv, 512
		Pool	3x3 conv, 512
xNet)		3x3 conv, 512	3x3 conv, 512
		3x3 conv, 512	3x3 conv, 512
		3x3 conv, 512	Pool
		Pool	3x3 conv, 512
	Softmax	3x3 conv, 512	3x3 conv, 512
	FC 1000	3x3 conv, 512	3x3 conv, 512
	FC 4096	3x3 conv, 512	3x3 conv, 512
	FC 4096	Pool	Pool
= 3x3	Pool	3x3 conv, 256	3x3 conv, 256
1)	3x3 conv, 256	3x3 conv, 256	3x3 conv, 256
1)	3x3 conv, 384	Pool	Pool
	Pool	3x3 conv, 128	3x3 conv, 128
	3x3 conv, 384	3x3 conv, 128	3x3 conv, 128
	Pool	Pool	Pool
	5x5 conv, 256	3x3 conv, 64	3x3 conv, 64
	11x11 conv, 96	3x3 conv, 64	3x3 conv, 64
	Input	Input	Input
	AlexNet	VGG16	VGG19

VGG Net

INPUT: [224x224x3] memory: 224*224*3=150K params: 0 CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*3)*64 = 1,728 CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*64)*64 = 36,864 POOL2: [112x112x64] memory: 112*112*64=800K params: 0 CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*64)*128 = 73,728 CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*128)*128 = 147,456 POOL2: [56x56x128] memory: 56*56*128=400K params: 0 CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*128)*256 = 294,912 CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824 CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824 POOL2: [28x28x256] memory: 28*28*256=200K params: 0 CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*256)*512 = 1,179,648 CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296 CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296 POOL2: [14x14x512] memory: 14*14*512=100K params: 0 CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296 CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296 CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296 POOL2: [7x7x512] memory: 7*7*512=25K params: 0 FC: [1x1x4096] memory: 4096 params: 7*7*512*4096 = 102,760,448 FC: [1x1x4096] memory: 4096 params: 4096*4096 = 16,777,216 FC: [1x1x1000] memory: 1000 params: 4096*1000 = 4,096,000

TOTAL memory: 24M * 4 bytes ~= 96MB / image (only forward! ~*2 for bwd) TOTAL params: 138M parameters

[Simonyan and Zisserman, 2014]

```
(not counting biases)
```

* slide from Fei-Dei Li, Justin Johnson, Serena Yeung, cs231n Stanford

VGG16

ILSVRC winner 2012

even deeper network with computational efficiency

- -22 layers
- Efficient "Inception" module
- No FC layers
- Only 5 million parameters!
- (12x less than AlexNet!)
- Better performance (@6.7 top 5 error)

[Szegedy et al., 2014]

these modules

Szegedy et al., 2014]

Idea: design good local topology ("network within network") and then stack

these modules

Apply parallel filter operations on the input from previous layer

 Multiple receptive field sizes for convolution (1x1, 3x3, 5x5)

- Pooling operation (3x3)

Concatenate all filter outputs together at output depth-wise Szegedy et al., 2014]

Idea: design good local topology ("network within network") and then stack

What's the problem?

Naive Inception module

these modules

Apply parallel filter operations on the input from previous layer

 Multiple receptive field sizes for convolution (1x1, 3x3, 5x5)

- Pooling operation (3x3)

Concatenate all filter outputs together at output depth-wise

Szegedy et al., 2014]

Idea: design good local topology ("network within network") and then stack

Convolutional Layer: 1x1 convolutions

56 x 56 x 64 **image**

Idea: design good local topology ("network within network") and then stack these modules

Naive Inception module

1x1 "bottleneck" layers

Inception module with dimension reduction

saves approximately 60% of computations

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[Szegedy et al., 2014]

Optimizing Deep Neural Networks

Consider multi-layer neural network with sigmoid activations and loss C

Source: http://neuralnetworksanddeeplearning.com/chap5.html

Optimizing **Deep** Neural Networks

 $\frac{\partial C}{\partial b_1} = \sigma'(z_1) \times w_2 \times \sigma'(z_2) \times w_3 \times \sigma'(z_3) \times w_4 \times \sigma'(z_4) \times \frac{\partial C}{\partial a_4}$

Source: http://neuralnetworksanddeeplearning.com/chap5.html

Optimizing **Deep** Neural Networks

Expression for **gradient** of bias in **Layer 1**:

Expression for gradient

 $\frac{\partial C}{\partial b_1} = \sigma'(z_1) \times w_2 \times \sigma'(z_2) \times w_3 \times \sigma'(z_3) \times w_4 \times \sigma'(z_4) \times \frac{\partial C}{\partial a_4}$

$$\frac{\partial C}{\partial b_1} = \sigma'(z_1) \, w_2 \sigma'(z_2) \, w_3 \sigma'(z_3) \, w_4 \sigma'(z_4) \frac{\partial C}{\partial a_4}$$

of bias in **Layer 3**:
$$\frac{\partial C}{\partial b_3} = \sigma'(z_3) w_4 \sigma'(z_4) \frac{\partial C}{\partial a_4}$$

Source: http://neuralnetworksanddeeplearning.com/chap5.html








Observations:

|weight| < 1 (due to initialization) max of derivative of sigmoid = 1/4 @ 0

 $\frac{\partial C}{\partial b_1} = \sigma'(z_1) \times w_2 \times \sigma'(z_2) \times w_3 \times \sigma'(z_3) \times w_4 \times \sigma'(z_4) \times \frac{\partial C}{\partial a_4}$ ^w4 $\rightarrow C$ $\xrightarrow{a_3} (b_3)$ (b4) $\xrightarrow{}$ (b_2) $\frac{\partial C}{\partial b_1} = \sigma'(z_1) w_2 \sigma'(z_2) w_3 \sigma'(z_3) w_4 \sigma'(z_4) \frac{\partial C}{\partial a_4}$ common terms $\frac{\partial C}{\partial b_3} = \sigma'(z_3) \, \overline{w_4 \sigma'(z_4)} \frac{\partial C}{\partial a_4}$





Observations:

|weight| < 1 (due to initialization) max of derivative of sigmoid = 1/4 @ 0

 $\frac{\partial C}{\partial b_1} = \sigma'(z_1) \times w_2 \times \sigma'(z_2) \times w_3 \times \sigma'(z_3) \times w_4 \times \sigma'(z_4) \times \frac{\partial C}{\partial a_4}$ $\begin{pmatrix} b_1 \end{pmatrix} \xrightarrow{w_2} \begin{pmatrix} b_2 \end{pmatrix} \xrightarrow{w_3} \begin{pmatrix} b_3 \end{pmatrix}$ $\longrightarrow C$







This is called vanishing gradient problem

 makes deep networks hard to train later layers learn faster than earlier ones

 $\frac{\partial C}{\partial b_1} = \sigma'(z_1) \times w_2 \times \sigma'(z_2) \times w_3 \times \sigma'(z_3) \times w_4 \times \sigma'(z_4) \times \frac{\partial C}{\partial a_4}$ $\underbrace{\frac{\partial C}{\partial b_1}}_{=\sigma'(z_1)} \underbrace{\sigma'(z_2)}_{w_2\sigma'(z_2)} \underbrace{\frac{\partial C}{w_3\sigma'(z_3)}}_{w_3\sigma'(z_3)} w_4\sigma'(z_4) \frac{\partial C}{\partial a_4}$ common terms

 $\frac{\partial C}{\partial b_3} = \sigma'(z_3) w_4 \sigma'(z_4) \frac{\partial C}{\partial a_4}$





Exploding gradient problem

- makes weights large (e.g., 100) - make bias such that pre-activation = 0

 $\frac{\partial C}{\partial b_1} = \sigma'(z_1) \times w_2 \times \sigma'(z_2) \times w_3 \times \sigma'(z_3) \times w_4 \times \sigma'(z_4) \times \frac{\partial C}{\partial a_4}$ $\begin{pmatrix} b_1 \end{pmatrix} \xrightarrow{w_2} \begin{pmatrix} b_2 \end{pmatrix} \xrightarrow{w_3} \begin{pmatrix} b_3 \end{pmatrix} \xrightarrow{w_4} \begin{pmatrix} b_4 \end{pmatrix} \longrightarrow C$ >1 >1 $\frac{\partial C}{\partial b_1} = \sigma'(z_1) \underbrace{w_2 \sigma'(z_2)}_{w_2 \sigma'(z_2)} \underbrace{w_3 \sigma'(z_3)}_{w_3 \sigma'(z_3)} w_4 \sigma'(z_4) \frac{\partial C}{\partial a_4}$ common terms $\frac{\partial C}{\partial b_3} = \sigma'(z_3) w_4 \sigma'(z_4) \frac{\partial C}{\partial a_4}$

GoogleLeNet

even deeper network with computational efficiency

- -22 layers
- Efficient "Inception" module
- No FC layers
- Only 5 million parameters!
- (12x less than AlexNet!)
- Better performance (@6.7 top 5 error)

[Szegedy et al., 2014]



ILSVRC winner 2012



ResNet

even deeper — **152 layers**! using residual connections

[He et al., 2015]







ResNet: Motivation



[He et al., 2015]

What happens when we continue to stacking deeper layers on a "plain" CNN



Whats the **problem**?



ResNet: Motivation

Hypothesis: deeper models are harder to optimize (optimization problem)

Observation: the deeper model should (conceptually) perform just as well (e.g., take shallower model and use identity for all remaining layers)

How do we implement this idea in practice

[He et al., 2015]



ResNet

Solution: use network to fit residual mapping instead of directly trying to fit a desired underlying mapping

H(x) = F(x) + X



[He et al., 2015]





ResNet

Full details

- Stacked **residual blocks**
- Every residual block consists of two 3x3 filters
- Periodically double # of filters and downsample spatially using stride of 2
- Additional convolutional layer in the beginning
- No FC layers at the end (only FC to output 1000 classes)

[He et al., 2015]







ILSVRC winner 2012



Regularization: Stochastic Depth

Effectively "dropout" but for layers

some layer (for each batch)



Huang et al., ECCV 2016]



One can view a sequence of outputs from residual layers as a **Dynamical** System



[Cheng et al., ICLR 2018]

One can view a sequence of outputs from residual layers as a **Dynamical** System



$\mathbf{Y}_{j+1} = \mathbf{Y}_j + G(\mathbf{Y}_j, \boldsymbol{\theta}_j)$



[Cheng et al., ICLR 2018]

One can view a sequence of outputs from residual layers as a **Dynamical** System



What happens if you take more layers and take smaller steps?

[Chen et al., NIPS 2018 **best paper**]

One can view a sequence of outputs from residual layers as a **Dynamical** System



What happens if you take more layers and take smaller steps?

You can actually treat a neural network as an **ODE**:

$$\frac{d\mathbf{h}(t)}{dt} = f(\mathbf{h}(t), t, \theta)$$

[Chen et al., NIPS 2018 best paper]

Comparing **Complexity**



An Analysis of Deep Neural Network Models for Practical Applications, 2017.

