THE UNIVERSITY OF BRITISH COLUMBIA

Topics in AI (CPSC 532S):
Multimodal Learning with Vision, Language and Sound

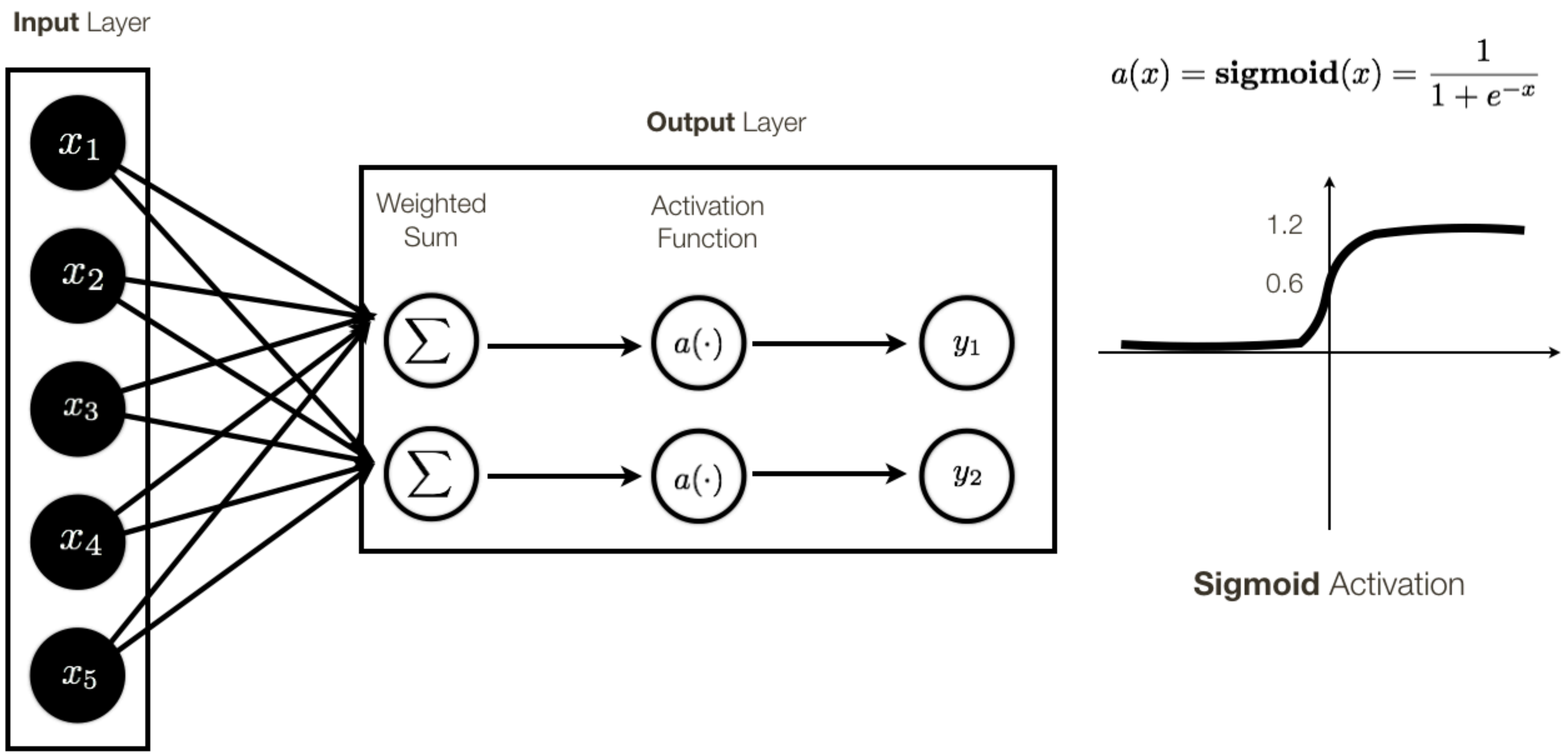
Lecture 3: Introduction to Deep Learning (continued)

Course **Logistics**

- **Assignment 1** … any questions?

Short **Review** …

- Introduced the basic building block of Neural Networks **(MLP/FC) layer**



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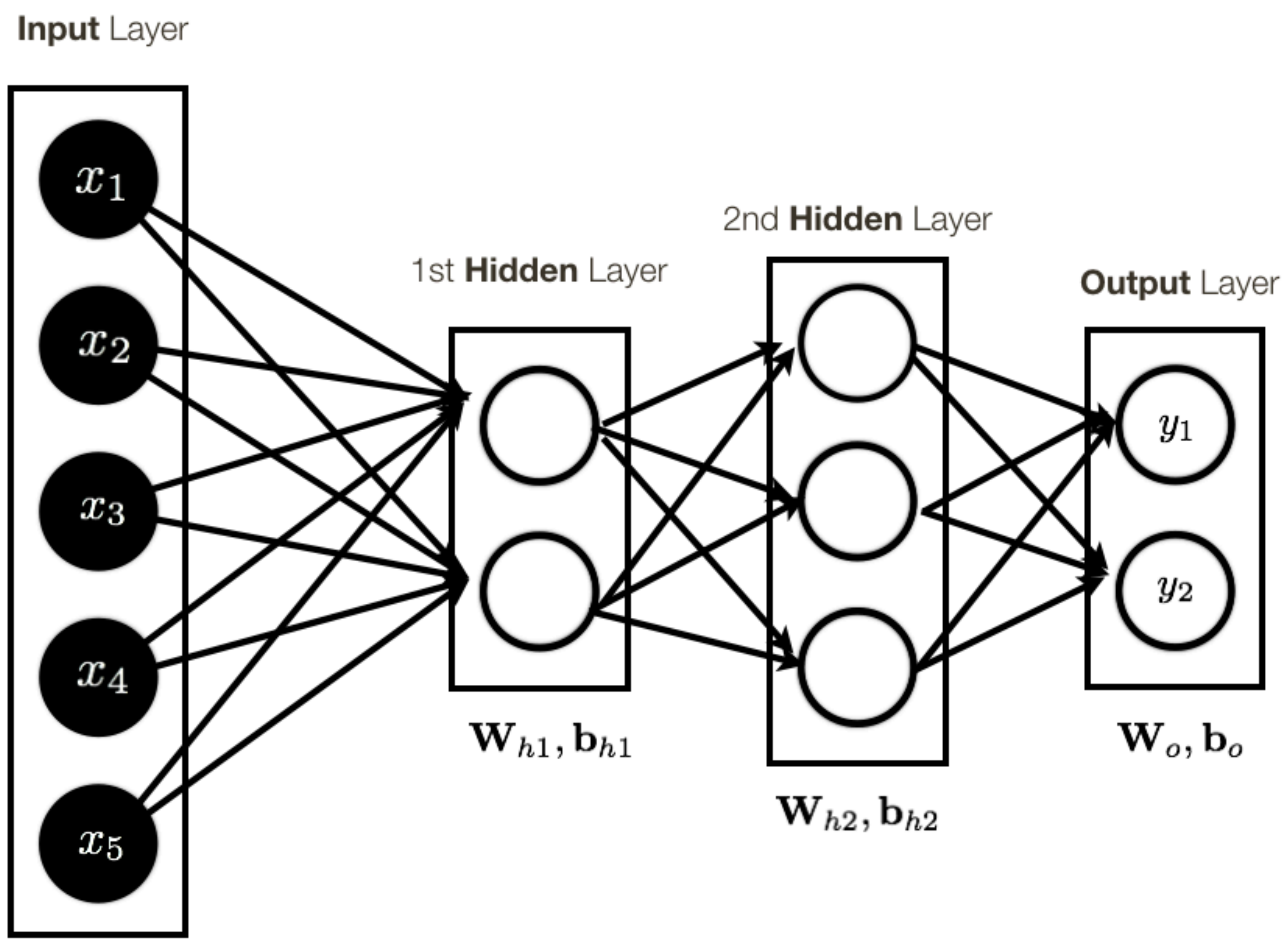
- How do we **stack these layers** up to make a Deep NN



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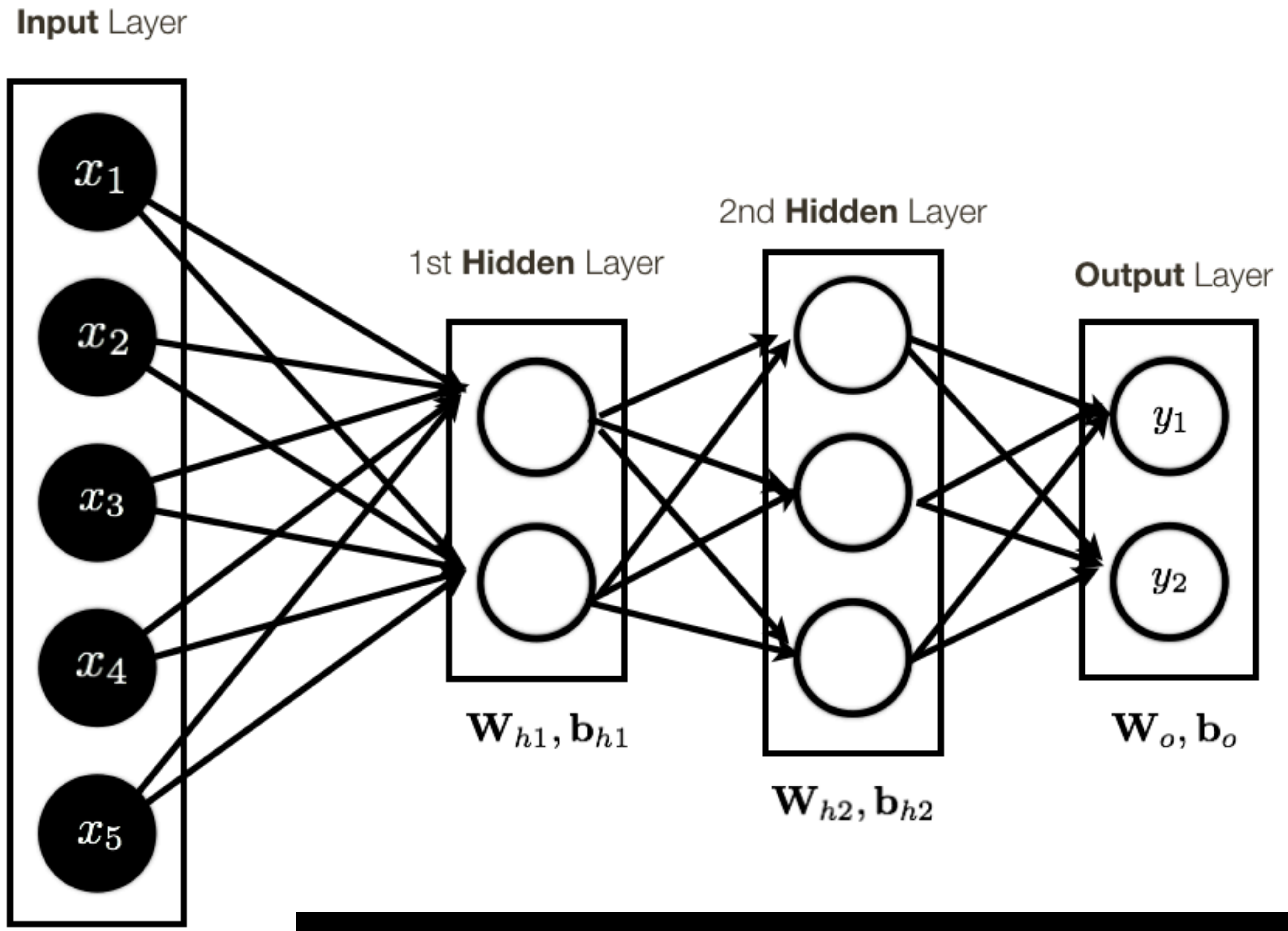

Note: output layer often does not contain activation, or has “activation” function of a different form, to account for the specific **output** we want to produce.

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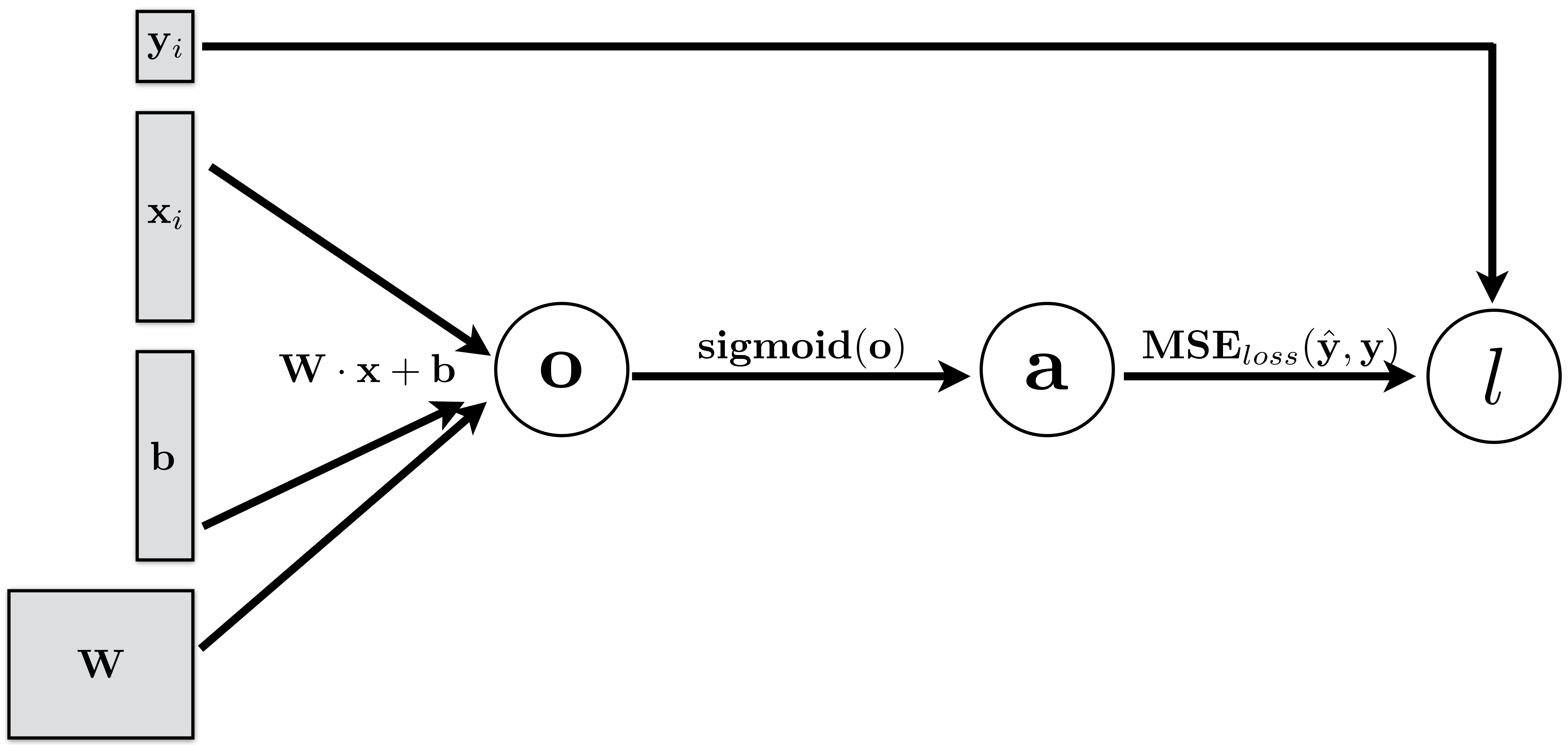
- Basic **NN operations** (implemented using **computational graph**)



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Prediction / Inference

Function evaluation

(a.k.a. **ForwardProp**)

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(Stochastic) Gradient Descent (needs **derivatives**)

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- **Symbolic** differential (intractable)
- AutoDiff **Forward** (computationally expensive)
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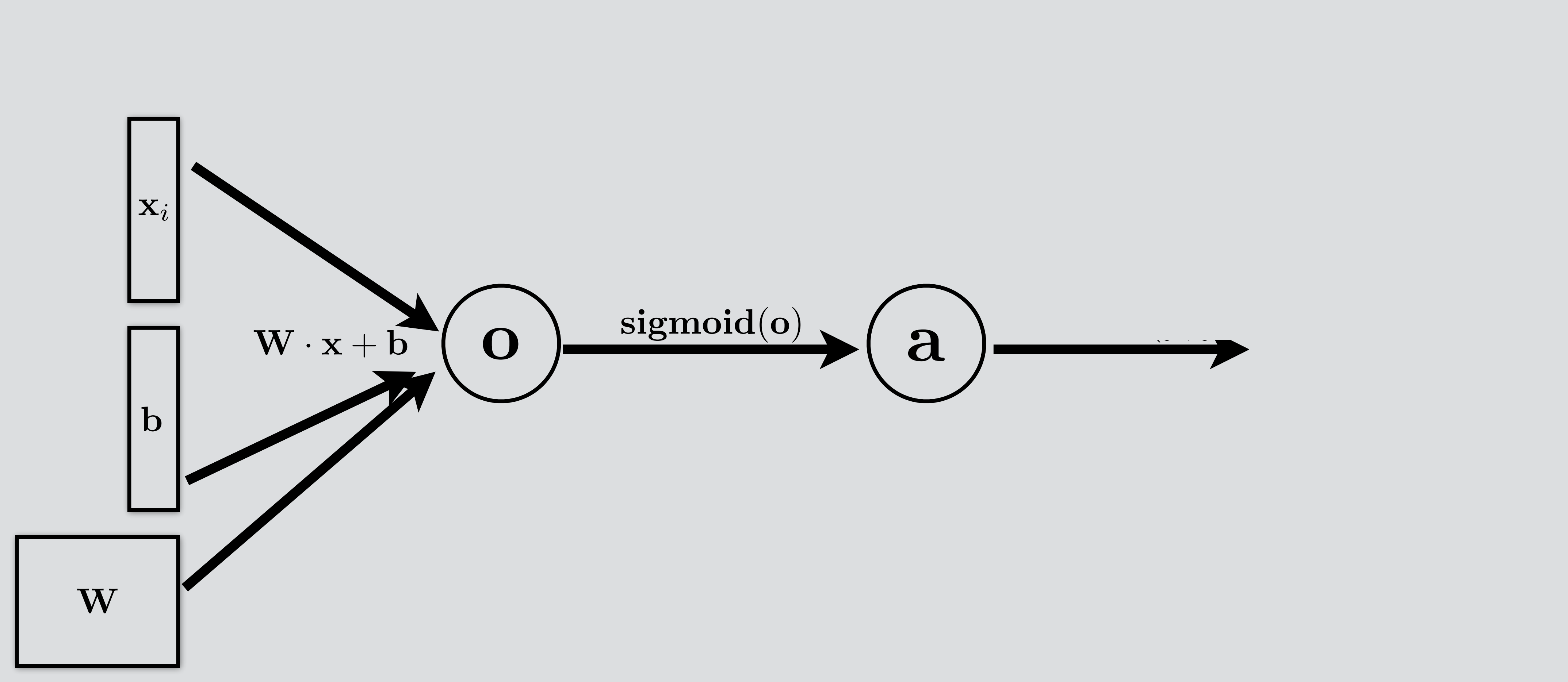
Short **Review** ...

Prediction / Inference



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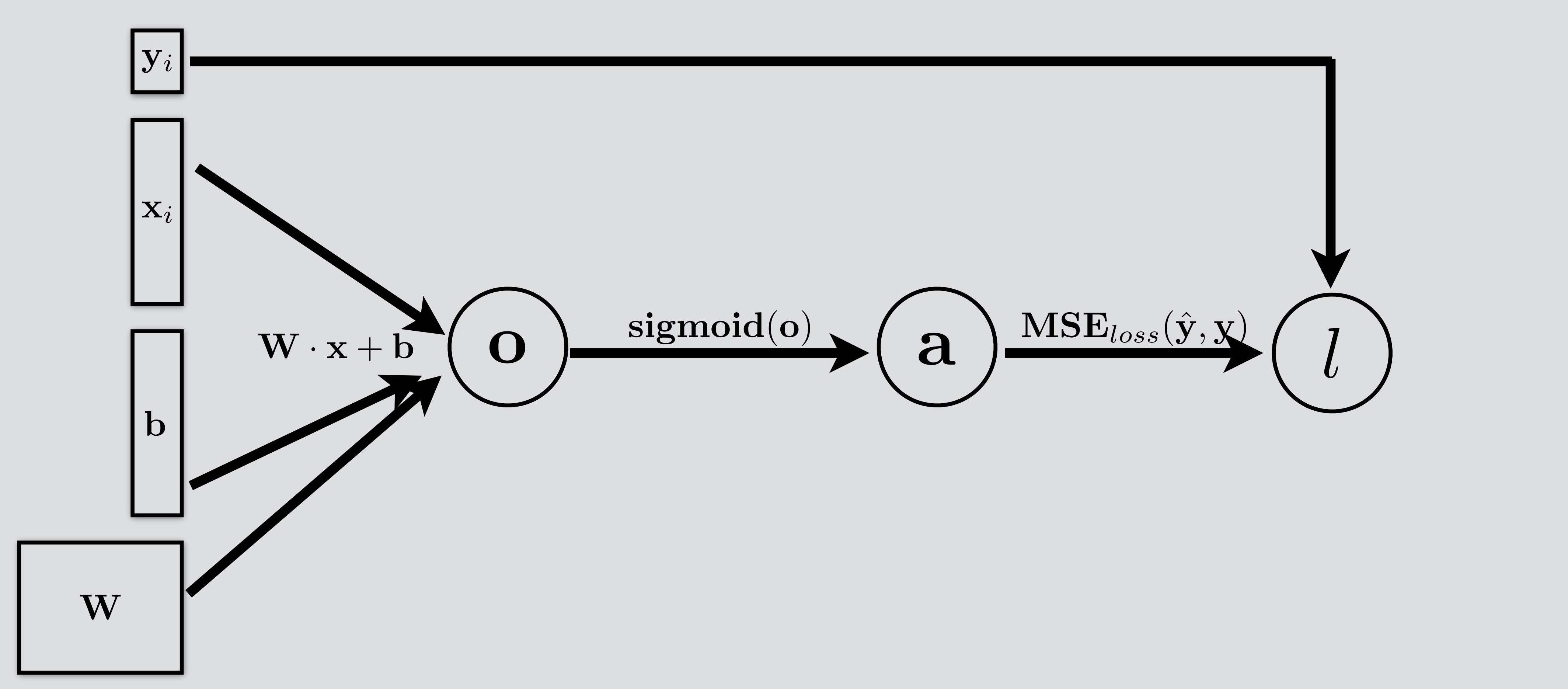

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- Different **activation functions** and saturation problem

Activation Function: Sigmoid

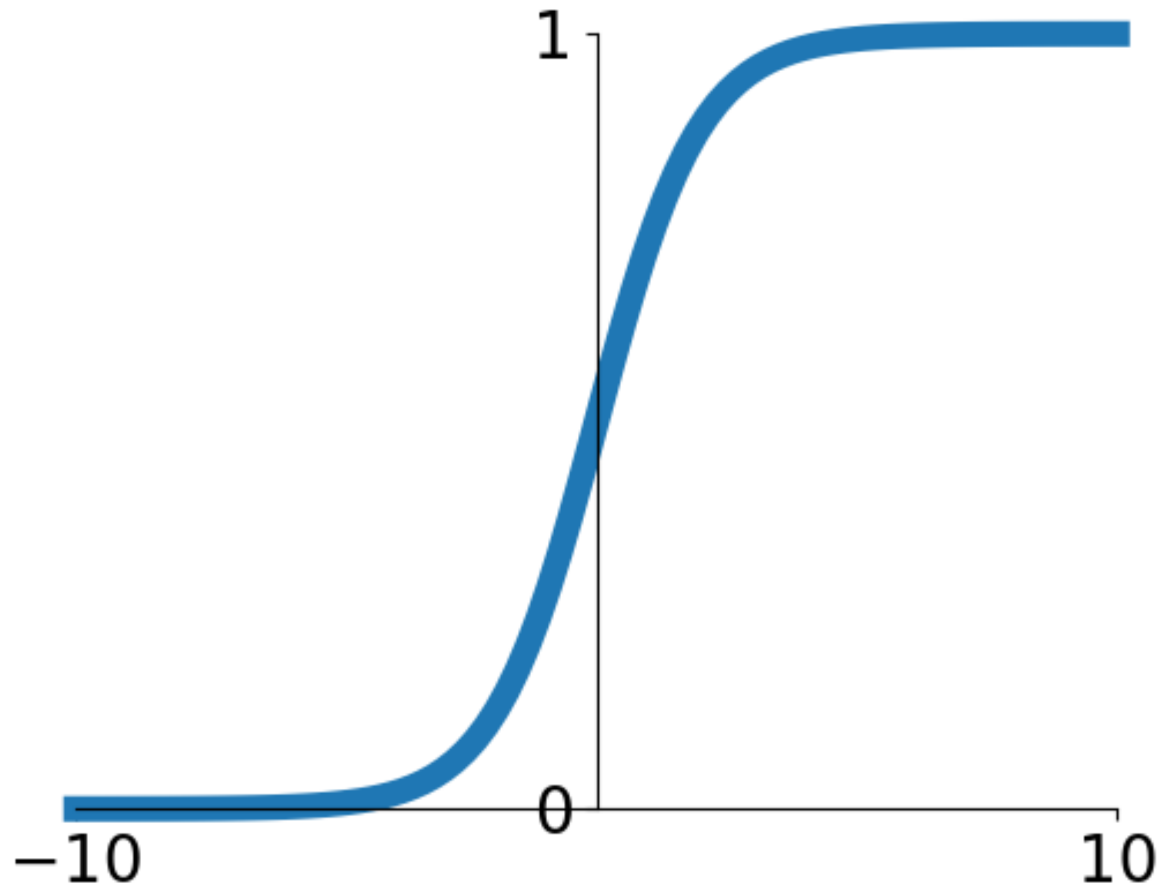
$$a(x) = \textbf{sigmoid}(x) = \frac{1}{1 + e^{-x}}$$

Pros:

- Squishes everything in the range [0,1]
- Can be interpreted as "probability"
- Has well defined gradient everywhere

Cons:

- Saturated neurons "kill" the gradients
- Non-zero centered
- Could be expensive to compute

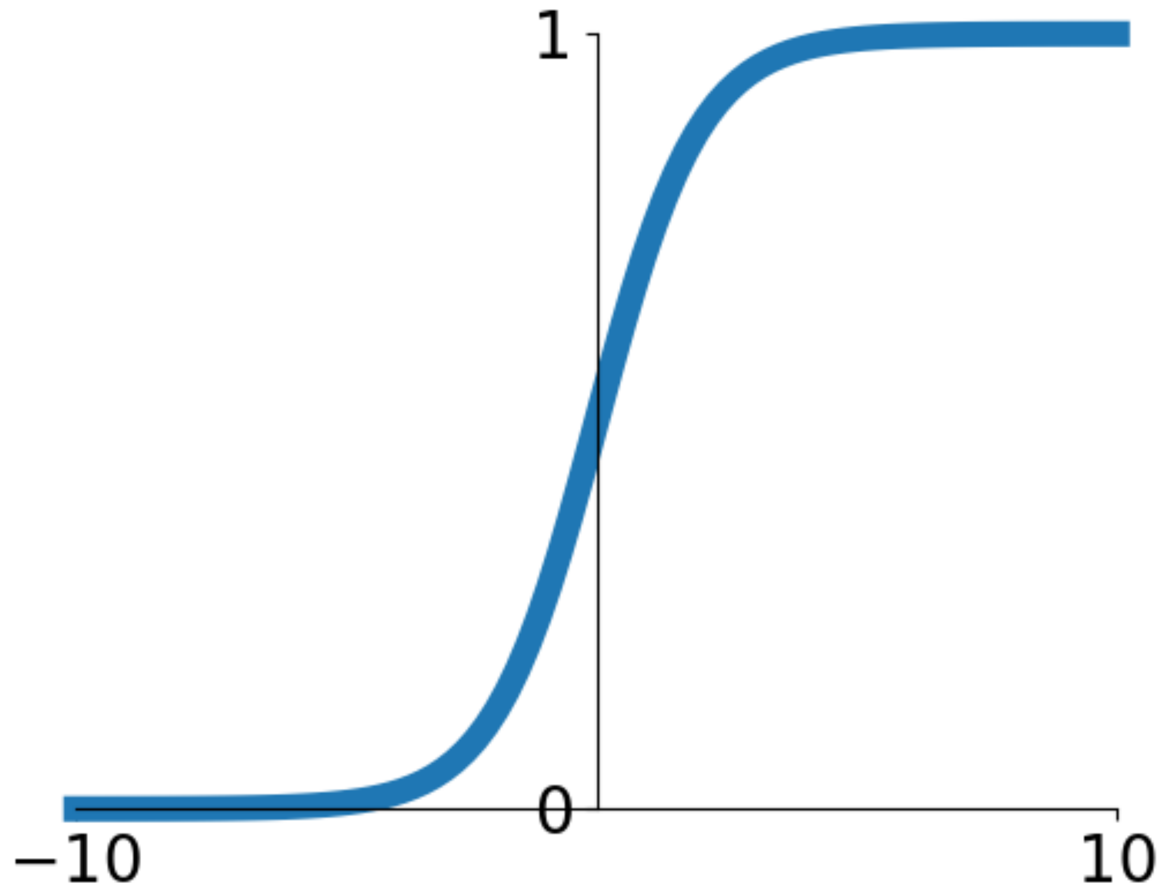


Sigmoid Activation

Activation Function: Sigmoid

Sigmoid Gate

$$a(x) = \mathbf{sigmoid}(x) = \frac{1}{1 + e^{-x}}$$



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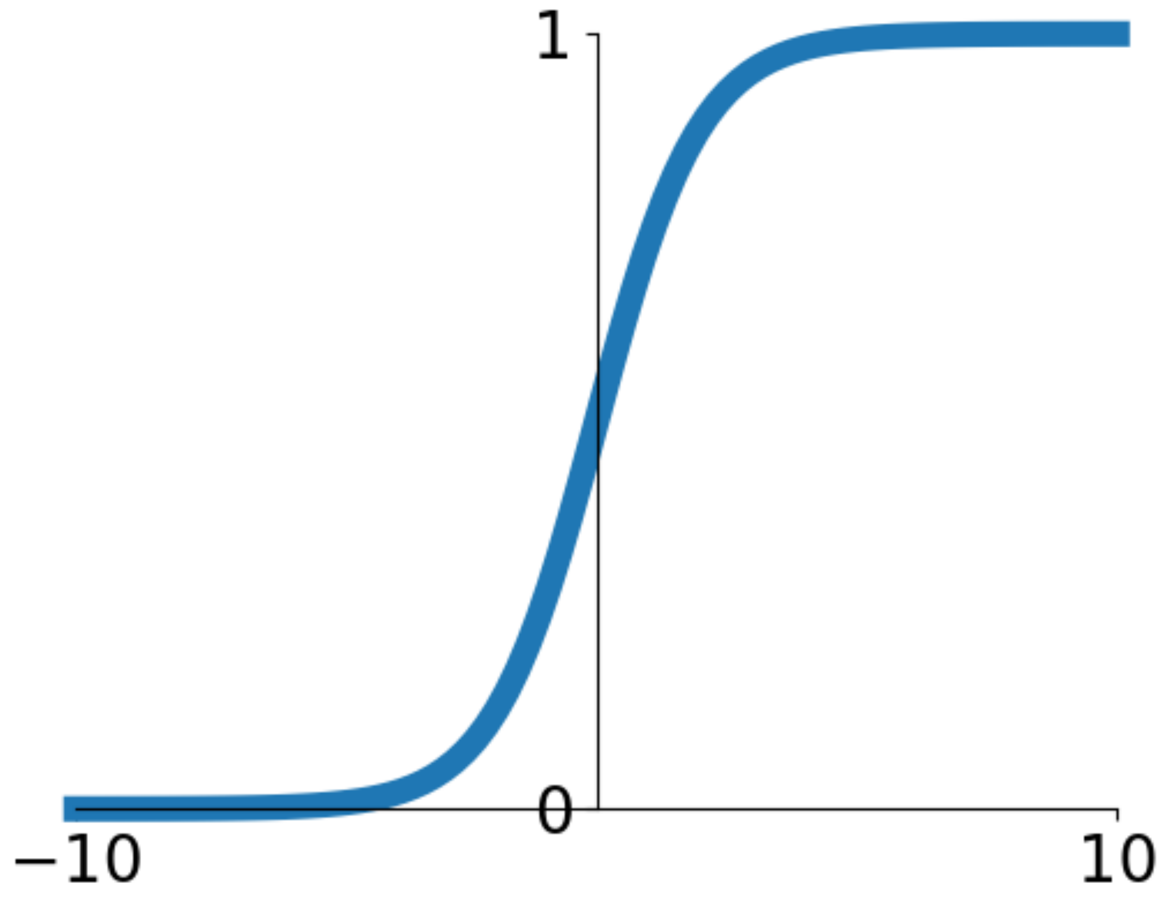

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$$a = \mathbf{sigmoid}(x) = \frac{1}{1+e^{-x}}$$

x

$$\frac{\partial \mathcal{L}}{\partial x} = \frac{\partial\ \mathbf{sigmoid}(x)}{\partial x} \frac{\partial \mathcal{L}}{\partial a}$$

$$\frac{\partial \mathcal{L}}{\partial a}$$

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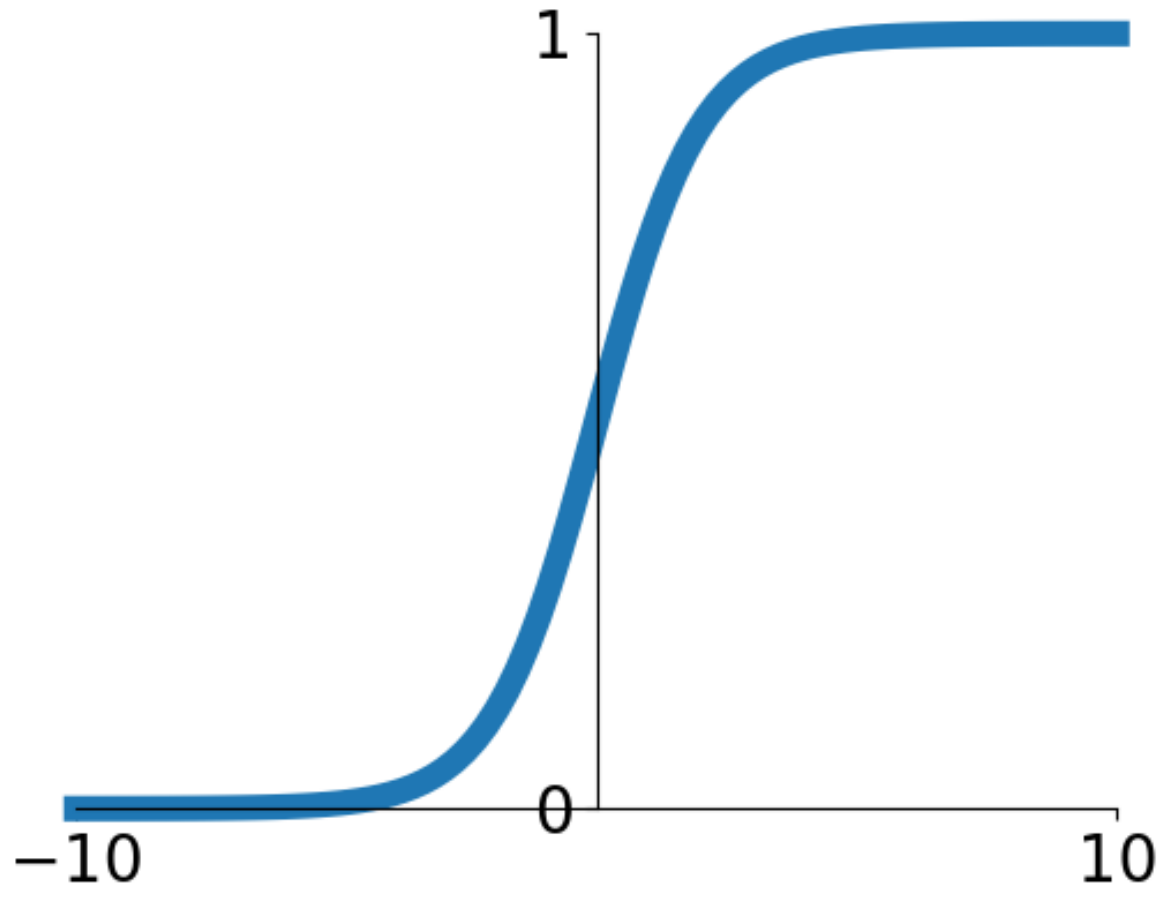

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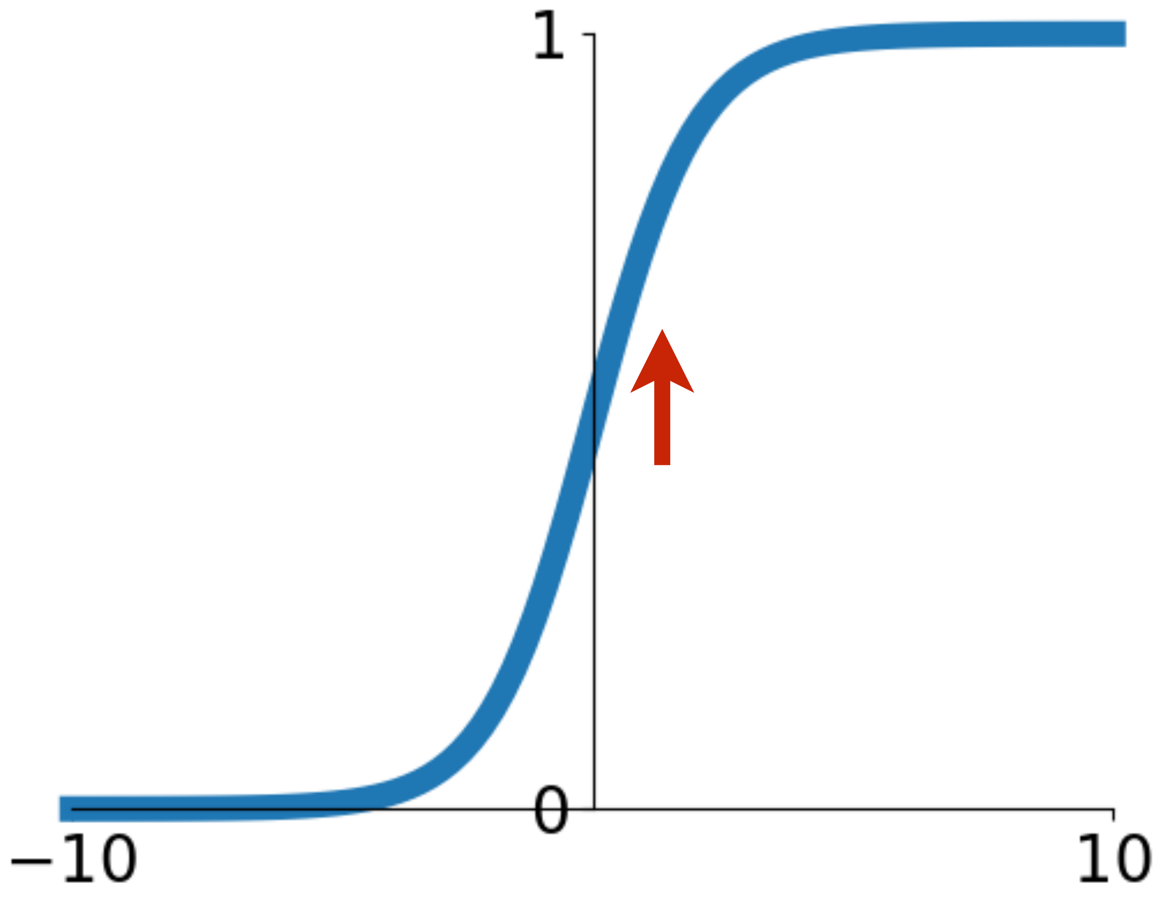

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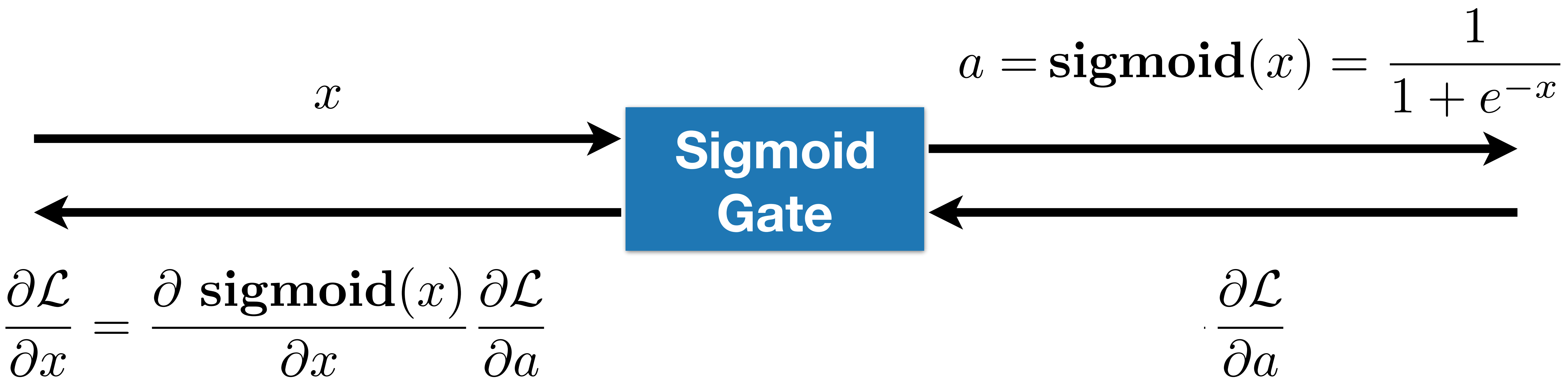

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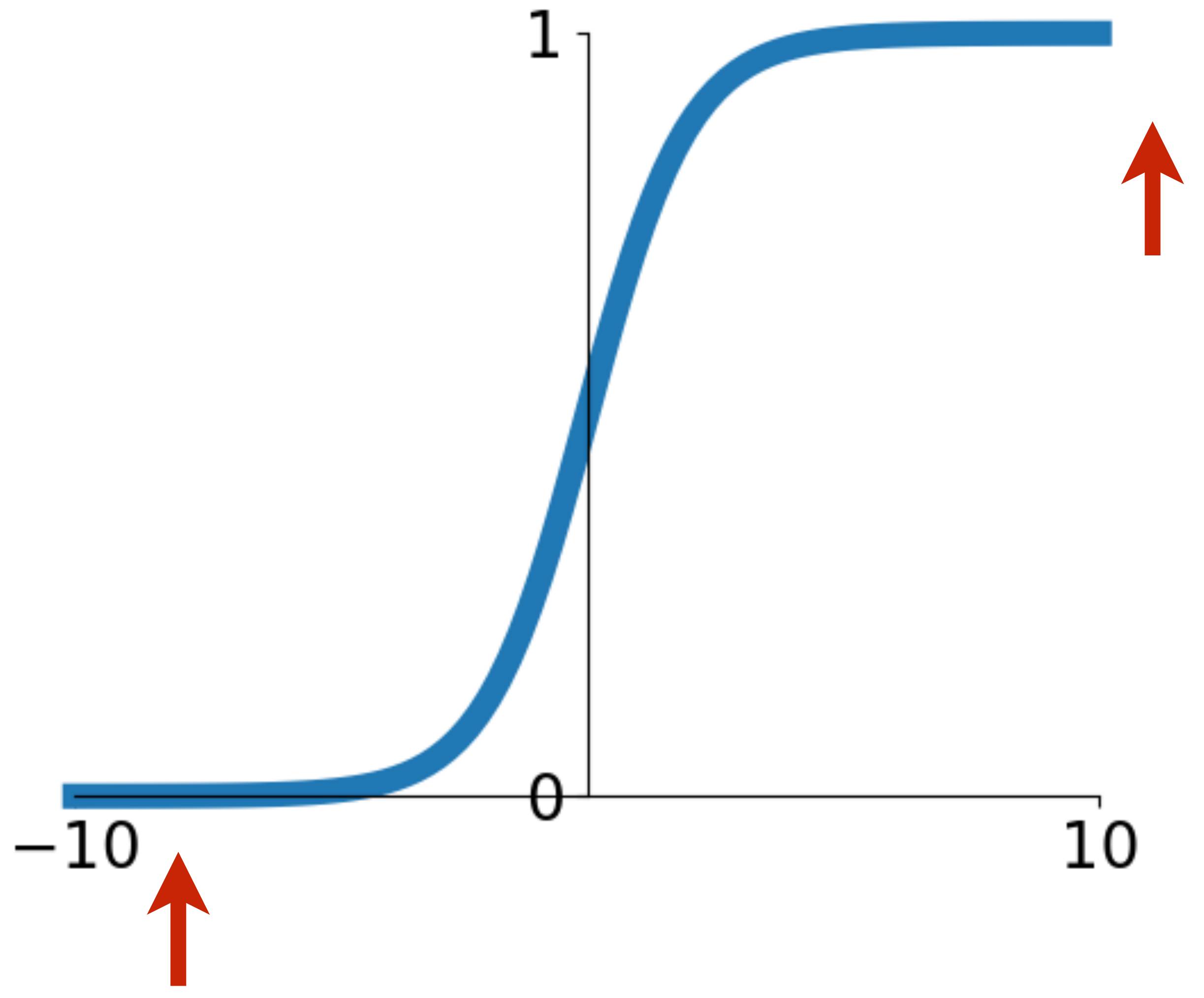

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0

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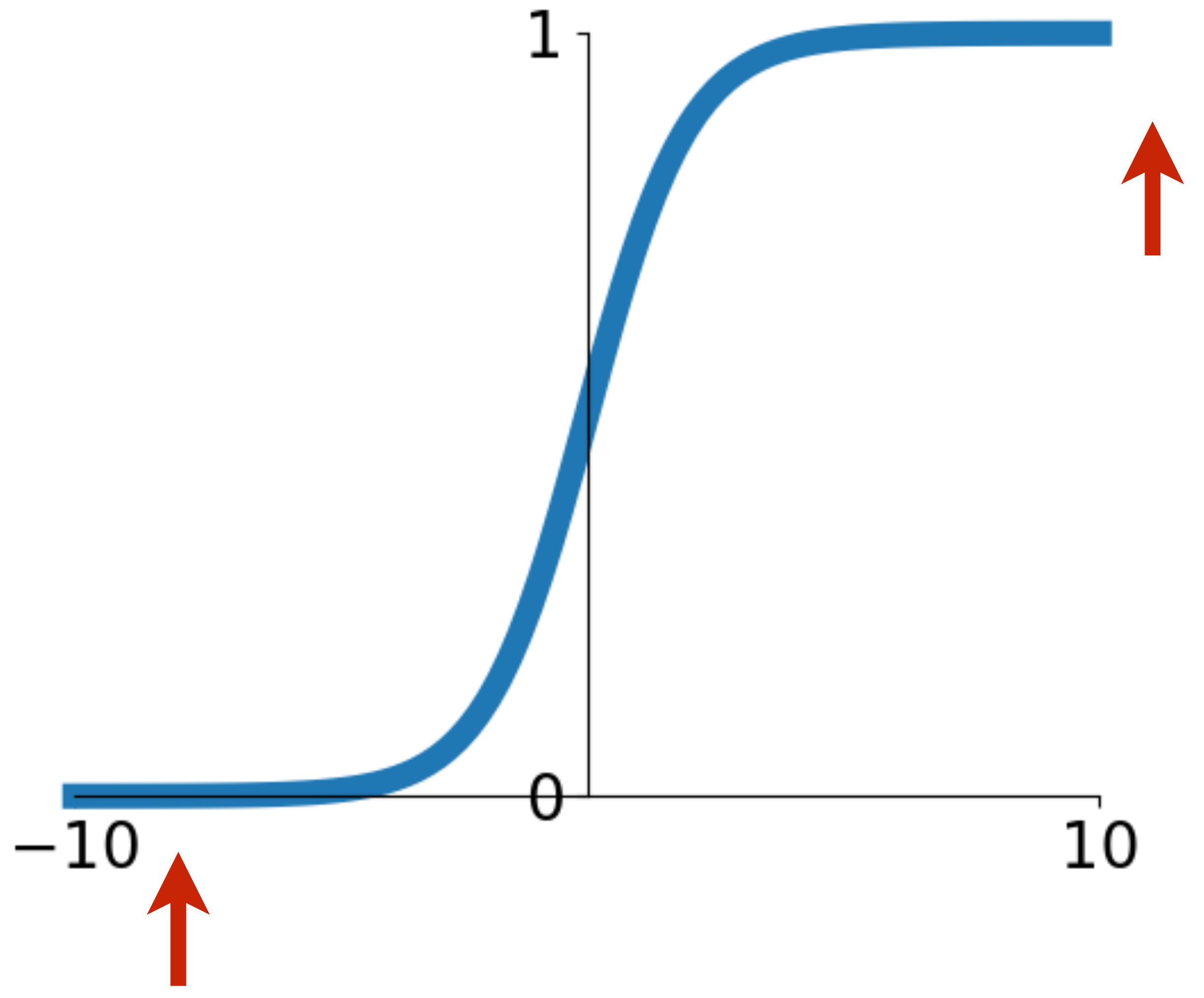

Sigmoid Activation

Cons:

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- Could be expensive to compute

* slide adopted from Li, Karpathy, Johnson’s **CS231n at Stanford**

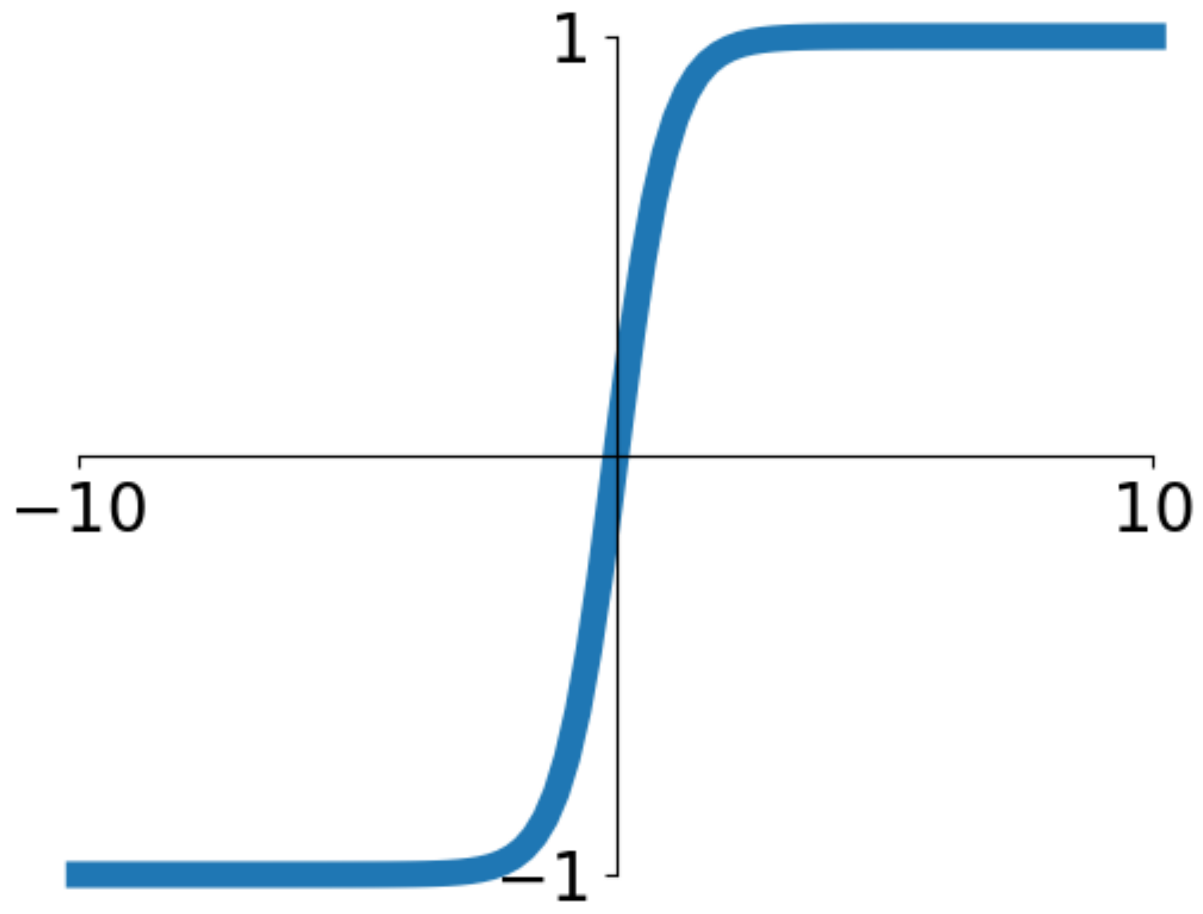
Activation Function: Tanh

$$a(x) = \mathbf{tanh}(x) = 2 \cdot \mathbf{sigmoid}(2x) - 1$$

$$a(x) = \mathbf{tanh}(x) = \frac{2}{1 + e^{-2x}} - 1$$

Pros:
- Squishes everything in the range [-1,1]
- Centered around zero
- Has well defined gradient everywhere

Cons:
- Saturated neurons "kill" the gradients
- Could be expensive to compute



Tanh Activation

Activation Function: Rectified Linear Unit (ReLU)

$$a(x) = max(0, x)$$

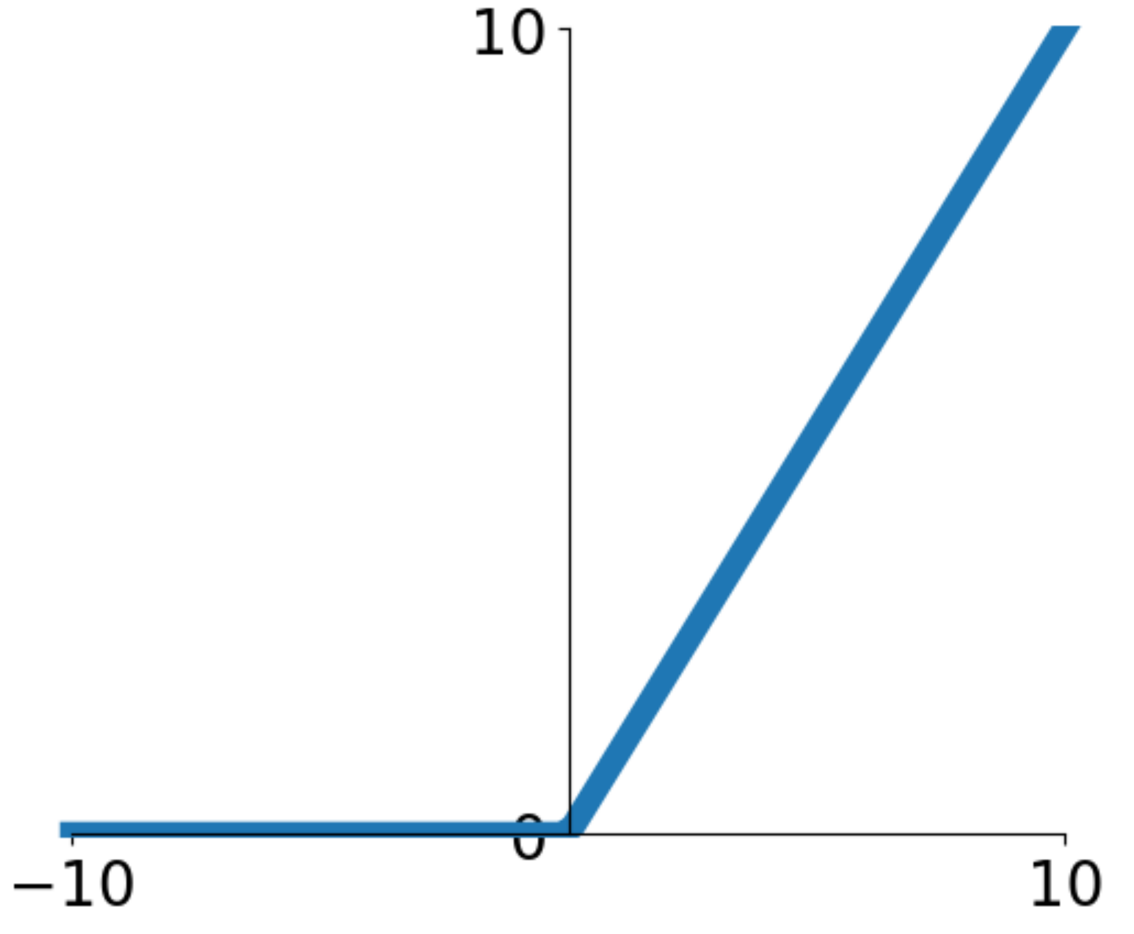
$$a'(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

Pros:

- Does not saturate (for x > 0)
- Computationally very efficient
- Converges faster in practice (e.g. 6 times faster)

Cons:

- Not zero centered



ReLU Activation

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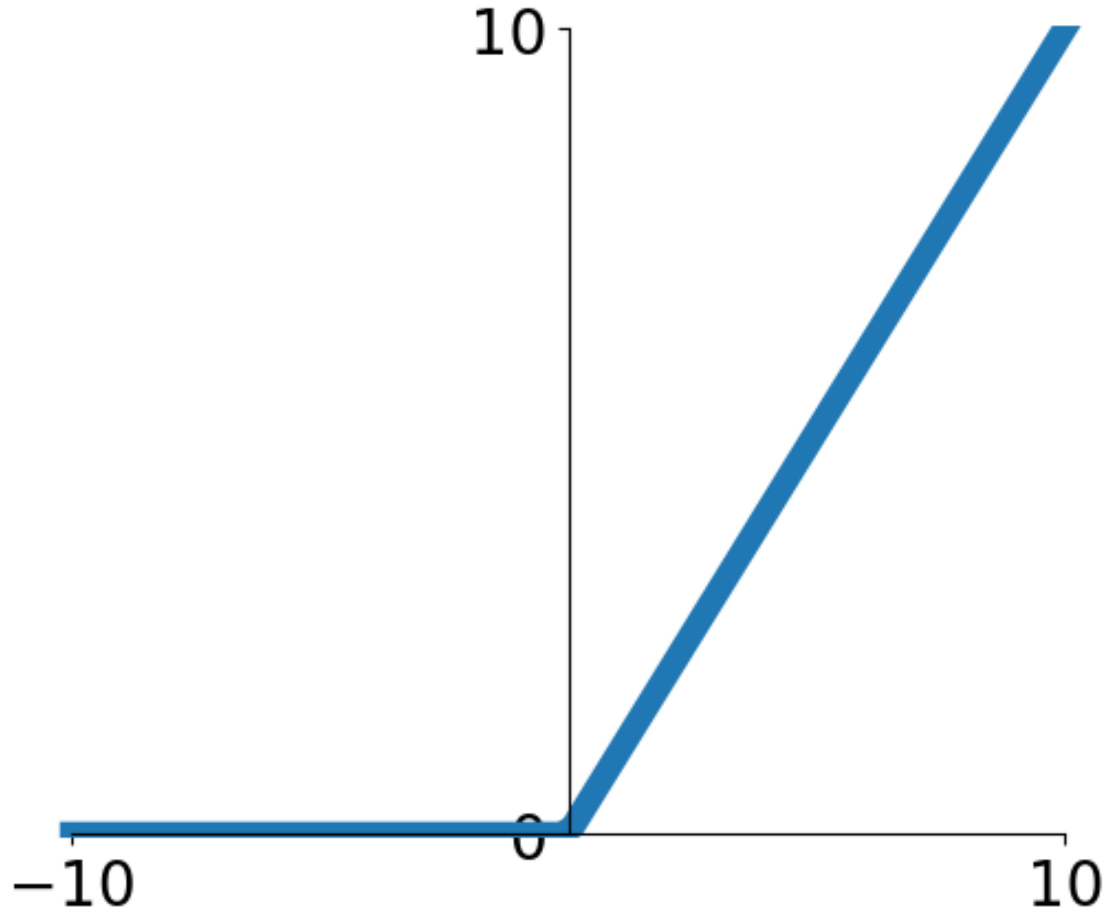
Question: What do ReLU layers accomplish?



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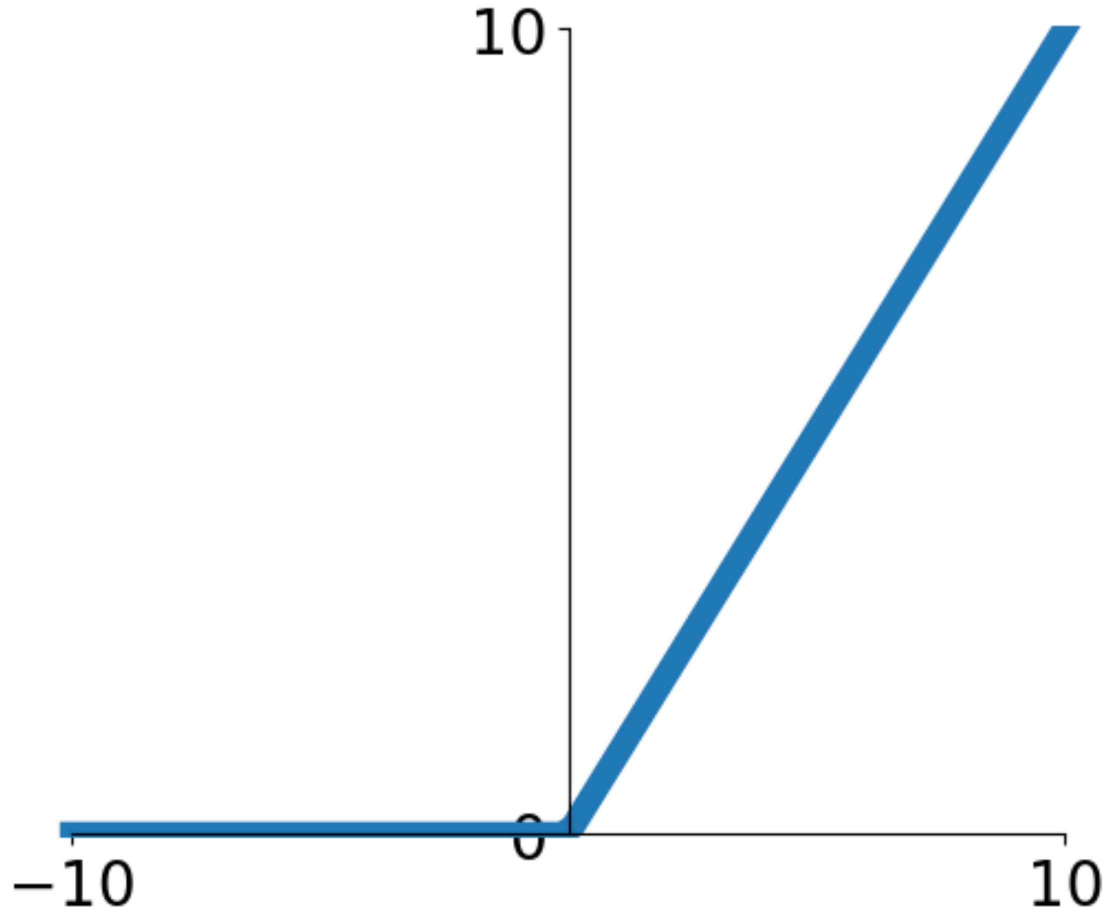
Question: What do ReLU layers accomplish?

Answer: Locally linear tiling, function is locally linear



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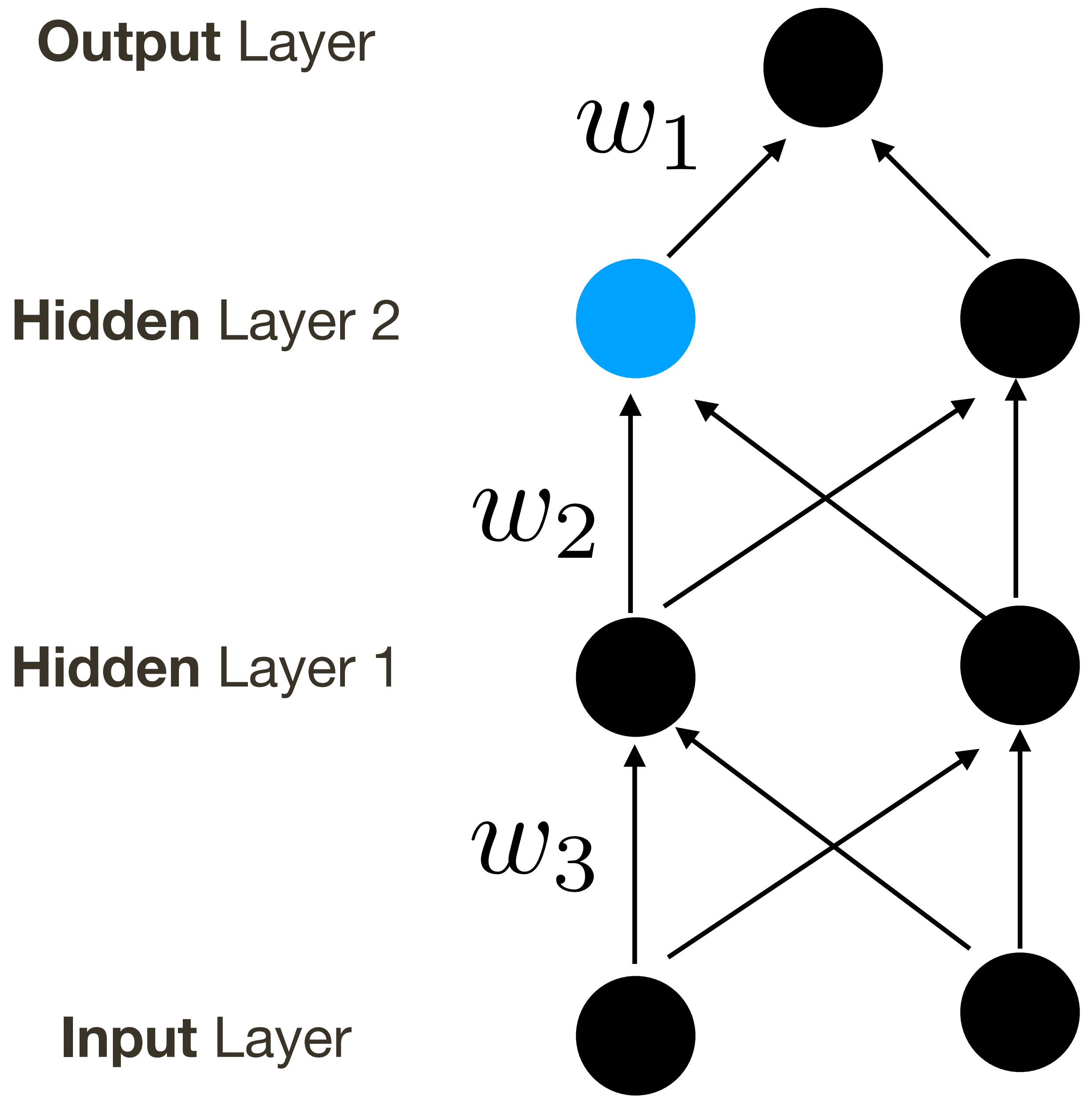
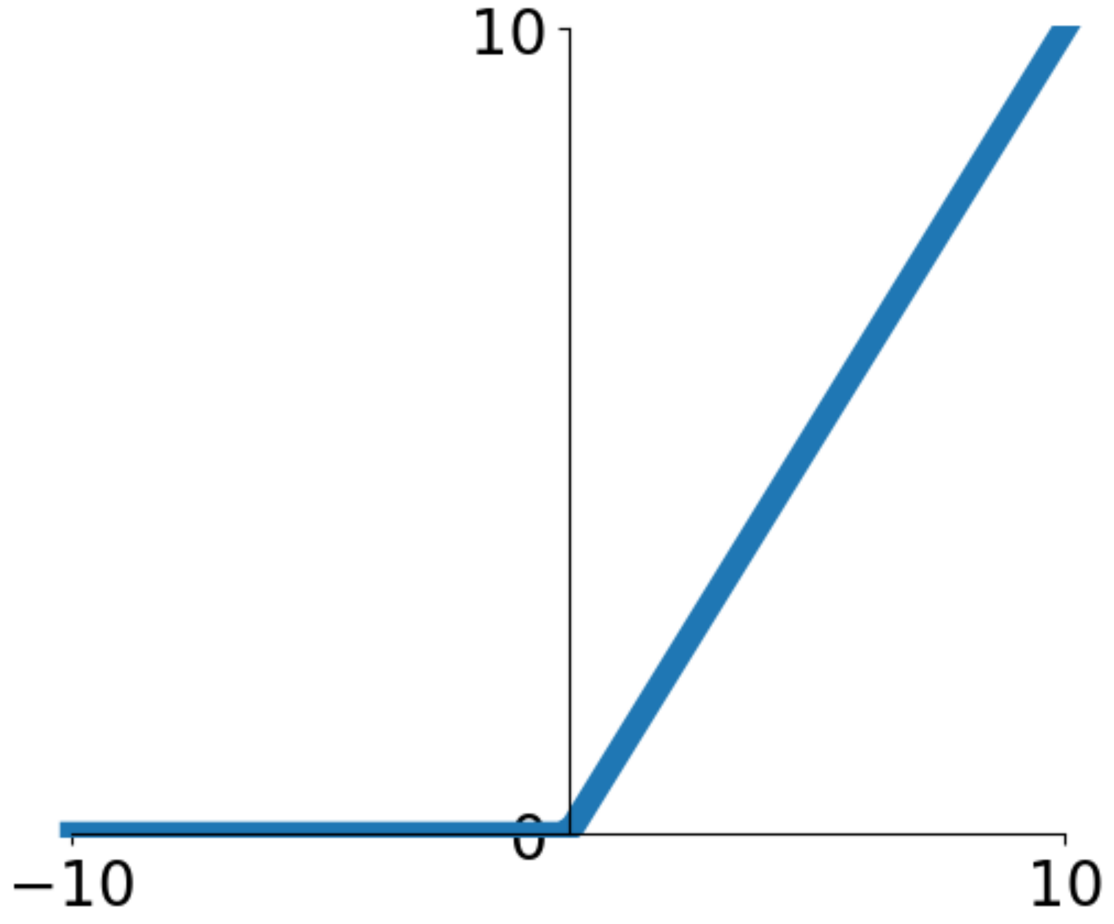
ReLU sparcifies activations and derivatives





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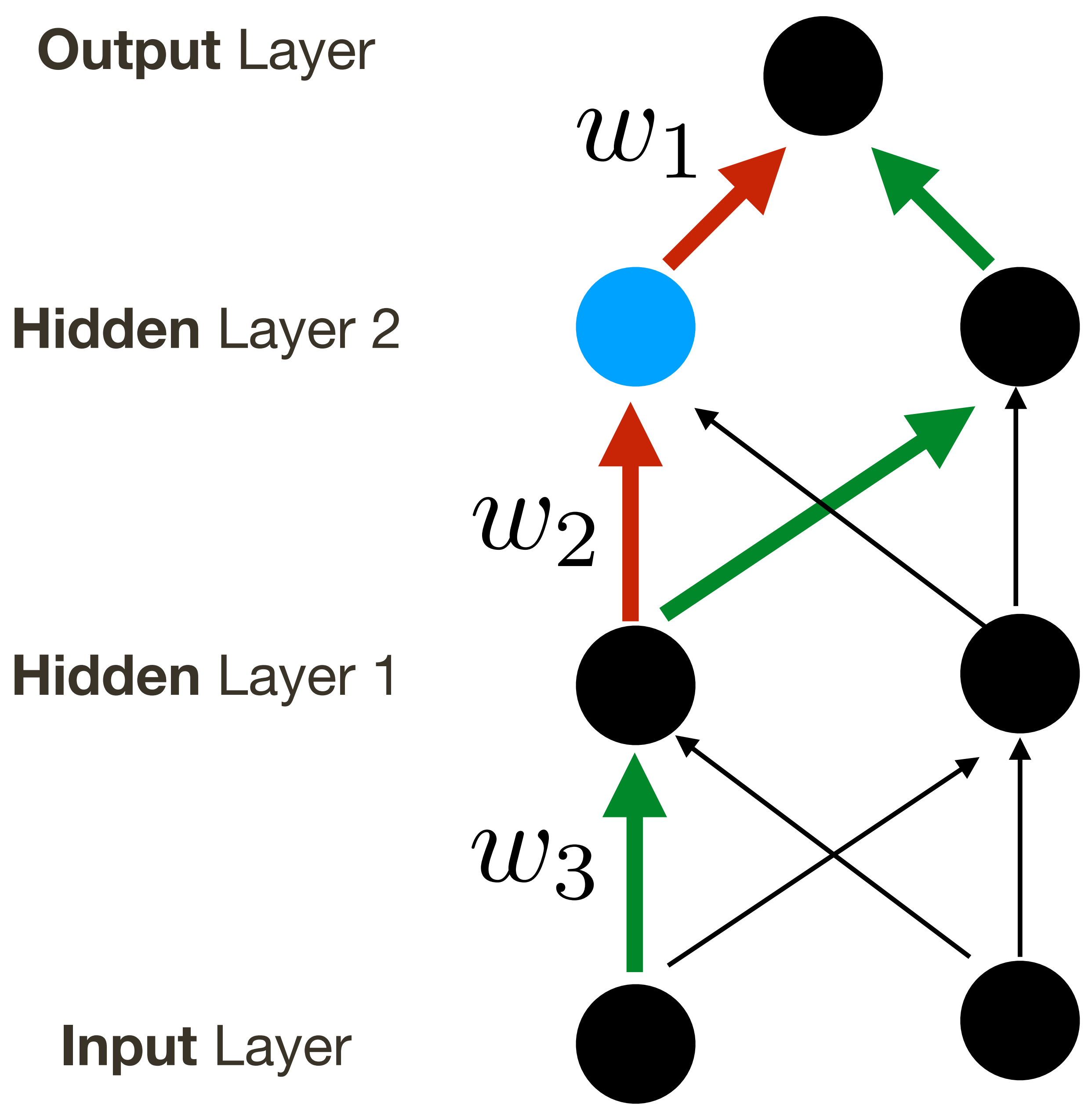

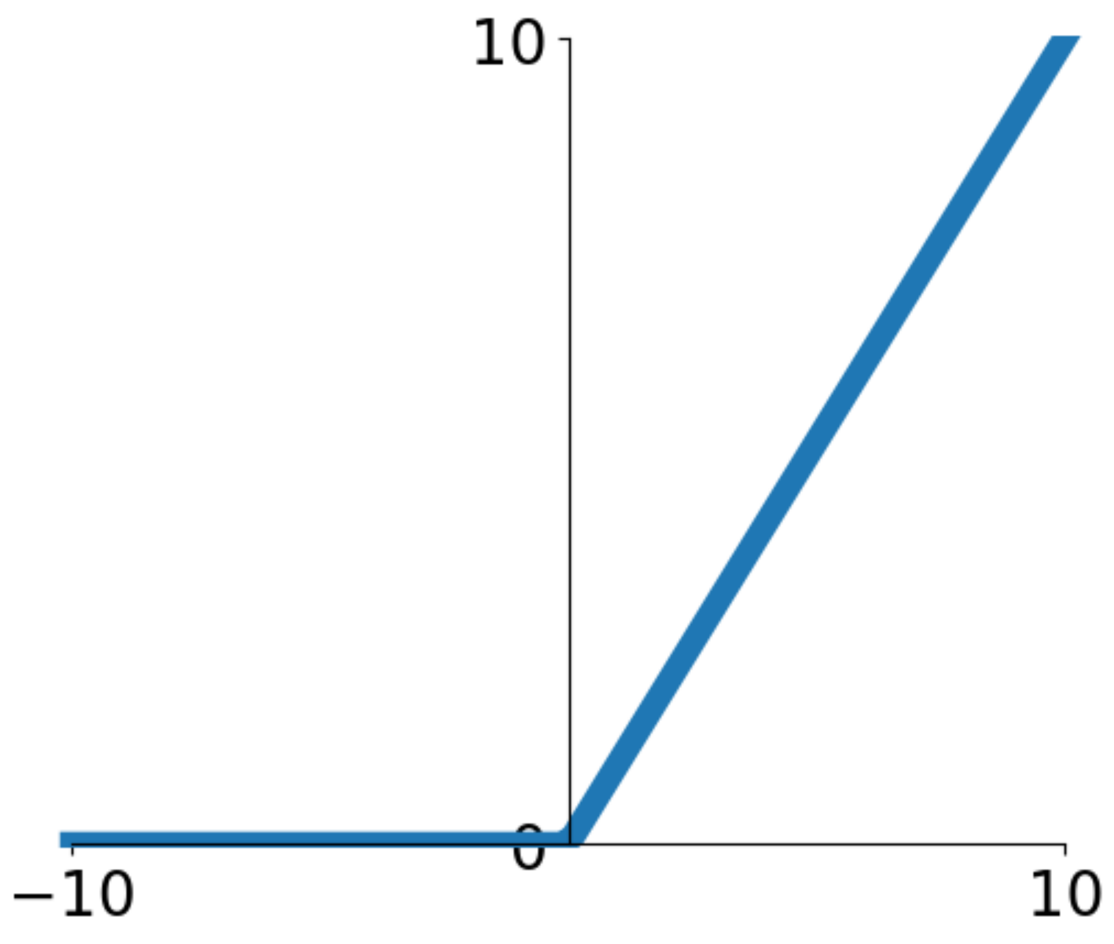

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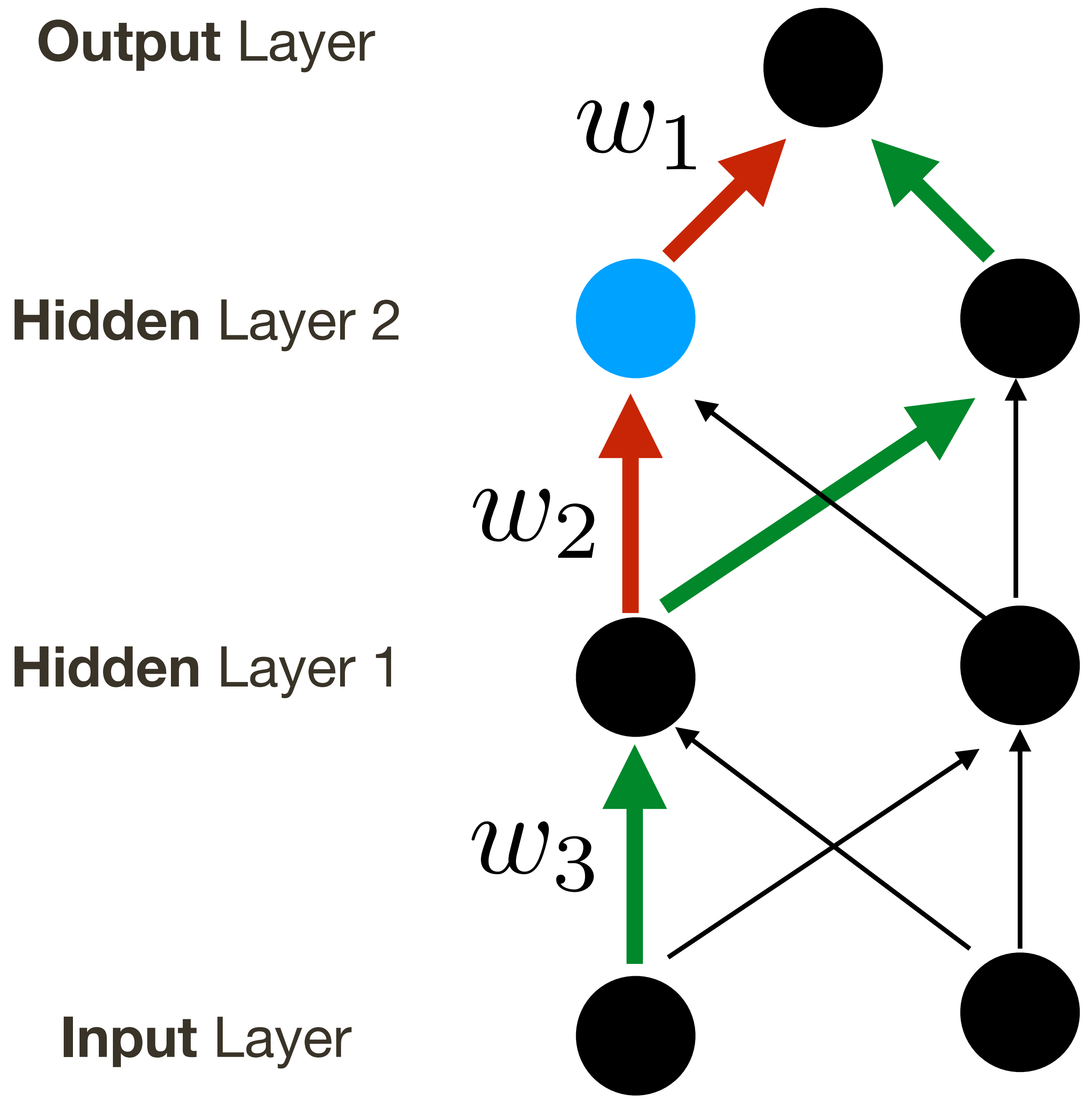

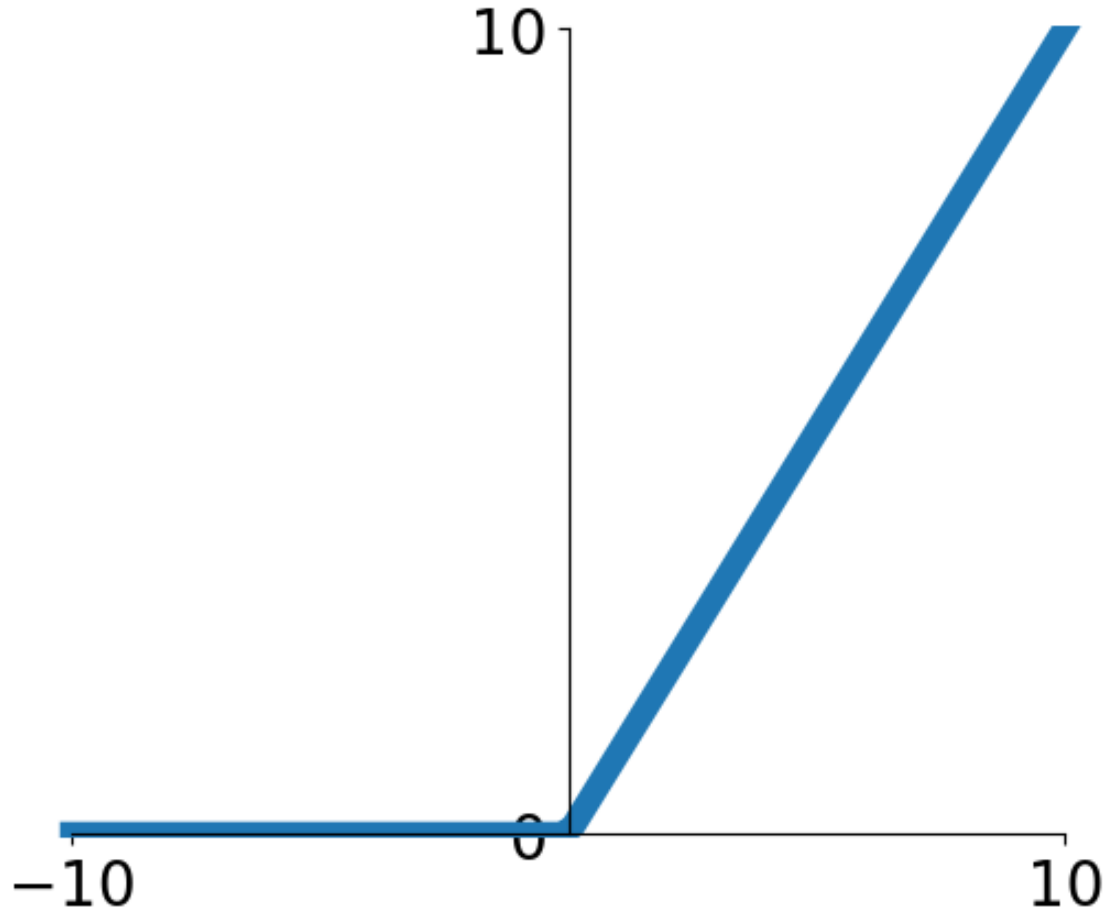

ReLU Activation

10%-20% of neurons end up being “**dead**” in most strained networks

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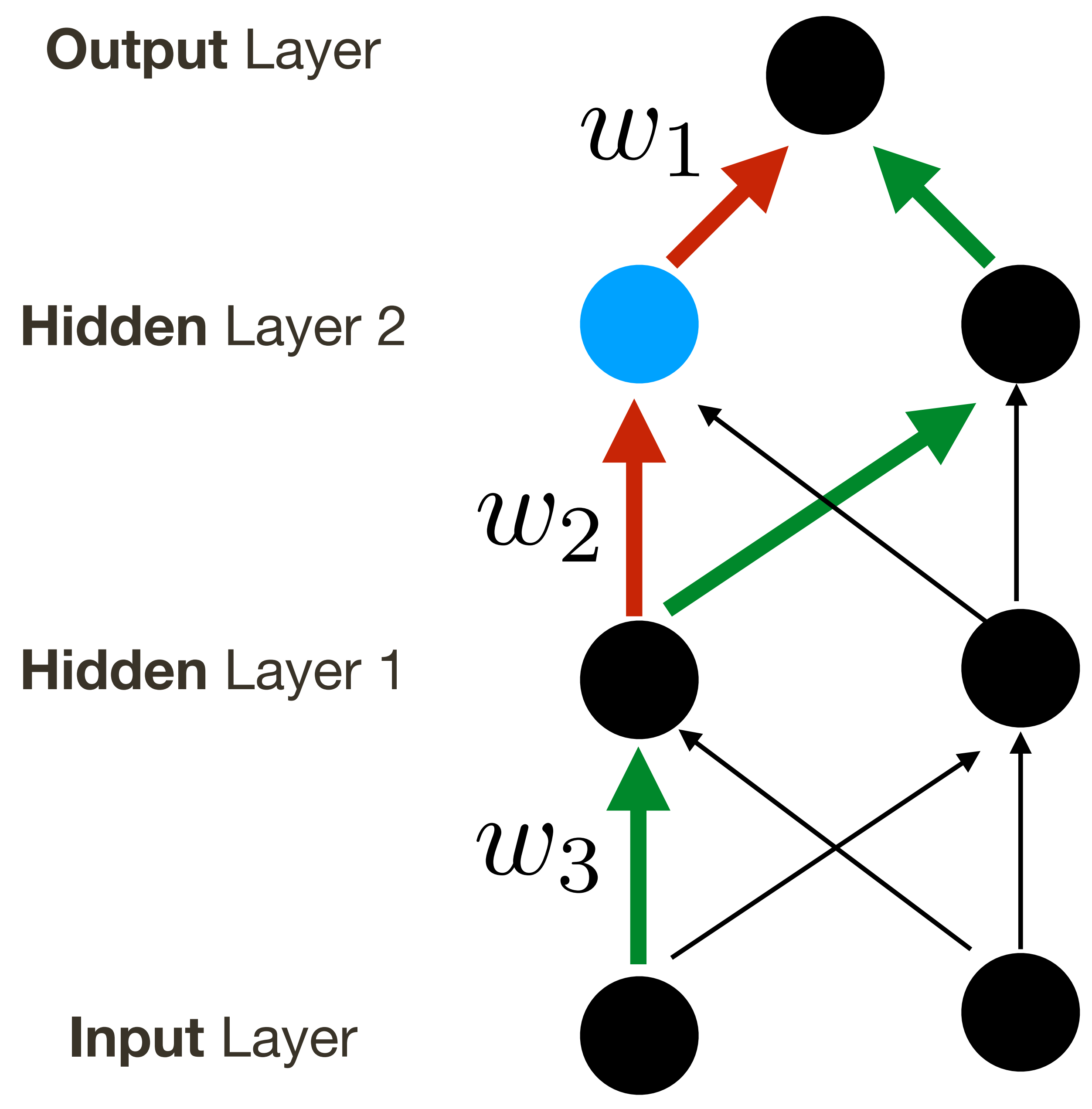

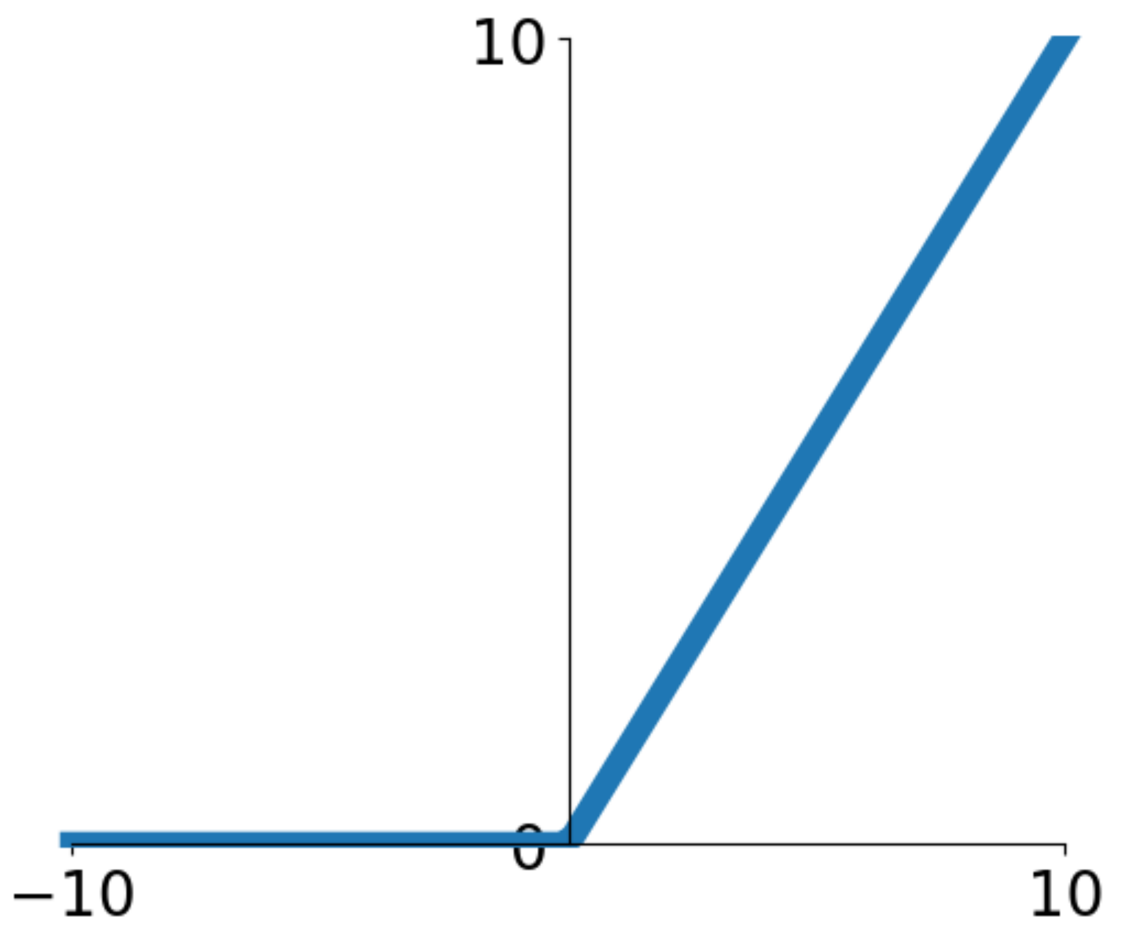

ReLU Activation

Trick: initialize bias for neurons with ReLU activation to small positive value (0.01)

Initialization

Many tricks for initializations exist. I will not really cover this.

You will partly see why soon …

Recall:

Conditions needed to prove NN is a universal approximator: Activation function needs to be well defined

$$\lim_{x \to \infty} a(x) = A$$

$$\lim_{x \to -\infty} a(x) = B$$

$$A \neq B$$

*slide adopted from http://neuralnetworksanddeeplearning.com/chap4.html

Recall:

Conditions needed to prove NN is a universal approximator: Activation function needs to be well defined

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Fun **Exercise:** Try to prove that network with ReLU is still a universal approximator (not too difficult if you think about it visually)

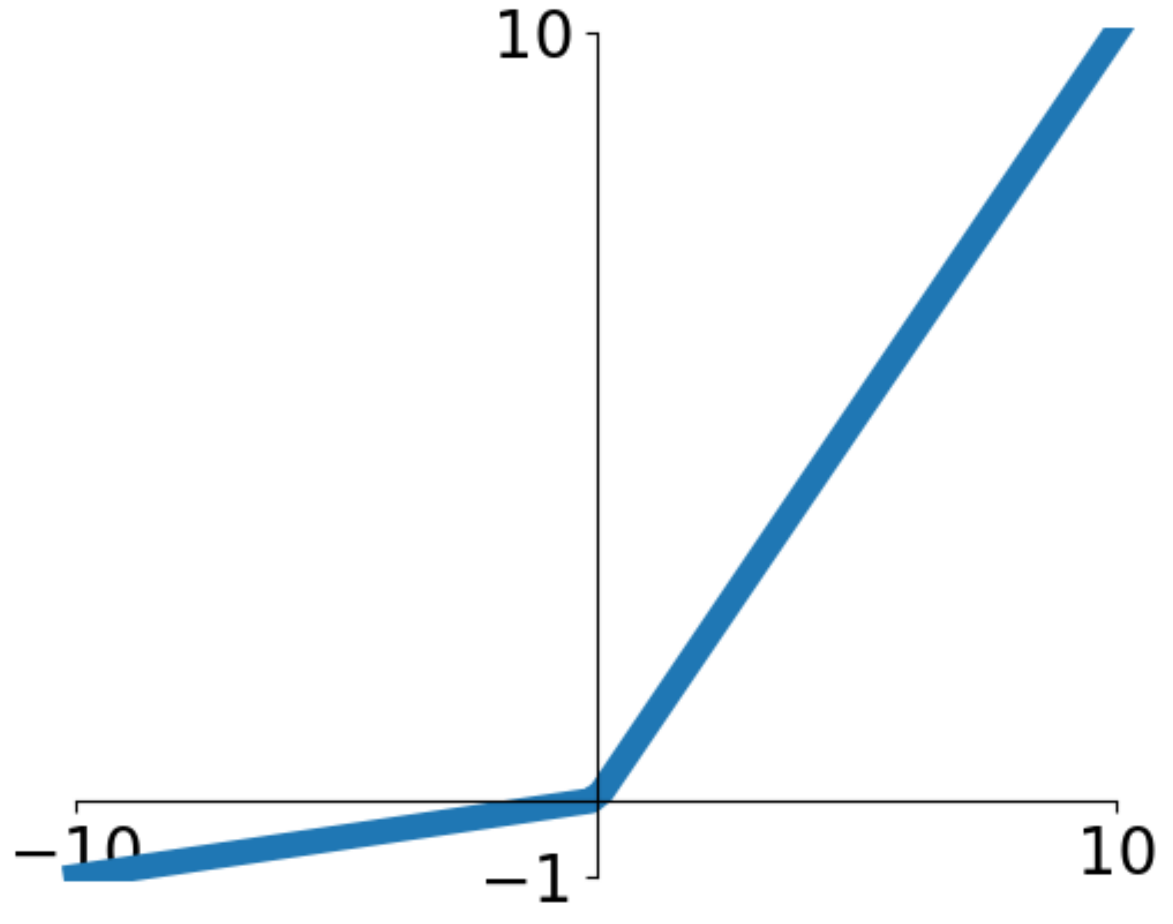
*slide adopted from http://neuralnetworksanddeeplearning.com/chap4.html

Activation Function: Leaky / Parametrized ReLU

Leaky: alpha is fixed to a small value (e.g., 0.01)

$$a(x) = \begin{cases} x & \text{if } x \geq 0 \\ \alpha x & \text{if } x < 0 \end{cases}$$

Parametrized: alpha is optimized as part of the network (BackProp through)

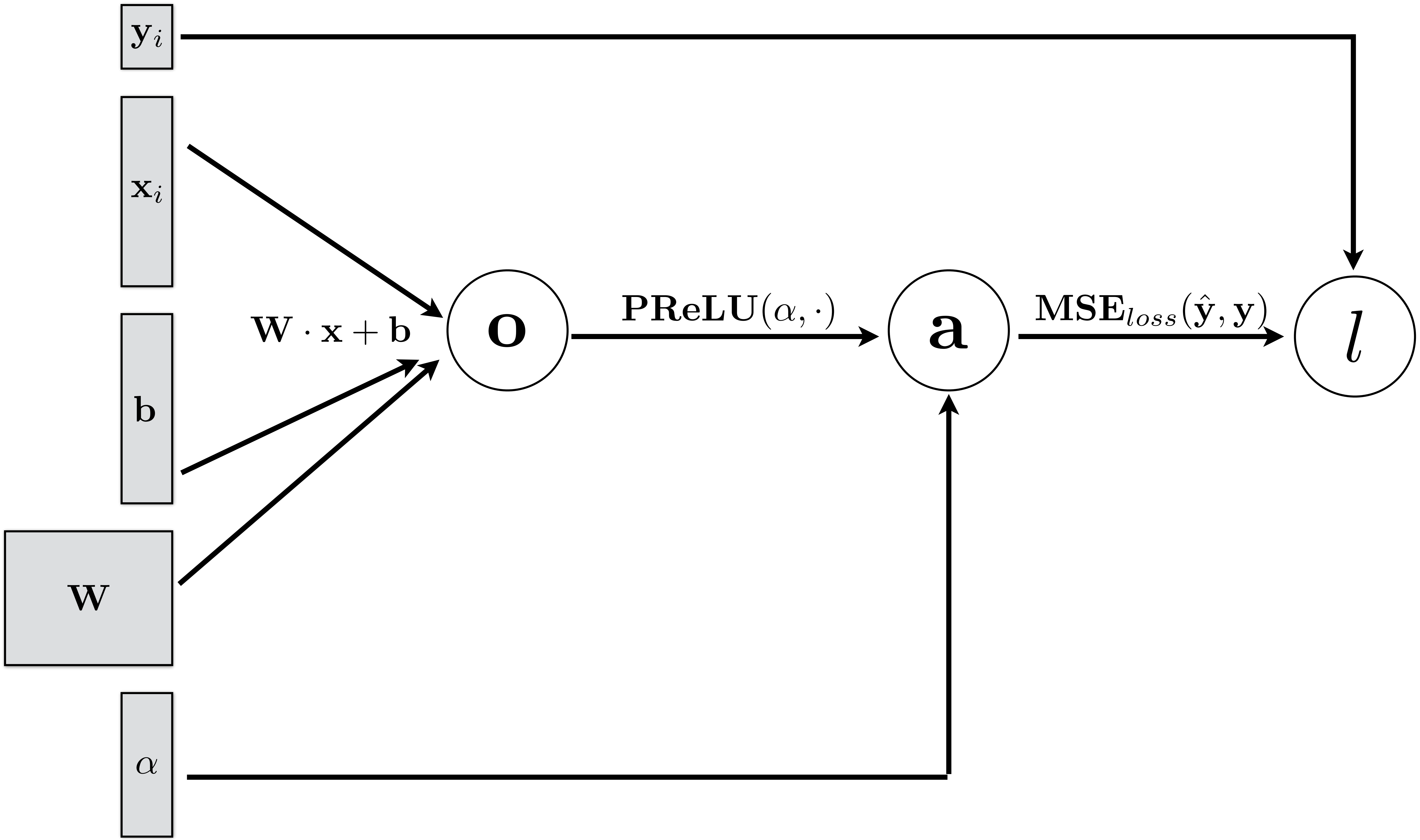


Leaky / Parametrized ReLU Activation

Pros:

- Does not saturate
- Computationally very efficient
- Converges faster in practice (e.g. 6x)

Computational Graph: 1-layer with PReLU



Activation Functions: Review

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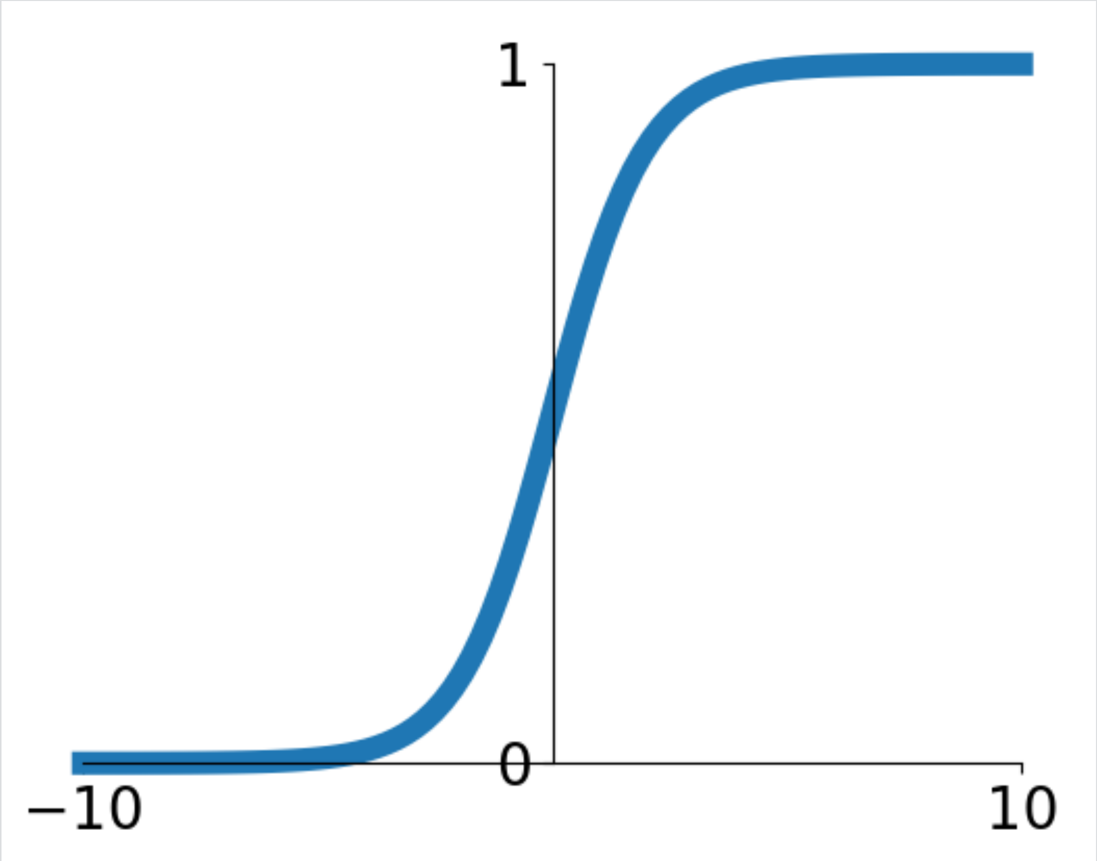

Sigmoid

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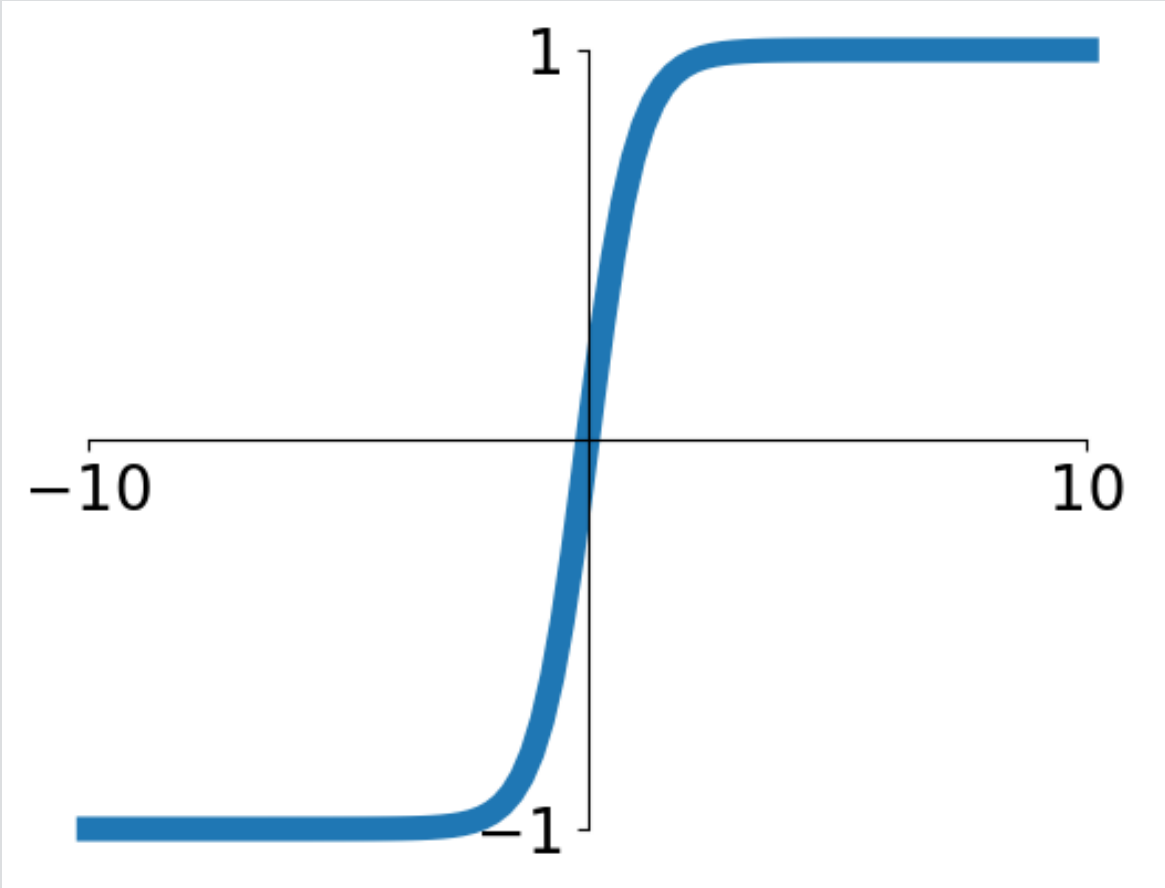

Tanh

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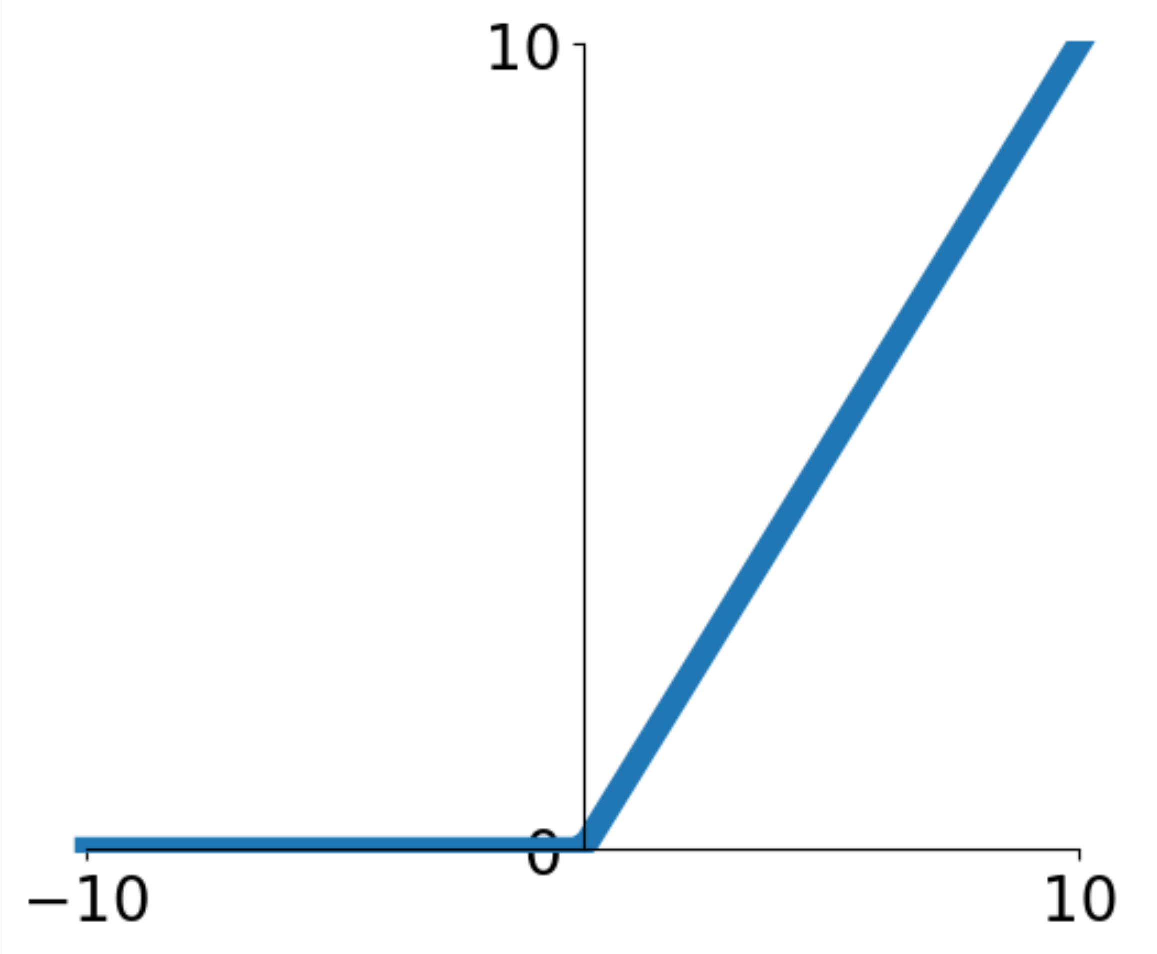

ReLU

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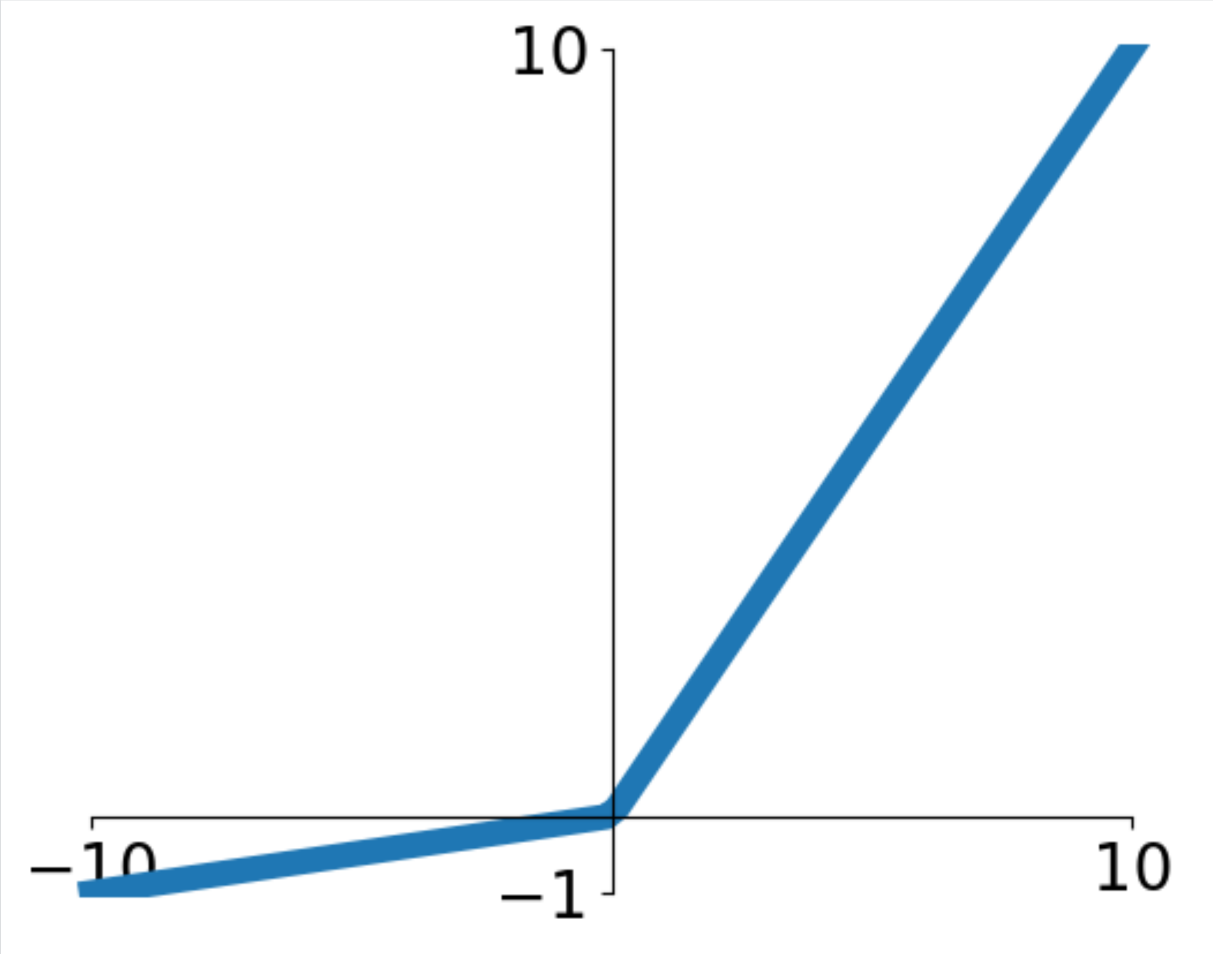
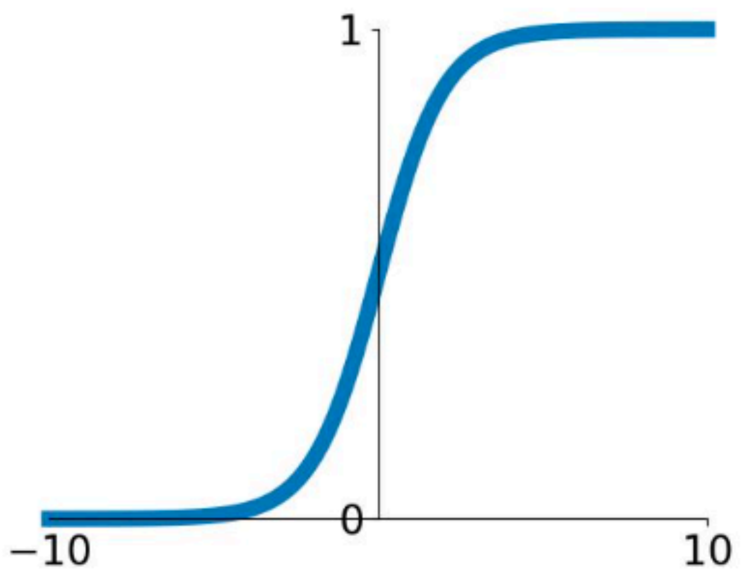


Leaky / Parametrized **ReLU**

Activation Functions: Review

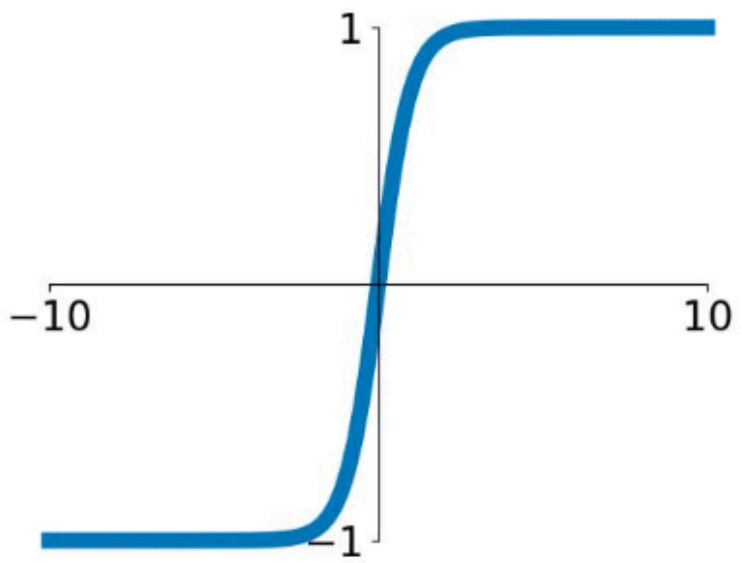
Sigmoid

$$\sigma(x) = \frac{1}{1+e^{-x}}$$



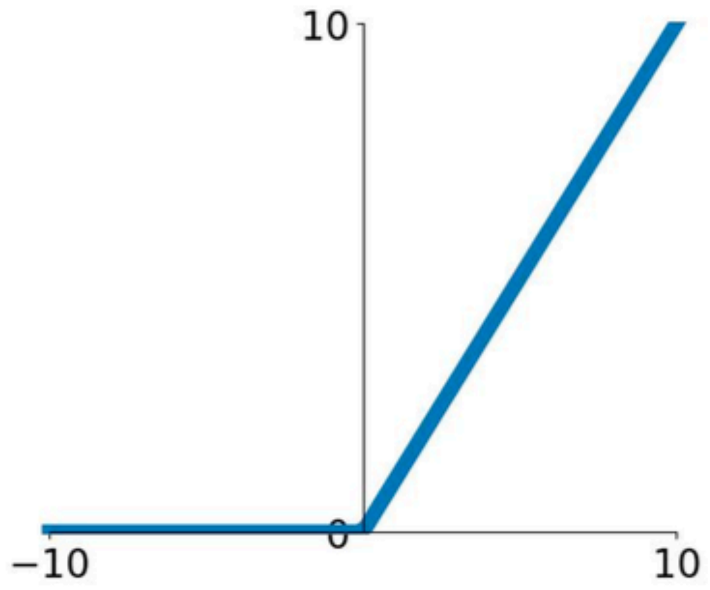
tanh

$$\tanh(x)$$



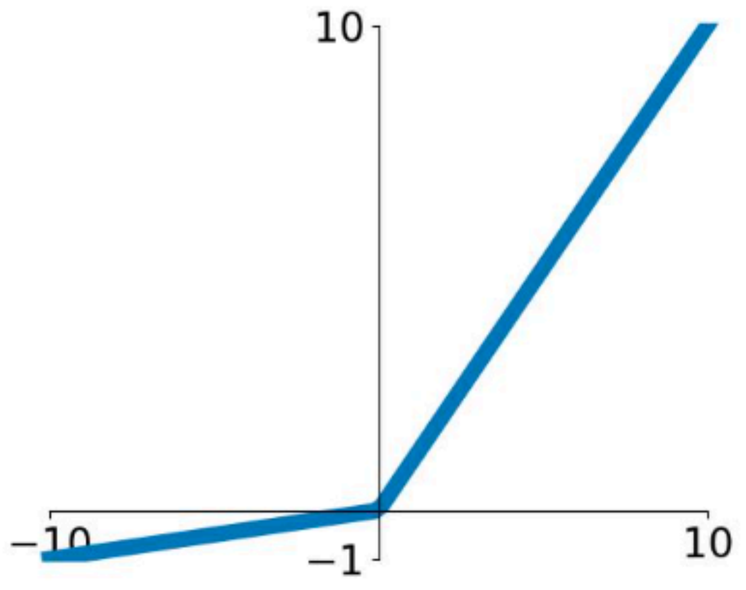
ReLU

$$\max(0, x)$$



Leaky ReLU

$$\max(0.1x, x)$$

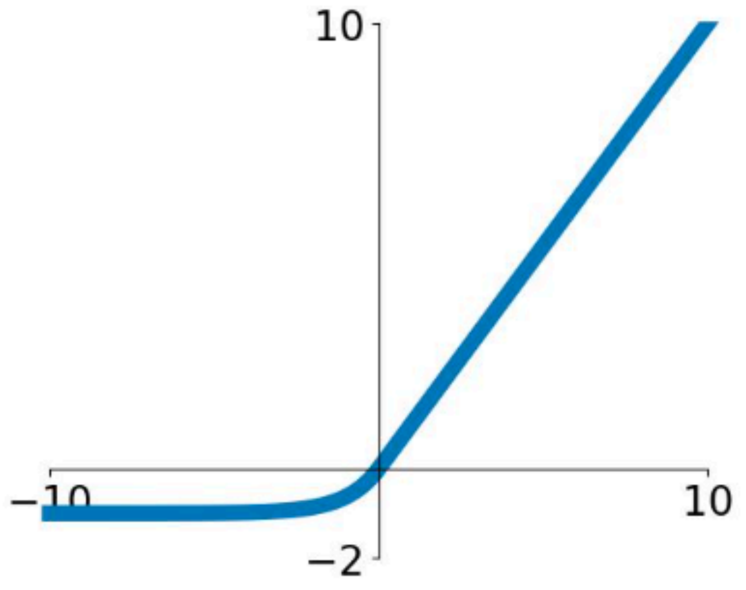


Maxout

$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

ELU

$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



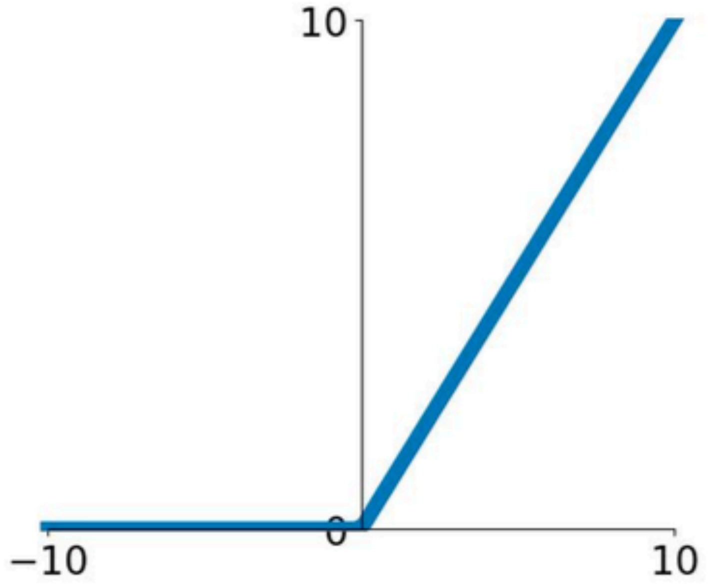
* slide adopted from Li, Karpathy, Johnson's **CS231n at Stanford**

Activation Functions: Review

Good “**default**” choice

ReLU

$\max(0, x)$



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Regularization: L2 or L1 on the weights

L2 Regularization: Learn a more (dense) distributed representation

$$R(\mathbf{W}) = ||\mathbf{W}||_2 = \sum_i \sum_j \mathbf{W}_{i,j}^2$$

L1 Regularization: Learn a sparse representation (few non-zero wight elements)

$$R(\mathbf{W}) = ||\mathbf{W}||_1 = \sum_i \sum_j |\mathbf{W}_{i,j}|$$

(others regularizers are also possible)

Example:

$$\mathbf{x} = [1, 1, 1, 1]$$

$$\mathbf{W}_1 = [1, 0, 0, 0]$$

$$\mathbf{W}_2 = \left[\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right]$$

$$\mathbf{W}_1 \cdot \mathbf{x}^T = \mathbf{W}_2 \cdot \mathbf{x}^T$$

two networks will have identical output

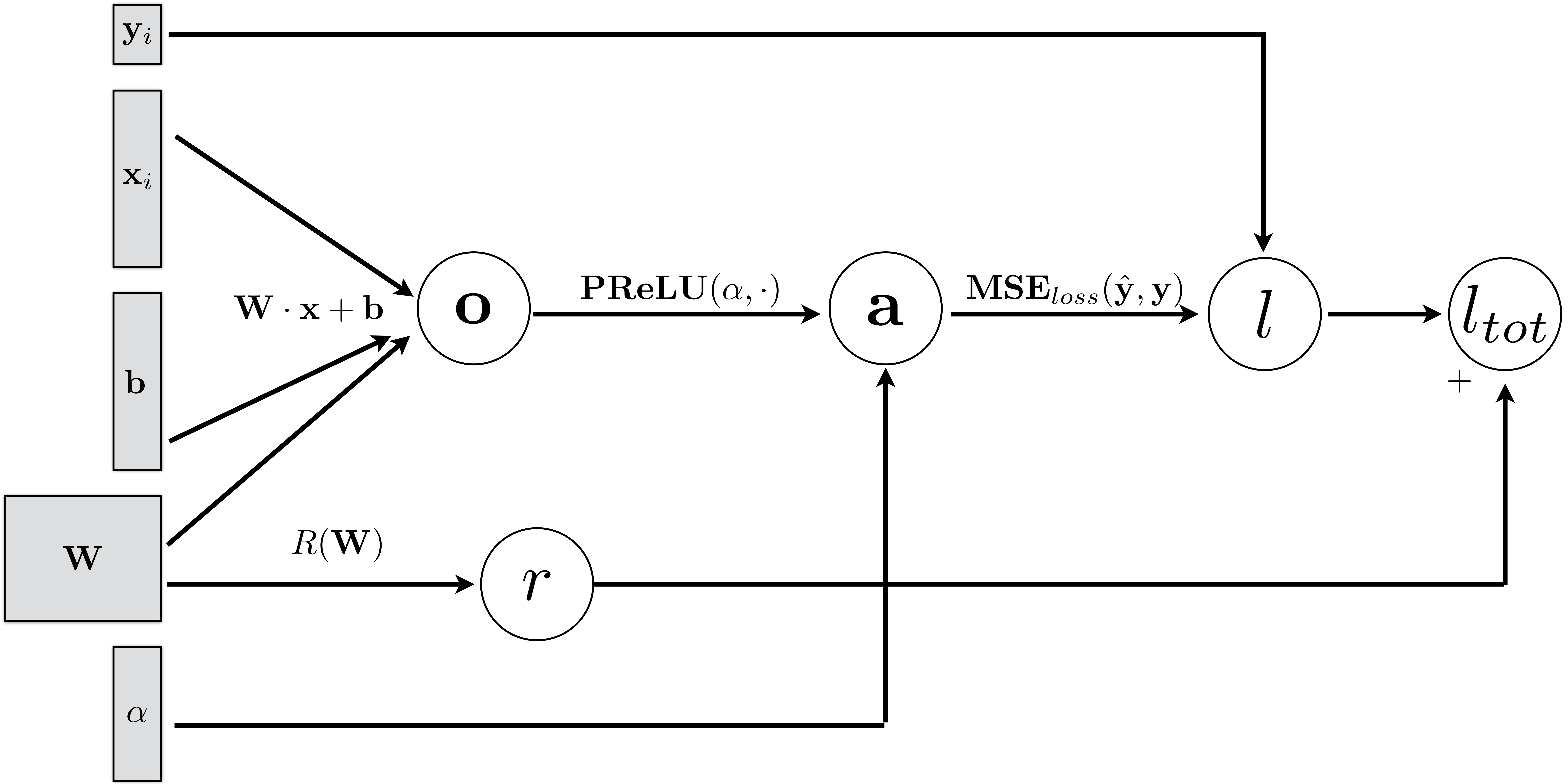
L2 Regularizer:

$$R_{L2}(\mathbf{W}_1) = 1$$
$$R_{L2}(\mathbf{W}_2) = 0.25 \leftarrow$$

L1 Regularizer:

$$R_{L1}(\mathbf{W}_1) = 1 \leftarrow$$
$$R_{L1}(\mathbf{W}_2) = 1 \leftarrow$$

Computational Graph: 1-layer with PReLU + Regularizer



Remember … **Initialization**

Many tricks for initializations exist. I will not really cover this.

Regularization: Batch Normalization

Normalize each mini-batch (using Batch Normalization layer) by subtracting empirically computed mean and dividing by variance for every dimension -> samples are approximately unit Gaussian

$$\bar{x}^{(k)} = \frac{x^{(k)} - \mathbb{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$

Benefit:

Improves learning (better gradients, higher learning rate)

[Ioffe and Szegedy, NIPS 2015]

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Why?

[Ioffe and Szegedy, NIPS 2015]

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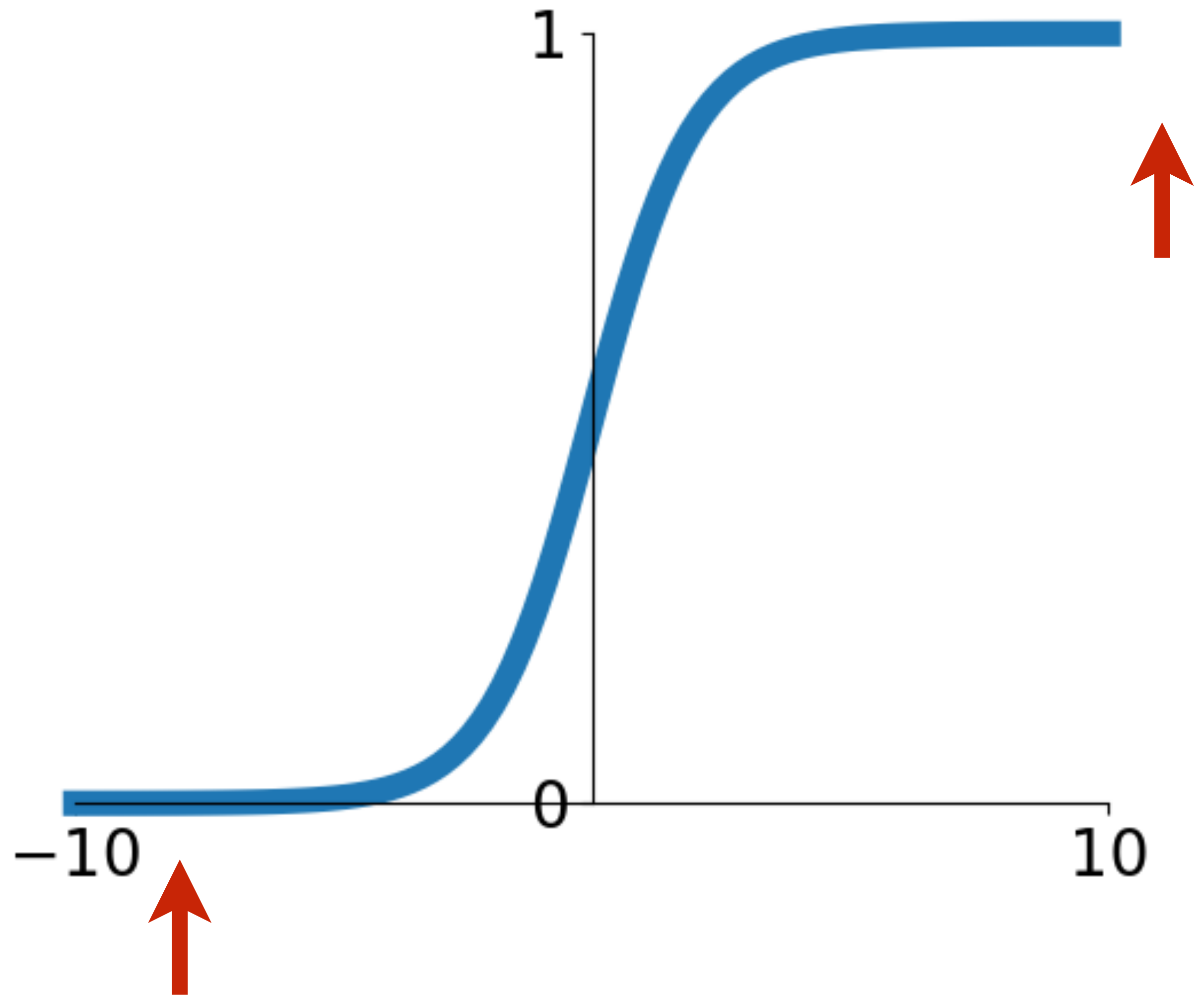
$$\frac{\partial \mathcal{L}}{\partial x} = \frac{\partial\, \mathbf{sigmoid}(x)}{\partial x} \frac{\partial \mathcal{L}}{\partial a}$$

$$\frac{\partial \mathcal{L}}{\partial a}$$

$$a(x) = \mathbf{sigmoid}(x) = \frac{1}{1+e^{-x}}$$



Sigmoid Activation

Cons:

- Saturated neurons **“kill” the gradients**
- Non-zero centered
- Could be expensive to compute

Regularization: Batch Normalization

Normalize each mini-batch (using Batch Normalization layer) by subtracting empirically computed mean and dividing by variance for every dimension -> samples are approximately unit Gaussian

$$\bar{x}^{(k)} = \frac{x^{(k)} - \mathbb{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$

Benefit:

Improves learning (better gradients, higher learning rate)

Typically inserted **before** activation layer

[Ioffe and Szegedy, NIPS 2015]

Activation Function: Sigmoid vs. Tanh

$$a(x) = \mathbf{tanh}(x) = 2 \cdot \mathbf{sigmoid}(2x) - 1$$

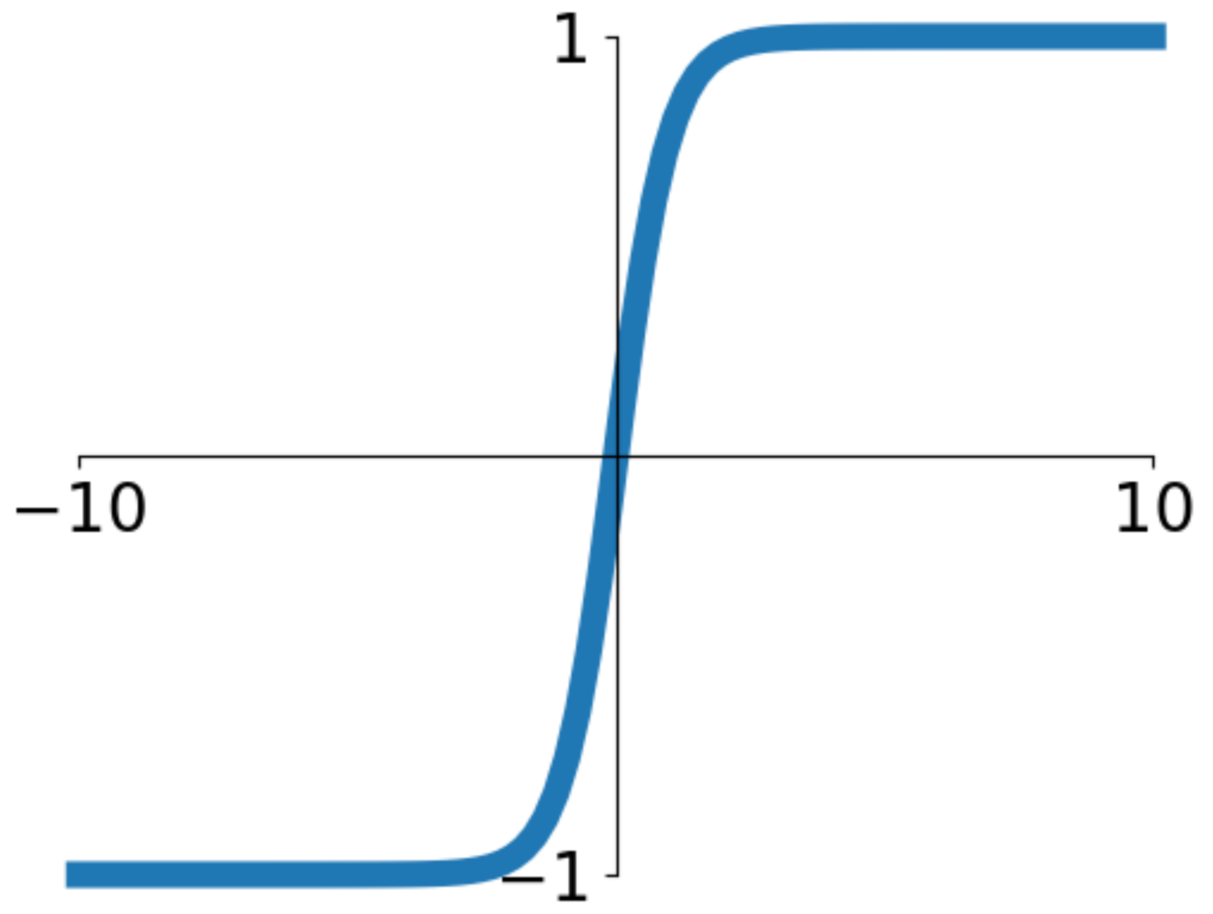
$$a(x) = \mathbf{tanh}(x) = \frac{2}{1 + e^{-2x}} - 1$$

Pros:

- Squishes everything in the range [-1,1]
- Centered around zero
- Has well defined gradient everywhere

Cons:

- Saturated neurons “kill” the gradients



Tanh Activation

Regularization: Batch Normalization

Normalize each mini-batch (using Batch Normalization layer) by subtracting empirically computed mean and dividing by variance for every dimension -> samples are approximately unit Gaussian

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In practice, also learn how to scale and offset:

$$y^{(k)} = \gamma^{(k)} \bar{x}^{(k)} + \beta^{(k)}$$

BN layer parameters

Benefit:

Improves learning (better gradients, higher learning rate, less reliance on initialization)

Typically inserted **before** activation layer

[Ioffe and Szegedy, NIPS 2015]

Regularization: Batch Normalization

Consider what happens at **runtime**, when you are only passing a single sample

$$\bar{x}^{(k)} = \frac{x^{(k)} - \mathbb{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$

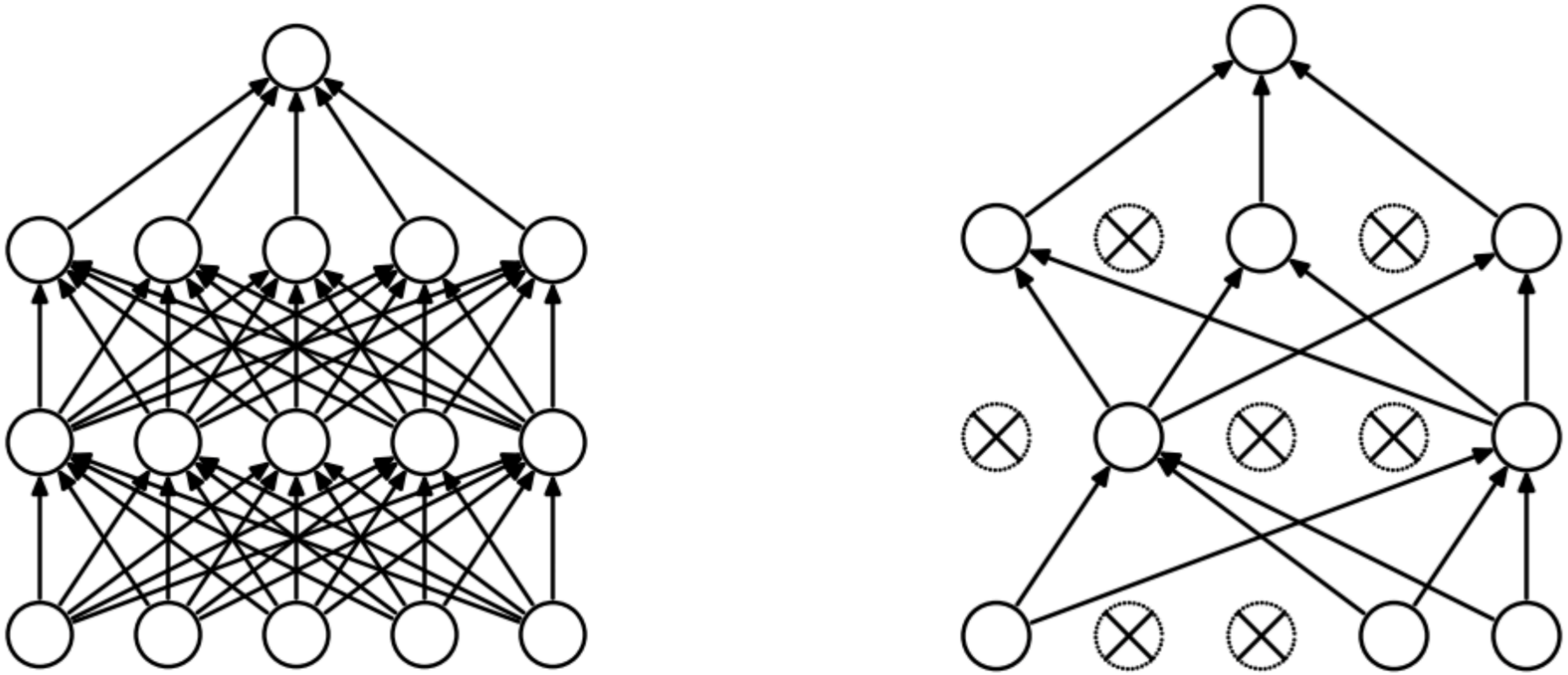
In practice, also learn how to scale and offset:

$$y^{(k)} = \underbrace{\gamma^{(k)}\bar{x}^{(k)} + \beta^{(k)}}_{\text{BN layer parameters}}$$

[Ioffe and Szegedy, NIPS 2015]

Regularization: Dropout

Randomly **set some neurons to zero** in the forward pass, with probability proportional to `dropout rate` (between 0 to 1)



Standar Neural Network

After Applying **Dropout**

[Srivastava et al, JMLR 2014]

* adopted from slides of **CS231n at Stanford**

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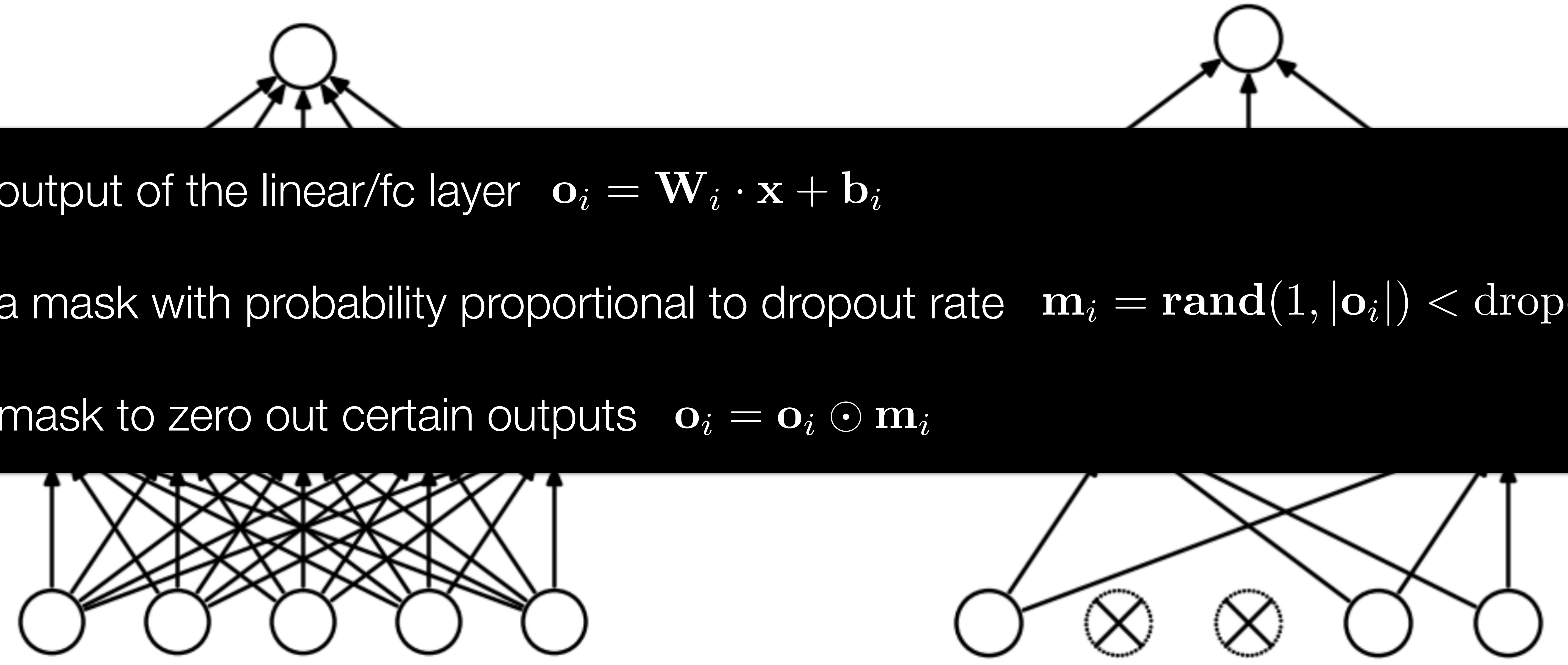

1. Compute output of the linear/fc layer $\mathbf{o}_i = \mathbf{W}_i \cdot \mathbf{x} + \mathbf{b}_i$

2. Compute a mask with probability proportional to dropout rate $\mathbf{m}_i = \mathbf{rand}(1, |\mathbf{o}_i|) < \text{dropout rate}$

3. Apply the mask to zero out certain outputs $\mathbf{o}_i = \mathbf{o}_i \odot \mathbf{m}_i$

Standar Neural Network

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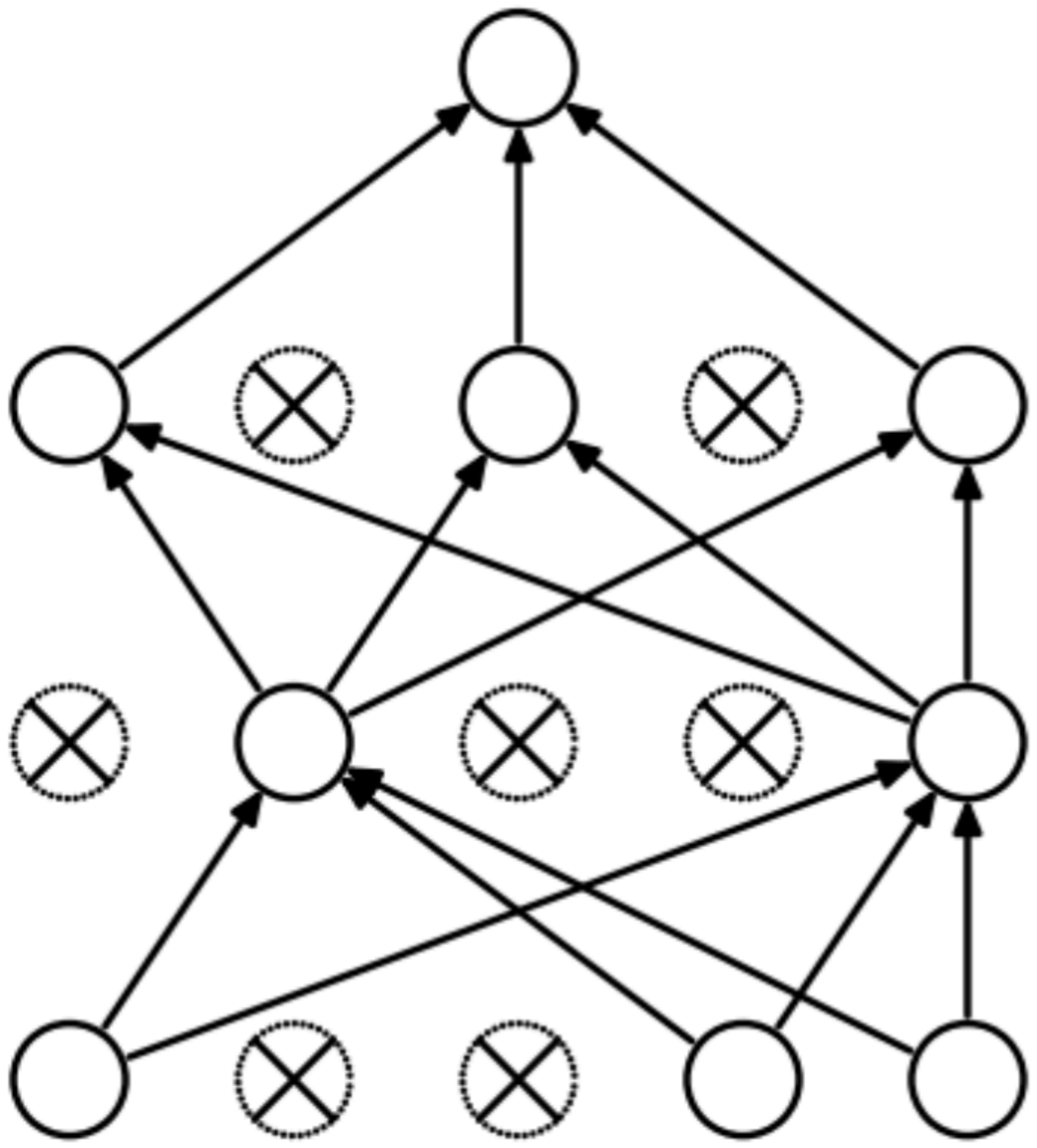
Why is this a good idea?



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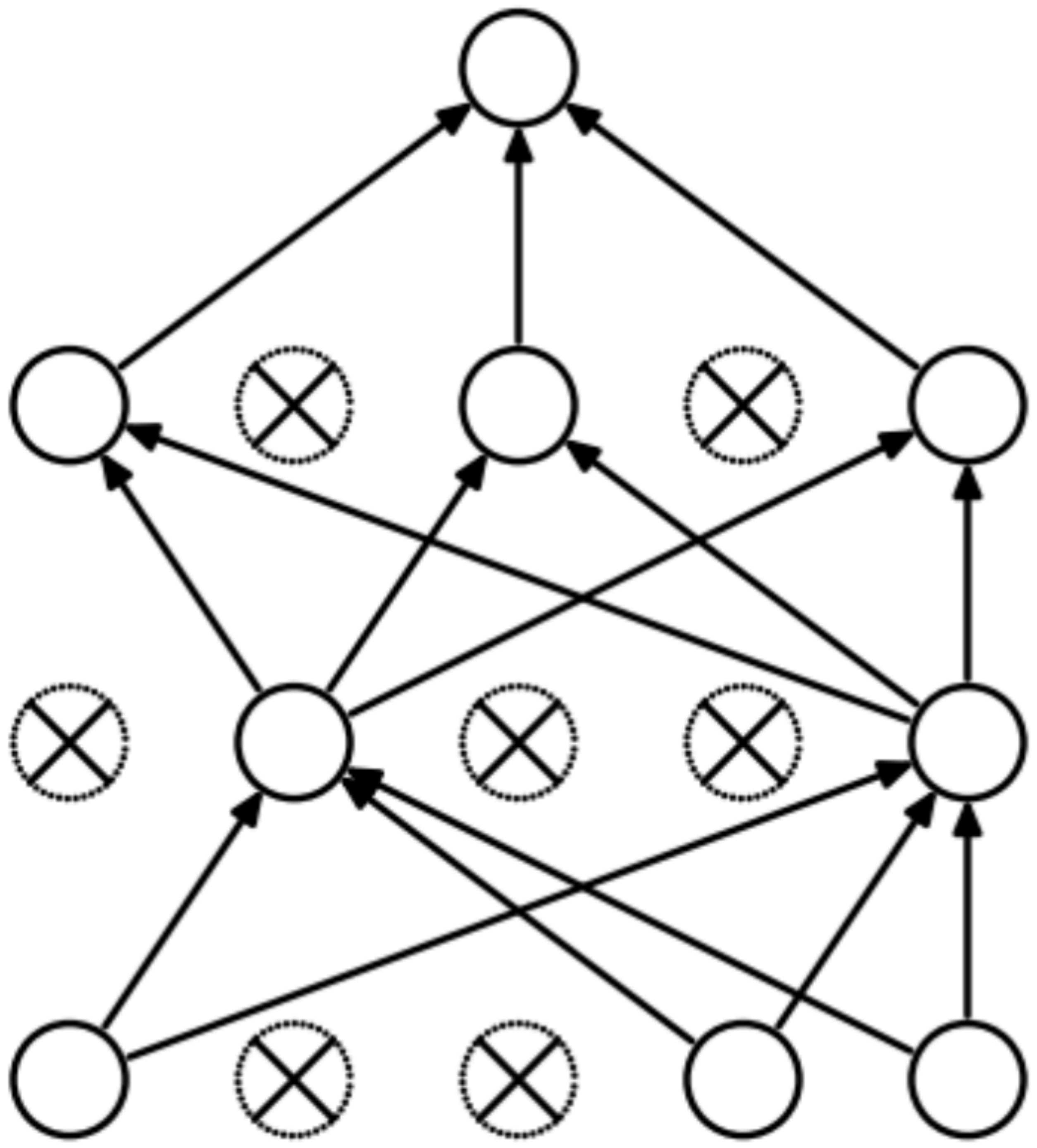


After Applying **Dropout**

Why is this a good idea?

Dropout is training an **ensemble of models** that share parameters

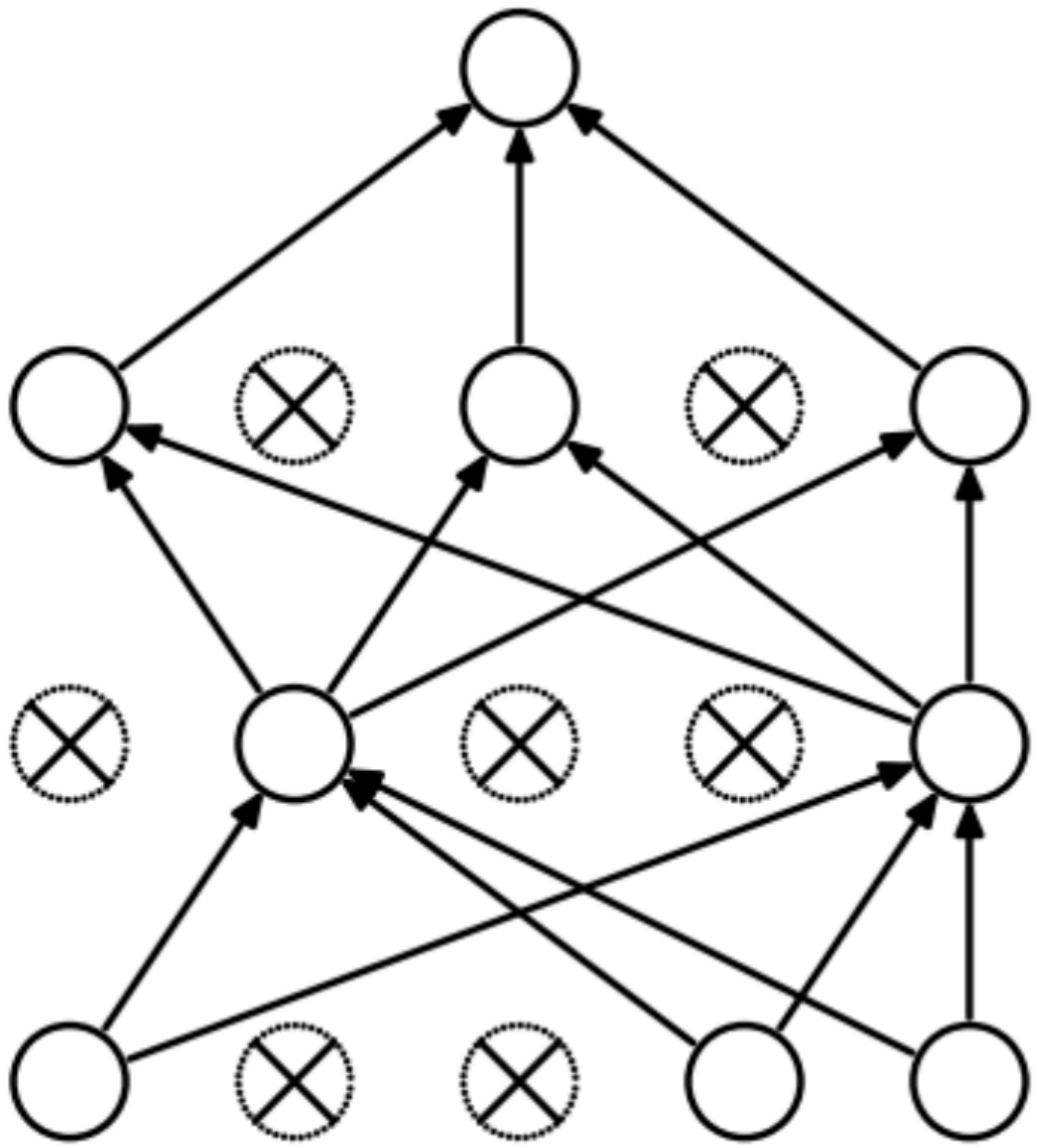
Each binary mask (generated in the forward pass) is one model that is trained on (approximately) one data point

[Srivastava et al, JMLR 2014]

* adopted from slides of **CS231n at Stanford**

Regularization: Dropout (at test time)

Randomly **set some neurons to zero** in the forward pass, with probability proportional to `dropout rate` (between 0 to 1)



After Applying **Dropout**

At test time, **integrate out all the models** in the ensemble

Monte Carlo approximation: many forward passes with different masks and average all predictions

[Srivastava et al, JMLR 2014]

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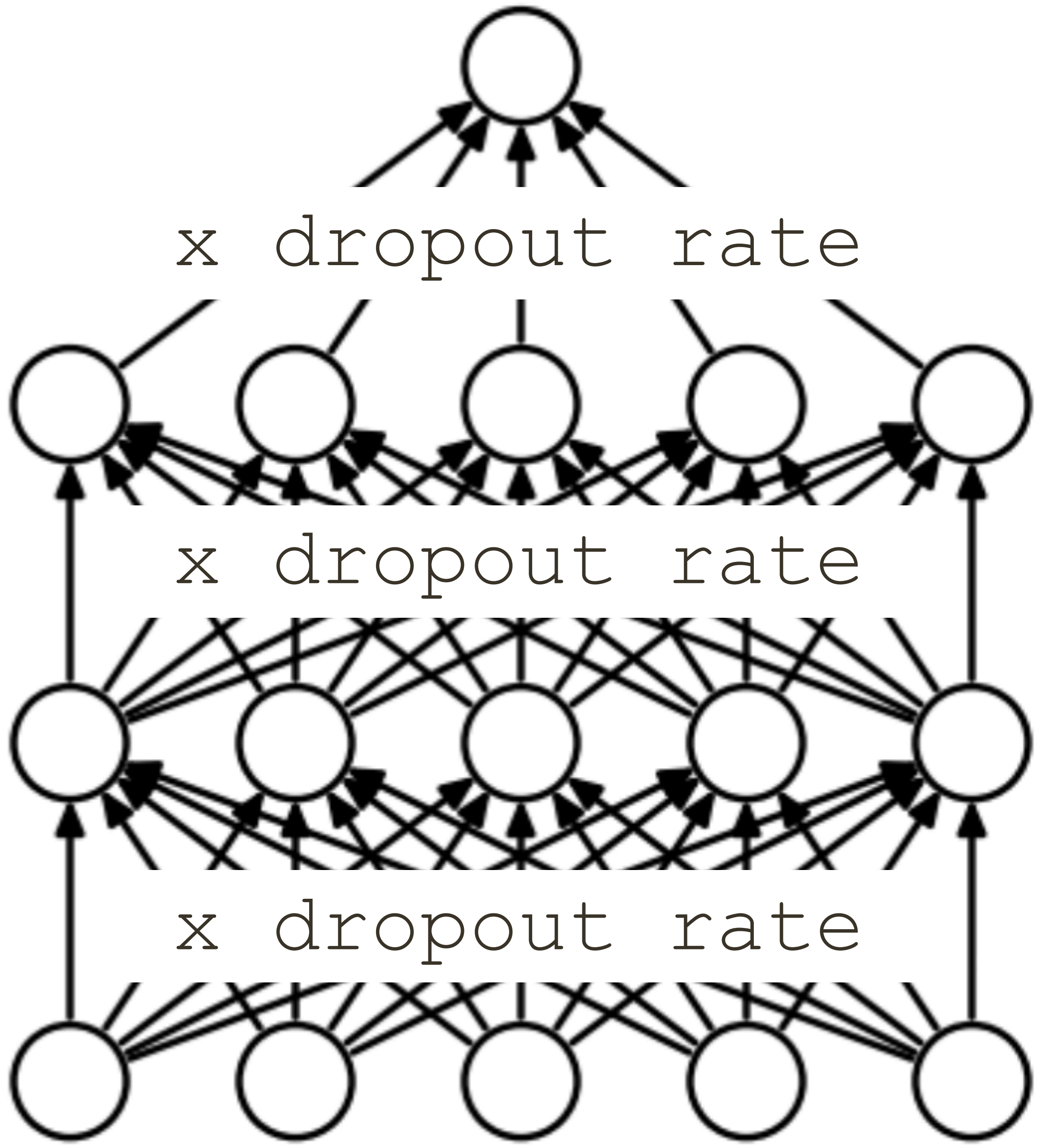

At test time, **integrate out all the models** in the ensemble

Monte Carlo approximation: many forward passes with different masks and average all predictions

Equivalent to forward pass with all connections on and **scaling of the outputs** by dropout rate

For derivation see Lecture 6 of **CS231n at Stanford**

[Srivastava et al, JMLR 2014]

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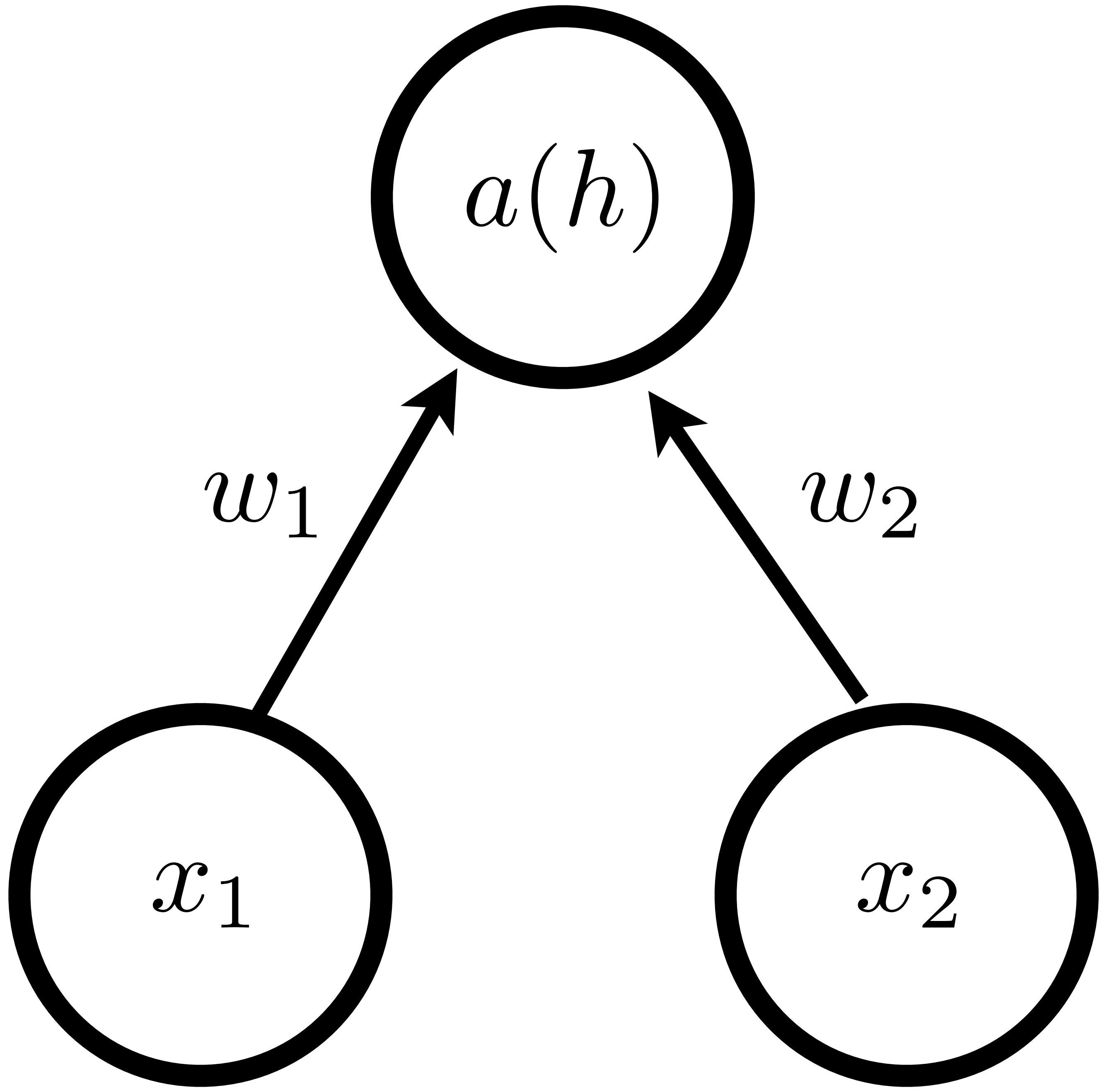
Regularization: Dropout (at test time)

Consider a single neuron



Regularization: Dropout (at test time)

At test time we want to compute **expectation** over input to activation function with respect to exponential number of masks

$$\mathbb{E}_{\mathbf{m}}[h] = \mathbb{E}_{\mathbf{m}}[(\mathbf{W} \cdot \mathbf{x}) \odot \mathbf{m}]$$

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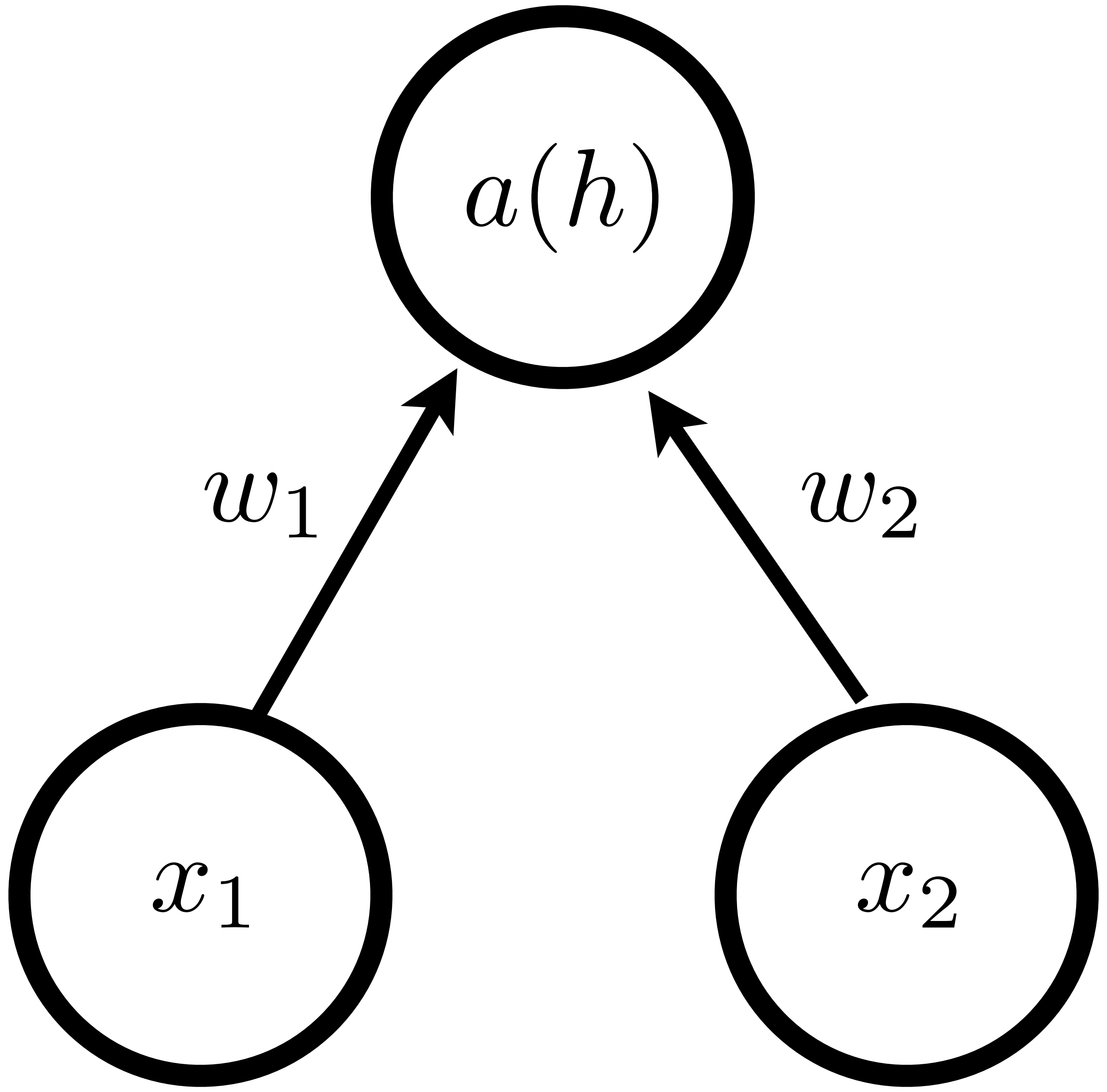

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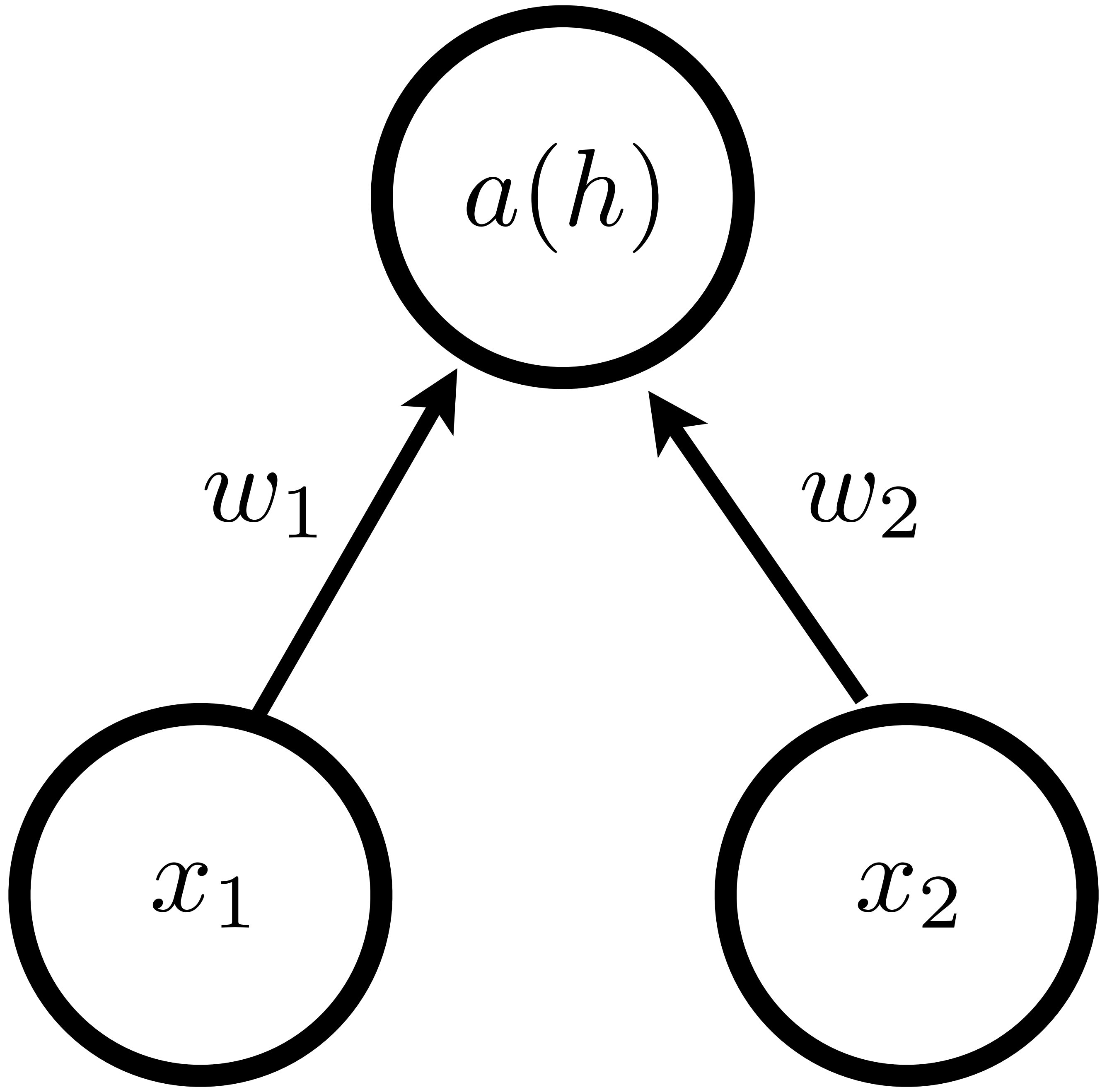

consider dropout rate of p = 0.5

$$\mathbb{E}_{\mathbf{m}}[h] = \mathbb{E}_{(m_1,m_2)}[w_1 x_1 m_1 + w_2 x_2 m_2]$$

$$= \frac{1}{4}(w_1 x_1 + w_2 x_2) + \frac{1}{4}(w_1 x_1) \frac{1}{4}(w_2 x_2) + \frac{1}{4}(0)$$

$$= \frac{1}{2}(w_1 x_1 + w_2 x_2)$$

Regularization: Dropout (without change in forward pass)

Randomly **set some neurons to zero** in the forward pass, with probability proportional to `dropout rate` (between 0 to 1)

1. Compute output of the linear/fc layer $\mathbf{o}_i = \mathbf{W}_i \cdot \mathbf{x} + \mathbf{b}_i$

2. Compute a mask with probability proportional to dropout rate $\mathbf{m}_i = \mathbf{rand}(1, |\mathbf{o}_i|) < \text{dropout rate}$

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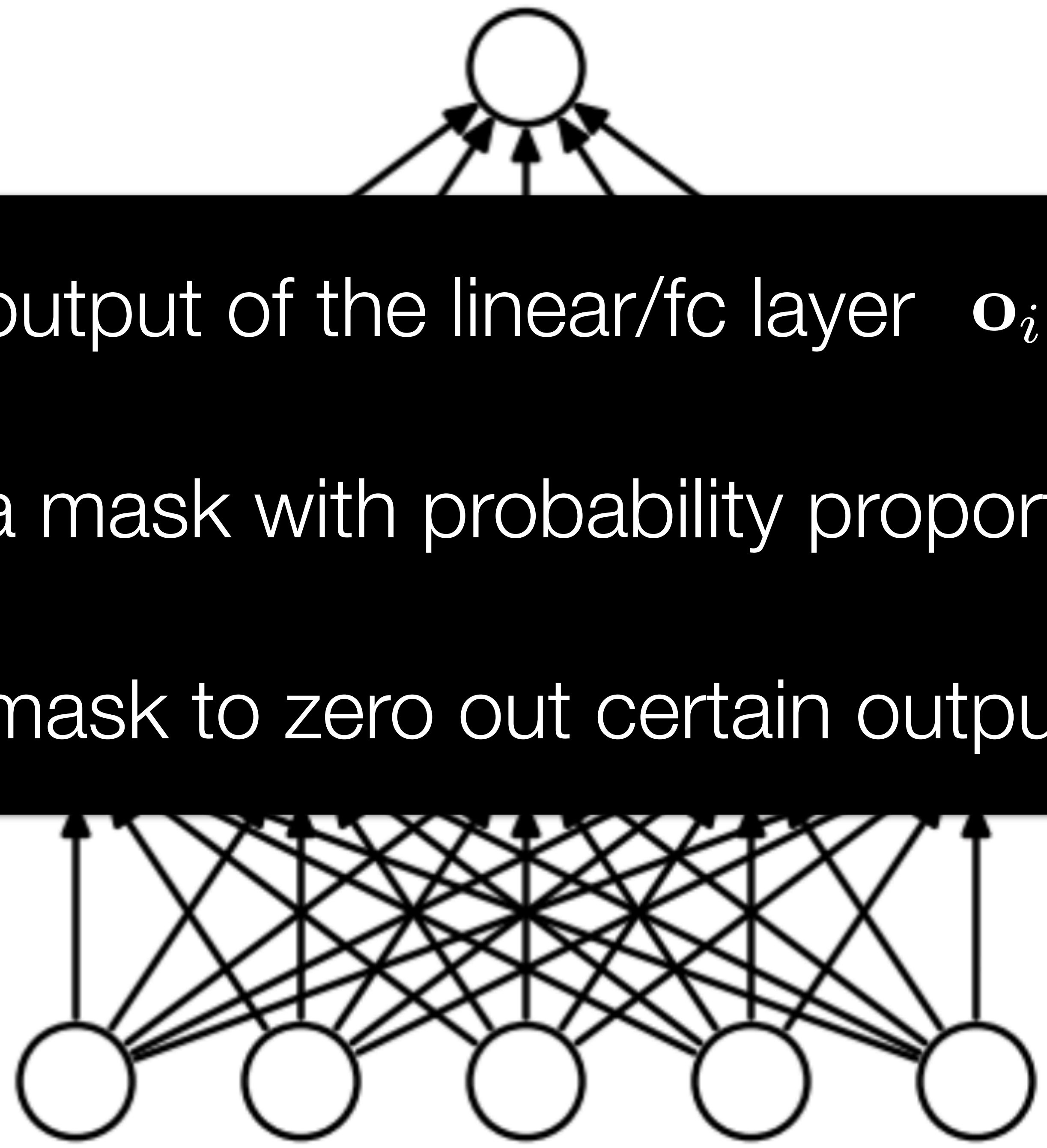

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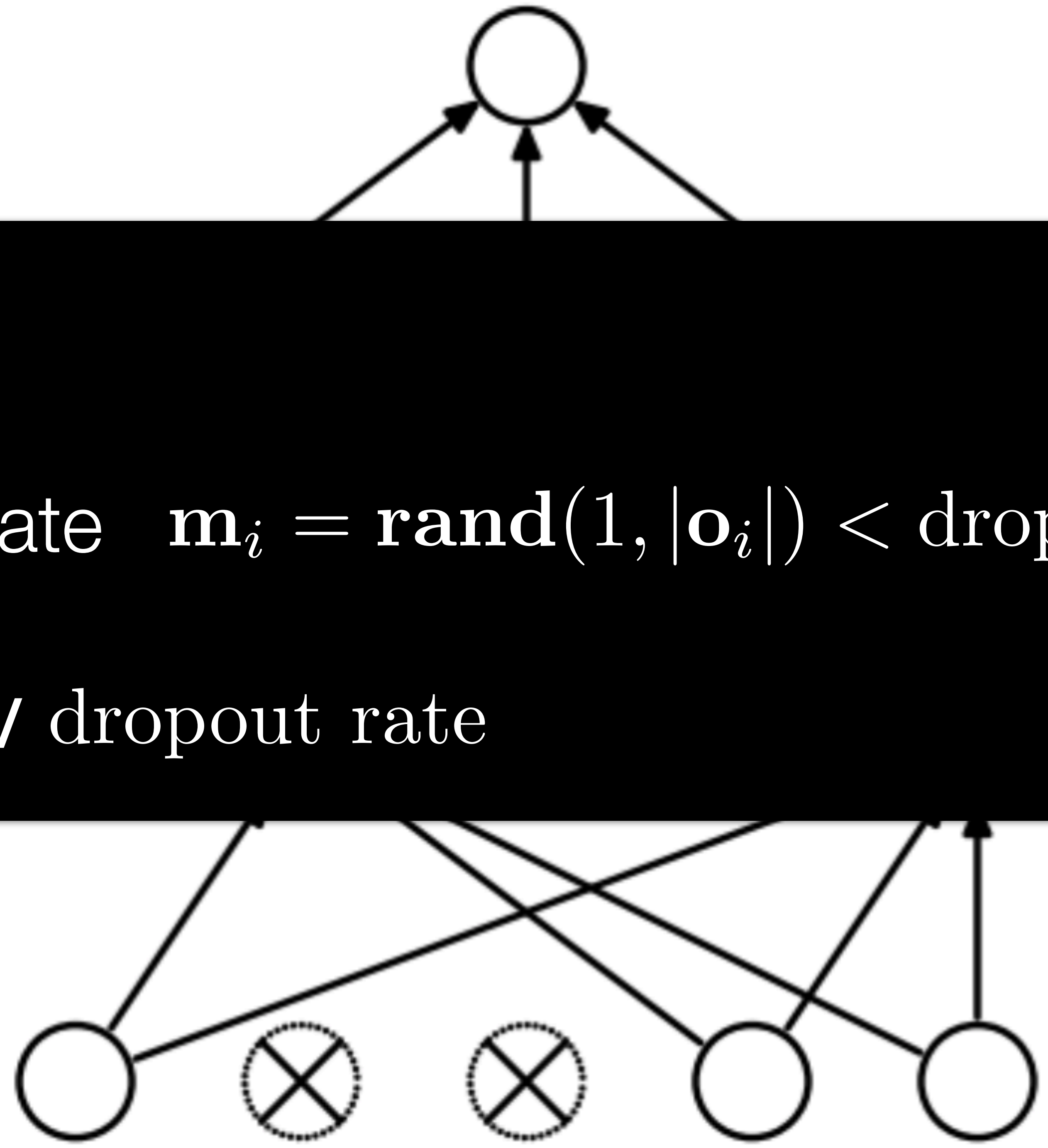

After Applying **Dropout**

[Srivastava et al, JMLR 2014]

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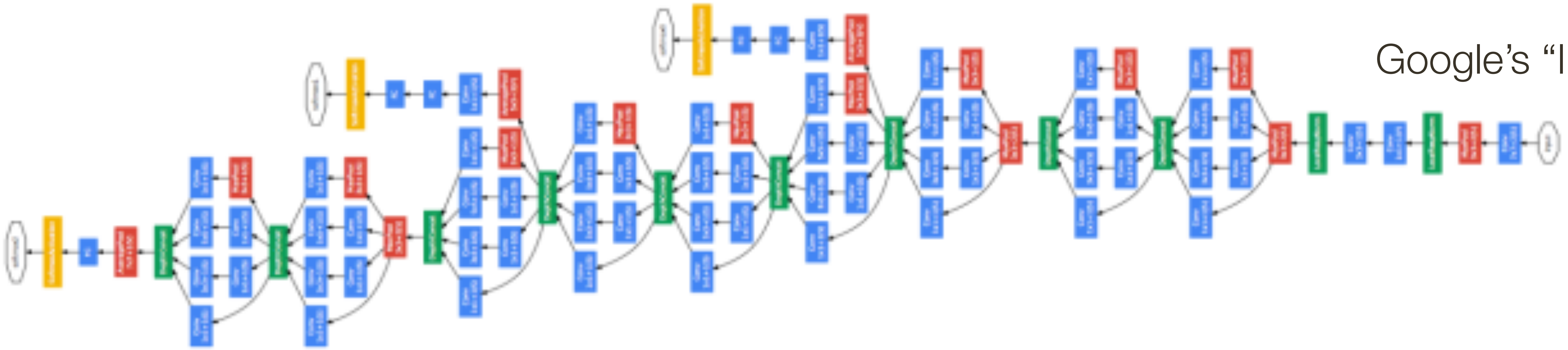
Deep Learning **Terminology**



Google's "Inception" network

- **Network structure:** number and types of layers, forms of activation functions, dimensionality of each layer and connections (defines computational graph)

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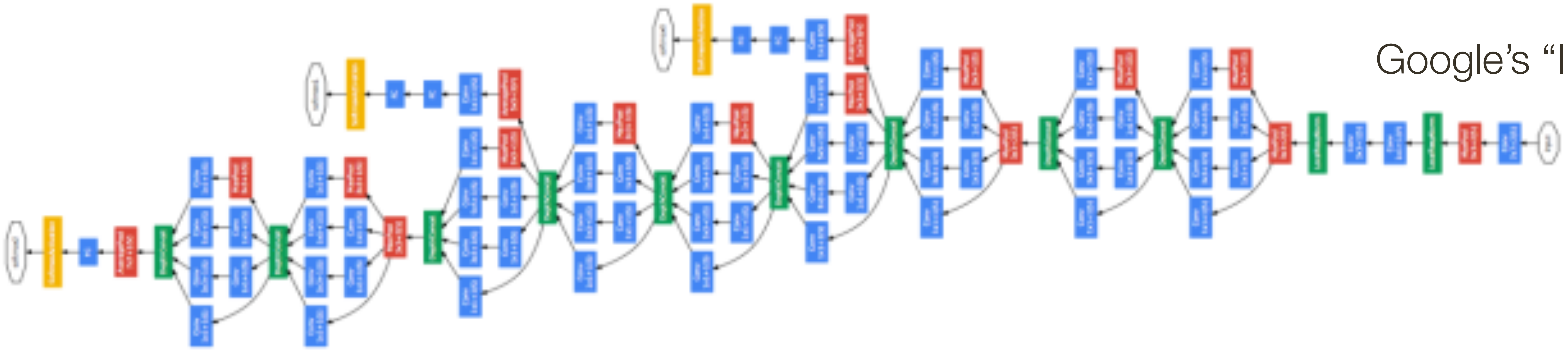

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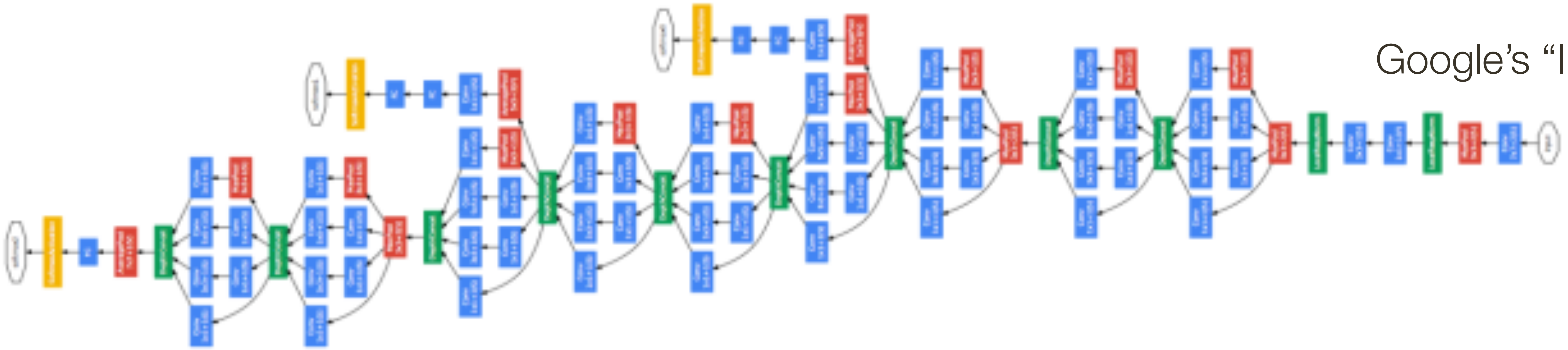

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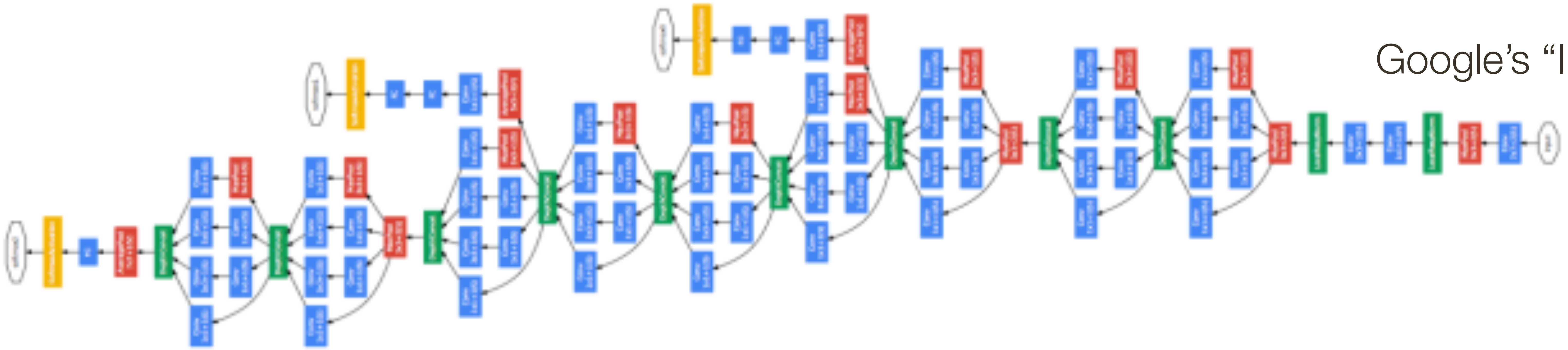

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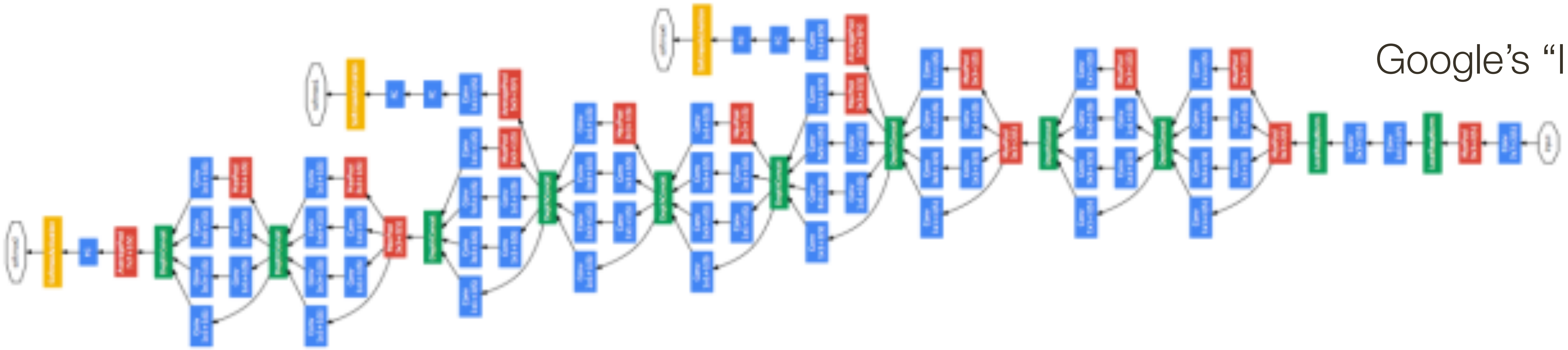

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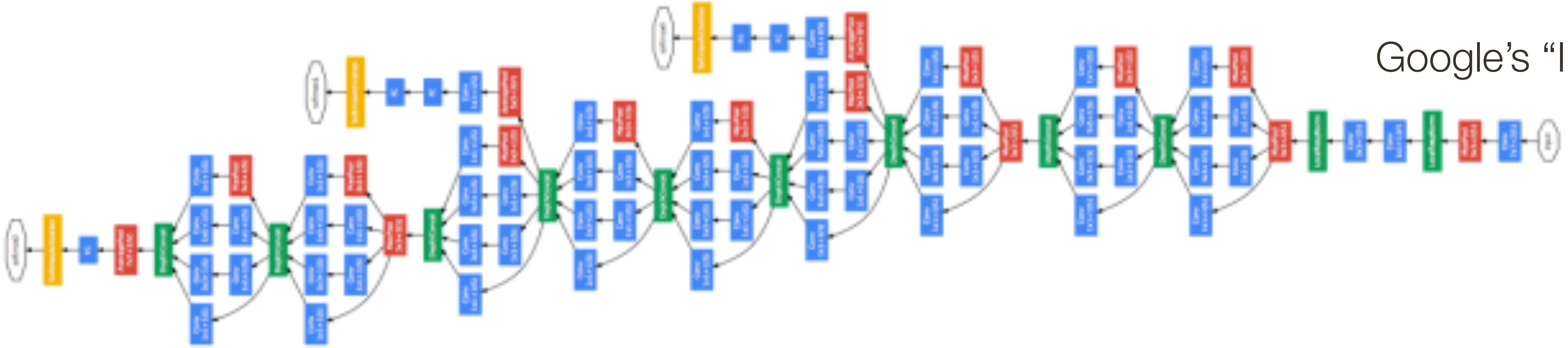

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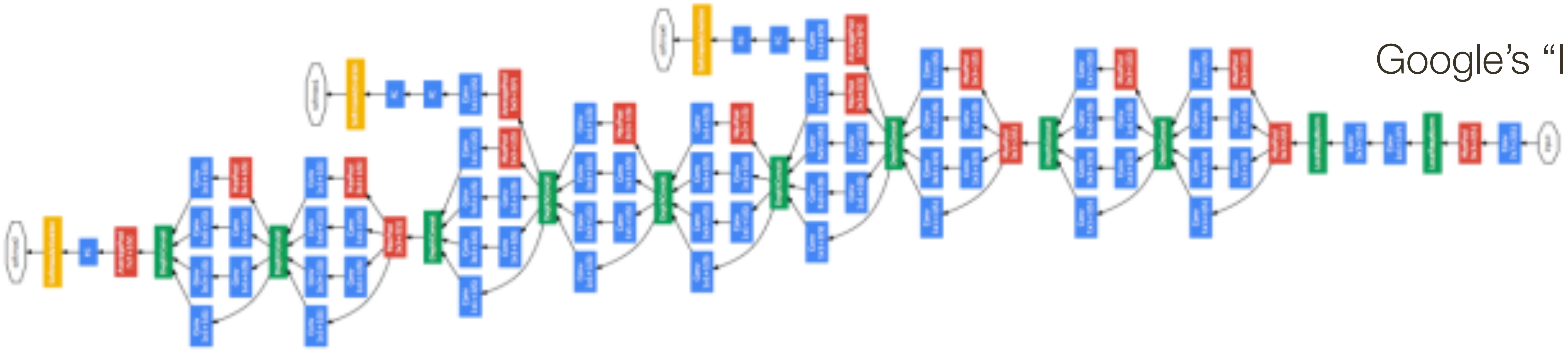

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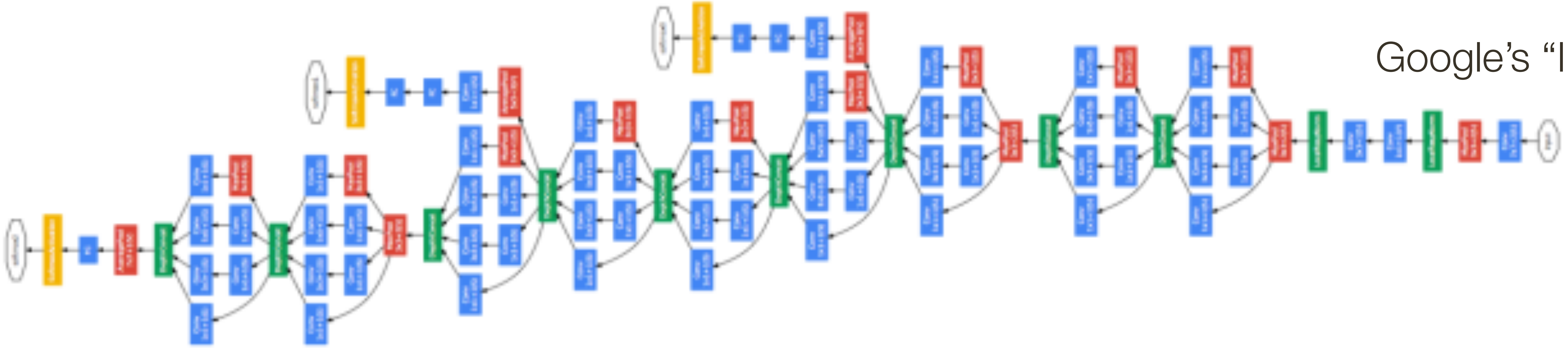

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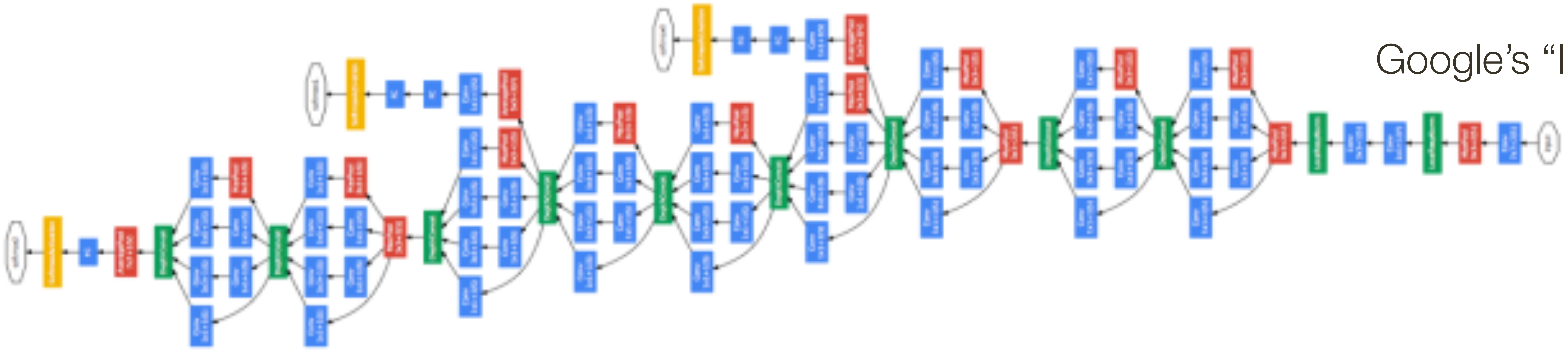

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grid search

Loss Functions …

This is where all the **fun** is … we will only look a most common ones

Multivariate **Regression**

Input: feature vector $\mathbf{x} \in \mathbb{R}^n$

Output: output vector $\mathbf{y} \in \mathbb{R}^m$

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Neural Network (input + intermediate hidden layers) $f(\mathbf{x}; \Theta) : \mathbb{R}^n \rightarrow \mathbb{R}^k$

with **sigmoid** activations: $\mathbf{0} \leq f(\mathbf{x}; \Theta) \leq \mathbf{1}$

with **Tanh** activations: $-\mathbf{1} \leq f(\mathbf{x}; \Theta) \leq \mathbf{1}$

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Neural Network (output): linear layer

$$\hat{\mathbf{y}} = g(\mathbf{x}; \mathbf{W}, \mathbf{b}) = \mathbf{W} f(\mathbf{x}; \Theta) + \mathbf{b} : \mathbb{R}^k \to \mathbb{R}^m$$

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Loss: $\mathcal{L}(\mathbf{y}, \hat{\mathbf{y}}) = ||\mathbf{y} - \hat{\mathbf{y}}||^2$

Binary Classification (Bernoulli)

Input: feature vector $\mathbf{x} \in \mathbb{R}^n$

Output: binary label $y \in \{0, 1\}$

Neural Network (input + intermediate hidden layers) $f(\mathbf{x}; \Theta) : \mathbb{R}^n \to \mathbb{R}$

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with **sigmoid** activations: $\mathbf{0} \leq f(\mathbf{x}; \Theta) \leq \mathbf{1}$

Neural Network (output): threshold hidden output (which is a sigmoid)

$$\hat{y} = 1[f(\mathbf{x}; \Theta) > 0.5]$$

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Problem: Not differentiable, probabilistic interpretation maybe desirable

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Neural Network (output): interpret sigmoid output as probability

$$p(y = 1) = f(\mathbf{x}; \Theta)$$

can interpret the score as the log-odds of $y = 1$ (a.k.a. the **logits**)

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Loss: similarity between two distributions

Binary Classification (Bernoulli)

Input: feature vector $\mathbf{x} \in \mathbb{R}^n$

Output: binary label $y \in \{0, 1\}$

We can measure similarity between distribution $p(x)$ and $q(x)$ using cross-entropy

$$H(p, q) = -\mathbb{E}_{x\sim p}[\log q(x)]$$

For discrete distributions this ends up being:

$$H(p, q) = -\sum_{x} p(x) \log q(x)$$

Loss: similarity between two distributions

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Loss: $\mathcal{L}(y, \hat{y}) = -y \log[f(\mathbf{x}; \Theta)] - (1 - y) \log[1 - f(\mathbf{x}; \Theta)]$

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$$\mathcal{L}(y, \hat{y}) = \begin{cases} -log[1 - f(\mathbf{x}; \Theta)] & y = 0 \\ -log[f(\mathbf{x}; \Theta)] & y = 1 \end{cases}$$

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Minimizing this **loss** is the same as maximizing **log likelihood** of data

Loss:

$$\mathcal{L}(y, \hat{y}) = \begin{cases} -log[1 - f(\mathbf{x}; \Theta)] & y = 0 \\ -log[f(\mathbf{x}; \Theta)] & y = 1 \end{cases}$$

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Neural Network (output): linear layer with one neuron and sigmoid activation

Multiclass Classification (e.g, ImageNet)

Input: feature vector $\mathbf{x} \in \mathbb{R}^n$

Output: muticlass label $\mathbf{y} \in \{0, 1\}^m$

(**one-hot** encoding)

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Multiclass Classification (e.g, ImageNet)

Input: feature vector $\mathbf{x} \in \mathbb{R}^n$

Output: muticlass label $\mathbf{y} \in \{0, 1\}^m$

(**one-hot** encoding)

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convert score into **probability**

normalize to sum up to 1 across classes

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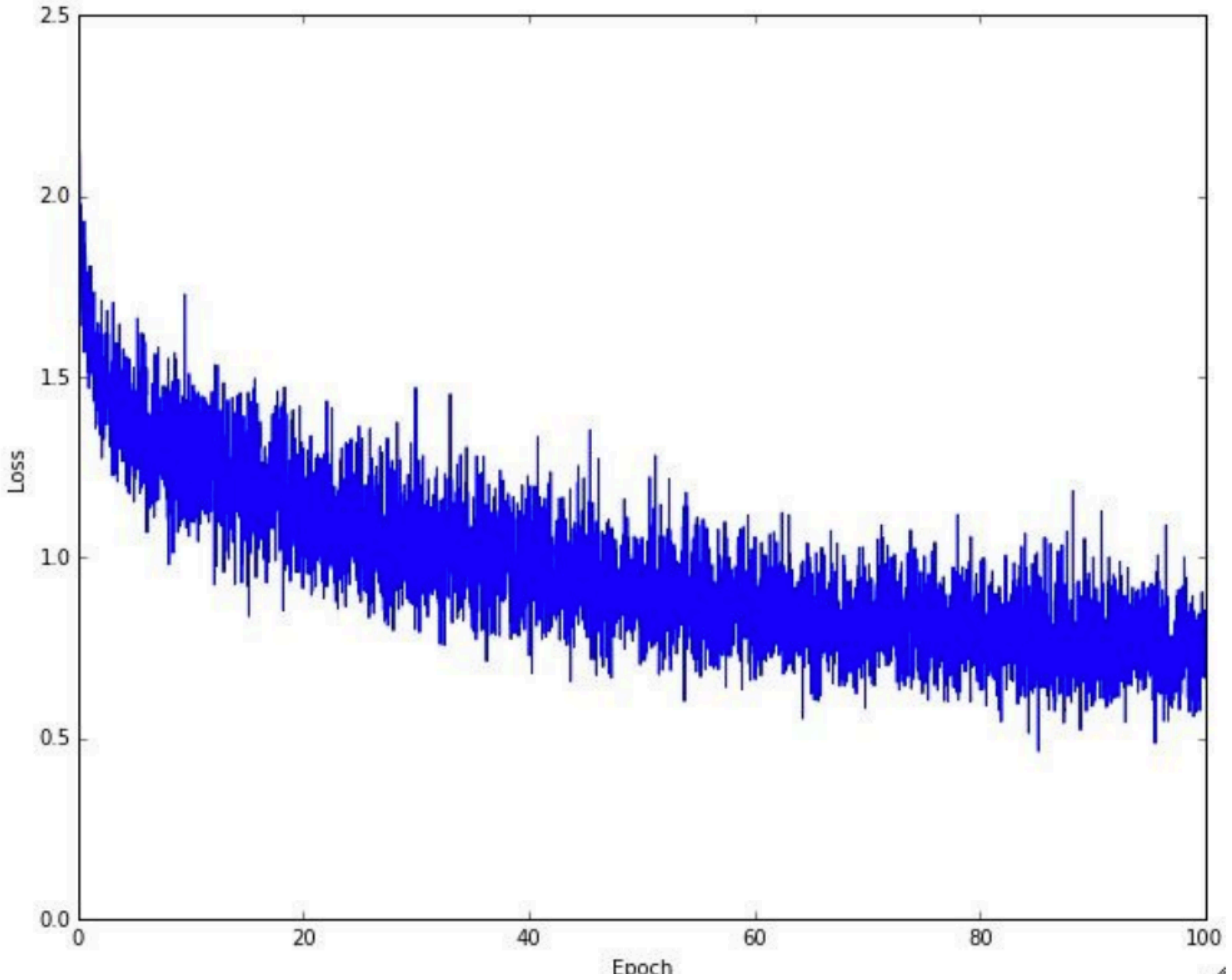
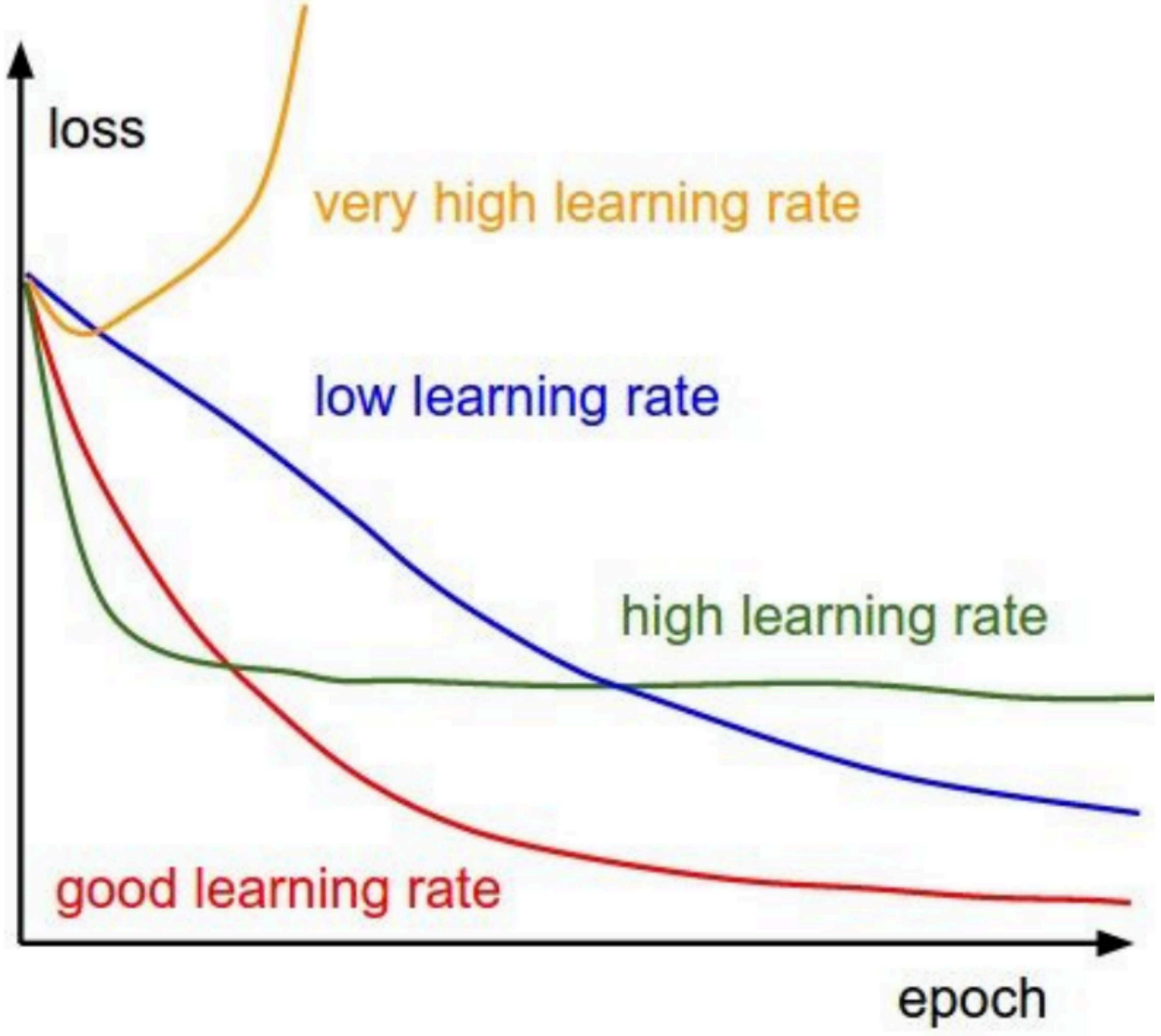
Special case for multi-class single label

Monitoring Learning: Visualizing the (training) loss





* slide from Li, Karpathy, Johnson's **CS231n at Stanford**

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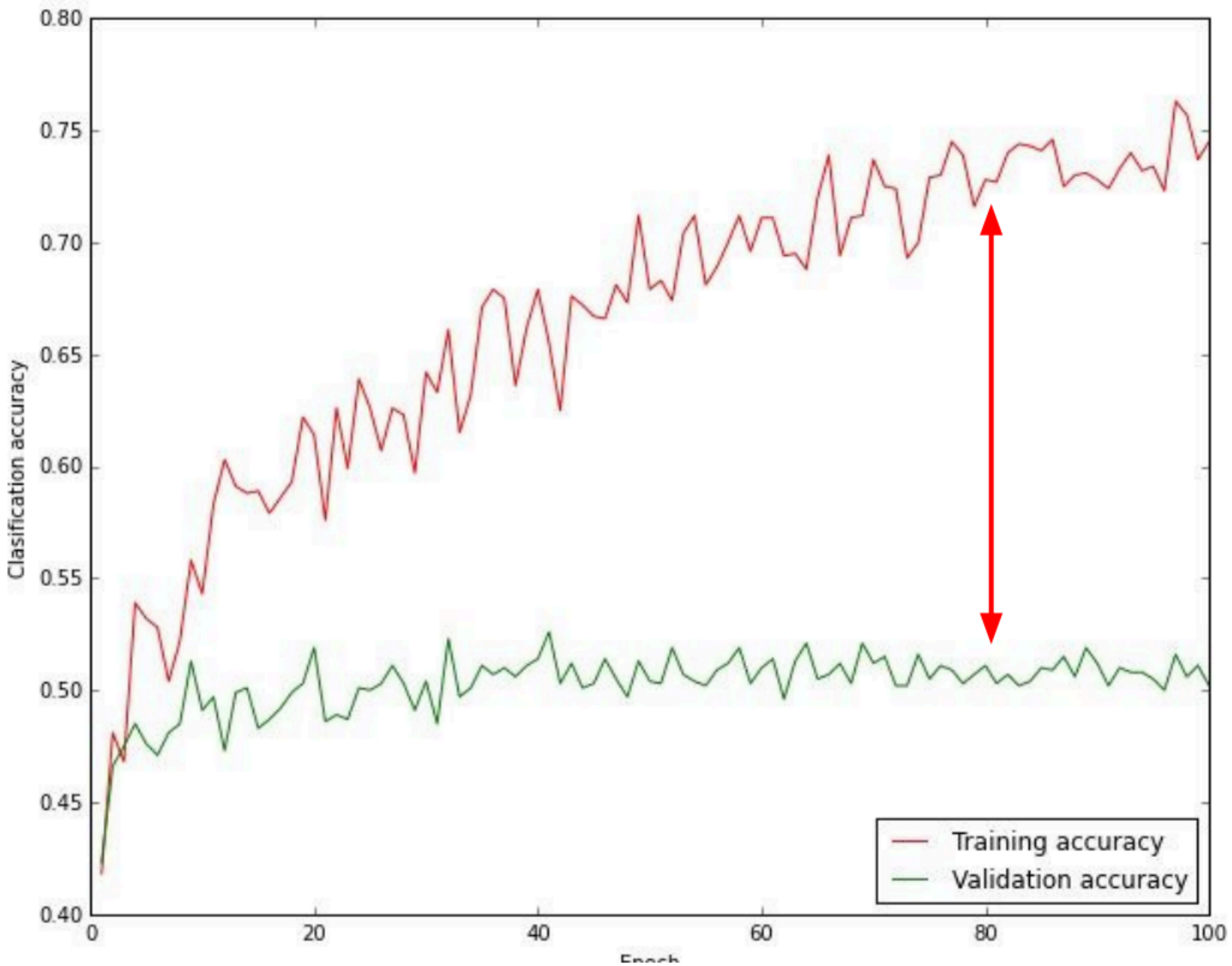

Big gap = overfitting

Solution: increase regularization

No gap = undercutting

Solution: increase model capacity

Small gap = **ideal**

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