THE UNIVERSITY OF BRITISH COLUMBIA

Topics in AI (CPSC 532S):
Multimodal Learning with Vision, Language and Sound

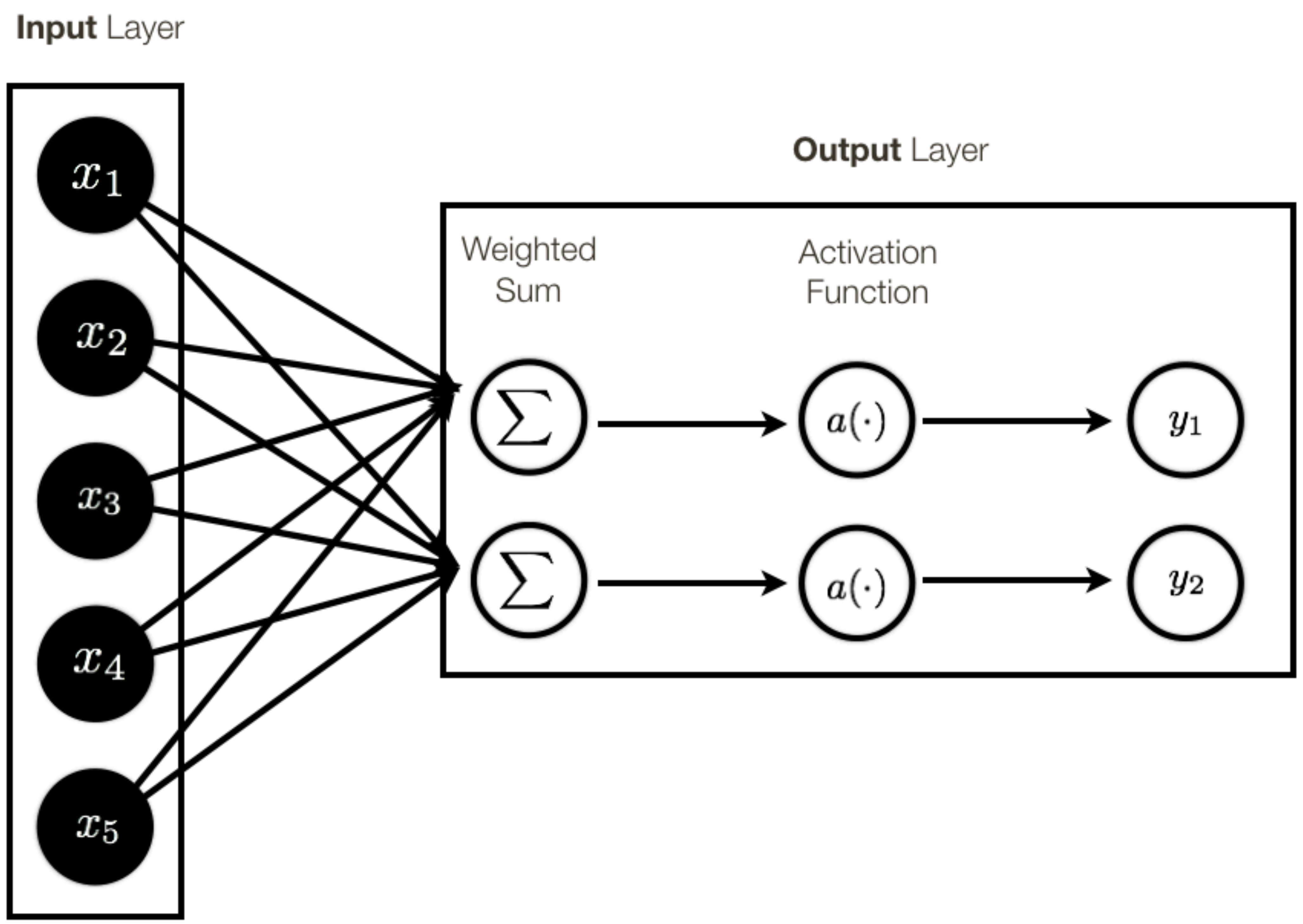
Lecture 3: Introduction to Deep Learning (continued)

Course **Logistics**

- Update on **course registrations:**
    - New Room (**SWING 409**) — 47 seats capacity (instead of 40)
    - Registrations (39 students currently registered) — approx. 8 seats left
- Microsoft **Azure** credits and tutorial next week
- **Assignment 1** … any questions?

Short **Review** …

- Introduced the basic building block of Neural Networks **(MLP/FC) layer**



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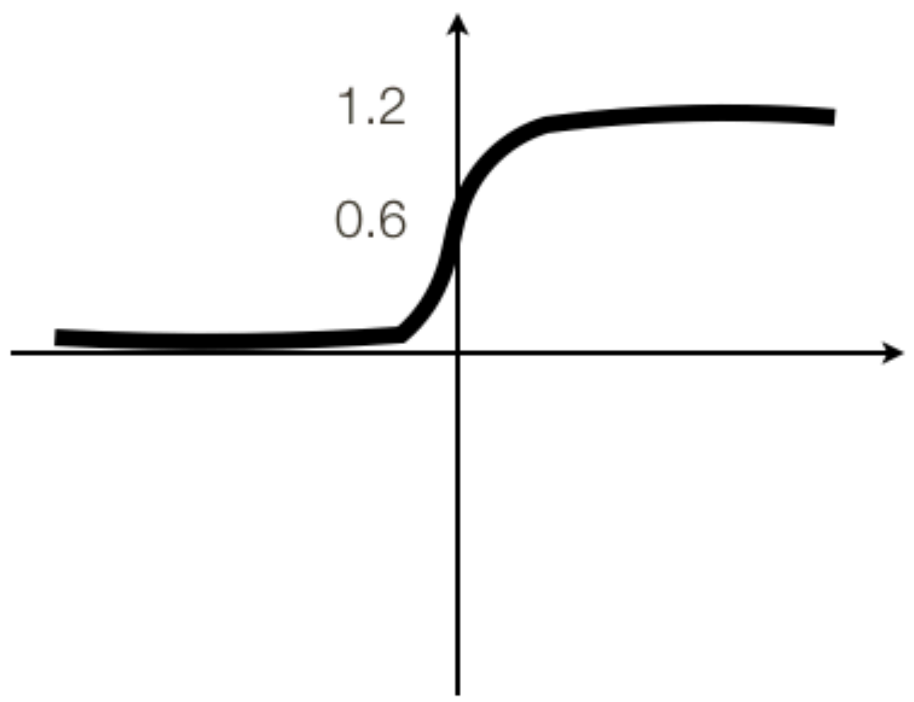

Sigmoid Activation

Short **Review** …

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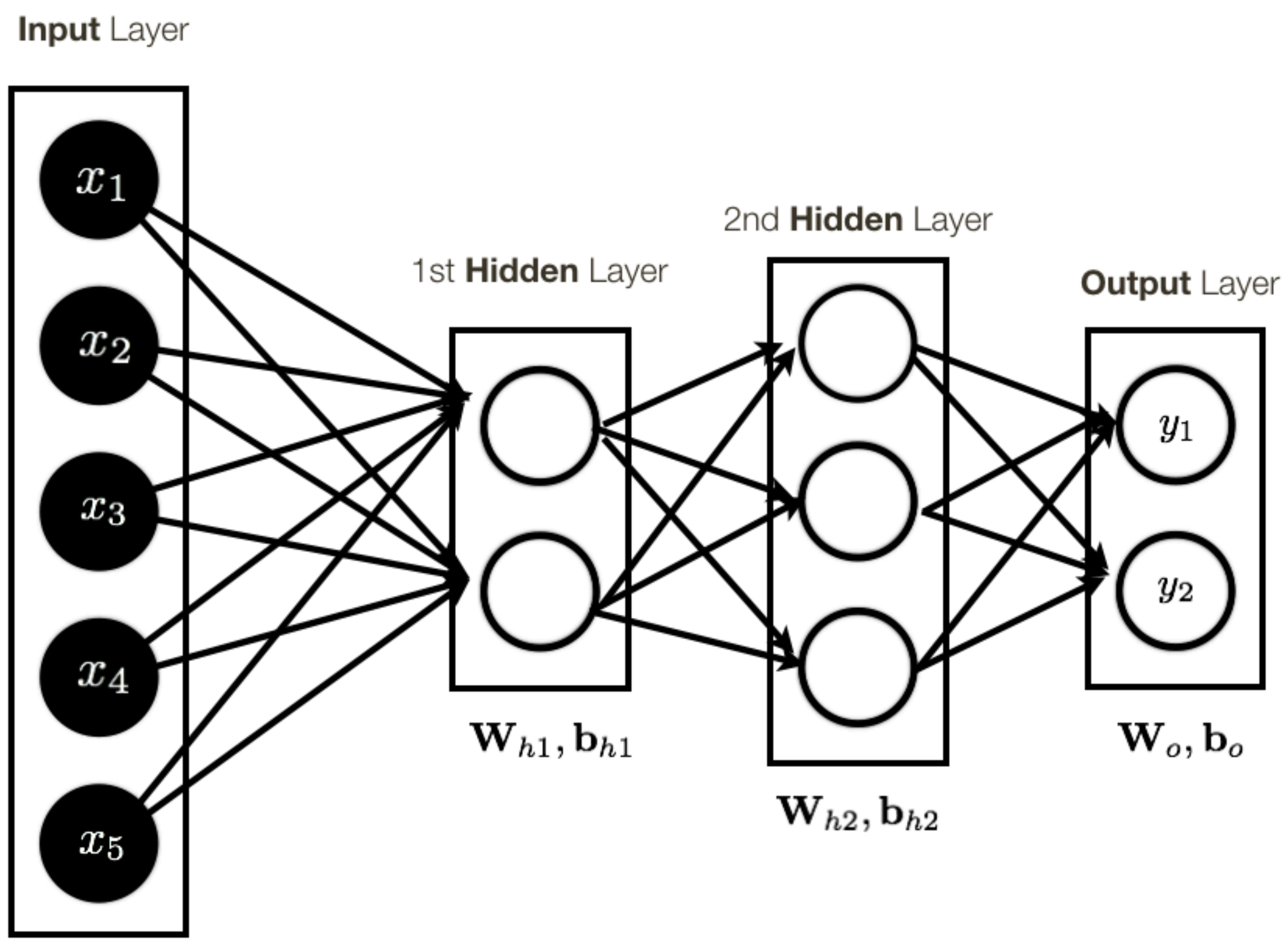
- How do we **stack these layers** up to make a Deep NN



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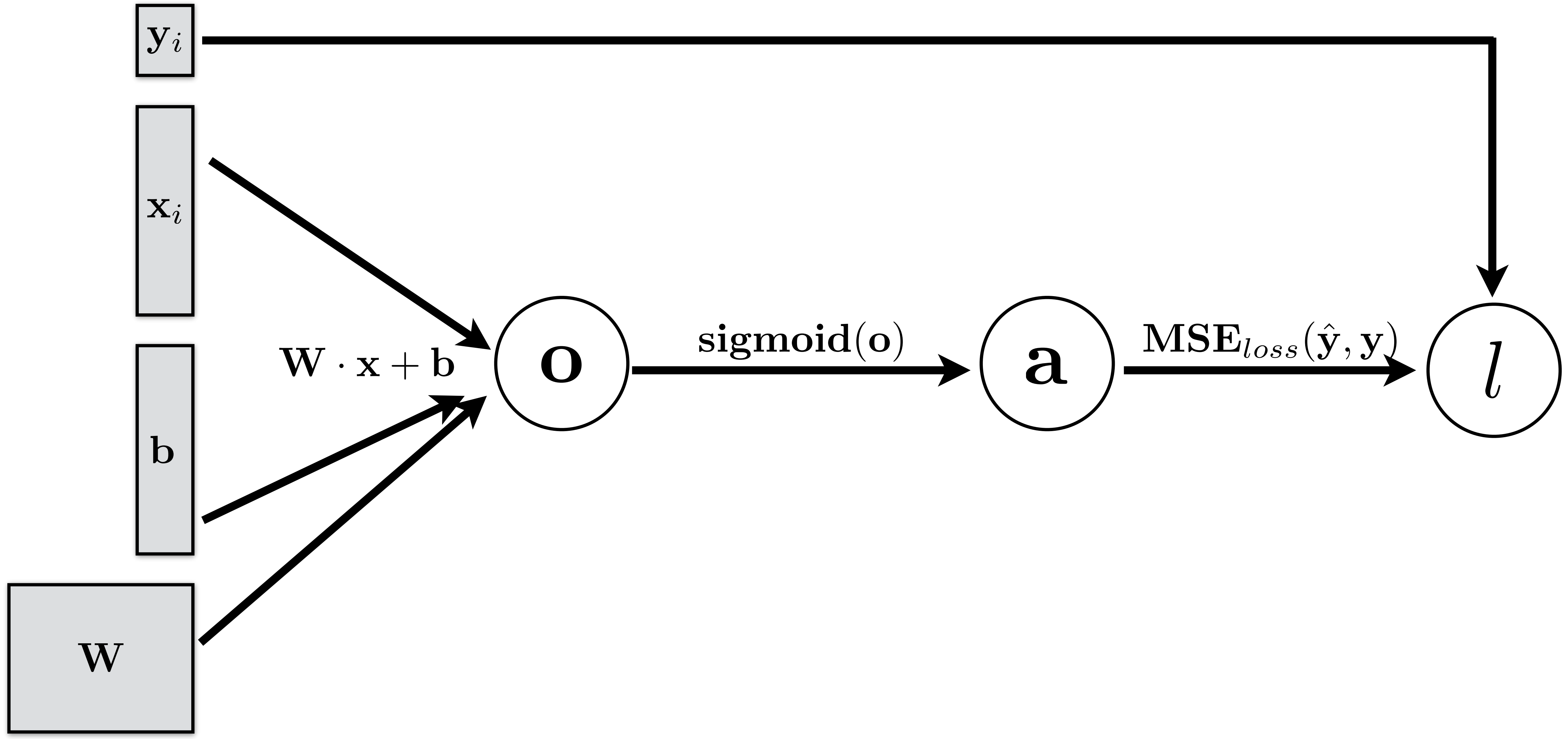

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Prediction / Inference

Function evaluation

(a.k.a. **ForwardProp**)

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Parameter **Learnings**

(Stochastic) Gradient Descent (needs **derivatives**)

- **Numerical** differentiation (not accurate)
- **Symbolic** differential (intractable)
- AutoDiff **Forward** (computationally expensive)
- AutoDiff **Backward / BackProp**

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- Different **activation functions** and saturation problem

Regularization: L2 or L1 on the weights

L2 Regularization: Learn a more (dense) distributed representation

$$R(\mathbf{W}) = ||\mathbf{W}||_2 = \sum_i \sum_j \mathbf{W}_{i,j}^2$$

L1 Regularization: Learn a sparse representation (few non-zero wight elements)

$$R(\mathbf{W}) = ||\mathbf{W}||_1 = \sum_i \sum_j |\mathbf{W}_{i,j}|$$

(others regularizers are also possible)

Example:

$$\mathbf{x} = [1, 1, 1, 1]$$

$$\mathbf{W}_1 = [1, 0, 0, 0]$$

$$\mathbf{W}_2 = \left[\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right]$$

$$\mathbf{W}_1 \cdot \mathbf{x}^T = \mathbf{W}_2 \cdot \mathbf{x}^T$$

two networks will have identical loss

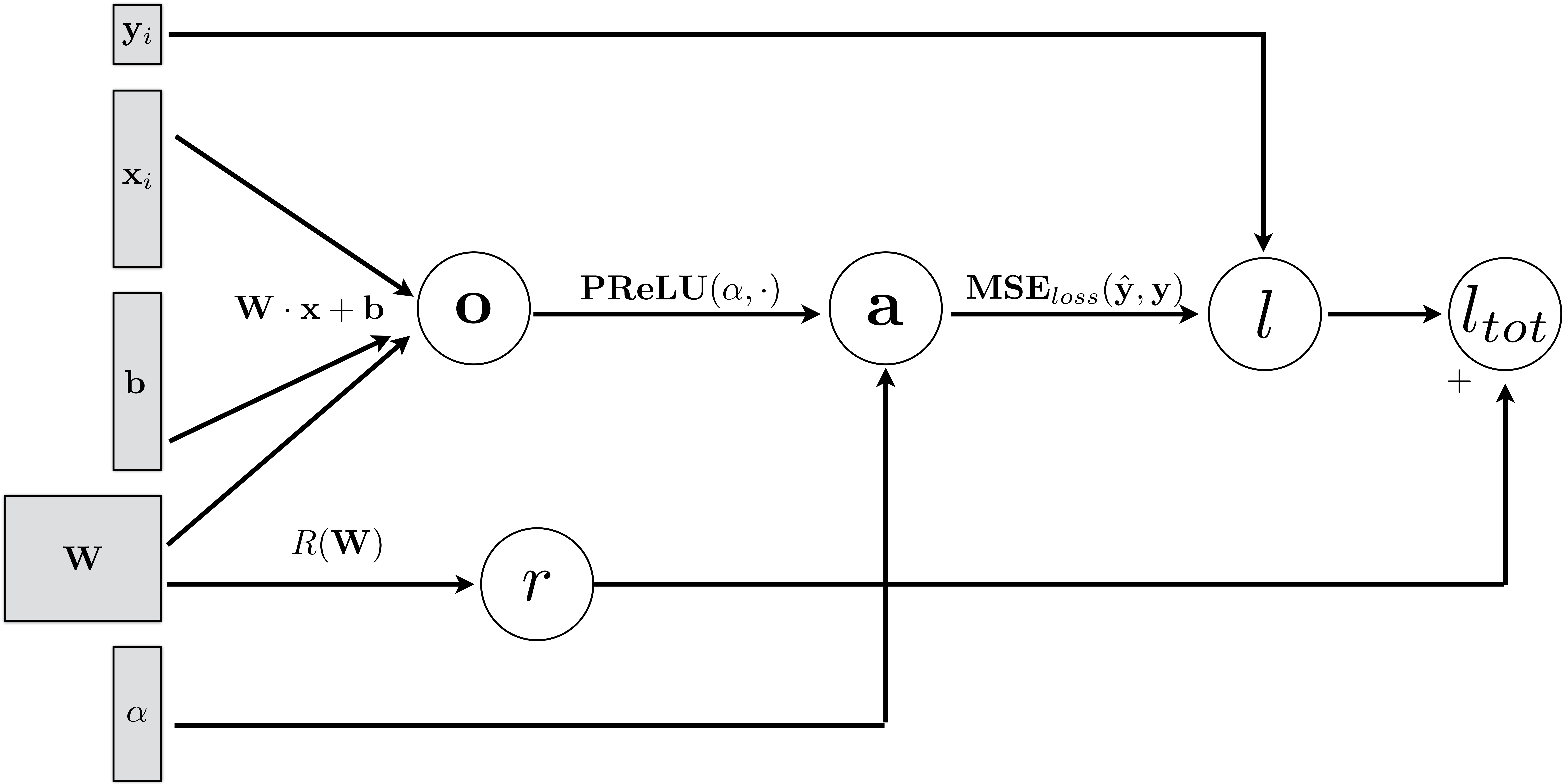
L2 Regularizer:

$$R_{L2}(\mathbf{W}_1) = 1$$
$$R_{L2}(\mathbf{W}_2) = 0.25 \leftarrow$$

L1 Regularizer:

$$R_{L1}(\mathbf{W}_1) = 1 \leftarrow$$
$$R_{L1}(\mathbf{W}_2) = 1 \leftarrow$$

Computational Graph: 1-layer with PReLU + Regularizer



Regularization: Batch Normalization

Normalize each mini-batch (using Batch Normalization layer) by subtracting empirically computed mean and dividing by variance for every dimension -> samples are approximately unit Gaussian

$$\bar{x}^{(k)} = \frac{x^{(k)} - \mathbb{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$

Benefit:

Improves learning (better gradients, higher learning rate)

[Ioffe and Szegedy, NIPS 2015]

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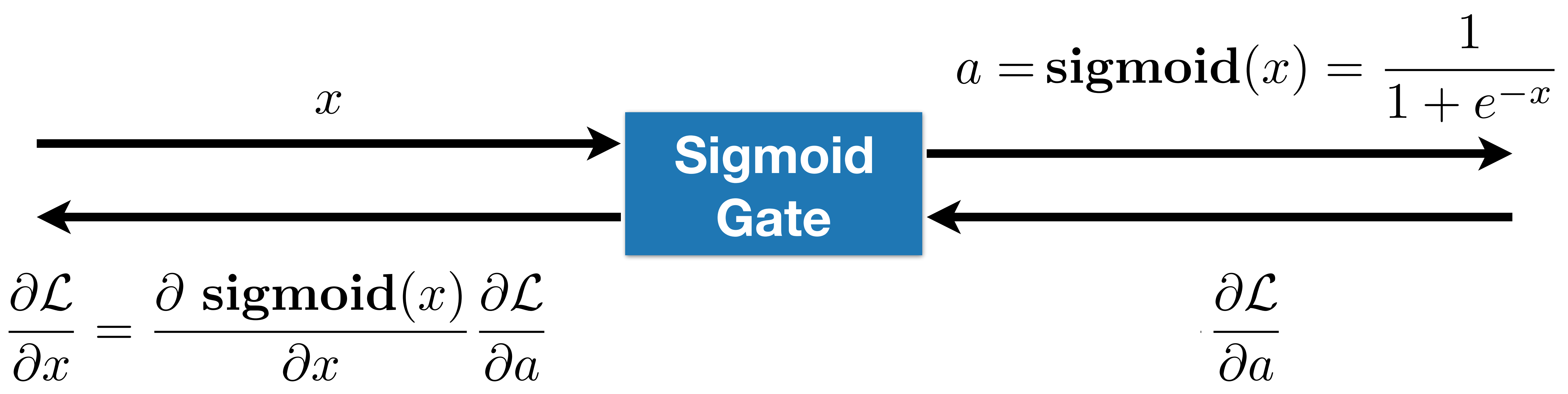
Why?

[Ioffe and Szegedy, NIPS 2015]

Activation Function: Sigmoid



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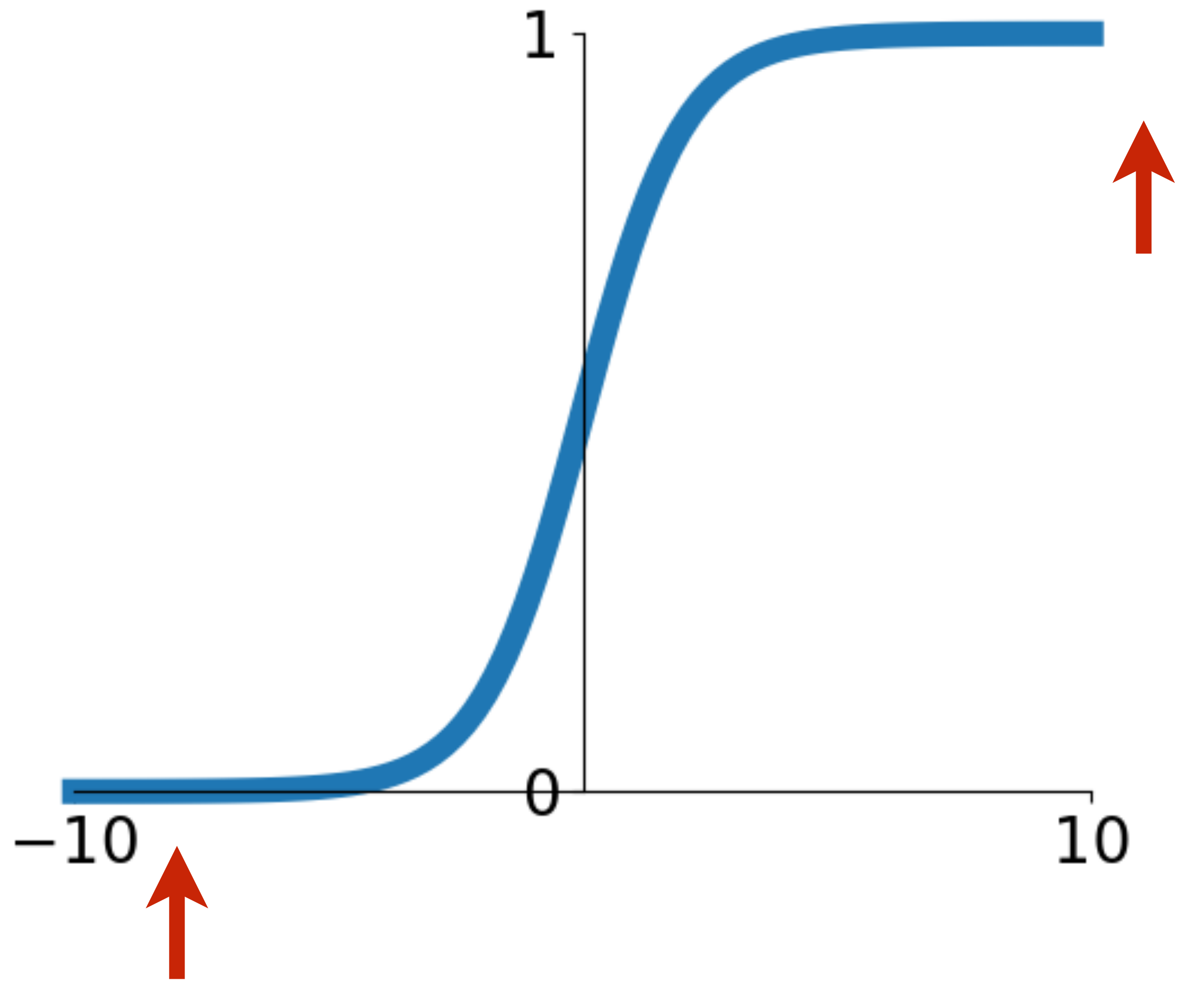

Sigmoid Activation

Cons:

- Saturated neurons **“kill” the gradients**
- Non-zero centered
- Could be expensive to compute

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Typically inserted **before** activation layer

[Ioffe and Szegedy, NIPS 2015]

Activation Function: Sigmoid vs. Tanh

$$a(x) = \mathbf{tanh}(x) = 2 \cdot \mathbf{sigmoid}(2x) - 1$$

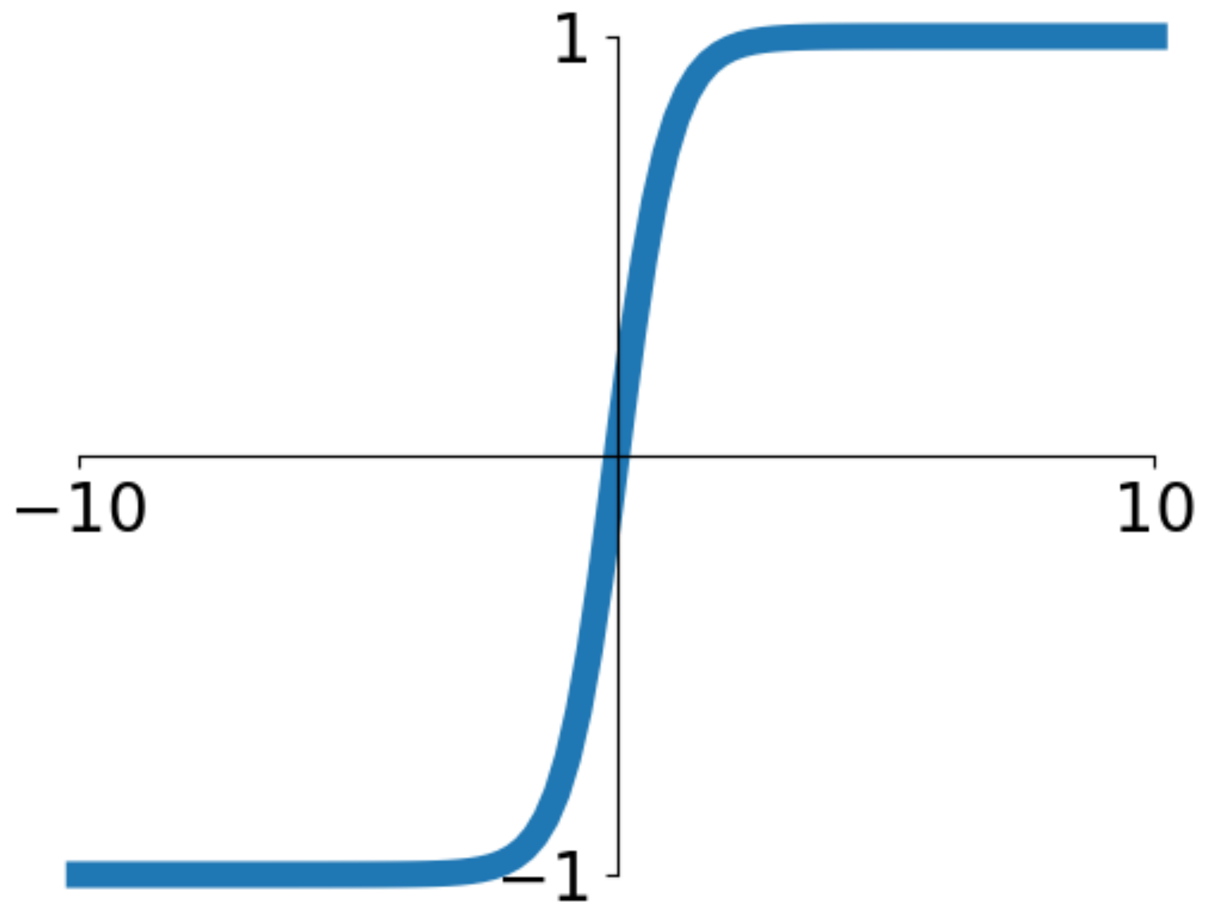
$$a(x) = \mathbf{tanh}(x) = \frac{2}{1 + e^{-2x}} - 1$$

Pros:

- Squishes everything in the range [-1,1]
- Centered around zero
- Has well defined gradient everywhere

Cons:

- Saturated neurons "kill" the gradients



Tanh Activation

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Normalize each mini-batch (using Batch Normalization layer) by subtracting empirically computed mean and dividing by variance for every dimension -> samples are approximately unit Gaussian

$$\bar{x}^{(k)} = \frac{x^{(k)} - \mathbb{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$

In practice, also learn how to scale and offset:

$$y^{(k)} = \gamma^{(k)}\bar{x}^{(k)} + \beta^{(k)}$$

BN layer parameters

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Improves learning (better gradients, higher learning rate)

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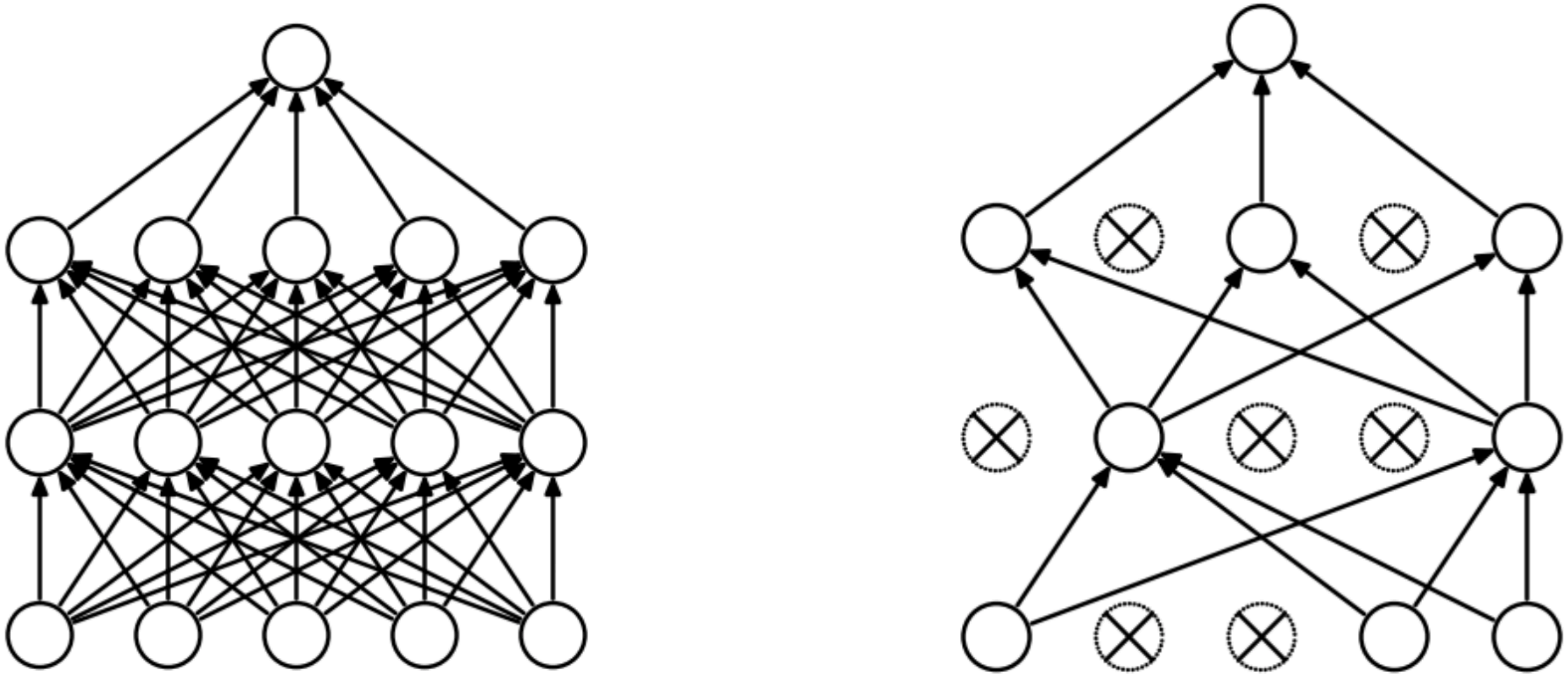
Regularization: Dropout

Randomly **set some neurons to zero** in the forward pass, with probability proportional to `dropout rate` (between 0 to 1)



Standar Neural Network

After Applying **Dropout**

[Srivastava et al, JMLR 2014]

* adopted from slides of **CS231n at Stanford**

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1. Compute output of the linear/fc layer $\mathbf{o}_i = \mathbf{W}_i \cdot \mathbf{x} + \mathbf{b}_i$

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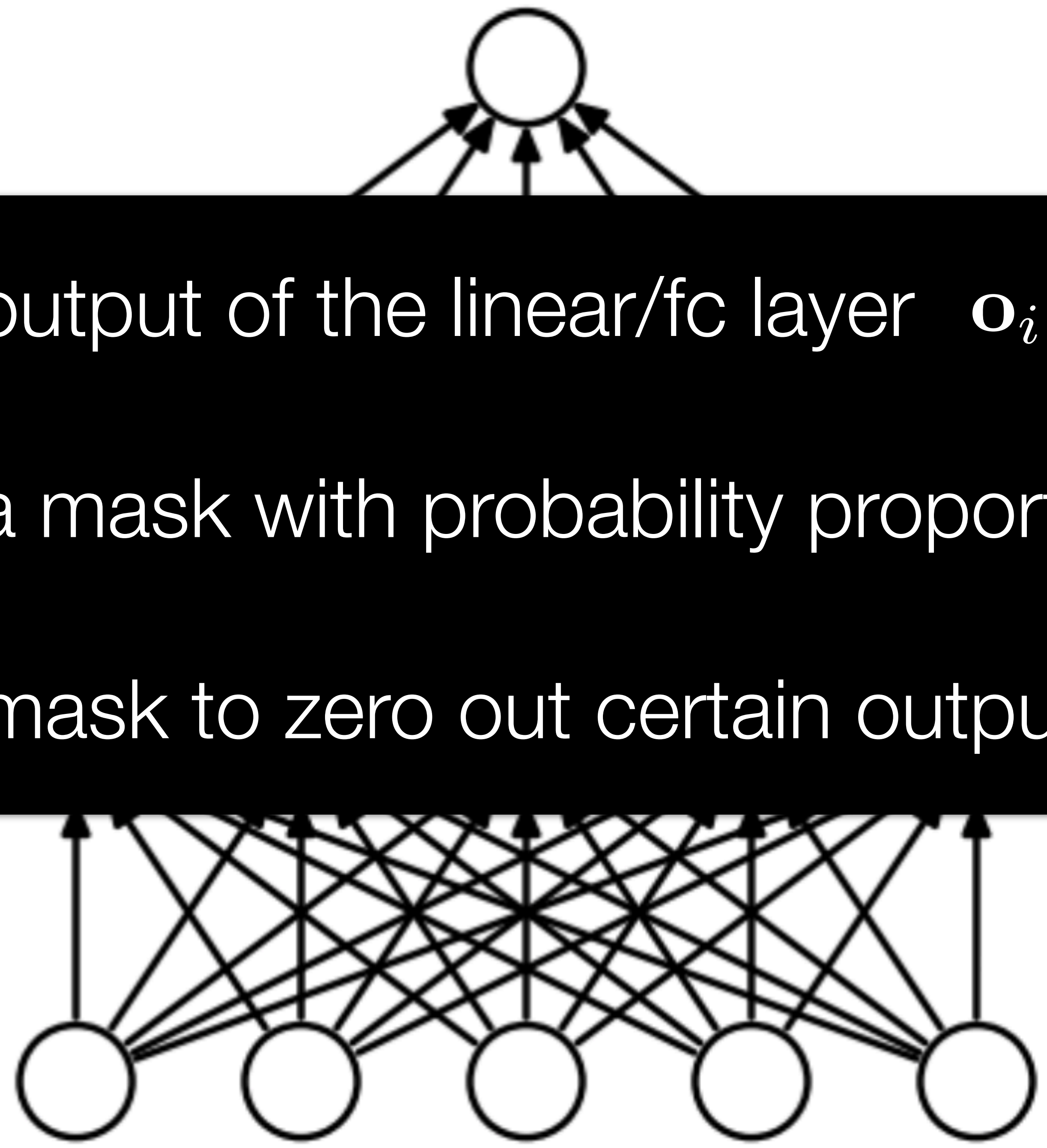
3. Apply the mask to zero out certain outputs $\mathbf{o}_i = \mathbf{o}_i \odot \mathbf{m}_i$



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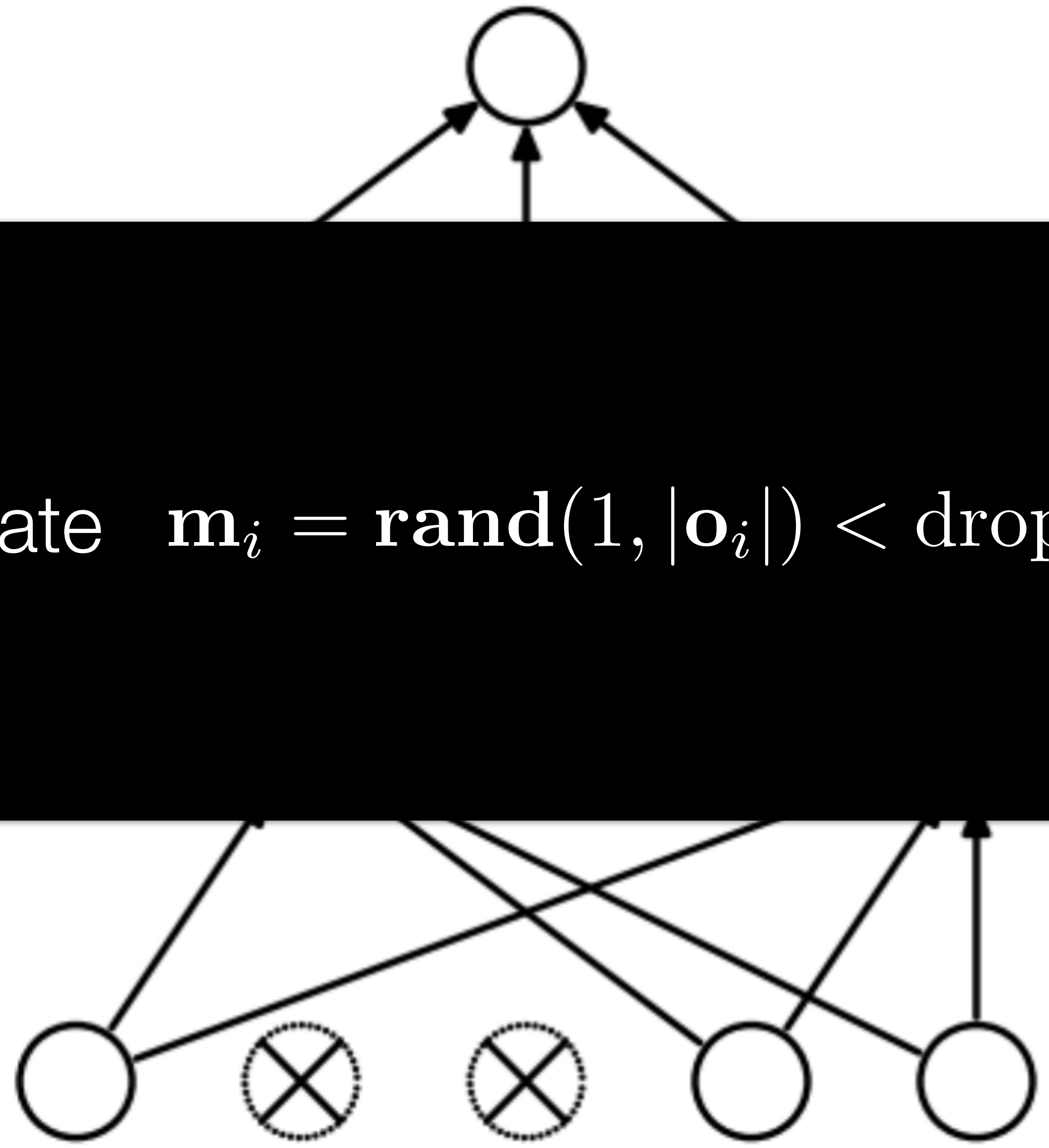

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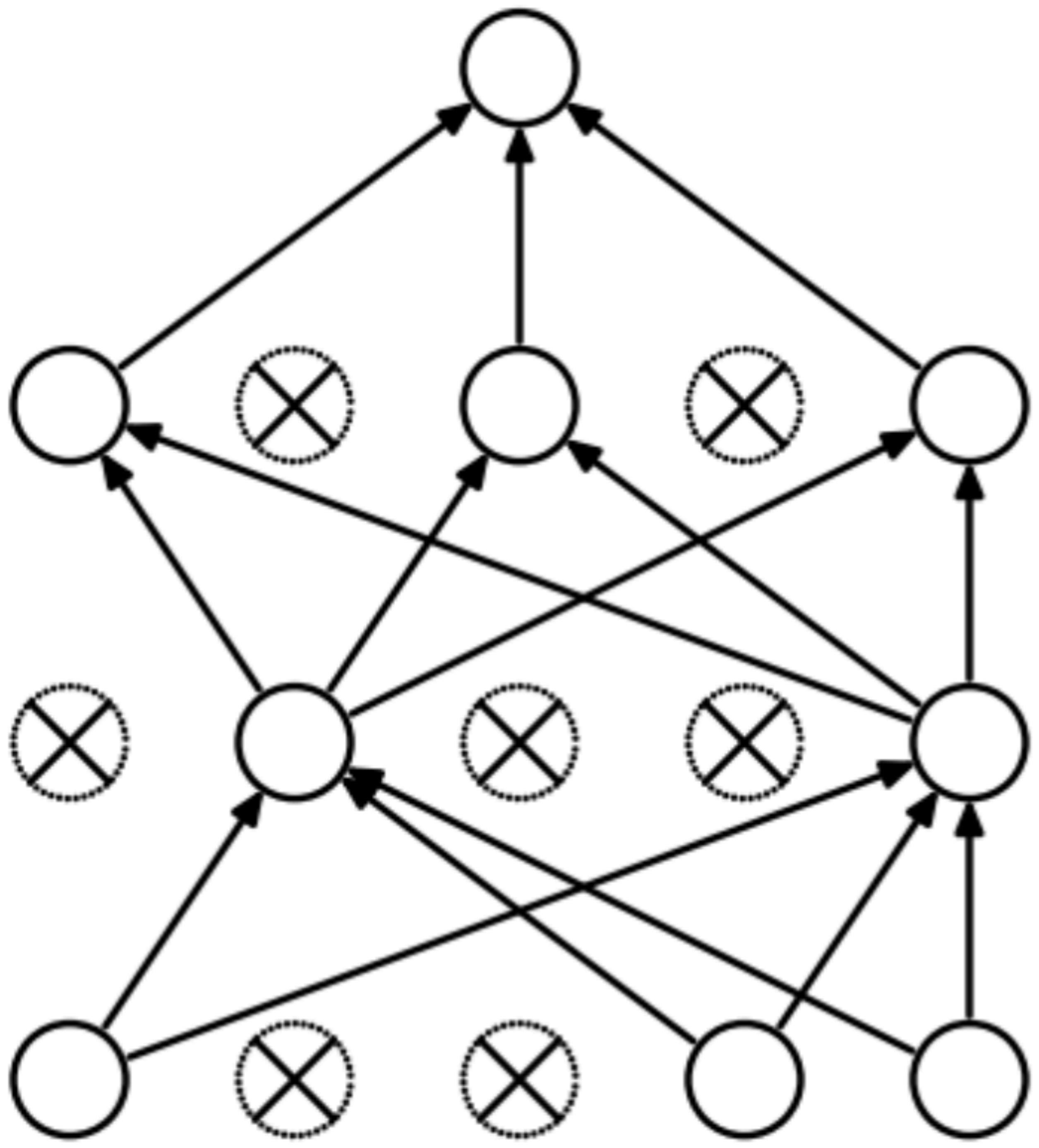

After Applying **Dropout**

Why is this a good idea?

[Srivastava et al, JMLR 2014]

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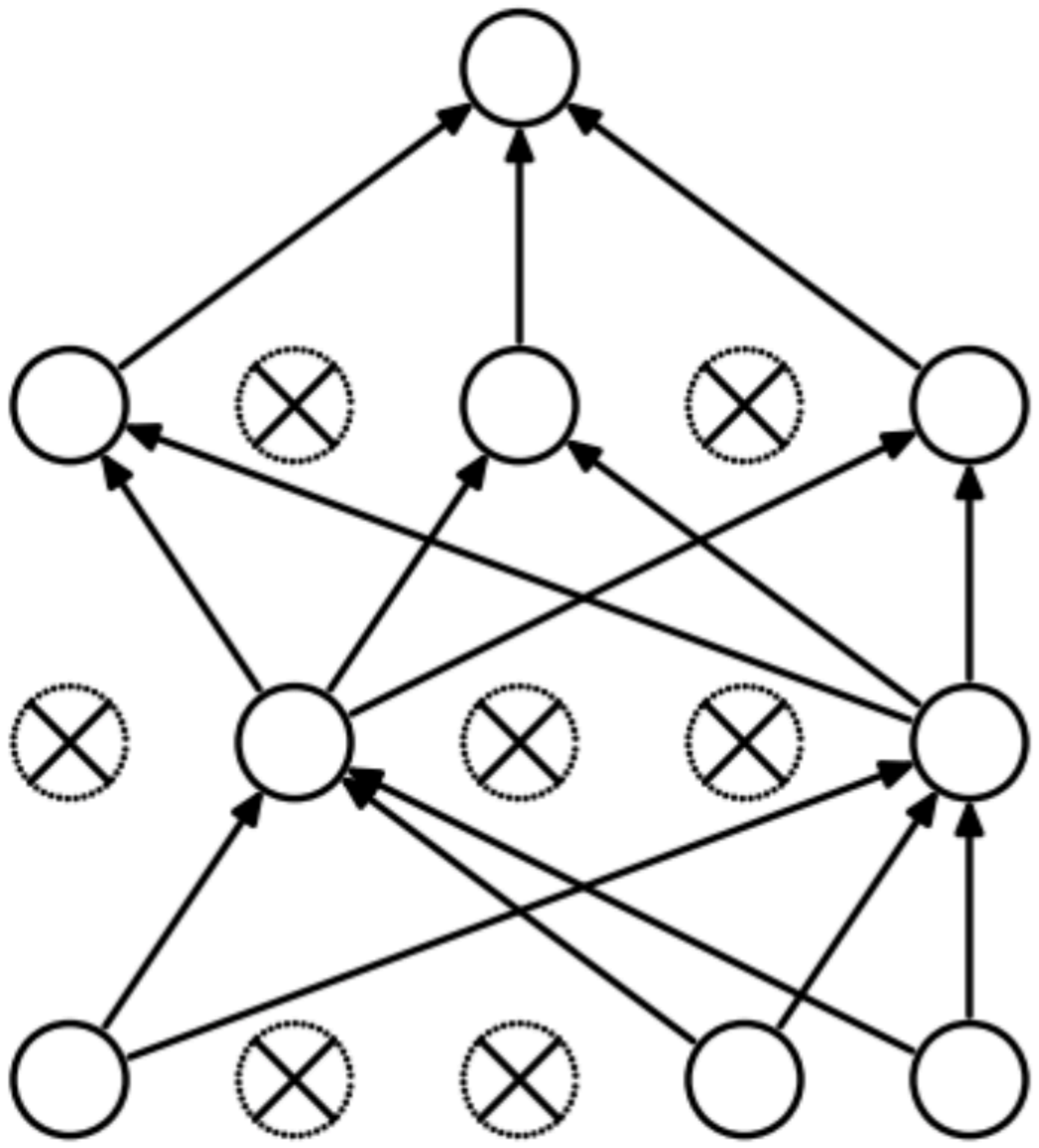

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Why is this a good idea?

Dropout is training an **ensemble of models** that share parameters

Each binary mask (generated in the forward pass) is one model that is trained on (approximately) one data point

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Regularization: Dropout (at test time)

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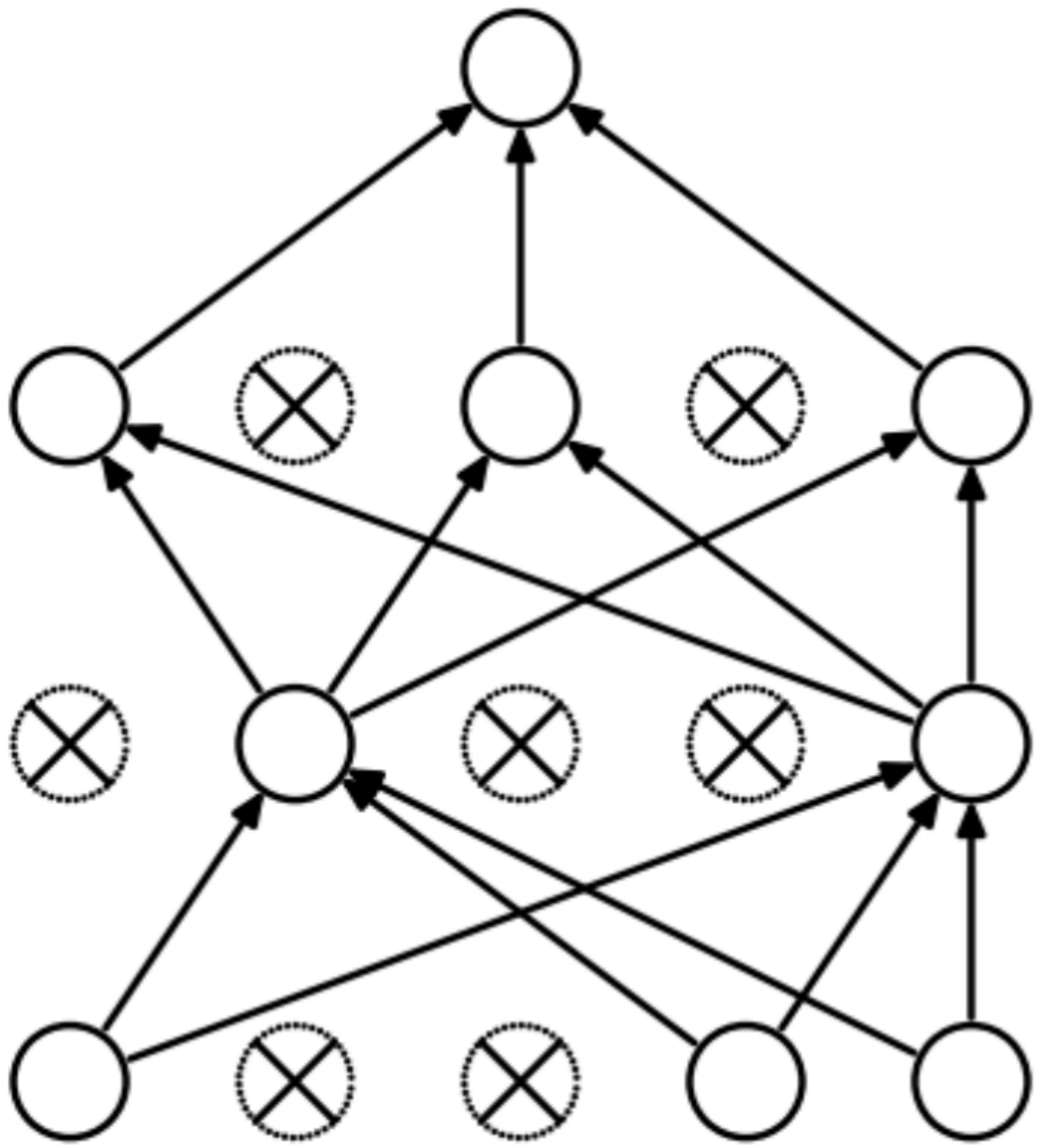

After Applying **Dropout**

At test time, **integrate out all the models** in the ensemble

Monte Carlo approximation: many forward passes with different masks and average all predictions

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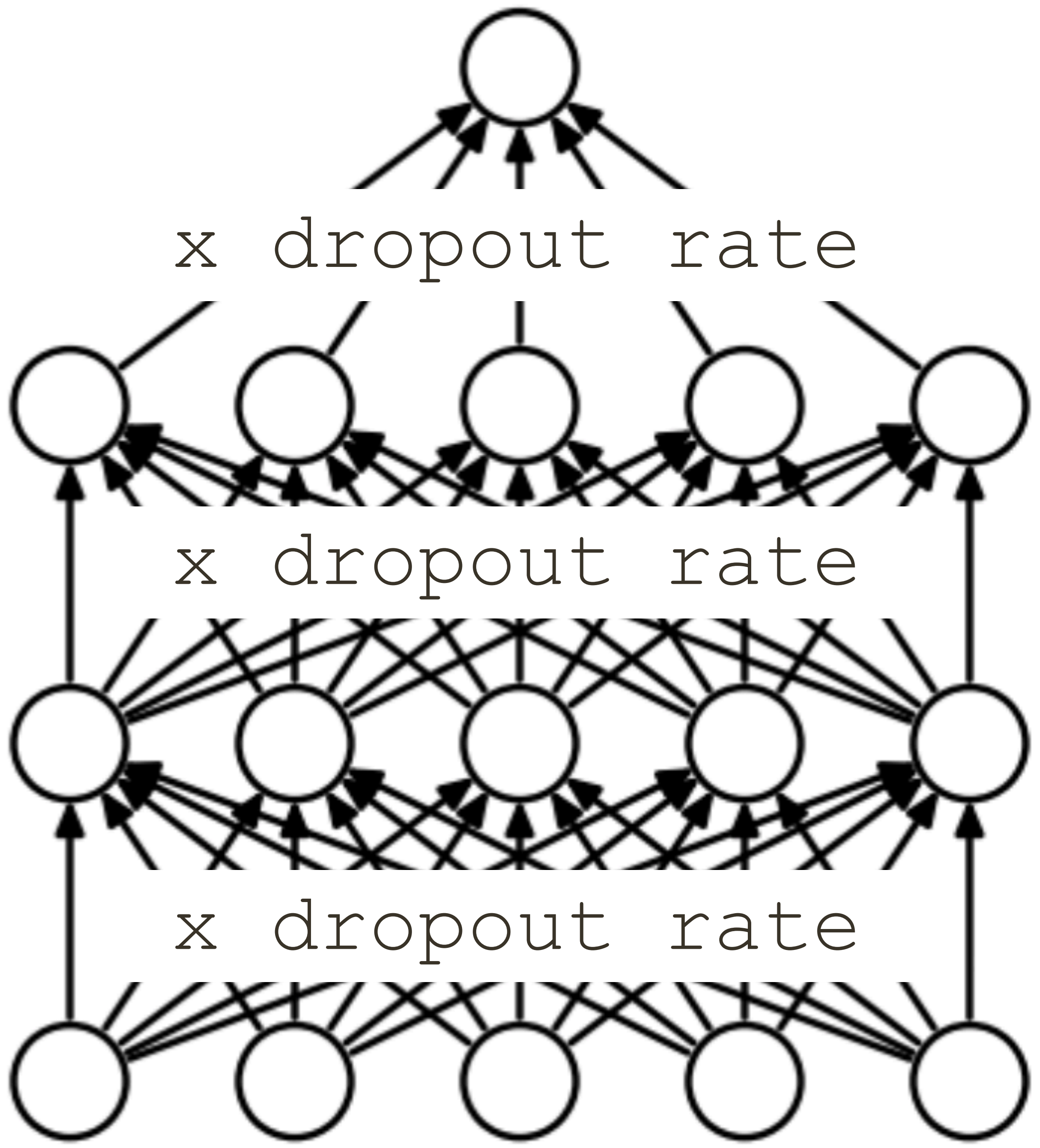

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Monte Carlo approximation: many forward passes with different masks and average all predictions

Equivalent to forward pass with all connections on and **scaling of the outputs** by dropout rate

[Srivastava et al, JMLR 2014]

For derivation see Lecture 6 of **CS231n at Stanford**

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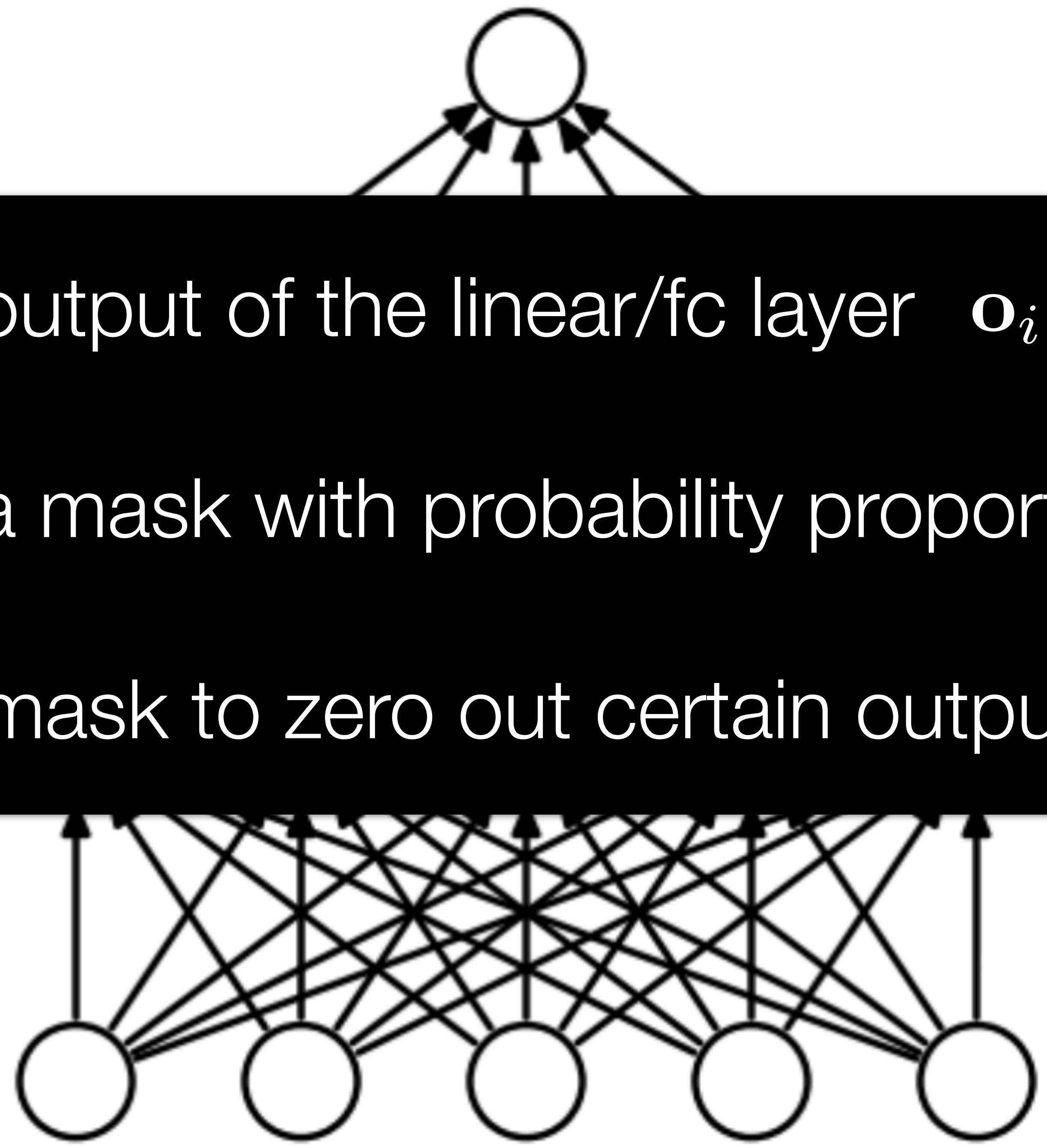

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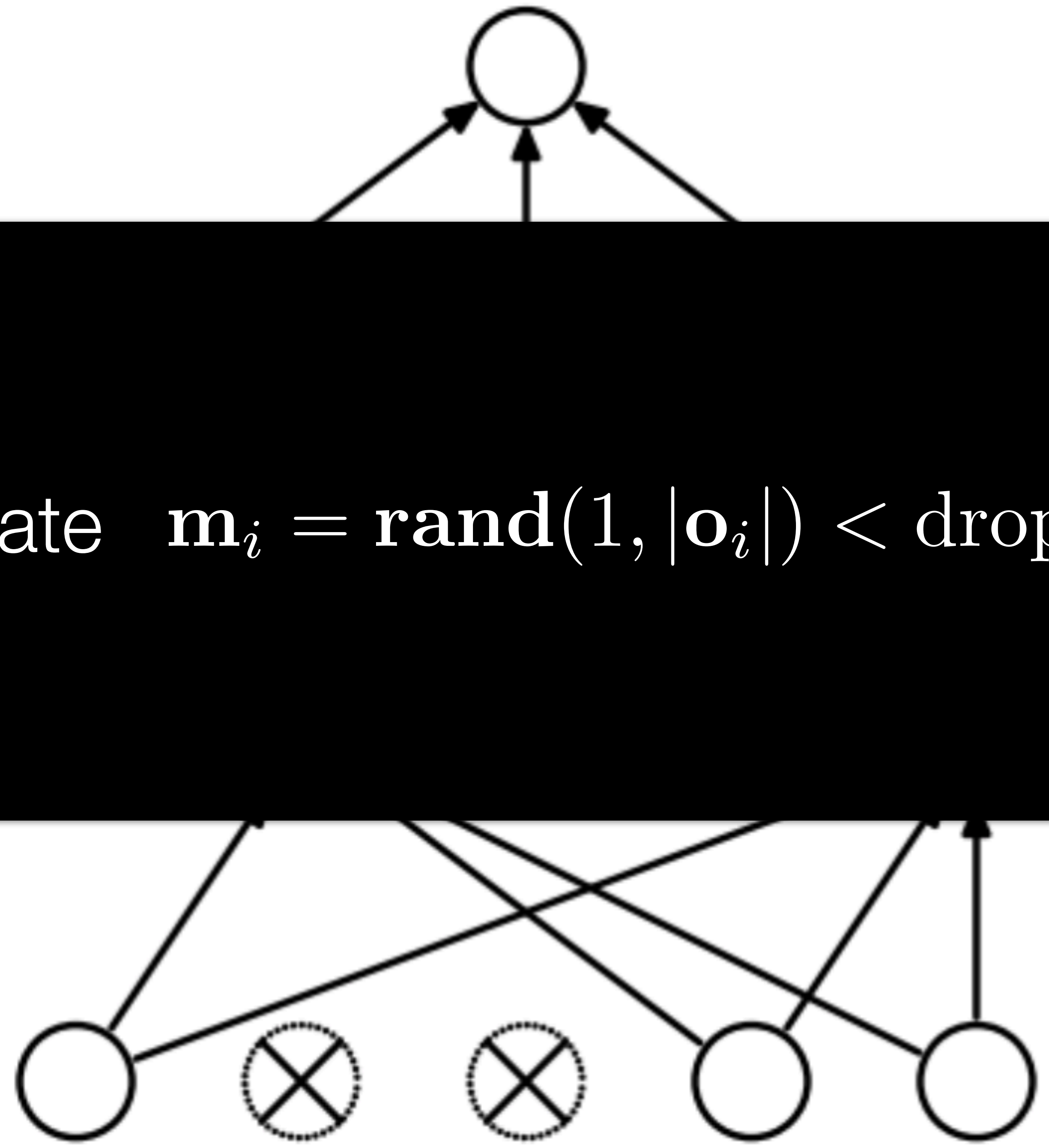

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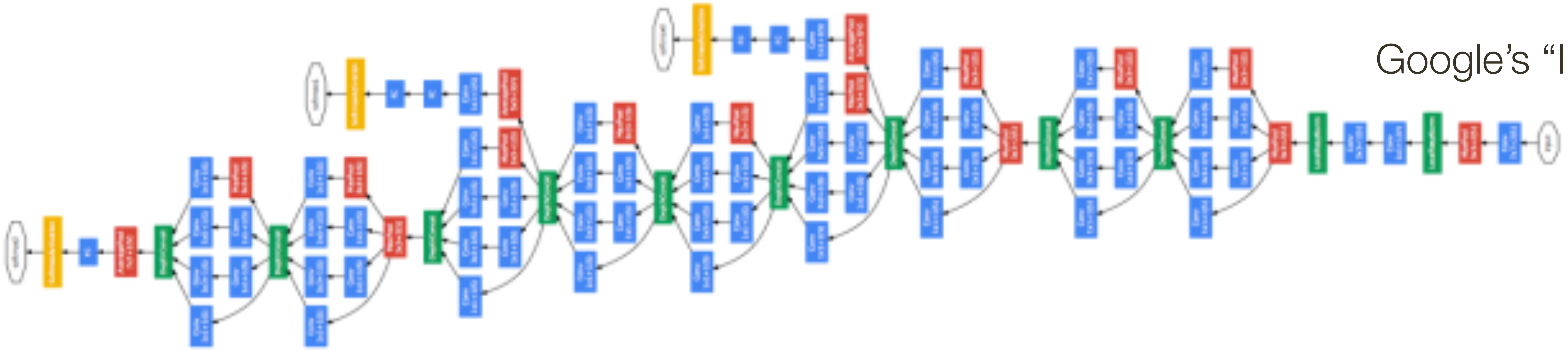
Deep Learning **Terminology**



Google's "Inception" network

- **Network structure:** number and types of layers, forms of activation functions, dimensionality of each layer and connections (defines computational graph)

generally kept fixed, requires some knowledge of the problem and NN to sensibly set

deeper = better

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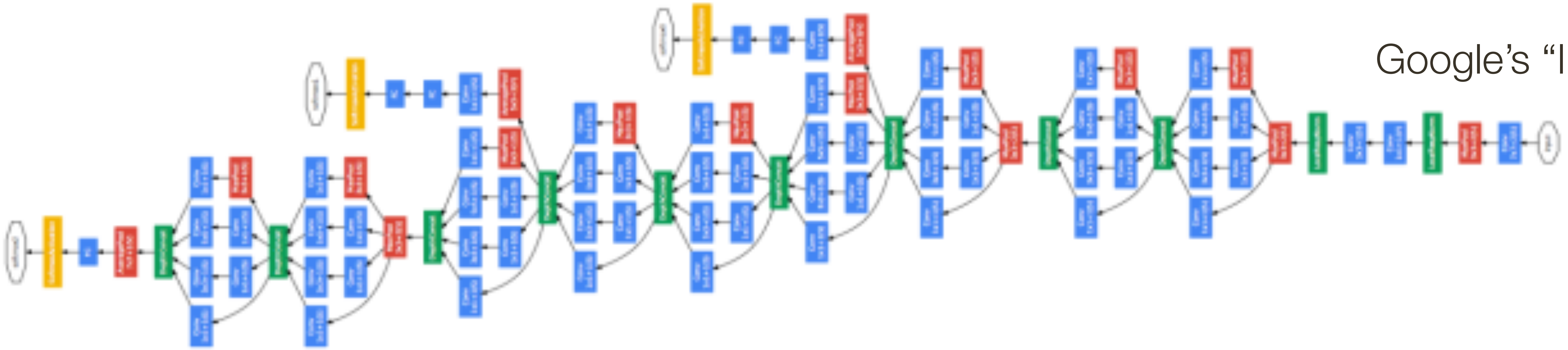

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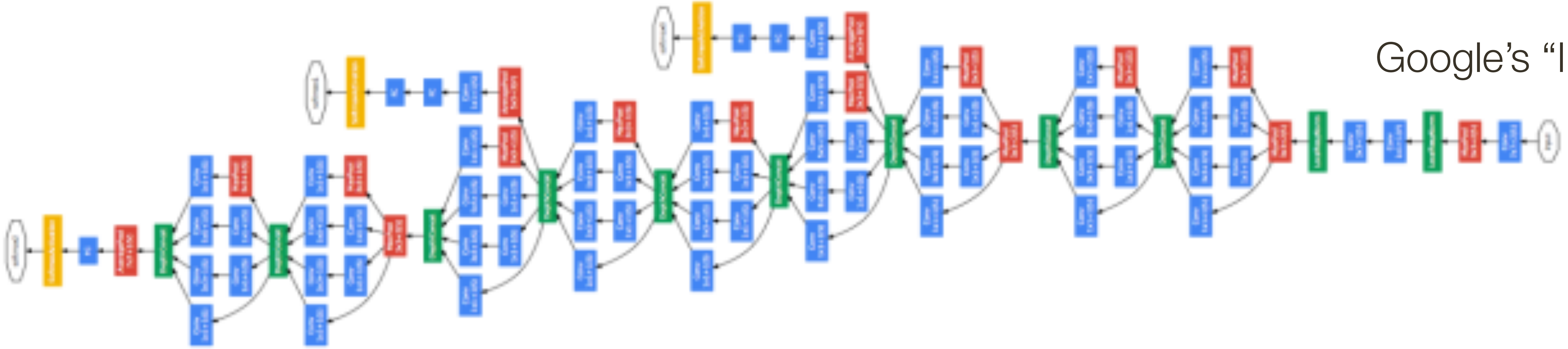

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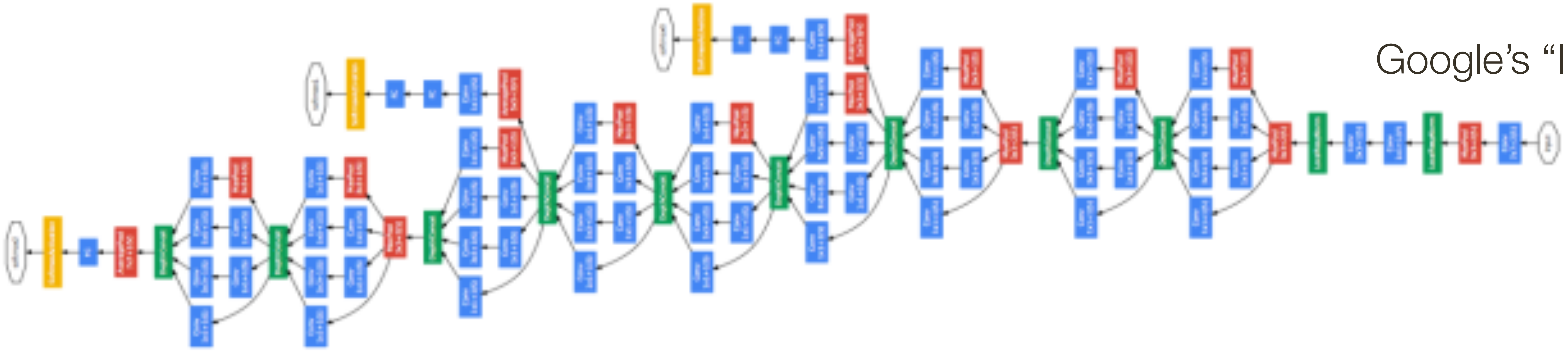

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- **Hyper-parameters:** parameters, including for optimization, that are not optimized directly as part of training (*e.g.*, `learning rate`, `batch size`, `drop-out rate`)

grid search

Loss Functions …

This is where all the **fun** is … we will only look a most common ones

Multivariate **Regression**

Input: feature vector $\mathbf{x} \in \mathbb{R}^n$

Output: output vector $\mathbf{y} \in \mathbb{R}^m$

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with **sigmoid** activations: $\mathbf{0} \leq f(\mathbf{x}; \Theta) \leq \mathbf{1}$

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Loss:

$$\mathcal{L}(\mathbf{y}, \hat{\mathbf{y}}) = ||\mathbf{y} - \hat{\mathbf{y}}||^2$$

Binary Classification (Bernoulli)

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Neural Network (output): threshold hidden output (which is a sigmoid)

$$\hat{y} = 1[f(\mathbf{x}; \Theta) > 0.5]$$

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Problem: Not differentiable, probabilistic interpretation maybe desirable

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Neural Network (output): interpret sigmoid output as probability

$$p(y = 1) = f(\mathbf{x}; \Theta)$$

can interpret the score as the log-odds of $y = 1$ (a.k.a. the **logits**)

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Loss: similarity between two distributions

Binary Classification (Bernoulli)

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We can measure similarity between distribution and using cross-entropy

$$H(p, q) = -\mathbb{E}_{x \sim p}[\log q(x)]$$

For discrete distributions this ends up being:

$$H(p, q) = -\sum_x p(x) \log q(x)$$

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Loss: $\mathcal{L}(y, \hat{y}) = -y \log[f(\mathbf{x}; \Theta)] - (1 - y) \log[1 - f(\mathbf{x}; \Theta)]$

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Minimizing this **loss** is the same as maximizing **log likelihood** of data

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with **ReLU** activations: $\mathbf{0} \leq f(\mathbf{x}; \Theta)$

Neural Network (output): linear layer with one neuron and sigmoid activation

Multiclass Classification (e.g, ImageNet)

Input: feature vector $\mathbf{x} \in \mathbb{R}^n$

Output: muticlass label $\mathbf{y} \in \{0, 1\}^m$

(**one-hot** encoding)

Multiclass Classification (e.g, ImageNet)

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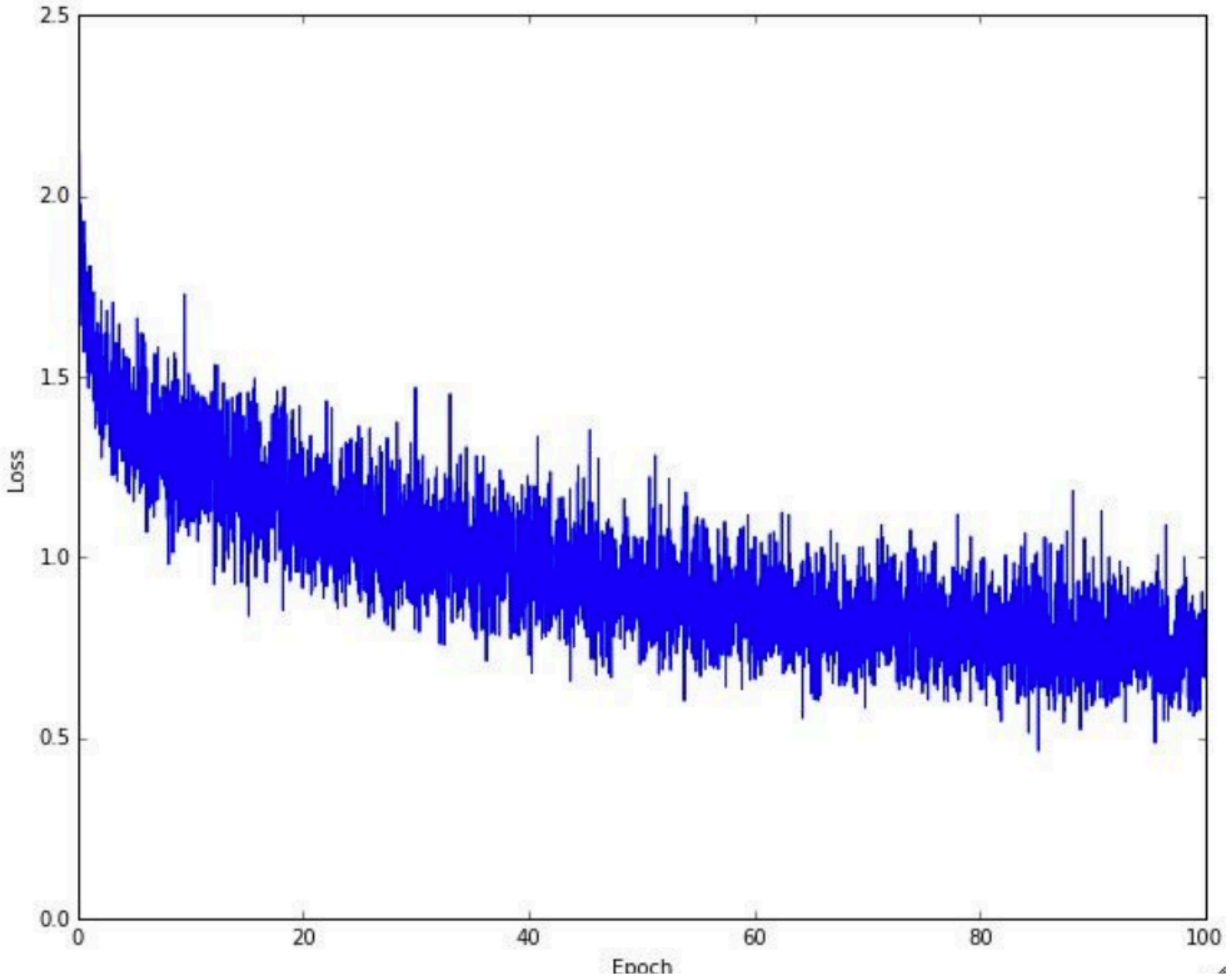
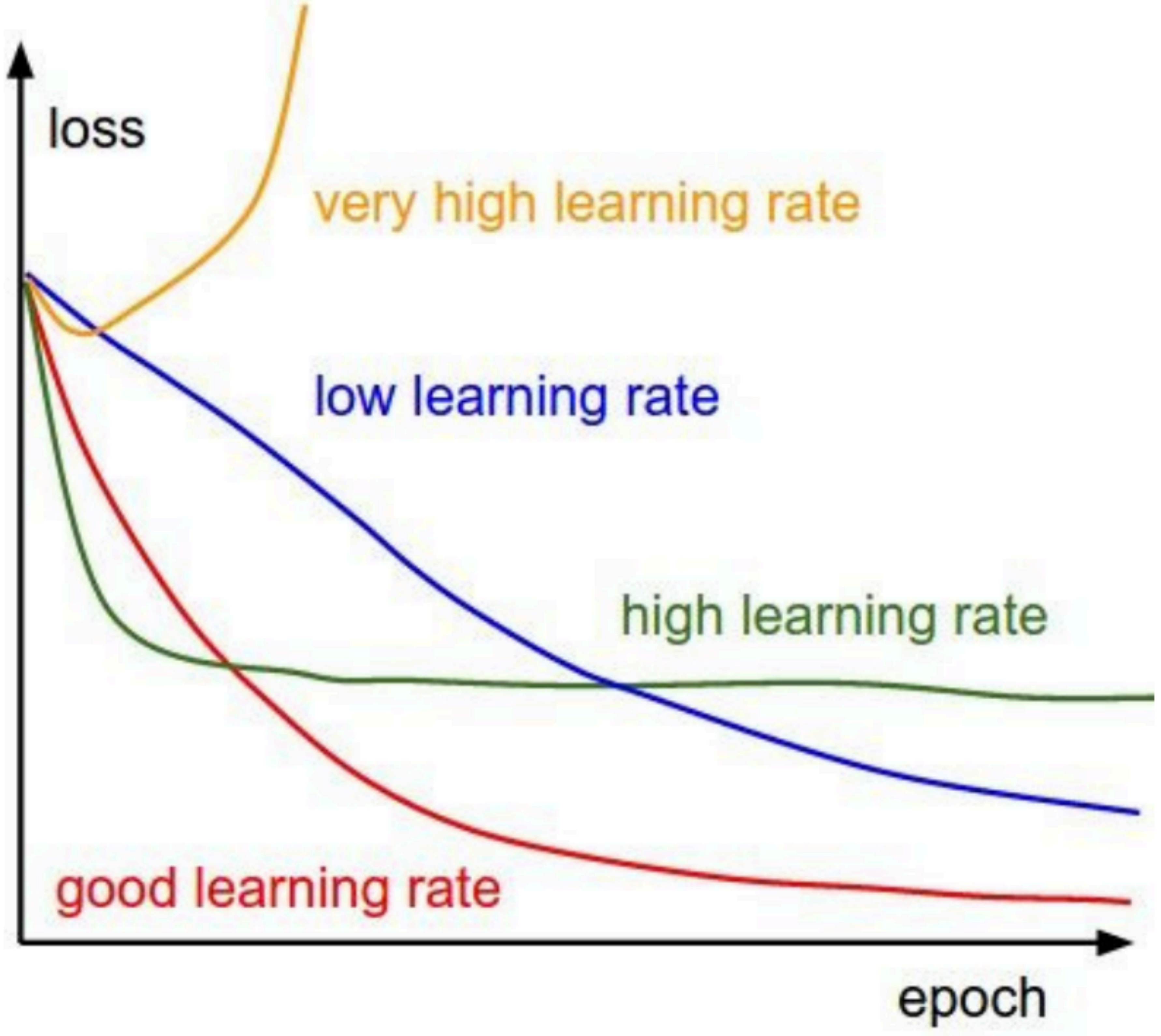
Special case for multi-class single label

Monitoring Learning: Visualizing the (training) loss





* slide from Li, Karpathy, Johnson's **CS231n at Stanford**

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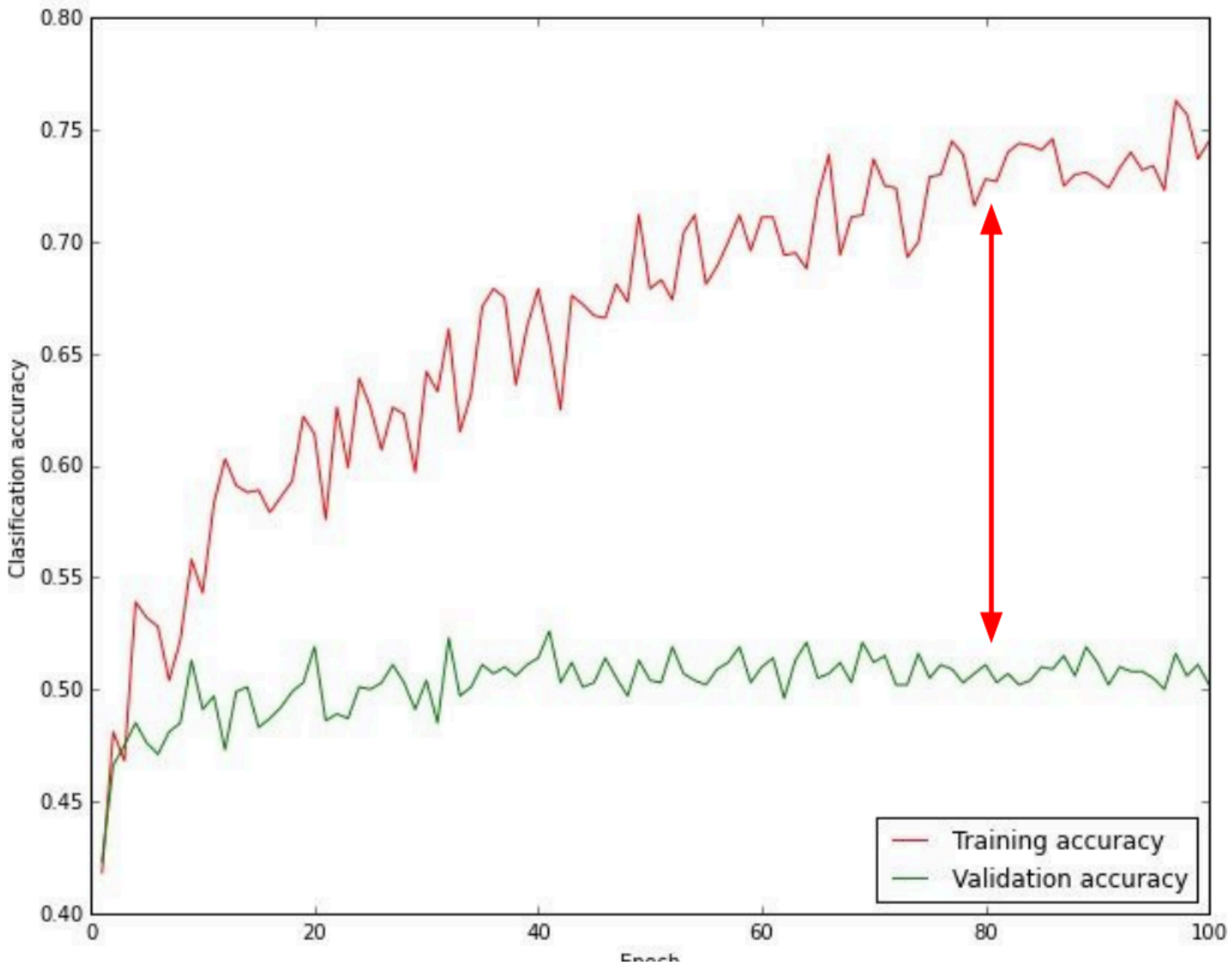

Big gap = overfitting

Solution: increase regularization

No gap = undercutting

Solution: increase model capacity

Small gap = **ideal**

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