THE UNIVERSITY OF BRITISH COLUMBIA

Topics in AI (CPSC 532S):
Multimodal Learning with Vision, Language and Sound

Lecture 11: Coordinated Representations and Joint Embeddings

Multimodal Representations

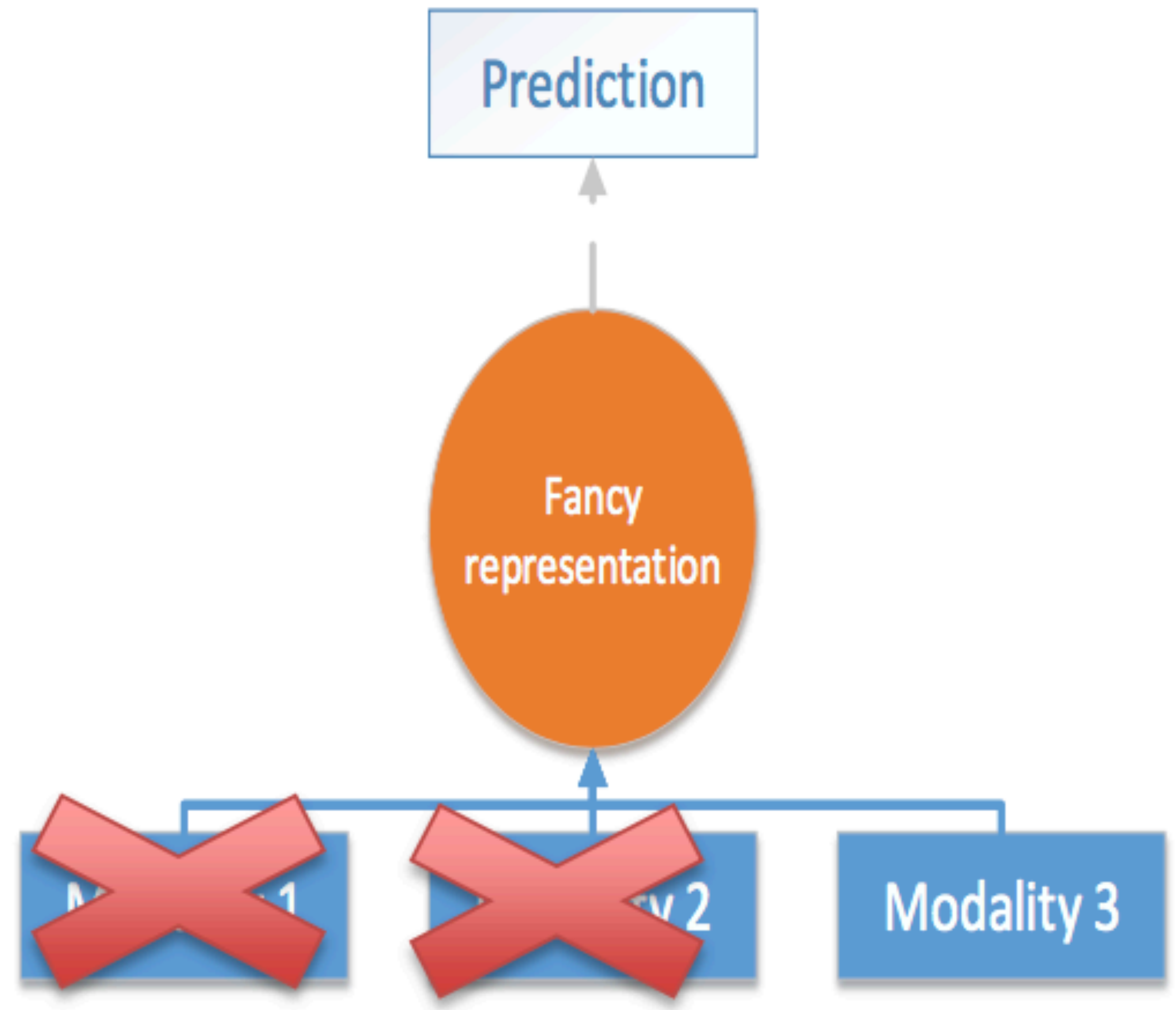
What is a **good** multimodal representation?

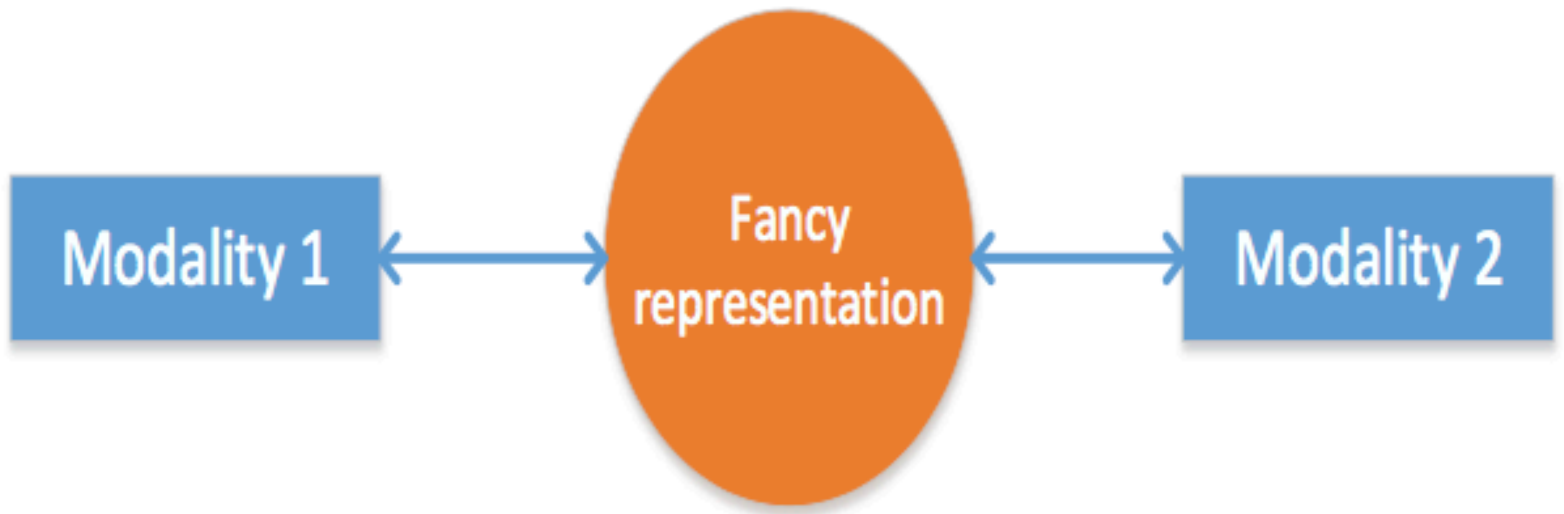
— Similarity in the representation (somehow) implies similarity in corresponding concepts
(we saw this in word2vec)

— Useful for various discriminative tasks (retrieval, mapping, fusion, etc.)

— Possible to obtain in absence of one or mere modalities

— Fill in missing modalities given others (map or translate between modalities)

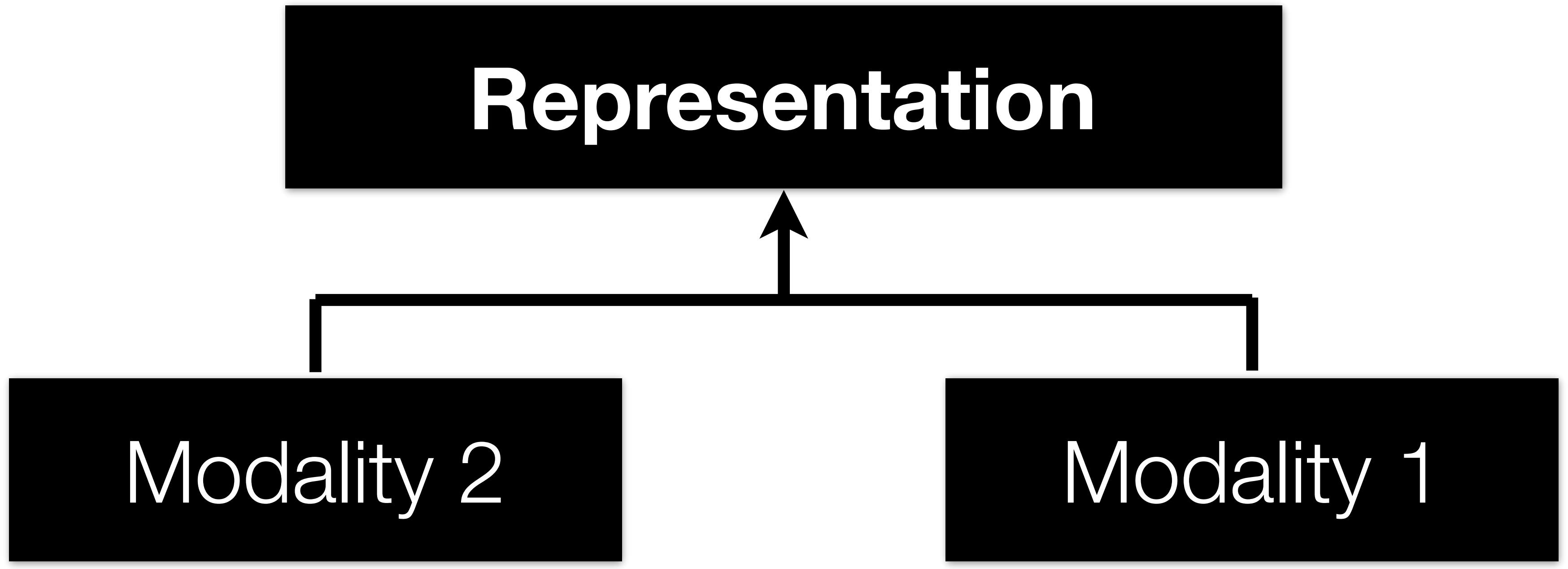




*slide from Louis-Philippe Morency

Multimodal Representation Types

Joint representations:



— Simplest version: modality concatenation (early fusion)

— Can be learned supervised or unsupervised

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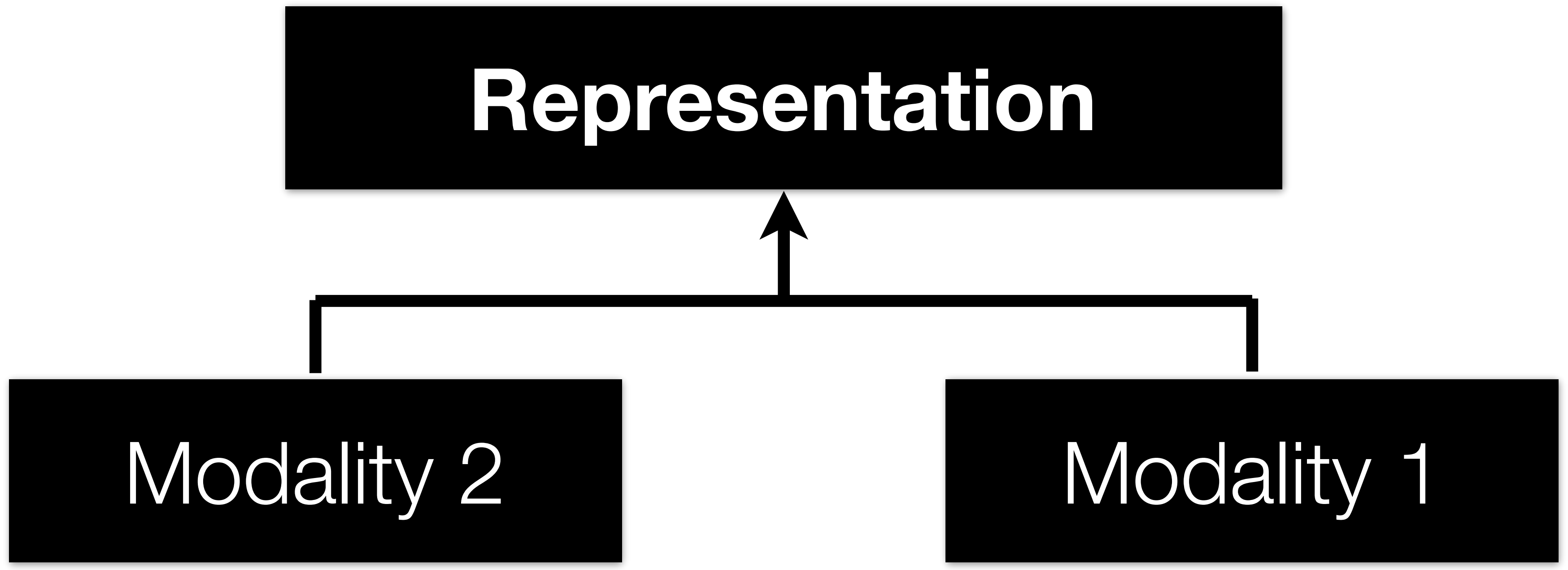

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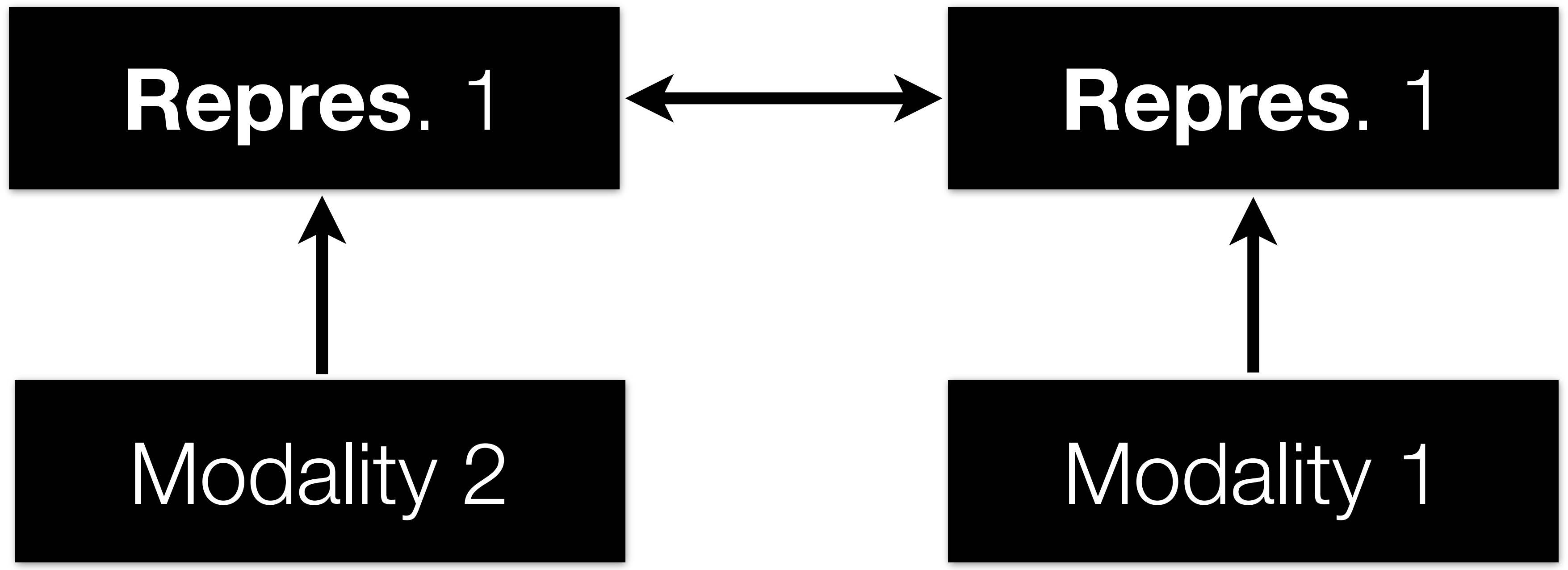
Coordinated representations:



— Similarity-based methods (e.g., cosine distance)

— Structure constraints (e.g., orthogonality, sparseness)

— Examples: CCA, joint embeddings

*slide from Louis-Philippe Morency

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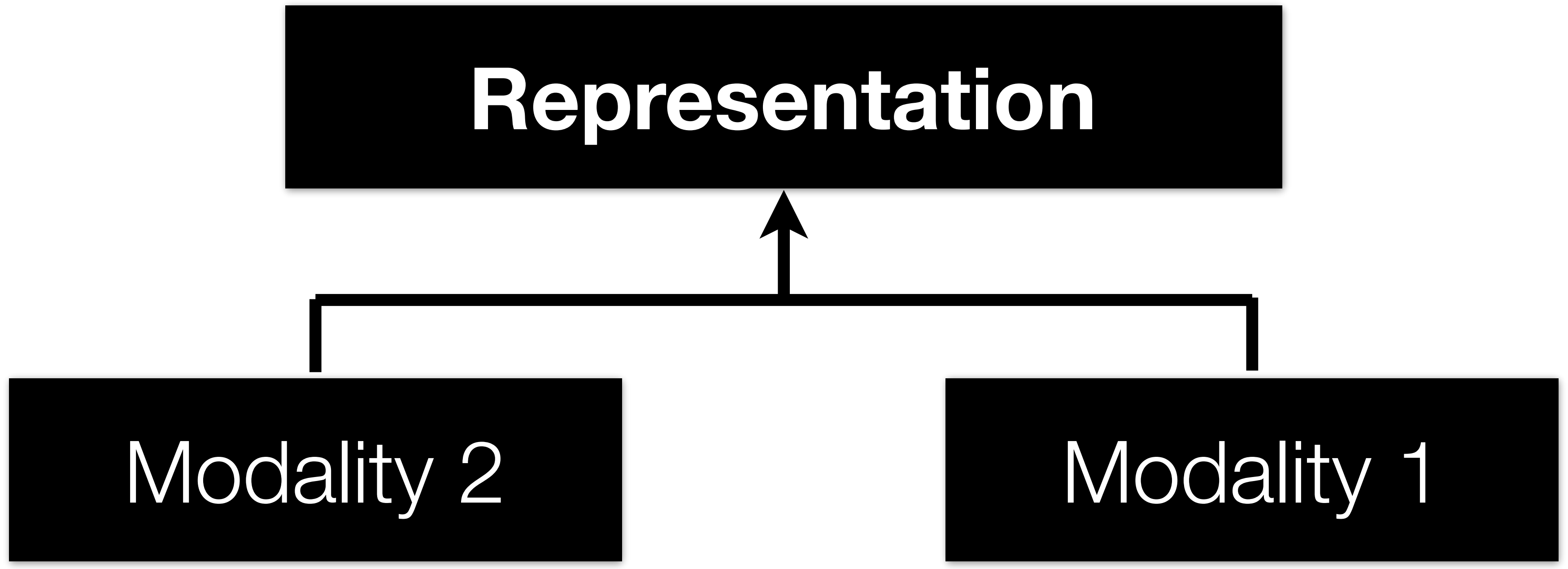

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Joint Representation: Deep Multimodal Autoencoders

[Ngiam et al., 2011]

Each modality can be pre-trained

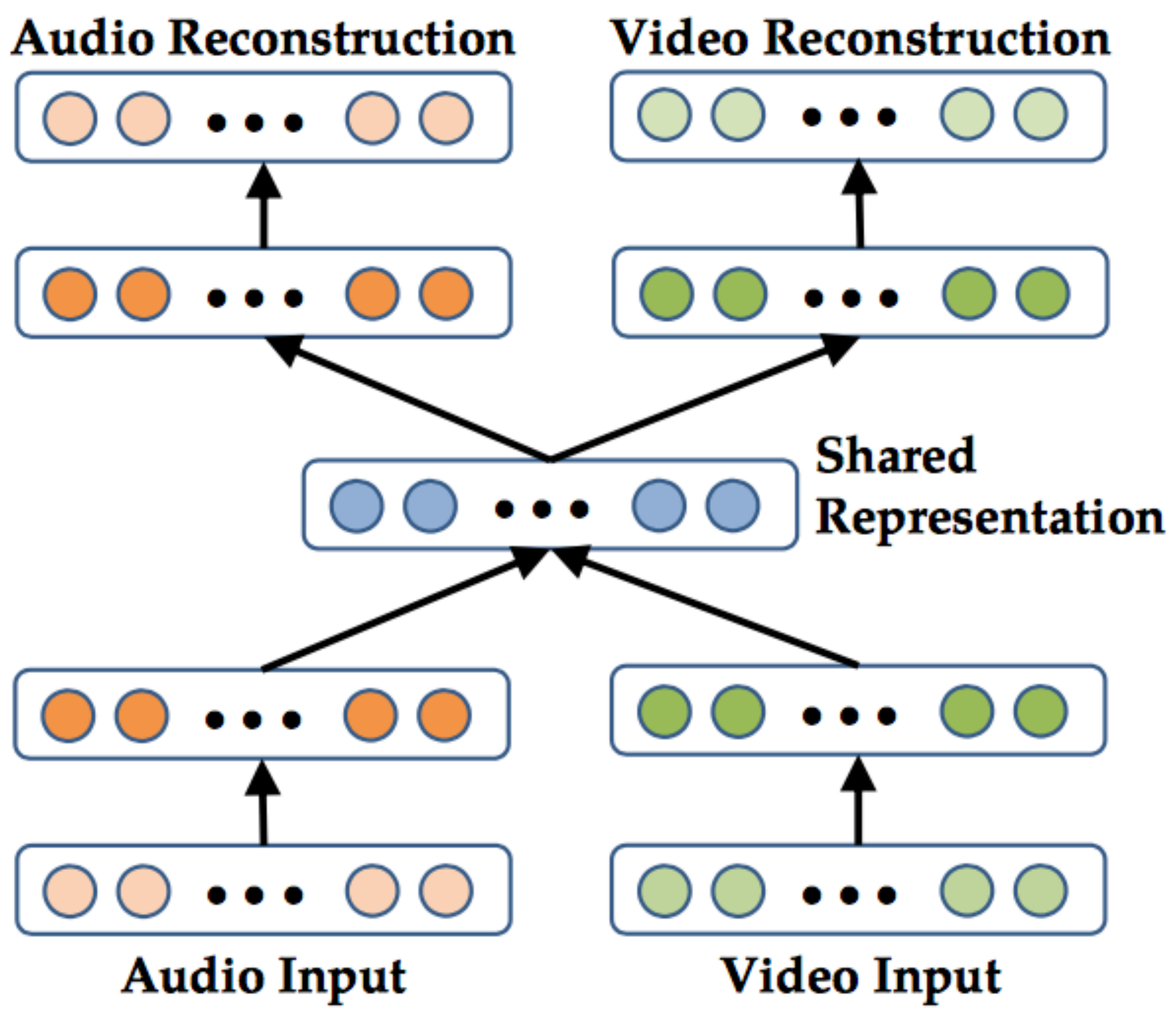
— using denoising autoencoder

To train the model, reconstruct both modalities using

— both Audio & Video

— just Audio

— just Video



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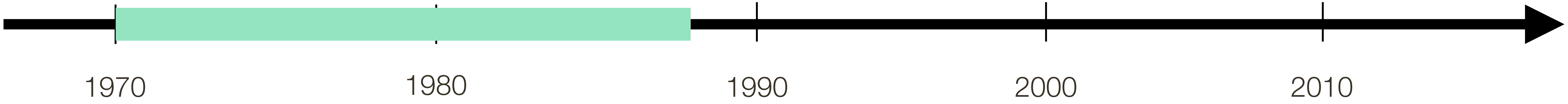
Multimodal Research: Historical Perspective

McGurk Effect (1976)



* video credit: **OK Science**

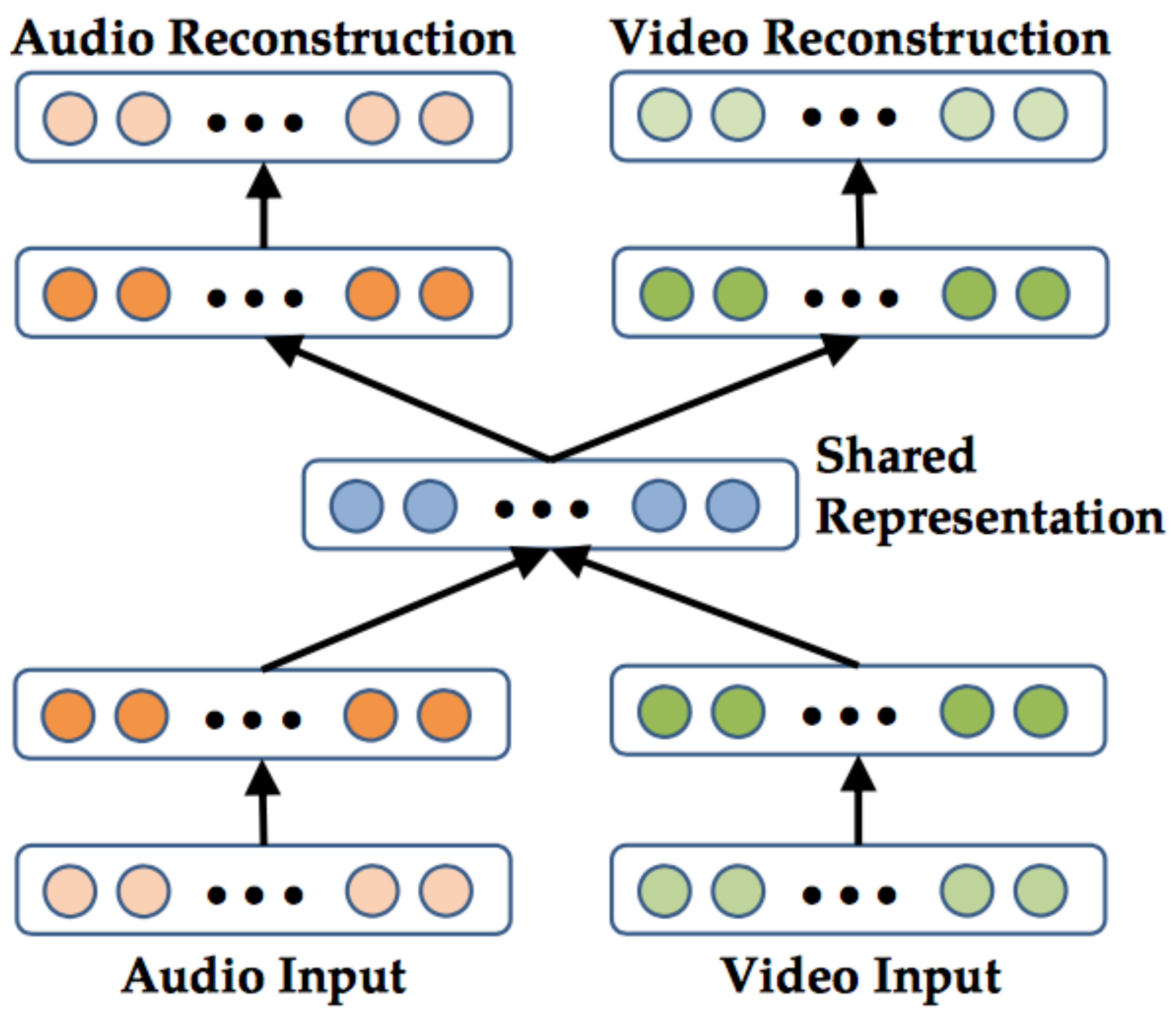
* Adopted from slides by Louis-Philippe Morency

Joint Representation: Deep Multimodal Autoencoders

[Ngiam et al., 2011]

Table 3: McGurk Effect

Audio / Visual Setting	**Model prediction**		
	/ga/	*/ba/*	*/da/*
Visual */ga/*, Audio */ga/*	82.6%	2.2%	15.2%
Visual */ba/*, Audio */ba/*	4.4%	89.1%	6.5%
Visual */ga/*, Audio */ba/*	28.3%	13.0%	58.7%



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Joint Representation: Deep Multimodal Autoencoders

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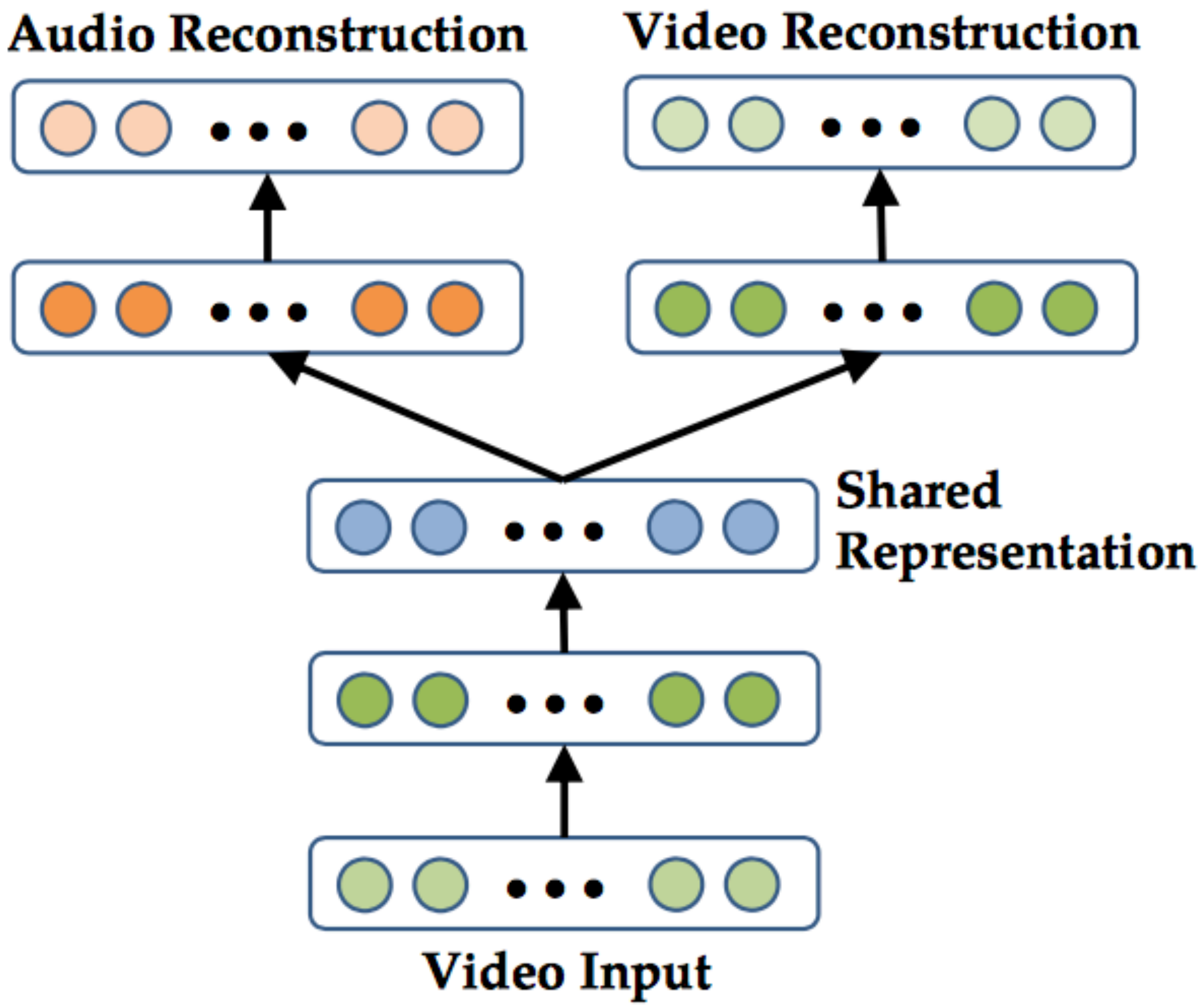
Useful when you know you may only be conditioning on one modality at test time

Can be regarded as a form of **regularization**



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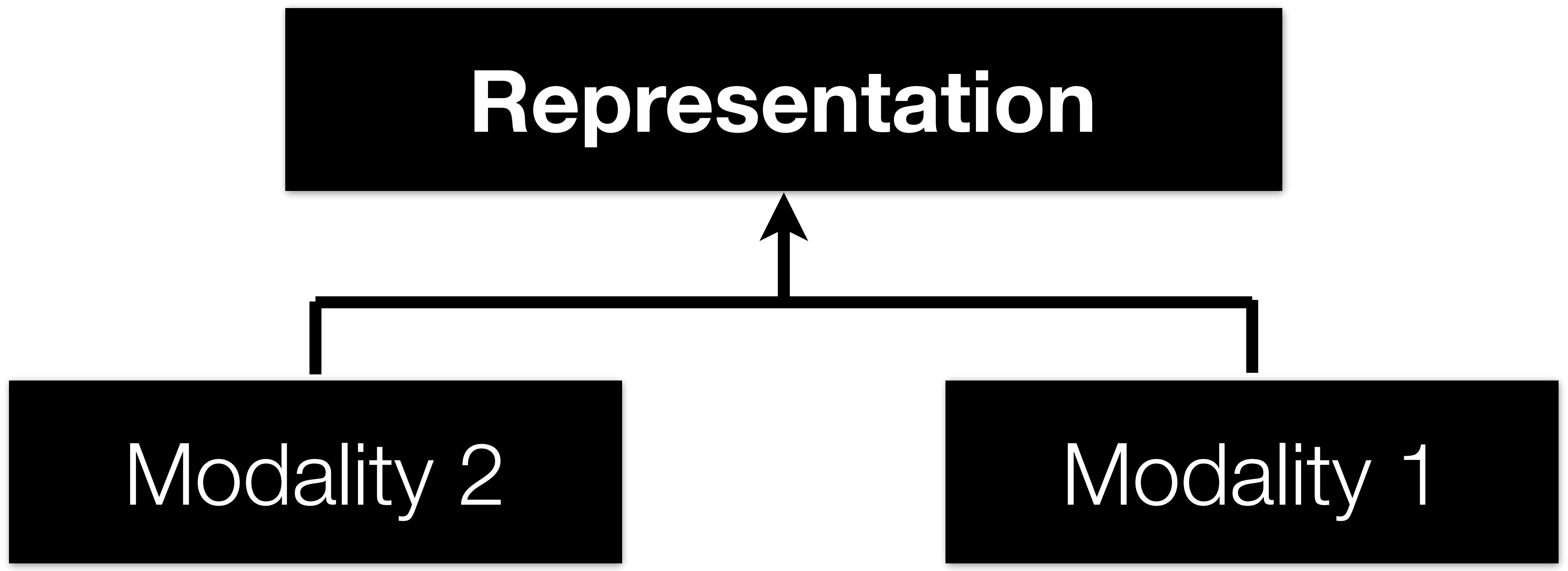

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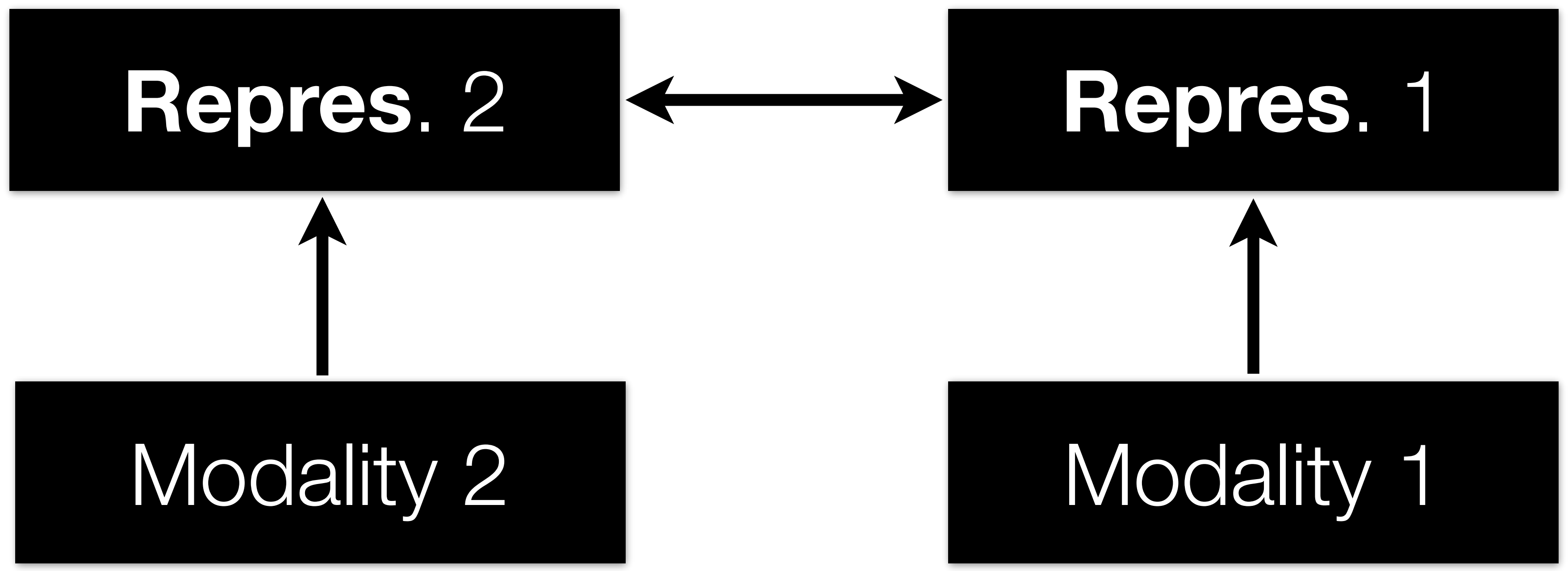

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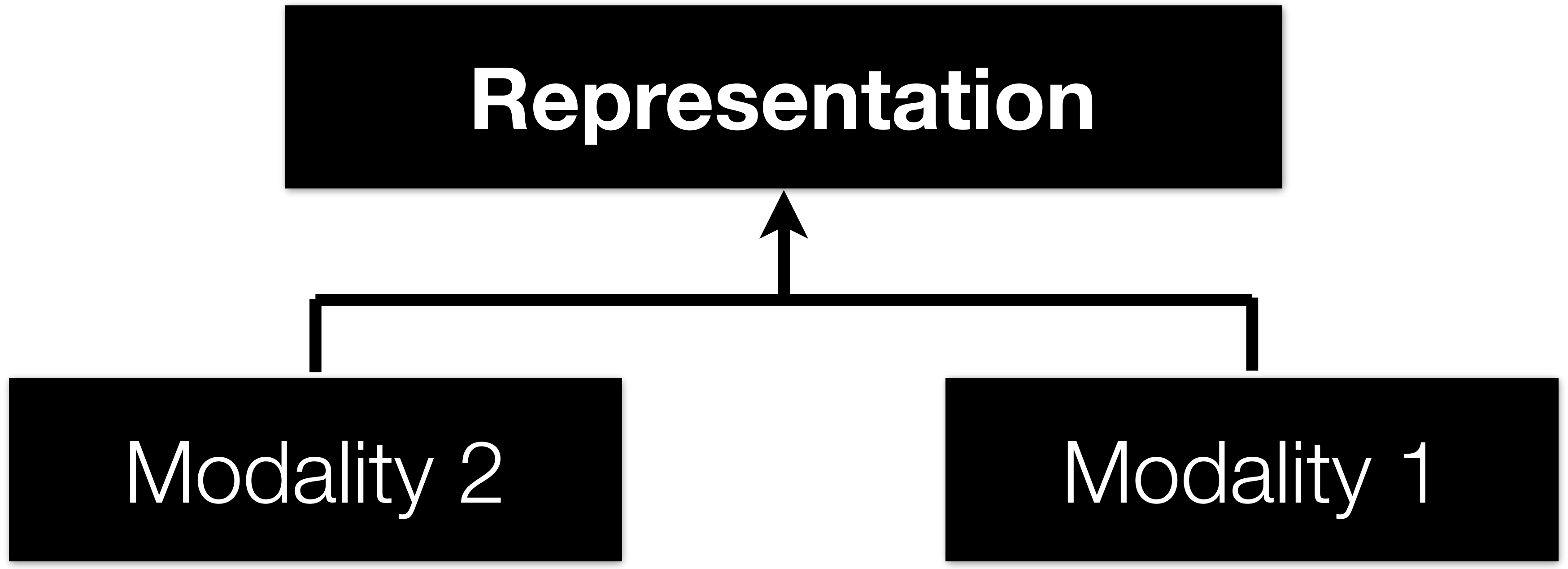

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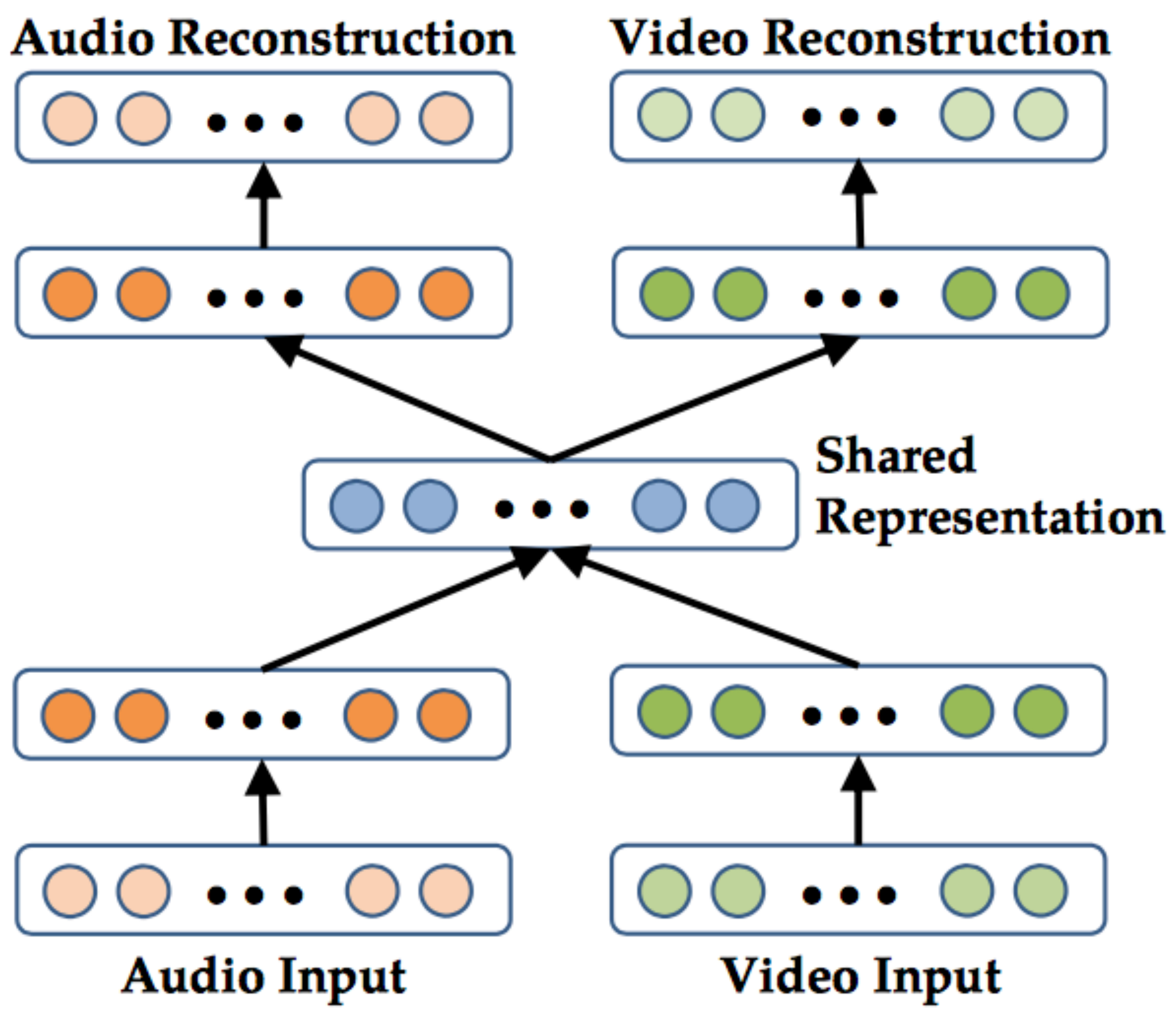

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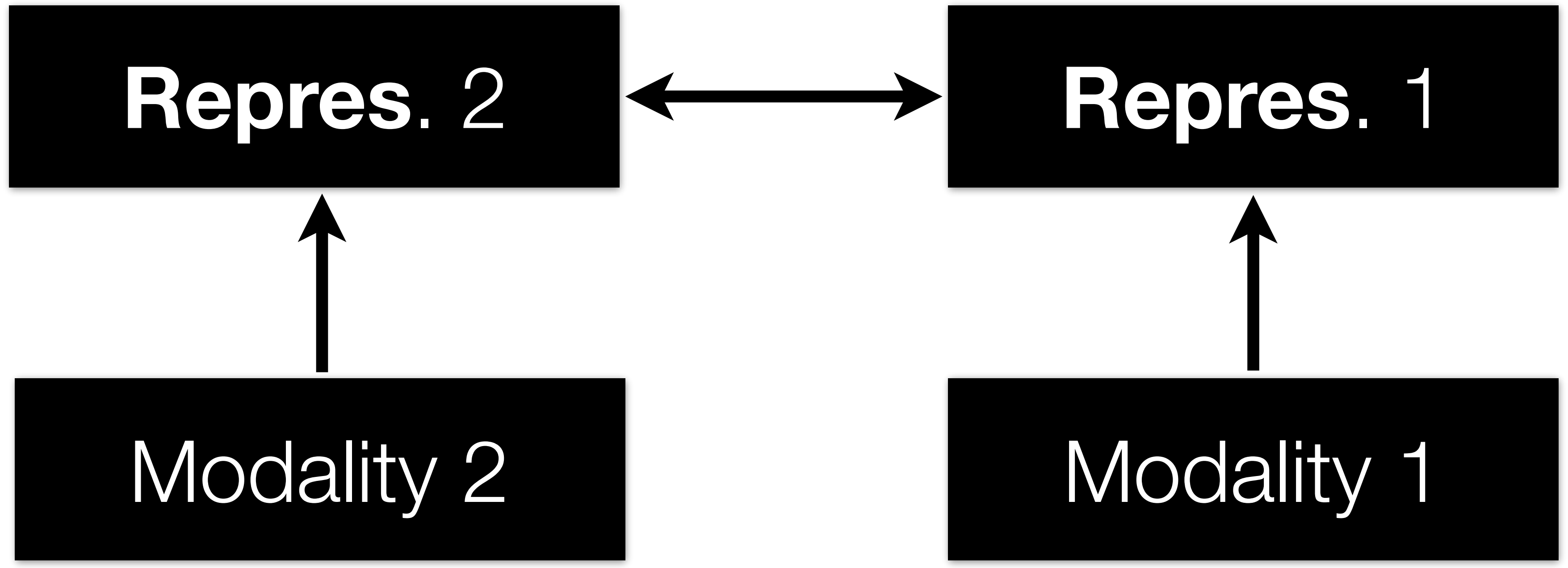

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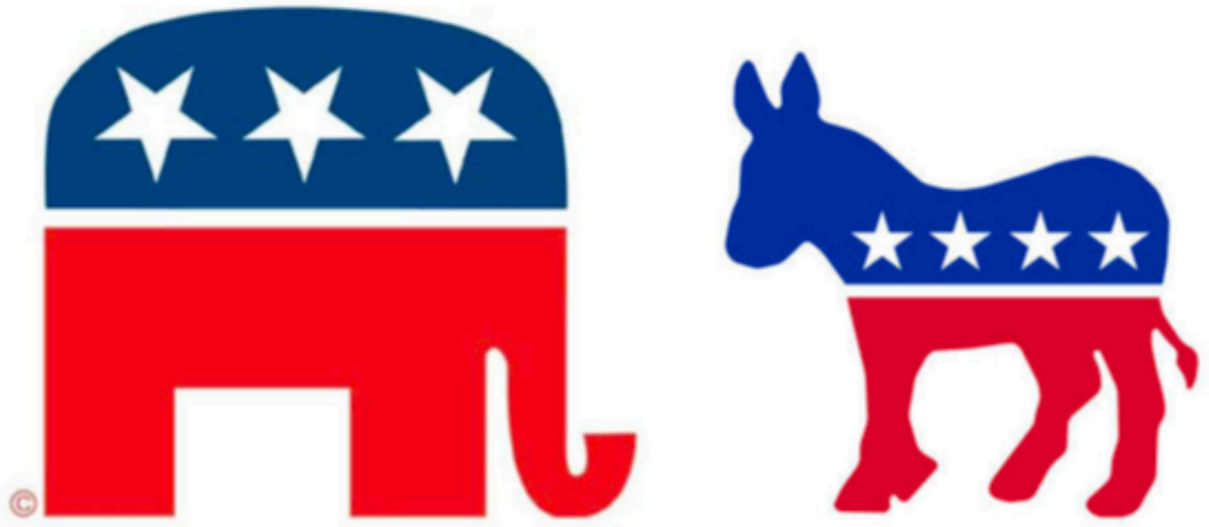
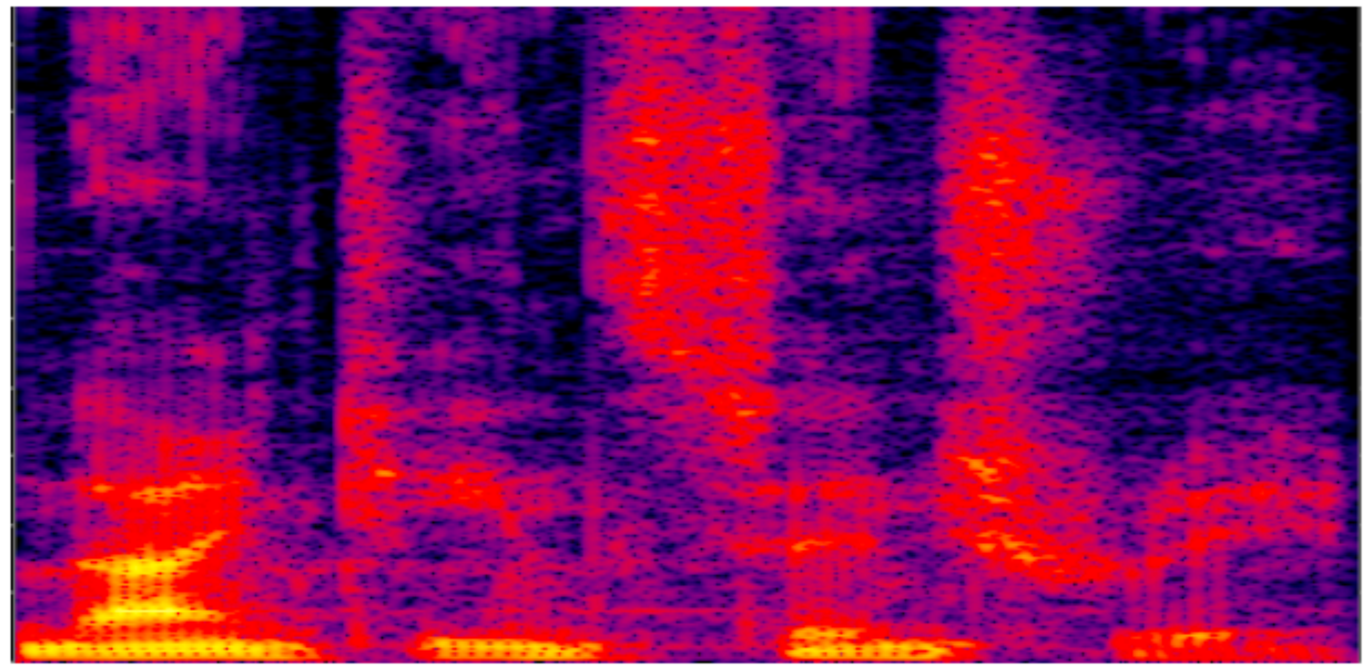
Data with **Multiple Views**

$x_1^{(i)}$	$x_2^{(i)}$
 demographic properties	 responses to survey
 audio features at time i	 video features at time i

*slide from Andrew, Arora, Bilmes, Livescu

Correlated Representations

Goal: Find representations $f_1(\mathbf{x}_1), f_2(\mathbf{x}_2)$ for each view that maximize correlation:

$$\mathbf{corr}(f_1(\mathbf{x}_1), f_2(\mathbf{x}_2)) = \frac{\mathbf{cov}(f_1(\mathbf{x}_1), f_2(\mathbf{x}_2))}{\sqrt{\mathbf{var}(f_1(\mathbf{x}_1)) \cdot \mathbf{var}(f_2(\mathbf{x}_2))}}$$

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Finding correlated representations can be **useful** for

— Gaining insights into the data

— Detecting of asynchrony in test data

— Removing noise uncorrelated across views

— Translation or retrieval across views

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Has been **applied widely** to problems in computer vision, speech, NLP, medicine, chemometrics, metrology, neurology, etc.

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CCA: Canonical Correlation Analysis

Classical technique to find **linear** correlated representations, i.e.,

$$f_1(\mathbf{x}_1) = \mathbf{W}_1^T \mathbf{x}_1 \qquad \mathbf{W}_1 \in \mathbb{R}^{d_1 \times k}$$

where

$$f_2(\mathbf{x}_2) = \mathbf{W}_2^T \mathbf{x}_2 \qquad \mathbf{W}_2 \in \mathbb{R}^{d_2 \times k}$$

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The first columns $(\mathbf{w}_{1,:1}, \mathbf{w}_{2,:1})$ of the matrices $\mathbf{W}_1$ and $\mathbf{W}_2$ are found to maximize the **correlation of the projections**:

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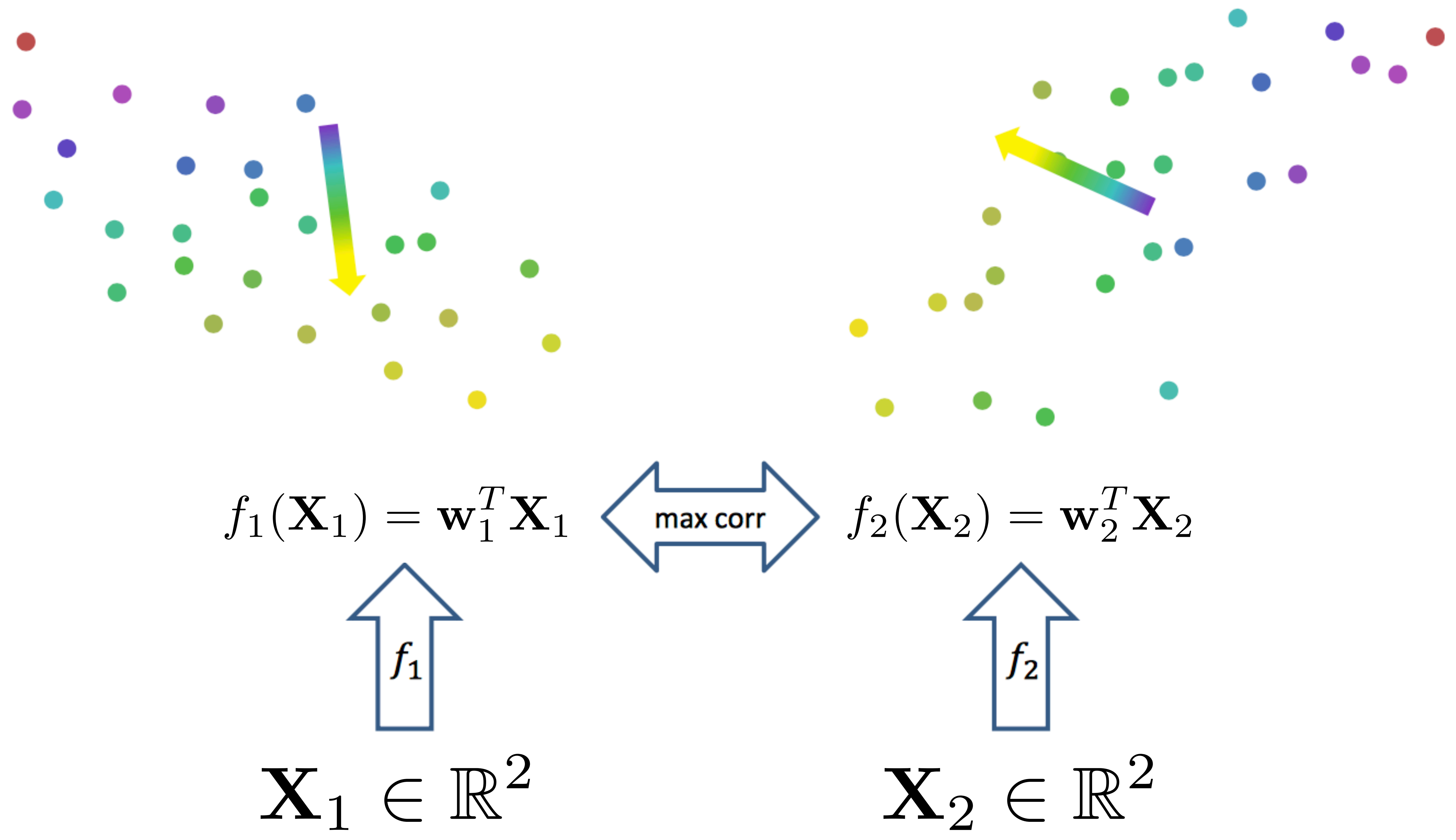
Subsequent pairs are constrained to be **uncorrelated with previous components** (i.e., for $j < i$)

$$\mathbf{corr}(\mathbf{w}_{1,:i}^T \mathbf{X}_1, \mathbf{w}_{1,:j}^T \mathbf{X}_1) = \mathbf{corr}(\mathbf{w}_{2,:i}^T \mathbf{X}_2, \mathbf{w}_{2,:j}^T \mathbf{X}_2) = 0$$

*slide from Andrew, Arora, Bilmes, Livescu

CCA Illustration



Two views of each instance have the same color

*slide from Andrew, Arora, Bilmes, Livescu

CCA: Canonical Correlation Analysis

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4. The optimal projection matrices are:

$$\mathbf{W}_1^* = \Sigma_{11}^{-1/2}\mathbf{U}_k$$

$$\mathbf{W}_2^* = \Sigma_{11}^{-1/2}\mathbf{V}_k$$

where $\mathbf{U}_k$ is the first k columns of $\mathbf{U}$.

*slide from Andrew, Arora, Bilmes, Livescu

KCCA: Kernel CCA

There maybe **non-linear** functions $f_1(\mathbf{x}_1), f_2(\mathbf{x}_2)$ that produce more highly correlated (better) representations than linear projections

Kernel CCA is a principal method for finding such function

— Learns functions from any reproducing kernel Hilbert space

— May use different kernels for each view

Using **RBF** (Gaussian) kernel in KCCA is akin to finding sets of instances that form clusters in both views

*slide from Andrew, Arora, Bilmes, Livescu

KCCA vs. **CCA**

Pros:

— More complex function space of KCCA can yield dramatically higher correlations

Cons:

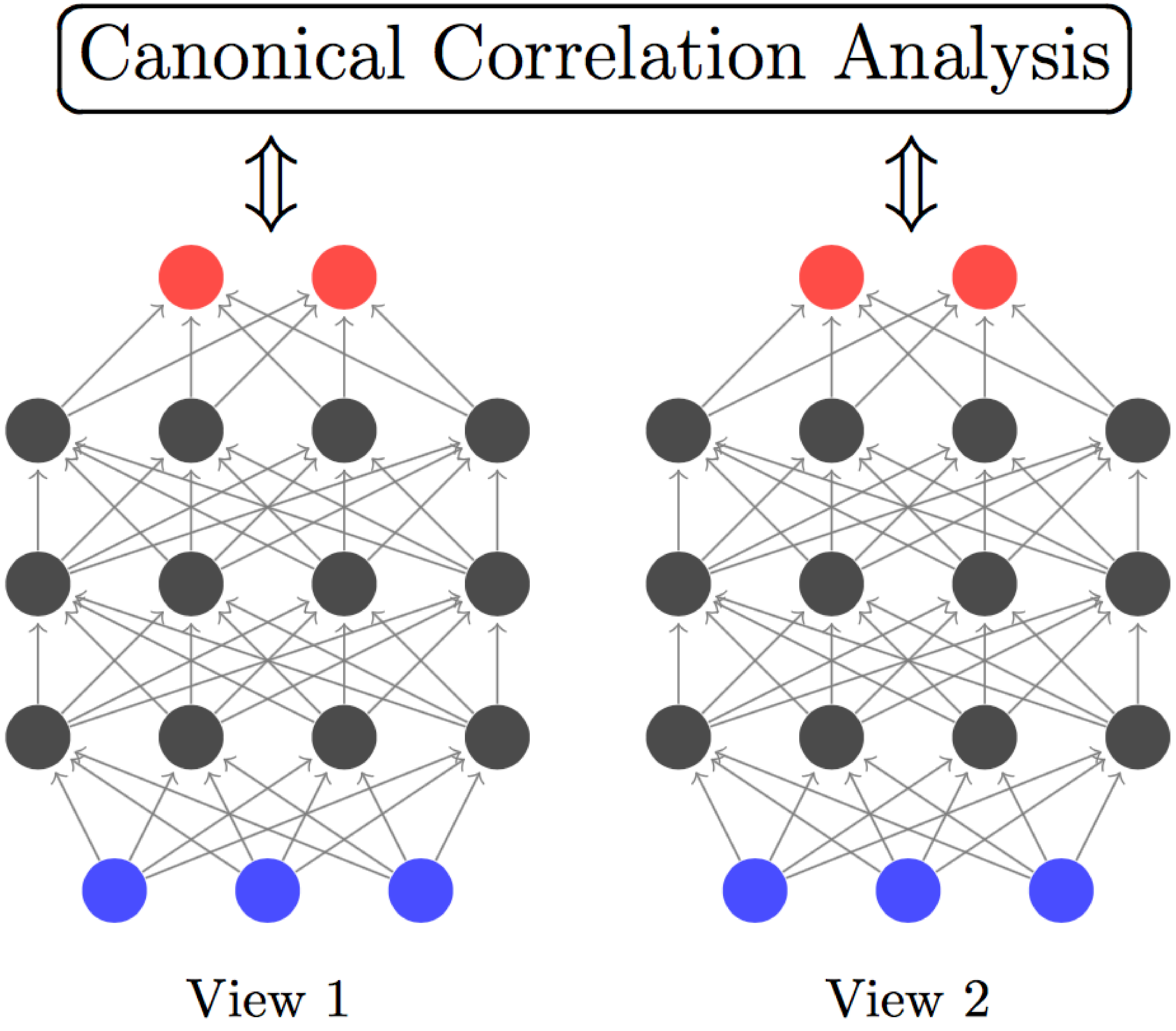
— KCCA is slower to train

— For KCCA training set must be stored and referenced at test time

— KCCA model is more difficult to interpret

*slide from Andrew, Arora, Bilmes, Livescu

Deep CCA



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Benefits of Deep CCA

Pros:

- Better suited for natural, real-world data
- **Parametric model**
  - The training set can be disregarded once the model is learned
  - Computational speed at test time is fast

*slide from Andrew, Arora, Bilmes, Livescu

Deep CCA: Training

Training a Deep CCA model:

1. **Pretrain** the layers of **each side** individually

2. **Jointly fine-tune** all parameters to maximize the total correlation of the output layers. Requires computing correlation gradient:
   - Forward propagate activations on both sides.
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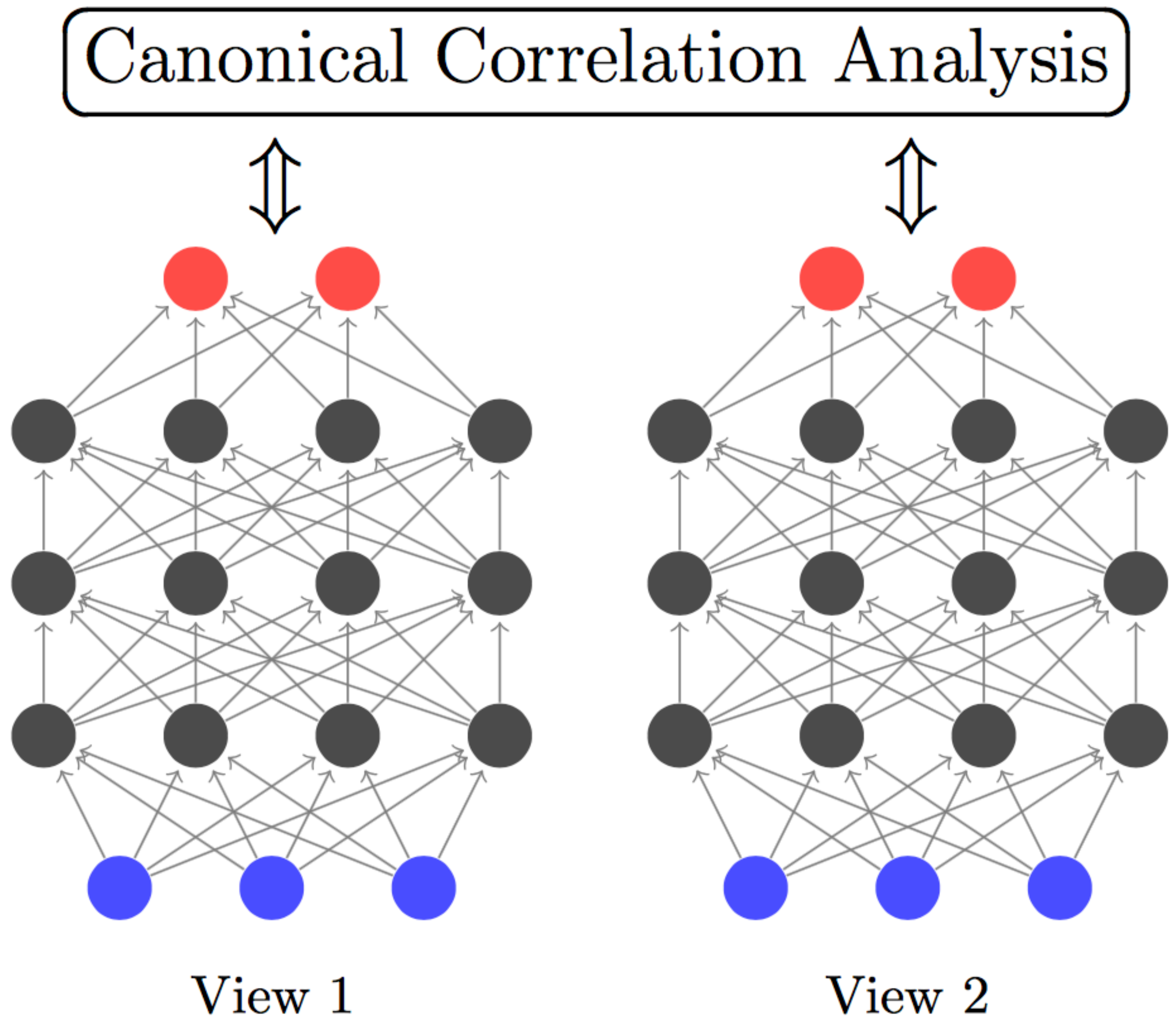

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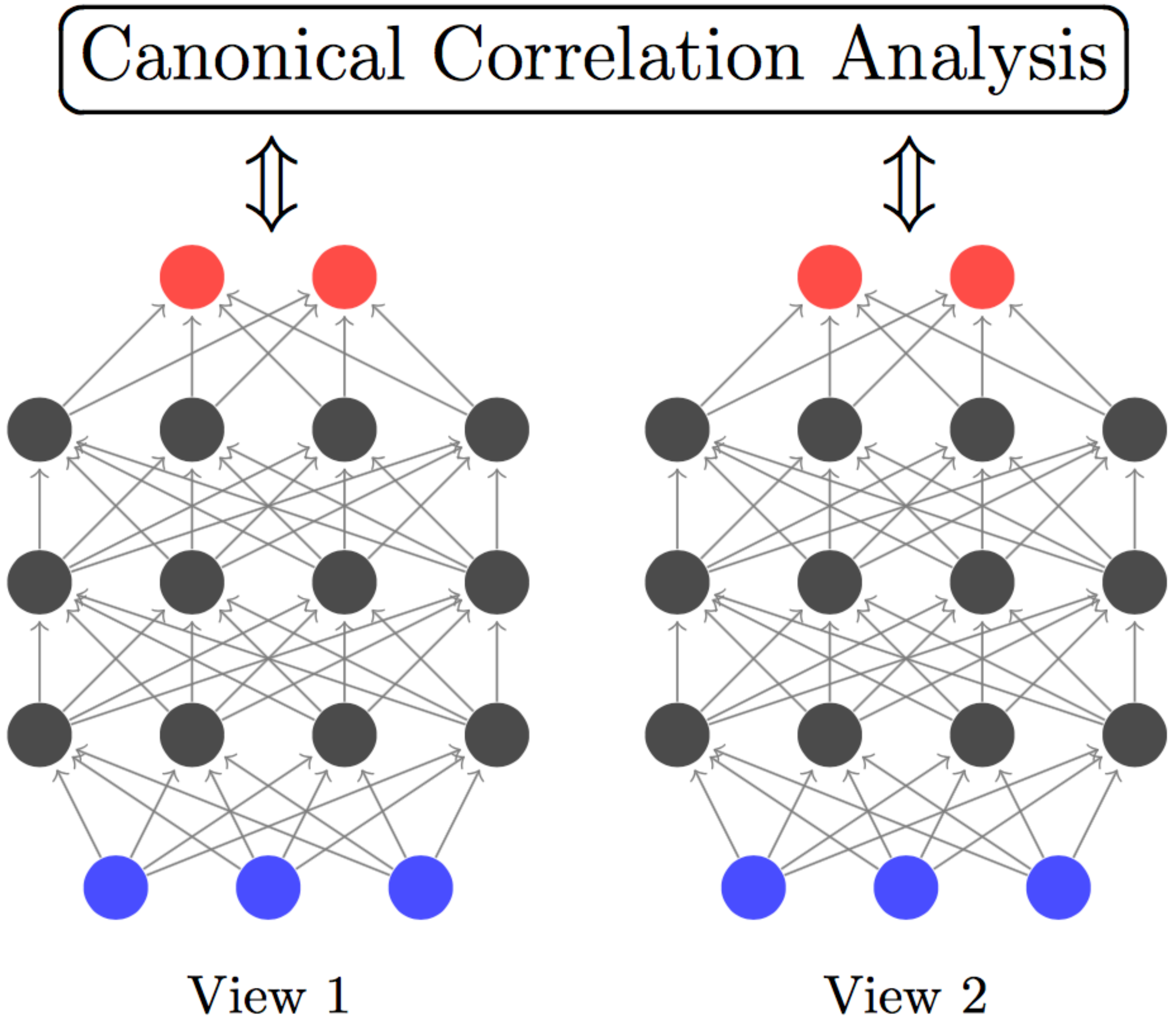

Correlation is a population objective, so instead of one instance (or minibatch) training, requires L-BFGS second-order method (with full-batch)

*slide from Andrew, Arora, Bilmes, Livescu

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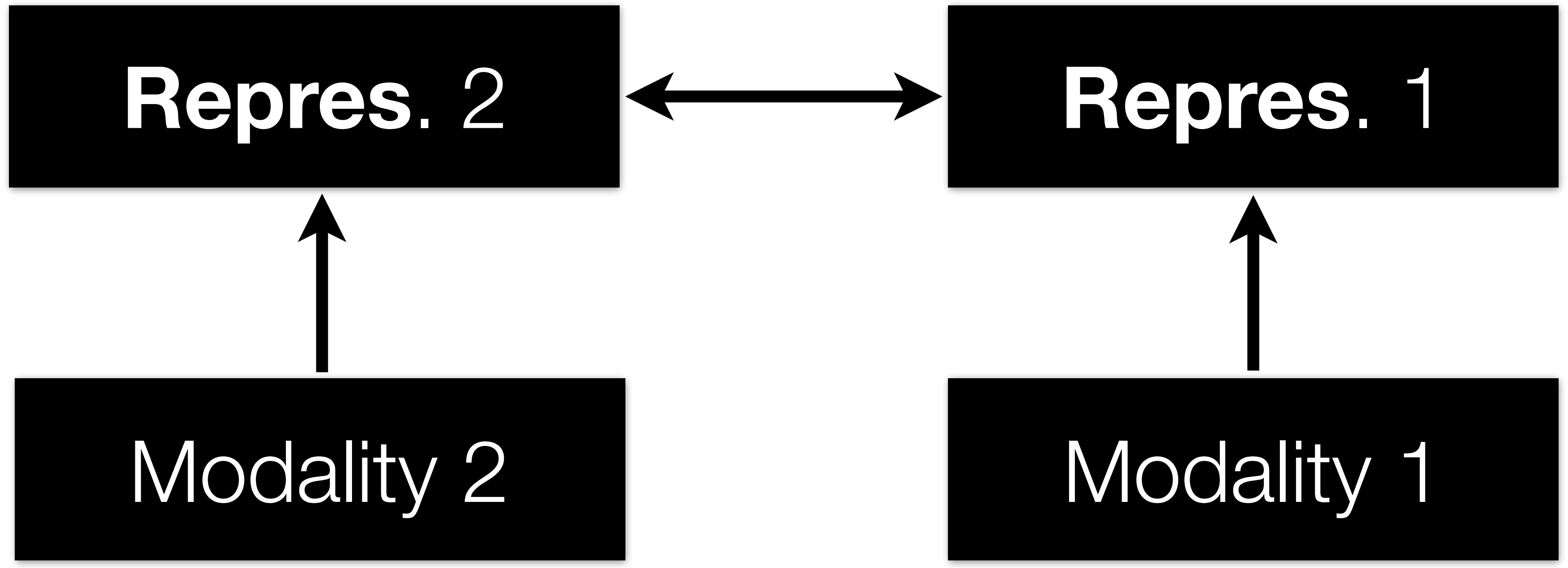

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Correlated Representations vs. **Joint Embeddings**

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Joint Embeddings: Models that minimize distance between ground truth pairs of samples:

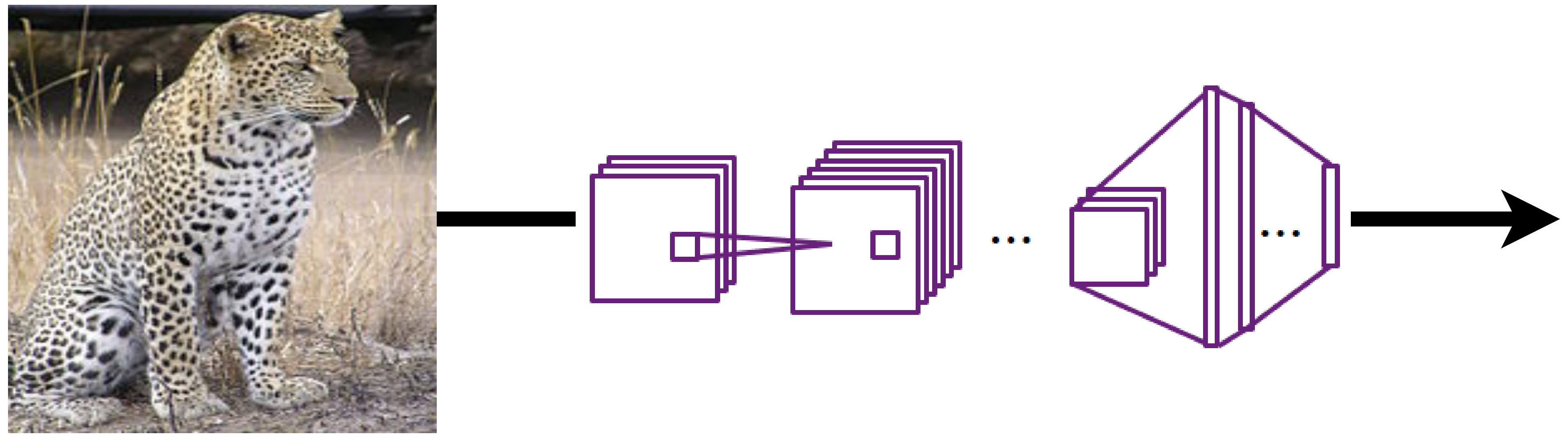
$$min_{f_1,f_2} D\left(f_1(\mathbf{x}_1^{(i)}), f_2(\mathbf{x}_2^{(i)})\right)$$

Object **Classification**

Category	Prediction
Dog	No
Cat	No
Couch	No
Flowers	No
Leopard	**Yes**
…	…

Problem: For each image predict which category it belongs to out of a fixed set

Object **Classification**



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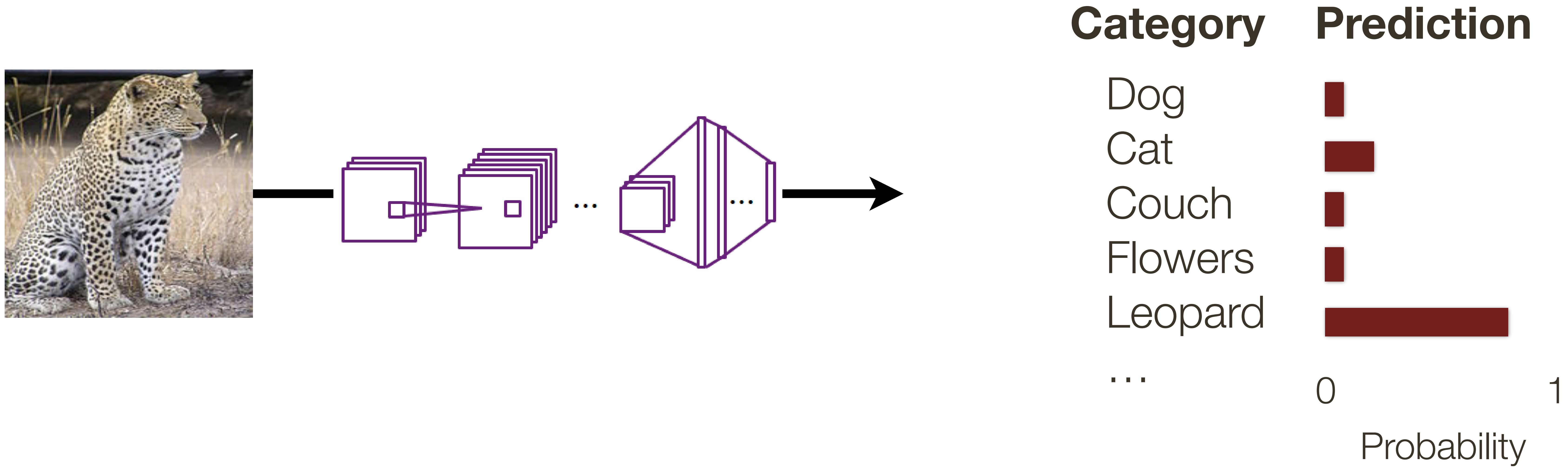

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Discriminative Embeddings

Images and **class labels** are embedded into the same space

$\mathbb{R}^d$

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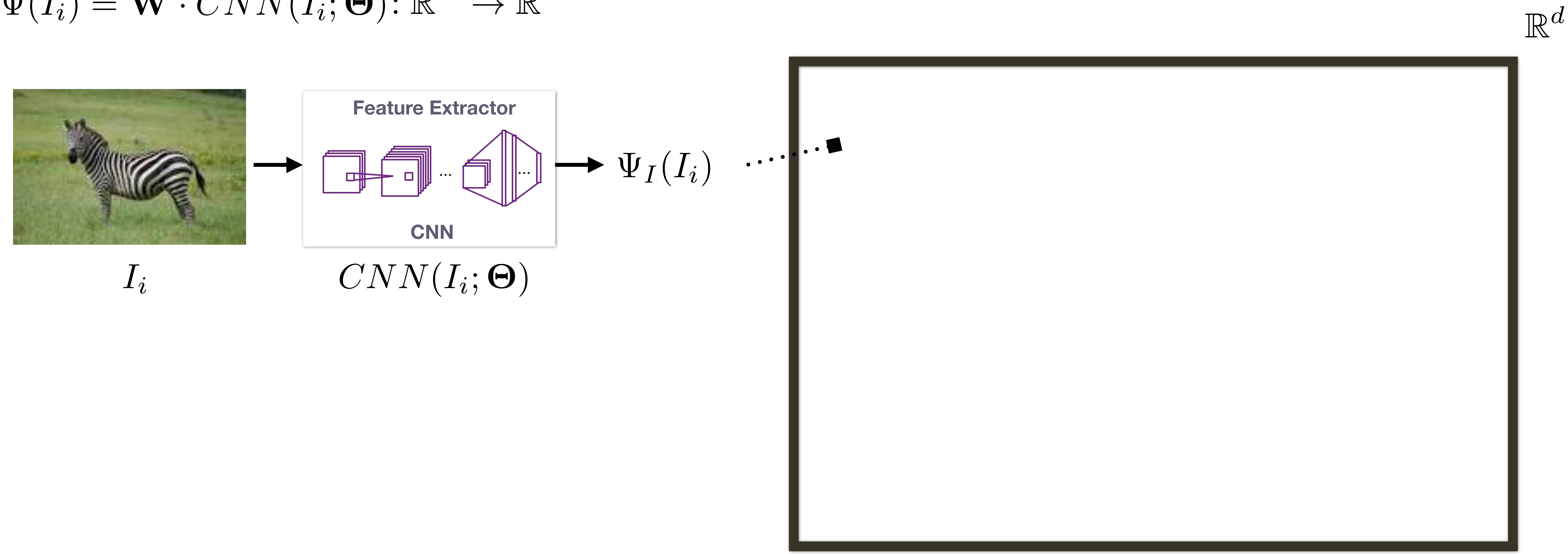
Image Embedding

$$\Psi(I_i) = \mathbf{W} \cdot CNN(I_i; \mathbf{\Theta}) \colon \mathbb{R}^D \rightarrow \mathbb{R}^d$$



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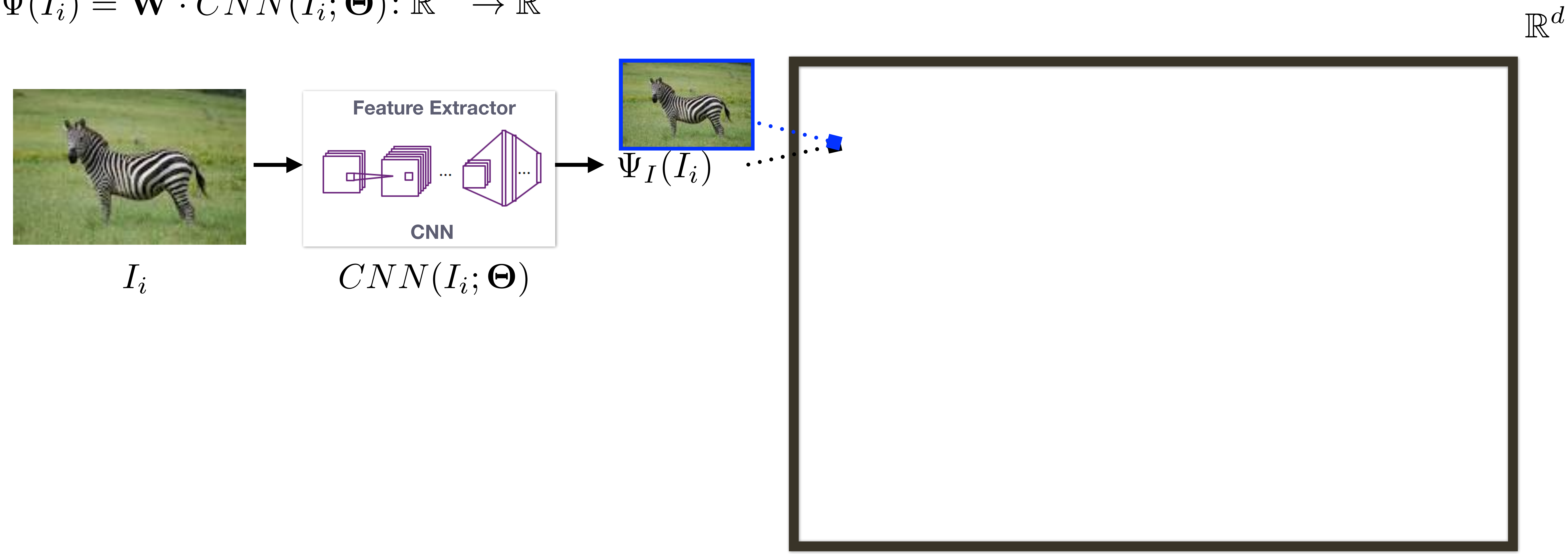

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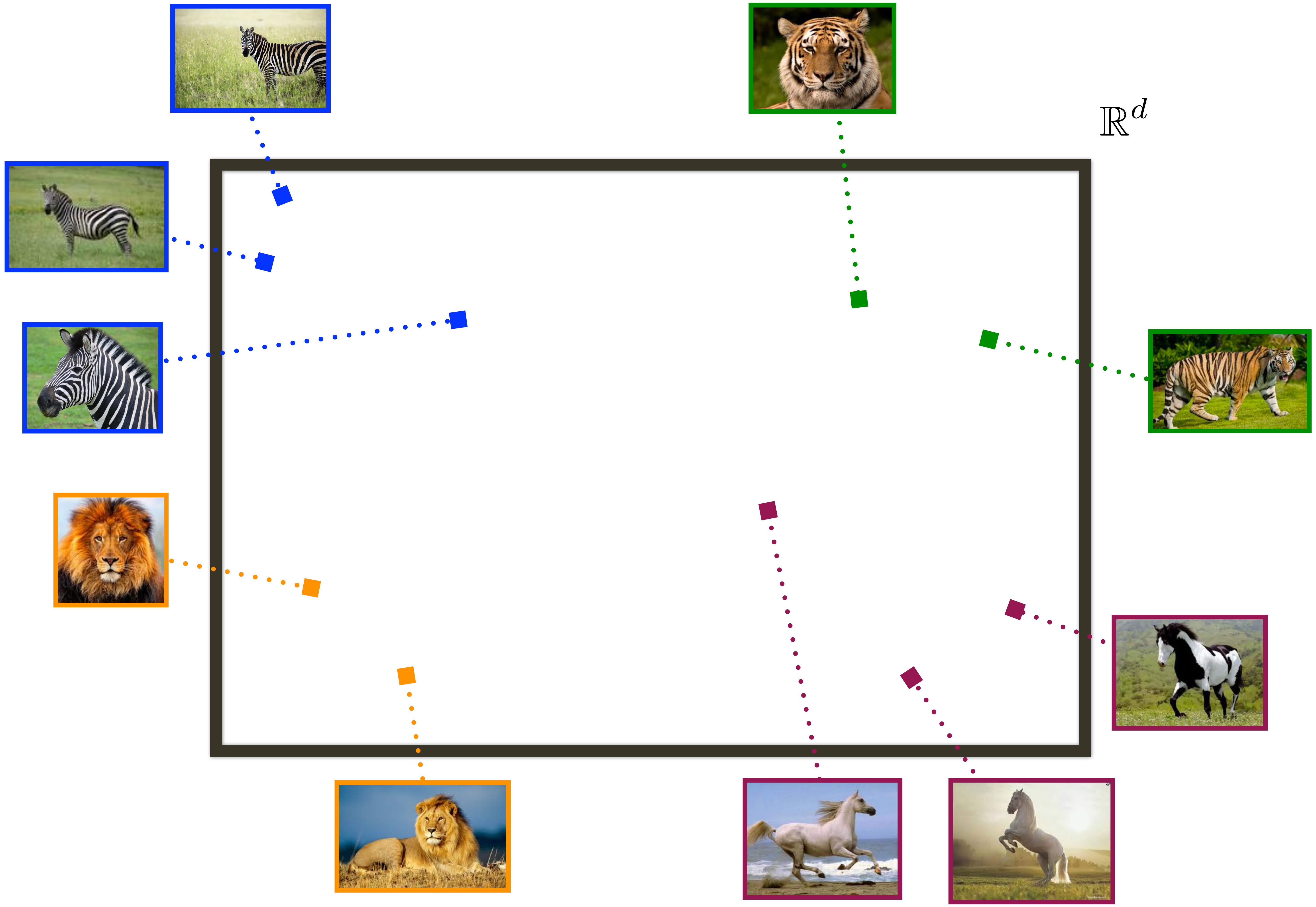

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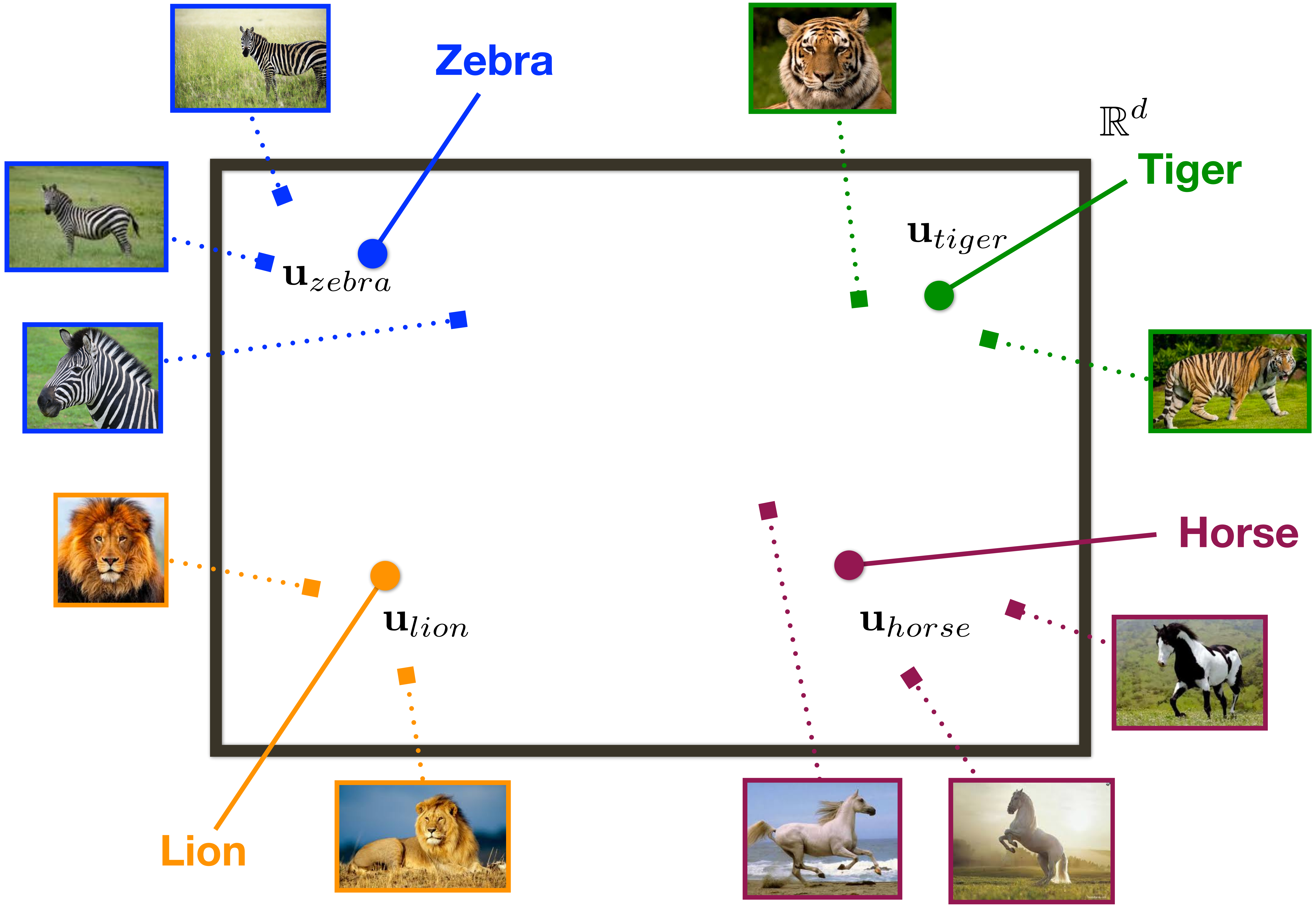
Label Embedding

$$\Psi_L(word_i) = \mathbf{u}_i : \{1, ..., L\} \rightarrow \mathbb{R}^d$$



Discriminative Embeddings

Images and **class labels** are embedded into the same space

Image Embedding

$$\Psi(I_i) = \mathbf{W} \cdot CNN(I_i; \mathbf{\Theta}) \colon \mathbb{R}^D \rightarrow \mathbb{R}^d$$

Label Embedding

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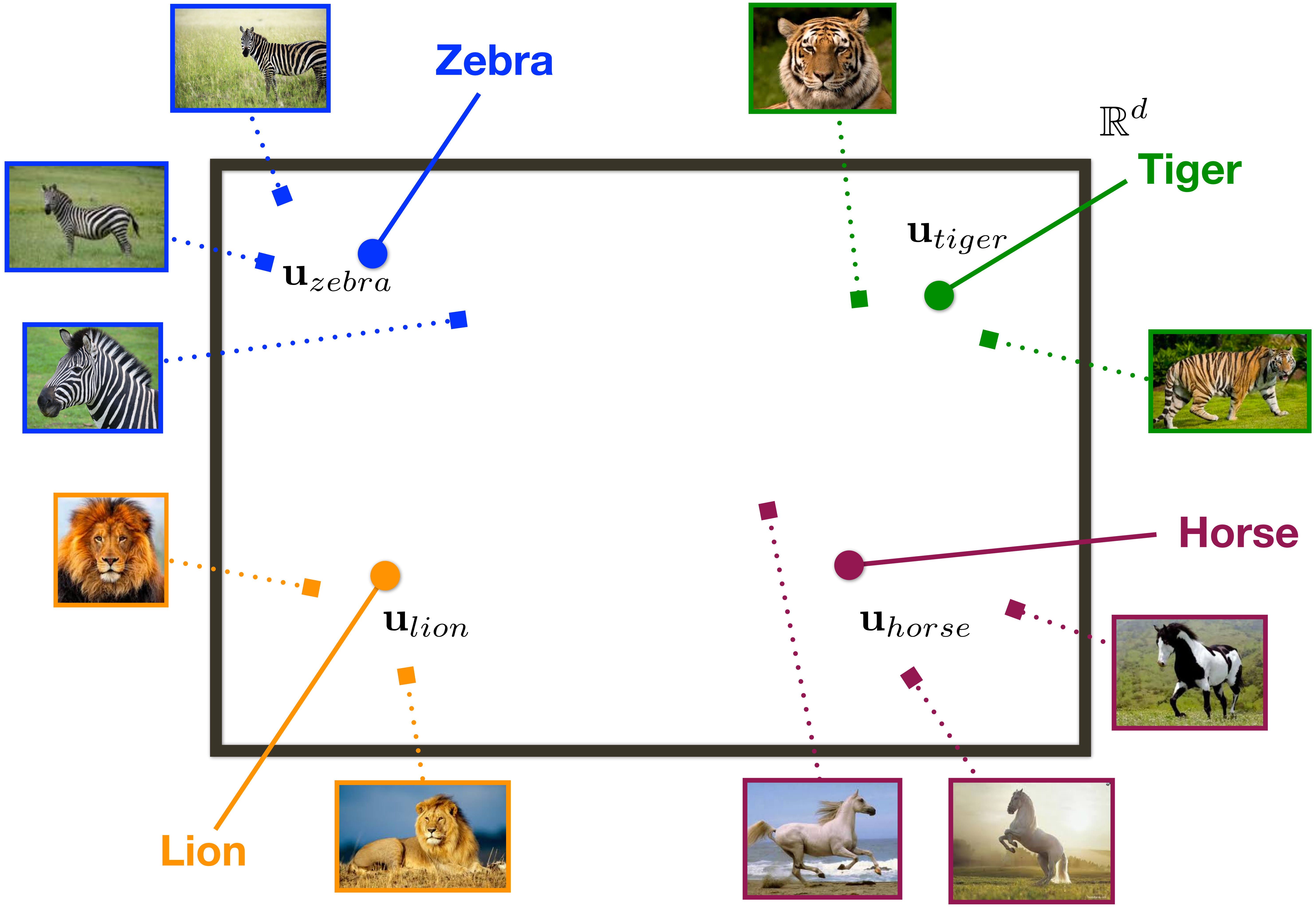
Similarity in Embedding Space

$$D(\mathbf{u}, \mathbf{u}') = ||\mathbf{u} - \mathbf{u}'||_2^2$$



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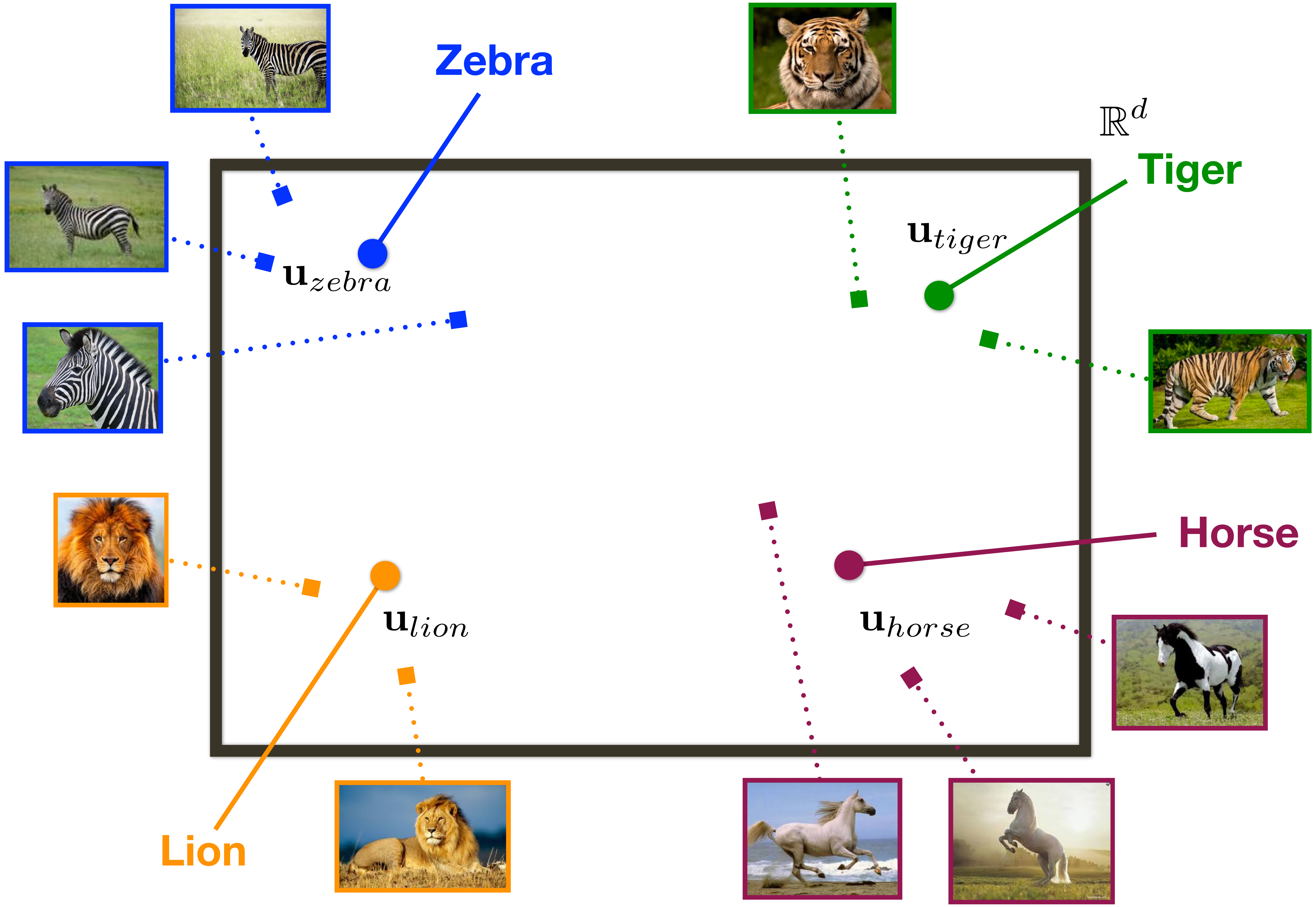

Discriminative Embeddings

Image Categorization / Annotation

which object category does image belong to?

Image Embedding

$$\Psi(I_i) = \mathbf{W} \cdot CNN(I_i; \mathbf{\Theta}) : \mathbb{R}^D \rightarrow \mathbb{R}^d$$

Label Embedding

$$\Psi_L(word_i) = \mathbf{u}_i : \{1, ..., L\} \rightarrow \mathbb{R}^d$$

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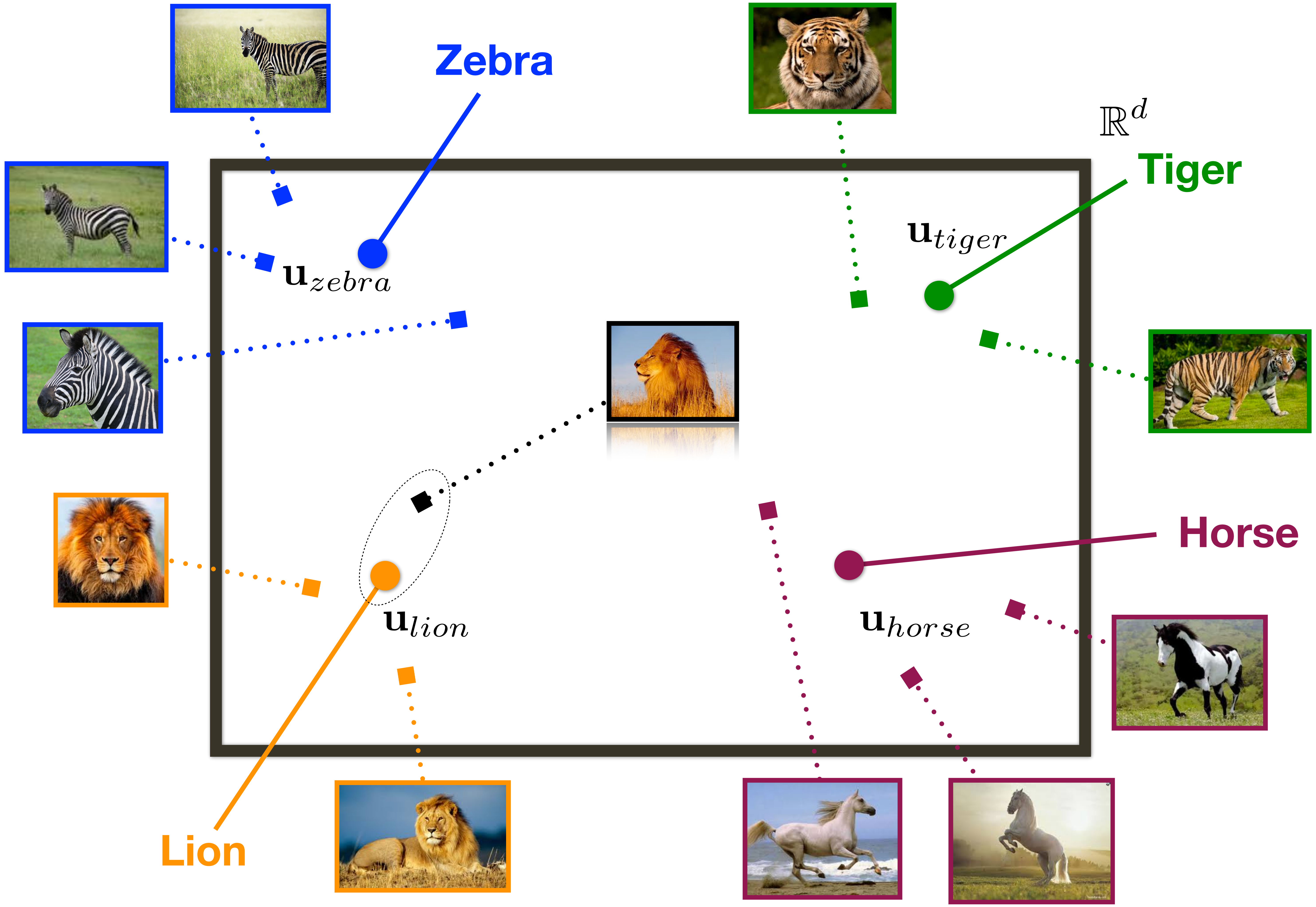

Discriminative Embeddings

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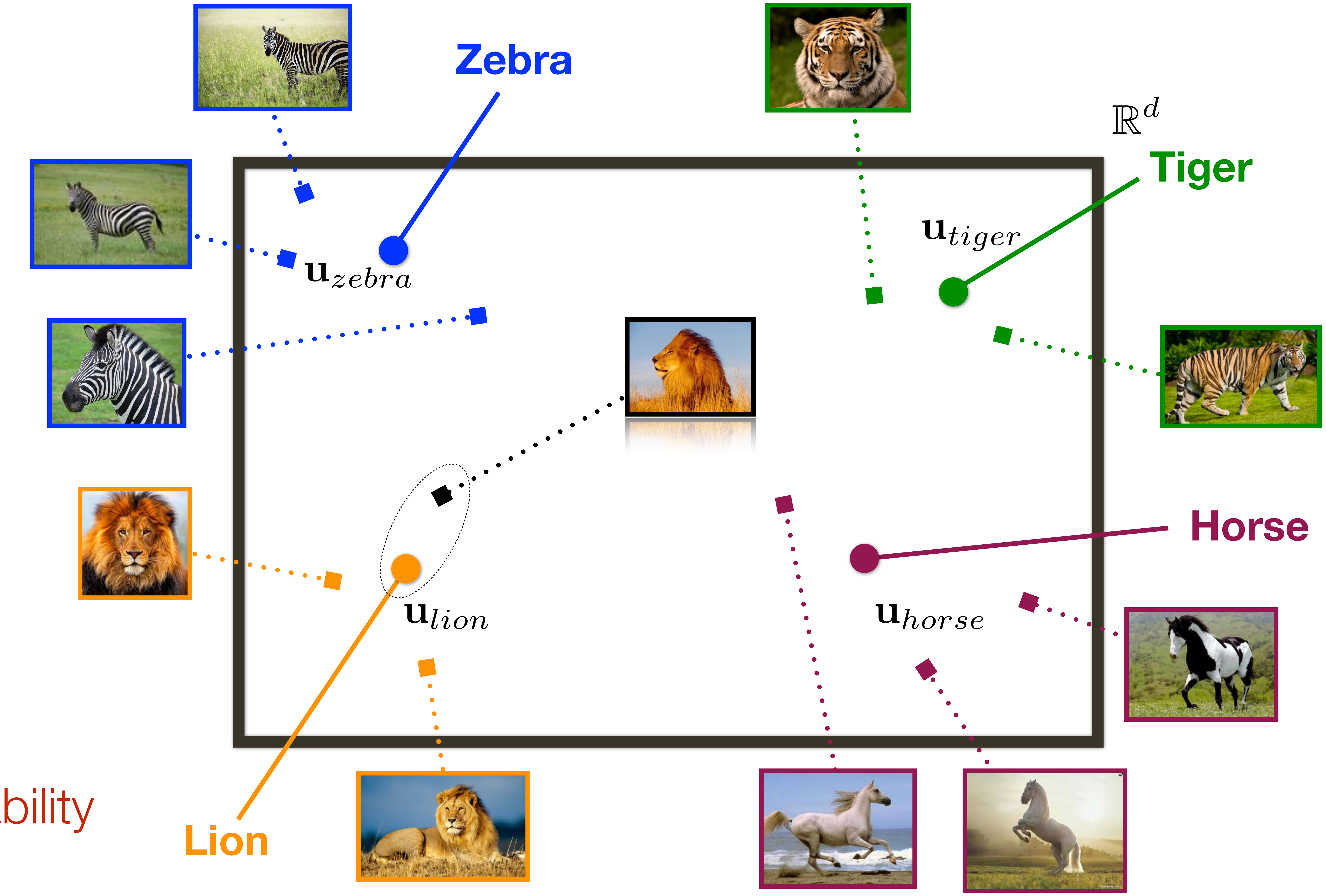
Similarity in Embedding Space

$$D(\mathbf{u}, \mathbf{u}') = ||\mathbf{u} - \mathbf{u}'||_2^2$$

Distance can be interpreted as probability

Image Categorization / Annotation

which object category does image belong to?



Discriminative Embeddings

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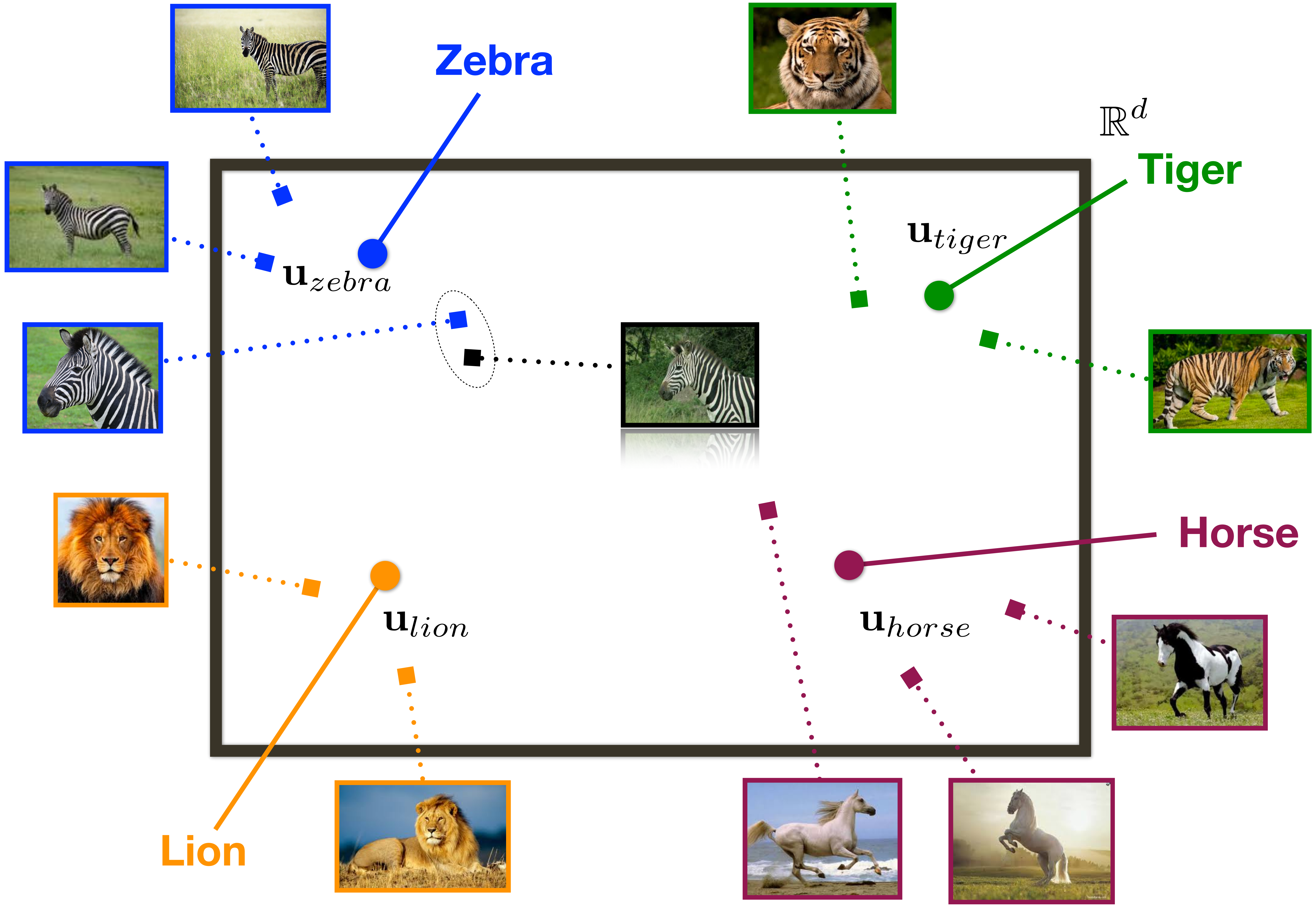
Similarity in Embedding Space

$$D(\mathbf{u}, \mathbf{u}') = ||\mathbf{u} - \mathbf{u}'||_2^2$$

Search by Image

most similar image to a query?



Discriminative Embeddings

Image Embedding

$$\Psi(I_i) = \mathbf{W} \cdot CNN(I_i; \mathbf{\Theta}) \colon \mathbb{R}^D \rightarrow \mathbb{R}^d$$

Label Embedding

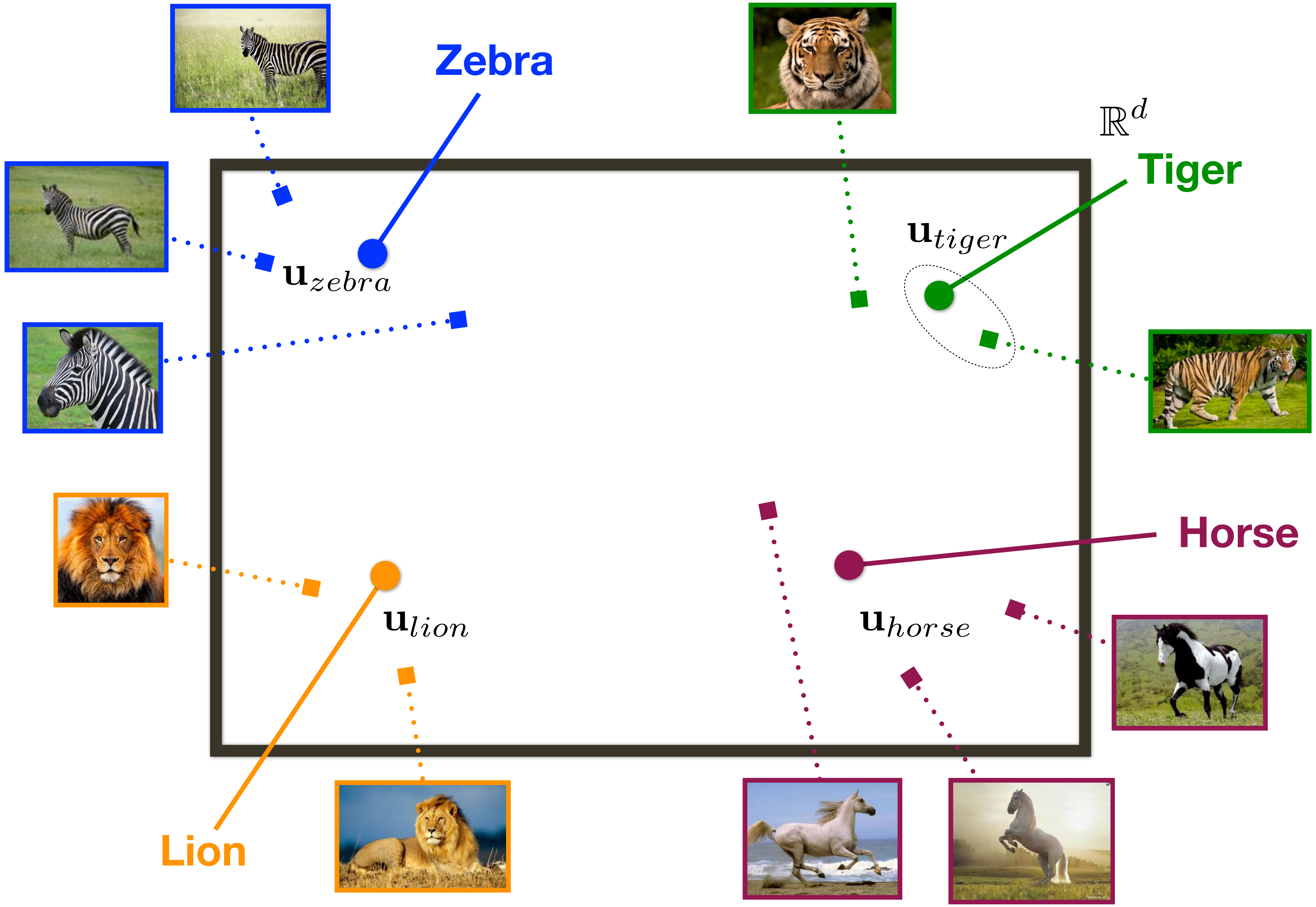
$$\Psi_L(word_i) = \mathbf{u}_i : \{1, ..., L\} \rightarrow \mathbb{R}^d$$

Similarity in Embedding Space

$$D(\mathbf{u}, \mathbf{u}') = ||\mathbf{u} - \mathbf{u}'||_2^2$$

Search by Label

most representative image for a label?



Discriminative Embeddings

Why not minimize distance directly?

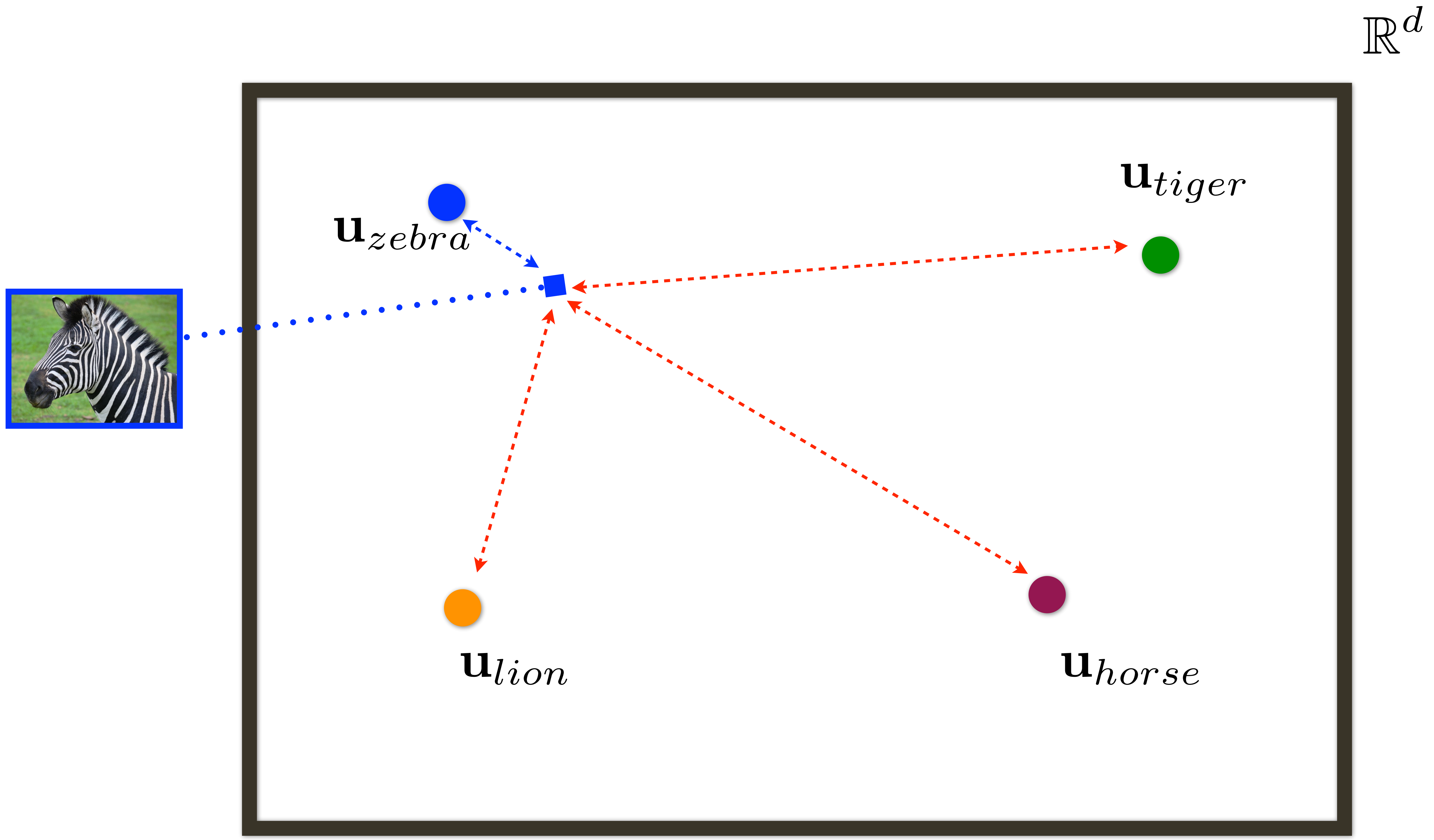
$$\mathcal{L}_C(\mathbf{W}, \mathbf{U}, I_i, y_i) = \sum [1 + D(\Psi(I_i), \mathbf{u}_{y_i}) - D(\Psi(I_i), \mathbf{u}_{y_c})]$$

Image Embedding

$$\Psi(I_i) = \mathbf{W} \cdot CNN(I_i; \boldsymbol{\Theta}) : \mathbb{R}^D \rightarrow \mathbb{R}^d$$

Label Embedding

$$\Psi_L(word_i) = \mathbf{u}_i : \{1, ..., L\} \rightarrow \mathbb{R}^d$$



Similarity in Embedding Space

$$D(\mathbf{u}, \mathbf{u}') = ||\mathbf{u} - \mathbf{u}'||_2^2$$

Objective Function:

$$\min_{\mathbf{W}, \mathbf{U}} \sum_i^N \mathcal{L}_C(\mathbf{W}, \mathbf{U}, I_i, y_i) + \lambda_1 ||\mathbf{W}||_F^2 + \lambda_2 ||\mathbf{U}||_F^2$$

[Bengio *et al*.,, NIPS'10]

[Weinberger, Chapelle, NIPS'09]

Discriminative Embeddings

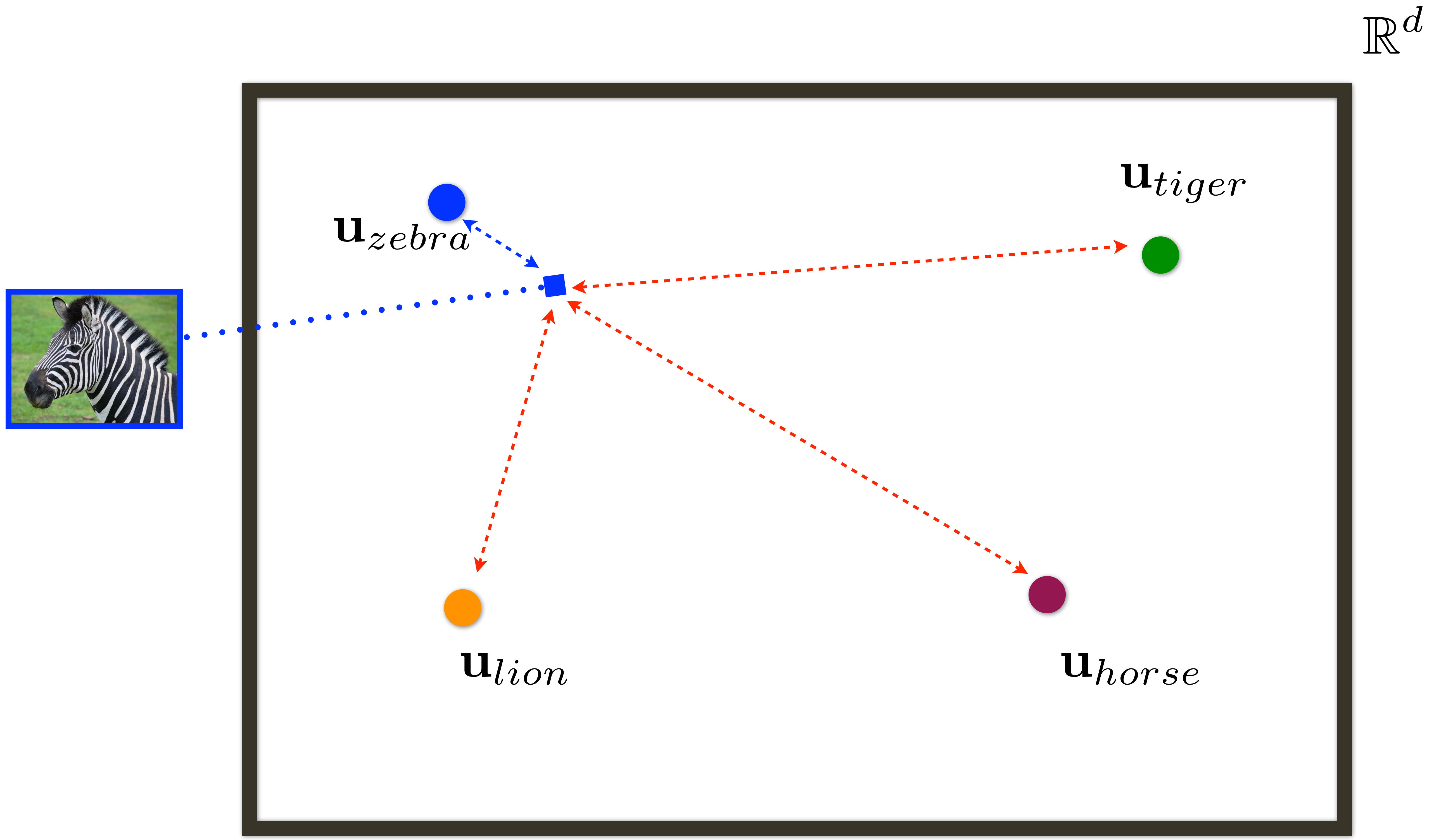
$$\mathcal{L}_C(\mathbf{W}, \mathbf{U}, I_i, y_i) = \sum max\{0, \alpha - \underline{D(\Psi(I_i), \mathbf{u}_{y_i})} + \underline{D(\Psi(I_i), \mathbf{u}_{y_c})}\}$$

Image Embedding

$$\Psi(I_i) = \mathbf{W} \cdot CNN(I_i; \boldsymbol{\Theta}) \colon \mathbb{R}^D \rightarrow \mathbb{R}^d$$

Label Embedding

$$\Psi_L(word_i) = \mathbf{u}_i : \{1, ..., L\} \rightarrow \mathbb{R}^d$$



Similarity in Embedding Space

$$D(\mathbf{u}, \mathbf{u}') = \frac{\mathbf{u}}{||\mathbf{u}||} \cdot \frac{\mathbf{u}'}{||\mathbf{u}'||}$$

Objective Function:

$$\min_{\mathbf{W}, \mathbf{U}} \sum_i^N \mathcal{L}_C(\mathbf{W}, \mathbf{U}, I_i, y_i) + \lambda_1 ||\mathbf{W}||_F^2 + \lambda_2 ||\mathbf{U}||_F^2$$

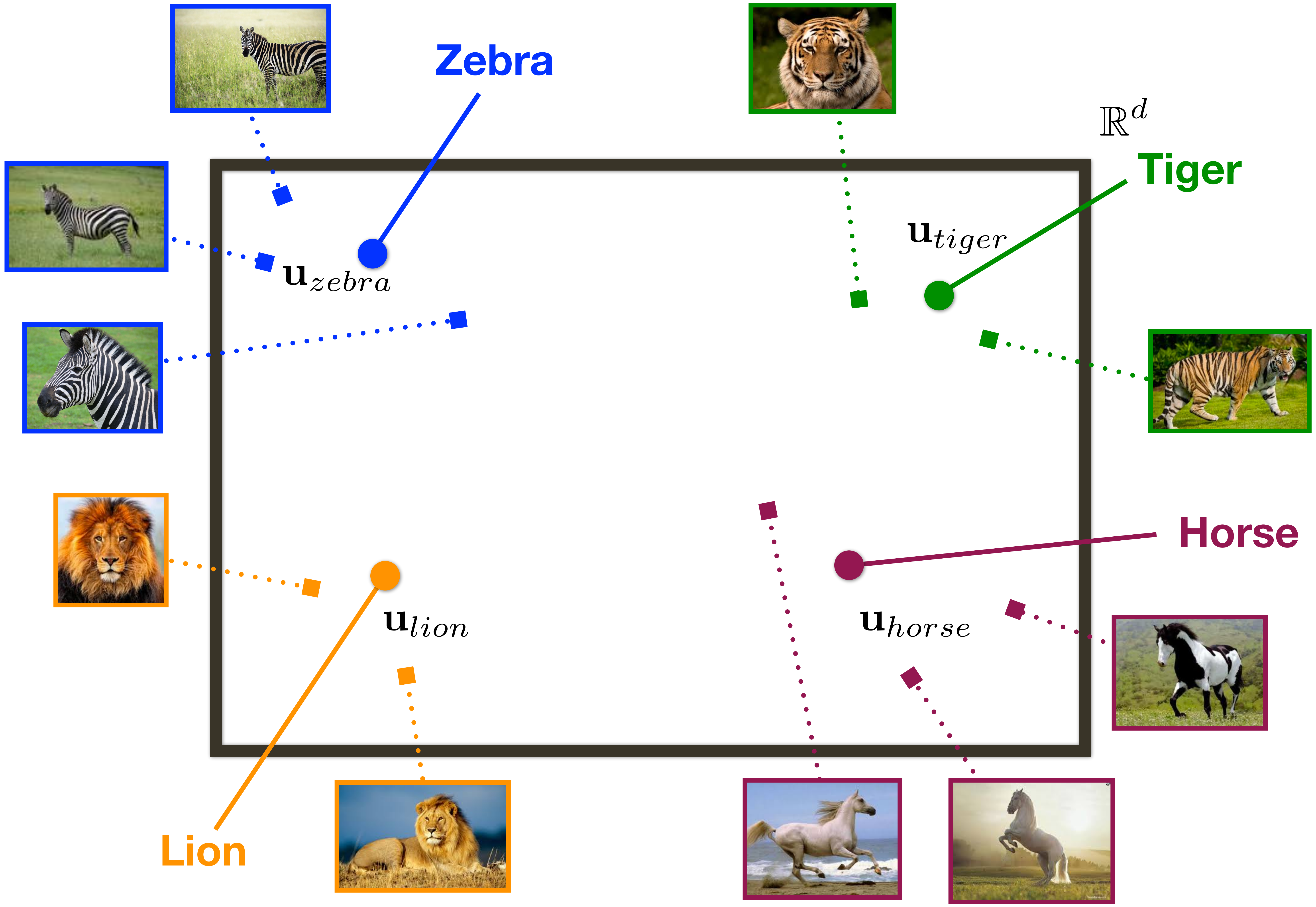
[Bengio *et al.*,, NIPS'10]

[Weinberger, Chapelle, NIPS'09]

Discriminative Embeddings

This is a very **convenient model**

Inducing semantics on the embedding space



Semantic Embeddings

Why adding **semantics is useful**?

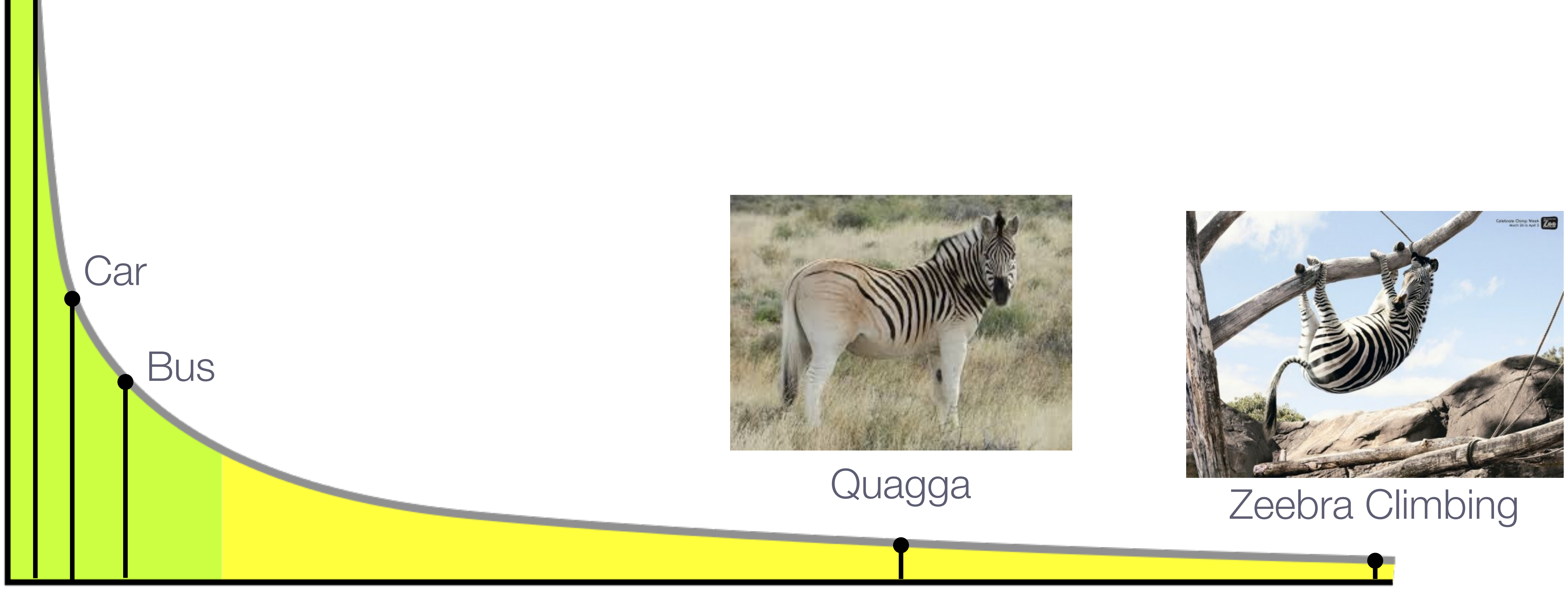
— Allows for transference of knowledge from classes that have a lot of data to those that have few (or no labeled instances)

— Can serve as additional regularization, so can be more efficient for learning.

Long Tail of Categories

Few most frequent categories contain most of the samples, most of the categories contain few samples

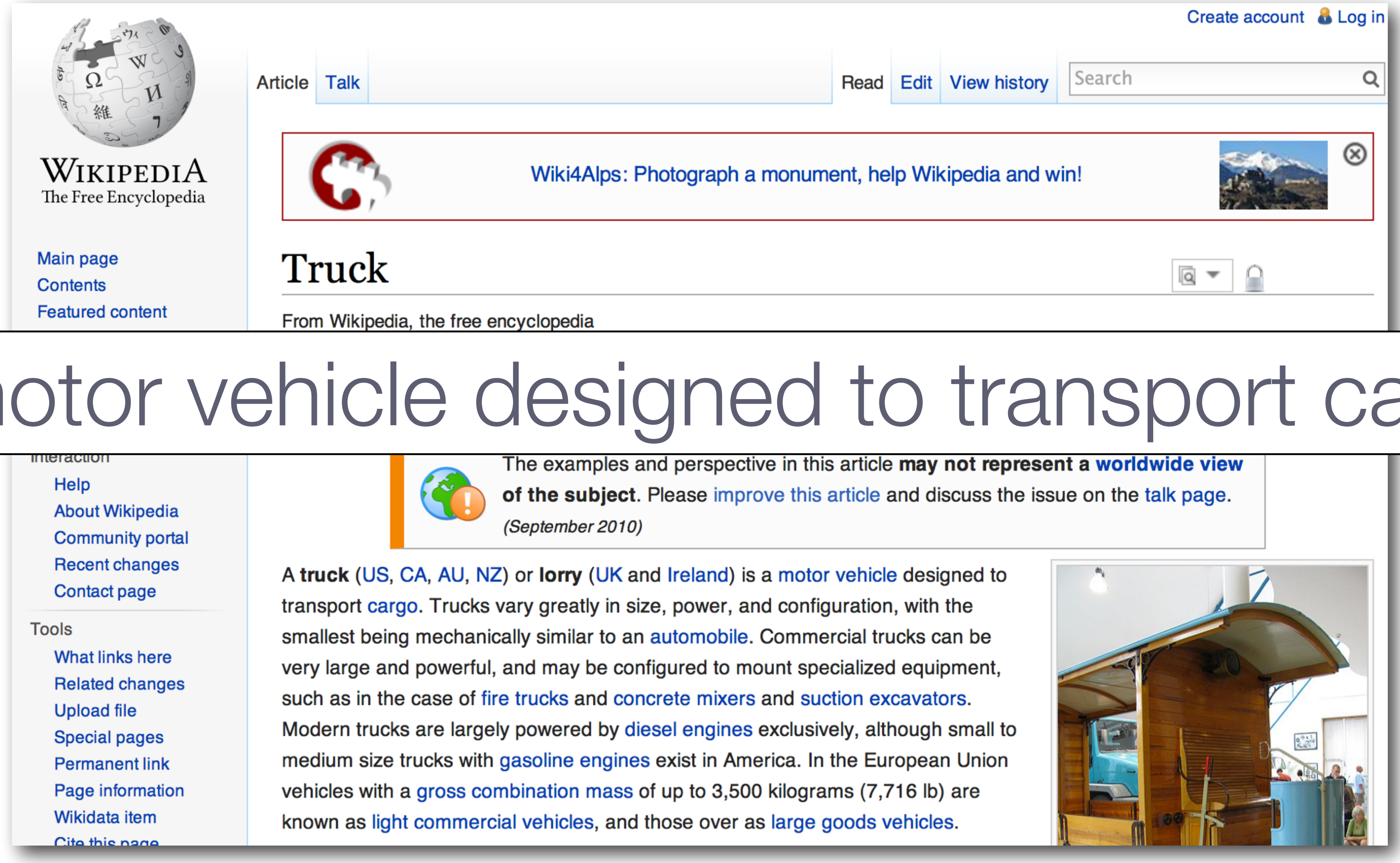
As granularity of categories increases, the amount of data per category decreases



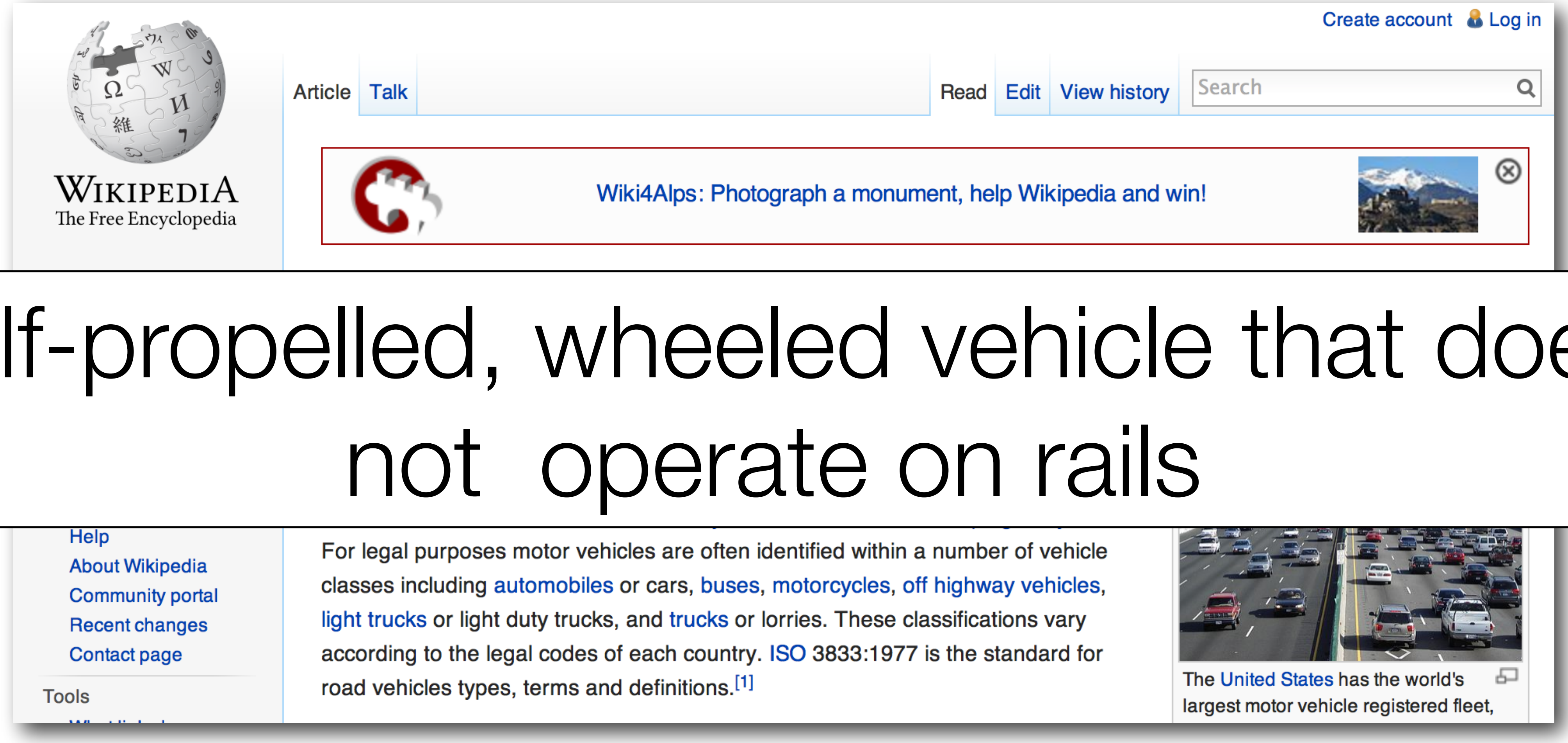
Inspiration from Human Structured Semantics

[Hwang et al., 2014]

Truck



motor vehicle designed to transport cargo



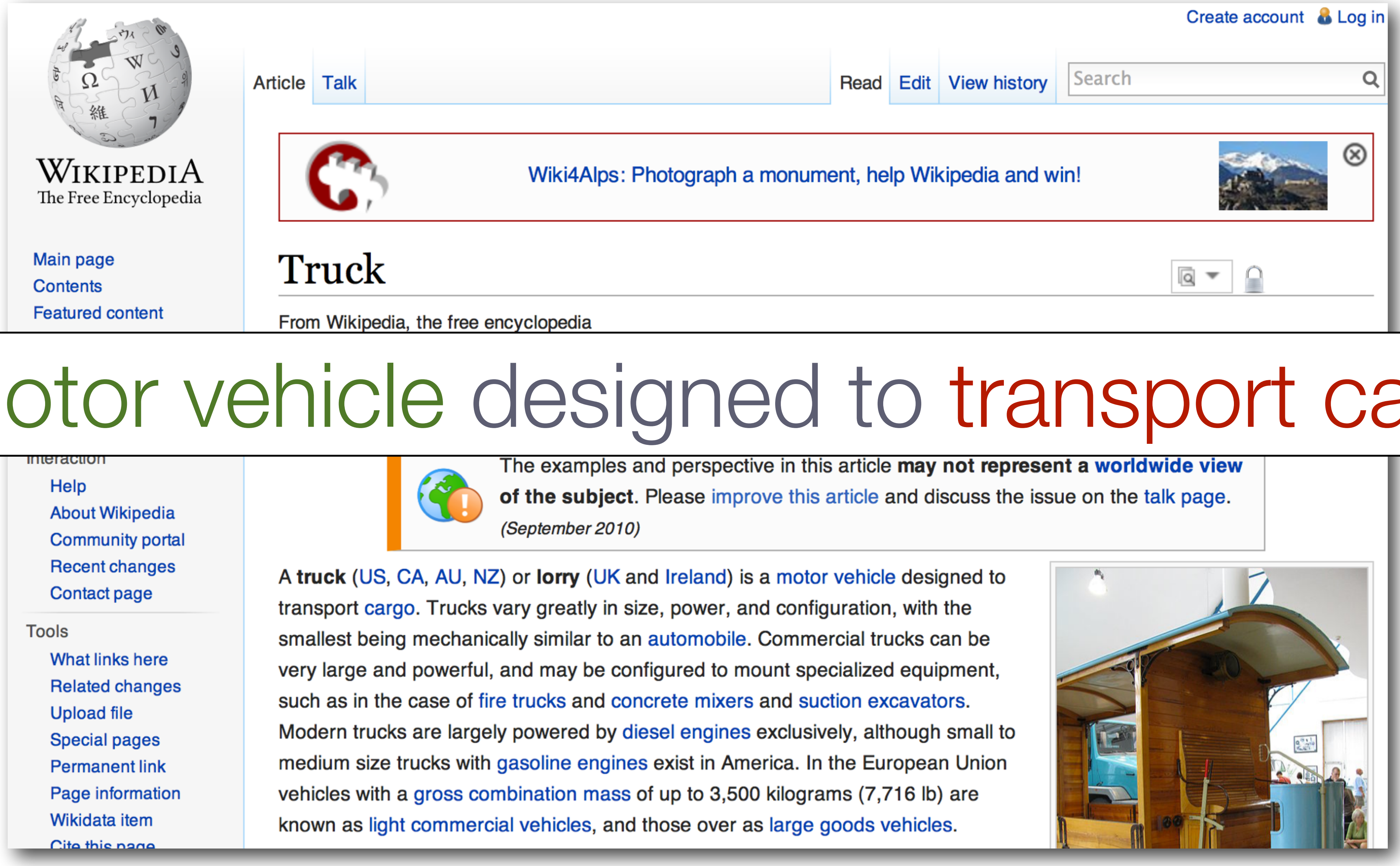
self-propelled, wheeled vehicle that does not operate on rails

Inspiration from Human Structured Semantics

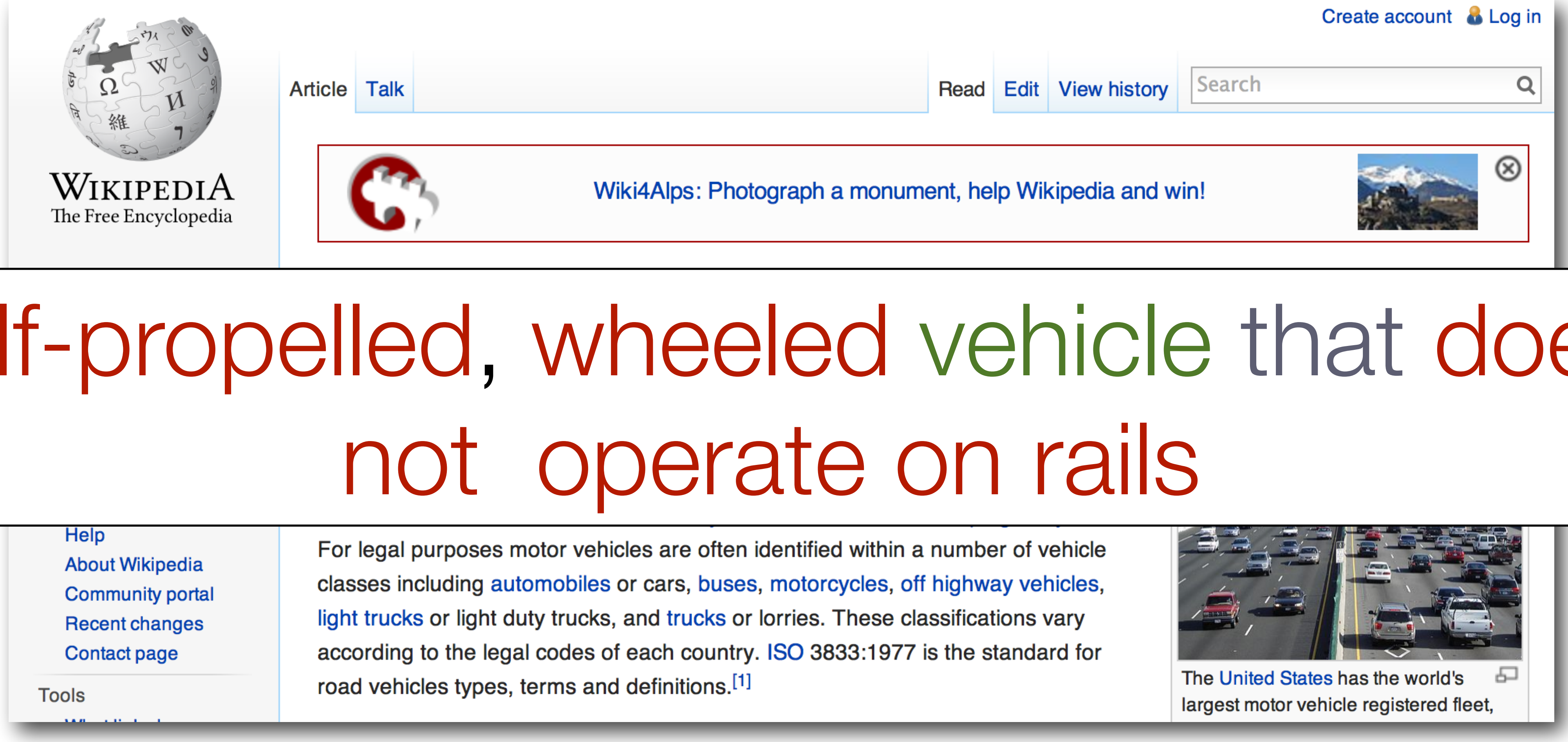
[Hwang et al., 2014]

Parent Category + Attributes

Truck

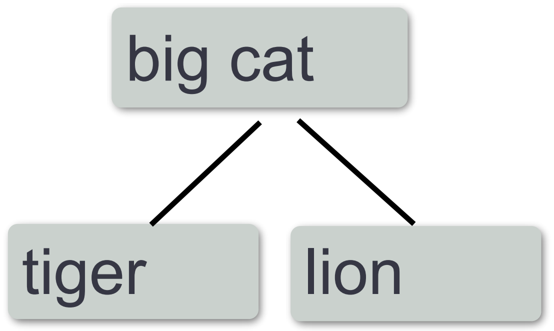


motor vehicle designed to transport cargo



self-propelled, wheeled vehicle that does not operate on rails

Unified Semantic Embedding



Adding regularization from **ontology / taxonomy** over labels

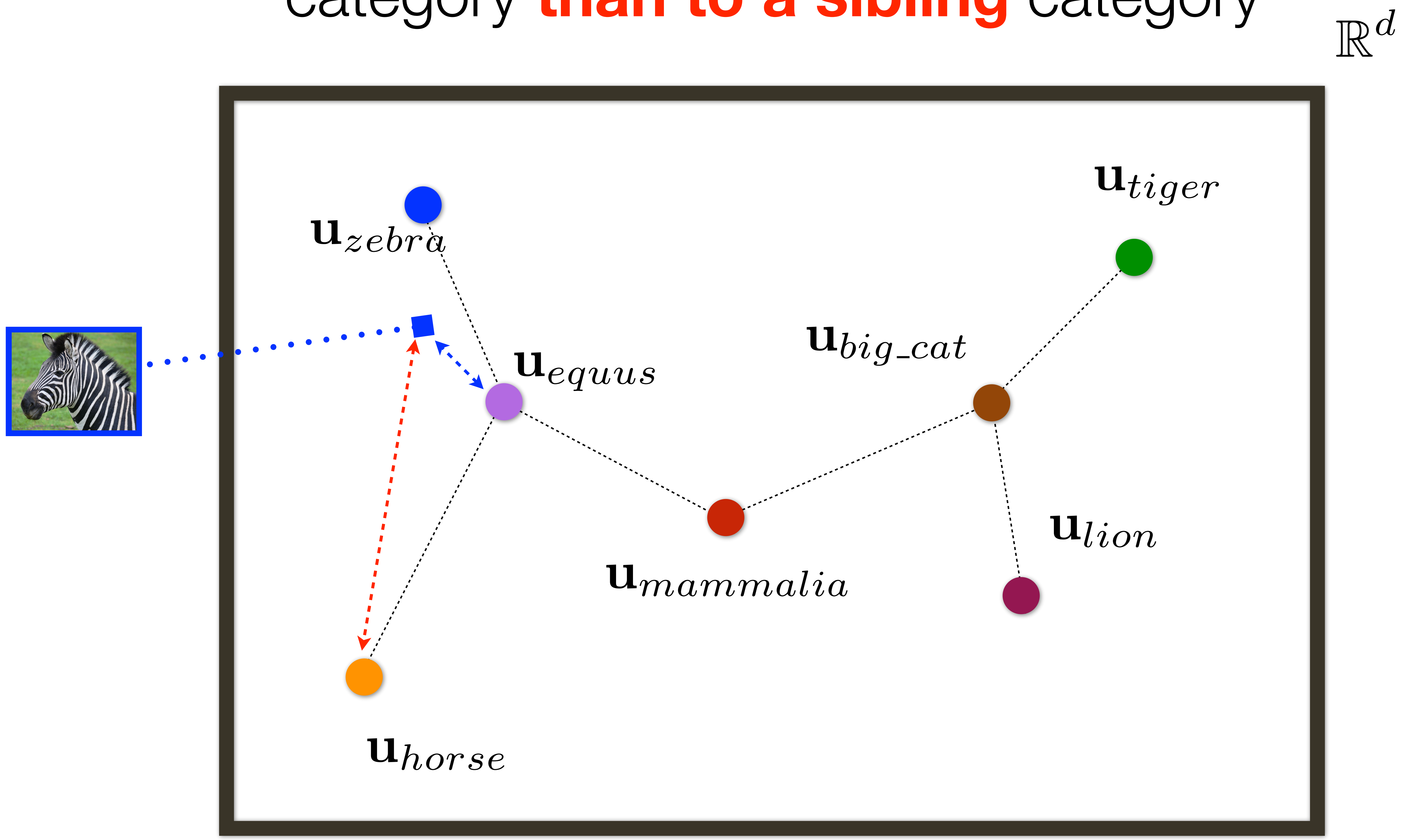
Image Embedding

$$\Psi_I(I_i) = \mathbf{W} \cdot CNN(I_i) : \mathbb{R}^D \to \mathbb{R}^d$$

Label Embedding

$$\Psi_L(word_i) = \mathbf{u}_i : \{1, ..., L\} \to \mathbb{R}^d$$

Each sample is **closer to the parent** category **than to a sibling** category



Similarity in Embedding Space

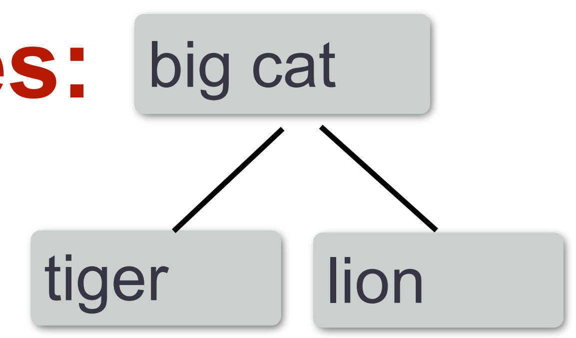
$$D(\mathbf{u}, \mathbf{u}') = ||\mathbf{u} - \mathbf{u}'||_2^2$$

Objective Function:

$$\min_{\mathbf{W},\mathbf{U},\mathcal{B}} \sum_i^N \mathcal{L}_C(\mathbf{W}, \mathbf{U}, I_i, y_i) + \lambda||\mathbf{W}||_F^2 + \lambda||\mathbf{U}||_F^2 + \mathcal{L}_S(\mathbf{W}, \mathbf{U}, I_i, y_i) + \mathcal{L}_A(\mathbf{W}, \mathbf{U}, I_i, y_i) + \mathcal{R}(\mathbf{U}, \mathcal{B})$$

Unified Semantic Embedding

Taxonomies:



Adding regularization from **ontology / taxonomy** over labels

Image Embedding

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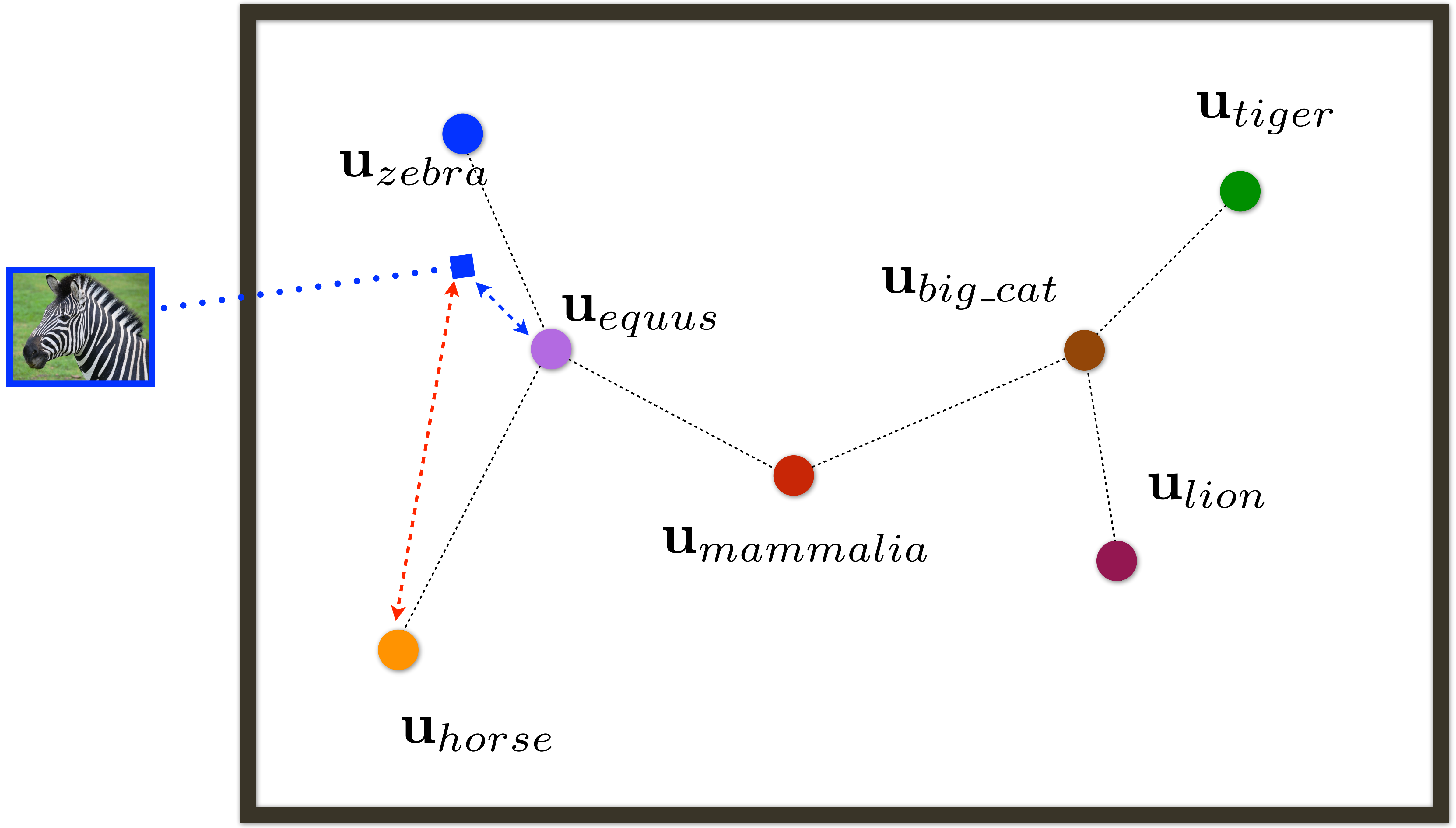
Label Embedding

$$\Psi_L(word_i) = \mathbf{u}_i : \{1, ..., L\} \rightarrow \mathbb{R}^d$$

$$\mathcal{L}_S(\boldsymbol{W}, \boldsymbol{U}, \boldsymbol{x}_i, y_i) = \sum_{s \in \mathcal{P}_{y_i}} \sum_{c \in \mathcal{S}_s} [1 + \|\boldsymbol{W}\boldsymbol{x}_i - \boldsymbol{u}_s\|_2^2 - \|\boldsymbol{W}\boldsymbol{x}_i - \boldsymbol{u}_c\|_2^2]_+$$



Similarity in Embedding Space

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Objective Function:

$$\min_{\mathbf{W},\mathbf{U},\mathbf{B}} \sum_i^N \mathcal{L}_C(\mathbf{W}, \mathbf{U}, I_i, y_i) + \mathcal{L}_S(\mathbf{W}, \mathbf{U}, I_i, y_i) + \mathcal{L}_A(\mathbf{W}, \mathbf{U}, I_i, y_i) + \lambda_1 ||\mathbf{W}||_F^2 + \lambda_2 ||\mathbf{U}||_F^2$$

Unified Semantic Embedding

Attributes : has(zebra, Stripes)

Attributes embedded as (basis) **vectors** in the semantic space

Image Embedding

$$\Psi_I(I_i) = \mathbf{W} \cdot CNN(I_i) : \mathbb{R}^D \rightarrow \mathbb{R}^d$$

Label Embedding

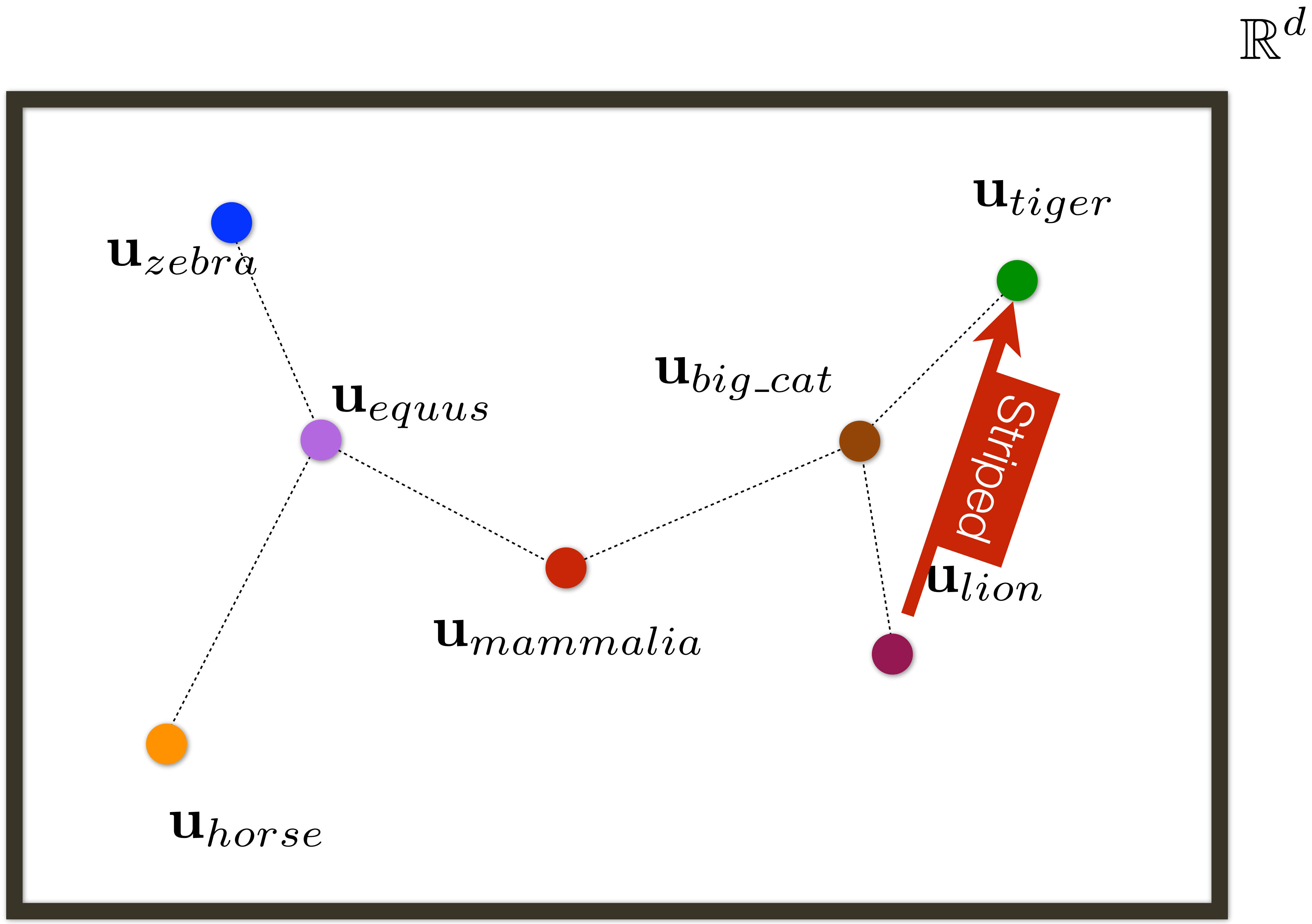
$$\Psi_L(word_i) = \mathbf{u}_i : \{1, ..., L\} \rightarrow \mathbb{R}^d$$

Attribute Embedding

$$\Psi_A(attr_i) = \mathbf{a}_i : \{1, ..., A\} \rightarrow \mathbb{R}^d, s.t. ||\mathbf{a}_i||^2 \leq 1$$

Similarity in Embedding Space

$$D(\mathbf{u}, \mathbf{u}') = ||\mathbf{u} - \mathbf{u}'||_2^2$$



Objective Function:

$$\min_{\mathbf{W},\mathbf{U},\mathbf{B}} \sum_i^N \mathcal{L}_C(\mathbf{W}, \mathbf{U}, I_i, y_i) + \mathcal{L}_S(\mathbf{W}, \mathbf{U}, I_i, y_i) + \mathcal{L}_A(\mathbf{W}, \mathbf{U}, I_i, y_i) + \mathcal{R}(\mathbf{U}, \mathcal{B}) + \lambda_1 ||\mathbf{W}||_F^2 + \lambda_2 ||\mathbf{U}||_F^2$$

Unified Semantic Embedding

[Hwang et al., 2014]

$$\mathcal{R}(\boldsymbol{U},\boldsymbol{B}) = \sum_{c}^{C} \|\boldsymbol{u}_c - \boldsymbol{u}_p - \boldsymbol{U}^A \boldsymbol{\beta}_c\|_2^2 + \gamma_2 \|\boldsymbol{\beta}_c + \boldsymbol{\beta}_o\|_2^2.$$

each category is a parent + sparse subset of attribute bases

Image Embedding

$$\Psi_I(I_i) = \mathbf{W} \cdot CNN(I_i) : \mathbb{R}^D \to \mathbb{R}^d$$

Label Embedding

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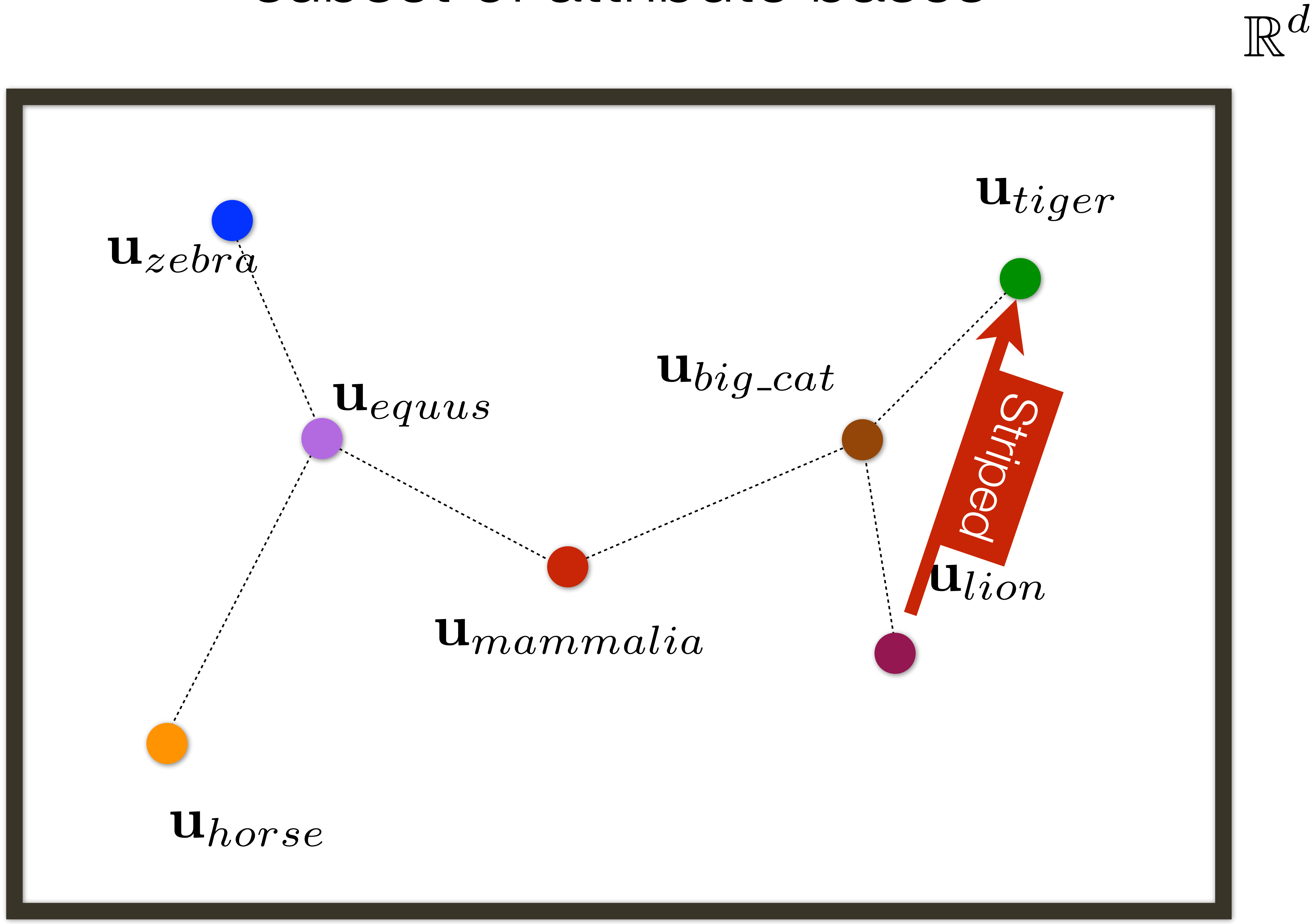

Objective Function:

$$\min_{\mathbf{W},\mathbf{U},\mathbf{B}} \sum_{i}^{N} \mathcal{L}_C(\mathbf{W},\mathbf{U},I_i,y_i) + \mathcal{L}_S(\mathbf{W},\mathbf{U},I_i,y_i) + \mathcal{L}_A(\mathbf{W},\mathbf{U},I_i,y_i) + \mathcal{R}(\mathbf{U},\mathcal{B}) + \lambda_1||\mathbf{W}||_F^2 + \lambda_2||\mathbf{U}||_F^2$$

Unified Semantic Embedding

[Hwang et al., 2014]

Image Embedding

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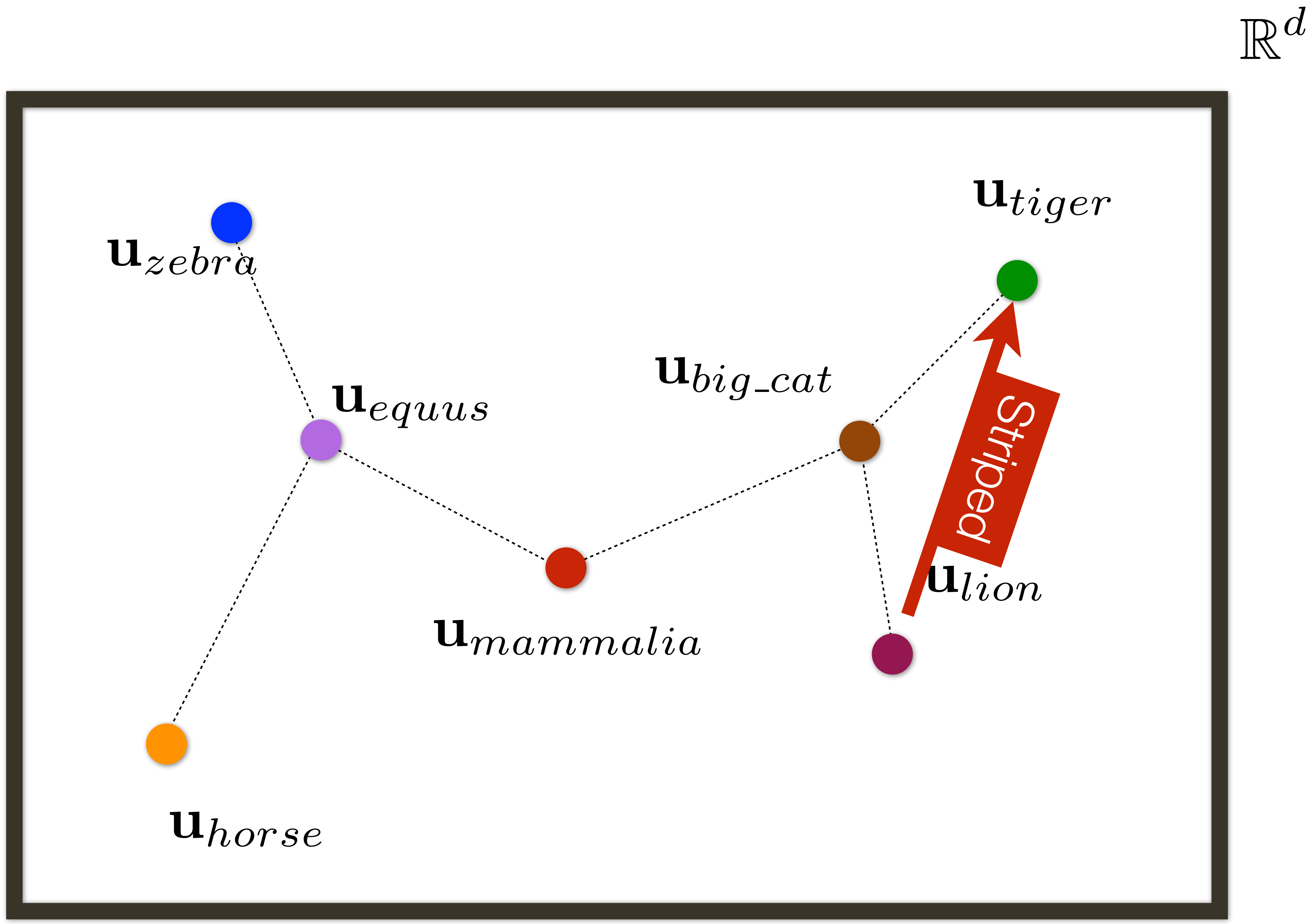
Objective Function:

$$\min_{\mathbf{W},\mathbf{U},\mathbf{B}} \sum_i^N \mathcal{L}_C(\mathbf{W}, \mathbf{U}, I_i, y_i) + \mathcal{L}_S(\mathbf{W}, \mathbf{U}, I_i, y_i) + \mathcal{L}_A(\mathbf{W}, \mathbf{U}, I_i, y_i) + \mathcal{R}(\mathbf{U}, \mathcal{B}) + \lambda_1||\mathbf{W}||_F^2 + \lambda_2||\mathbf{U}||_F^2$$

Alternating optimization



Experiments: Animals with Attributes (AwA) Dataset

(we assume no association between classes and attributes)

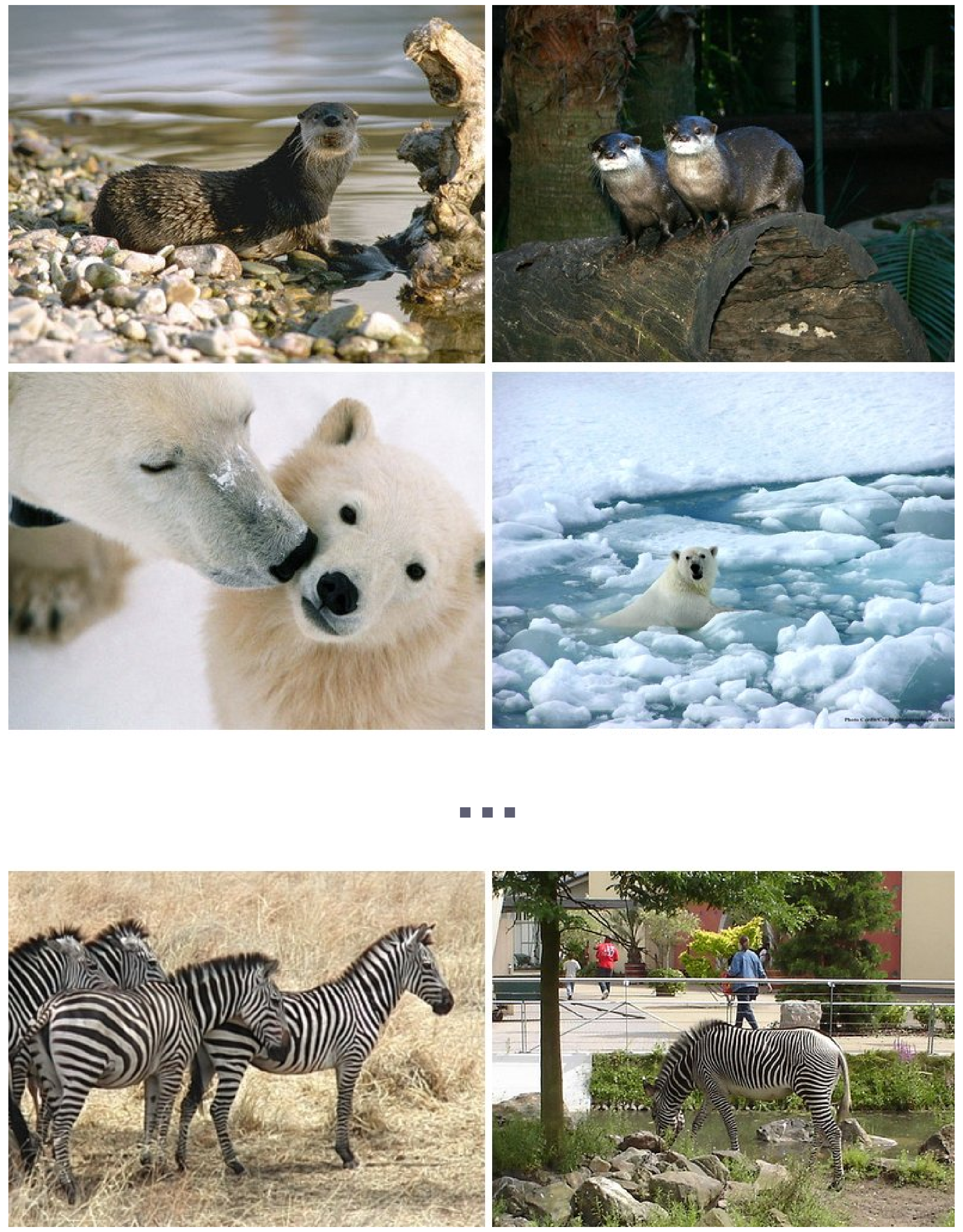
Labeled Images

Otter

Polar Bear

...

Zebra



30,475 Images

50 Animal Classes

Semantic Attributes

black
white
blue
brown
gray
orange
red
yellow
patches

...

paws
longlegs
longneck
tail
chew teeth
meat teeth
buck teeth
horns
claws
tusks

85 Attributes

Class Ontology

WordNet
A lexical database for English

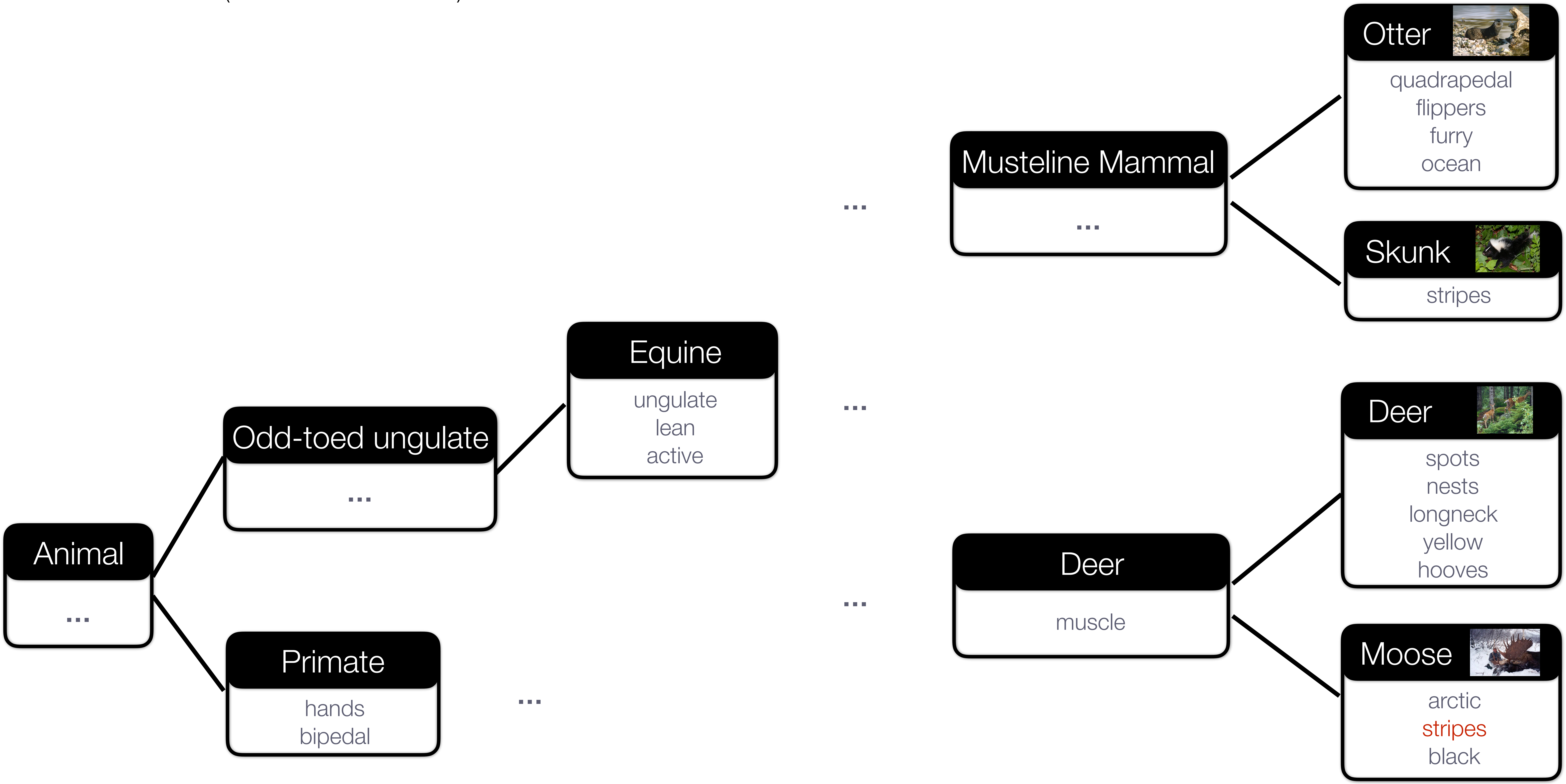
50 Animal Classes are Leaves

[Lampert, Nickisch, Harmeling, CVPR'09]

Experiments

[Hwang et al., 2014]

Results with AWA (with latent attributes)



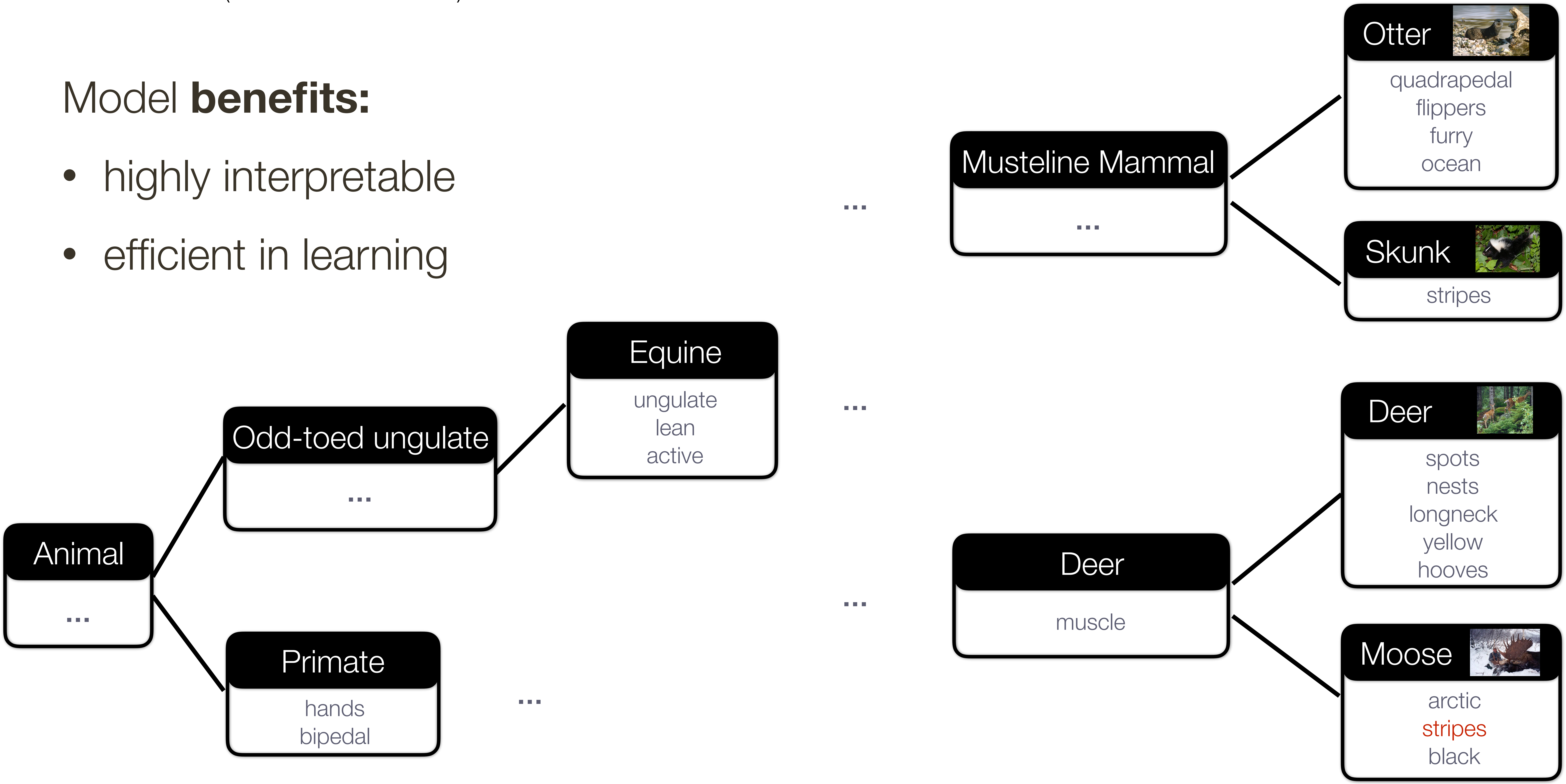
Experiments

Results with AWA (with latent attributes)

[Hwang et al., 2014]

Model **benefits:**

- highly interpretable
- efficient in learning



Experiments

Results with AWA (with latent attributes)

[Hwang et al., 2014]

Model **benefits:**

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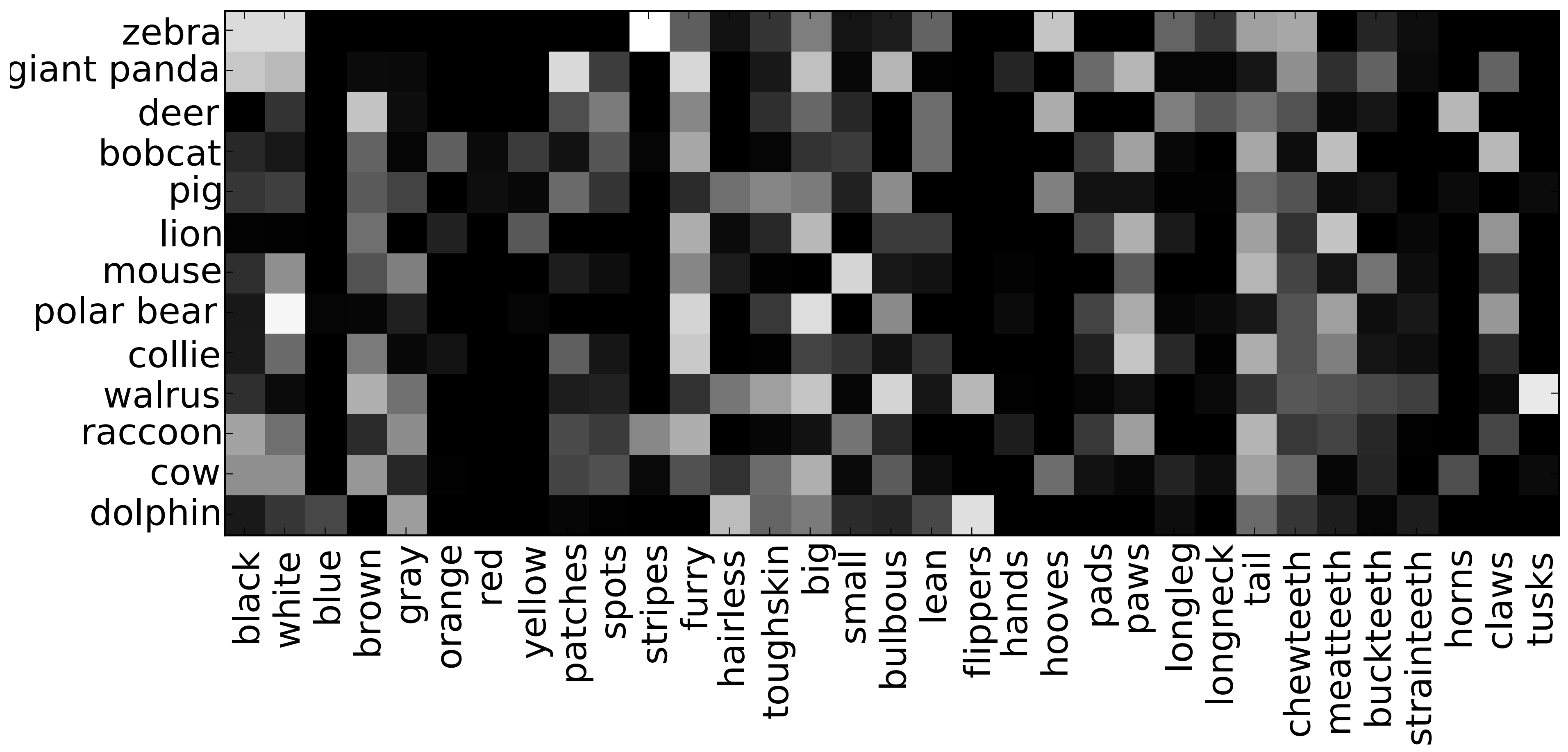
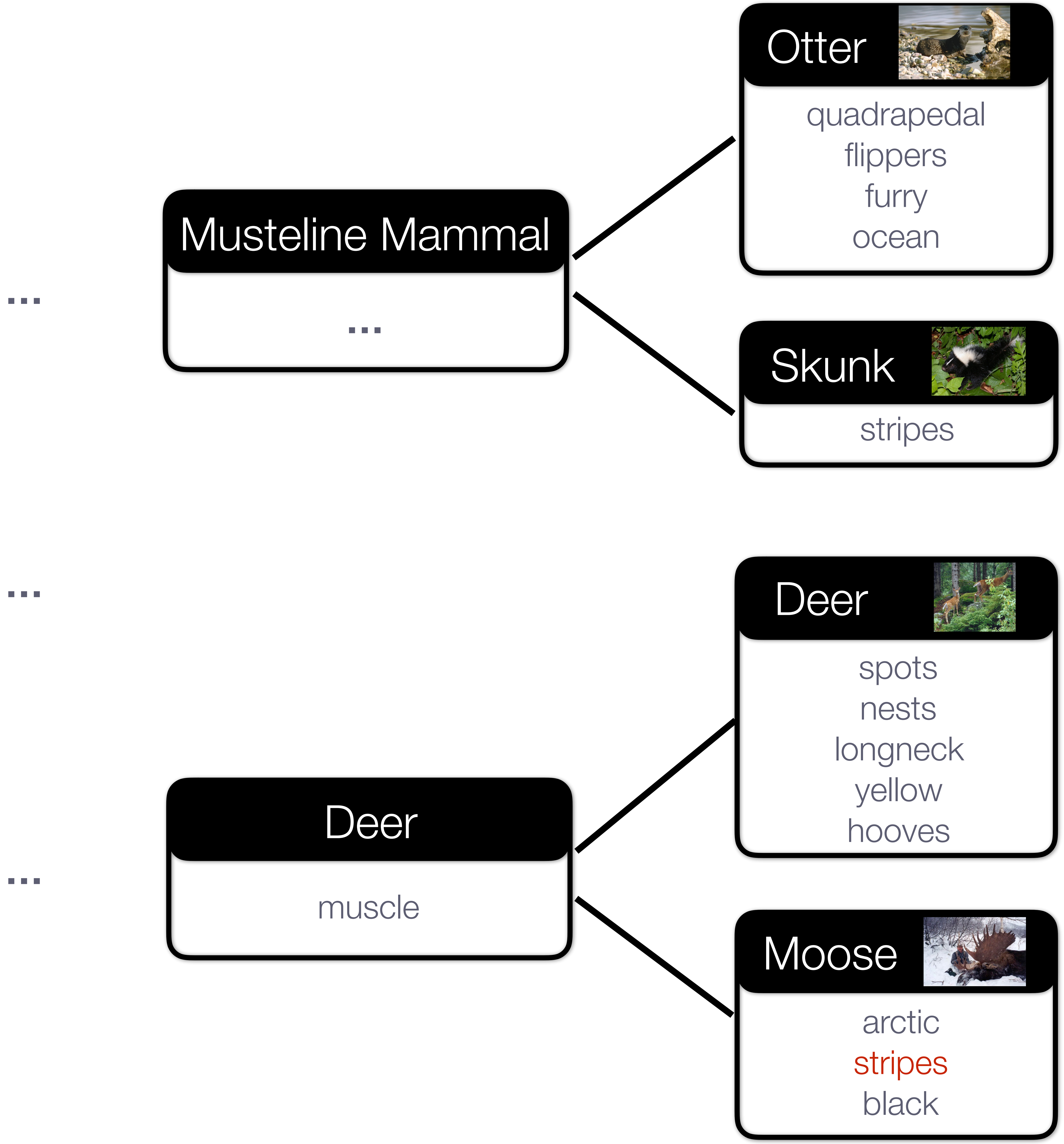


alternative attribute-based representations



Experiments

[Hwang et al., 2014]

Results with AWA (with latent attributes)

	Method	Flat hit @ k (%) 1	2	5	Hierarchical precision @ k (%) 2	5
No semantics	Ridge Regression	38.39 ± 1.48	48.61 ± 1.29	62.12 ± 1.20	38.51 ± 0.61	41.73 ± 0.54
	NCM **[1]**	43.49 ± 1.23	57.45 ± 0.91	75.48 ± 0.58	45.25 ± 0.52	50.32 ± 0.47
	LME	44.76 ± 1.77	58.08 ± 2.05	75.11 ± 1.48	44.84 ± 0.98	49.87 ± 0.39
Implicit semantics	LMTE **[2]**	38.92 ± 1.12	49.97 ± 1.16	63.35 ± 1.38	38.67 ± 0.46	41.72 ± 0.45
	ALE **[3]**	36.40 ± 1.03	50.43 ± 1.92	70.25 ± 1.97	42.52 ± 1.17	52.46 ± 0.37
	HLE **[3]**	33.56 ± 1.64	45.93 ± 2.56	64.66 ± 1.77	46.11 ± 2.65	**56.79 ± 2.05**
	AHLE **[3]**	38.01 ± 1.69	52.07 ± 1.19	71.53 ± 1.41	44.43 ± 0.66	54.39 ± 0.55
Explicit semantics	LME-MTL-S	45.03 ± 1.32	57.73 ± 1.75	74.43 ± 1.26	46.05 ± 0.89	51.08 ± 0.36
	LME-MTL-A	45.55 ± 1.71	58.60 ± 1.76	74.97 ± 1.15	44.23 ± 0.95	48.52 ± 0.29
USE	USE-No Reg.	45.93 ± 1.76	59.37 ± 1.32	74.97 ± 1.15	47.13 ± 0.62	51.04 ± 0.46
	USE-Reg.	**46.42 ± 1.33**	**59.54 ± 0.73**	**76.62 ± 1.45**	**47.39 ± 0.82**	53.35 ± 0.30

Variants of our Unified Semantic Embedding (**USE**) model:

- **Ontology**
- **Attributes**
- **Parent + Sparse Attributes**

[1] Mensink, Varbeek, Perronnin, Csurka Chapelle, TPAMI'13

[2] Weinberger, Chapelle, NIPS'09

[3] Akata, Perronnin, Harchaoui, Schmid, CVPR'13

Experiments

[Hwang et al., 2014]

Results with AWA (with latent attributes)

	Method	1
No semantics	Ridge Regression NCM [1] LME	38.93
Implicit semantics	LMTE [2] ALE [3] HLE [3] AHLE [3]	
Explicit semantics	LME-MTL-S LME-MTL-A	
USE	USE-No Reg.	44.87 (+5.9%)
	USE-Reg.	**49.87** (+5.0%)

Variants of our Unified Semantic Embedding (**USE**) model:

- Ontology
- Attributes
- Parent + Sparse Attributes

with **2 samples/category**

[1] Mensink, Varbeek, Perronnin, Csurka Chapelle, TPAMI'13

[2] Weinberger, Chapelle, NIPS'09

[3] Akata, Perronnin, Harchaoui, Schmid, CVPR'13