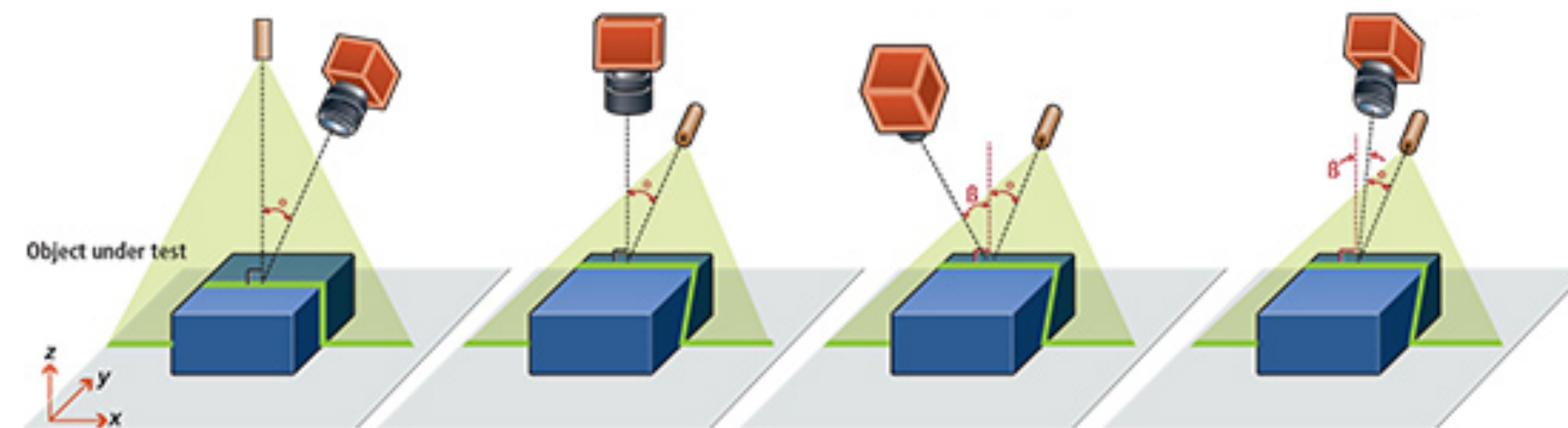


CPSC 425: Computer Vision



Lecture 2: Image Formation

(unless otherwise stated slides are taken or adopted from **Bob Woodham, Jim Little** and **Fred Tung**)

Menu for Today (September 9, 2025)

Topics:

- Image Formation
- Cameras and Lenses
- Projection
- Human Eye ??

Readings:

- **Today's** Lecture: Szeliski Chapter 2, Forsyth & Ponce (2nd ed.) 1.1.1 — 1.1.3
- **Next** Lecture: Forsyth & Ponce (2nd ed.) 4.1, 4.5

Reminders:

- Complete **Assignment 0** (ungraded) by Wednesday, **September 11**
- **Assignment 1** (graded) will be out Wednesday, **September 11**
- Please sign up for **Piazza** (if you have not done it)



Today’s “**fun**” Example

Today's “**fun**” Example



Photo credit: reddit user [Liammm](#)

Today's “**fun**” Example: **Eye Sink Illusion**

Pereidolia



Photo credit: reddit user [Liammm](#)

Salvador Dali — **Pareidolia**



Salvador Dali — Pareidolia



Lecture 2: Goal

To understand how images are formed
(and develop relevant mathematical
concepts and abstractions)

What is **Computer Vision**?

Computer vision, broadly speaking, is a research field aimed to enable computers to **process and interpret visual data**, as sighted humans can.

Image (or video)

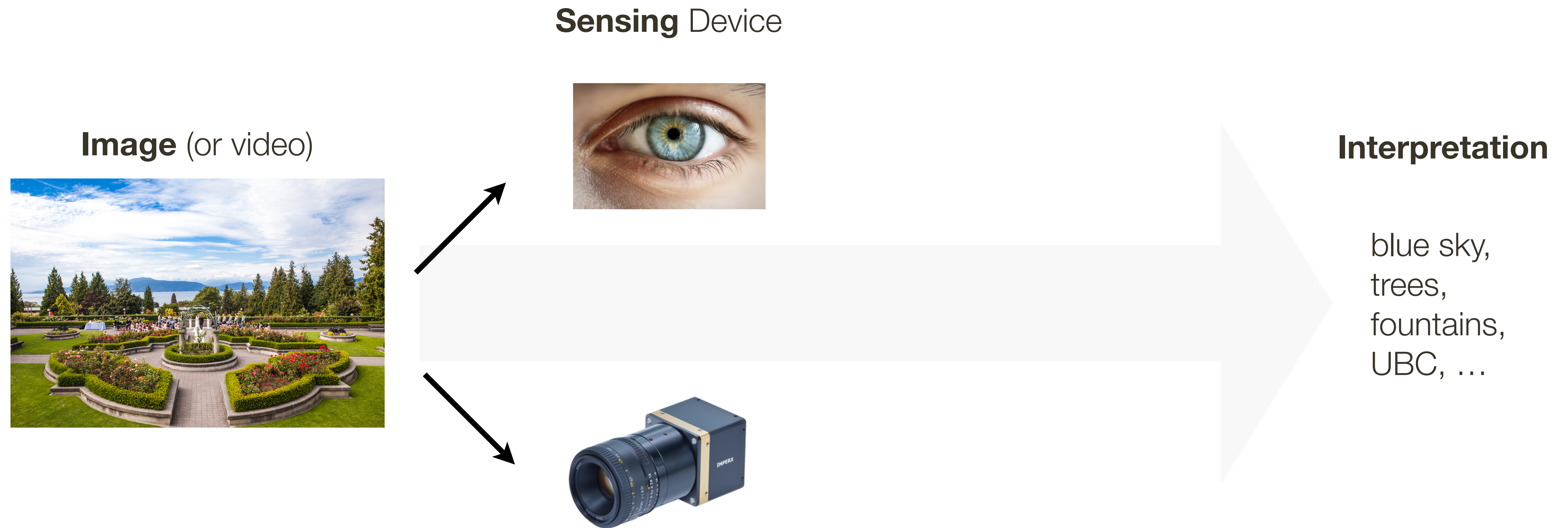


Interpretation

blue sky,
trees,
fountains,
UBC, ...

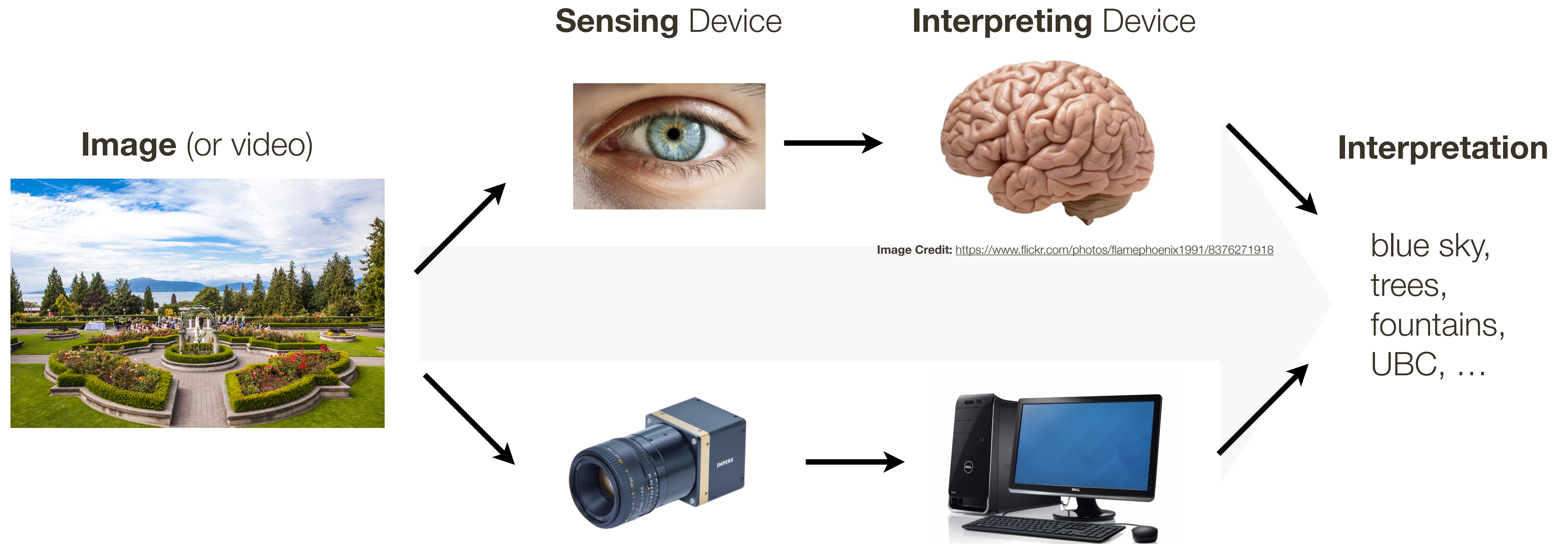
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What is **Computer Vision**?

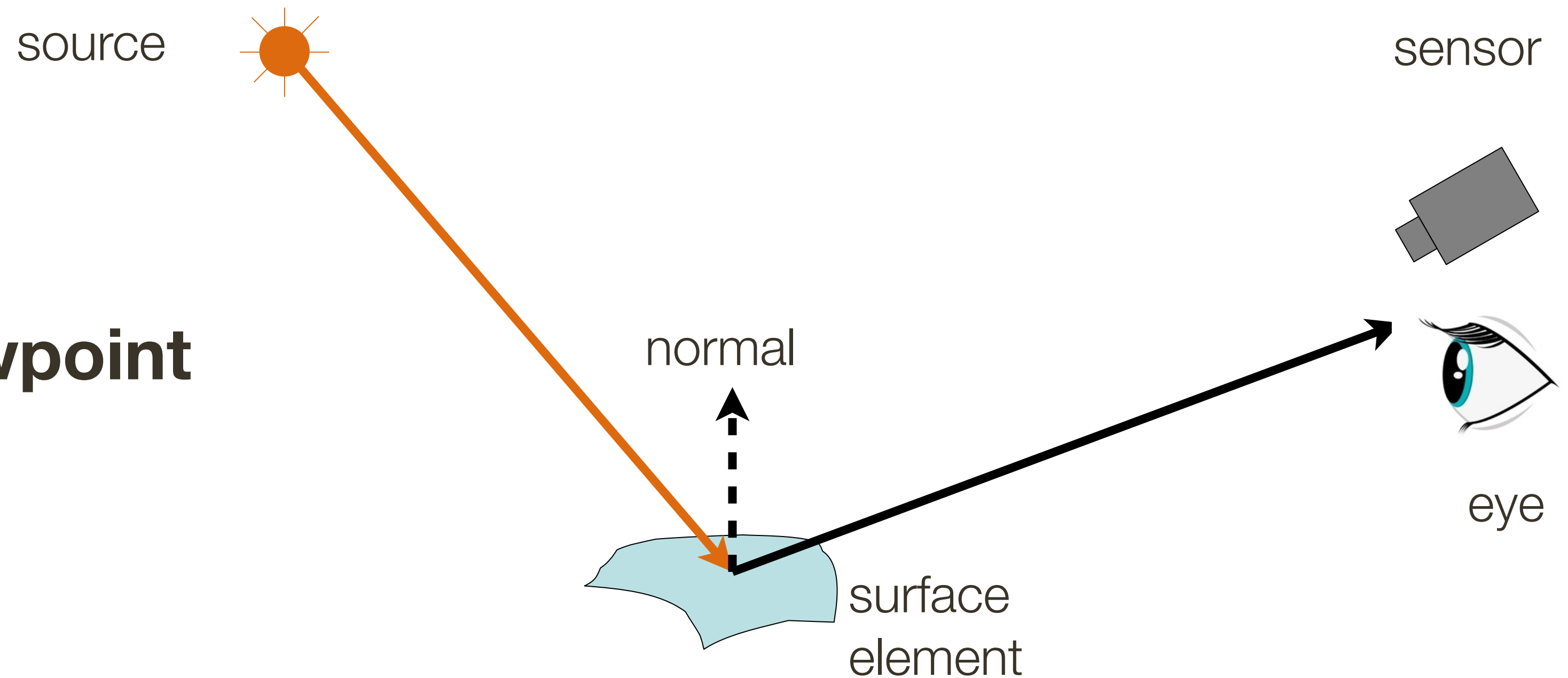
Computer vision, broadly speaking, is a research field aimed to enable computers to **process and interpret visual data**, as sighted humans can.



Overview: Image Formation, Cameras and Lenses

The **image formation process** that produces a particular image depends on

- **Lightening** condition
- Scene **geometry**
- **Surface** properties
- Camera **optics** and **viewpoint**

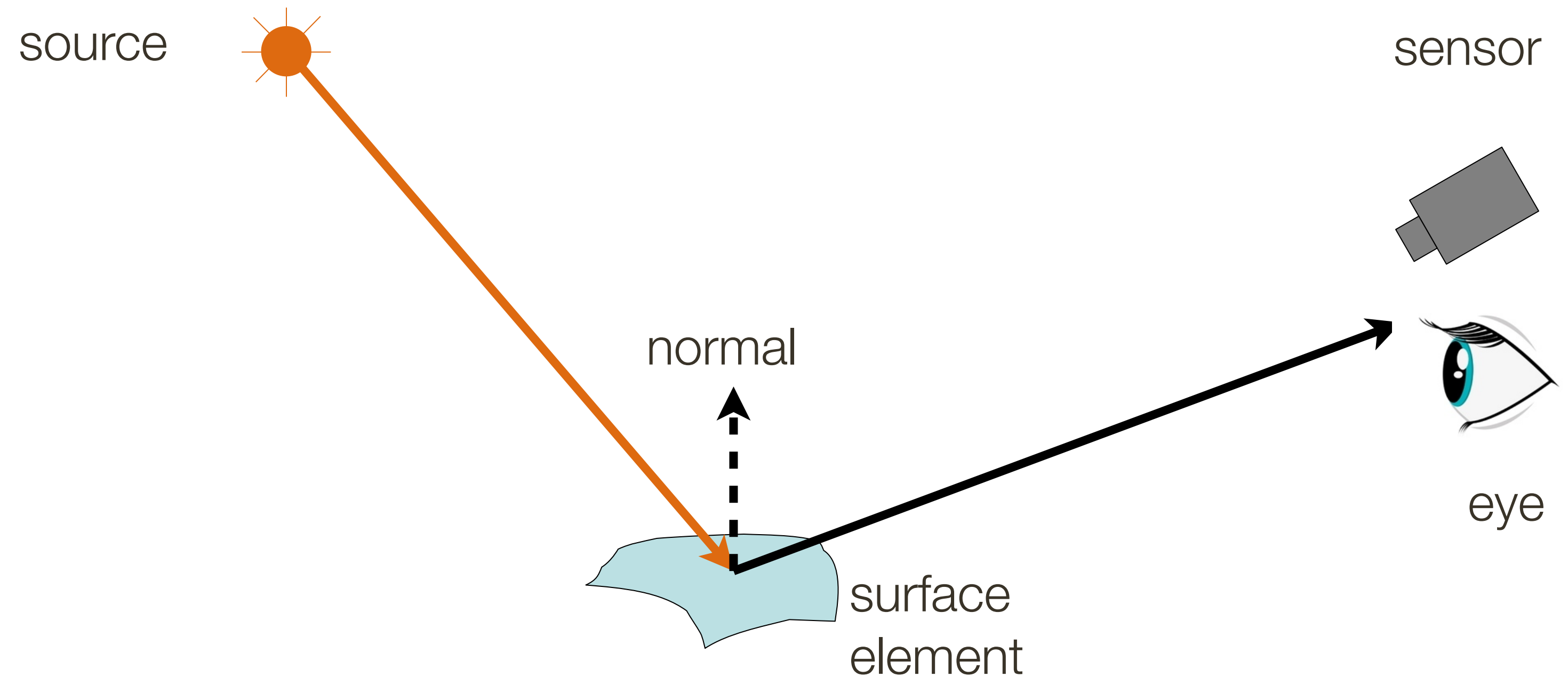


Sensor (or eye) **captures amount of light** reflected from the object

Overview: Image Formation, Cameras and Lenses

The **image formation process** that produces a particular image depends on

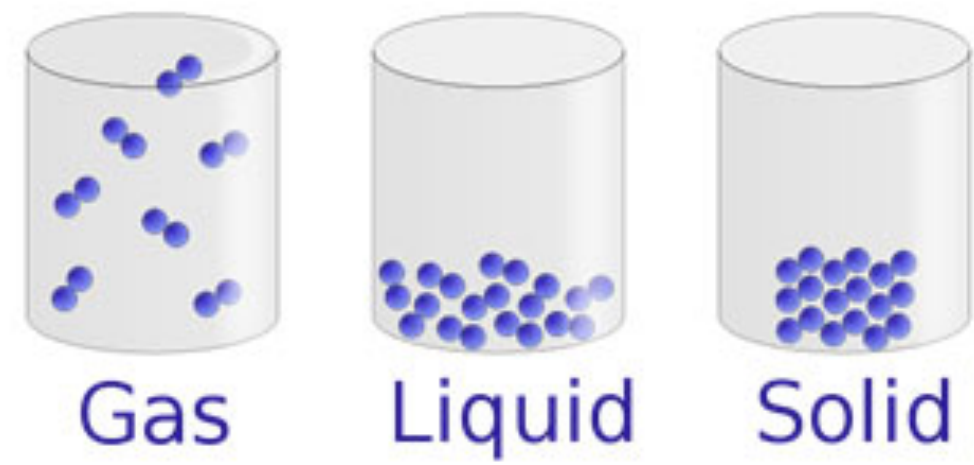
— **Lightening** condition



Sensor (or eye) **captures amount of light** reflected from the object

Light

Behaves like **particles**?

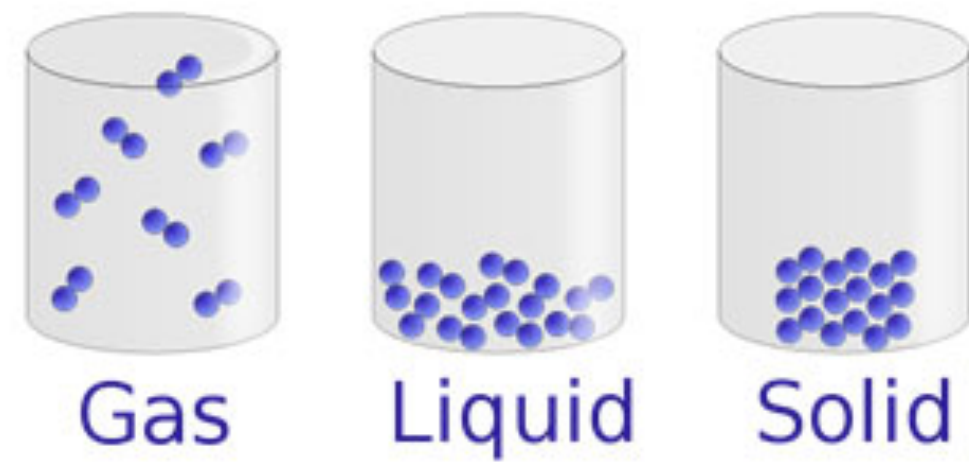


Behaves as **waves**?

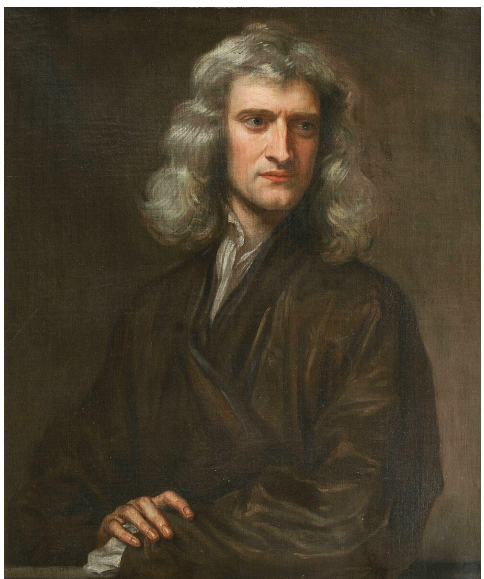


Light

Behaves like **particles**? photons



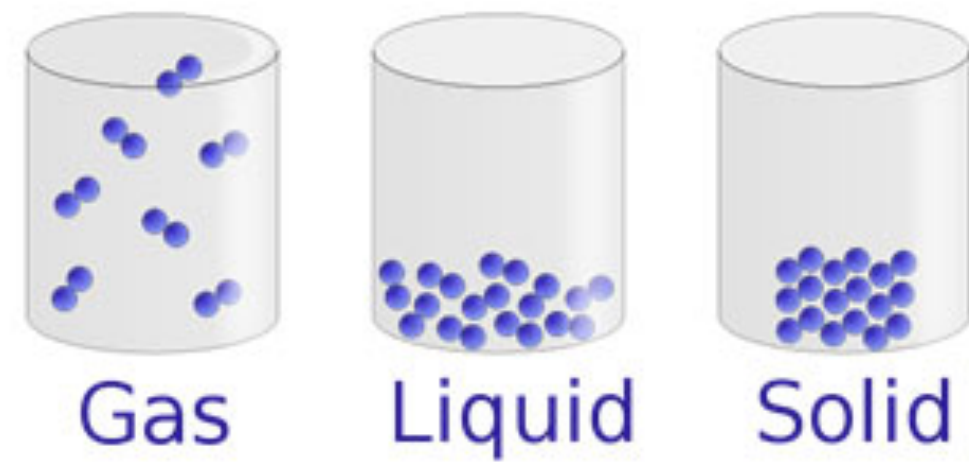
Behaves as **waves**?



Sir Isaac Newton

Light

Behaves like **particles**? photons



Behaves as **waves**?



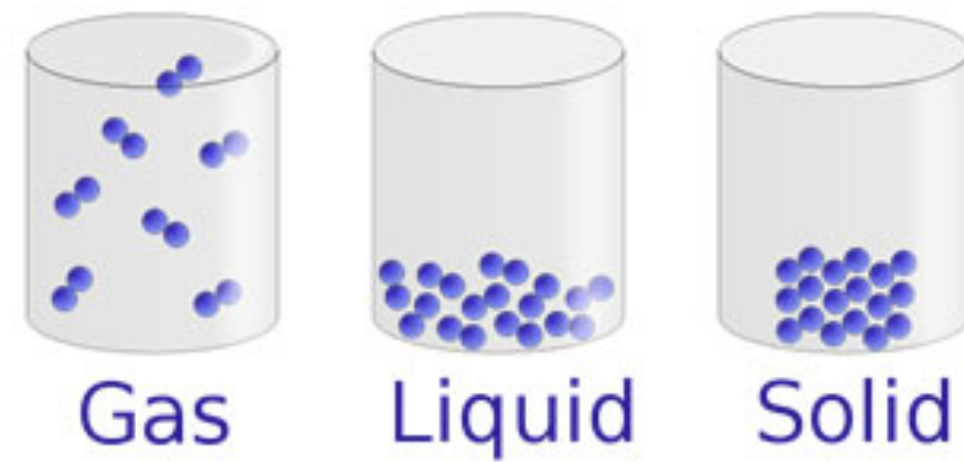
Sir Isaac Newton



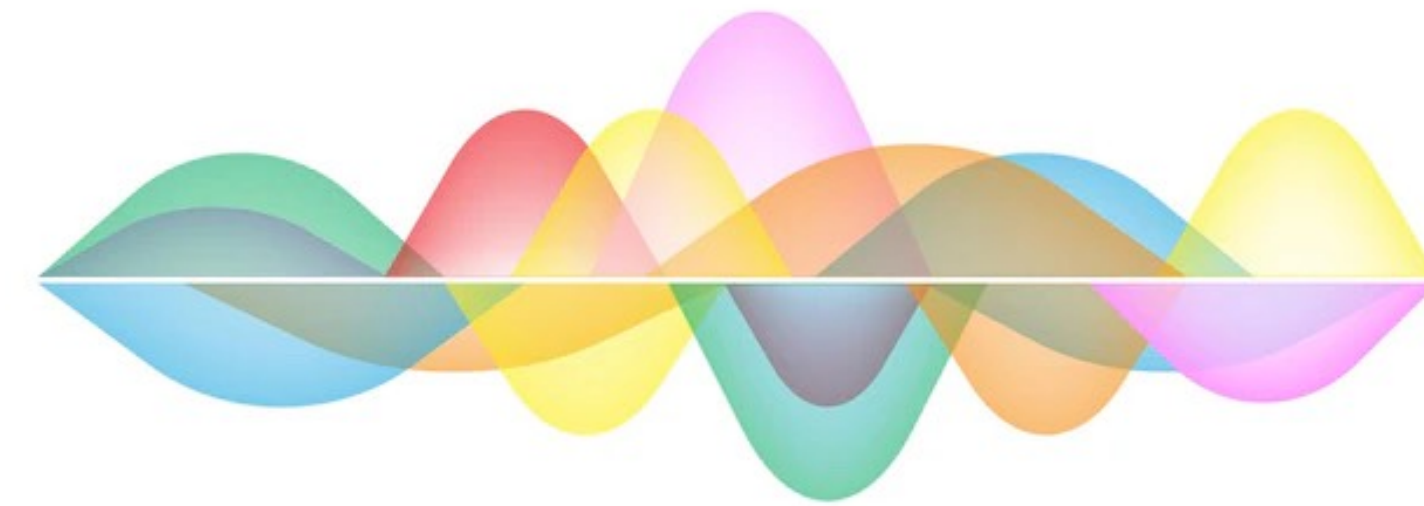
Christiaan Huygens

Light

Behaves like **particles**? photons



Behaves as **waves**?



Wave-particle Duality: light exhibit particle or wave properties according to the experimental circumstances

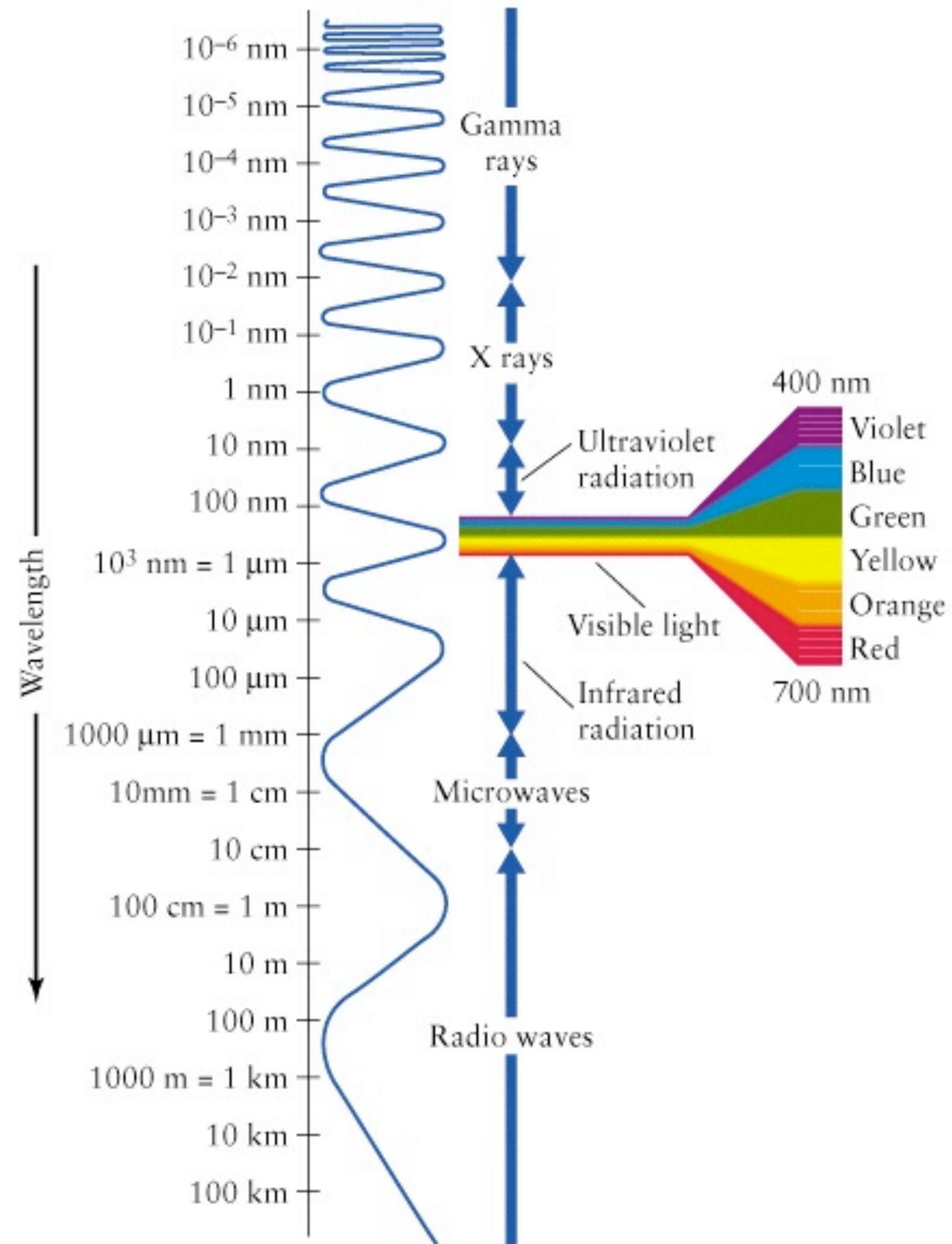


Sir Isaac Newton



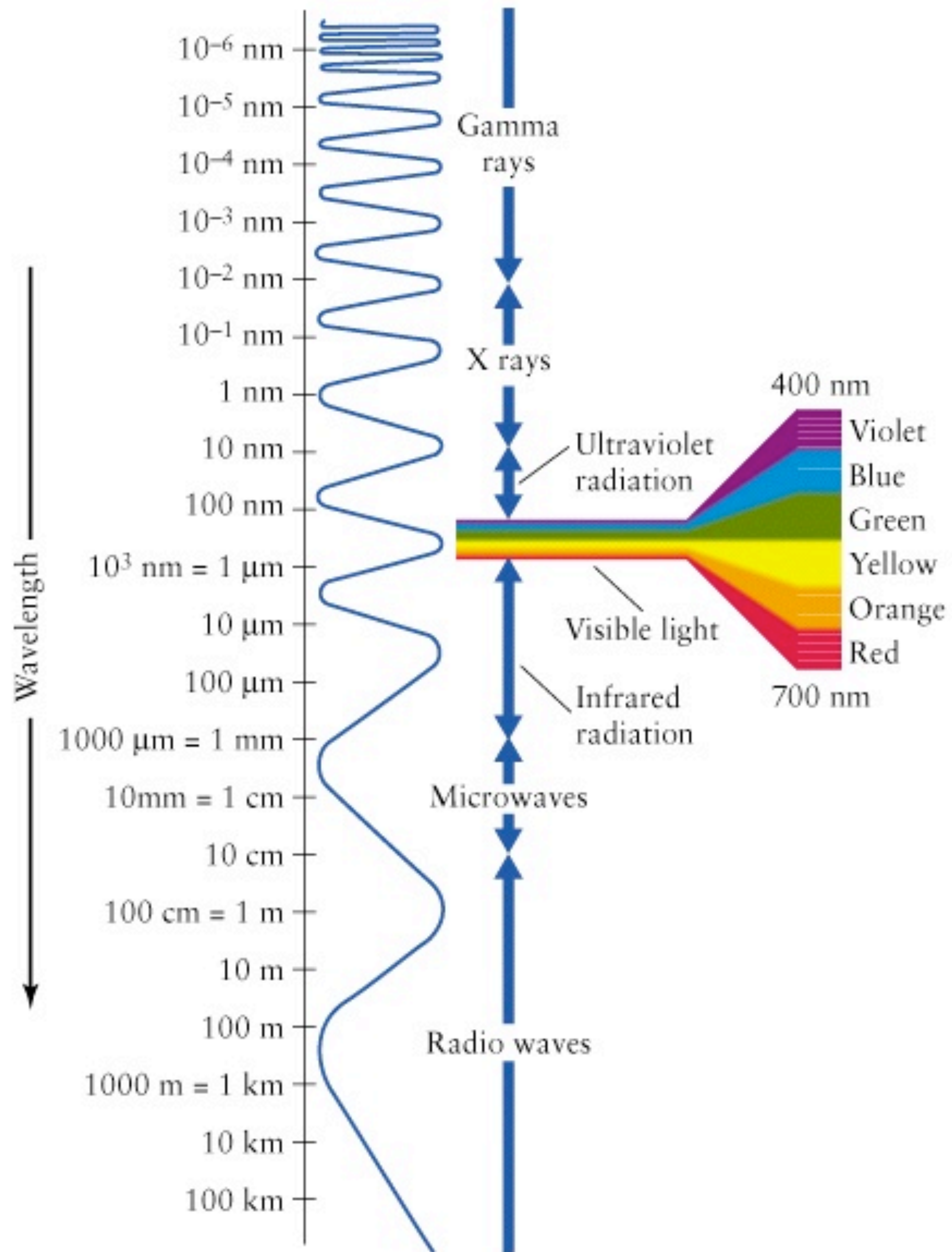
Christiaan Huygens

Light and Color: A Short Preview



Visible light is electromagnetic radiation in the 400nm-700nm band of wavelengths

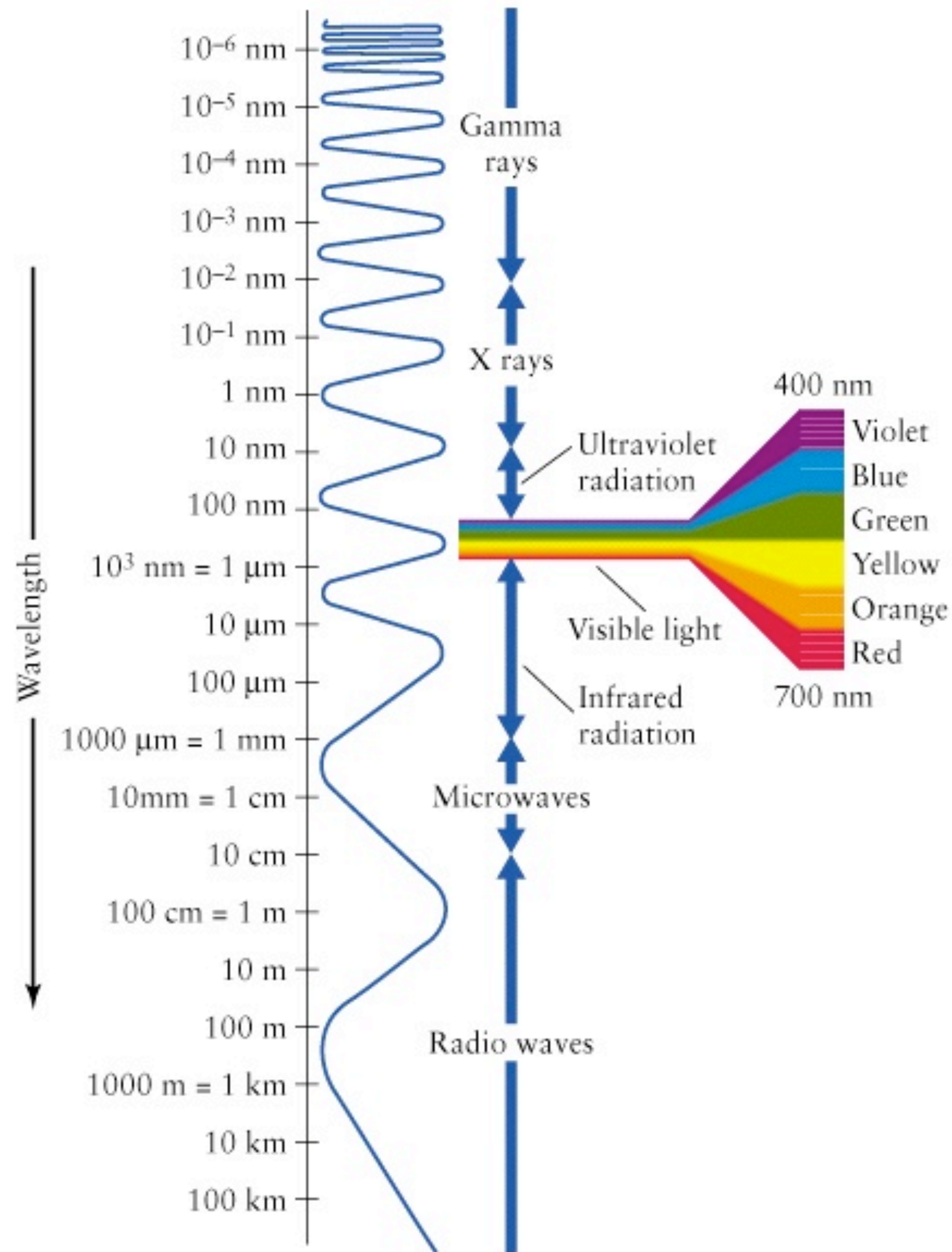
Light and Color: A Short Preview



Visible light is electromagnetic radiation in the 400nm-700nm band of wavelengths

- Black is the absence of light
- Sunlight is a spectrum of light

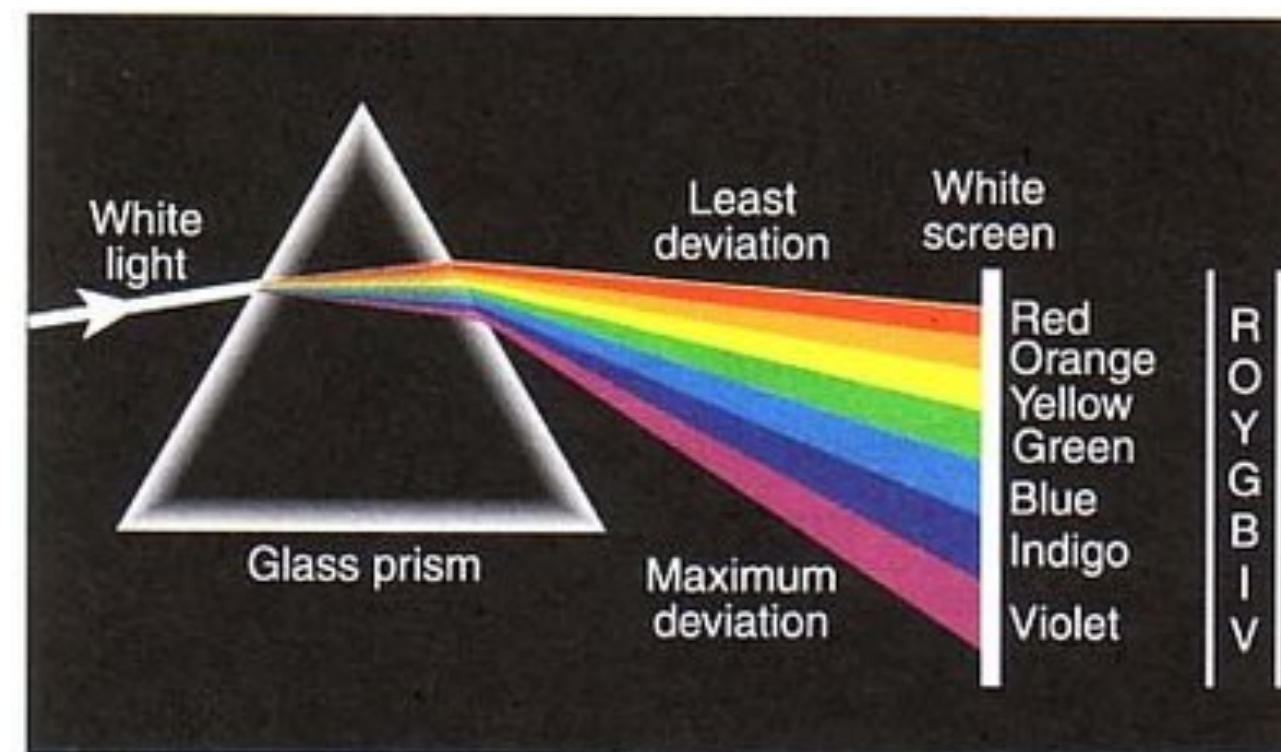
Light and Color: A Short Preview



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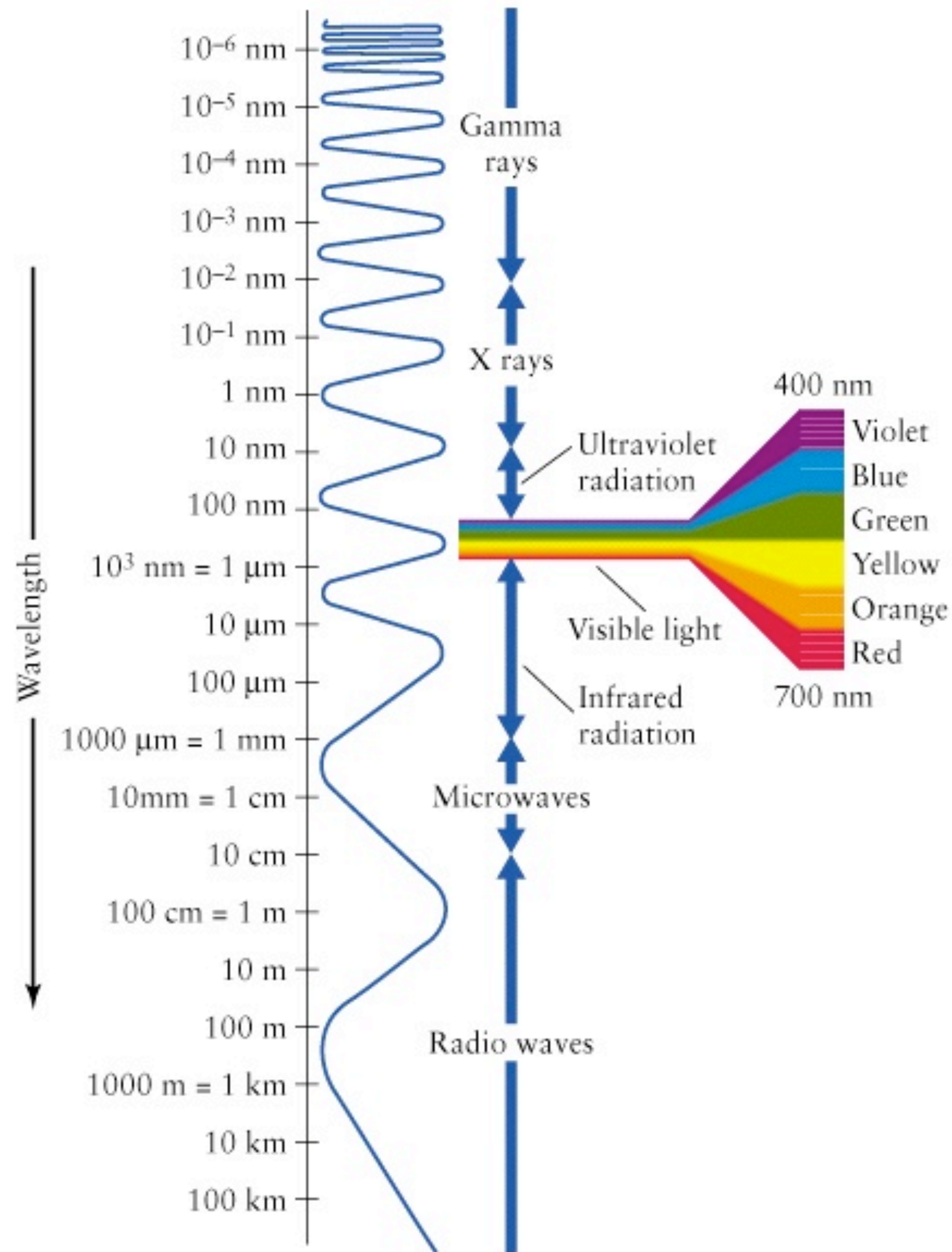
- Black is the absence of light
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Newton's Prism experiment showed that white light is composed of all frequencies



Source: Chong Kum Ying et. al, 2006; Focus Excel Science; Form 4; pg 170

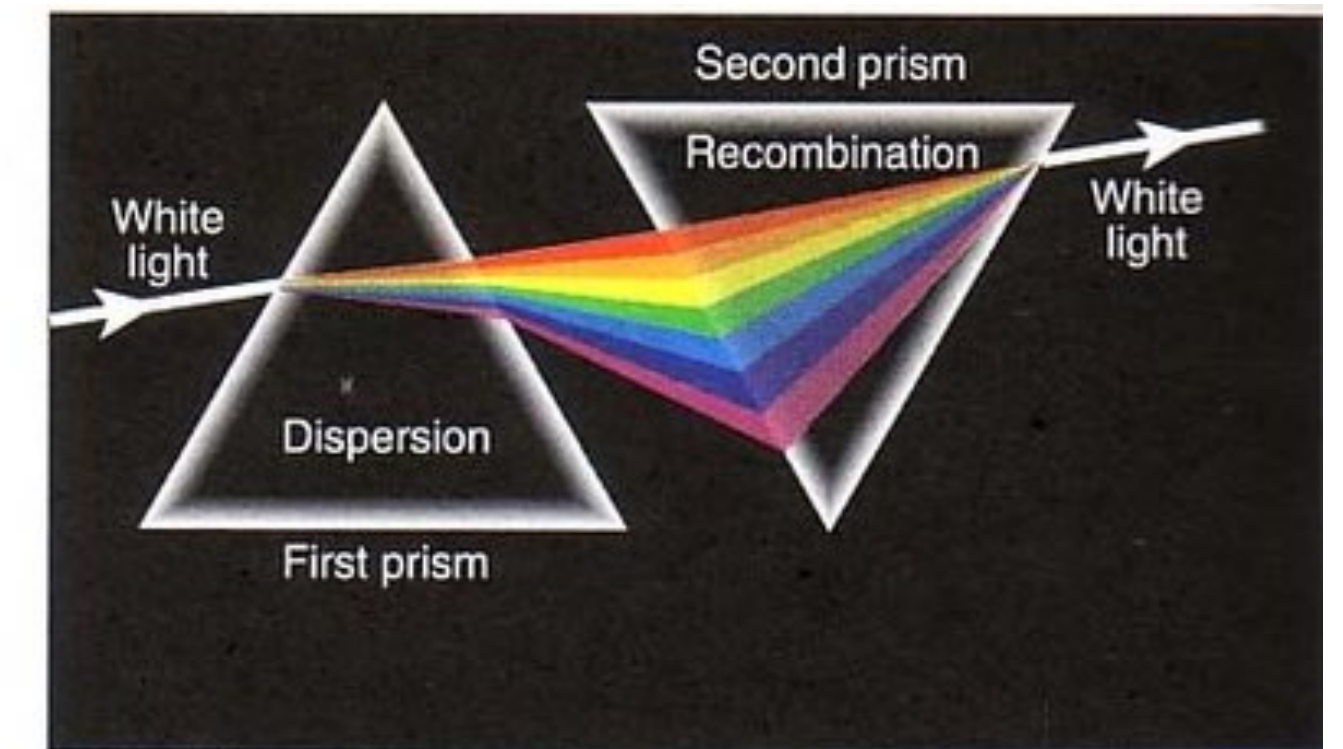
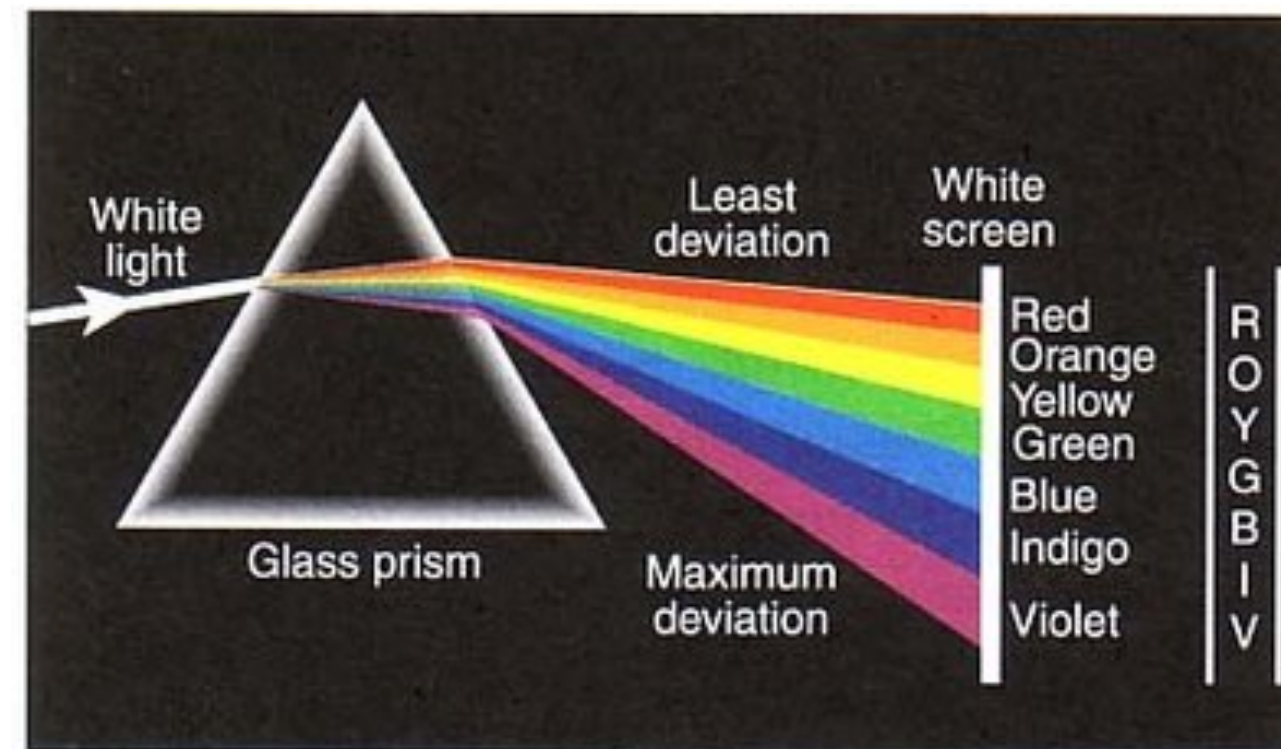
Light and Color: A Short Preview



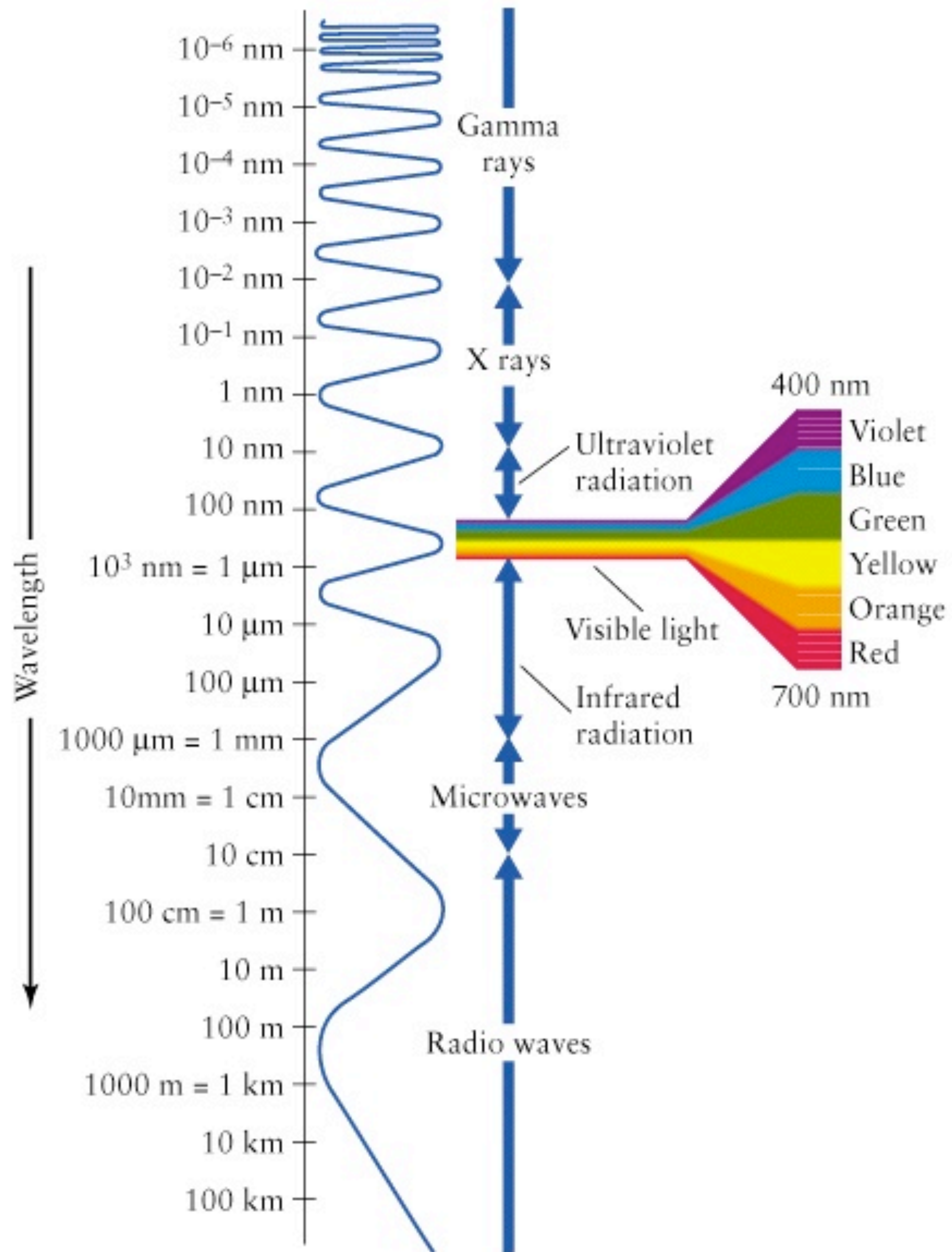
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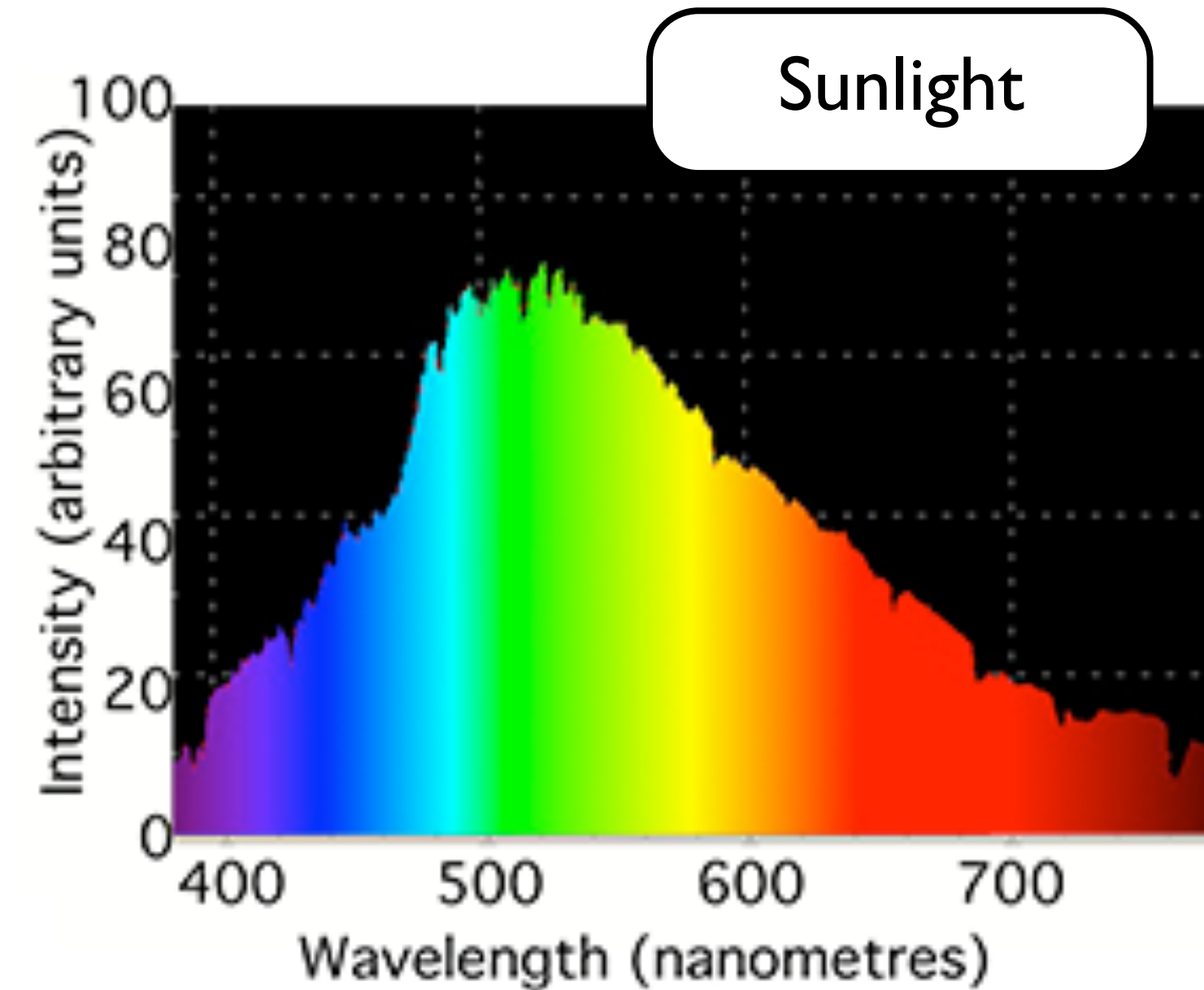
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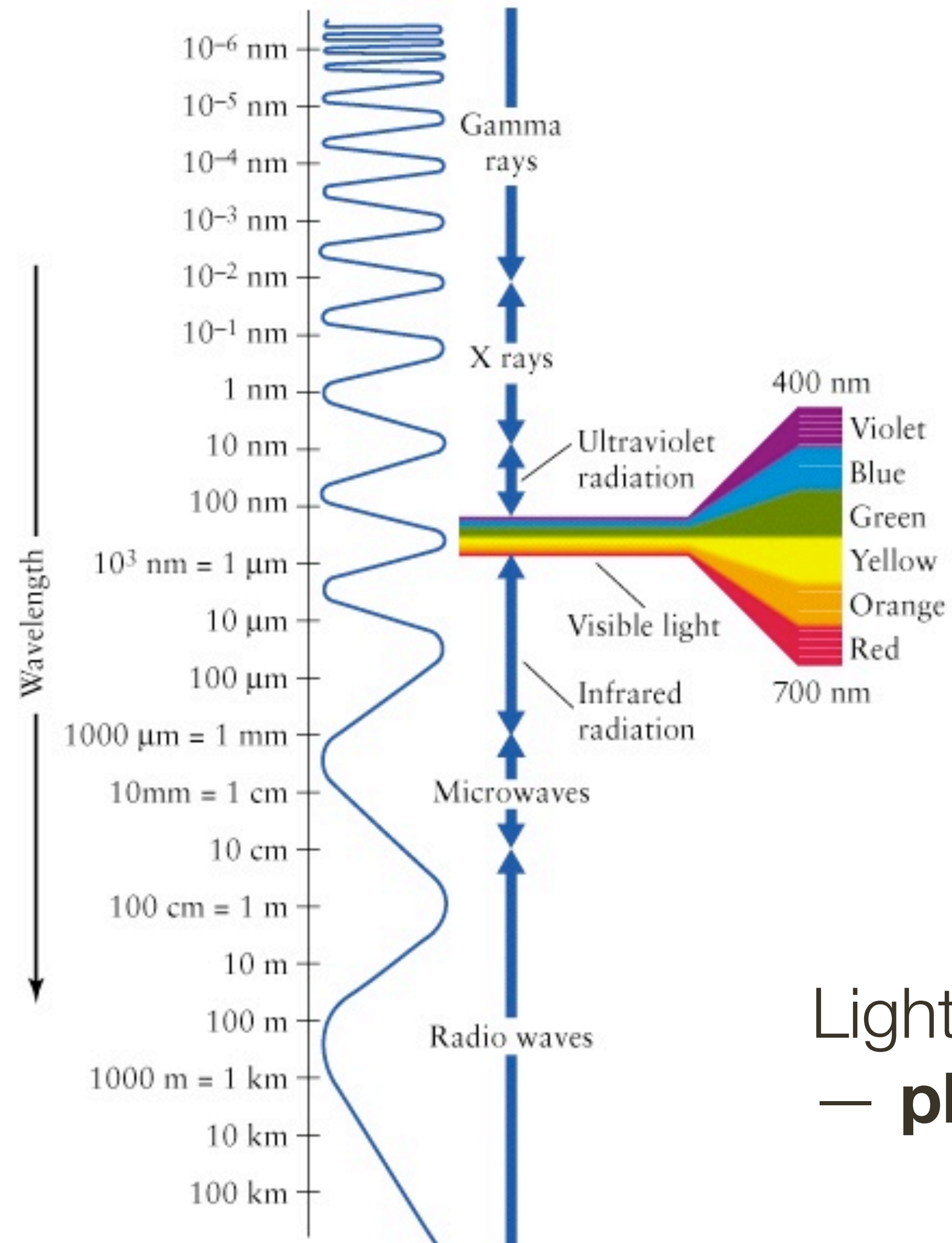
Light and Color: A Short Preview



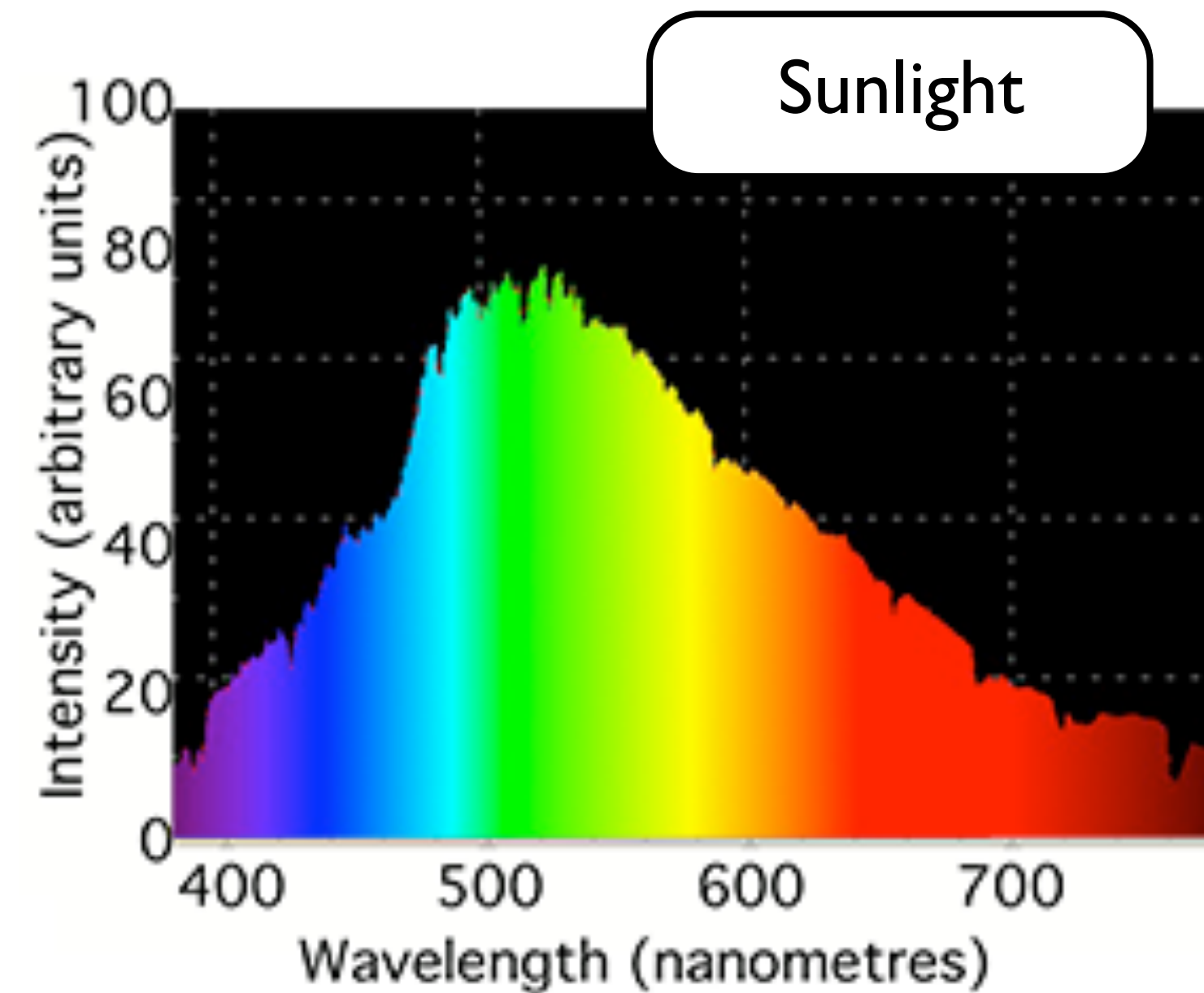
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Light and Color: A Short Preview

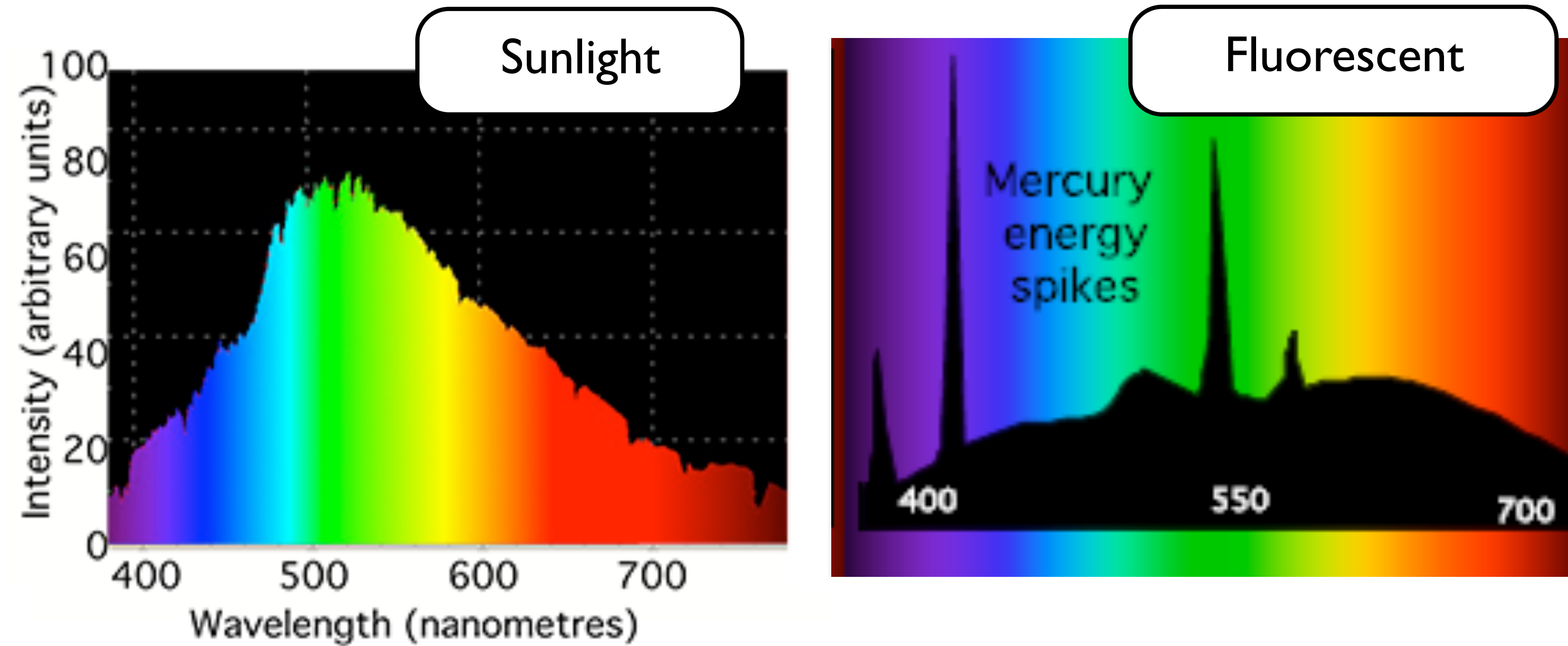


Visible light is electromagnetic radiation in the 400nm-700nm band of wavelengths



Light also behaves as particles with specific wavelengths — **photons**; that travel in straight lines within a medium

Spectral Power Distribution



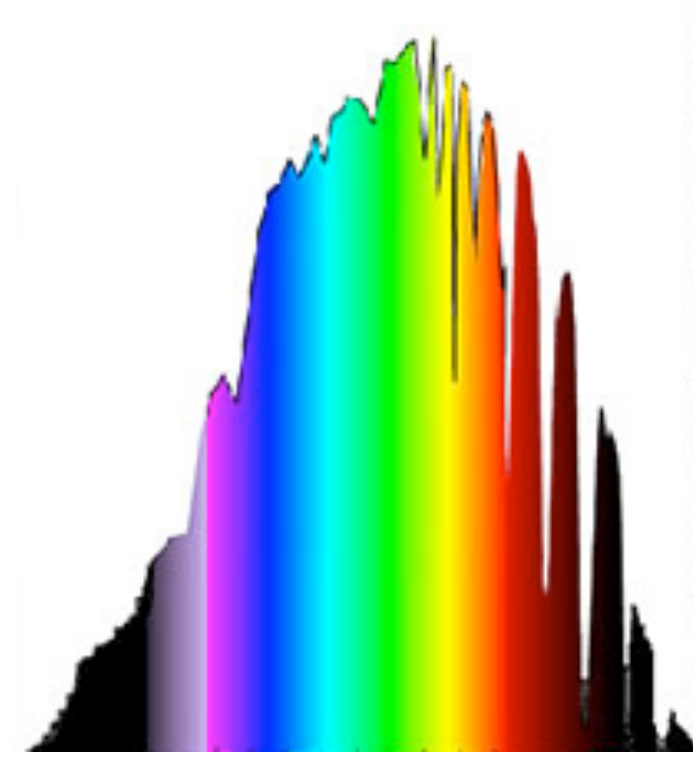
The **spectral distribution of energy** in a light ray determines its colour

— e.g., you can have pure yellow or mixture of red and green

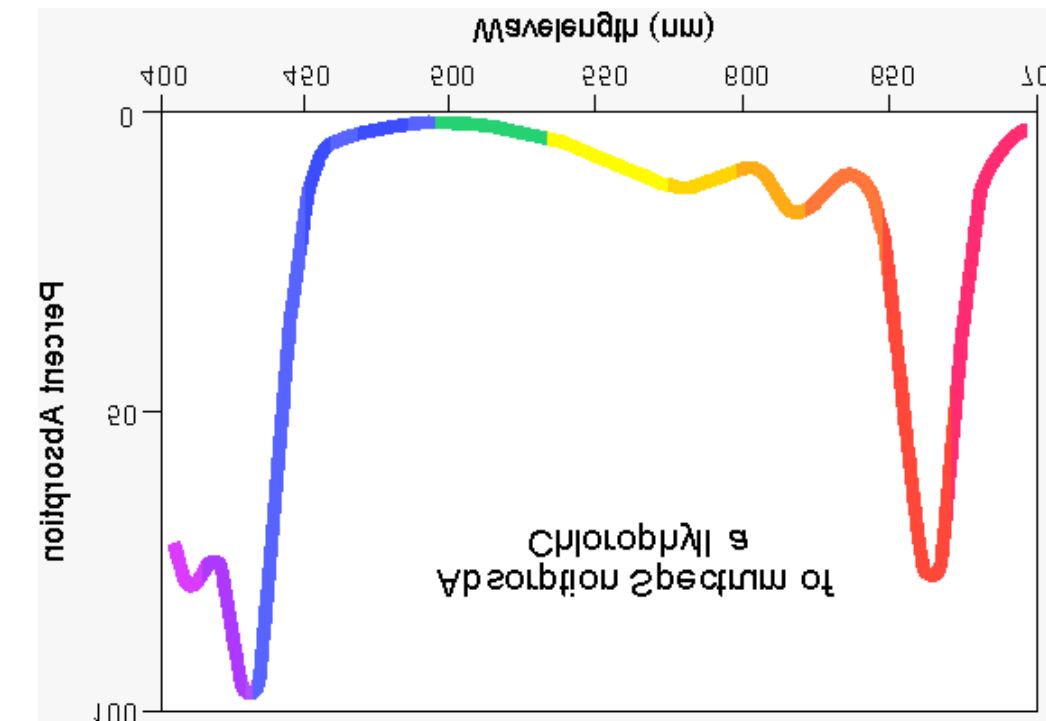
Surface **reflects** light energy according to a spectral distribution as well

The combination of **incident** and **reflectance** spectra determines **observed colour**

Spectral Reflectance Example



$$E(\lambda)$$



$$S(\lambda)$$



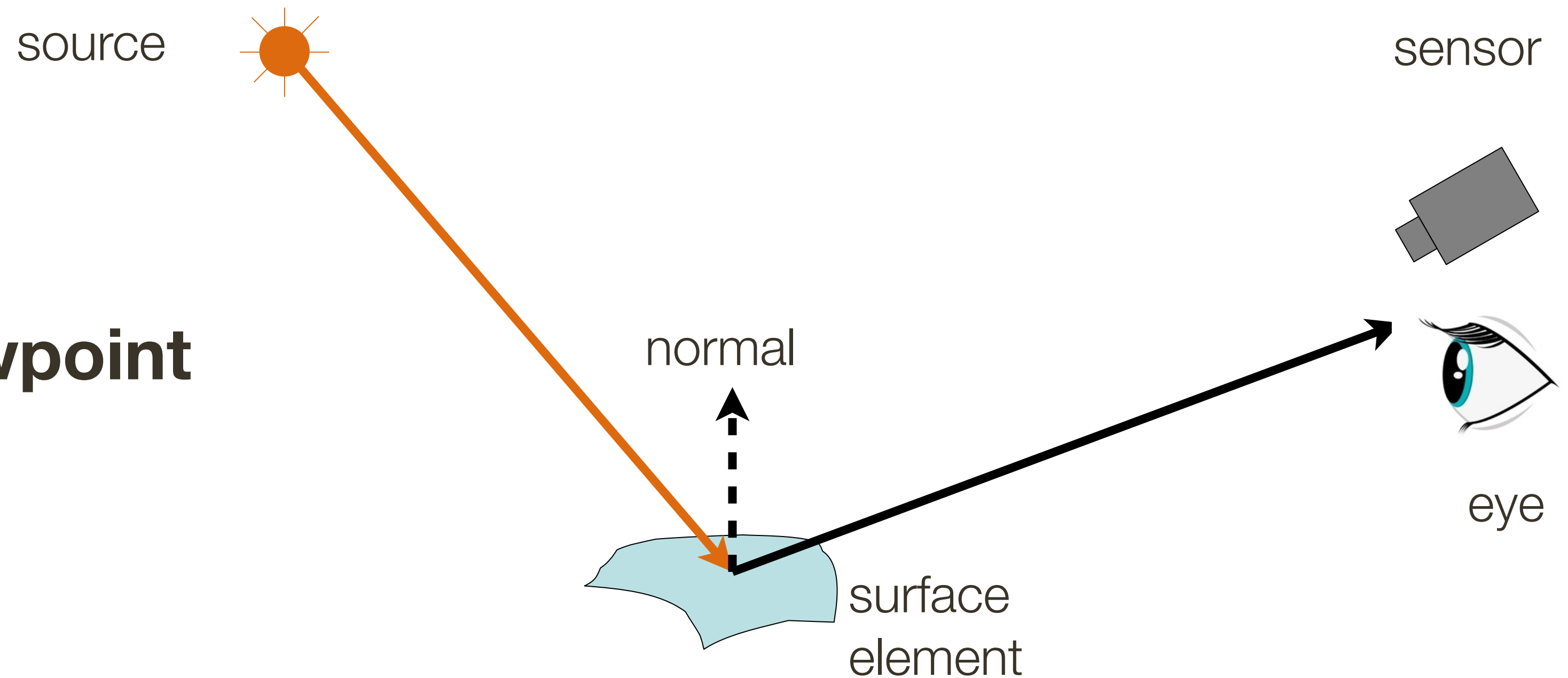
$$E(\lambda)S(\lambda)$$



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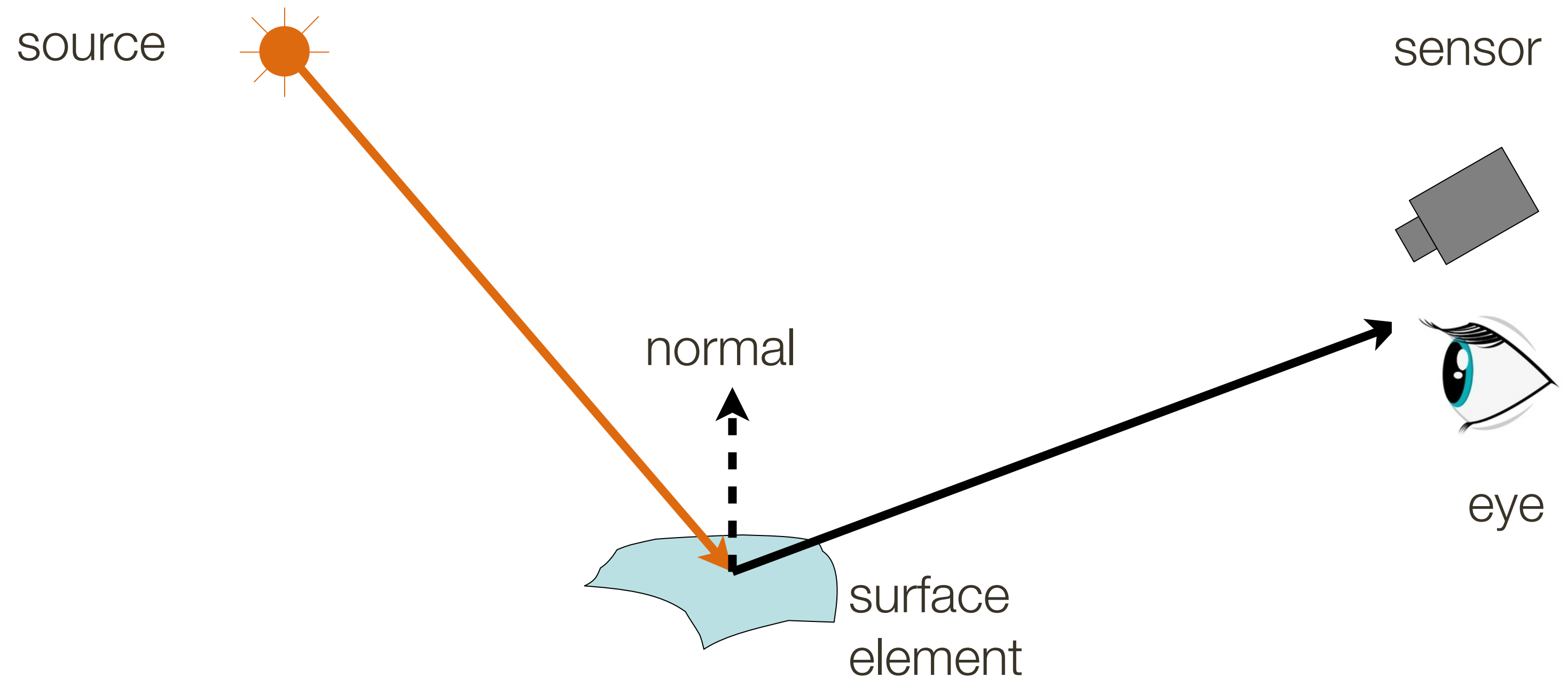


Sensor (or eye) **captures amount of light** reflected from the object

Overview: Image Formation, Cameras and Lenses

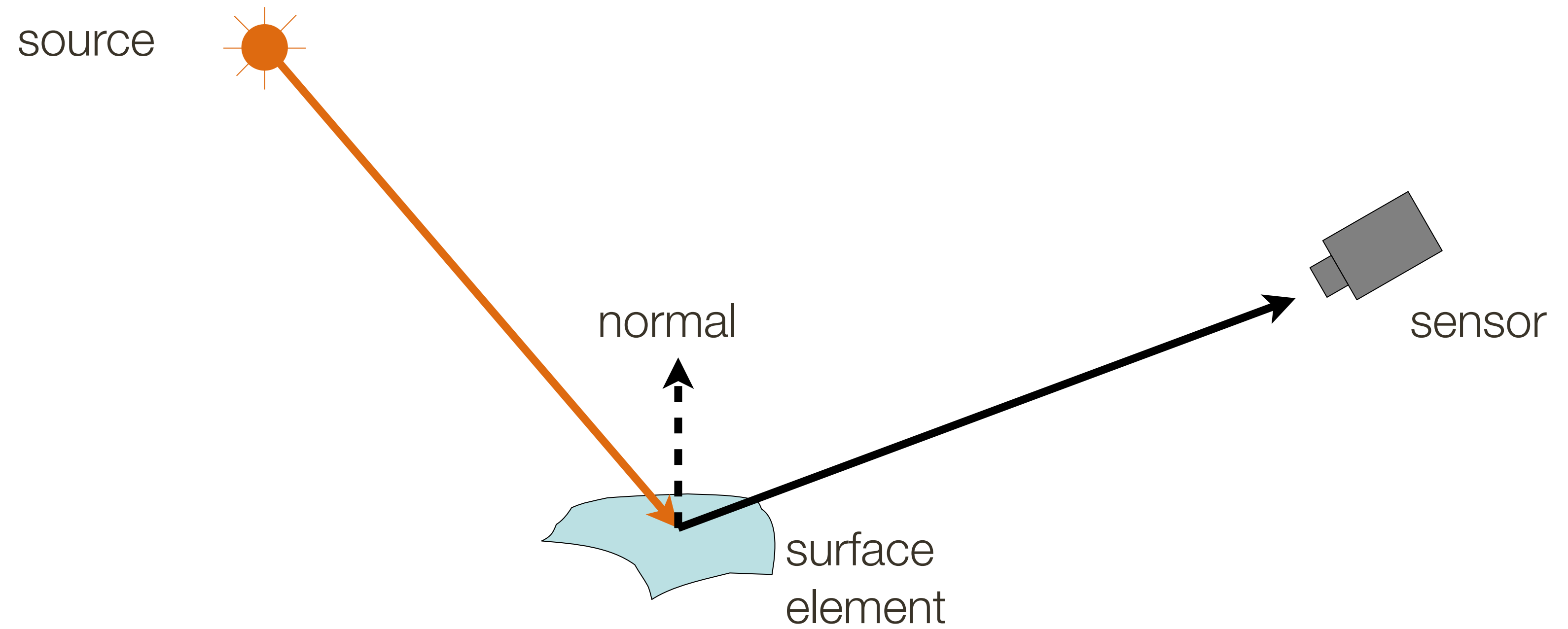
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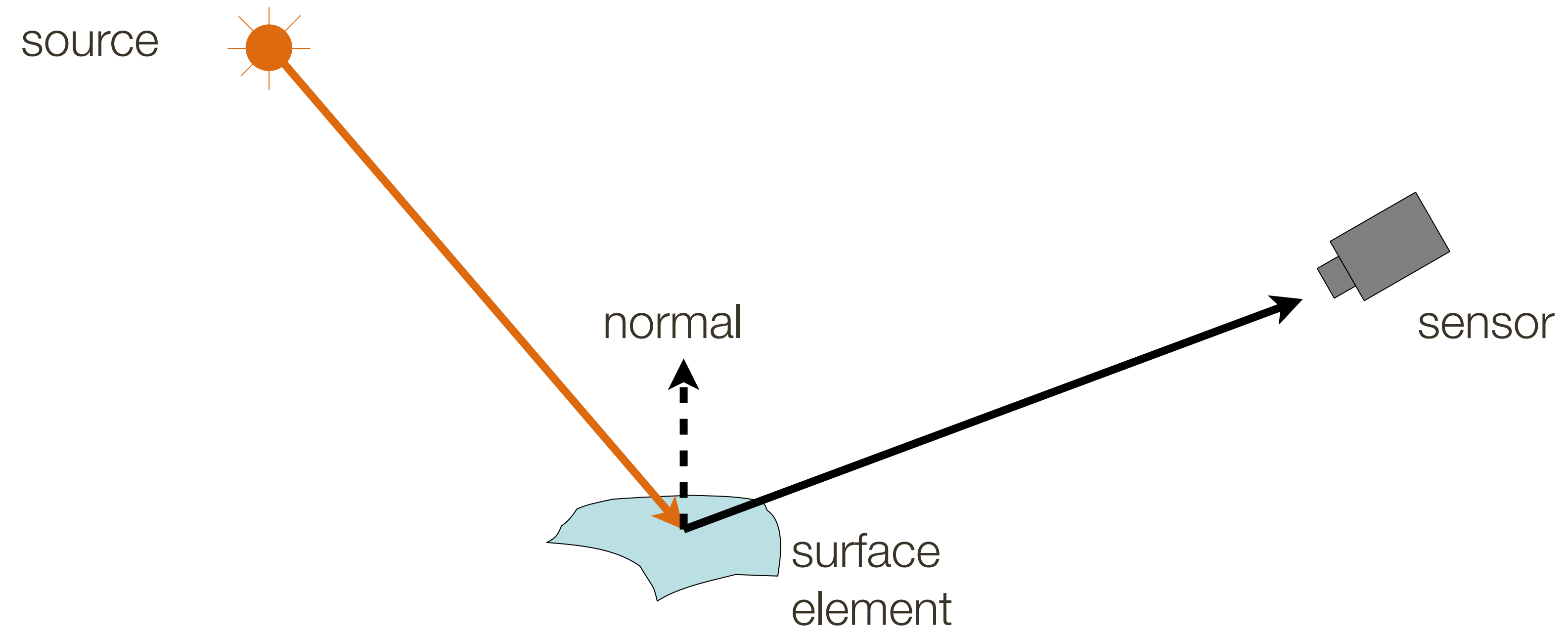


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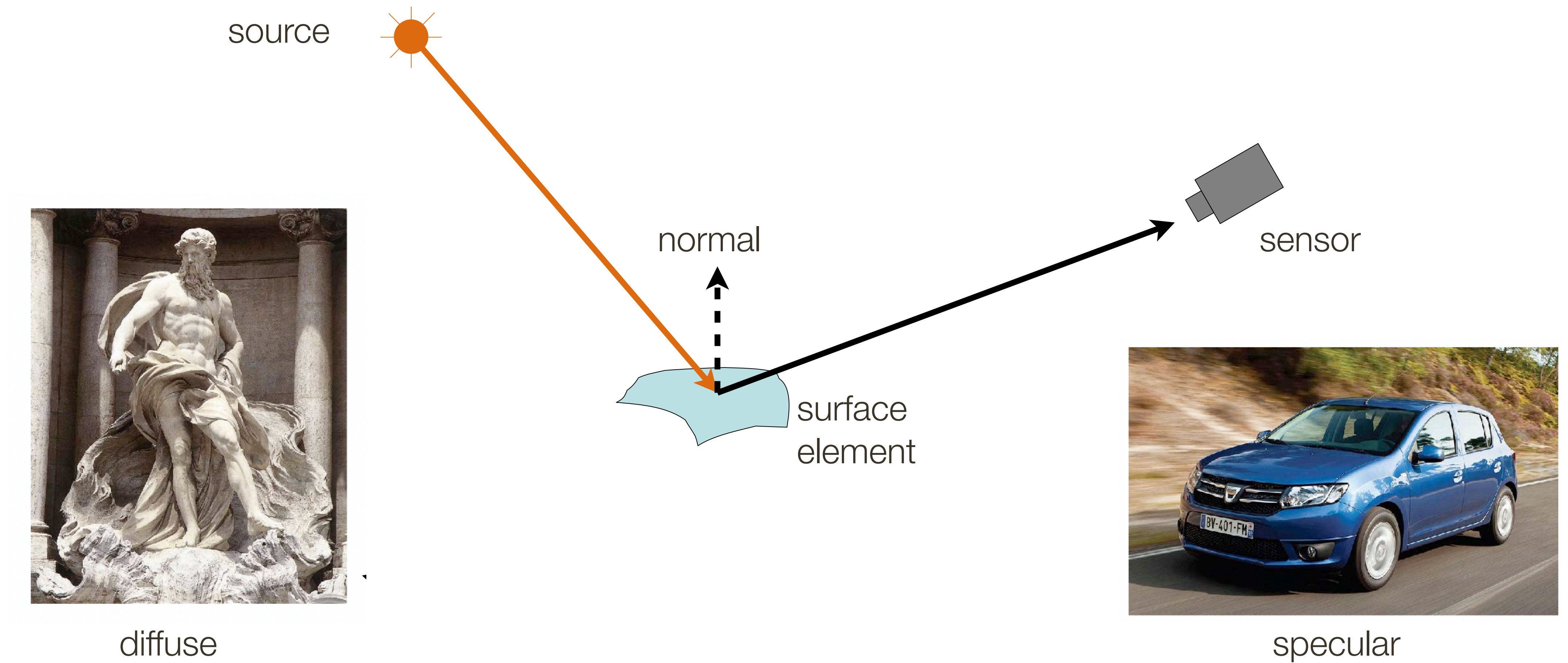
(small) **Graphics** Review



(small) **Graphics** Review

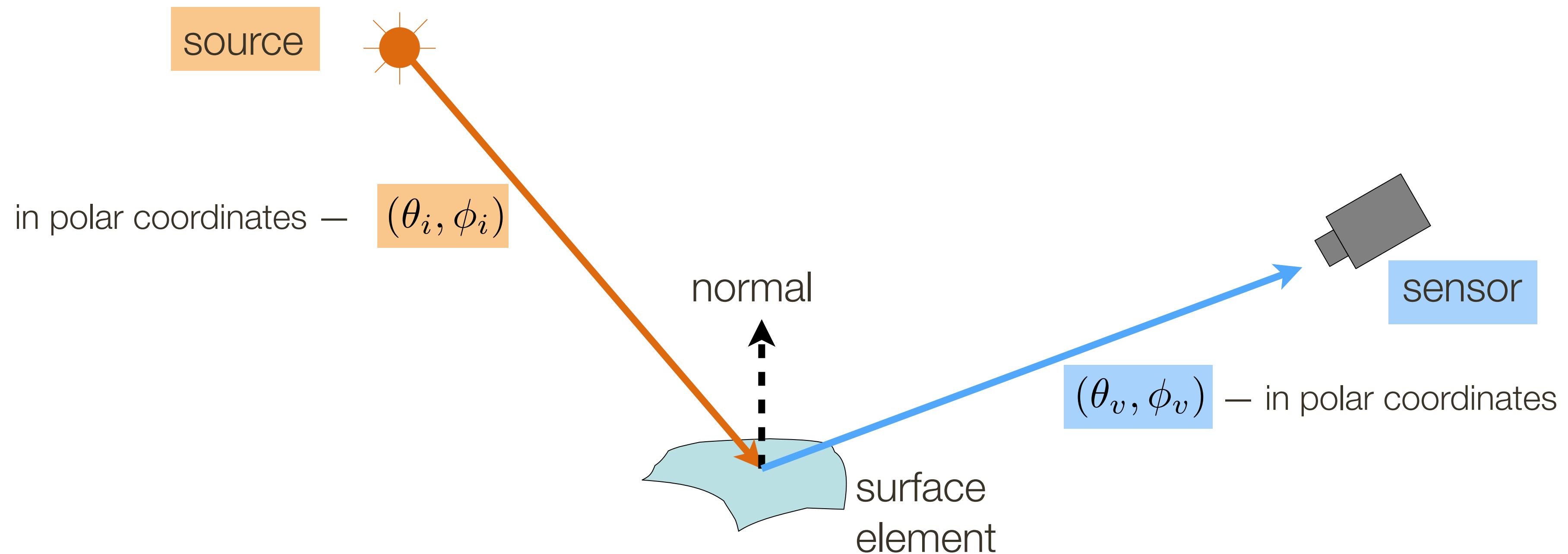


(small) **Graphics** Review



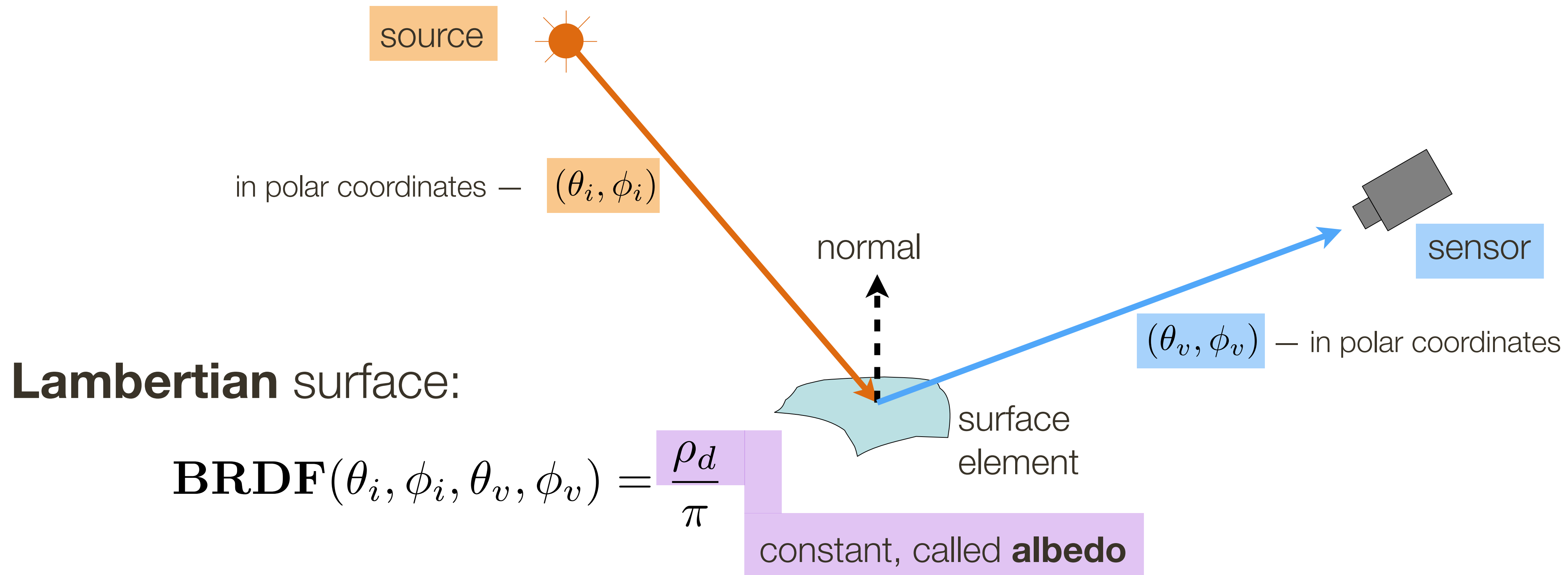
(small) **Graphics** Review

Surface reflection depends on both the **viewing** (θ_v, ϕ_v) and **illumination** (θ_i, ϕ_i) direction, with Bidirectional Reflection Distribution Function: **BRDF** $(\theta_i, \phi_i, \theta_v, \phi_v)$



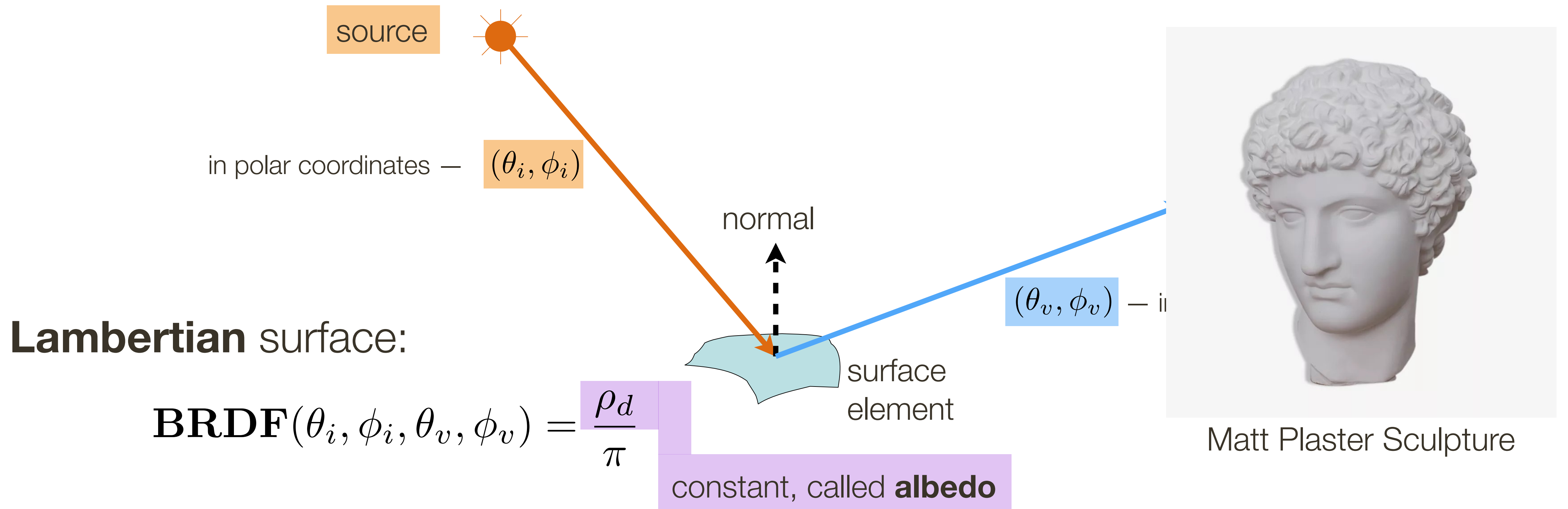
(small) Graphics Review

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(small) Graphics Review

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(small) Graphics Review

Surface reflection depends on both the **viewing** (θ_v, ϕ_v) and **illumination** (θ_i, ϕ_i) direction, with Bidirectional Reflection Distribution Function: **BRDF** $(\theta_i, \phi_i, \theta_v, \phi_v)$

source

(θ_i, ϕ_i)

normal

(θ_v, ϕ_v)

surface element

Lambertian surface:

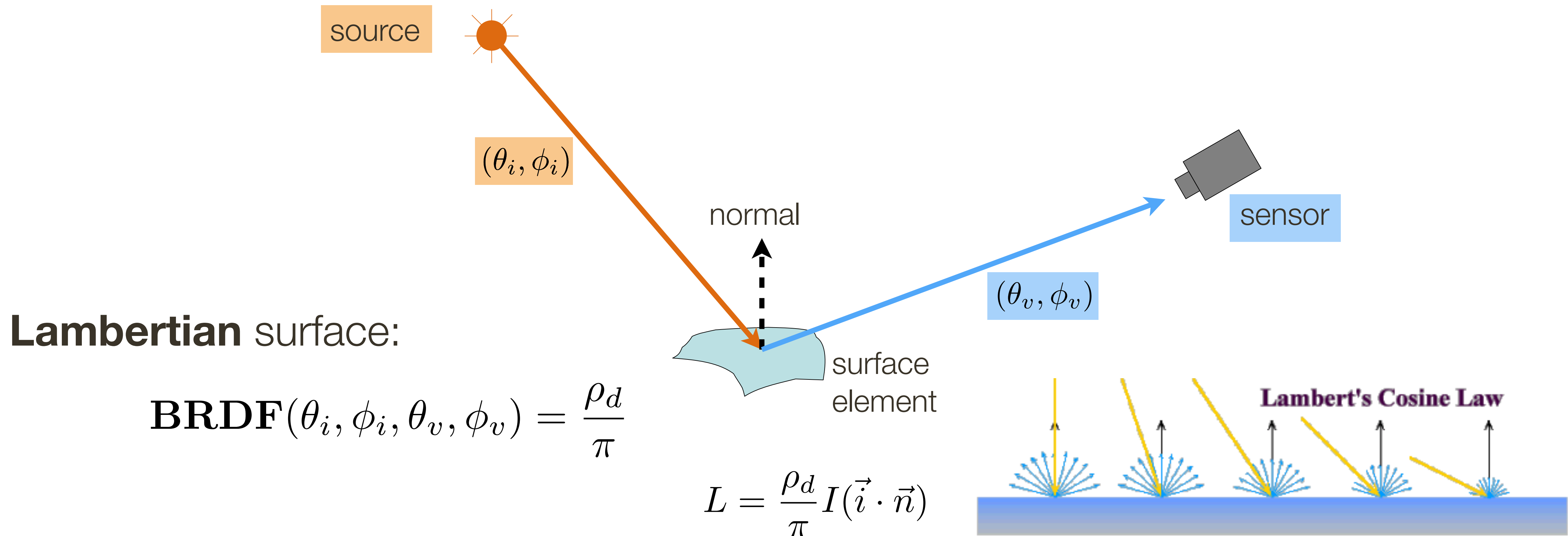
$$\text{BRDF}(\theta_i, \phi_i, \theta_v, \phi_v) = \frac{\rho_d}{\pi}$$

constant, called **albedo**

Surface type	Typical value
Fresh asphalt	0.03 – 0.04
Open ocean	0.06
Conifer forest (summer)	0.08 – 0.15
Worn asphalt	0.12
Deciduous trees	0.15 – 0.18
Sand	0.15 – 0.45
Tundra	0.18 – 0.25
Agricultural crops	0.18 – 0.25
Bare soil	0.17
Green grass	0.20 – 0.25
Dessert sand	0.30 – 0.40
Snow	0.40 – 0.90
Ocean ice	0.50 – 0.70
Fresh snow	0.80 – 0.90

(small) Graphics Review

Surface reflection depends on both the **viewing** (θ_v, ϕ_v) and **illumination** (θ_i, ϕ_i) direction, with Bidirectional Reflection Distribution Function: **BRDF** $(\theta_i, \phi_i, \theta_v, \phi_v)$



(small) Graphics Review

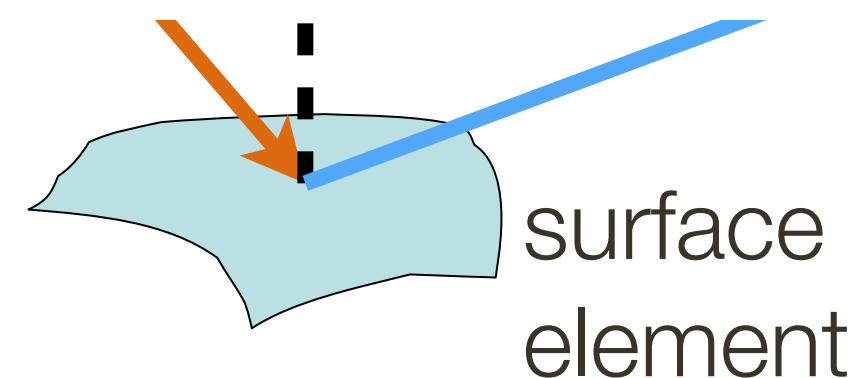
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To sum up: For a perfect **lambertian** surface reflected light is

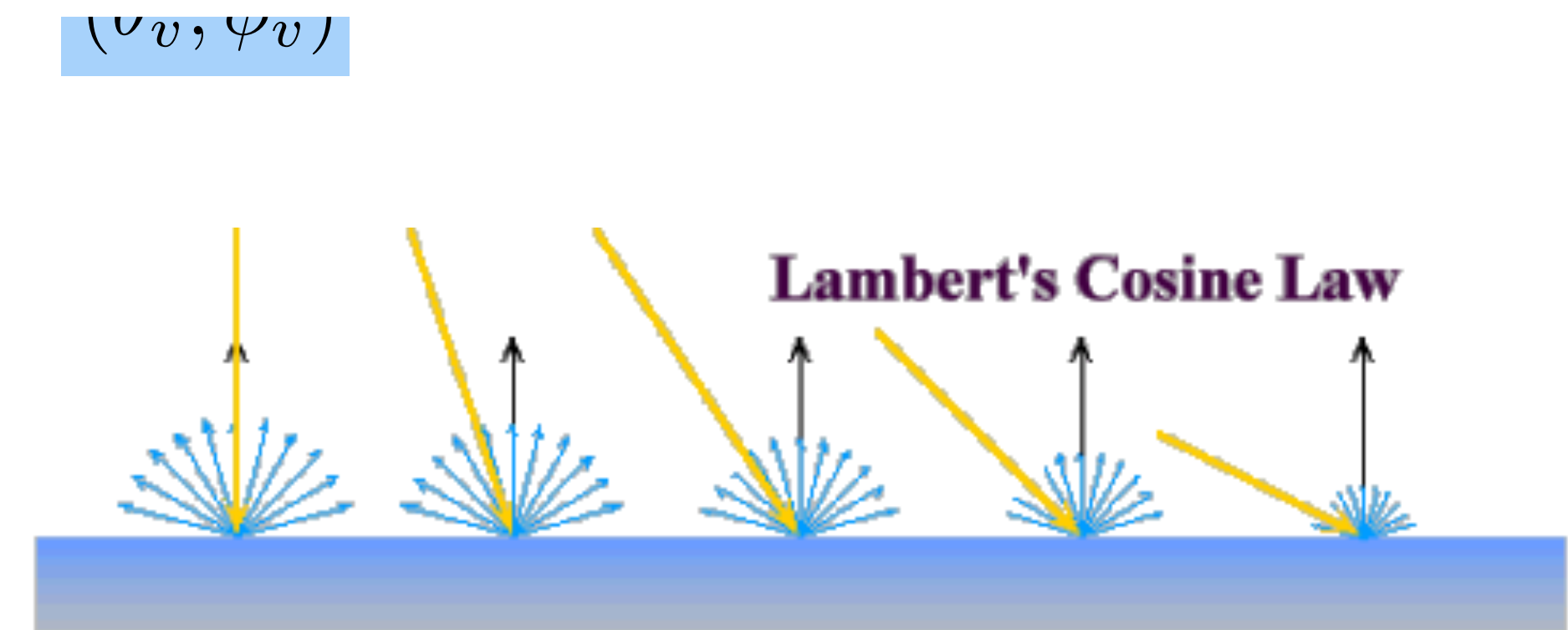
- (1) amount and color of incident light — I
- (2) fraction of light being reflected (material) — ρ_d
- (3) angle between the light and the surface (geometry) — $(\vec{i} \cdot \vec{n})$

Lambertian surface:

$$\text{BRDF}(\theta_i, \phi_i, \theta_v, \phi_v) = \frac{\rho_d}{\pi}$$

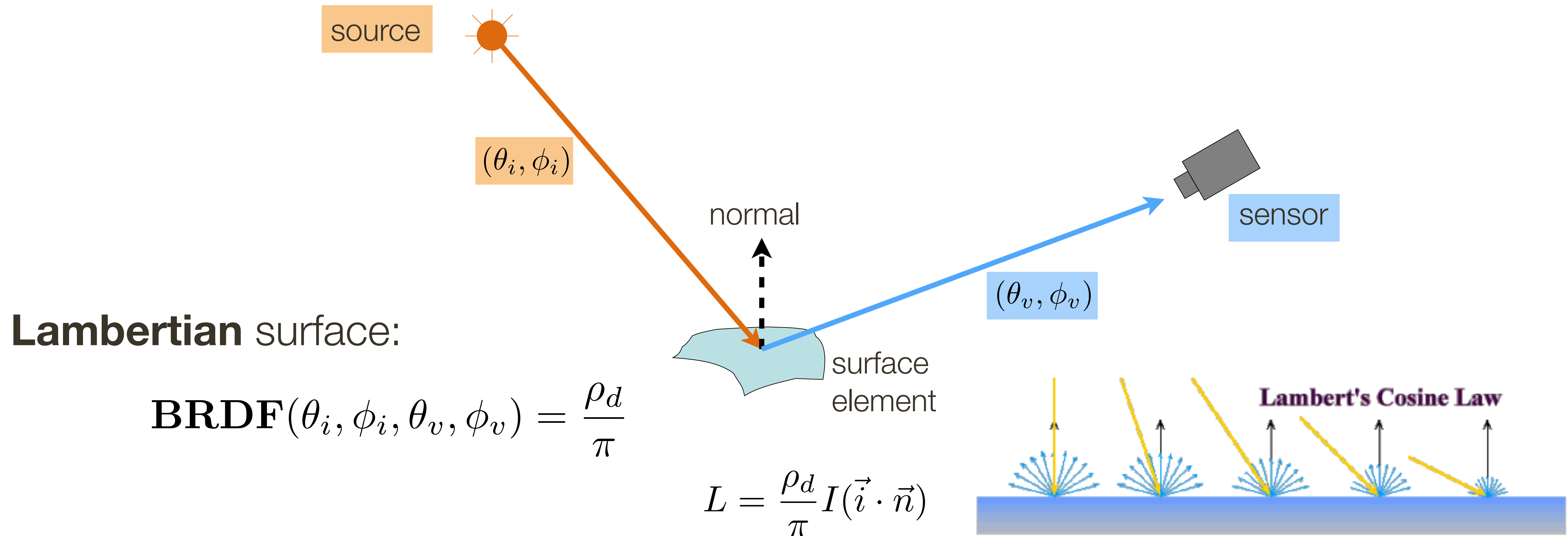


$$L = \frac{\rho_d}{\pi} I(\vec{i} \cdot \vec{n})$$



(small) Graphics Review

Question: What are the simplifying assumptions we are making here?



(small) Graphics Review

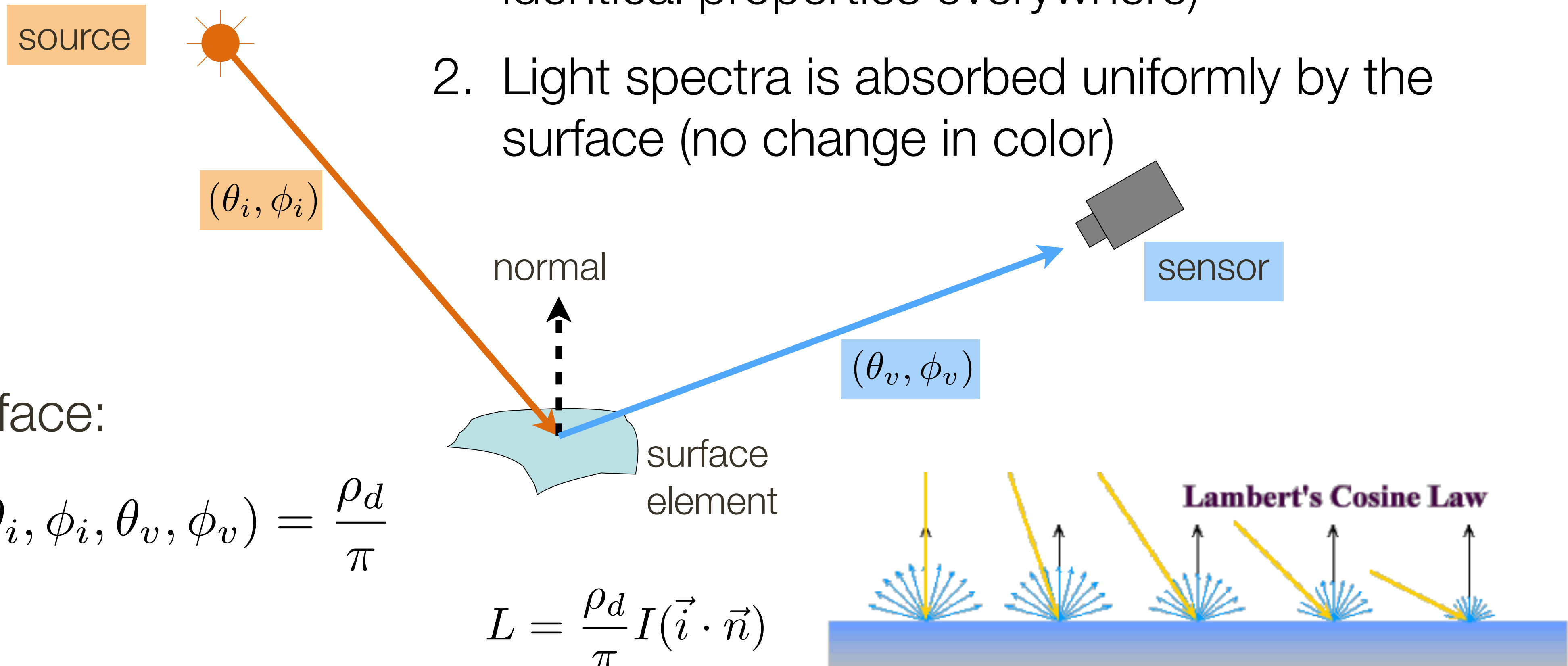
Question: What are the simplifying assumptions we are making here?

1. BRDF is the same everywhere (i.e., surface has identical properties everywhere)
2. Light spectra is absorbed uniformly by the surface (no change in color)

Lambertian surface:

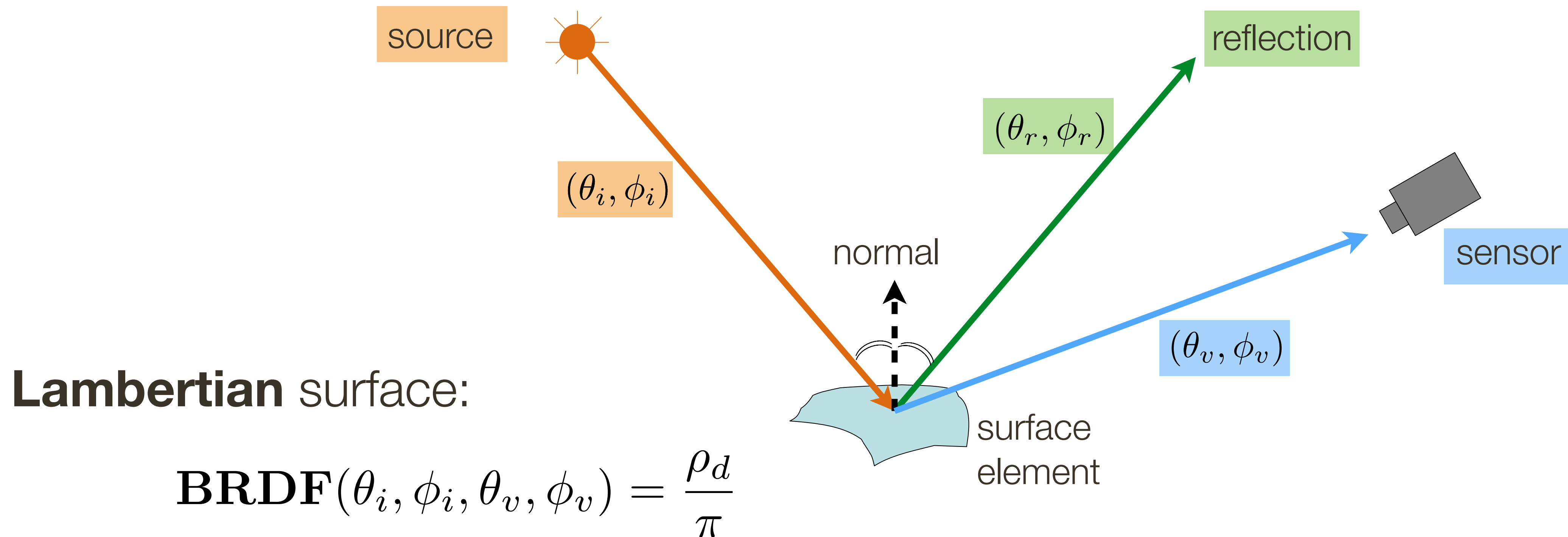
$$\text{BRDF}(\theta_i, \phi_i, \theta_v, \phi_v) = \frac{\rho_d}{\pi}$$

$$L = \frac{\rho_d}{\pi} I(\vec{i} \cdot \vec{n})$$



(small) Graphics Review

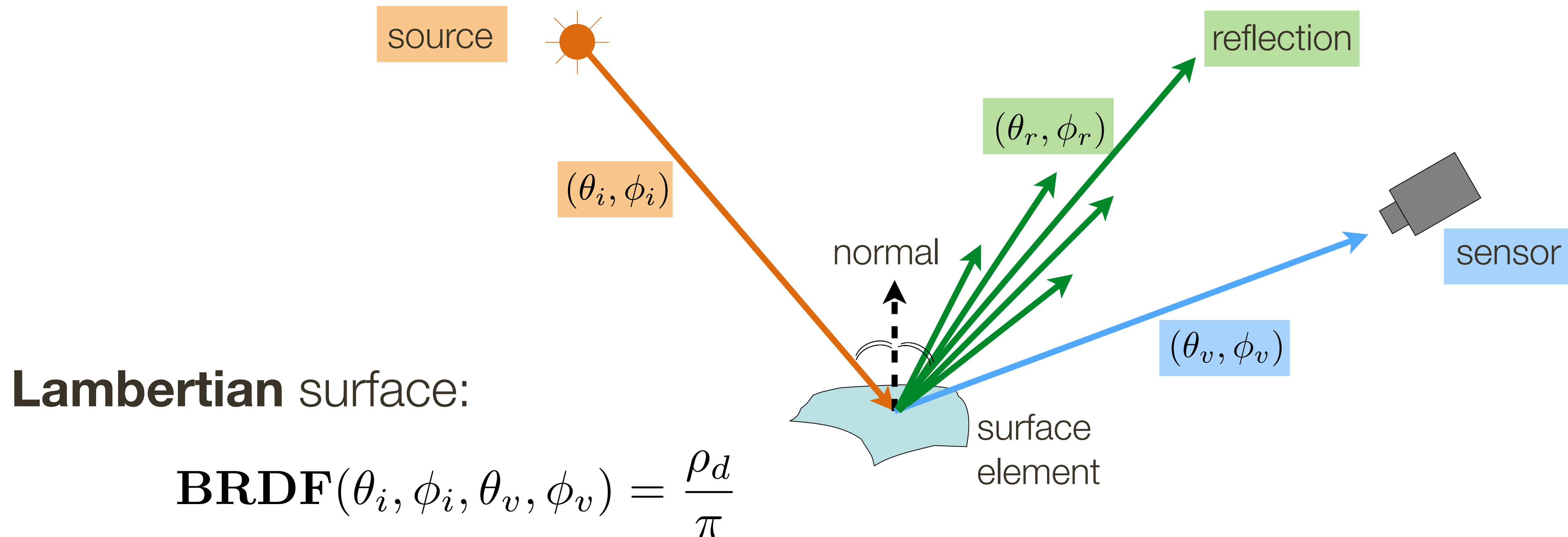
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Mirror surface: all incident light reflected in one directions $(\theta_v, \phi_v) = (\theta_r, \phi_r)$

(small) Graphics Review

Surface reflection depends on both the **viewing** (θ_v, ϕ_v) and **illumination** (θ_i, ϕ_i) direction, with Bidirectional Reflection Distribution Function: **BRDF** $(\theta_i, \phi_i, \theta_v, \phi_v)$

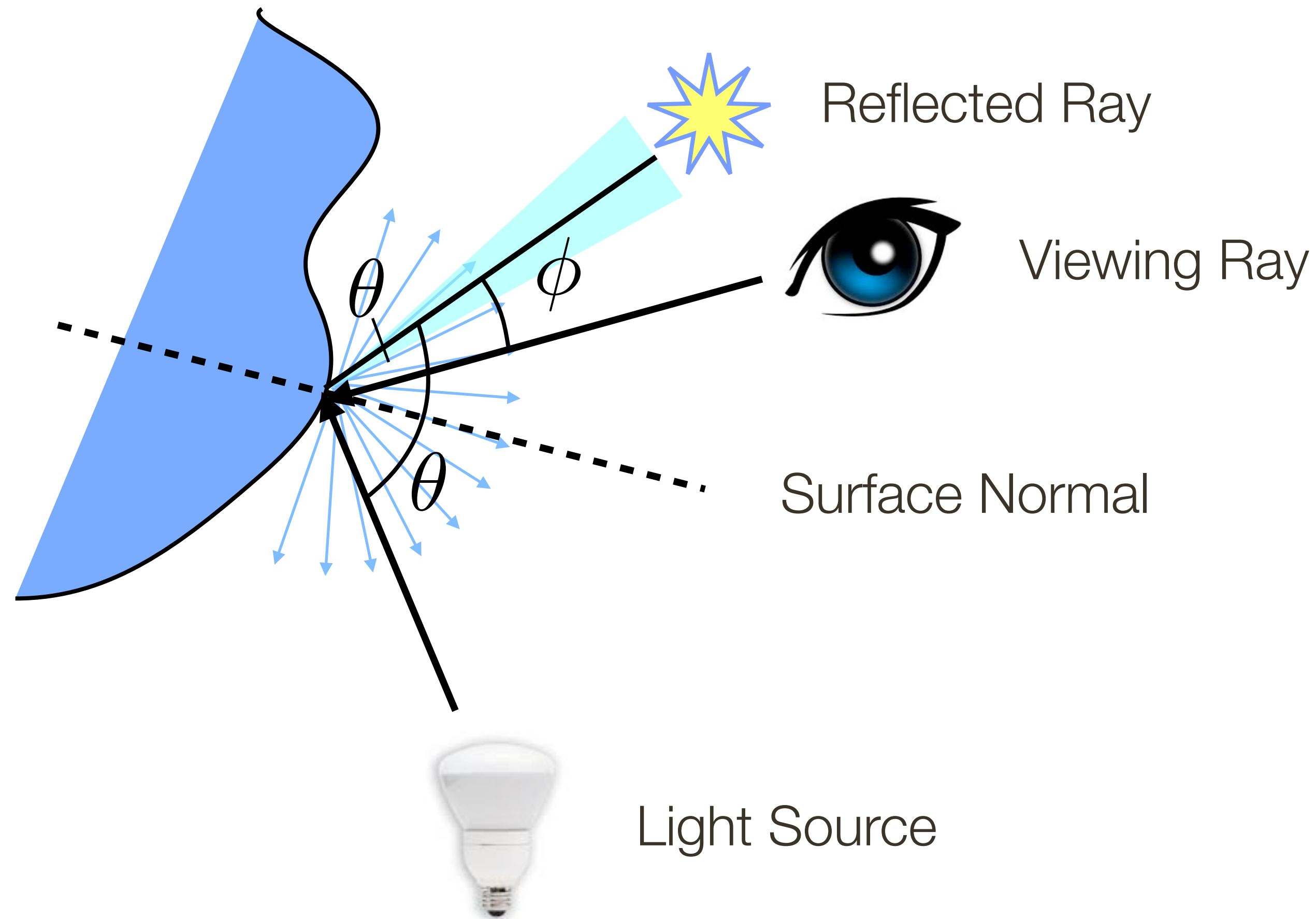


Mirror surface: all incident light reflected in one directions $(\theta_v, \phi_v) = (\theta_r, \phi_r)$

Phong Illumination Model

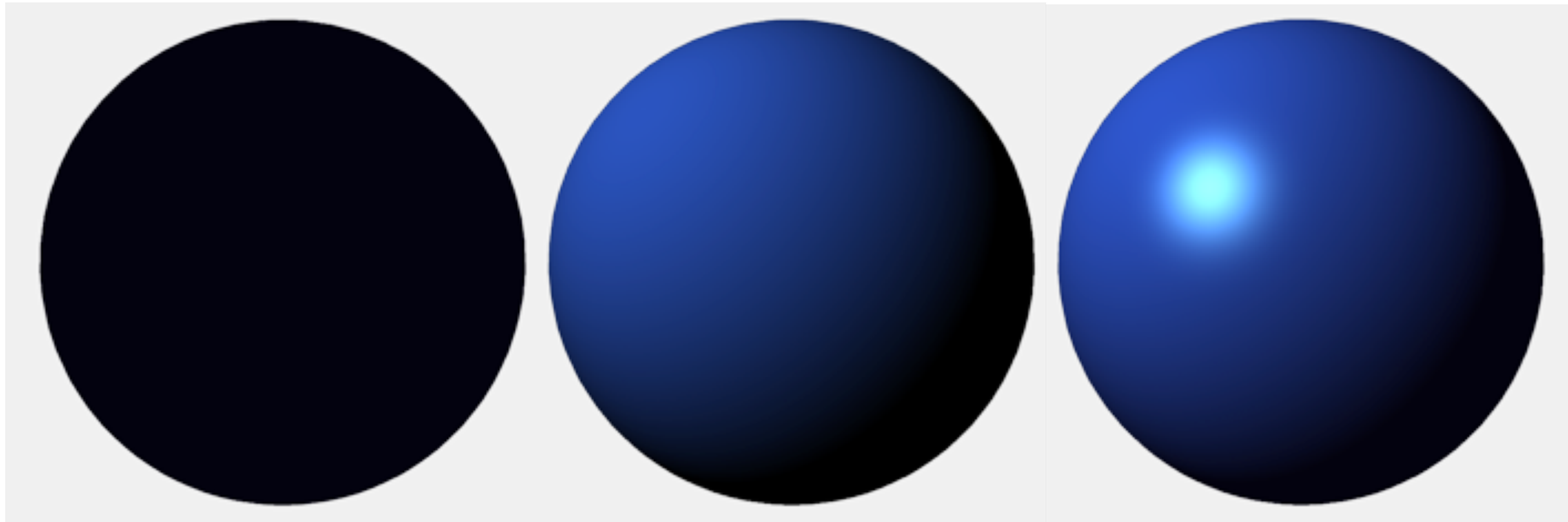
Includes **ambient**, **diffuse** and **specular** reflection

$$I = k_a i_a + k_d i_d \cos \theta + k_s i_s \cos^\alpha \phi$$



Phong Illumination Model

Includes **ambient**, **diffuse** and **specular** reflection



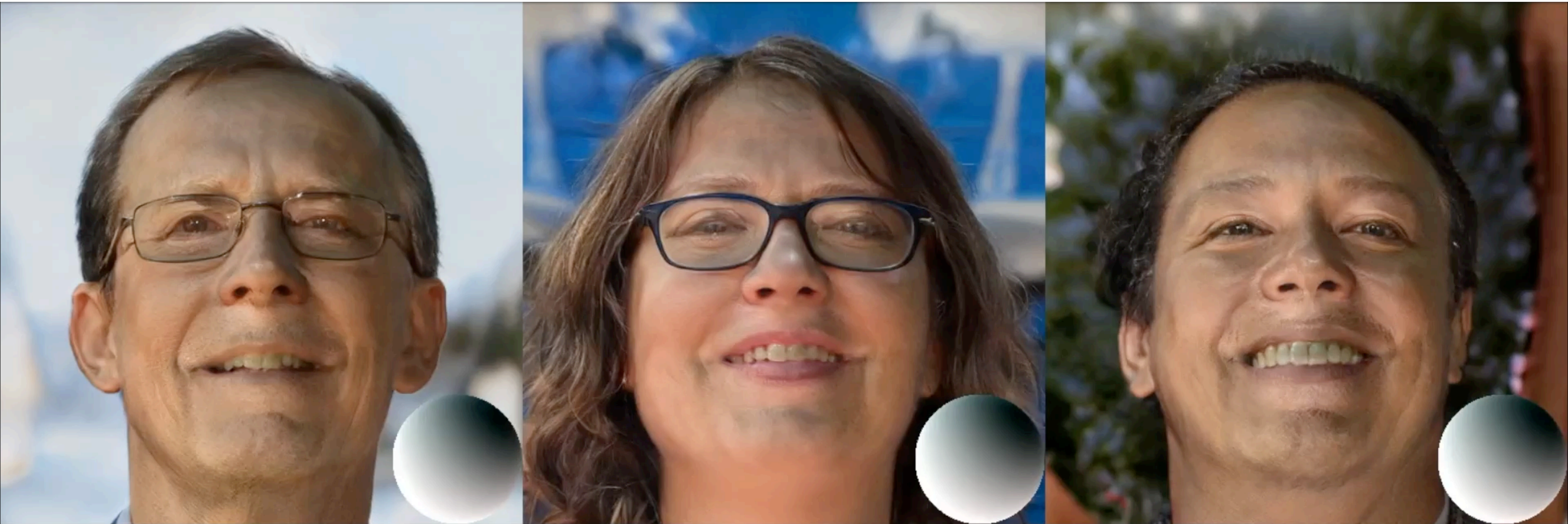
Ambient

+Diffuse

+Specular

Phong Illumination Model

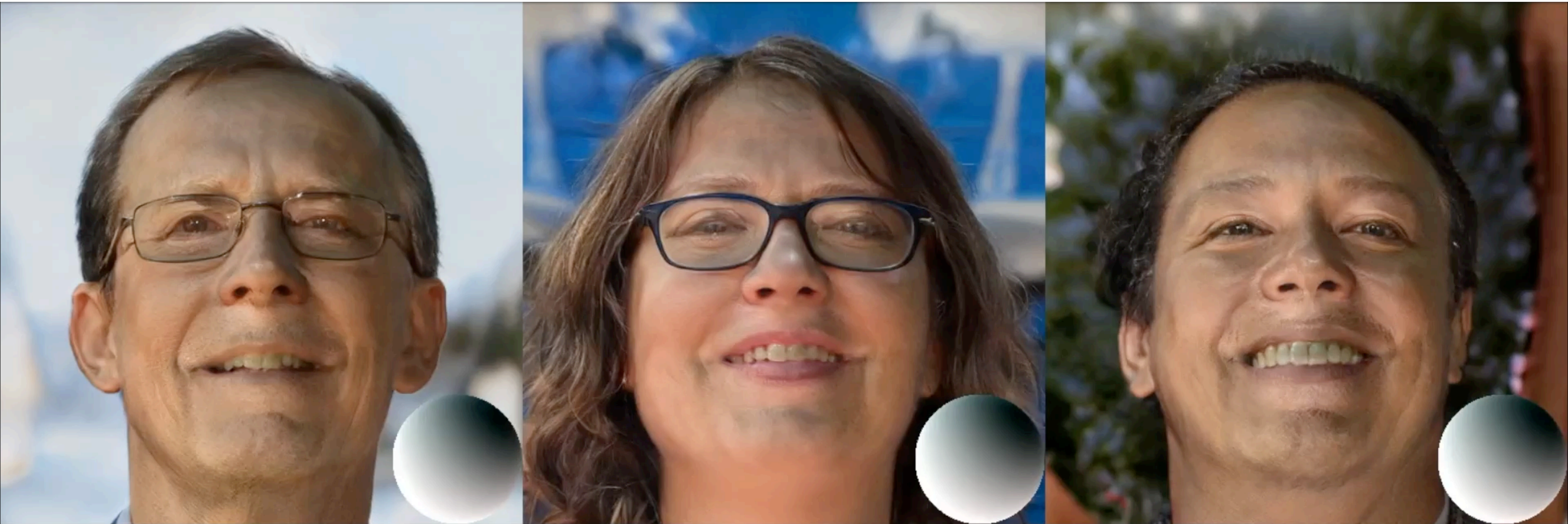
Motivating example that uses this reflection model



View + Light Control

Phong Illumination Model

Motivating example that uses this reflection model

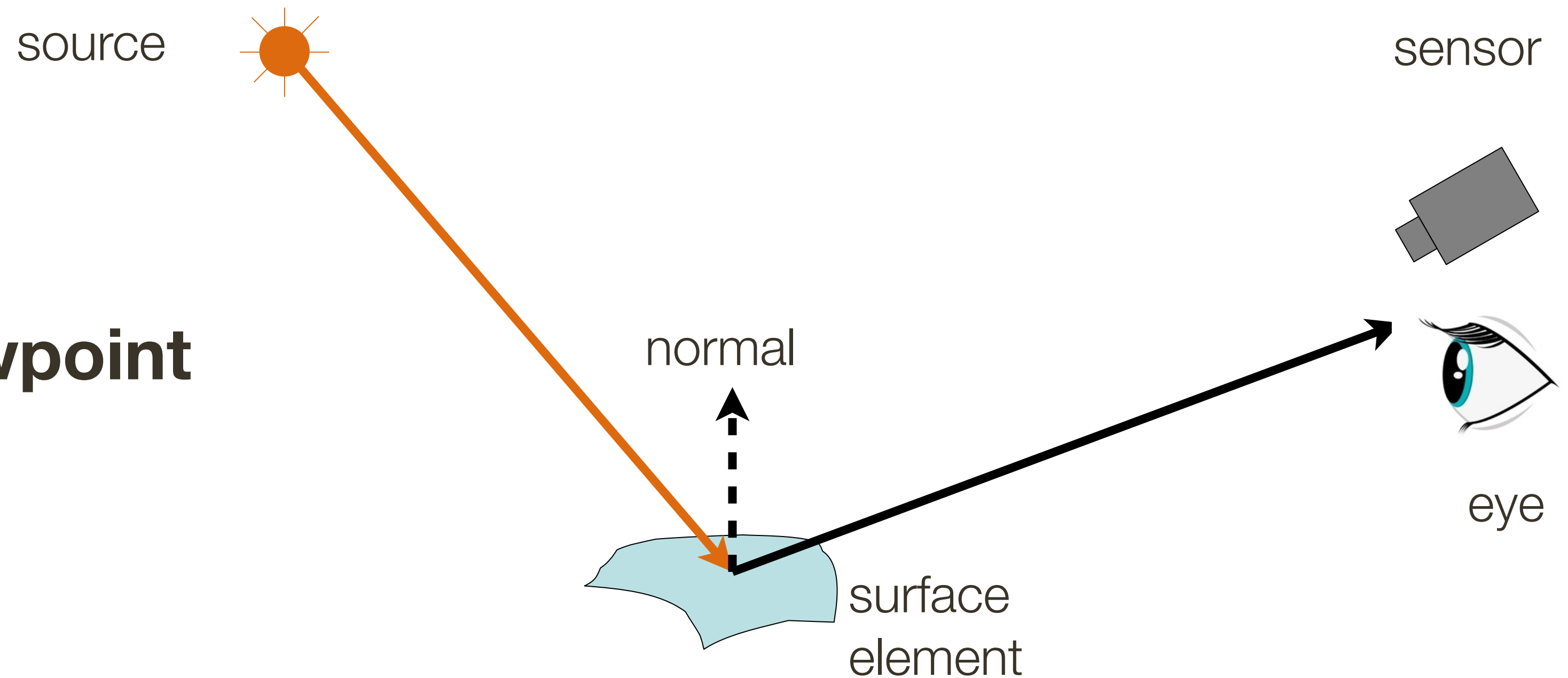


View + Light Control

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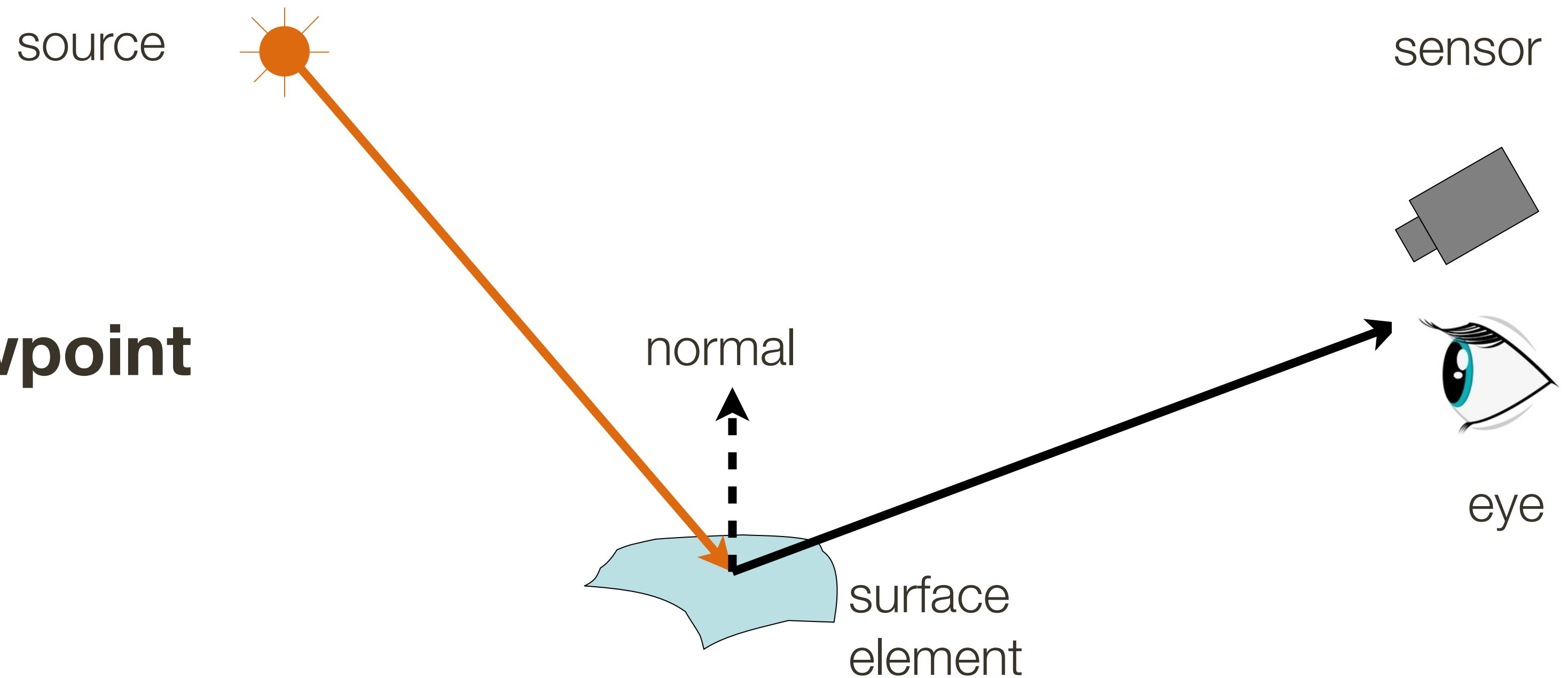


Sensor (or eye) **captures amount of light** reflected from the object

Overview: Image Formation, Cameras and Lenses

The **image formation process** that produces a particular image depends on

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Sensor (or eye) **captures amount of light** reflected from the object

Cameras

Old school **film** camera



Digital CCD/CMOS camera

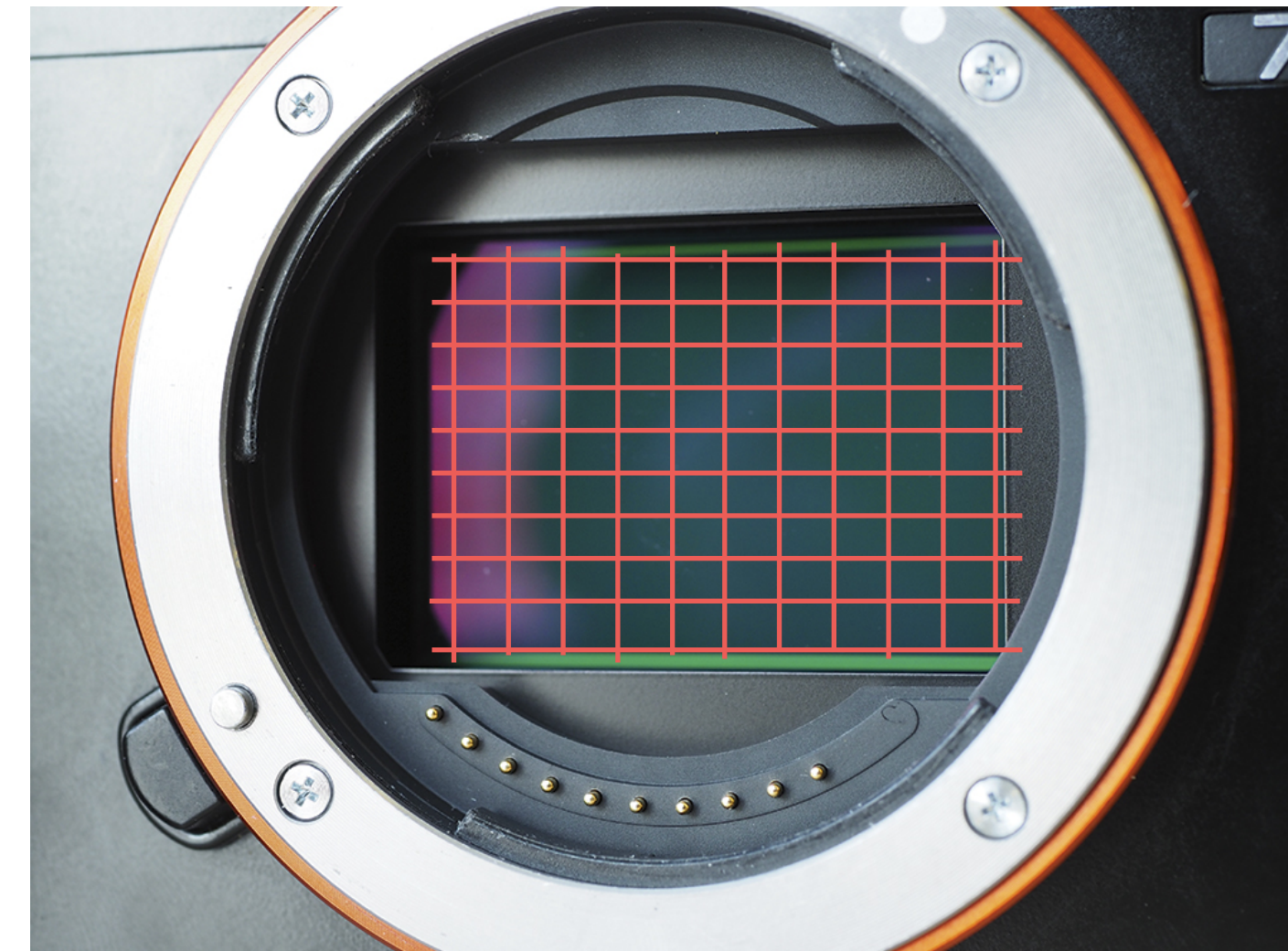


Cameras

Old school **film** camera



Digital CCD/CMOS camera



Let's say we have a **sensor** ...

Digital CCD/CMOS camera



Let's say we have a **sensor** ...

Digital CCD/CMOS camera



Let's say we have a **sensor** ...

Digital CCD/CMOS camera



digital sensor
(CCD or
CMOS)

... and the **object** we would like to photograph

What would an image taken like this look like?

real-world
object



digital sensor
(CCD or
CMOS)



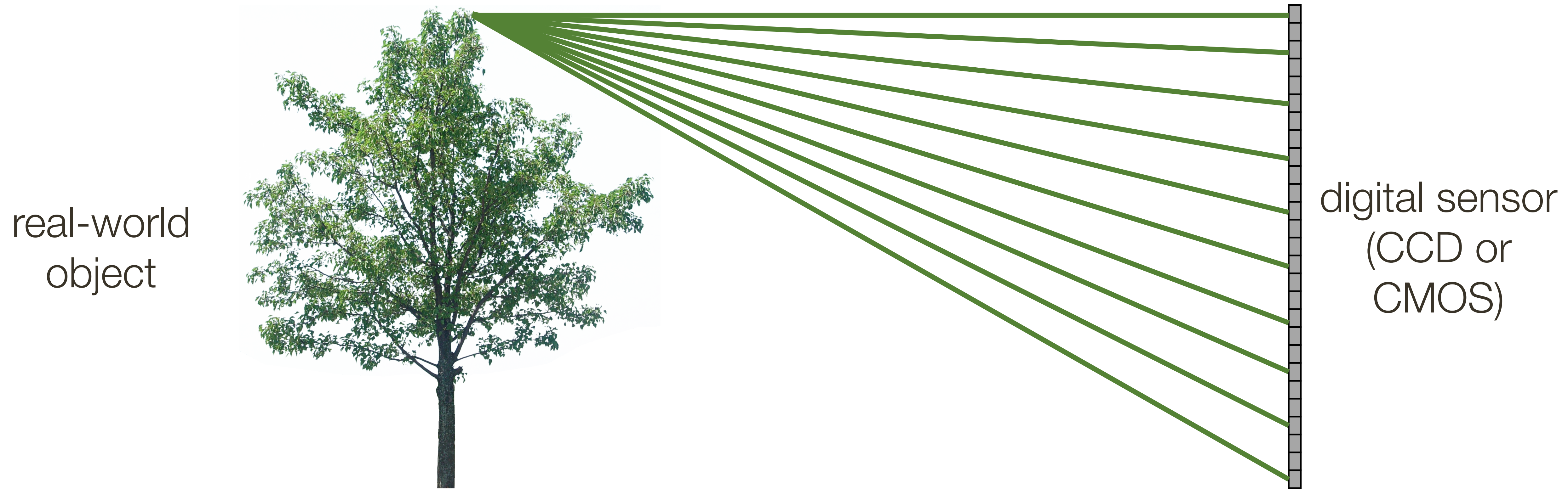
Bare-sensor imaging

real-world
object

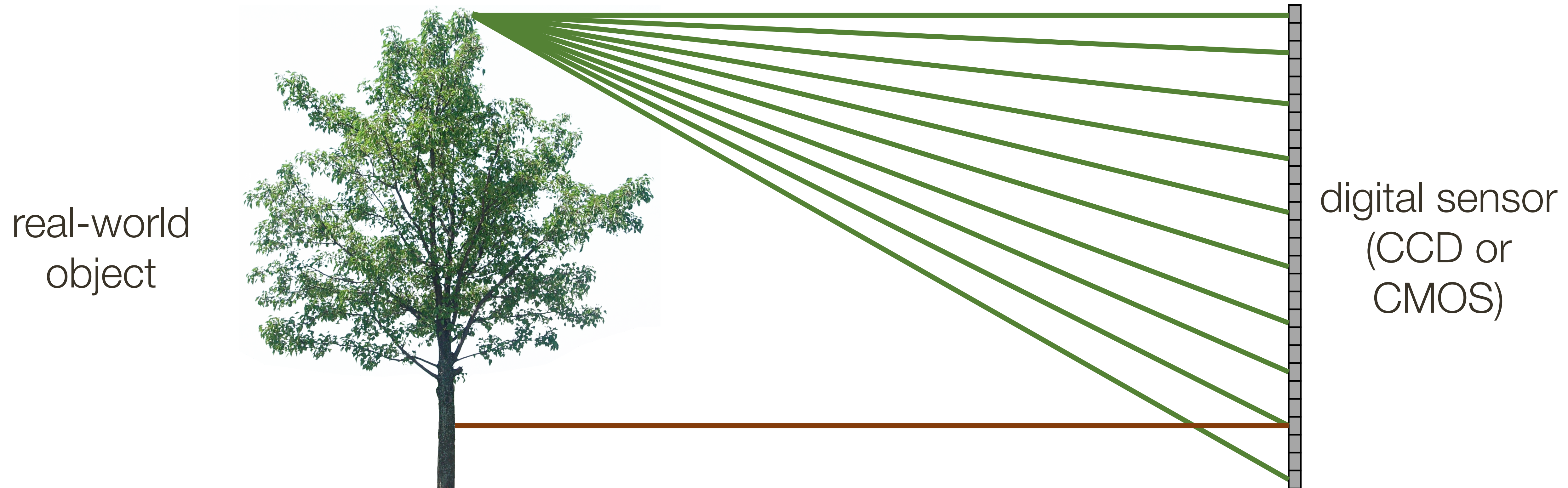


digital sensor
(CCD or
CMOS)

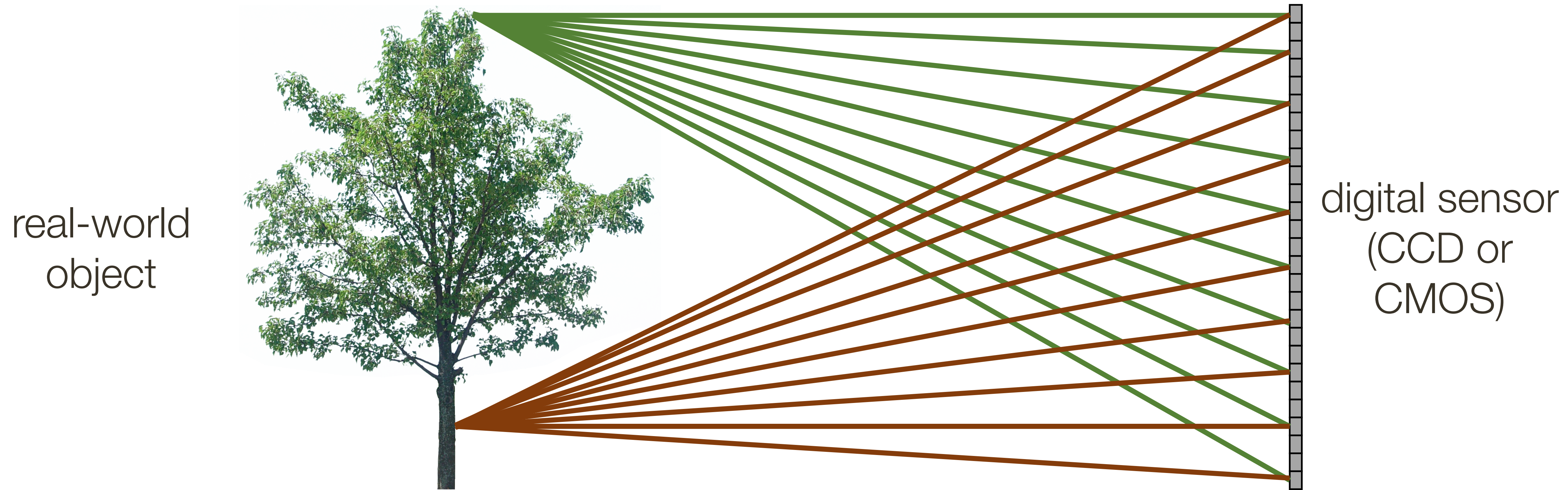
Bare-sensor imaging



Bare-sensor imaging



Bare-sensor imaging



All scene points contribute to all sensor pixels

Bare-sensor imaging



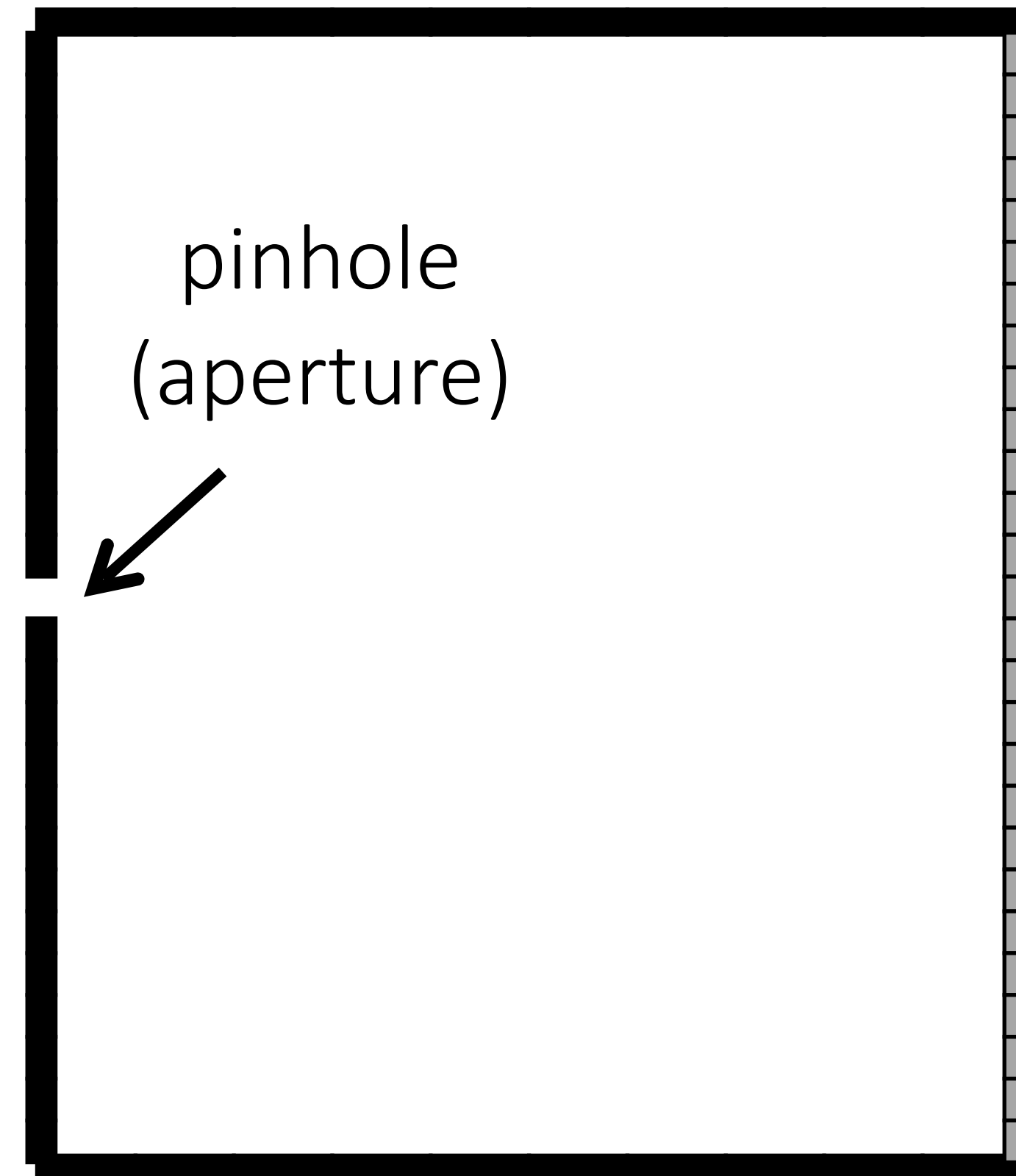
All scene points contribute to all sensor pixels

Pinhole Camera

real-world
object



barrier (diaphragm)



digital sensor
(CCD or
CMOS)

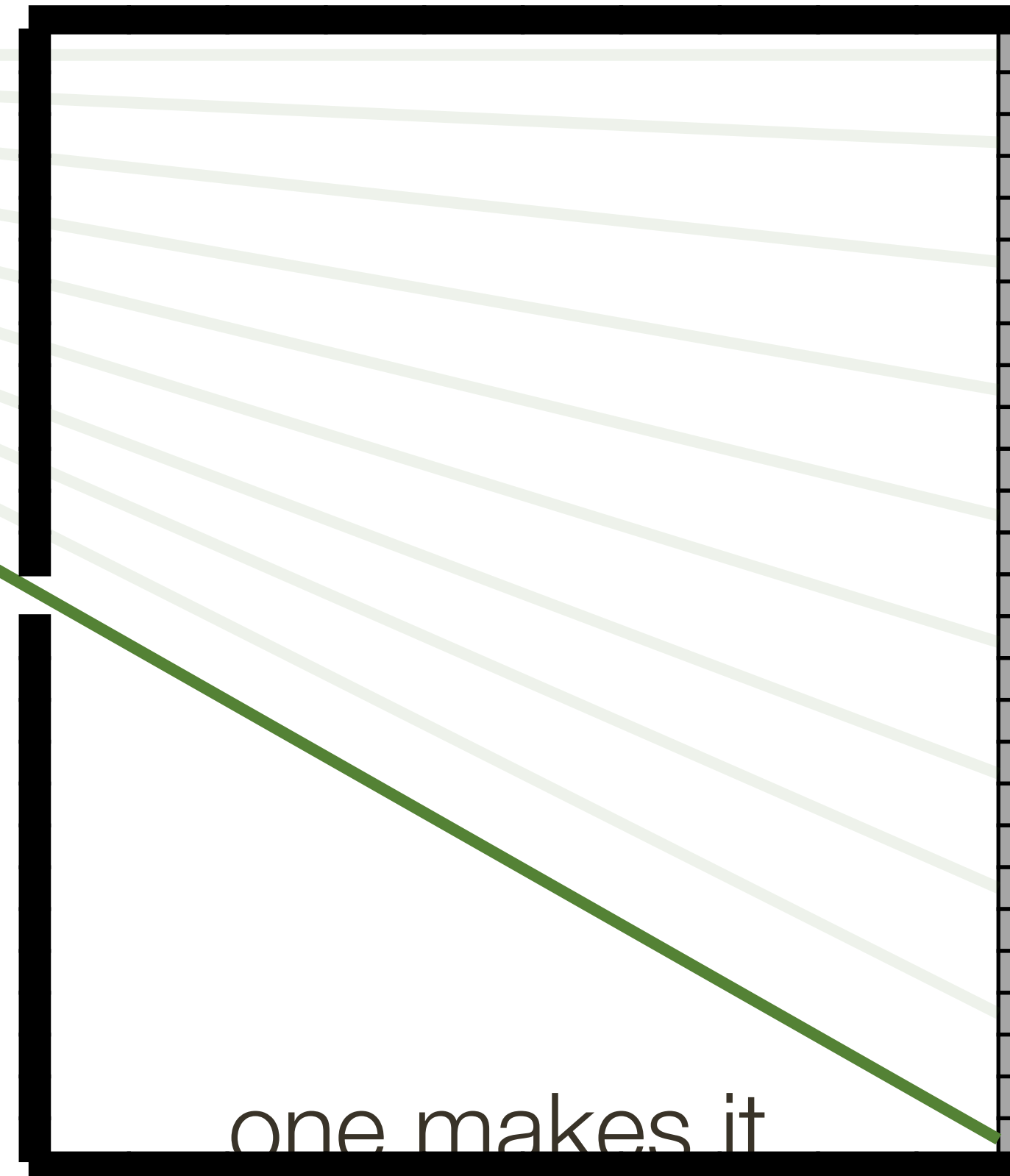
What would an image taken like this look like?

Pinhole Camera

real-world
object



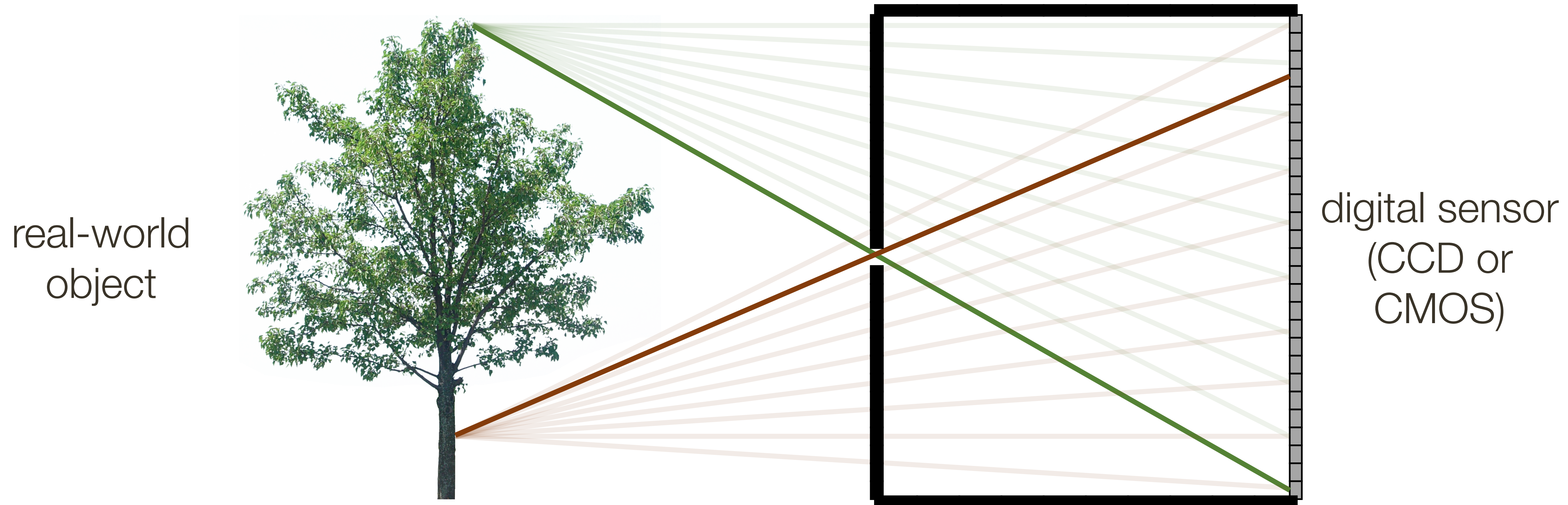
most rays are
blocked



one makes it
through

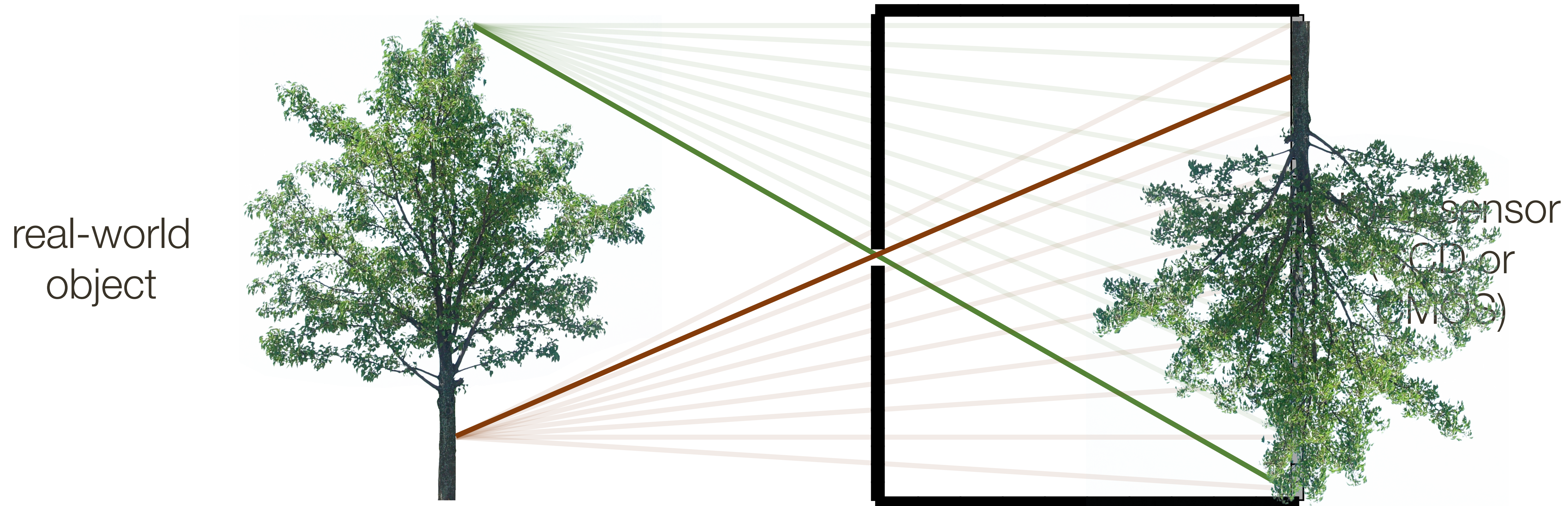
digital sensor
(CCD or
CMOS)

Pinhole Camera



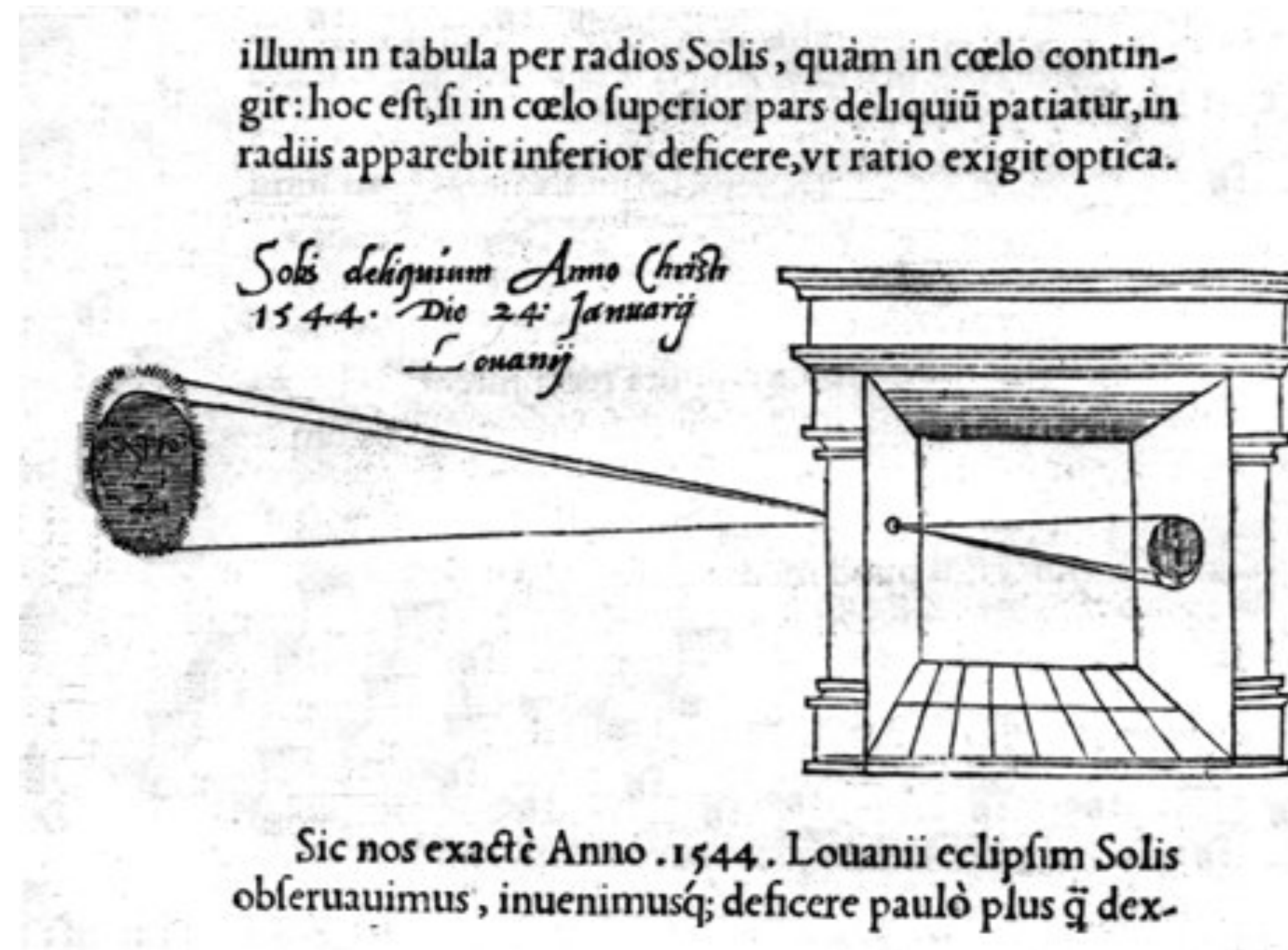
Each scene point contributes to only one sensor pixel

Pinhole Camera



Each scene point contributes to only one sensor pixel

Camera Obscura (latin for “dark chamber”)



Reinerus Gemma-Frisius observed an eclipse of the sun at Louvain on January 24, 1544. He used this illustration in his book, “De Radio Astronomica et Geometrica,” 1545. It is thought to be the first published illustration of a camera obscura.

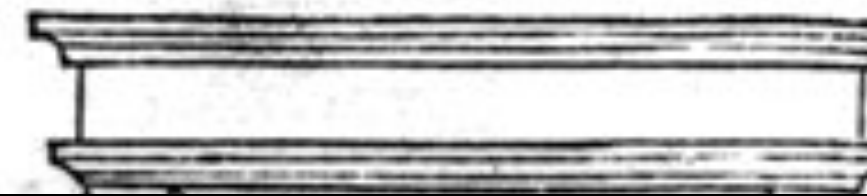
Credit: John H., Hammond, “Th Camera Obscure, A Chronicle”

Camera Obscura (latin for “dark chamber”)



illum in tabula per radios Solis, quam in cœlo contin-
git: hoc est, si in cœlo superior pars deliquiū patiatur, in
radiis apparebit inferior deficere, vt ratio exigit optica.

*Solis deliquium Anno Christi
1544. Die 24. Januarij
Louanij*



principles behind the pinhole camera or camera obscura were first mentioned by Chinese philosopher Mozi (Mo-Ti) (470 to 390 BCE)



Sic nos exactè Anno .1544. Louanii eclipsim Solis
obseruauimus, inuenimusq; deficere paulò plus q̃ dex-

Reinerus Gemma-Frisius observed an eclipse of the sun at Louvain on January 24, 1544. He used this illustration in his book, “De Radio Astronomica et Geometrica,” 1545. It is thought to be the first published illustration of a camera obscura.

Credit: John H., Hammond, “Th Camera Obscure, A Chronicle”

First **Photograph** on Record

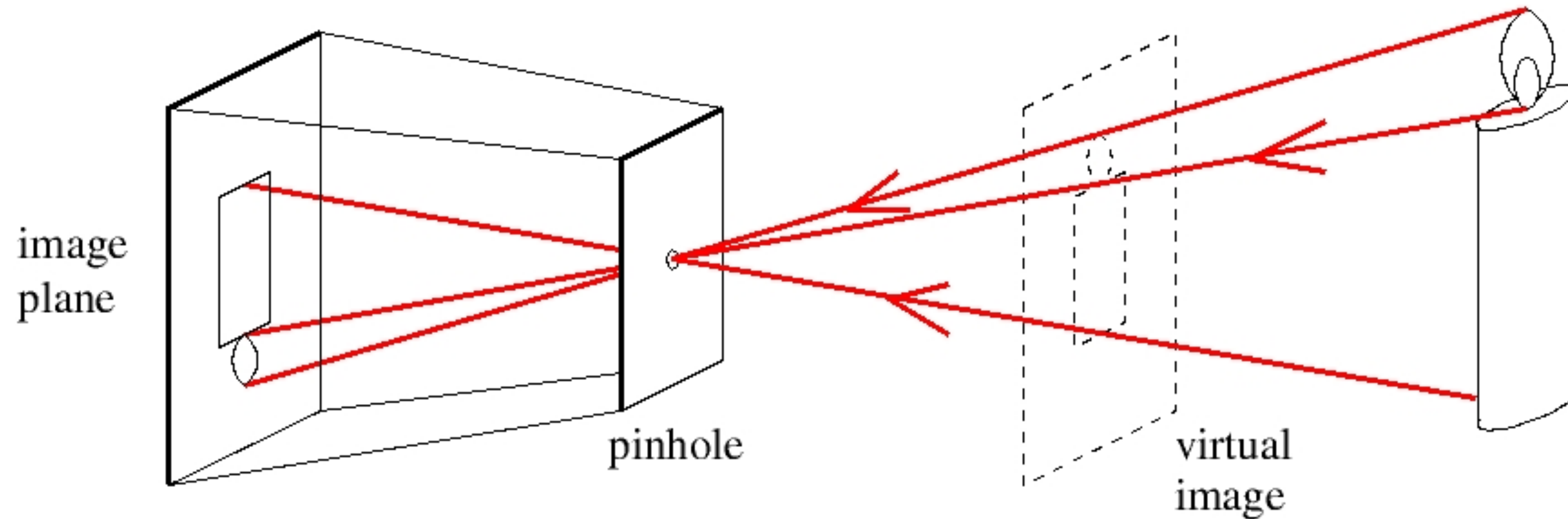
La table servie



Credit: Nicéphore Niepce, 1822

Pinhole Camera

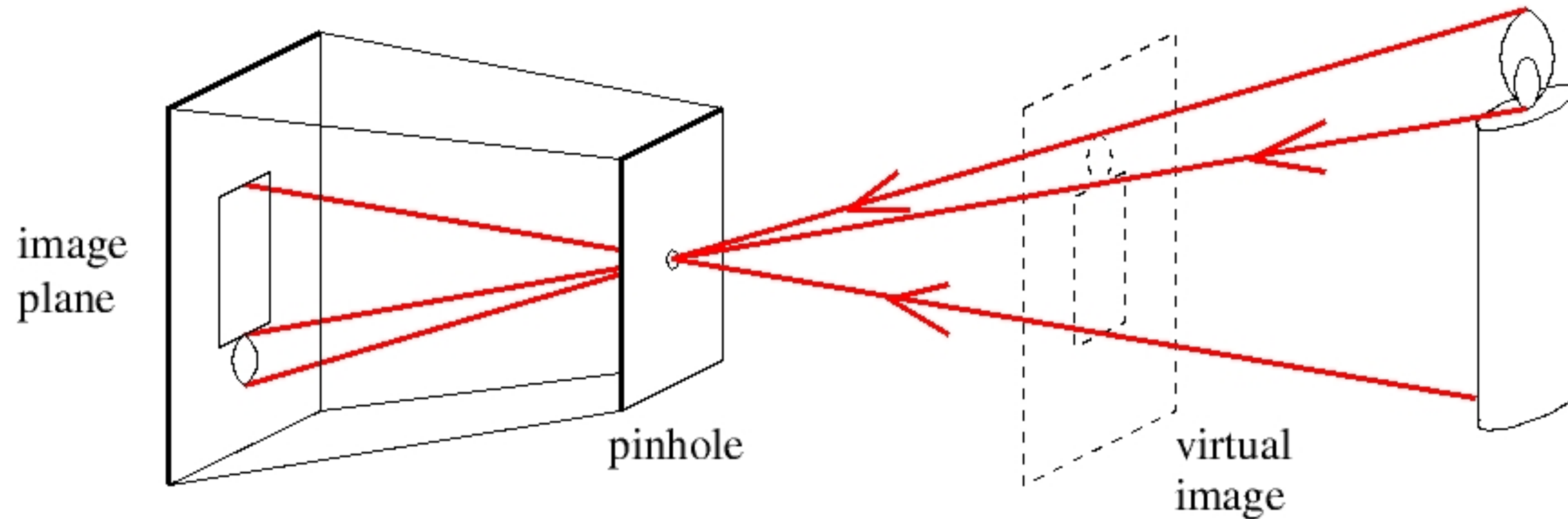
A pinhole camera is a box with a small hole (**aperture**) in it



Forsyth & Ponce (2nd ed.) Figure 1.2

Pinhole Camera

A pinhole camera is a box with a small hole (**aperture**) in it



Forsyth & Ponce (2nd ed.) Figure 1.2

Image Formation



Forsyth & Ponce (2nd ed.) Figure 1.1

Credit: US Navy, Basic Optics and Optical Instruments. Dover, 1969

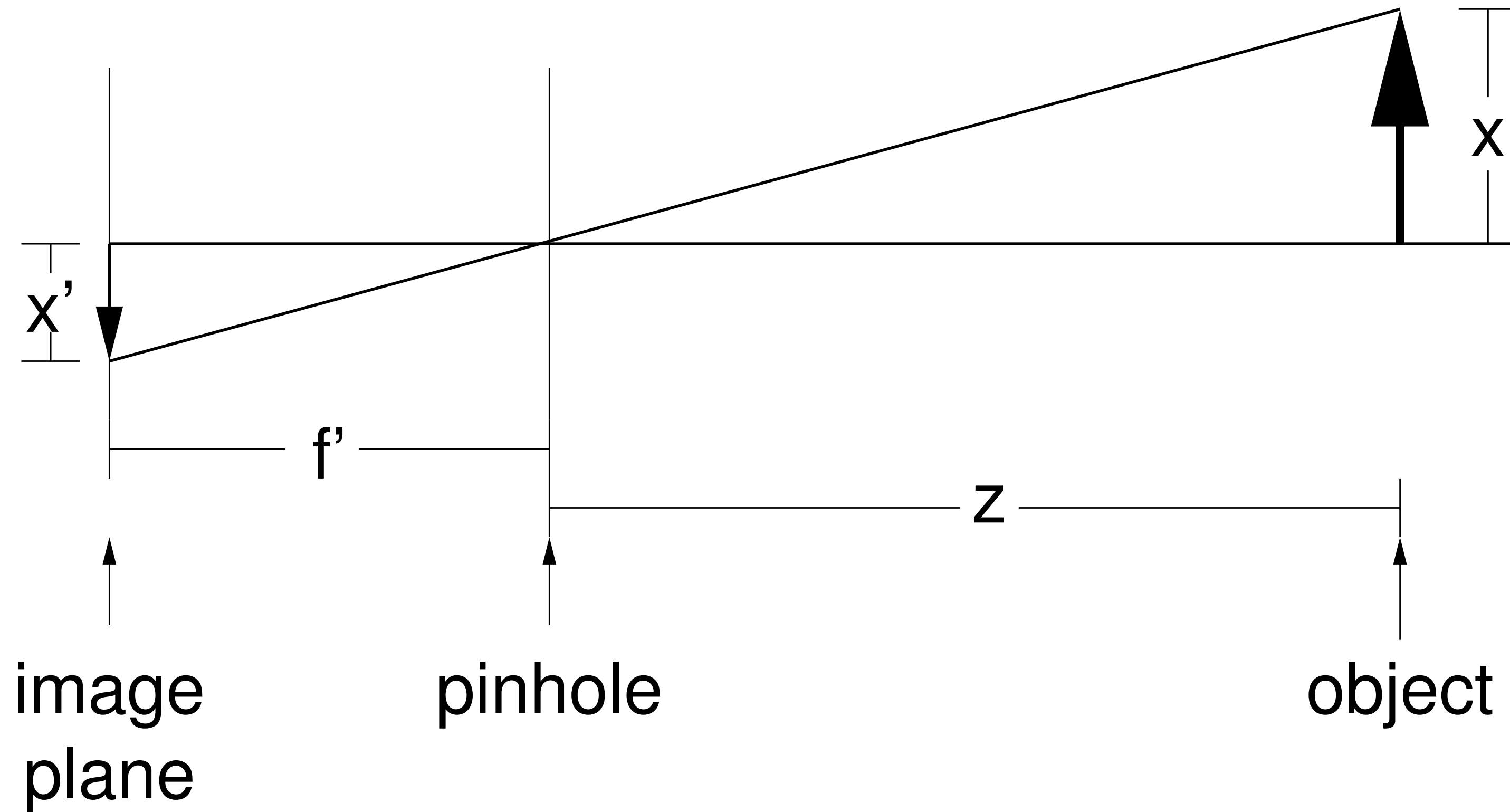
Accidental Pinhole Camera



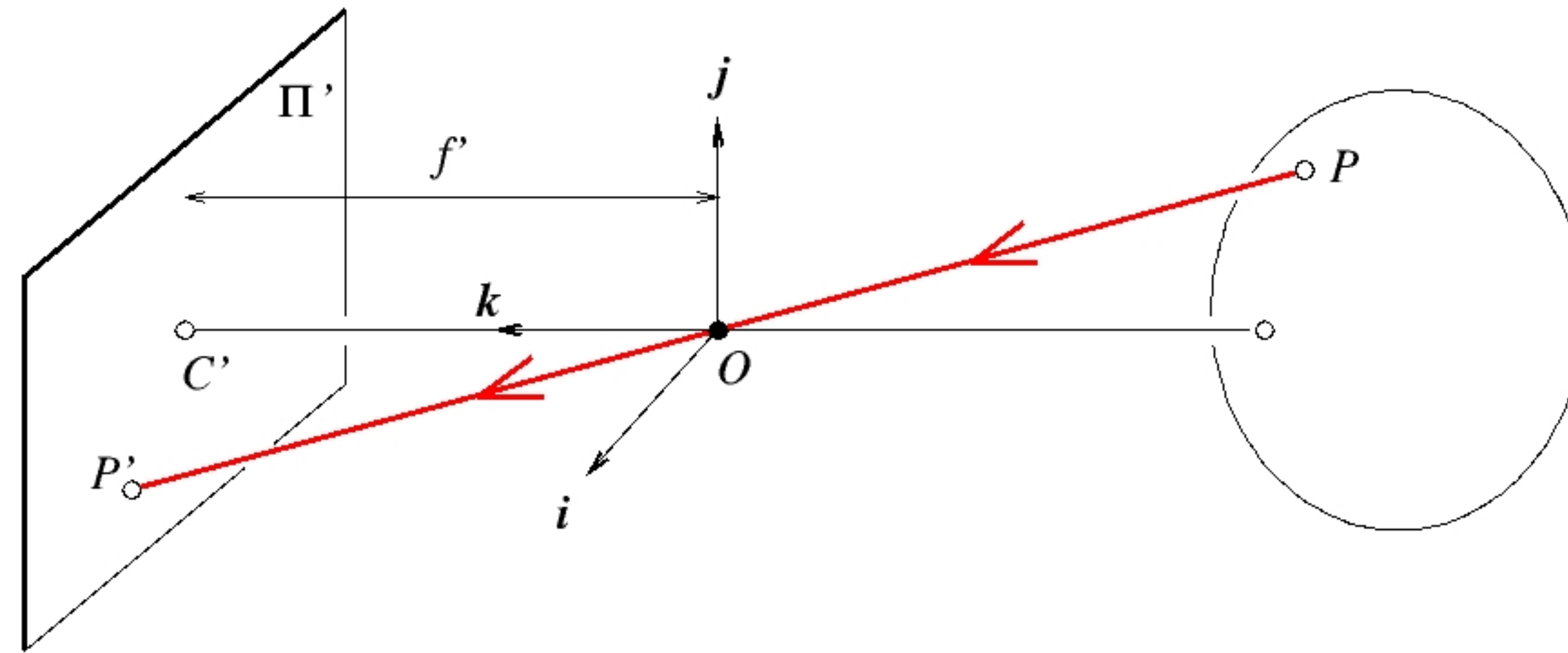
Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

Pinhole Camera (Simplified 1D)

f' is the **focal length** of the camera



Perspective Projection



3D object point

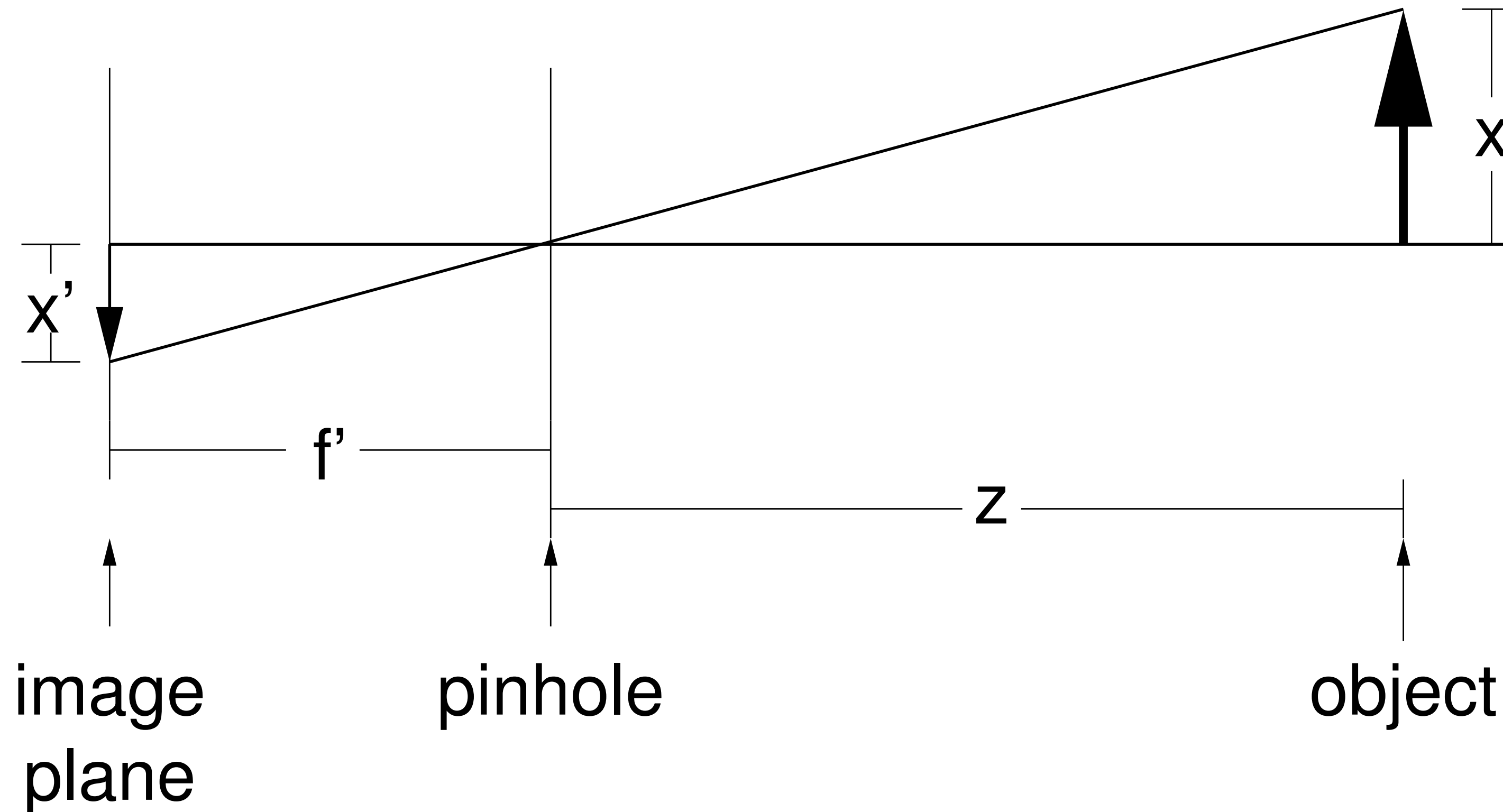
Forsyth & Ponce (1st ed.) Figure 1.4

$P = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ projects to 2D image point $P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$ where

$$\begin{aligned} x' &= f' \frac{x}{z} \\ y' &= f' \frac{y}{z} \end{aligned}$$

Pinhole Camera (Simplified 1D)

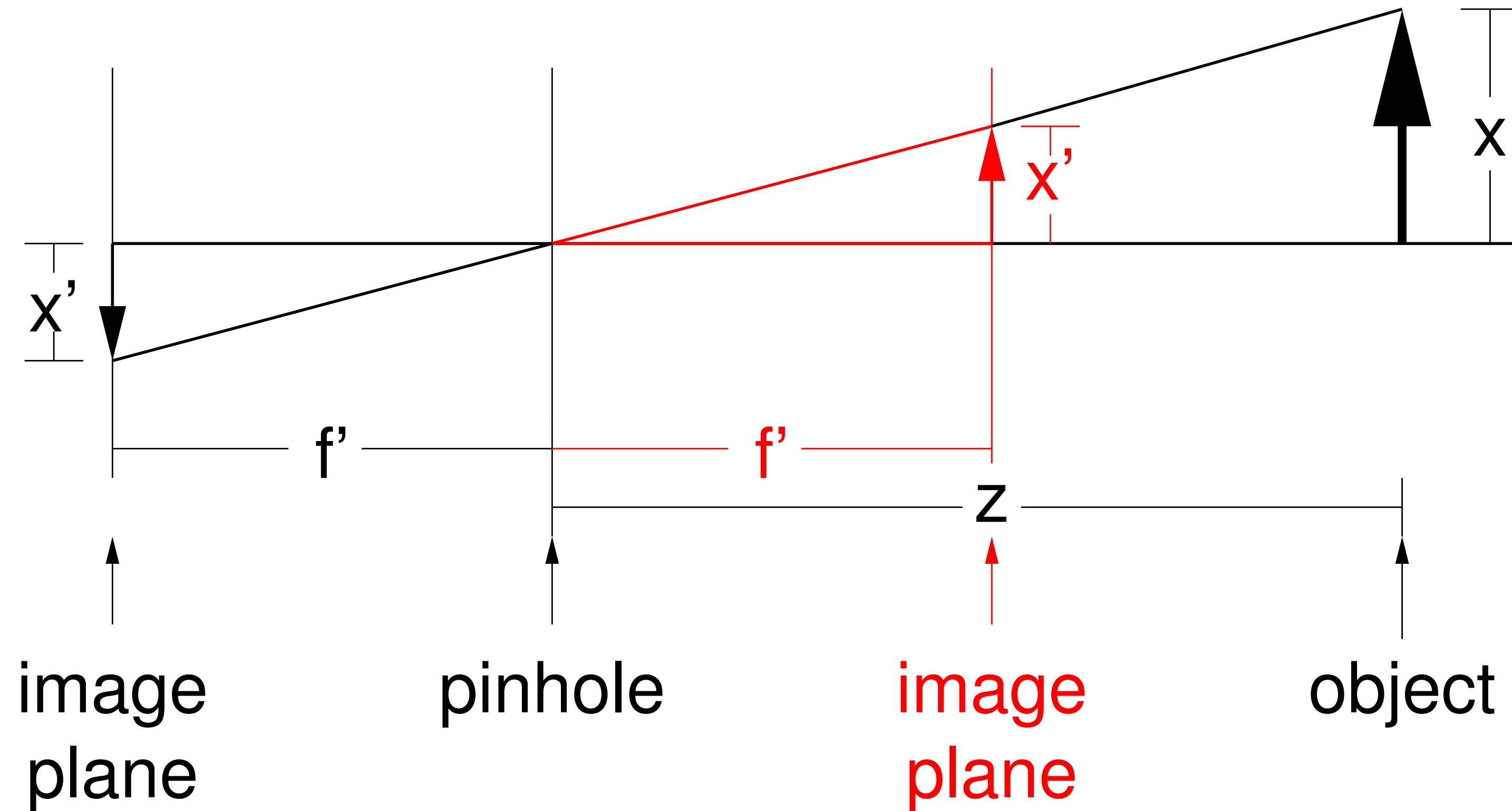
f' is the **focal length** of the camera



Note: In a pinhole camera we can adjust the focal length, all this will do is change the **size** of the resulting image

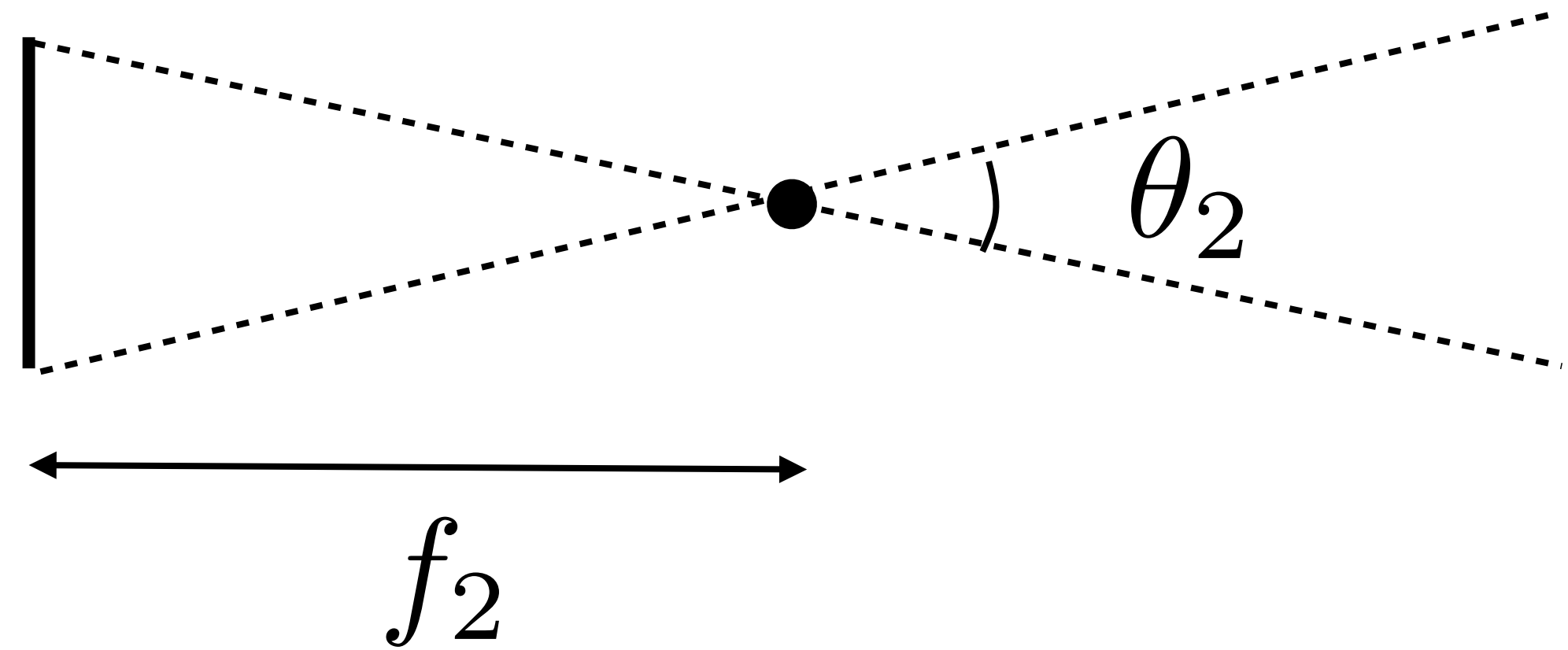
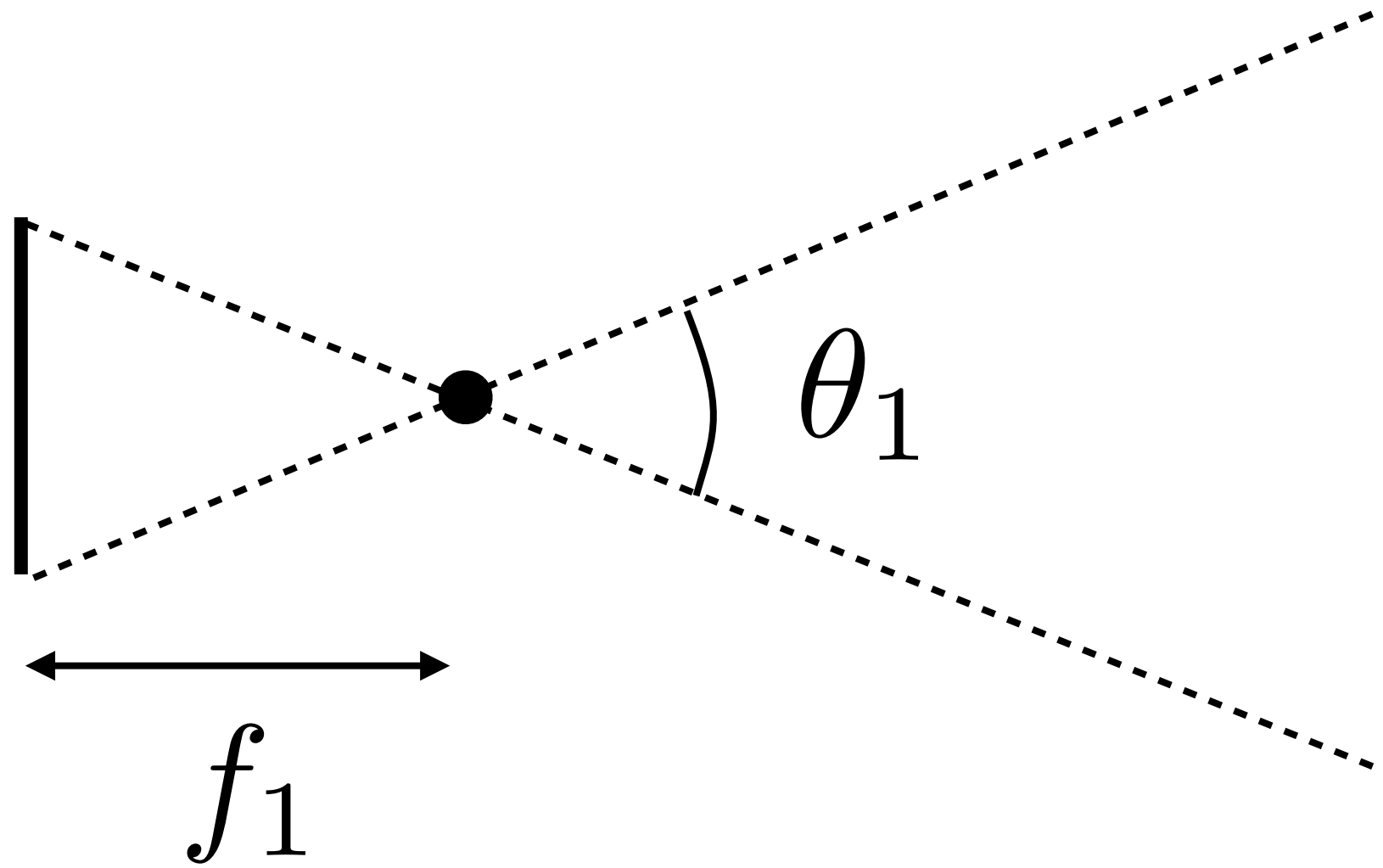
Pinhole Camera (Simplified 1D)

It is convenient to think of the **image plane** which is in from of the pinhole



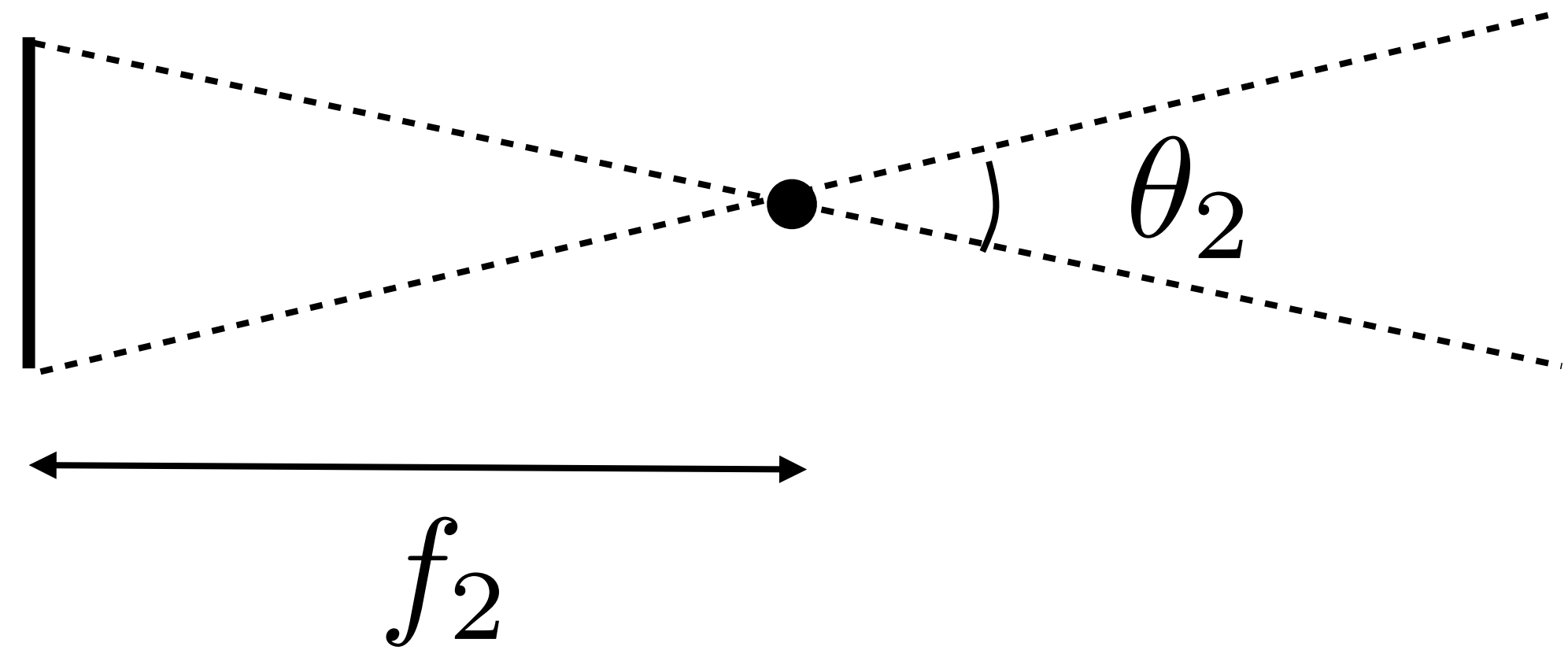
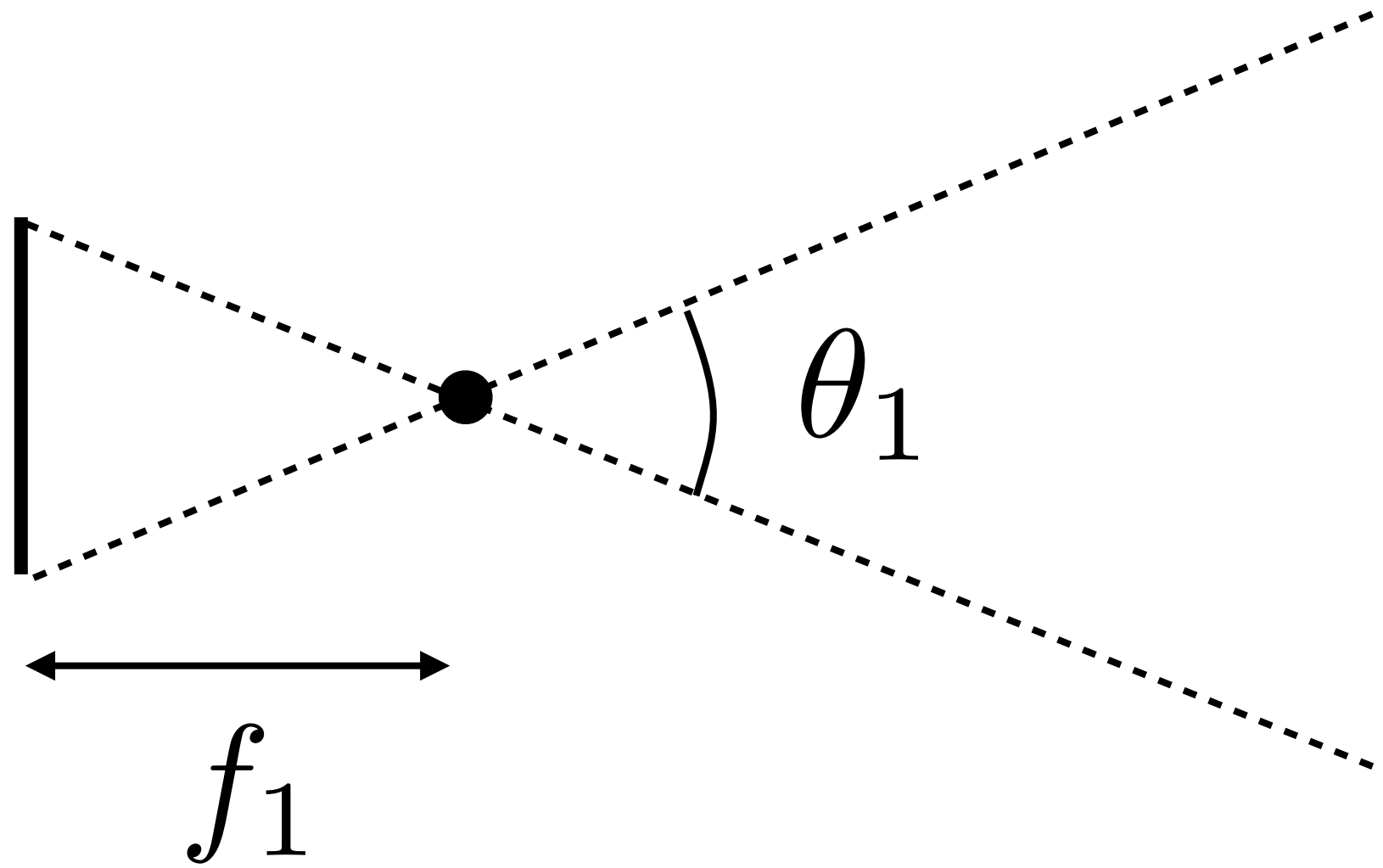
Focal Length

For a fixed sensor size, focal length determines the **field of view** (FoV)



Focal Length

For a fixed sensor size, focal length determines the **field of view** (FoV)



Exercise: What is the field of view of a Sensor size full frame (35mm) camera with a Focal length 50mm lens? 100mm lens?

Focal Length



28 mm



35 mm



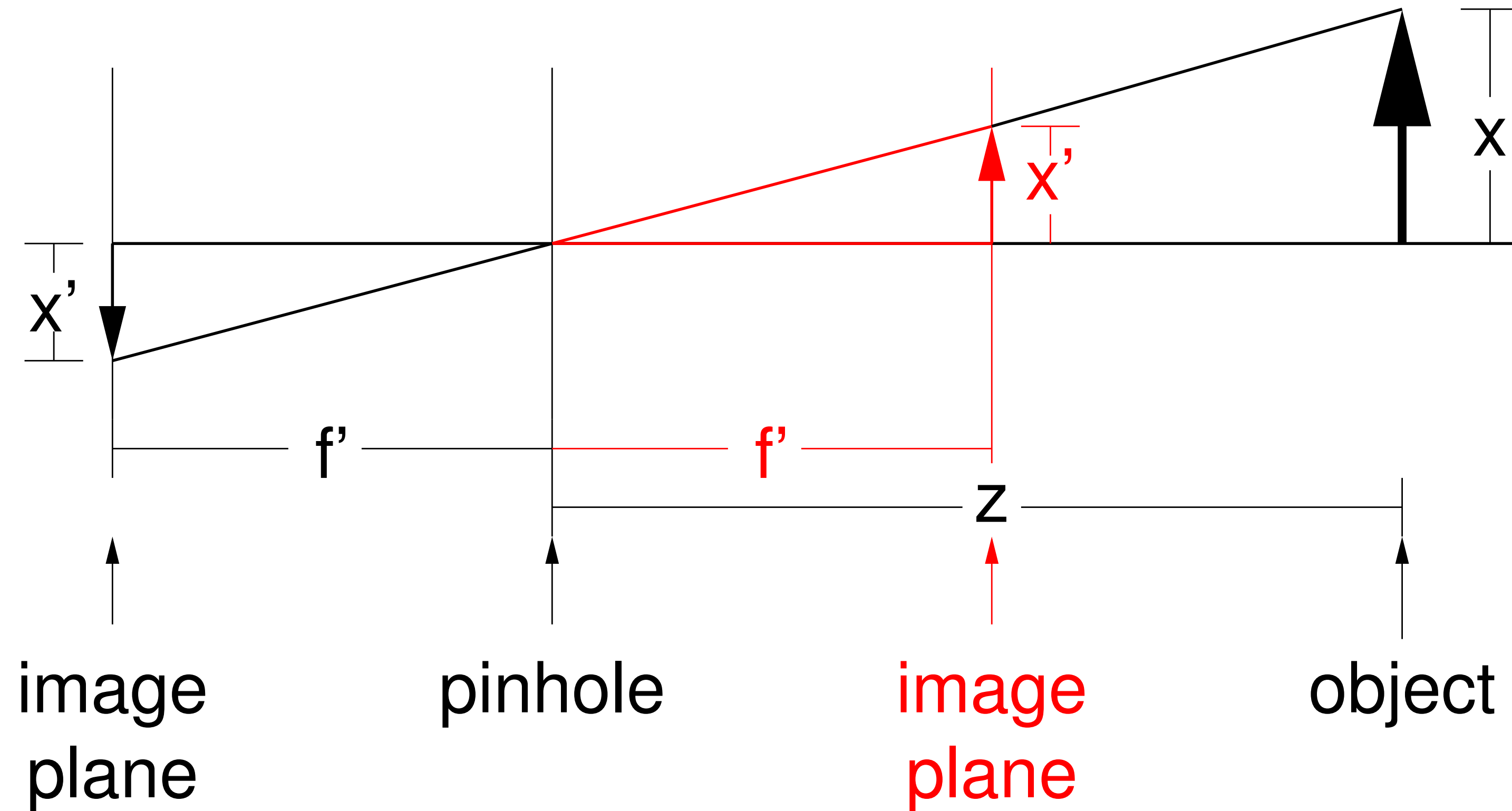
50 mm



70 mm

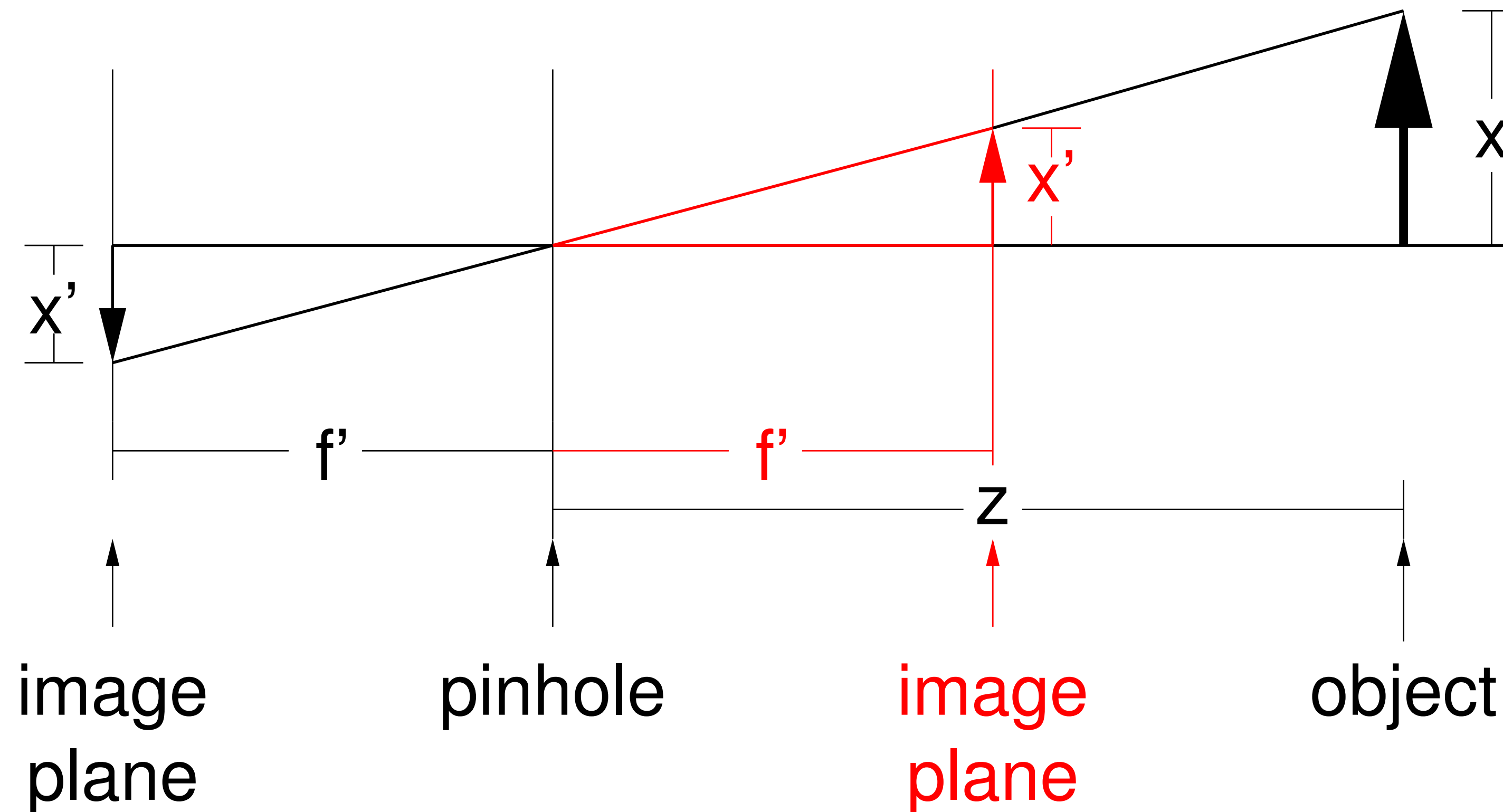
Pinhole Camera (Simplified 1D)

It is convenient to think of the **image plane** which is in from of the pinhole



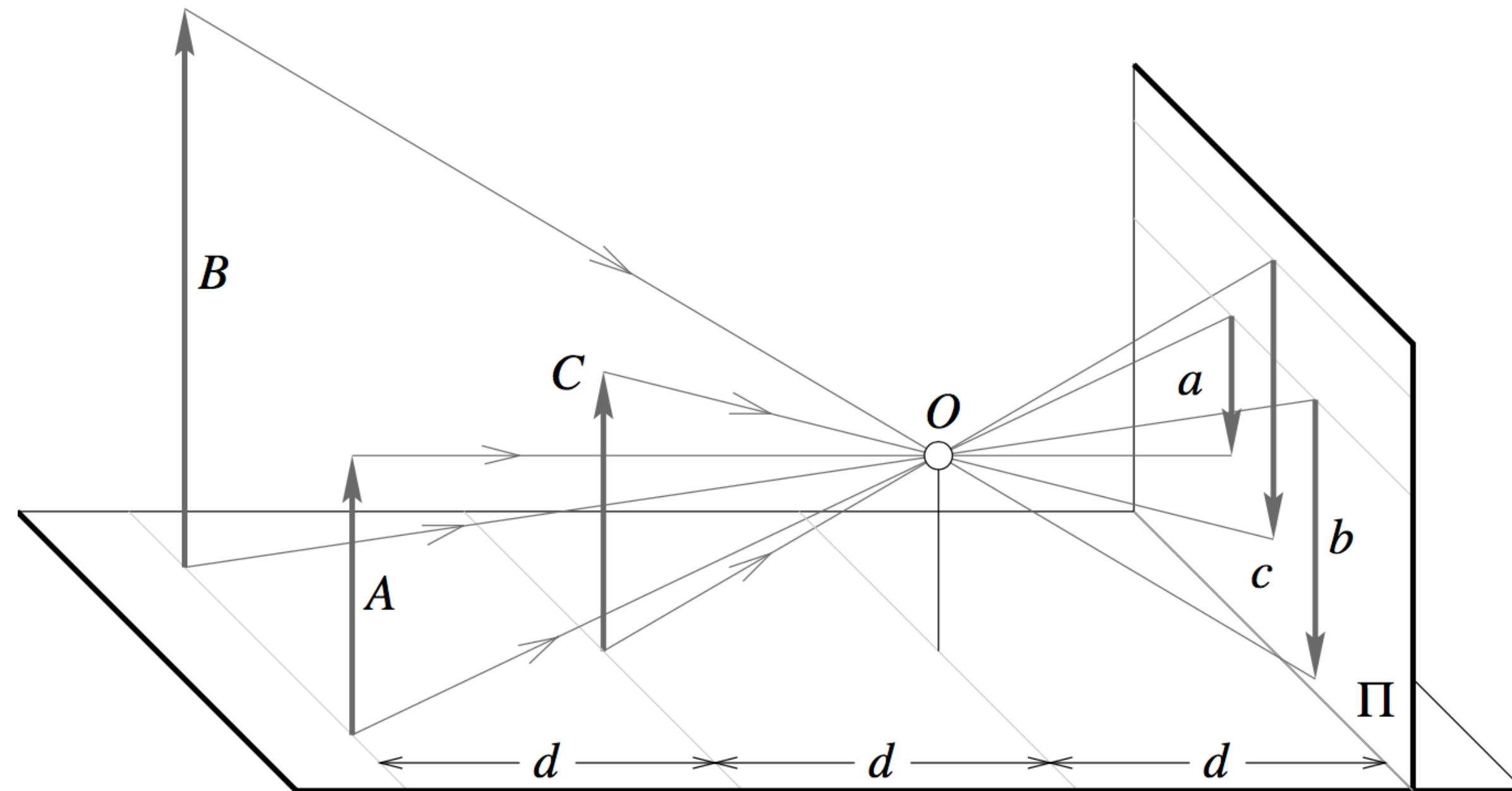
Pinhole Camera (Simplified 1D)

It is convenient to think of the **image plane** which is in from of the pinhole



What happens if object moves towards the camera? Away from the camera?

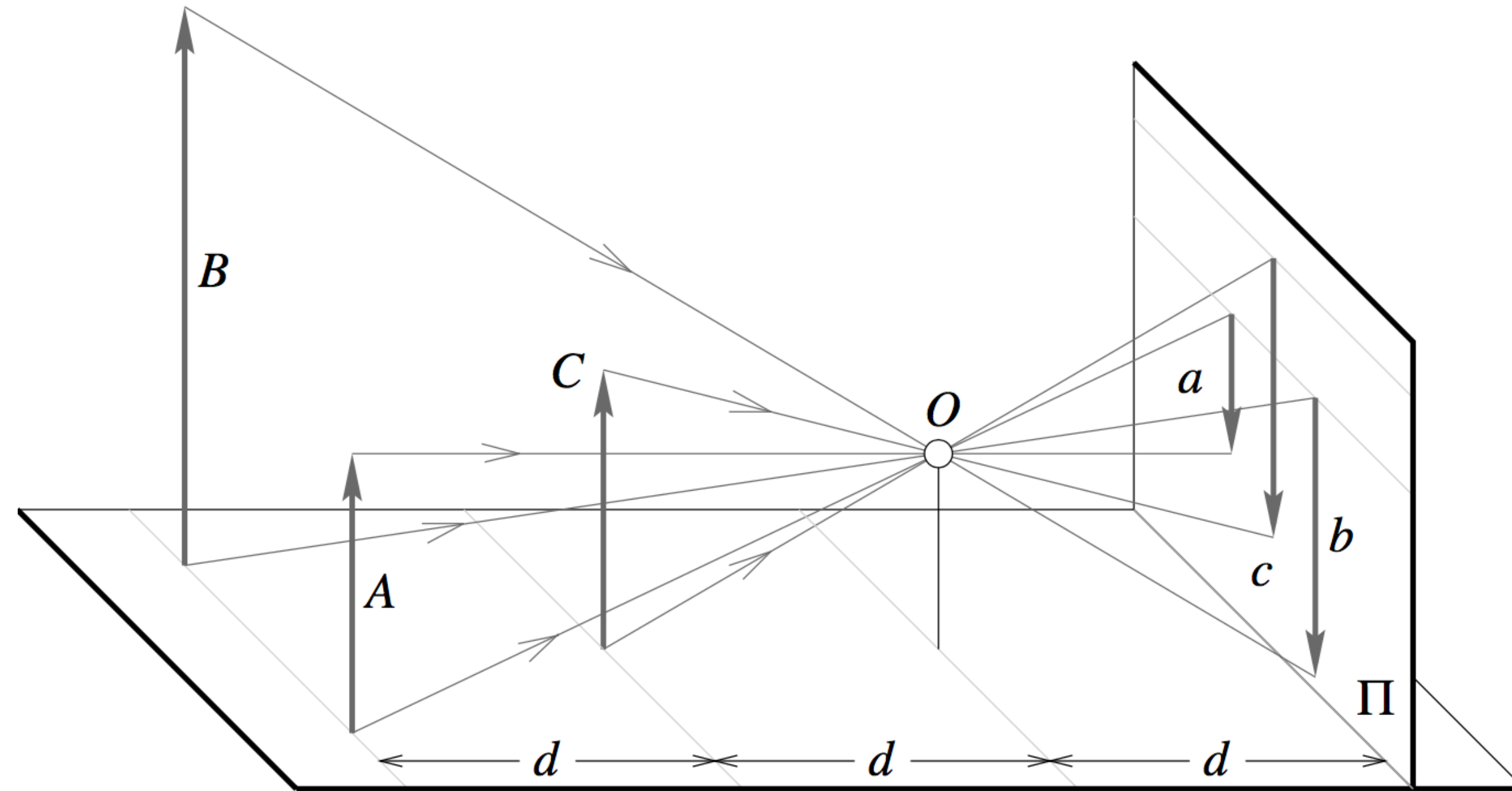
Perspective Effects



Forsyth & Ponce (2nd ed.) Figure 1.3a

Perspective Effects

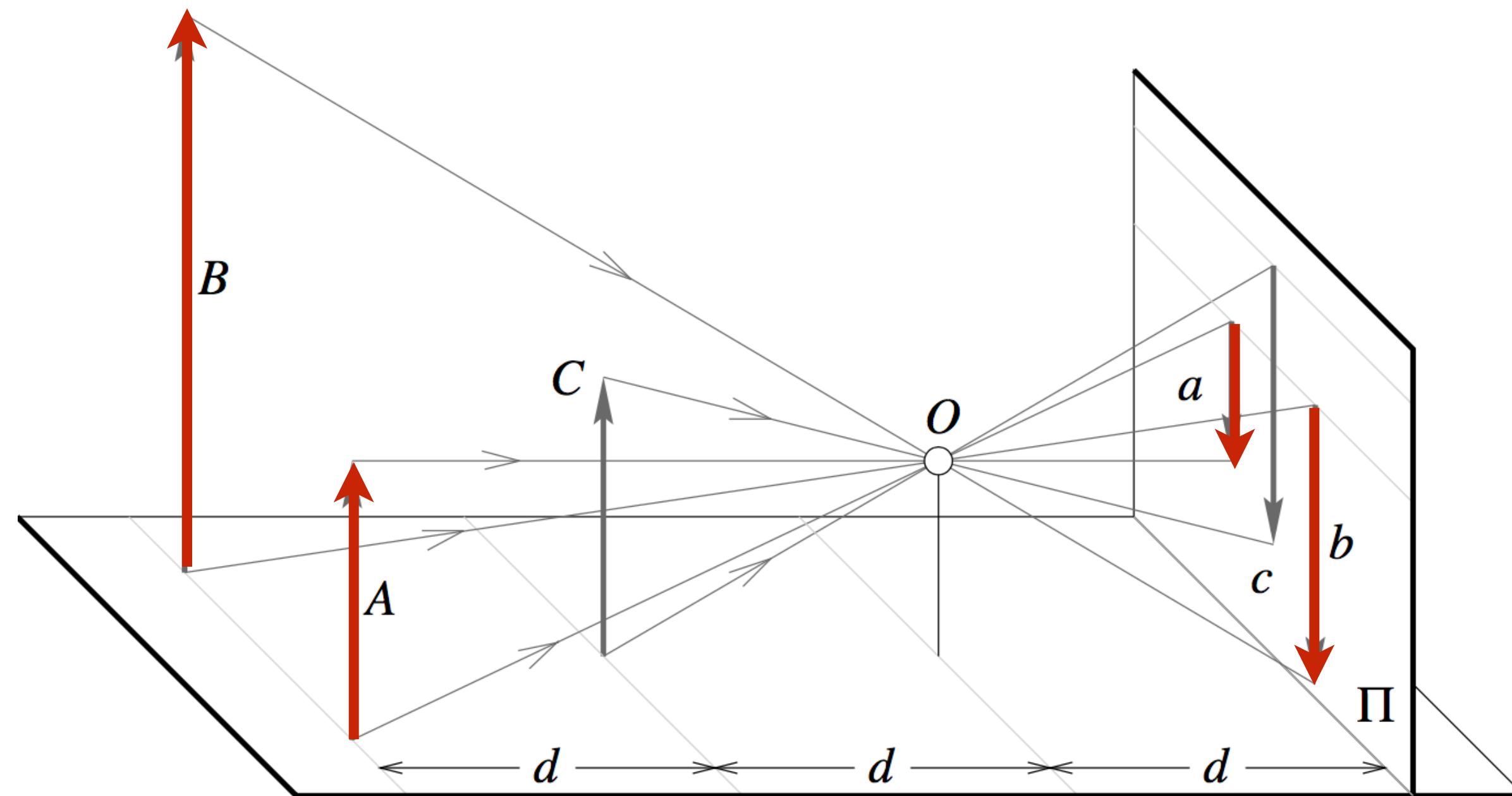
Far objects appear **smaller** than close ones



Forsyth & Ponce (2nd ed.) Figure 1.3a

Perspective Effects

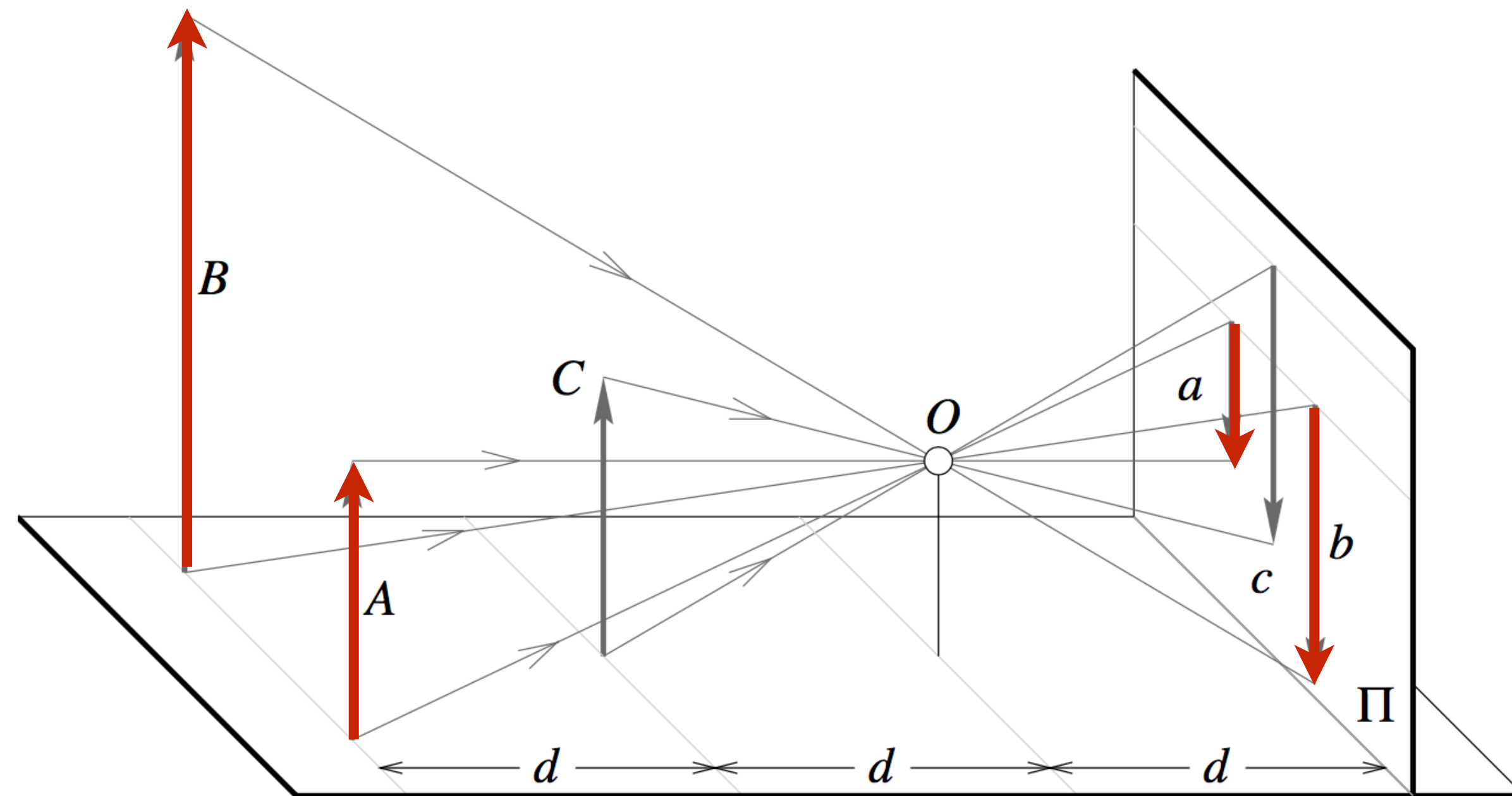
Far objects appear **smaller** than close ones



Forsyth & Ponce (2nd ed.) Figure 1.3a

Perspective Effects

Far objects appear **smaller** than close ones

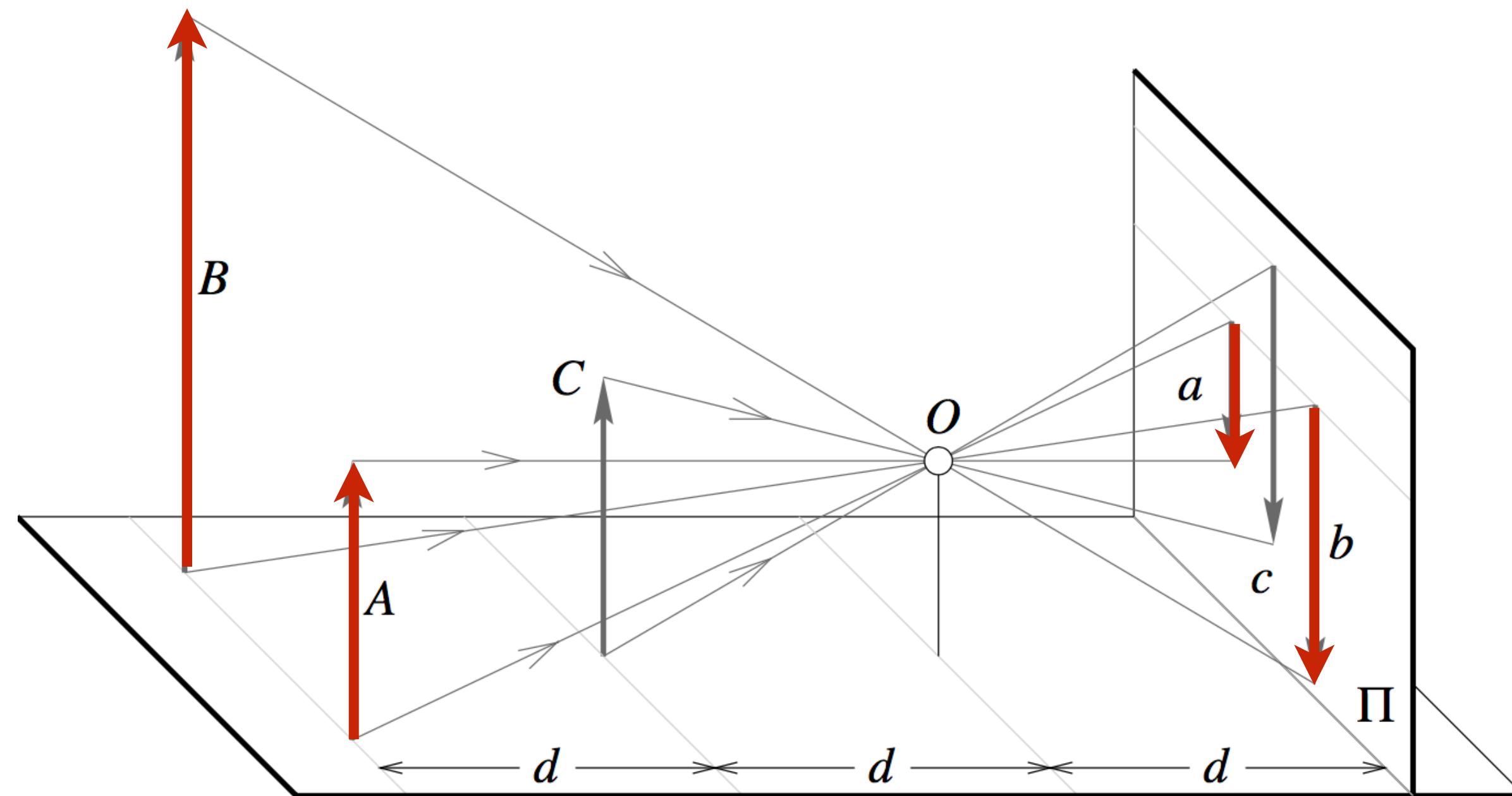


Forsyth & Ponce (2nd ed.) Figure 1.3a

Relative size of objects that are equally far from camera is preserved
(e.g., **A** is half size of **B**, so **a** will be half the size of **b** in the image plane)

Perspective Effects

Far objects appear **smaller** than close ones

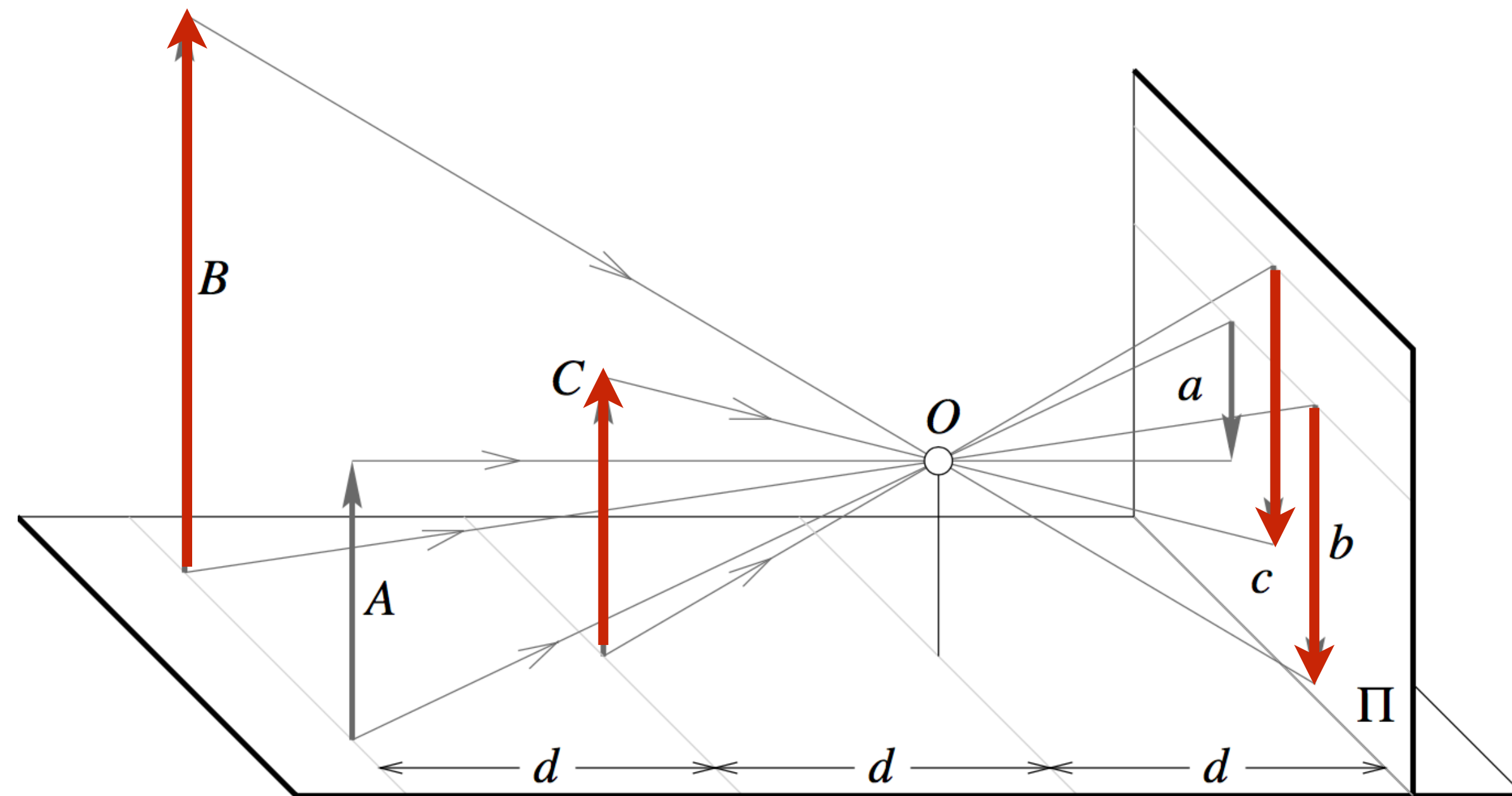


Forsyth & Ponce (2nd ed.) Figure 1.3a

Size is **inversely** proportions to distance

Perspective Effects

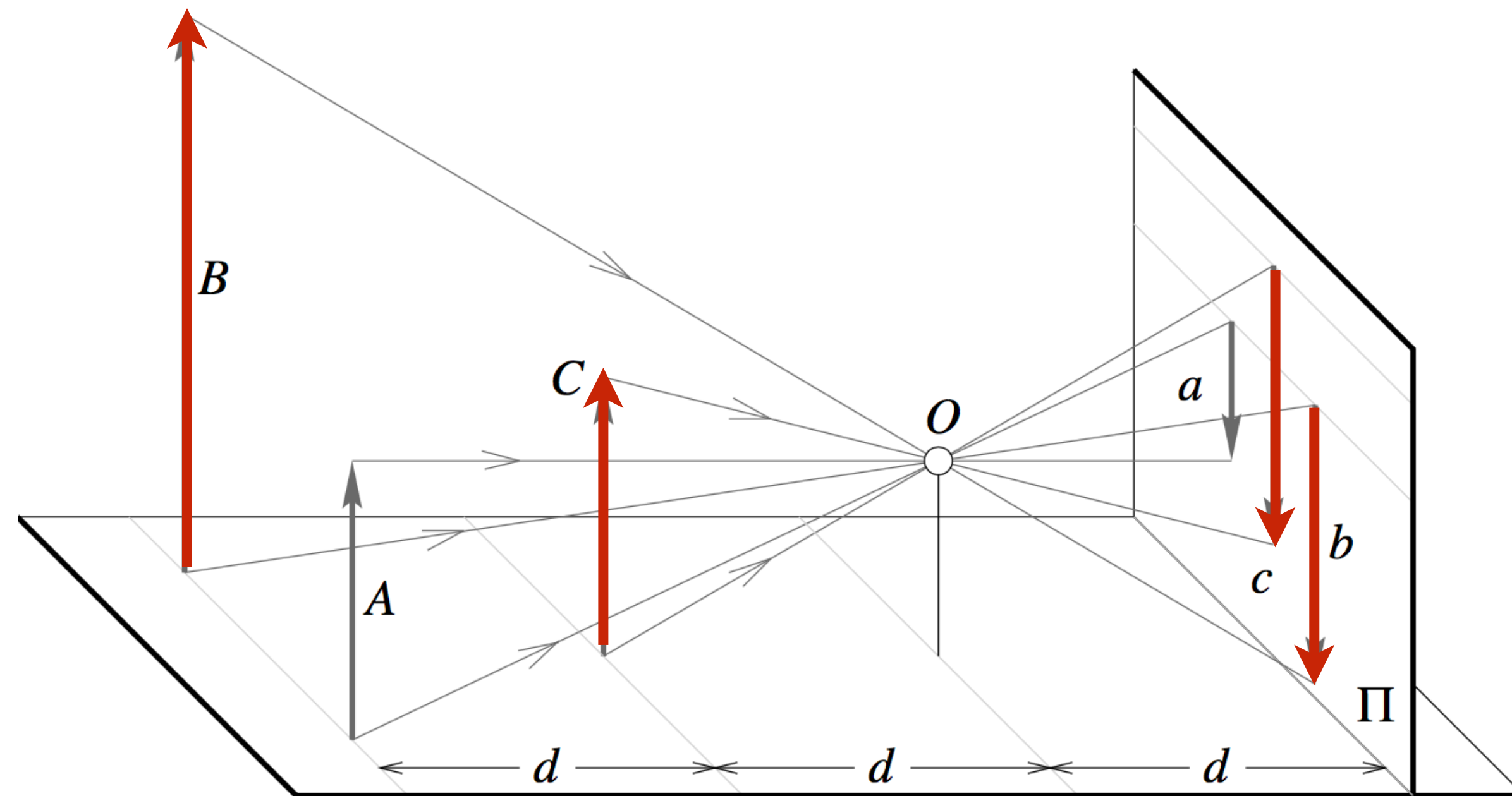
Far objects appear **smaller** than close ones



Forsyth & Ponce (2nd ed.) Figure 1.3a

Perspective Effects

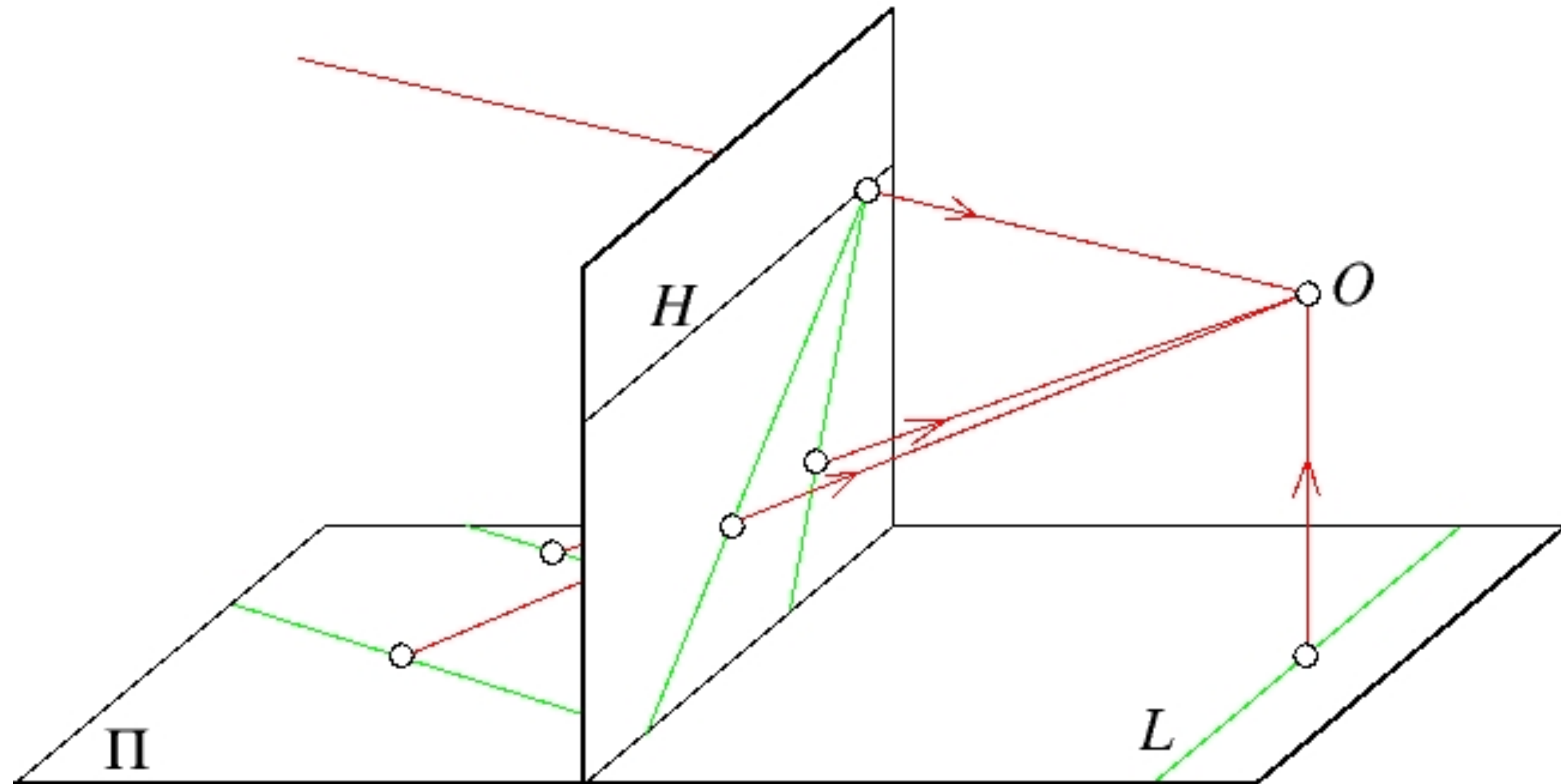
Far objects appear **smaller** than close ones



Forsyth & Ponce (2nd ed.) Figure 1.3a

(e.g., **C** is half size of **B** and half closer, resulting in **c** and **b** being same size in projection)

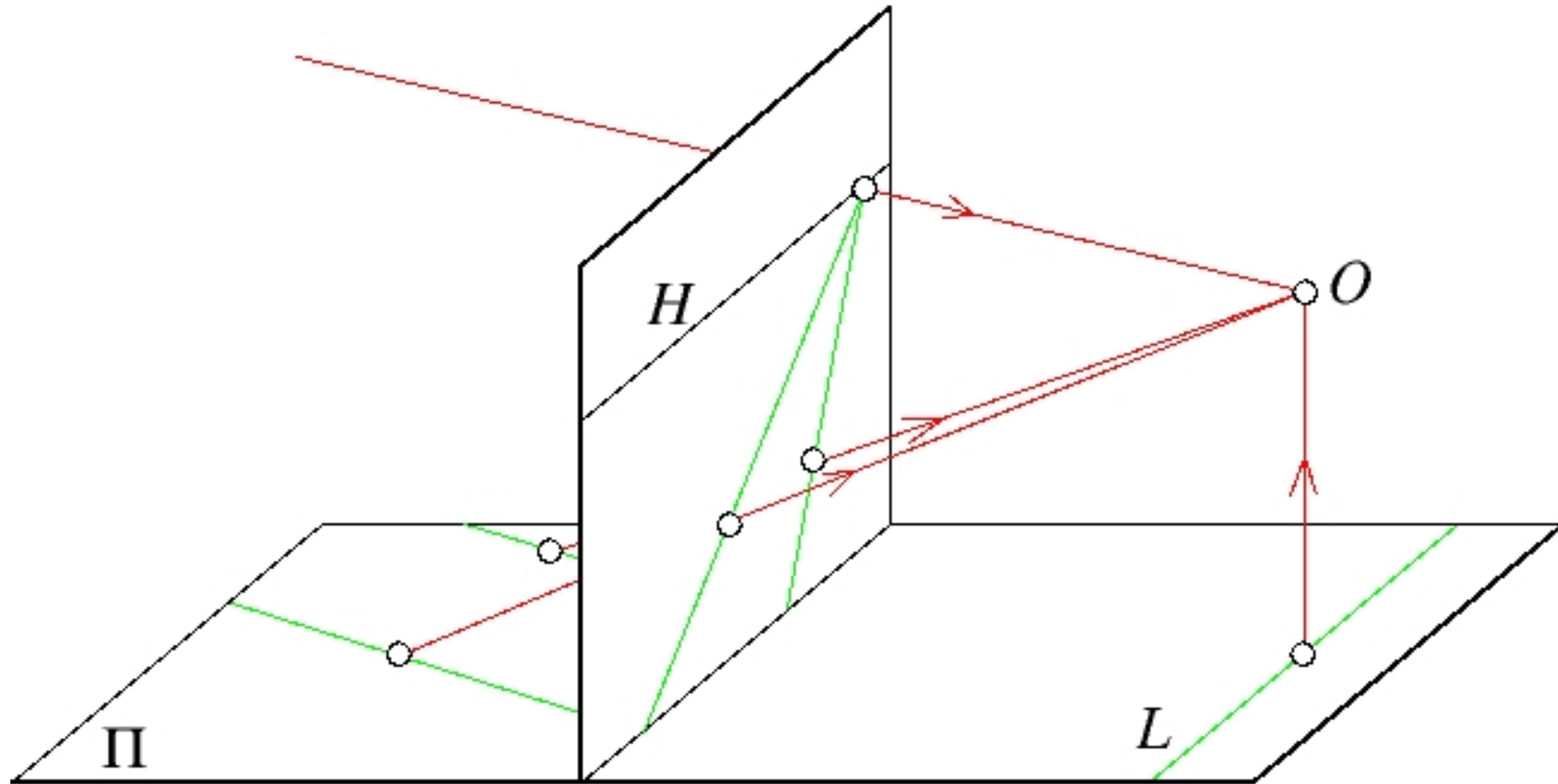
Perspective Effects



Forsyth & Ponce (1st ed.) Figure 1.3b

Perspective Effects

Parallel lines meet at a point (**vanishing point**)



Forsyth & Ponce (1st ed.) Figure 1.3b

Vanishing Points

Each set of parallel lines meet at a different point

— the point is called **vanishing point**

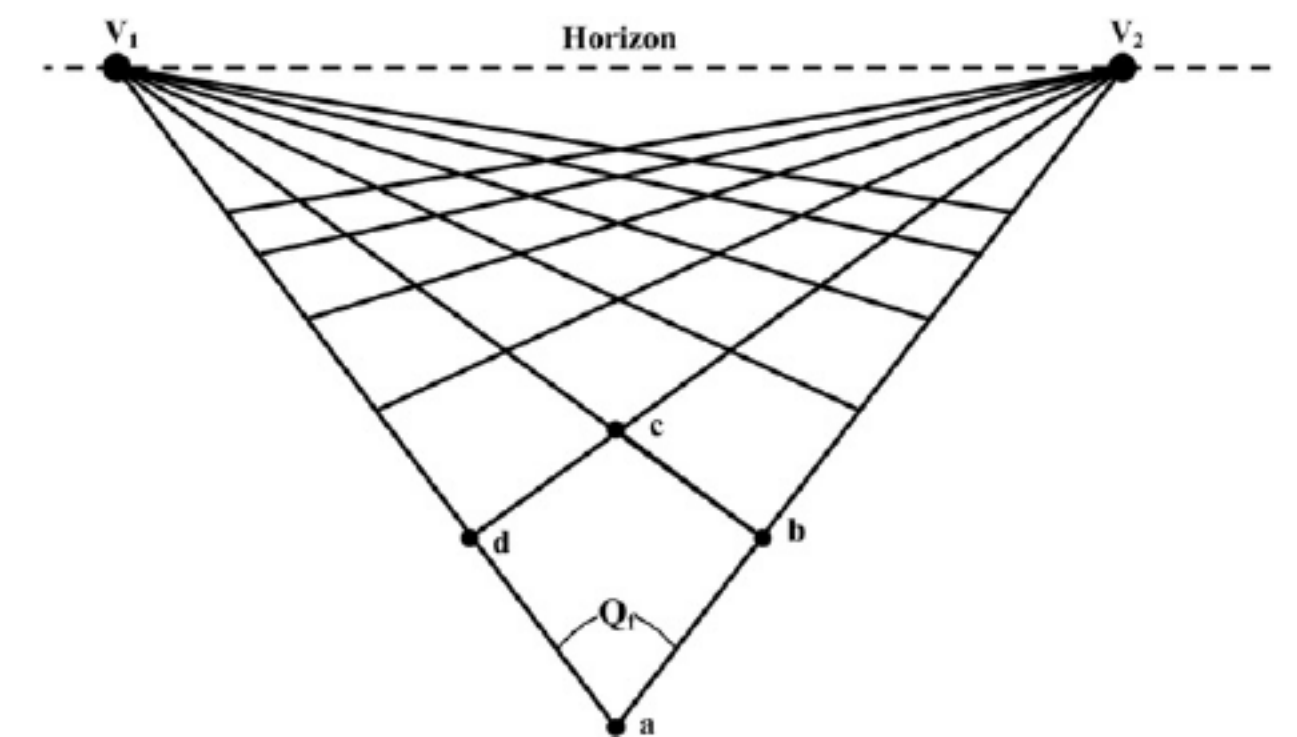
Vanishing Points

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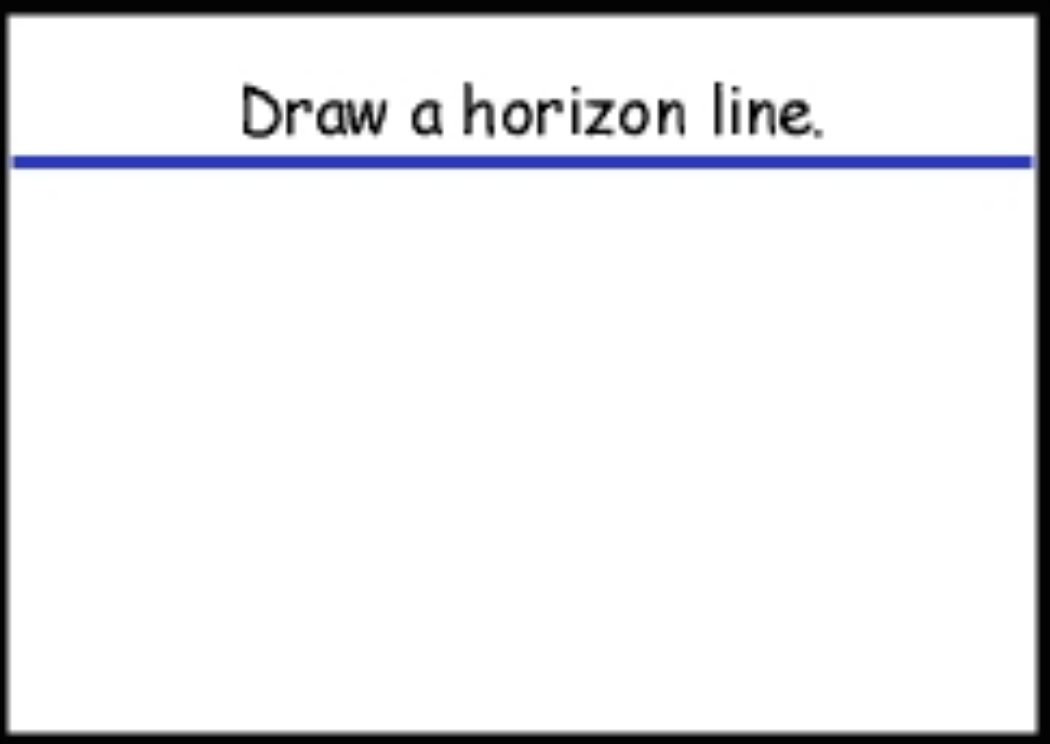
Sets of parallel lines on the same plane lead to **collinear** vanishing points

— the line is called a **horizon** for that plane

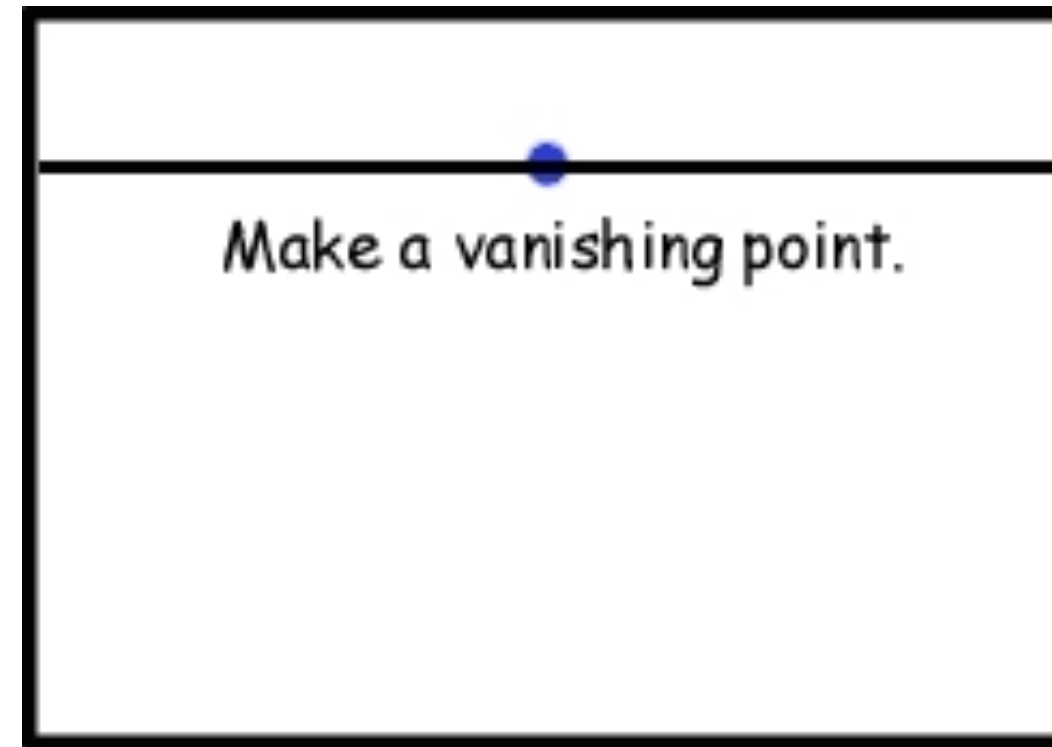
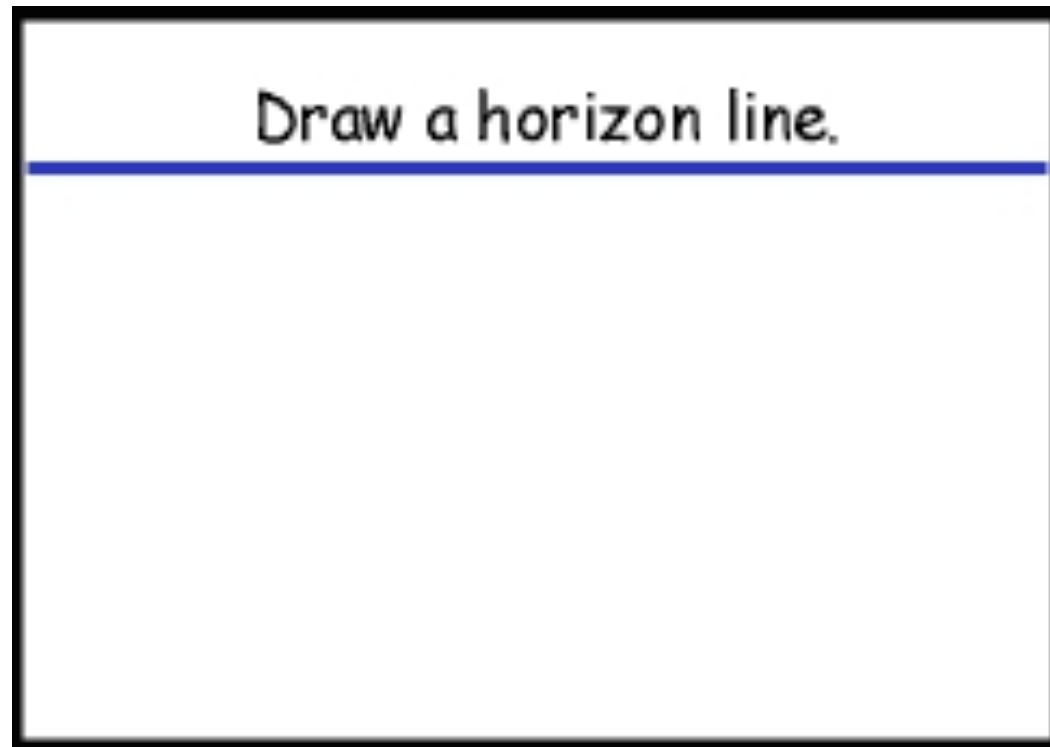




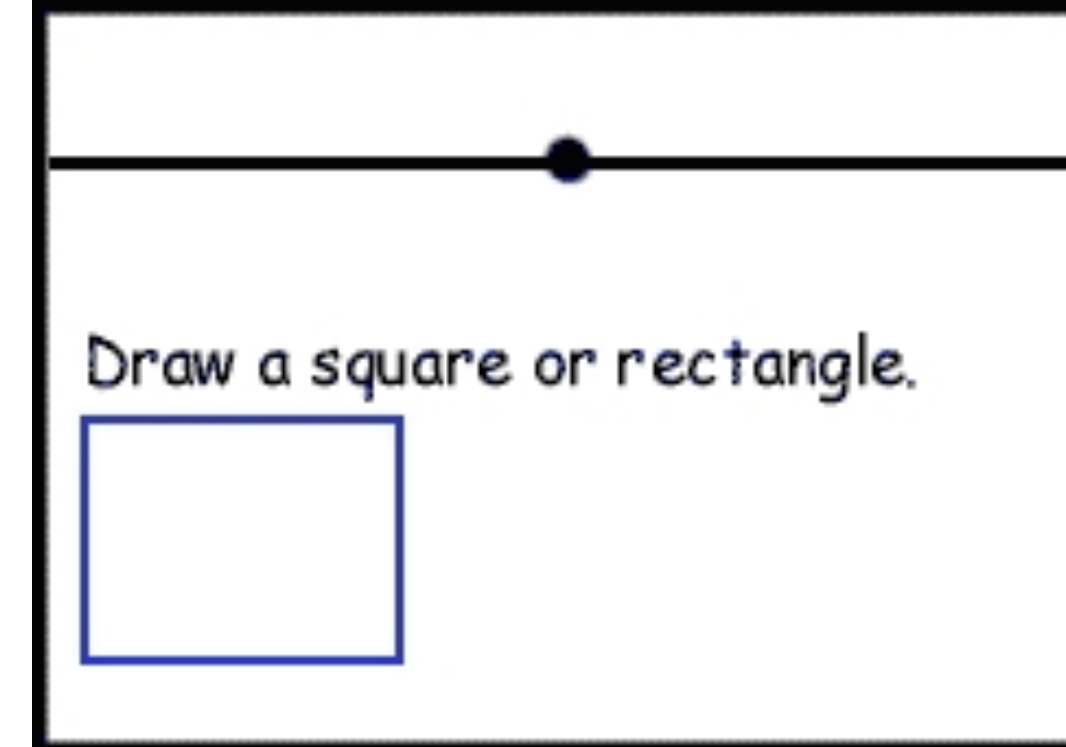
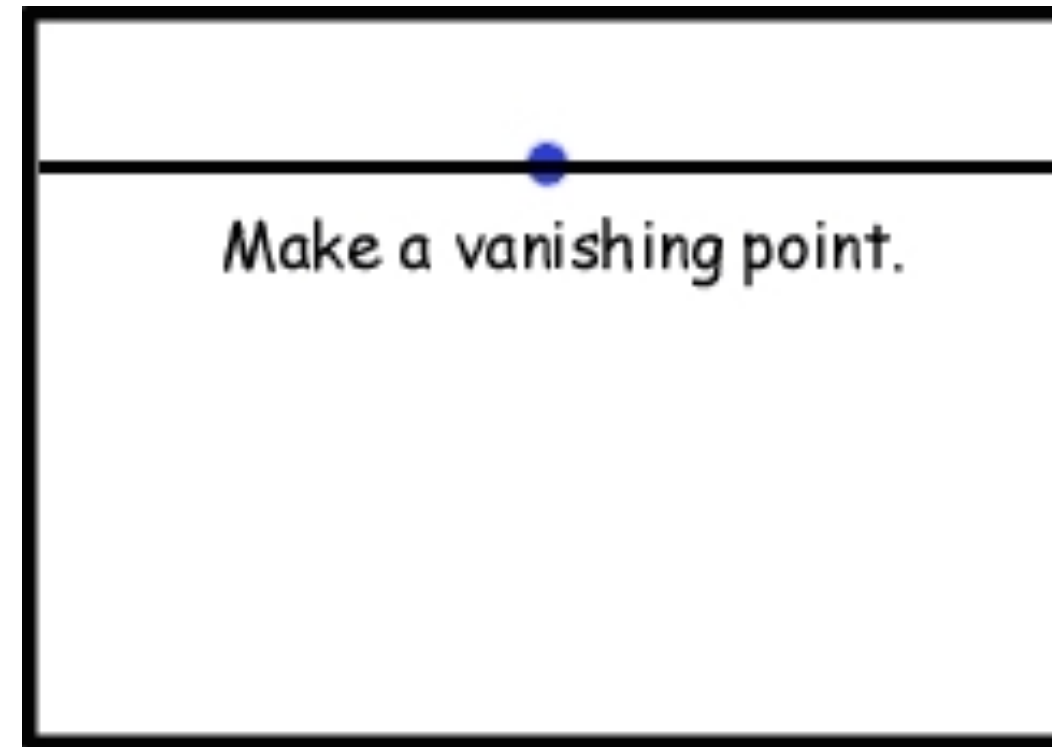
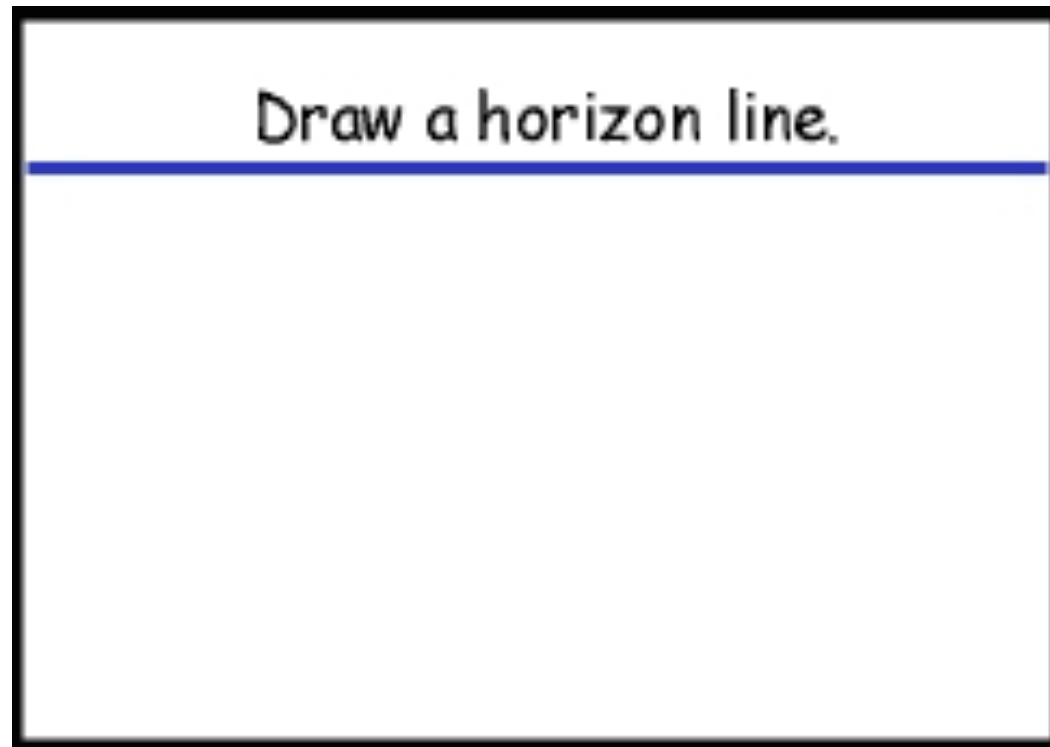
Vanishing Points



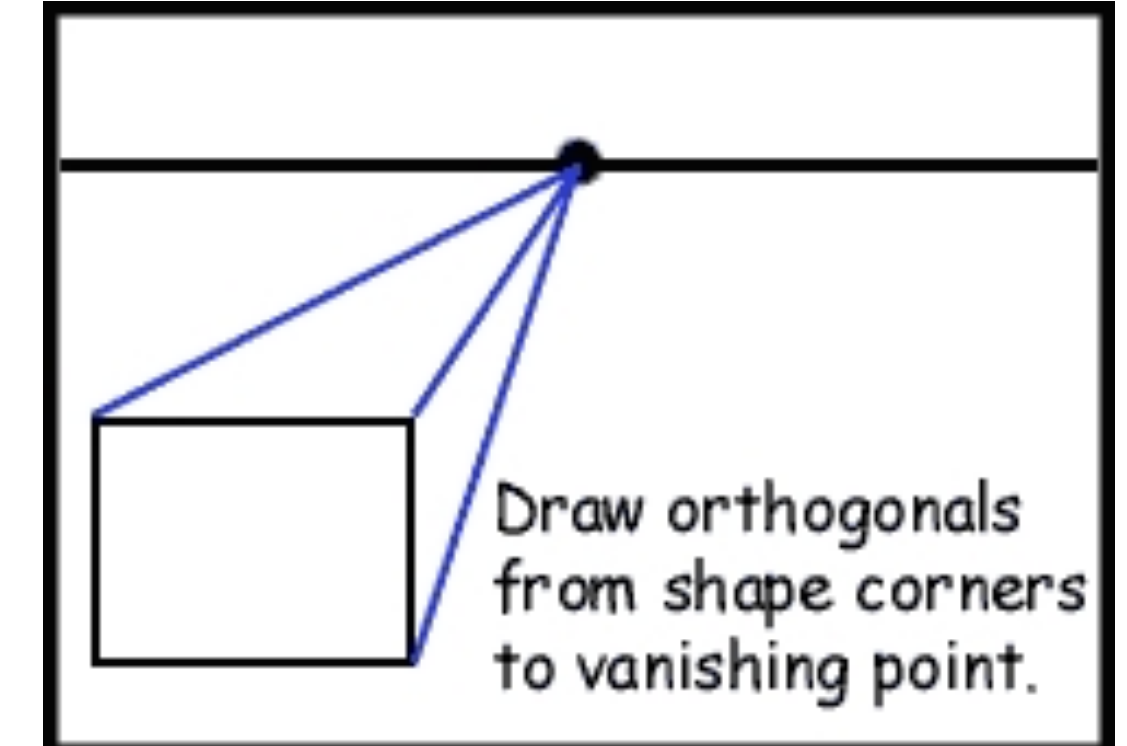
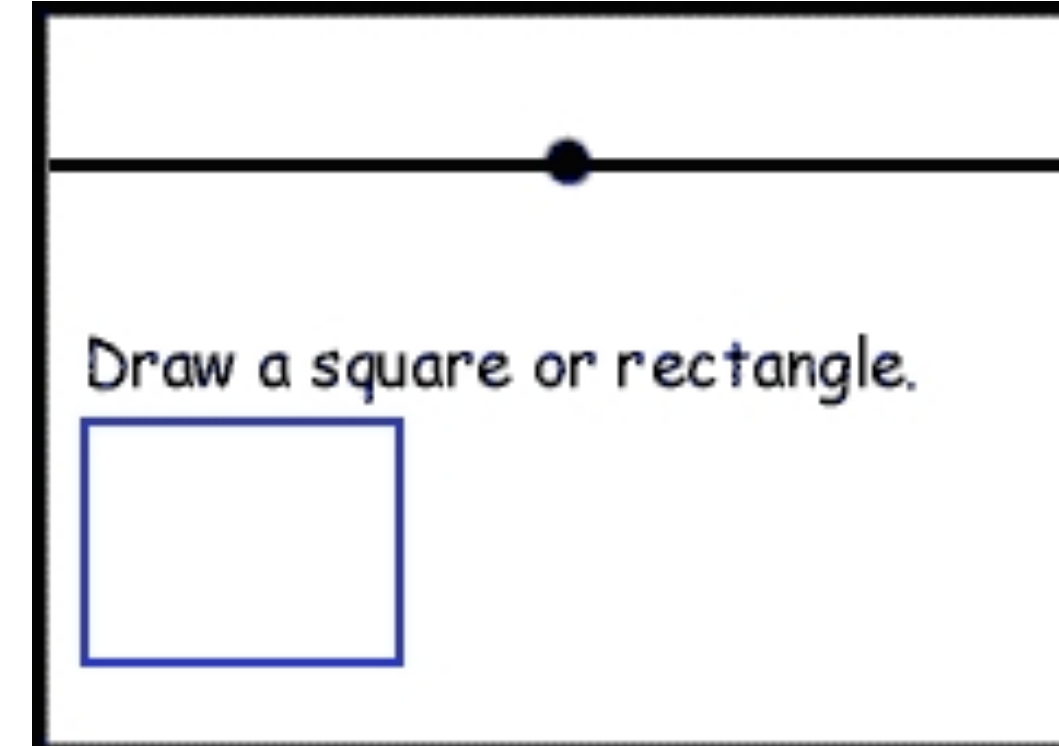
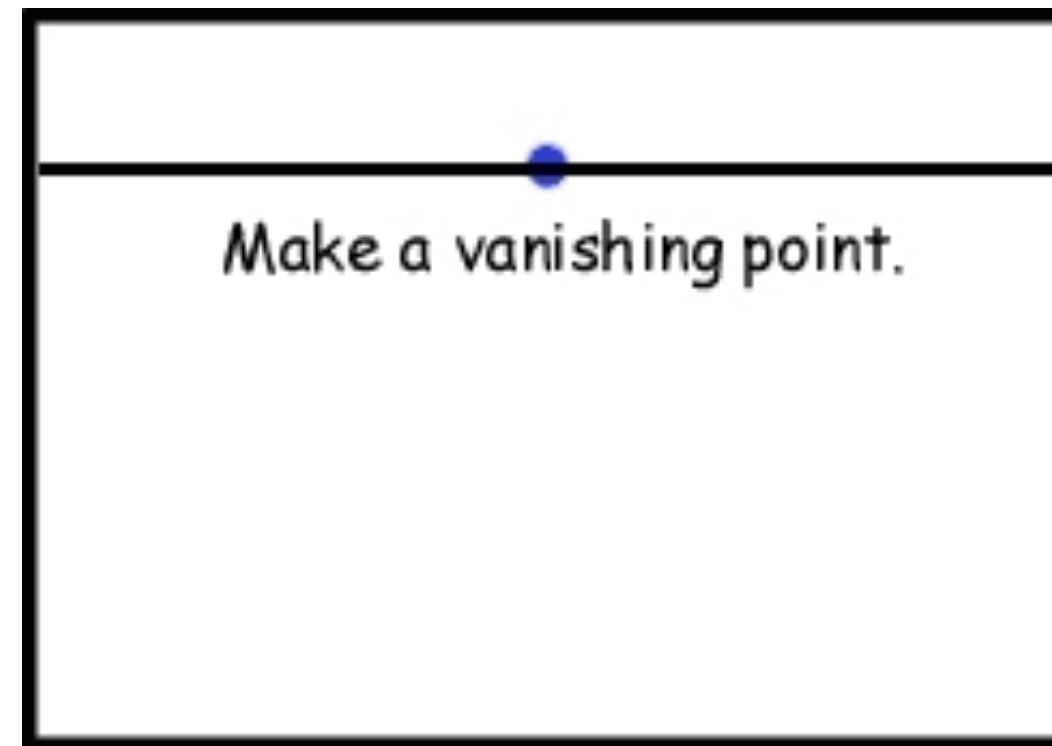
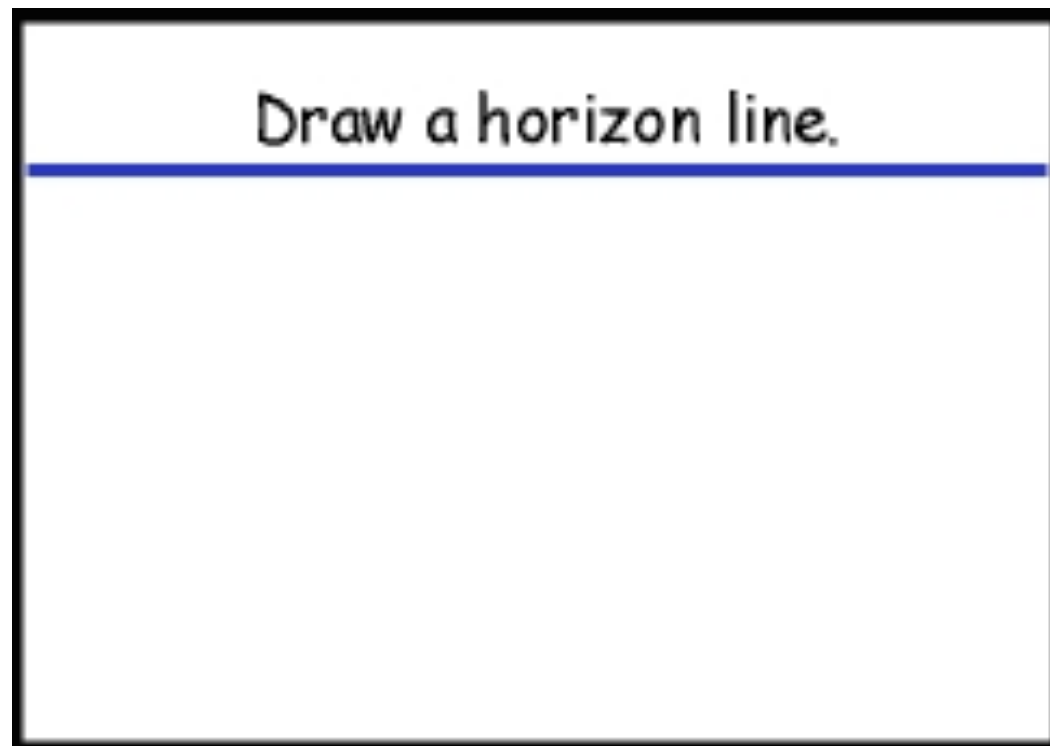
Vanishing Points



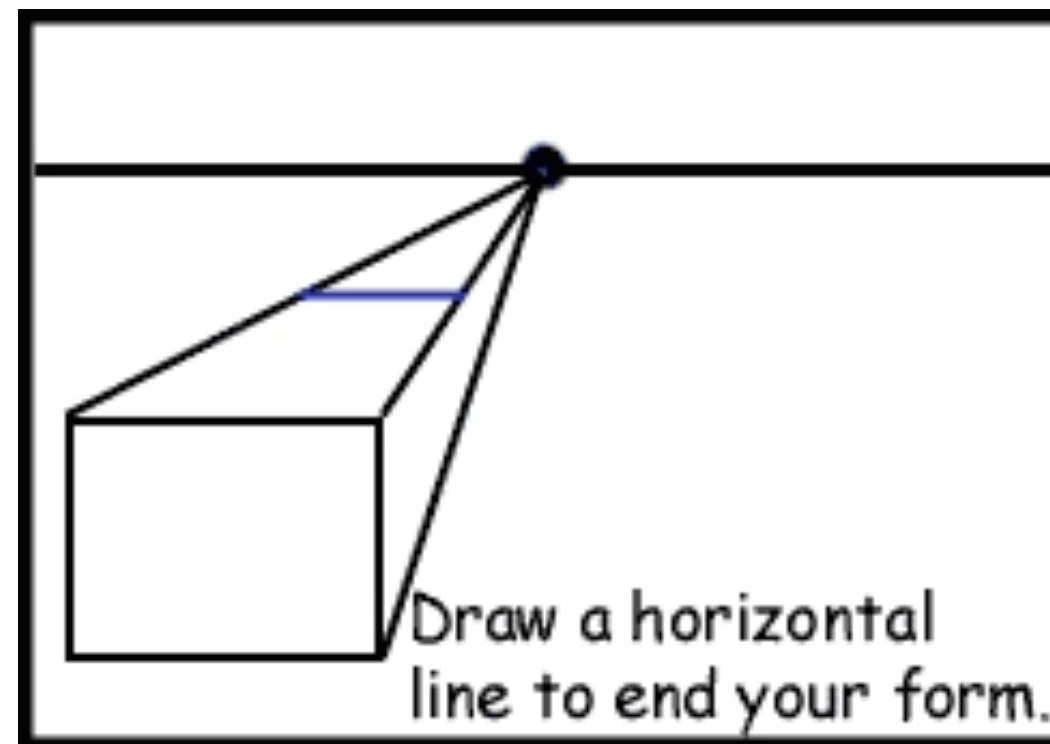
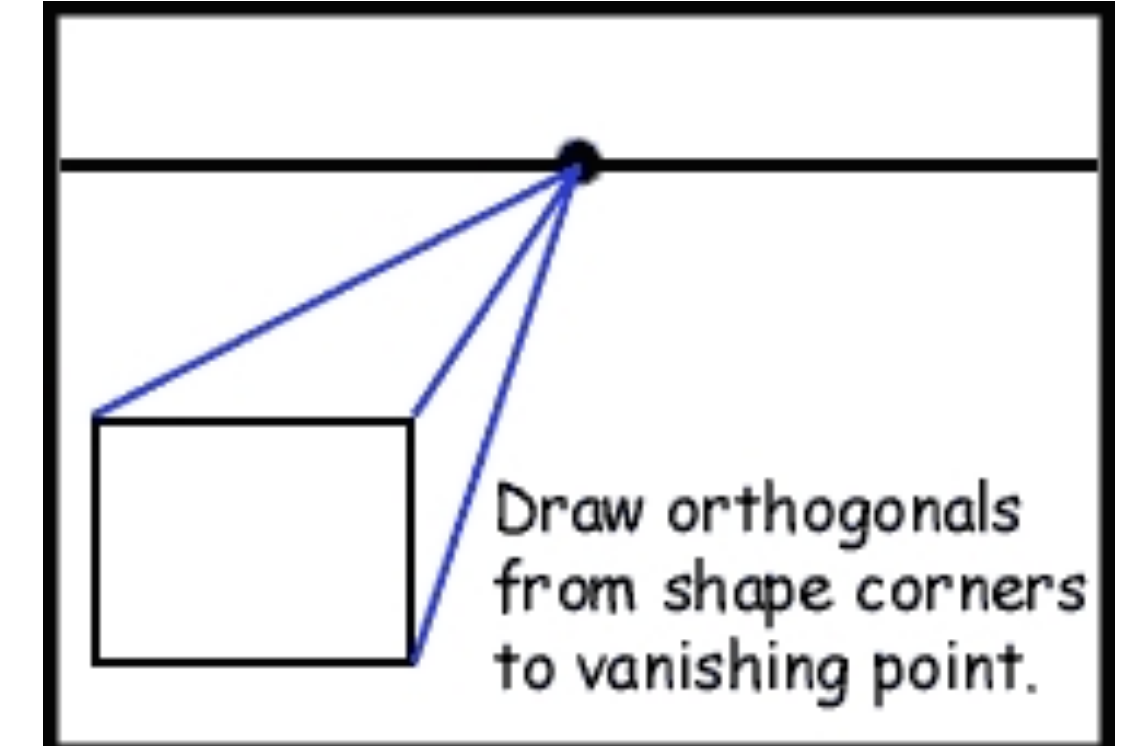
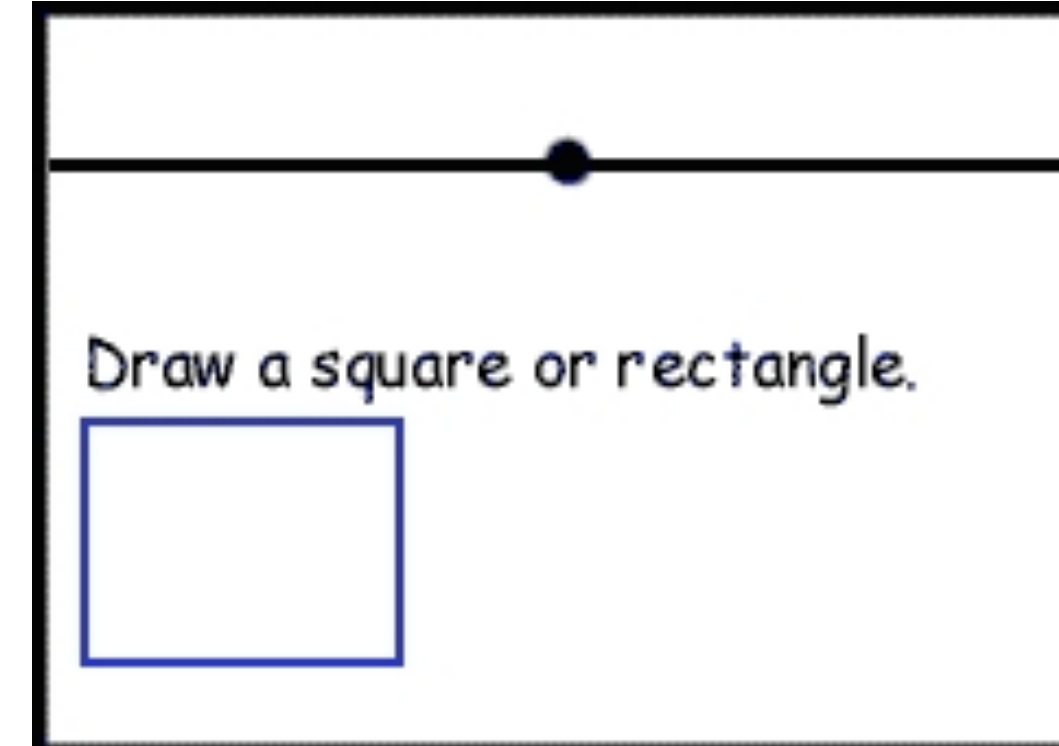
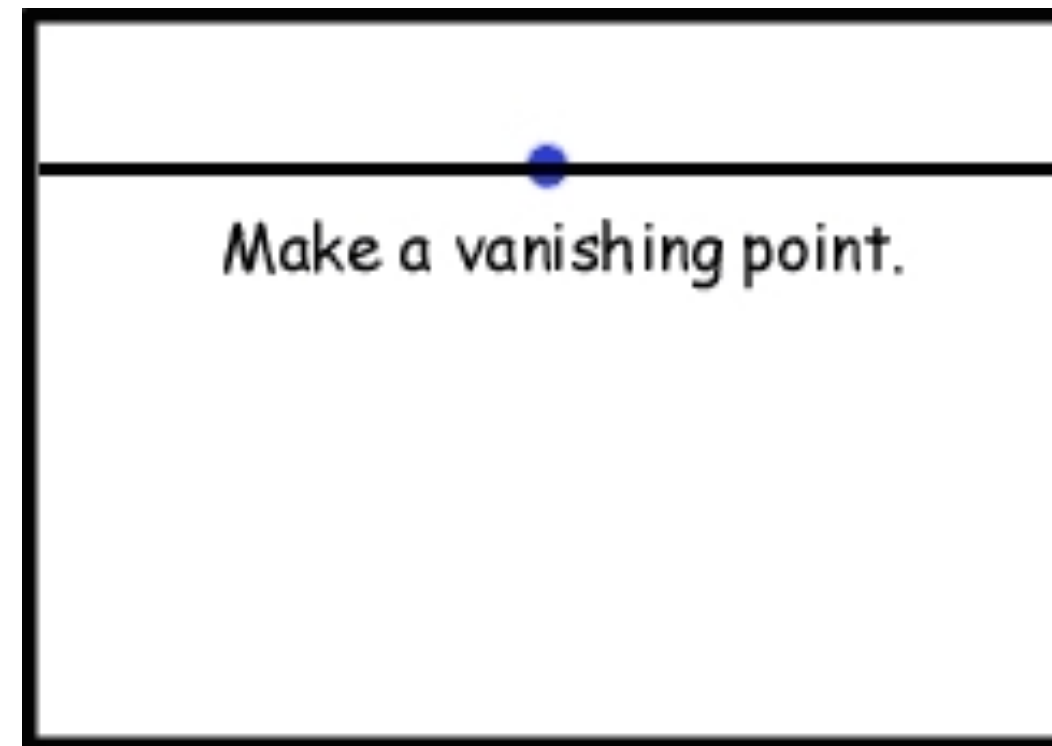
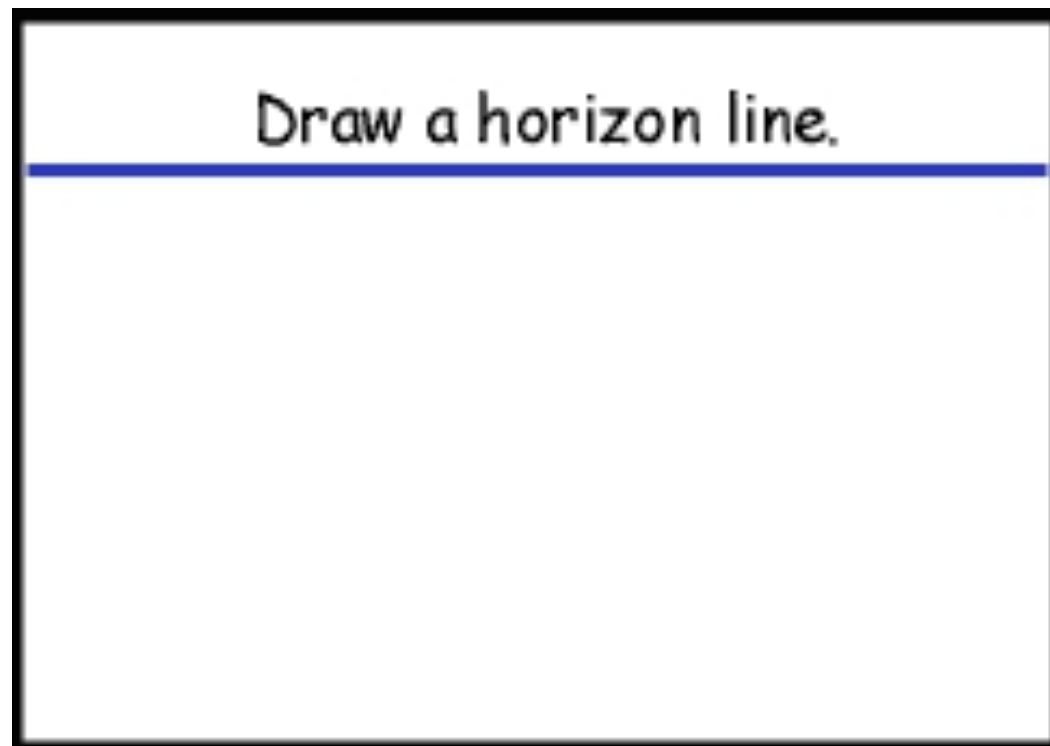
Vanishing Points



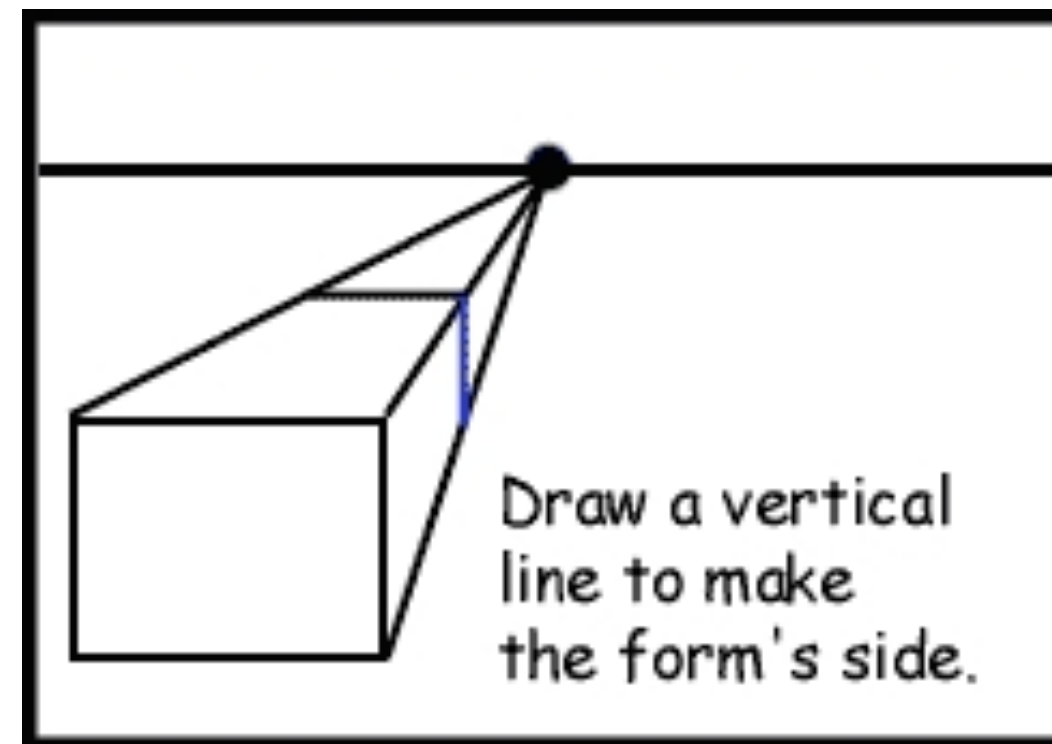
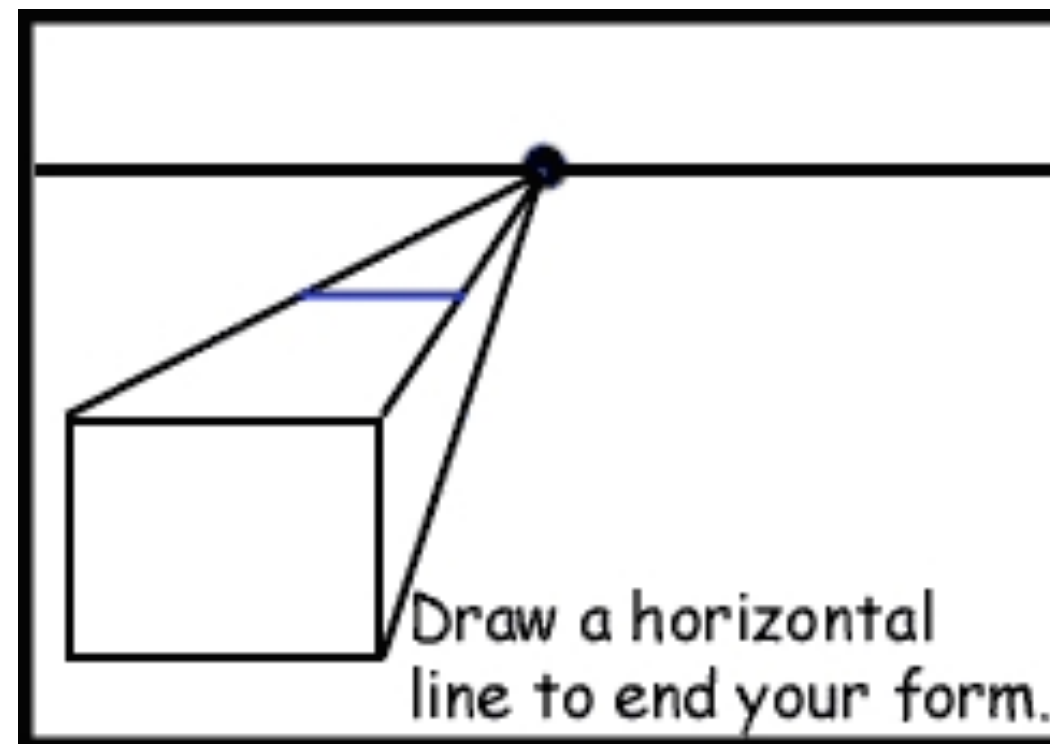
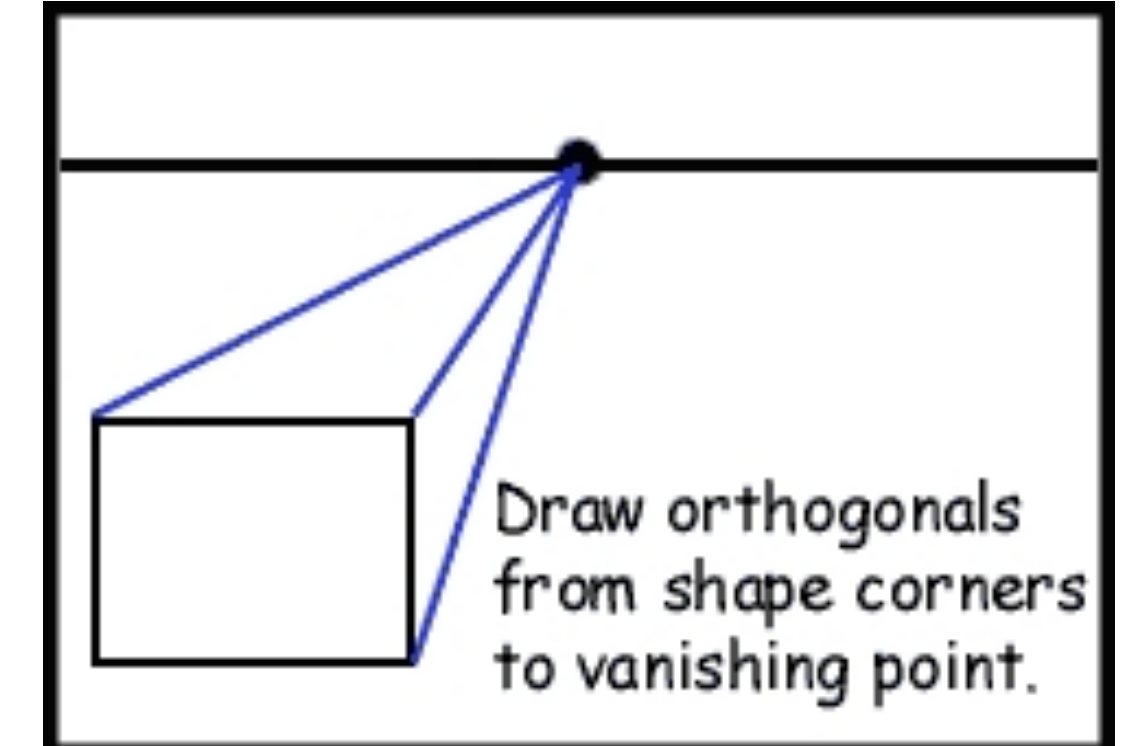
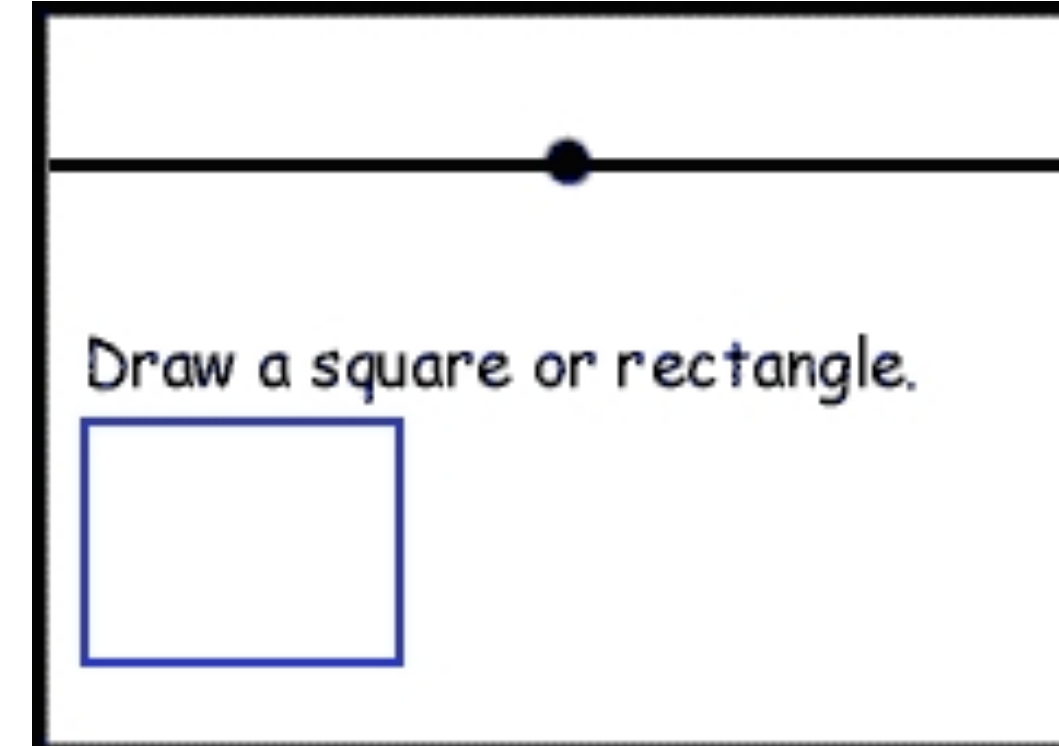
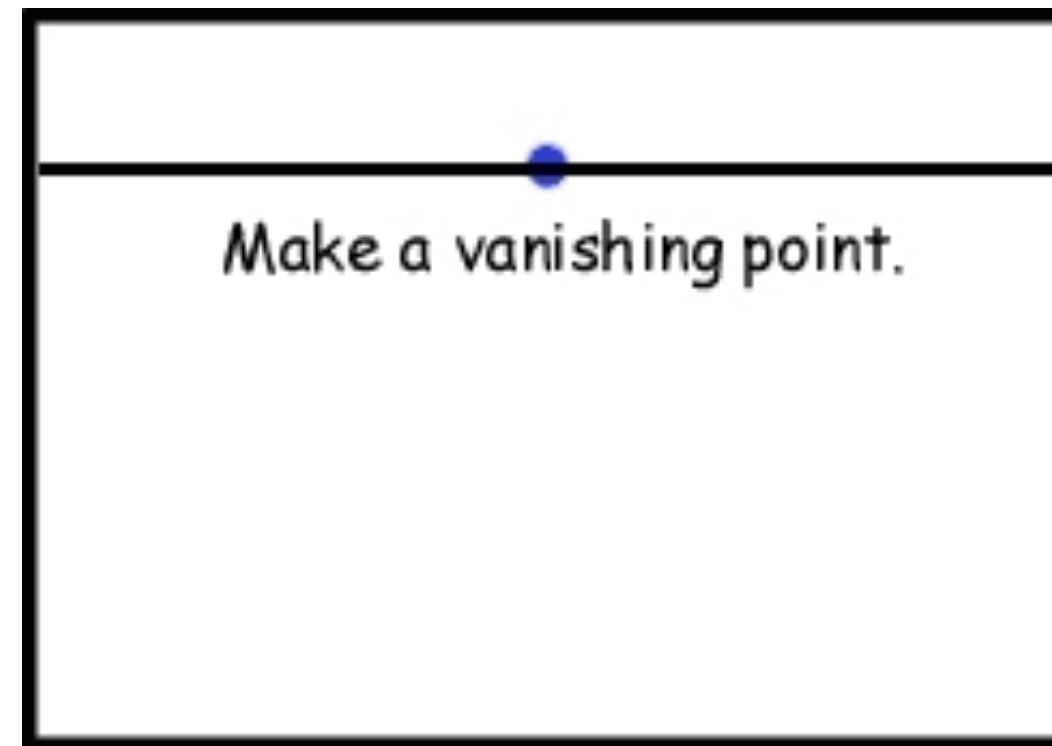
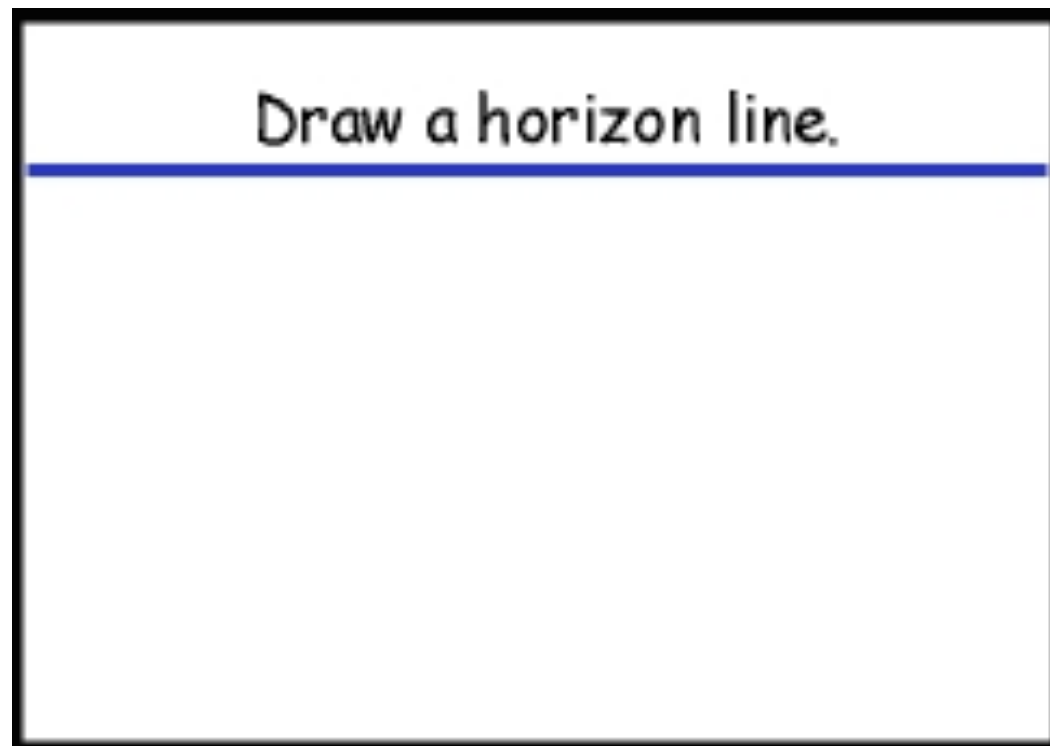
Vanishing Points



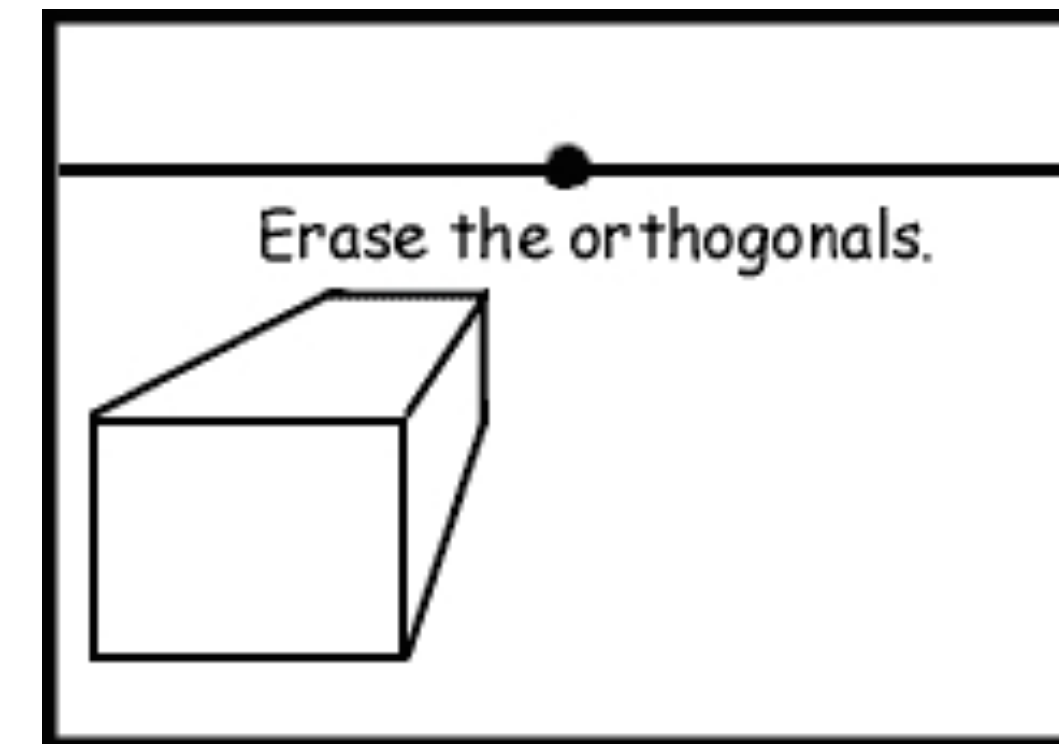
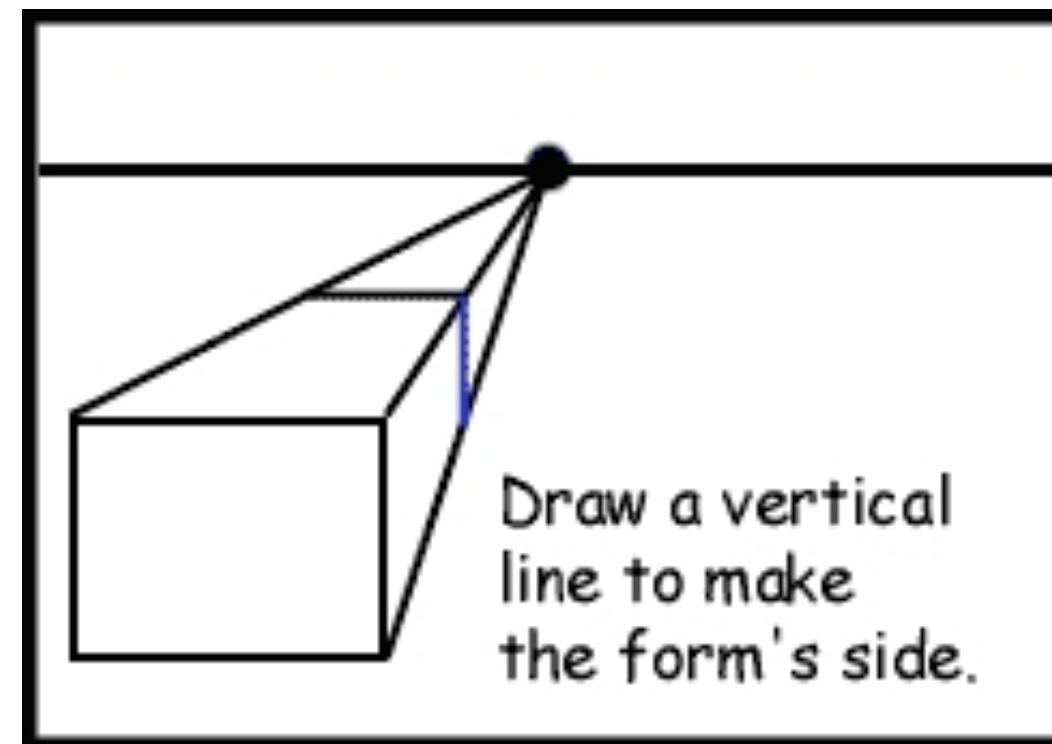
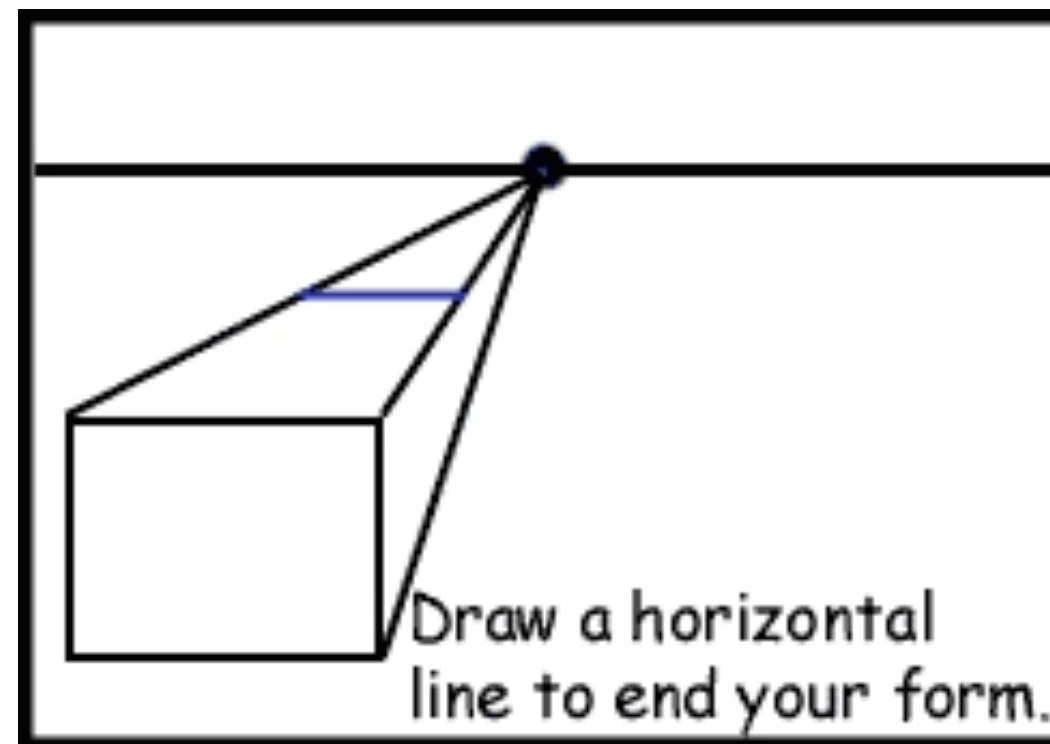
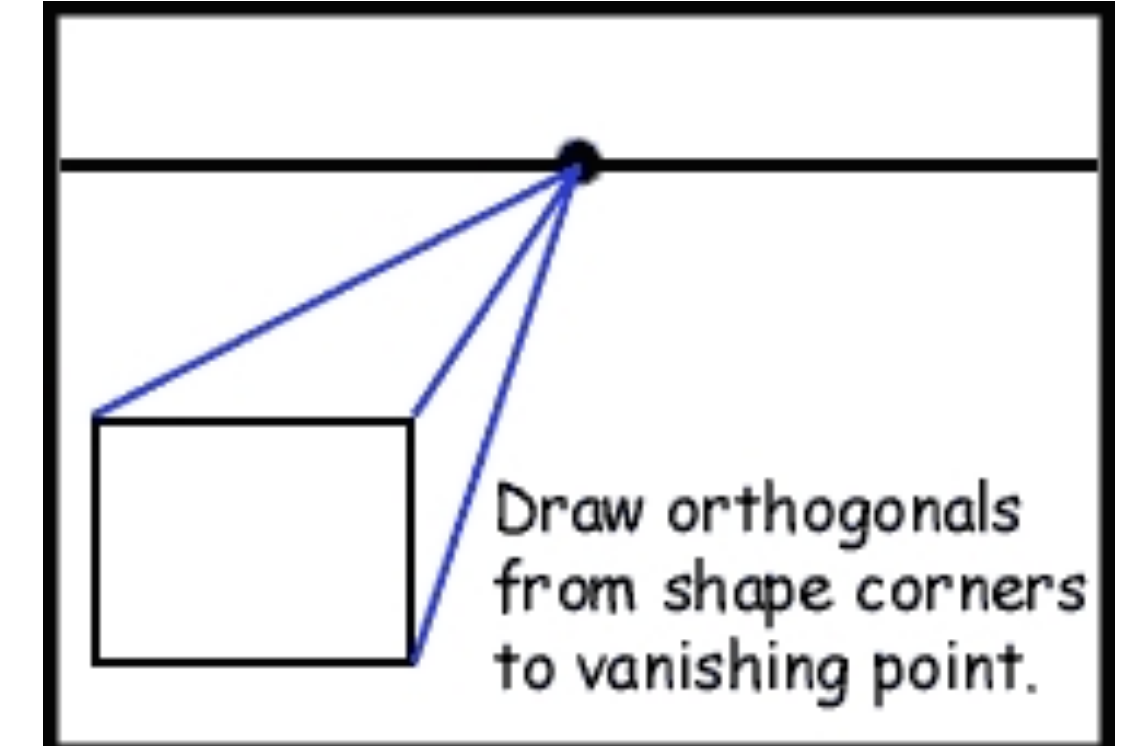
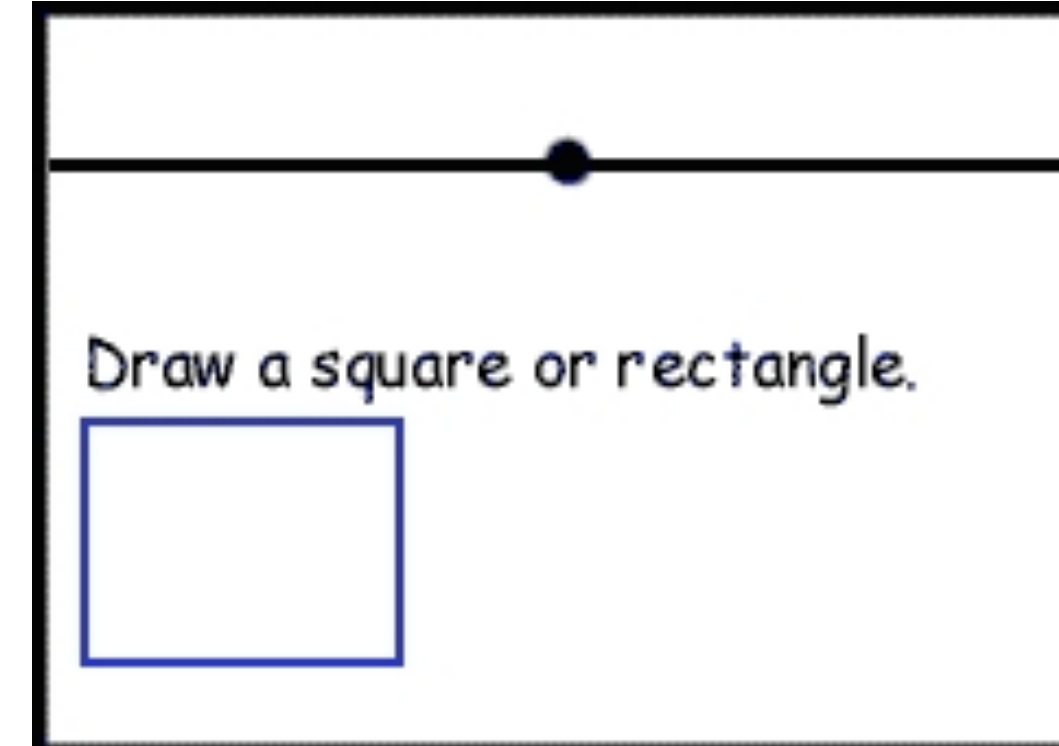
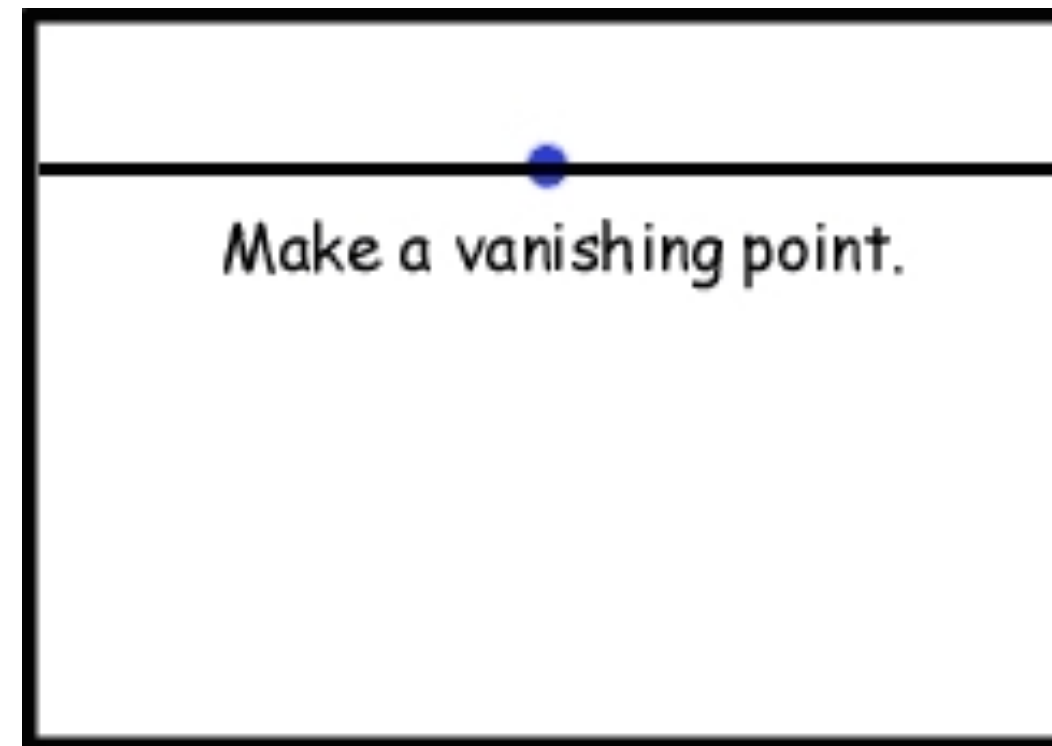
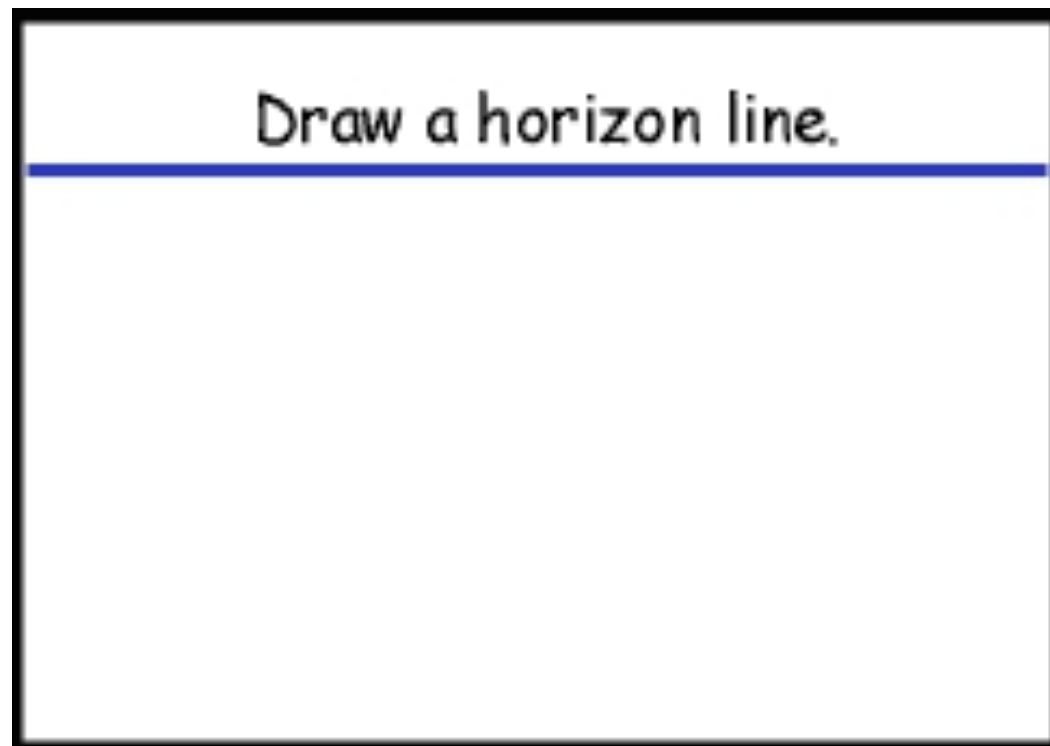
Vanishing Points



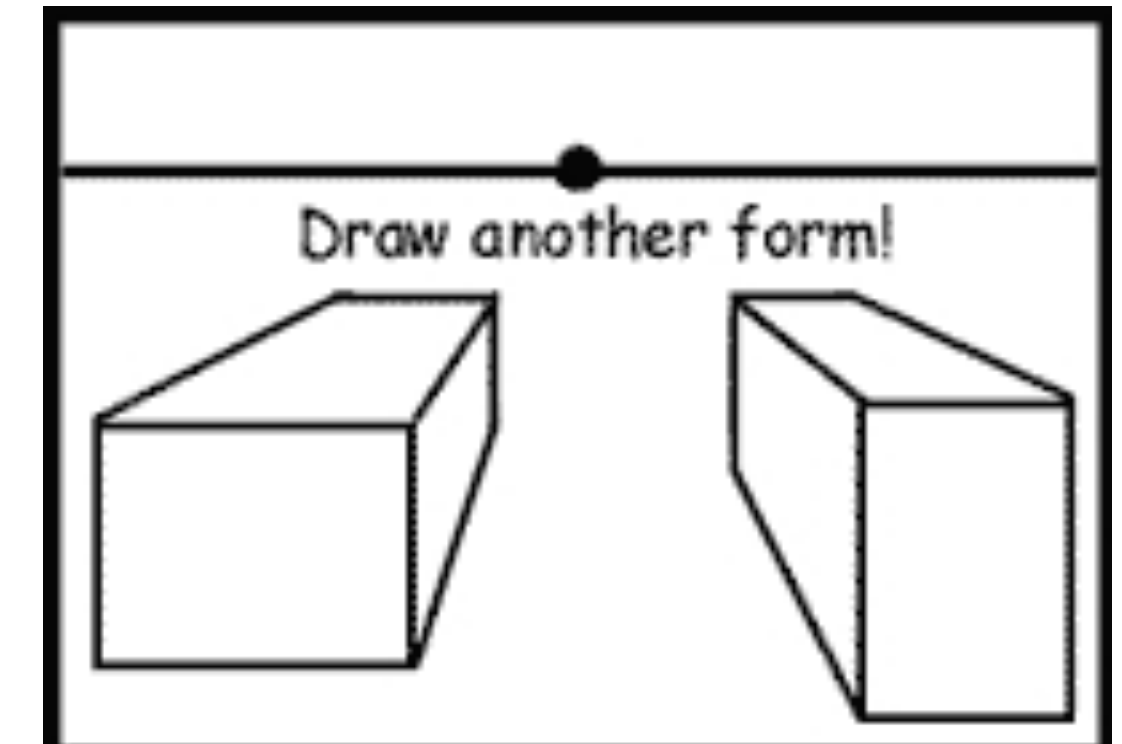
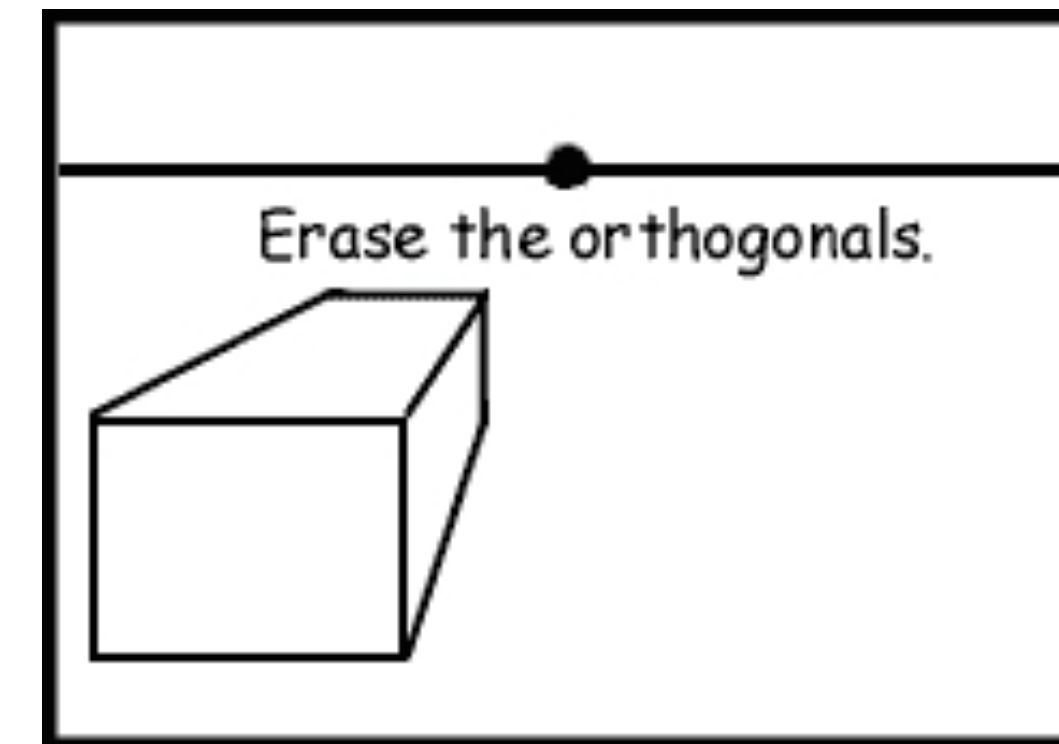
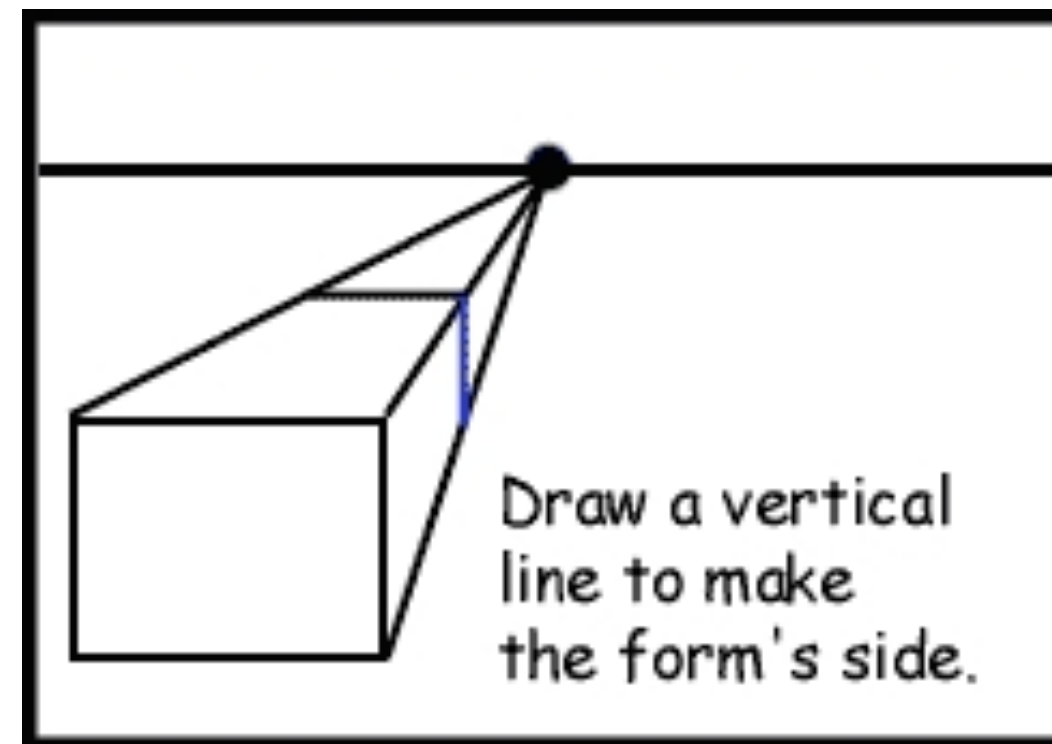
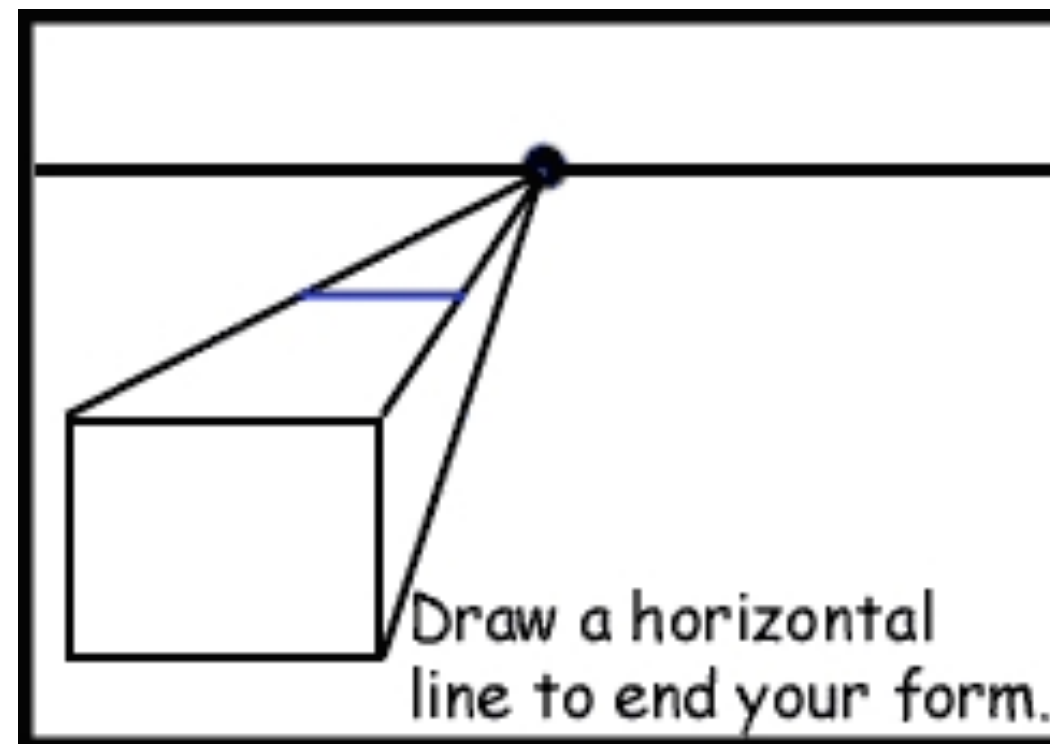
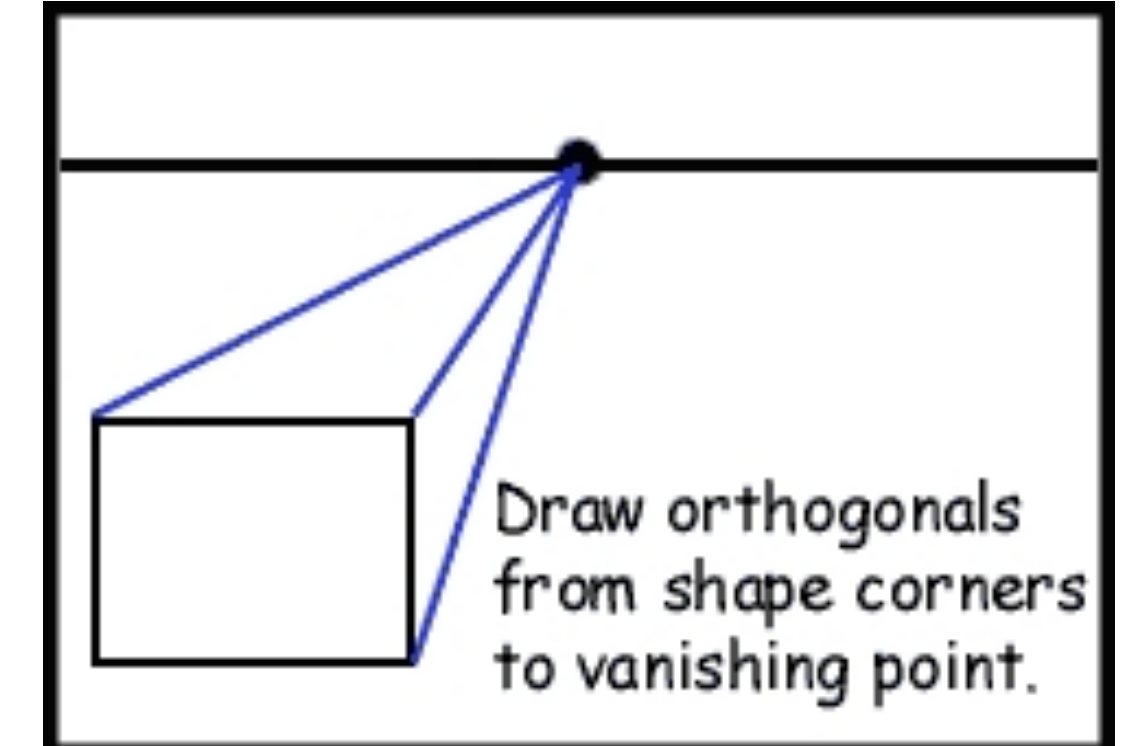
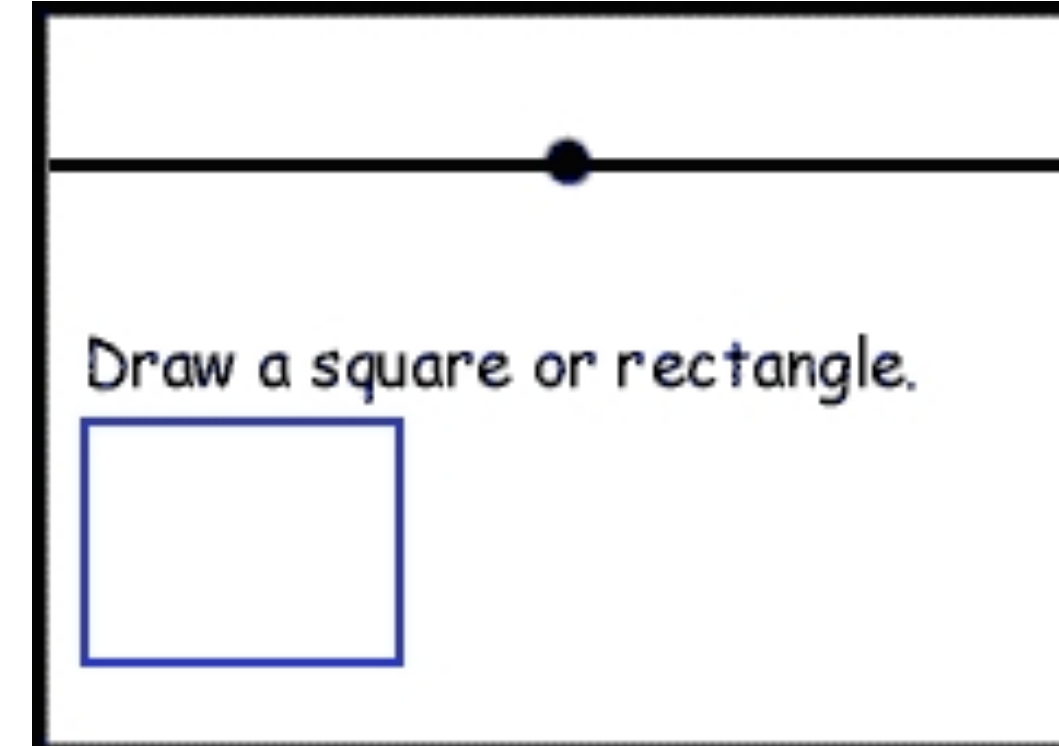
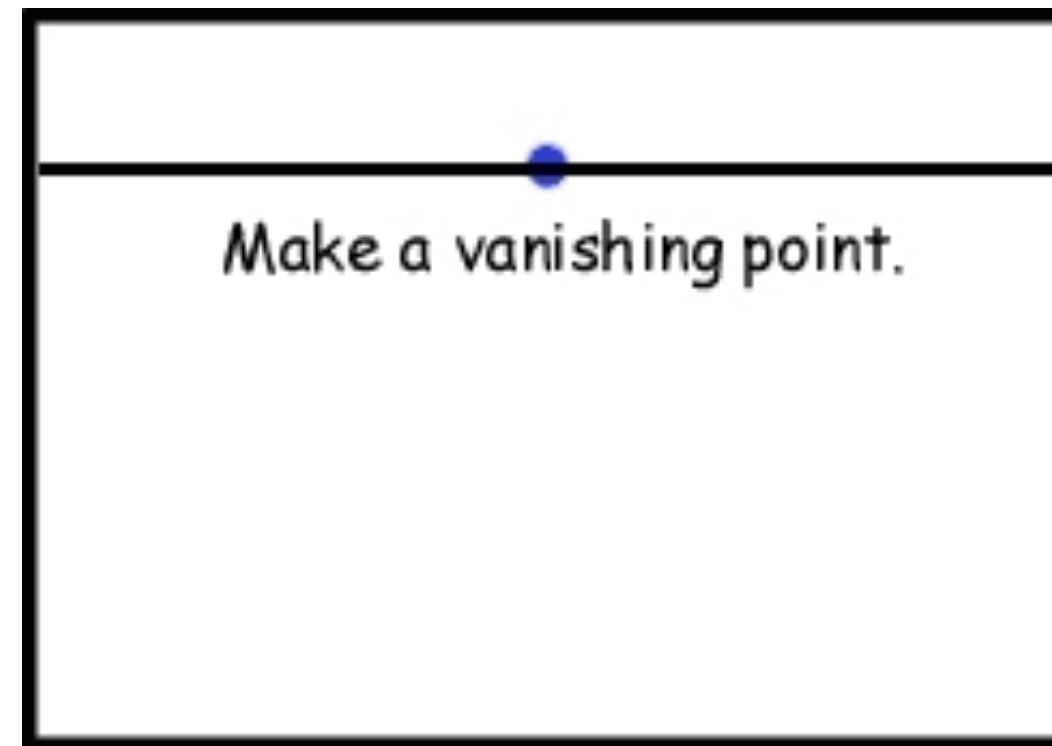
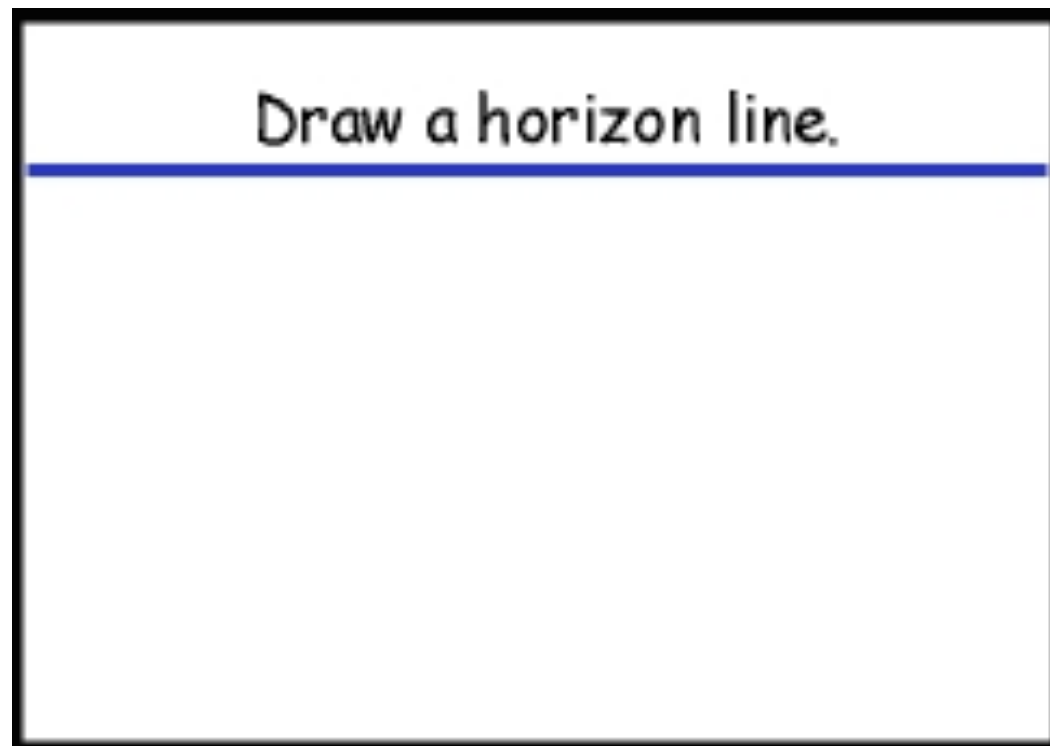
Vanishing Points



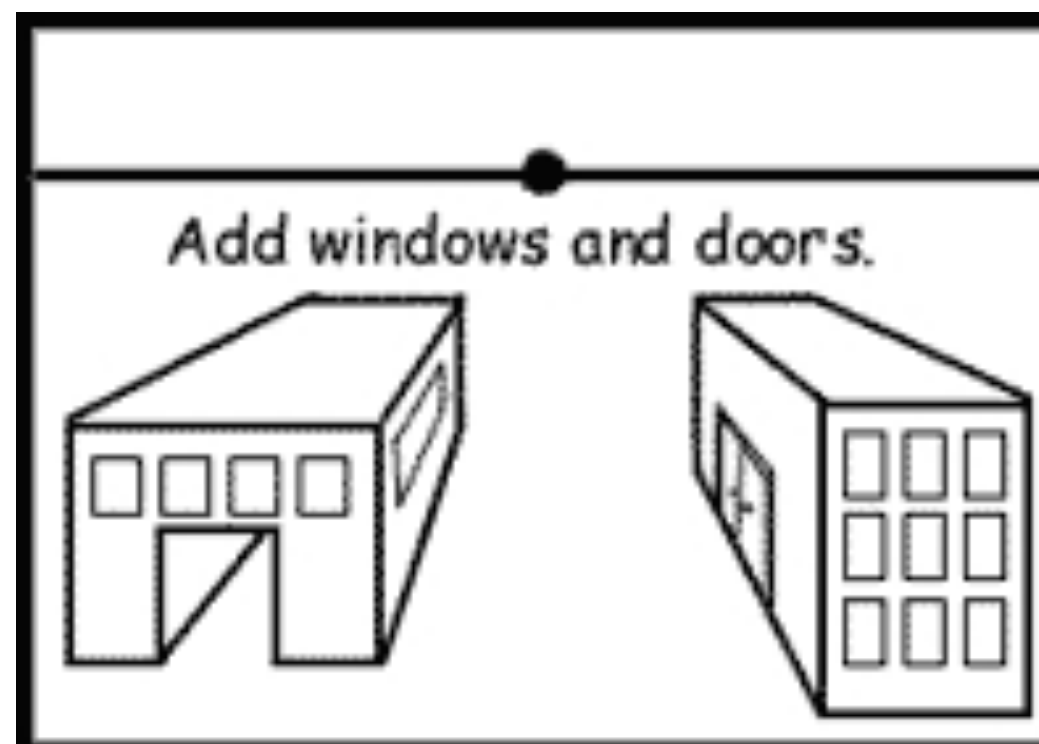
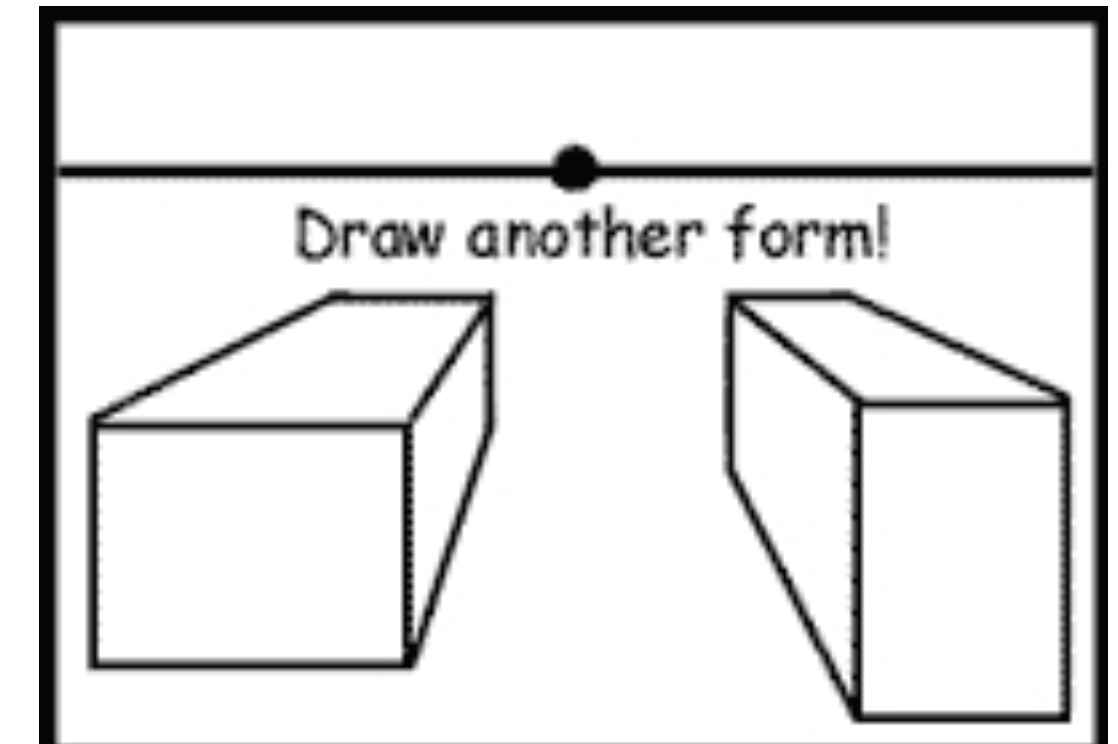
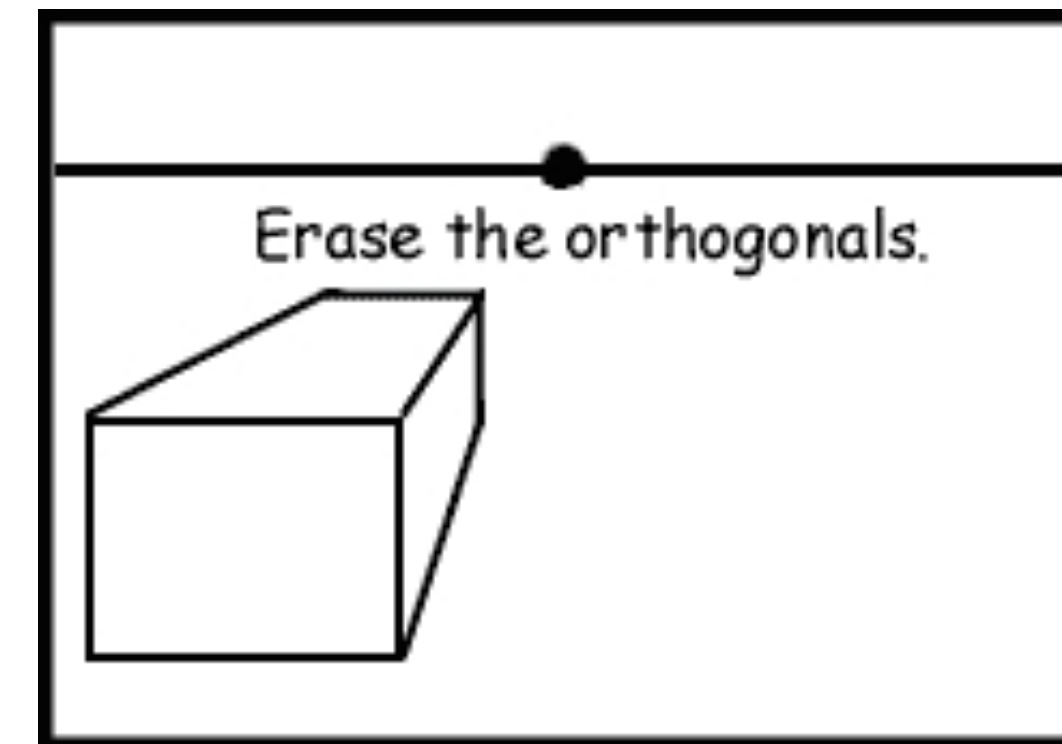
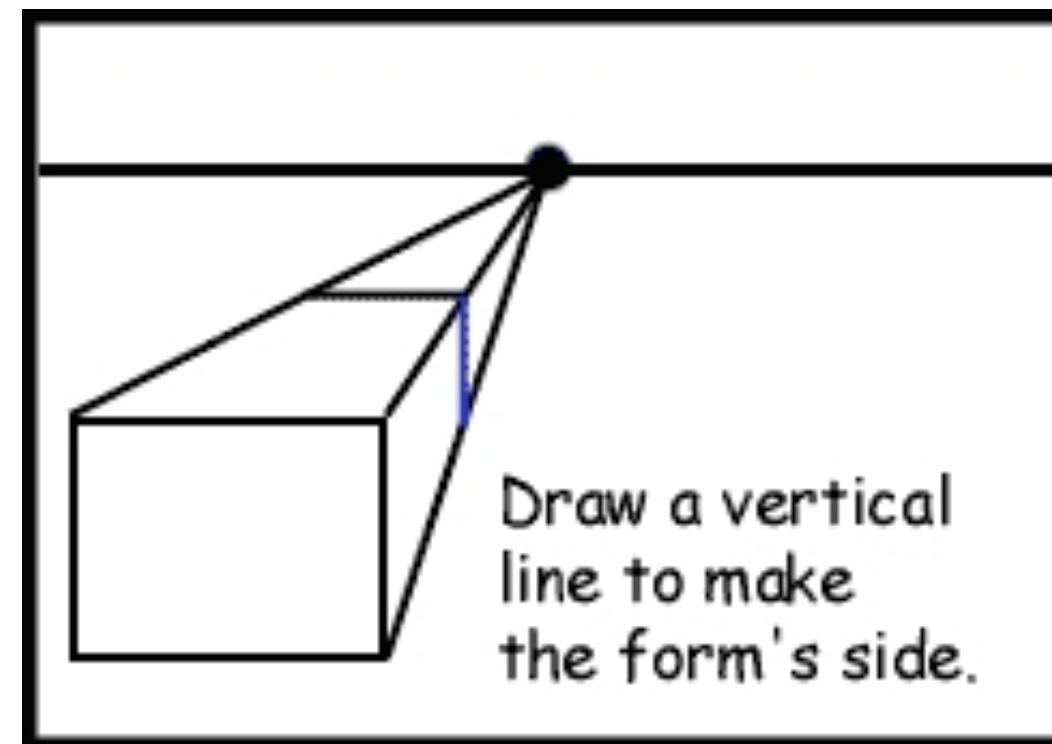
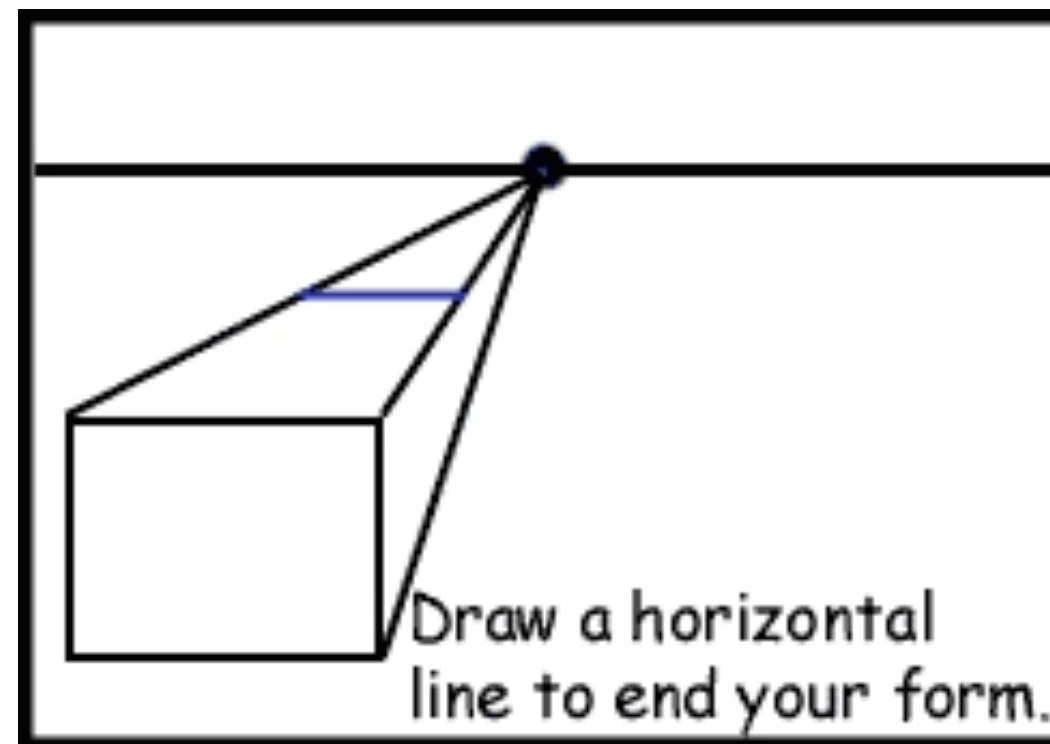
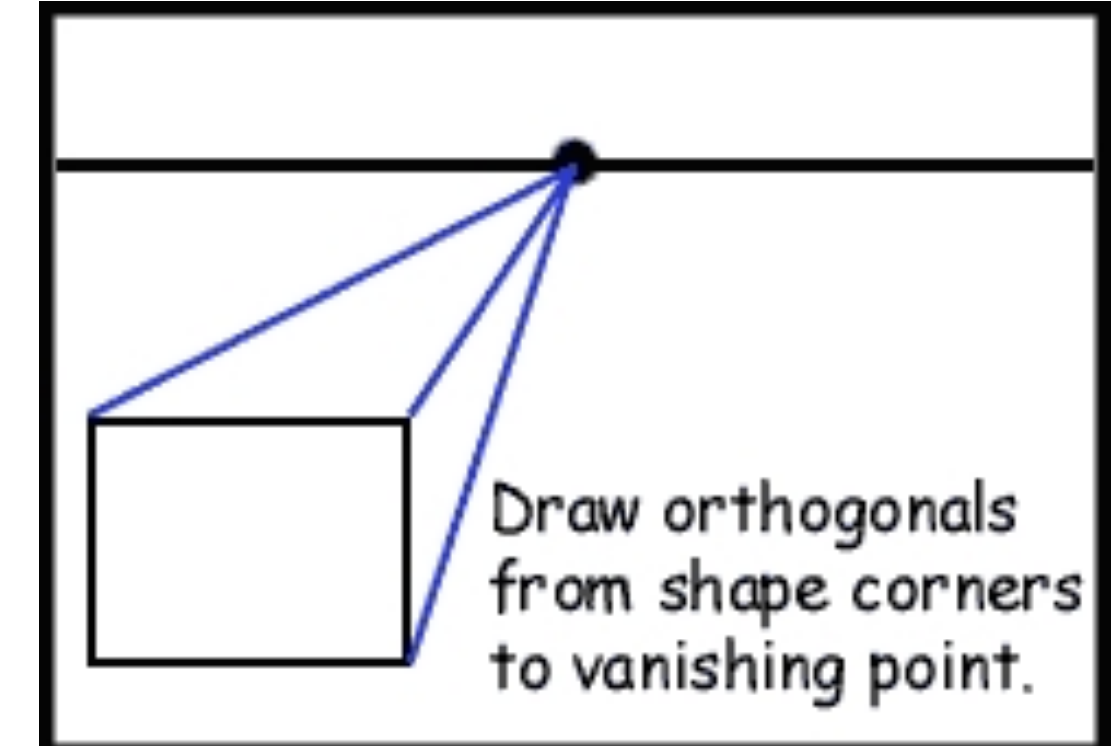
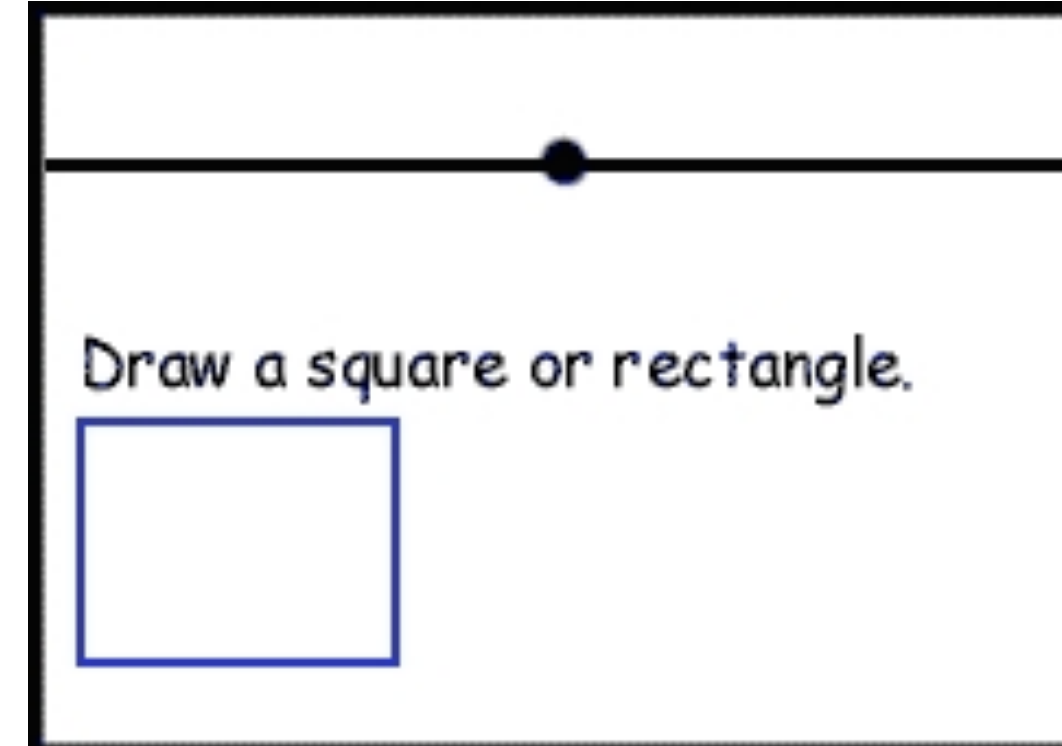
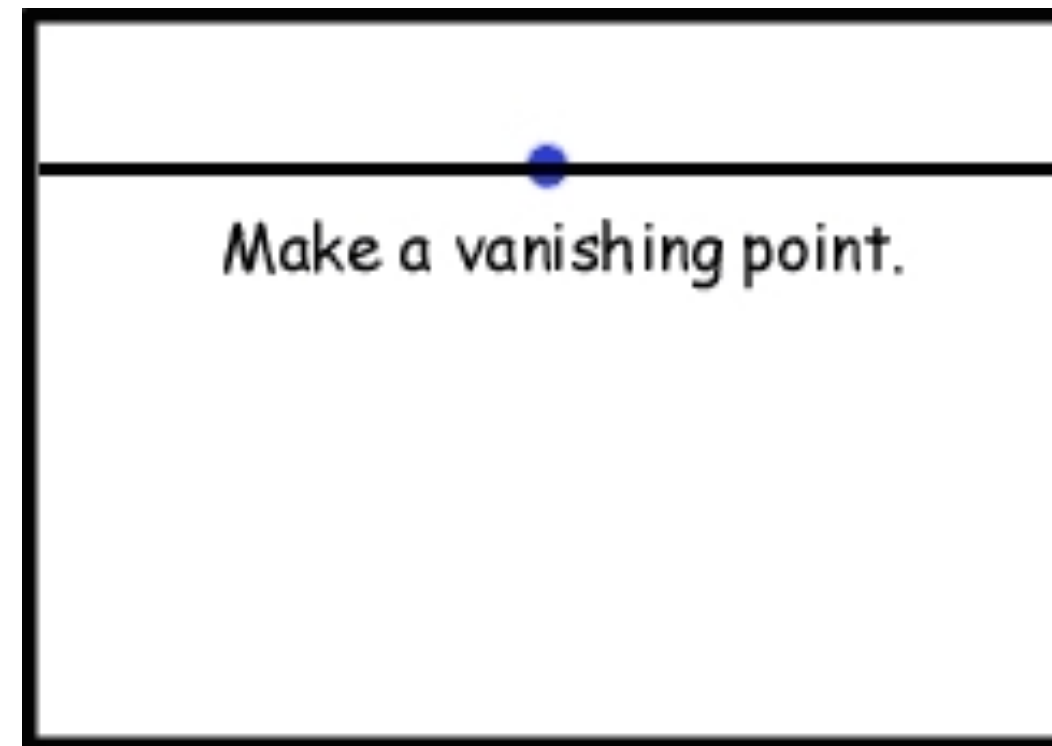
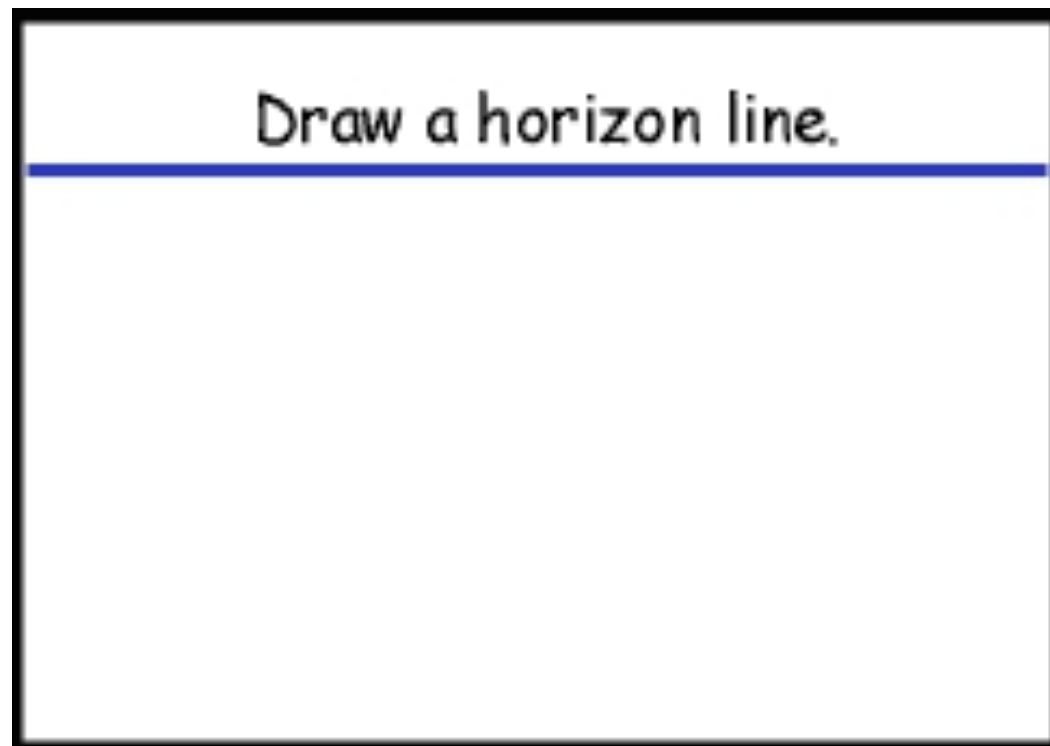
Vanishing Points



Vanishing Points



Vanishing Points



Vanishing Points

Each set of parallel lines meet at a different point

— the point is called **vanishing point**

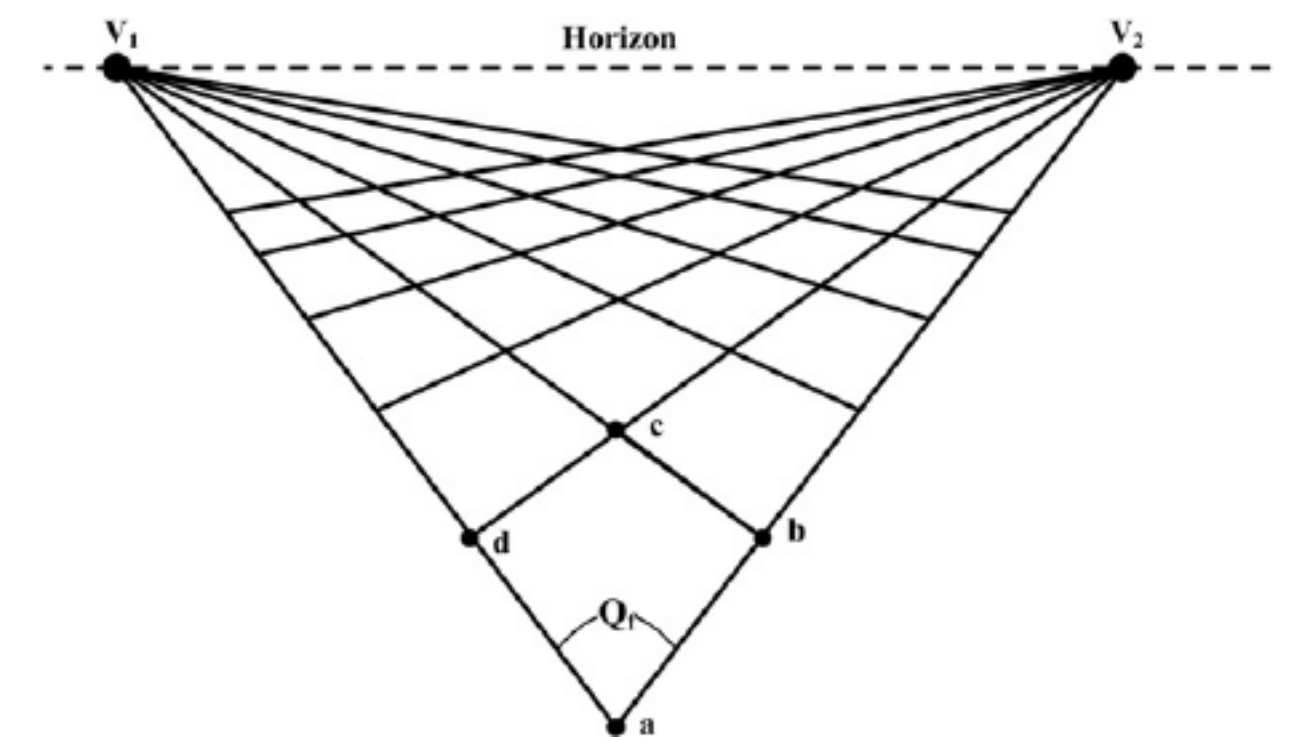
Sets of parallel lines on the same plane lead to **collinear** vanishing points

— the line is called a **horizon** for that plane

Good way to **spot fake images**

— scale and perspective do not work

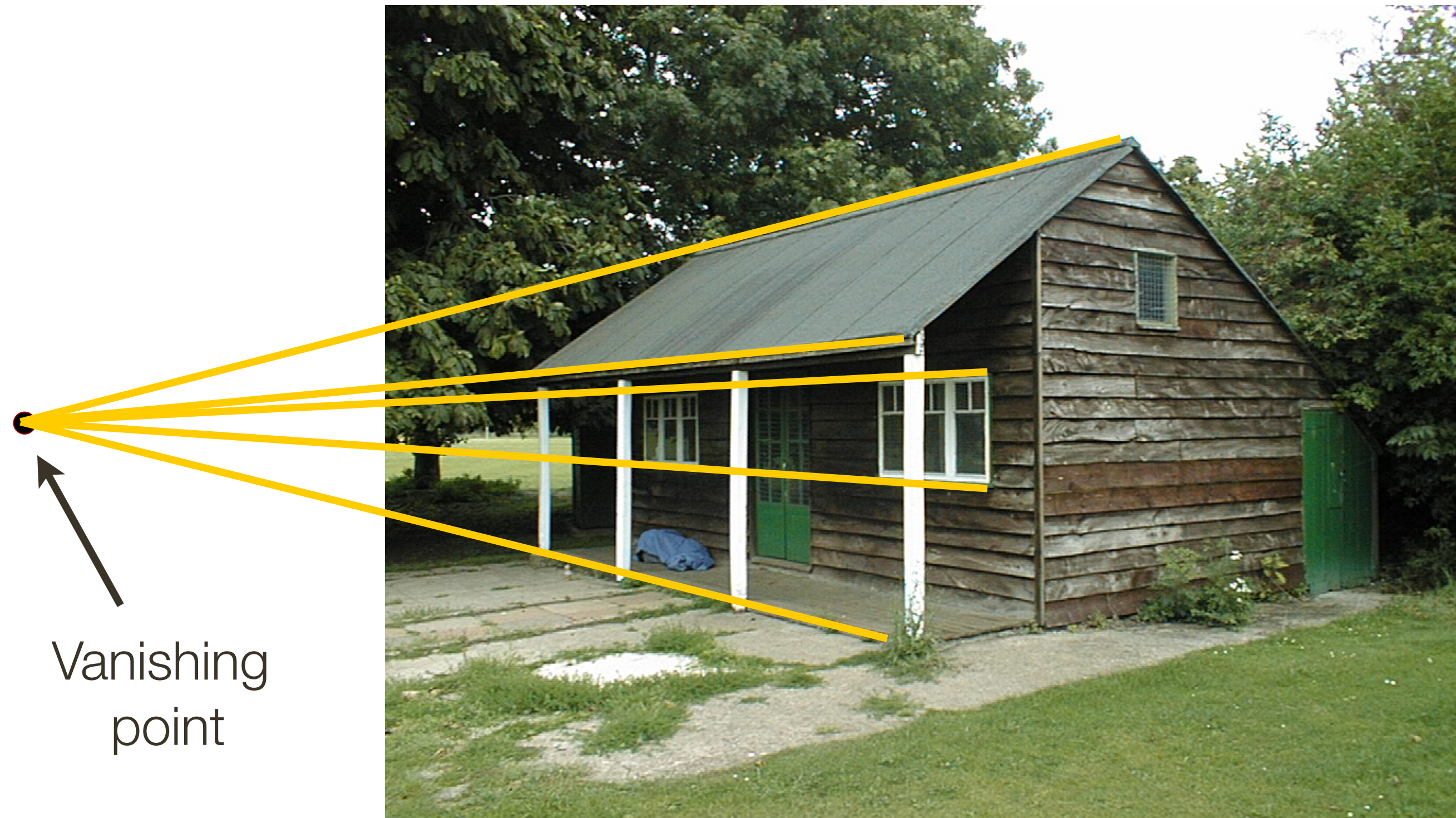
— vanishing points behave badly



Vanishing Points



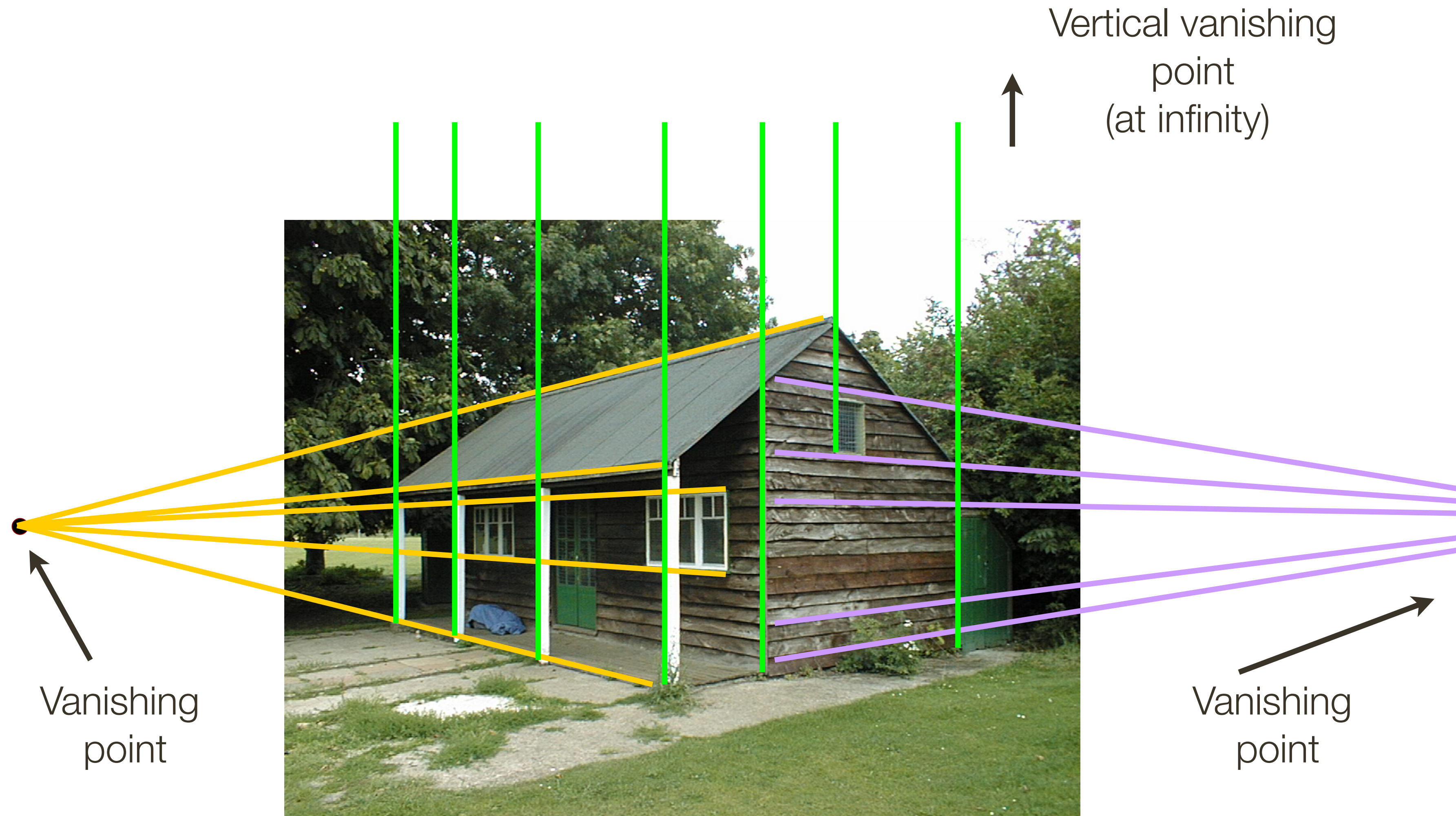
Vanishing Points



Vanishing Points

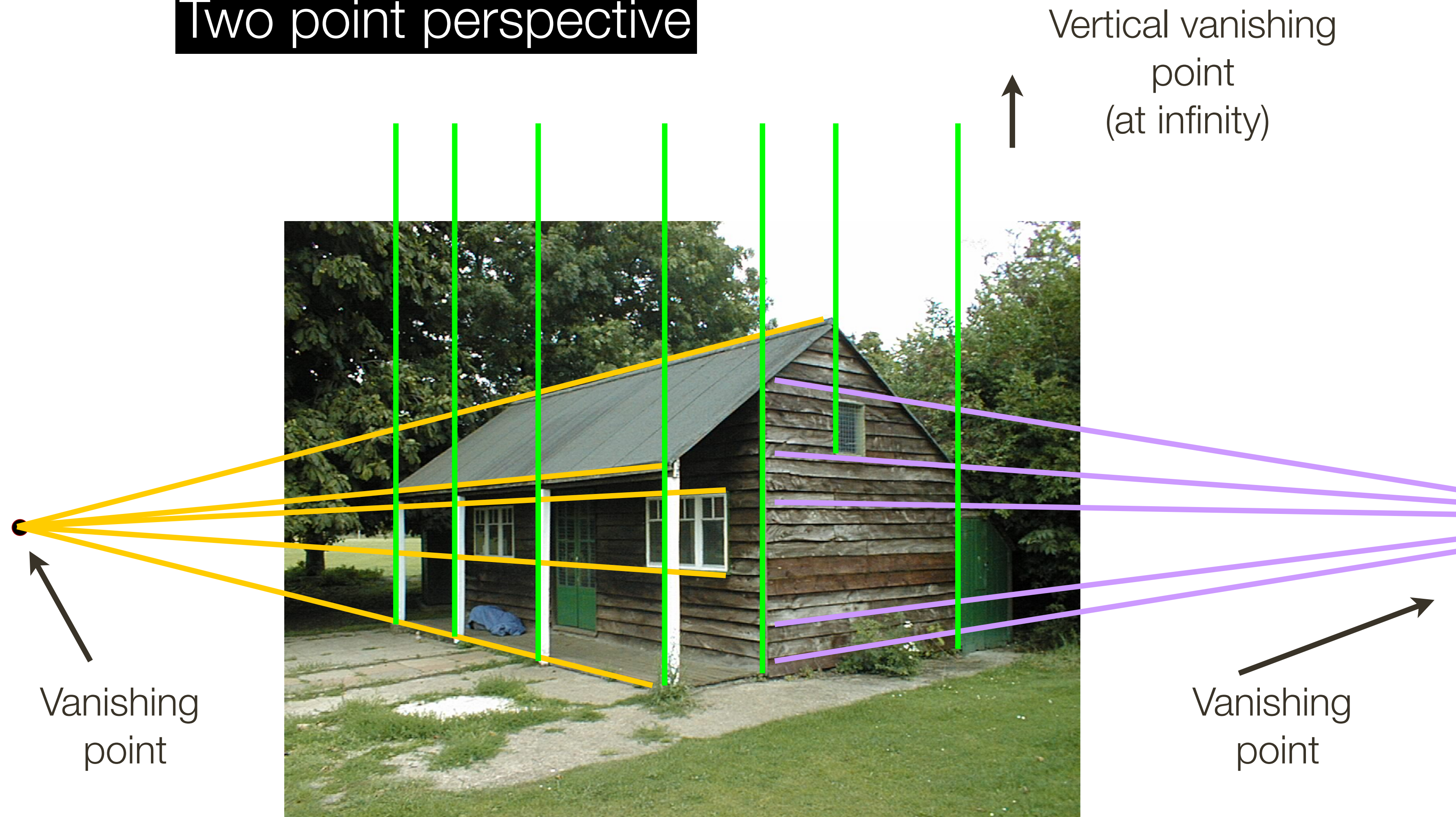


Vanishing Points



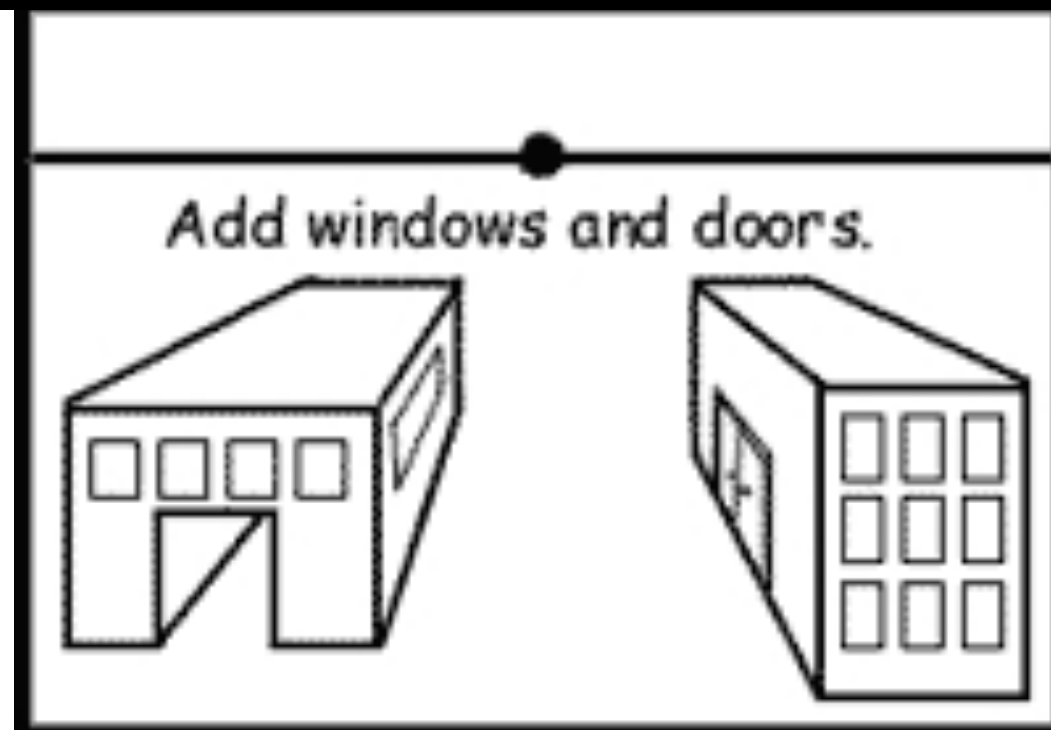
Vanishing Points

Two point perspective

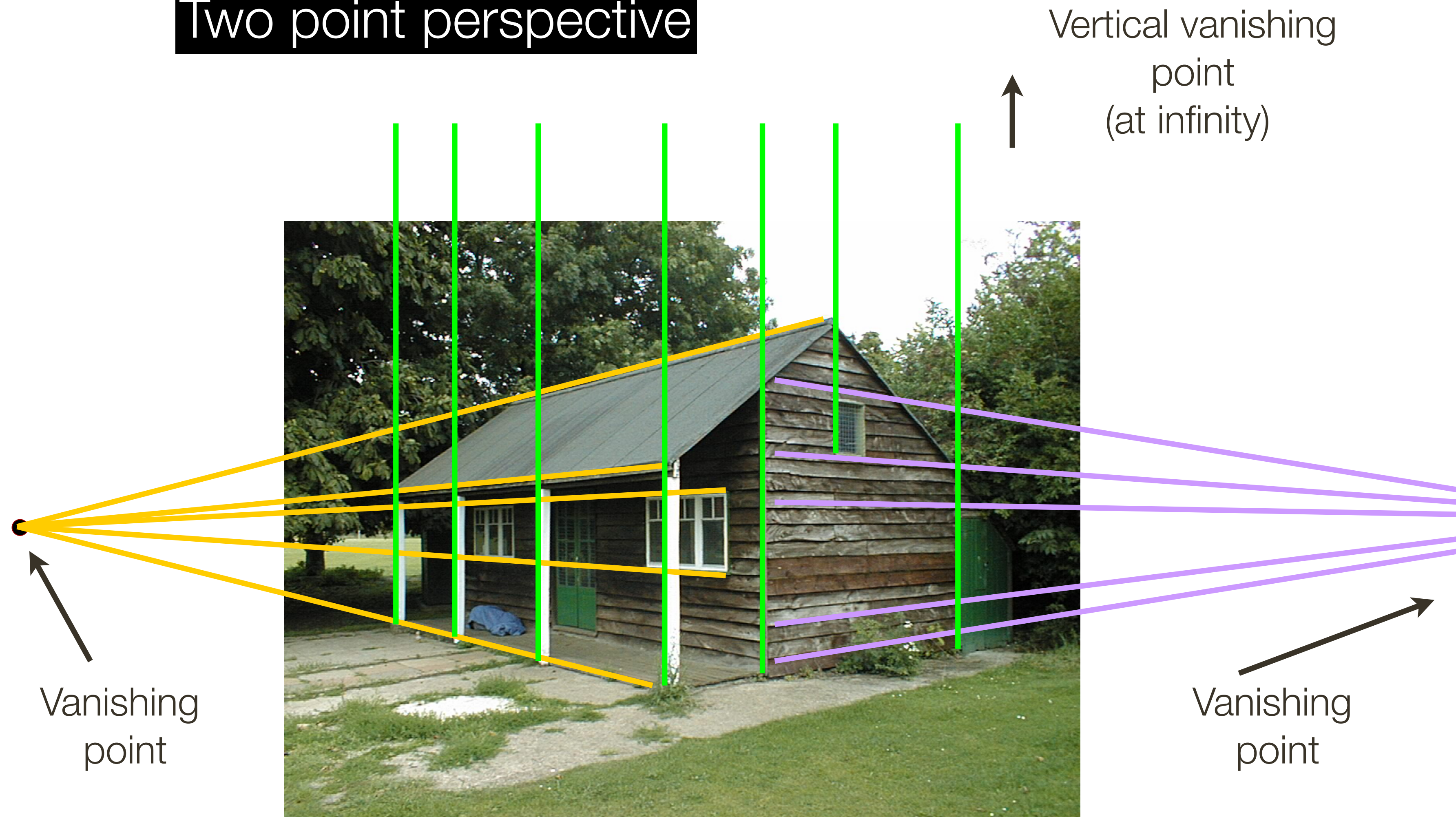


Vanishing Points

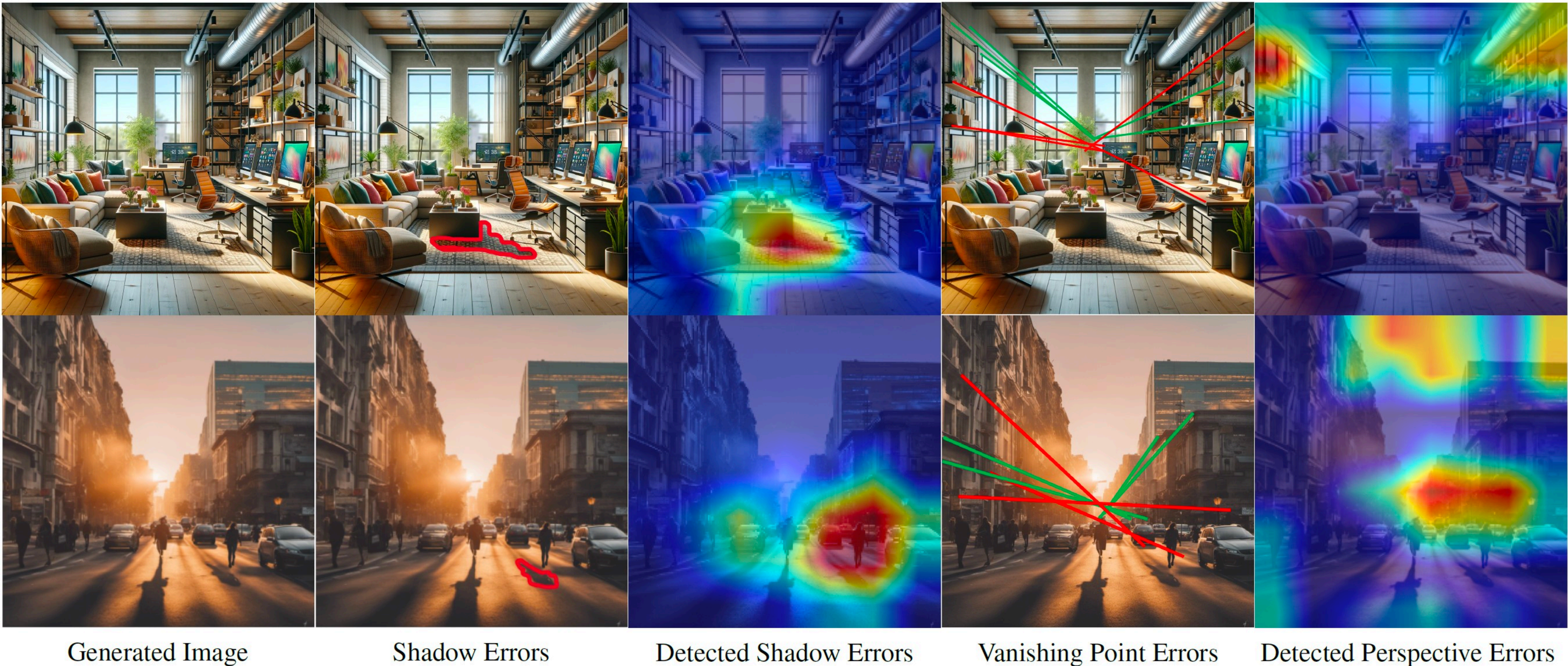
One point perspective



Two point perspective



Spotting fake images with **Vanishing** Points



Generated Image

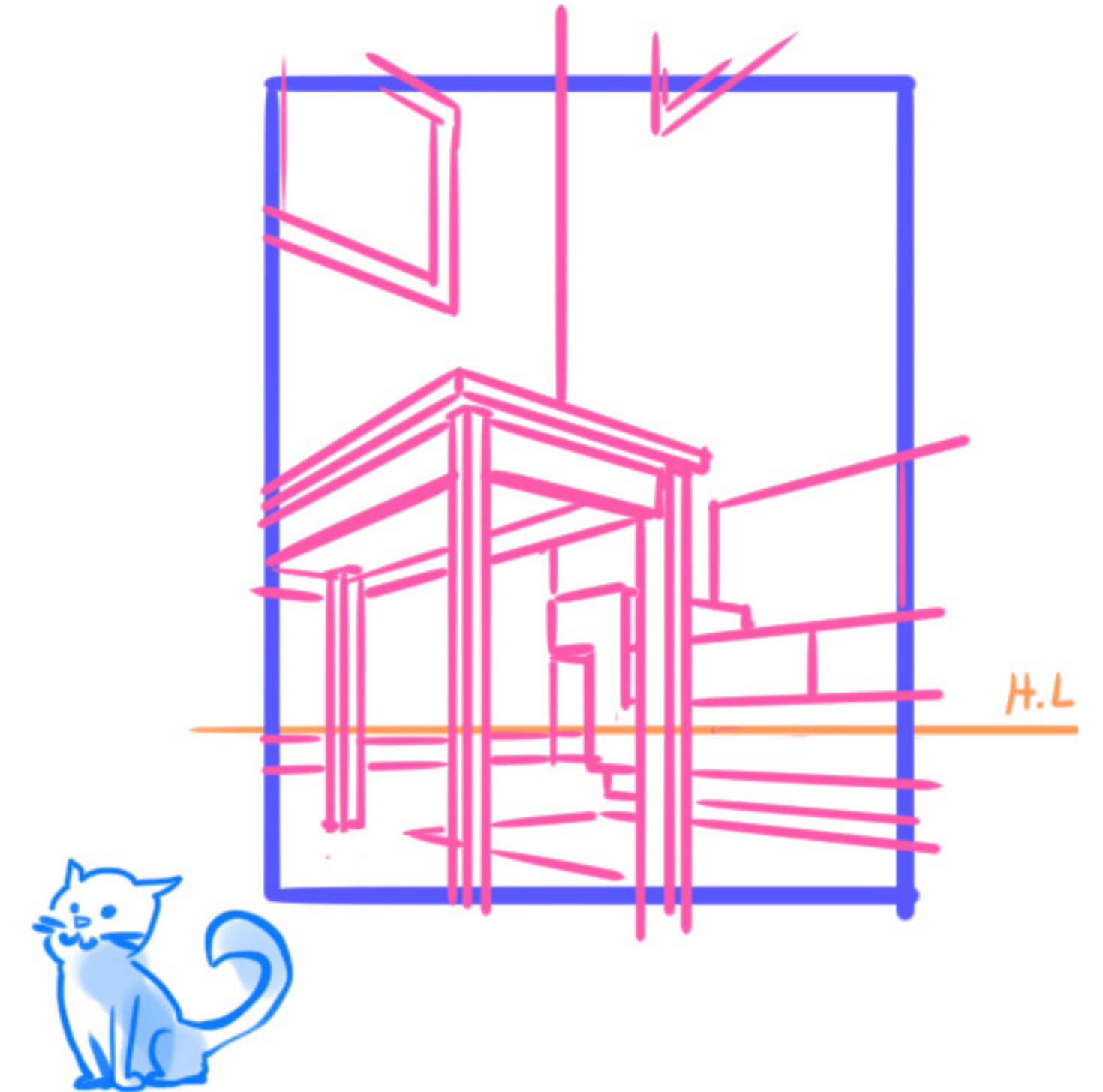
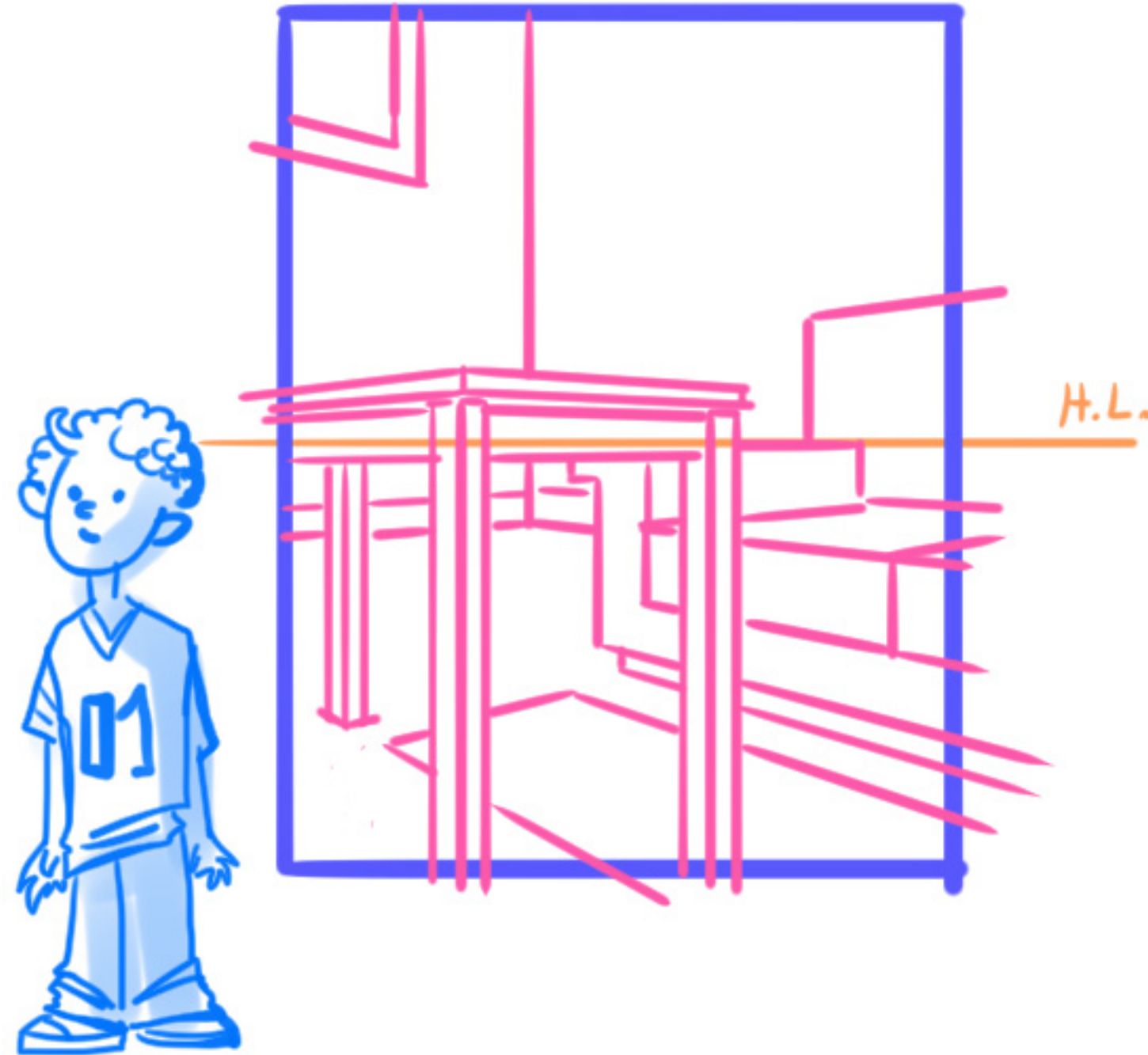
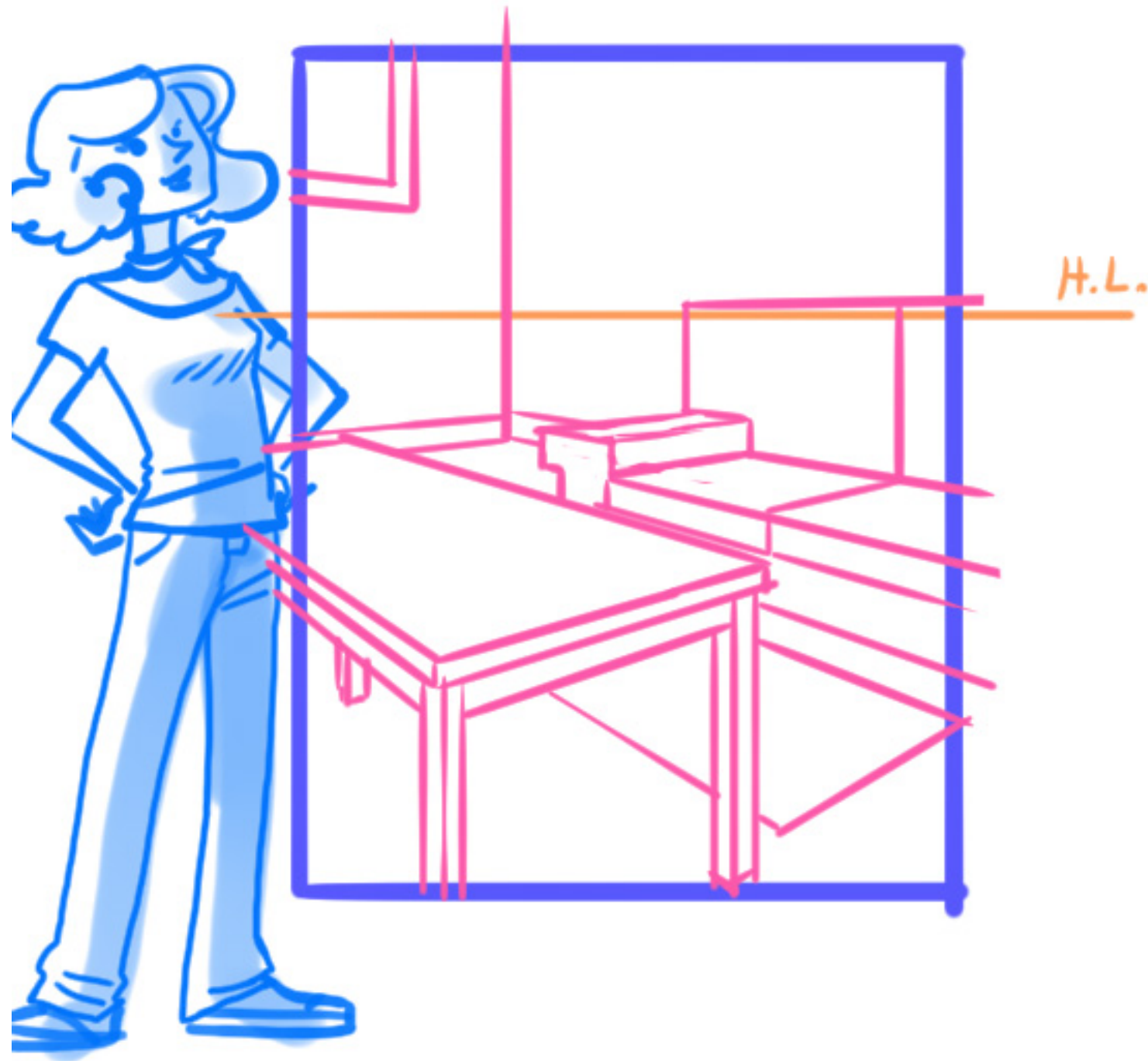
Shadow Errors

Detected Shadow Errors

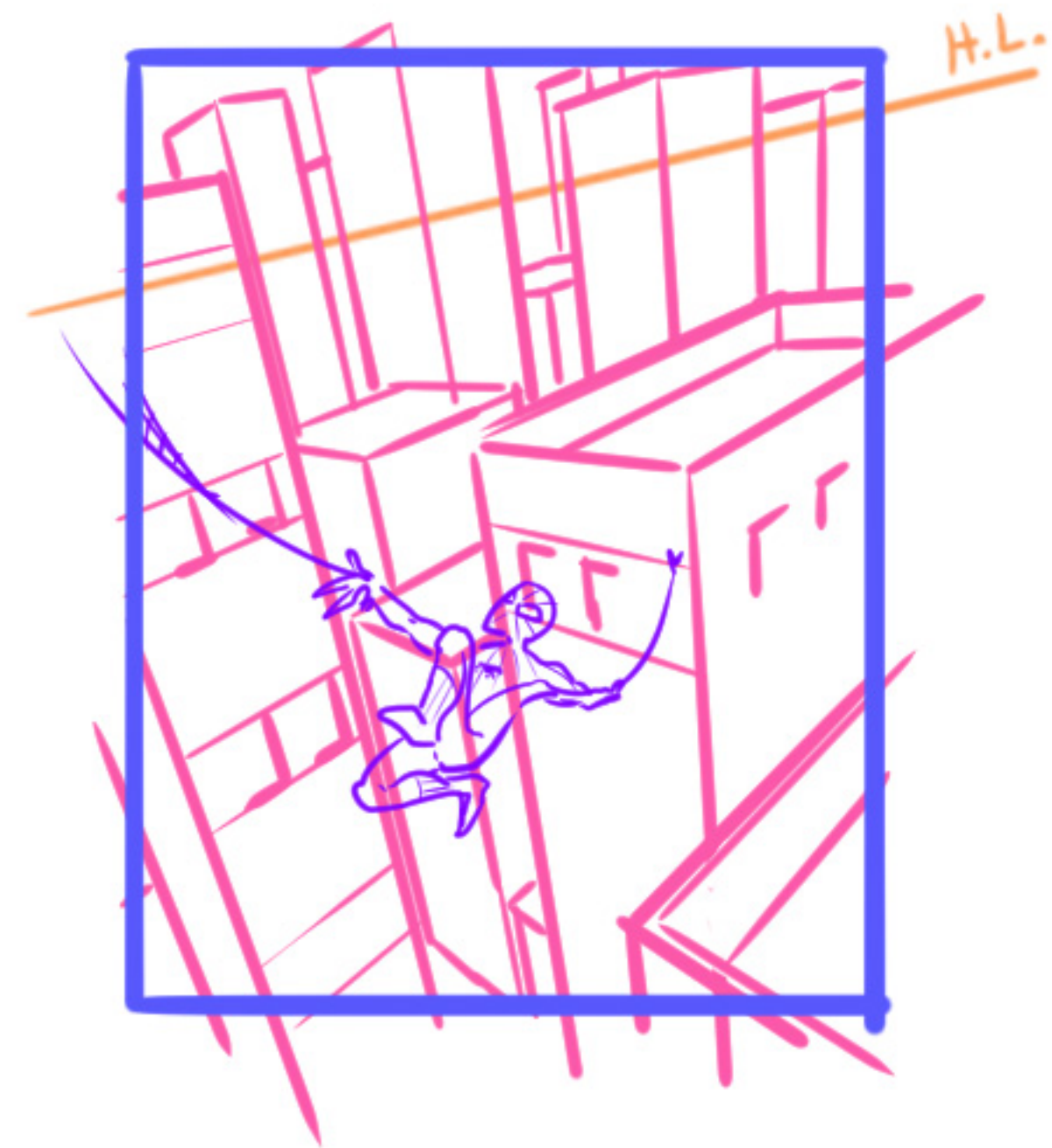
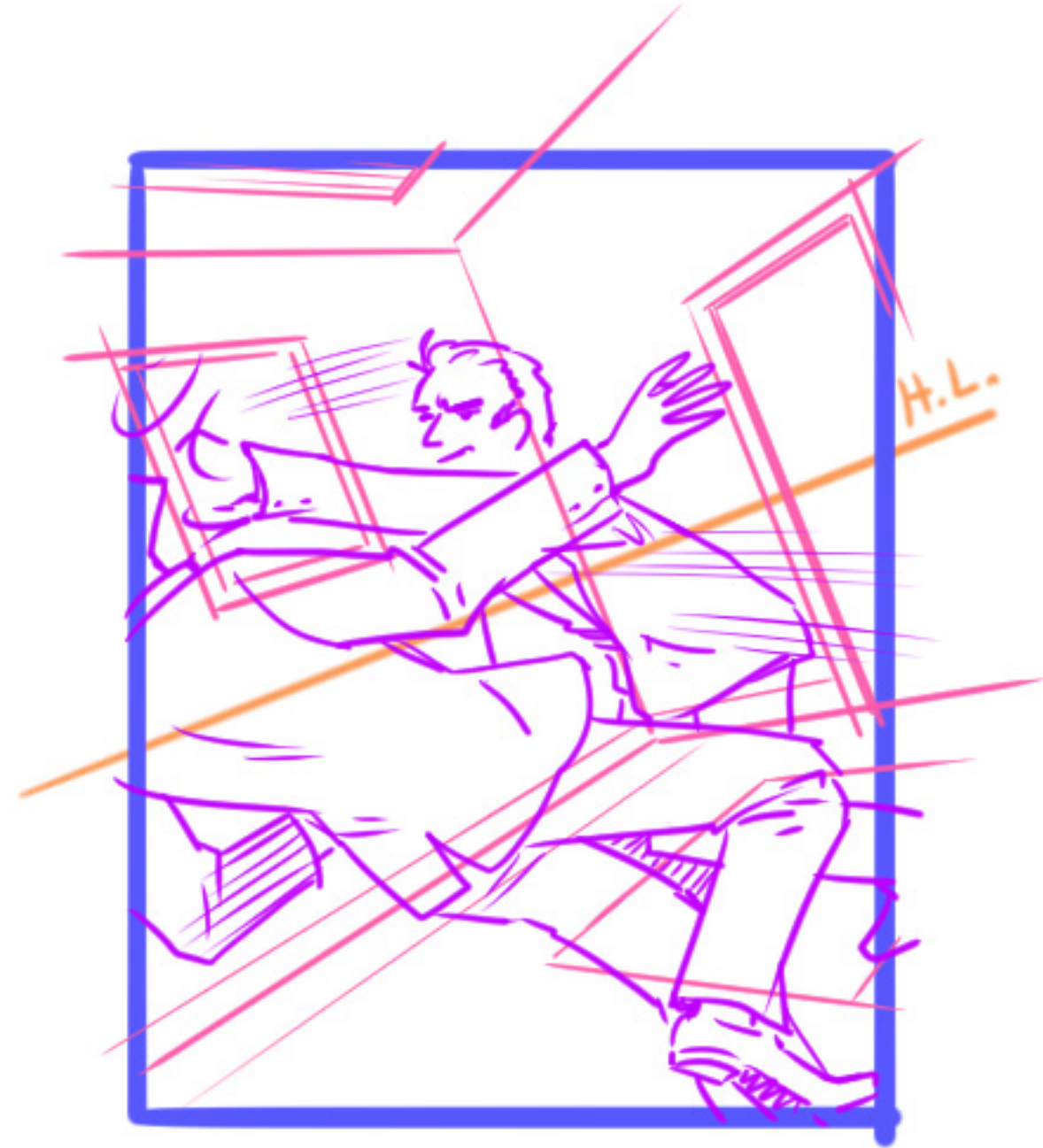
Vanishing Point Errors

Detected Perspective Errors

Perspective Aside



Perspective Aside



Properties of Projection

- **Points** project to **points**
- **Lines** project to **lines**
- **Planes** project to the **whole** or **half** image
- Angles are **not** preserved

Properties of Projection

- **Points** project to **points**
- **Lines** project to **lines**
- **Planes** project to the **whole** or **half** image
- Angles are **not** preserved

Degenerate cases

- Line through focal point projects to a point
- Plane through focal point projects to a line

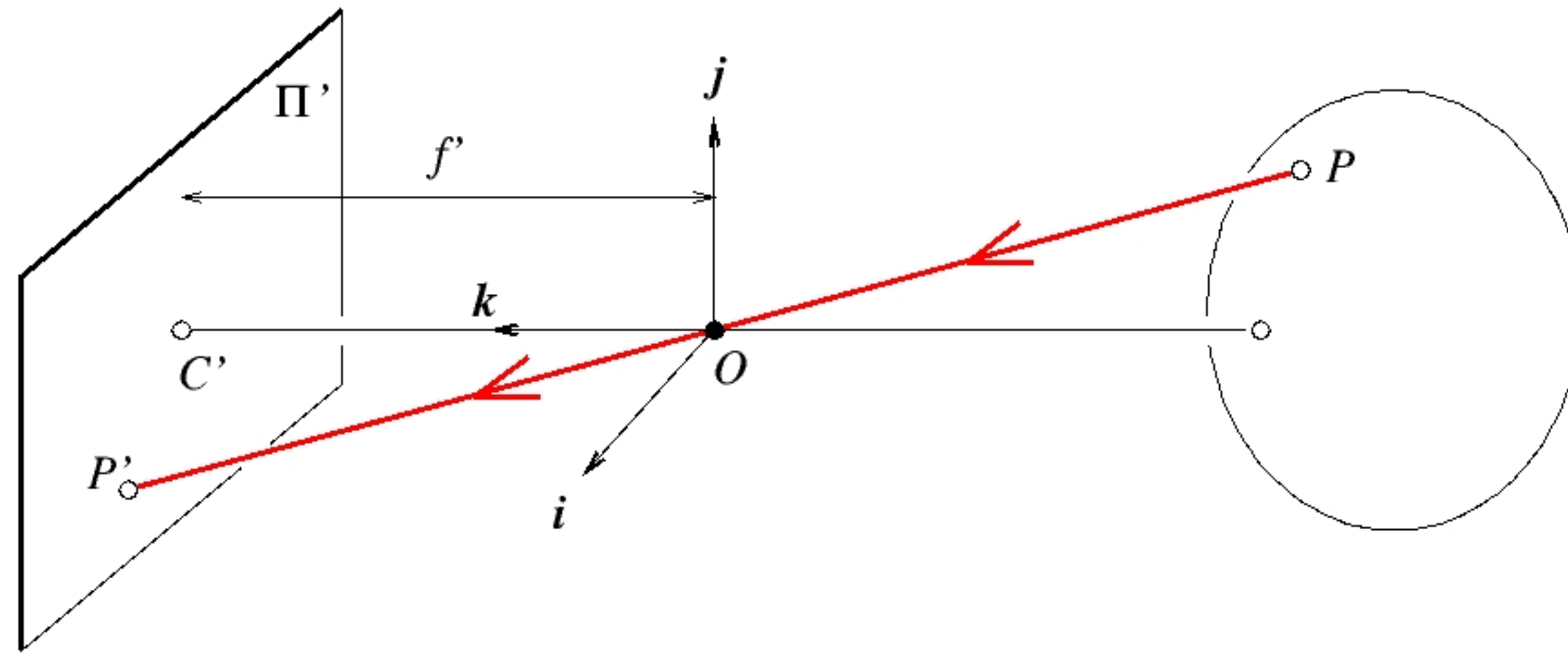
Projection Illusion



Projection Illusion



Perspective Projection



3D object point

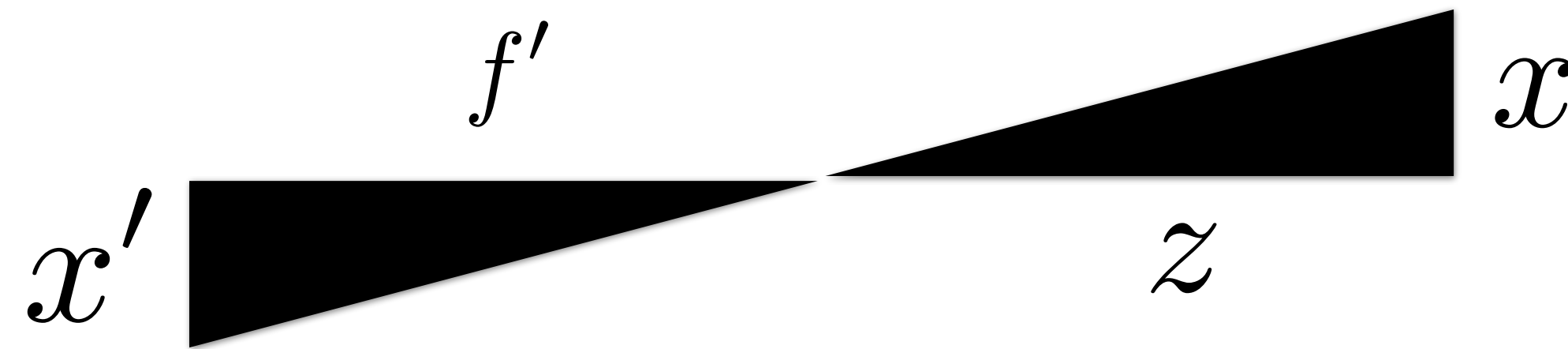
Forsyth & Ponce (1st ed.) Figure 1.4

$P = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ projects to 2D image point $P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$ where

$$\begin{aligned} x' &= f' \frac{x}{z} \\ y' &= f' \frac{y}{z} \end{aligned}$$

Note: this assumes world coordinate frame at the optical center (pinhole) and aligned with the image plane, image coordinate frame aligned with the camera coordinate frame

Perspective Projection: Proof



3D object point

Forsyth & Ponce (1st ed.) Figure 1.4

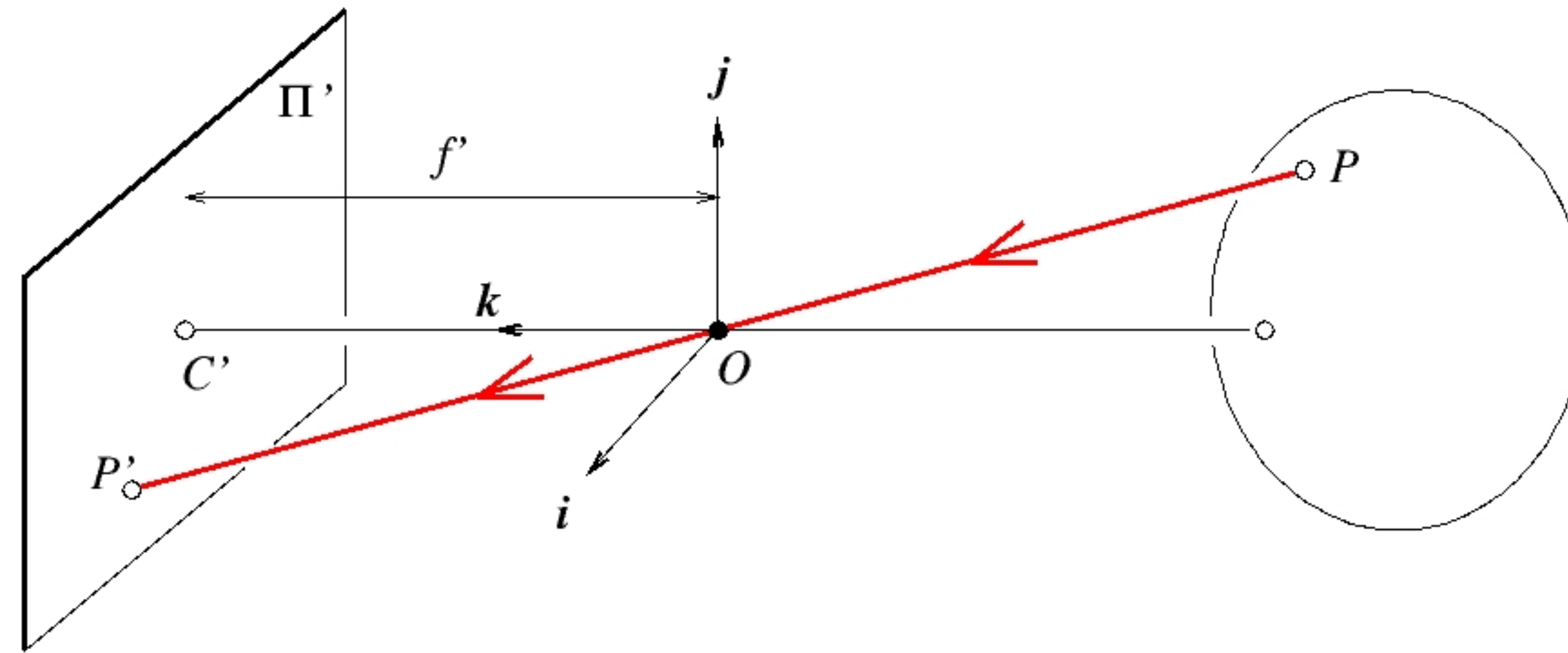
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Note: this assumes world coordinate frame at the optical center (pinhole) and aligned with the image plane, image coordinate frame aligned with the camera coordinate frame

Aside: Camera Matrix

Camera Matrix



$$\mathbf{C} = \begin{bmatrix} f' & 0 & 0 & 0 \\ 0 & f' & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

3D object point

Forsyth & Ponce (1st ed.) Figure 1.4

$$P = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \text{ projects to 2D image point } P' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \text{ where } \boxed{P' = \mathbf{C}P}$$

Note: this assumes world coordinate frame at the optical center (pinhole) and aligned with the image plane, image coordinate frame aligned with the camera coordinate frame

Aside: Camera Matrix

Camera Matrix

$$\begin{aligned}x' &= f' \frac{x}{z} \\ y' &= f' \frac{y}{z}\end{aligned}$$

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$$\begin{bmatrix} f' & 0 & 0 & 0 \\ 0 & f' & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} f'x \\ f'y \\ z \end{bmatrix} = \begin{bmatrix} \frac{f'x}{z} \\ \frac{f'y}{z} \\ 1 \end{bmatrix}$$

$$P = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \text{ projects to 2D image point } P' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \text{ where } \boxed{P' = \mathbf{C}P}$$

Aside: Camera Matrix

Camera Matrix

$$\mathbf{C} = \begin{bmatrix} f' & 0 & 0 & 0 \\ 0 & f' & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Pixels are squared / lens is perfectly symmetric

Sensor and pinhole perfectly aligned

Coordinate system centered at the pinhole

$$P = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \text{ projects to 2D image point } P' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \text{ where } P' = \mathbf{C}P$$

Aside: Camera Matrix

Camera Matrix

$$\mathbf{C} = \begin{bmatrix} f'_x & 0 & 0 & 0 \\ 0 & f'_y & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

~~Pixels are squared / lens is perfectly symmetric~~

Sensor and pinhole perfectly aligned

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$$P = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \text{ projects to 2D image point } P' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \text{ where } P' = \mathbf{C}P$$

Aside: Camera Matrix

Camera Matrix

$$\mathbf{C} = \begin{bmatrix} f'_x & 0 & 0 & c_x \\ 0 & f'_y & 0 & c_y \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

~~Pixels are squared / lens is perfectly symmetric~~

~~Sensor and pinhole perfectly aligned~~

Coordinate system centered at the pinhole

$$P = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \text{ projects to 2D image point } P' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \text{ where } P' = \mathbf{C}P$$

Aside: Camera Matrix

Camera Matrix

$$\mathbf{C} = \begin{bmatrix} f'_x & 0 & 0 & c_x \\ 0 & f'_y & 0 & c_y \\ 0 & 0 & 1 & 0 \end{bmatrix} \mathbb{R}_{4 \times 4}$$

~~Pixels are squared / lens is perfectly symmetric~~

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$$P = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \text{ projects to 2D image point } P' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \text{ where } P' = \mathbf{C}P$$

Aside: Camera Matrix

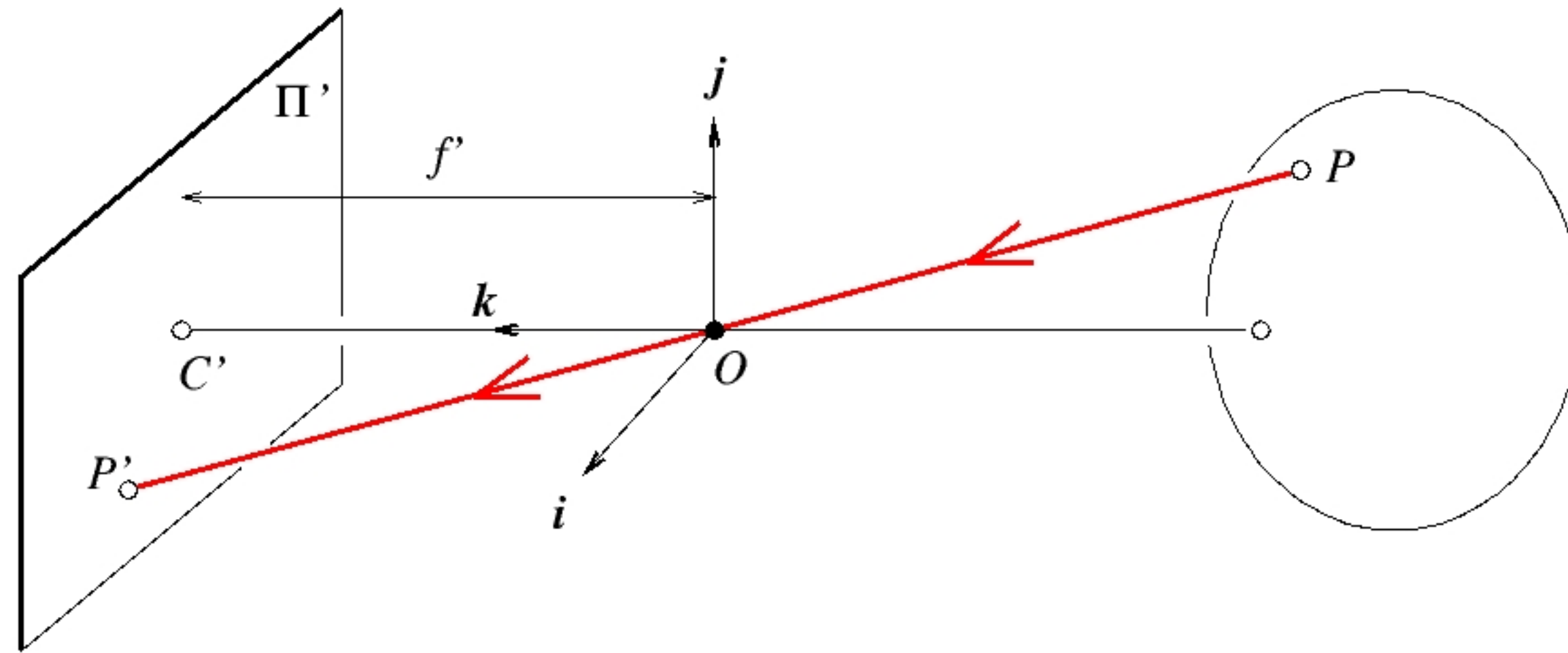
Camera Matrix

$$\mathbf{C} = \begin{bmatrix} f'_x & 0 & 0 & c_x \\ 0 & f'_y & 0 & c_y \\ 0 & 0 & 1 & 0 \end{bmatrix} \mathbb{R}_{4 \times 4}$$

Camera calibration is the process of estimating parameters of the camera matrix based on set of 3D-2D correspondences (usually requires a pattern whose structure and size is known)

$$P = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \text{ projects to 2D image point } P' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \text{ where } P' = \mathbf{C}P$$

Perspective Projection



3D object point

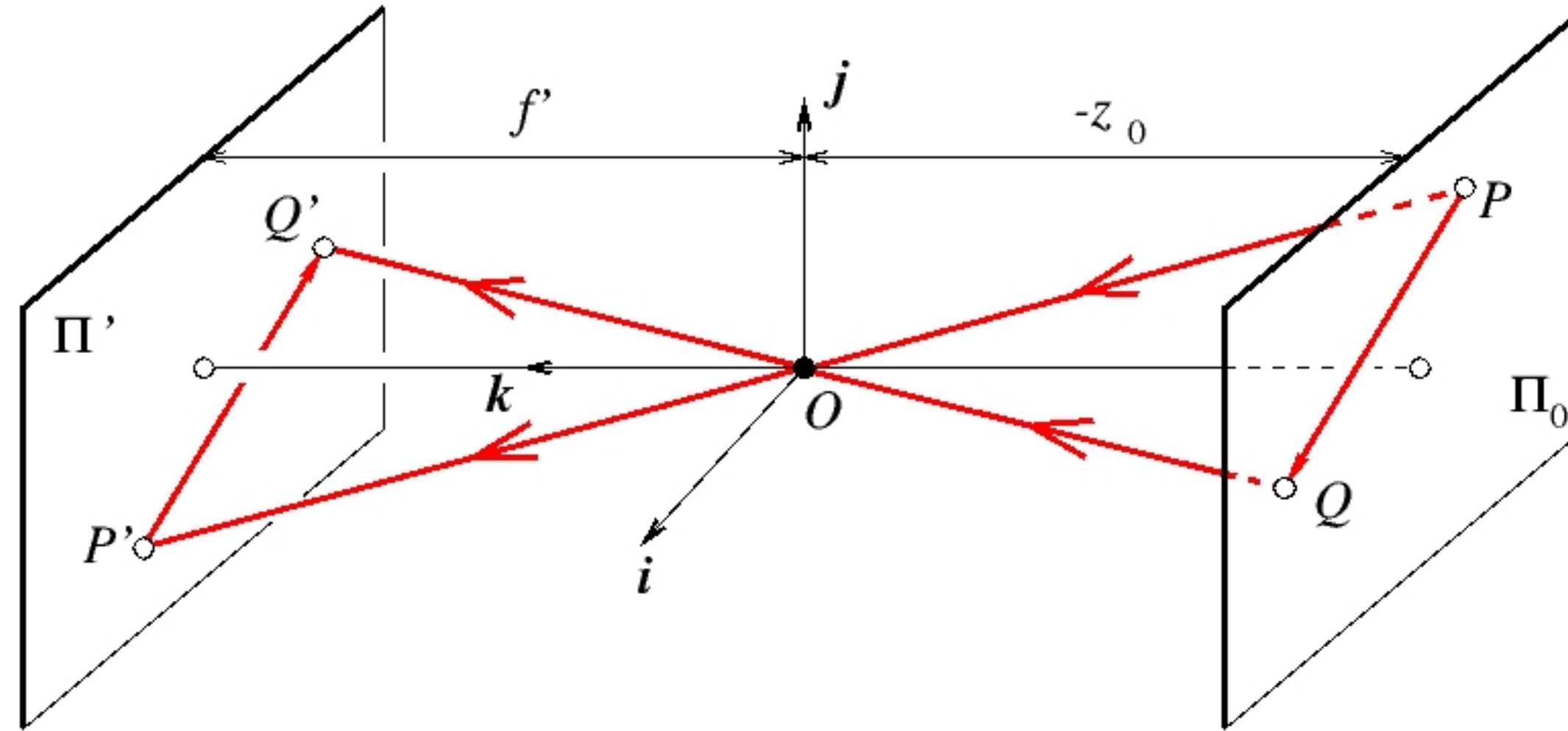
Forsyth & Ponce (1st ed.) Figure 1.4

$P = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ projects to 2D image point $P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$ where

$$\begin{aligned} x' &= f' \frac{x}{z} \\ y' &= f' \frac{y}{z} \end{aligned}$$

Note: this assumes world coordinate frame at the optical center (pinhole) and aligned with the image plane, image coordinate frame aligned with the camera coordinate frame

Weak Perspective

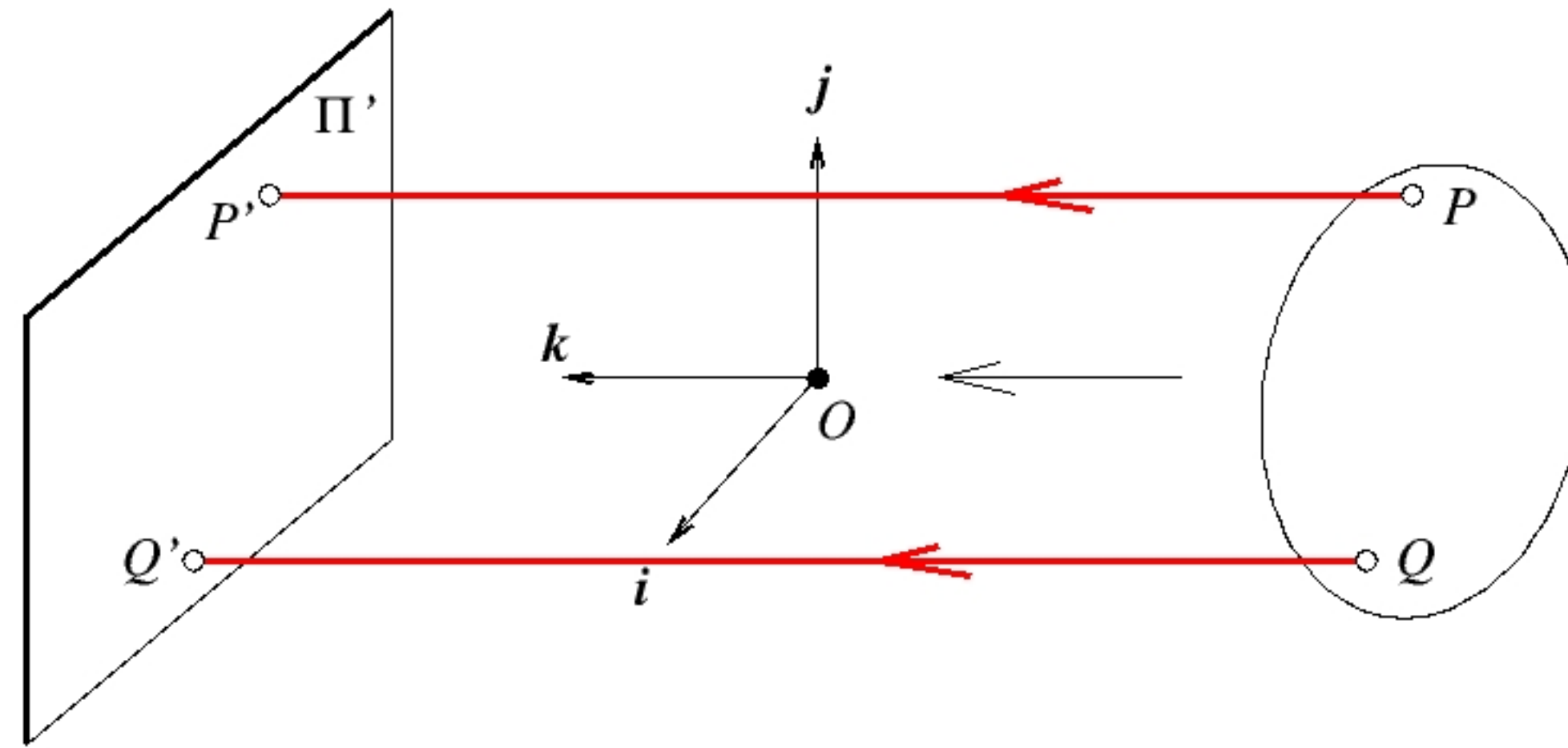


Forsyth & Ponce (1st ed.) Figure 1.5

3D object point $P = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ in Π_0 projects to 2D image point $P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$

where $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} mx \\ my \end{bmatrix}$ and $m = \frac{f'}{z_0}$

Orthographic Projection



Forsyth & Ponce (1st ed.) Figure 1.6

3D object point $P = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ projects to 2D image point $P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$

where

$$\begin{array}{rcl} x' & = & x \\ y' & = & y \end{array}$$

Summary of **Projection Equations**

3D object point $P = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ projects to 2D image point $P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$ where

Perspective

$$\begin{aligned} x' &= f' \frac{x}{z} \\ y' &= f' \frac{y}{z} \end{aligned}$$

Weak Perspective

$$\begin{aligned} x' &= m x \\ y' &= m y \end{aligned} \quad m = \frac{f'}{z_0}$$

Orthographic

$$\begin{aligned} x' &= x \\ y' &= y \end{aligned}$$

Projection Models: Pros and Cons

Weak perspective (including orthographic) has simpler mathematics

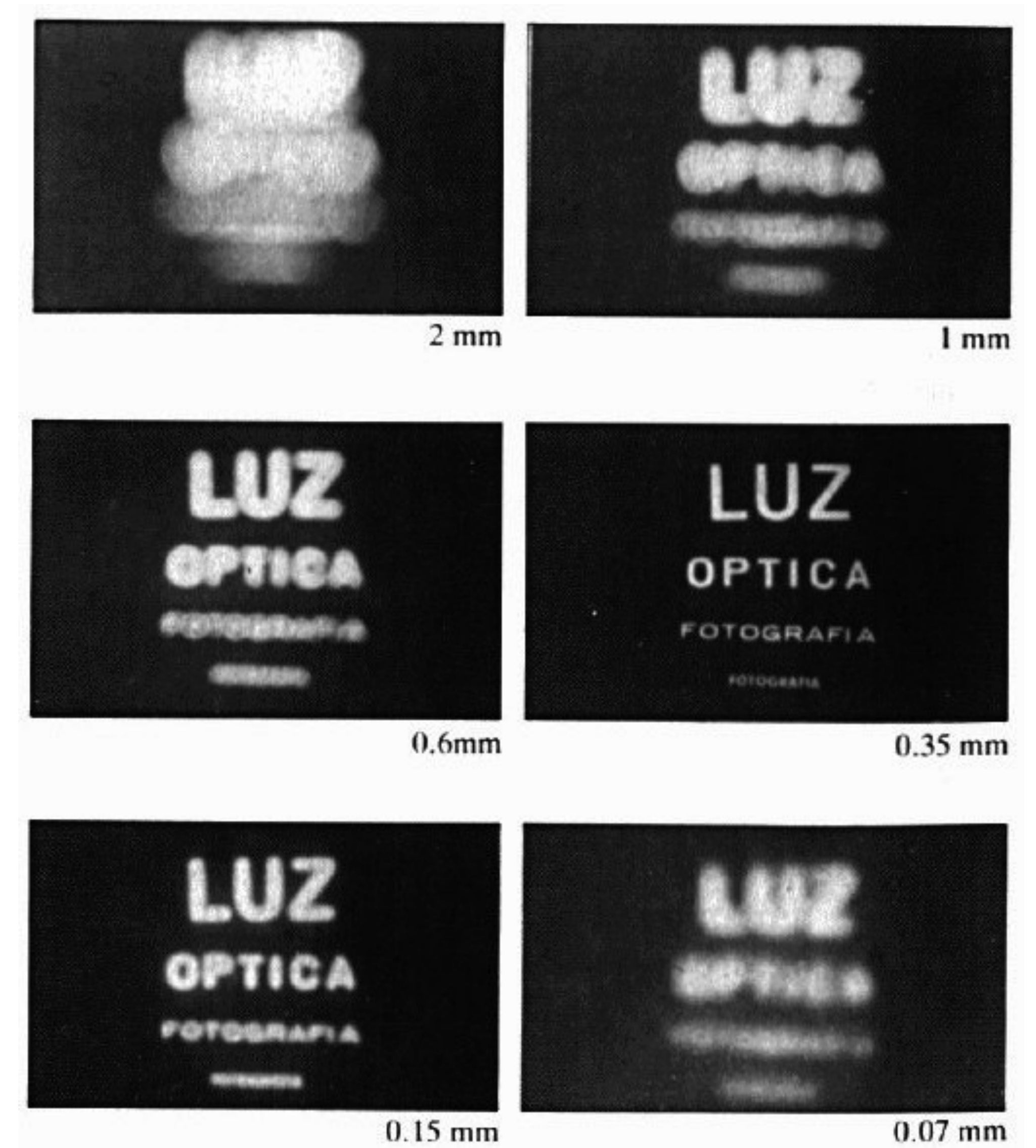
- accurate when object is small and/or distant
- useful for recognition

Perspective is more accurate for real scenes

When **maximum accuracy** is required, it is necessary to model additional details of a particular camera

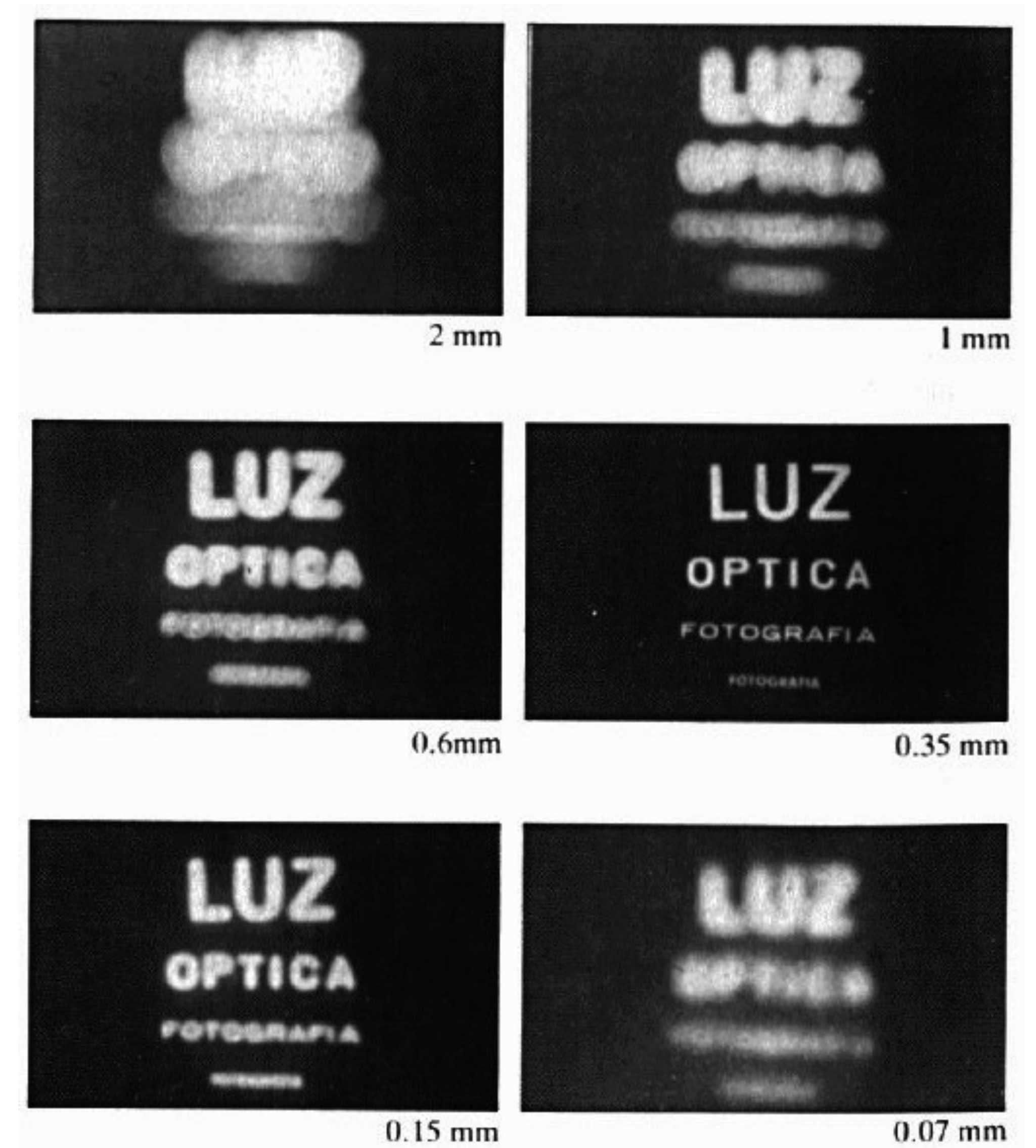
- use perspective projection with additional parameters (e.g., lens distortion)

Why **Not** a Pinhole Camera?



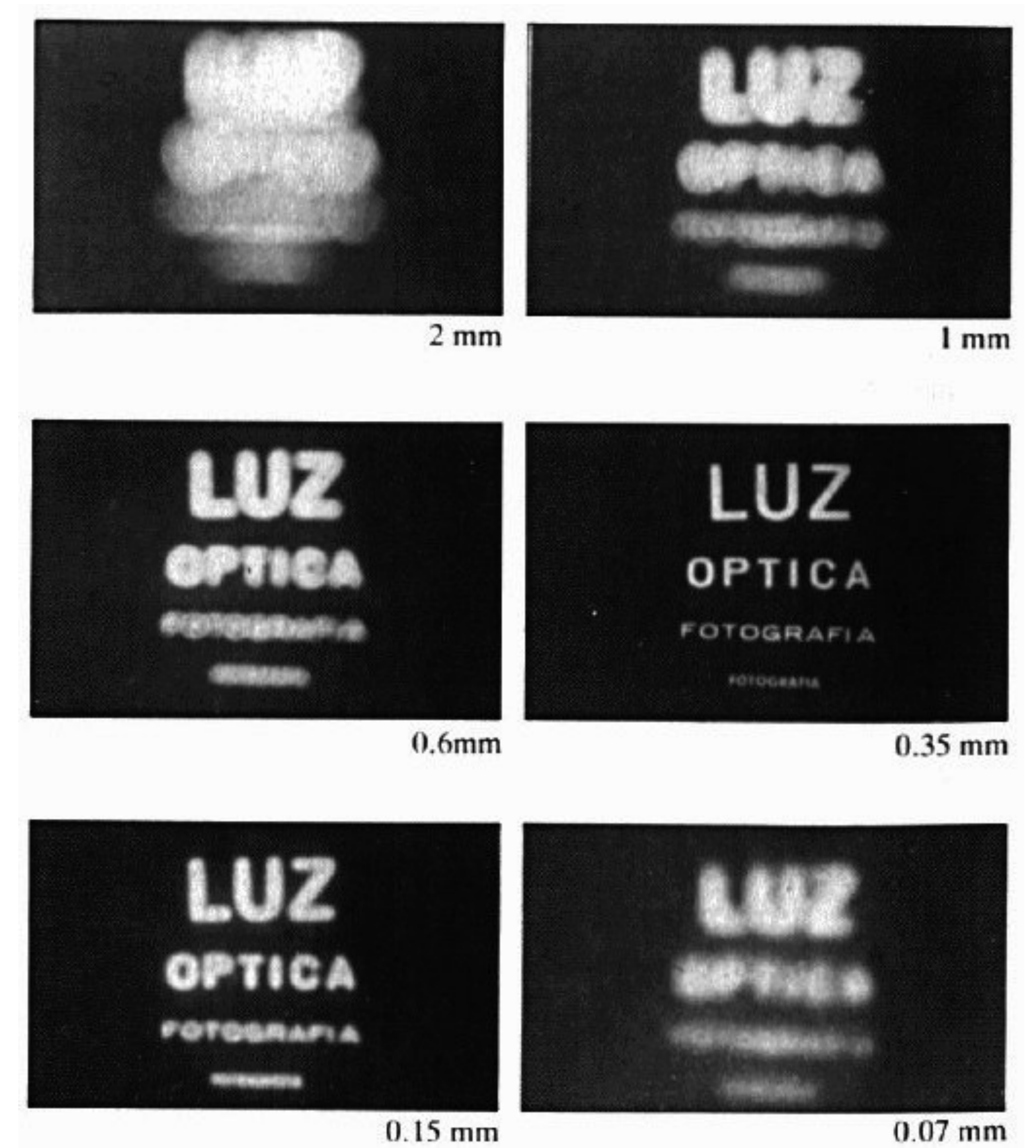
Why **Not** a Pinhole Camera?

- If pinhole is **too big** then many directions are averaged, blurring the image



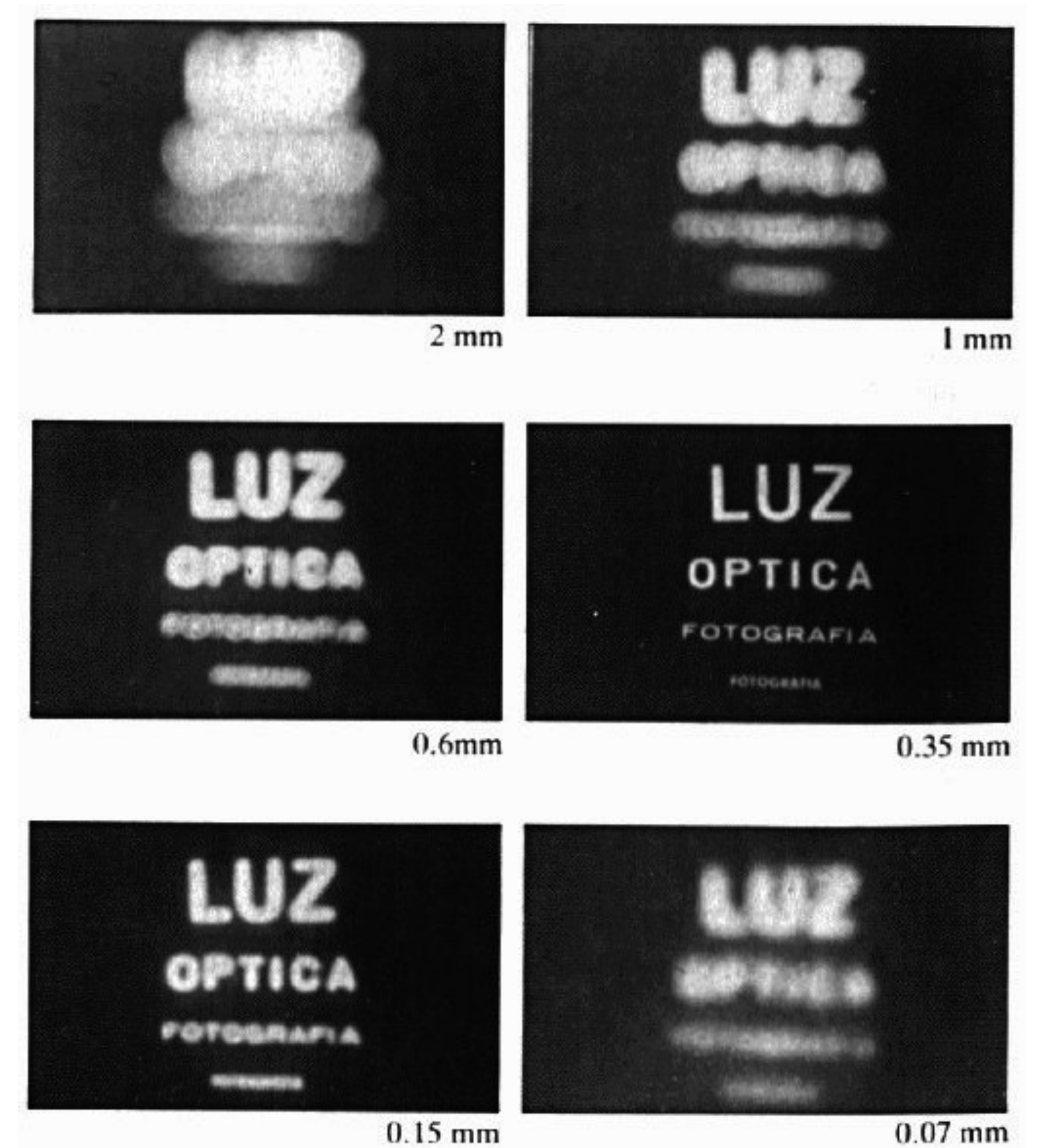
Why **Not** a Pinhole Camera?

- If pinhole is **too big** then many directions are averaged, blurring the image
- If pinhole is **too small** then diffraction becomes a factor, also blurring the image



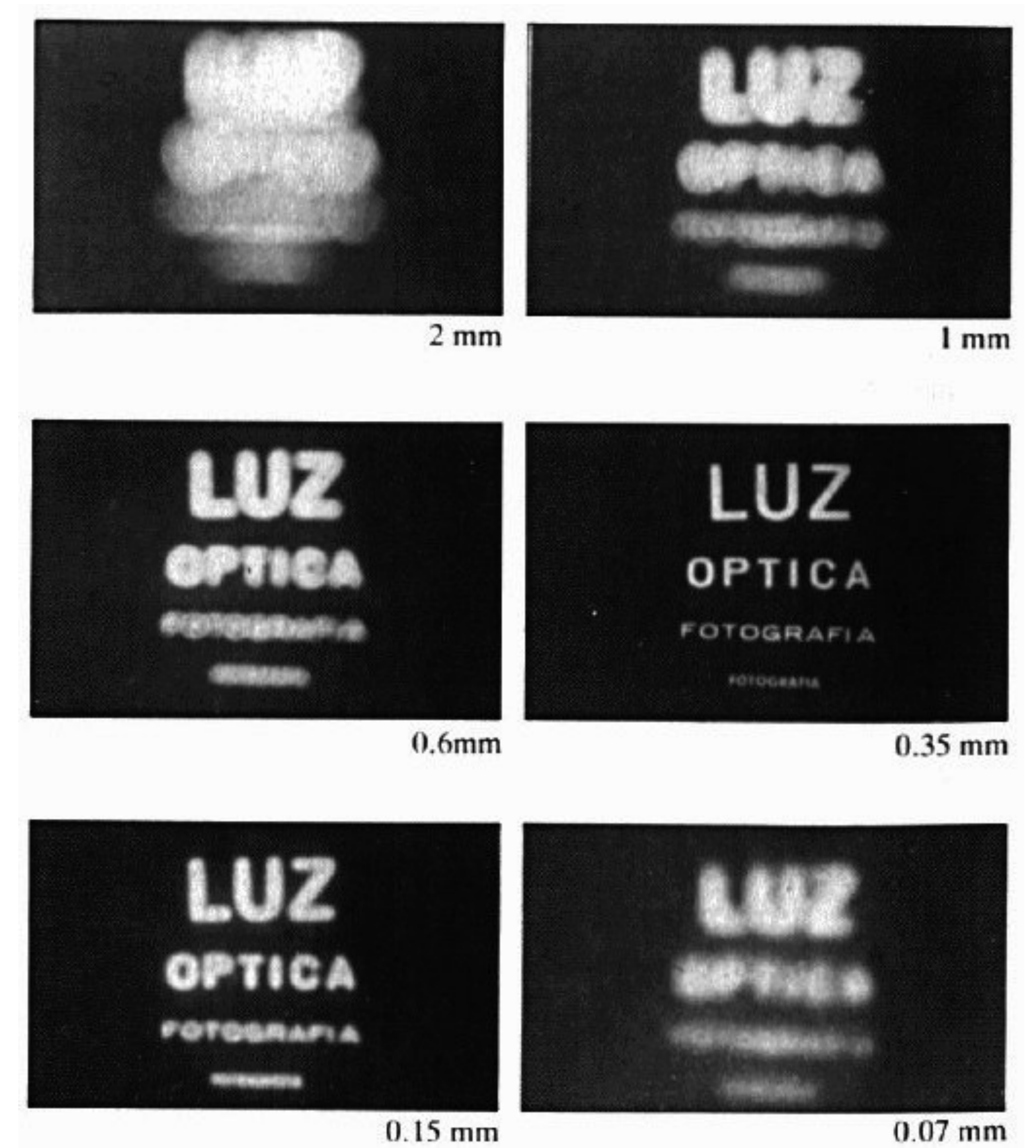
Why **Not** a Pinhole Camera?

- If pinhole is **too big** then many directions are averaged, blurring the image
- If pinhole is **too small** then diffraction becomes a factor, also blurring the image
- Generally, pinhole cameras are **dark**, because only a very small set of rays from a particular scene point hits the image plane



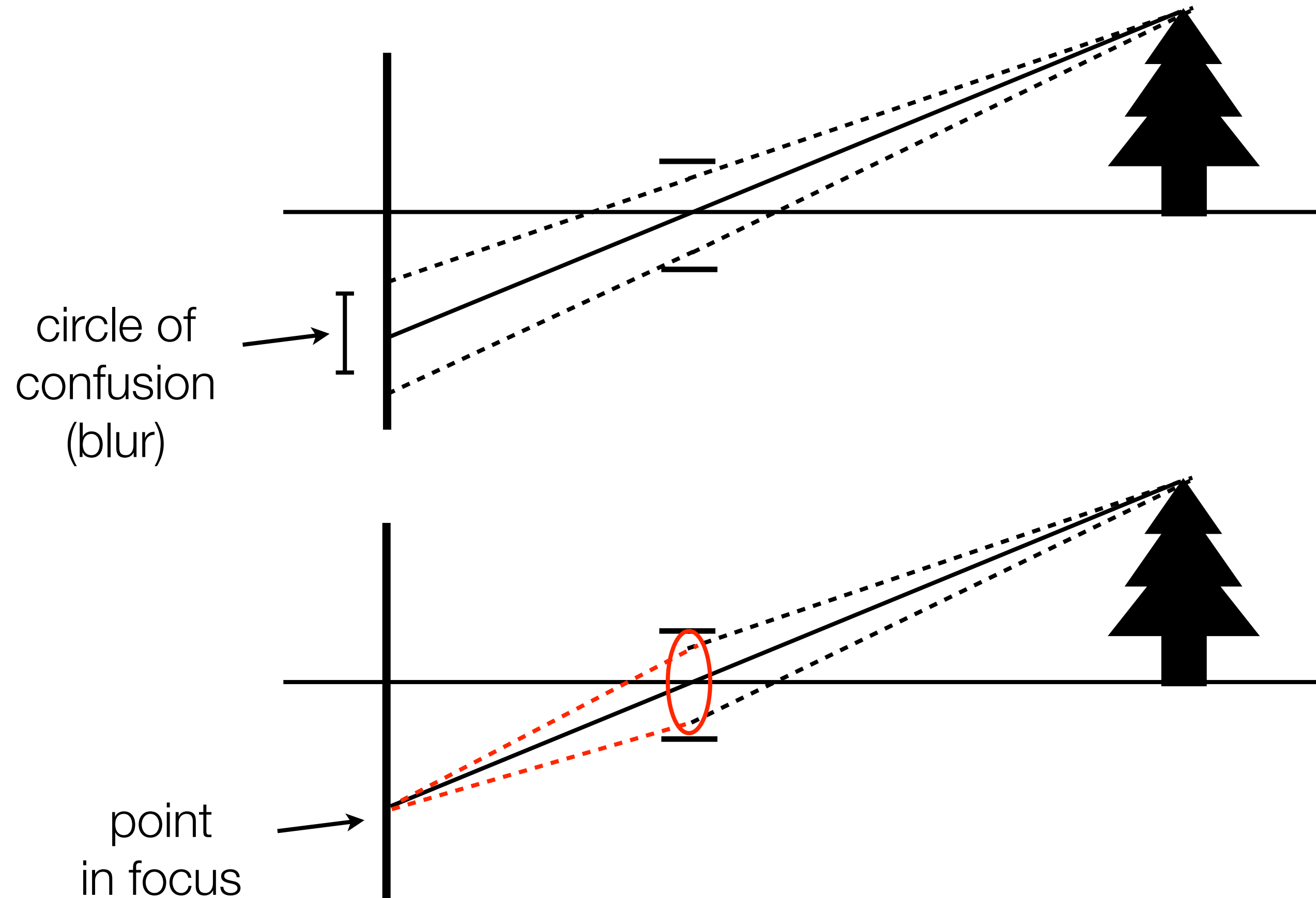
Why **Not** a Pinhole Camera?

- If pinhole is **too big** then many directions are averaged, blurring the image
- If pinhole is **too small** then diffraction becomes a factor, also blurring the image
- Generally, pinhole cameras are **dark**, because only a very small set of rays from a particular scene point hits the image plane
- Pinhole cameras are **slow**, because only a very small amount of light from a particular scene point hits the image plane per unit time



Reason for **Lenses**

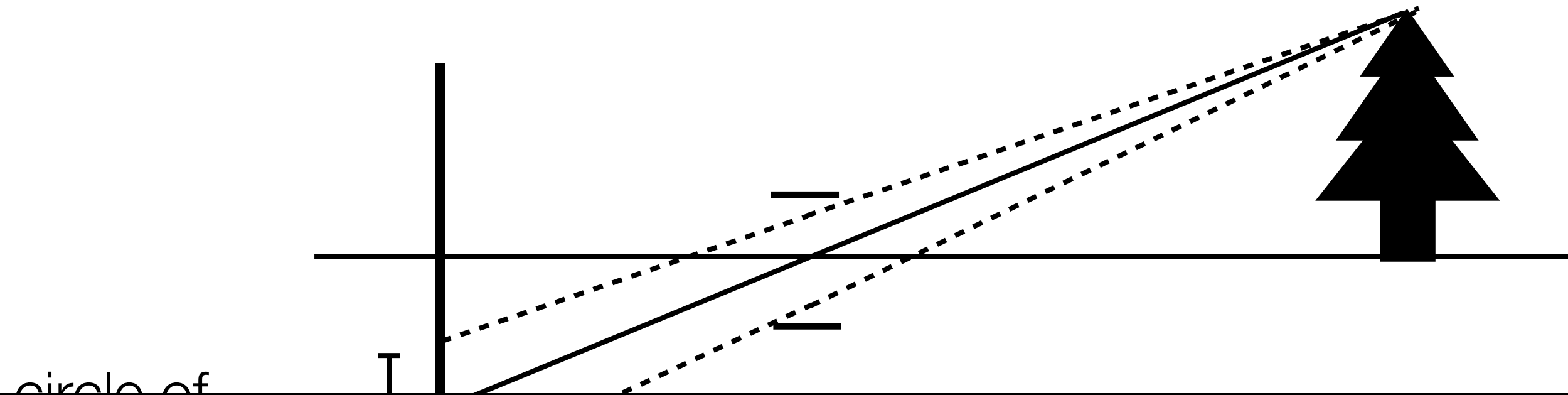
A real camera must have a finite aperture to get enough light, but this causes blur in the image



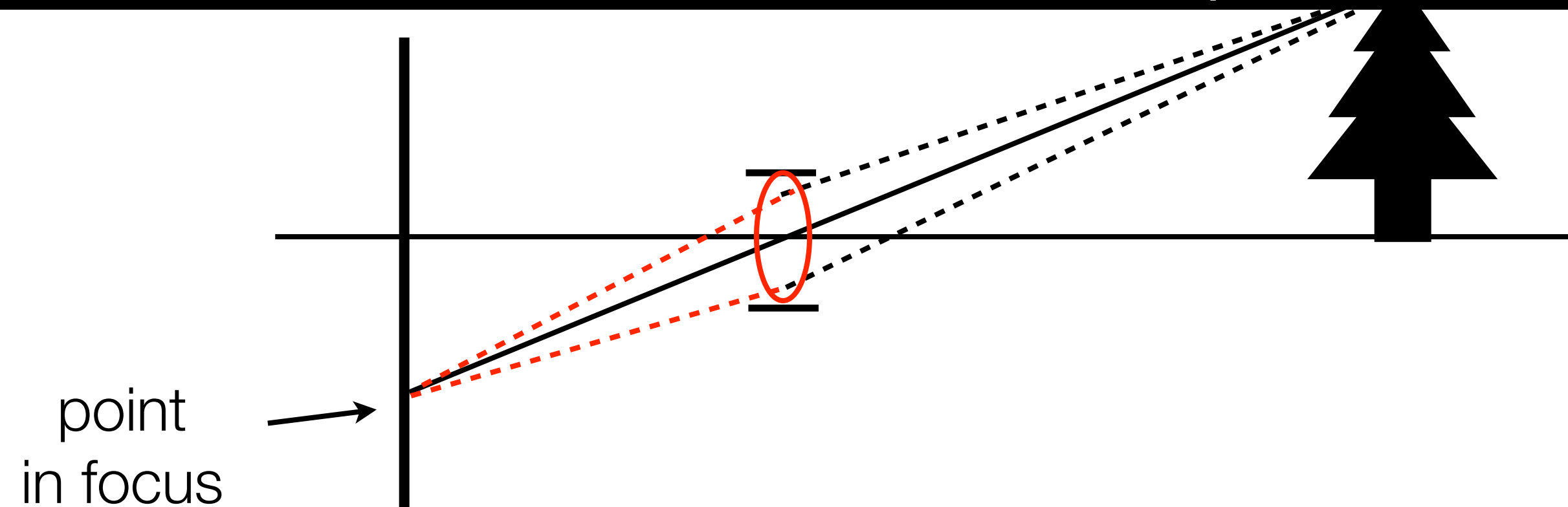
Solution: use a **lens** to focus light onto the image plane

Reason for **Lenses**

A real camera must have a finite aperture to get enough light, but this causes blur in the image

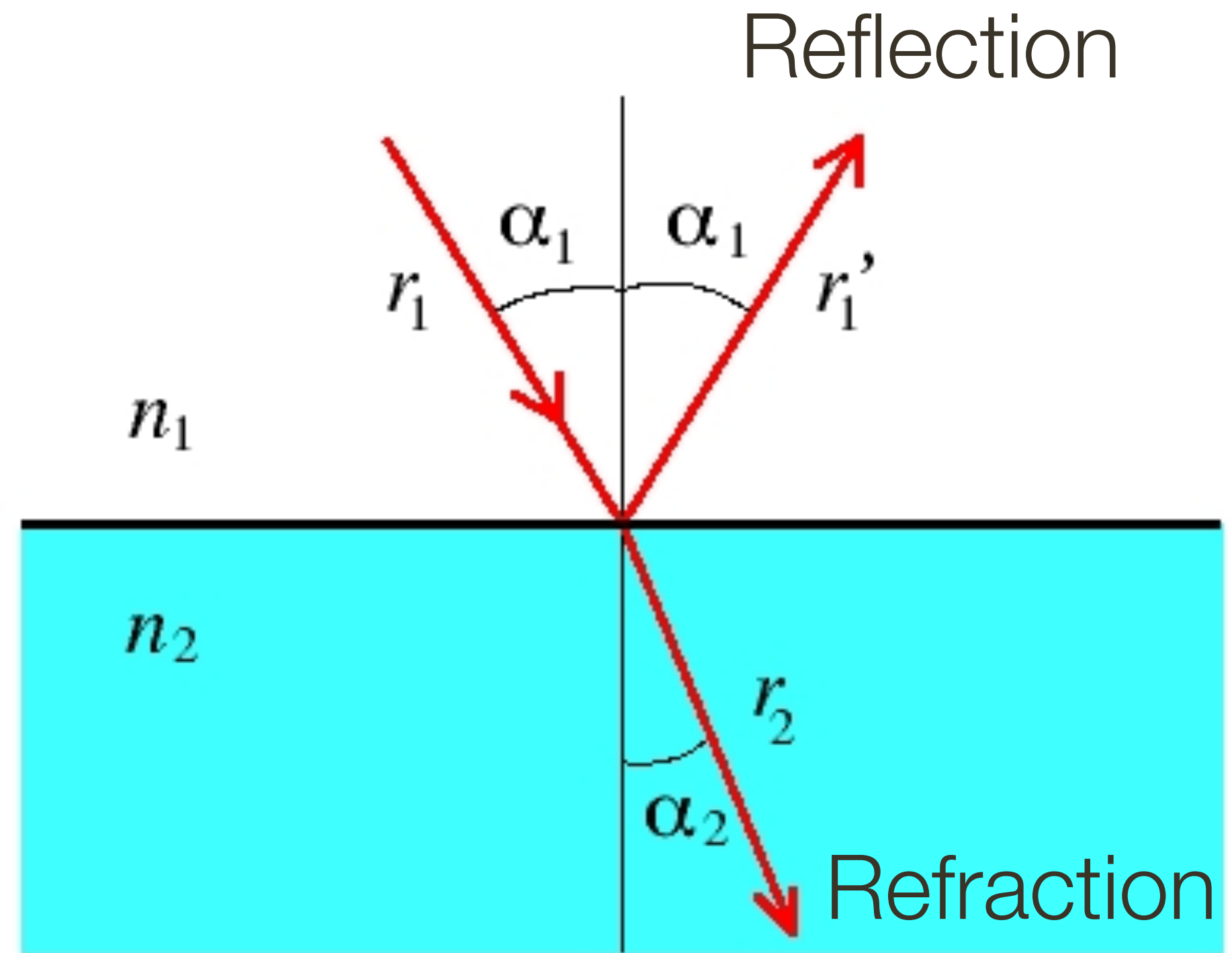


The role of a lens is to **capture more light** while preserving, as much as possible, the abstraction of an ideal pinhole camera.



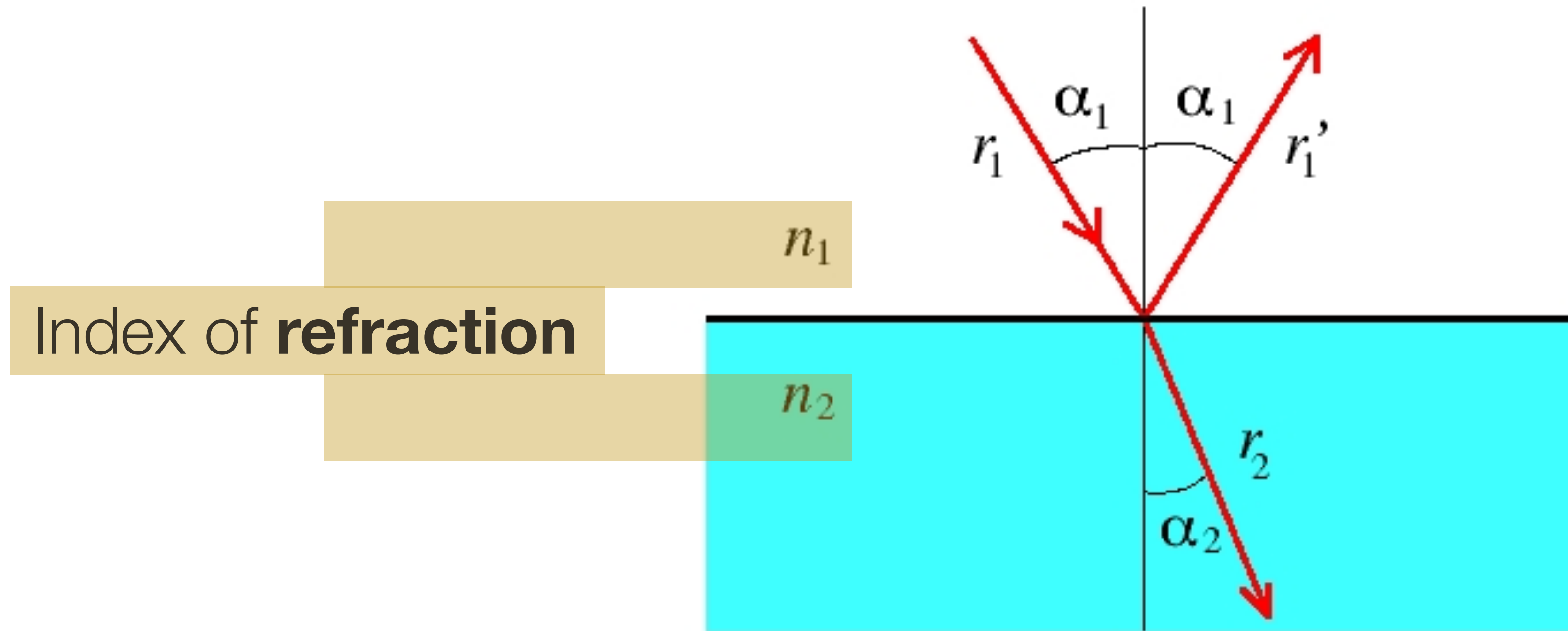
Solution: use a **lens** to focus light onto the image plane

Snell's Law



$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$

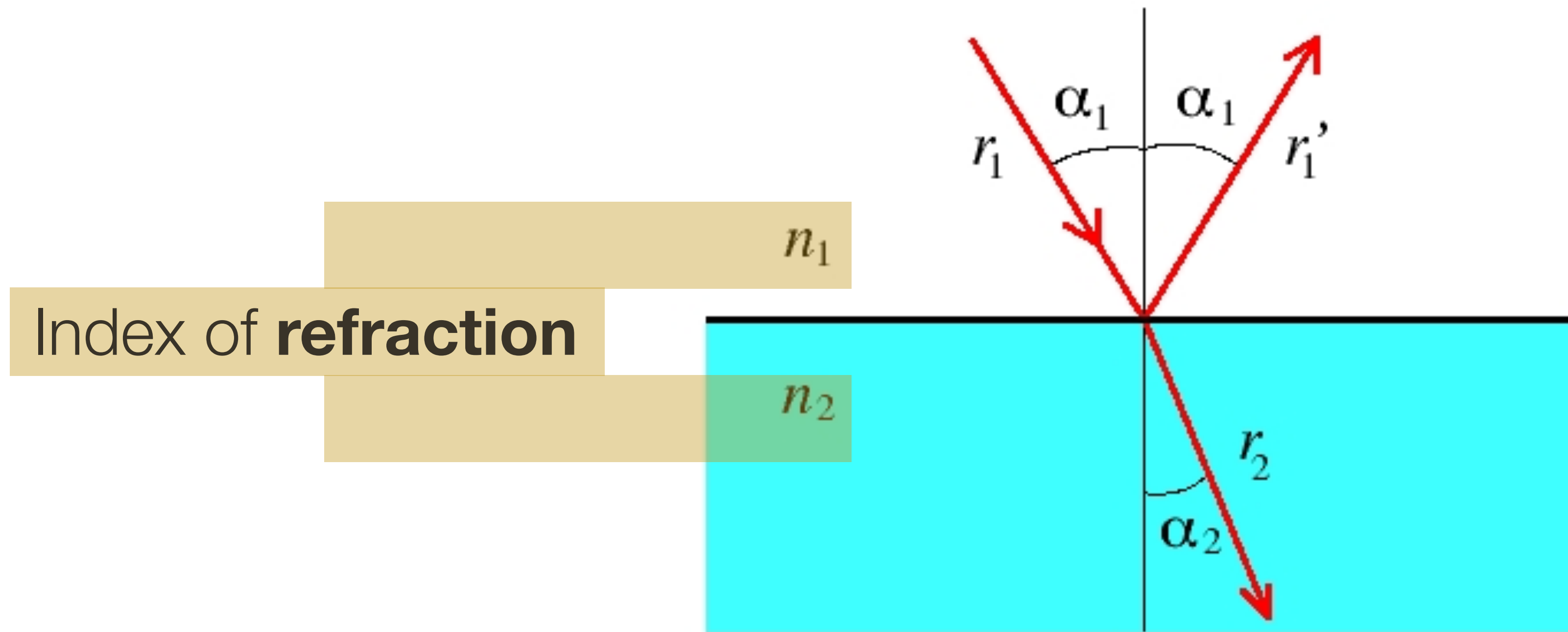
Snell's Law



$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$

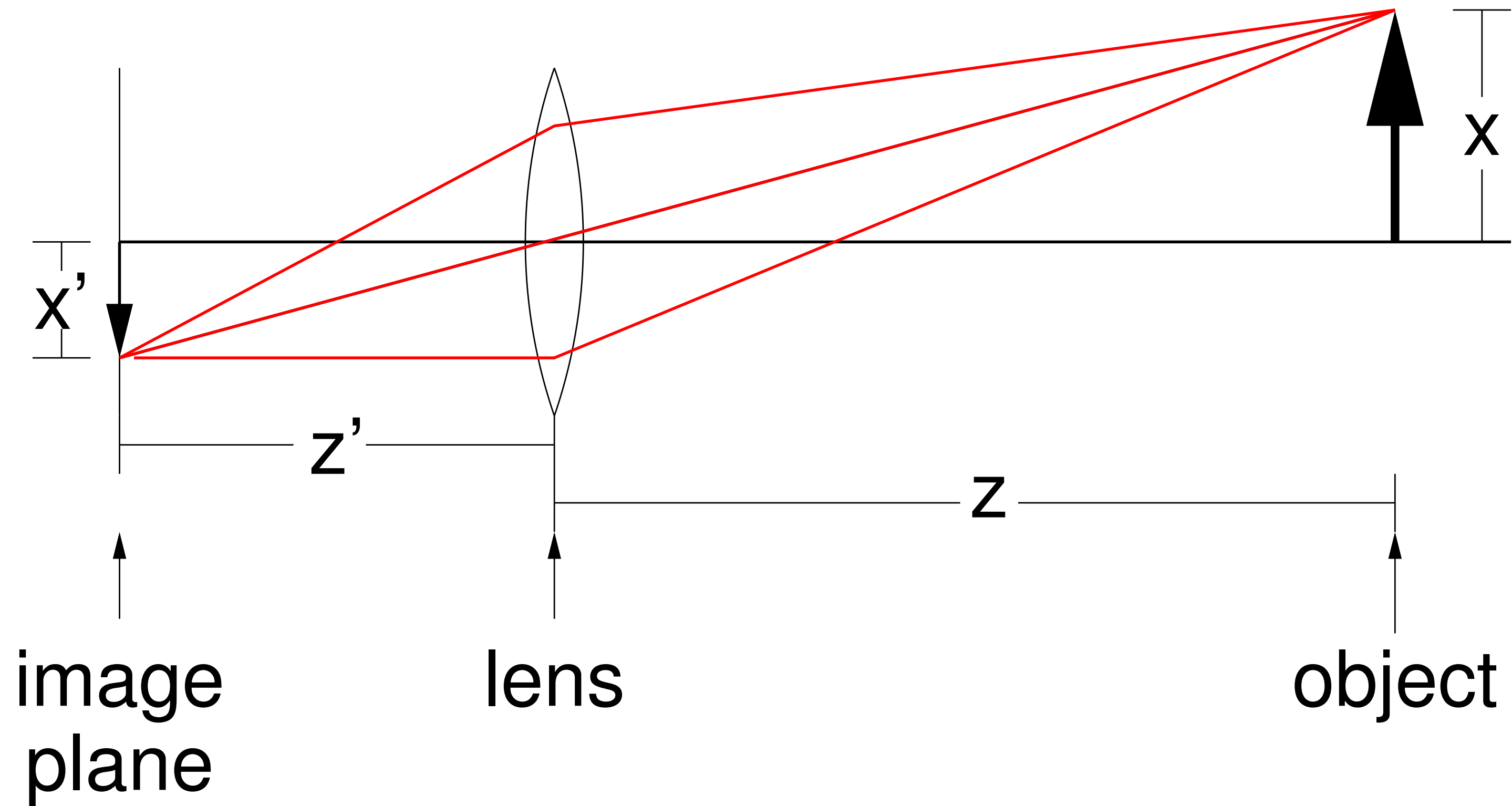
Snell's Law

Exercise: Would it make sense to make the lens from material whose index of refraction equals to air? Why?

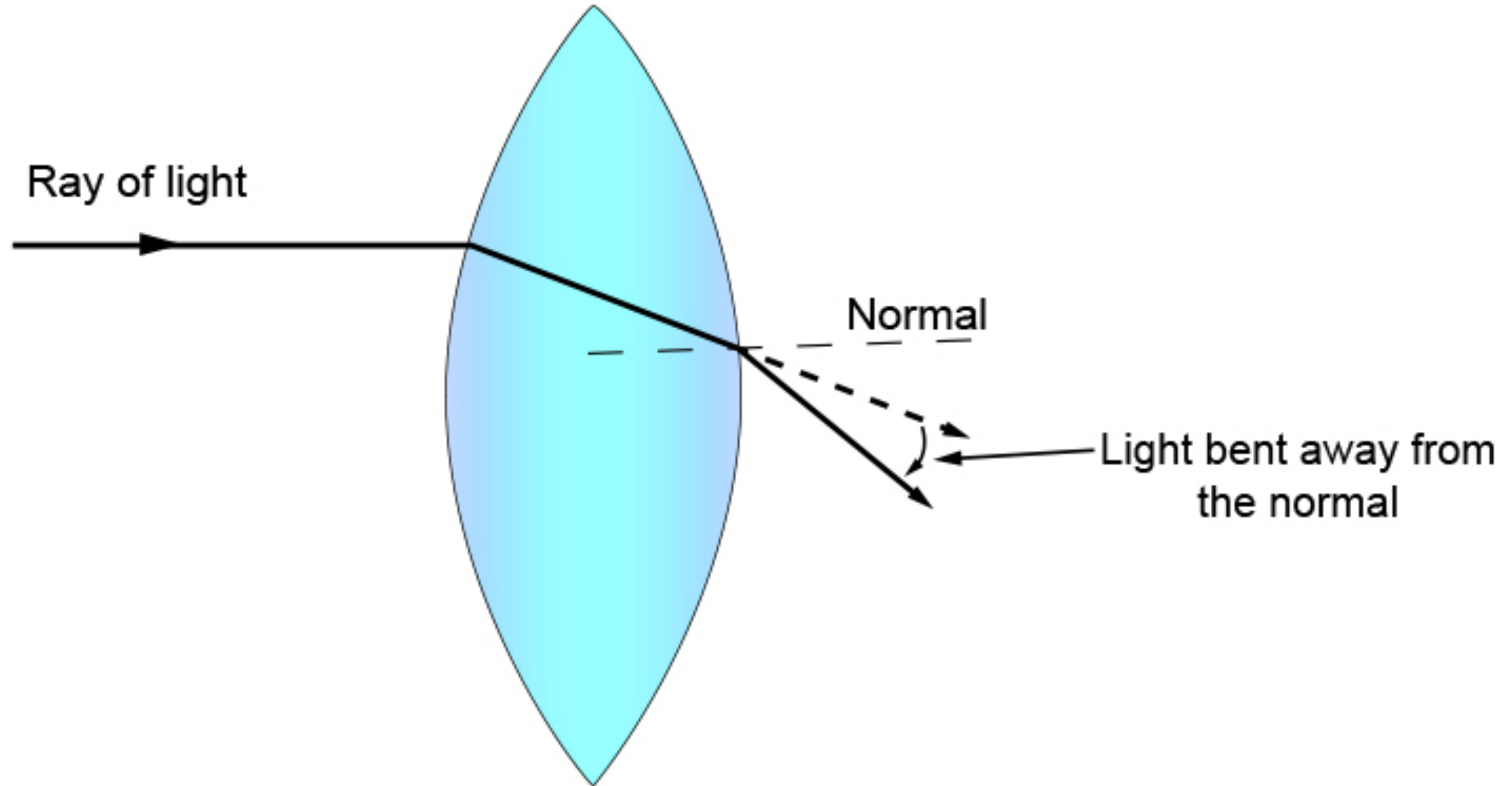


$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$

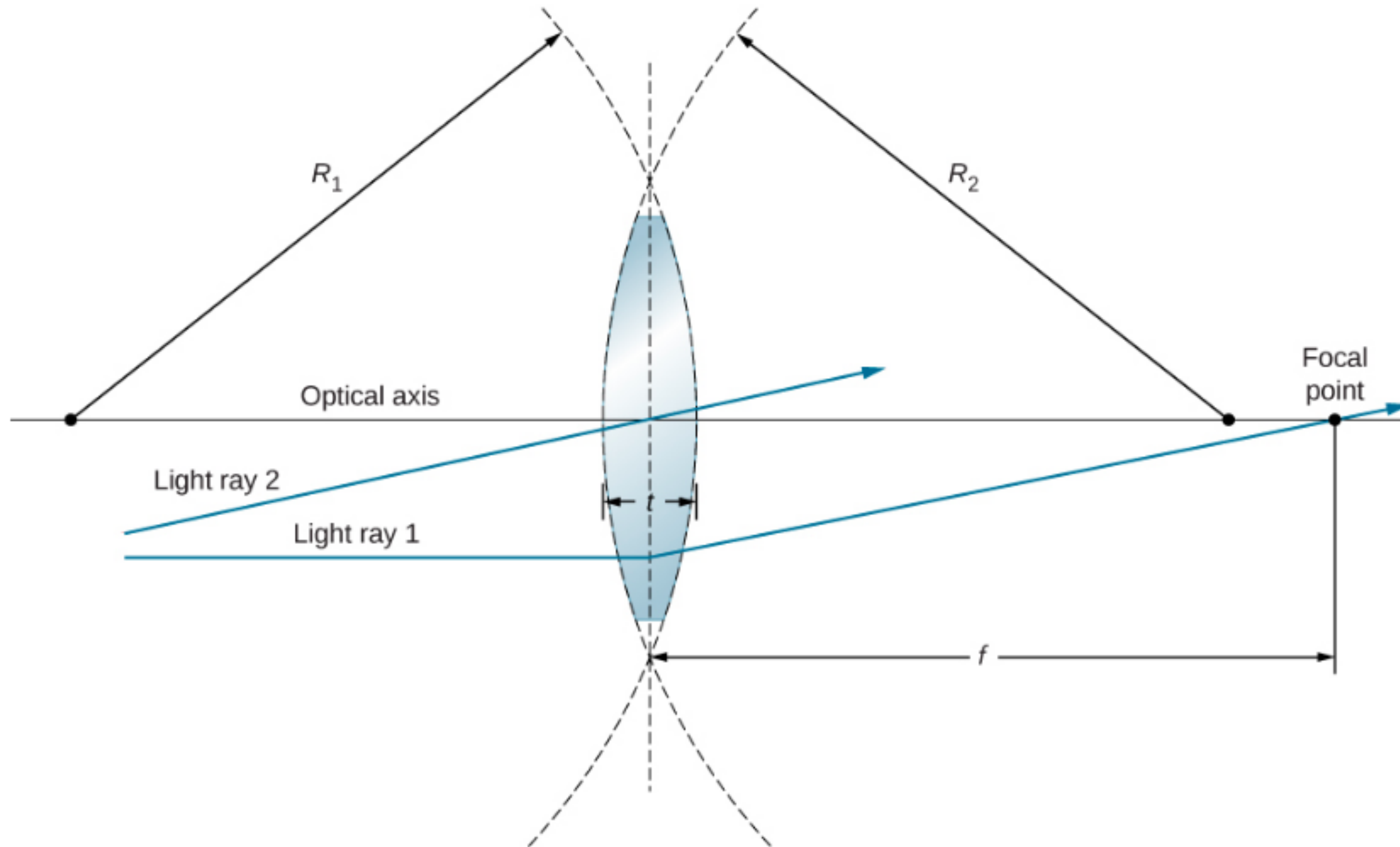
Pinhole Model **with Lens**



General Lens

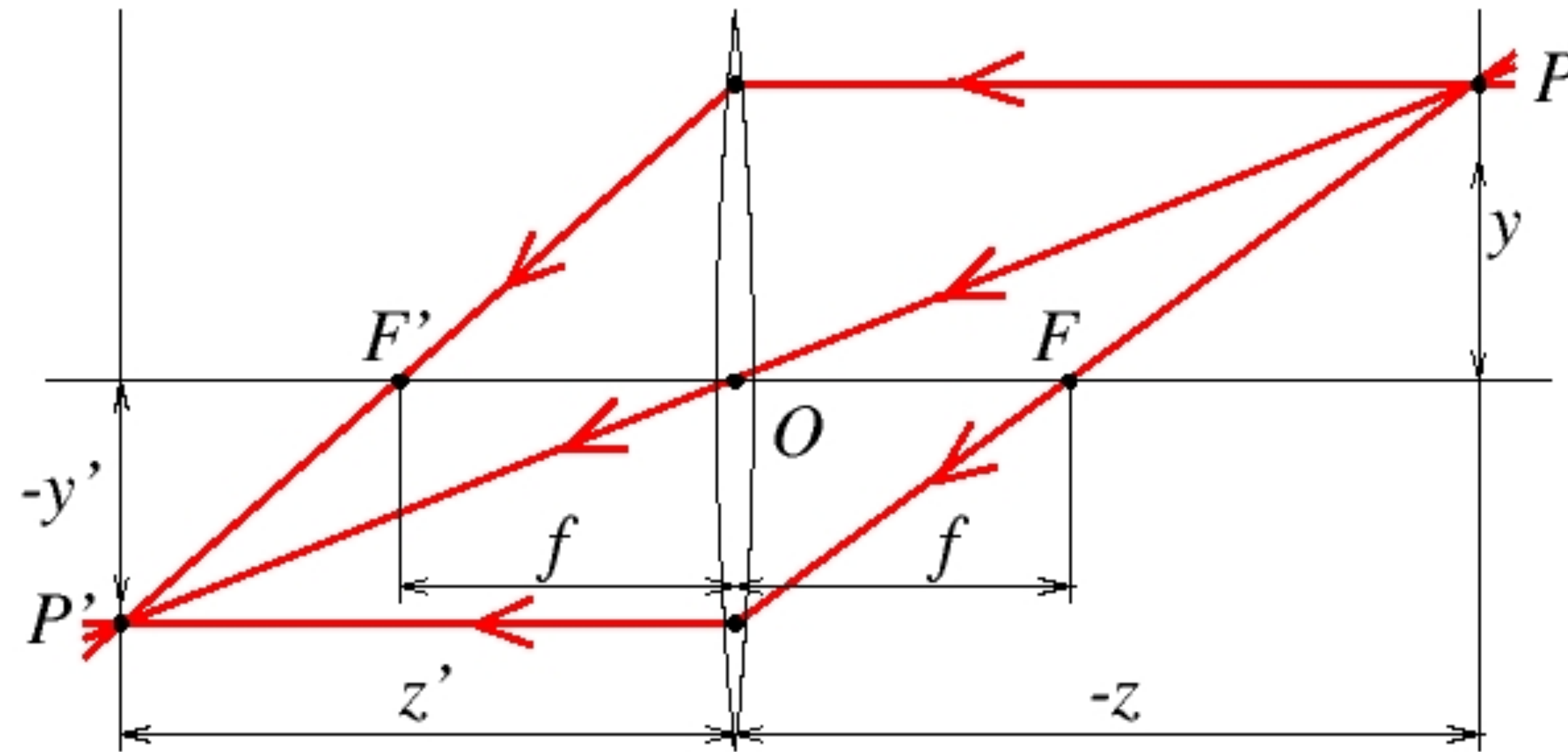


Thin Lens



[https://phys.libretexts.org/Bookshelves/University_Physics/Book%3A_University_Physics_\(OpenStax\)/Map%3A_University_Physics_III_-_Optics_and_Modern_Physics_\(OpenStax\)/02%3A_Geometric_Optics_and_Image_Formation/2.05%3A_Thin_Lenses](https://phys.libretexts.org/Bookshelves/University_Physics/Book%3A_University_Physics_(OpenStax)/Map%3A_University_Physics_III_-_Optics_and_Modern_Physics_(OpenStax)/02%3A_Geometric_Optics_and_Image_Formation/2.05%3A_Thin_Lenses)

Thin Lens Equation

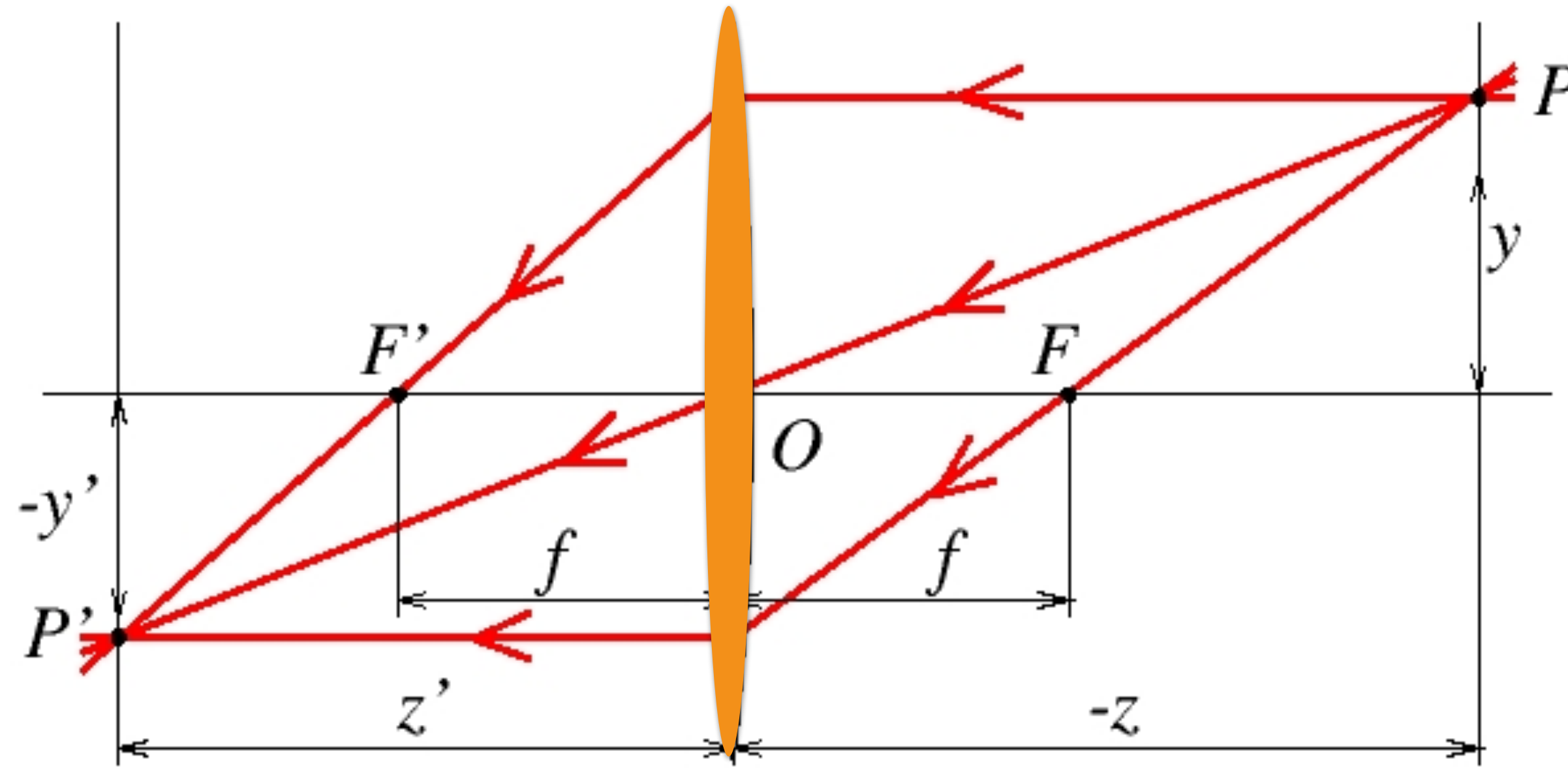


Forsyth & Ponce (1st ed.) Figure 1.9

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

Thin Lens Equation

Focal Length: Property of the lens (geometry and refraction index)

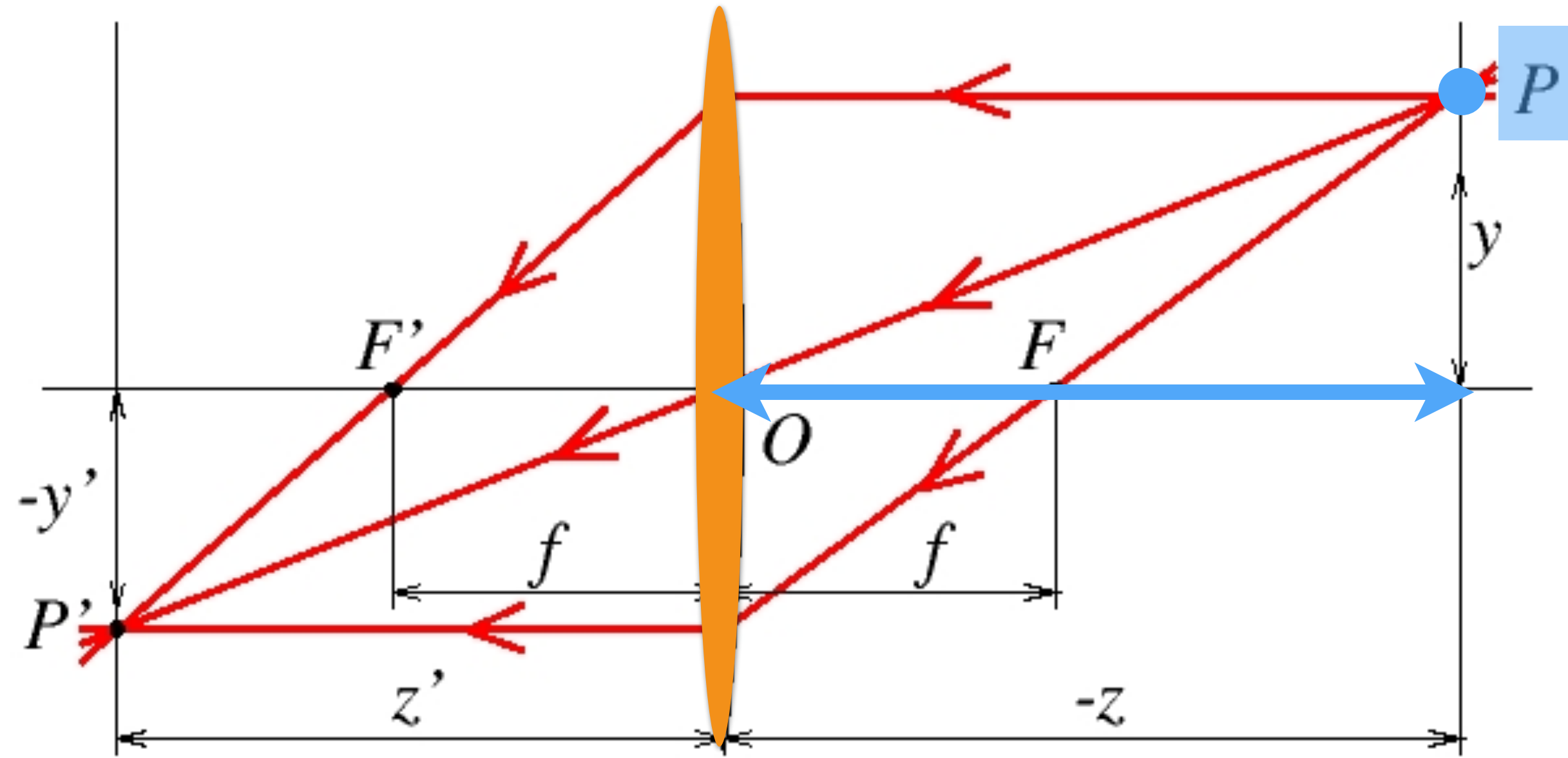


Forsyth & Ponce (1st ed.) Figure 1.9

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

Thin Lens Equation

Focal Length: Property of the lens (geometry and refraction index)



Depth of the point (P) in the world

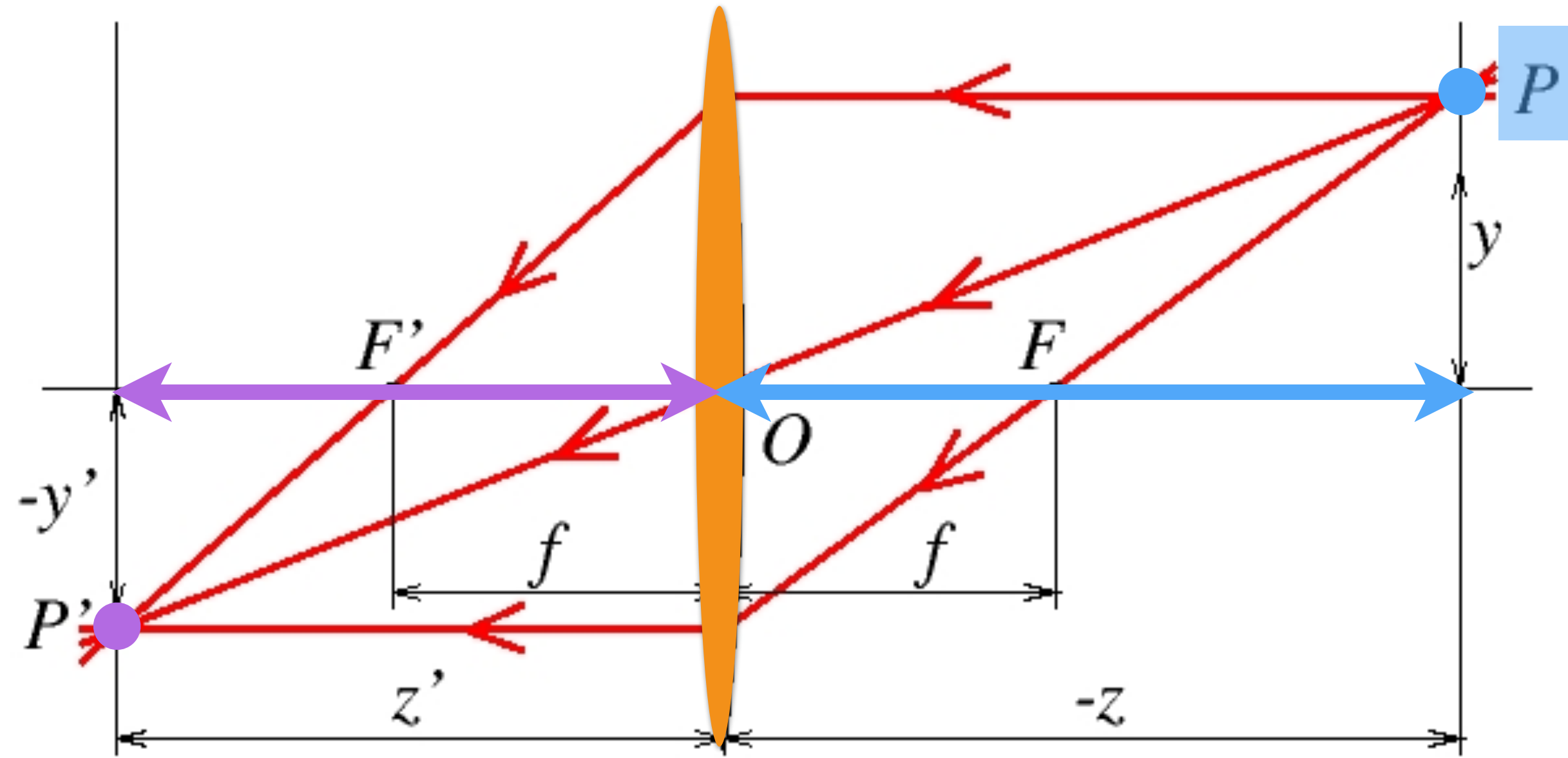
Forsyth & Ponce (1st ed.) Figure 1.9

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

Thin Lens Equation

Focal Length: Property of the lens (geometry and refraction index)

Location of the imaging plane where the projection of this point (P) will be in focus

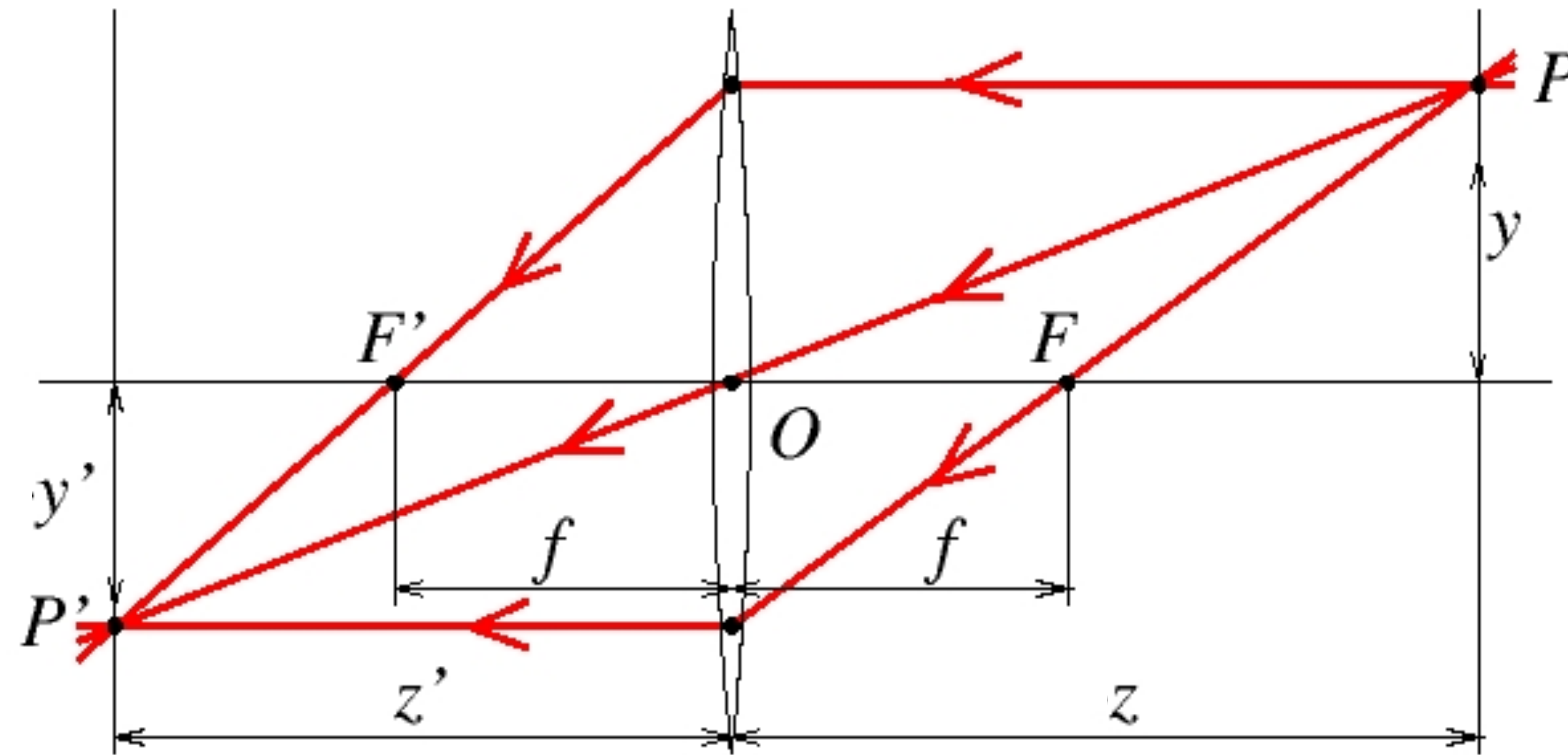


Depth of the point (P) in the world

Forsyth & Ponce (1st ed.) Figure 1.9

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

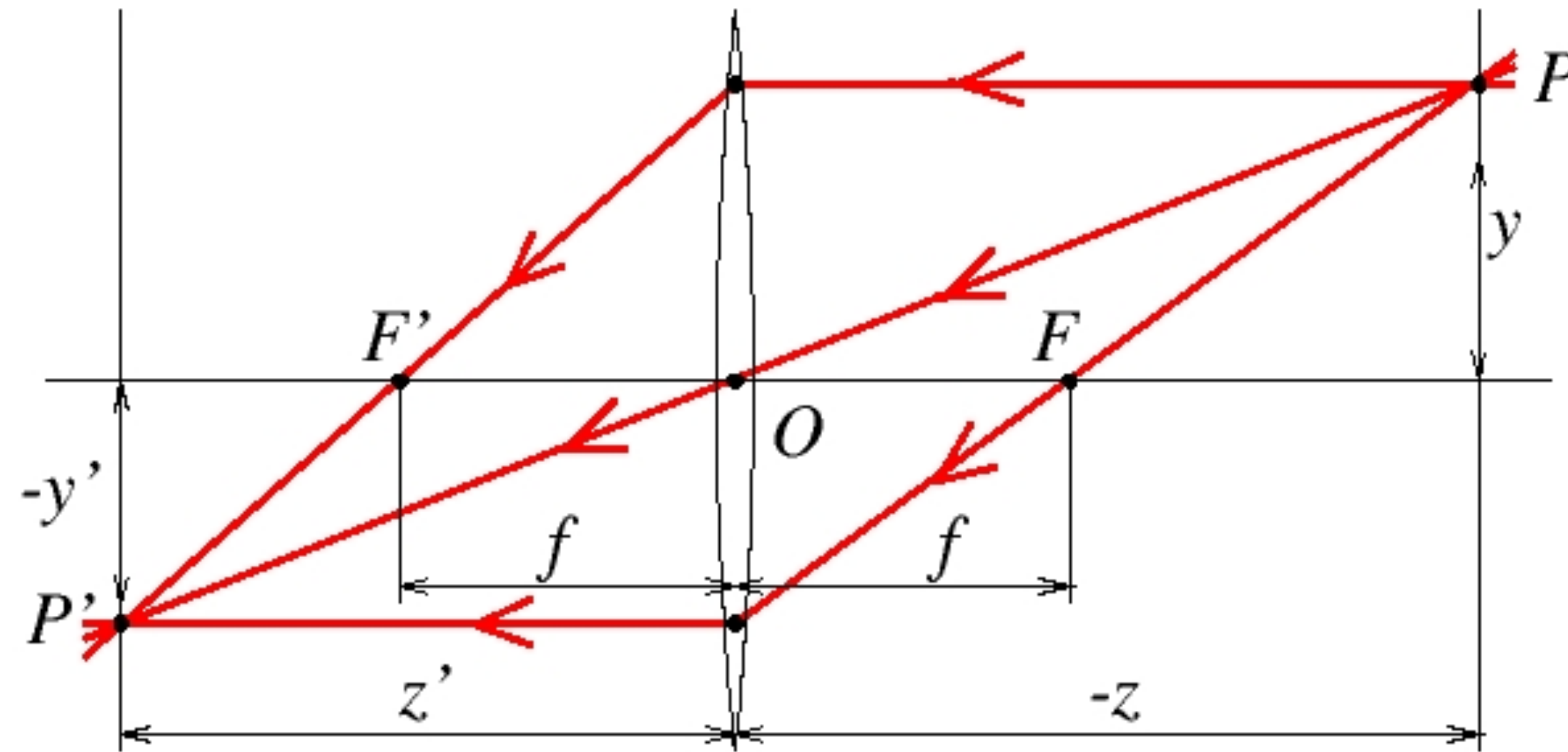
Thin Lens Equation



Forsyth & Ponce (1st ed.) Figure 1.9

$$\frac{1}{z'} + \frac{1}{z} = \frac{1}{f}$$

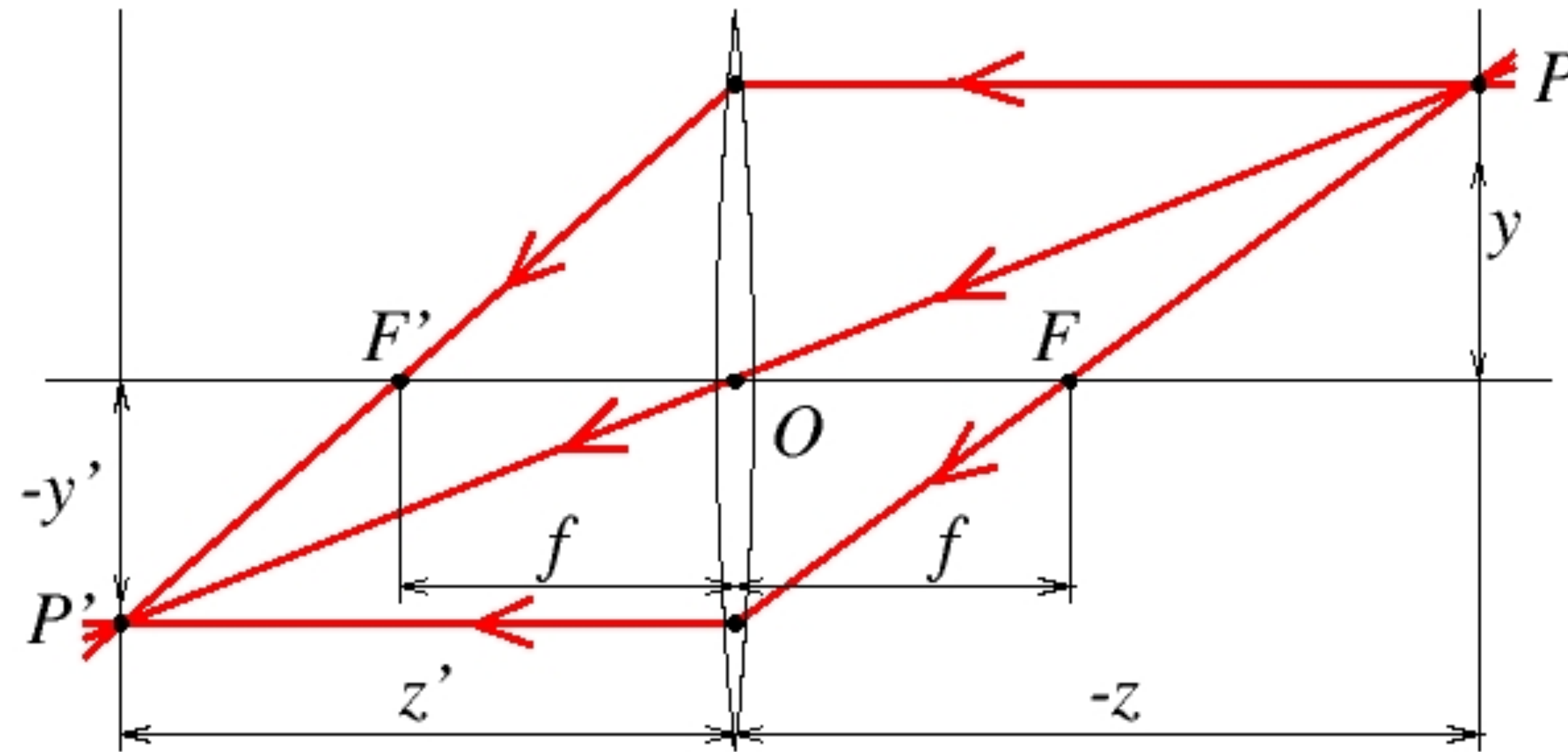
Thin Lens Equation



Forsyth & Ponce (1st ed.) Figure 1.9

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

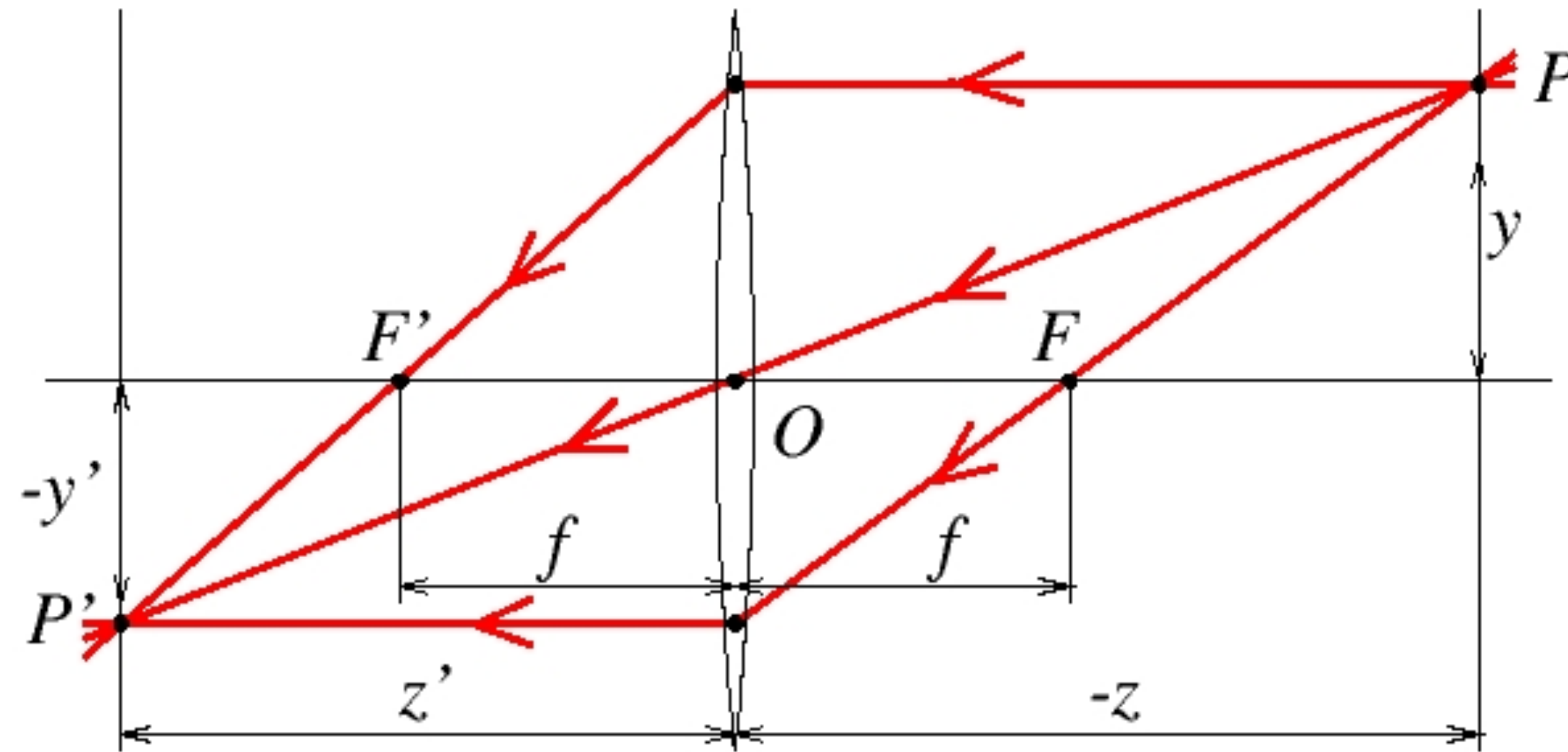
Possible Uses of Thin Lens Abstraction



Forsyth & Ponce (1st ed.) Figure 1.9

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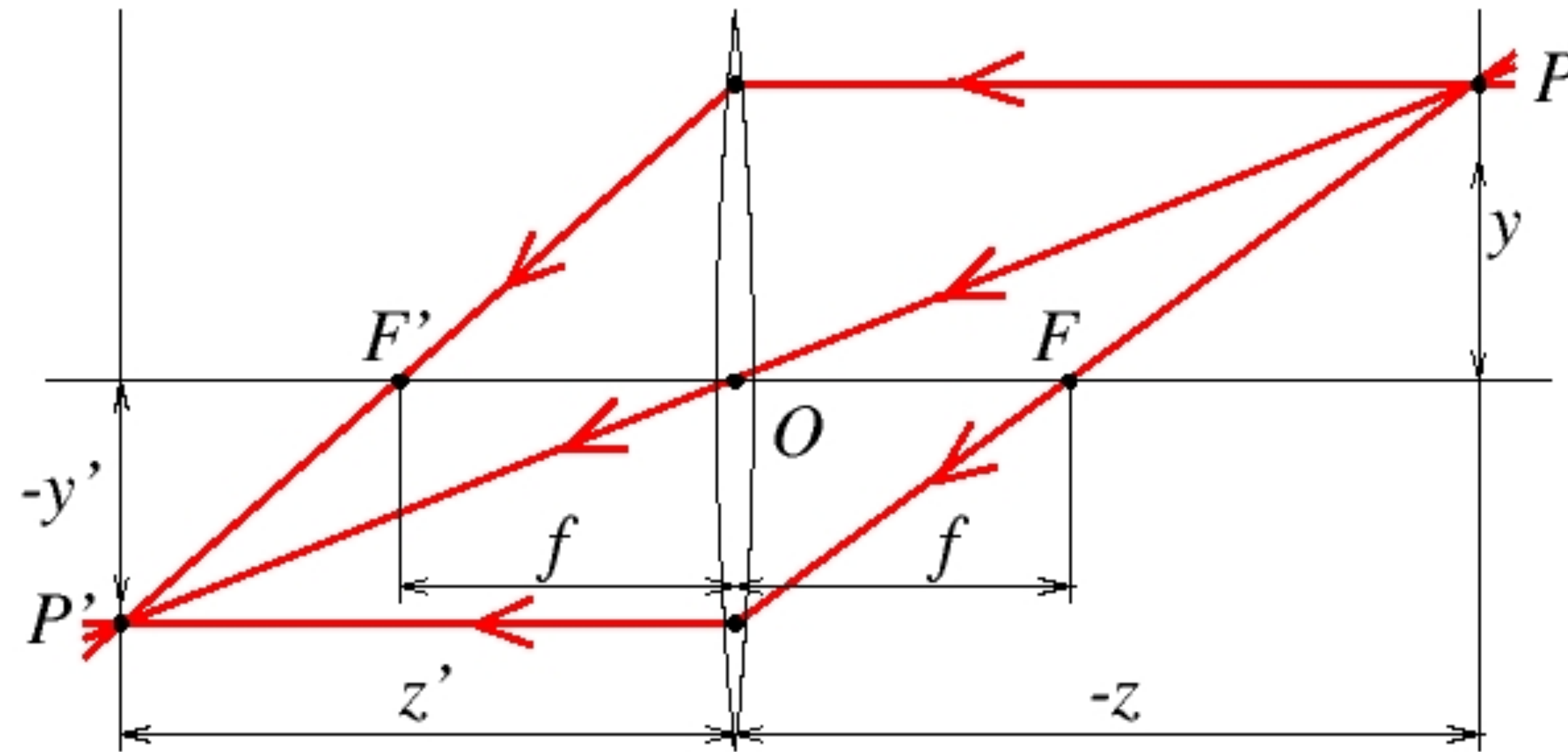
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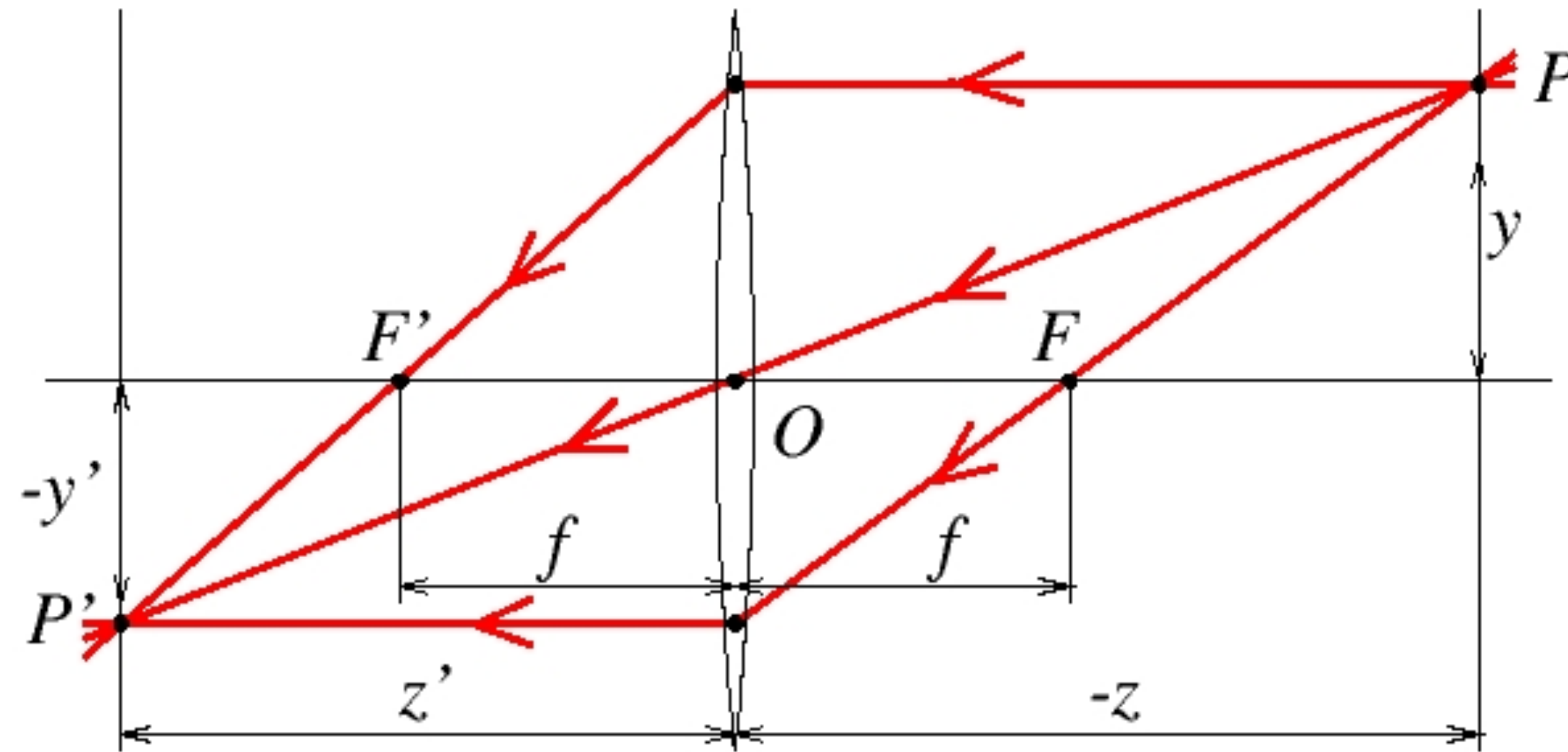
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Possible Uses of Thin Lens Abstraction



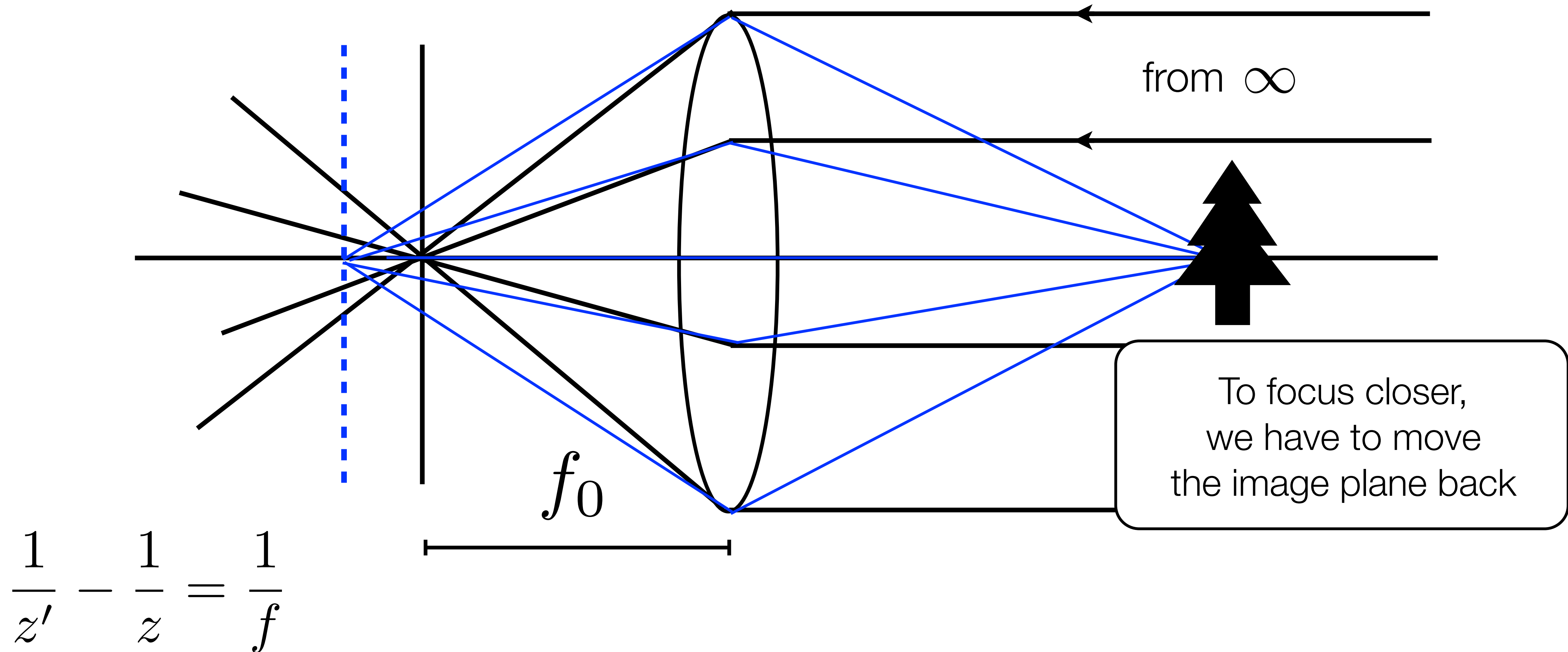
Forsyth & Ponce (1st ed.) Figure 1.9

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

Lens Basics

A lens focuses parallel rays (from points at infinity) at focal length of the lens

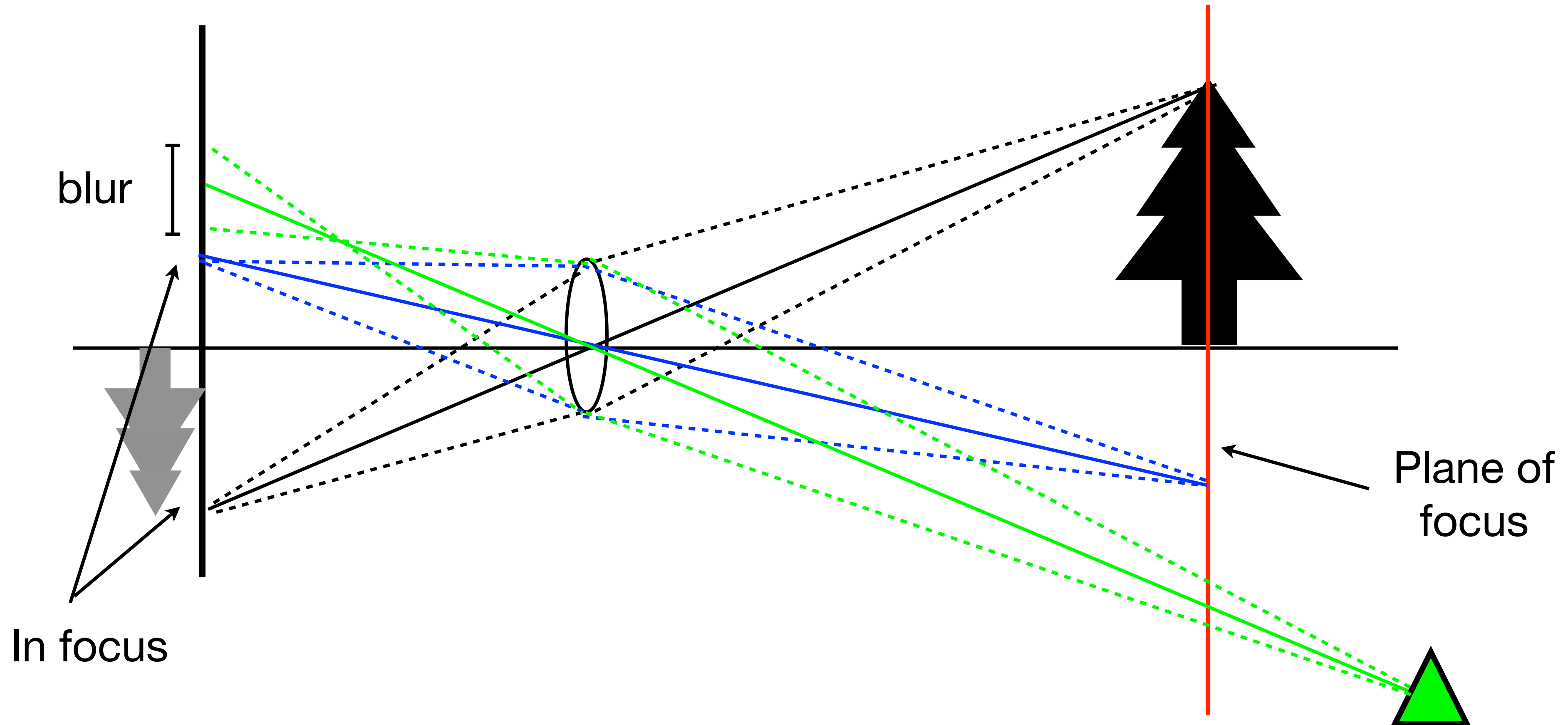
Rays passing through the center of the lens are not bent



Lens Basics

Lenses focus all rays from a (parallel to lens) plane in the world

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$



Objects off the plane are blurred depending on the distance

Pinhole Camera with a Lens

Perspective Projection: location in the image where a 3D world point projects

$$\begin{aligned}x' &= f' \frac{x}{z} \\ y' &= f' \frac{y}{z}\end{aligned}$$

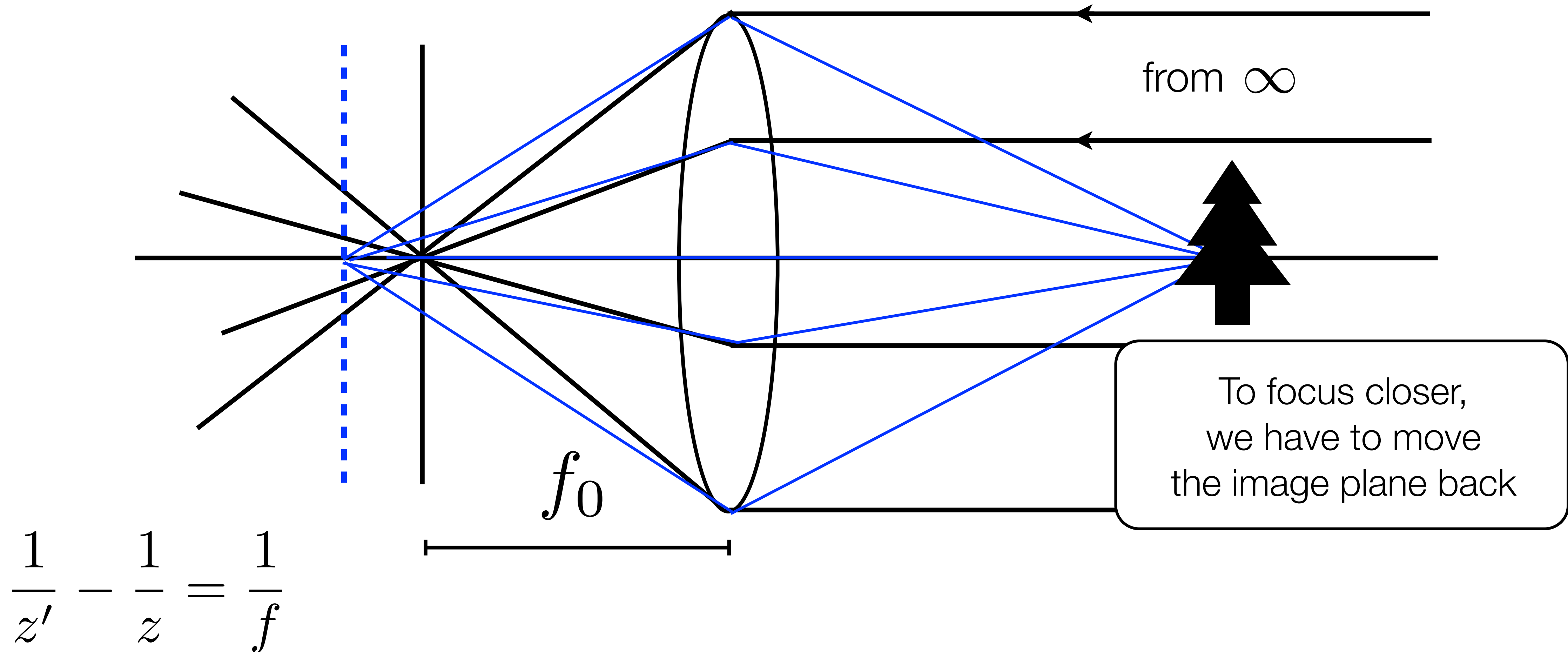
Thin Lens Equation: depth of the imaging plane itself where this point will be in focus

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

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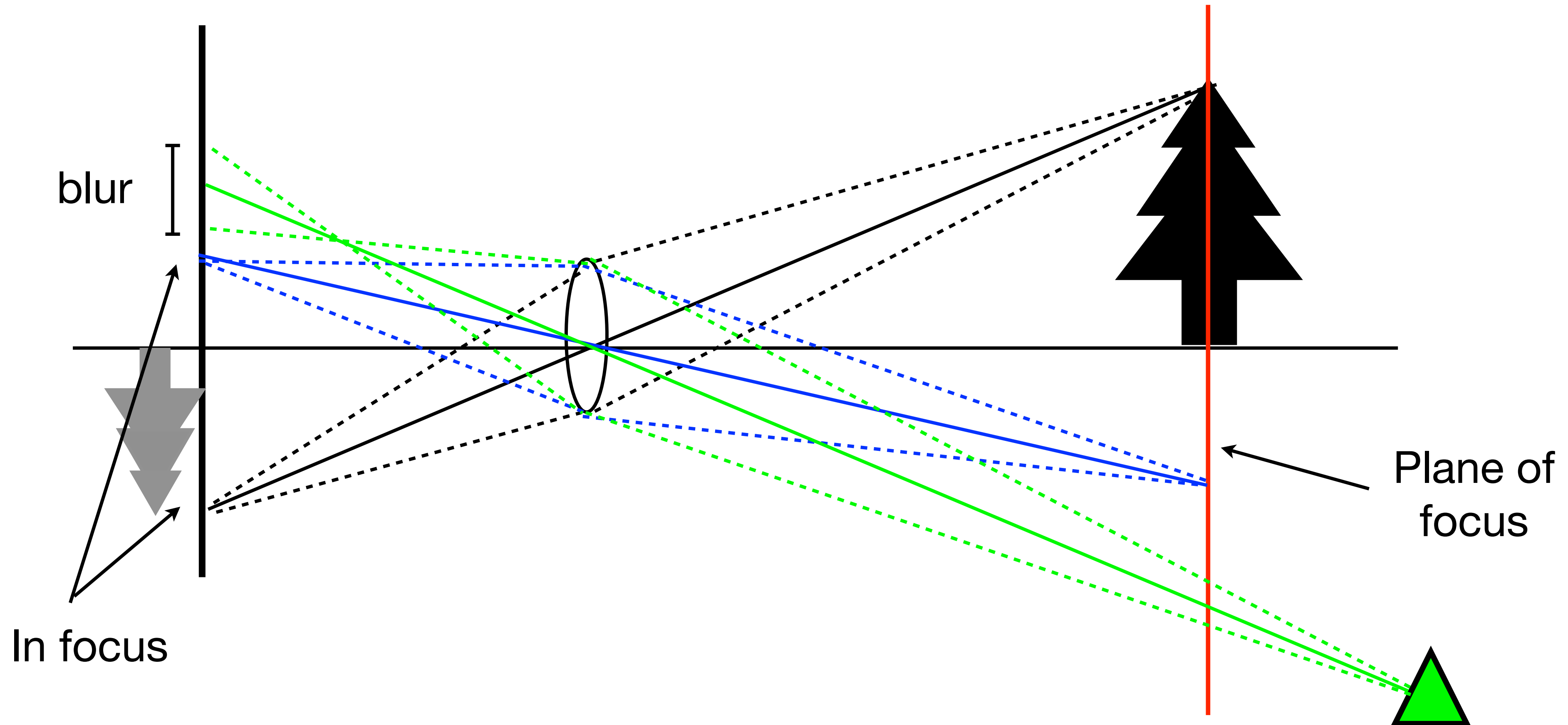
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Objects off the plane are blurred depending on the distance

Perspective Projection + Thin Lens Examples

Where would the focusing plane be for various positions of the object?

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

Perspective Projection + Thin Lens Examples

Where would the focusing plane be for various positions of the object?

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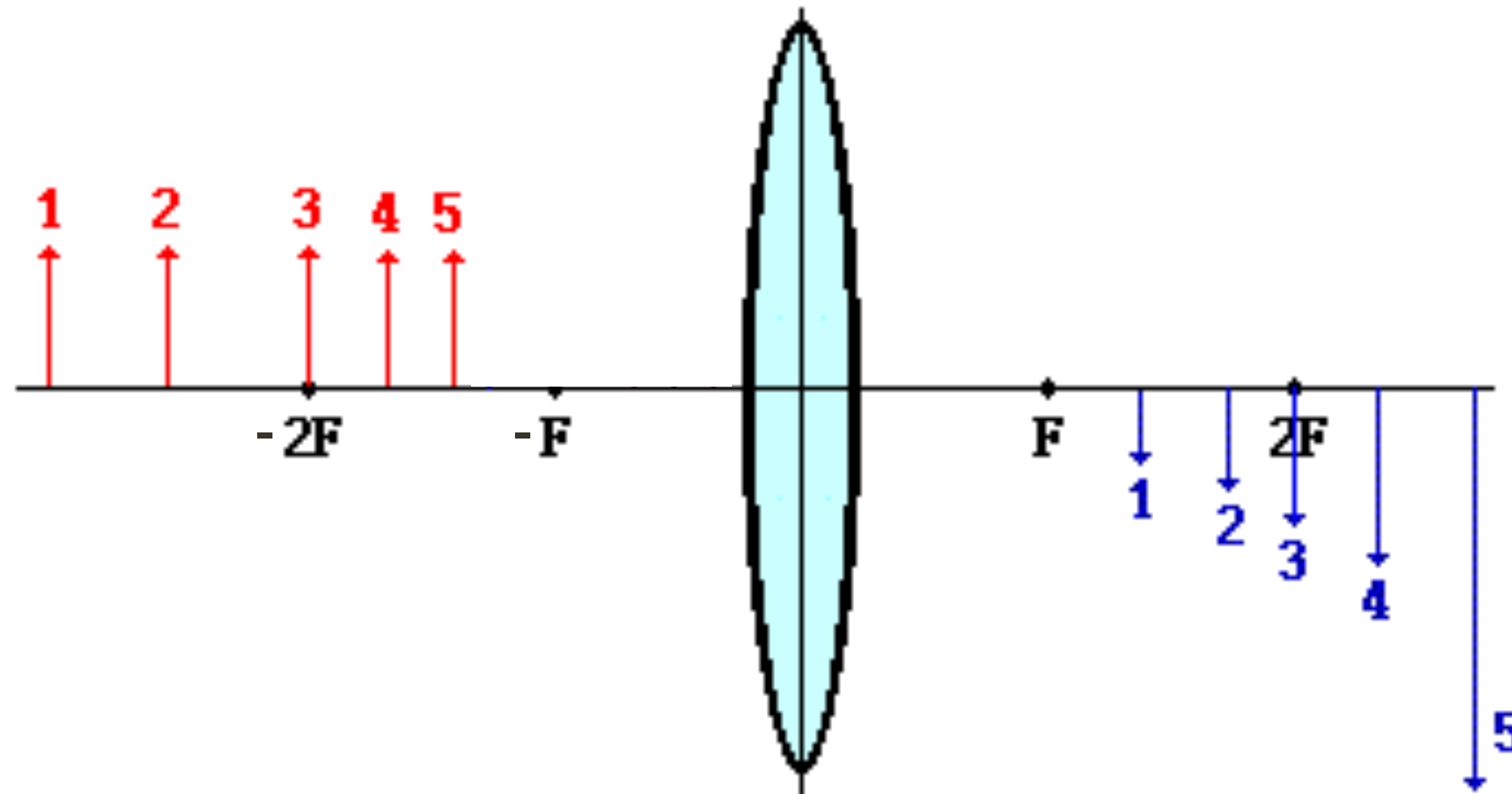
$$z' = \frac{zf}{z + f}$$

Perspective Projection + Thin Lens Examples

Where would the focusing plane be for various positions of the object?

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

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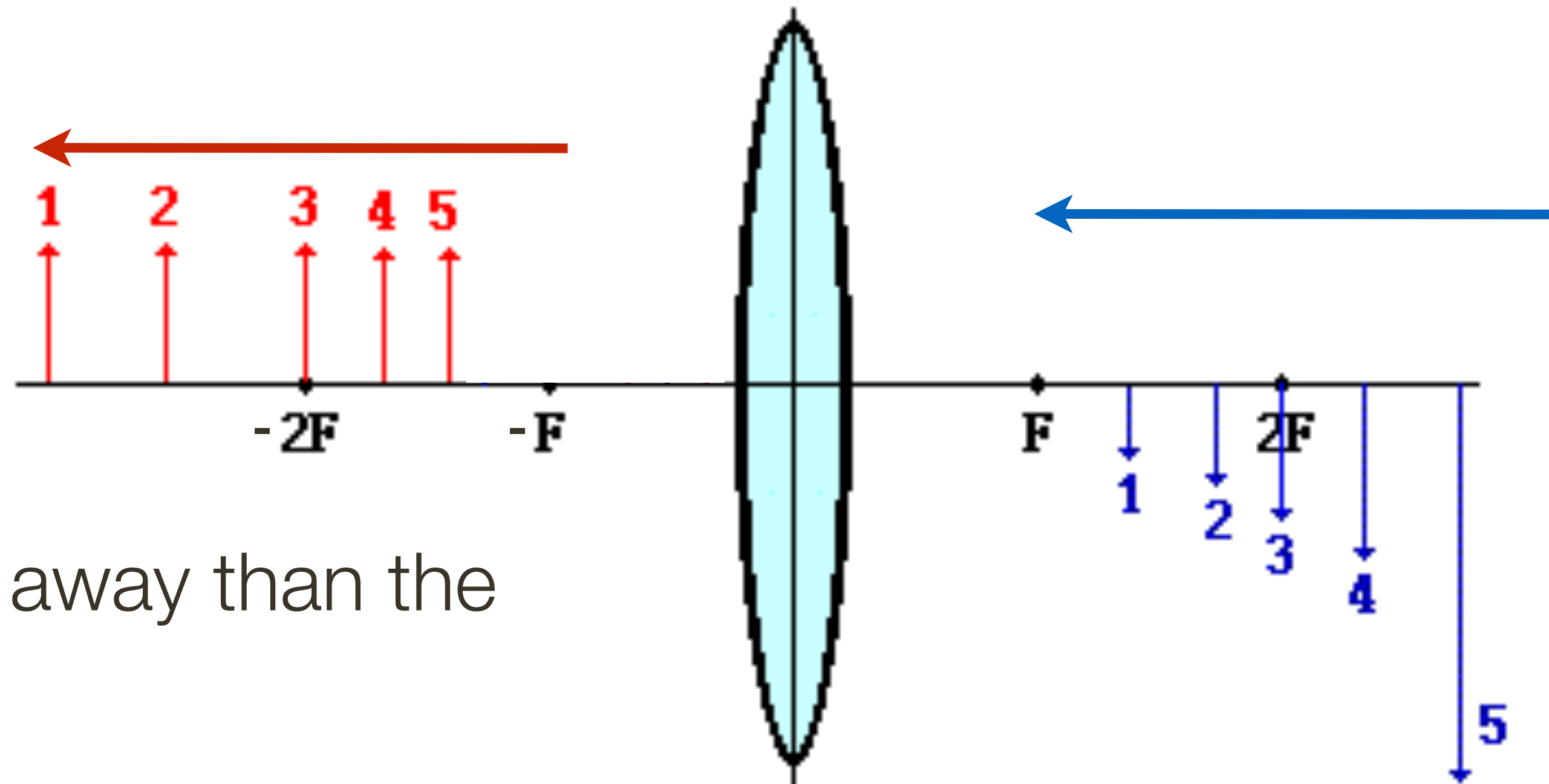


Perspective Projection + Thin Lens Examples

Where would the focusing plane be for various positions of the object?

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

$$z' = \frac{zf}{z + f}$$



Objects **further** away than the **focal length**

Perspective Projection + Thin Lens Examples

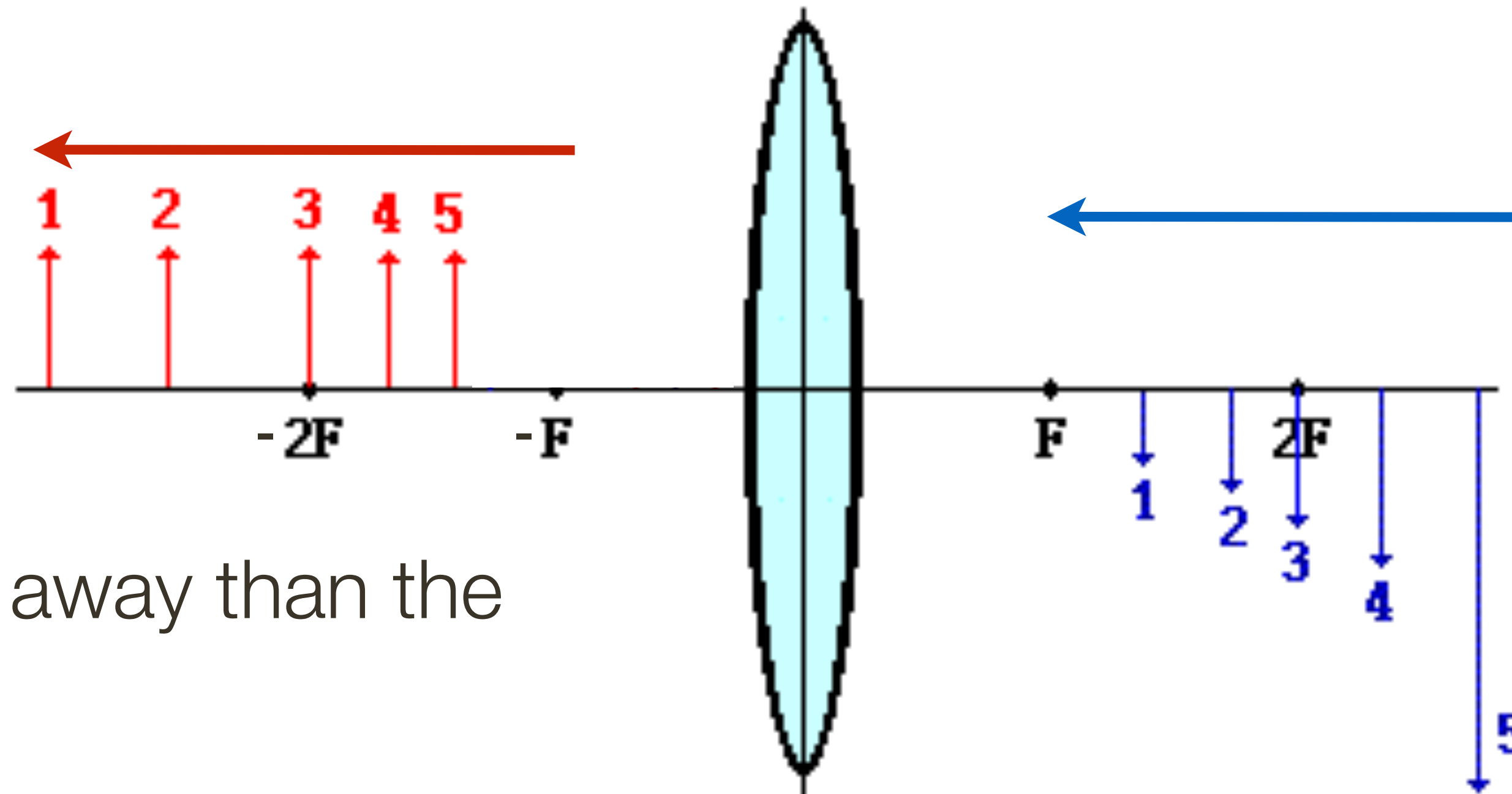
Where would the focusing plane be for various positions of the object?

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

$$z' = \frac{zf}{z + f}$$

$$\lim_{z \rightarrow -\infty} \frac{zf}{z + f} = f$$

L'Hopital's Rule



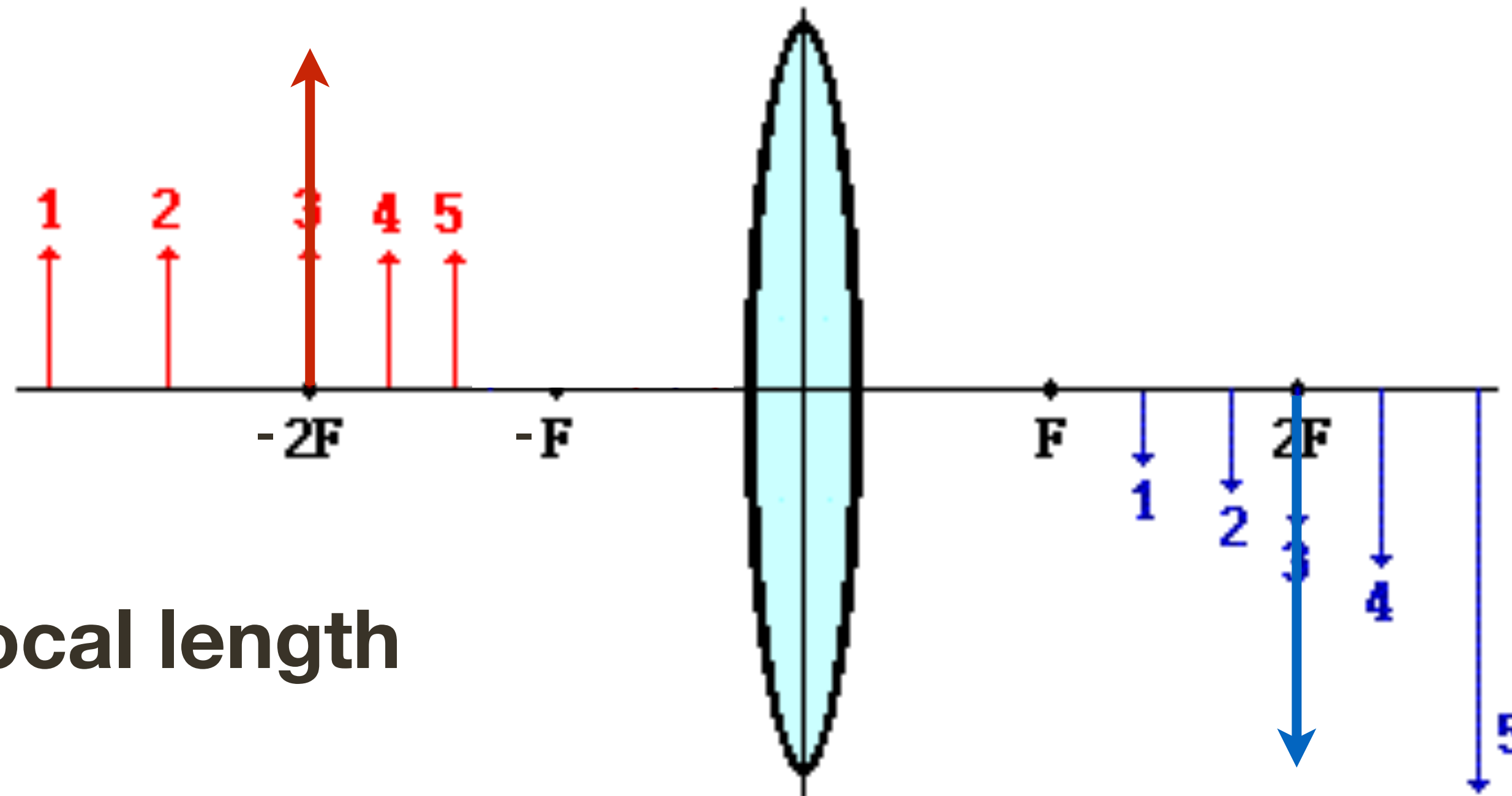
Objects **further** away than the **focal length**

Perspective Projection + Thin Lens Examples

Where would the focusing plane be for various positions of the object?

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

$$z' = \frac{zf}{z + f}$$



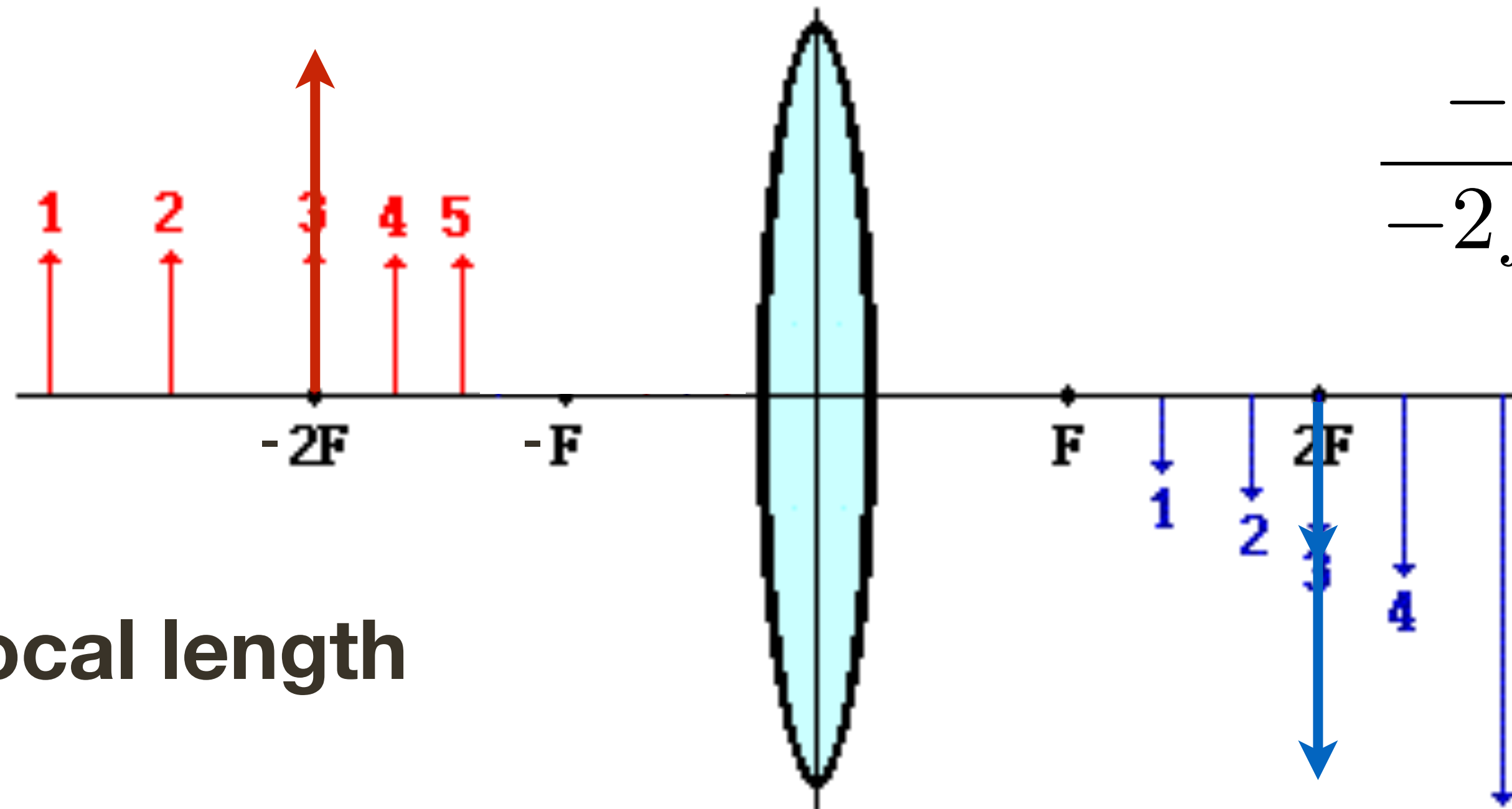
Objects at 2 x **focal length**

Perspective Projection + Thin Lens Examples

Where would the focusing plane be for various positions of the object?

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

$$z' = \frac{zf}{z + f}$$



$$\frac{-2f^2}{-2f + f} = \frac{-2f^2}{-f} = 2f$$

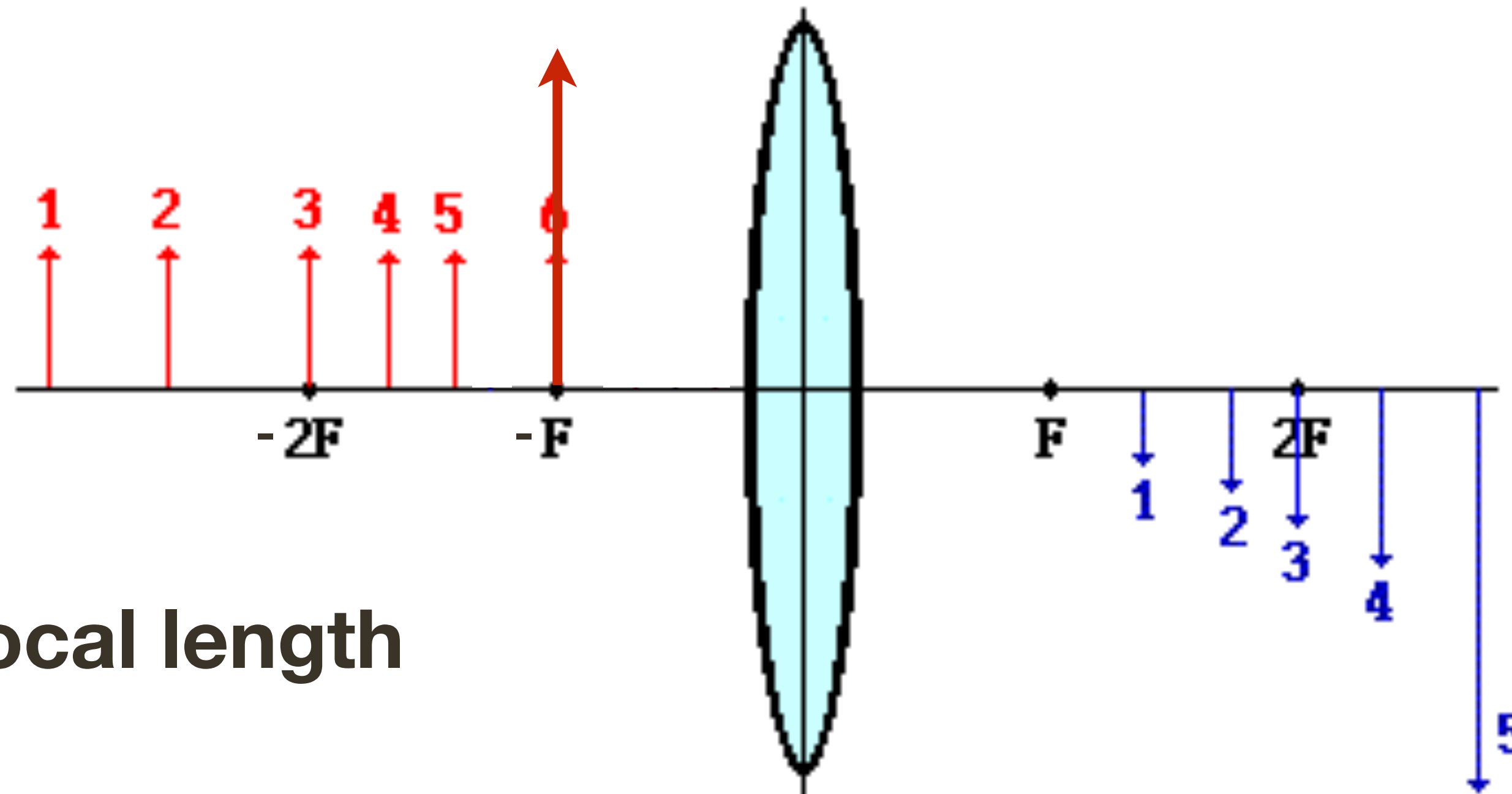
Objects at 2 x **focal length**

Perspective Projection + Thin Lens Examples

Where would the focusing plane be for various positions of the object?

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

$$z' = \frac{zf}{z + f}$$



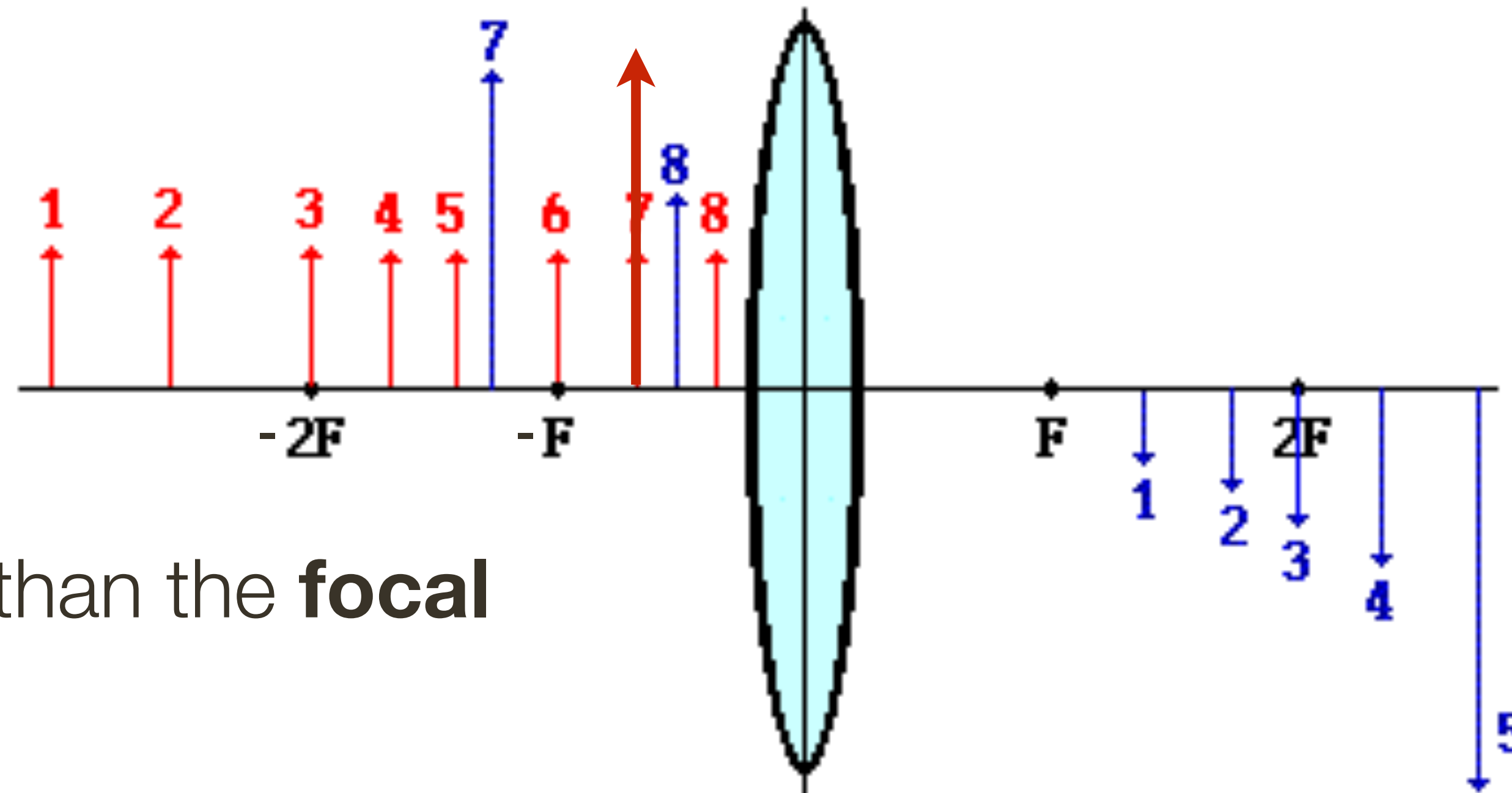
Objects at the **focal length**

Perspective Projection + Thin Lens Examples

Where would the focusing plane be for various positions of the object?

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

$$z' = \frac{zf}{z + f}$$



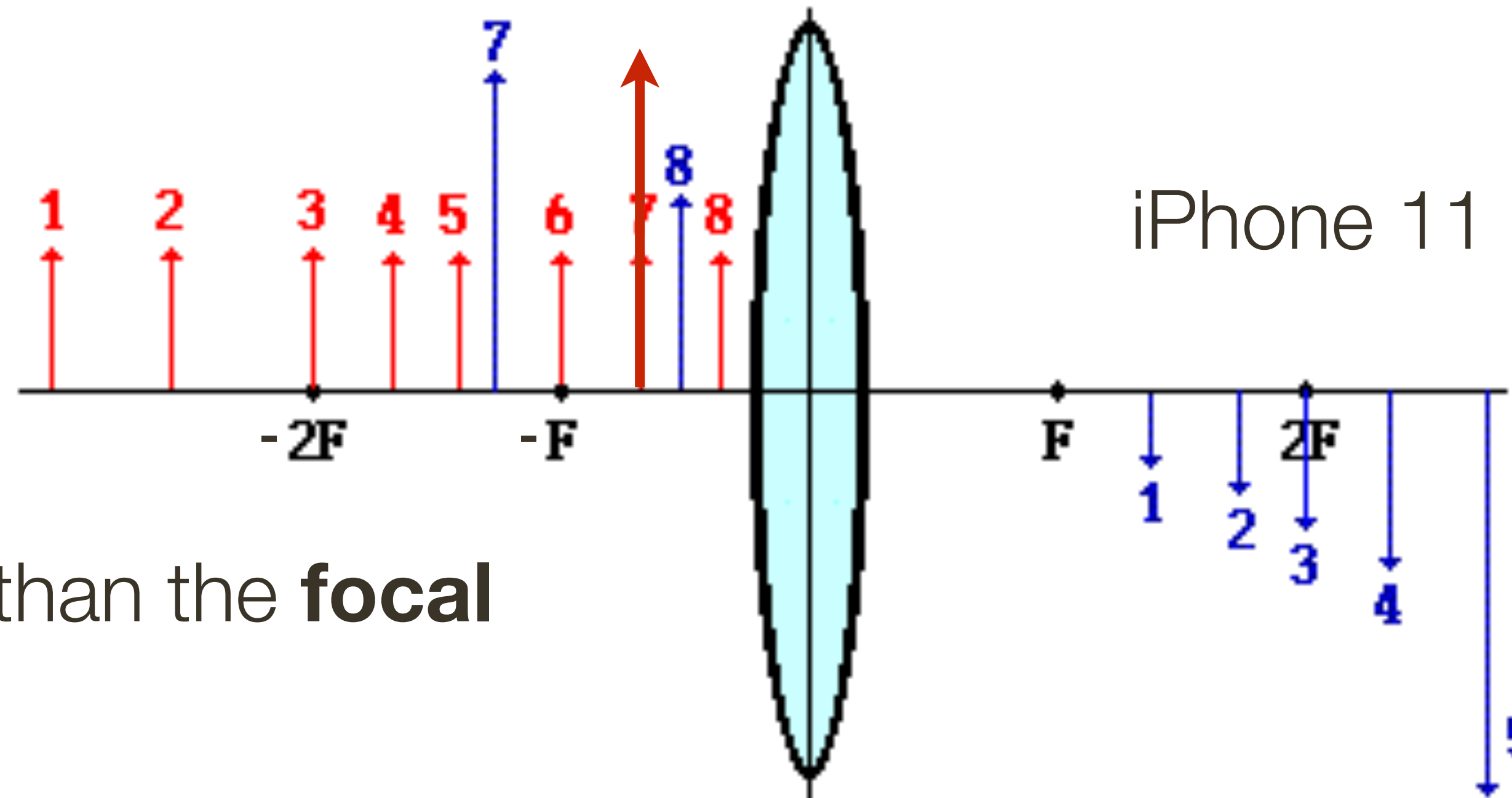
Objects **closer** than the **focal length**

Perspective Projection + Thin Lens Examples

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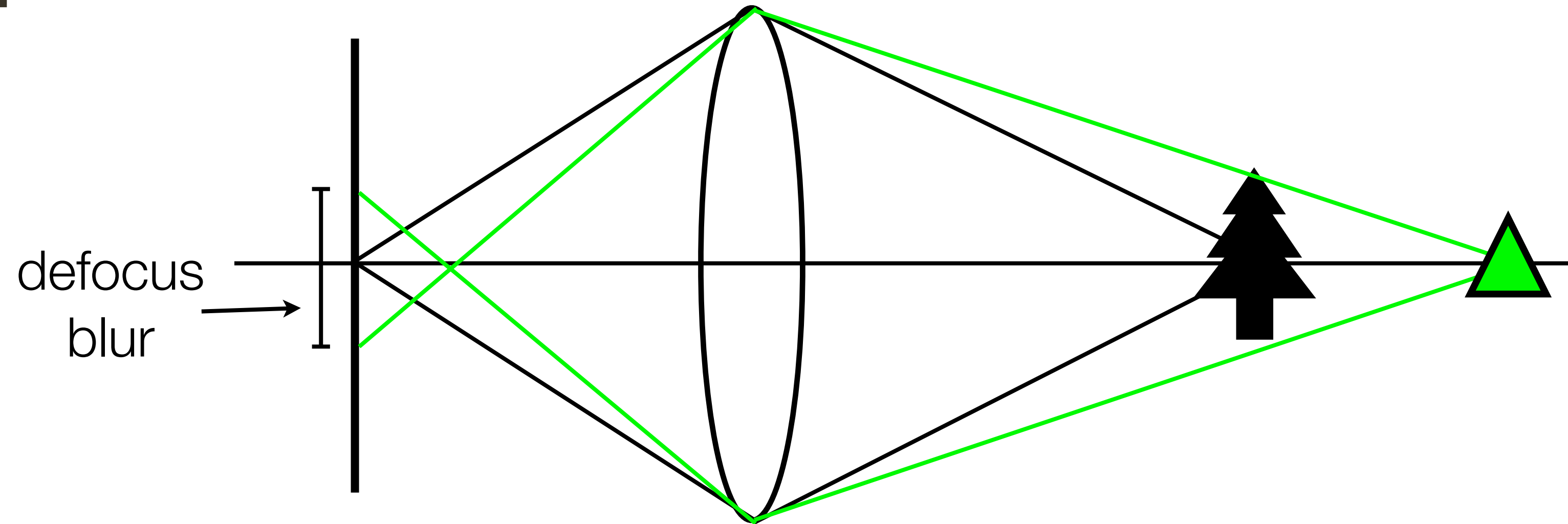
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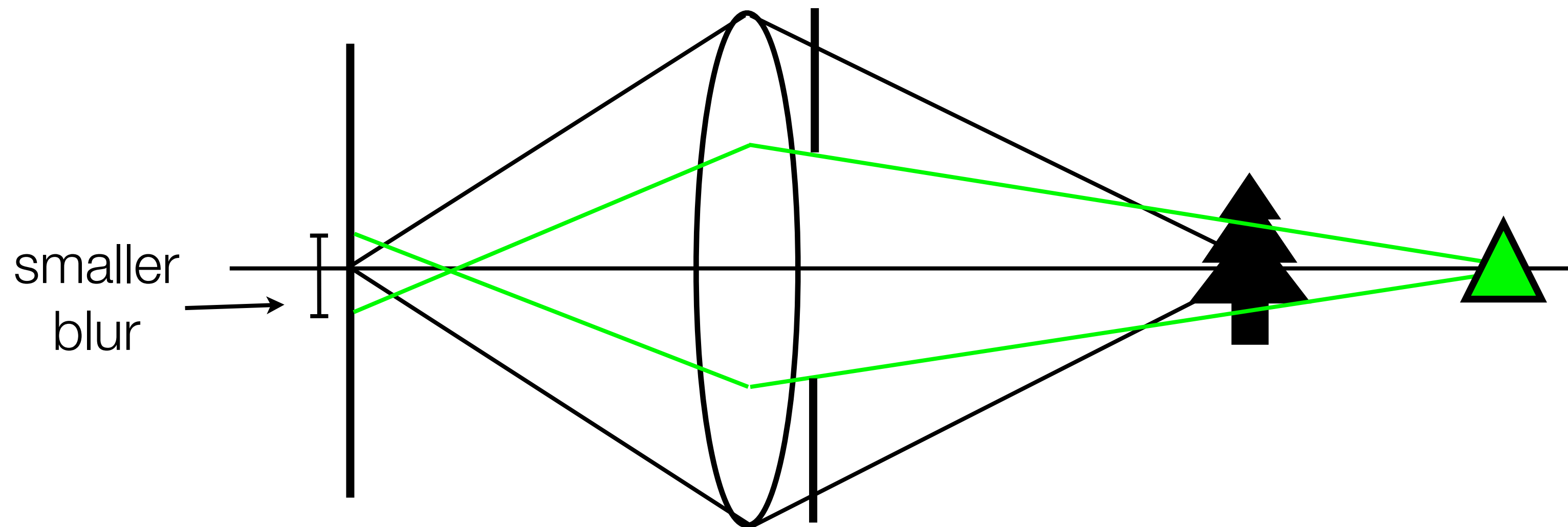
iPhone 11 Camera Lens: **26mm**

Objects **closer** than the **focal length**

Effect of **Aperture** Size



Smaller aperture $\Rightarrow$ smaller blur, larger **depth of field**



Depth of Field



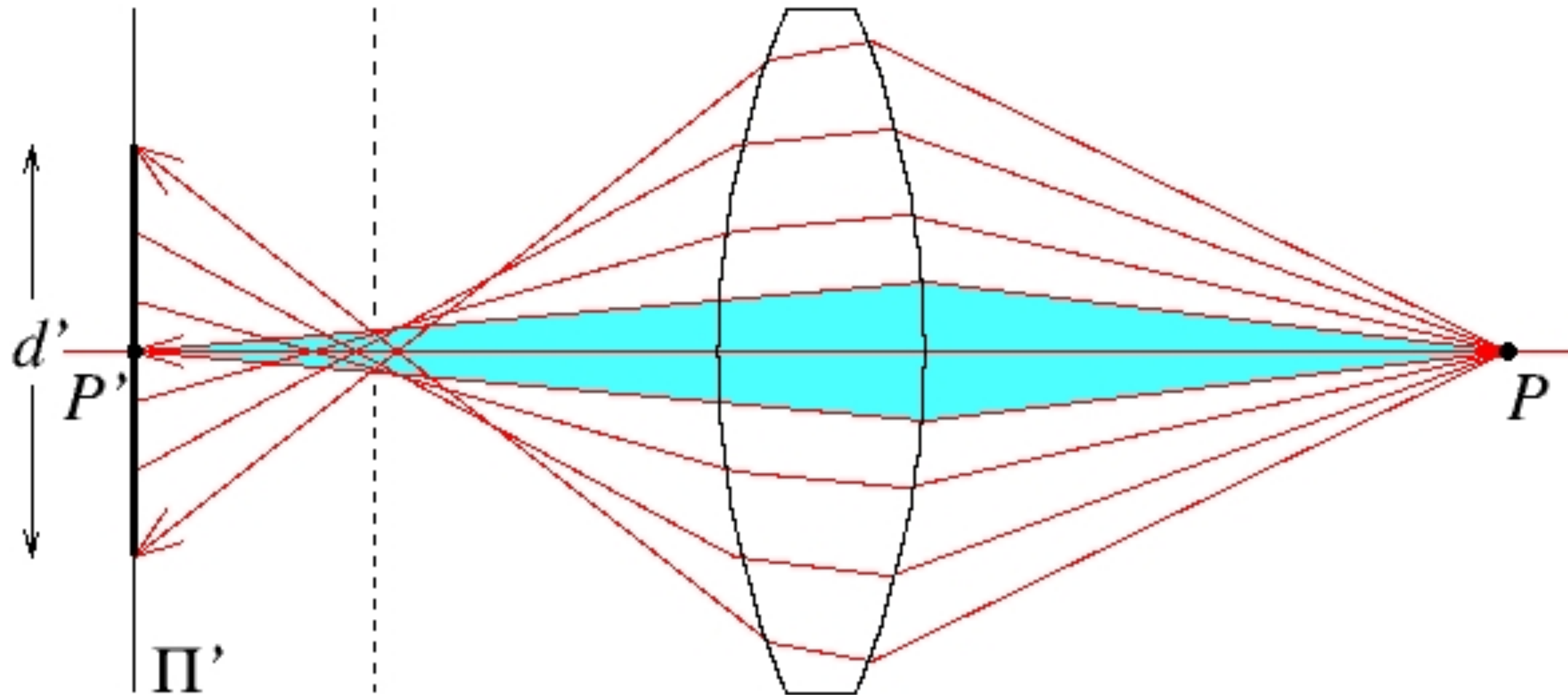
Aperture size = f/N , $\Rightarrow$ large N = small aperture

Real Lenses



- Real Lenses have multiple stages of positive and negative elements with differing refractive indices
- This can help deal with issues such as chromatic aberration (different colours bent by different amounts), vignetting (light fall off at image edge) and sharp imaging across the zoom range

Spherical **Aberration**



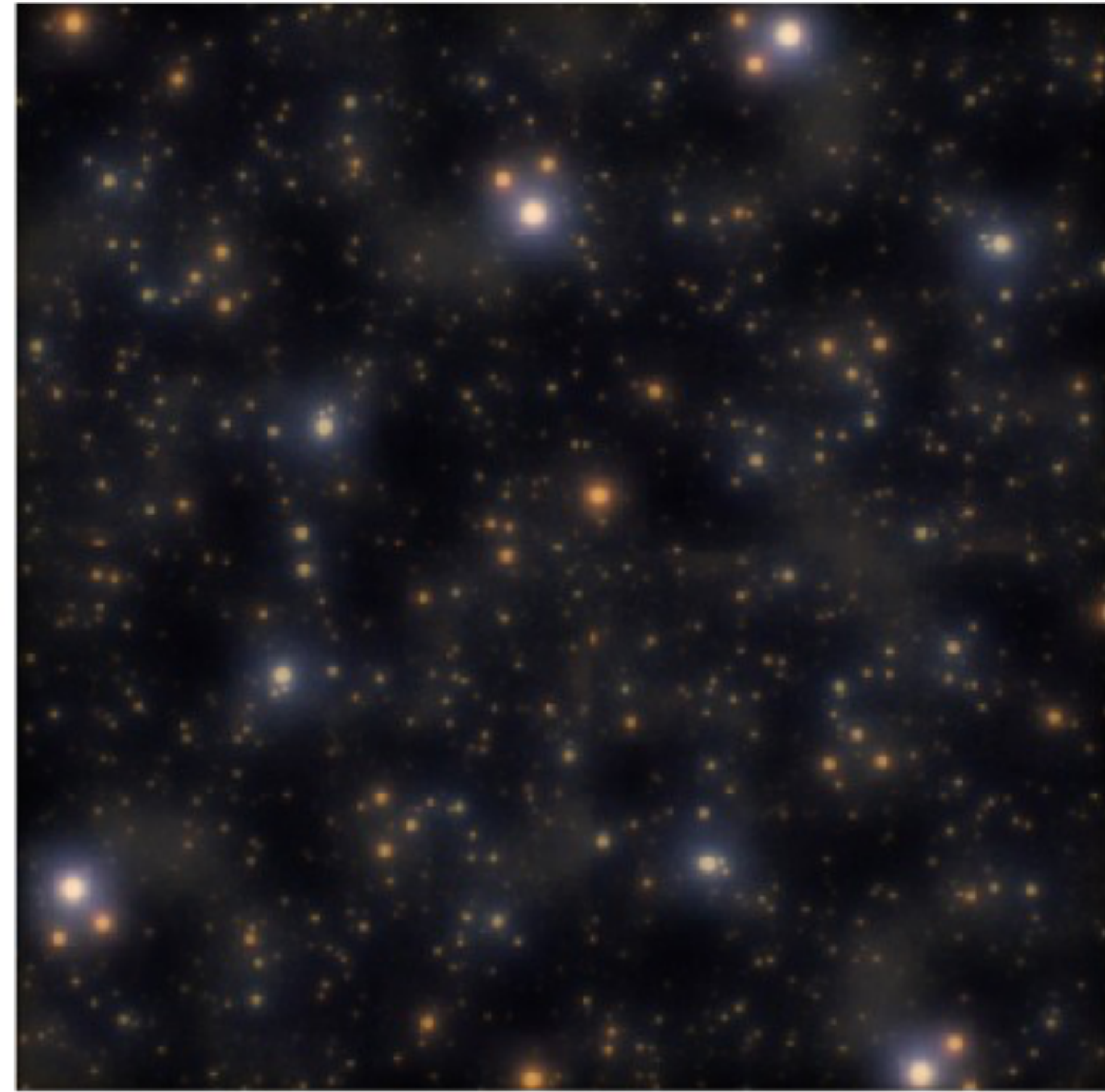
Forsyth & Ponce (1st ed.) Figure 1.12a

Spherical **Aberration**

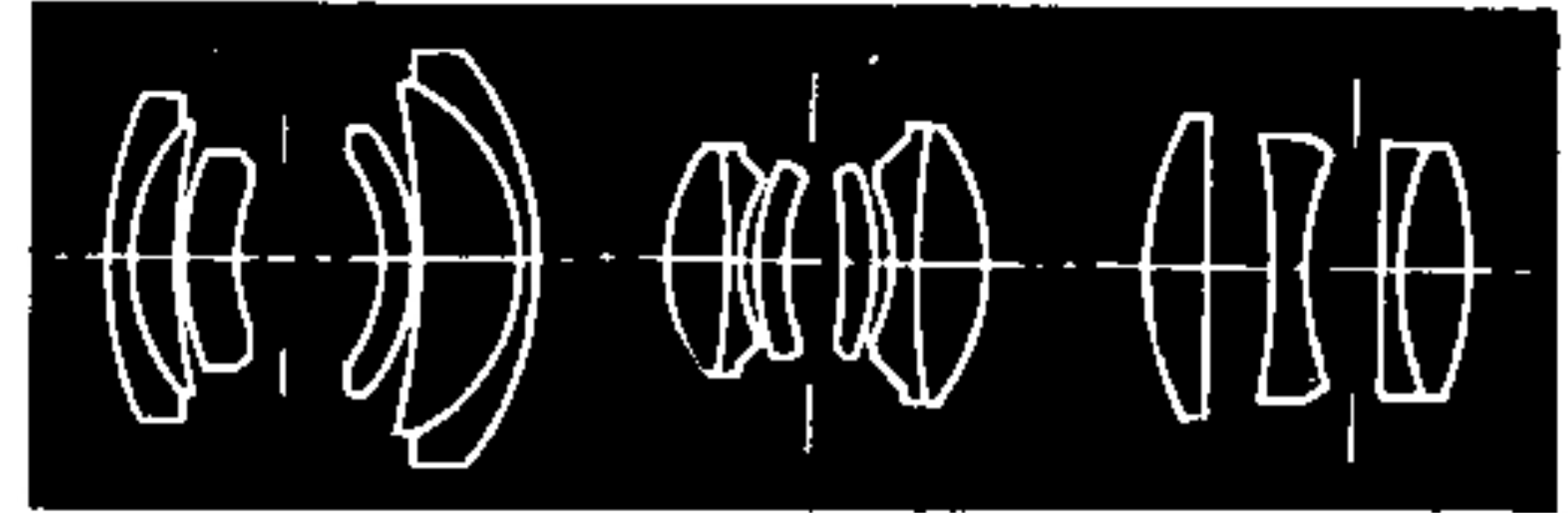
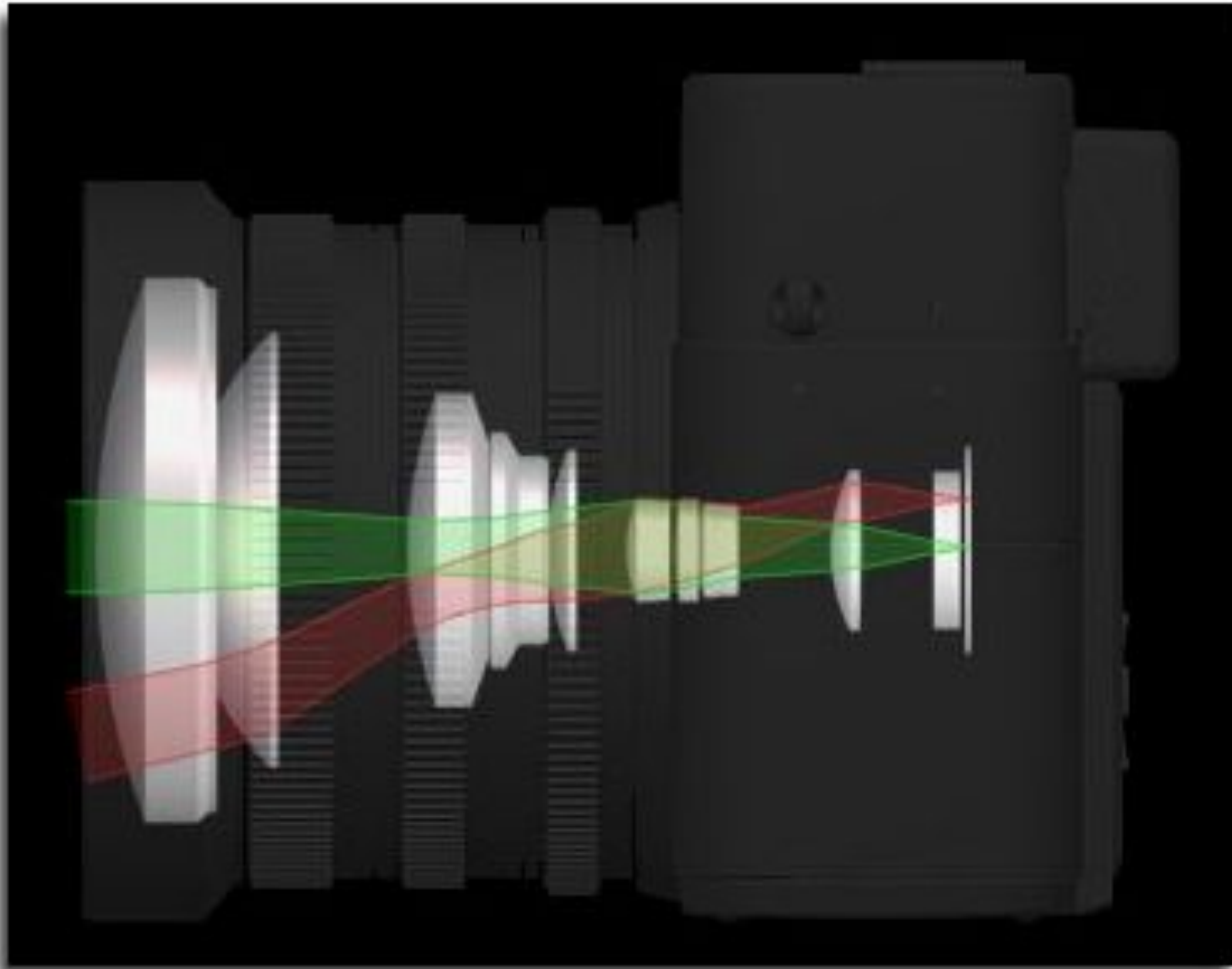
Un-aberrated image



Image from lens with Spherical Aberration



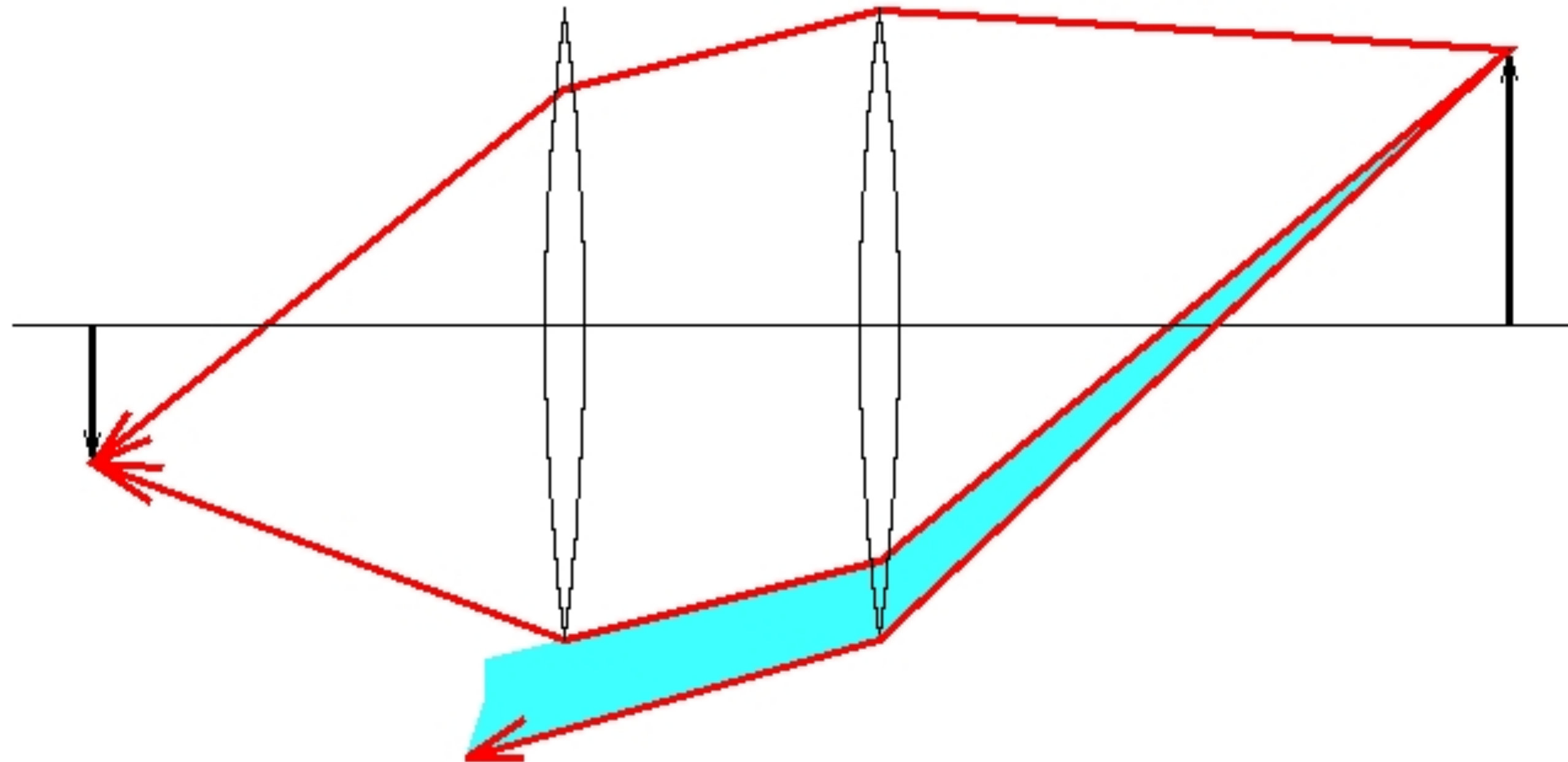
Compound **Lens Systems**



A modern camera lens may contain multiple components, including aspherical elements

Vignetting

Vignetting in a two-lens system



Forsyth & Ponce (2nd ed.) Figure 1.12

The shaded part of the beam **never reaches** the second lens

Vignetting



Image Credit: Cambridge in Colour

Chromatic **Aberration**

- Index of **refraction depends on wavelength**, λ , of light
- Light of different colours follows different paths
- Therefore, not all colours can be in equal focus

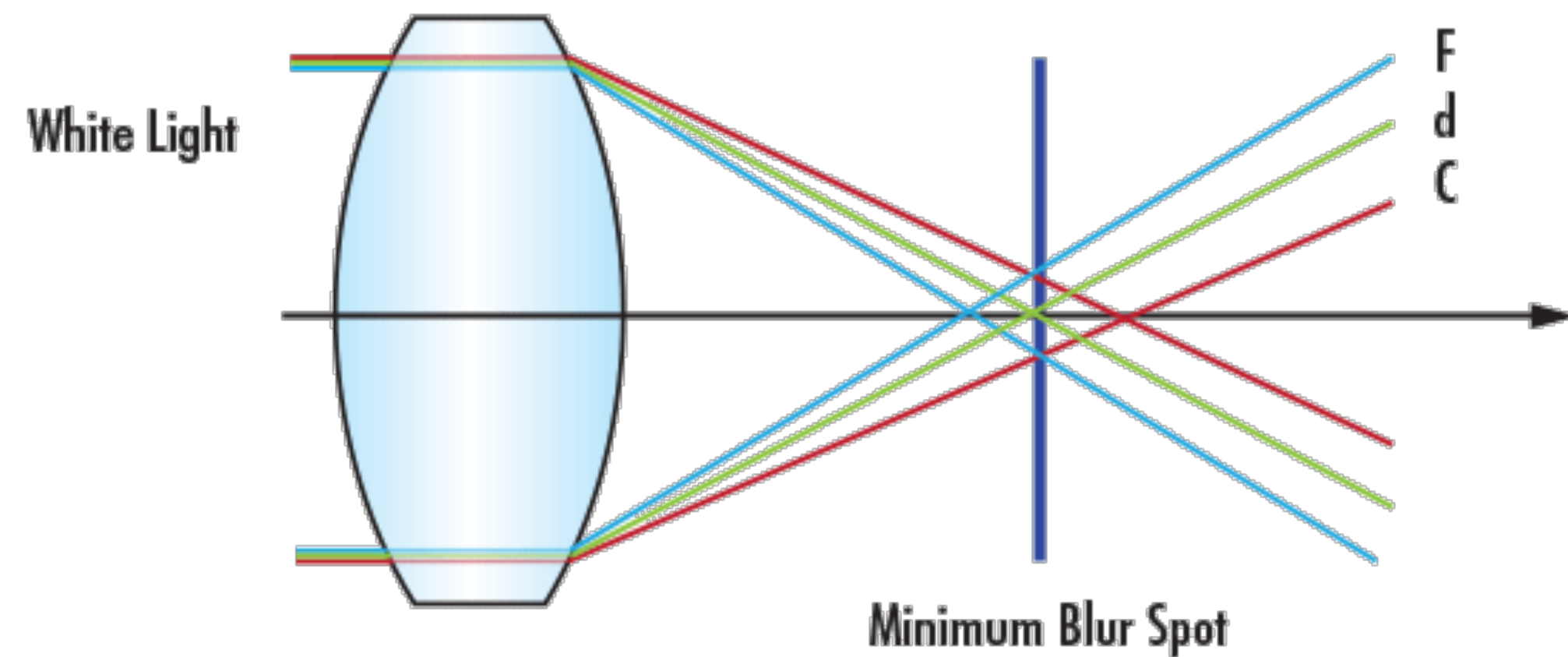


Image Credit: Trevor Darrell

Other (Possibly Significant) **Lens Effects**

Chromatic **aberration**

- Index of refraction depends on wavelength, λ , of light
- Light of different colours follows different paths
- Therefore, not all colours can be in equal focus

Scattering at the lens surface

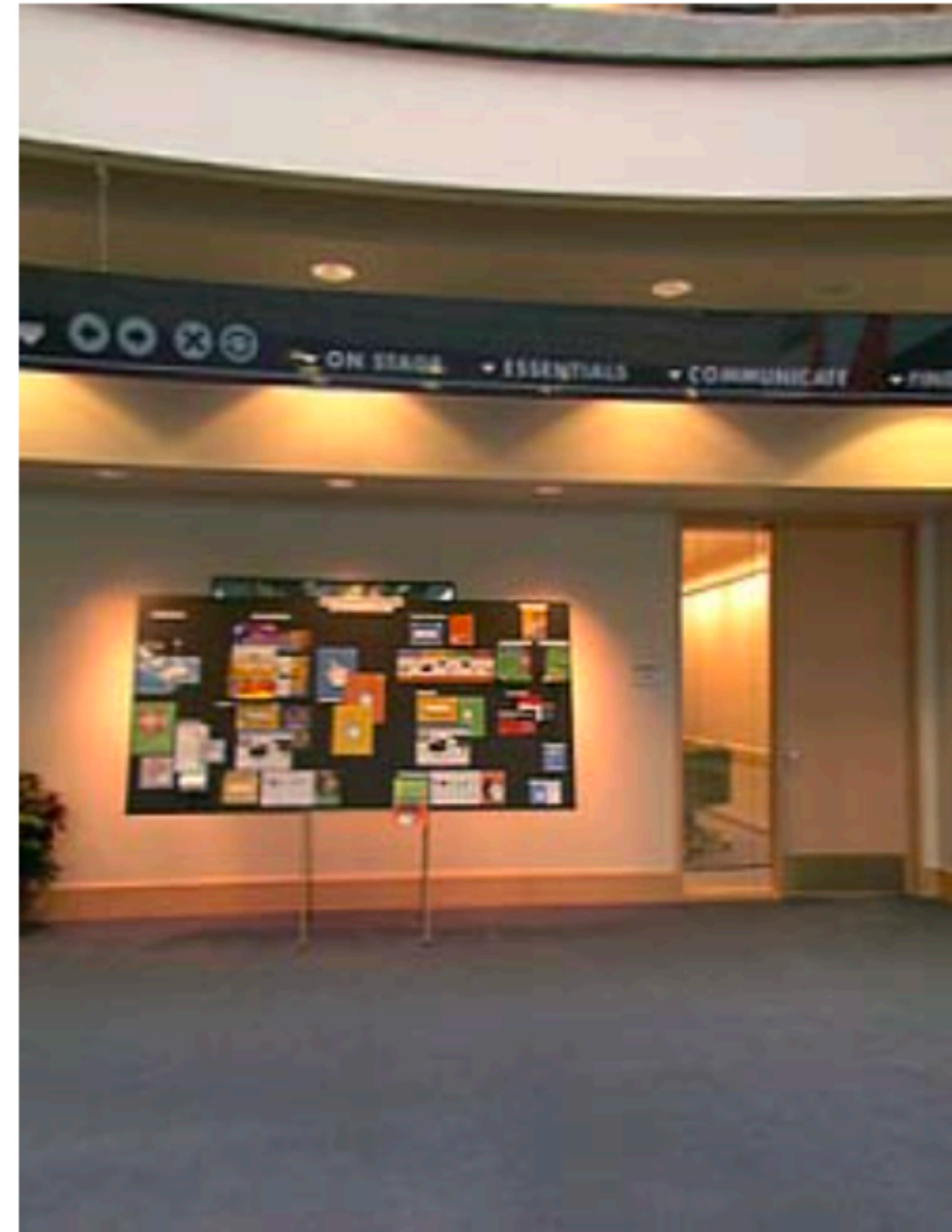
- Some light is reflected at each lens surface

There are other **geometric phenomena/distortions**

- pincushion distortion
- barrel distortion
- etc

Lens Distortion

Fish-eye Lens

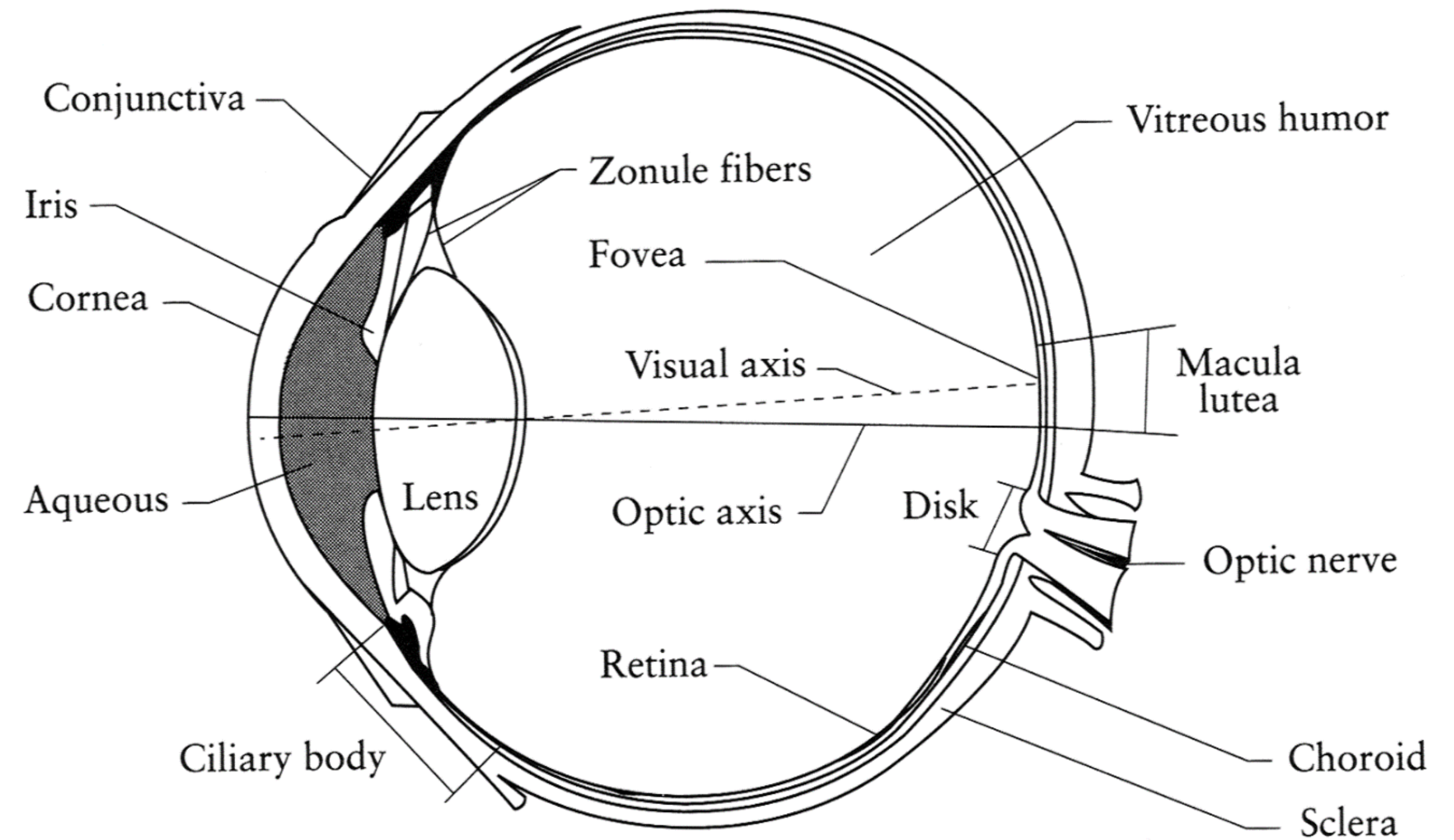


Szeliski (1st ed.) Figure 2.13

Lines in the world are no longer lines on the image, they are curves!

Human Eye

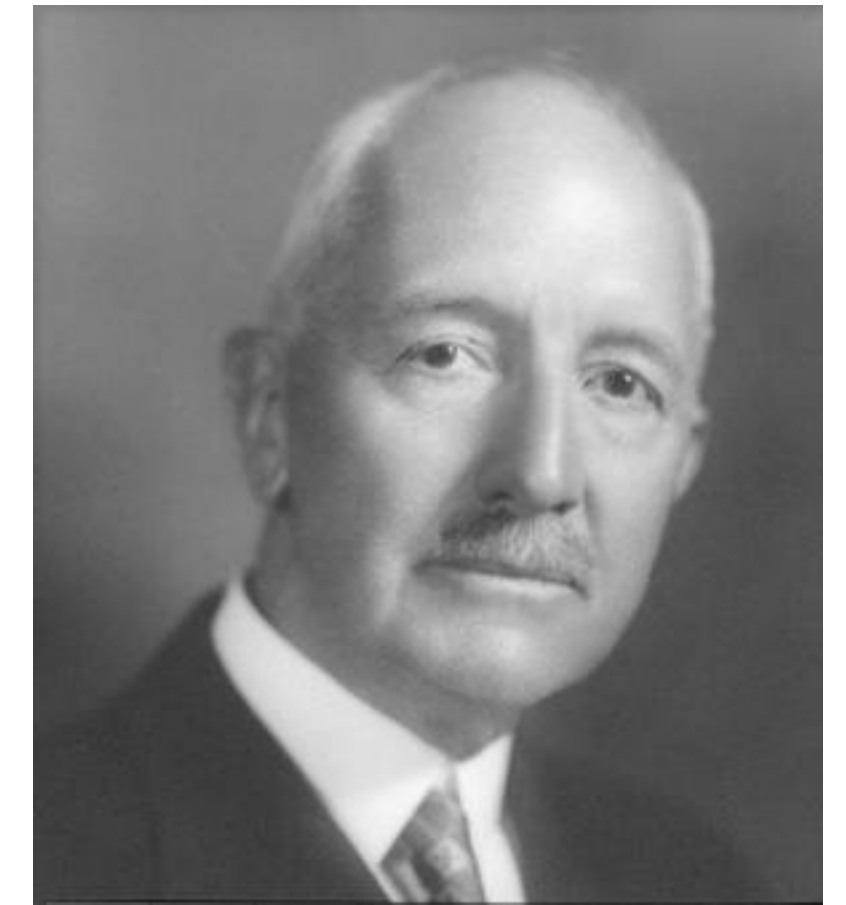
- The eye has an **iris** (like a camera)
- **Focusing** is done by changing shape of lens
- When the eye is properly focused, light from an object outside the eye is imaged on the **retina**
- The retina contains light receptors called **rods** and **cones**



pupil = pinhole / aperture

retina = film / digital sensor

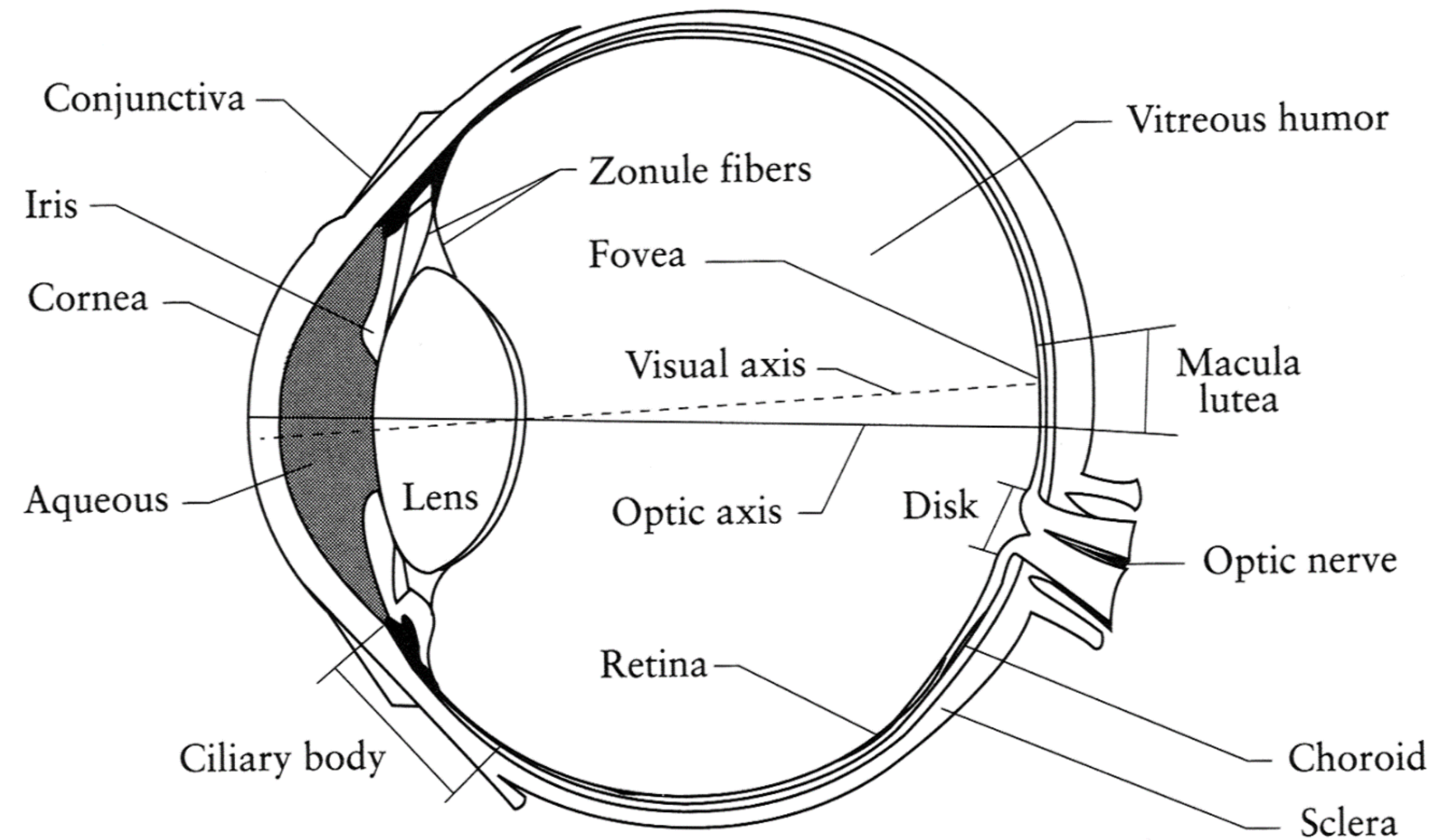
Fun **Aside**



George M. Stratton

Human Eye

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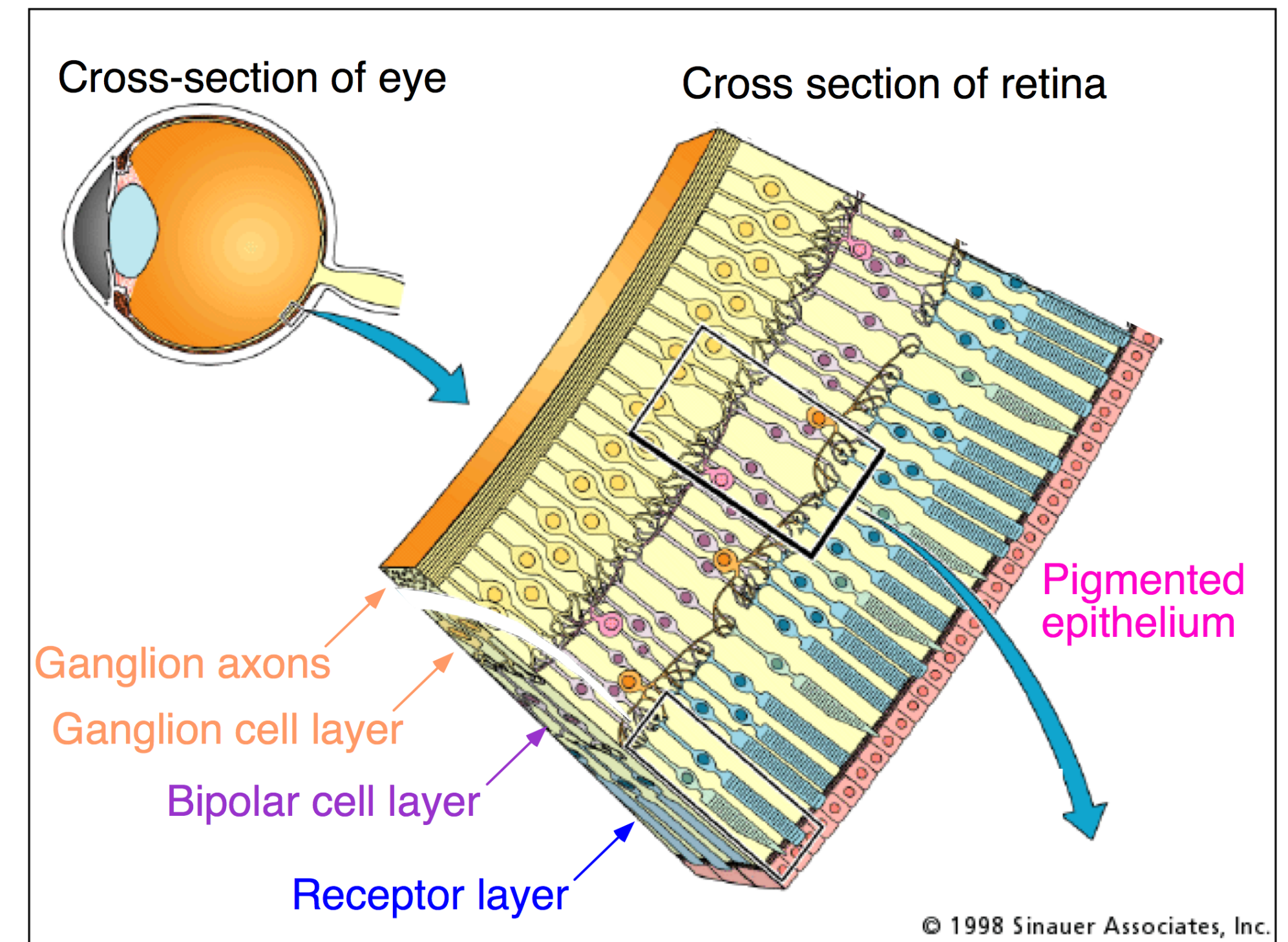


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Human Eye

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Two-types of **Light Sensitive Receptors**

Rods

75-150 million rod-shaped receptors

not involved in color vision, gray-scale vision only

operate at night

highly sensitive, can responding to a single photon

yield relatively poor spatial detail

Cones

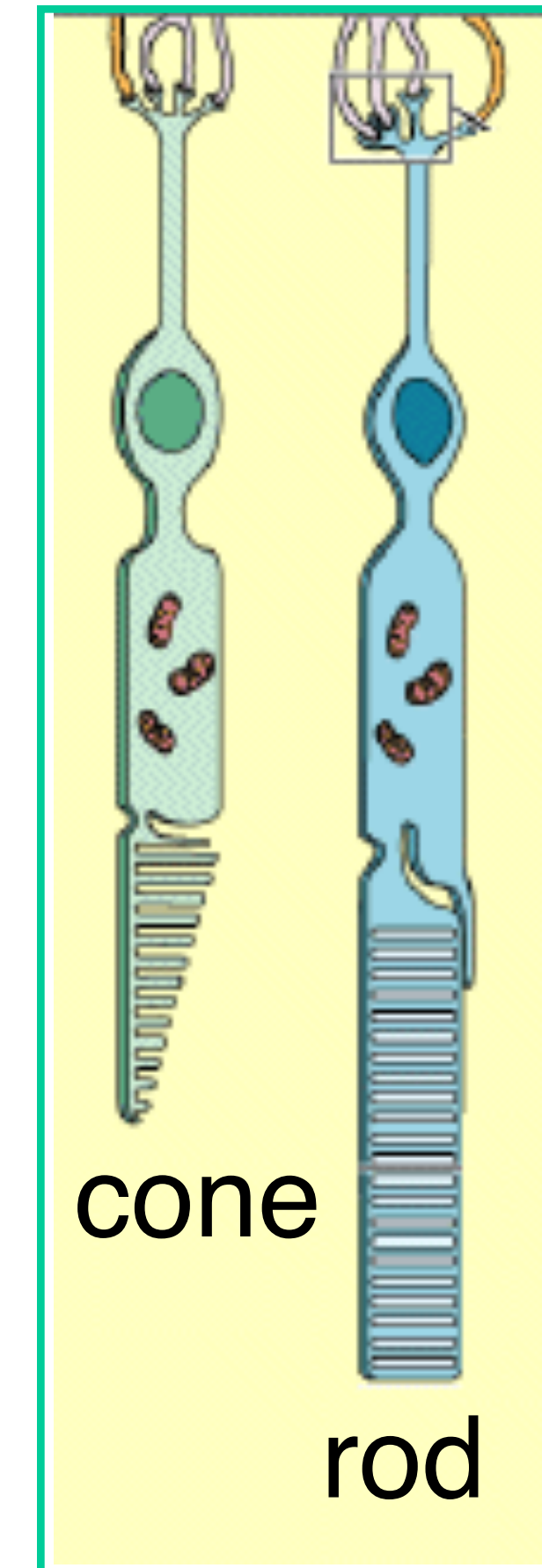
6-7 million cone-shaped receptors

color vision

operate in high light

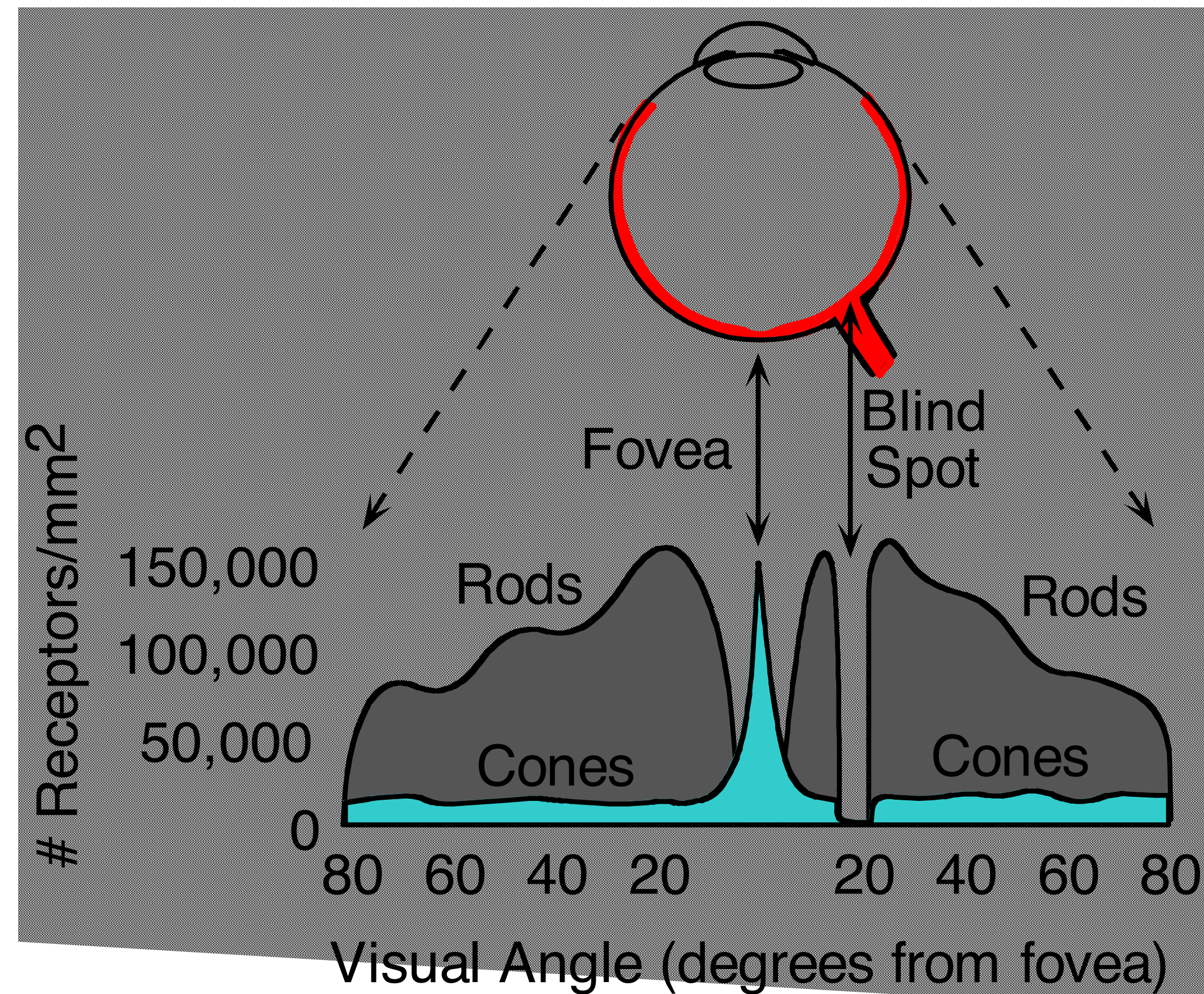
less sensitive

yield higher resolution



Human Eye

Density of rods and cones



Lecture **Summary**

- We discussed a “physics-based” approach to image formation. Basic abstraction is the **pinhole camera**.
- **Lenses overcome limitations** of the pinhole model while trying to preserve it as a useful abstraction
- Projection equations: **perspective**, weak perspective, orthographic
- Thin lens equation
- Some “aberrations and **distortions**” persist (e.g. spherical aberration, vignetting)
- The **human eye** functions much like a camera