THE UNIVERSITY OF BRITISH COLUMBIA

CPSC 425: Computer Vision


Midterm Review

(unless otherwise stated slides are taken or adopted from **Bob Woodham, Jim Little** and **Fred Tung**)

Midterm Details

Closed book, (simple) calculators allowed

Format similar to posted practice problems

- Part A: Multiple-part true/false
- Part B: Short answer

No coding questions

No complex math questions

Midterm Review: Study materials

Lectures 1–12 **slides**

Assigned **readings** from Szeliski / Forsyth and Ponce

Assignments 1–2

iClicker questions

Practice quizzes on Canvas

Practice problems / solutions on Canvas

Overview: Image Formation, Cameras and Lenses

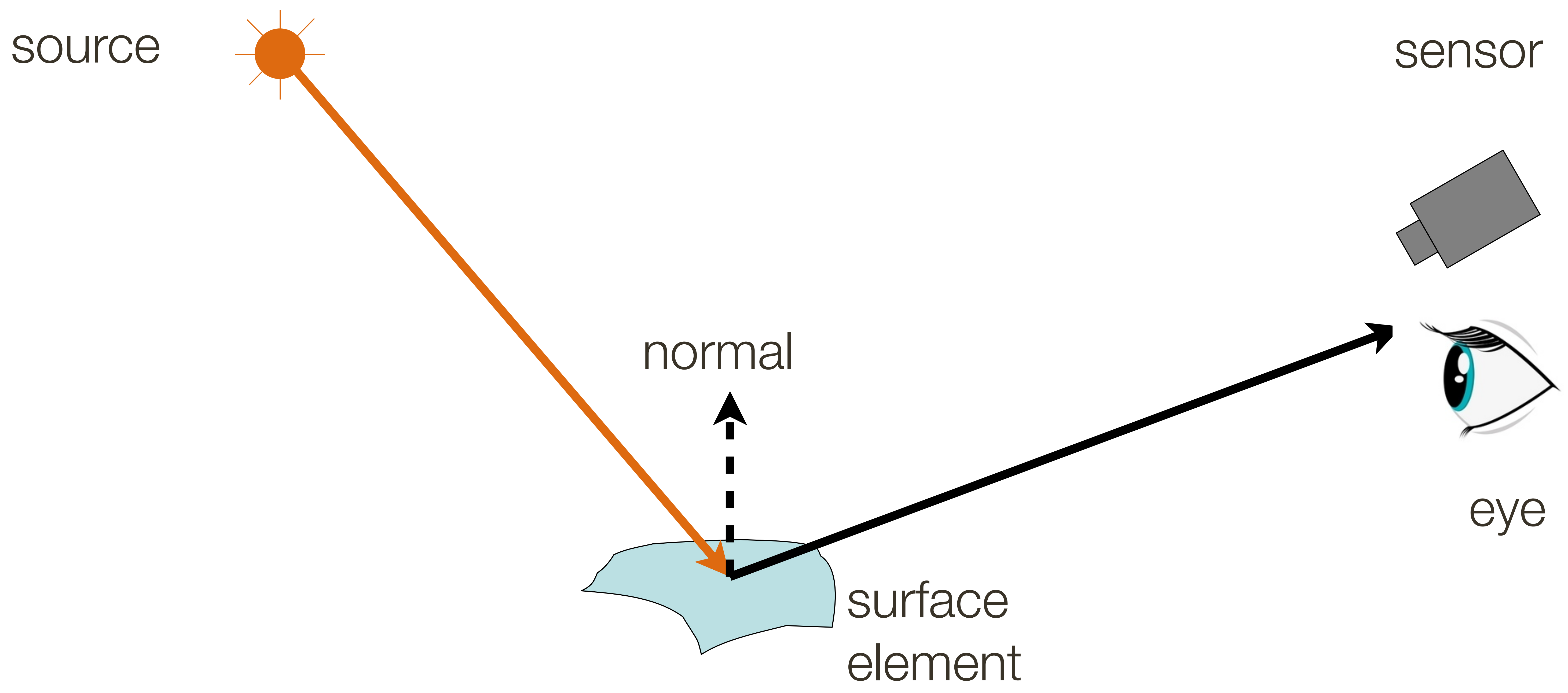
The **image formation process** that produces a particular image depends on

— **Lightening** condition

— Scene **geometry**

— **Surface** properties

— Camera **optics**

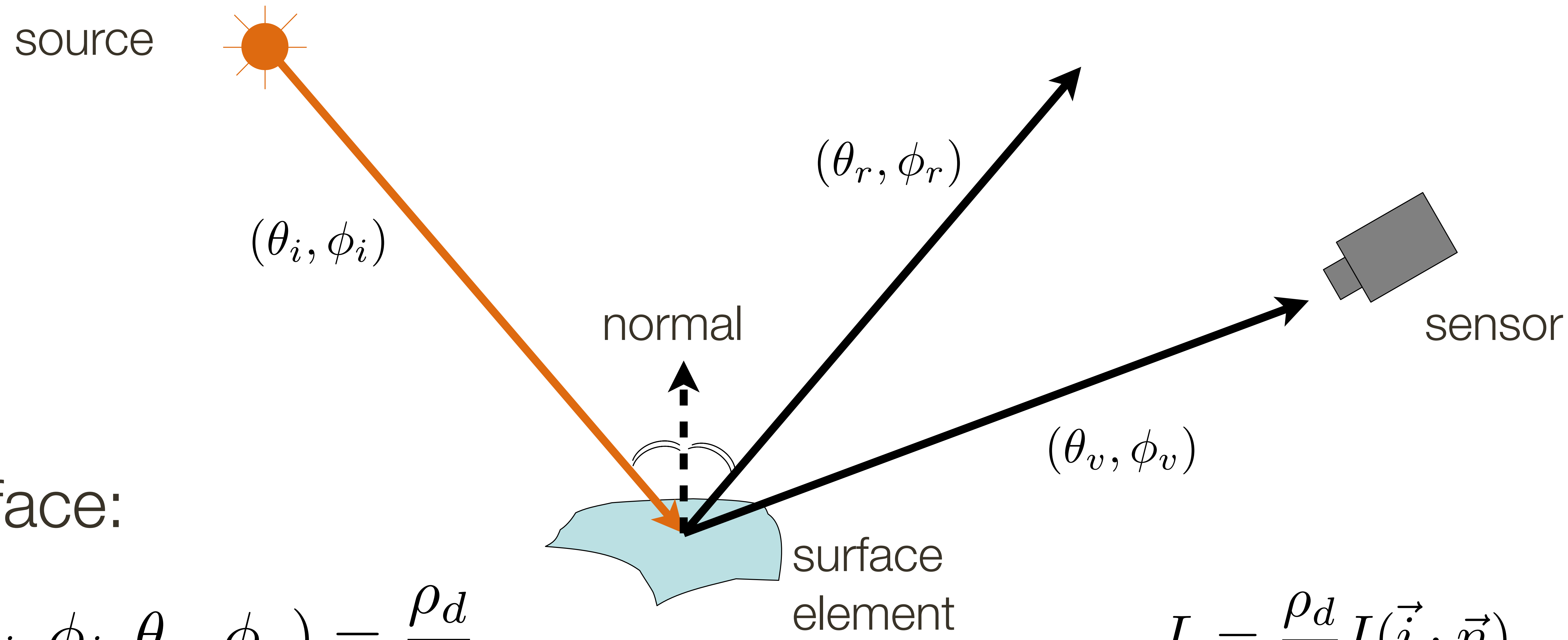


Sensor (or eye) **captures amount of light** reflected from the object

Diffuse vs **Specular** Surfaces

(small) **Graphics** Review

Surface reflection depends on both the **viewing** (θ_v, ϕ_v) and **illumination** (θ_i, ϕ_i) direction, with Bidirectional Reflection Distribution Function: $\mathbf{BRDF}(\theta_i, \phi_i, \theta_v, \phi_v)$



Lambertian surface:

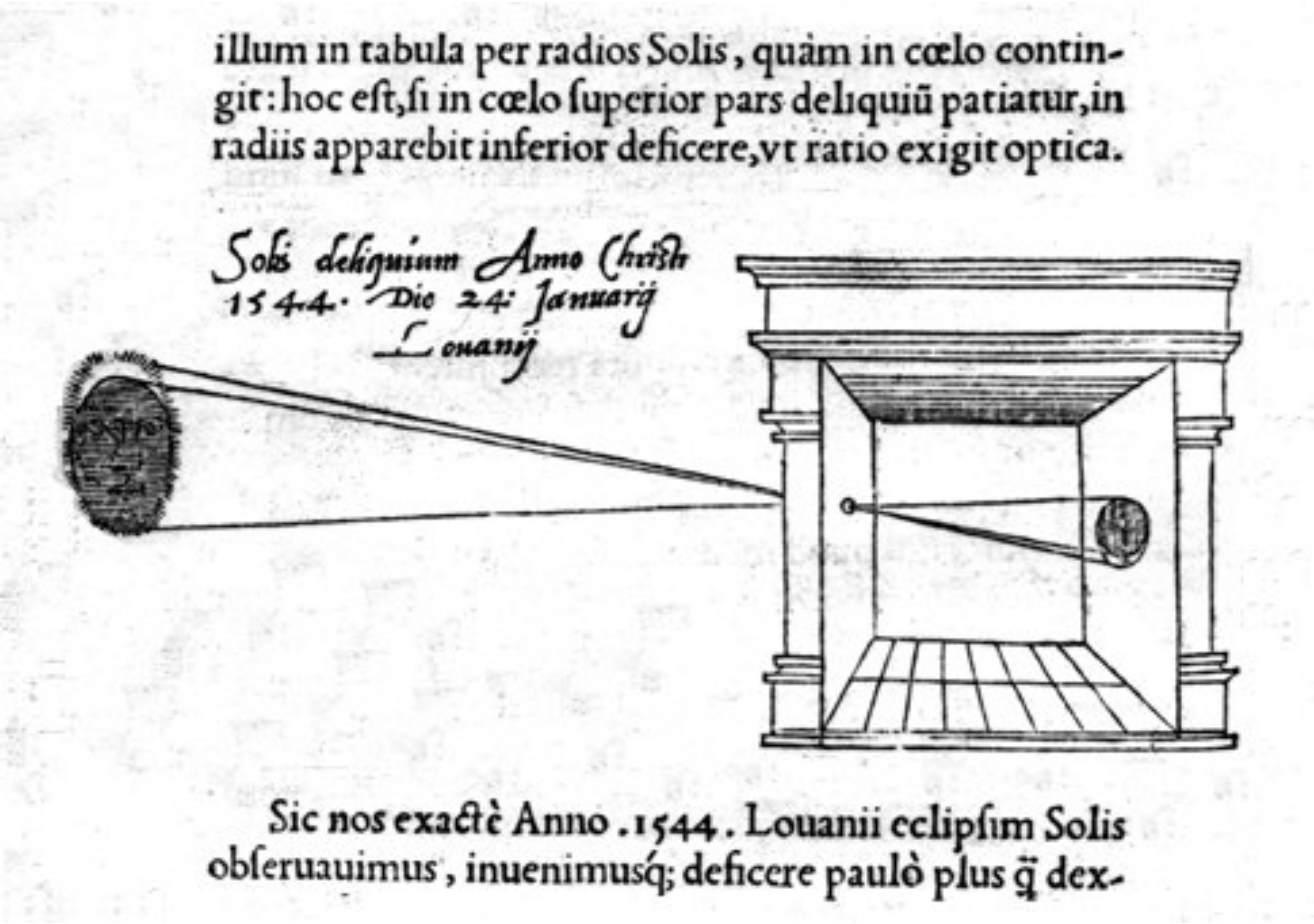
$$\mathbf{BRDF}(\theta_i, \phi_i, \theta_v, \phi_v) = \frac{\rho_d}{\pi}$$

$$L = \frac{\rho_d}{\pi} I(\vec{i} \cdot \vec{n})$$

Mirror surface: all incident light reflected in one directions $(\theta_v, \phi_v) = (\theta_r, \phi_r)$

Slide adopted from: Ioannis (Yannis) Gkioulekas (CMU)

Camera Obscura (latin for “dark chamber”)



Reinerus Gemma-Frisius observed an eclipse of the sun at Louvain on January 24, 1544. He used this illustration in his book, “De Radio Astronomica et Geometrica,” 1545. It is thought to be the first published illustration of a camera obscura.

Credit: John H., Hammond, “Th Camera Obscure, A Chronicle”

Camera Obscura (latin for "dark chamber")

illum in tabula per radios Solis, quàm in cœlo contin-
git: hoc eſt, ſi in cœlo ſuperior pars deliquiũ patiatur, in
radiis apparebit inferior deficere, vt ratio exigit optica.

Solis deliquium Anno Christi
1544. Die 24. Januarij
Louanij

Sic nos exactè Anno .1544. Louanii eclipſim Solis
obſeruauimus, inuenimusq; deficere paulò plus q̃ dex-

principles behind the pinhole camera or camera obscura were first mentioned by Chinese philosopher Mozi (Mo-Ti) (470 to 390 BCE)

Reinerus Gemma-Frisius observed an eclipse of the sun at Louvain on January 24, 1544. He used this illustration in his book, "De Radio Astronomica et Geometrica," 1545. It is thought to be the first published illustration of a camera obscura.

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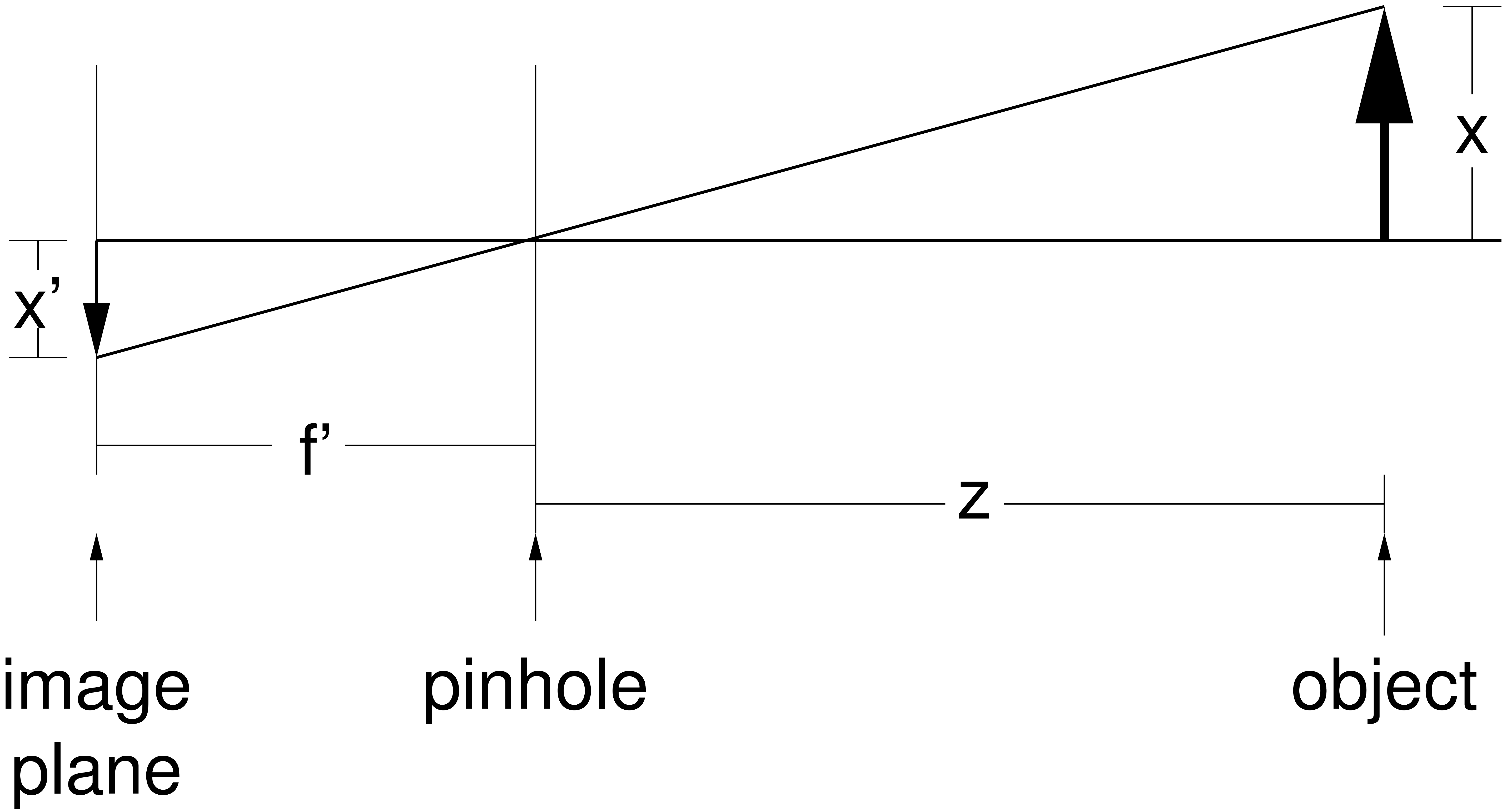
Pinhole Camera (Simplified)

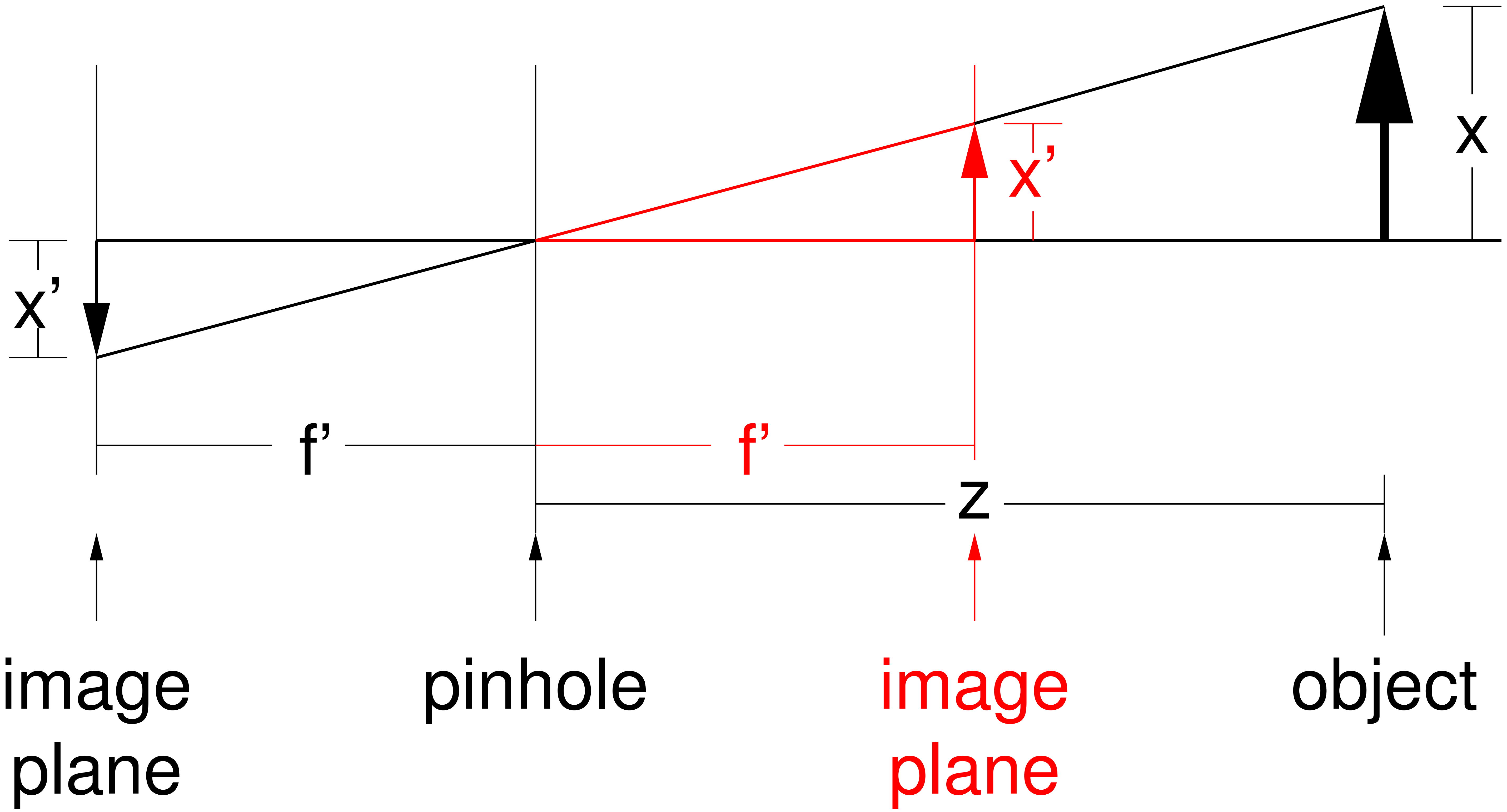
f' is the **focal length** of the camera



Note: In a pinhole camera we can adjust the focal length, all this will do is change the **size** of the resulting image

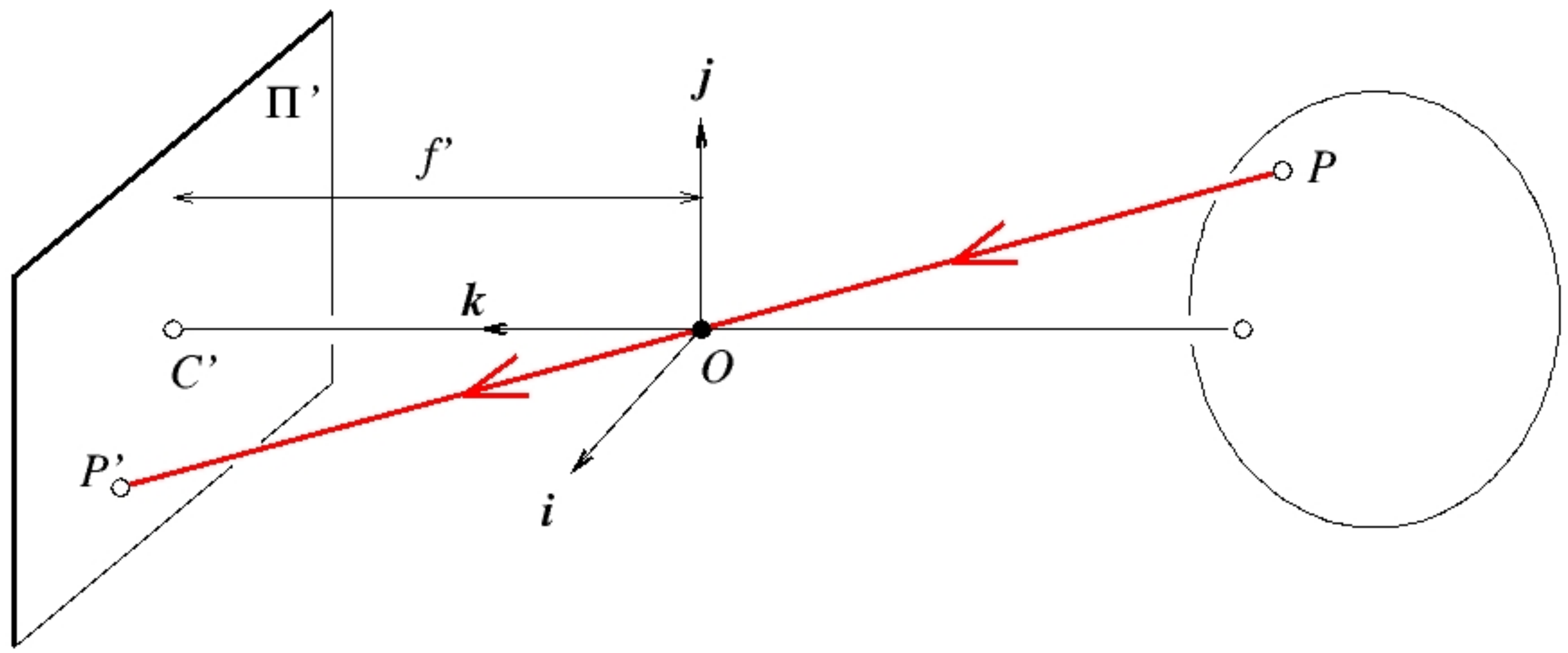
Pinhole Camera (Simplified)

It is convenient to think of the **image plane** which is in from of the pinhole



What happens if object moves towards the camera? Away from the camera?

Perspective Projection



3D object point

Forsyth & Ponce (1st ed.) Figure 1.4

$$P = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$ projects to 2D image point $$P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$$ where

$$\boxed{\begin{aligned} x' &= f' \frac{x}{z} \\ y' &= f' \frac{y}{z} \end{aligned}}$$

Note: this assumes world coordinate frame at the optical center (pinhole) and aligned with the image plane, image coordinate frame aligned with the camera coordinate frame

Summary of **Projection Equations**

3D object point $P = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ projects to 2D image point $P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$ where

Perspective	$x' = f' \dfrac{x}{z}$ $y' = f' \dfrac{y}{z}$
Weak Perspective	$x' = m\,x$ $y' = m\,y$ $m = \dfrac{f'}{z_0}$
Orthographic	$x' = x$ $y' = y$

Sample Question: Image Formation

True of **false**: A pinhole camera uses an orthographic projection.

Why **Not** a Pinhole Camera?

— If pinhole is **too big** then many directions are averaged, blurring the image

— If pinhole is **too small** then diffraction becomes a factor, also blurring the image

— Generally, pinhole cameras are **dark**, because only a very small set of rays from a particular scene point hits the image plane

— Pinhole cameras are **slow**, because only a very small amount of light from a particular scene point hits the image plane per unit time

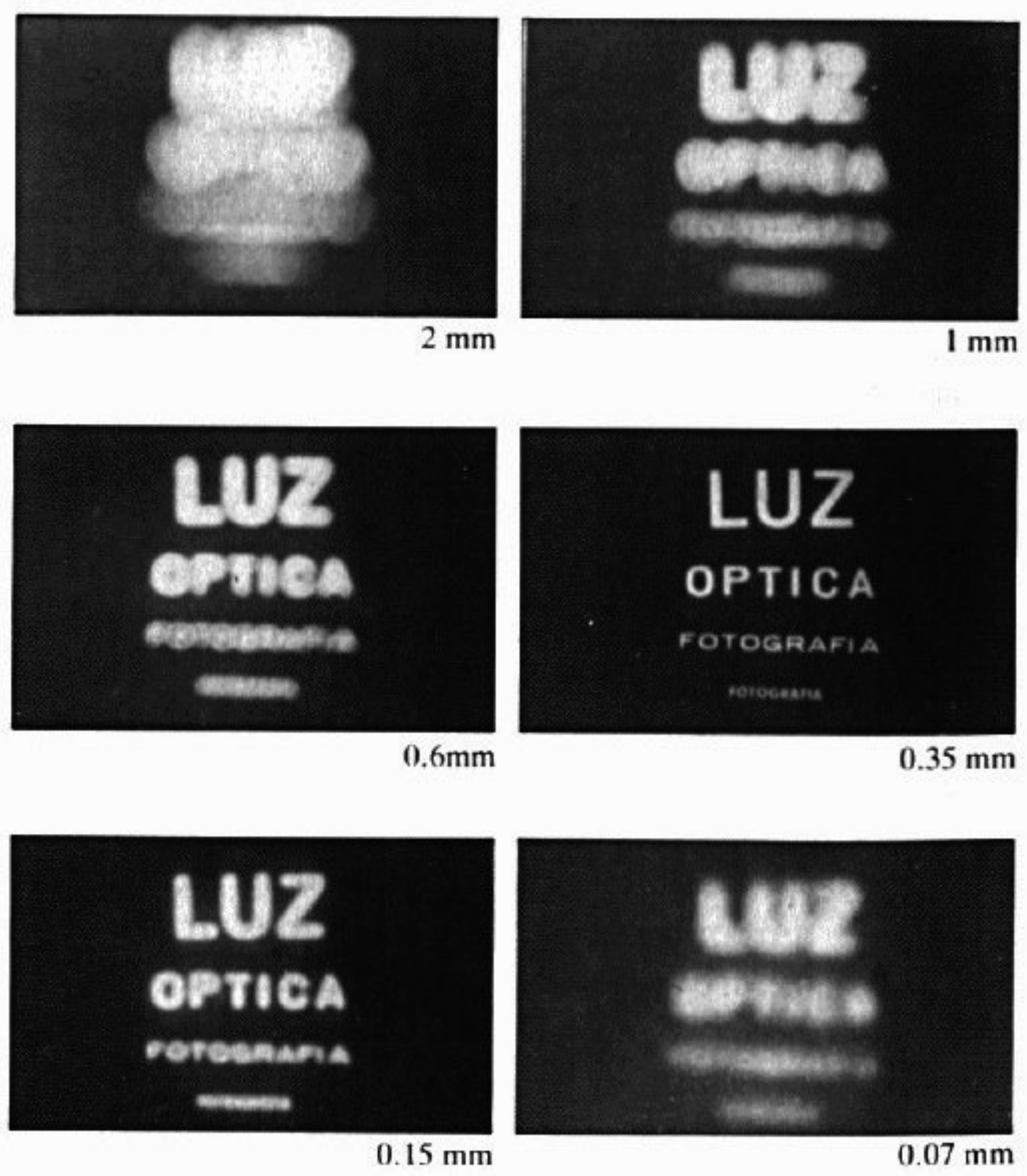
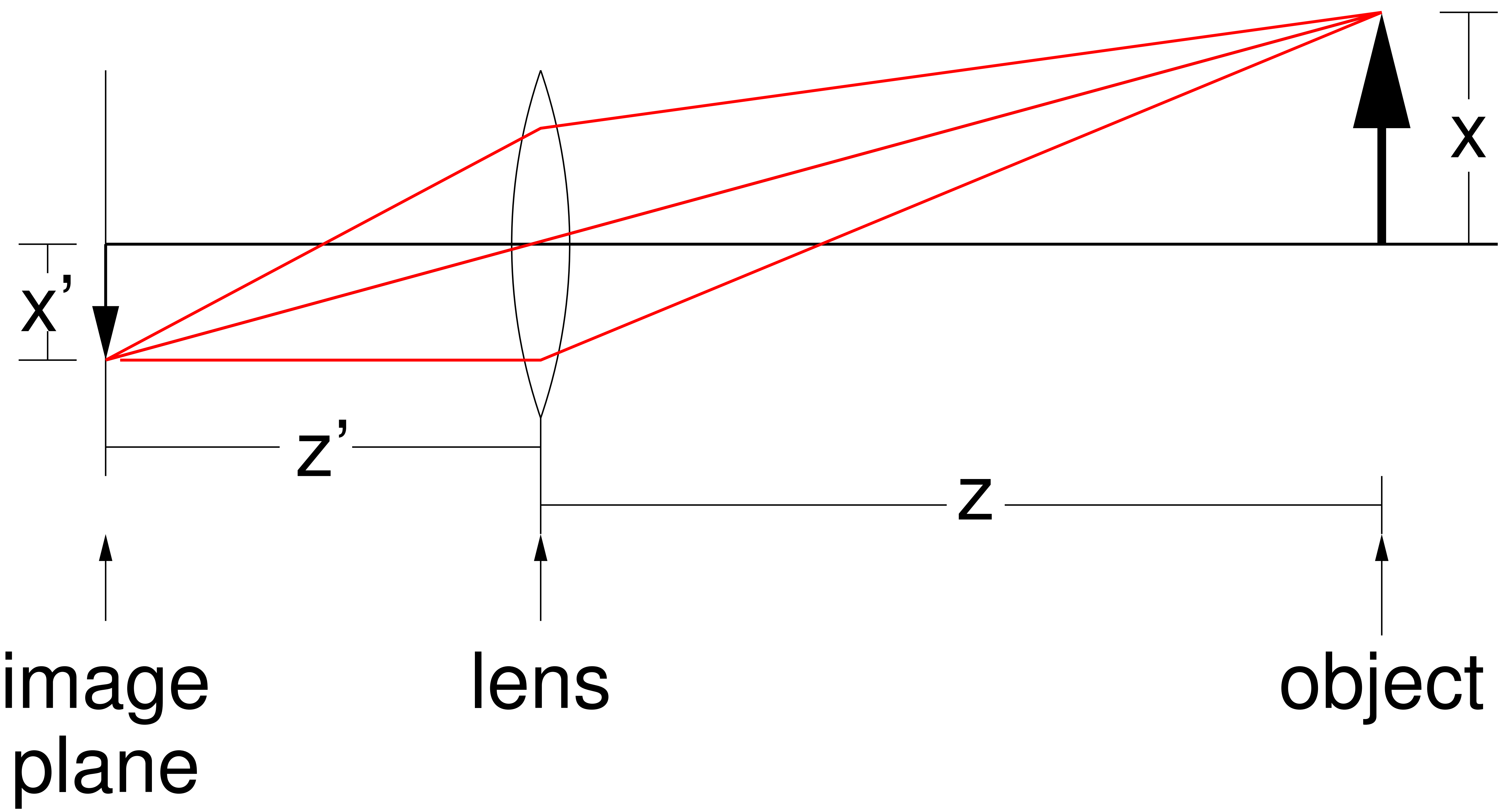


Image Credit: Credit: E. Hecht. “Optics,” Addison-Wesley, 1987

Pinhole Model (Simplified) **with Lens**



Vignetting

Image Credit: Cambridge in Colour

Chromatic **Aberration**

- Index of **refraction depends on wavelength**, λ, of light
- Light of different colours follows different paths
- Therefore, not all colours can be in equal focus

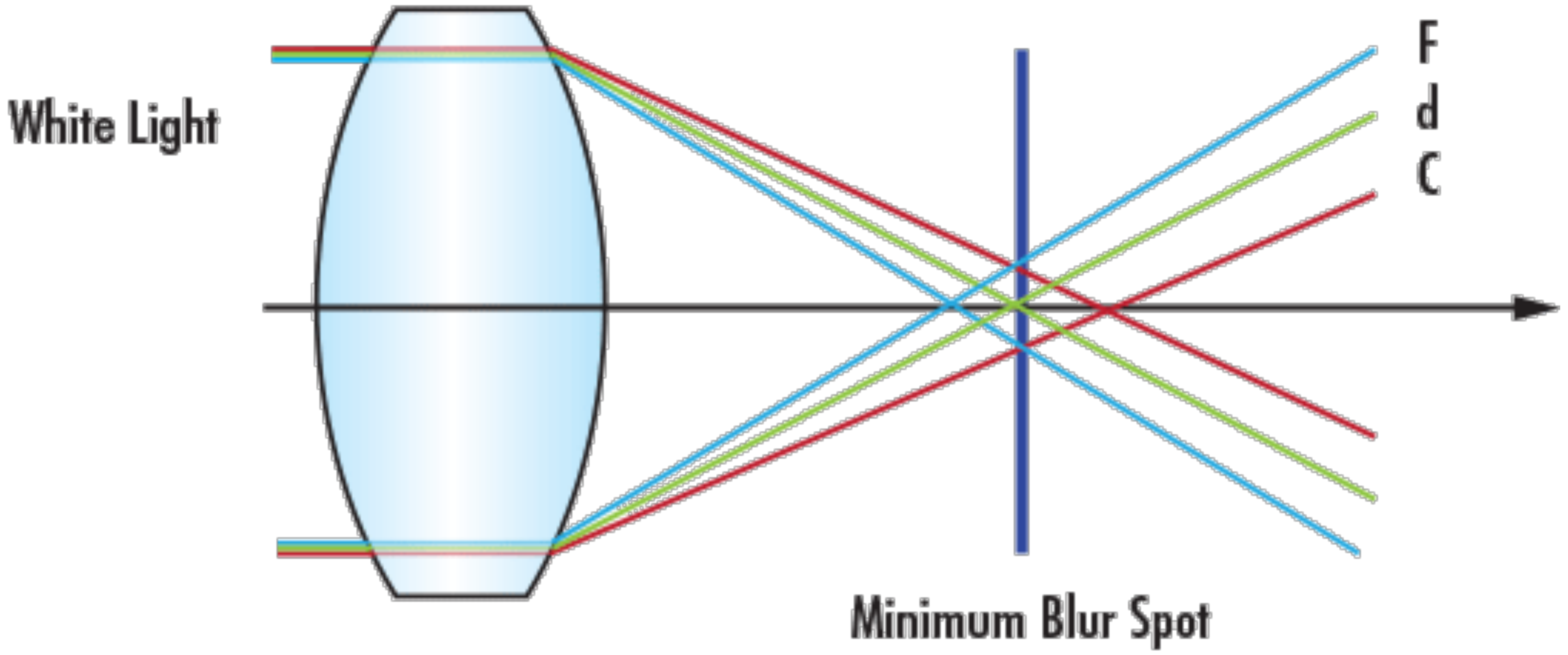
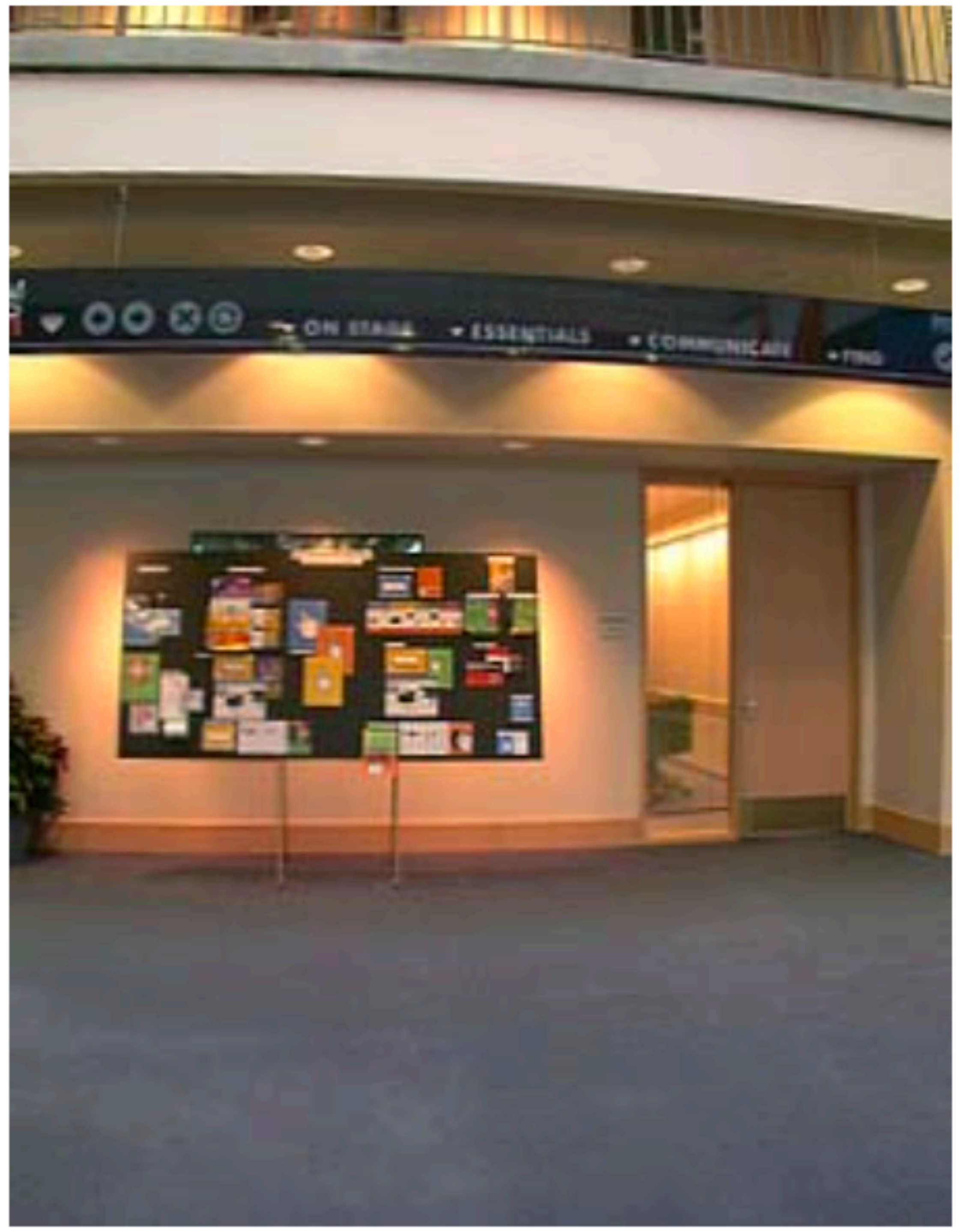
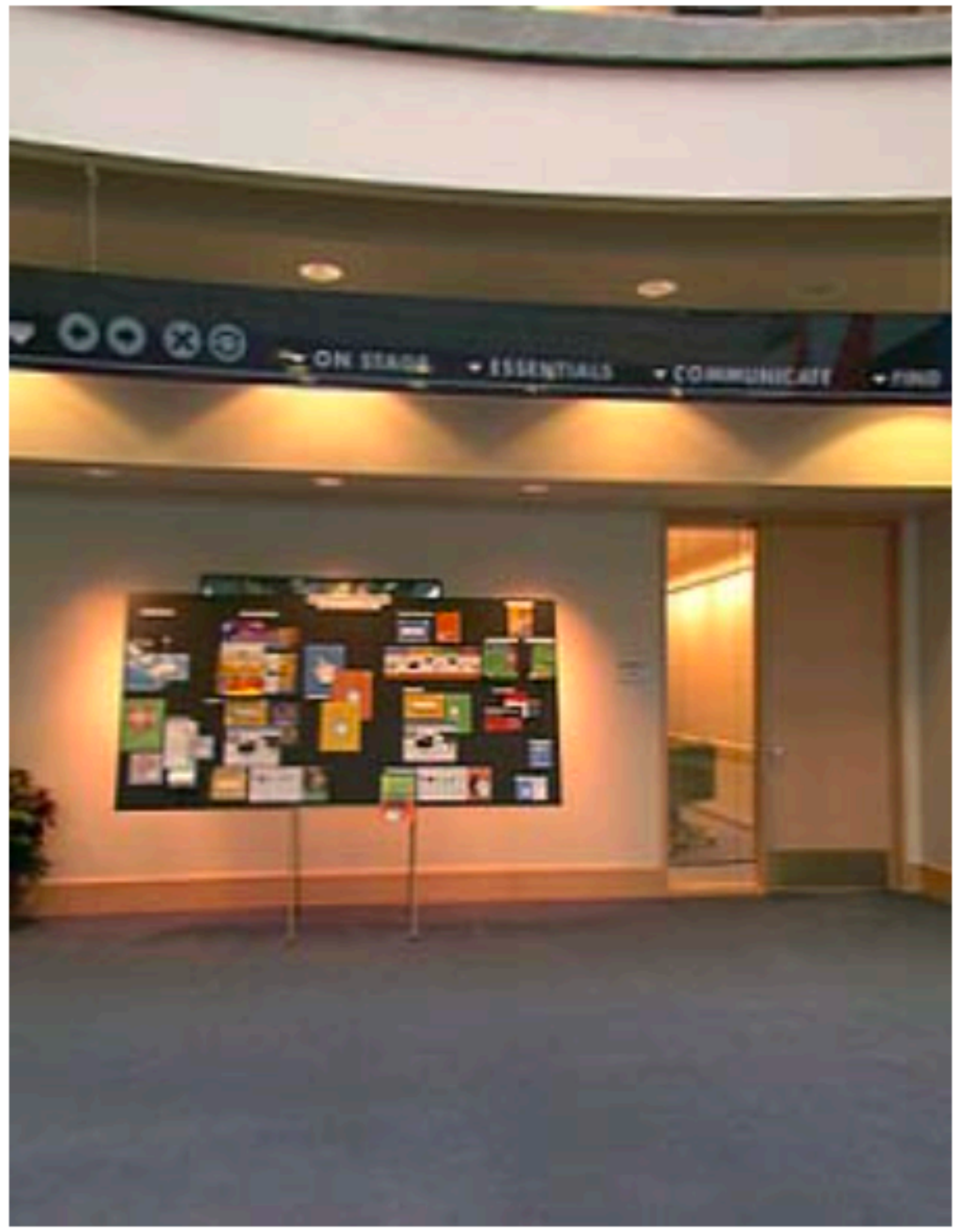


Image Credit: Trevor Darrell

Lens **Distortion**





Fish-eye Lens

Szeliski (1st ed.) Figure 2.13

Lines in the world are no longer lines on the image, they are curves!

Sample Question: Cameras and Lenses

True of **false**: Snell's Law describes how much light is reflected and how much passes through the boundary between two materials.

Linear **Filters**

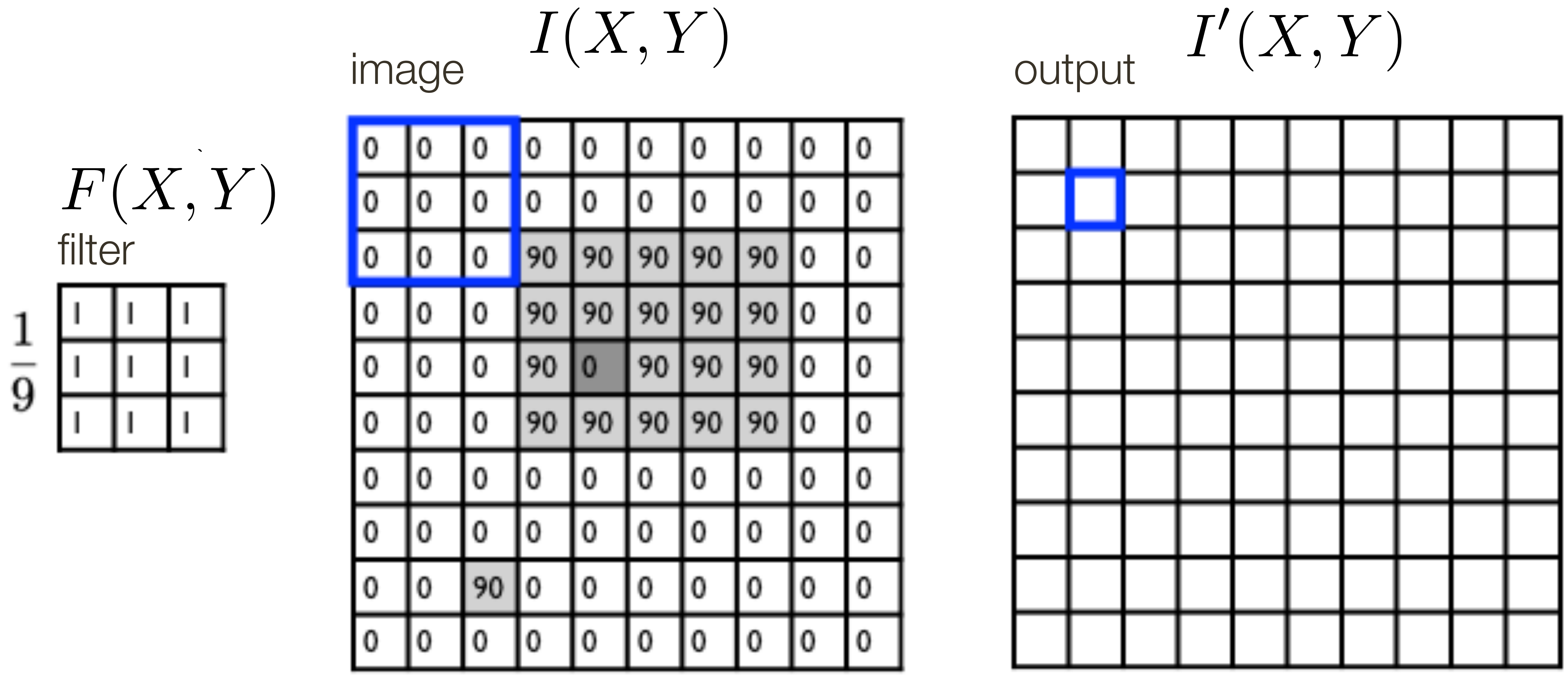
$$\boxed{I'(X,Y)} = \sum_{j=-k}^{k} \sum_{i=-k}^{k} \boxed{F(I,J)}\boxed{I(X+i,Y+j)}$$

output — filter — image (signal)

For a give X and Y, superimpose the filter on the image centered at (X,Y)

Compute the new pixel value, $I'(X,Y)$, as the sum of $m \times m$ values, where each value is the product of the original pixel value in $I(X,Y)$ and the corresponding values in the filter

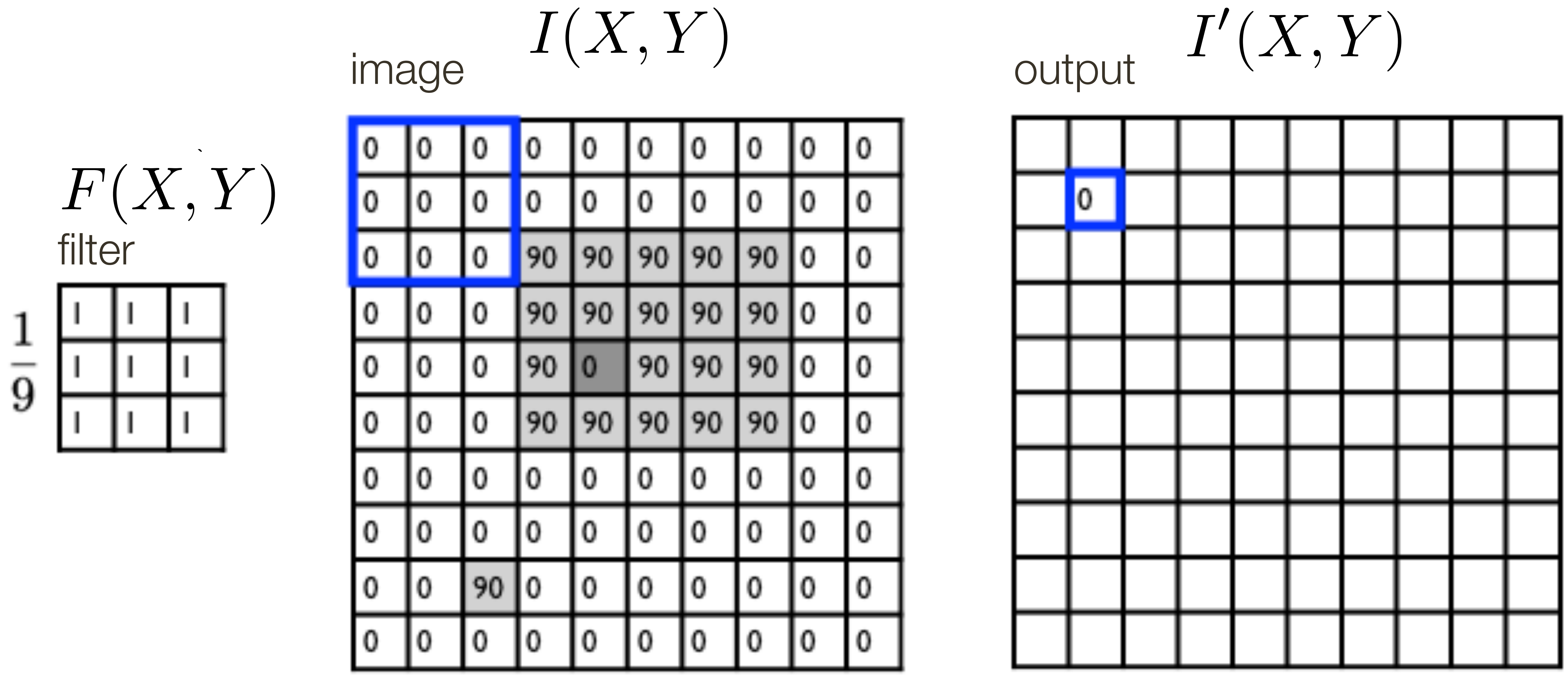
Linear Filter **Example**



$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k} \sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}} \underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filter **Example**



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Linear Filter **Example**

filter $F(X,Y)$

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0								

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k} \sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}} \underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

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Linear Filter **Example**

filter $F(X,Y)$

$$\frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10							

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k} \sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}} \underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

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Linear Filter **Example**

$F(X,Y)$

filter

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10							

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k} \sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}} \underbrace{I(X+i, Y+j)}_{\text{image (signal)}}$$

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Linear Filter **Example**

$F(X,Y)$

filter

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10	20						

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k} \sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}} \underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

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Linear Filter **Example**

$F(X,Y)$

filter

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10	20						

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k} \sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}} \underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

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Linear Filter **Example**

$F(X,Y)$

filter

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10	20	30					

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k} \sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}} \underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

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Linear Filter **Example**

$F(X,Y)$

filter

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10	20	30	30				

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k}\sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}}\,\underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

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Linear Filter **Example**

$F(X,Y)$

filter

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10	20	30	30	30			

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k} \sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}} \underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

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Linear Filter **Example**

$F(X,Y)$

filter

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10	20	30	30	30	20		

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k}\sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}}\,\underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filter **Example**

$F(X,Y)$

filter

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10	20	30	30	30	20	10	

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k}\sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}}\, \underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filter **Example**

$F(X,Y)$

filter

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

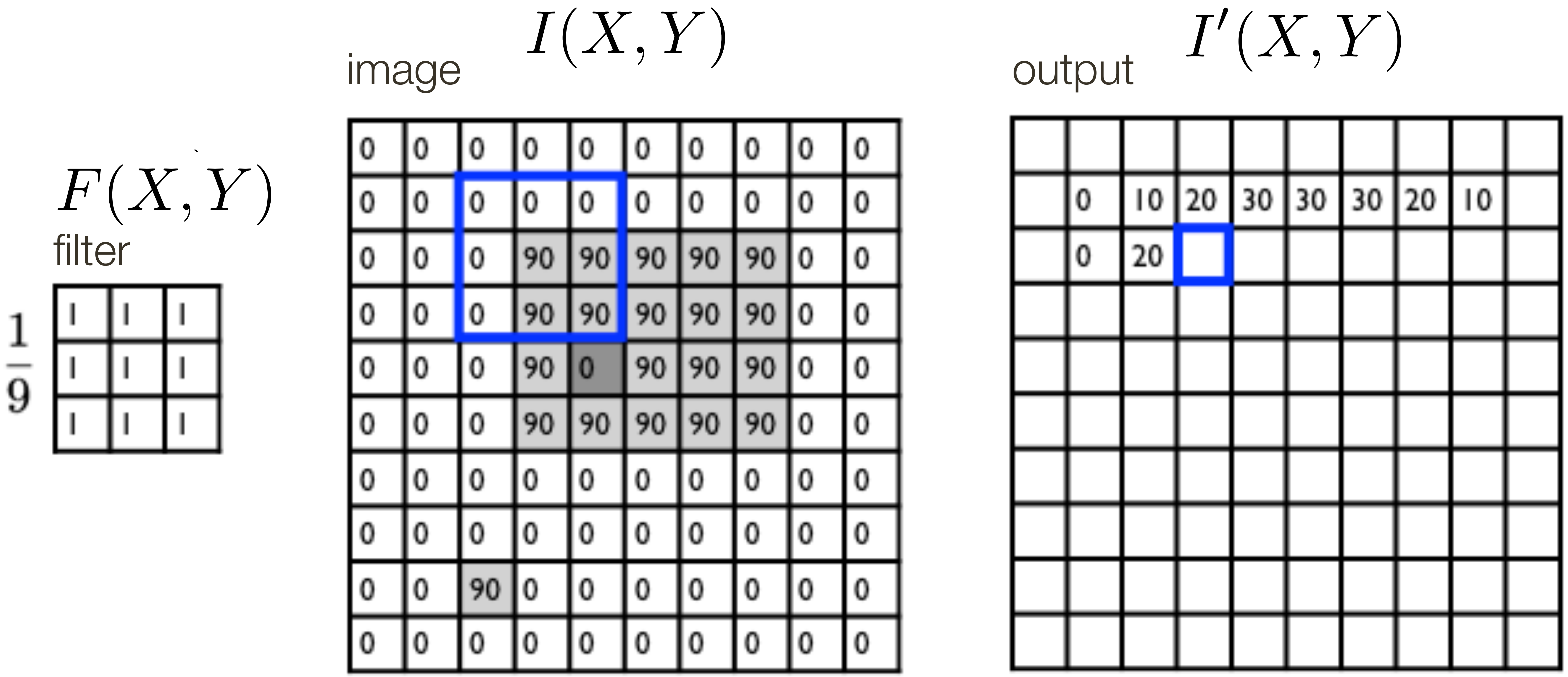
output $I'(X,Y)$

	0	10	20	30	30	30	20	10	
	0								

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k}\sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}}\underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filter **Example**



$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k} \sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}} \underbrace{I(X+i, Y+j)}_{\text{image (signal)}}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filter **Example**

$F(X,Y)$

filter

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10	20	30	30	30	20	10	
	0	20	40						

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k}\sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}}\,\underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

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Linear Filter **Example**

$F(X,Y)$

filter

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0								

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k}\sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}}\,\underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filter **Example**

$F(X,Y)$

filter

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30							

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k} \sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}} \underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filter **Example**

$F(X,Y)$

filter

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	0	10	20	30	30	30	20	10	
	10	10	10	10	0	0	0	0	
	10								

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k}\sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}}\,\underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filter **Example**

$F(X,Y)$

filter

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	0	10	20	30	30	30	20	10	
	10	10	10	10	0	0	0	0	
	10	10	10	10	0	0	0	0	

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k} \sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}} \underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filter **Example**

filter $F(X,Y)$

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

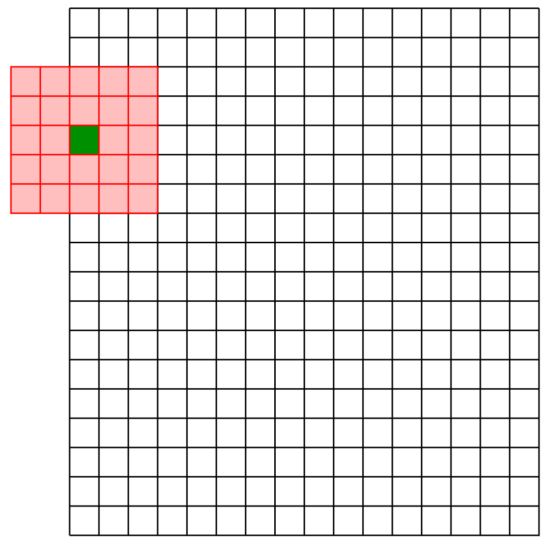
output $I'(X,Y)$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	0	10	20	30	30	30	20	10	
	10	10	10	10	0	0	0	0	
	10	10	10	10	0	0	0	0	

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k}\sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}}\,\underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filters: **Boundary** Effects



Three standard ways to deal with boundaries:

1. **Ignore these locations:** Make the computation undefined for the top and bottom k rows and the leftmost and rightmost k columns

2. **Pad the image with zeros**: Return zero whenever a value of I is required at some position outside the defined limits of X and Y

3. **Assume periodicity**: The top row wraps around to the bottom row; the leftmost column wraps around to the rightmost column

Linear **Filters**

— The **correlation** of $F(X,Y)$ and $I(X,Y)$ is:

$$\underbrace{\boxed{I'(X,Y)}}_{\text{output}} = \sum_{j=-k}^{k} \sum_{i=-k}^{k} \underbrace{\boxed{F(I,J)}}_{\text{filter}} \underbrace{\boxed{I(X+i, Y+j)}}_{\text{image (signal)}}$$

— **Visual interpretation**: Superimpose the filter F on the image I at (X,Y), perform an element-wise multiply, and sum up the values

— **Convolution** is like **correlation** except filter "flipped"

if $F(X,Y) = F(-X,-Y)$ then correlation = convolution.

Linear System: **Characterization** Theorem

Any linear, shift invariant operation can be expressed as a convolution

Linear System: **Characterization** Theorem

Any linear, shift invariant operation can be expressed as a convolution

(‘if and only if’ result)

Low-pass Filtering = "Smoothing"

Gaussian Filter

$$\frac{1}{256}\begin{array}{|c|c|c|c|c|}\hline 1 & 4 & 6 & 4 & 1 \\ \hline 4 & 16 & 24 & 16 & 4 \\ \hline 6 & 24 & 36 & 24 & 6 \\ \hline 4 & 16 & 24 & 16 & 4 \\ \hline 1 & 4 & 6 & 4 & 1 \\ \hline \end{array}$$

All of these filters are **Low-pass Filters**

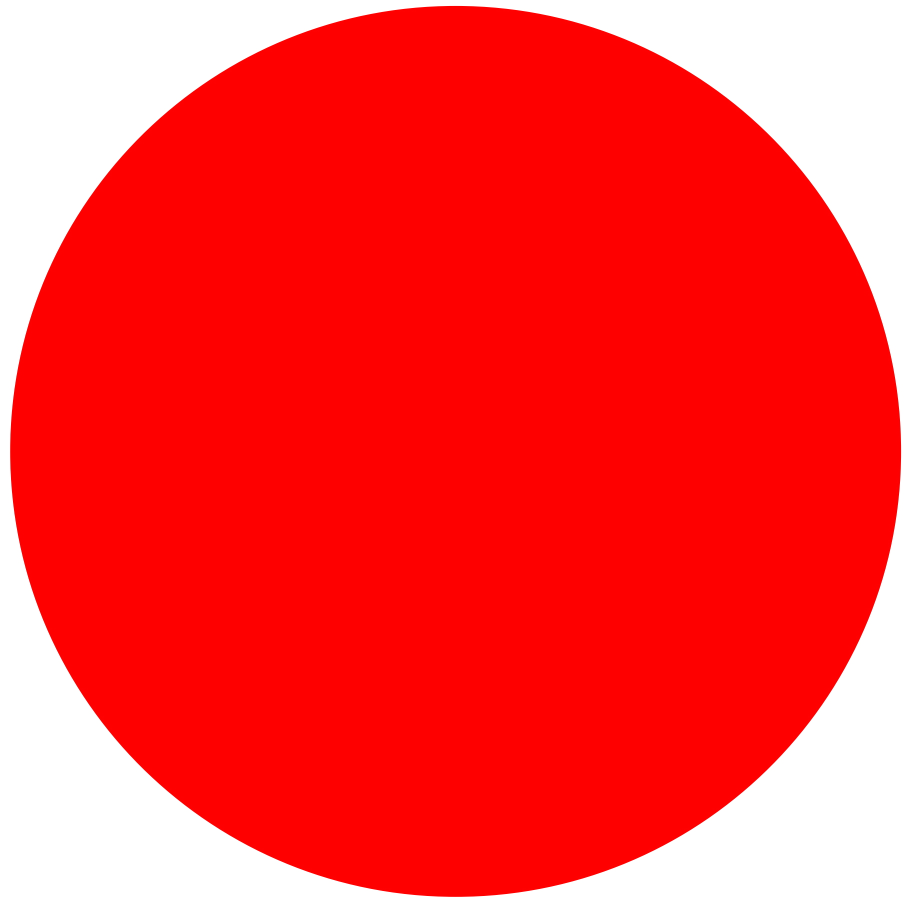
Low-pass filter: Low pass filter filters out all of the high frequency content of the image, only low frequencies remain

Other smoothing filters

Box Filter

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

Pillbox Filter

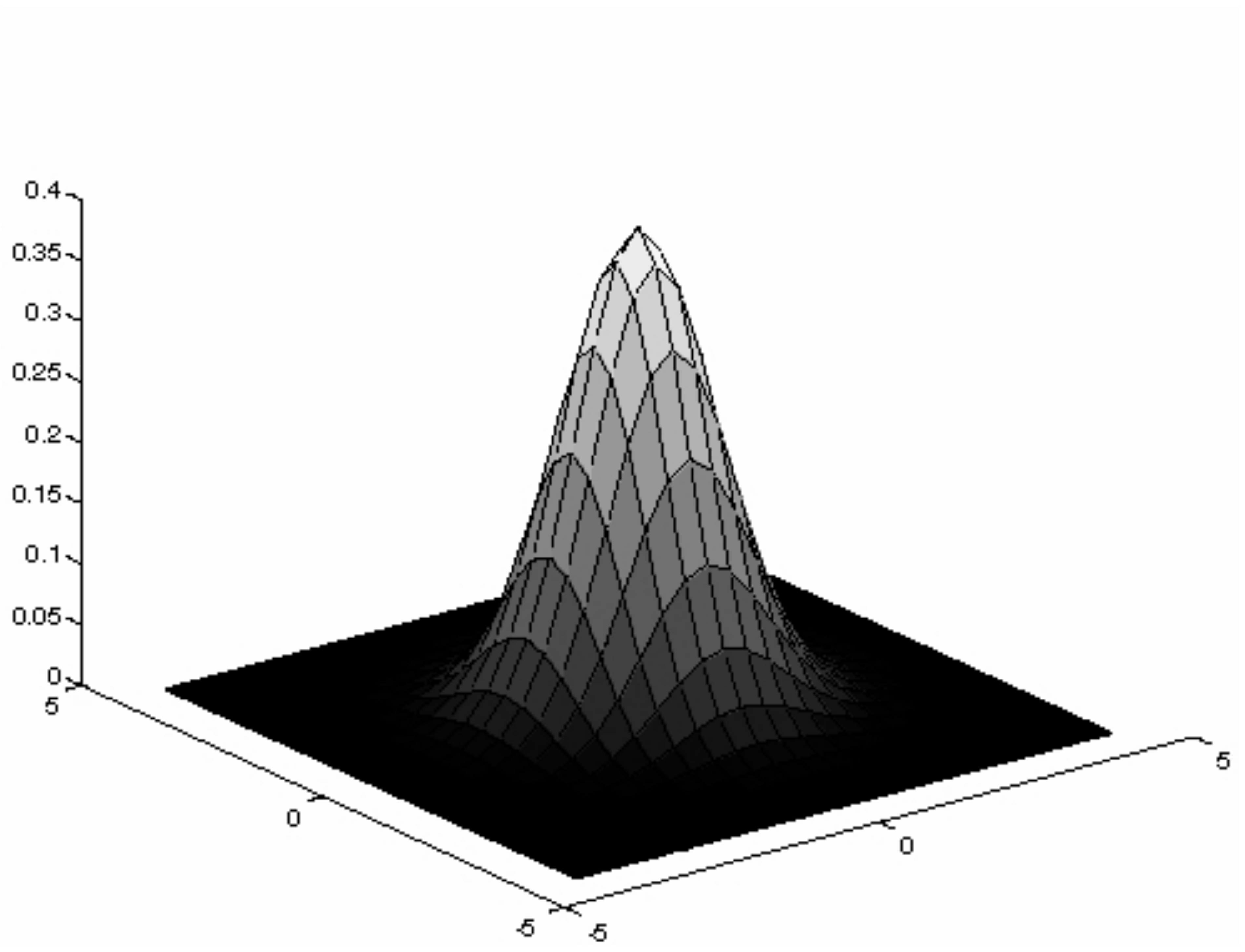


Smoothing with a Gaussian

Idea: Weight contributions of pixels by spatial proximity (nearness)

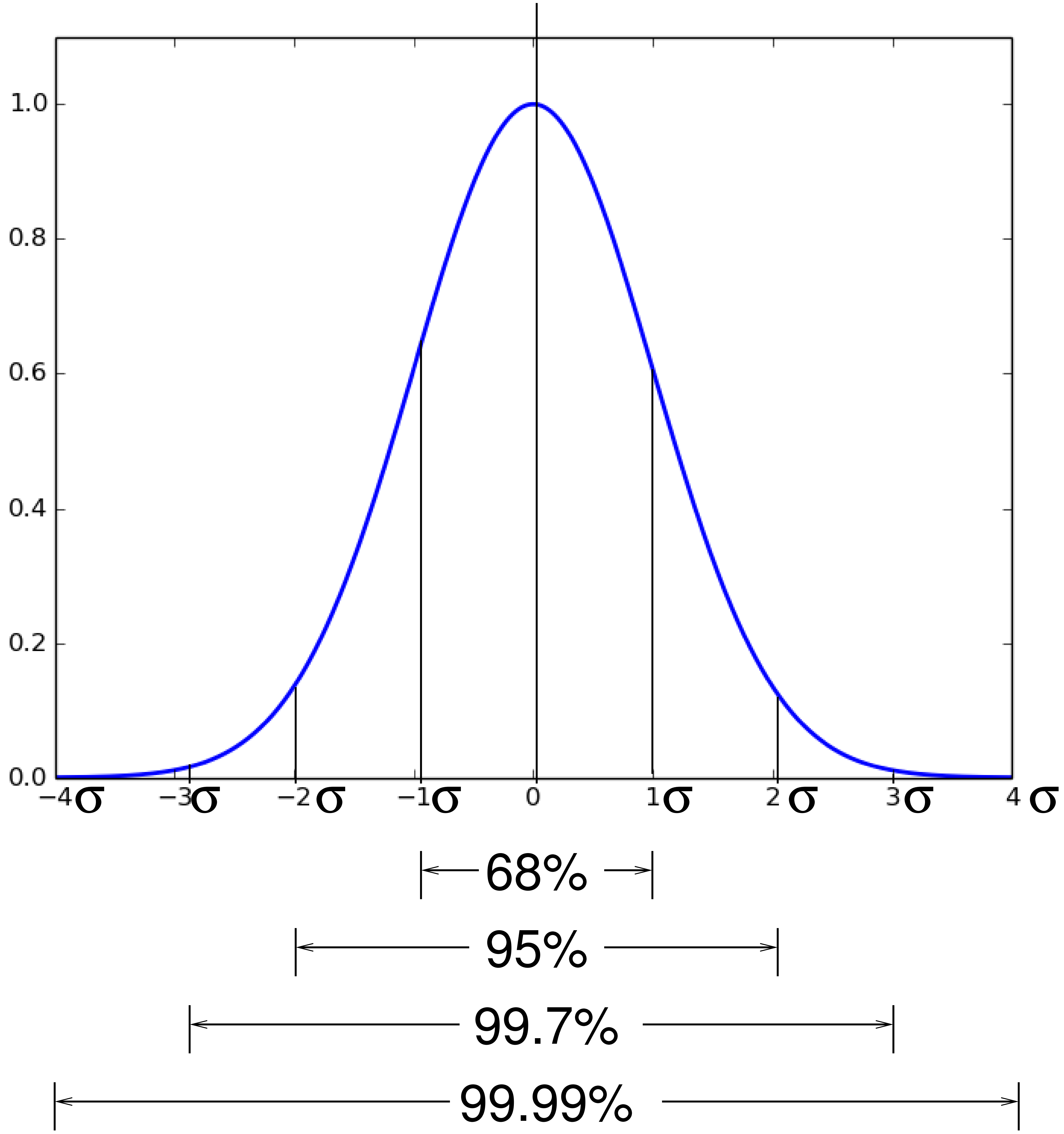
2D **Gaussian** (continuous case):

$$G_\sigma(x, y) = \frac{1}{2\pi\sigma^2} \exp^{-\frac{x^2+y^2}{2\sigma^2}}$$



Forsyth & Ponce (2nd ed.)
Figure 4.2

Gaussian: Area Under the Curve



Efficient Implementation: **Separability**

A 2D function of $\mathbf{x}$ and $\mathbf{y}$ is **separable** if it can be written as the product of two functions, one a function only of $\mathbf{x}$ and the other a function only of $\mathbf{y}$

Both the 2D box filter and the 2D Gaussian filter are separable

Both can be implemented as two 1D convolutions:
— First, convolve each row with a 1D filter
— Then, convolve each column with a 1D filter
— Aside: or vice versa

The **2D Gaussian** is the only (non trivial) 2D function that is both separable and rotationally invariant.

Linear Filters: Additional Properties

Let $\otimes$ denote convolution. Let $I(X,Y)$ be a digital image. Let F and G be digital filters

— Convolution is **associative**. That is,

$$G \otimes (F \otimes I(X,Y)) = (G \otimes F) \otimes I(X,Y)$$

— Convolution is **symmetric**. That is,

$$(G \otimes F) \otimes I(X,Y) = (G \otimes F) \otimes I(X,Y)$$

Convolving $I(X,Y)$ with filter F and then convolving the result with filter G can be achieved in single step, namely convolving $I(X,Y)$ with filter $G \otimes F = F \otimes G$

Bilateral Filter

An edge-preserving non-linear filter

Like a Gaussian filter:

— The filter weights depend on spatial distance from the center pixel
— Pixels nearby (in space) should have greater influence than pixels far away

Unlike a Gaussian filter:

— The filter weights also depend on range distance from the center pixel
— Pixels with similar brightness value should have greater influence than pixels with dissimilar brightness value

Bilateral Filter

Gaussian filter: weights of neighbor at a spatial offset (x, y) away from the center pixel $I(X, Y)$ given by:

$$G_\sigma(x, y) = \frac{1}{2\pi\sigma^2} \exp^{-\frac{x^2+y^2}{2\sigma^2}}$$

(with appropriate normalization)

Bilateral filter: weights of neighbor at a spatial offset (x, y) away from the center pixel $I(X, Y)$ given by a product:

domain kernel	$\exp^{-\frac{x^2+y^2}{2\sigma_d^2}}$	$\exp^{-\frac{(I(X+x,Y+y)-I(X,Y))^2}{2\sigma_r^2}}$	**range** kernel

(with appropriate normalization)

Bilateral Filter Application: Denoising

Noisy Image

Gaussian Filter

Bilateral Filter

Slide Credit: Alexander Wong

Sample Question: Filters

What does the following 3×3 linear, shift invariant filter compute when applied to an image?

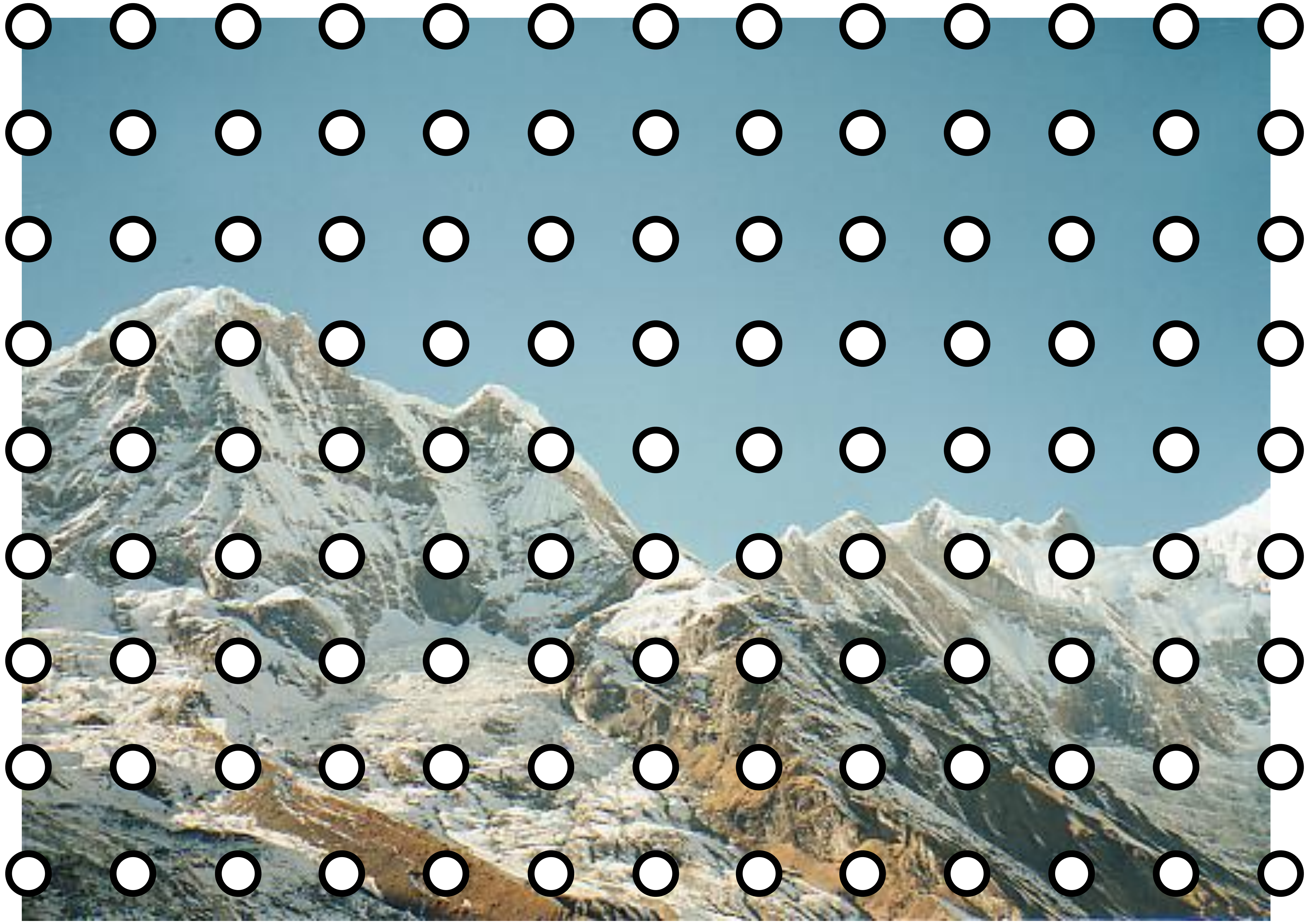
$$\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Optional subtitle

Optional subtitle

Resampling Images

- Naive method: form new image by selecting every *n*th pixel



Aliasing Example

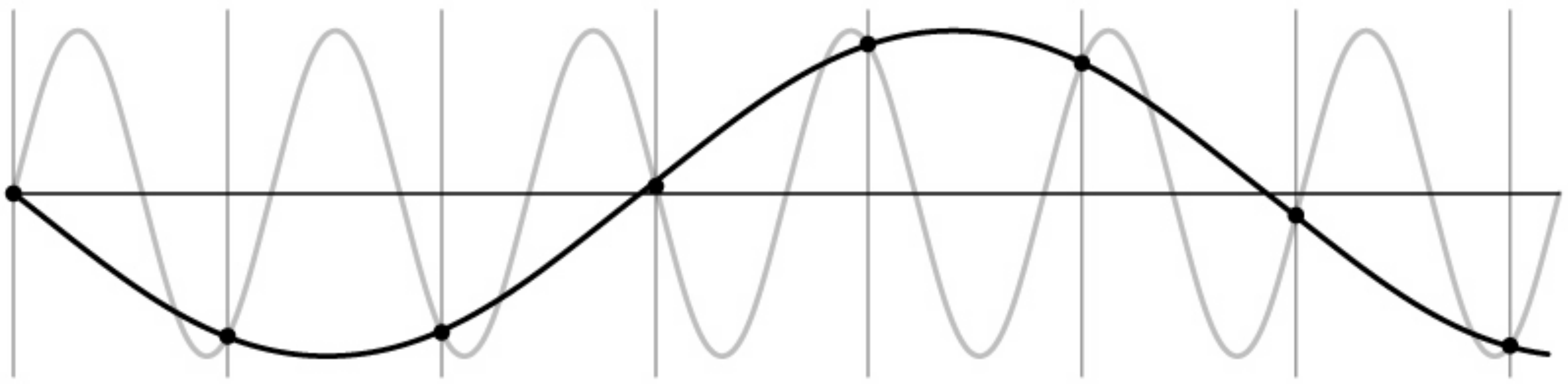
- Sampling every 5th pixel, while shifting rightwards 1 pixel at a time

Aliasing Example

- Sampling every 5th pixel, while shifting rightwards 1 pixel at a time

Example: A Simple Sine Wave

How do we discretize the signal?



Signal can be confused with one at lower frequency
— This is called “Aliasing”

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Nyquist Sampling Theorem

To avoid aliasing a signal must be sampled at twice the maximum frequency:

$$f_s > 2 \times f_{max}$$

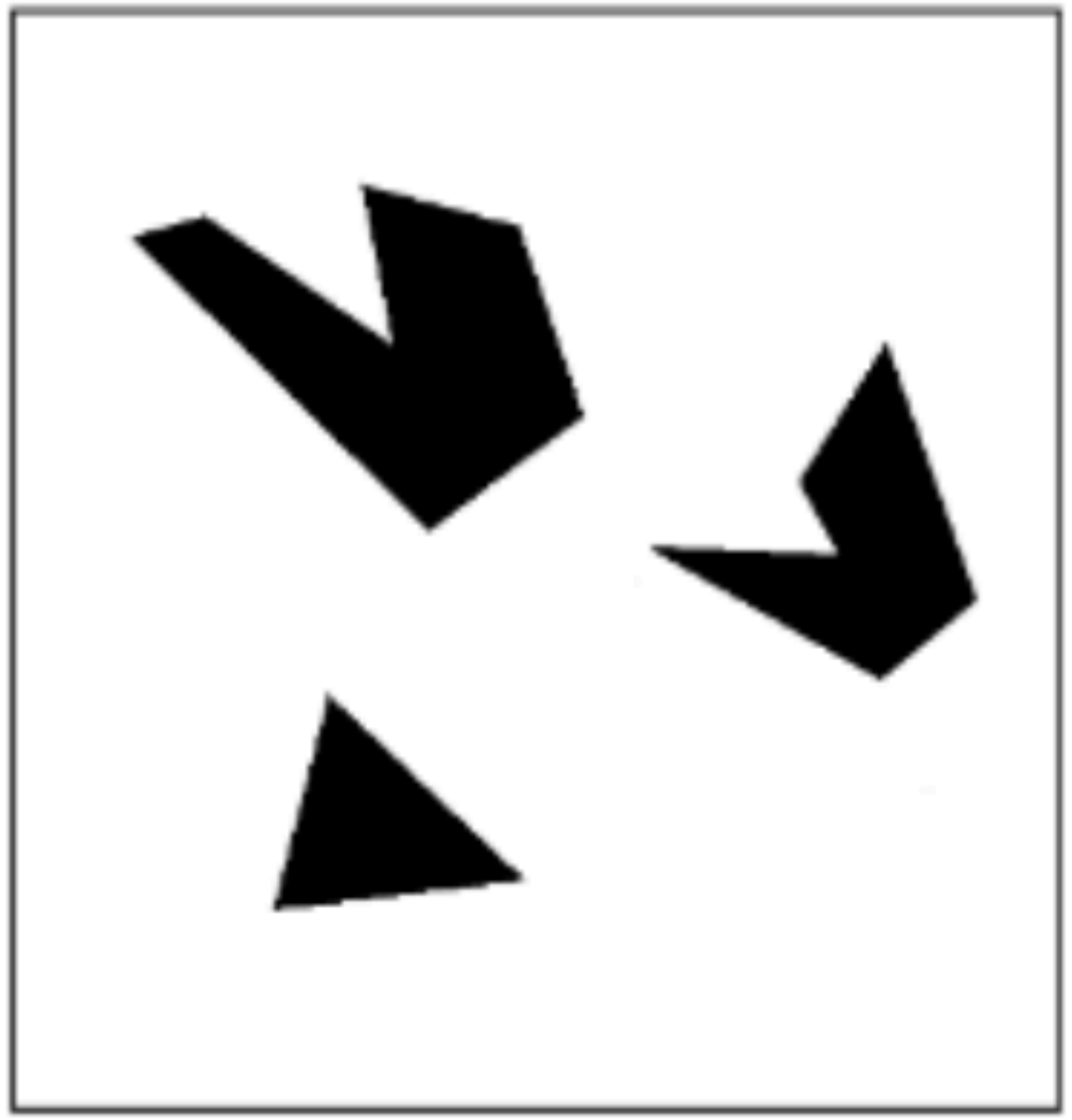
where f_s is the sampling frequency, and f_{max} is the maximum frequency present in the signal

Futhermore, Nyquist's theorem states that a signal is **exactly recoverable** from its **samples** if sampled at the **Nyquist rate** (or higher)

Note: that a signal must be **bandlimited** for this to apply (i.e., it has a maximum frequency)

Optional subtitle

Template Matching



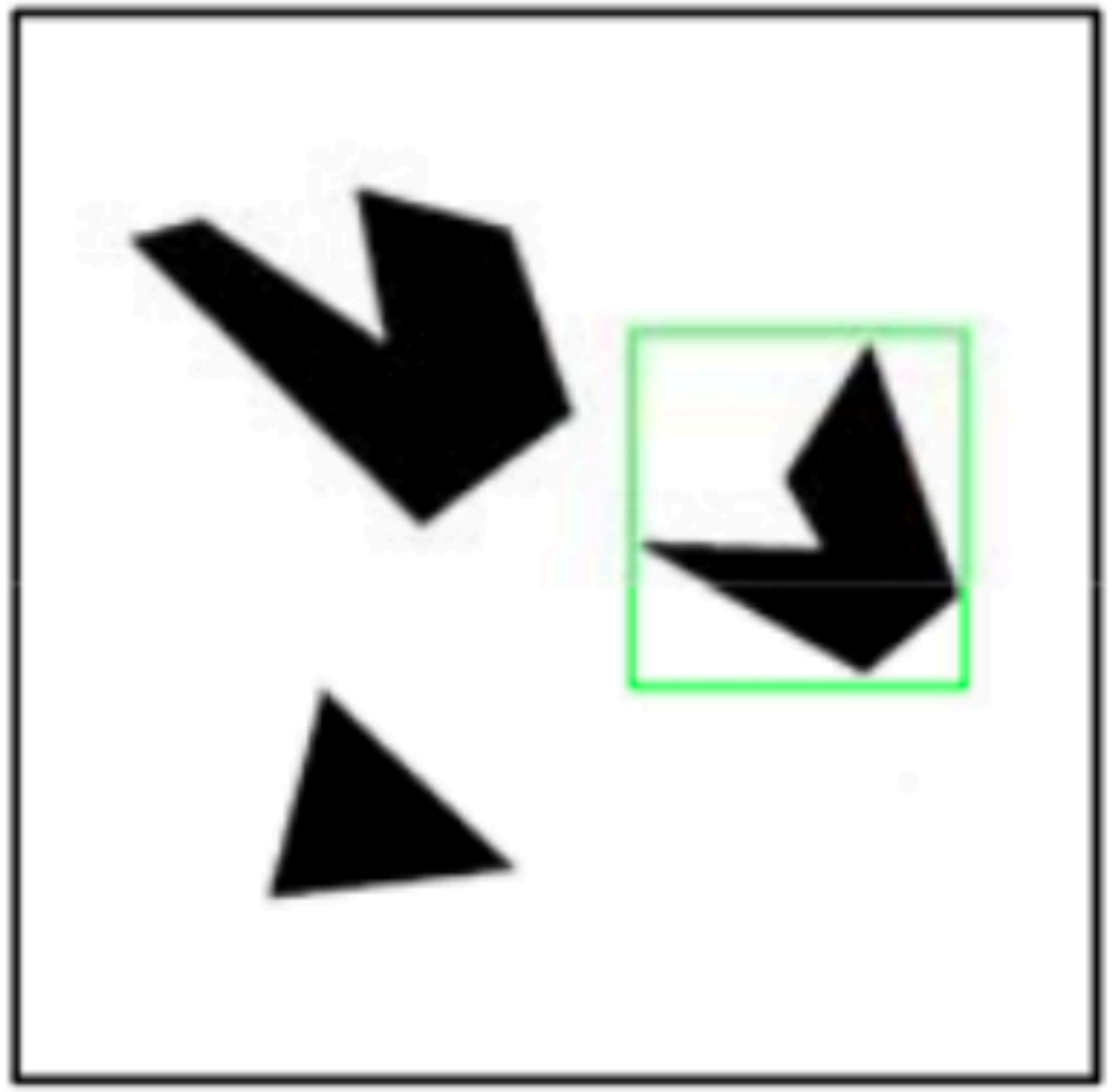
Scene

Template (mask)

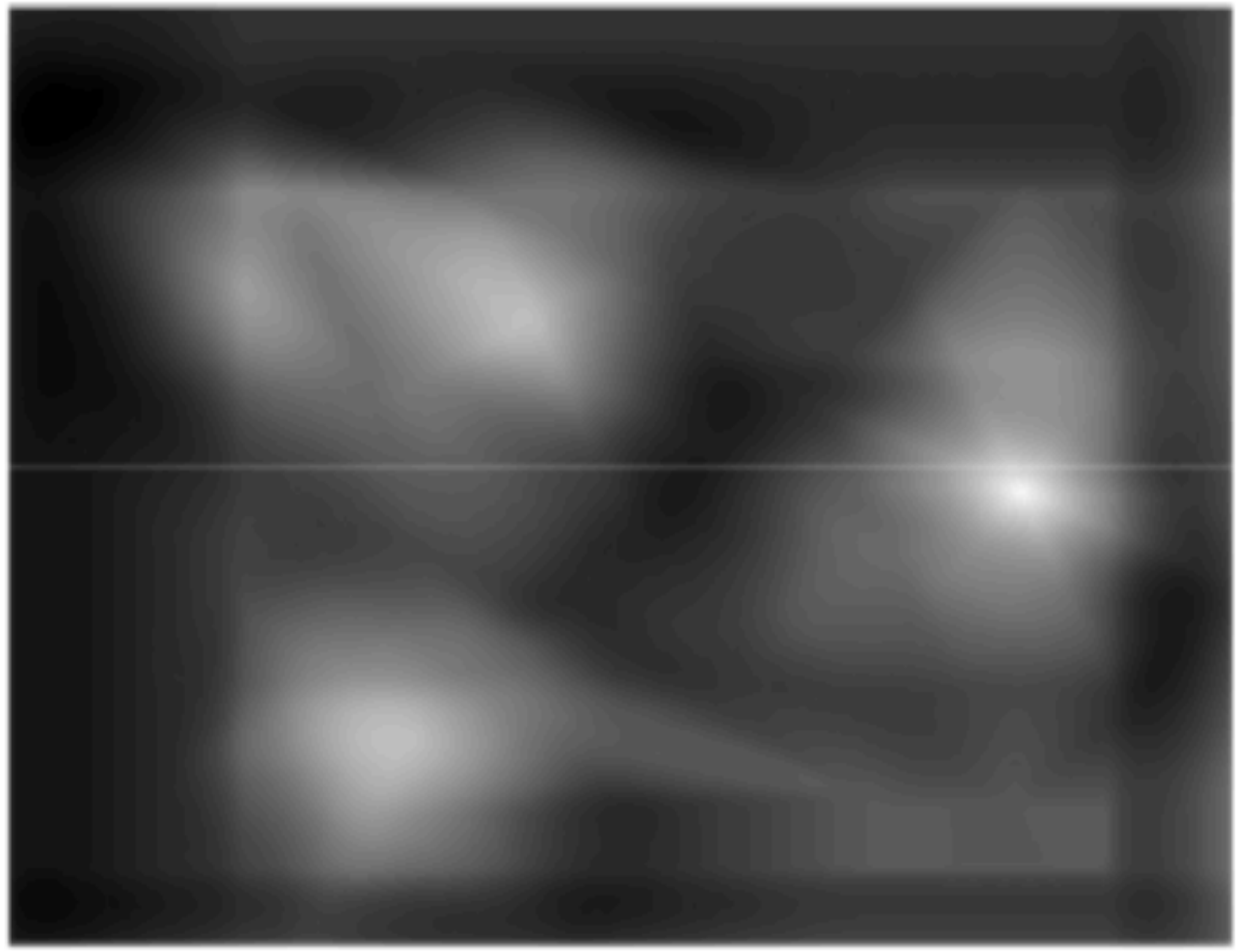
A toy example

Slide Credit: Kristen Grauman

Template Matching



Detected template



Correlation map

Slide Credit: Kristen Grauman

Template Matching

Similarity measures between a filter $\mathbf{J}$ local image region $\mathbf{I}$

Correlation, CORR = $\mathbf{I} \cdot \mathbf{J} = \mathbf{I}^T \mathbf{J}$

Normalised Correlation, NCORR = $\dfrac{\mathbf{I}^T \mathbf{J}}{|\mathbf{I}||\mathbf{J}|} = \cos\theta$

Sum Squared Difference, SSD = $|\mathbf{I} - \mathbf{J}|^2$

Normalized correlation varies between –1 and 1, attains the value 1 when the filter and image region are identical (up to a scale factor)

Minimising SSD and maximizing Normalized Correlation are equivalent if $|\mathbf{I}| = |\mathbf{J}| = 1$

Template Matching

Convolve image with template, find local maxima

Template Matching

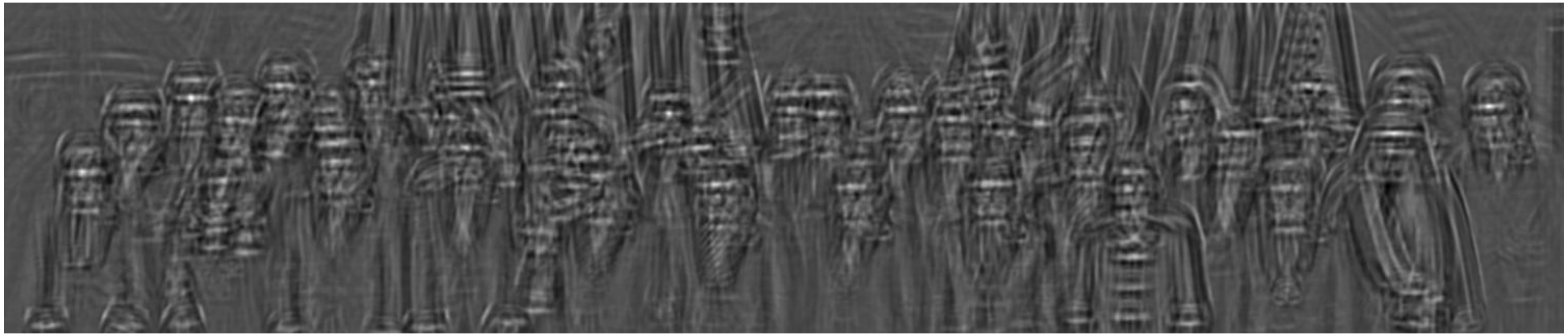
Convolve image with template, find local maxima

$*$  $\rightarrow$

Template Matching

Convolve image with template, find local maxima

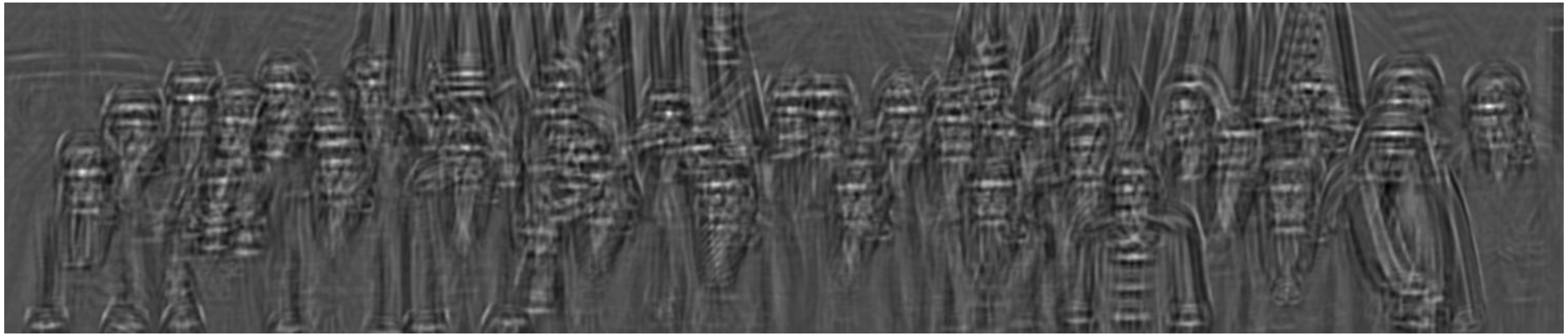
$*$  $\rightarrow$



Template Matching

Convolve image with template, find local maxima

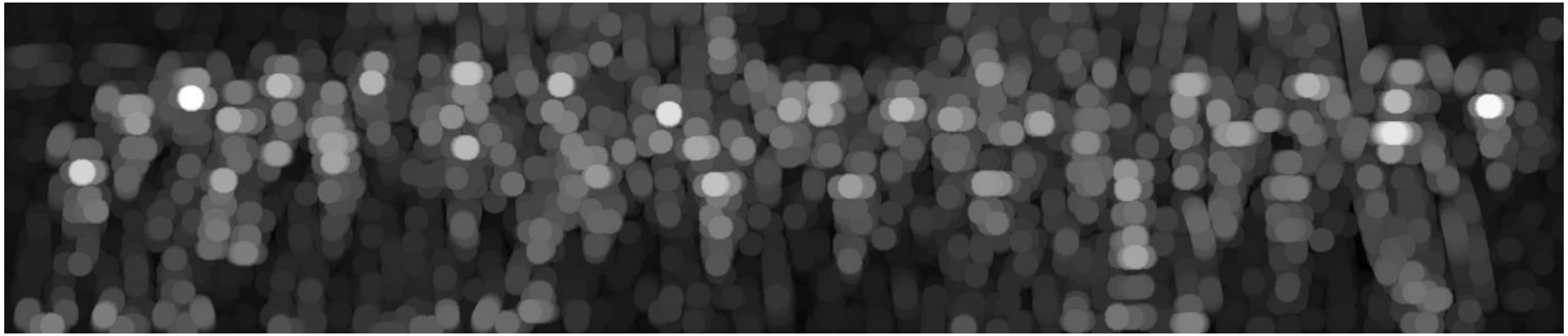
$*$  $\longrightarrow$ Non-max suppress $\longrightarrow$



Template Matching

Convolve image with template, find local maxima

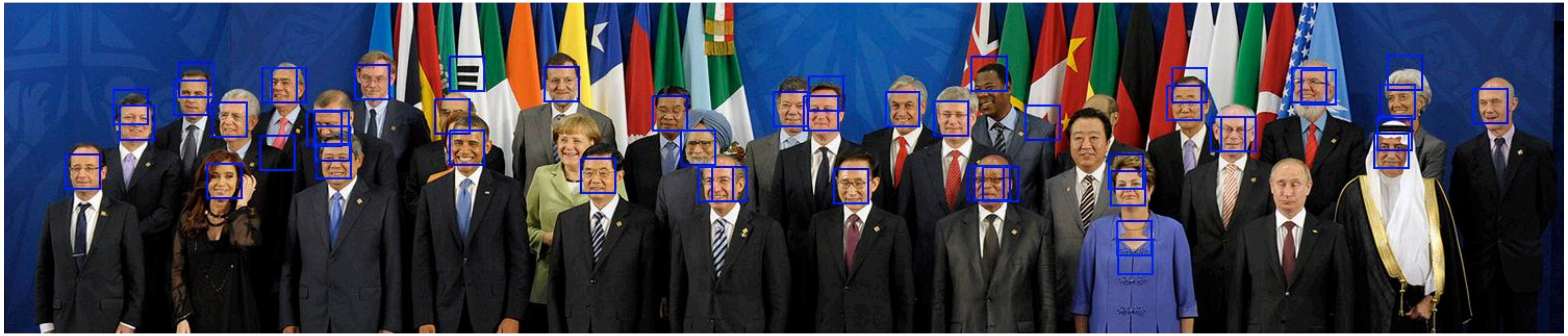
$*$  $\longrightarrow$ Non-max suppress $\longrightarrow$



Template Matching

Convolve image with template, find local maxima

$*$  $\rightarrow$ Non-max suppress + threshold $\rightarrow$



Template Matching

When might **template matching fail**?

— Different scales 

— Different orientation 

— Lighting conditions 

— Left vs. Right hand 

— Partial Occlusions 

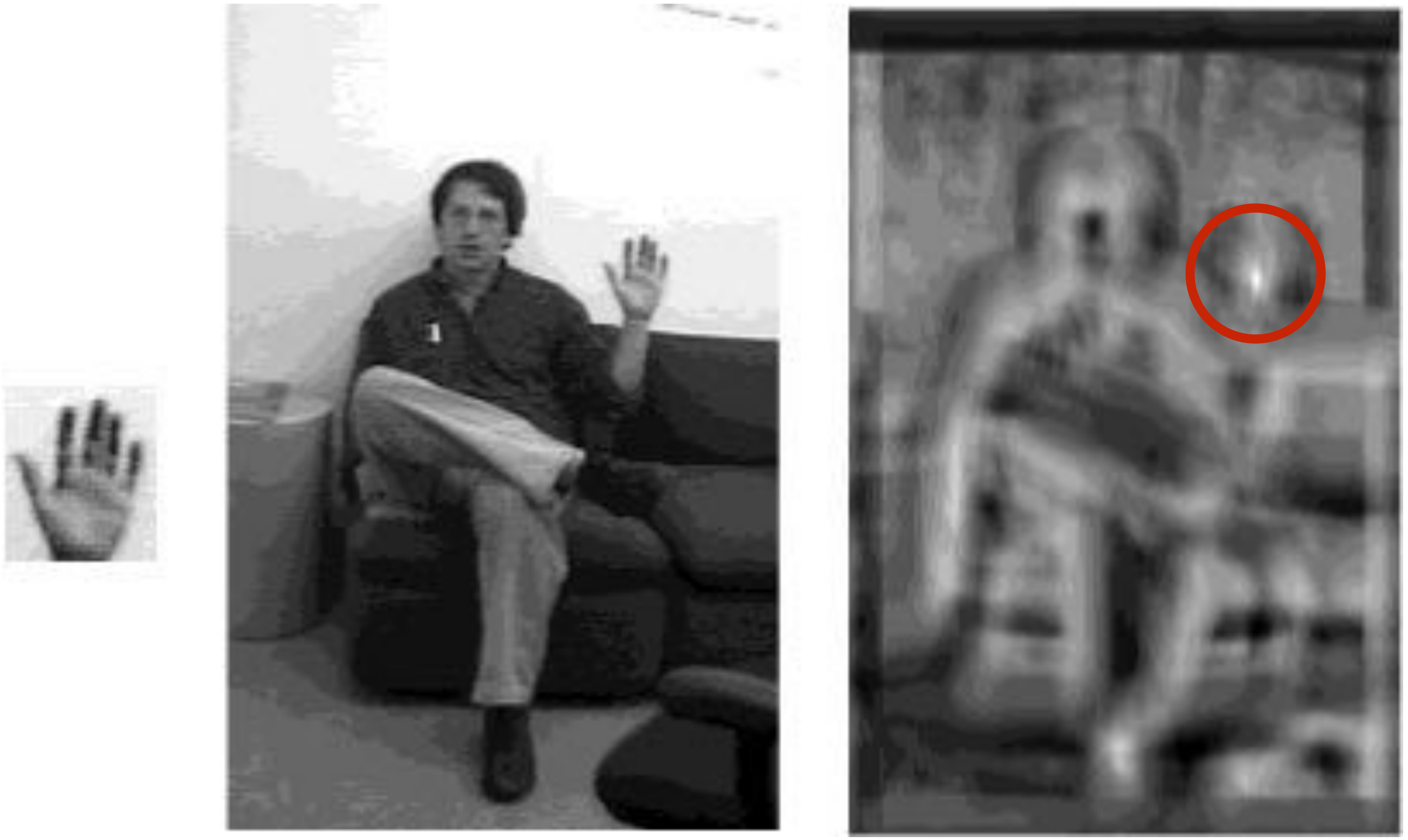
— Different Perspective

— Motion / blur

Example 1:

Template (left), image (middle), normalized correlation (right)

Note peak value at the true position of the hand



Credit: W. Freeman et al., “Computer Vision for Interactive Computer Graphics,” IEEE Computer Graphics and Applications, 1998

Sample Question: Template Matching

True or **false**: Normalized correlation is robust to a constant scaling in the image brightness.

Scaled Representations: Goals

to find **template matches** at all scales

— template size constant, image scale varies

— finding hands or faces when we don't know what size they are in the image

efficient search for image–to–image correspondences

— look first at coarse scales, refine at finer scales

— much less cost (but may miss best match)

to examine all **levels of detail**

— find edges with different amounts of blur

— find textures with different spatial frequencies (i.e., different levels of detail)

$G1$

Blur with a Gaussian kernel, then select every 2nd pixel

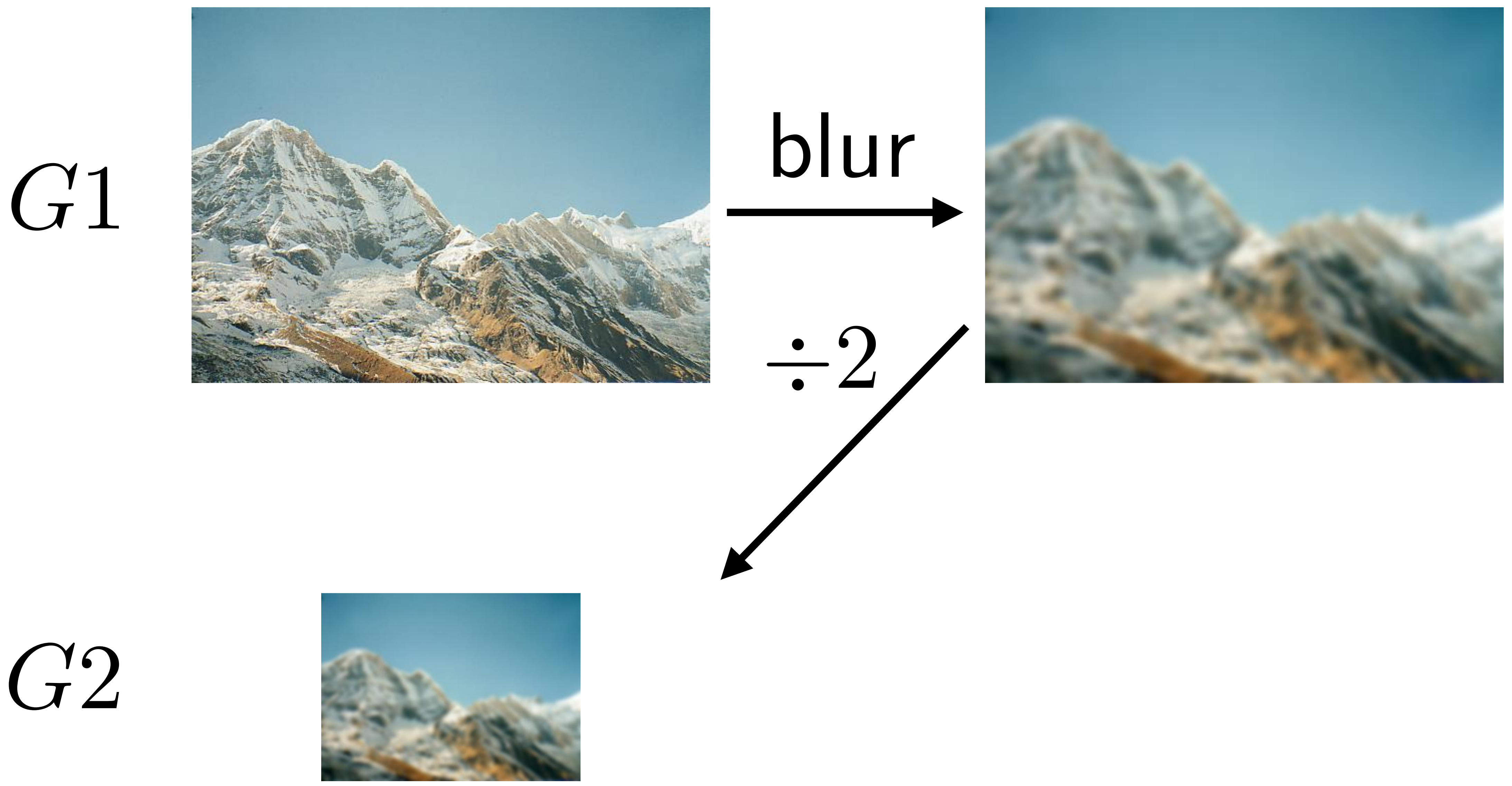
$$I_s(x, y) = I(x, y) * g_\sigma(x, y)$$

$G1$

blur $\longrightarrow$

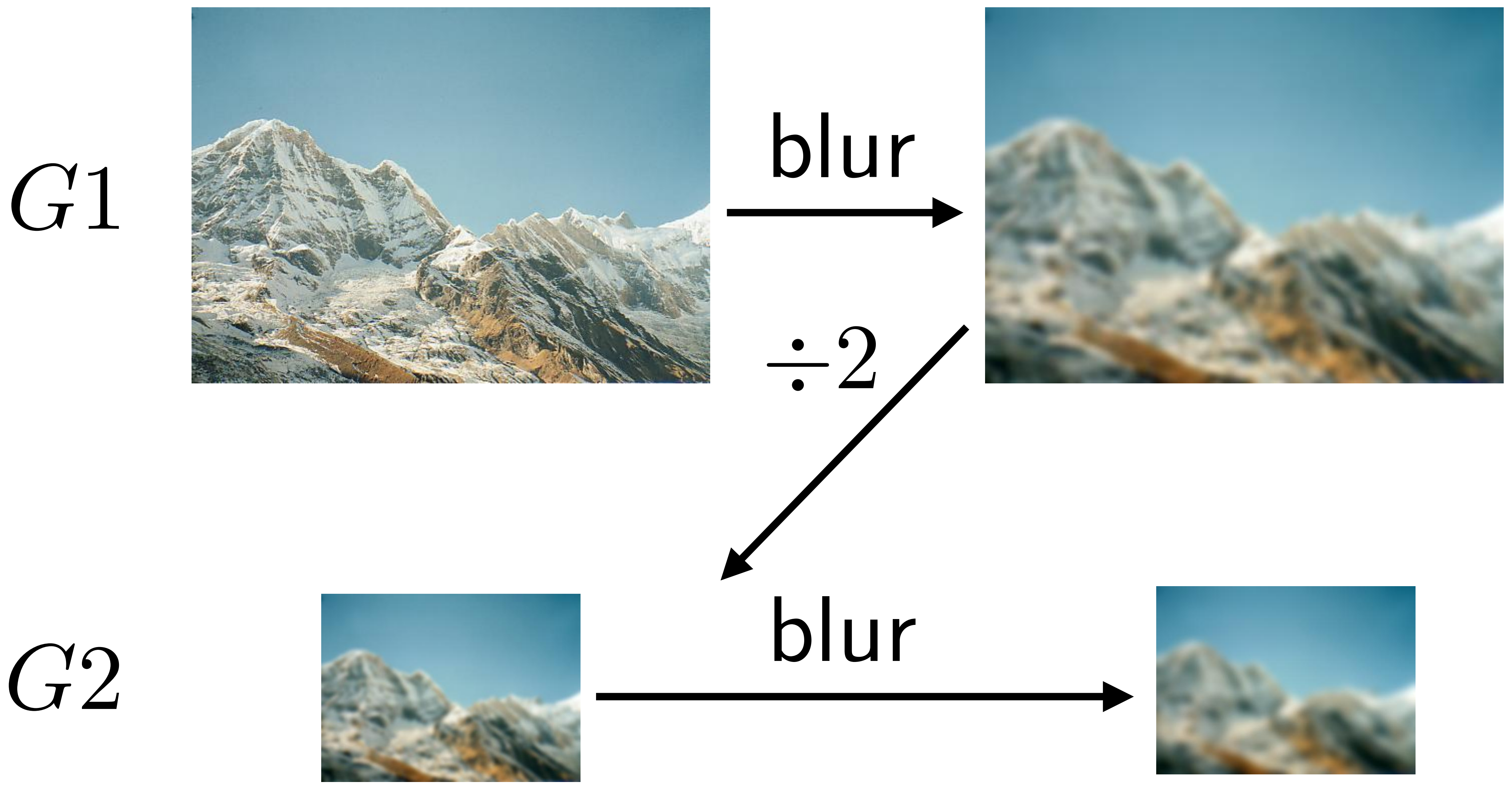
Blur with a Gaussian kernel, then select every 2nd pixel

$$I_s(x, y) = I(x, y) * g_\sigma(x, y)$$



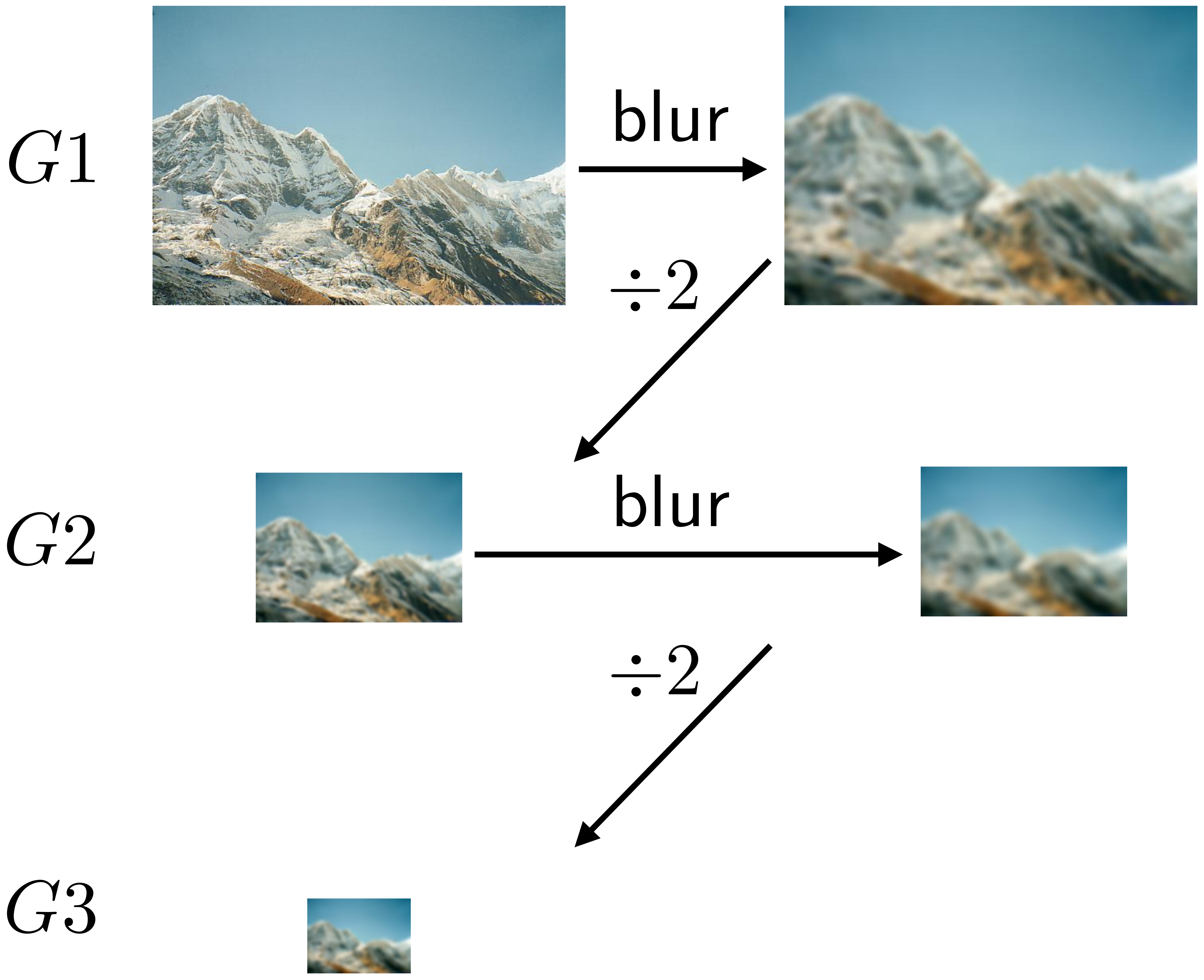
Blur with a Gaussian kernel, then select every 2nd pixel

$$I_s(x, y) = I(x, y) * g_\sigma(x, y)$$



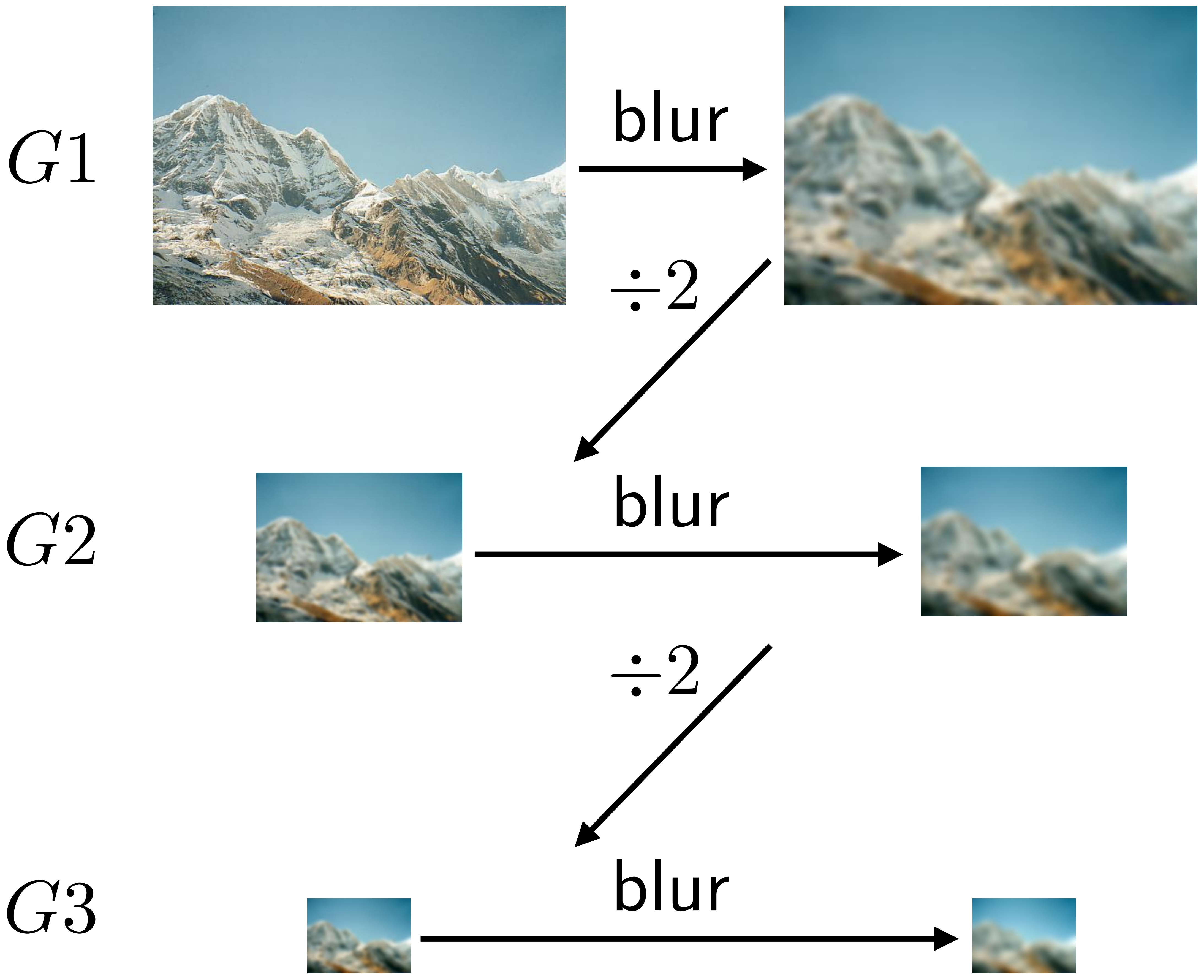
Blur with a Gaussian kernel, then select every 2nd pixel

$$I_s(x, y) = I(x, y) * g_\sigma(x, y)$$



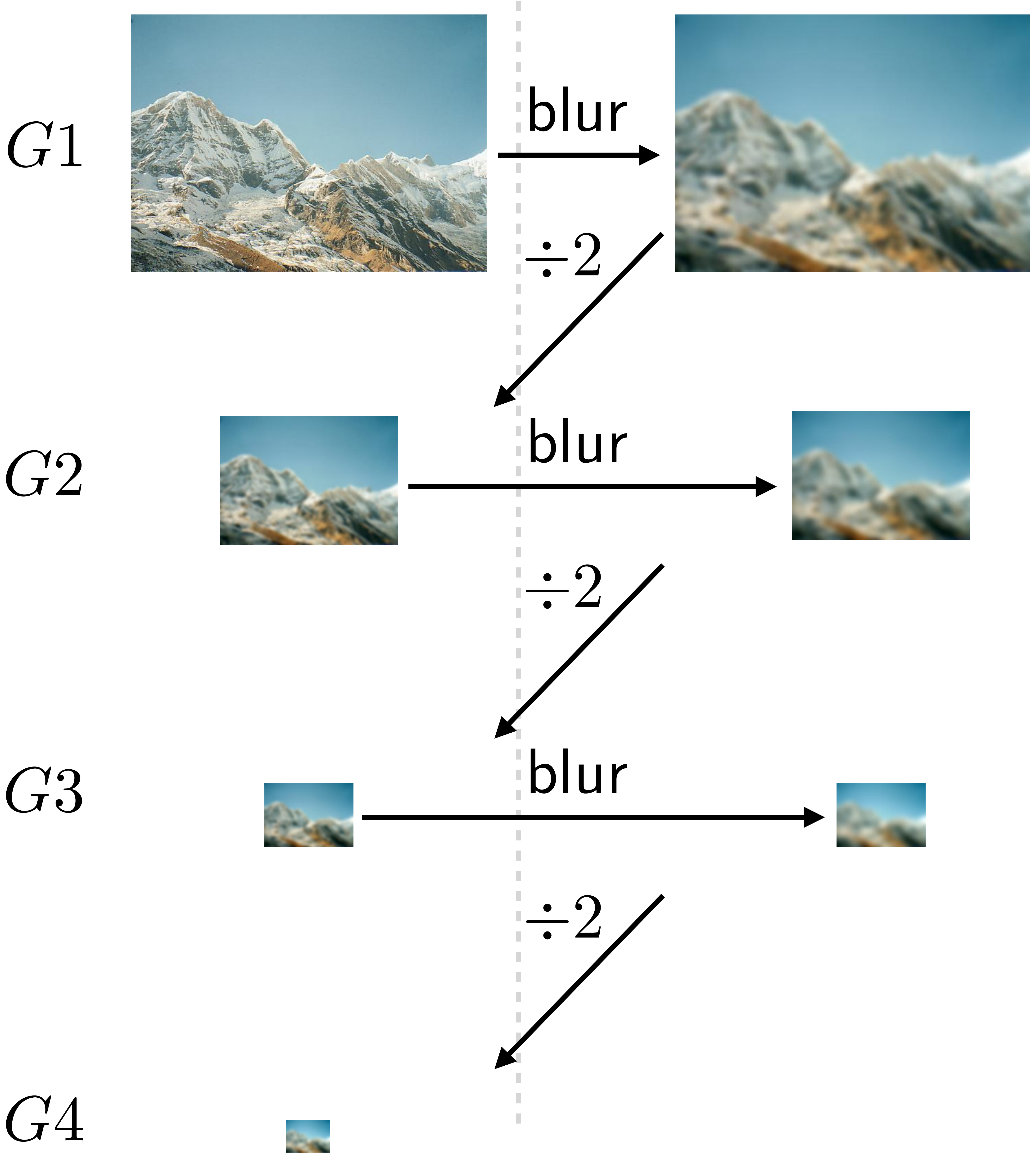
Blur with a Gaussian kernel, then select every 2nd pixel

$$I_s(x,y) = I(x,y) * g_\sigma(x,y)$$



Blur with a Gaussian kernel, then select every 2nd pixel

$$I_s(x,y) = I(x,y) * g_\sigma(x,y)$$



Blur with a Gaussian kernel, then select every 2nd pixel

$$I_s(x, y) = I(x, y) * g_\sigma(x, y)$$

Gaussian Pyramid

$G1$

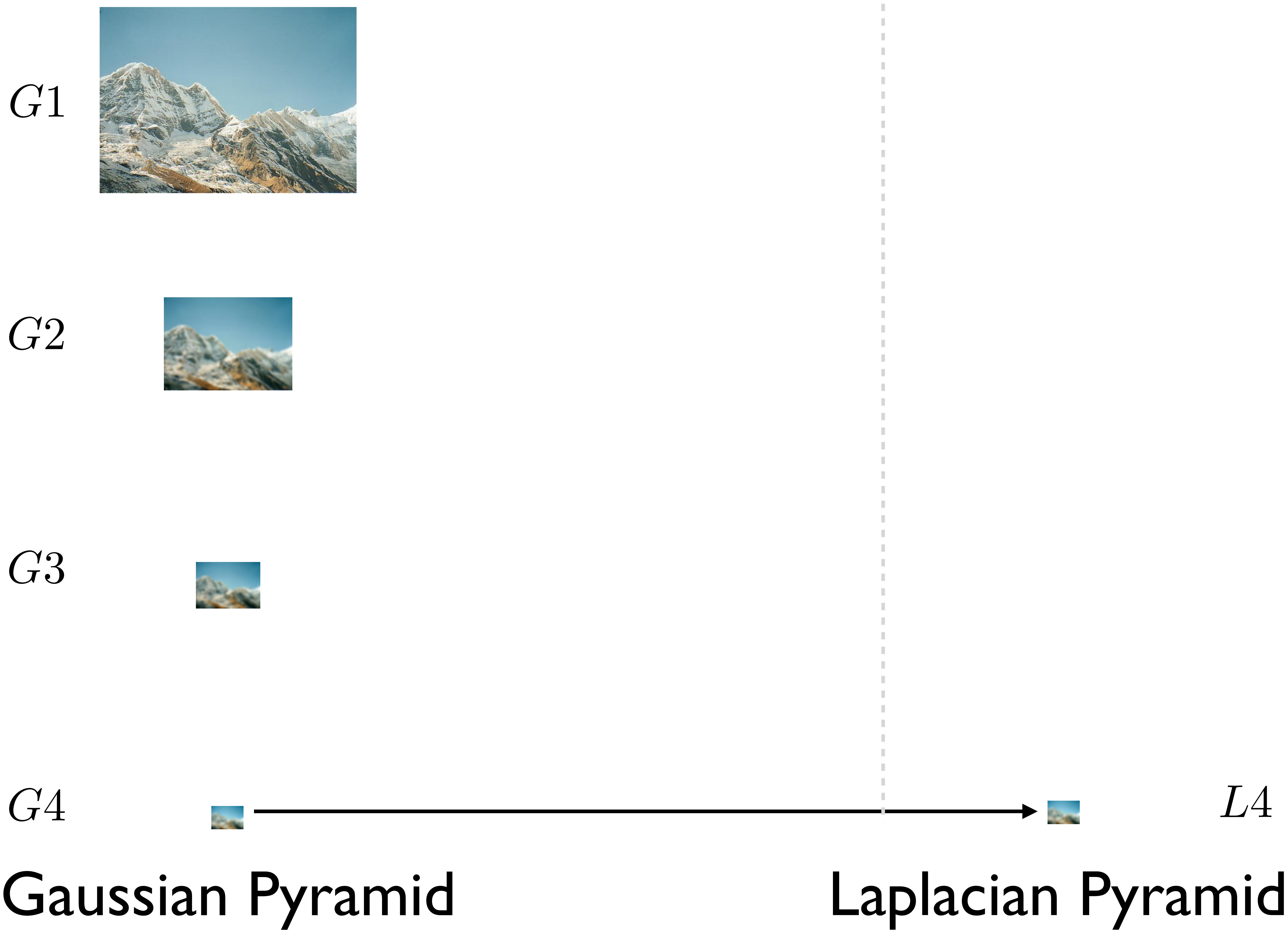
$G2$

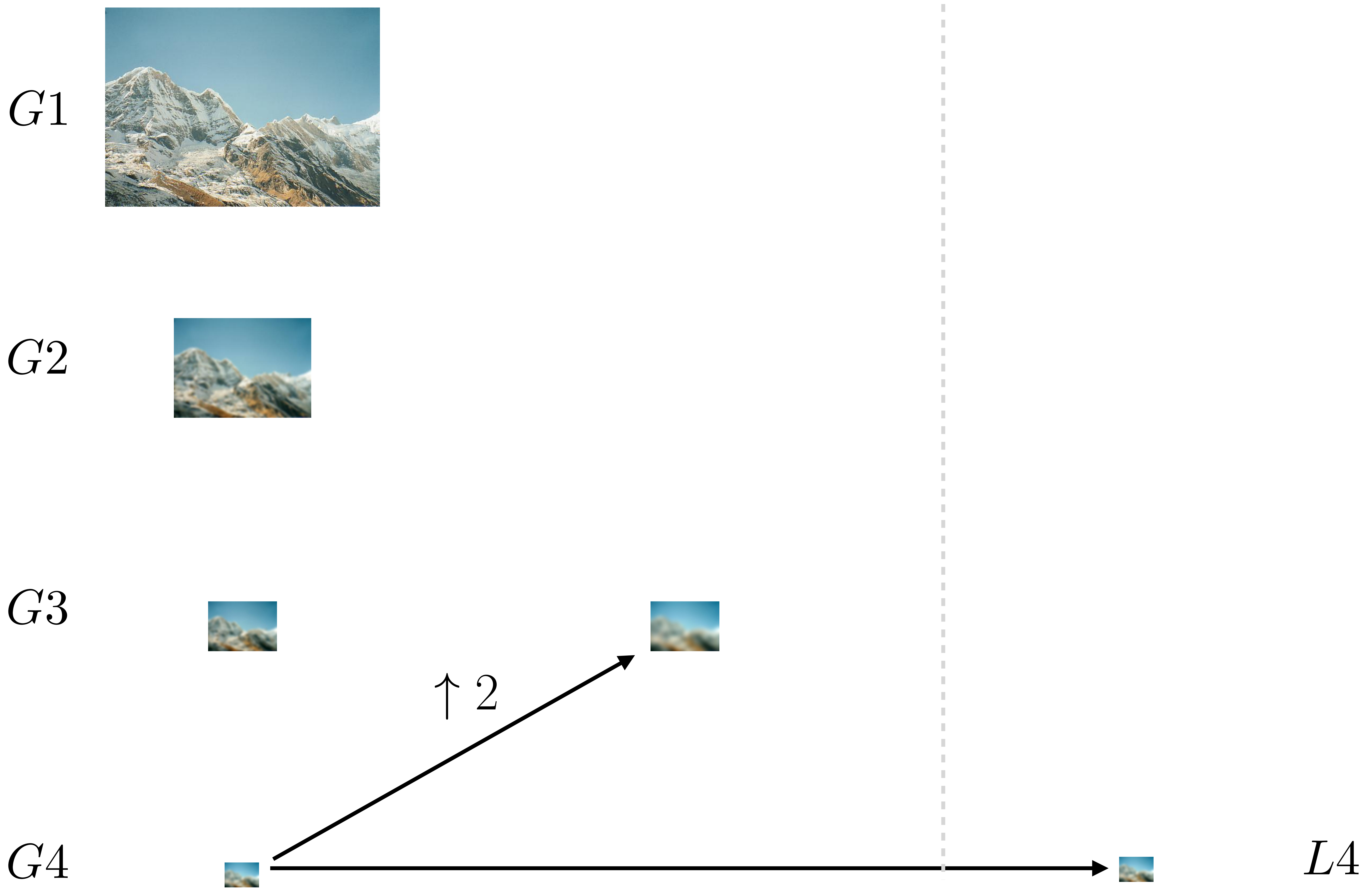
$G3$

$G4$

Gaussian Pyramid

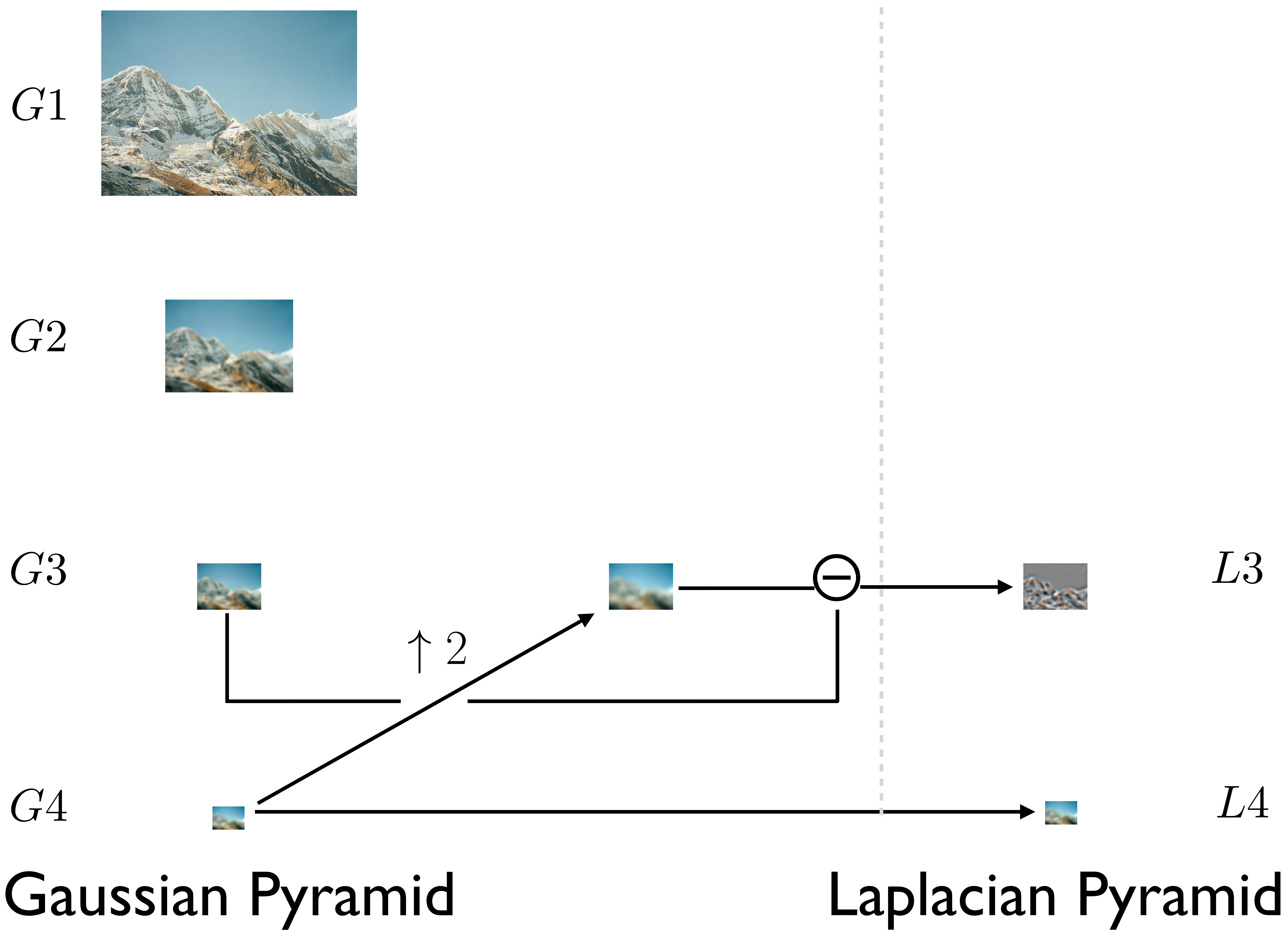
Laplacian Pyramid


$G1$
$G2$
$G3$
$G4$
$L4$
Gaussian Pyramid
Laplacian Pyramid

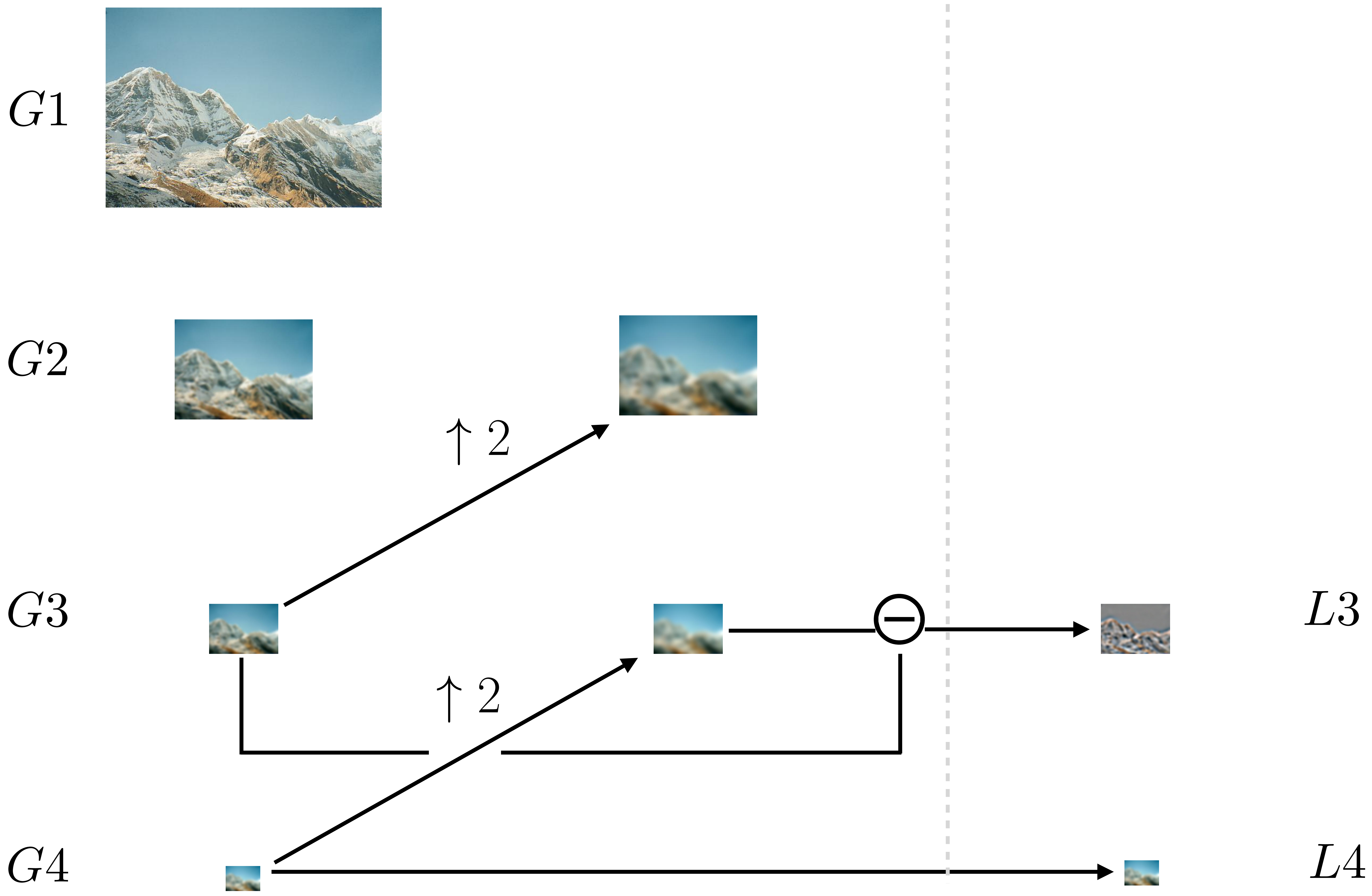


Gaussian Pyramid

Laplacian Pyramid

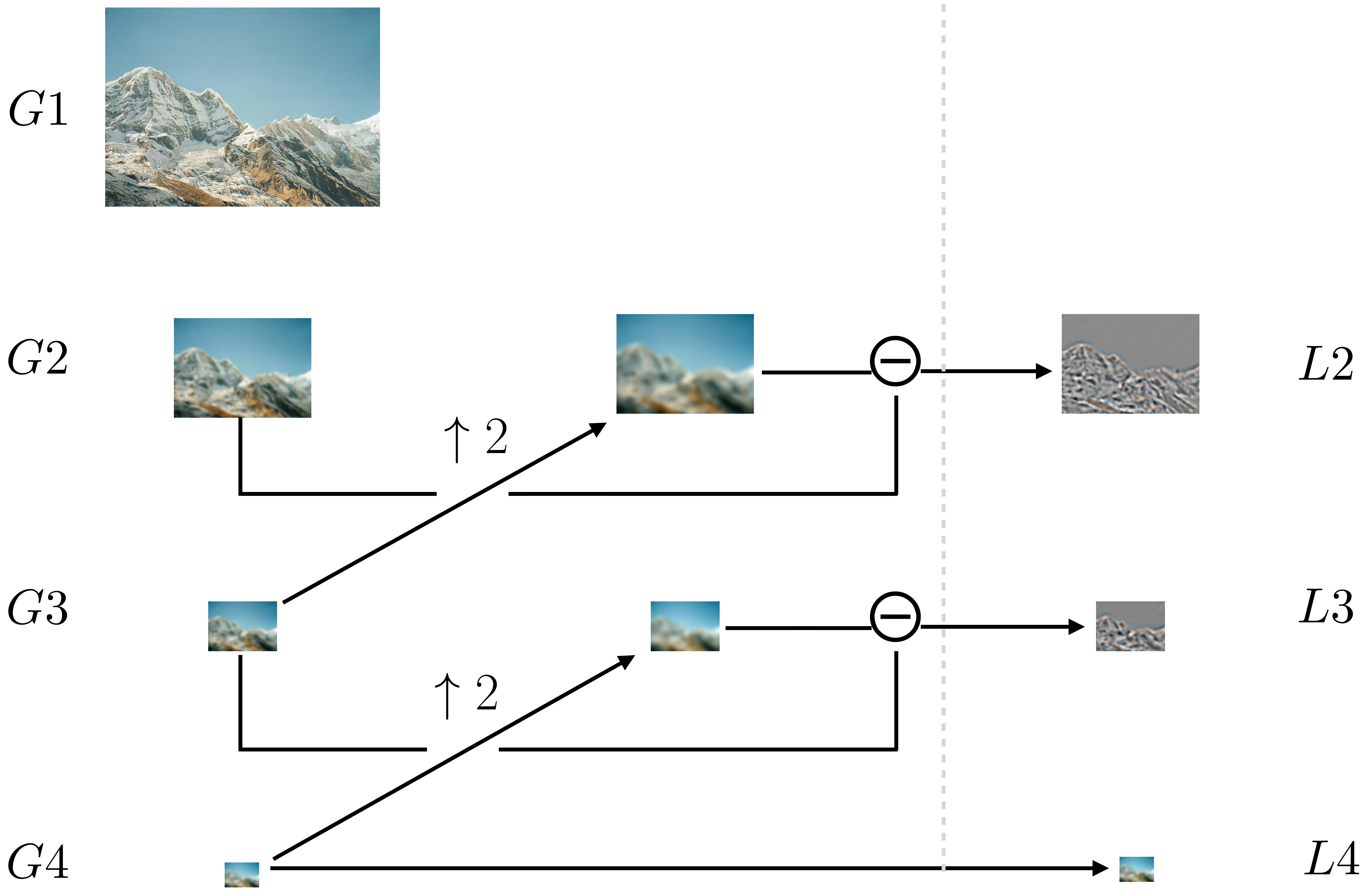


$G1$
$G2$
$G3$
$\uparrow 2$
$G4$
$L3$
$L4$
Gaussian Pyramid
Laplacian Pyramid

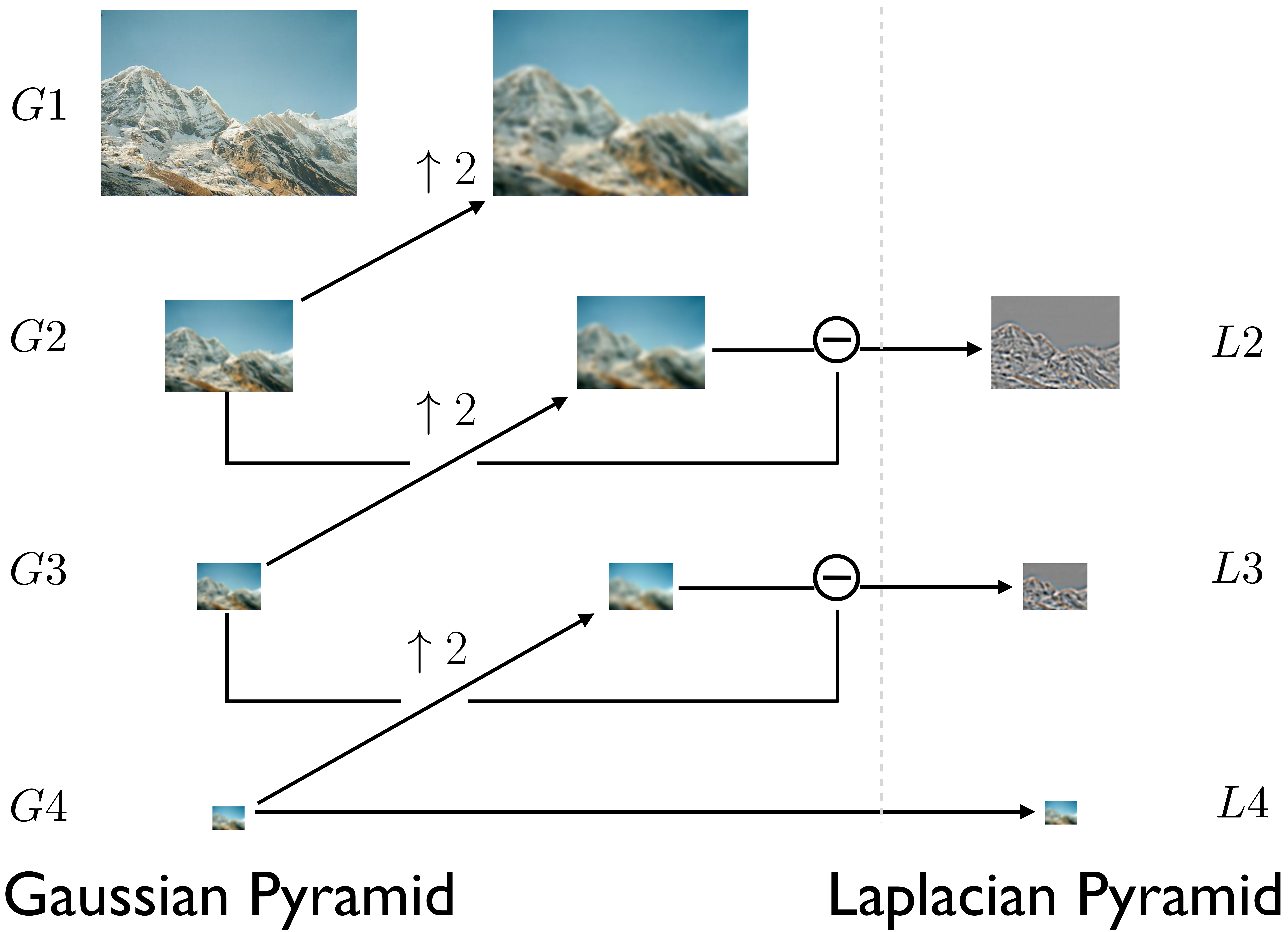


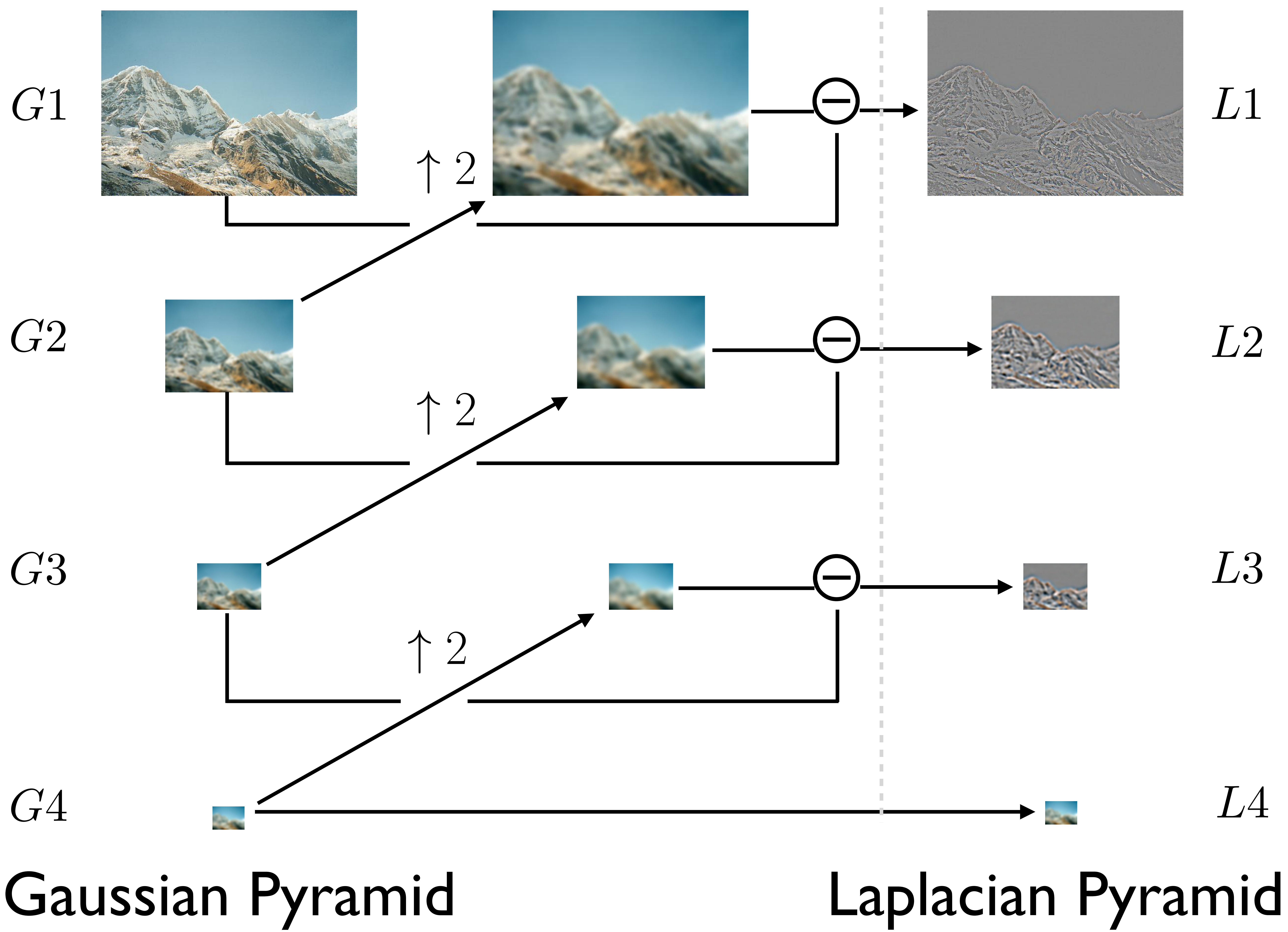
Gaussian Pyramid

Laplacian Pyramid



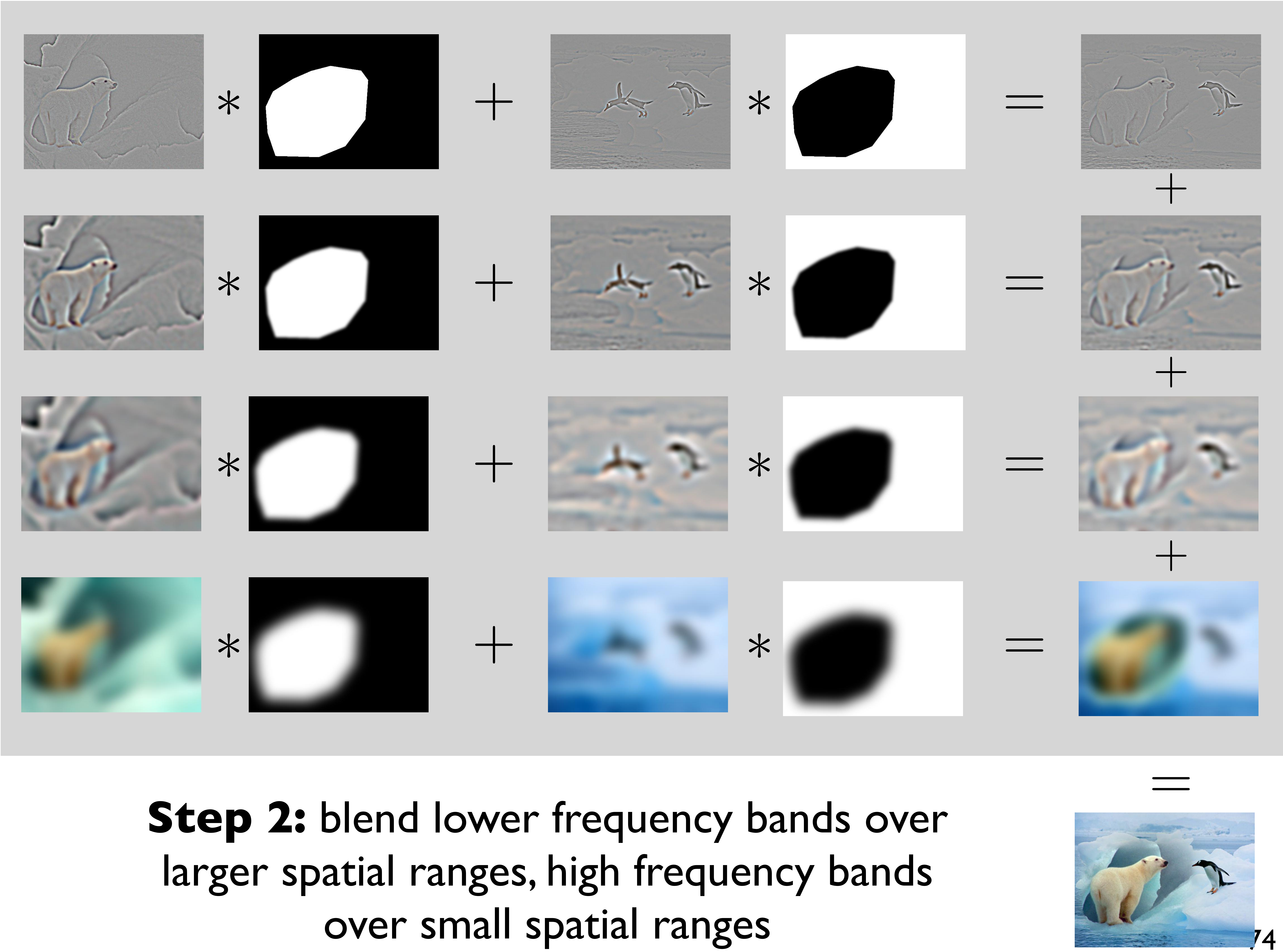
Gaussian Pyramid Laplacian Pyramid


$G1$
$\uparrow 2$
$G2$
$\uparrow 2$
$L2$
$G3$
$\uparrow 2$
$L3$
$G4$
$L4$
Gaussian Pyramid
Laplacian Pyramid



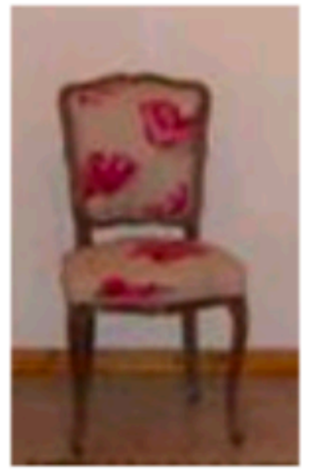
$G1$
$G2$
$G3$
$G4$
$\uparrow 2$
$\uparrow 2$
$\uparrow 2$
$L1$
$L2$
$L3$
$L4$
Gaussian Pyramid
Laplacian Pyramid

Pyramid Blending

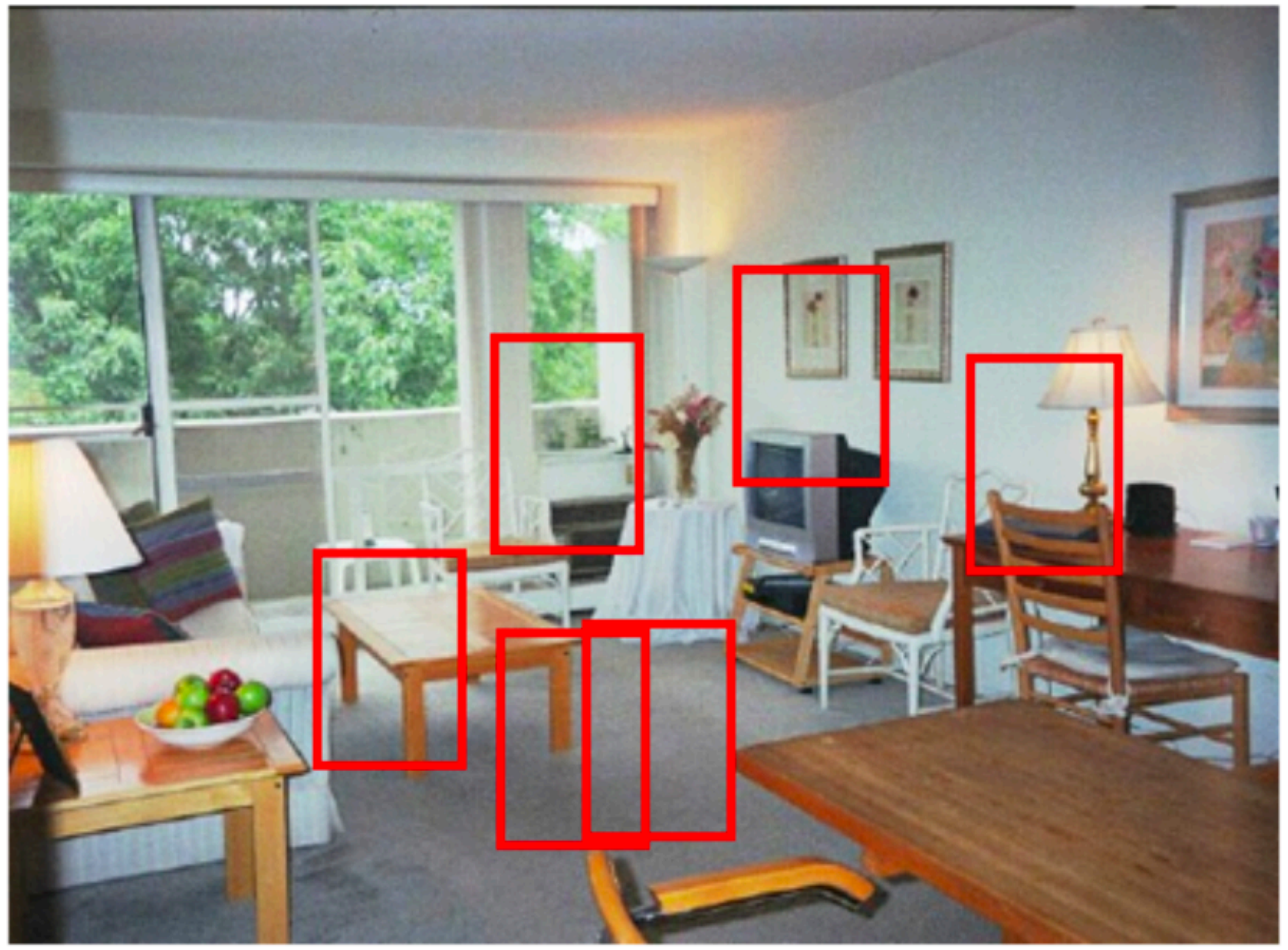


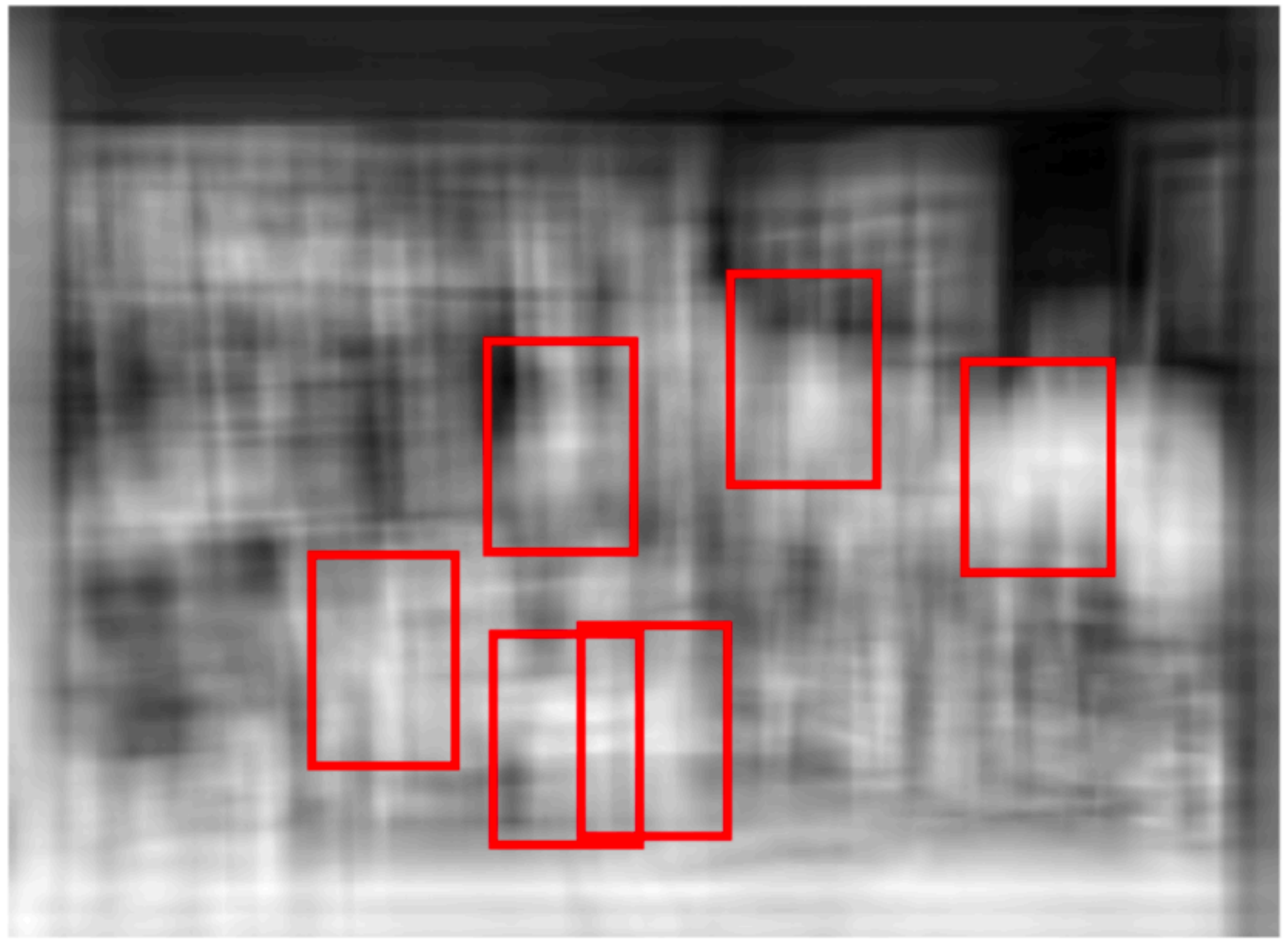
Step 2: blend lower frequency bands over larger spatial ranges, high frequency bands over small spatial ranges

From Template Matching to **Local Feature Detection**



Find the chair in this image





Pretty much garbage

Simple template matching is not going to make it

Slide Credit: Li Fei-Fei, Rob Fergus, and Antonio Torralba

Estimating **Derivatives**

Recall, for a 2D (continuous) function, f(x,y)

$$\frac{\partial f}{\partial x} = \lim_{\epsilon \to 0} \frac{f(x+\epsilon, y) - f(x,y)}{\epsilon}$$

Differentiation is linear and shift invariant, and therefore can be implemented as a convolution

A (discrete) approximation is

$$\frac{\partial f}{\partial x} \approx \frac{F(X+1, y) - F(x,y)}{\Delta x}$$

-1	1

Estimating **Derivatives**

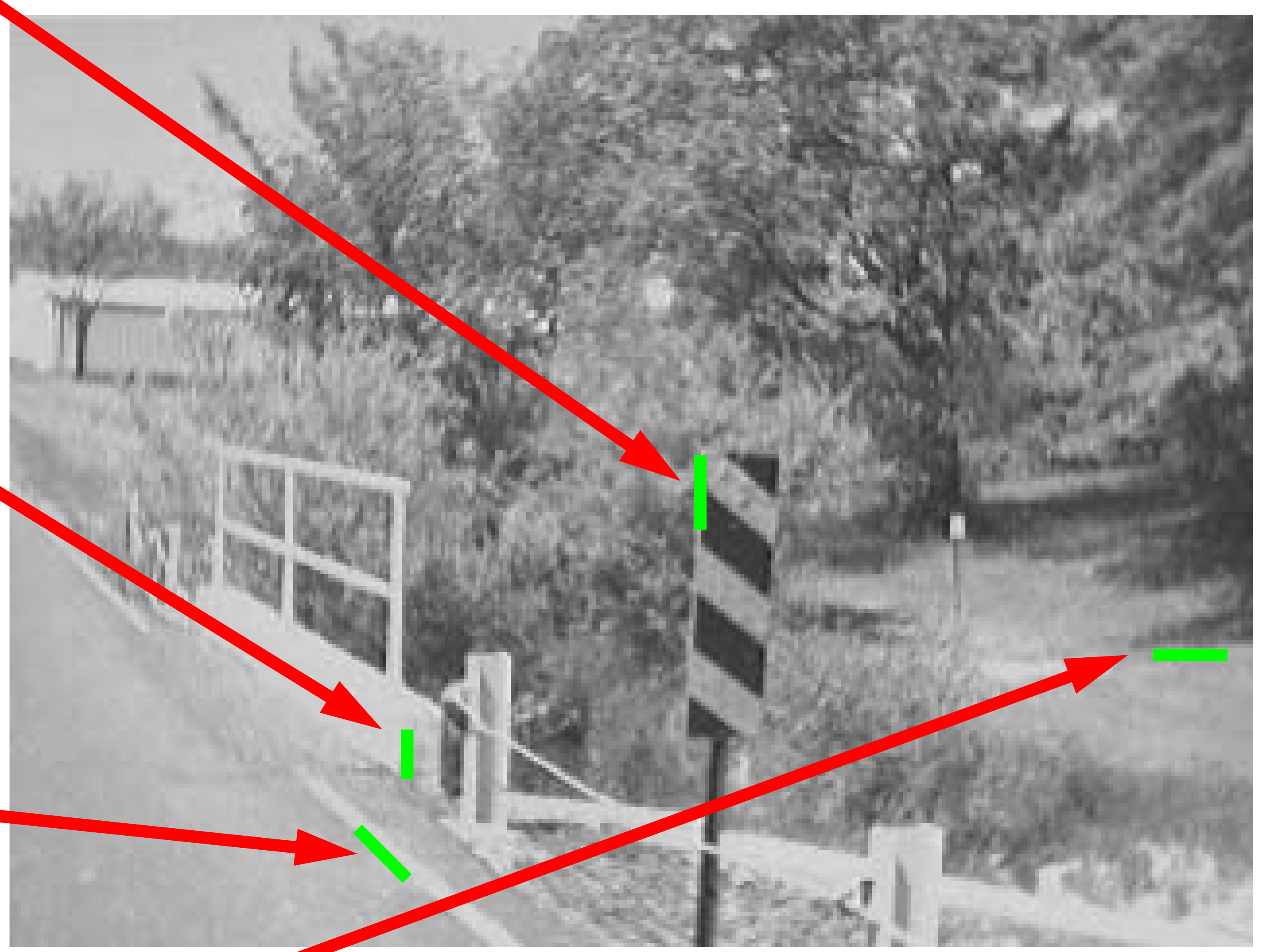
A similar definition (and approximation) holds for $\dfrac{\partial f}{\partial y}$

Image **noise** tends to result in pixels not looking exactly like their neighbours, so simple “finite differences” are sensitive to noise.

The usual way to deal with this problem is to **smooth** the image prior to derivative estimation.

What Causes **Edges**?

- Depth discontinuity
- Surface orientation discontinuity
- Reflectance discontinuity (i.e., change in surface material properties)
- Illumination discontinuity (e.g., shadow)



Slide Credit: Christopher Rasmussen

Smoothing and Differentiation

Edge: a location with high gradient (derivative)

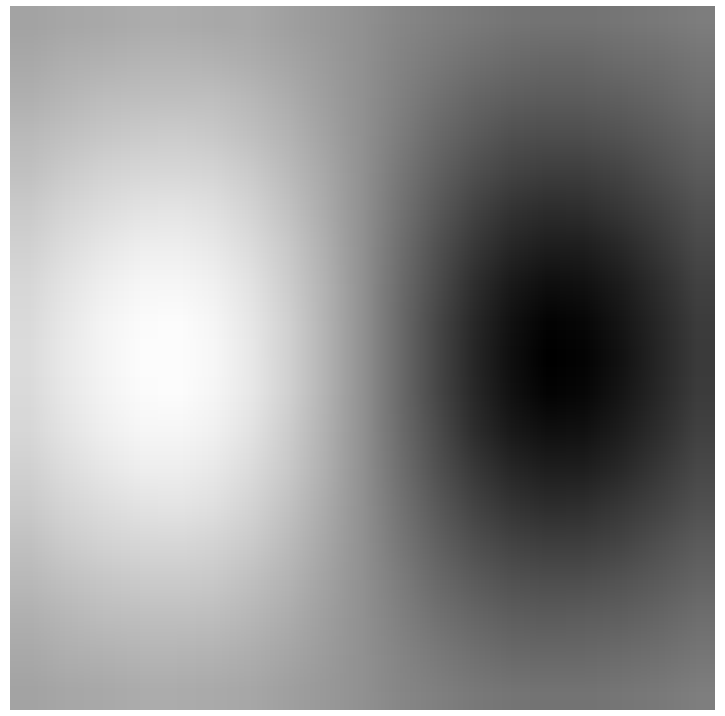
Need smoothing to reduce noise prior to taking derivative

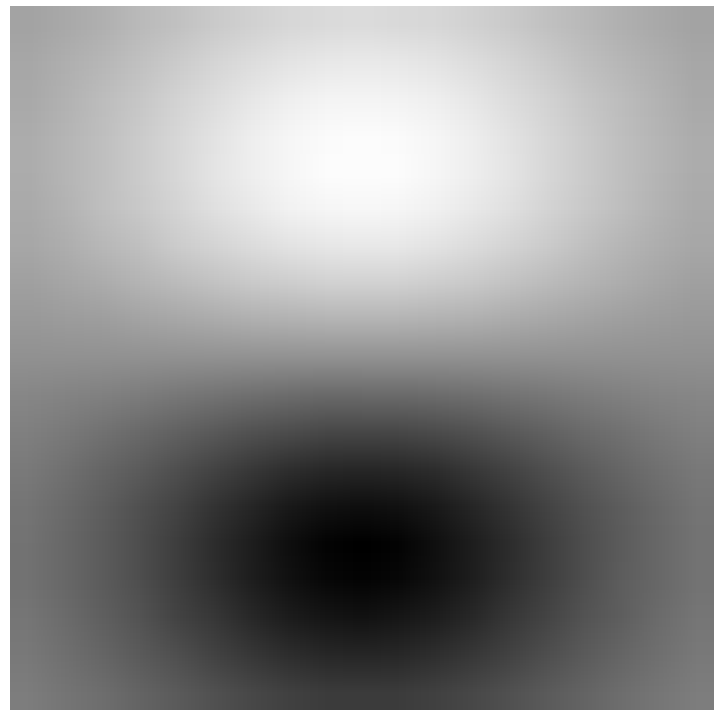
Need two derivatives, in x and y direction

We can use **derivative of Gaussian** filters
— because differentiation is convolution, and
— convolution is associative

Let $\otimes$ denote convolution

$$D \otimes (G \otimes I(X,Y)) = (D \otimes G) \otimes I(X,Y)$$





Gradient **Magnitude**

Let $I(X, Y)$ be a (digital) image

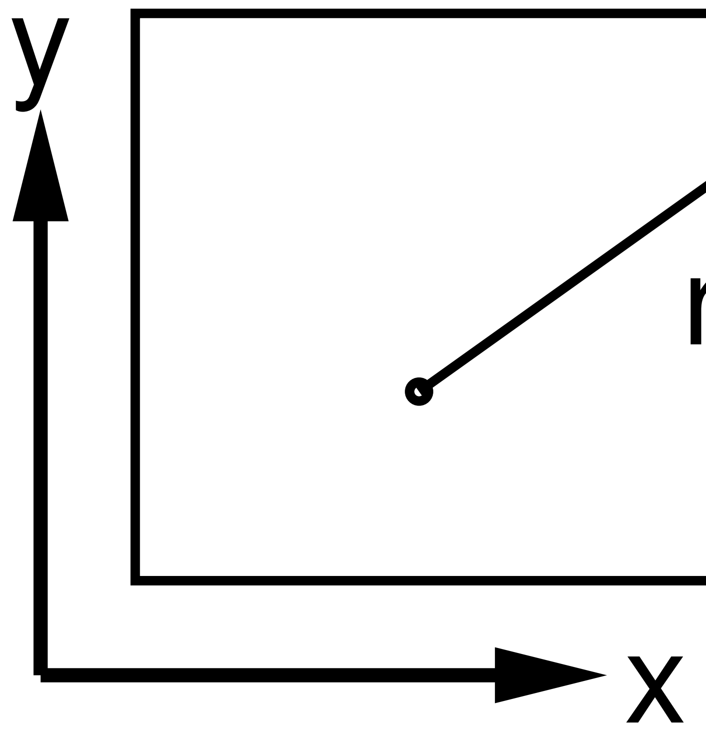
Let $I_x(X, Y)$ and $I_y(X, Y)$ be estimates of the partial derivatives in the x and y directions, respectively.

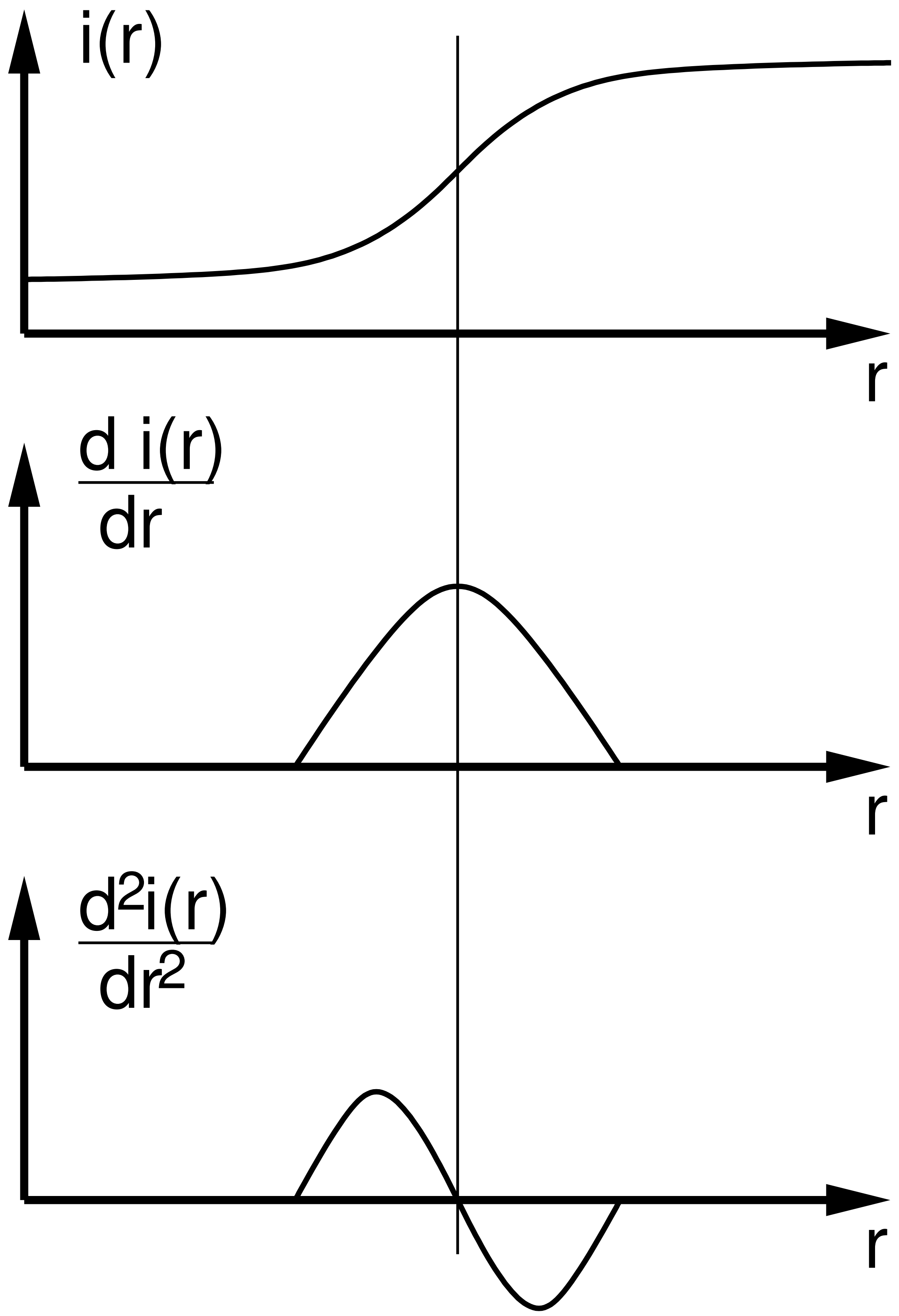
Call these estimates I_x and I_y (for short) The vector $[I_x, I_y]$ is the **gradient**

The scalar $\sqrt{I_x^2 + I_y^2}$ is the **gradient magnitude**

The **gradient direction** is given by: $\theta = \tan^{-1}\left(\frac{I_y}{I_x}\right)$

Two Generic Approaches for **Edge** Detection





Two generic approaches to **edge point detection**:

— (significant) local extrema of a first derivative operator

— zero crossings of a second derivative operator

Marr / Hildreth **Laplacian of Gaussian**

A "**zero crossings** of a second derivative operator" approach

Steps:

1. Gaussian for smoothing

2. Laplacian (∇^2) for differentiation where

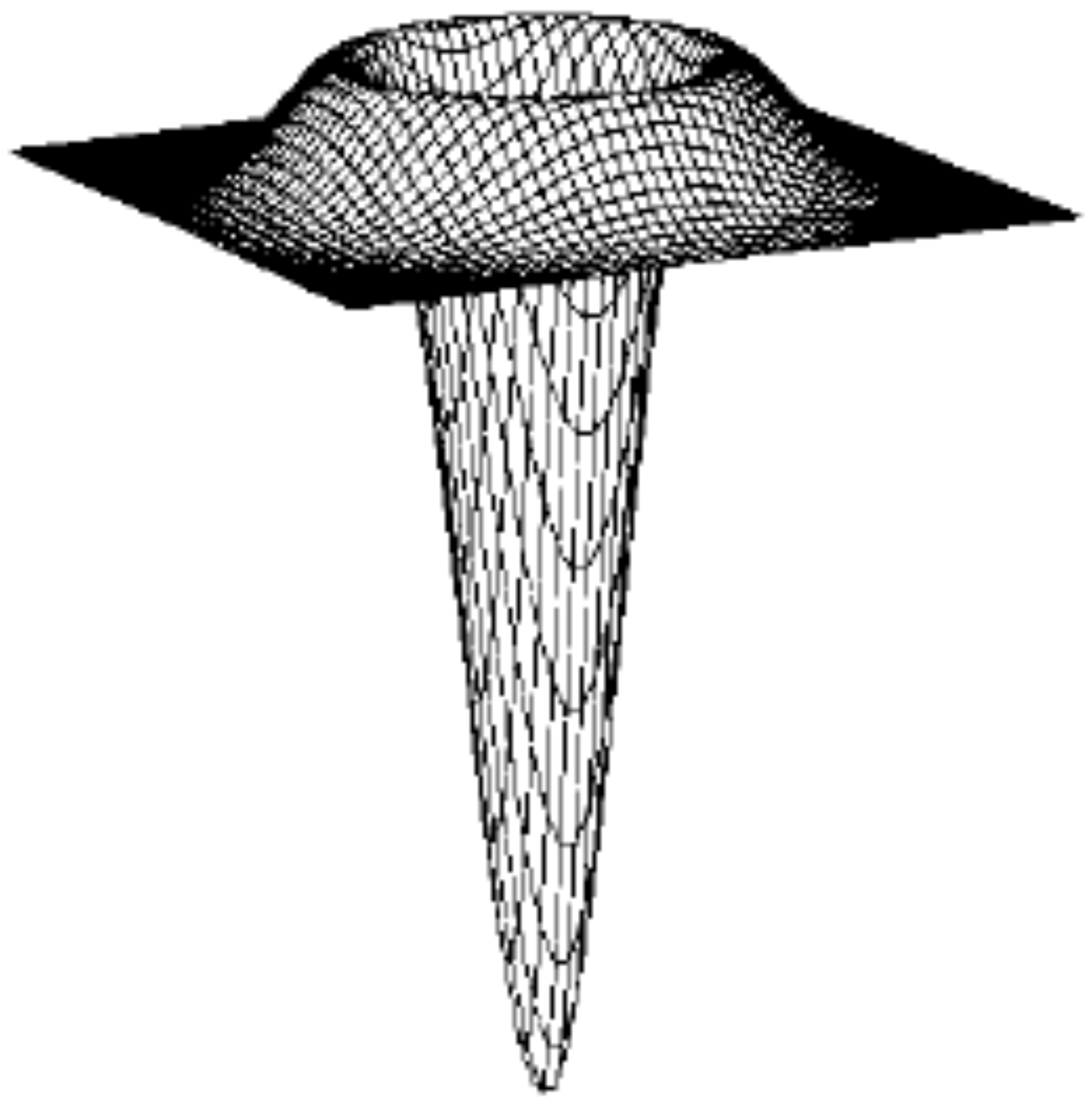
$$\nabla^2 f(x,y) = \frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}$$

3. Locate zero-crossings in the Laplacian of the Gaussian ($\nabla^2 G$) where

$$\nabla^2 G(x,y) = \frac{-1}{2\pi\sigma^4}\left[2 - \frac{x^2+y^2}{\sigma^2}\right]\exp^{-\frac{x^2+y^2}{2\sigma^2}}$$

Marr / Hildreth **Laplacian of Gaussian**

Here's a 3D plot of the Laplacian of the Gaussian ($\nabla^2 G$)



. . . with its characteristic "Mexican hat" shape

Canny Edge Detector

Steps:

1. Apply **directional derivatives** of Gaussian

2. Compute **gradient magnitude** and **gradient direction**

3. **Non-maximum** suppression
   — thin multi-pixel wide “ridges” down to single pixel width

4. **Linking** and thresholding
   — Low, high edge-strength thresholds
   — Accept all edges over low threshold that are connected to edge over high threshold

Sample Question: Edges

Why is non-maximum suppression applied in the Canny edge detector?

What is a **corner**?

Image Credit: John Shakespeare, Sydney Morning Herald

We can think of a corner as any locally distinct 2D image feature that (hopefully) corresponds to a distinct position on an 3D object of interest in the scene.

Why are corners **distinct**?

A corner can be **localized reliably**.

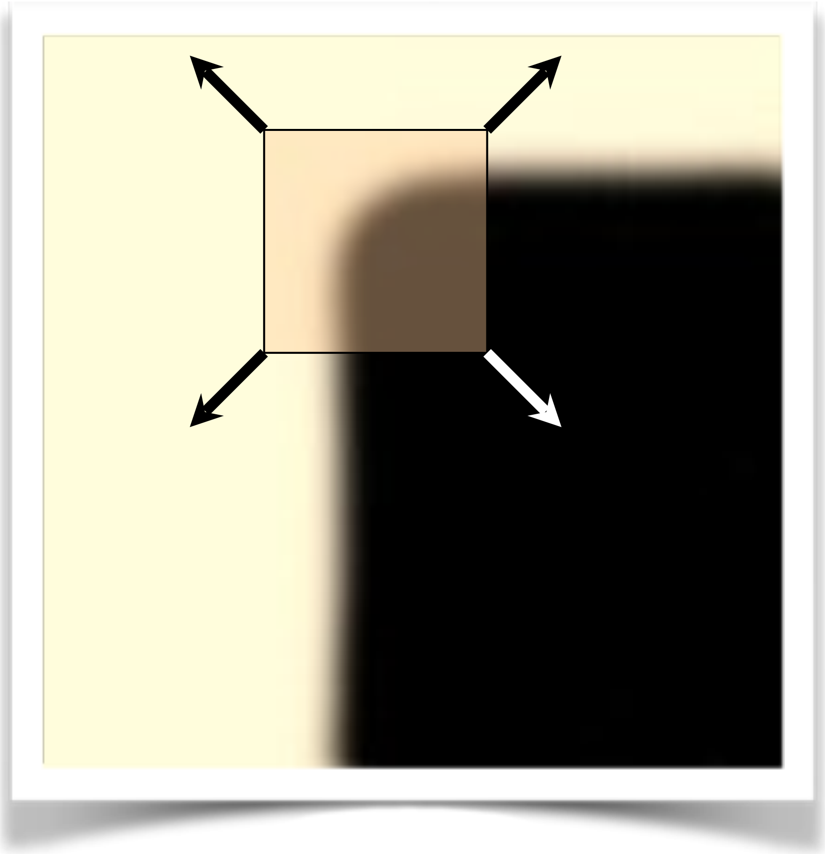
Thought experiment:

— Place a small window over a patch of constant image value. If you slide the window in any direction, the image in the window will not change.

— Place a small window over an edge. If you slide the window in the direction of the edge, the image in the window will not change

→ Cannot estimate location along an edge (a.k.a., **aperture** problem)

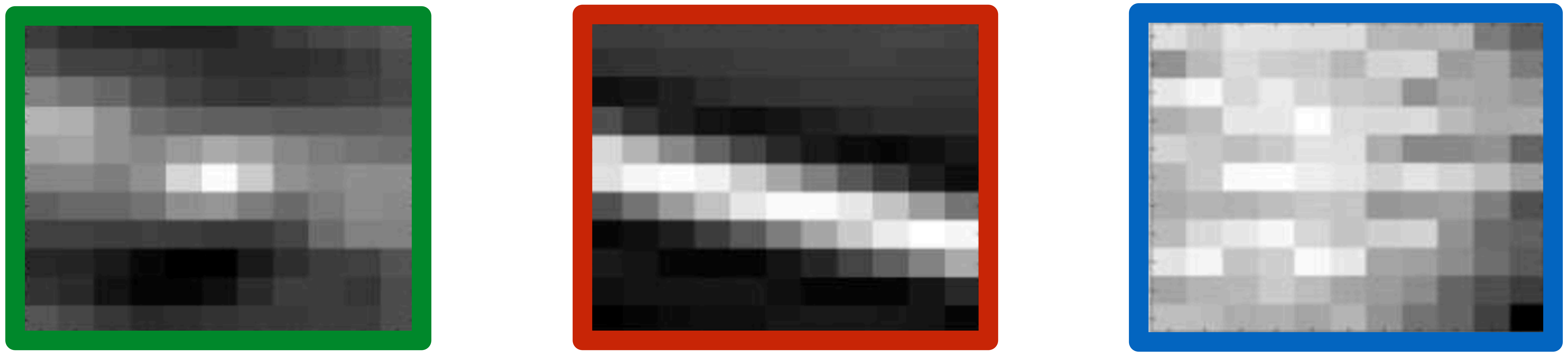
— Place a small window over a corner. If you slide the window in any direction, the image in the window changes.



"**corner**": significant change in all directions

Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

Autocorrelation



Szeliski, Figure 4.5

Corner Detection

Edge detectors perform poorly at corners

Observations:

— The gradient is ill defined exactly at a corner

— Near a corner, the gradient has two (or more) distinct values

Harris Corner Detection

1. Compute image gradients over small region

2. Compute the covariance matrix

3. Compute eigenvectors and eigenvalues

4. Use threshold on eigenvalues to detect corners

$$I_x = \frac{\partial I}{\partial x} \qquad I_y = \frac{\partial I}{\partial y}$$

$$\begin{bmatrix} \sum\limits_{p \in P} I_x I_x & \sum\limits_{p \in P} I_x I_y \\ \sum\limits_{p \in P} I_y I_x & \sum\limits_{p \in P} I_y I_y \end{bmatrix}$$

Slide Adopted: Ioannis (Yannis) Gkioulekas (CMU)

2. Compute the **covariance matrix** (a.k.a. 2nd moment matrix)

Sum over small region around the corner

Gradient with respect to x, times gradient with respect to y

$$C = \begin{bmatrix} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{bmatrix}$$

Matrix is **symmetric**

Harris Corner Detection

— Filter image with **Gaussian**

— Compute magnitude of the x and y **gradients** at each pixel

— Construct C in a window around each pixel

  — Harris uses a **Gaussian window**

— Solve for product of the λ's

— If λ's both are big (product reaches local maximum above threshold) then we have a corner

  — Harris also checks that ratio of λs is not too high

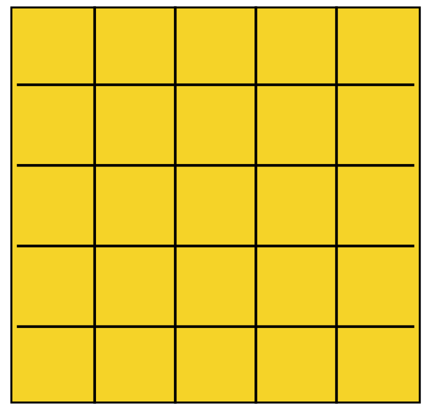
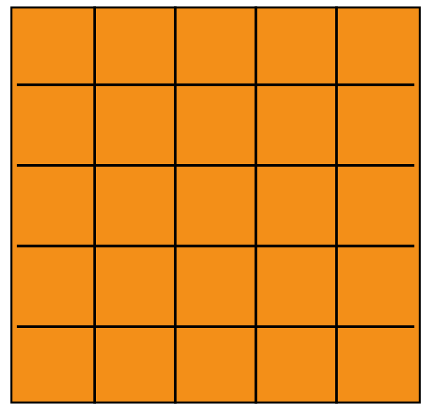
Compute the **covariance matrix** (a.k.a. 2nd moment matrix)

Sum over small region around the corner

Gradient with respect to x, times gradient with respect to y

$$C = \begin{bmatrix} \sum\limits_{p \in P} I_x I_x & \sum\limits_{p \in P} I_x I_y \\ \sum\limits_{p \in P} I_y I_x & \sum\limits_{p \in P} I_y I_y \end{bmatrix}$$

$$I_x = \frac{\partial I}{\partial x} \qquad I_y = \frac{\partial I}{\partial y}$$

$\sum\limits_{p \in P} I_x I_y$ =sum( .* )

array of x gradients

array of y gradients

Example: Harris Corner Detection

Lets compute a measure of “corner-ness” for the green pixel:

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

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0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

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0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

$$I_x = \frac{\partial I}{\partial x}$$

0	0	0	0	0	0	
-1	1	0	0	-1	1	
-1	0	0	0	1	0	
-1	0	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	

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0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

$$I_x = \frac{\partial I}{\partial x}$$

0	0	0	0	0	0	
-1	1	0	0	-1	1	
-1	0	0	0	1	0	
-1	0	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	

$$I_y = \frac{\partial I}{\partial y}$$

0	-1	0	0	0	-1	0
0	0	-1	-1	-1	1	0
0	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Example: Harris Corner Detection

Lets compute a measure of “corner-ness” for the green pixel:

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

$$\sum \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix} = 3$$

$$I_x = \frac{\partial I}{\partial x}$$

0	0	0	0	0	0	
-1	1	0	0	-1	1	
-1	0	0	0	1	0	
-1	0	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	

$$I_y = \frac{\partial I}{\partial y}$$

0	-1	0	0	0	-1	0
0	0	-1	-1	-1	1	0
0	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Example: Harris Corner Detection

Lets compute a measure of “corner-ness” for the green pixel:

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

$$\mathbf{C} = \begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}$$

$$I_x = \frac{\partial I}{\partial x}$$

0	0	0	0	0	0	
-1	1	0	0	-1	1	
-1	0	0	0	1	0	
-1	0	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	

$$I_y = \frac{\partial I}{\partial y}$$

0	-1	0	0	0	-1	0
0	0	-1	-1	-1	1	0
0	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Example: Harris Corner Detection

Lets compute a measure of “corner-ness” for the green pixel:

$$\mathbf{C} = \begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix} => \lambda_1 = 1.4384; \lambda_2 = 5.5616$$

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

$$I_x = \frac{\partial I}{\partial x}$$

0	0	0	0	0	0	
-1	1	0	0	-1	1	
-1	0	0	0	1	0	
-1	0	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	

$$I_y = \frac{\partial I}{\partial y}$$

0	-1	0	0	0	-1	0
0	0	-1	-1	-1	1	0
0	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Example: Harris Corner Detection

Lets compute a measure of “corner-ness” for the green pixel:

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

$$\mathbf{C} = \begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix} => \lambda_1 = 1.4384; \lambda_2 = 5.5616$$

$$\det(\mathbf{C}) - 0.04\text{trace}^2(\mathbf{C}) = 6.04$$

$$I_x = \frac{\partial I}{\partial x}$$

0	0	0	0	0	0	
-1	1	0	0	-1	1	
-1	0	0	0	1	0	
-1	0	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	

$$I_y = \frac{\partial I}{\partial y}$$

0	-1	0	0	0	-1	0
0	0	-1	-1	-1	1	0
0	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Example: Harris Corner Detection

Lets compute a measure of “corner-ness” for the green pixel:

$$\mathbf{C} = \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix} => \lambda_1 = 3; \lambda_2 = 0$$

$$\det(\mathbf{C}) - 0.04\text{trace}^2(\mathbf{C}) = -0.36$$

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

$$I_x = \frac{\partial I}{\partial x}$$

0	0	0	0	0	0	
-1	1	0	0	-1	1	
-1	0	0	0	1	0	
-1	0	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	

$$I_y = \frac{\partial I}{\partial y}$$

0	-1	0	0	0	-1	0
0	0	-1	-1	-1	1	0
0	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Example: Harris Corner Detection

Lets compute a measure of “corner-ness” for the green pixel:

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

$$\mathbf{C} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} => \lambda_1 = 3; \lambda_2 = 2$$

$$\det(\mathbf{C}) - 0.04\mathrm{trace}^2(\mathbf{C}) = 5$$

$$I_x = \frac{\partial I}{\partial x}$$

0	0	0	0	0	0	
-1	1	0	0	-1	1	
-1	0	0	0	1	0	
-1	0	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	

$$I_y = \frac{\partial I}{\partial y}$$

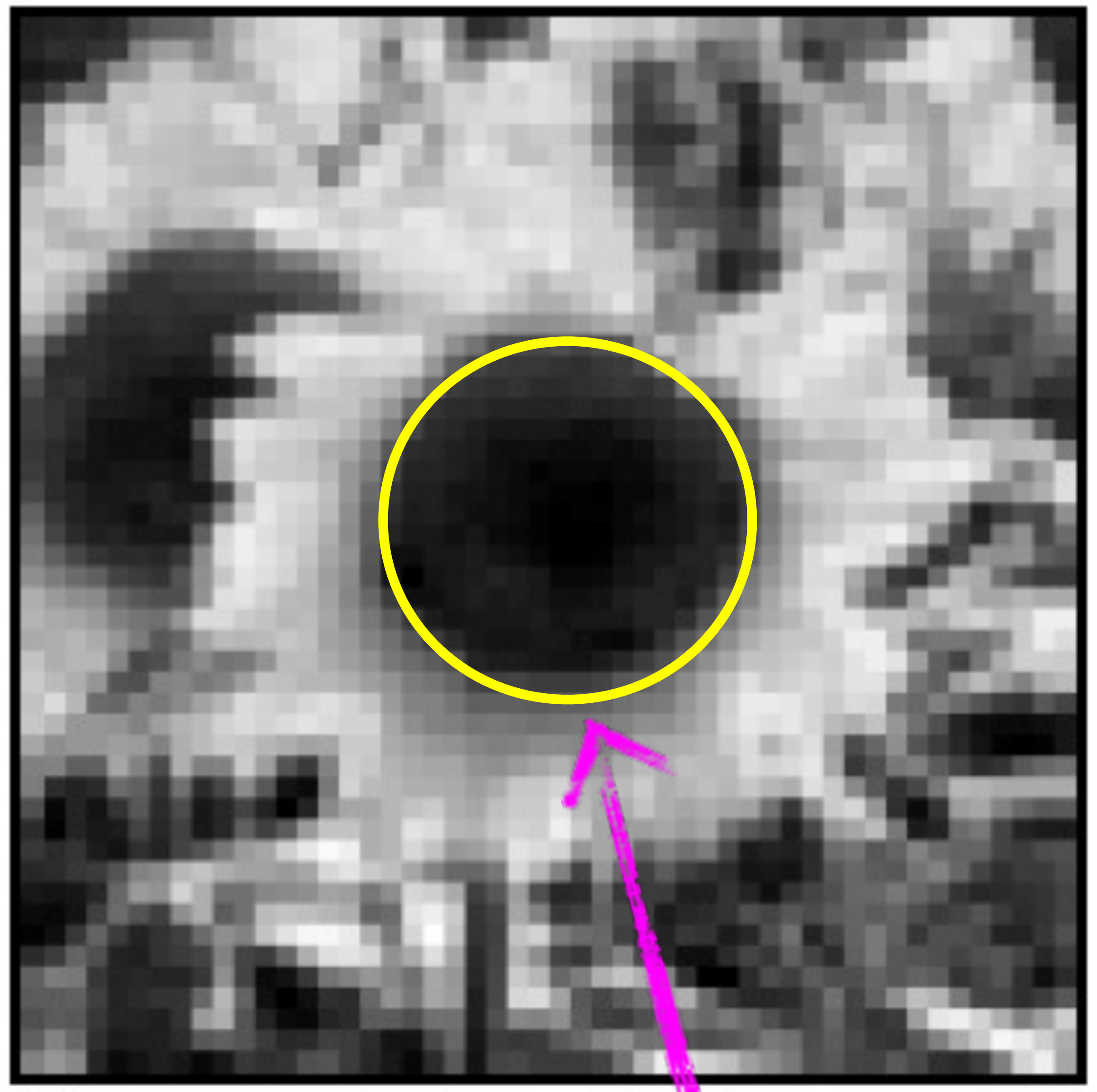
0	-1	0	0	0	-1	0
0	0	-1	-1	-1	1	0
0	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

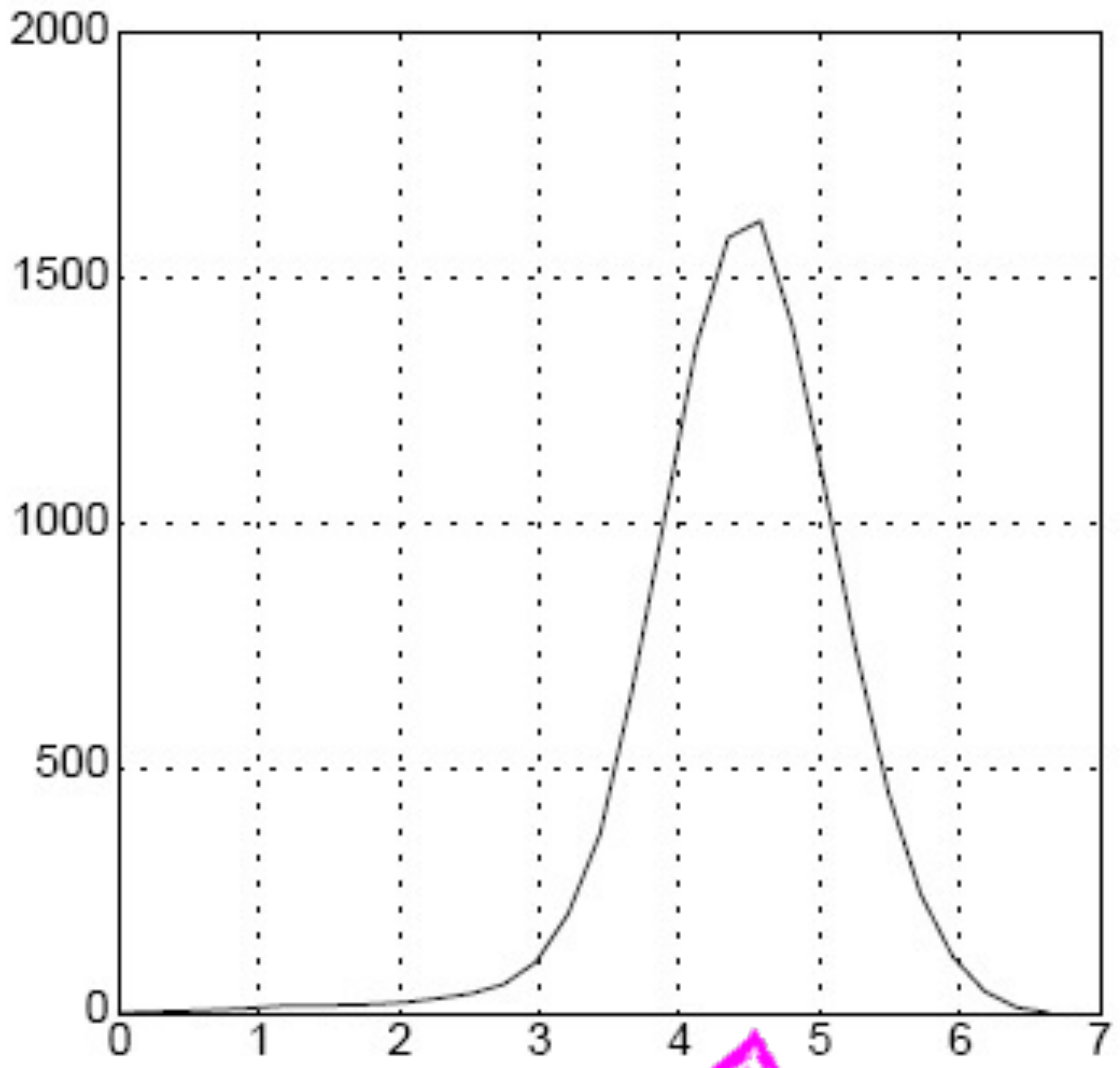
Properties: NOT Invariant to Scale Changes

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Characteristic Scale

characteristic scale - the scale that produces peak filter response





characteristic scale

we need to search over characteristic scales

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Sample Questions: Corners

The Harris corner detector is stable under some image transformations (features are considered stable if the same locations on an object are typically selected in the transformed image).

True or **false**: The Harris corner detector is stable under image blur.

Texture

We will look at two main questions:

1. How do we represent texture?
   → Texture **analysis**

2. How do we generate new examples of a texture?
   → Texture **synthesis**

Texture **Synthesis**

Why might we want to synthesize texture?

1. To fill holes in images (**inpainting**)

— Art directors might want to remove telephone wires. Restorers might want to remove scratches or marks.

— We need to find something to put in place of the pixels that were removed

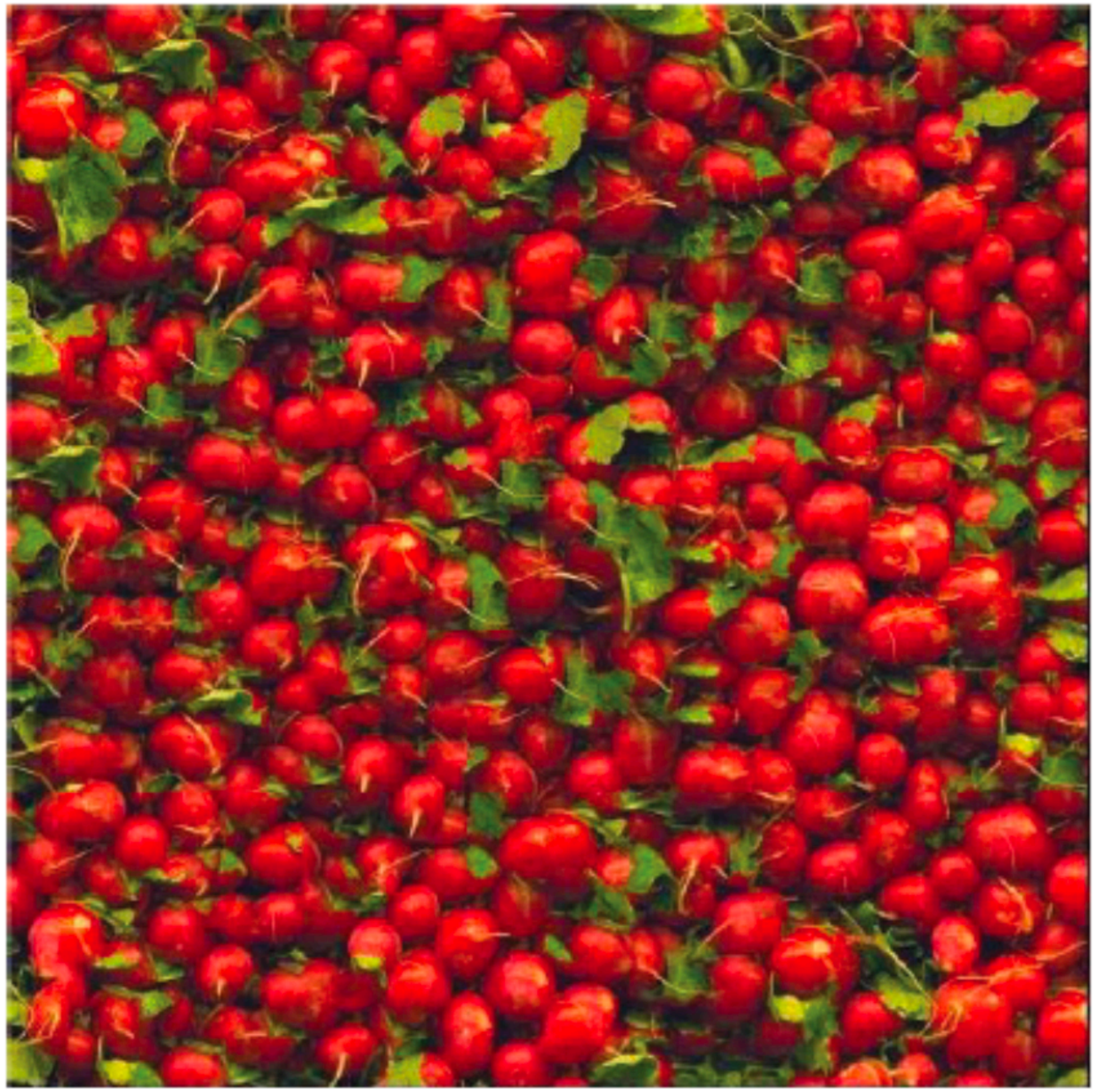
— We synthesize regions of texture that fit in and look convincing

2. To produce large quantities of texture for computer graphics

— Good textures make object models look more realistic

Texture **Synthesis**

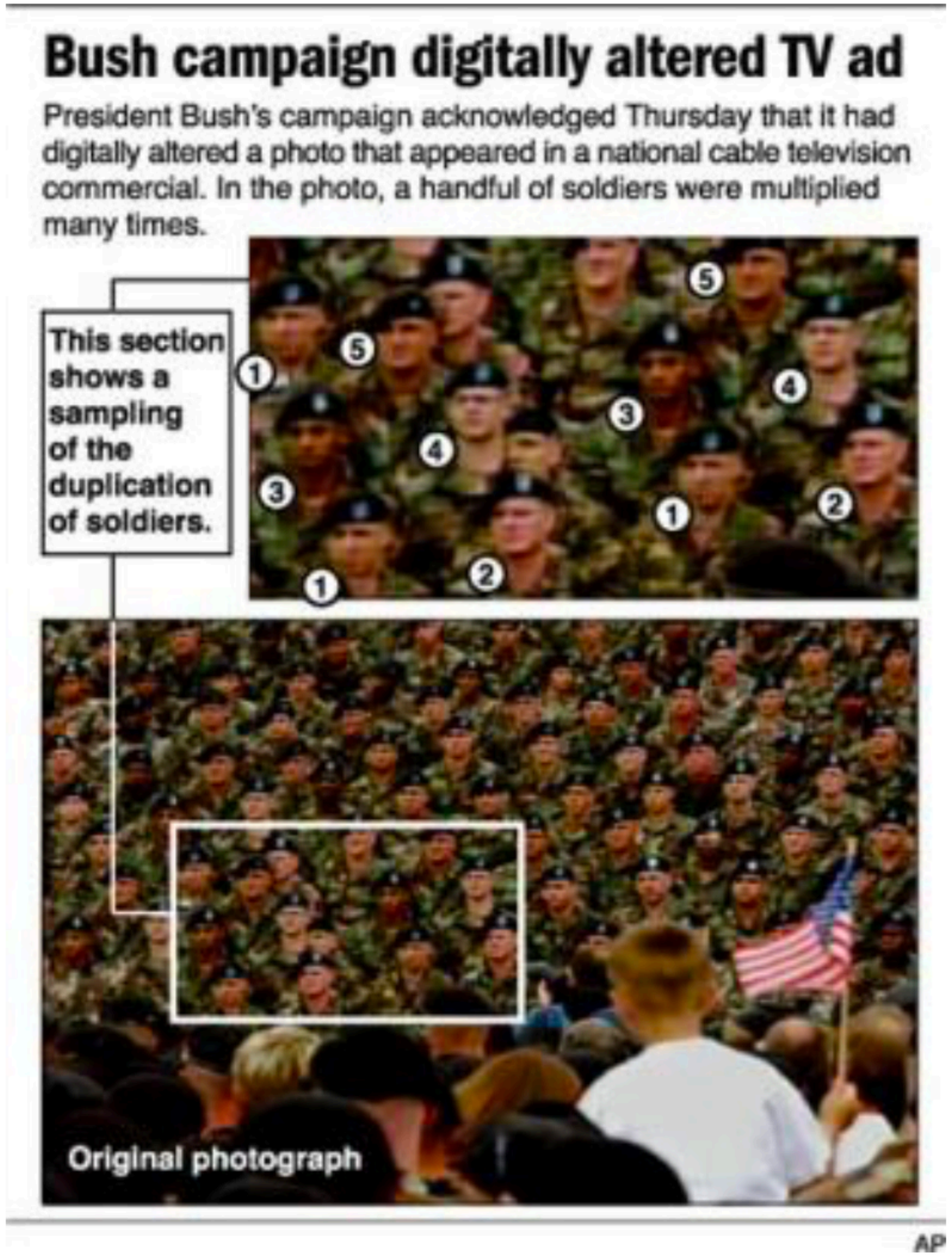
radishes



lots more radishes

Szeliski, Fig. 10.49

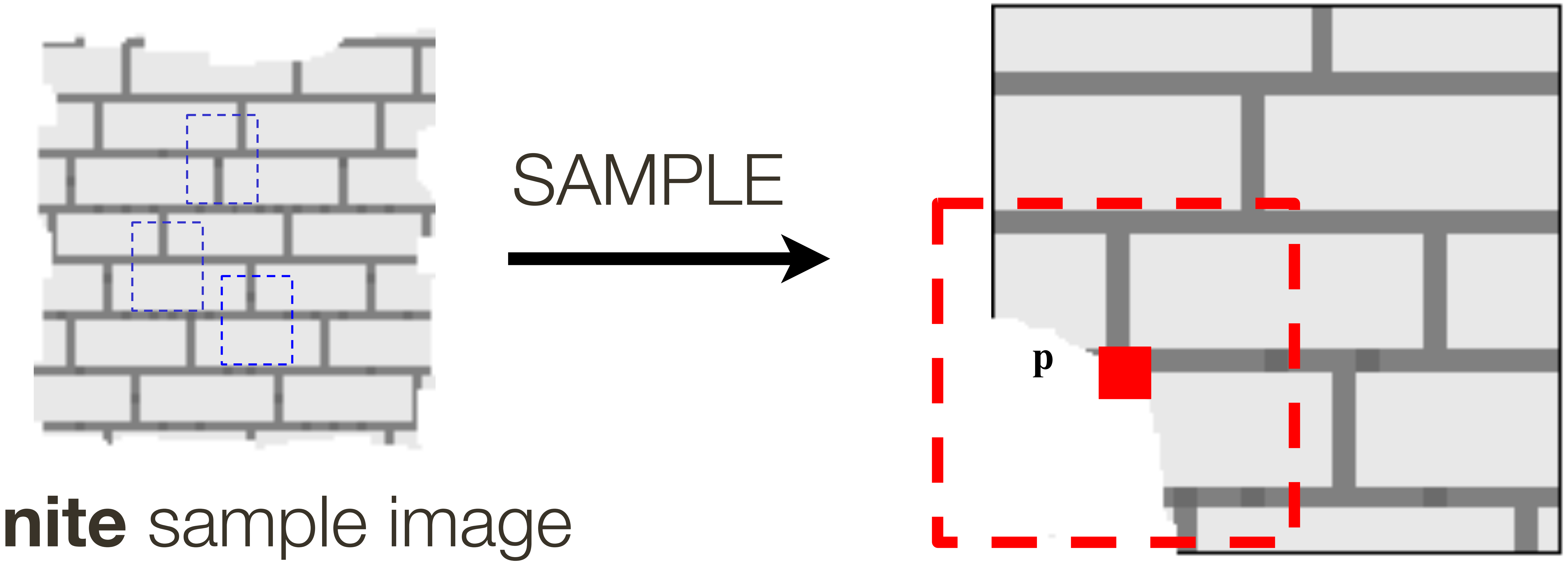
Texture **Synthesis**


Bush campaign digitally altered TV ad

President Bush's campaign acknowledged Thursday that it had digitally altered a photo that appeared in a national cable television commercial. In the photo, a handful of soldiers were multiplied many times.

Photo Credit: Associated Pres

Efros and Leung: Synthesizing One Pixel



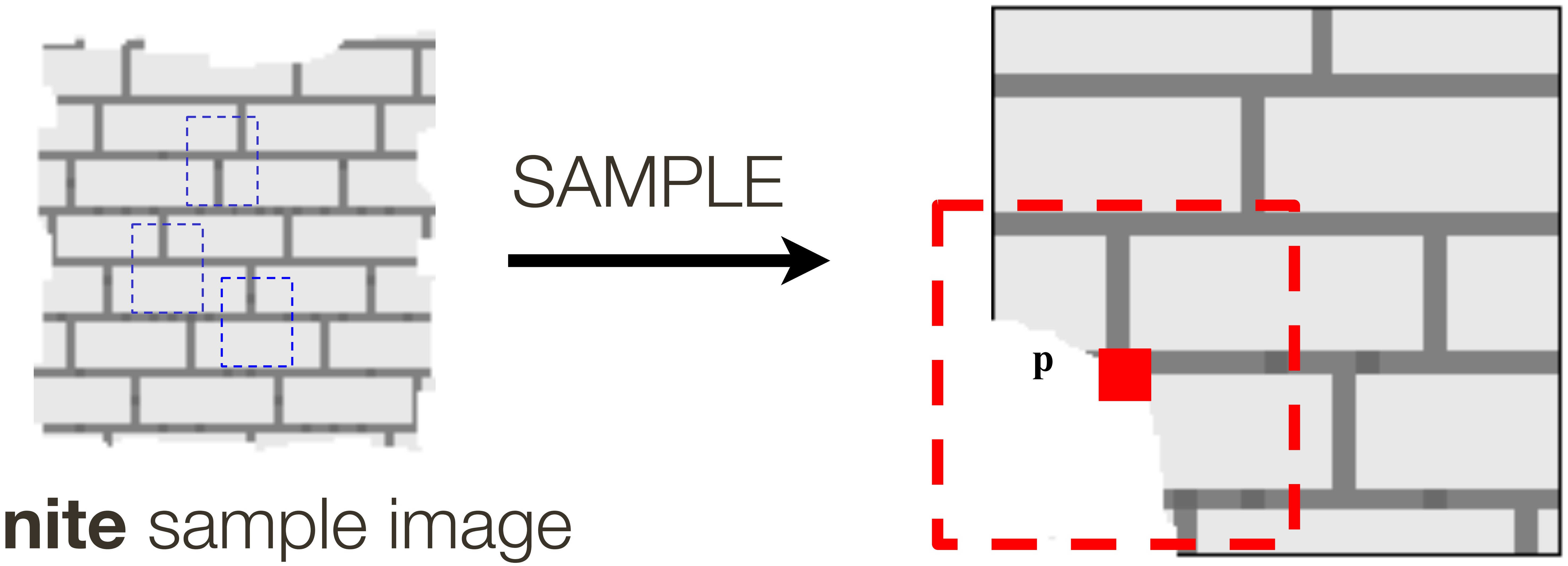
Infinite sample image

— What is **conditional** probability distribution of *p*, given the neighbourhood window?

— Directly search the input image for all such neighbourhoods to produce a **histogram** for *p*

— To **synthesize** *p*, pick one match at random

Efros and Leung: Synthesizing One Pixel



Infinite sample image

— Since the sample image is finite, an exact neighbourhood match might not be present

— Find the **best match** using SSD error, weighted by Gaussian to emphasize local structure, and take all samples within some distance from that match

Efros and Leung: Synthesizing Many Pixels

For multiple pixels, "grow" the texture in layers

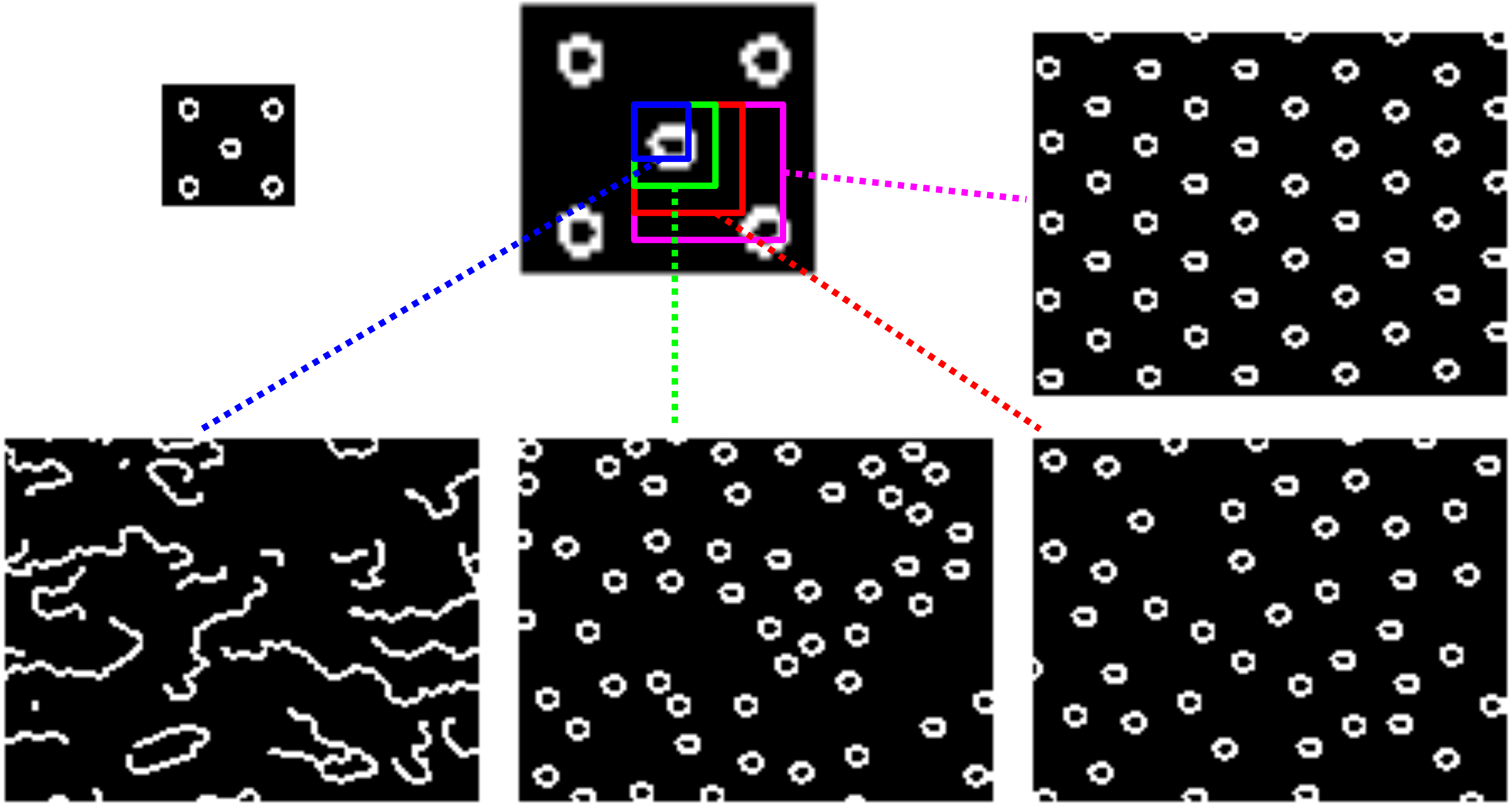
— In the case of hole-filling, start from the edges of the hole

For an interactive demo, see

https://una-dinosauria.github.io/efros-and-leung-js/

(written by Julieta Martinez, a previous CPSC 425 TA)

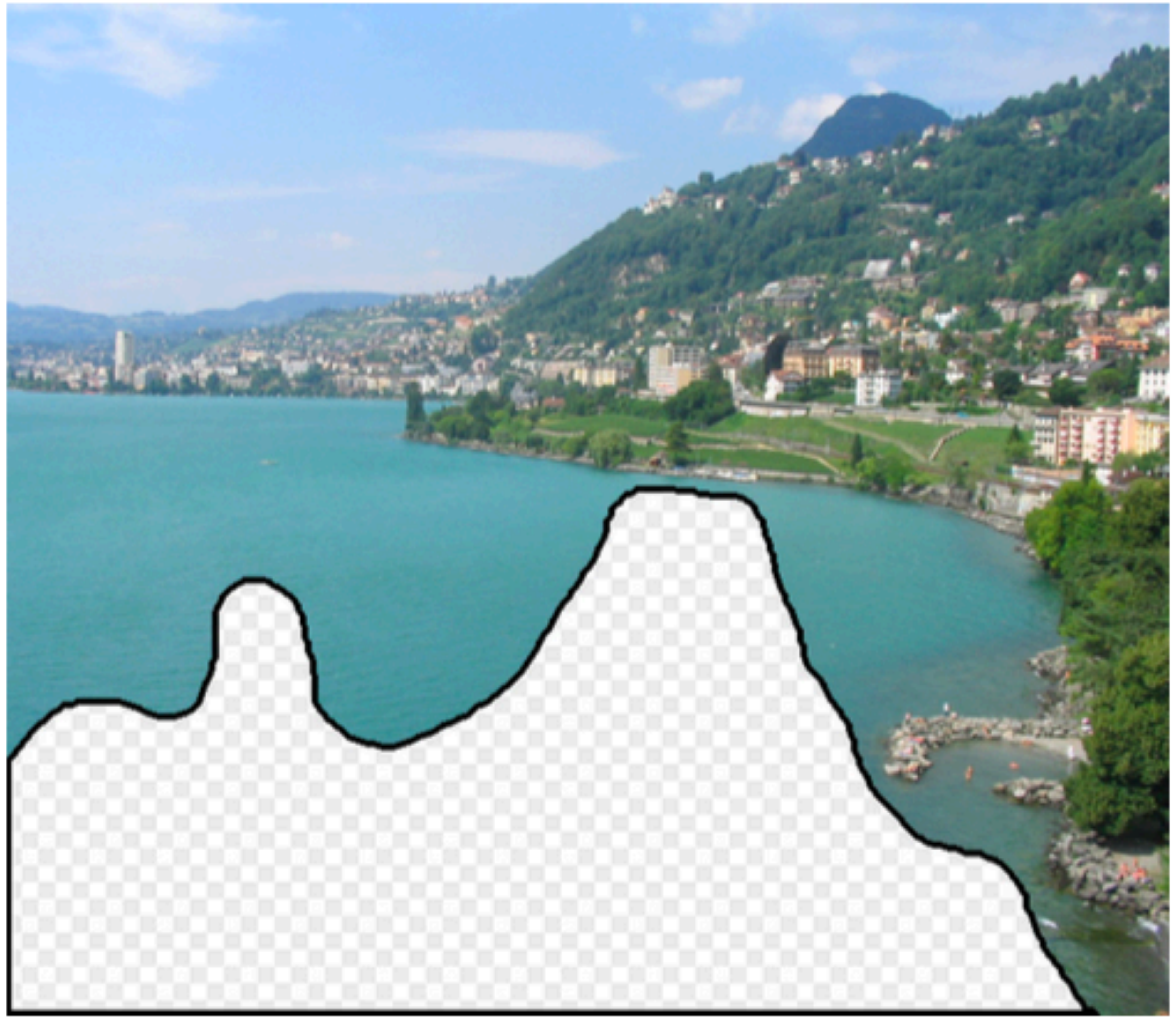
Randomness Parameter



Slide Credit: http://graphics.cs.cmu.edu/people/efros/research/NPS/efros-iccv99.ppt

"**Big** Data" Meets Inpainting

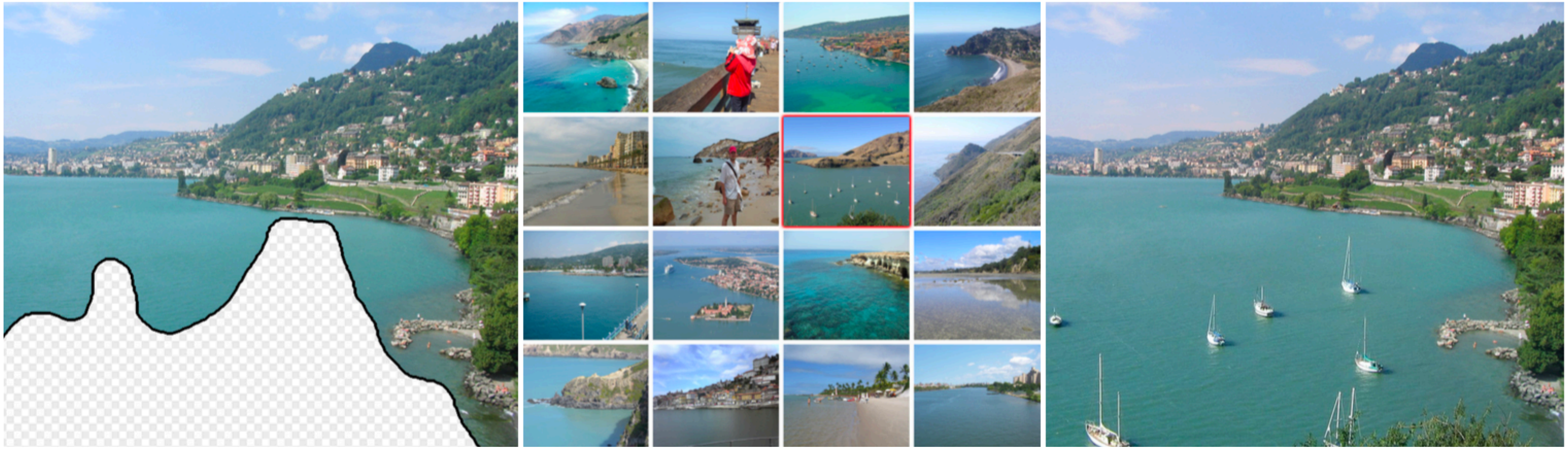
Original Image



Input

Figure Credit: Hays and Efros 2007

“Big Data” Meets Inpainting



Input | Scene Matches | Output

Figure Credit: Hays and Efros 2007

“**Big** Data” Meets Inpainting

Algorithm sketch (Hays and Efros 2007):

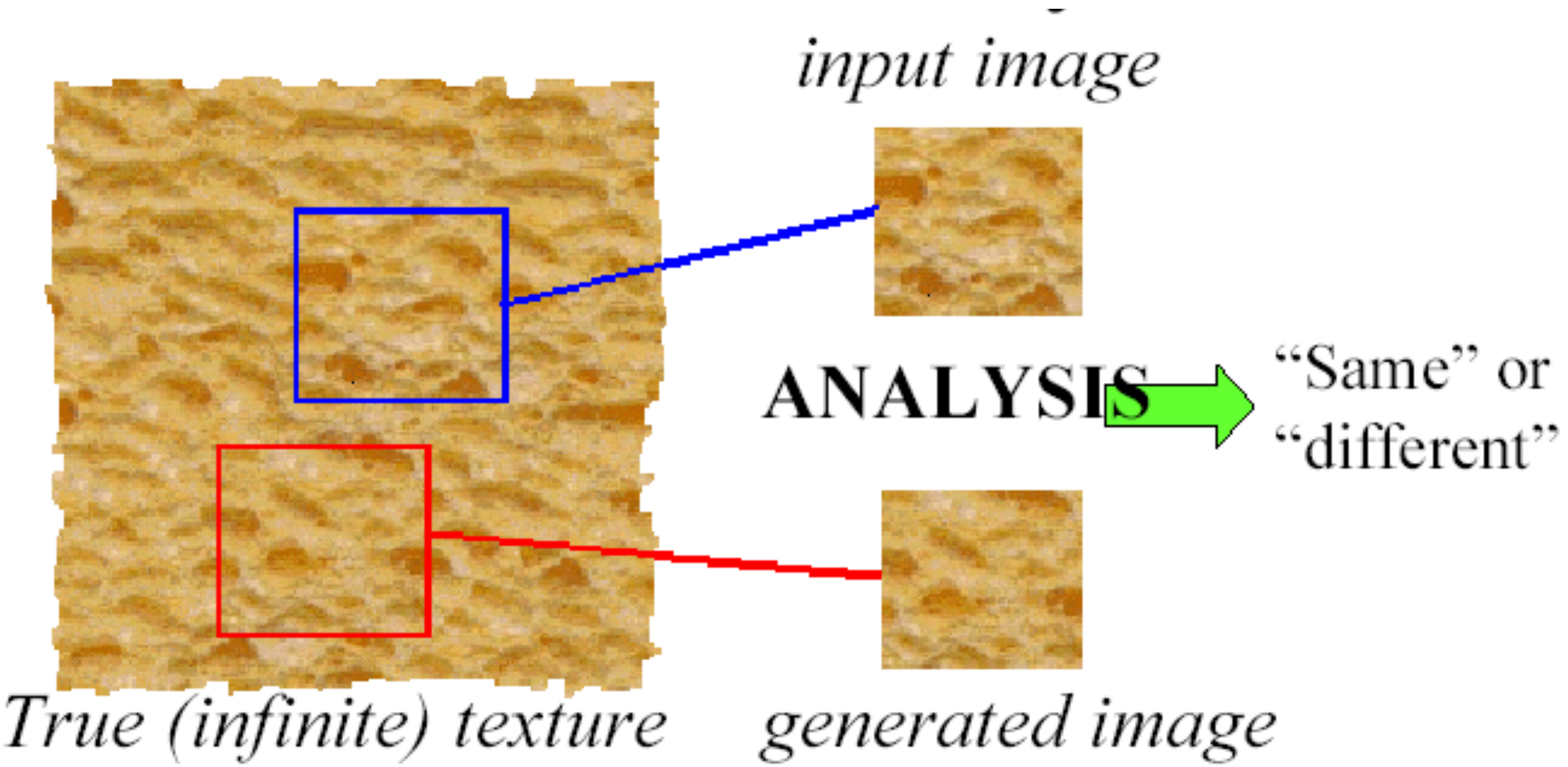
1. Create a short list of a few hundred “best matching" images based on global image statistics

2. Find patches in the short list that match the context surrounding the image region we want to fill

3. Blend the match into the original image

Purely **data-driven**, requires no manual labeling of images

Goal of Texture **Analysis**



Compare textures and decide if they're mae of the same "stuff"

Credit: Bill Freeman

Texture **Representation**

Observation: Textures are made up of generic sub-elements, repeated over a region with similar statistical properties

Idea: Find the sub-elements with filters, then represent each point in the image with a summary of the pattern of sub-elements in the local region

Texture **Representation**

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Question: What filters should we use?

Answer: Human vision suggests spots and oriented edge filters at a variety of different orientations and scales

Texture **Representation**

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Question: What filters should we use?

Answer: Human vision suggests spots and oriented edge filters at a variety of different orientations and scales

Question: How do we “summarize”?

Answer: Compute the mean or maximum of each filter response over the region
— Other statistics can also be useful

Texture **Representation**

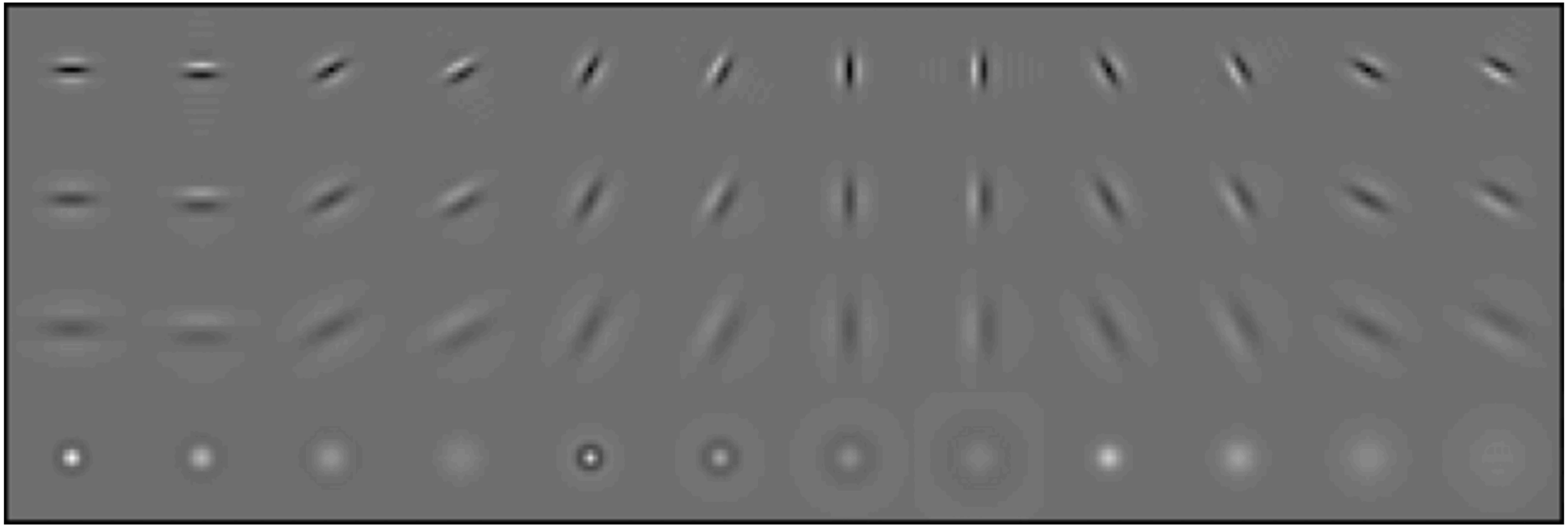
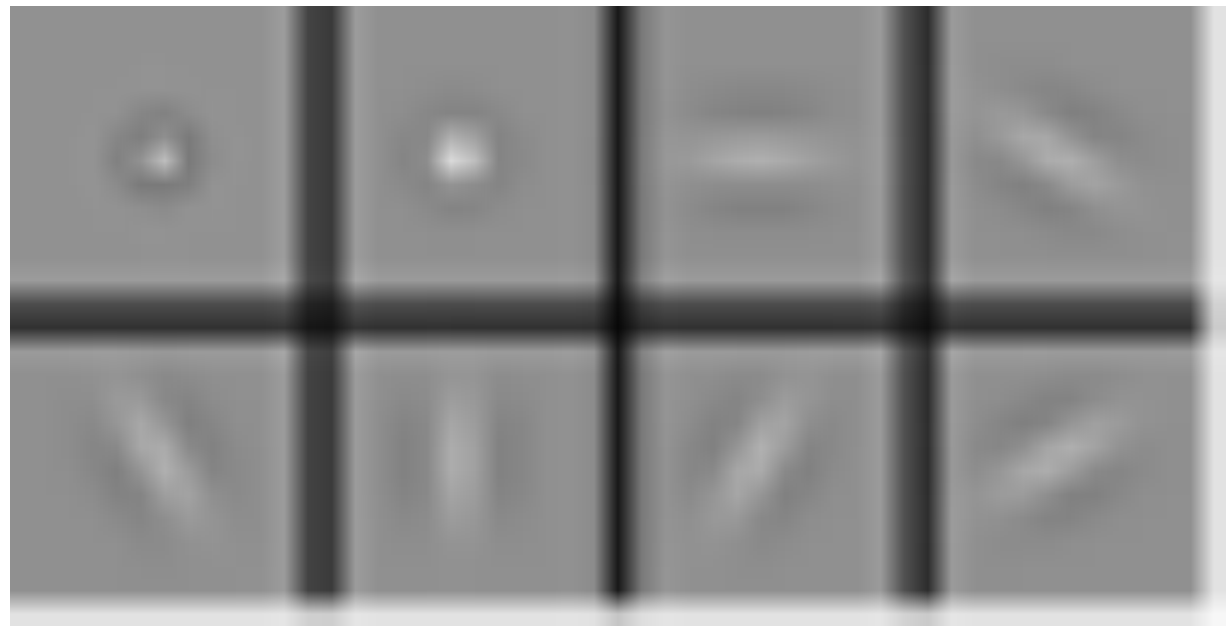
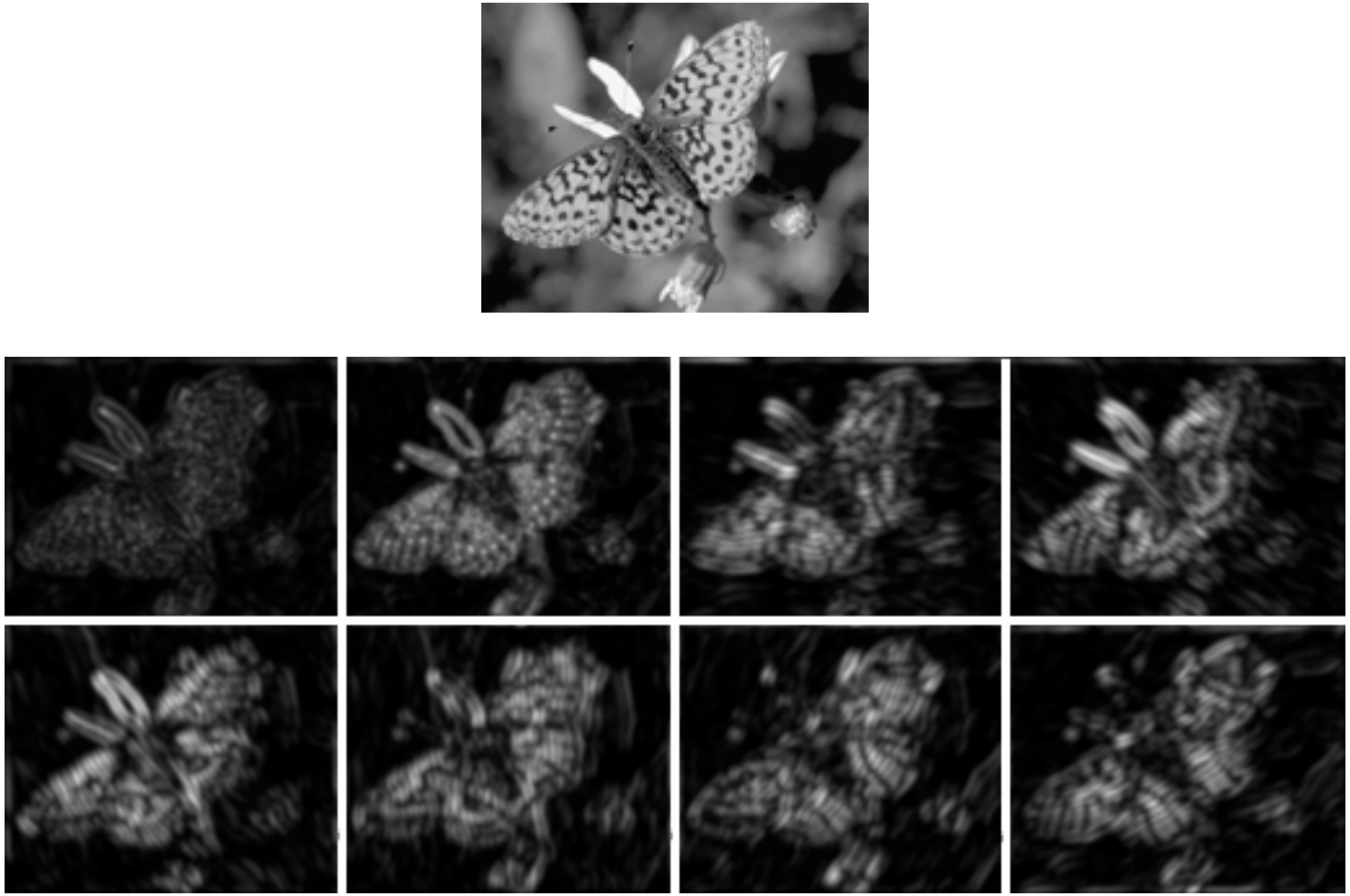


Figure Credit: Leung and Malik, 2001

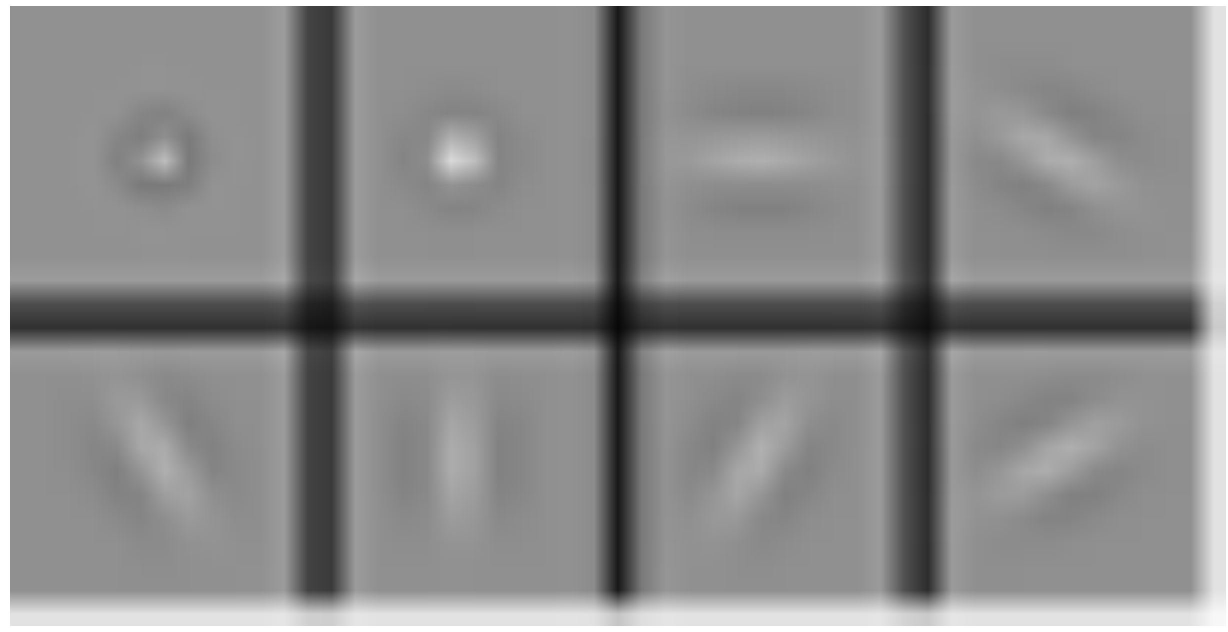
Spots and Bars (Fine Scale)

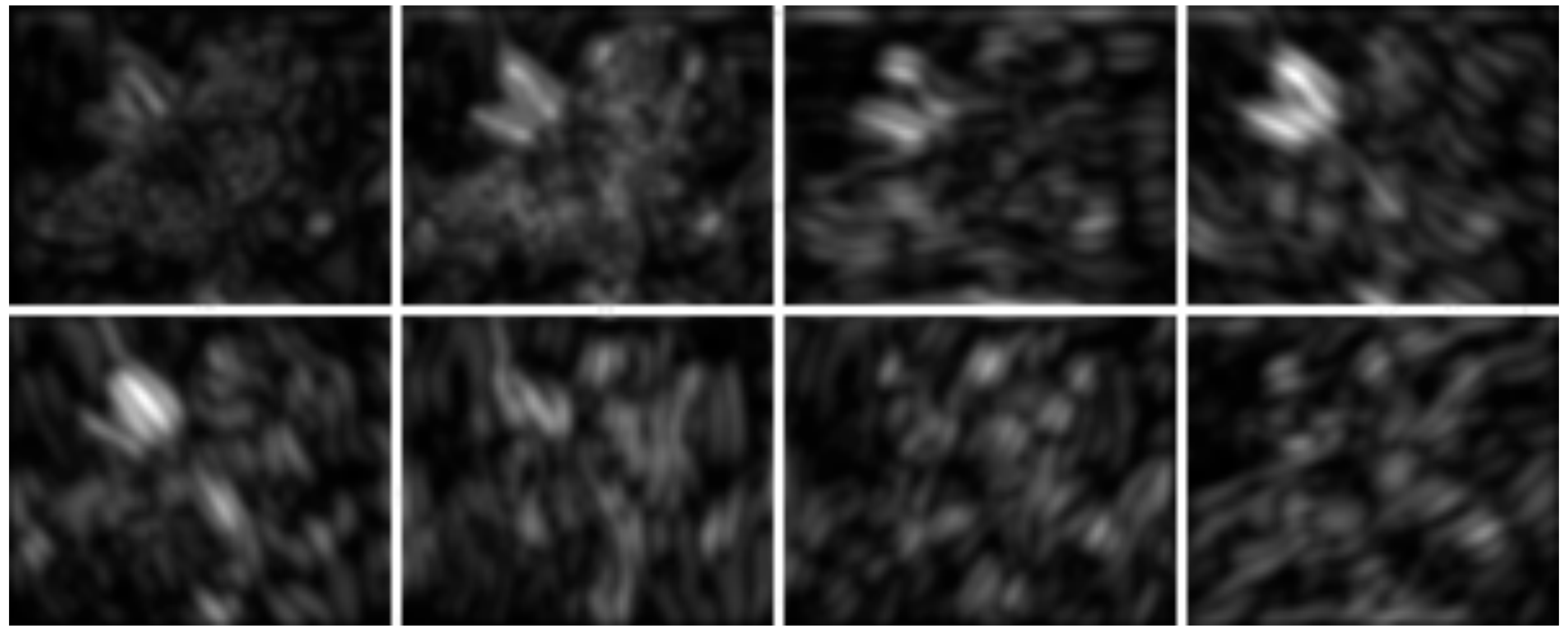




Forsyth & Ponce (1st ed.) Figures 9.3–9.4

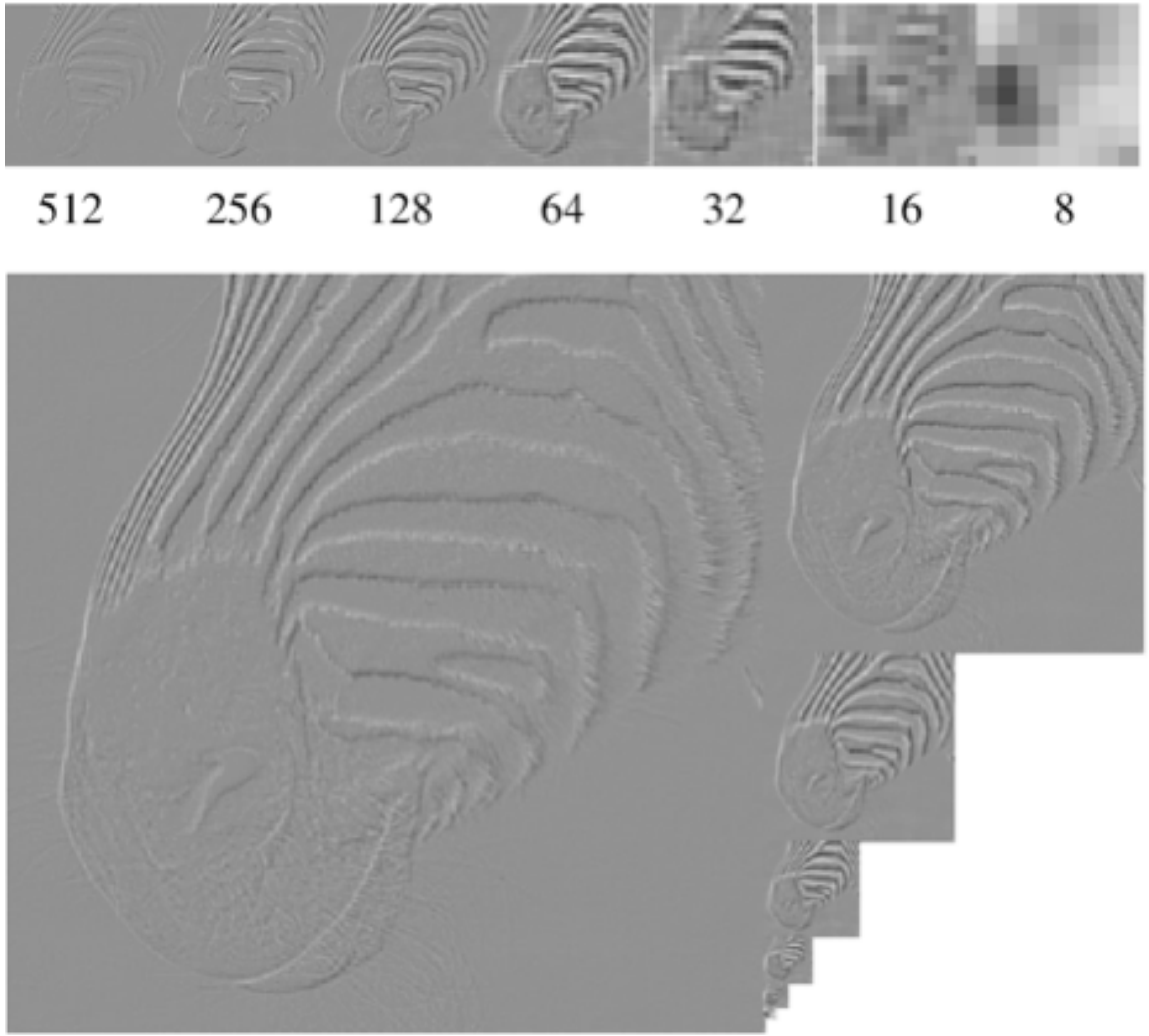
Spots and Bars (Coarse Scale)





Forsyth & Ponce (1st ed.) Figures 9.3 and 9.5

Laplacian Pyramid



Laplacian Pyramid

Building a **Laplacian** pyramid:

— Create a Gaussian pyramid

— Take the difference between one Gaussian pyramid level and the next (before subsampling)

Properties

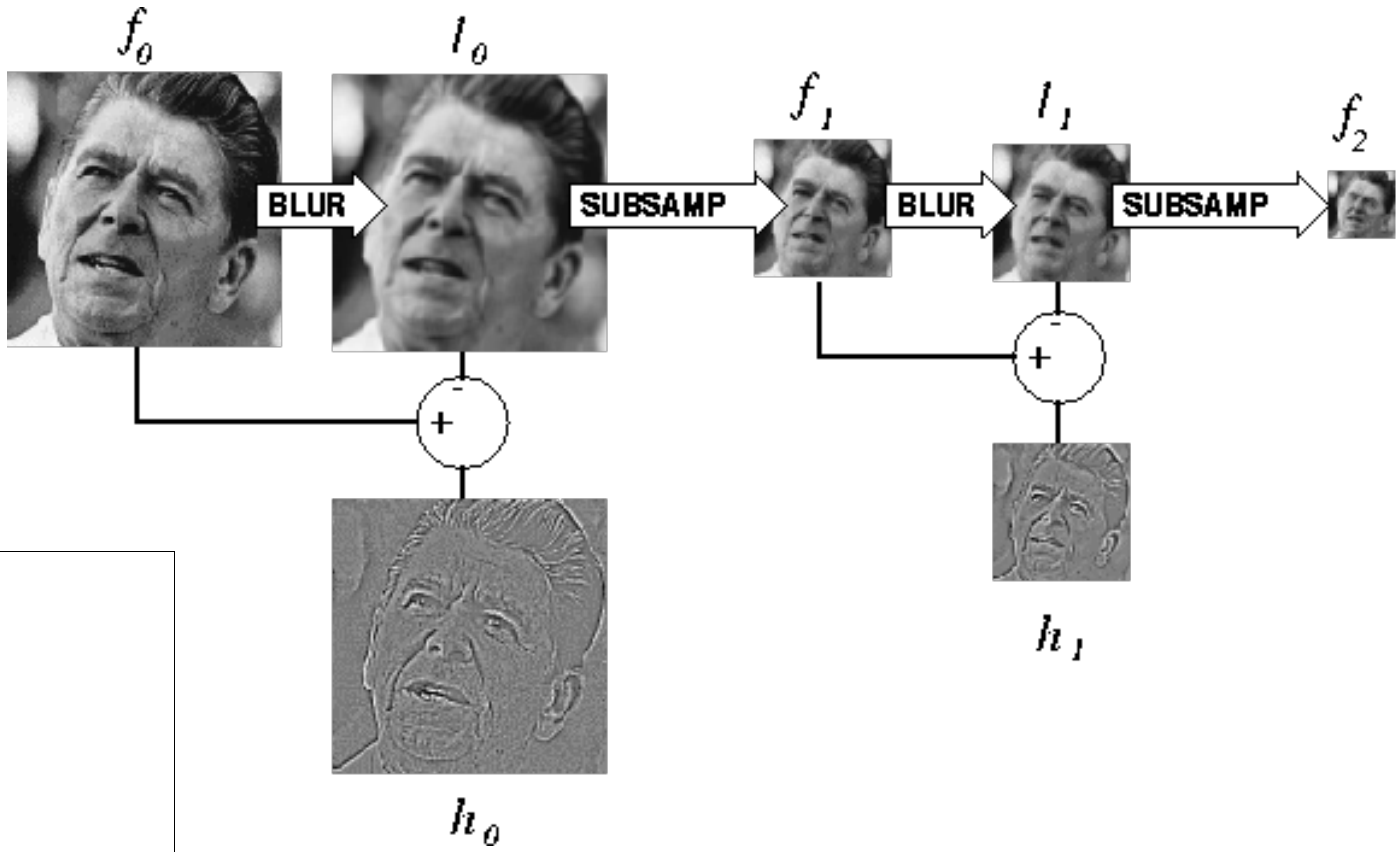
— Also known as the difference-of-Gaussian (DOG) function, a close approximation to the Laplacian

— It is a band pass filter – each level represents a different band of spatial frequencies

Reconstructing the original image:

— Reconstruct the Gaussian pyramid starting at top

Constructing a **Laplacian** Pyramid

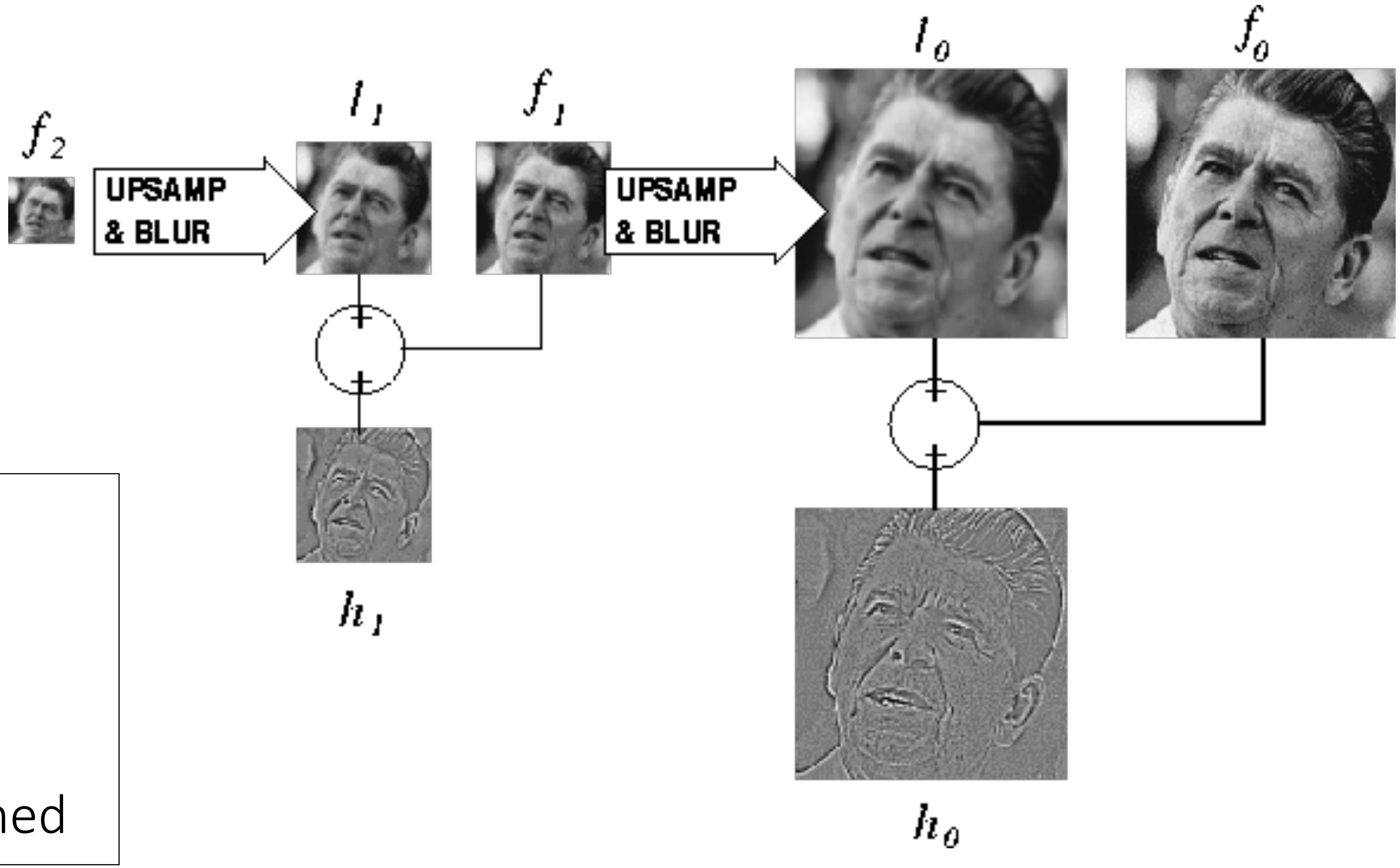


Algorithm

repeat:
  filter
  compute residual
  subsample
until min resolution reached

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Reconstructing the Original Image



Algorithm

repeat:

upsample

sum with residual

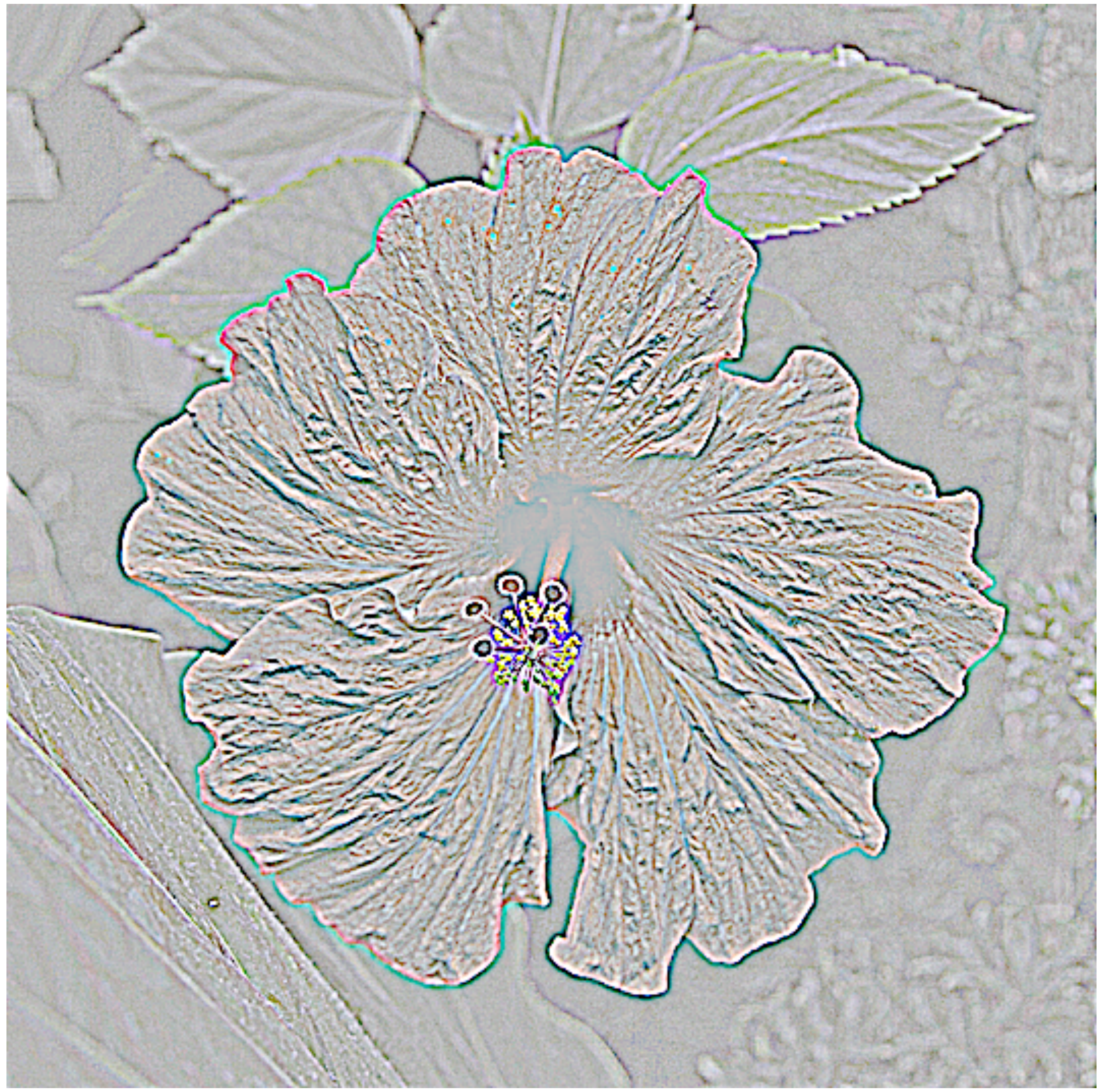
until orig resolution reached

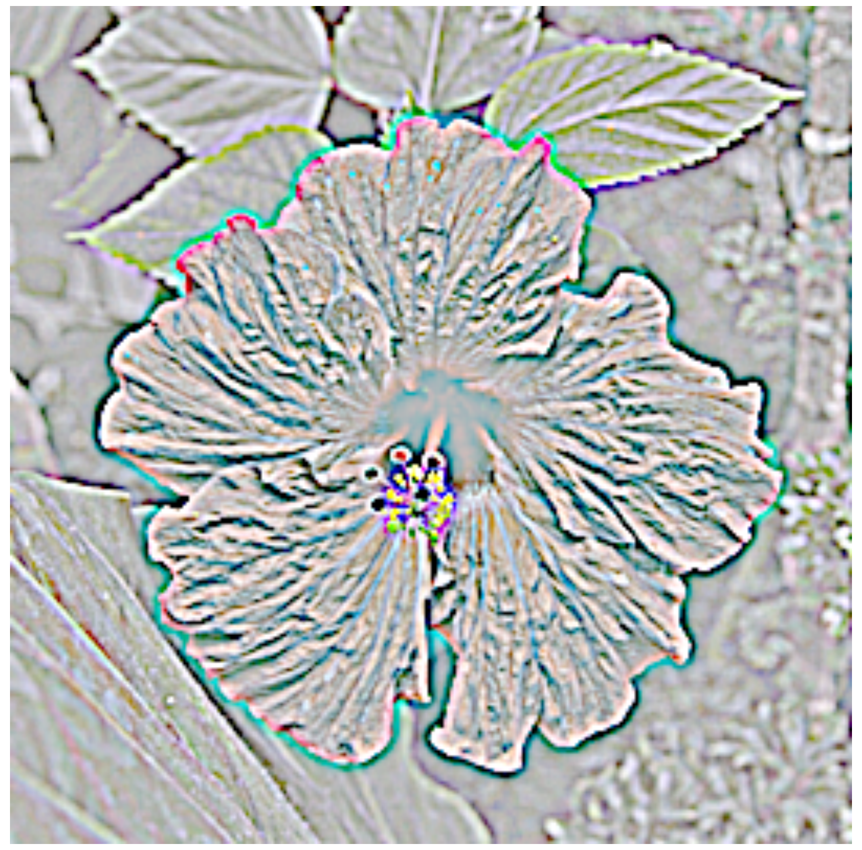
Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Gaussian vs **Laplacian** Pyramid




Shown in opposite order for space

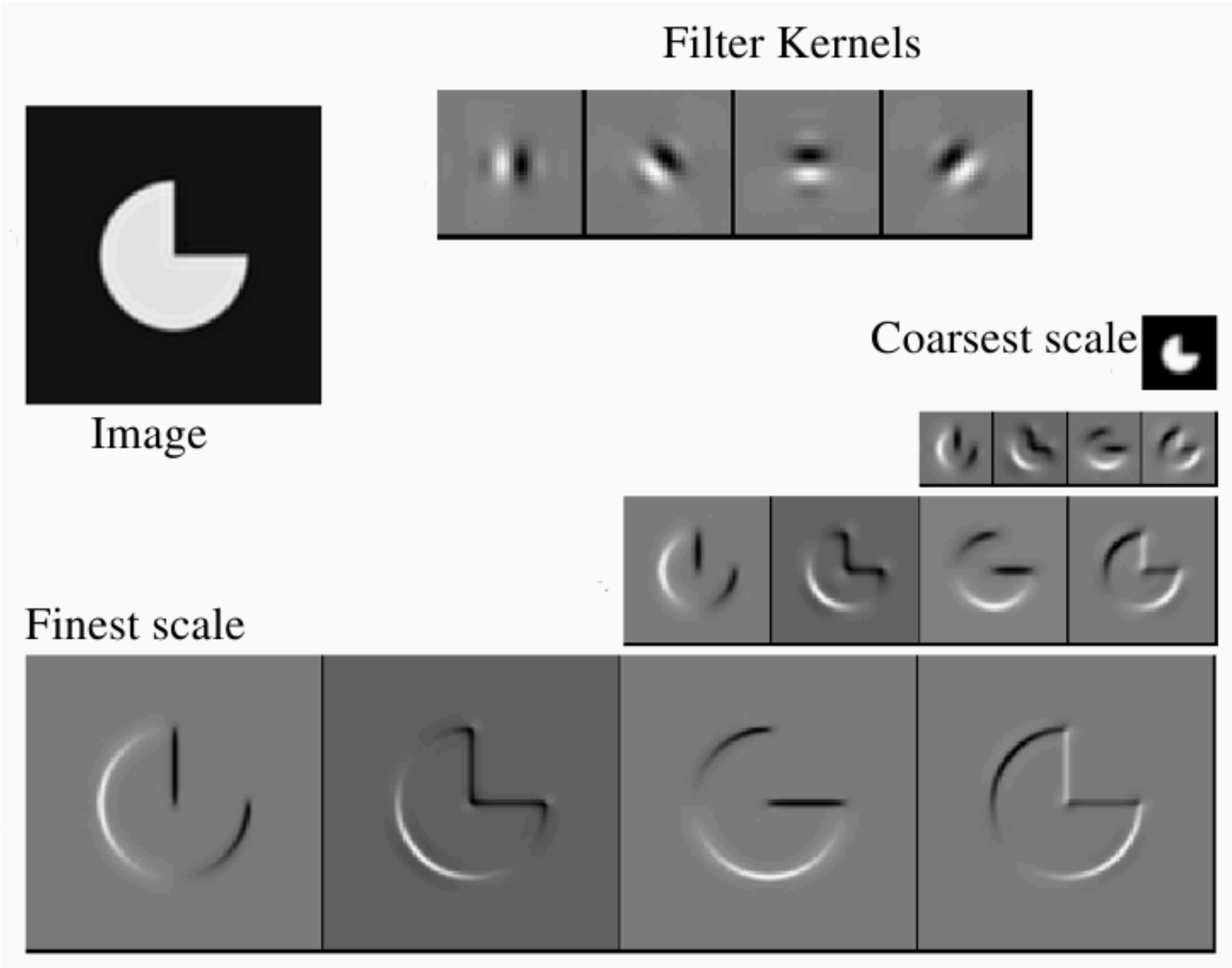




Which one takes more space to store?


Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Oriented Pyramids



Forsyth & Ponce (1st ed.) Figure 9.13

Final Texture Representaation

Steps:

1. Form a Laplacian and oriented pyramid (or equivalent set of responses to filters at different scales and orientations)

2. Square the output (makes values positive)

3. Average responses over a neighborhood by blurring with a Gaussian

4. Take statistics of responses

— Mean of each filter output

— Possibly standard deviation of each filter

Sample Question: Texture

How does the top-most image in a Laplacian pyramid differ from the others?