

# Topics in AI (CPSC 532S): Multimodal Learning with Vision, Language and Sound

Lecture 18: Deep Reinforcement Learning

# Types of Learning

#### Supervised training

- Learning from the teacher
- Training data includes desired output

#### **Unsupervised** training

Training data does not include desired output

#### Reinforcement learning

Learning to act under evaluative feedback (rewards)

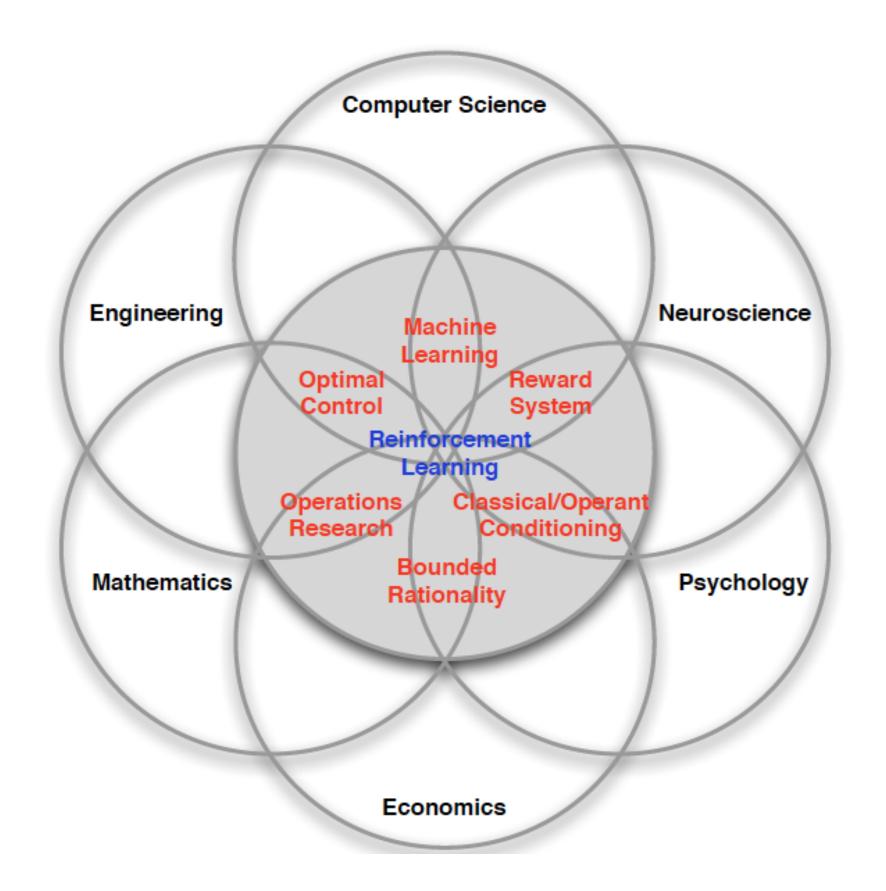
### What is Reinforcement Learning

**Agent-oriented learning** — learning by interacting with an environment to achieve a goal

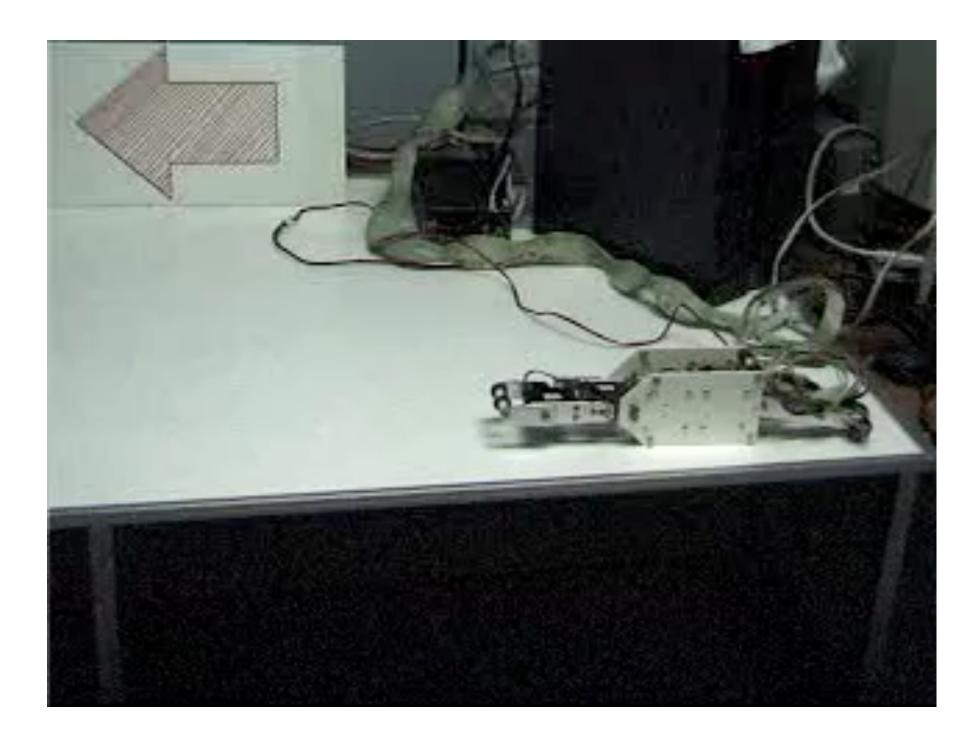
More realizing and ambitious than other kinds of machine learning

Learning by trial and error, with only delayed evaluative feedback (reward)

- The kind go machine learning most like natural learning
- Learning that can tell for itself when it is right or wrong



# Example: Hajime Kimura's RL Robot



Before

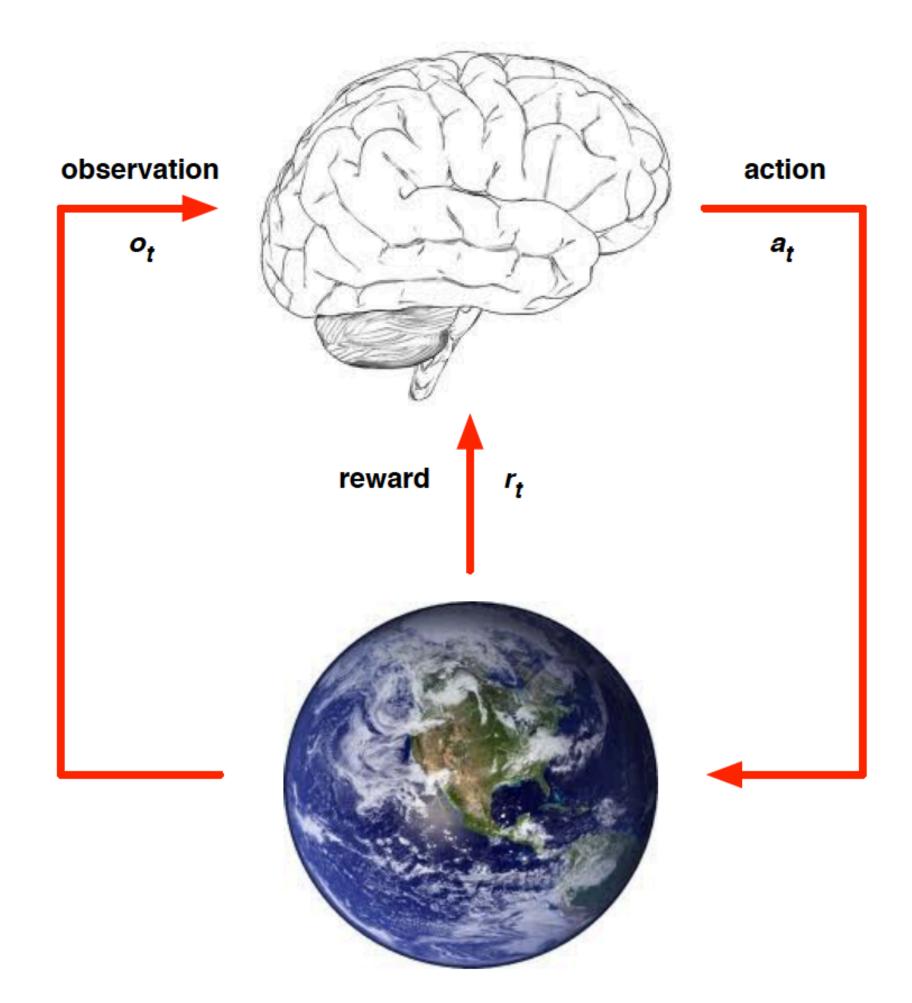


After

### Challenges of RL

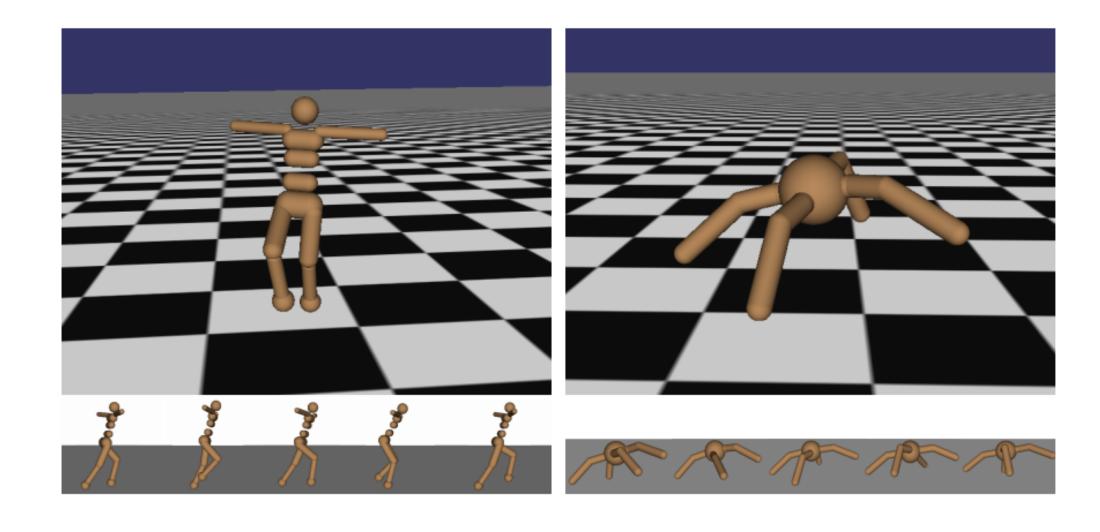
- Evaluative feedback (reward)
- Sequentiality, delayed consequences
- Need for trial and error, to explore as well as exploit
- Non-stationarity
- The fleeting nature of time and online data

### How does RL work?



- At each step *t* the agent:
  - Executes action a<sub>t</sub>
  - $\triangleright$  Receives observation  $o_t$
  - $\triangleright$  Receives scalar reward  $r_t$
- ► The environment:
  - $\triangleright$  Receives action  $a_t$
  - ightharpoonup Emits observation  $o_{t+1}$
  - ightharpoonup Emits scalar reward  $r_{t+1}$

### Robot Locomotion



Objective: Make the robot move forward

State: Angle and position of the joints

Action: Torques applied on joints

Reward: 1 at each time step upright +

forward movement

<sup>\*</sup> slide from Fei-Dei Li, Justin Johnson, Serena Yeung, cs231n Stanford

### Atari Games



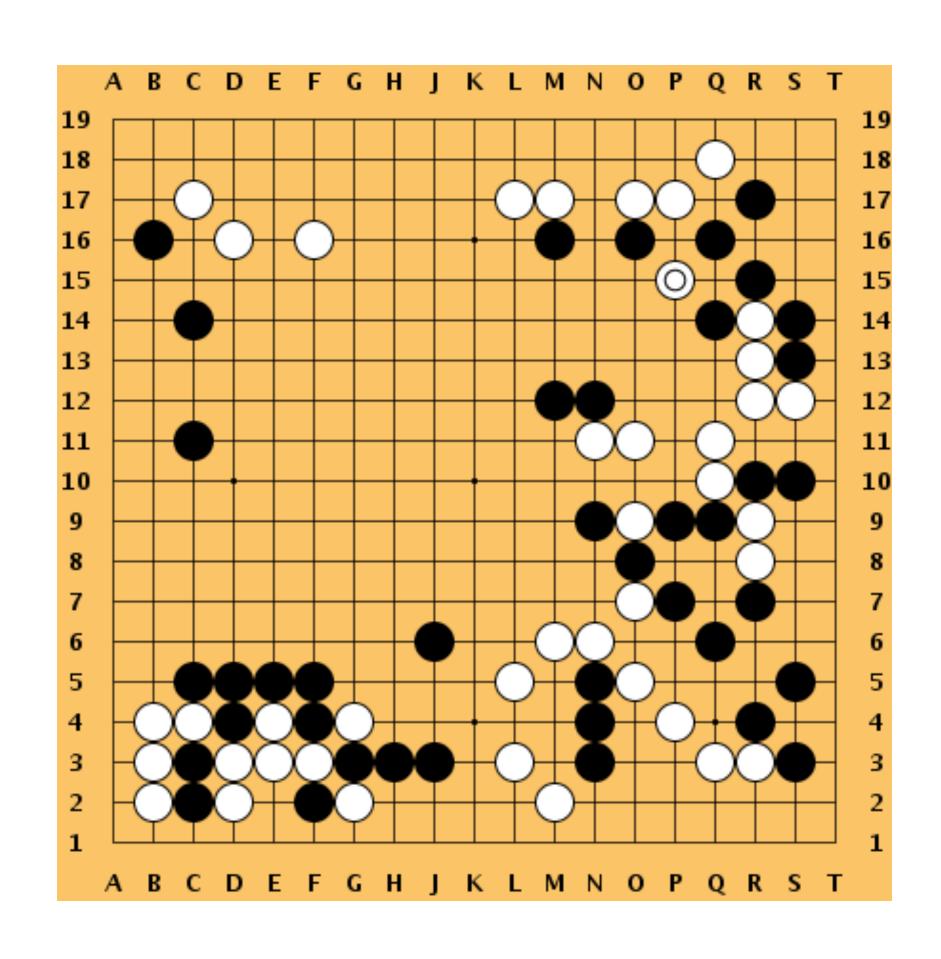
Objective: Complete the game with the highest score

State: Raw pixel inputs of the game state

Action: Game controls e.g. Left, Right, Up, Down

Reward: Score increase/decrease at each time step

# Go Game (AlphaGo)



Objective: Win the game!

State: Position of all pieces

Action: Where to put the next piece down

Reward: 1 if win at the end of the game, 0

otherwise

### Markov Decision Processes

— Mathematical formulation of the RL problem

#### Defined by:

S: set of possible states

 $\mathcal{A}$ : set of possible actions

R: distribution of reward given (state, action) pair

r : transition probability i.e. distribution over next state given (state, action) pair

 $\gamma$ : discount factor

### Markov Decision Processes

Mathematical formulation of the RL problem

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- Life is trajectory:  $...S_t, A_t, R_{t+1}, S_{t+1}, A_{t+1}, R_{t+2}, S_{t+2}, ...$ 

### Markov Decision Processes

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- Life is trajectory: ...  $S_t$ ,  $A_t$ ,  $R_{t+1}$ ,  $S_{t+1}$ ,  $A_{t+1}$ ,  $R_{t+2}$ ,  $S_{t+2}$ , ...
- Markov property: Current state completely characterizes the state of the world

$$p(r, s'|s, a) = Prob[R_{t+1} = r, S_{t+1} = s' | S_t = s, A_t = a]$$

# Components of the RL Agent

#### **Policy**

— How does the agent behave?

#### **Value Function**

— How good is each state and/or action pair?

#### Model

Agent's representation of the environment

### Policy

- The policy is how the agent acts
- Formally, map from states to actions:

Deterministic policy:  $a = \pi(s)$ Stochastic policy:  $\pi(a|s) = \mathbb{P}[A_t = a|S_t = s]$  e.g. State Action  $A \longrightarrow 2$   $B \longrightarrow 1$ 

What is a good policy?

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Maximizes current reward? Sum of all future rewards?

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Discounted future rewards!

What is a good policy?

Maximizes current reward? Sum of all future rewards?

#### Discounted future rewards!

Formally: 
$$\pi^* = \arg\max_{\pi} \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | \pi\right]$$

with 
$$s_0 \sim p(s_0), a_t \sim \pi(\cdot|s_t), s_{t+1} \sim p(\cdot|s_t, a_t)$$

# Components of the RL Agent



— How does the agent behave?

#### **Value Function**

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### Value Function

A value function is a prediction of future reward

"State Value Function" or simply "Value Function"

- How good is a state?
- Am I screwed? Am I winning this game?

"Action Value Function" or **Q-function** 

- How good is a state action-pair?
- Should I do this now?

### Value Function and Q-value Function

Following a policy produces sample trajectories (or paths) s<sub>0</sub>, a<sub>0</sub>, r<sub>0</sub>, s<sub>1</sub>, a<sub>1</sub>, r<sub>1</sub>, ...

— The **value function** (how good is the state) at state s, is the expected cumulative reward from state s (and following the policy thereafter):

$$V^{\pi}(s) = \mathbb{E}\left[\sum_{t \geq 0} \gamma^t r_t | s_0 = s, \pi
ight]$$

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— The **Q-value function** (how good is a state-action pair) at state s and action a, is the expected cumulative reward from taking action a in state s (and following the policy thereafter):

$$Q^{\pi}(s,a) = \mathbb{E}\left[\sum_{t \geq 0} \gamma^t r_t | s_0 = s, a_0 = a, \pi
ight]$$

# Components of the RL Agent



— How does the agent behave?

# ✓ Value Function

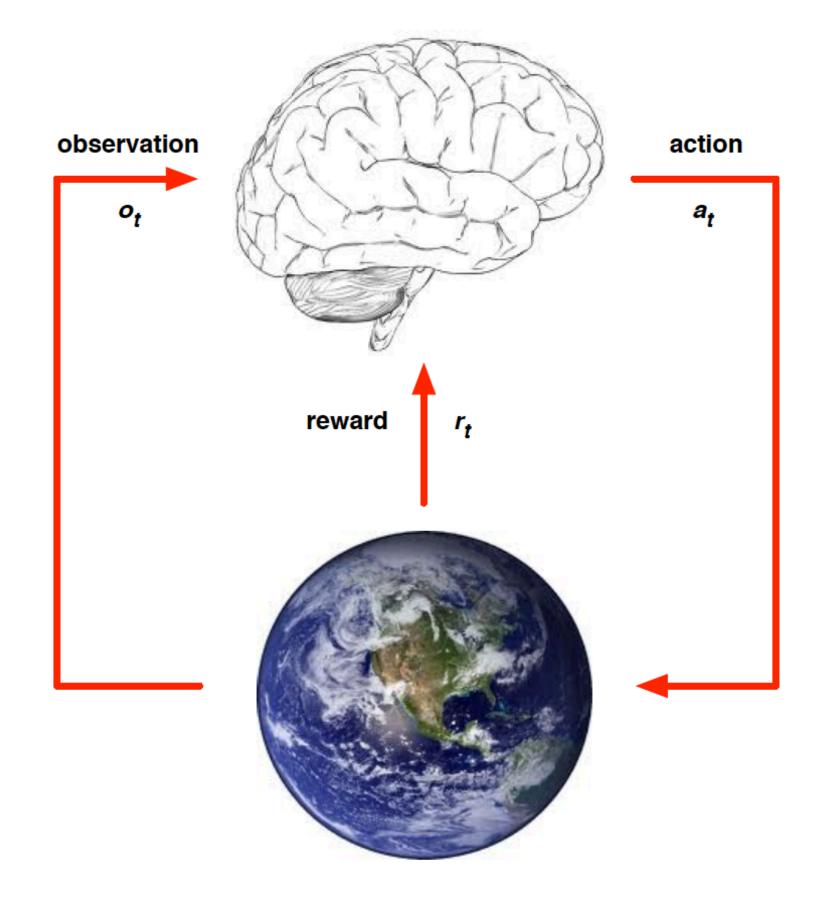
— How good is each state and/or action pair?

#### Model

Agent's representation of the environment

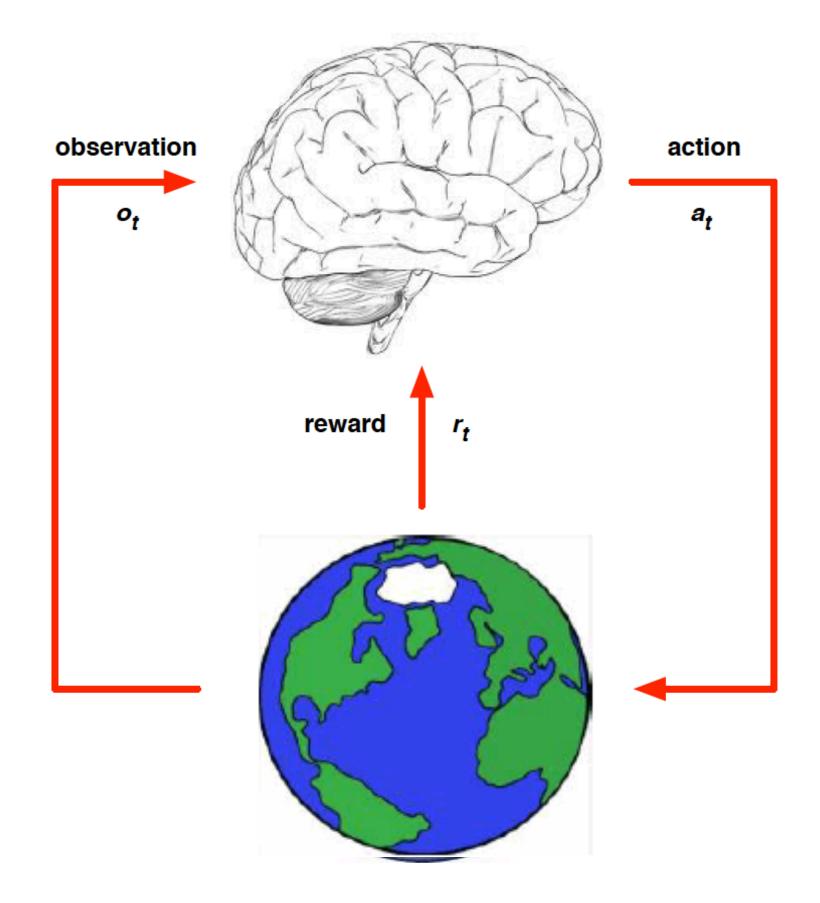
### Model

Model predicts what the world will do next



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# Components of the RL Agent



— How does the agent behave?

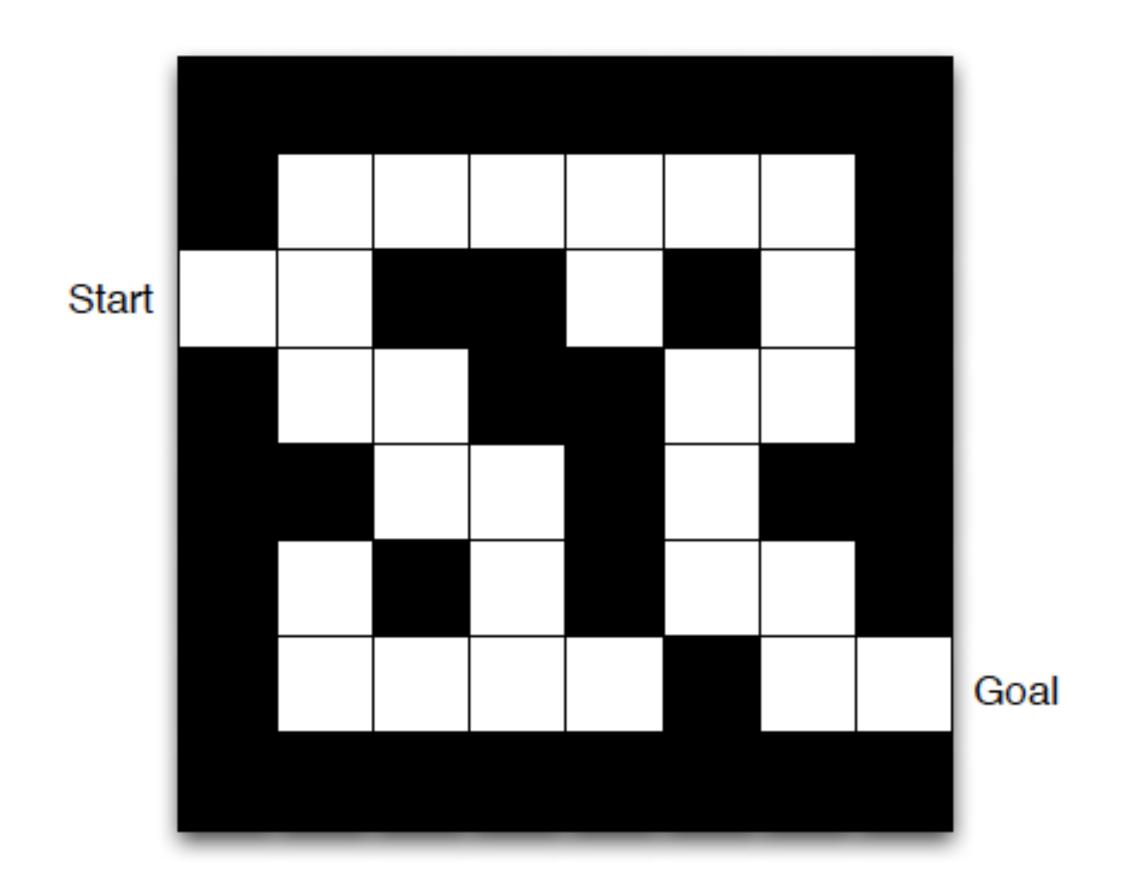
# **✓ Value Function**

— How good is each state and/or action pair?

# ✓ Mode

Agent's representation of the environment

### Maze Example

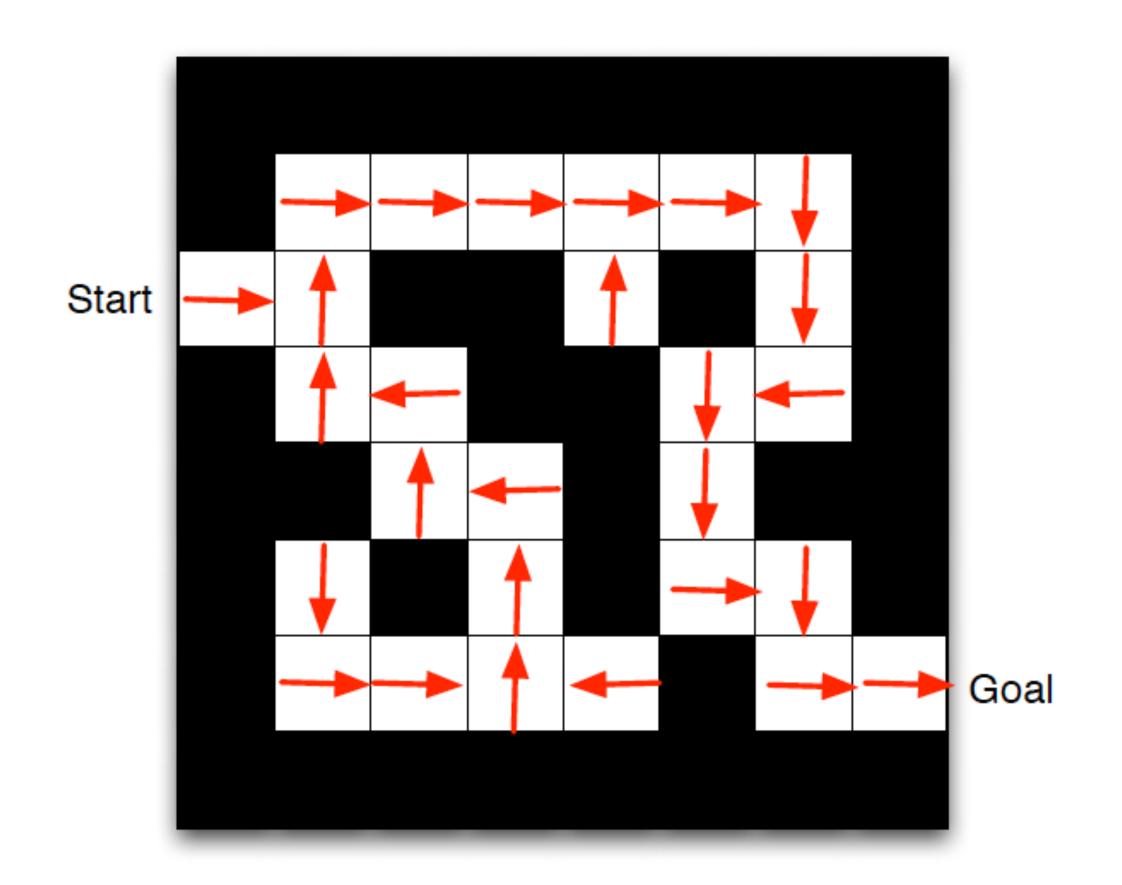


Reward: -1 per time-step

Actions: N, E, S, W

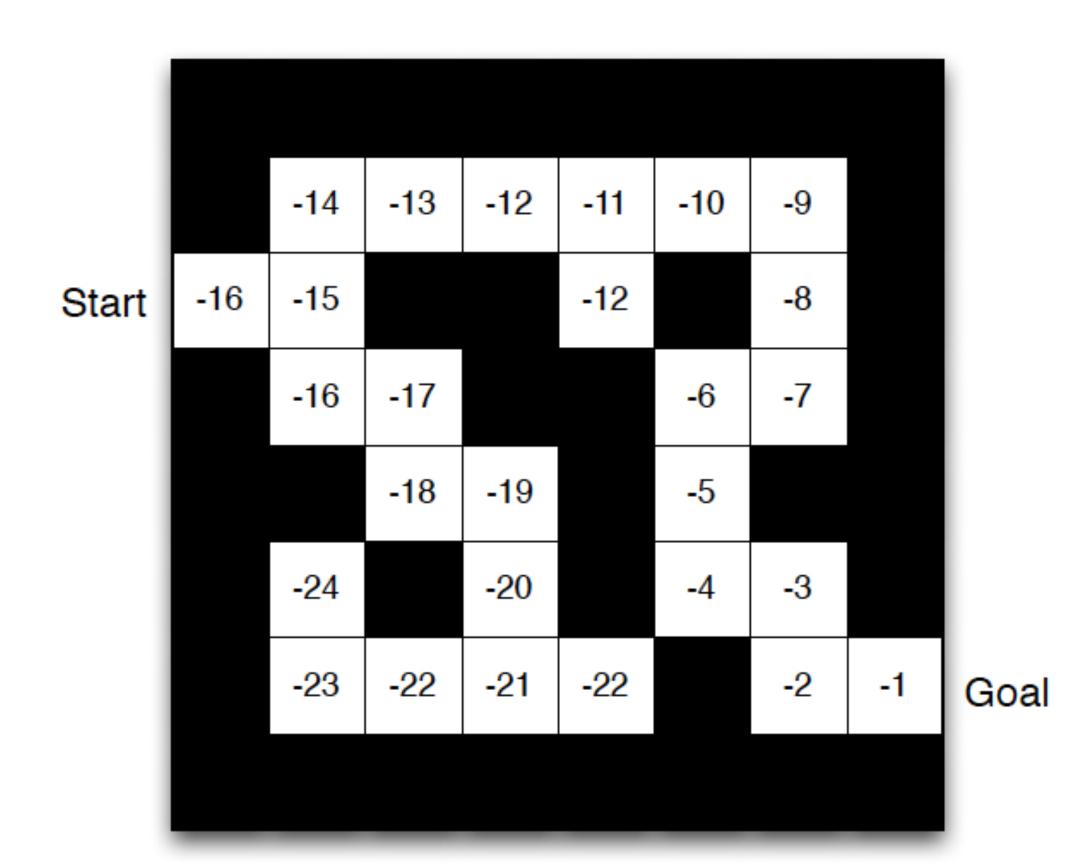
States: Agent's location

# Maze Example: Policy



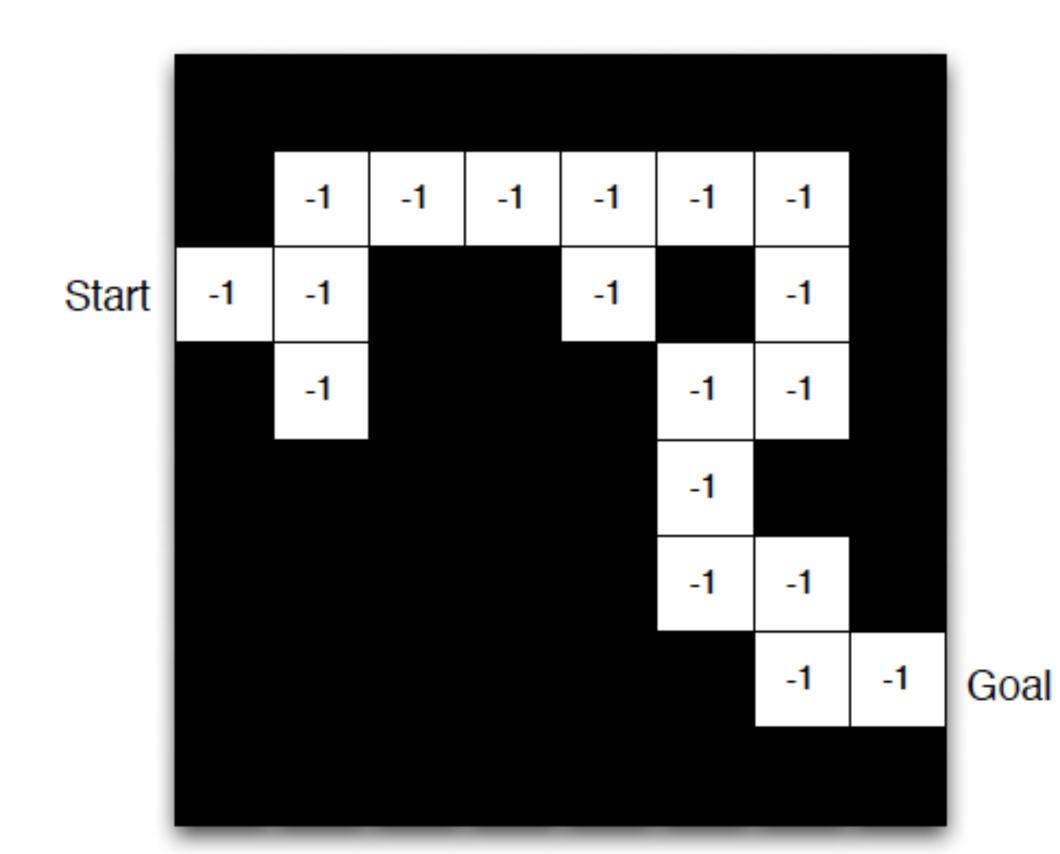
Arrows represent a policy  $\pi(s)$  for each state s

# Maze Example: Value



Numbers represent value  $\mathbf{v}_{\pi}(s)$  of each state s

### Maze Example: Model



Grid layout represents transition model

Numbers represent the immediate reward for each state (same for all states)

# Components of the RL Agent

#### **Policy**

— How does the agent behave?

#### **Value Function**

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#### Model

Agent's representation of the environment

### Approaches to RL: Taxonomy

#### Model-free RL

#### Value-based RL

- Estimate the optimal action-value function  $Q^*(s,a)$
- No policy (implicit)

#### Policy-based RL

- Search directly for the optima policy  $\pi^*$
- No value function

#### Model-based RL

- Build a model of the world
- Plan (e.g., by look-ahead) using model

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#### Actor-critic RL

- Value function
- Policy function

### Deep RL

#### Value-based RL

— Use neural nets to represent Q function  $Q(s,a;\theta)$   $Q(s,a;\theta^*) \approx Q^*(s,a)$ 

$$Q(s, a; \theta^*) \approx Q^*(s, a)$$

#### Policy-based RL

Use neural nets to represent the policy

$$\pi_{\theta^*} pprox \pi^*$$

#### Model-based RL

Use neural nets to represent and learn the model

# Approaches to RL

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- Estimate the optimal action-value function  $Q^*(s,a)$
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Optimal Q-function is the maximum achievable value

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Optimal value maximizes over all future decisions

$$Q^*(s,a) = r_{t+1} + \gamma \max_{a_{t+1}} r_{t+2} + \gamma^2 \max_{a_{t+2}} r_{t+3} + \dots$$
$$= r_{t+1} + \gamma \max_{a_{t+1}} Q^*(s_{t+1}, a_{t+1})$$

## Optimal Value Function

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$$= r_{t+1} + \gamma \max_{a_{t+1}} Q^*(s_{t+1}, a_{t+1})$$

Formally, Q\* satisfied Bellman Equations

$$Q^*(s,a) = \mathbb{E}_{s'}\left[r + \gamma \max_{a'} Q^*(s',a') \mid s,a\right]$$

Value iteration algorithm: Use Bellman equation as an iterative update

$$Q_{i+1}(s, a) = \mathbb{E}\left[r + \gamma \max_{a'} Q_i(s', a') | s, a\right]$$

Q<sub>i</sub> will converge to Q\* as i -> infinity

Value iteration algorithm: Use Bellman equation as an iterative update

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#### What's the problem with this?

**Not scalable**. Must compute Q(s,a) for every state-action pair. If state is e.g. game pixels, computationally infeasible to compute for entire state space!

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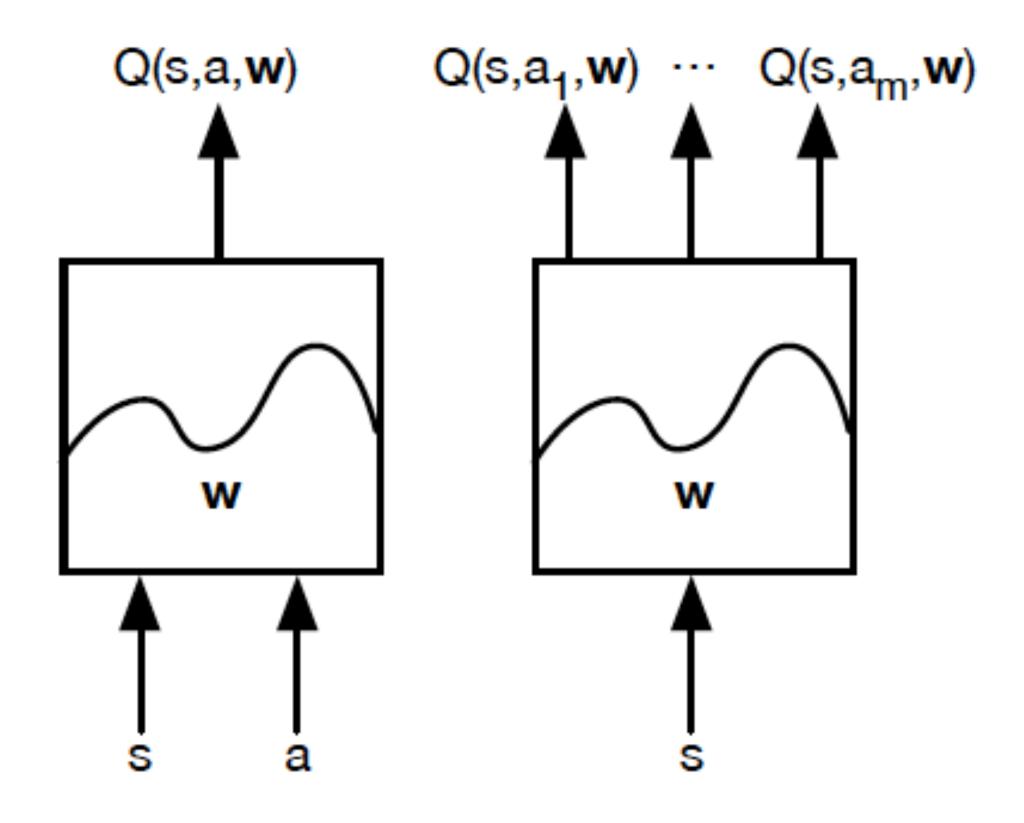
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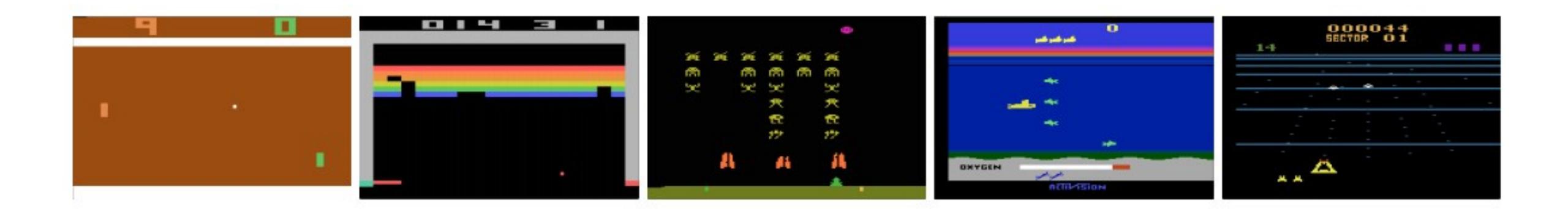
**Not scalable**. Must compute Q(s,a) for every state-action pair. If state is e.g. game pixels, computationally infeasible to compute for entire state space!

Solution: use a function approximator to estimate Q(s,a). E.g. a neural network!

#### Q-Networks

$$Q(s, a, \mathbf{w}) \approx Q^*(s, a)$$



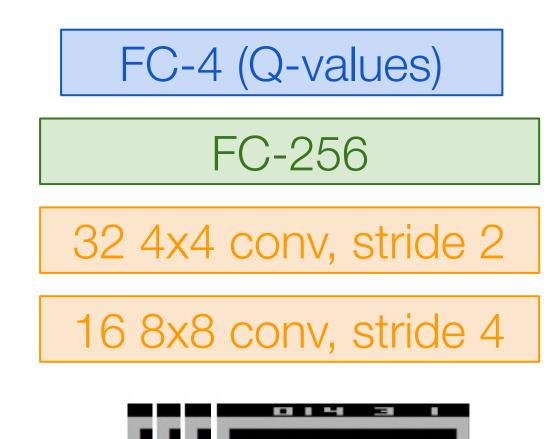


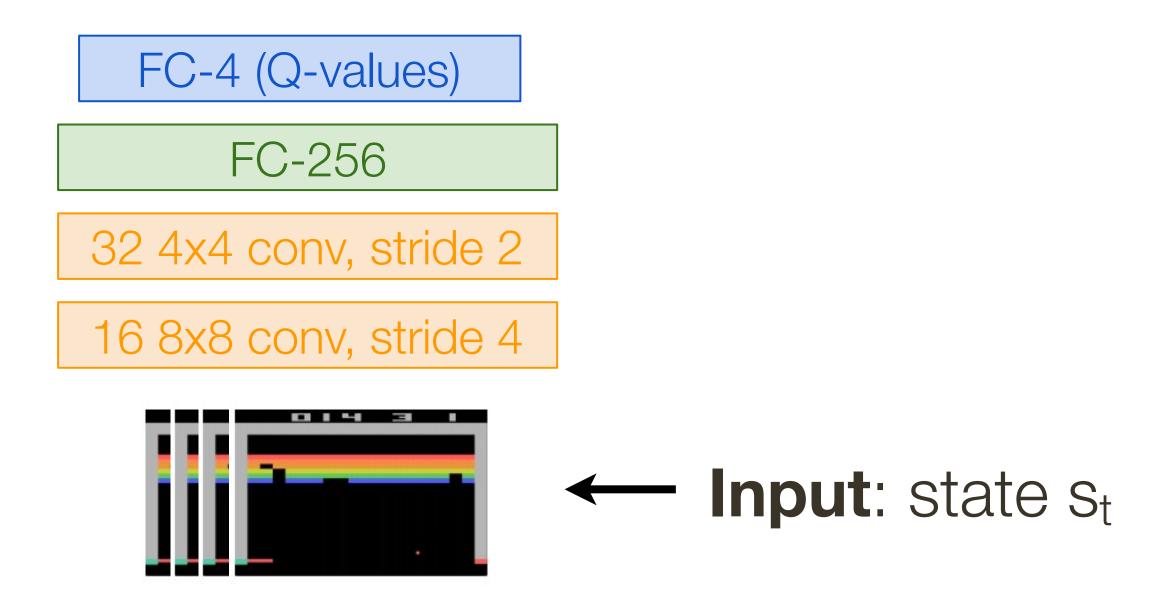
Objective: Complete the game with the highest score

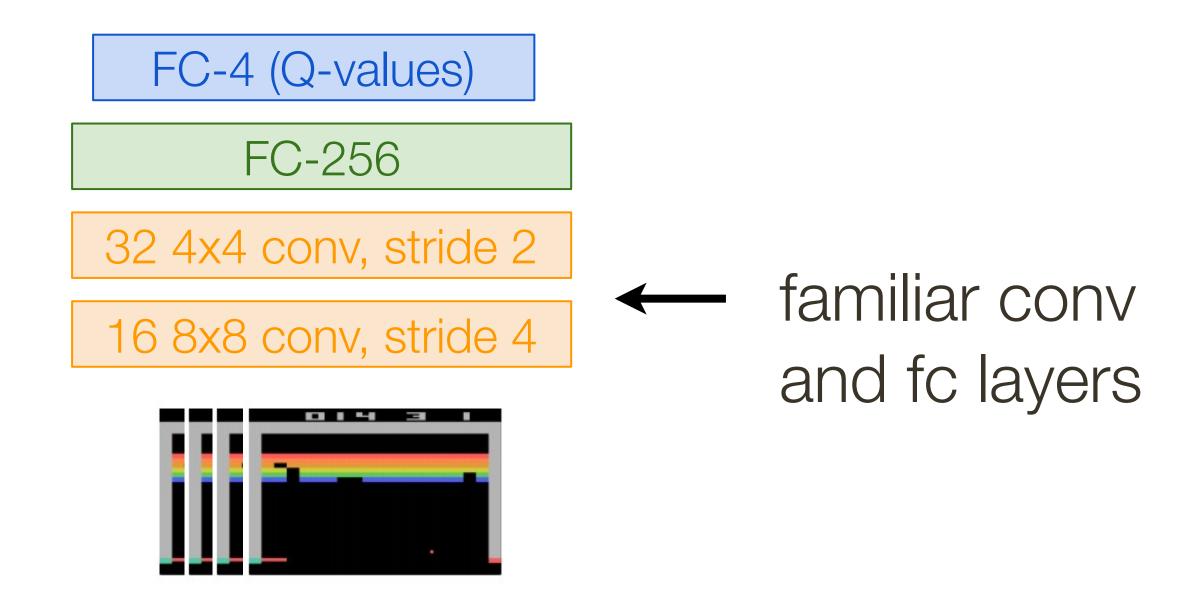
State: Raw pixel inputs of the game state

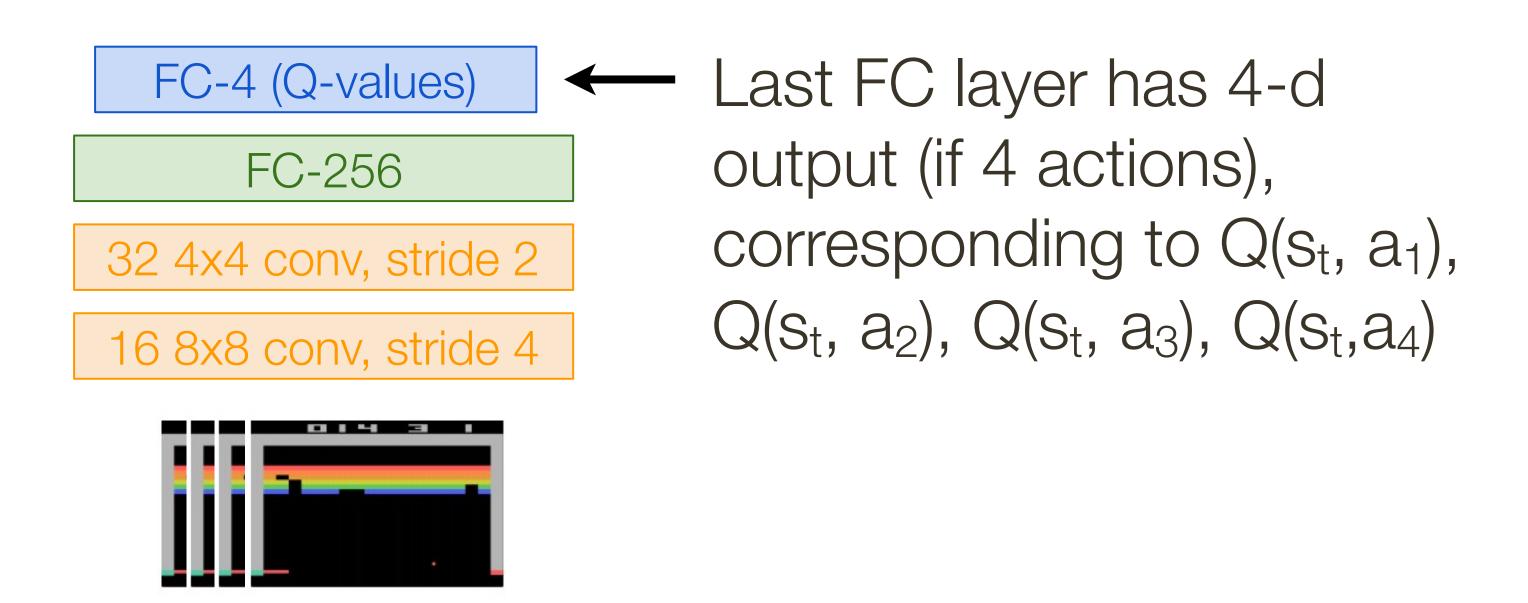
Action: Game controls e.g. Left, Right, Up, Down

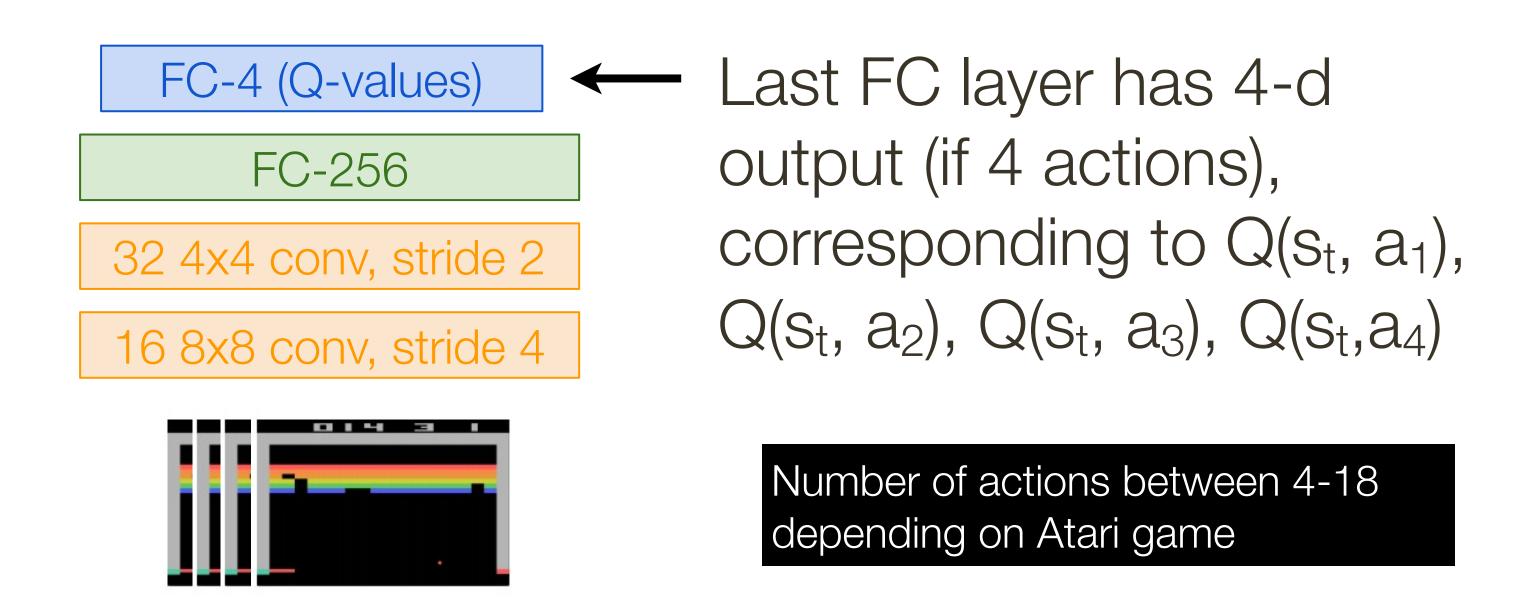
Reward: Score increase/decrease at each time step



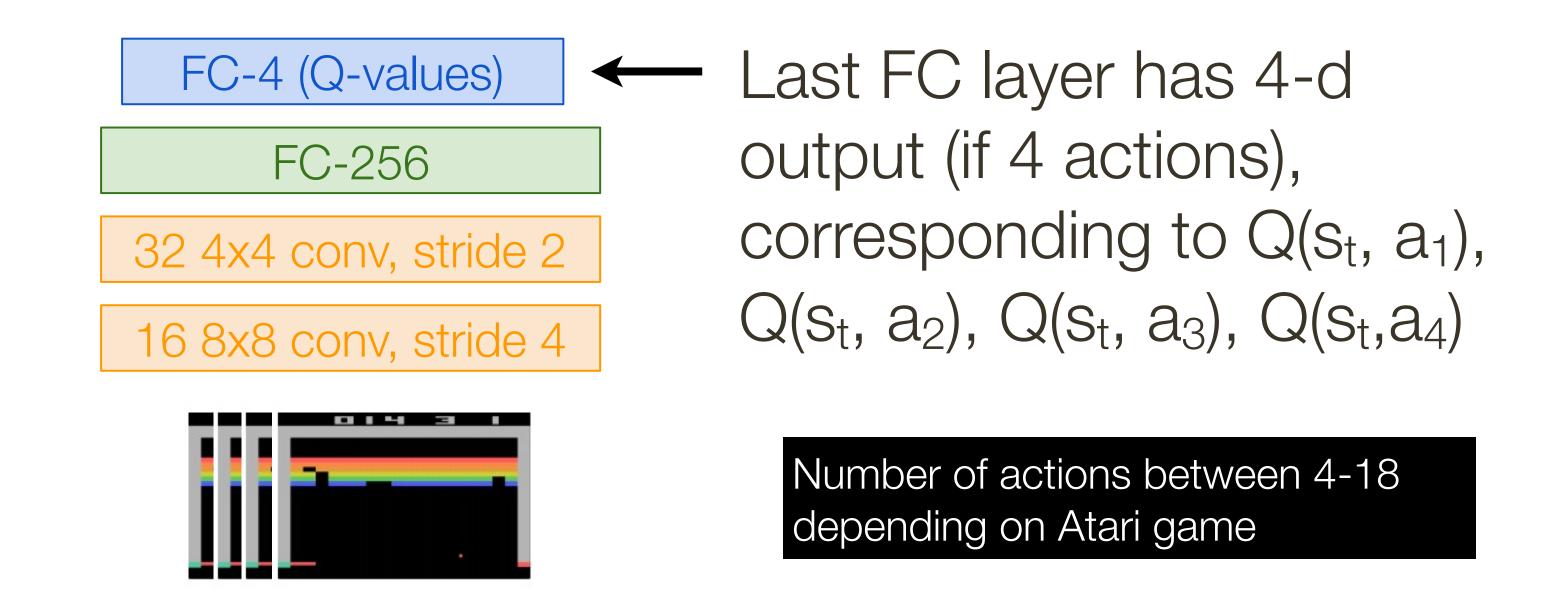








A single feedforward pass to compute Q-values for all actions from the current state => efficient!



Remember: want to find a Q-function that satisfies the Bellman Equation:

$$Q^*(s, a) = \mathbb{E}[r + \gamma \max_{a'} Q^*(s', a') \mid s, a]$$

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#### Forward Pass:

Loss function: 
$$L_i(\theta_i) = \mathbb{E}\left[(y_i - Q(s, a; \theta_i)^2)\right]$$
 where  $y_i = \mathbb{E}[r + \gamma \max_{a'} Q^*(s', a') \mid s, a]$ 

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Gradient update (with respect to Q-function parameters  $\theta$ ):

$$\nabla_{\theta_i} L_i(\theta_i) = \mathbb{E}\left[r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) - Q(s, a; \theta_i)) \nabla_{\theta_i} Q(s, a; \theta_i)\right]$$

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Iteratively try to make the Q-value close to the target value ( $y_i$ ) it should have, if Q-function corresponds to optimal Q\* (and optimal policy  $\pi^*$ )

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# Training the Q-Network: Experience Replay

#### Learning from batches of consecutive samples is problematic:

- Samples are correlated => inefficient learning
- Current Q-network parameters determines next training samples (e.g. if maximizing action is to move left, training samples will be dominated by samples from left-hand size)
- => can lead to bad feedback loops

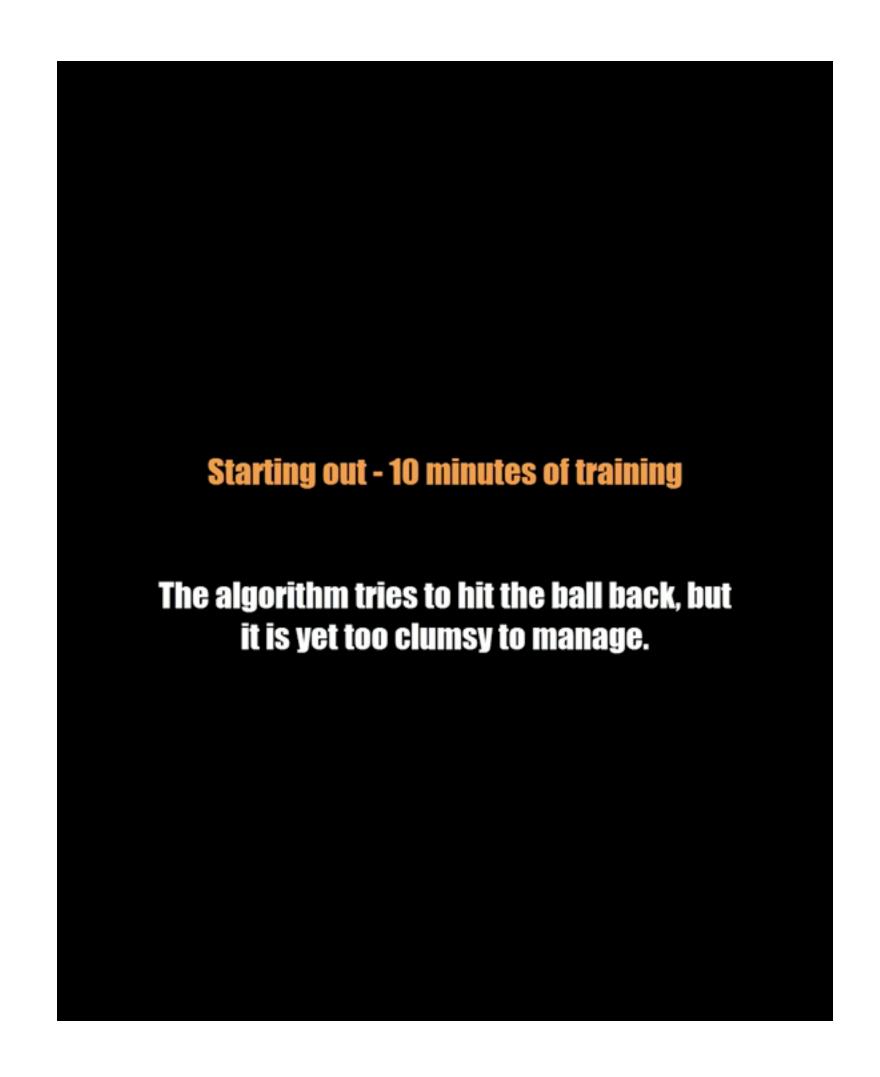
#### Address these problems using experience replay

- Continually update a replay memory table of transitions ( $s_t$ ,  $a_t$ ,  $r_t$ ,  $s_{t+1}$ ) as game (experience) episodes are played
- Train Q-network on random minibatches of transitions from the replay memory, instead of consecutive samples

## Experience Replay

To remove correlations, build data-set from agent's own experience

# Example: Atari Playing



#### Deep RL

#### Value-based RL

— Use neural nets to represent Q function  $Q(s, a; \theta)$ 

$$Q(s, a; \theta)$$
  
 $Q(s, a; \theta^*) \approx Q^*(s, a)$ 

#### Policy-based RL

— Use neural nets to represent the policy  $\pi_{\theta}$ 

$$\pi_{\theta^*} \approx \pi^*$$

#### Model-based RL

— Use neural nets to represent and learn the model

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Formally, let's define a class of parameterized policies:

For each policy, define its value:

$$J(\theta) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | \pi_{\theta}\right]$$

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How can we do this?

Gradient ascent on policy parameters!

# REINFORCE algorithm

Expected reward:

$$J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)} [r(\tau)]$$
$$= \int_{\tau} r(\tau) p(\tau;\theta) d\tau$$

Where  $r(\tau)$  is the reward of a trajectory  $\tau=(s_0,a_0,r_0,s_1,\ldots)$ 

# REINFORCE algorithm

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Now let's differentiate this:  $\nabla_{\theta}J(\theta)=\int_{ au}r( au)\nabla_{\theta}p( au; heta)\mathrm{d} au$ 

Intractable! Expectation of gradient is problematic when p depends on  $\theta$ 

# REINFORCE algorithm

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Now let's differentiate this:  $\nabla_{\theta}J(\theta)=\int_{ au}r( au)\nabla_{\theta}p( au;\theta)\mathrm{d} au$ 

However, we can use a nice trick:  $\nabla_{\theta} p(\tau;\theta) = p(\tau;\theta) \frac{\nabla_{\theta} p(\tau;\theta)}{p(\tau;\theta)} = p(\tau;\theta) \nabla_{\theta} \log p(\tau;\theta)$ 

If we inject this back:

$$\nabla_{\theta} J(\theta) = \int_{\tau} (r(\tau) \nabla_{\theta} \log p(\tau; \theta)) p(\tau; \theta) d\tau$$
$$= \mathbb{E}_{\tau \sim p(\tau; \theta)} [r(\tau) \nabla_{\theta} \log p(\tau; \theta)]$$

Can estimate with Monte Carlo sampling

#### **Gradient estimator:**

$$\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} r(\tau) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$

#### Interpretation:

- If  $r(\tau)$  is high, push up the probabilities of the actions seen
- If  $r(\tau)$  is low, push down the probabilities of the actions seen

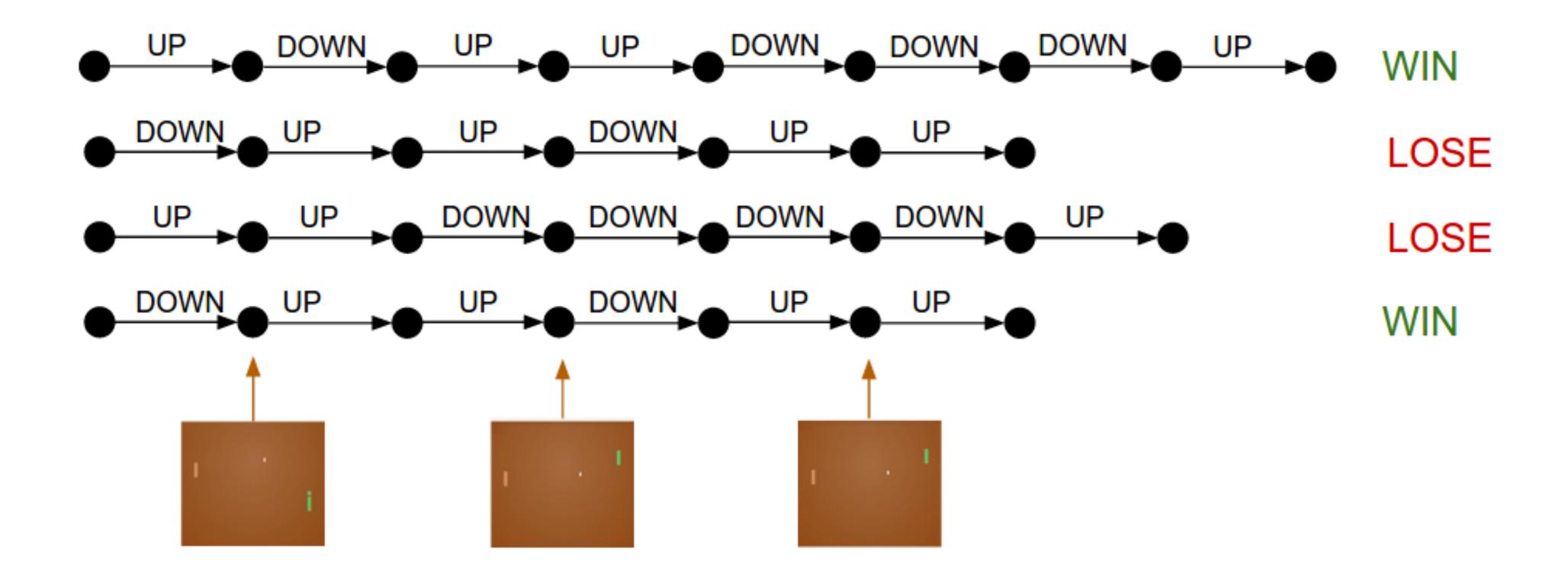
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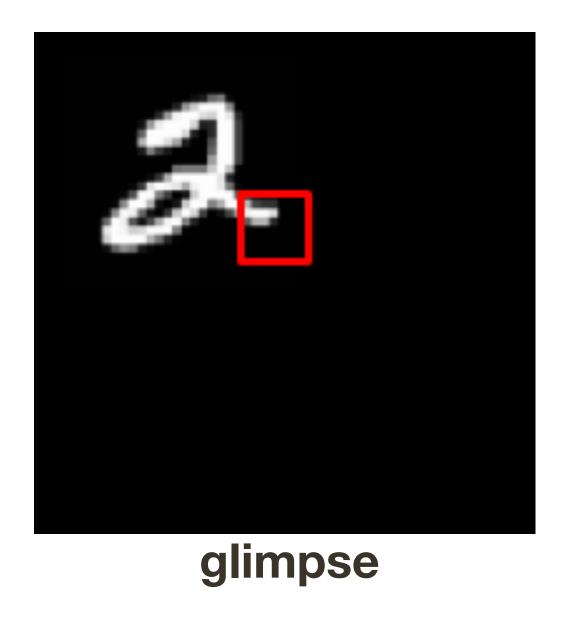
Might seem simplistic to say that if a trajectory is good then all its actions were good. **But in expectation, it averages out!** 

However, this also suffers from high variance because credit assignment is really hard. Can we help the estimator?

Objective: Image Classification

Take a sequence of "glimpses" selectively focusing on regions of the image, to predict class

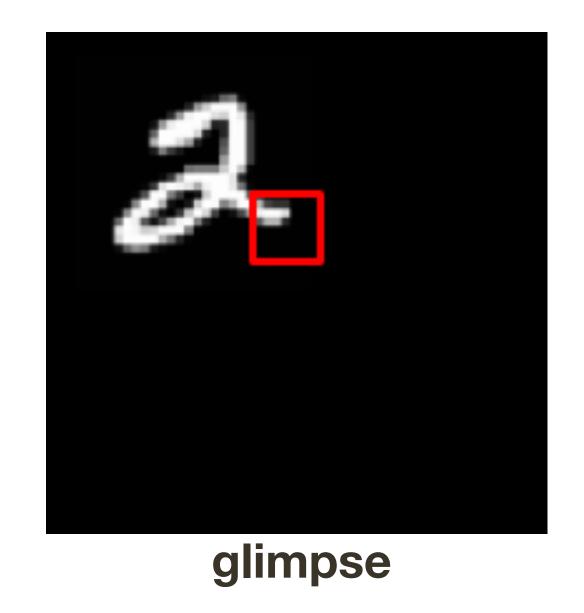
- Inspiration from human perception and eye movements
- Saves computational resources => scalability
- Able to ignore clutter / irrelevant parts of image



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- Saves computational resources => scalability
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State: Glimpses seen so far

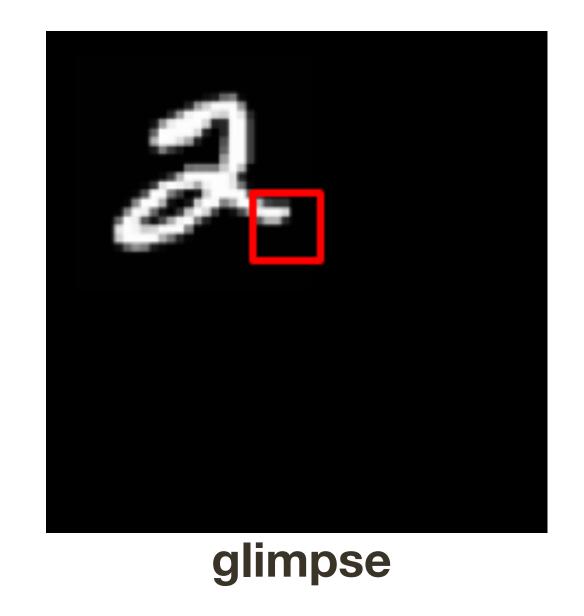
Action: (x,y) coordinates (center of glimpse) of where to look next in image

Reward: 1 at the final timestep if image correctly classified, 0 otherwise

Objective: Image Classification

Take a sequence of "glimpses" selectively focusing on regions of the image, to predict class

- Inspiration from human perception and eye movements
- Saves computational resources => scalability
- Able to ignore clutter / irrelevant parts of image

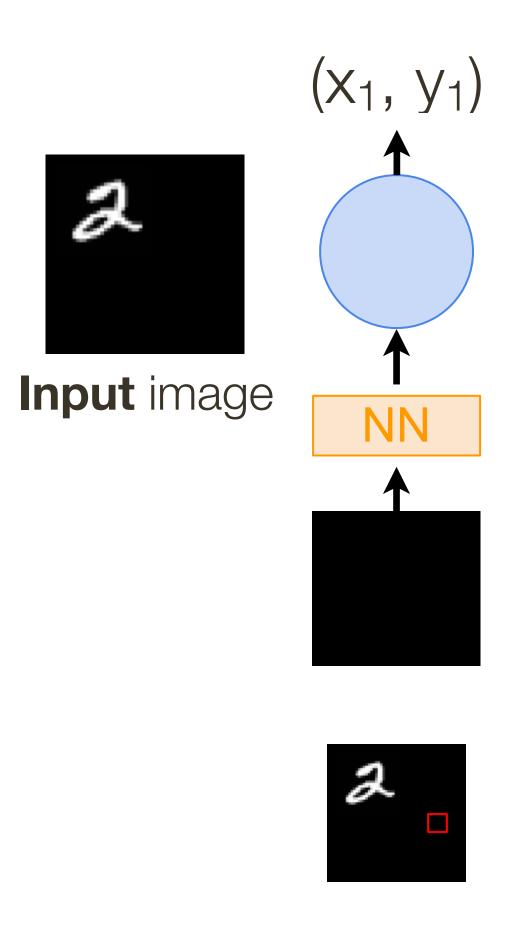


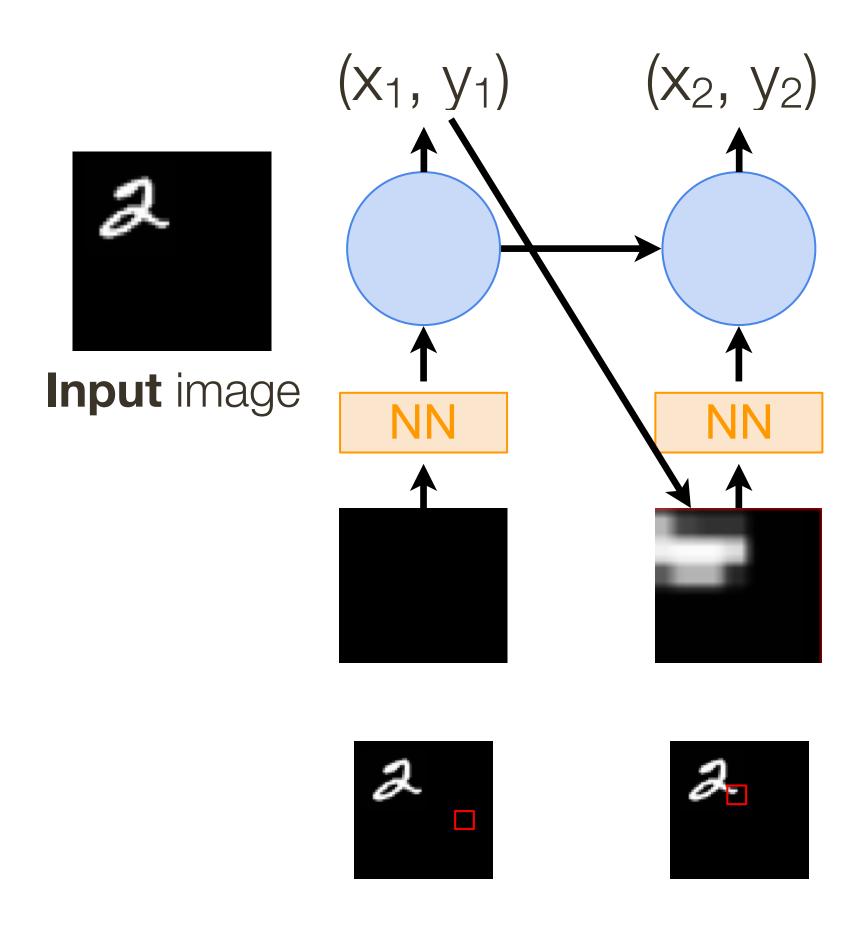
State: Glimpses seen so far

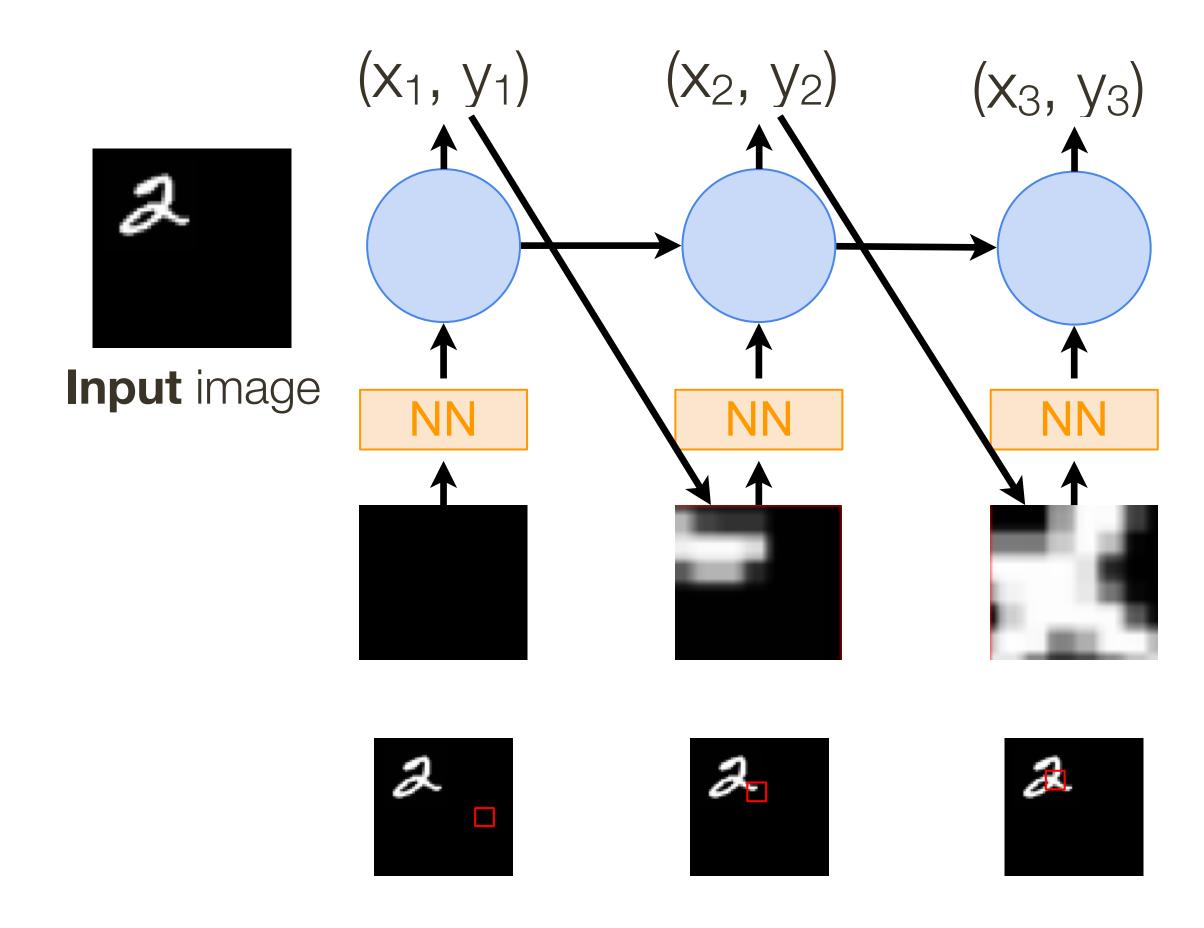
Action: (x,y) coordinates (center of glimpse) of where to look next in image

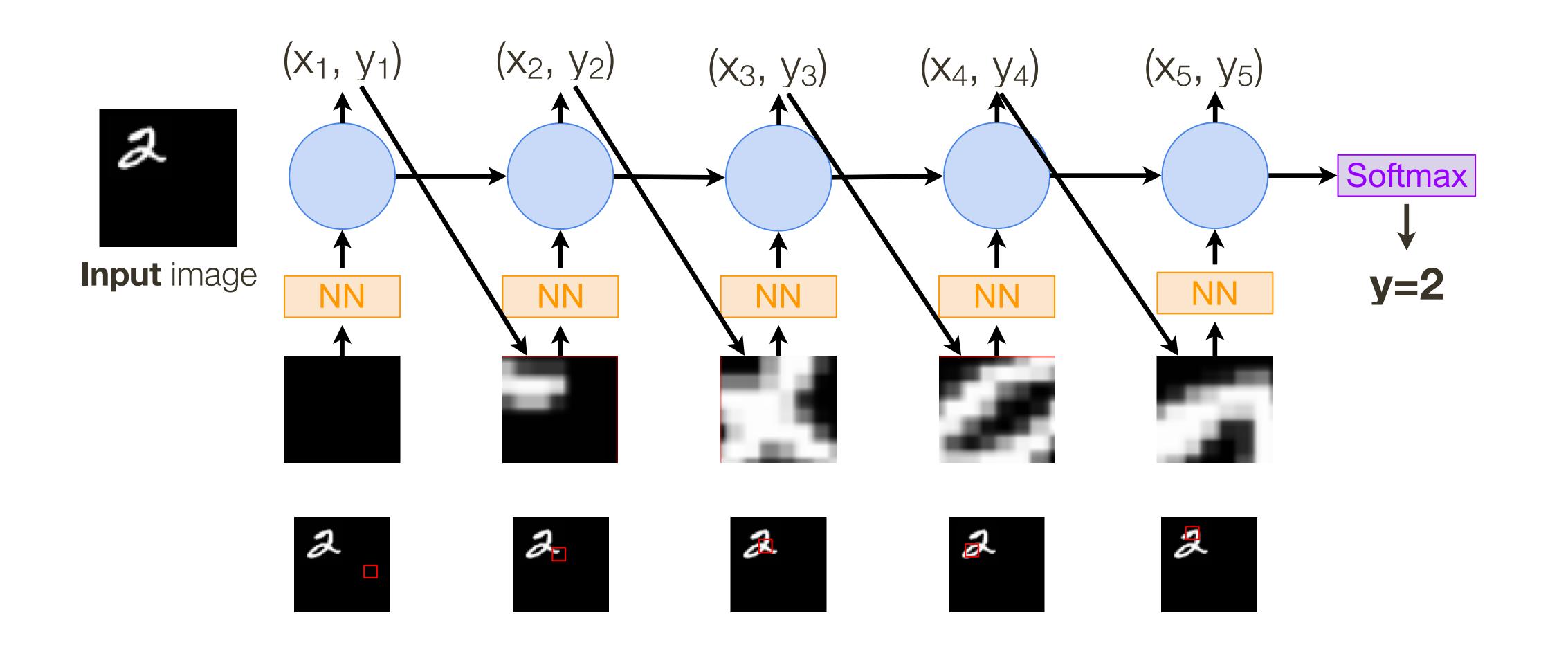
Reward: 1 at the final timestep if image correctly classified, 0 otherwise

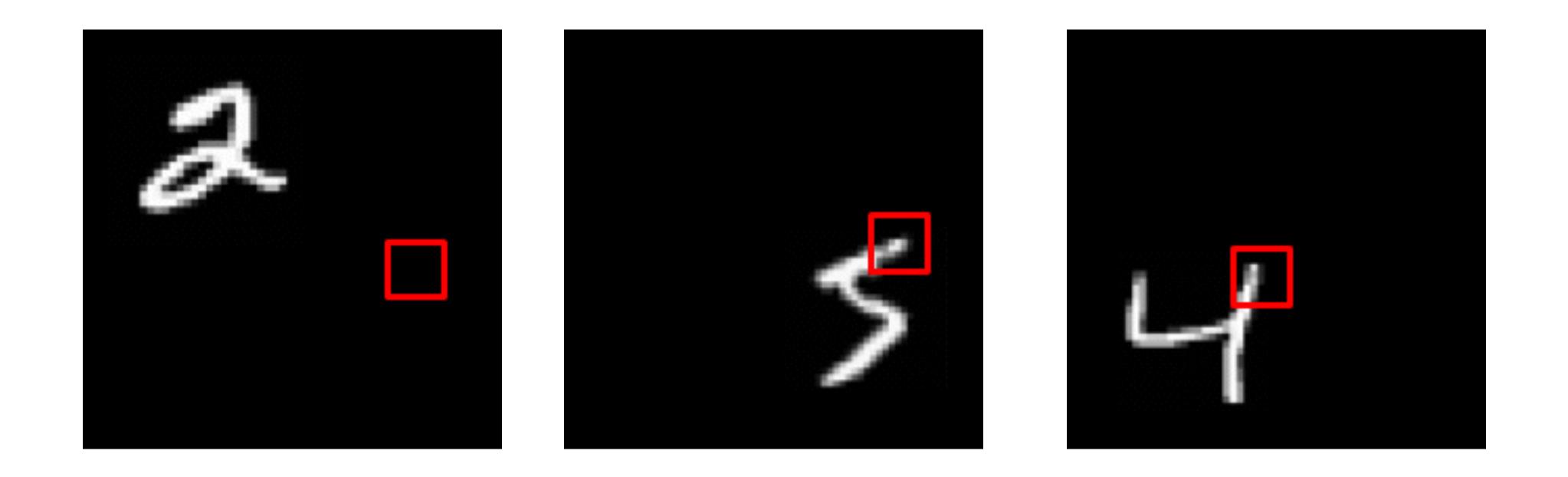
Glimpsing is a **non-differentiable operation** => learn policy for how to take glimpse actions using REINFORCE Given state of glimpses seen so far, use RNN to model the state and output next action











Has also been used in many other tasks including fine-grained image recognition, image captioning, and visual question-answering!

#### Summary

Policy gradients: very general but suffer from high variance so requires a lot of samples. *Challenge*: sample-efficiency

**Q-learning**: does not always work but when it works, usually more sample-efficient. *Challenge*: exploration

#### Guarantees:

- Policy Gradients: Converges to a local minima of  $J(\theta)$ , often good enough!
- Q-learning: Zero guarantees since you are approximating Bellman equation with a complicated function approximator