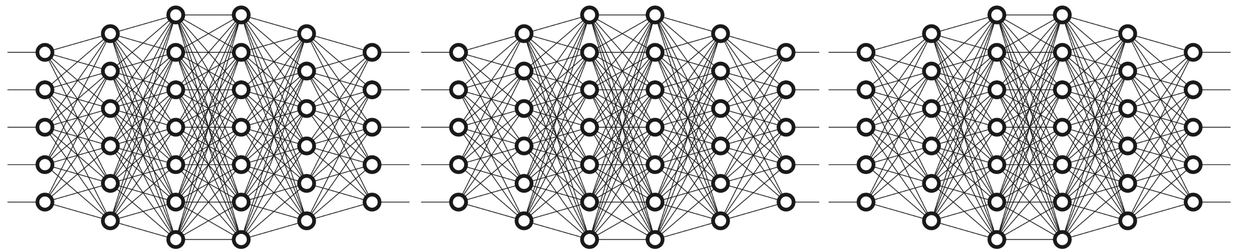
THE UNIVERSITY OF BRITISH COLUMBIA

CPSC 425: Computer Vision



Lecture 21: Neural Networks (cont), CNNs

Menu for Today

Topics:

- Backpropagation
- Convolutional Layers
- Pooling Layer
- Quiz 5?

Redings:

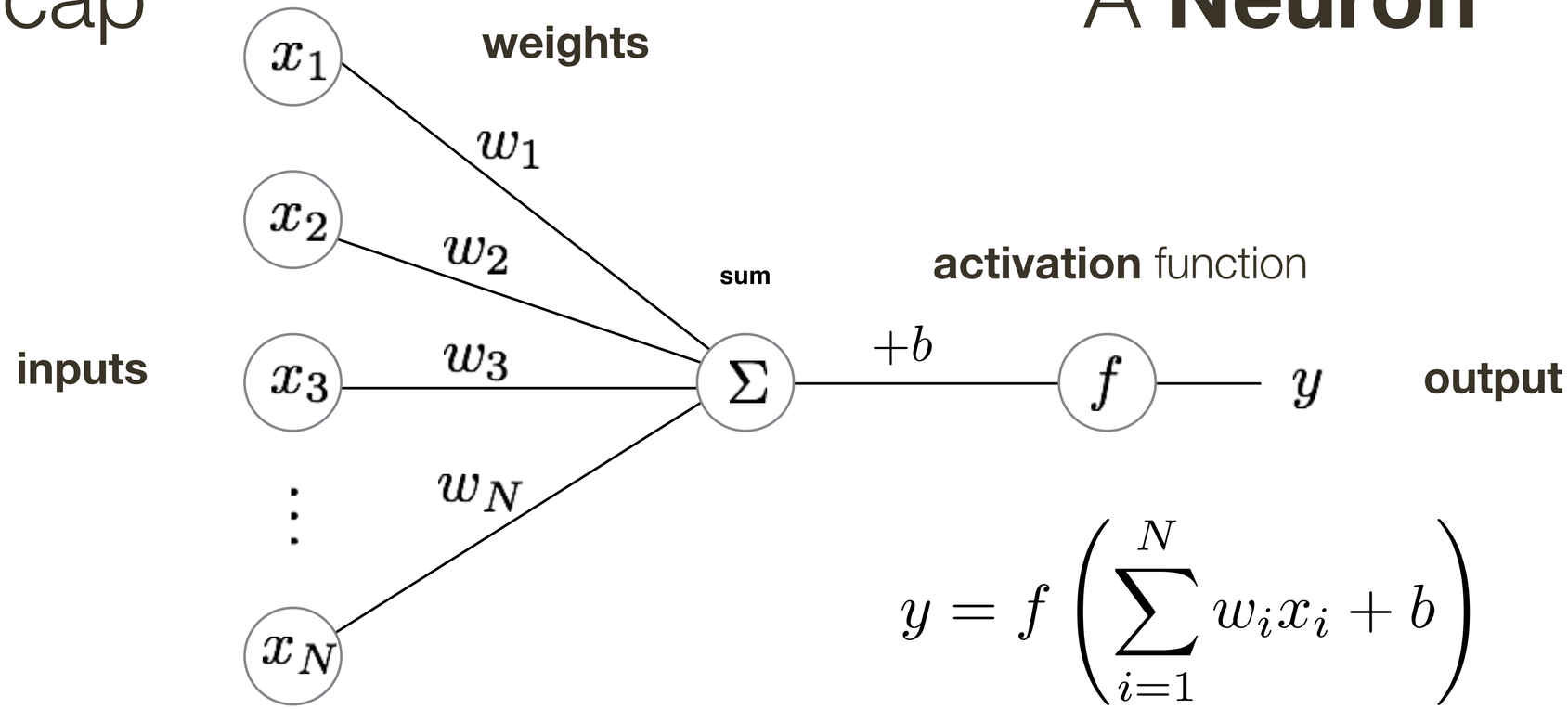
- **Today's** Lecture: N/A
- **Next** Lecture: N/A

Reminders:

- **Assignment 6**: Deep Learning will be out **tomorrow**
- **Material** for **Final Prep** will make available on Canvas this weekend

Lecture 20: Re-cap

A **Neuron**



$$y = f\left(\sum_{i=1}^{N} w_i x_i + b\right)$$

— The basic unit of computation in a neural network is a neuron.

— A neuron accepts some number of input signals, computes their weighted sum, and applies an **activation function** (or **non-linearity**) to the sum.

— Common activation functions include sigmoid and rectified linear unit (ReLU)

Lecture 20: Re-cap **Neural** Network

A neural network comprises neurons connected in an acyclic graph

The outputs of neurons can become inputs to other neurons

Neural networks typically contain multiple layers of neurons

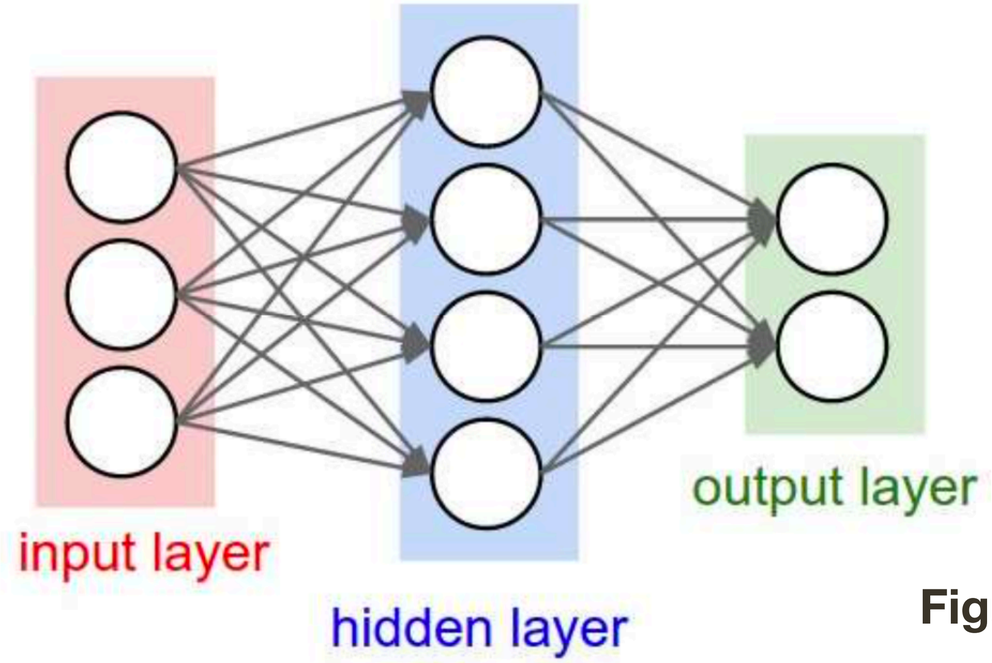
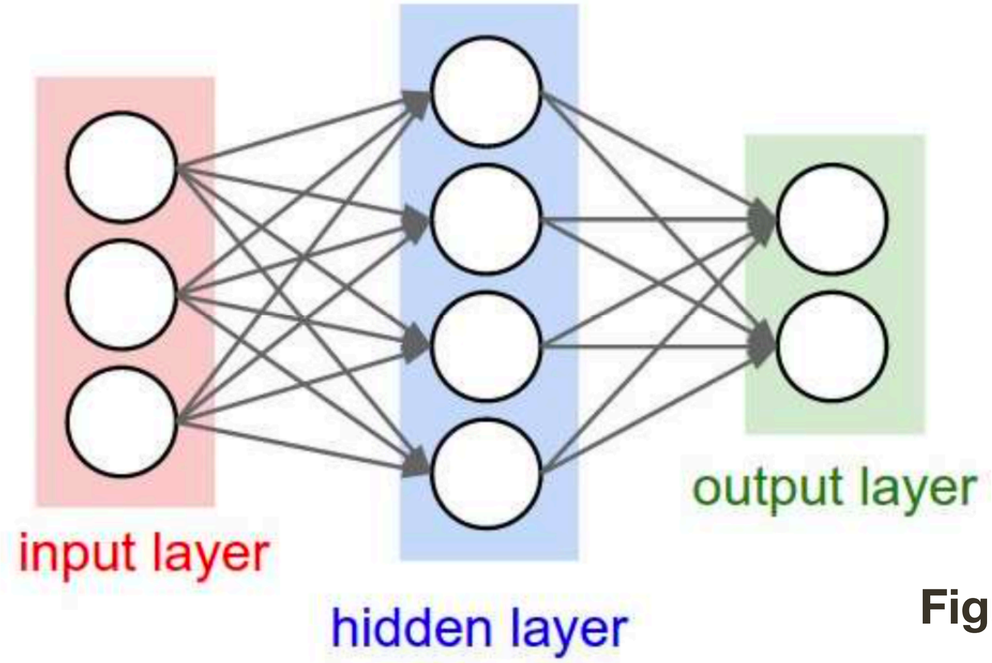


Figure credit: Fei-Fei and Karpathy

Example of a neural network with three inputs, a single hidden layer of four neurons, and an output layer of two neurons

Lecture 20: Re-cap **Neural** Network

Note: each neuron will have its own vector of weights and a bias, its easier to think of all neurons in a layer as a single entity with a matrix of weights (size = number of inputs x number of neurons) and a vector of biases (size = number of neurons)



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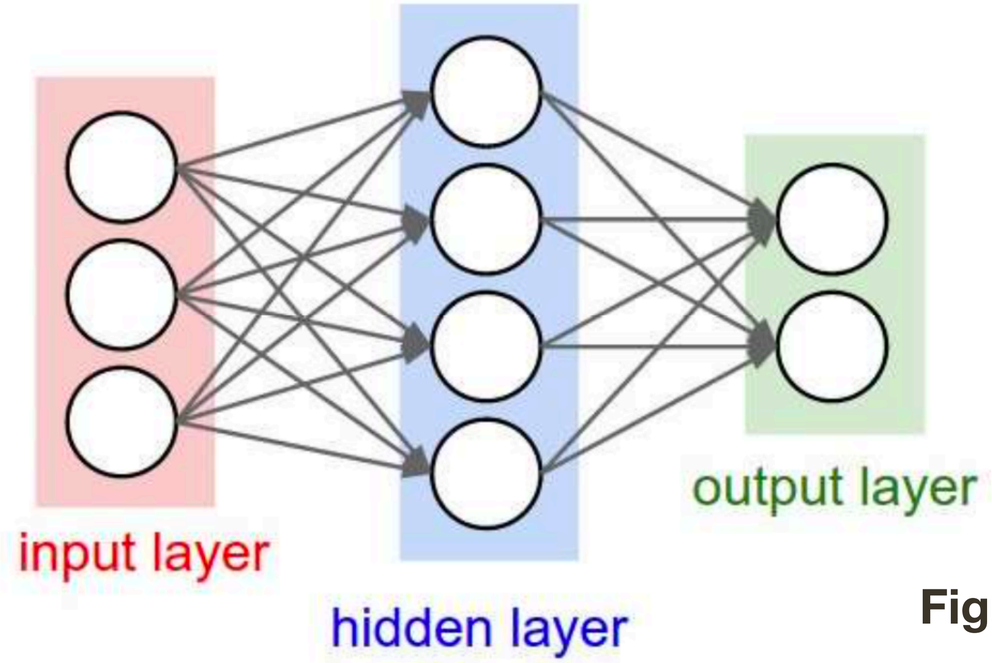

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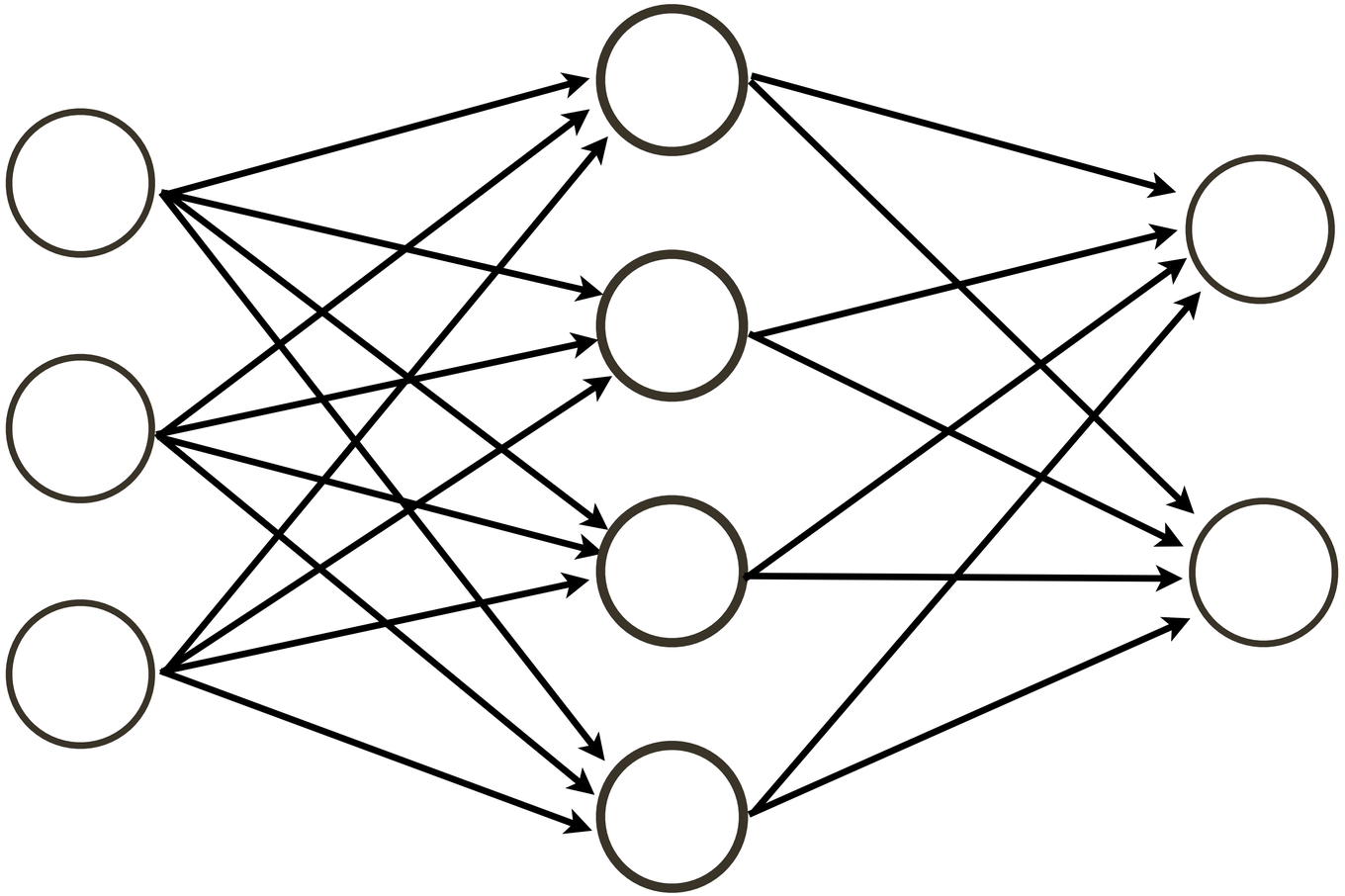
$$\hat{\mathbf{y}} = f(\mathbf{x}, \mathbf{W}_1, \mathbf{W}_2, \mathbf{b}_1, \mathbf{b}_2) = \sigma\left(\mathbf{W}_2^{(2\times4)}\sigma\left(\mathbf{W}_1^{(4\times3)}\mathbf{x} + \mathbf{b}_1^{(4)}\right) + \mathbf{b}_2^{(2)}\right)$$

Activation Function

Why can’t we have **linear** activation functions? Why have non-linear activations?



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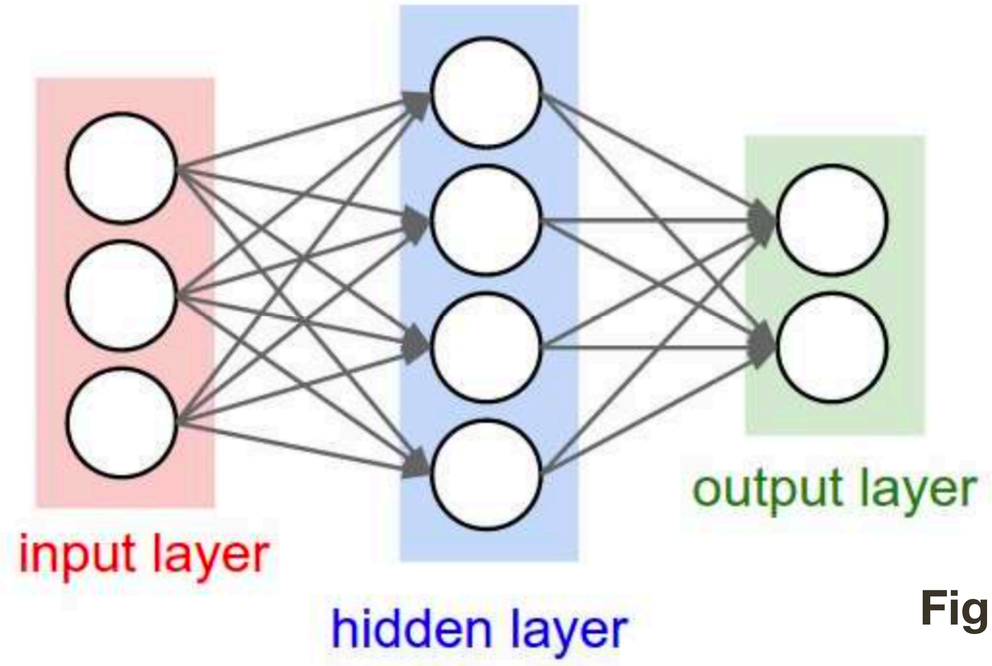

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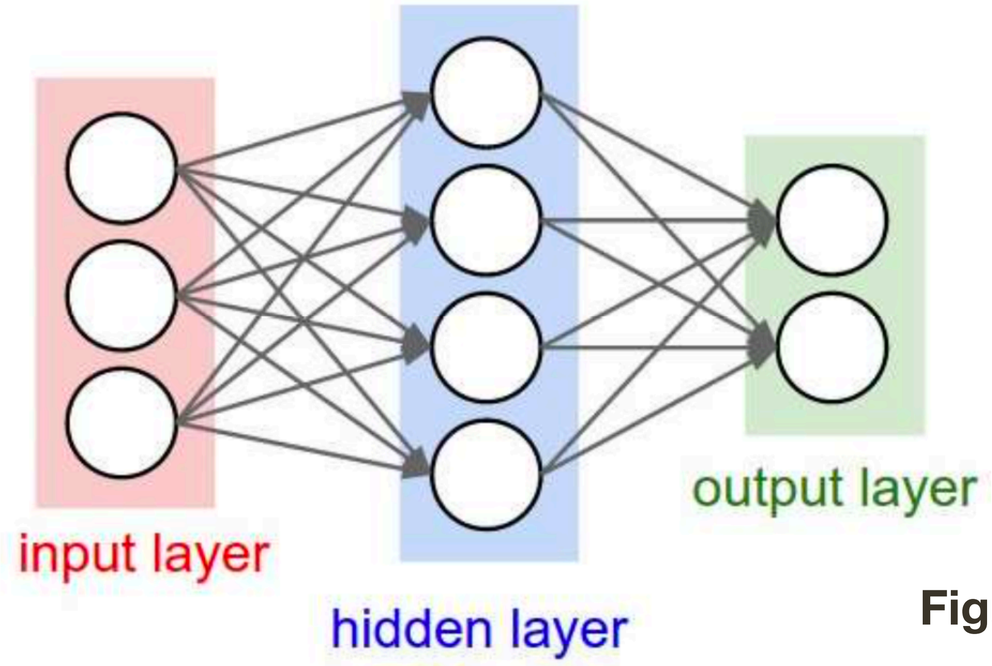
$$= \mathbf{W}_2^{(2\times4)}\left(\mathbf{W}_1^{(4\times3)}\mathbf{x} + \mathbf{b}_1^{(4)}\right) + \mathbf{b}_2^{(2)}$$



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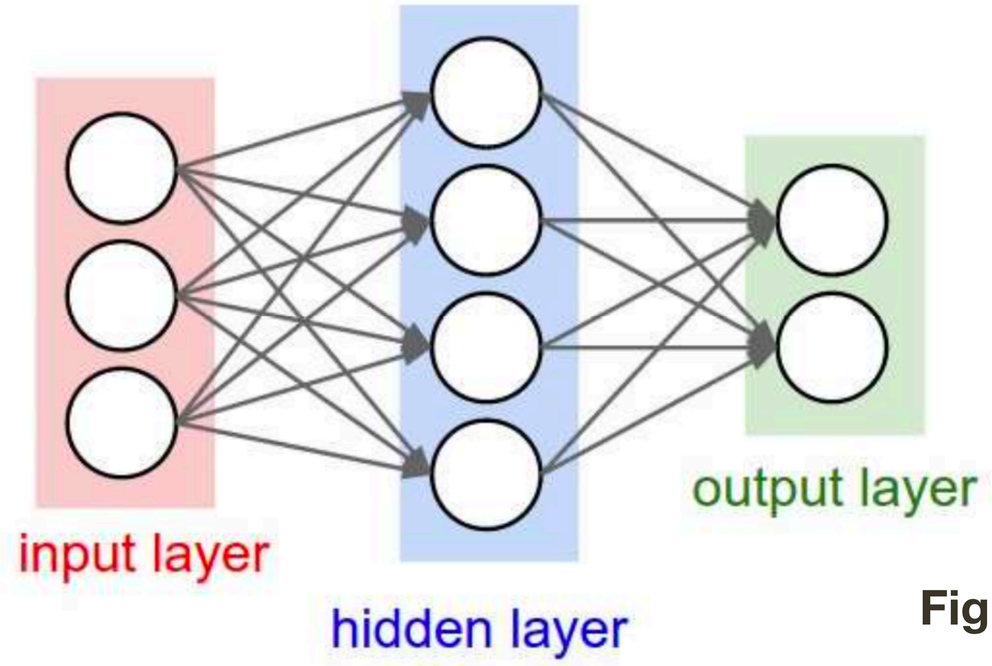

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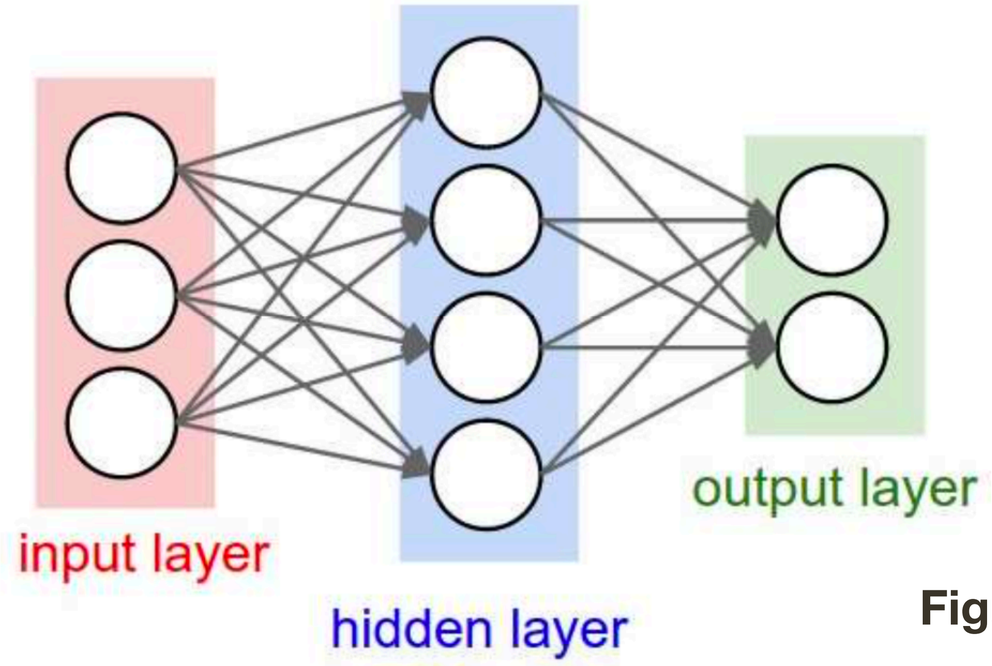

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Light Theory: Neural Network as Universal Approximator

Conditions needed for proof to hold: Activation function needs to be well defined

$$\lim_{x \to \infty} a(x) = A$$

$$\lim_{x \to -\infty} a(x) = B$$

$$A \neq B$$

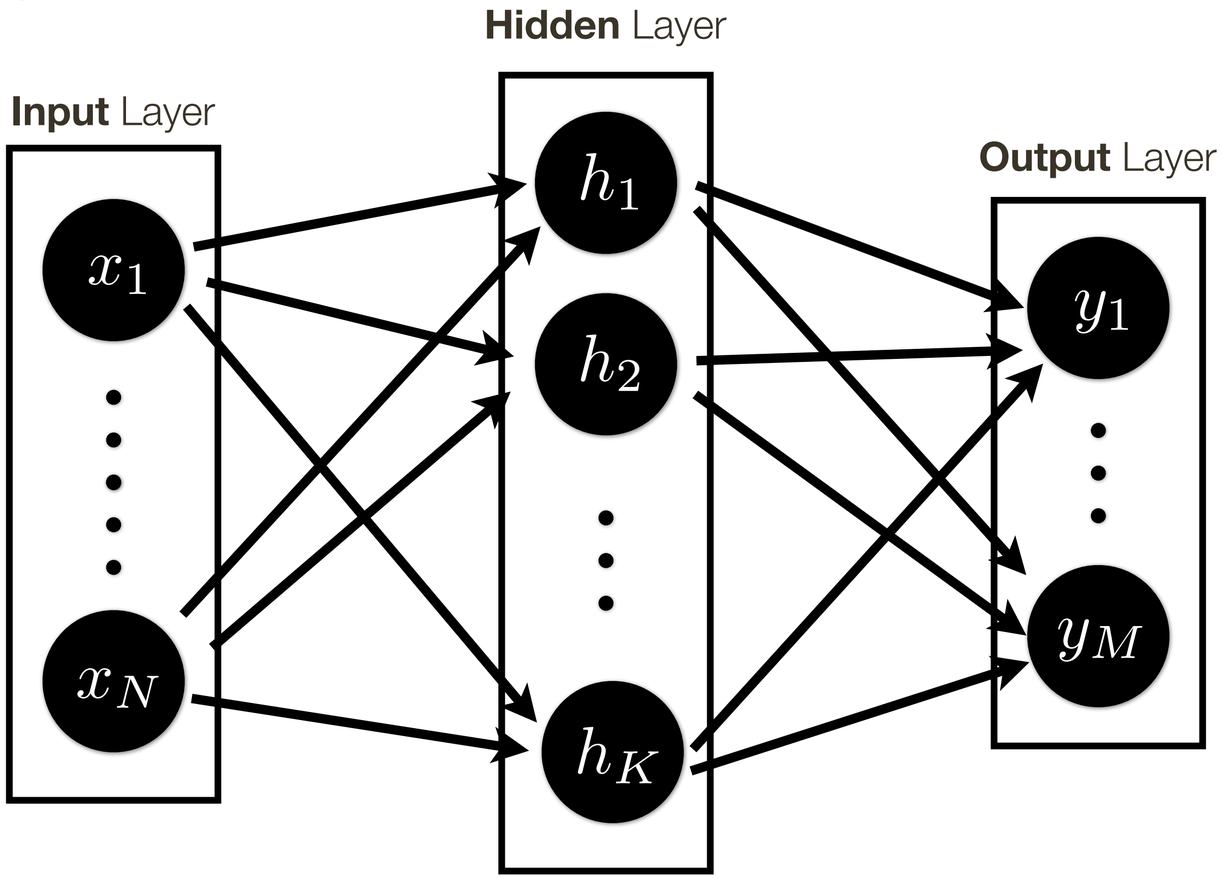
Note: This gives us another way to provably say that linear activation function cannot produce a neural network which is an universal approximator.

*slide adopted from http://neuralnetworksanddeeplearning.com/chap4.html

Light Theory: Neural Network as Universal Approximator

Universal Approximation Theorem: Single hidden layer can approximate any continuous function with compact support to arbitrary accuracy, when the width goes to infinity.

[Hornik *et al*., 1989]



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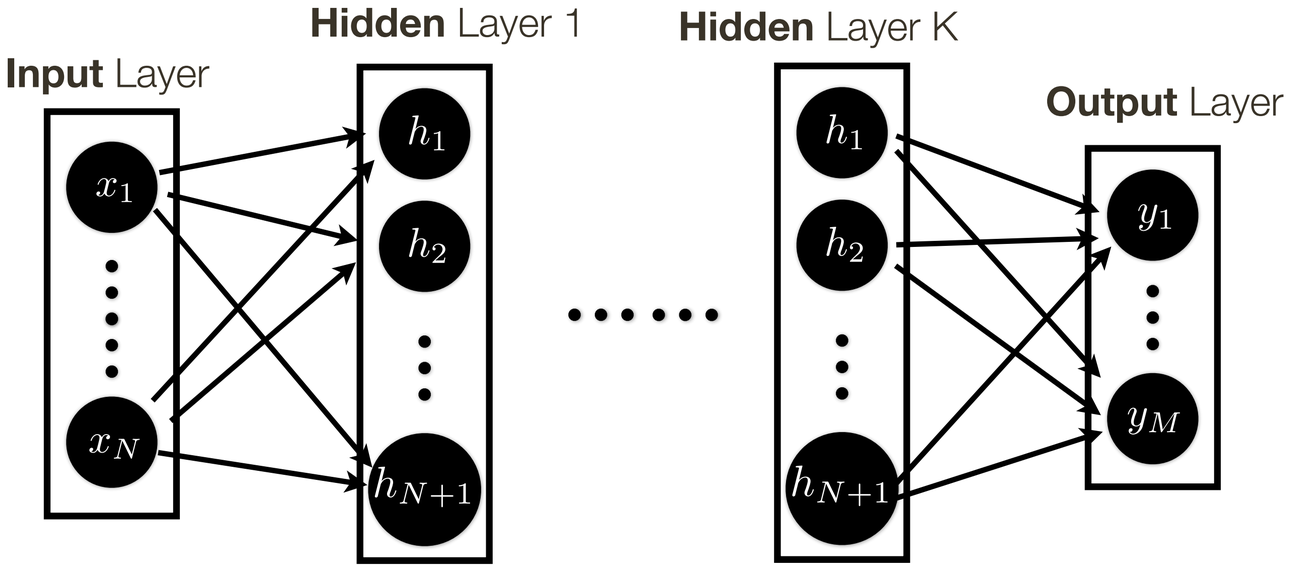
Universal Approximation Theorem (revised): A network of infinite depth with a hidden layer of size $d+1$ neurons, where d is the dimension of the input space, can approximate any continuous function.

[Lu *et al*., NIPS 2017]



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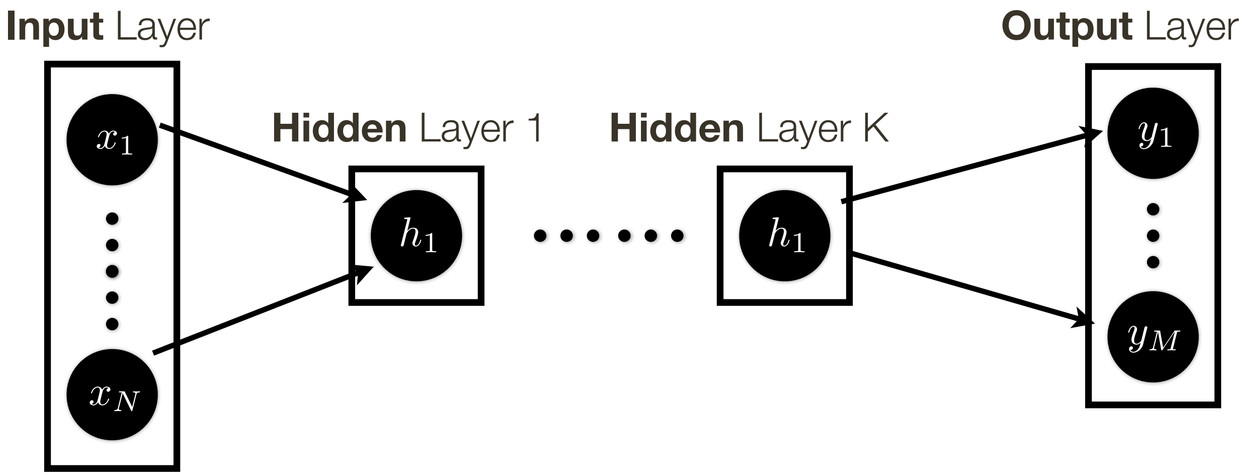

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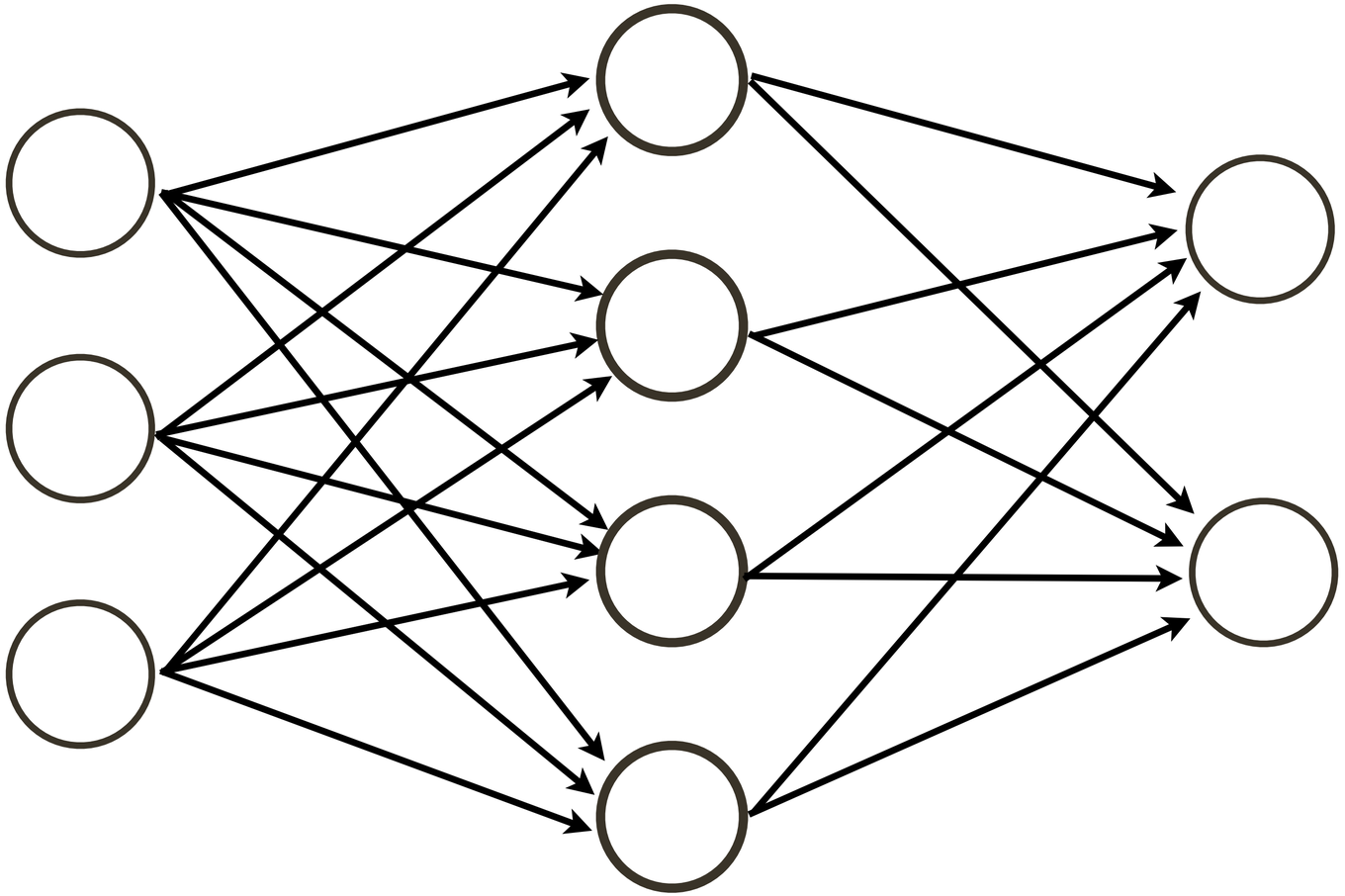
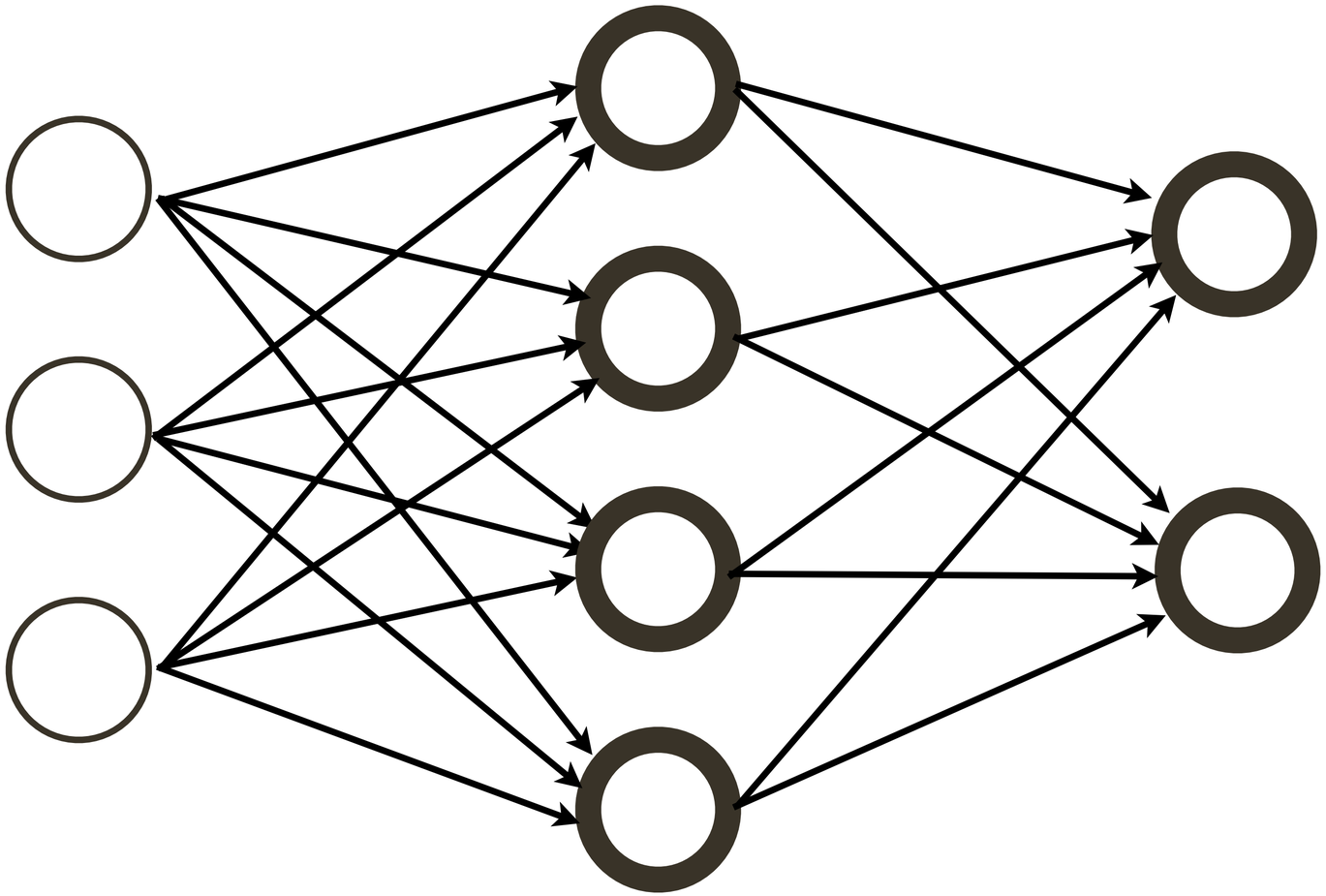
Neural Network

How many neurons?



Neural Network

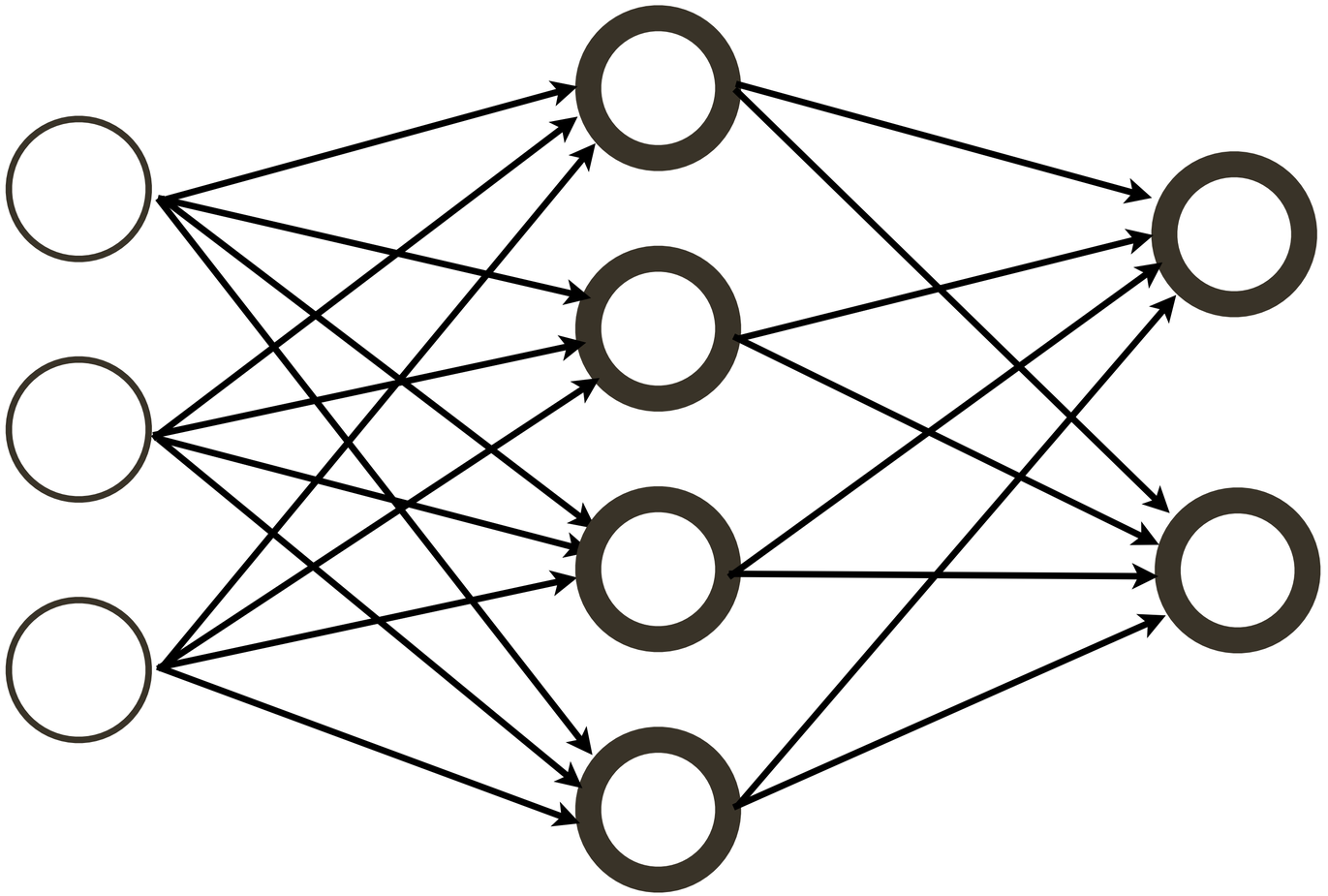
How many neurons? 4+2 = 6



Neural Network

How many neurons? 4+2 = 6

How many weights?

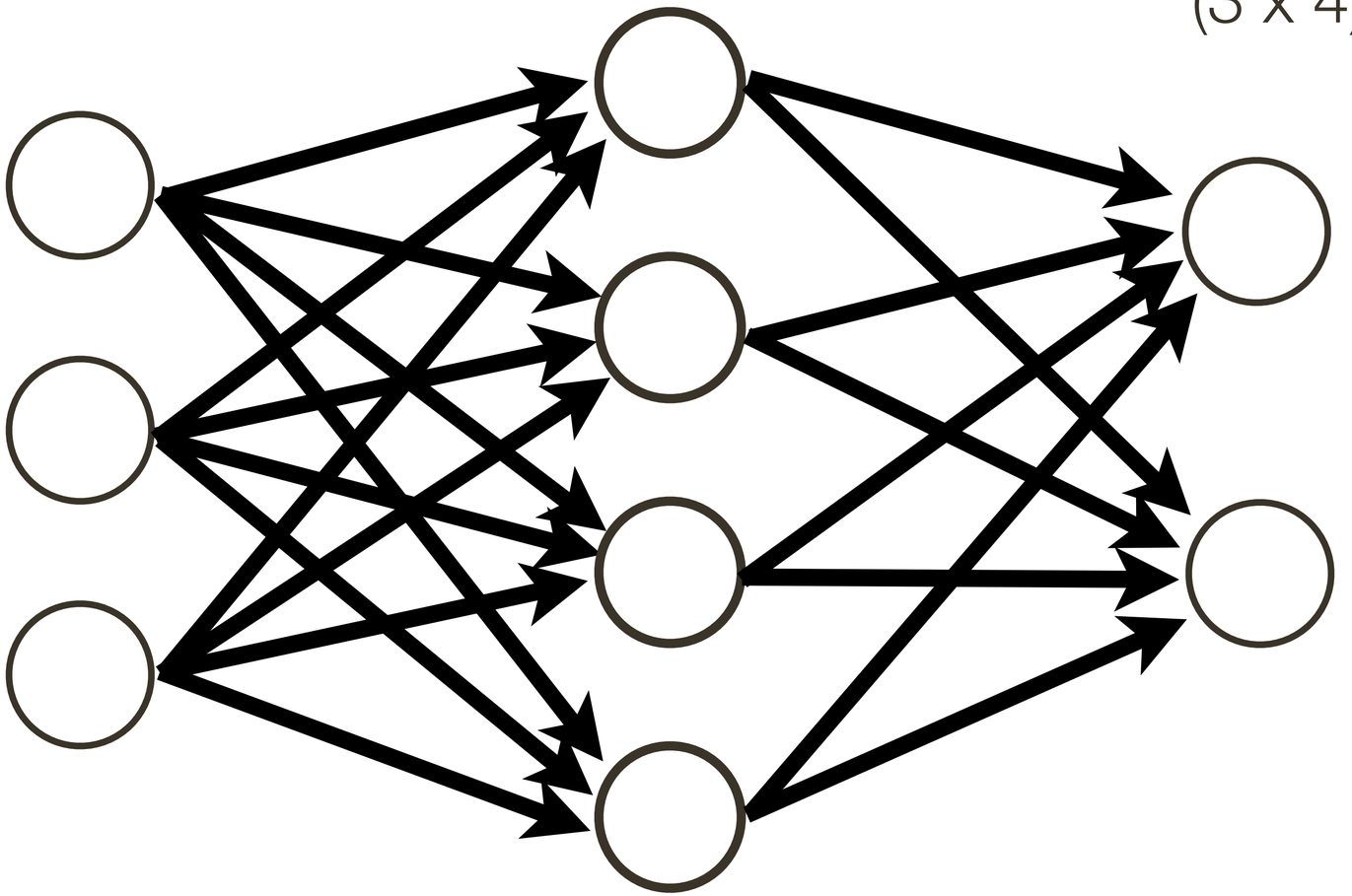


Neural Network

How many neurons? 4+2 = 6

How many weights?

(3 x 4) + (4 x 2) = 20



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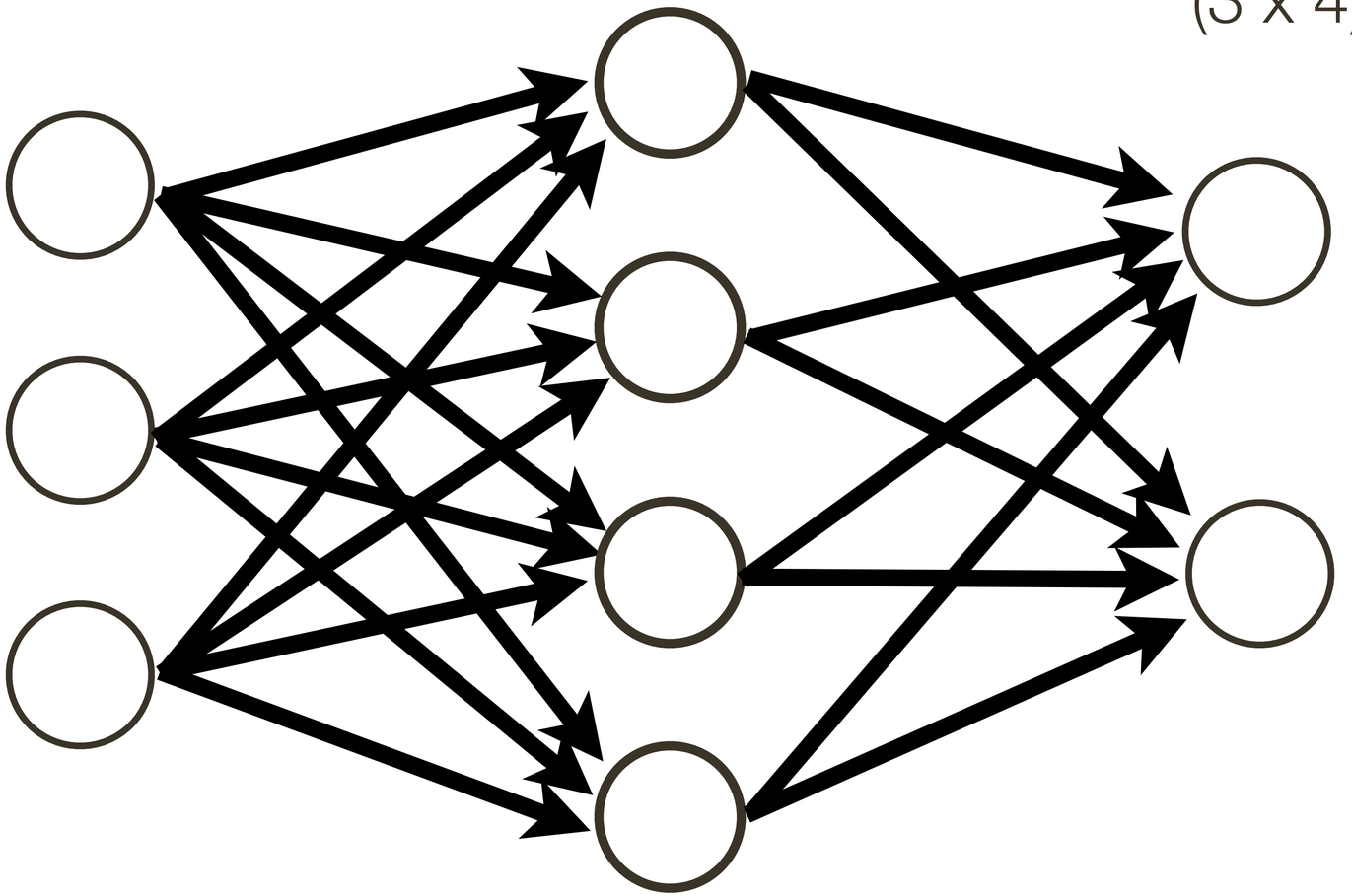

How many learnable parameters?

Neural Network

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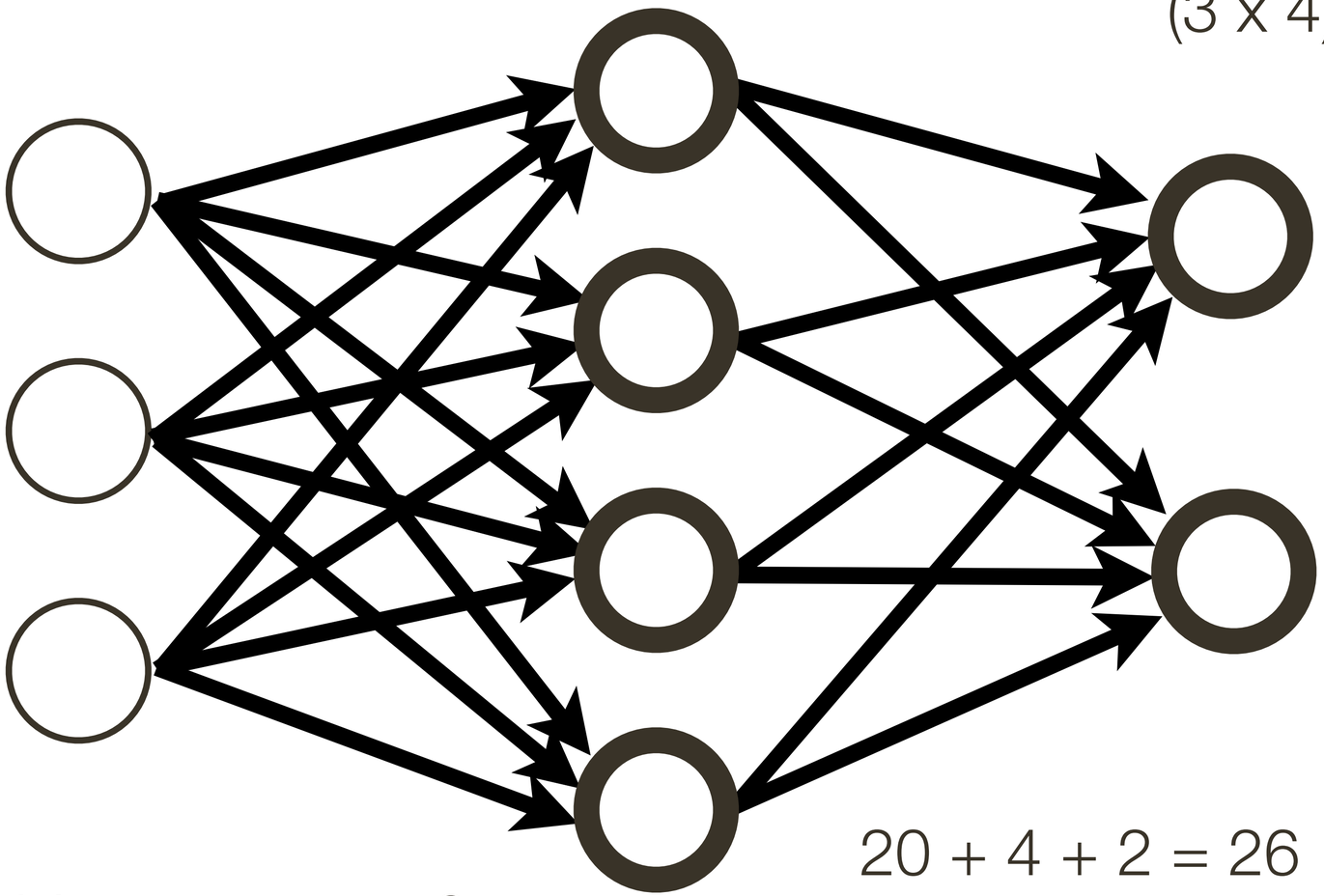


20 + 4 + 2 = 26

bias terms

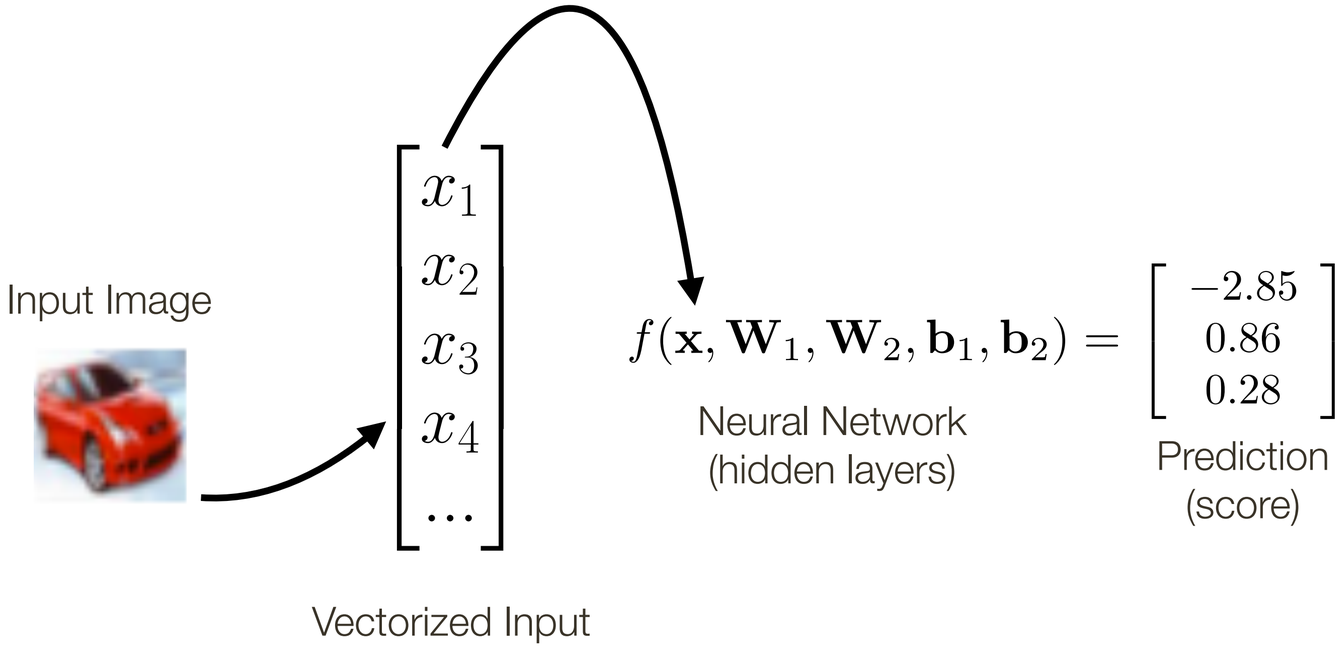
How many learnable parameters?

Neural Networks

Modern **convolutional neural networks** contain 10-20 layers and on the order of 100 million parameters

Training a neural network requires estimating a large number of parameters

Training a Neural Network



Training a Neural Network

Input Image

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \dots \end{bmatrix}$$

Vectorized Input

$$f(\mathbf{x}, \mathbf{W}_1, \mathbf{W}_2, \mathbf{b}_1, \mathbf{b}_2) = \begin{bmatrix} -2.85 \\ 0.86 \\ 0.28 \end{bmatrix}$$

Neural Network
(hidden layers)

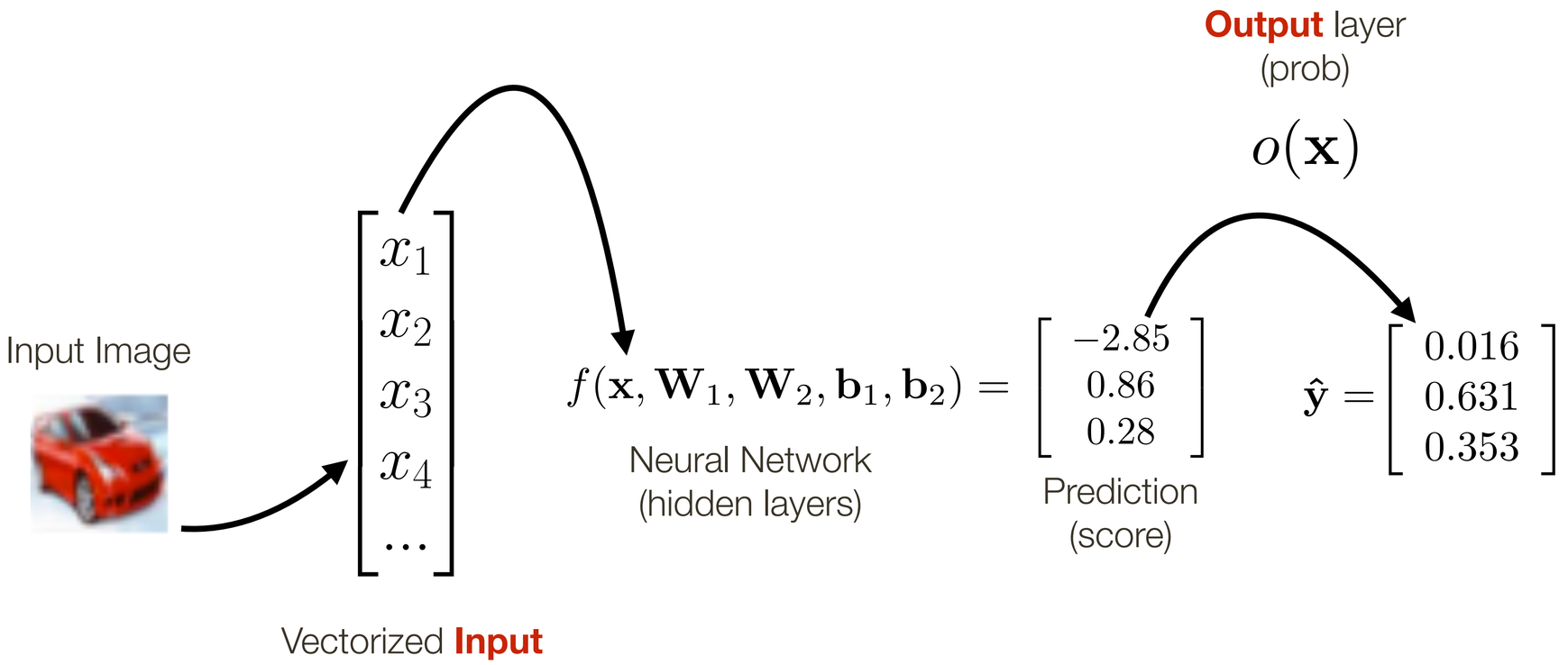
Prediction
(score)

Output layer
(prob)

$$o(\mathbf{x})$$

$$\hat{\mathbf{y}} = \begin{bmatrix} 0.016 \\ 0.631 \\ 0.353 \end{bmatrix}$$

Training a Neural Network



Input and **output** layers (size and form) are dictated by the problem, intermediate hidden layers have few constraints and can be *anything*

Training a Neural Network

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Inference: $o(f(\mathbf{x}, \cdots))$

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Output layer
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$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

True label

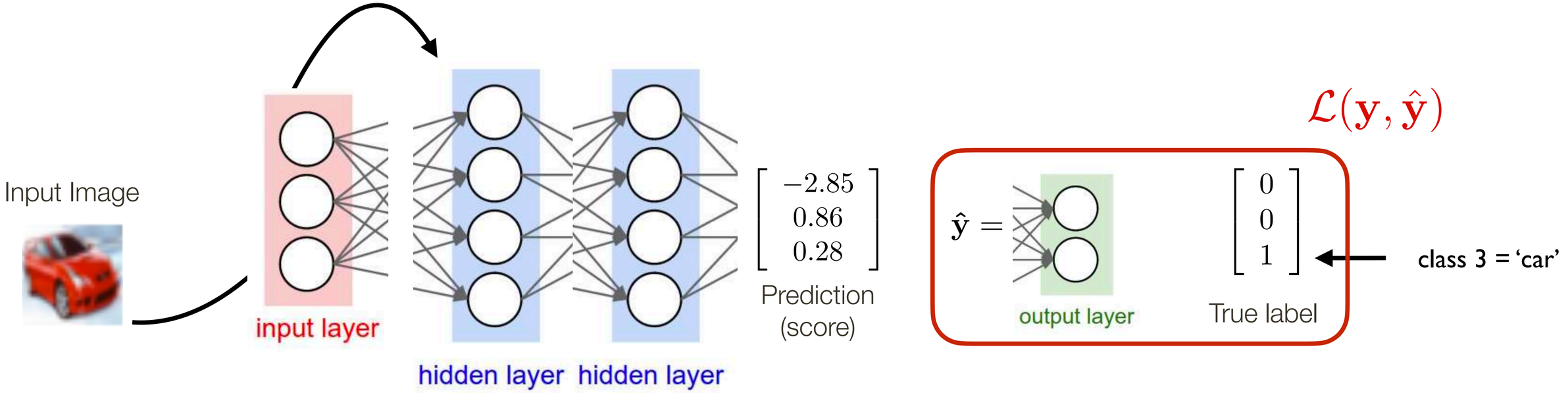
$$\mathcal{L}(\mathbf{y}, \hat{\mathbf{y}})$$

class 3 = 'car'

Inference: $o(f(\mathbf{x}, \cdots))$

Learning: $\mathcal{L}(\mathbf{y}, o(f(\mathbf{x}, \cdots)))$

Training a Neural Network



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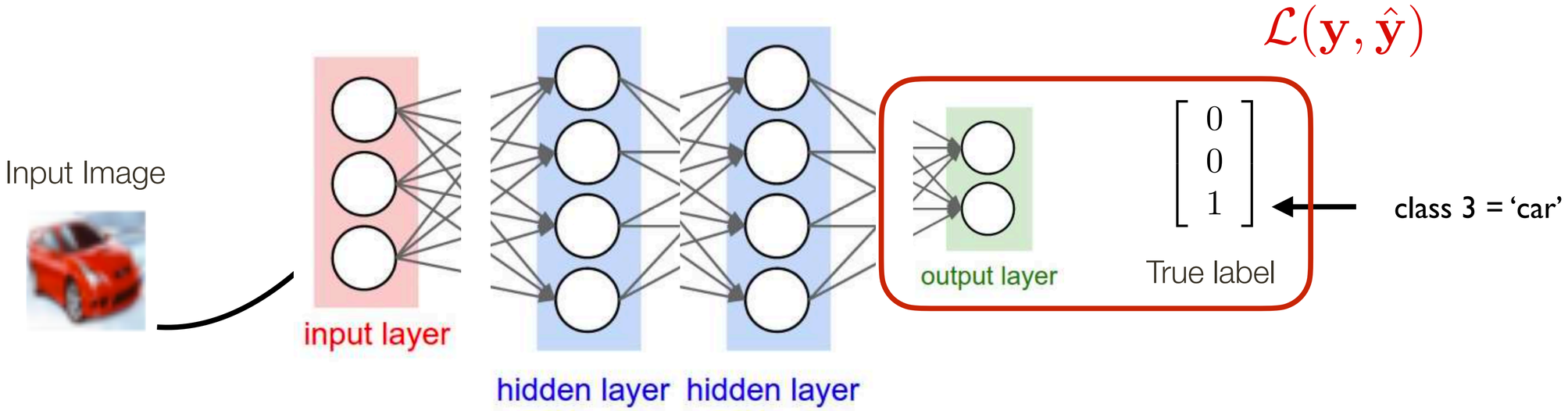
Training a Neural Network



Inference: $o(f(\mathbf{x}, \cdots))$

Learning: $\mathcal{L}(\mathbf{y}, o(f(\mathbf{x}, \cdots)))$

Training a Neural Network

When training a neural network, the final output will be some loss (error) function

— e.g. cross-entropy loss: $\mathcal{L} = -\sum_i y_i \log(\hat{y}_i) \quad \hat{y}_i = \frac{e^{f_{y_i}}}{\sum_j e^{f_{y_j}}}$

which defines loss for i-th training example with true class index y_i; and f_j is the j-th element of the vector of class scores coming from neural net.

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Consider neural net which takes input vector $\mathbf{x}_i$ and predicts scores for 3 classes, with true class being class 3:

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$$
\begin{aligned}
& f \\
c_1 &= -2.85 \\
c_2 &= 0.86 \\
c_3 &= 0.28
\end{aligned}
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$$
\begin{array}{cc}
 & f \\
c_1 = -2.85 & \\
c_2 = 0.86 & \xrightarrow{\exp} \\
c_3 = 0.28 &
\end{array}
\begin{array}{c}
 \\
0.058 \\
2.36 \\
1.32
\end{array}
$$

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\begin{array}{c}
f \\
c_1 = -2.85 \\
c_2 = 0.86 \\
c_3 = 0.28
\end{array}
\xrightarrow{\exp}
\begin{array}{c}
0.058 \\
2.36 \\
1.32
\end{array}
\xrightarrow{\text{Normalize to sum to 1}}
\begin{array}{c}
0.016 \\
0.631 \\
0.353
\end{array}
$$

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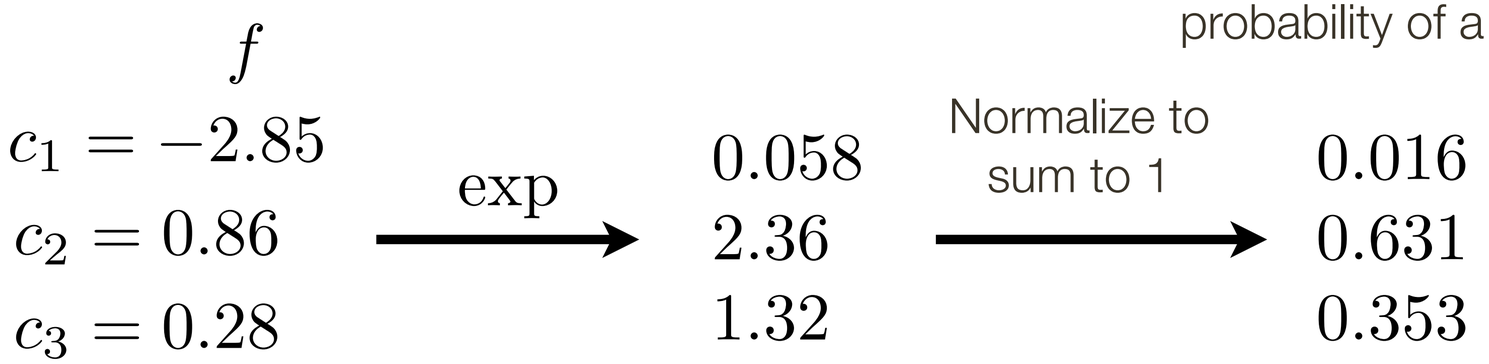

Training a Neural Network

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probability of a class

Training a Neural Network

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\begin{array}{c}
f \\
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c_3 = 0.28
\end{array}
\xrightarrow{\exp}
\begin{array}{c}
0.058 \\
2.36 \\
1.32
\end{array}
\xrightarrow[\text{sum to 1}]{\text{Normalize to}}
\begin{array}{c}
\text{probability of a class} \\
0.016 \\
0.631 \\
0.353
\end{array}
\qquad \mathcal{L} = -\log(0.353) = 1.04
$$

Training a Neural Network

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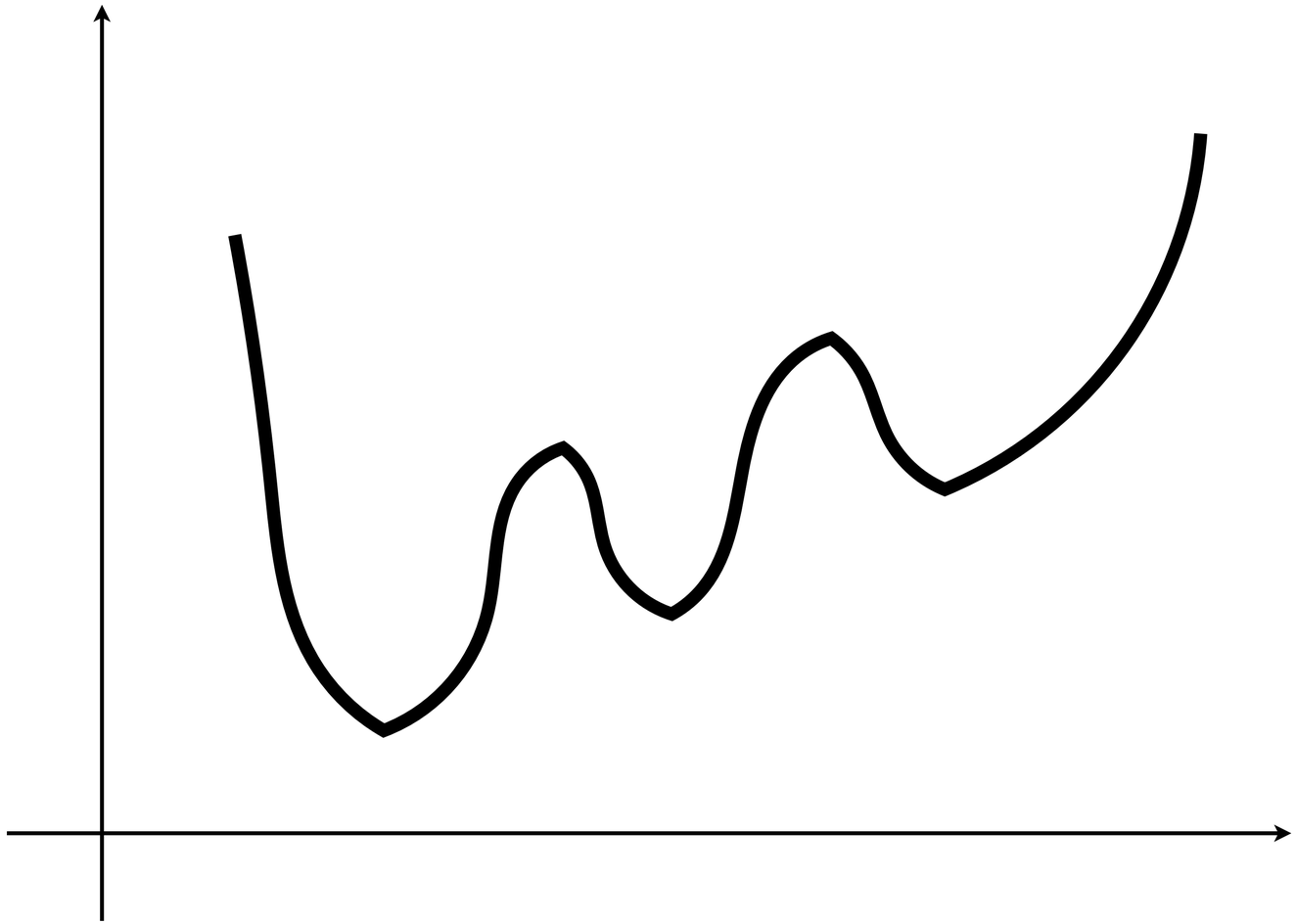
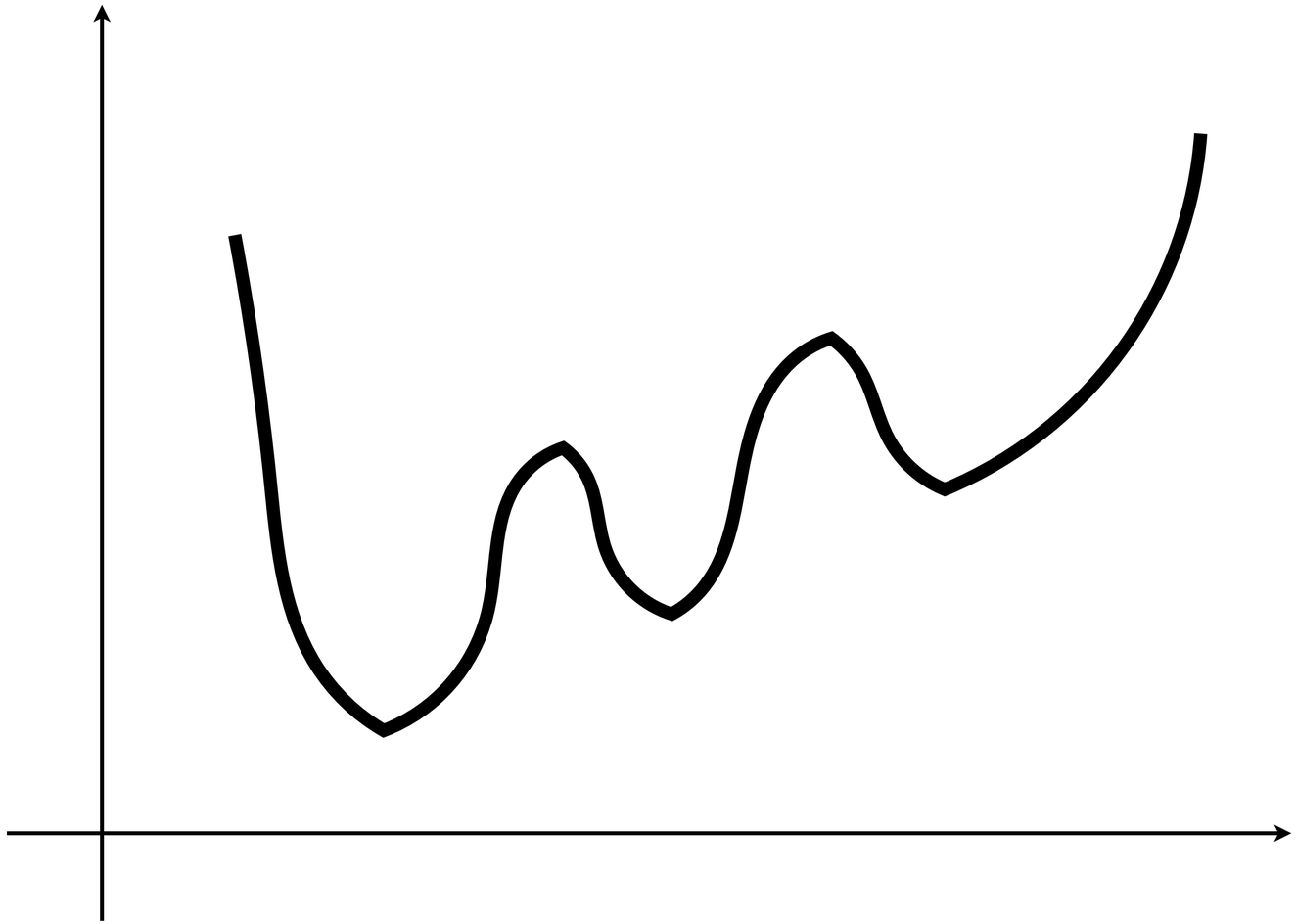
We want to compute the **gradient** of the loss with respect to the network parameters so that we can incrementally adjust the network parameters

Gradient Descent



*slide adopted from V. Ordonex

Gradient Descent



1. Start from random value of $\mathbf{W}_0, \mathbf{b}_0$

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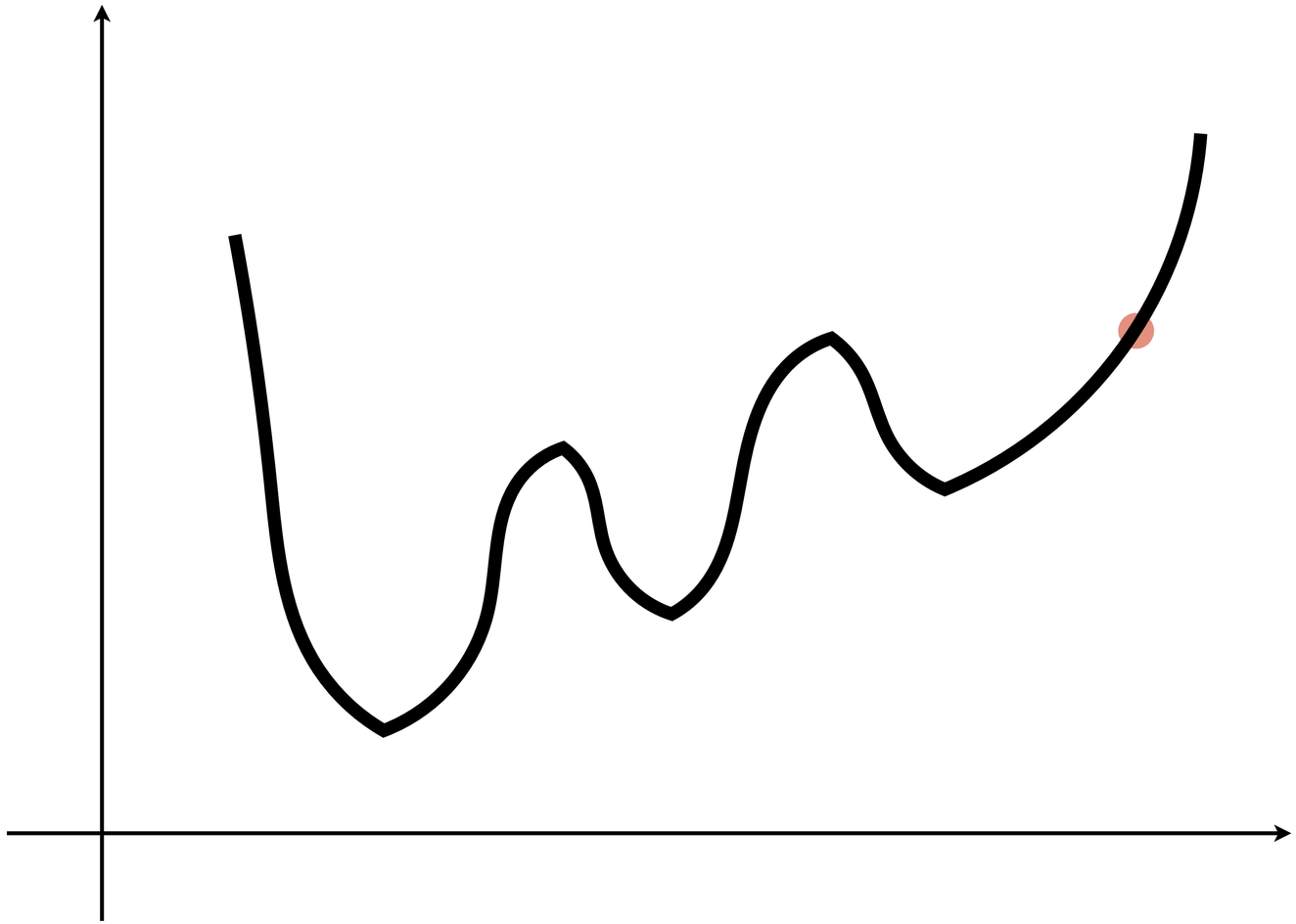
Gradient Descent



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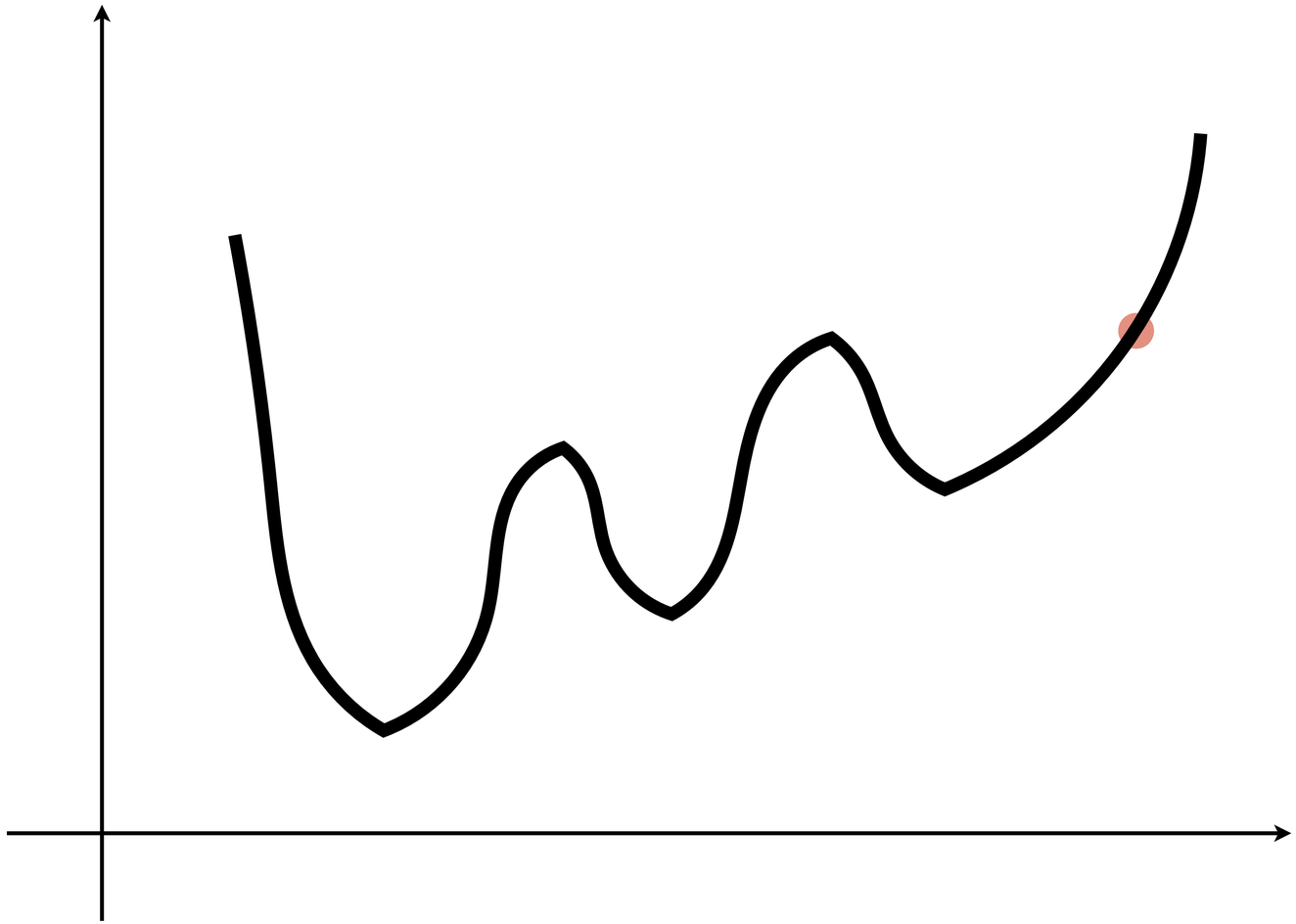

1. Start from random value of $\mathbf{W}_0, \mathbf{b}_0$

For $k = 0$ to max number of iterations

2. Compute gradient of the loss with respect to previous (initial) parameters:

$$\nabla \mathcal{L}(\mathbf{W}, \mathbf{b})|_{\mathbf{W}=\mathbf{W}_k, \mathbf{b}=\mathbf{b}_k}$$

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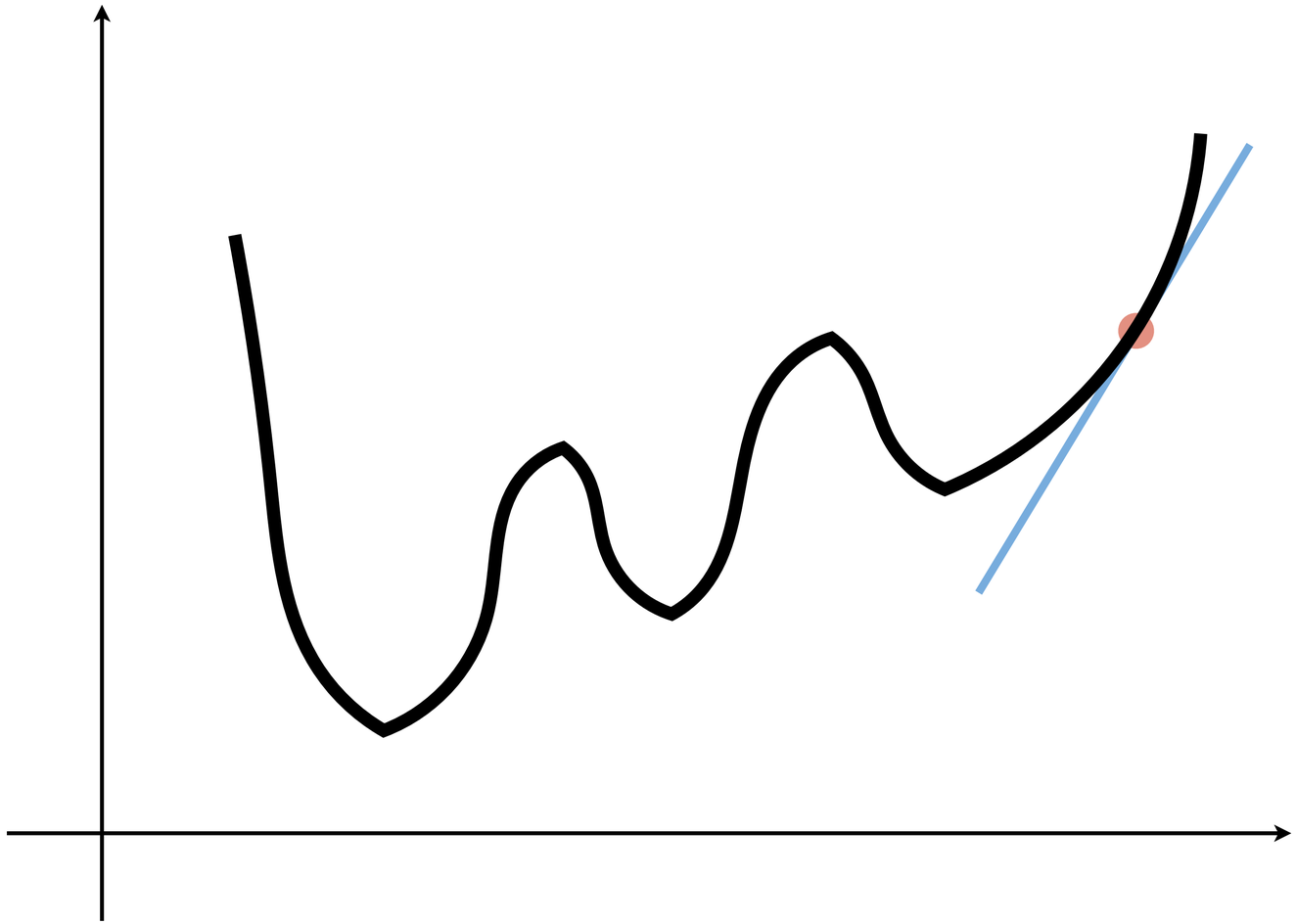
Gradient Descent



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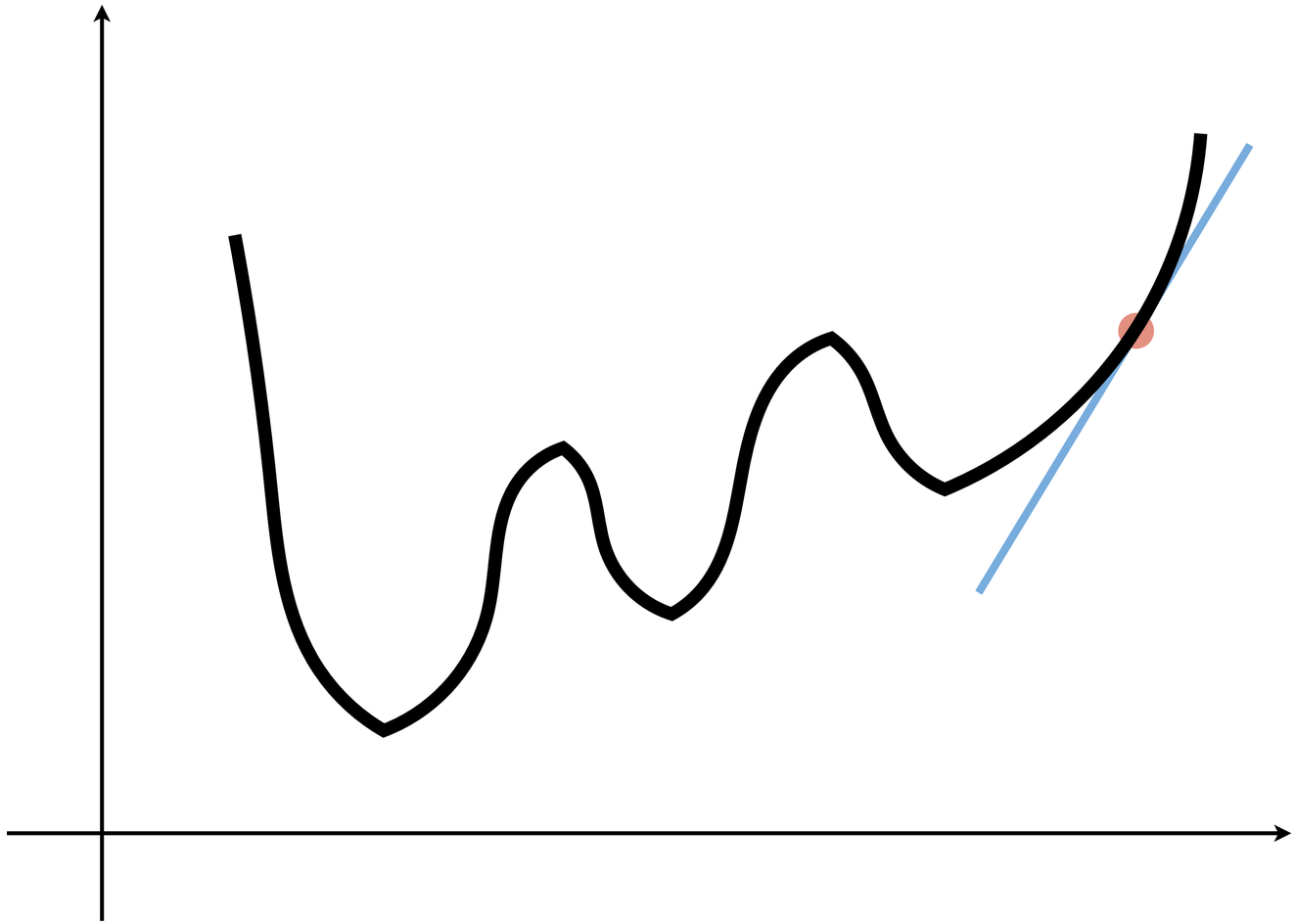

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3. Re-estimate the parameters

$$\mathbf{W}_{k+1} = \mathbf{W}_k - \lambda \left. \frac{\partial \mathcal{L}(\mathbf{W}, \mathbf{b})}{\partial \mathbf{W}} \right|_{\mathbf{W}=\mathbf{W}_k, \mathbf{b}=\mathbf{b}_k}$$

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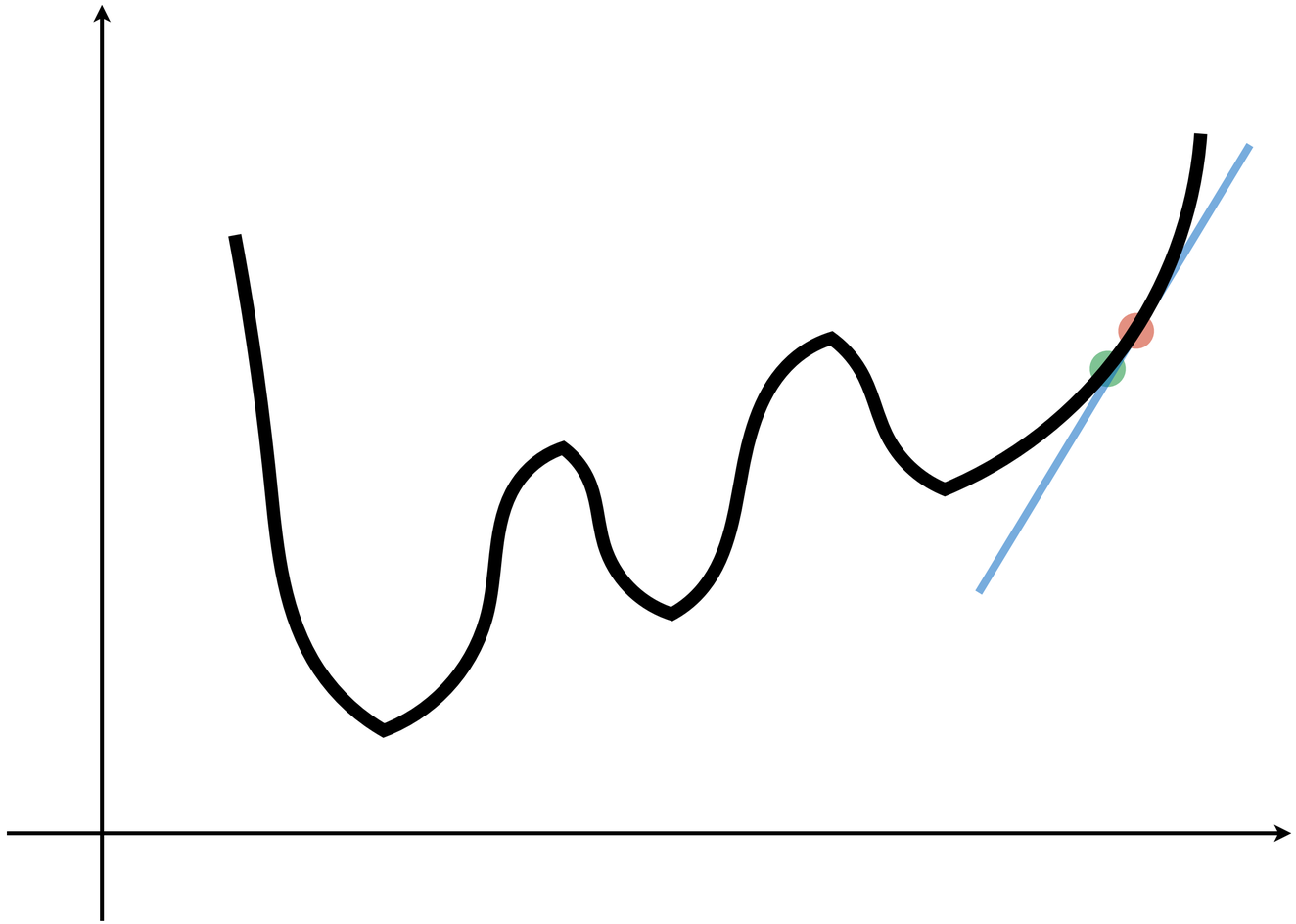

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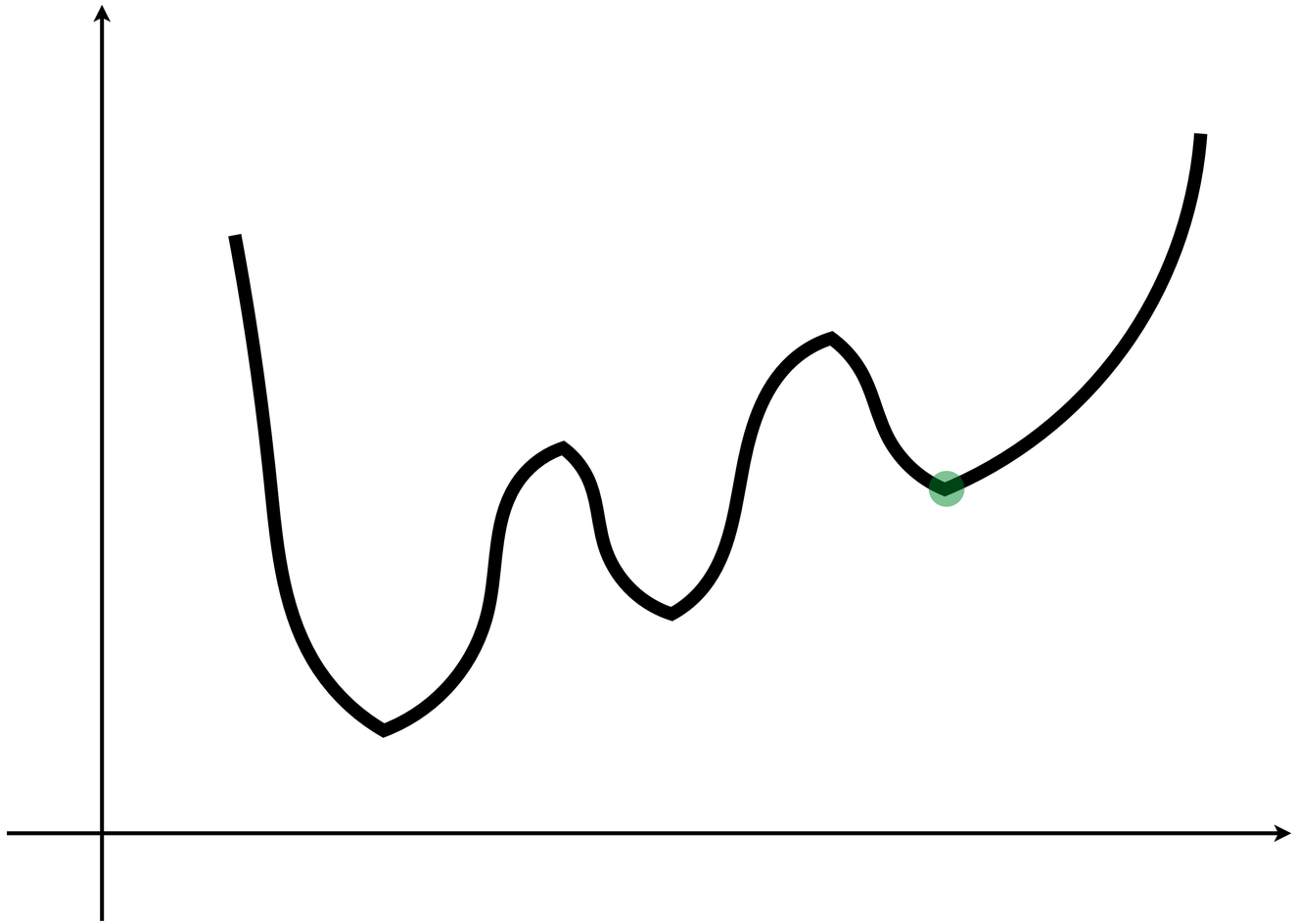

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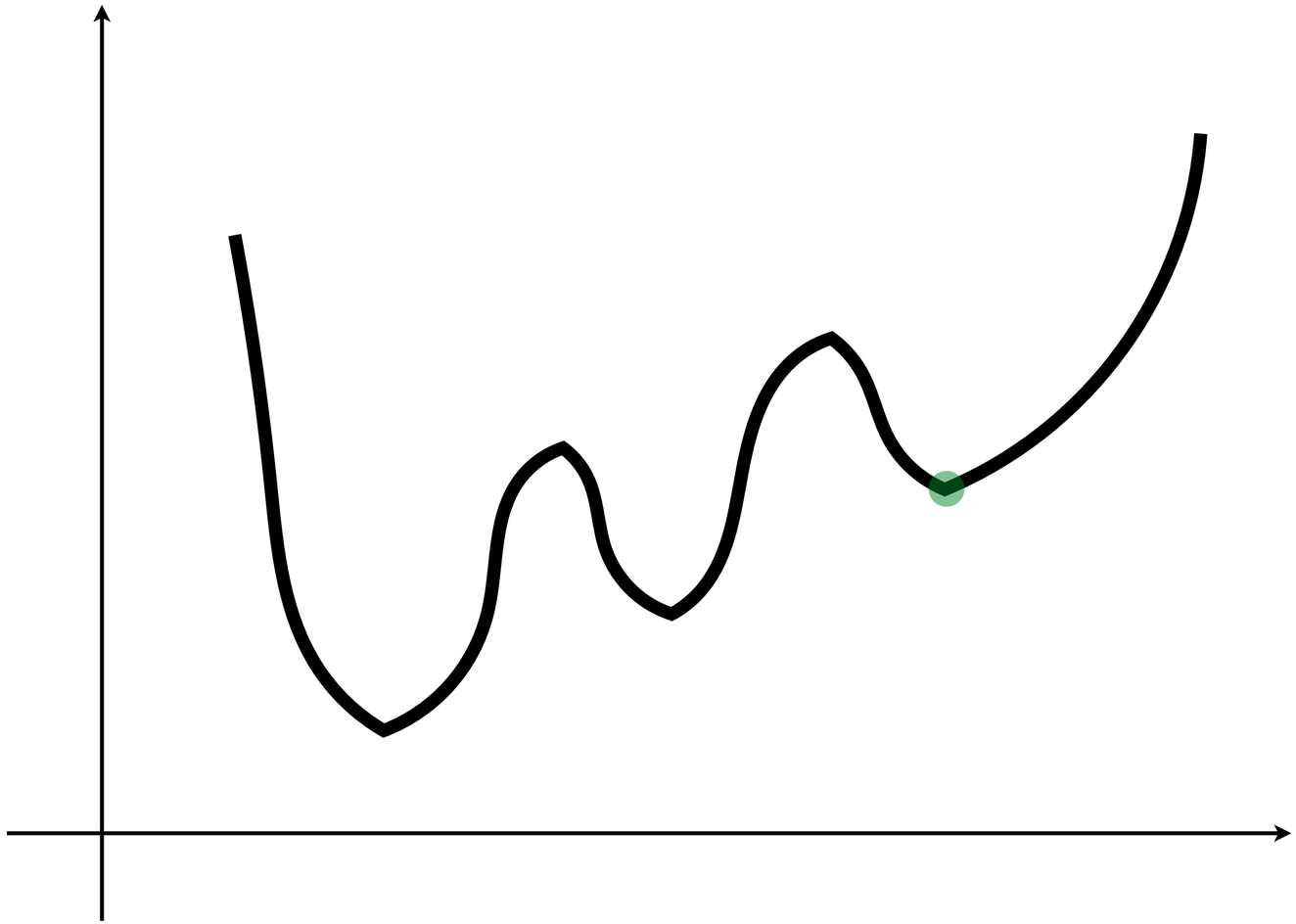

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λ - is the learning rate

*slide adopted from V. Ordonex

Gradient Descent

Loss: $$\mathcal{L} = \sum_{i=1}^{|\mathcal{D}_{train}|} ||\mathbf{y}_i - \hat{\mathbf{y}}_i|| = \sum_{i=1}^{|\mathcal{D}_{train}|} ||\mathbf{y}_i - f(\mathbf{x}_i, \mathbf{W}_1, \mathbf{W}_2, \mathbf{b}_1, \mathbf{b}_2)||$$

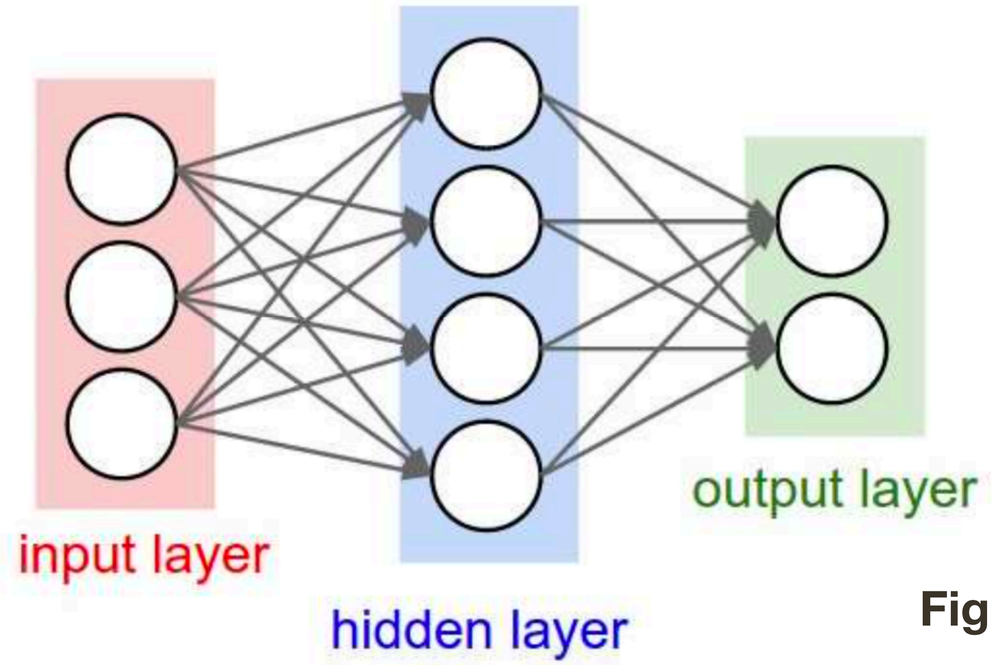

Figure credit: Fei-Fei and Karpathy

$$\hat{\mathbf{y}} = f(\mathbf{x}, \mathbf{W}_1, \mathbf{W}_2, \mathbf{b}_1, \mathbf{b}_2) = \sigma\left(\mathbf{W}_2^{(2\times4)}\sigma\left(\mathbf{W}_1^{(4\times3)}\mathbf{x} + \mathbf{b}_1^{(4)}\right) + \mathbf{b}_2^{(2)}\right)$$

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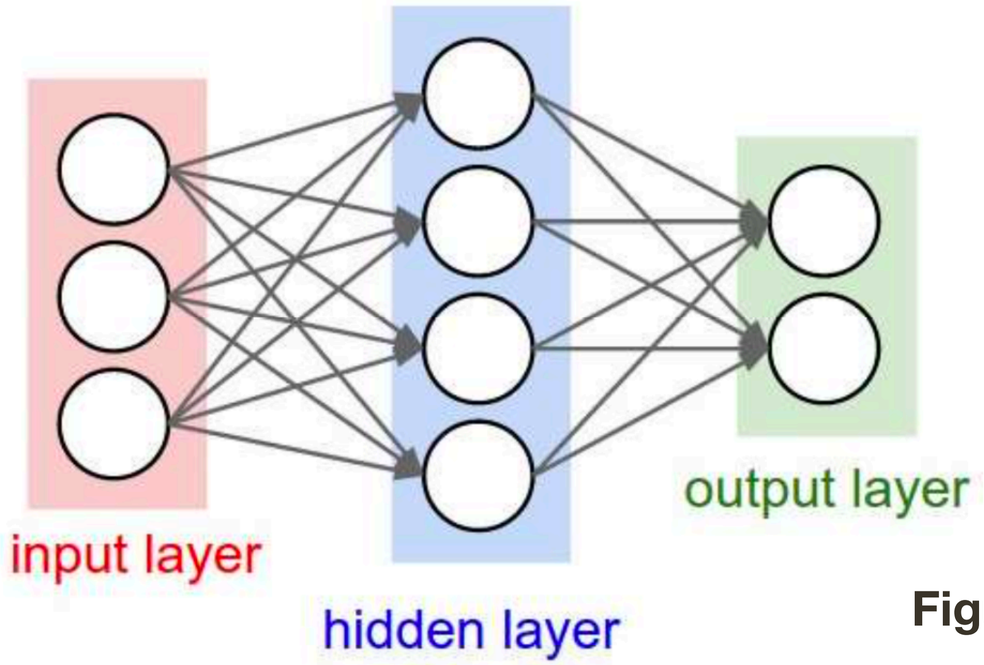
Gradient Descent

$$\mathbf{W}_{1,i,j} = \mathbf{W}_{1,i,j} - \lambda \frac{\partial \mathcal{L}(\mathbf{y}, \hat{\mathbf{y}})}{\partial \mathbf{W}_{1,i,j}}$$

$$\mathbf{b}_{1,i} = \mathbf{b}_{1,i} - \lambda \frac{\partial \mathcal{L}(\mathbf{y}, \hat{\mathbf{y}})}{\partial \mathbf{b}_{1,i}}$$



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Stochastic Gradient Descent

$$\frac{\partial \mathcal{L}}{\partial \mathbf{W}_{1,i,j}} = \frac{\partial}{\partial \mathbf{W}_{1,i,j}} \sum_{i=1}^{|\mathcal{D}_{train}|} [\mathbf{y}_i - f(\mathbf{x}_i, \mathbf{W}_1, \mathbf{W}_2, \mathbf{b}_1, \mathbf{b}_2)]^2$$

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Problem: For large datasets computing sum is expensive

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Solution: Compute approximate gradient with mini-batches of much smaller size (as little as 1-example sometimes)

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Problem: How do we compute the actual gradient?

Numerical Differentiation

$\mathbf{1}_i$ - Vector of all zeros, except for one 1 in i-th location

$\mathbf{1}_{ij}$ - Matrix of all zeros, except for one 1 in (i,j)-th location

We can approximate the gradient numerically, using:

$$\frac{\partial \mathcal{L}(\mathbf{W}, \mathbf{b})}{\partial w_{ij}} \approx \lim_{h \to 0} \frac{\mathcal{L}(\mathbf{W} + h\mathbf{1}_{ij}, \mathbf{b}) - \mathcal{L}(\mathbf{W}, \mathbf{b})}{h} \qquad \frac{\partial \mathcal{L}(\mathbf{W}, \mathbf{b})}{\partial b_j} \approx \lim_{h \to 0} \frac{\mathcal{L}(\mathbf{W}, \mathbf{b} + h\mathbf{1}_j) - \mathcal{L}(\mathbf{W}, \mathbf{b})}{h}$$

Even better, we can use central differencing:

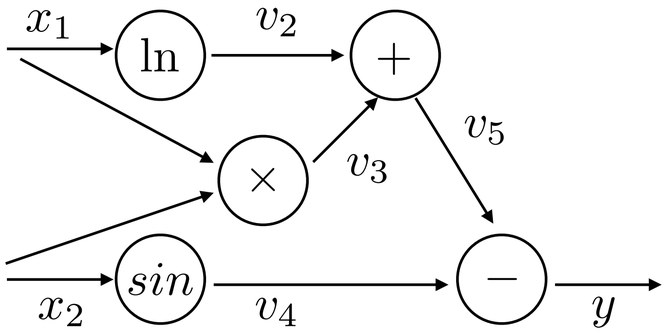
$$\frac{\partial \mathcal{L}(\mathbf{W}, \mathbf{b})}{\partial w_{ij}} \approx \lim_{h \to 0} \frac{\mathcal{L}(\mathbf{W} + h\mathbf{1}_{ij}, \mathbf{b}) - \mathcal{L}(\mathbf{W} + h\mathbf{1}_{ij}, \mathbf{b})}{2h} \qquad \frac{\partial \mathcal{L}(\mathbf{W}, \mathbf{b})}{\partial b_j} \approx \lim_{h \to 0} \frac{\mathcal{L}(\mathbf{W}, \mathbf{b} + h\mathbf{1}_j) - \mathcal{L}(\mathbf{W}, \mathbf{b} + h\mathbf{1}_j)}{2h}$$

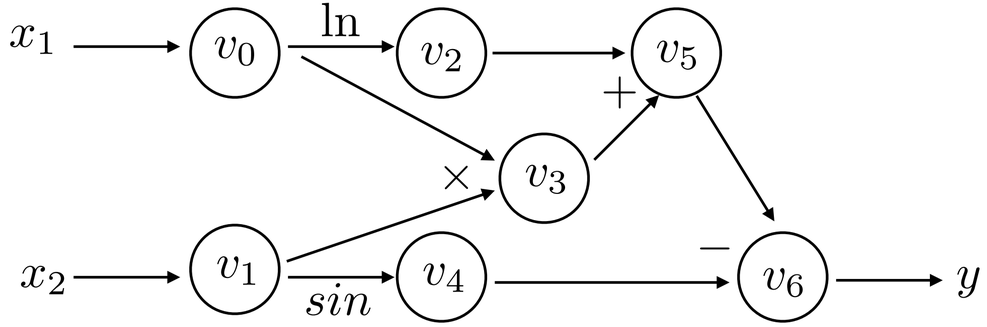
However, both of theses suffer from rounding errors and are not good enough for learning (they are very good tools for checking the correctness of implementation though, e.g., use $h = 0.000001$).

Symbolic Differentiation

$$y = f(x_1, x_2) = \ln(x_1) + x_1 x_2 - sin(x_2)$$

Input function is represented as **computational graph** (a symbolic tree)





Implements differentiation rules for composite functions:

Sum Rule

$$\frac{\mathrm{d}\left(f(x) + g(x)\right)}{\mathrm{d}x} = \frac{\mathrm{d}f(x)}{\mathrm{d}x} + \frac{\mathrm{d}g(x)}{\mathrm{d}x}$$

Product Rule

$$\frac{\mathrm{d}\left(f(x) \cdot g(x)\right)}{\mathrm{d}x} = \frac{\mathrm{d}f(x)}{\mathrm{d}x} g(x) + f(x) \frac{\mathrm{d}g(x)}{\mathrm{d}x}$$

Chain Rule

$$\frac{\mathrm{d}(f(g(x)))}{\mathrm{d}x} = \frac{\mathrm{d}f(g(x))}{\mathrm{d}g(x)} \cdot \frac{\mathrm{d}g(x)}{\mathrm{d}x}$$

*slide adopted from T. Chen, H. Shen, A. Krishnamurthy CSE 599G1 lecture at UWashington

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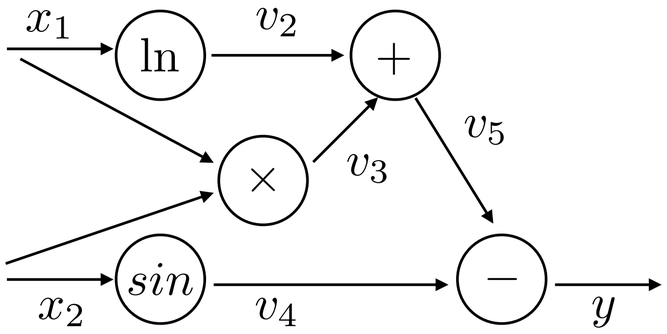

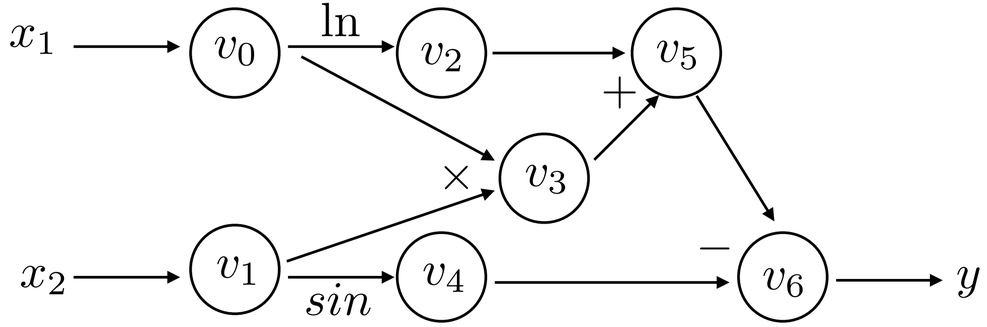

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Problem: For complex functions, expressions can be exponentially large; also difficult to deal with piece-wise functions (creates many symbolic cases)

*slide adopted from T. Chen, H. Shen, A. Krishnamurthy CSE 599G1 lecture at UWashington

Automatic Differentiation (AutoDiff)

$$y = f(x_1, x_2) = \ln(x_1) + x_1 x_2 - sin(x_2)$$

Intuition: Interleave symbolic differentiation and simplification

Key Idea: apply symbolic differentiation at the elementary operation level, evaluate and keep intermediate results

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Success of **deep learning** owes A LOT to success of AutoDiff algorithms
(also to advances in parallel architectures, and large datasets, …)

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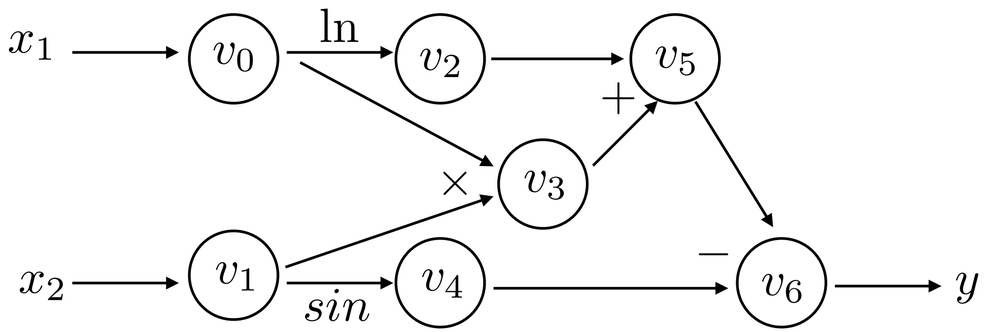

Each **node** is an input, intermediate, or output variable

Computational graph (a DAG) with variable ordering from topological sort.

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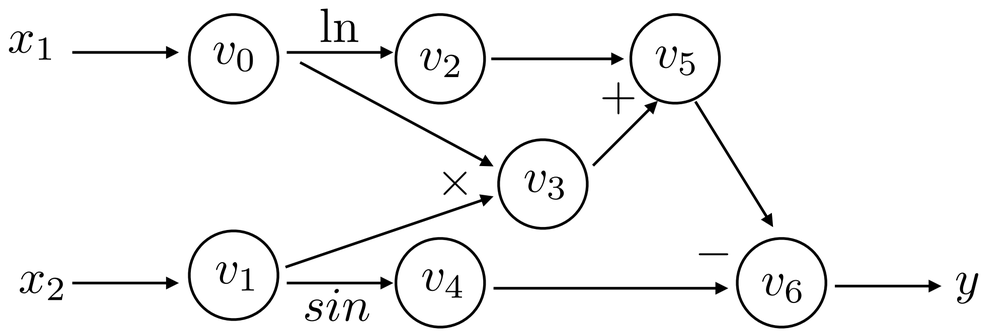

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Computational graph is governed by these equations

$$v_0 = x_1$$
$$v_1 = x_2$$
$$v_2 = \ln(v_0)$$
$$v_3 = v_0 \cdot v_1$$
$$v_4 = sin(v_1)$$
$$v_5 = v_2 + v_3$$
$$v_6 = v_5 - v_4$$
$$y = v_6$$

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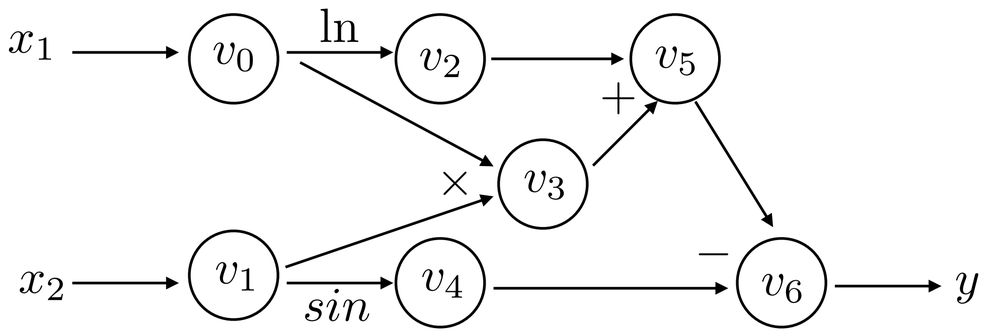

Lets see how we can **evaluate a function** using computational graph (DNN inferences)

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y &= v_6
\end{aligned}$$

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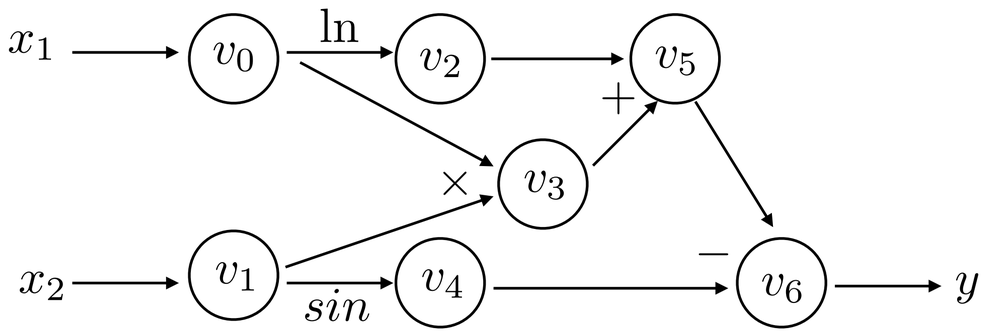

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Lets see how we can **evaluate a function** using computational graph (DNN inferences)

Forward Evaluation Trace:

	$f(2,5)$
$v_0 = x_1$	
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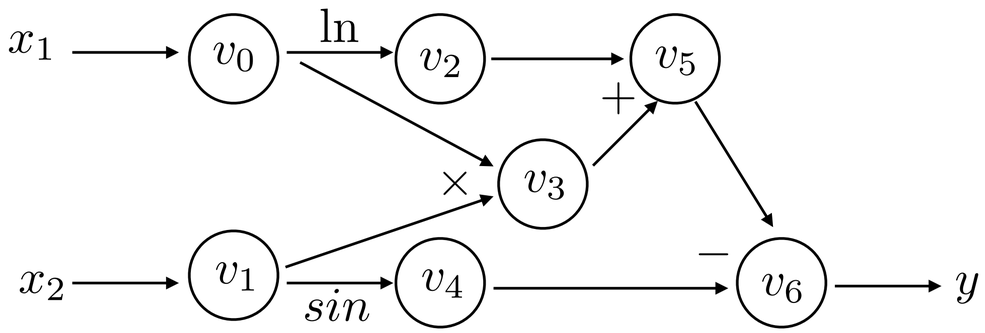

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Lets see how we can **evaluate a function** using computational graph (DNN inferences)

Forward Evaluation Trace:

	$f(2,5)$
$v_0 = x_1$	2
$v_1 = x_2$	
$v_2 = \ln(v_0)$	
$v_3 = v_0 \cdot v_1$	
$v_4 = sin(v_1)$	
$v_5 = v_2 + v_3$	
$v_6 = v_5 - v_4$	
$y = v_6$	

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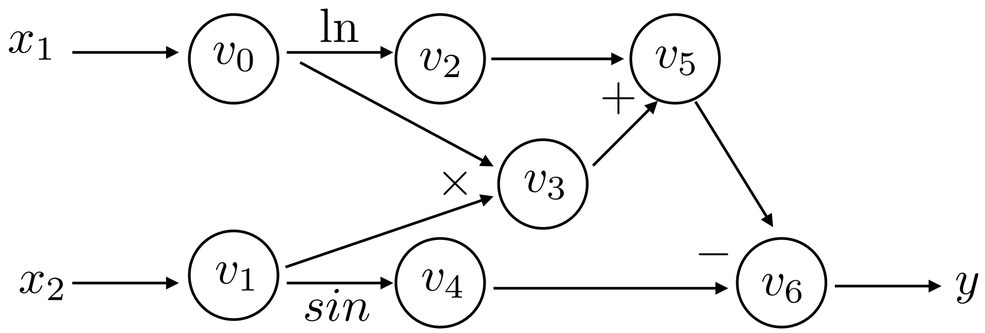

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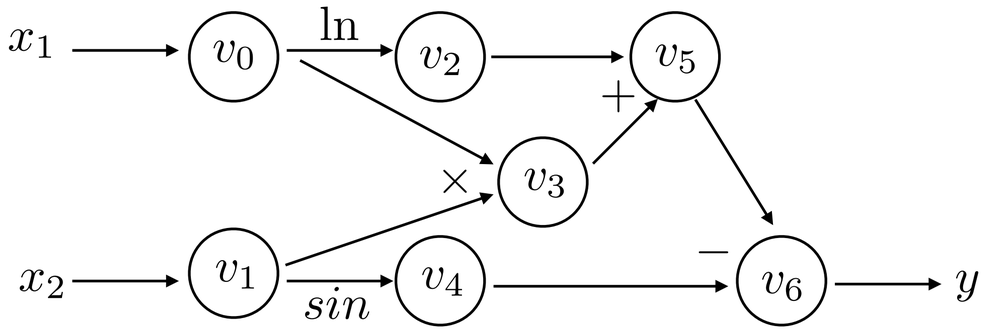

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	$f(2,5)$
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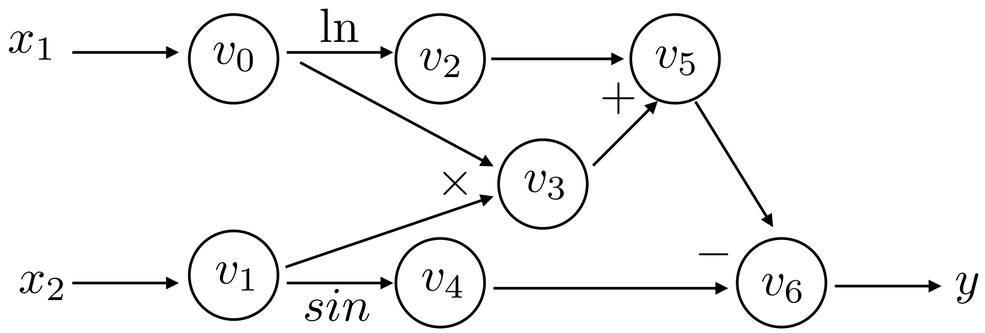
$v_1 = x_2$	5
$v_2 = \ln(v_0)$	ln(2) = 0.693
$v_3 = v_0 \cdot v_1$	
$v_4 = sin(v_1)$	
$v_5 = v_2 + v_3$	
$v_6 = v_5 - v_4$	
$y = v_6$	

Automatic Differentiation (AutoDiff)

$$y = f(x_1, x_2) = \ln(x_1) + x_1 x_2 - sin(x_2)$$



Each **node** is an input, intermediate, or output variable

Computational graph (a DAG) with variable ordering from topological sort.

Lets see how we can **evaluate a function** using computational graph (DNN inferences)

Forward Evaluation Trace:

	$f(2,5)$
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$v_1 = x_2$	5
$v_2 = \ln(v_0)$	ln(2) = 0.693
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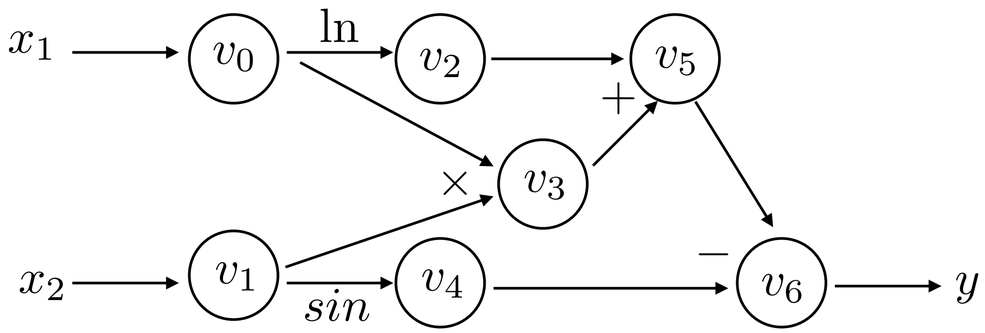

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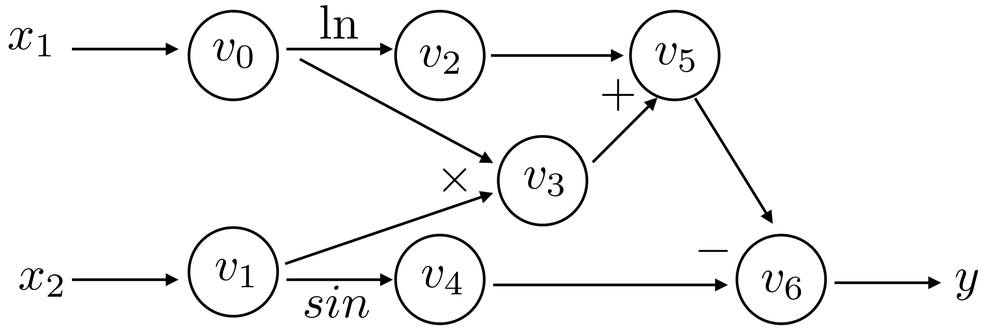

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AutoDiff - **Forward Mode**

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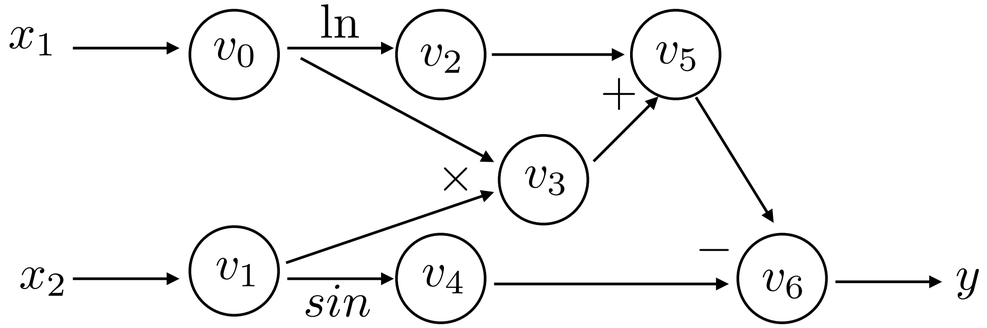

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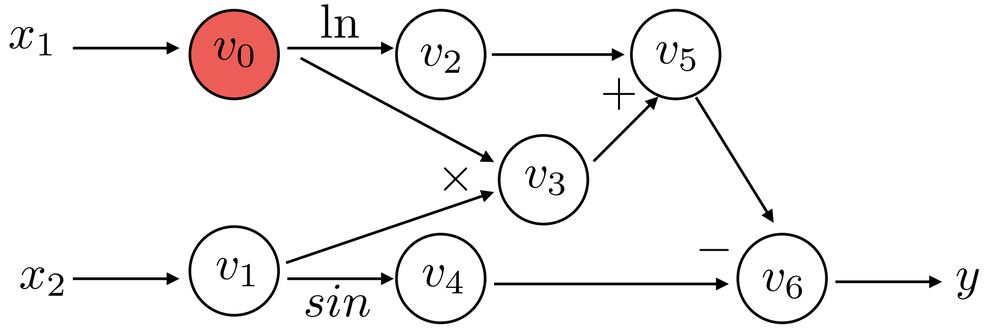
Lets see how we can **evaluate a derivative** using computational graph (DNN learning)

$$\left.\frac{\partial f(x_1, x_2)}{\partial x_1}\right|_{(x_1=2, x_2=5)}$$

We will do this with **forward mode** first, by introducing a derivative of each variable node with respect to the input variable.

AutoDiff - **Forward Mode**

$$y = f(x_1, x_2) = \ln(x_1) + x_1 x_2 - sin(x_2)$$



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Forward Derivative Trace:

	$\frac{\partial f(x_1, x_2)}{\partial x_1}\Big\|_{(x_1=2, x_2=5)}$

AutoDiff - **Forward Mode**

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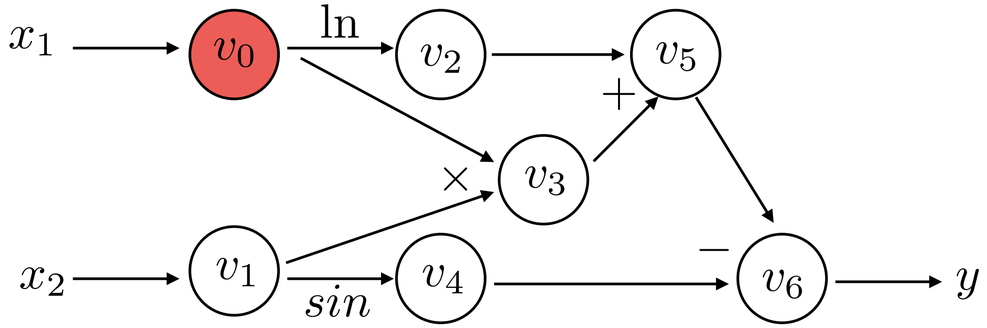

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Forward Derivative Trace:

	$\frac{\partial f(x_1, x_2)}{\partial x_1}\Big\|_{(x_1=2, x_2=5)}$
$\frac{\partial v_0}{\partial x_1}$	

AutoDiff - **Forward Mode**

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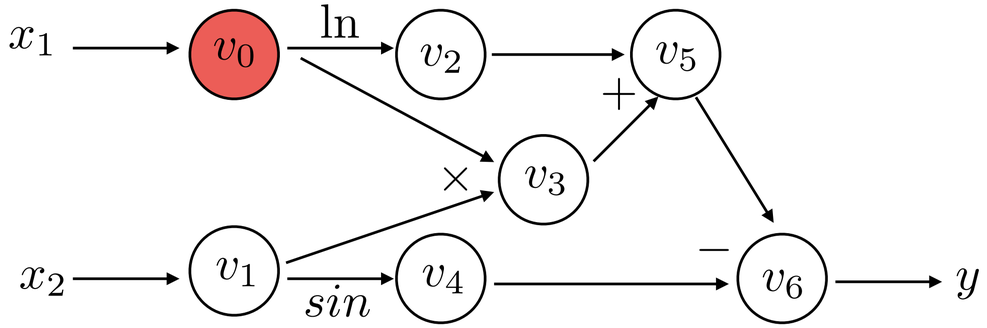

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Forward Derivative Trace:

	$\frac{\partial f(x_1, x_2)}{\partial x_1}\Big\|_{(x_1=2, x_2=5)}$
$\frac{\partial v_0}{\partial x_1}$	1

AutoDiff - **Forward Mode**

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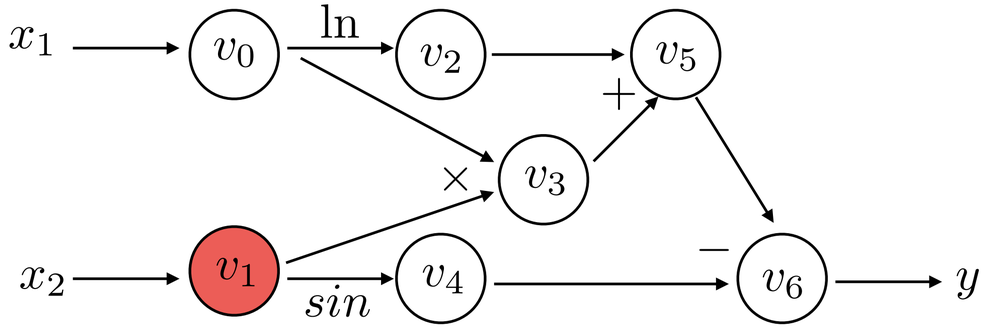

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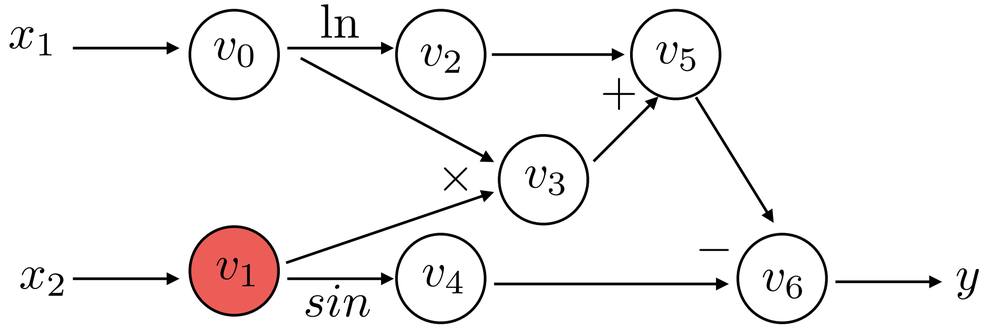

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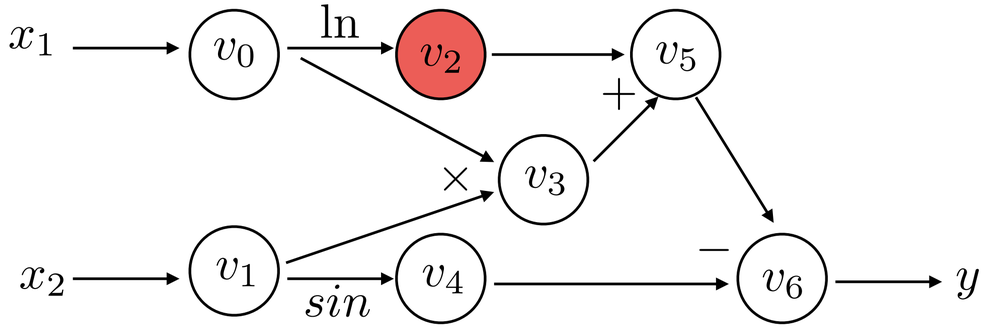

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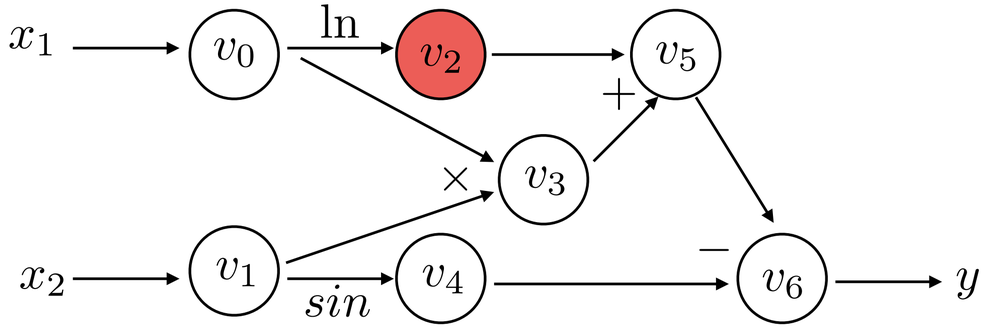

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Chain Rule

AutoDiff - **Forward Mode**

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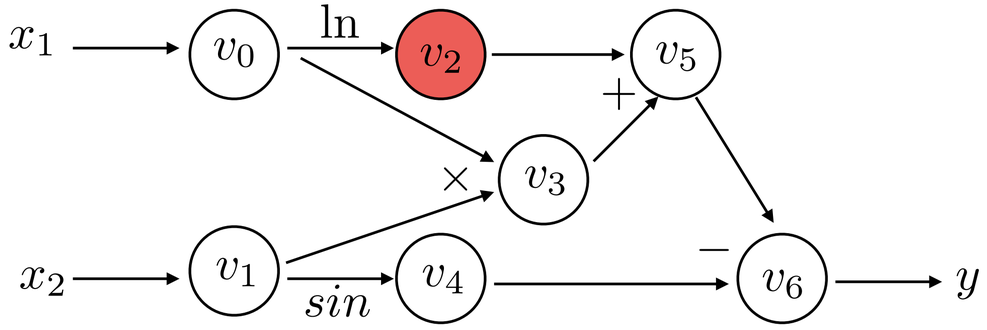

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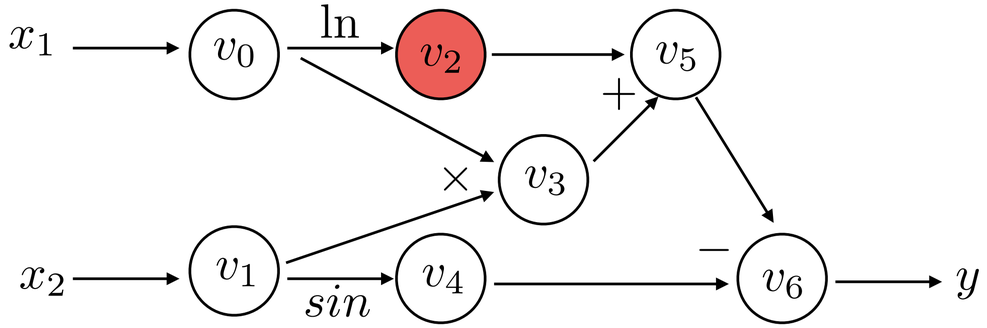

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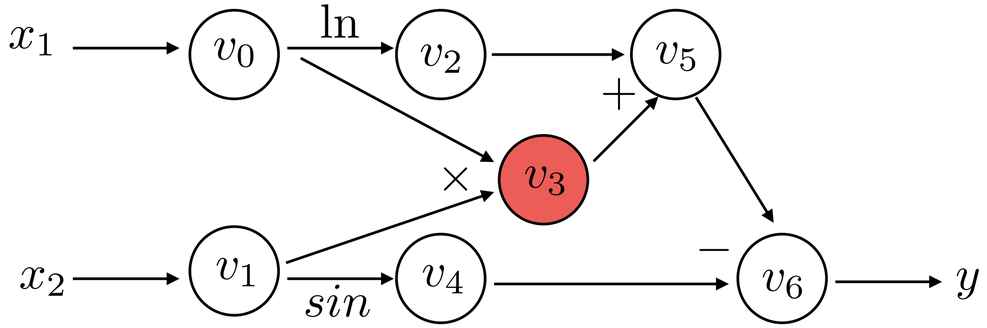

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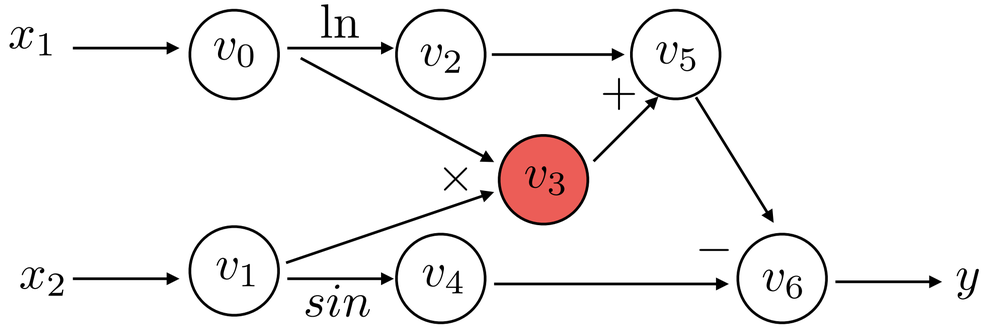

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Product Rule

AutoDiff - **Forward Mode**

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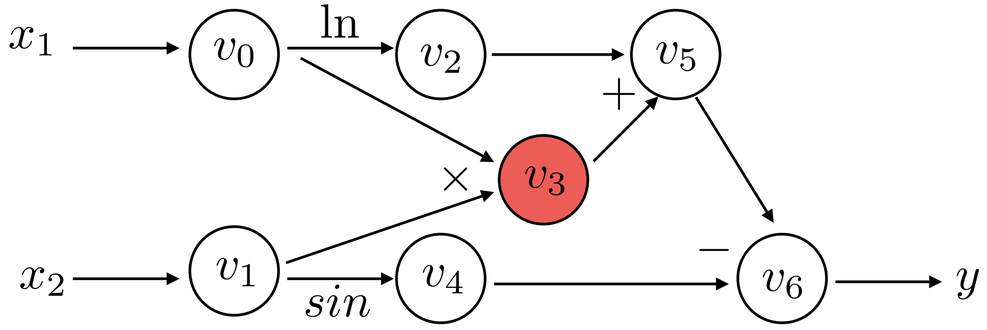

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$\frac{\partial v_3}{\partial x_1} = \frac{\partial v_0}{\partial x_1} \cdot v_1 + v_0 \cdot \frac{\partial v_1}{\partial x_1}$	

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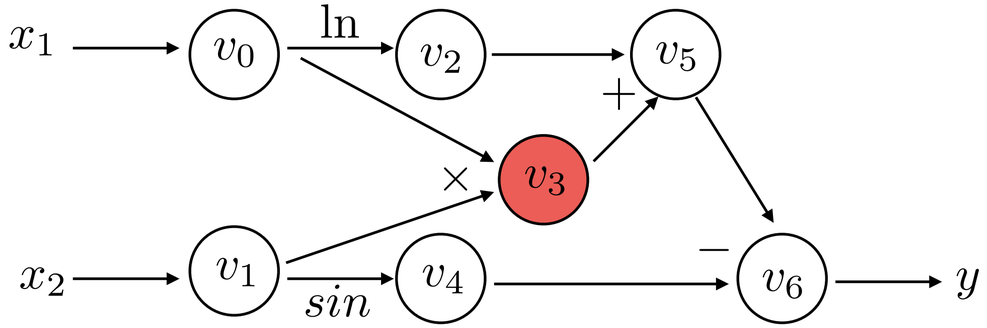

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$\frac{\partial v_2}{\partial x_1} = \frac{1}{v_0}\frac{\partial v_0}{\partial x_1}$	1/2 * 1 = 0.5
$\frac{\partial v_3}{\partial x_1} = \frac{\partial v_0}{\partial x_1} \cdot v_1 + v_0 \cdot \frac{\partial v_1}{\partial x_1}$	1*5 + 2*0 = 5

Product Rule

AutoDiff - **Forward Mode**

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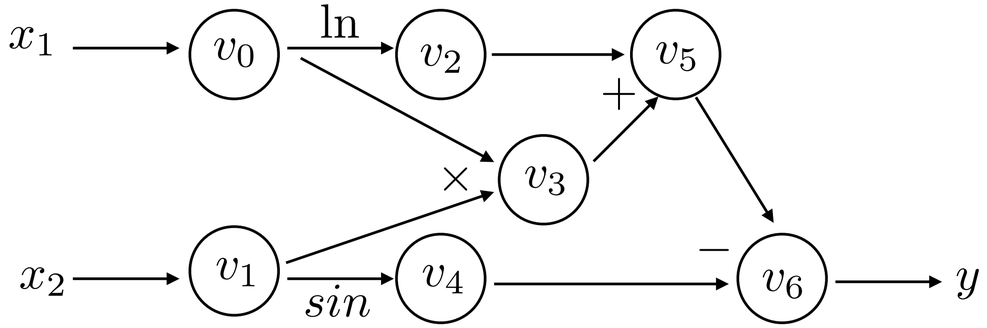

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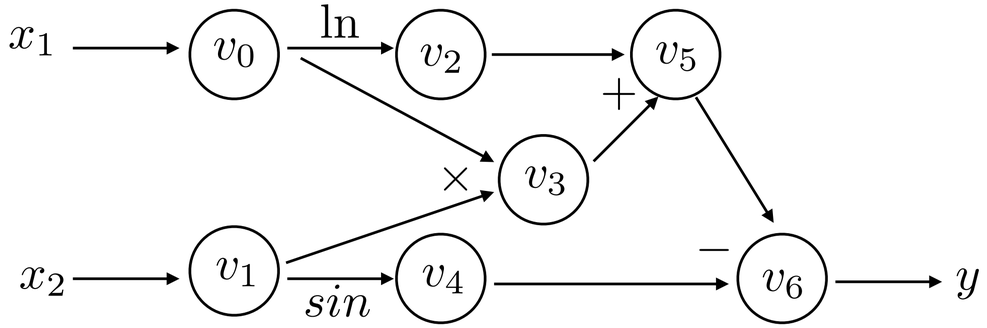
$\frac{\partial v_3}{\partial x_1} = \frac{\partial v_0}{\partial x_1} \cdot v_1 + v_0 \cdot \frac{\partial v_1}{\partial x_1}$	1*5 + 2*0 = 5
$\frac{\partial v_4}{\partial x_1} = \frac{\partial v_1}{\partial x_1} cos(v_1)$	0 * cos(5) = 0
$\frac{\partial v_5}{\partial x_1} = \frac{\partial v_2}{\partial x_1} + \frac{\partial v_3}{\partial x_1}$	0.5 + 5 = 5.5
$\frac{\partial v_6}{\partial x_1} = \frac{\partial v_5}{\partial x_1} - \frac{\partial v_4}{\partial x_1}$	5.5 - 0 = 5.5
$\frac{\partial y}{\partial x_1} = \frac{\partial v_6}{\partial x_1}$	5.5

AutoDiff - **Forward Mode**

$$y = f(x_1, x_2) = \ln(x_1) + x_1 x_2 - sin(x_2)$$



We now have:

$$\left.\frac{\partial f(x_1, x_2)}{\partial x_1}\right|_{(x_1=2, x_2=5)} = 5.5$$

Forward Derivative Trace:

	$\left.\frac{\partial f(x_1, x_2)}{\partial x_1}\right\|_{(x_1=2, x_2=5)}$
$\frac{\partial v_0}{\partial x_1}$	1
$\frac{\partial v_1}{\partial x_1}$	0
$\frac{\partial v_2}{\partial x_1} = \frac{1}{v_0}\frac{\partial v_0}{\partial x_1}$	1/2 * 1 = 0.5
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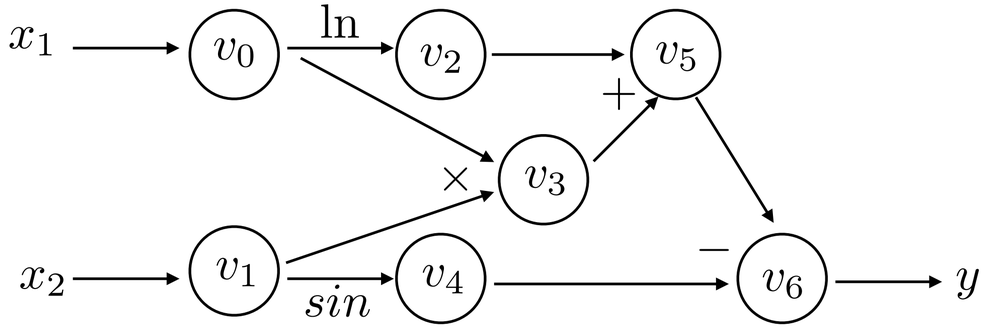

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Still need:

$$\left.\frac{\partial f(x_1, x_2)}{\partial x_2}\right|_{(x_1=2, x_2=5)}$$

Forward Derivative Trace:

	$\left.\frac{\partial f(x_1, x_2)}{\partial x_1}\right\|_{(x_1=2, x_2=5)}$
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$\frac{\partial v_1}{\partial x_1}$	0
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$\frac{\partial y}{\partial x_1} = \frac{\partial v_6}{\partial x_1}$	5.5

AutoDiff - **Forward Mode**

Forward mode needs m forward passes to get a full Jacobian (all gradients of output with respect to each input), where m is the number of inputs

$$\mathbf{y} = f(\mathbf{x}) : \mathbb{R}^m \rightarrow \mathbb{R}^n$$

*slide adopted from T. Chen, H. Shen, A. Krishnamurthy CSE 599G1 lecture at UWashington

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Problem: DNN typically has large number of inputs:

image as an input, plus all the weights and biases of layers = millions of inputs!

and very few outputs (many DNNs have $n = 1$)

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Problem: DNN typically has large number of inputs:

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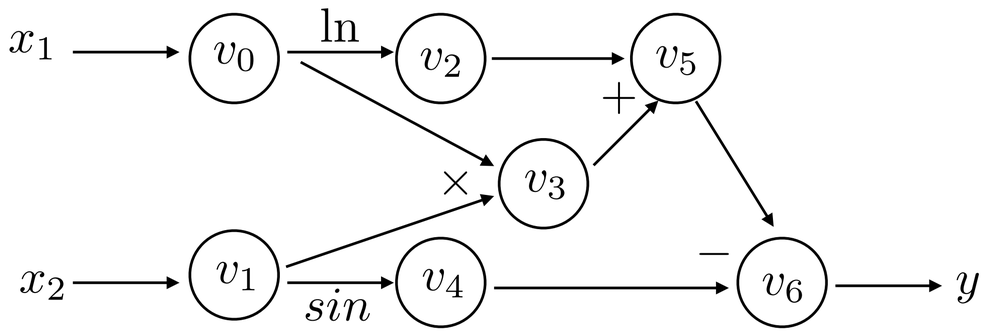
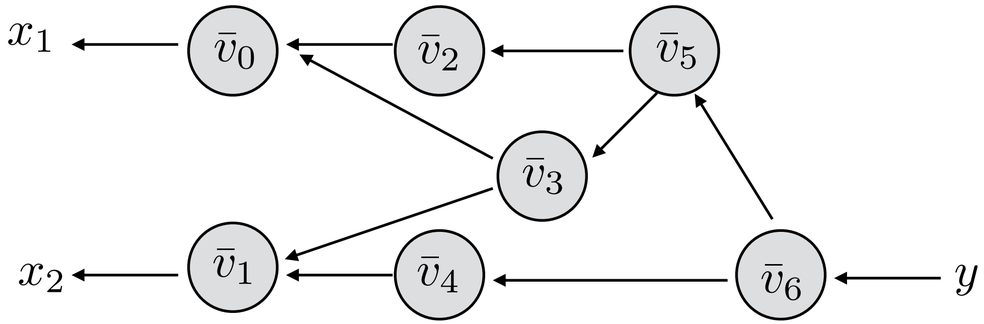
Automatic differentiation in **reverse mode** computes all gradients in n backwards passes (so for most DNNs in a single back pass — **back propagation**)

*slide adopted from T. Chen, H. Shen, A. Krishnamurthy CSE 599G1 lecture at UWashington

AutoDiff - **Reverse Mode**





Forward Evaluation Trace:

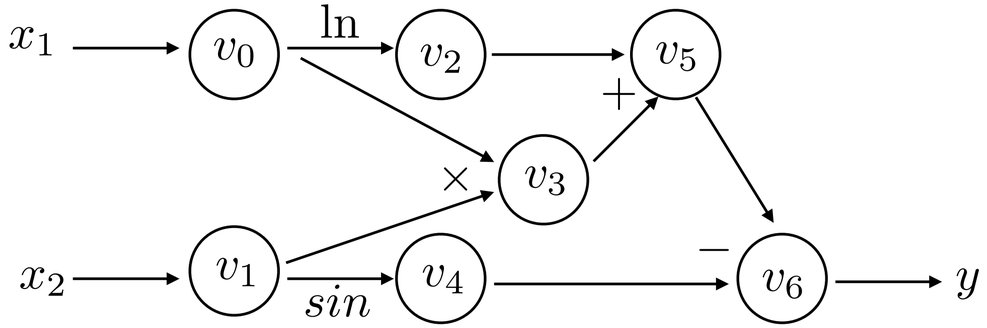
	$f(2,5)$
$v_0 = x_1$	2
$v_1 = x_2$	5
$v_2 = \ln(v_0)$	ln(2) = 0.693
$v_3 = v_0 \cdot v_1$	2 x 5 = 10
$v_4 = sin(v_1)$	sin(5) = 0.959
$v_5 = v_2 + v_3$	0.693 + 10 = 10.693
$v_6 = v_5 - v_4$	10.693 + 0.959 = 11.652
$y = v_6$	11.652

Traverse the original graph in the *reverse* topological order and for each node in the original graph introduce an **adjoint node**, which computes derivative of the output with respect to the local node (using Chain rule):

$$\bar{v}_i = \frac{\partial y_j}{\partial v_i} = \sum_{k \in \mathrm{pa}(i)} \frac{\partial v_k}{\partial v_i} \frac{\partial y_j}{\partial v_k} = \sum_{k \in \mathrm{pa}(i)} \frac{\partial v_k}{\partial v_i} \bar{v}_k$$

"**local**" derivative

AutoDiff - **Reverse Mode**



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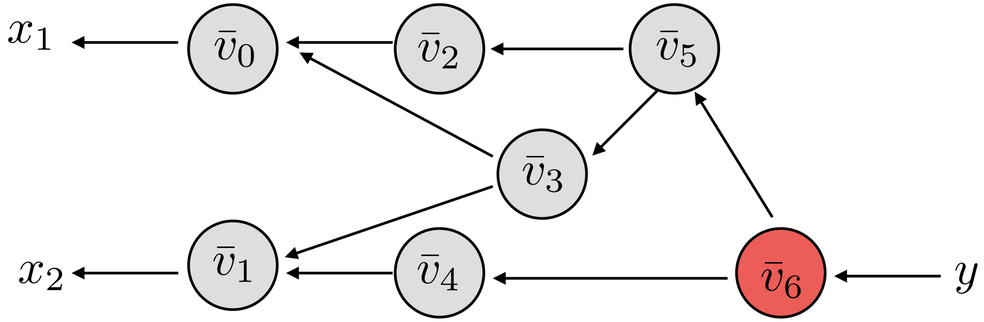
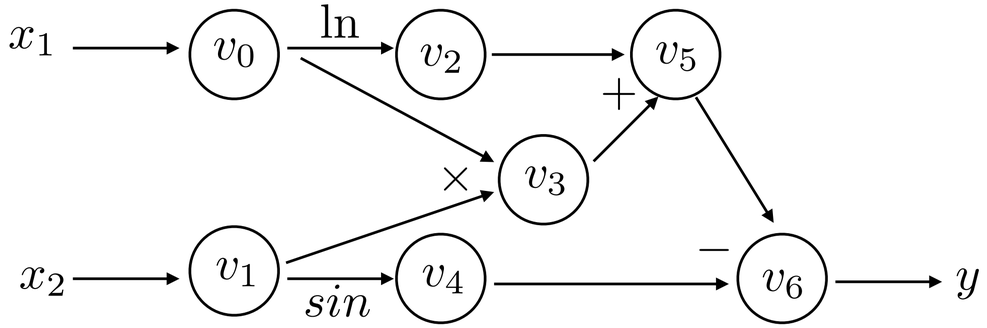


Backwards Derivative Trace:

$$\bar{v}_6 = \frac{\partial y}{\partial v_6}$$

AutoDiff - **Reverse Mode**



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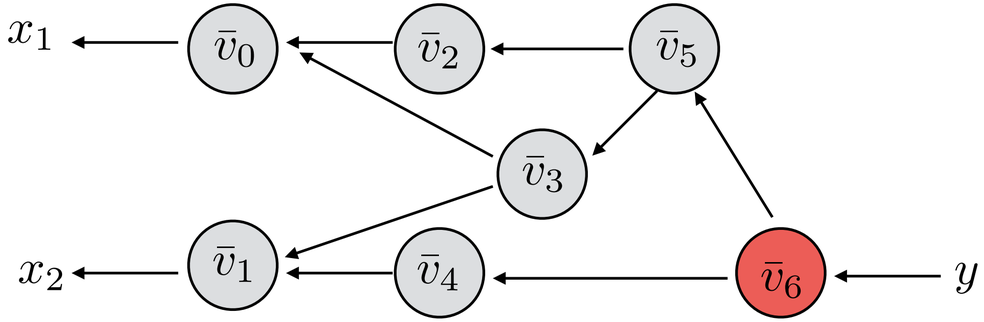

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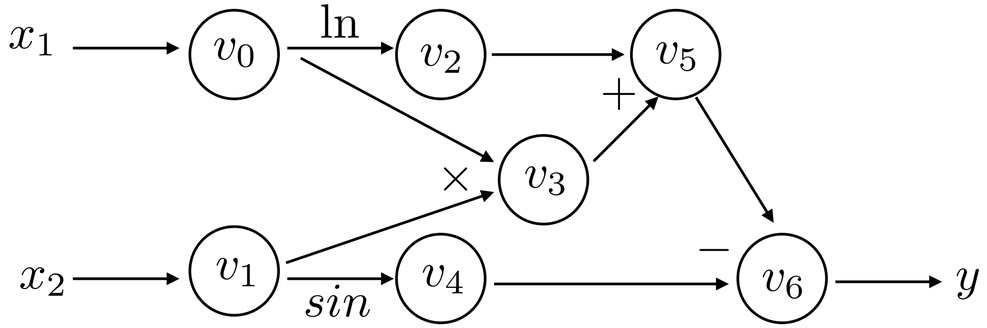
AutoDiff - **Reverse Mode**



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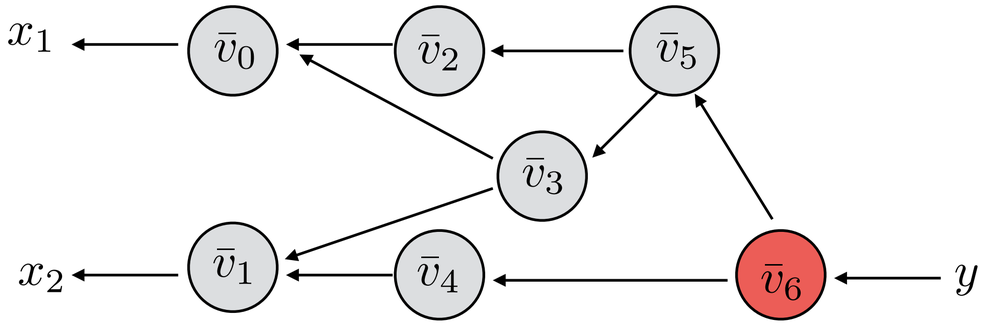
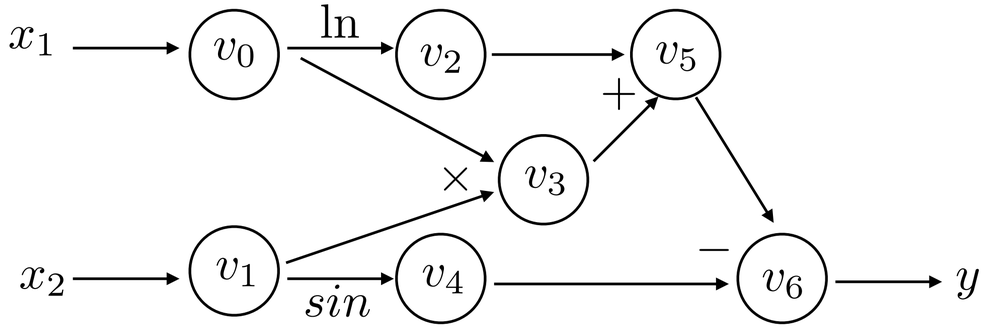


Backwards Derivative Trace:

$\bar{v}_6 = \dfrac{\partial y}{\partial v_6}$	1

AutoDiff - **Reverse Mode**



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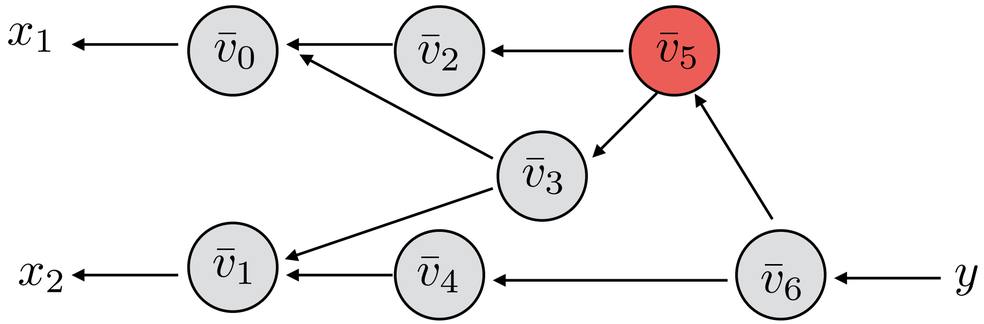
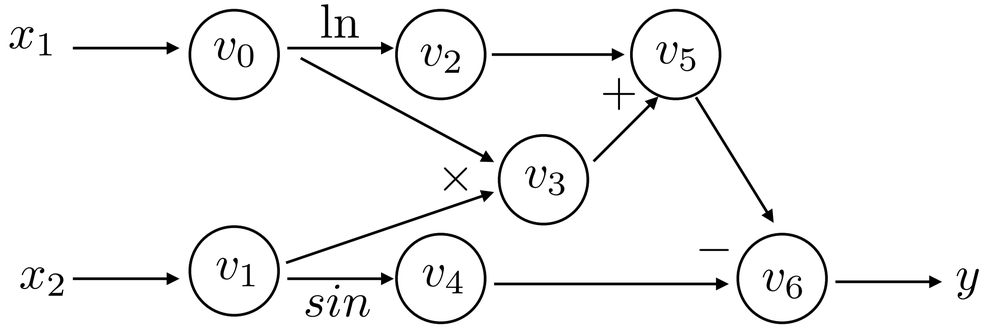


Backwards Derivative Trace:

$\bar{v}_5 = \bar{v}_6 \frac{\partial v_6}{\partial v_5}$	
$\bar{v}_6 = \frac{\partial y}{\partial v_6}$	1

AutoDiff - **Reverse Mode**



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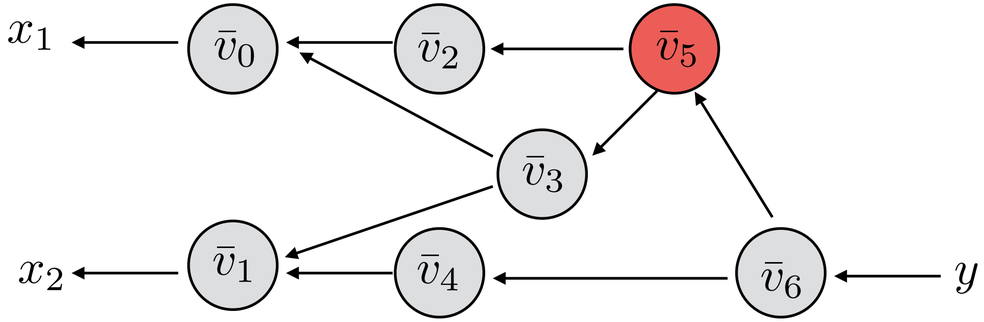

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$\bar{v}_5 = \bar{v}_6 \frac{\partial v_6}{\partial v_5}$	
$\bar{v}_6 = \frac{\partial y}{\partial v_6}$	1

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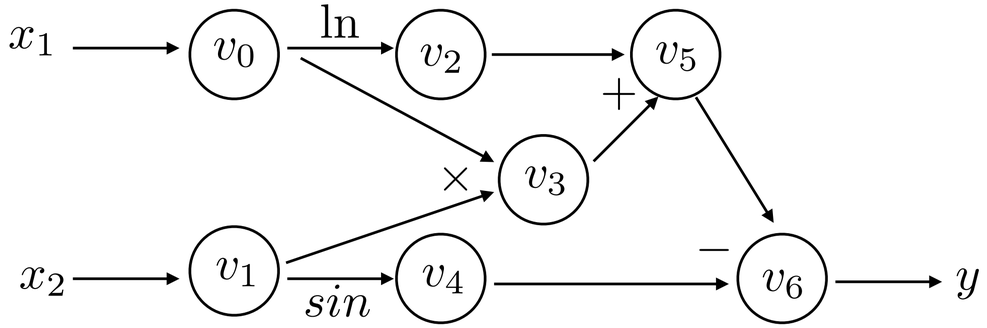

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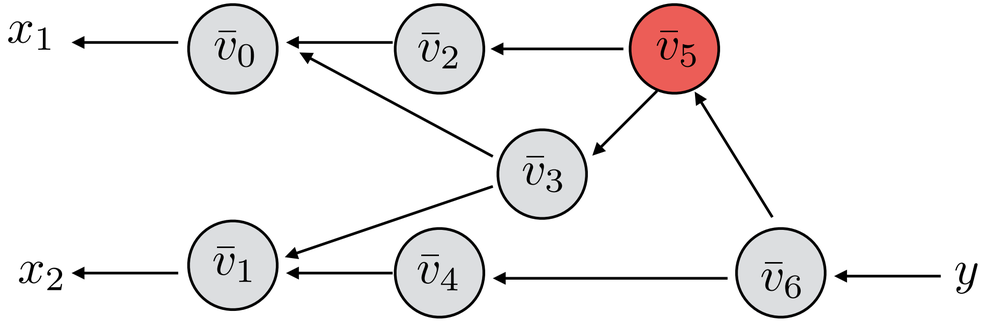

Backwards Derivative Trace:

$\bar{v}_5 = \bar{v}_6 \dfrac{\partial v_6}{\partial v_5} = \bar{v}_6 \cdot 1$	
$\bar{v}_6 = \dfrac{\partial y}{\partial v_6}$	1

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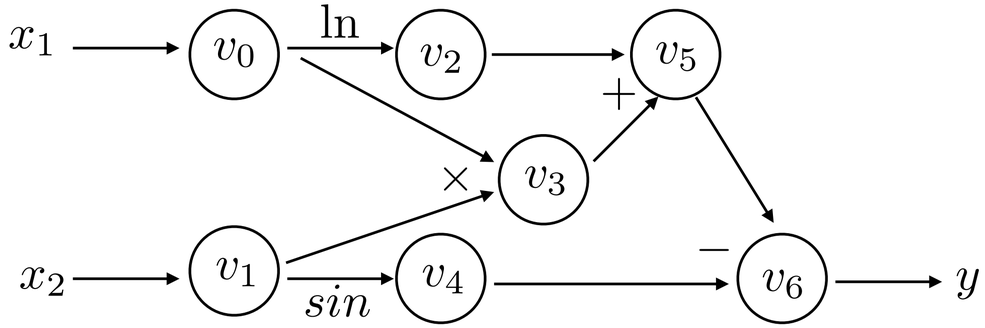

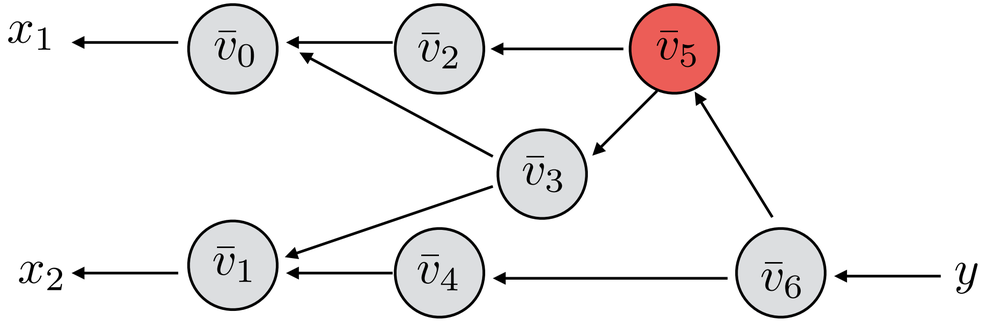

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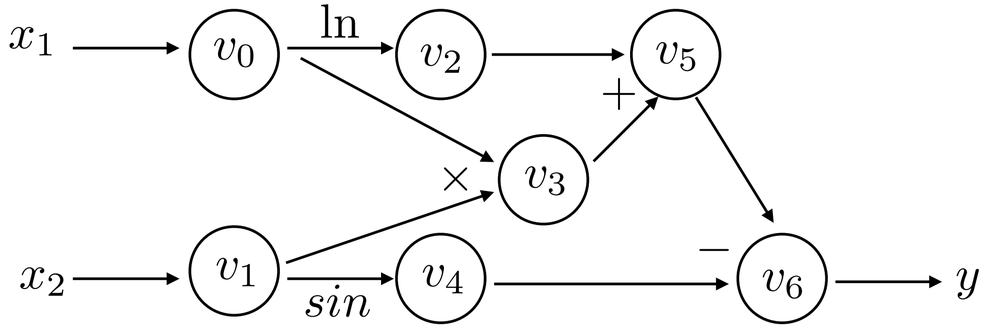

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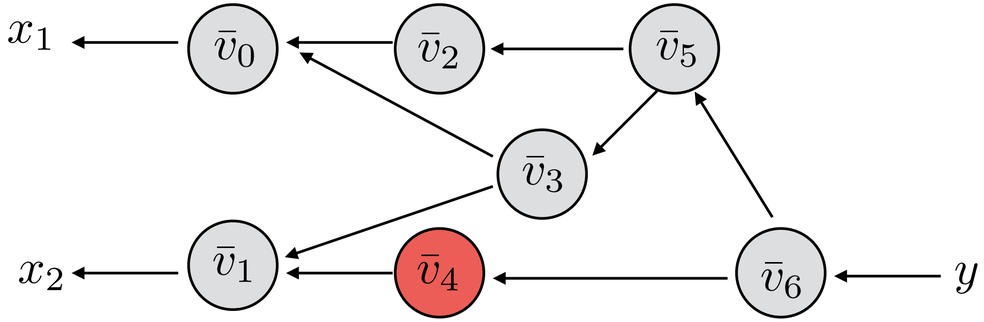

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$\bar{v}_4 = \bar{v}_6 \frac{\partial v_6}{\partial v_4}$	
$\bar{v}_5 = \bar{v}_6 \frac{\partial v_6}{\partial v_5} = \bar{v}_6 \cdot 1$	1x1 = 1
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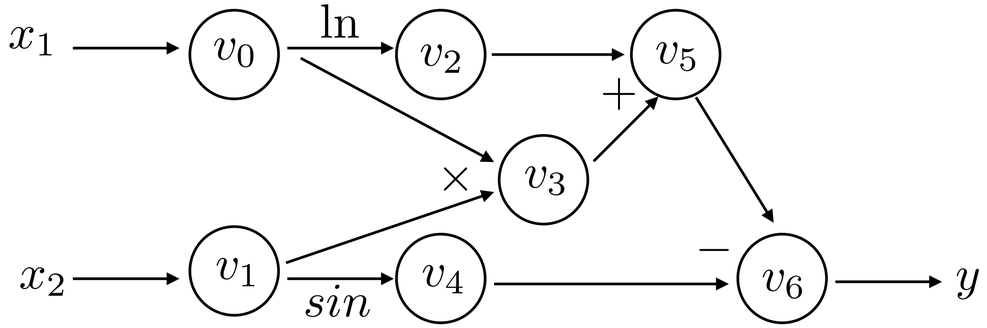
AutoDiff - **Reverse Mode**



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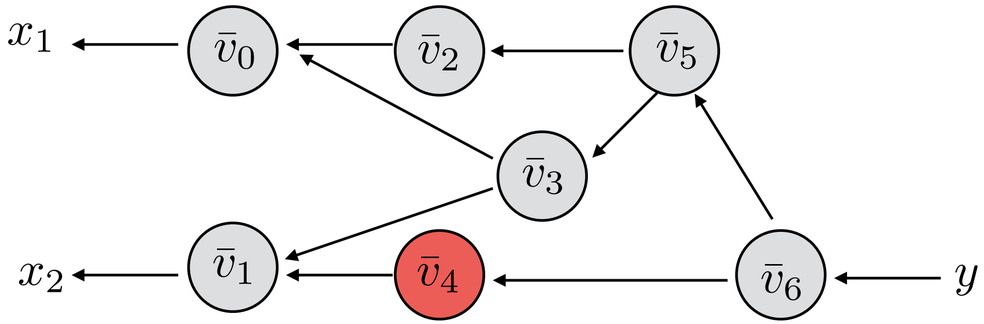

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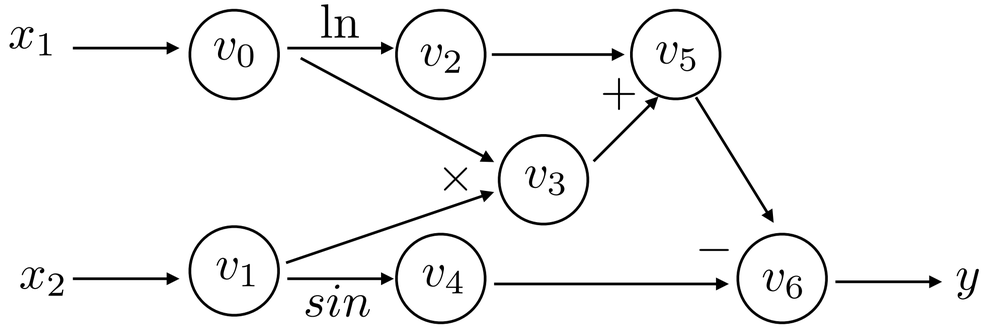

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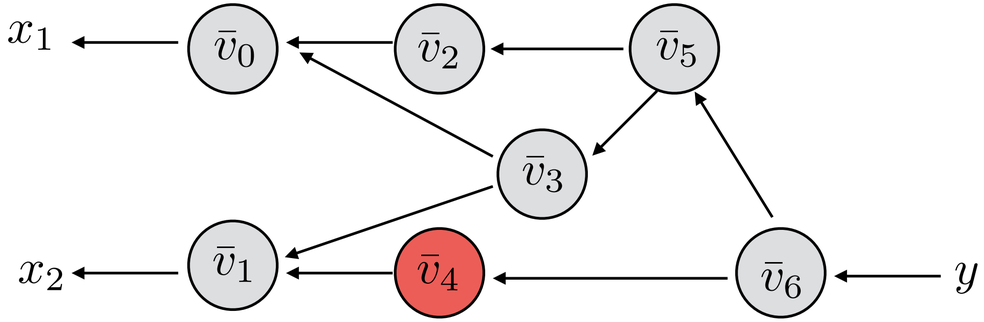

Backwards Derivative Trace:

$\bar{v}_4 = \bar{v}_6 \frac{\partial v_6}{\partial v_4} = \bar{v}_6 \cdot (-1)$	
$\bar{v}_5 = \bar{v}_6 \frac{\partial v_6}{\partial v_5} = \bar{v}_6 \cdot 1$	1x1 = 1
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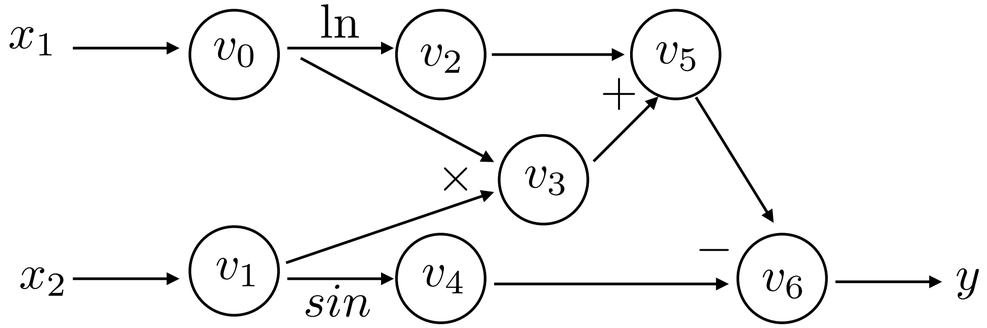
AutoDiff - **Reverse Mode**



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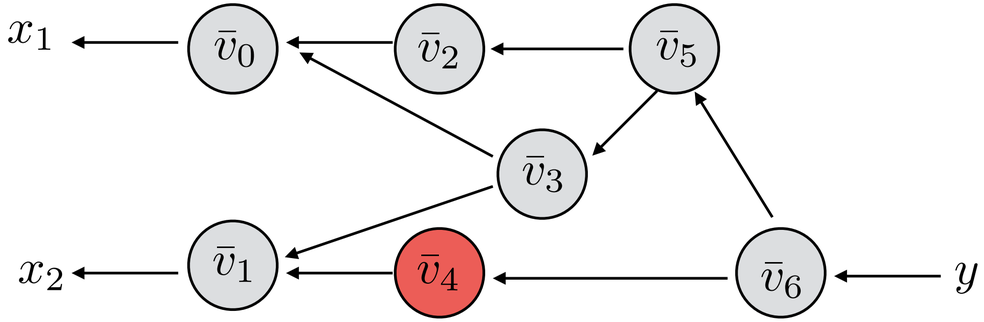
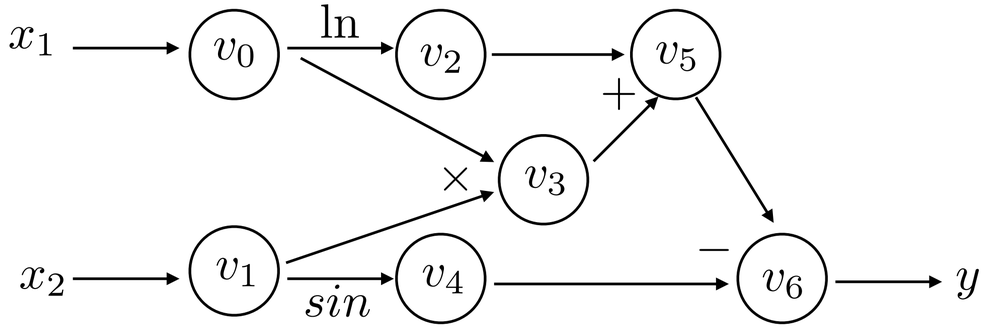


Backwards Derivative Trace:

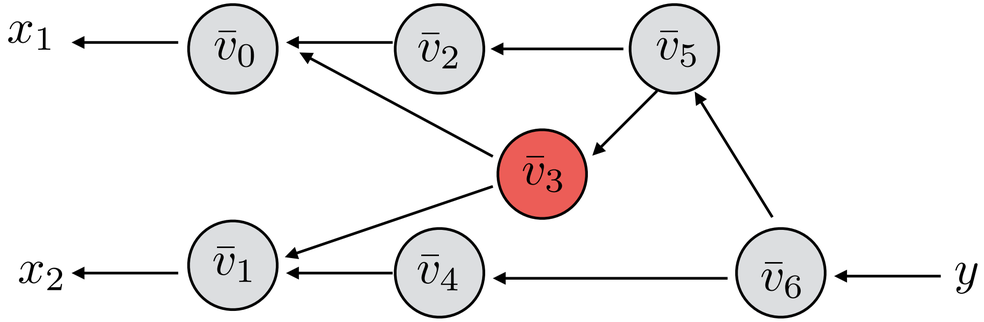
$\bar{v}_4 = \bar{v}_6 \frac{\partial v_6}{\partial v_4} = \bar{v}_6 \cdot (-1)$	1x-1 = -1
$\bar{v}_5 = \bar{v}_6 \frac{\partial v_6}{\partial v_5} = \bar{v}_6 \cdot 1$	1x1 = 1
$\bar{v}_6 = \frac{\partial y}{\partial v_6}$	1

AutoDiff - **Reverse Mode**



Forward Evaluation Trace:

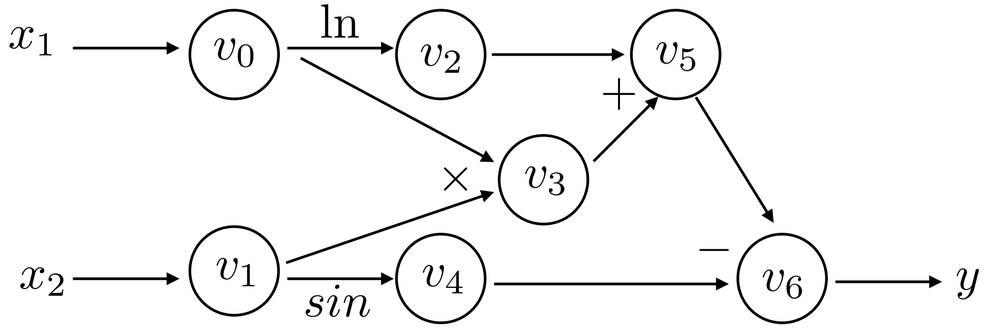
	$f(2,5)$
$v_0 = x_1$	2
$v_1 = x_2$	5
$v_2 = \ln(v_0)$	ln(2) = 0.693
$v_3 = v_0 \cdot v_1$	2 x 5 = 10
$v_4 = sin(v_1)$	sin(5) = 0.959
$v_5 = v_2 + v_3$	0.693 + 10 = 10.693
$v_6 = v_5 - v_4$	10.693 + 0.959 = 11.652
$y = v_6$	11.652



Backwards Derivative Trace:

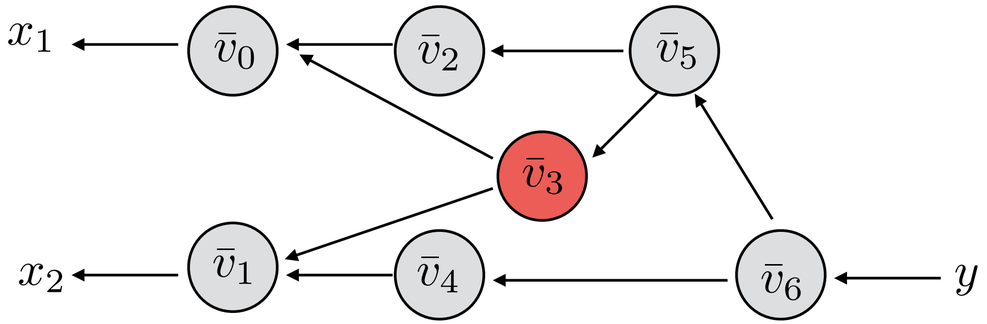
$\bar{v}_3 = \bar{v}_5 \frac{\partial v_5}{\partial v_3}$	
$\bar{v}_4 = \bar{v}_6 \frac{\partial v_6}{\partial v_4} = \bar{v}_6 \cdot (-1)$	1x-1 = -1
$\bar{v}_5 = \bar{v}_6 \frac{\partial v_6}{\partial v_5} = \bar{v}_6 \cdot 1$	1x1 = 1
$\bar{v}_6 = \frac{\partial y}{\partial v_6}$	1

AutoDiff - **Reverse Mode**



Forward Evaluation Trace:

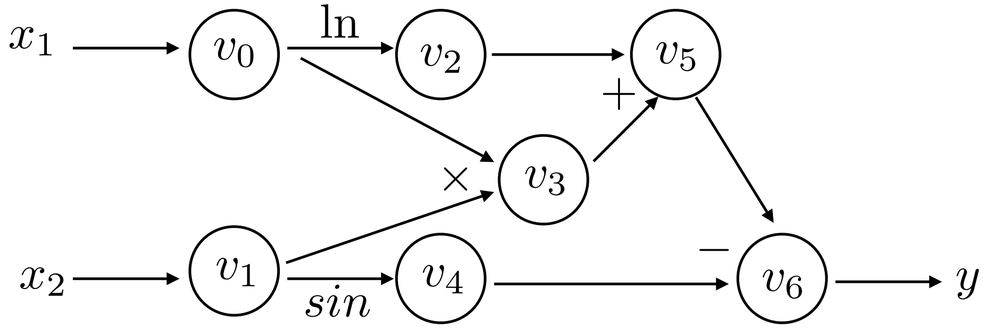
	$f(2,5)$
$v_0 = x_1$	2
$v_1 = x_2$	5
$v_2 = \ln(v_0)$	ln(2) = 0.693
$v_3 = v_0 \cdot v_1$	2 x 5 = 10
$v_4 = sin(v_1)$	sin(5) = 0.959
$v_5 = v_2 + v_3$	0.693 + 10 = 10.693
$v_6 = v_5 - v_4$	10.693 + 0.959 = 11.652
$y = v_6$	11.652



Backwards Derivative Trace:

$\bar{v}_3 = \bar{v}_5 \frac{\partial v_5}{\partial v_3}$	
$\bar{v}_4 = \bar{v}_6 \frac{\partial v_6}{\partial v_4} = \bar{v}_6 \cdot (-1)$	1x-1 = -1
$\bar{v}_5 = \bar{v}_6 \frac{\partial v_6}{\partial v_5} = \bar{v}_6 \cdot 1$	1x1 = 1
$\bar{v}_6 = \frac{\partial y}{\partial v_6}$	1

AutoDiff - **Reverse Mode**



Forward Evaluation Trace:

	$f(2,5)$
$v_0 = x_1$	2
$v_1 = x_2$	5
$v_2 = \ln(v_0)$	ln(2) = 0.693
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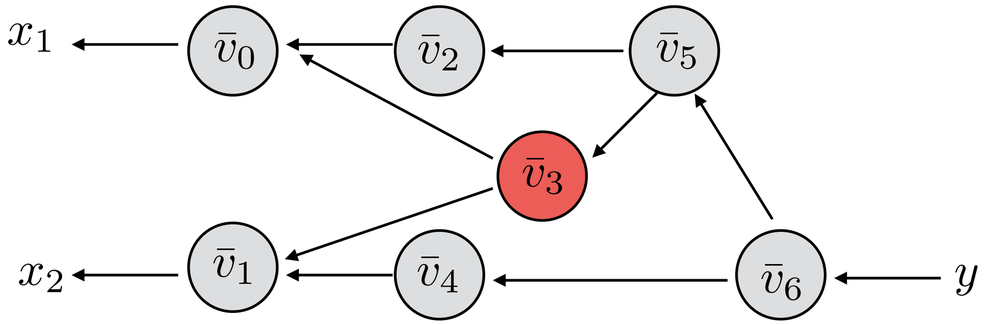
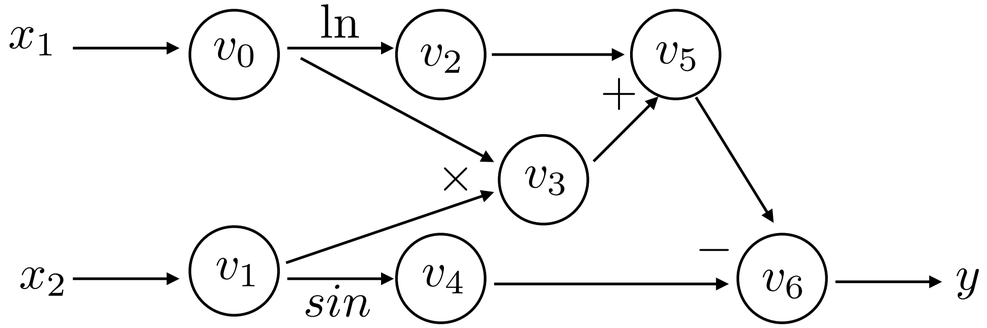


Backwards Derivative Trace:

$\bar{v}_3 = \bar{v}_5 \frac{\partial v_5}{\partial v_3} = \bar{v}_5 \cdot (1)$	
$\bar{v}_4 = \bar{v}_6 \frac{\partial v_6}{\partial v_4} = \bar{v}_6 \cdot (-1)$	1x-1 = -1
$\bar{v}_5 = \bar{v}_6 \frac{\partial v_6}{\partial v_5} = \bar{v}_6 \cdot 1$	1x1 = 1
$\bar{v}_6 = \frac{\partial y}{\partial v_6}$	1

AutoDiff - **Reverse Mode**



Forward Evaluation Trace:

	$f(2,5)$
$v_0 = x_1$	2
$v_1 = x_2$	5
$v_2 = \ln(v_0)$	ln(2) = 0.693
$v_3 = v_0 \cdot v_1$	2 x 5 = 10
$v_4 = sin(v_1)$	sin(5) = 0.959
$v_5 = v_2 + v_3$	0.693 + 10 = 10.693
$v_6 = v_5 - v_4$	10.693 + 0.959 = 11.652
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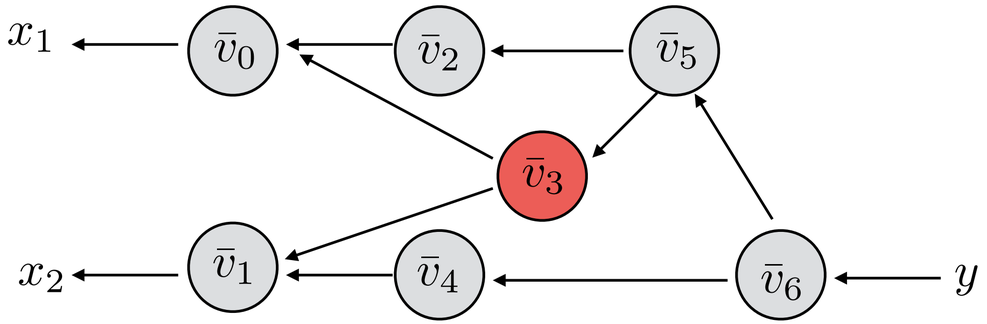
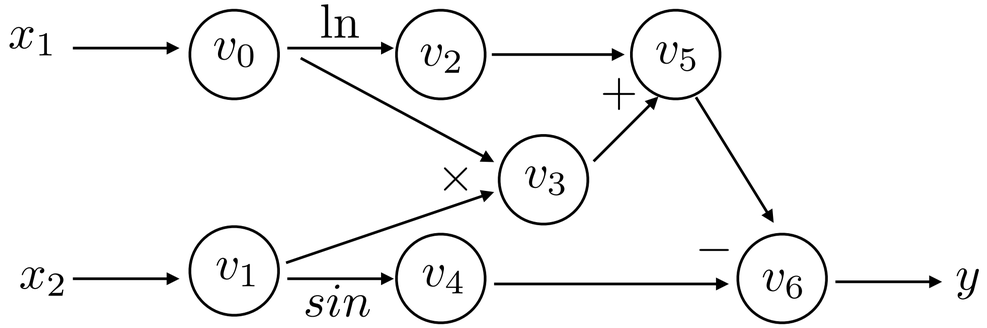


Backwards Derivative Trace:

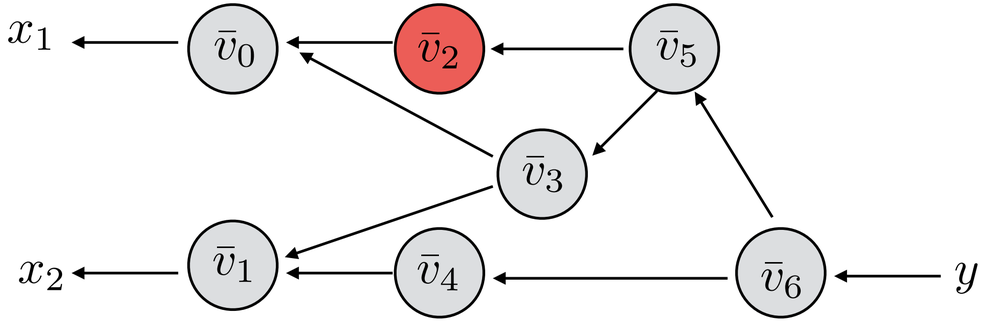
$\bar{v}_3 = \bar{v}_5 \frac{\partial v_5}{\partial v_3} = \bar{v}_5 \cdot (1)$	1x1 = 1
$\bar{v}_4 = \bar{v}_6 \frac{\partial v_6}{\partial v_4} = \bar{v}_6 \cdot (-1)$	1x-1 = -1
$\bar{v}_5 = \bar{v}_6 \frac{\partial v_6}{\partial v_5} = \bar{v}_6 \cdot 1$	1x1 = 1
$\bar{v}_6 = \frac{\partial y}{\partial v_6}$	1

AutoDiff - **Reverse Mode**



Forward Evaluation Trace:

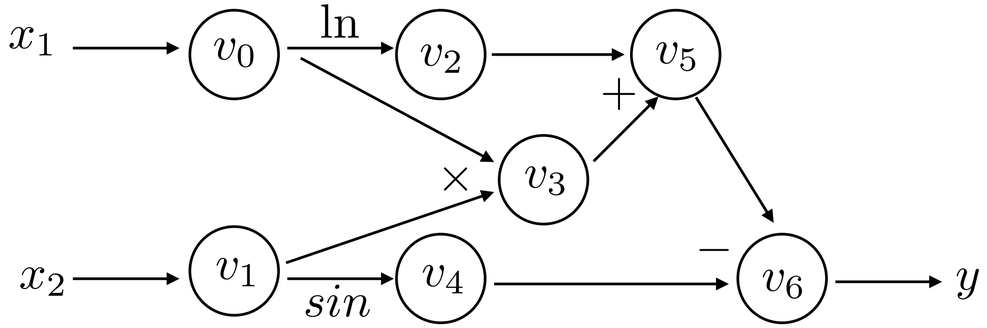
	$f(2,5)$
$v_0 = x_1$	2
$v_1 = x_2$	5
$v_2 = \ln(v_0)$	ln(2) = 0.693
$v_3 = v_0 \cdot v_1$	2 x 5 = 10
$v_4 = sin(v_1)$	sin(5) = 0.959
$v_5 = v_2 + v_3$	0.693 + 10 = 10.693
$v_6 = v_5 - v_4$	10.693 + 0.959 = 11.652
$y = v_6$	11.652



Backwards Derivative Trace:

$\bar{v}_2 = \bar{v}_5 \frac{\partial v_5}{\partial v_2}$	
$\bar{v}_3 = \bar{v}_5 \frac{\partial v_5}{\partial v_3} = \bar{v}_5 \cdot (1)$	1x1 = 1
$\bar{v}_4 = \bar{v}_6 \frac{\partial v_6}{\partial v_4} = \bar{v}_6 \cdot (-1)$	1x-1 = -1
$\bar{v}_5 = \bar{v}_6 \frac{\partial v_6}{\partial v_5} = \bar{v}_6 \cdot 1$	1x1 = 1
$\bar{v}_6 = \frac{\partial y}{\partial v_6}$	1

AutoDiff - **Reverse Mode**



Forward Evaluation Trace:

	$f(2,5)$
$v_0 = x_1$	2
$v_1 = x_2$	5
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$v_3 = v_0 \cdot v_1$	2 x 5 = 10
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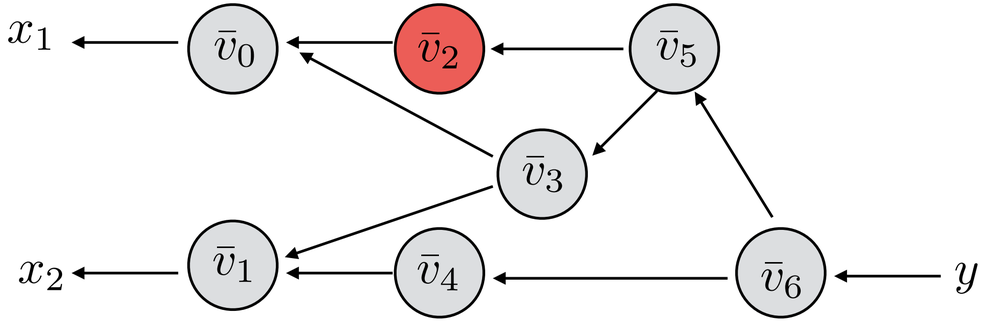
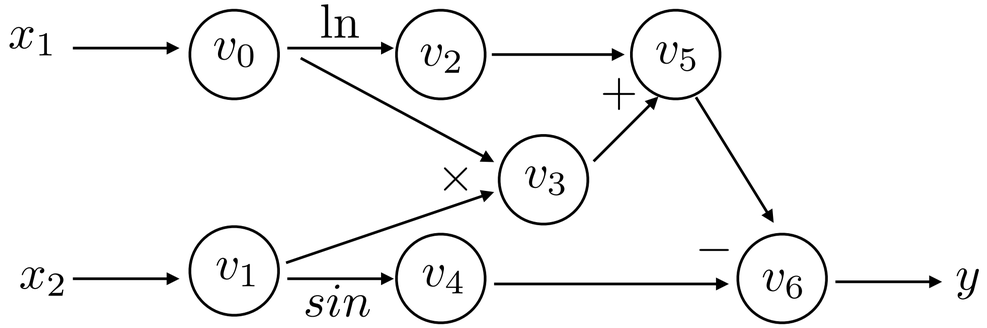


Backwards Derivative Trace:

$\bar{v}_2 = \bar{v}_5 \frac{\partial v_5}{\partial v_2}$	
$\bar{v}_3 = \bar{v}_5 \frac{\partial v_5}{\partial v_3} = \bar{v}_5 \cdot (1)$	1x1 = 1
$\bar{v}_4 = \bar{v}_6 \frac{\partial v_6}{\partial v_4} = \bar{v}_6 \cdot (-1)$	1x-1 = -1
$\bar{v}_5 = \bar{v}_6 \frac{\partial v_6}{\partial v_5} = \bar{v}_6 \cdot 1$	1x1 = 1
$\bar{v}_6 = \frac{\partial y}{\partial v_6}$	1

AutoDiff - **Reverse Mode**



Forward Evaluation Trace:

	$f(2,5)$
$v_0 = x_1$	2
$v_1 = x_2$	5
$v_2 = \ln(v_0)$	ln(2) = 0.693
$v_3 = v_0 \cdot v_1$	2 x 5 = 10
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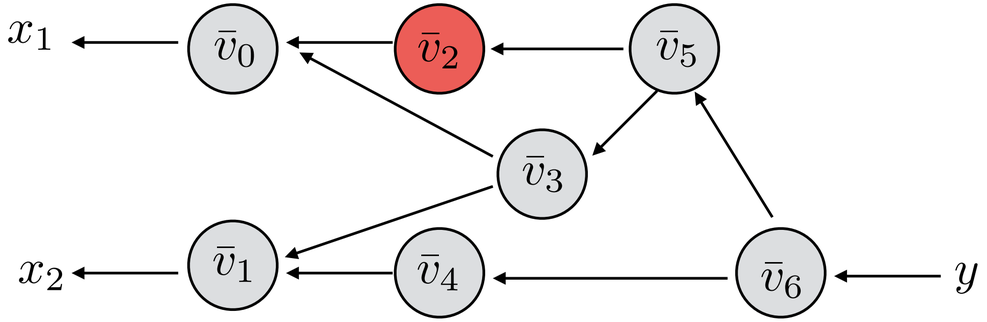
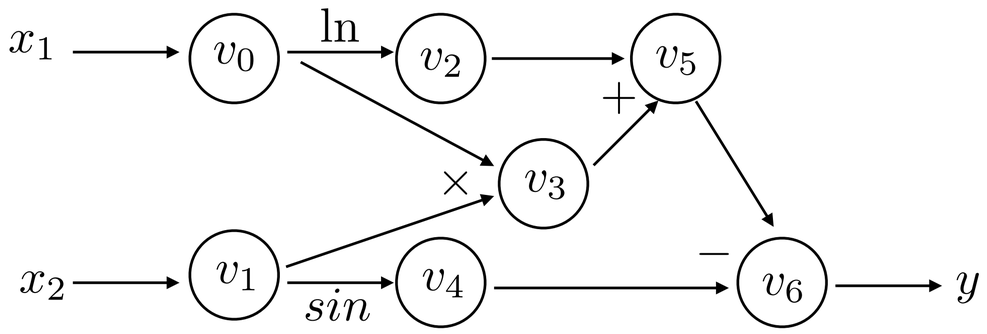
$v_5 = v_2 + v_3$	0.693 + 10 = 10.693
$v_6 = v_5 - v_4$	10.693 + 0.959 = 11.652
$y = v_6$	11.652



Backwards Derivative Trace:

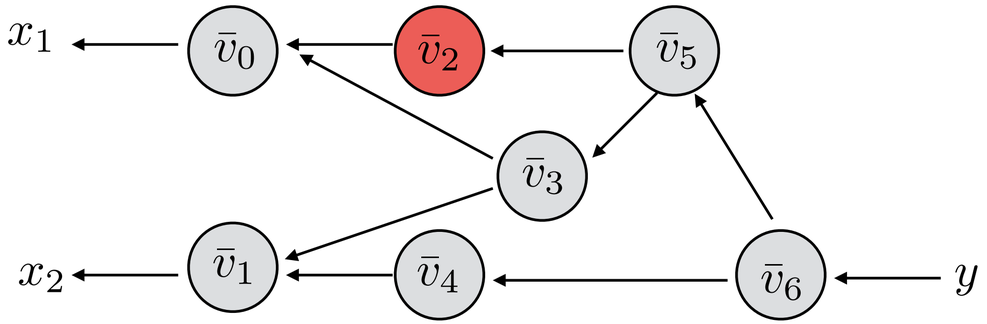
$\bar{v}_2 = \bar{v}_5 \frac{\partial v_5}{\partial v_2} = \bar{v}_5 \cdot (1)$	
$\bar{v}_3 = \bar{v}_5 \frac{\partial v_5}{\partial v_3} = \bar{v}_5 \cdot (1)$	1x1 = 1
$\bar{v}_4 = \bar{v}_6 \frac{\partial v_6}{\partial v_4} = \bar{v}_6 \cdot (-1)$	1x-1 = -1
$\bar{v}_5 = \bar{v}_6 \frac{\partial v_6}{\partial v_5} = \bar{v}_6 \cdot 1$	1x1 = 1
$\bar{v}_6 = \frac{\partial y}{\partial v_6}$	1

AutoDiff - **Reverse Mode**



Forward Evaluation Trace:

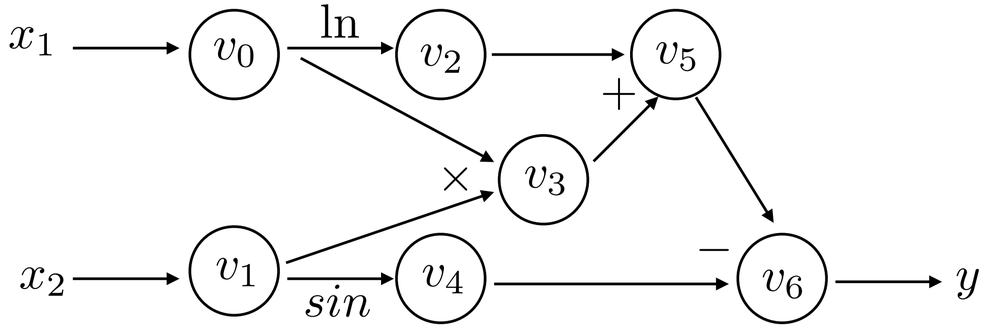
	$f(2,5)$
$v_0 = x_1$	2
$v_1 = x_2$	5
$v_2 = \ln(v_0)$	ln(2) = 0.693
$v_3 = v_0 \cdot v_1$	2 x 5 = 10
$v_4 = sin(v_1)$	sin(5) = 0.959
$v_5 = v_2 + v_3$	0.693 + 10 = 10.693
$v_6 = v_5 - v_4$	10.693 + 0.959 = 11.652
$y = v_6$	11.652



Backwards Derivative Trace:

$\bar{v}_2 = \bar{v}_5 \frac{\partial v_5}{\partial v_2} = \bar{v}_5 \cdot (1)$	1x1 = 1
$\bar{v}_3 = \bar{v}_5 \frac{\partial v_5}{\partial v_3} = \bar{v}_5 \cdot (1)$	1x1 = 1
$\bar{v}_4 = \bar{v}_6 \frac{\partial v_6}{\partial v_4} = \bar{v}_6 \cdot (-1)$	1x-1 = -1
$\bar{v}_5 = \bar{v}_6 \frac{\partial v_6}{\partial v_5} = \bar{v}_6 \cdot 1$	1x1 = 1
$\bar{v}_6 = \frac{\partial y}{\partial v_6}$	1

AutoDiff - **Reverse Mode**



Forward Evaluation Trace:

	$f(2,5)$
$v_0 = x_1$	2
$v_1 = x_2$	5
$v_2 = \ln(v_0)$	ln(2) = 0.693
$v_3 = v_0 \cdot v_1$	2 x 5 = 10
$v_4 = sin(v_1)$	sin(5) = 0.959
$v_5 = v_2 + v_3$	0.693 + 10 = 10.693
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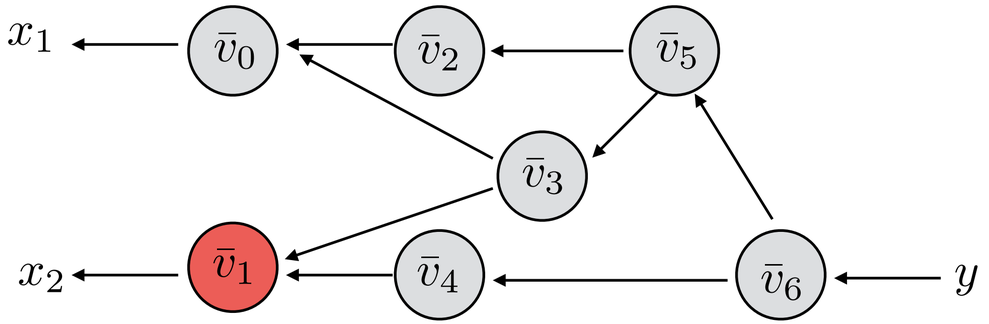
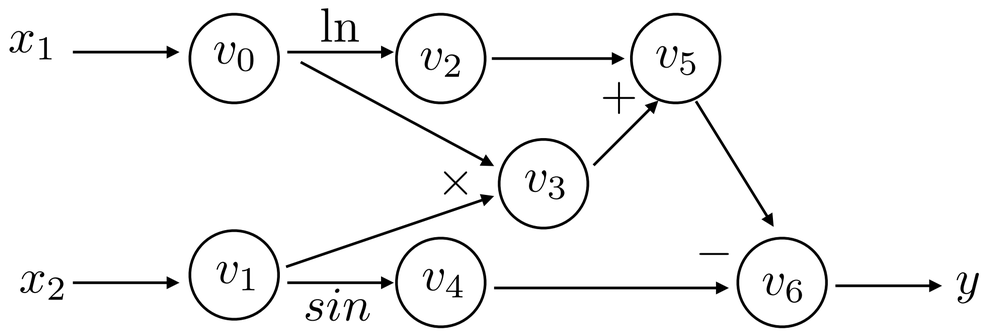


Backwards Derivative Trace:

$\bar{v}_1$	
$\bar{v}_2 = \bar{v}_5 \frac{\partial v_5}{\partial v_2} = \bar{v}_5 \cdot (1)$	1x1 = 1
$\bar{v}_3 = \bar{v}_5 \frac{\partial v_5}{\partial v_3} = \bar{v}_5 \cdot (1)$	1x1 = 1
$\bar{v}_4 = \bar{v}_6 \frac{\partial v_6}{\partial v_4} = \bar{v}_6 \cdot (-1)$	1x-1 = -1
$\bar{v}_5 = \bar{v}_6 \frac{\partial v_6}{\partial v_5} = \bar{v}_6 \cdot 1$	1x1 = 1
$\bar{v}_6 = \frac{\partial y}{\partial v_6}$	1

AutoDiff - **Reverse Mode**



Forward Evaluation Trace:

	$f(2,5)$
$v_0 = x_1$	2
$v_1 = x_2$	5
$v_2 = \ln(v_0)$	ln(2) = 0.693
$v_3 = v_0 \cdot v_1$	2 x 5 = 10
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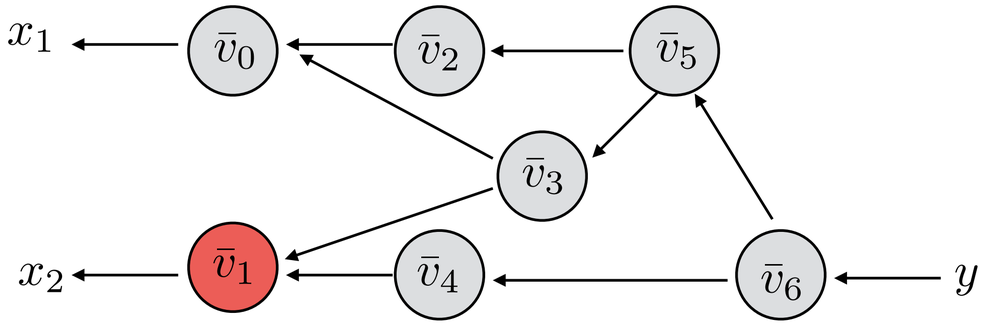
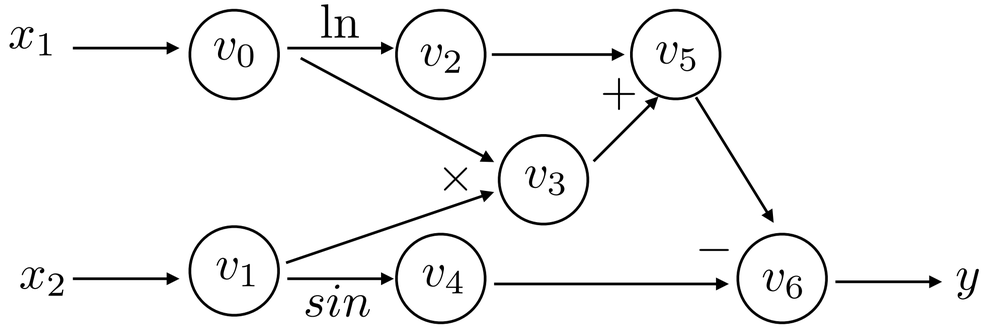


Backwards Derivative Trace:

$\bar{v}_1 = \bar{v}_3 \frac{\partial v_3}{\partial v_1} + \bar{v}_4 \frac{\partial v_4}{\partial v_1}$	
$\bar{v}_2 = \bar{v}_5 \frac{\partial v_5}{\partial v_2} = \bar{v}_5 \cdot (1)$	1x1 = 1
$\bar{v}_3 = \bar{v}_5 \frac{\partial v_5}{\partial v_3} = \bar{v}_5 \cdot (1)$	1x1 = 1
$\bar{v}_4 = \bar{v}_6 \frac{\partial v_6}{\partial v_4} = \bar{v}_6 \cdot (-1)$	1x-1 = -1
$\bar{v}_5 = \bar{v}_6 \frac{\partial v_6}{\partial v_5} = \bar{v}_6 \cdot 1$	1x1 = 1
$\bar{v}_6 = \frac{\partial y}{\partial v_6}$	1

AutoDiff - **Reverse Mode**



Forward Evaluation Trace:

	$f(2,5)$
$v_0 = x_1$	2
$v_1 = x_2$	5
$v_2 = \ln(v_0)$	ln(2) = 0.693
$v_3 = v_0 \cdot v_1$	2 x 5 = 10
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$v_5 = v_2 + v_3$	0.693 + 10 = 10.693
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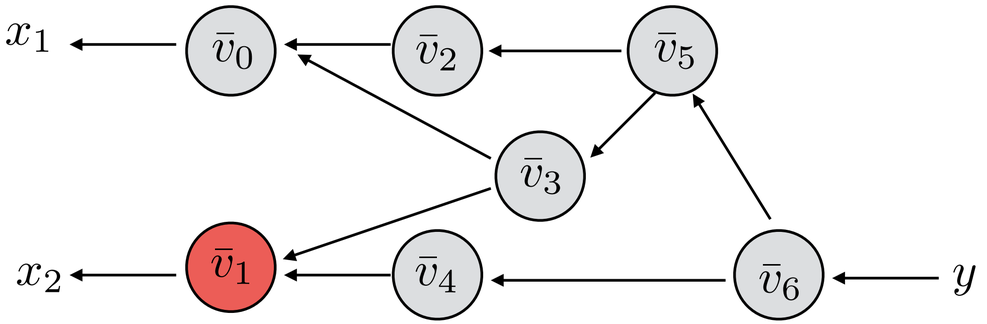
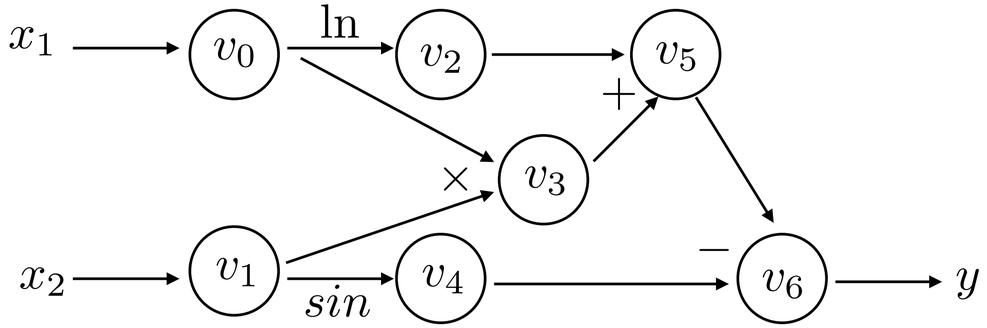


Backwards Derivative Trace:

$\bar{v}_1 = \bar{v}_3 \frac{\partial v_3}{\partial v_1} + \bar{v}_4 \frac{\partial v_4}{\partial v_1}$	
$\bar{v}_2 = \bar{v}_5 \frac{\partial v_5}{\partial v_2} = \bar{v}_5 \cdot (1)$	1x1 = 1
$\bar{v}_3 = \bar{v}_5 \frac{\partial v_5}{\partial v_3} = \bar{v}_5 \cdot (1)$	1x1 = 1
$\bar{v}_4 = \bar{v}_6 \frac{\partial v_6}{\partial v_4} = \bar{v}_6 \cdot (-1)$	1x-1 = -1
$\bar{v}_5 = \bar{v}_6 \frac{\partial v_6}{\partial v_5} = \bar{v}_6 \cdot 1$	1x1 = 1
$\bar{v}_6 = \frac{\partial y}{\partial v_6}$	1

AutoDiff - **Reverse Mode**



Forward Evaluation Trace:

	$f(2,5)$
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$v_1 = x_2$	5
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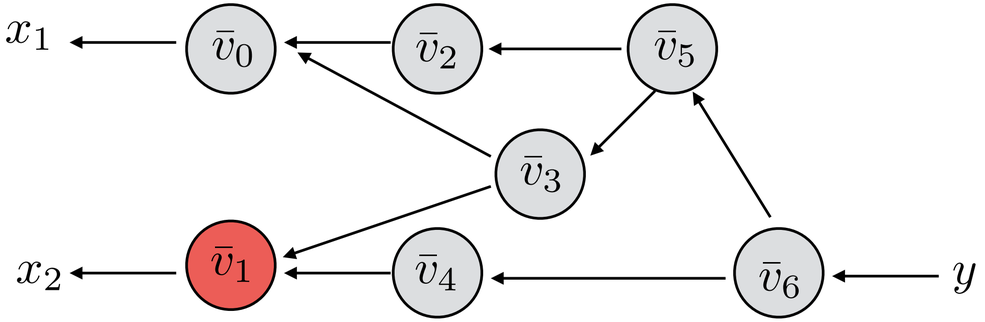
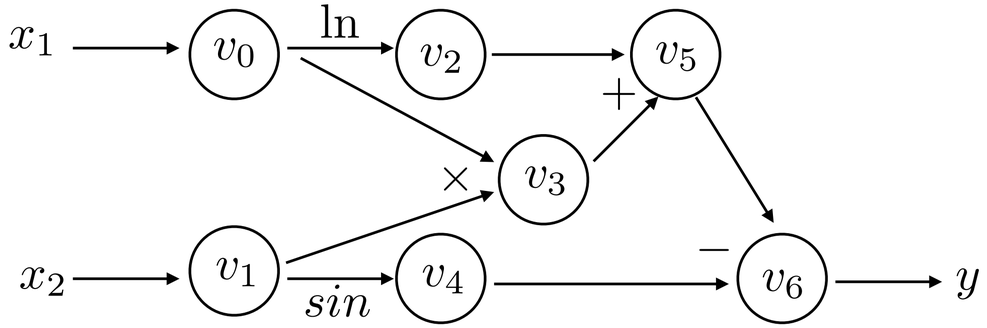


Backwards Derivative Trace:

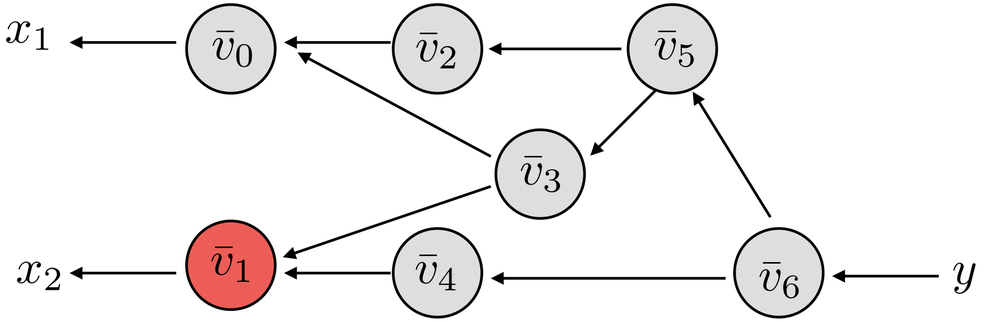
$\bar{v}_1 = \bar{v}_3 \frac{\partial v_3}{\partial v_1} + \bar{v}_4 \frac{\partial v_4}{\partial v_1} = \bar{v}_3 v_0 + \bar{v}_4 cos(v_1)$	
$\bar{v}_2 = \bar{v}_5 \frac{\partial v_5}{\partial v_2} = \bar{v}_5 \cdot (1)$	1x1 = 1
$\bar{v}_3 = \bar{v}_5 \frac{\partial v_5}{\partial v_3} = \bar{v}_5 \cdot (1)$	1x1 = 1
$\bar{v}_4 = \bar{v}_6 \frac{\partial v_6}{\partial v_4} = \bar{v}_6 \cdot (-1)$	1x-1 = -1
$\bar{v}_5 = \bar{v}_6 \frac{\partial v_6}{\partial v_5} = \bar{v}_6 \cdot 1$	1x1 = 1
$\bar{v}_6 = \frac{\partial y}{\partial v_6}$	1

AutoDiff - **Reverse Mode**



Forward Evaluation Trace:

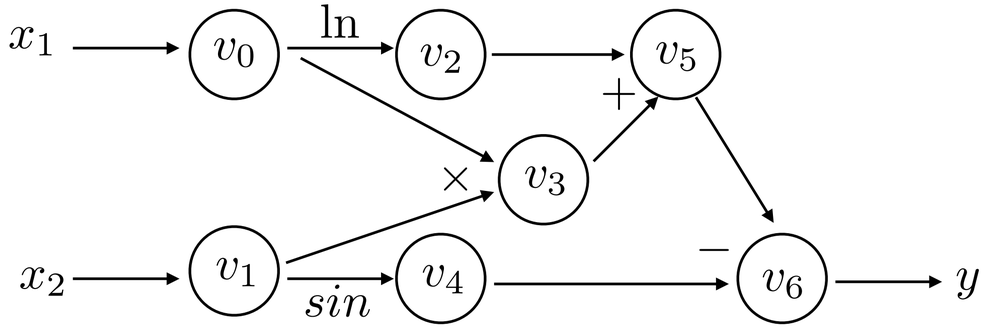
	$f(2,5)$
$v_0 = x_1$	2
$v_1 = x_2$	5
$v_2 = \ln(v_0)$	ln(2) = 0.693
$v_3 = v_0 \cdot v_1$	2 x 5 = 10
$v_4 = sin(v_1)$	sin(5) = 0.959
$v_5 = v_2 + v_3$	0.693 + 10 = 10.693
$v_6 = v_5 - v_4$	10.693 + 0.959 = 11.652
$y = v_6$	11.652



Backwards Derivative Trace:

$\bar{v}_1 = \bar{v}_3 \frac{\partial v_3}{\partial v_1} + \bar{v}_4 \frac{\partial v_4}{\partial v_1} = \bar{v}_3 v_0 + \bar{v}_4 cos(v_1)$	1.716
$\bar{v}_2 = \bar{v}_5 \frac{\partial v_5}{\partial v_2} = \bar{v}_5 \cdot (1)$	1x1 = 1
$\bar{v}_3 = \bar{v}_5 \frac{\partial v_5}{\partial v_3} = \bar{v}_5 \cdot (1)$	1x1 = 1
$\bar{v}_4 = \bar{v}_6 \frac{\partial v_6}{\partial v_4} = \bar{v}_6 \cdot (-1)$	1x-1 = -1
$\bar{v}_5 = \bar{v}_6 \frac{\partial v_6}{\partial v_5} = \bar{v}_6 \cdot 1$	1x1 = 1
$\bar{v}_6 = \frac{\partial y}{\partial v_6}$	1

AutoDiff - **Reverse Mode**



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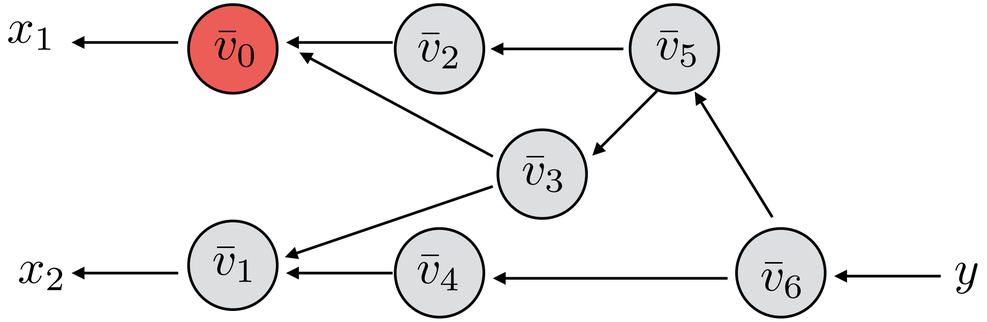

Backwards Derivative Trace:

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$\bar{v}_1 = \bar{v}_3 \frac{\partial v_3}{\partial v_1} + \bar{v}_4 \frac{\partial v_4}{\partial v_1} = \bar{v}_3 v_0 + \bar{v}_4 cos(v_1)$	1.716
$\bar{v}_2 = \bar{v}_5 \frac{\partial v_5}{\partial v_2} = \bar{v}_5 \cdot (1)$	1x1 = 1
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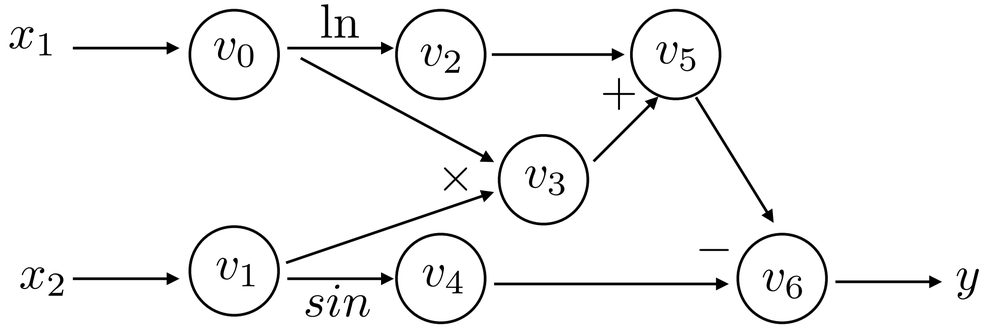
$\bar{v}_6 = \frac{\partial y}{\partial v_6}$	1

AutoDiff - **Reverse Mode**



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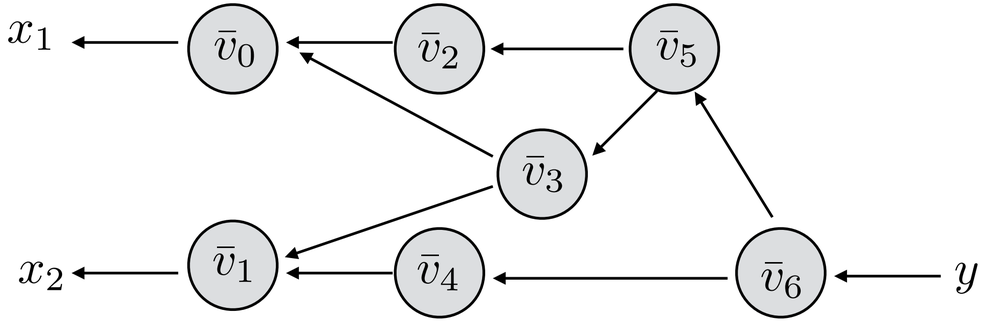

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$\bar{v}_1 = \bar{v}_3 \frac{\partial v_3}{\partial v_1} + \bar{v}_4 \frac{\partial v_4}{\partial v_1} = \bar{v}_3 v_0 + \bar{v}_4 cos(v_1)$	**1.716**
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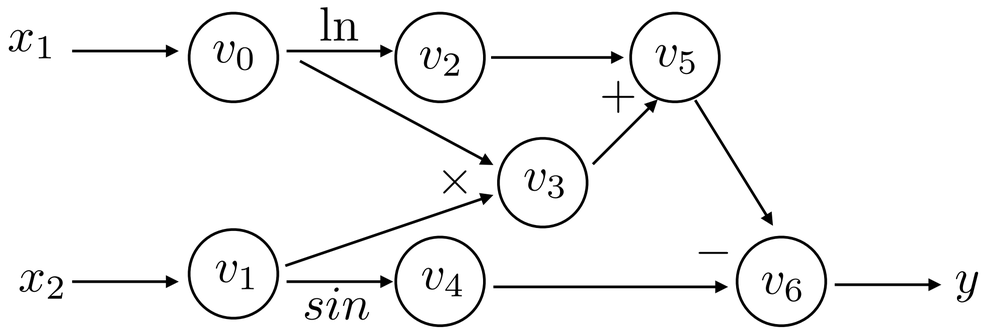
$\bar{v}_6 = \frac{\partial y}{\partial v_6}$	1

Automatic Differentiation (AutoDiff)

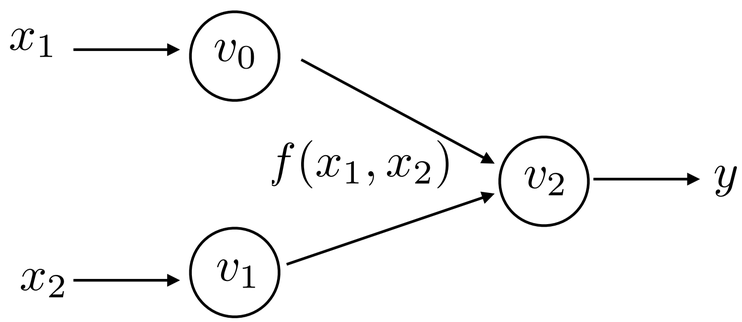
$$y = f(x_1, x_2) = \ln(x_1) + x_1 x_2 - sin(x_2)$$

AutoDiff can be done at various **granularities**

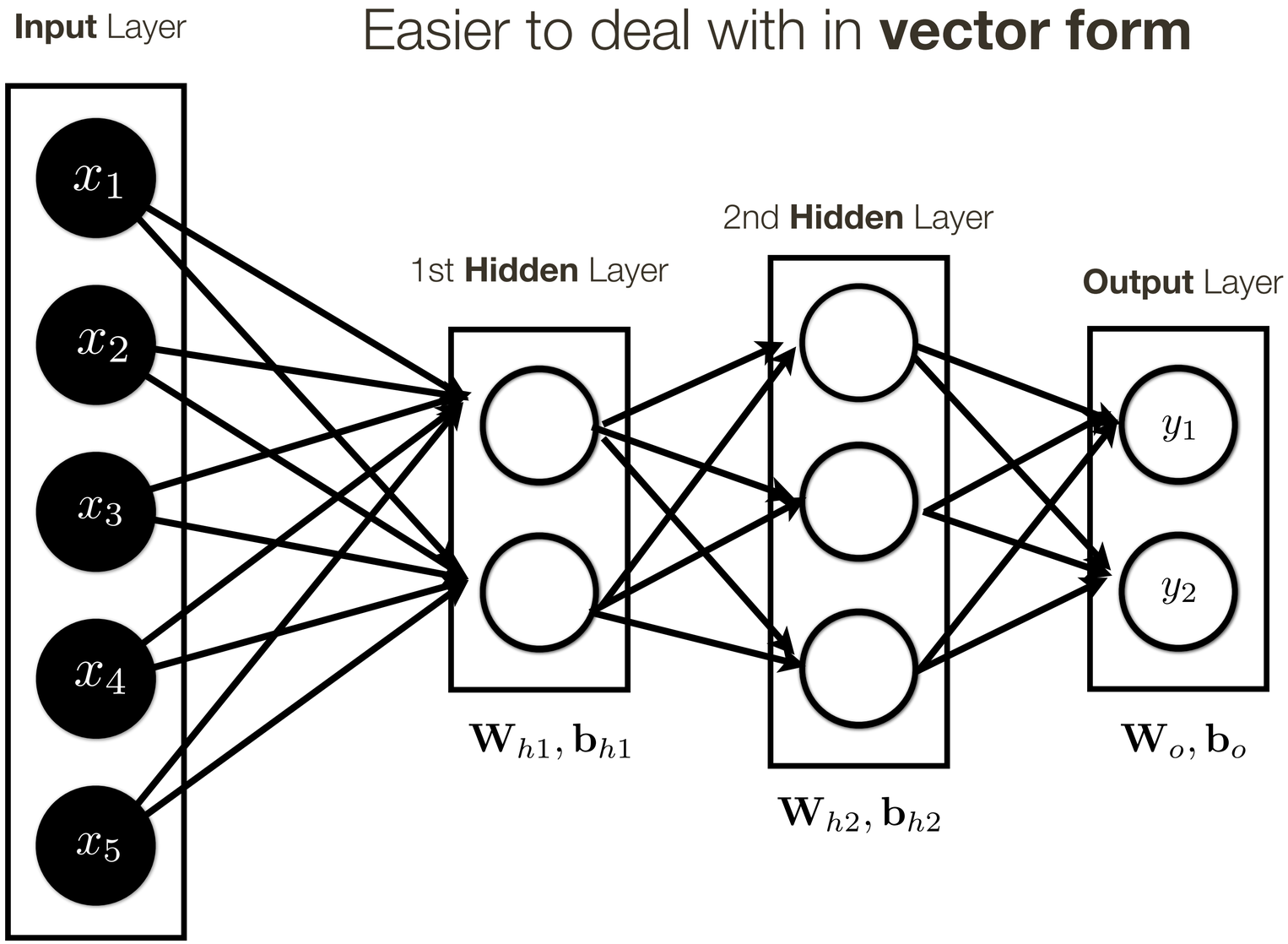
Elementary function granularity:



Complex function granularity:

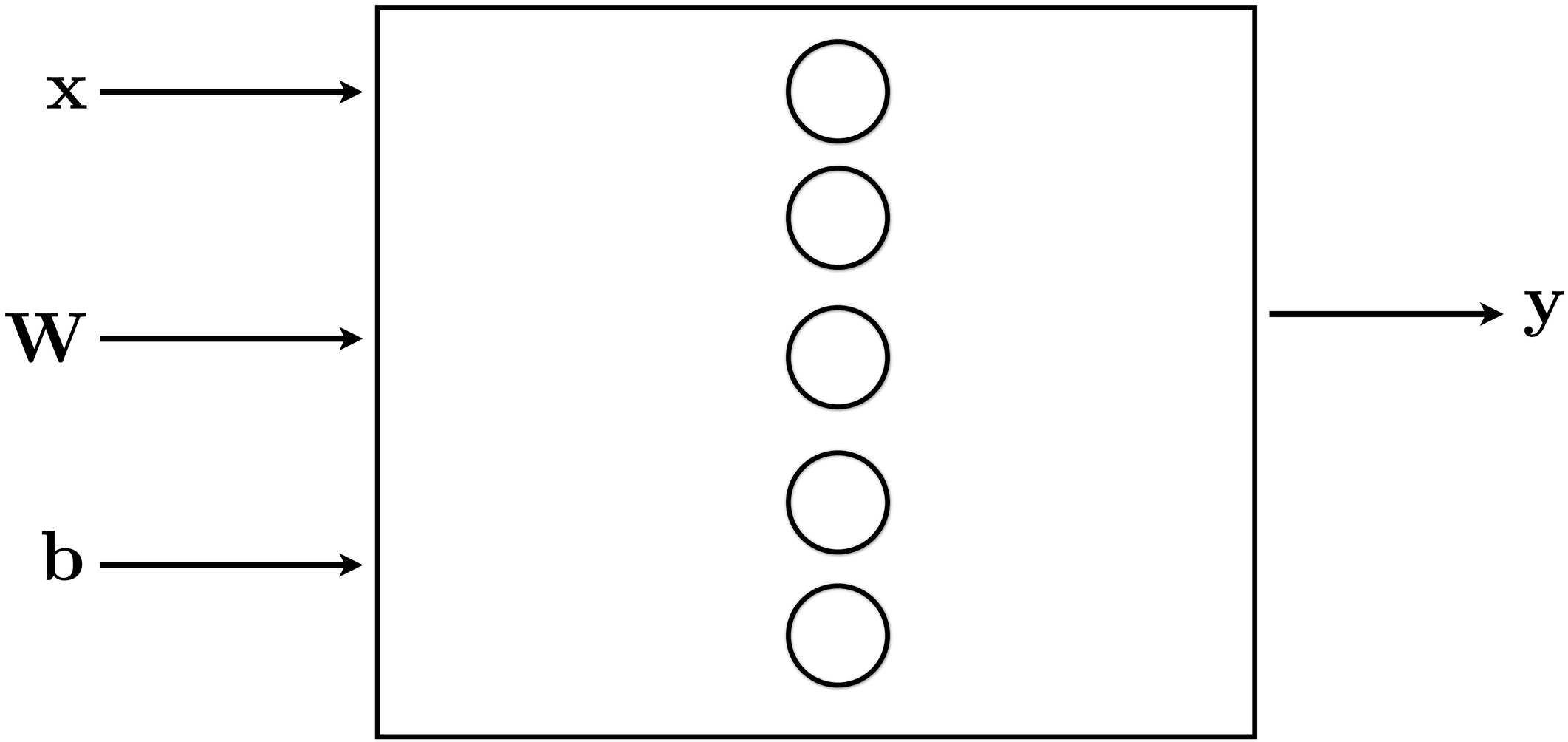


Backpropagation Practical Issues



Backpropagation Practical Issues

$$\mathbf{y} = f(\mathbf{W}, \mathbf{b}, \mathbf{x}) = \mathbf{sigmoid}(\mathbf{W} \cdot \mathbf{x} + \mathbf{b})$$

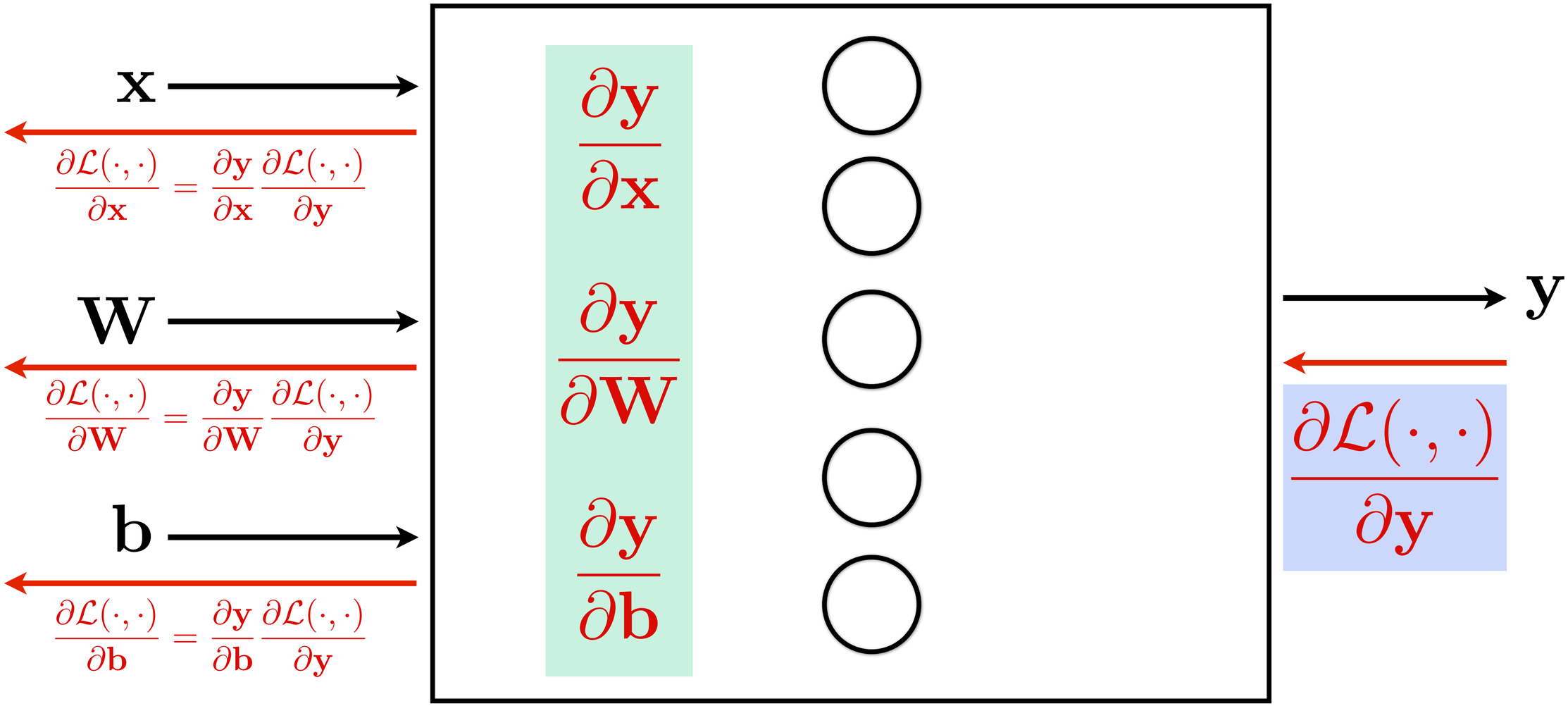


Backpropagation Practical Issues

"**local**" Jacobians
(matrix of partial derivatives, e.g. size |x| x |y|)

"**backprop**" Gradient

$$\mathbf{y} = f(\mathbf{W}, \mathbf{b}, \mathbf{x}) = \mathbf{sigmoid}(\mathbf{W} \cdot \mathbf{x} + \mathbf{b})$$



Backpropagation Practical Issues

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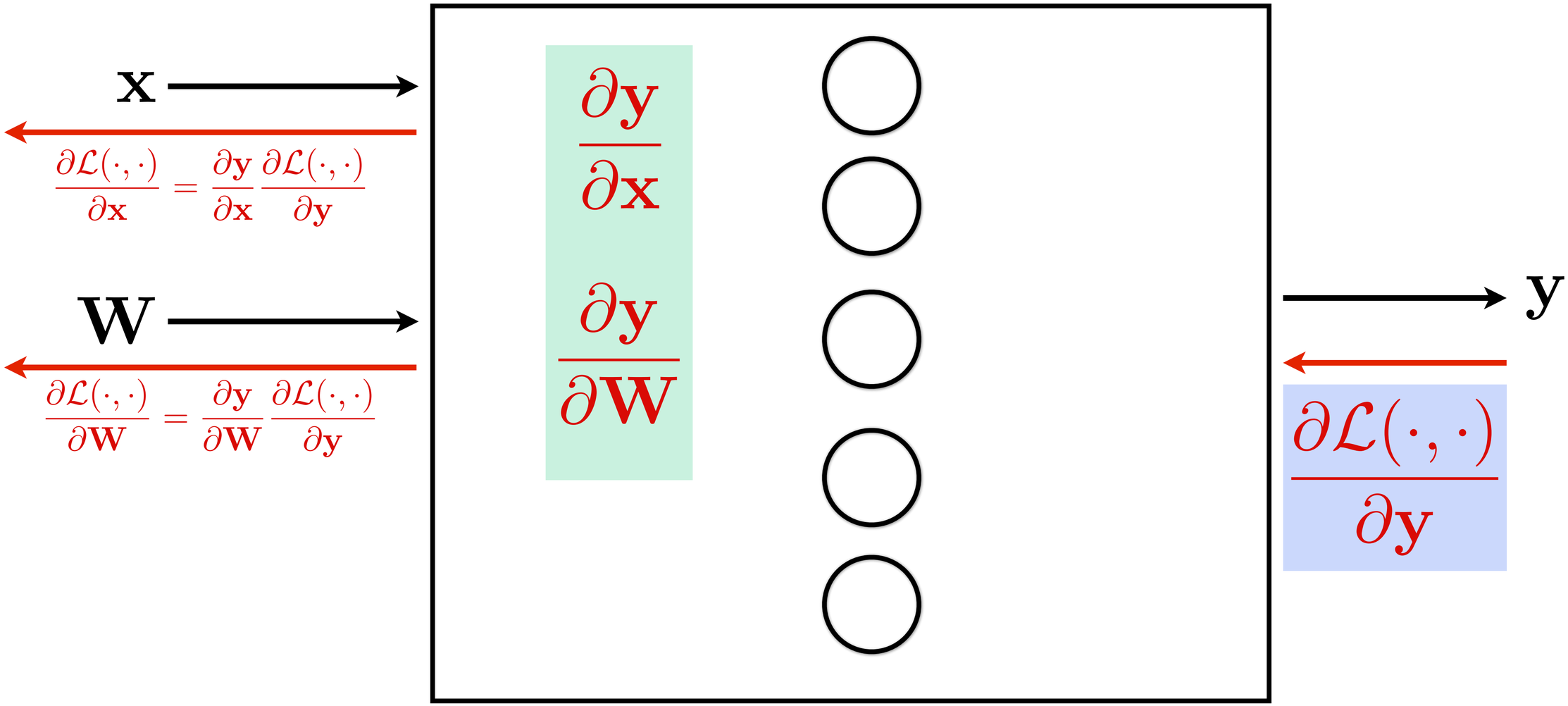
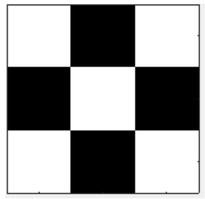
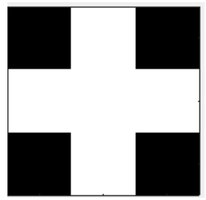
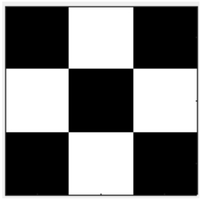


Example: Let's Build (world smallest) Neural Network

Lets create a neural network that will be able to differentiate (classify) these patterns of simple 3x3 pixel images







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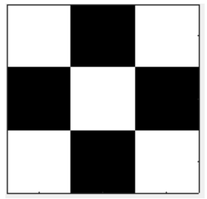
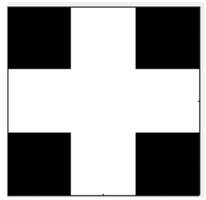
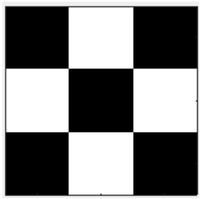
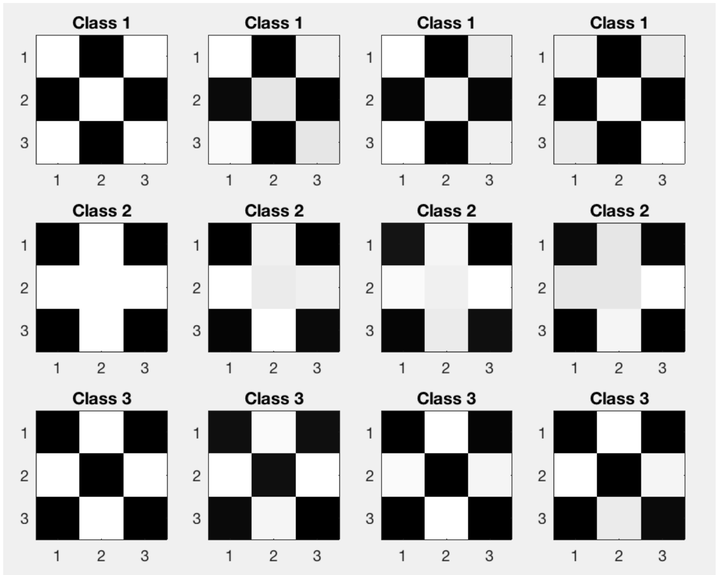






We will need some labeled data



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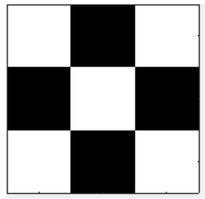

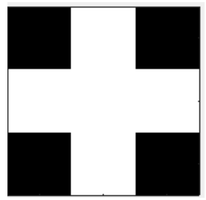

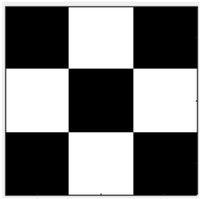

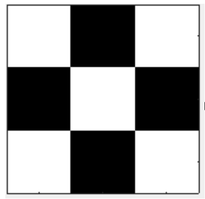

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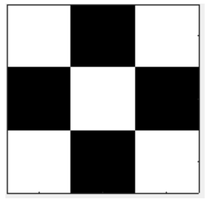
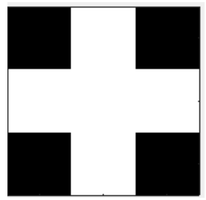
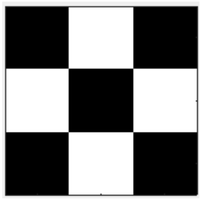
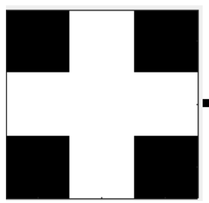




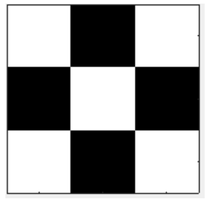


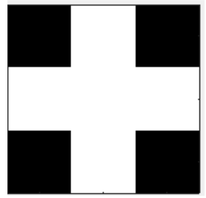


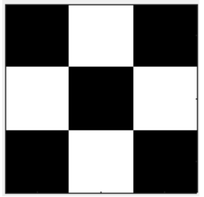
Class **2**

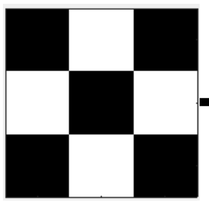
Example: Let's Build (world smallest) Neural Network

Lets create a neural network that will be able to differentiate (classify) these patterns of simple 3x3 pixel images









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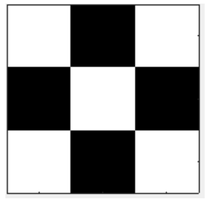
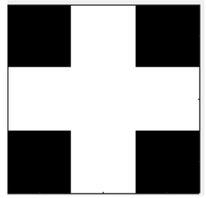
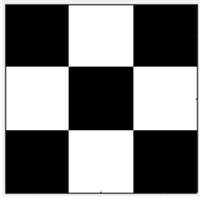

What do we need to do?

First, lets re-formulate the problem

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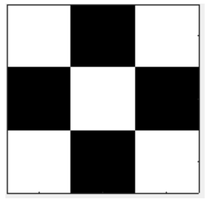
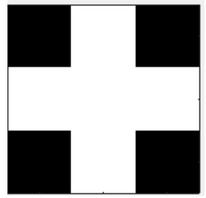
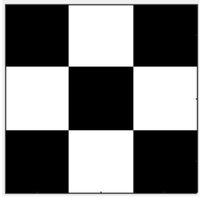

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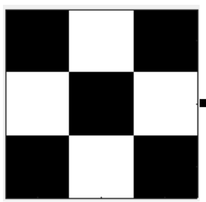

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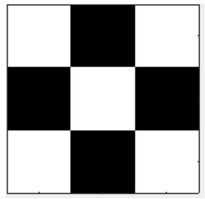
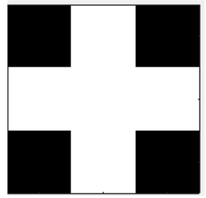
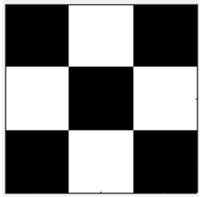

Now, lets build a **network**!

How many inputs should the network have? How neuron outputs?

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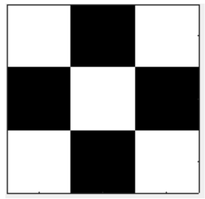
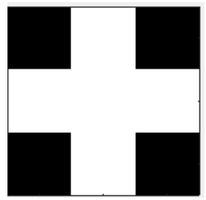
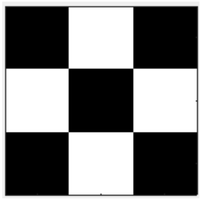
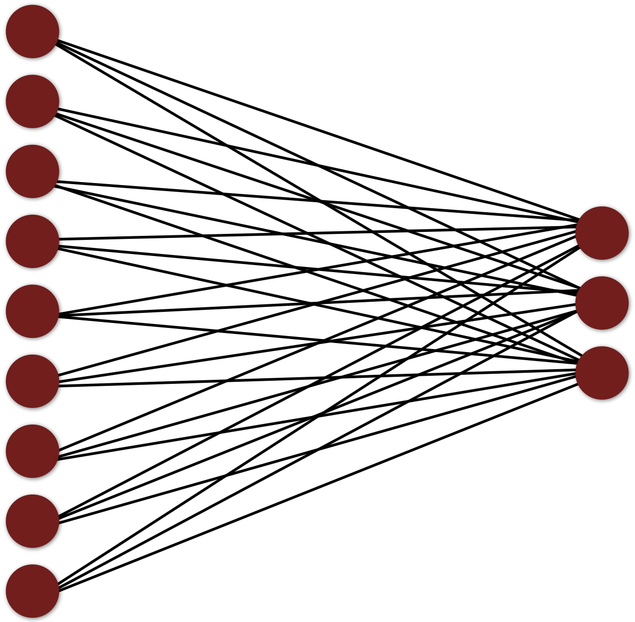






Input Layer

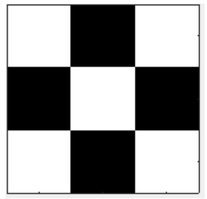
Output Layer



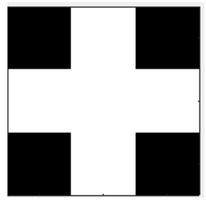
What else is missing for us to train it?

Example: Let's Build (world smallest) Neural Network

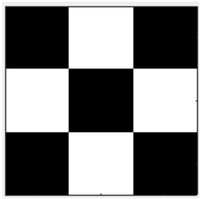
Lets create a neural network that will be able to differentiate (classify) these patterns of simple 3x3 pixel images



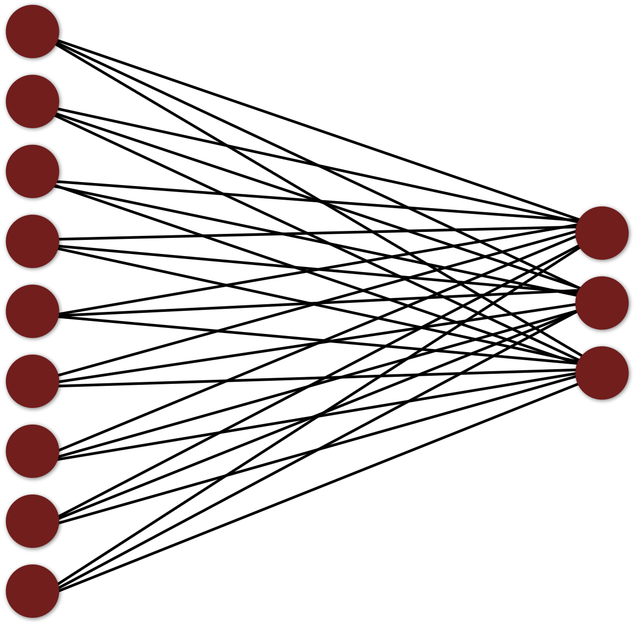
Input Layer



Output Layer



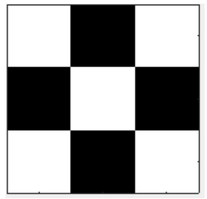
Loss



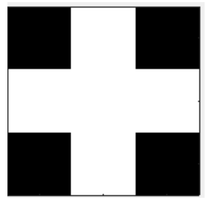
$$L_i = -\log\left(\frac{e^{f_{y_i}}}{\sum_j e^{f_{y_j}}}\right)$$

Example: Let's Build (world smallest) Neural Network

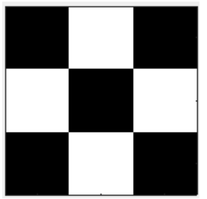
Lets create a neural network that will be able to differentiate (classify) these patterns of simple 3x3 pixel images



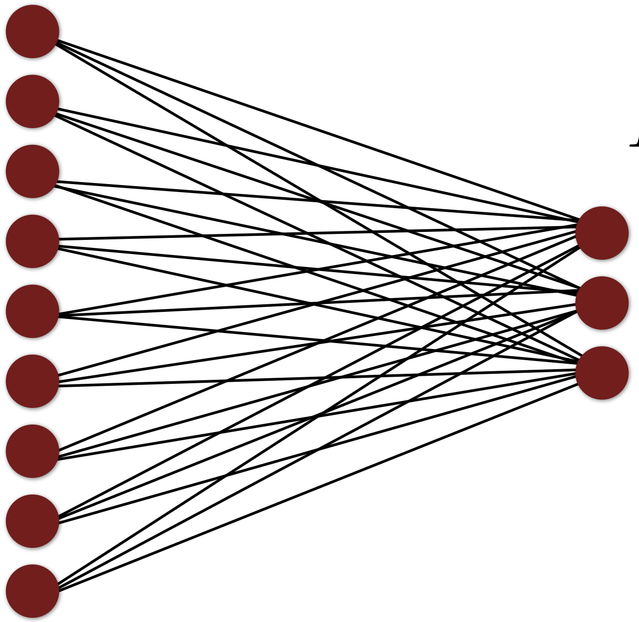
Input Layer



Output Layer

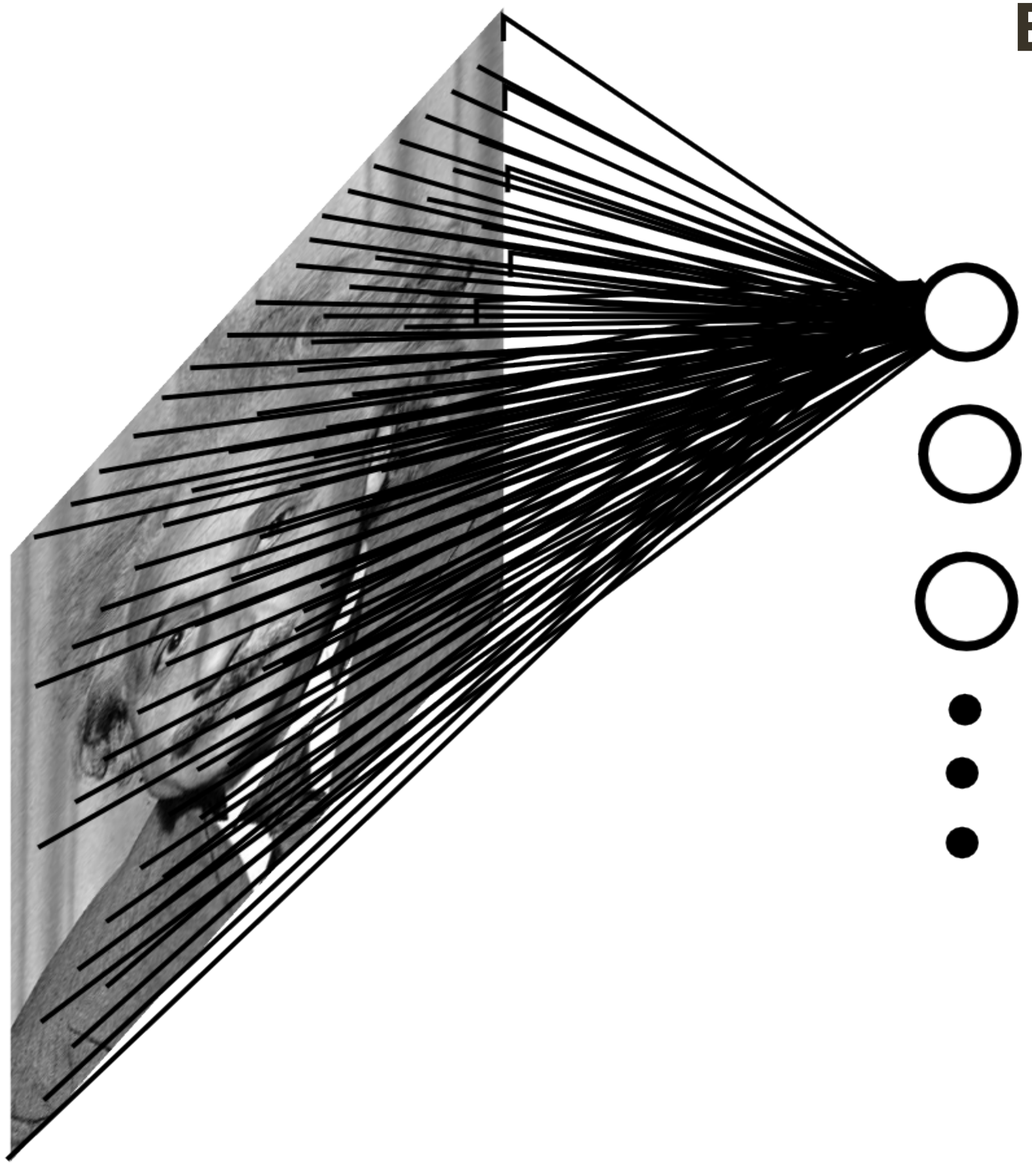


Loss



$$L_1 = -log\left(\frac{e^{\sum_{i=1}^{9}\sigma(w_{1,i}x_i+b_1)}}{\sum_{j=1}^{3}e^{\sum_{i=1}^{9}\sigma(w_{1,i}x_i+b_1)}}\right)$$

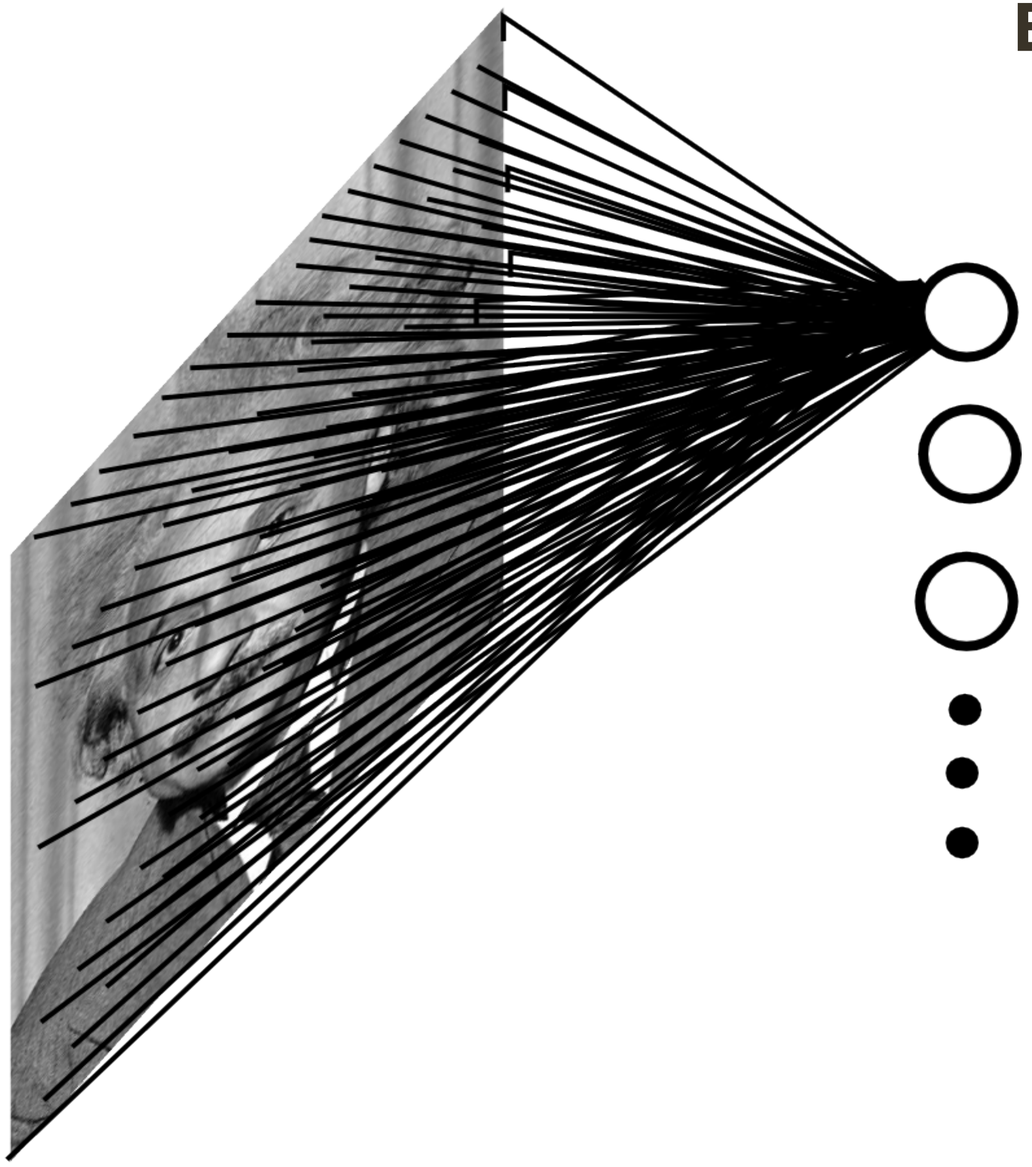
Fully Connected Layer



Example: 200 x 200 image (small)
x 40K hidden units

* slide from Marc'Aurelio Renzato

Fully Connected Layer

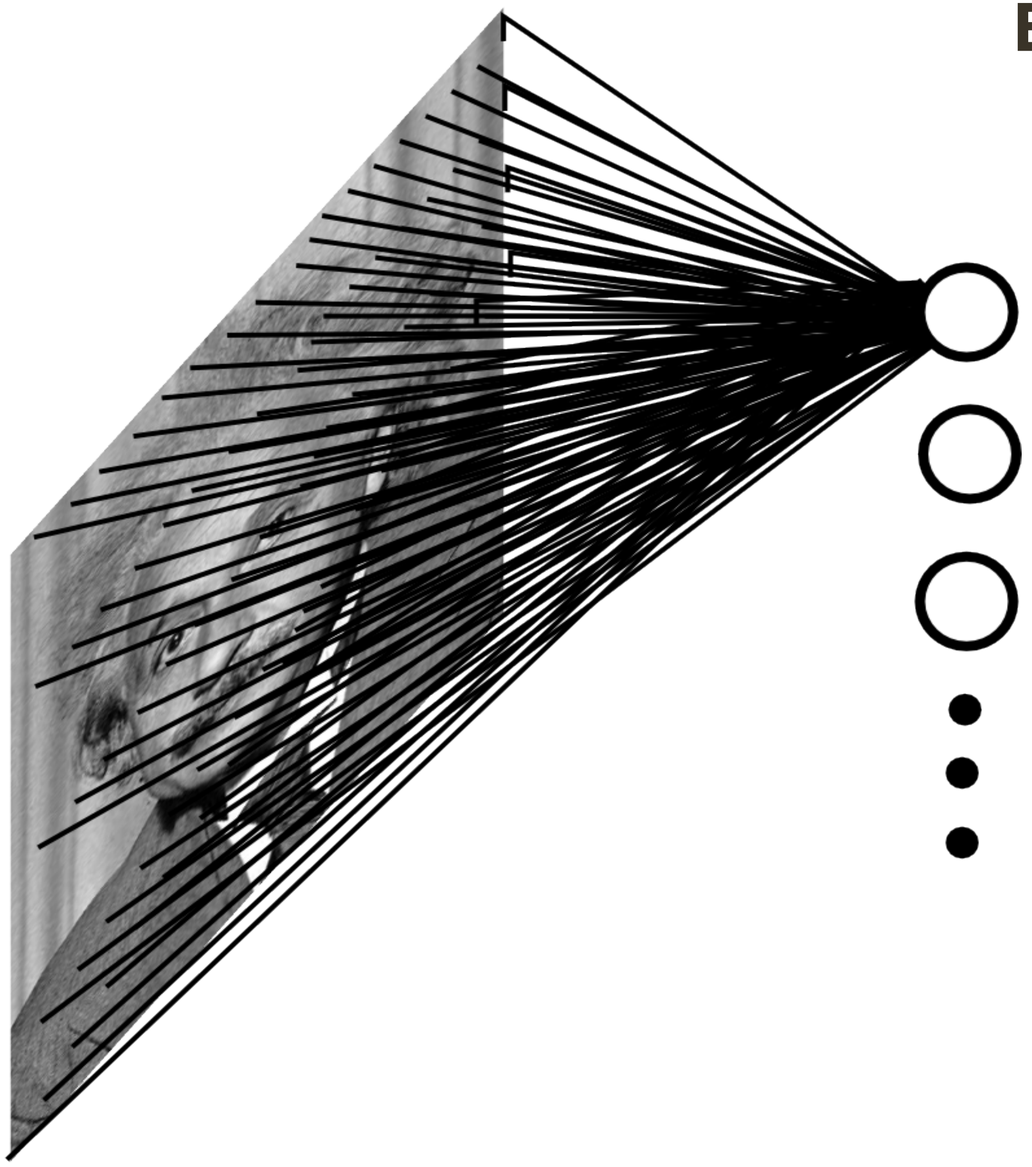


Example: 200 x 200 image (small)
x 40K hidden units

= ~ **2 Billion** parameters (for one layer!)

* slide from Marc'Aurelio Renzato

Fully Connected Layer



Example: 200 x 200 image (small)
x 40K hidden units

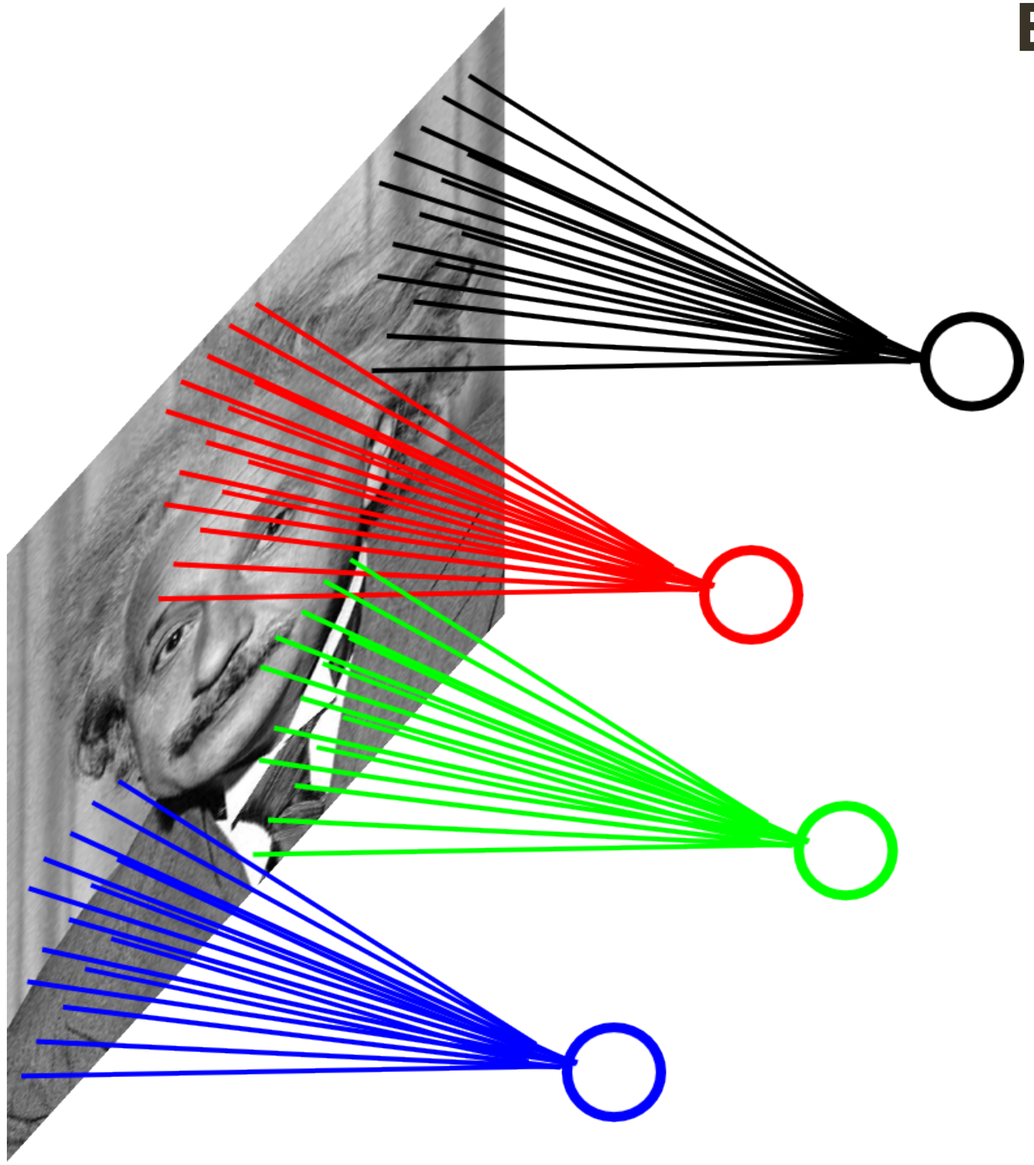
= ~ **2 Billion** parameters (for one layer!)

Spatial correlations are generally local

Waste of resources + we don't have enough data to train networks this large

* slide from Marc'Aurelio Renzato

Locally Connected Layer



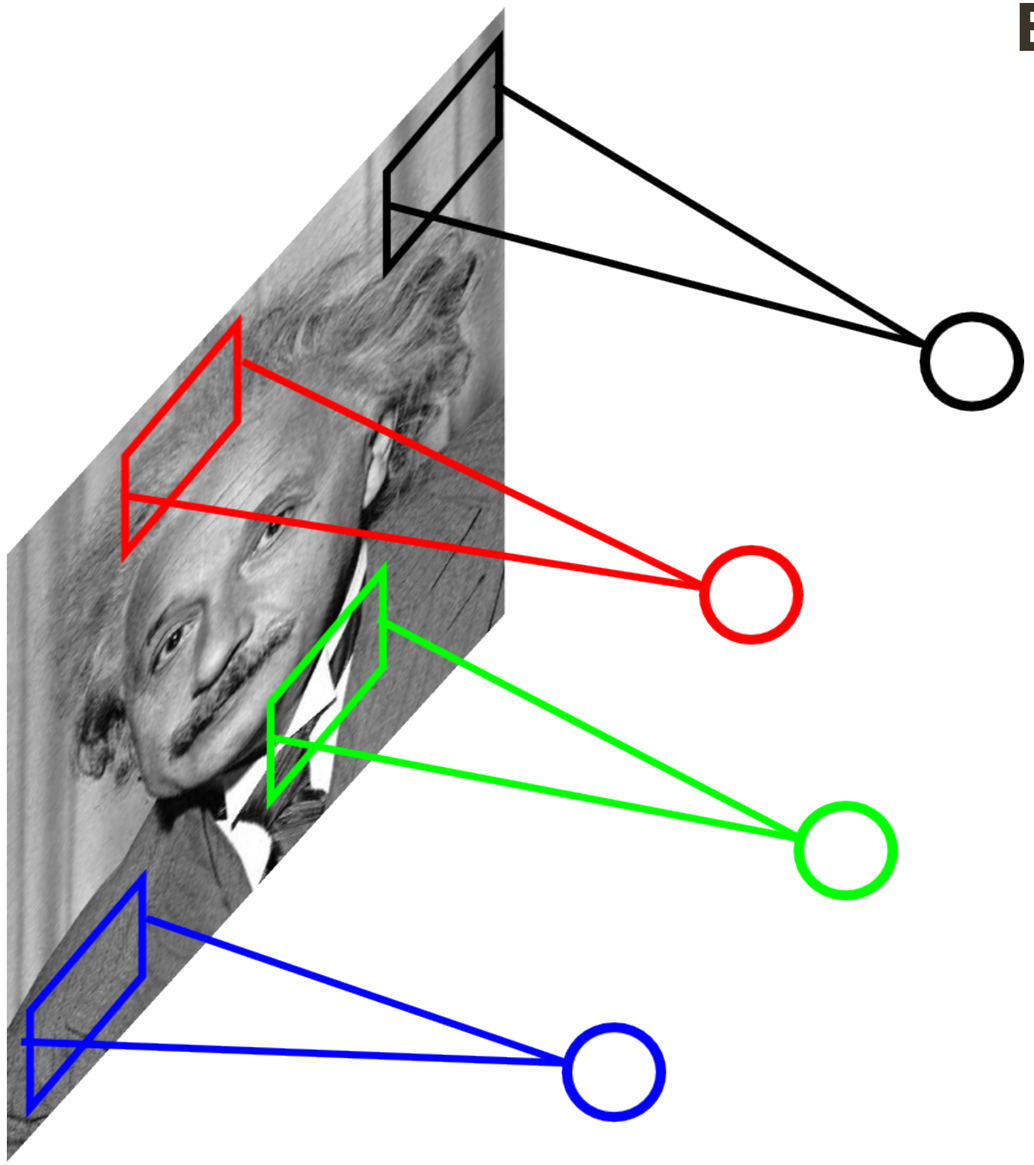
Example: 200 x 200 image (small)
x 40K hidden units

Filter size: 10 x 10

= ~ **4 Million** parameters

* slide from Marc'Aurelio Renzato

Locally Connected Layer



Example: 200 x 200 image (small)
x 40K hidden units

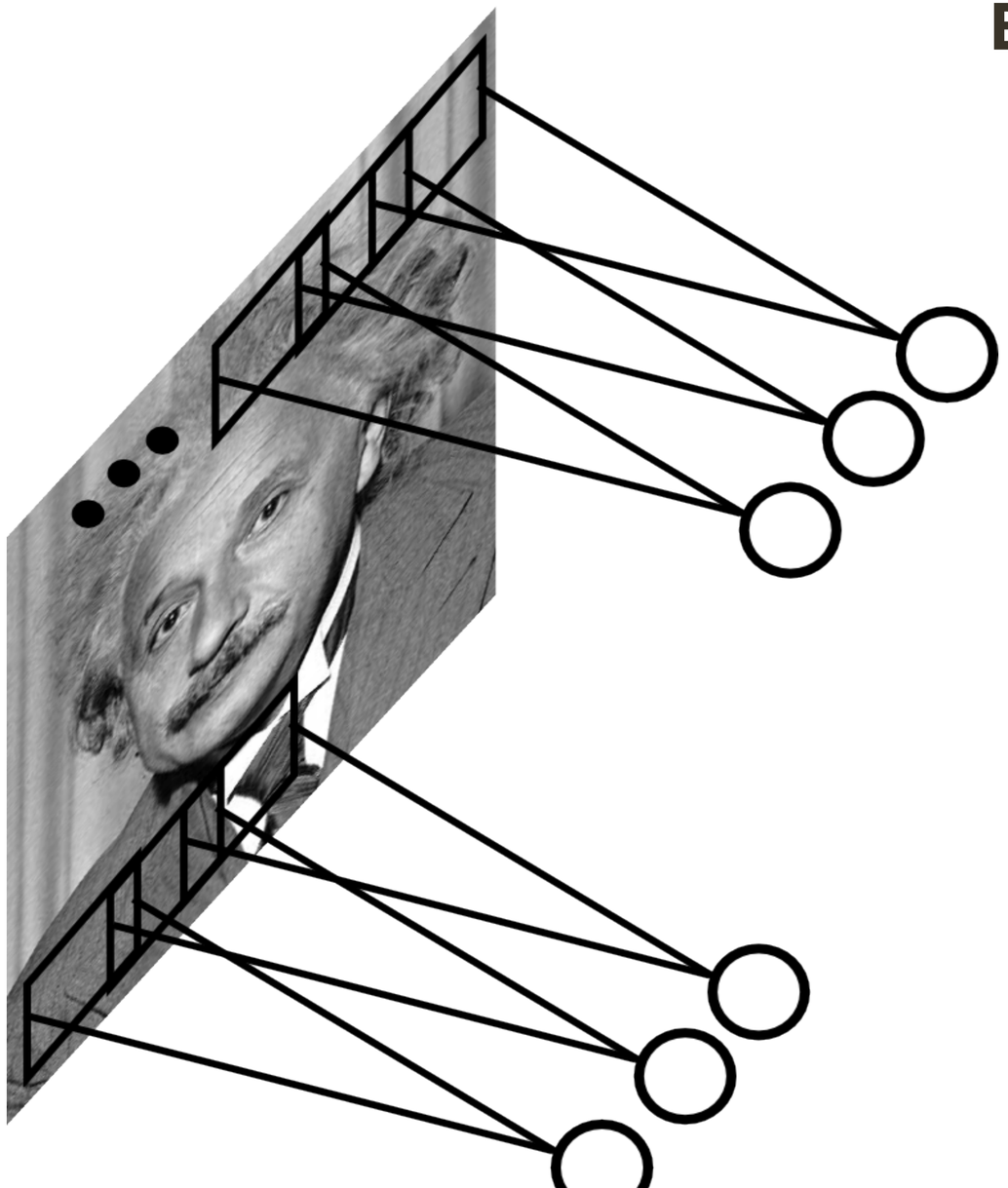
Filter size: 10 x 10

= ~ **4 Million** parameters

Stationarity — statistics is similar at different locations

* slide from Marc'Aurelio Renzato

Convolutional Layer



Example: 200 x 200 image (small)
x 40K hidden units

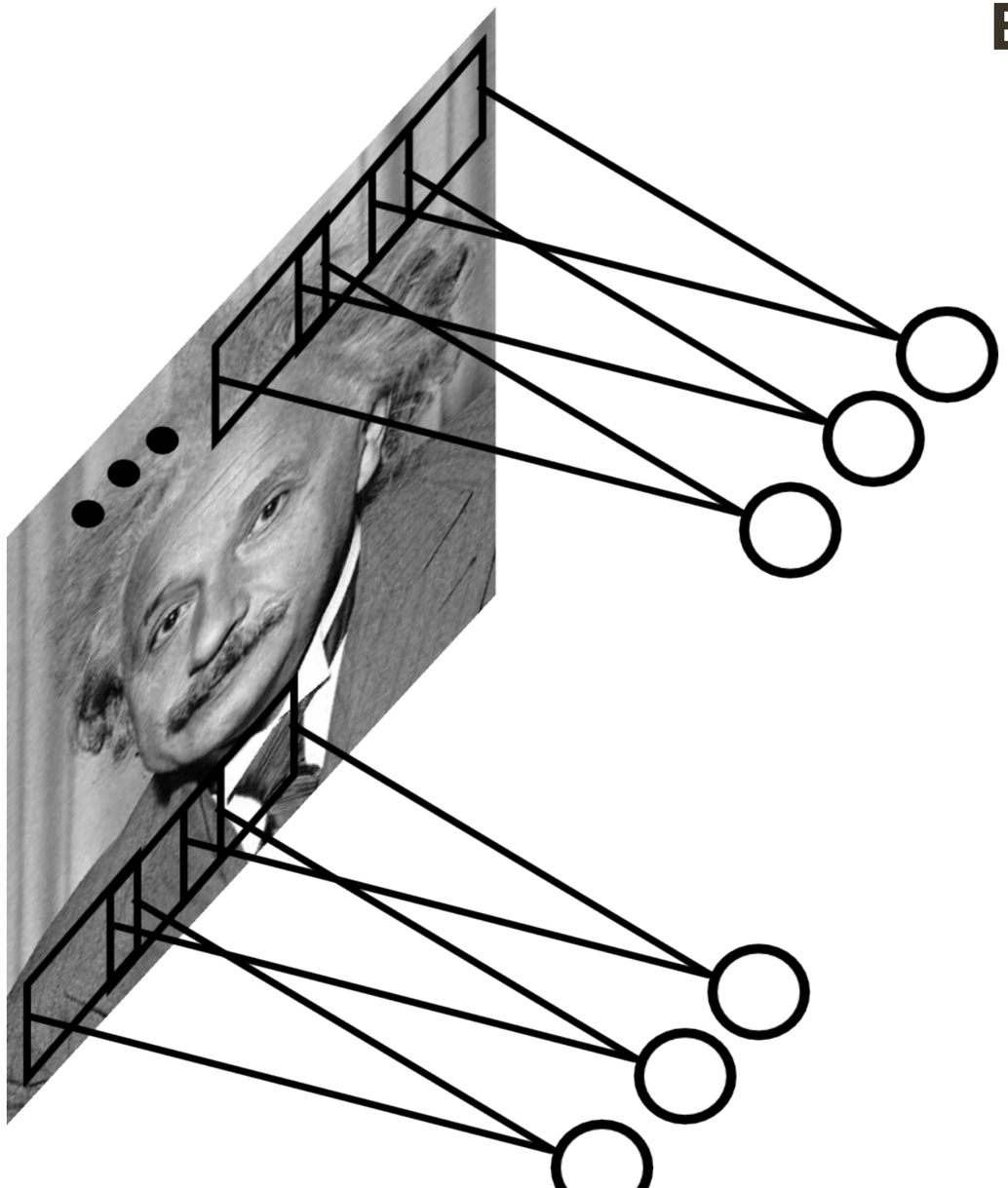
Filter size: 10 x 10

= ~ **4 Million** parameters

Share the same parameters across the locations (assuming input is stationary)

* slide adopted from Marc'Aurelio Renzato

Convolutional Layer



Example: 200 x 200 image (small)
x 40K hidden units

Filter size: 10 x 10

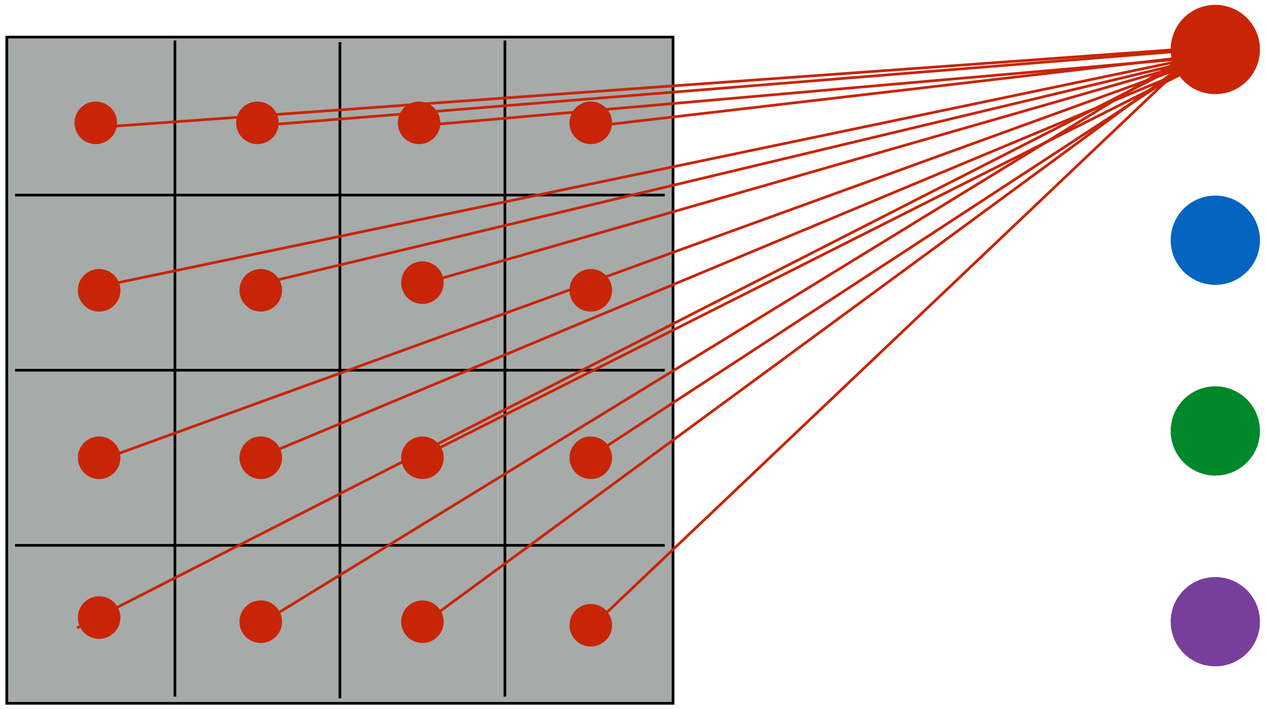
= ~ **4 Million** parameters

= 100+1 parameters

Share the same parameters across the locations (assuming input is stationary)

* slide adopted from Marc'Aurelio Renzato

Fully Connected Layer

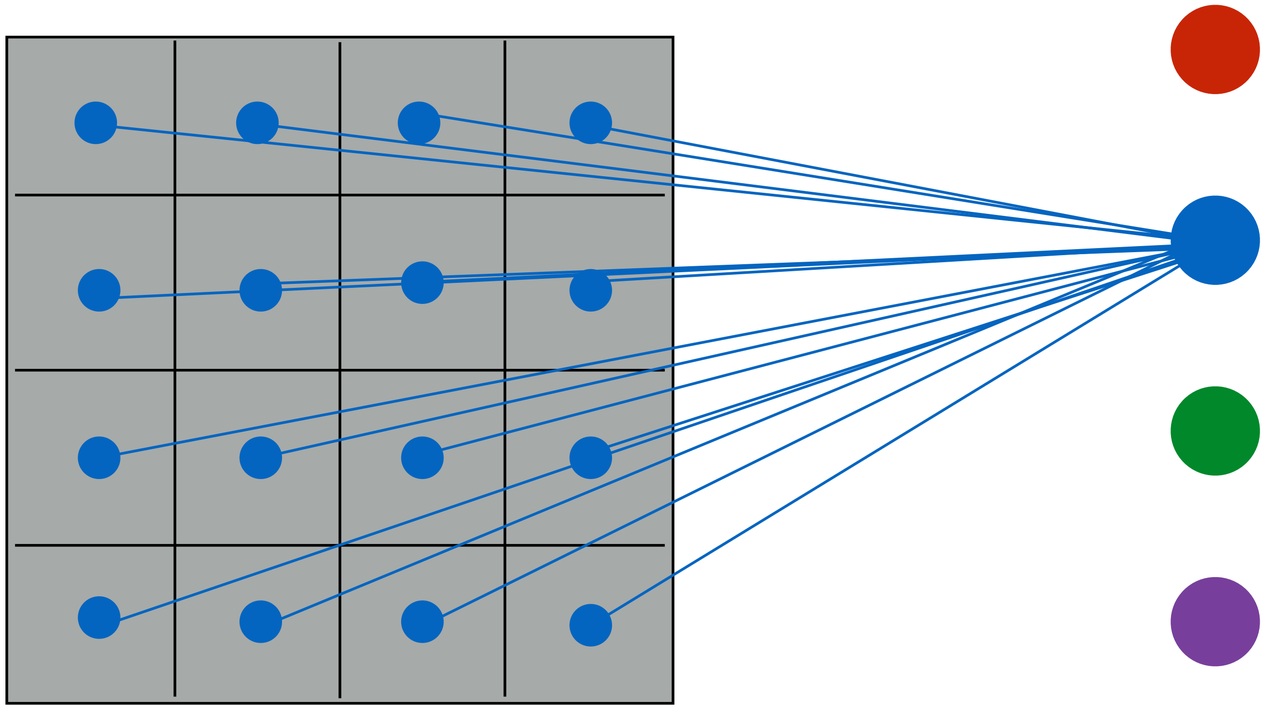


Fully Connected Layer

$$\sigma\left(\sum_{i=1}^{4}\sum_{j=1}^{4}\mathbf{W}_{1,i,j}\mathcal{I}(i,j)+b_1\right)$$

Linear Layer

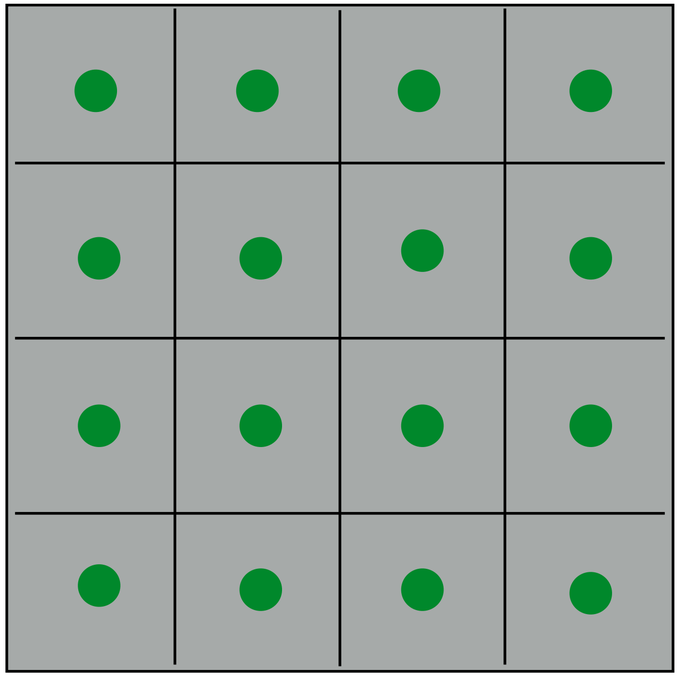
Fully Connected Layer



$$\sigma\left(\sum_{i=1}^{4}\sum_{j=1}^{4}\mathbf{W}_{1,i,j}\mathcal{I}(i,j)+b_1\right)$$

$$\sigma\left(\sum_{i=1}^{4}\sum_{j=1}^{4}\mathbf{W}_{2,i,j}\mathcal{I}(i,j)+b_2\right)$$

Fully Connected Layer

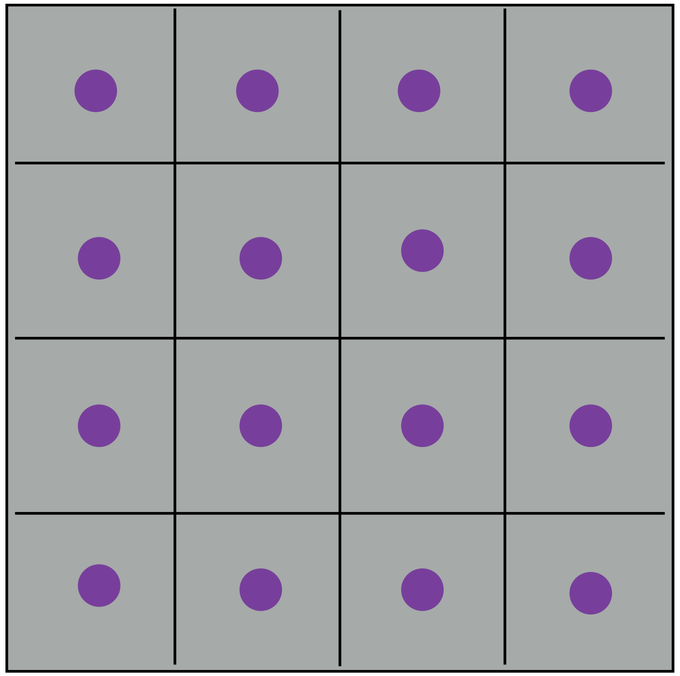


$$\sigma\left(\sum_{i=1}^{4}\sum_{j=1}^{4}\mathbf{W}_{1,i,j}\mathcal{I}(i,j)+b_1\right)$$

$$\sigma\left(\sum_{i=1}^{4}\sum_{j=1}^{4}\mathbf{W}_{2,i,j}\mathcal{I}(i,j)+b_2\right)$$

$$\sigma\left(\sum_{i=1}^{4}\sum_{j=1}^{4}\mathbf{W}_{3,i,j}\mathcal{I}(i,j)+b_3\right)$$

Fully Connected Layer



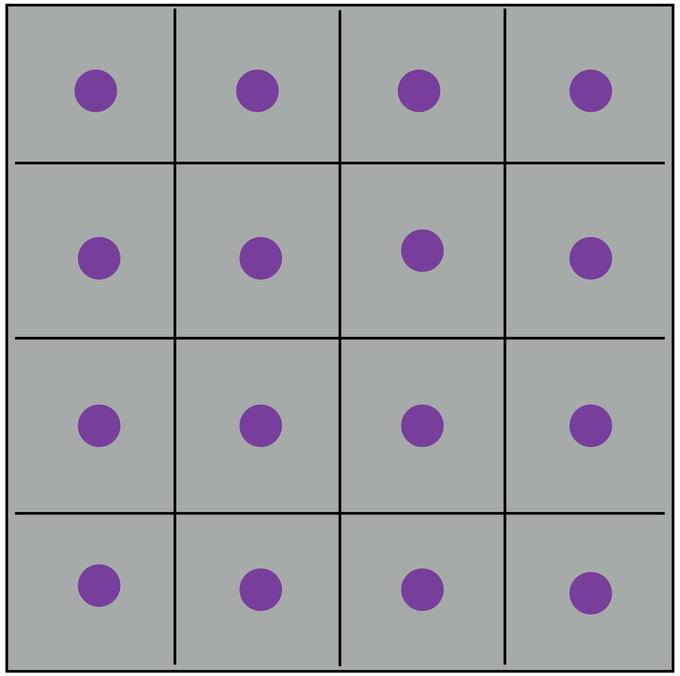
$$\sigma\left(\sum_{i=1}^{4}\sum_{j=1}^{4}\mathbf{W}_{1,i,j}\mathcal{I}(i,j)+b_1\right)$$

$$\sigma\left(\sum_{i=1}^{4}\sum_{j=1}^{4}\mathbf{W}_{2,i,j}\mathcal{I}(i,j)+b_2\right)$$

$$\sigma\left(\sum_{i=1}^{4}\sum_{j=1}^{4}\mathbf{W}_{3,i,j}\mathcal{I}(i,j)+b_3\right)$$

$$\sigma\left(\sum_{i=1}^{4}\sum_{j=1}^{4}\mathbf{W}_{4,i,j}\mathcal{I}(i,j)+b_4\right)$$

Fully Connected Layer



$$\sigma\left(\sum_{i=1}^{4}\sum_{j=1}^{4}\mathbf{W}_{1,i,j}\mathcal{I}(i,j)+b_1\right)$$

4 x 4 + 1 = 17

$$\sigma\left(\sum_{i=1}^{4}\sum_{j=1}^{4}\mathbf{W}_{2,i,j}\mathcal{I}(i,j)+b_2\right)$$

4 x 4 + 1 = 17

$$\sigma\left(\sum_{i=1}^{4}\sum_{j=1}^{4}\mathbf{W}_{3,i,j}\mathcal{I}(i,j)+b_3\right)$$

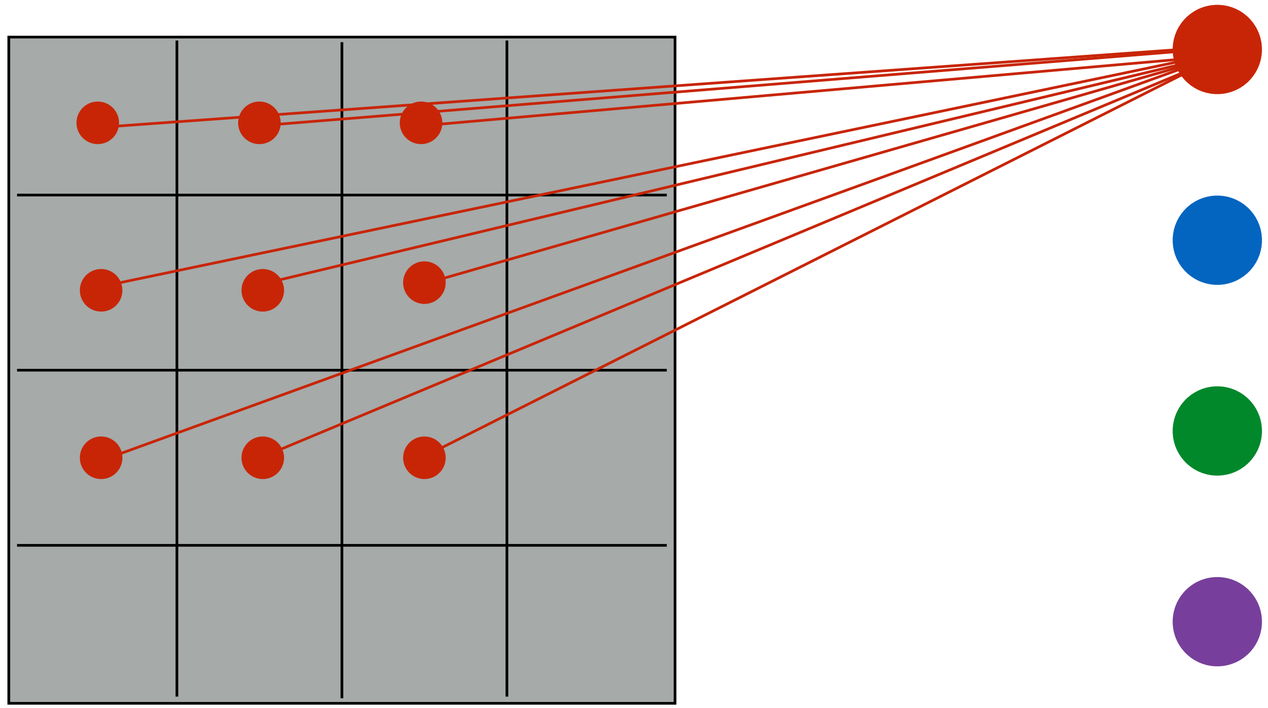
4 x 4 + 1 = 17

$$\sigma\left(\sum_{i=1}^{4}\sum_{j=1}^{4}\mathbf{W}_{4,i,j}\mathcal{I}(i,j)+b_4\right)$$

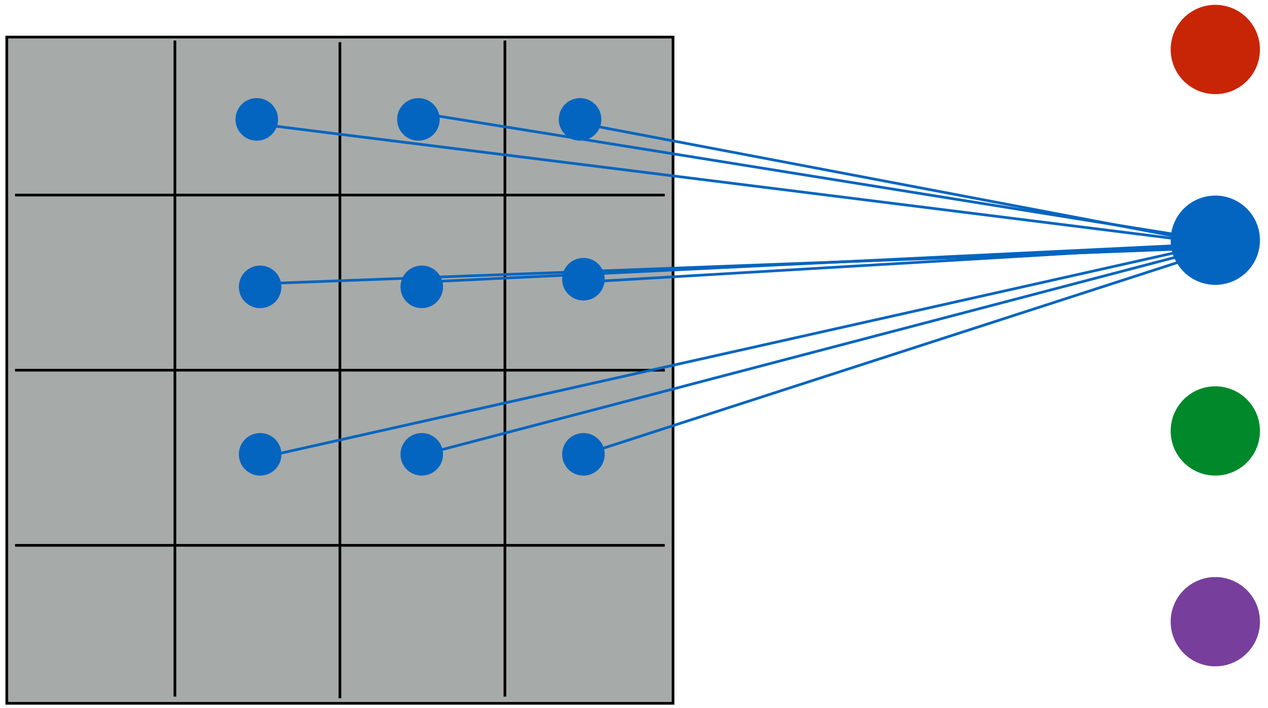
4 x 4 + 1 = 17

Locally Connected Layer

$$\sigma\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{W}_{1,i,j}\mathcal{I}(i,j)+b_1\right)$$



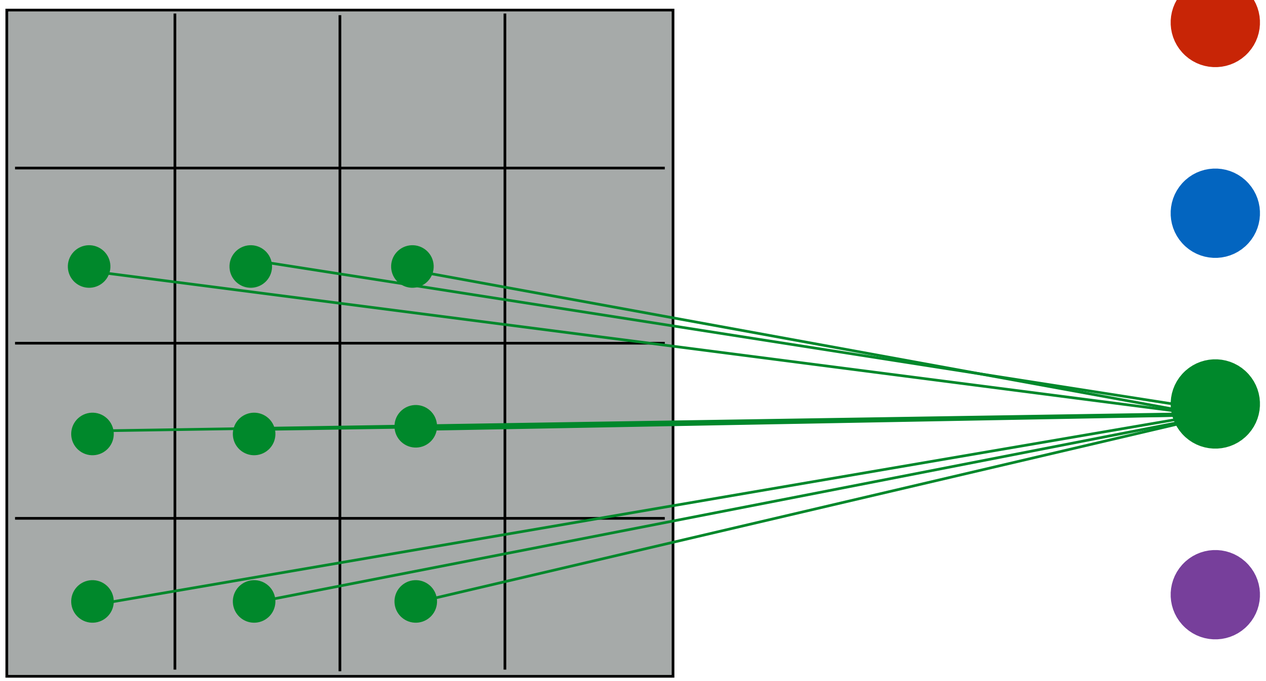
Locally Connected Layer



$$\sigma\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{W}_{1,i,j}\mathcal{I}(i,j)+b_1\right)$$

$$\sigma\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{W}_{2,i,j}\mathcal{I}(i+1,j)+b_2\right)$$

Locally Connected Layer

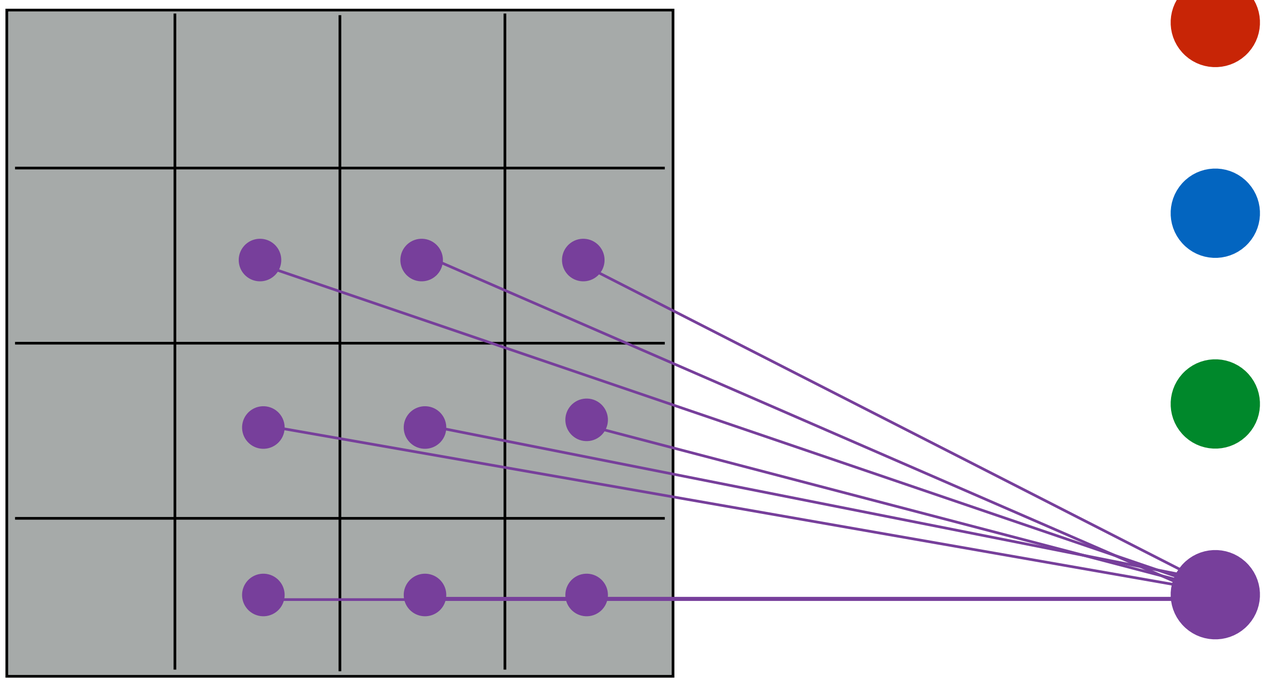


$$\sigma\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{W}_{1,i,j}\mathcal{I}(i,j)+b_1\right)$$

$$\sigma\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{W}_{2,i,j}\mathcal{I}(i+1,j)+b_2\right)$$

$$\sigma\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{W}_{3,i,j}\mathcal{I}(i,j+1)+b_3\right)$$

Locally Connected Layer



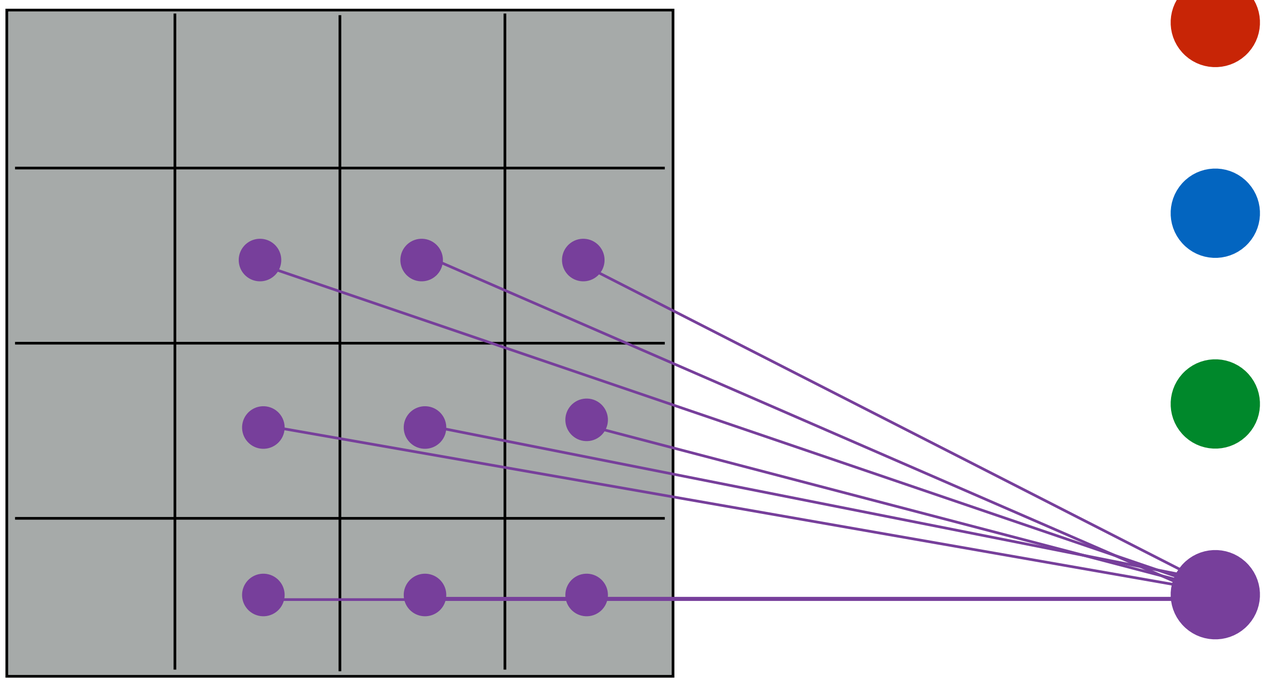
$$\sigma\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{W}_{1,i,j}\mathcal{I}(i,j)+b_1\right)$$

$$\sigma\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{W}_{2,i,j}\mathcal{I}(i+1,j)+b_2\right)$$

$$\sigma\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{W}_{3,i,j}\mathcal{I}(i,j+1)+b_3\right)$$

$$\sigma\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{W}_{4,i,j}\mathcal{I}(i+1,j+1)+b_4\right)$$

Locally Connected Layer



$$\sigma\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{W}_{1,i,j}\mathcal{I}(i,j)+b_1\right)$$

3 x 3 + 1 = 10

$$\sigma\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{W}_{2,i,j}\mathcal{I}(i+1,j)+b_2\right)$$

3 x 3 + 1 = 10

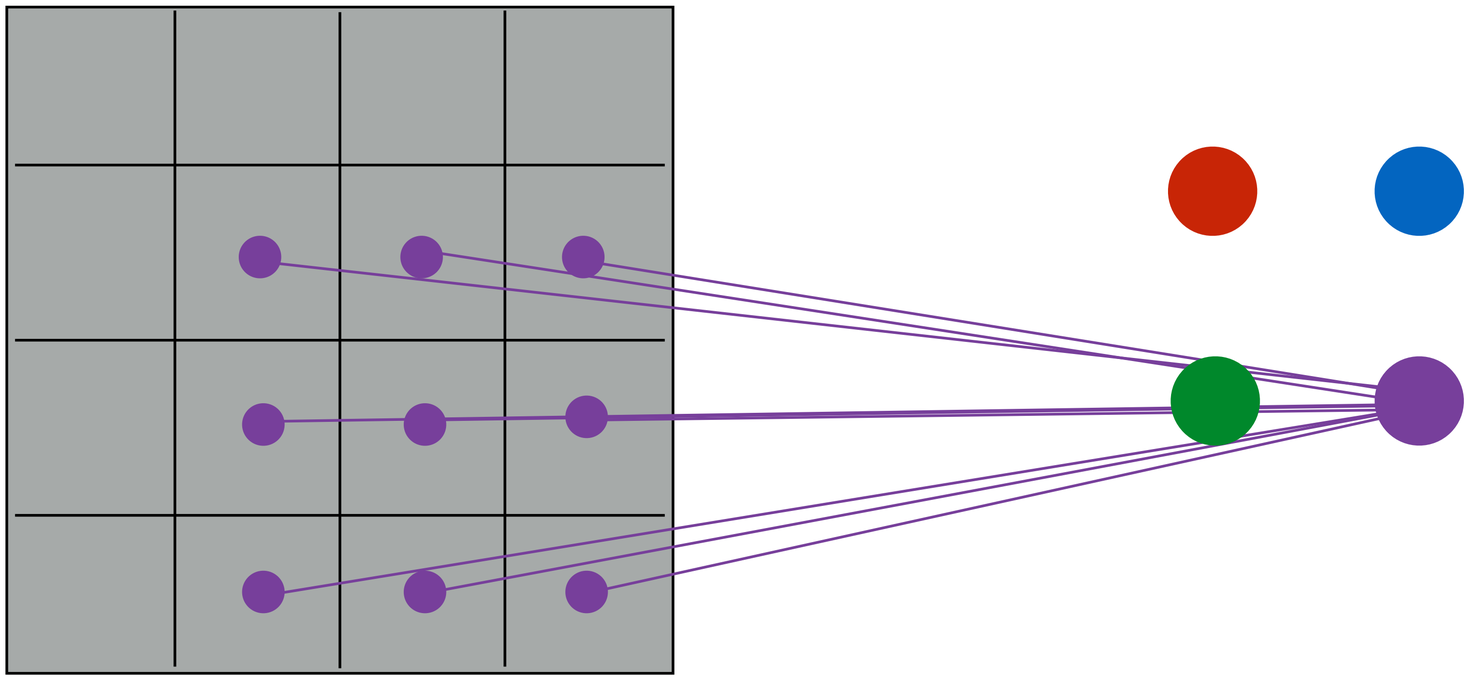
$$\sigma\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{W}_{3,i,j}\mathcal{I}(i,j+1)+b_3\right)$$

3 x 3 + 1 = 10

$$\sigma\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{W}_{4,i,j}\mathcal{I}(i+1,j+1)+b_4\right)$$

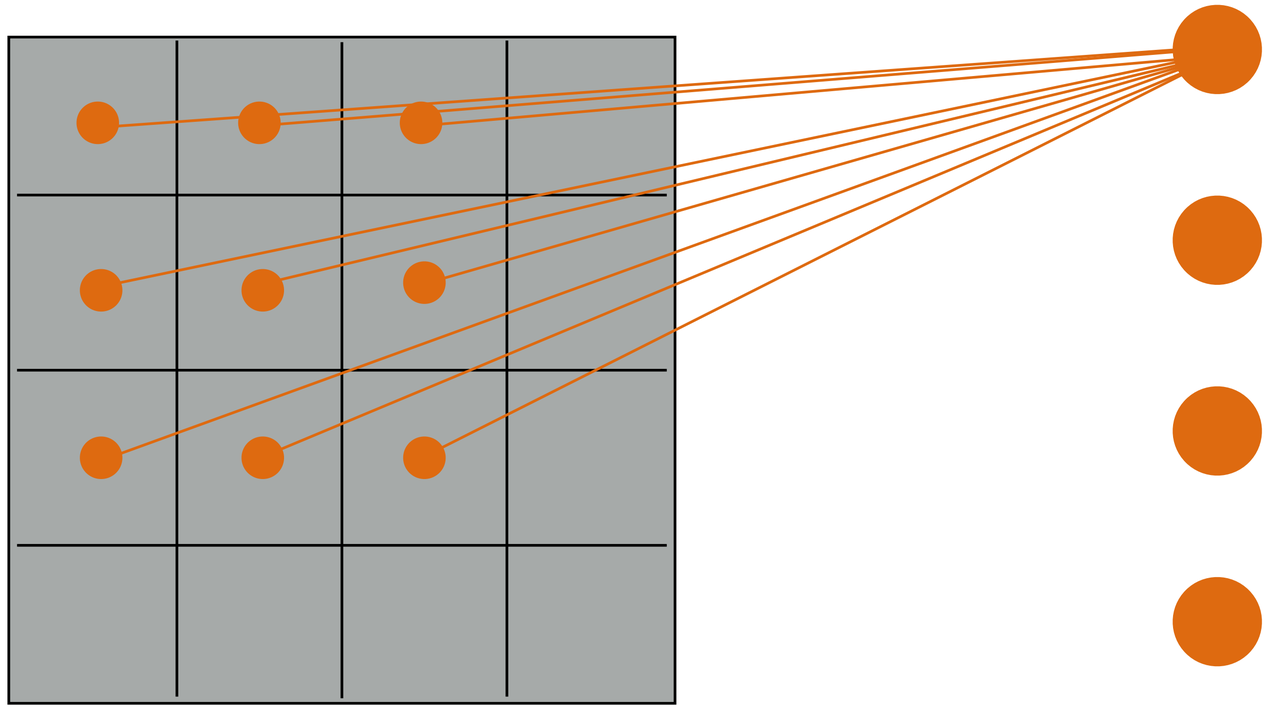
3 x 3 + 1 = 10

Locally Connected Layer

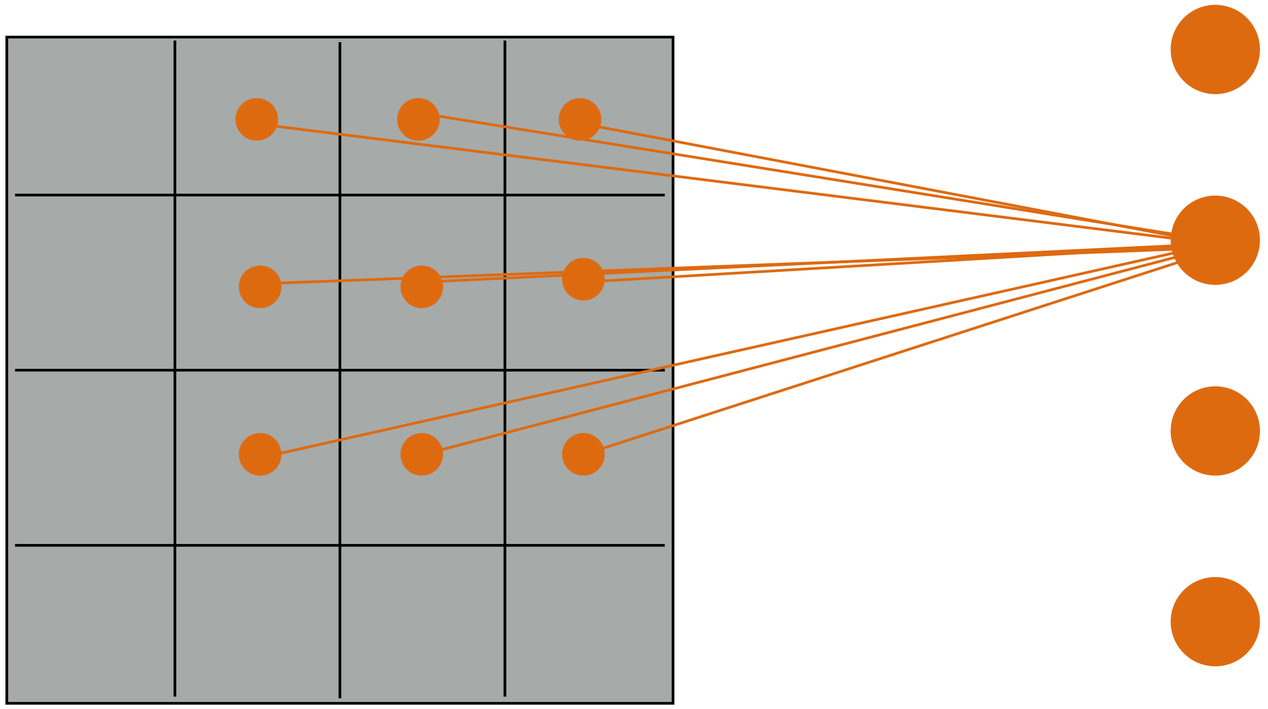


Convolutional Layer

$$\sigma\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{W}_{i,j}\mathcal{I}(i,j)+b\right)$$



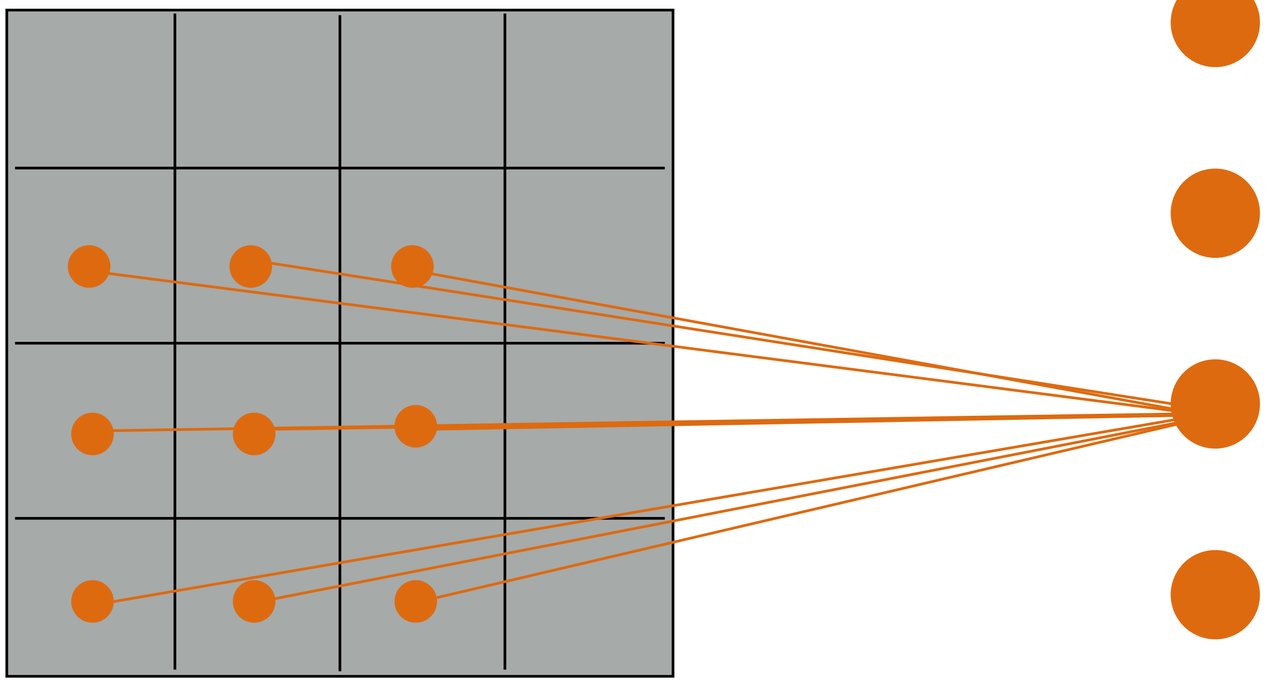
Convolutional Layer



$$\sigma\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{W}_{i,j}\mathcal{I}(i,j)+b\right)$$

$$\sigma\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{W}_{i,j}\mathcal{I}(i+1,j)+b\right)$$

Convolutional Layer

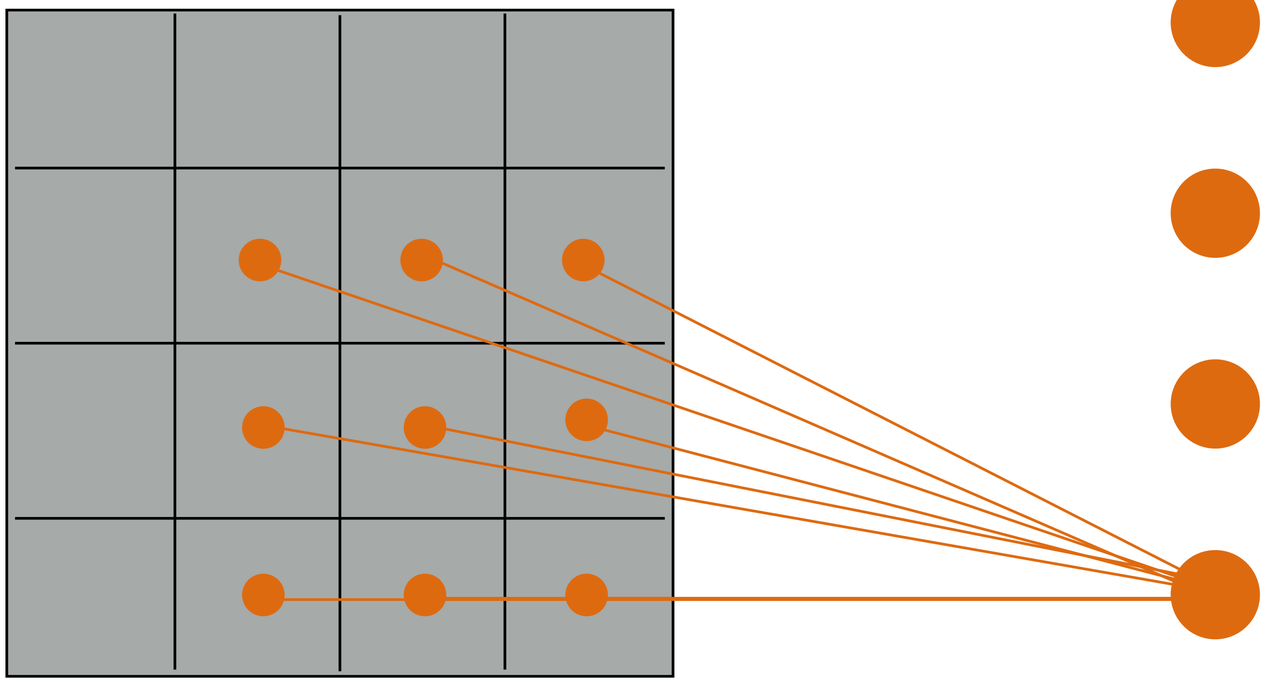


$$\sigma\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{W}_{i,j}\mathcal{I}(i,j)+b\right)$$

$$\sigma\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{W}_{i,j}\mathcal{I}(i+1,j)+b\right)$$

$$\sigma\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{W}_{i,j}\mathcal{I}(i,j+1)+b\right)$$

Convolutional Layer



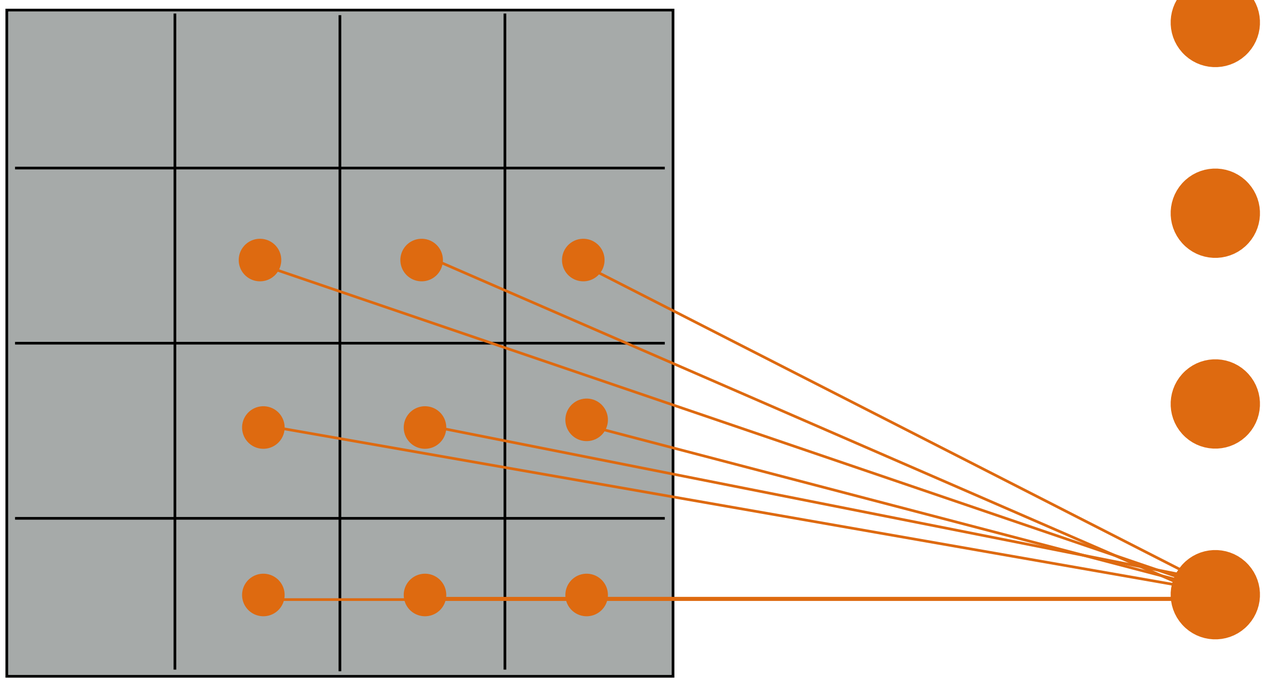
$$\sigma\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{W}_{i,j}\mathcal{I}(i,j)+b\right)$$

$$\sigma\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{W}_{i,j}\mathcal{I}(i+1,j)+b\right)$$

$$\sigma\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{W}_{i,j}\mathcal{I}(i,j+1)+b\right)$$

$$\sigma\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{W}_{i,j}\mathcal{I}(i+1,j+1)+b\right)$$

Convolutional Layer



$$\sigma\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{W}_{i,j}\mathcal{I}(i,j)+b\right)$$

3 x 3 + 1 = 10

$$\sigma\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{W}_{i,j}\mathcal{I}(i+1,j)+b\right)$$

0 x 0 + 0 = 0

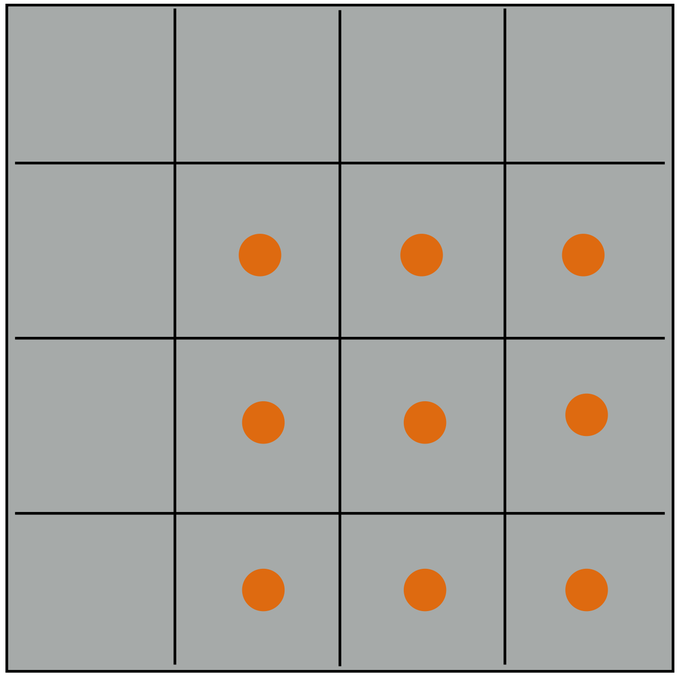
$$\sigma\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{W}_{i,j}\mathcal{I}(i,j+1)+b\right)$$

0 x 0 + 0 = 0

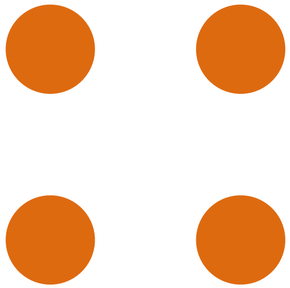
$$\sigma\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{W}_{i,j}\mathcal{I}(i+1,j+1)+b\right)$$

0 x 0 + 0 = 0

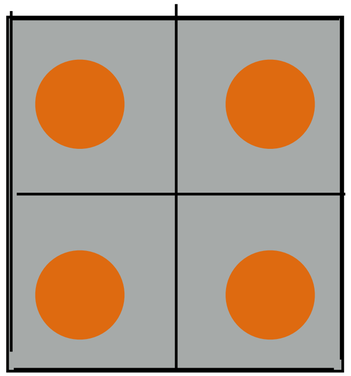
Convolutional Layer: Interpretation #1



Multiple neurons that share weights

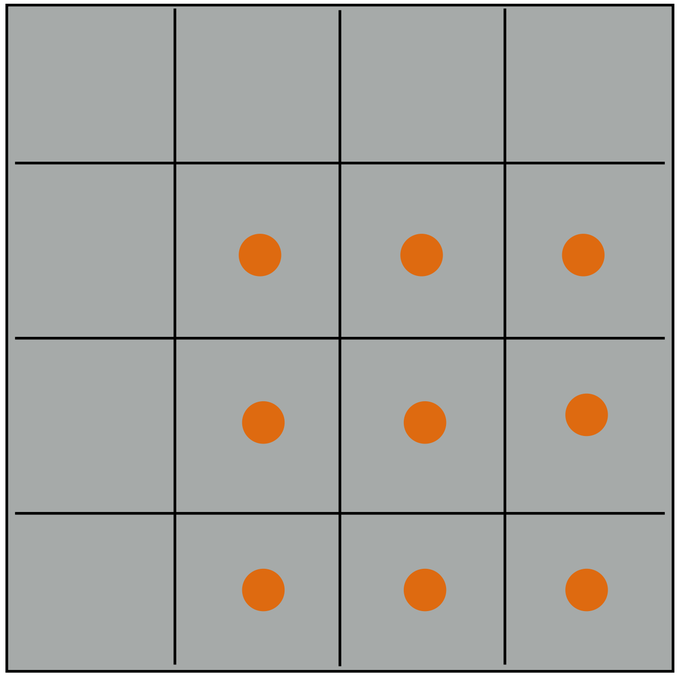


neurons



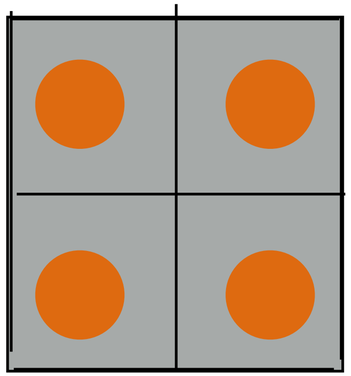
output

Convolutional Layer: Interpretation #2



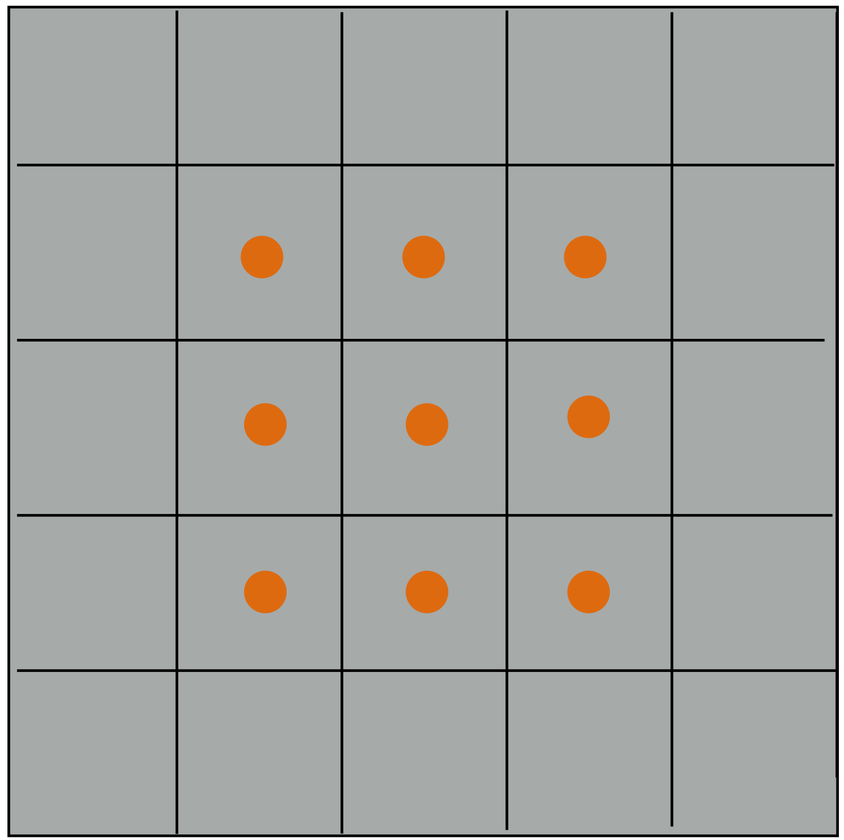
One neuron applied as convolution (by shifting)

neurons



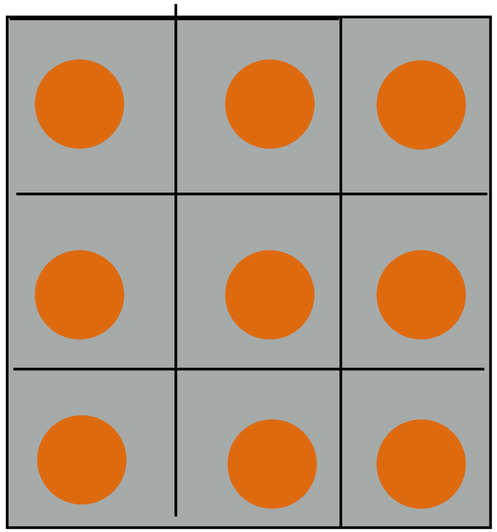
output

Convolutional Layer: Interpretation #2



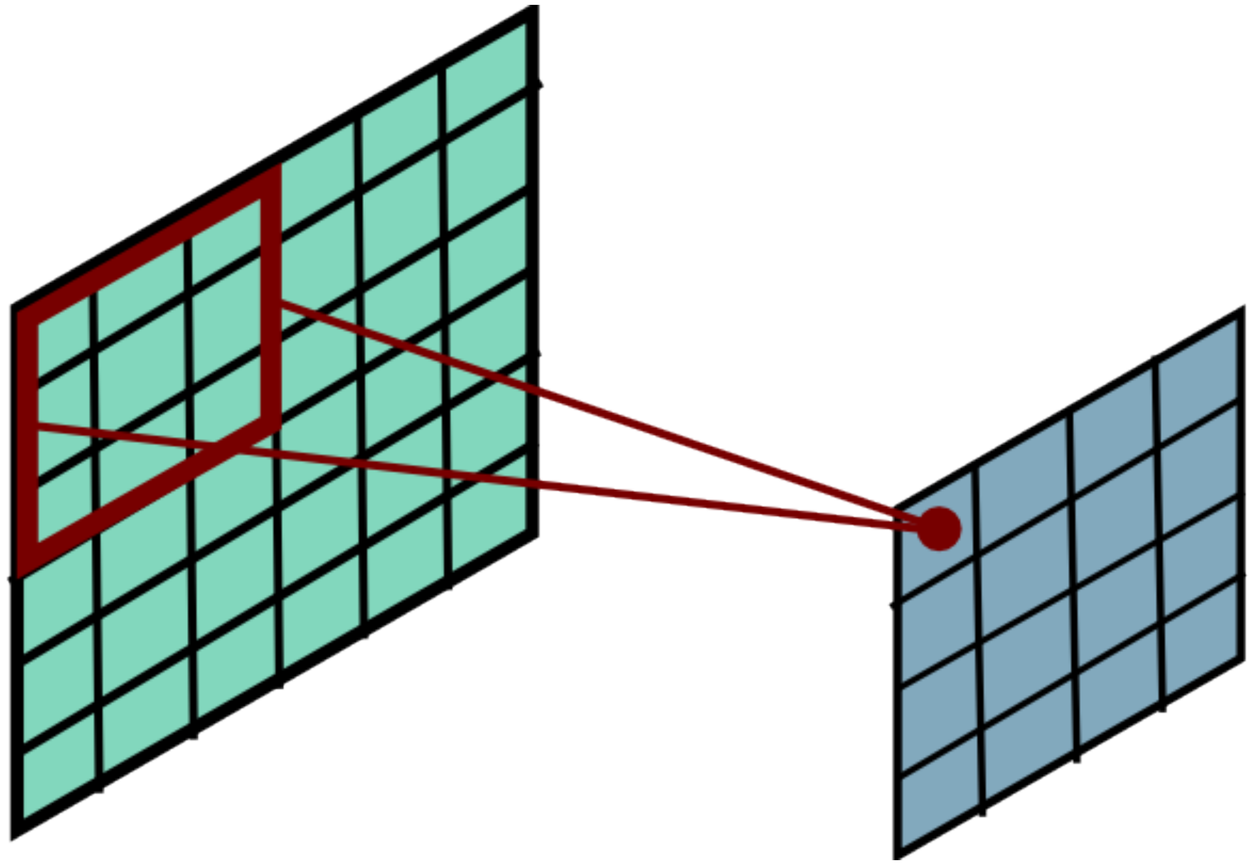
One neuron applied as convolution (by shifting)

neurons



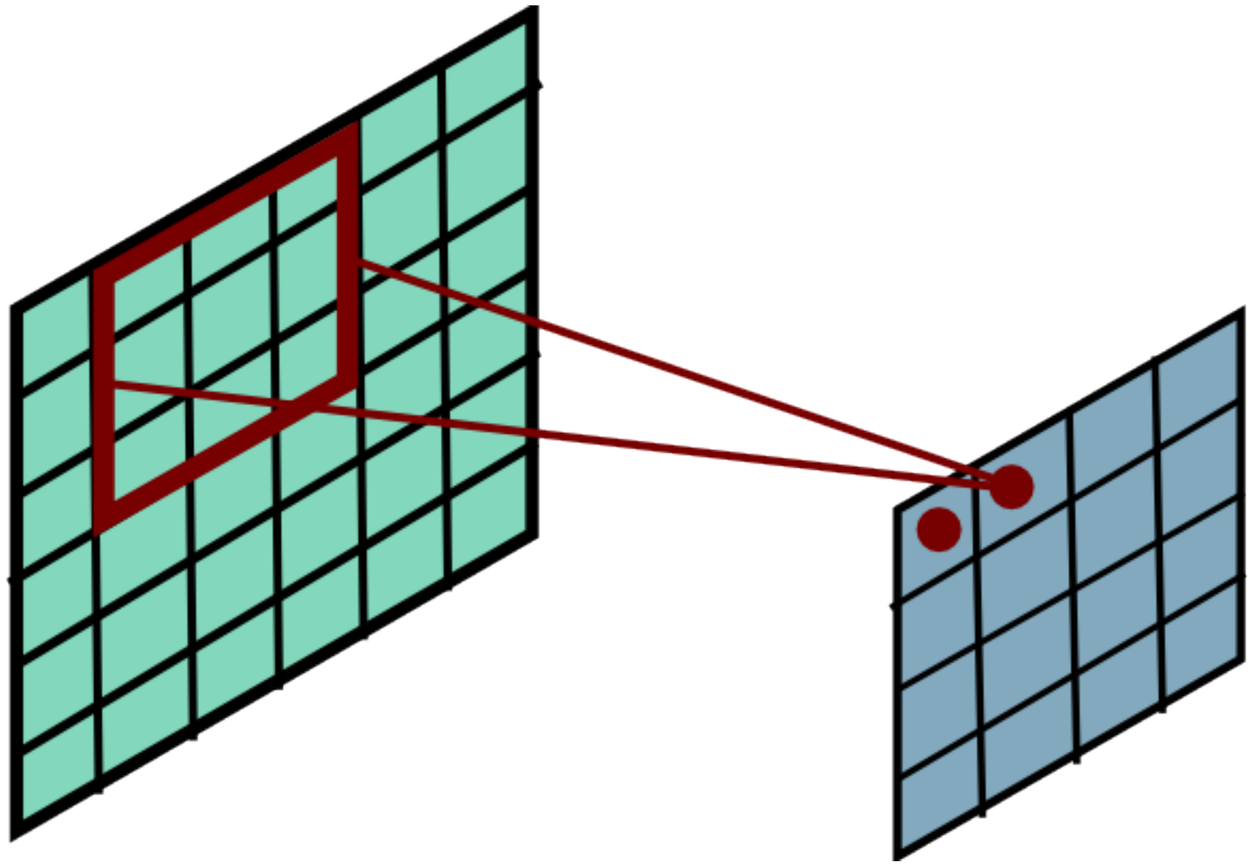
output

Convolutional Layer



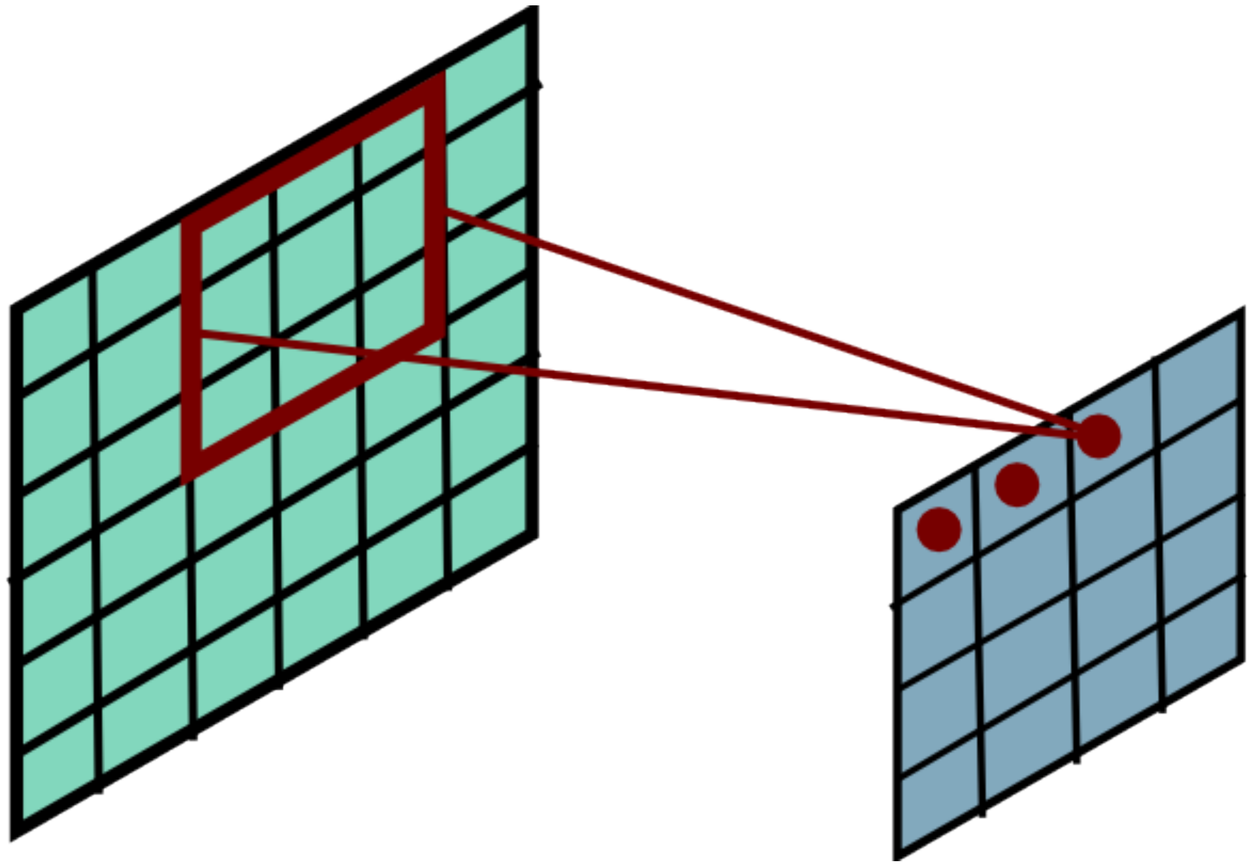
* slide from Marc'Aurelio Renzato

Convolutional Layer



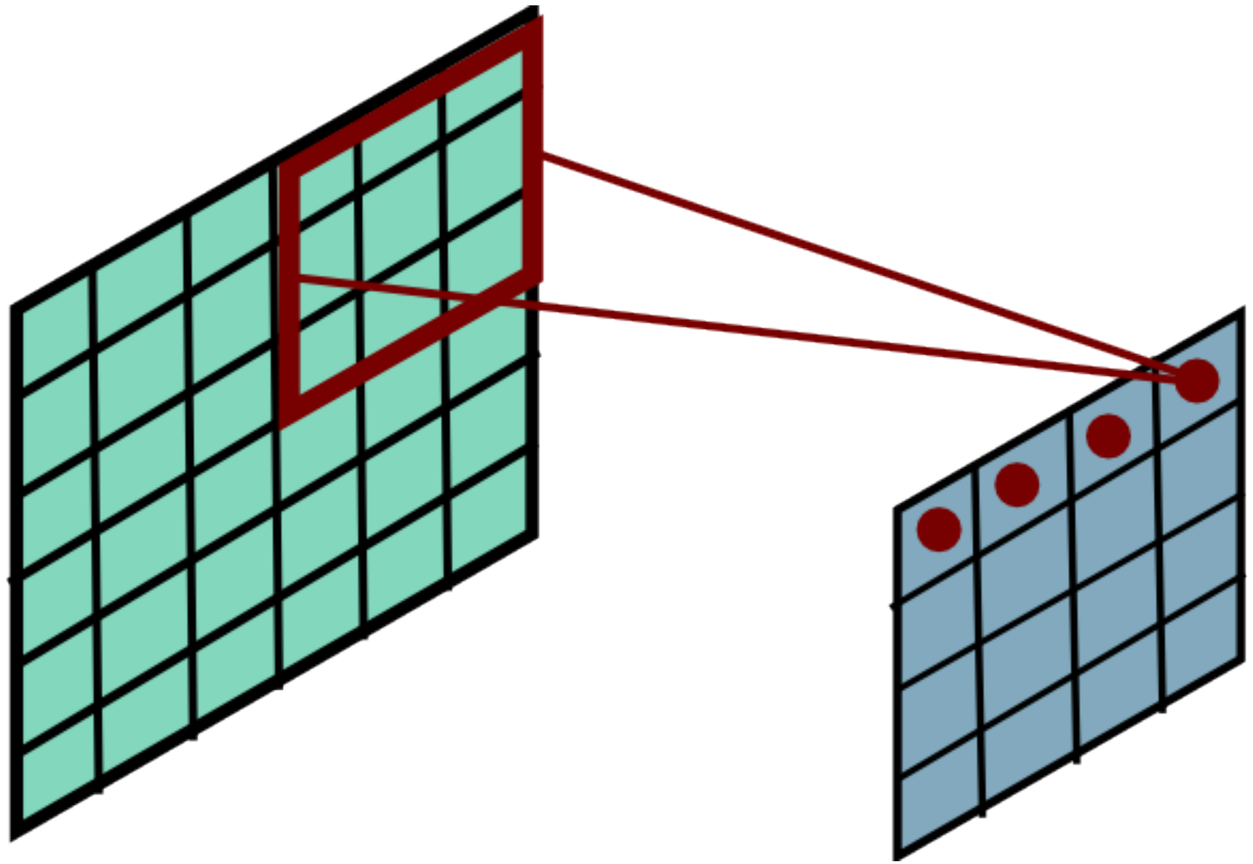
* slide from Marc'Aurelio Renzato

Convolutional Layer



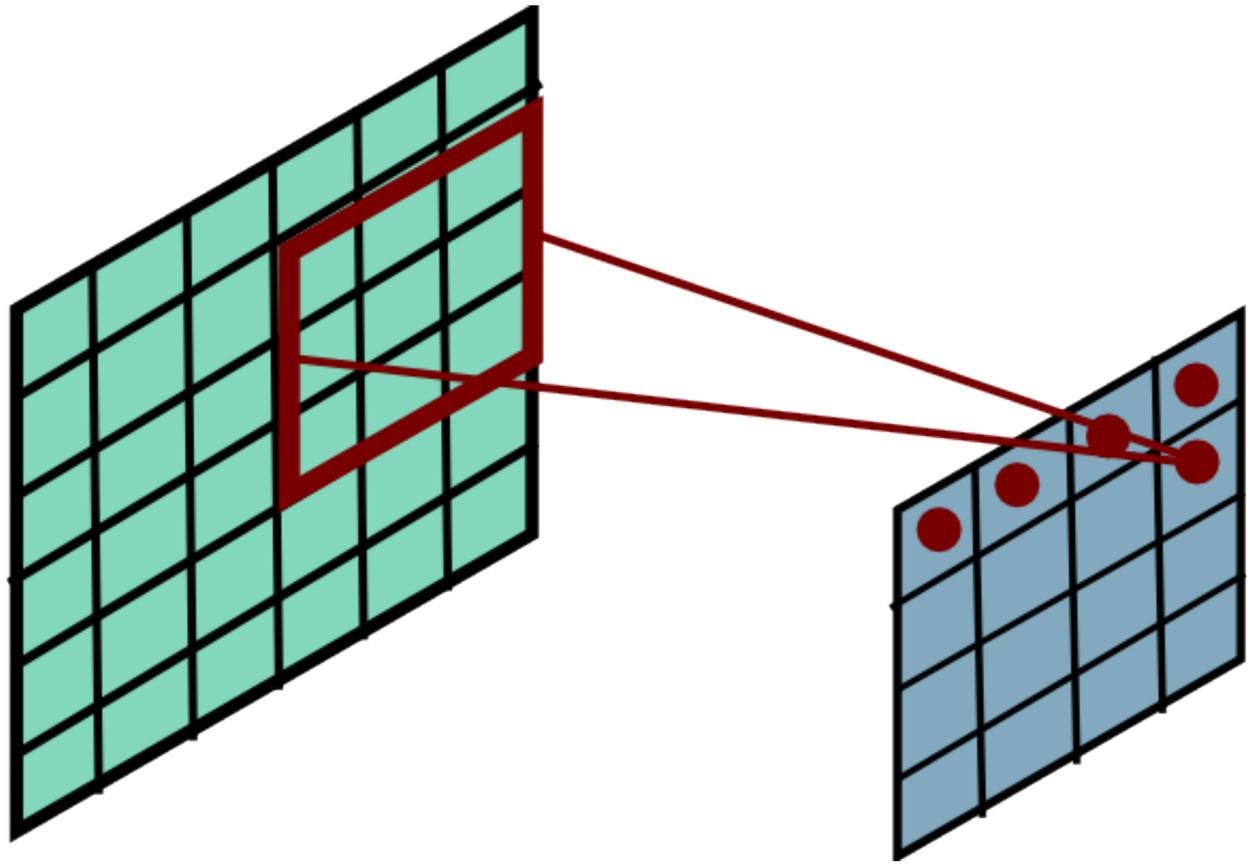
* slide from Marc'Aurelio Renzato

Convolutional Layer



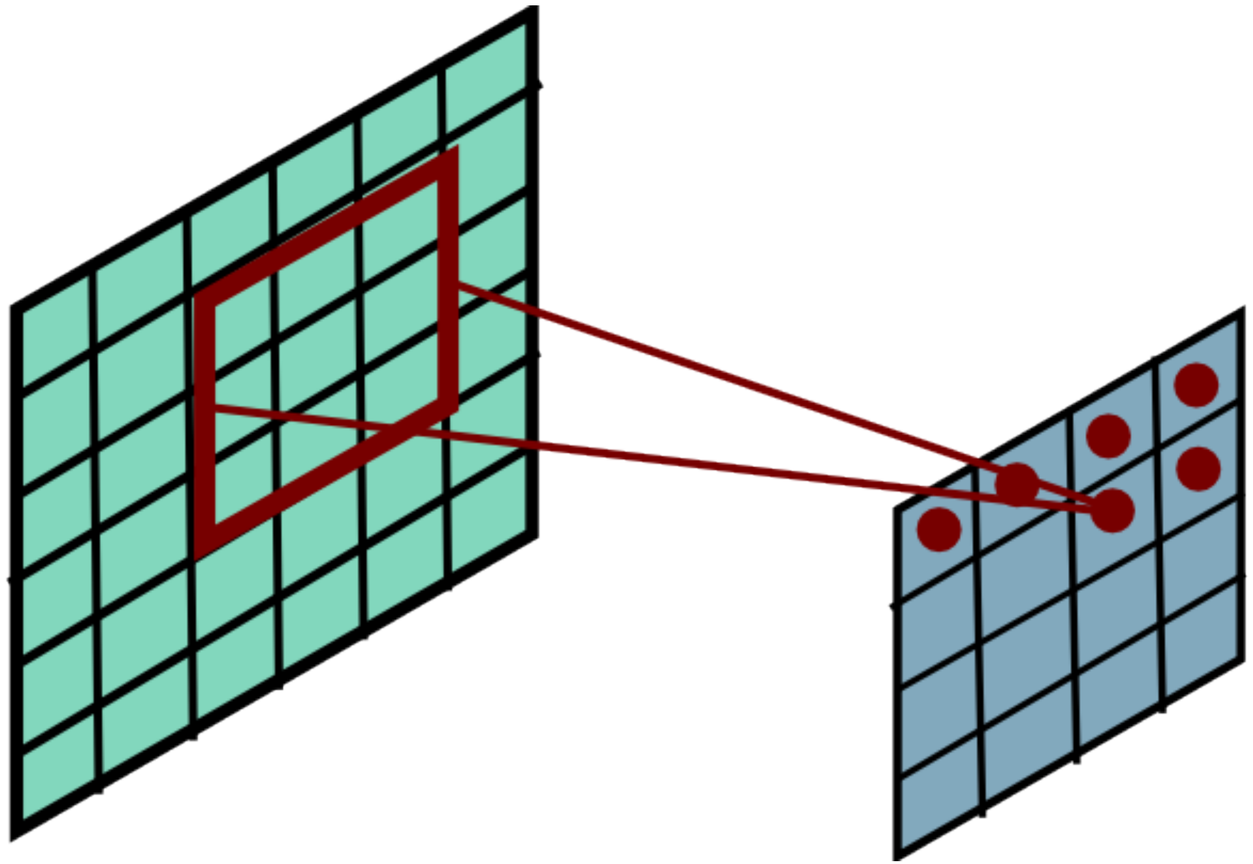
* slide from Marc'Aurelio Renzato

Convolutional Layer



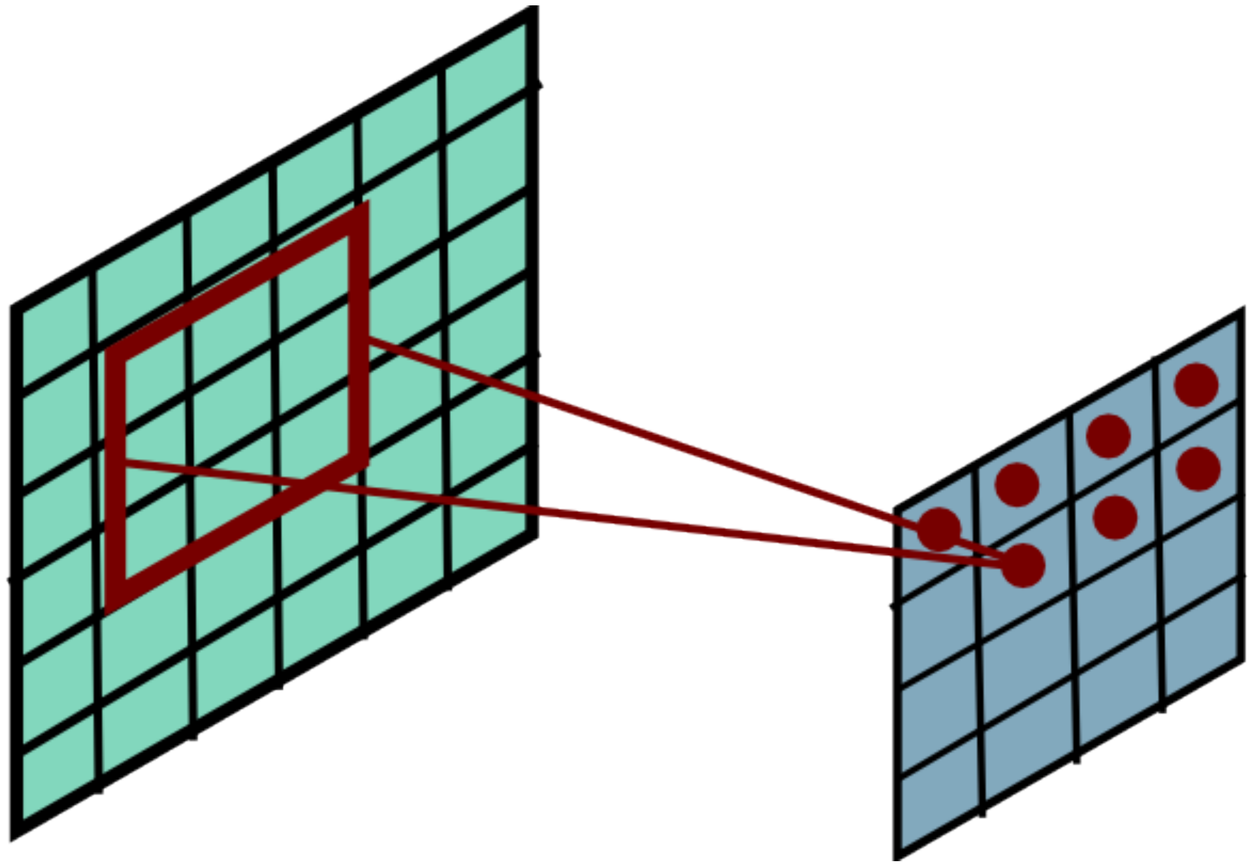
* slide from Marc'Aurelio Renzato

Convolutional Layer



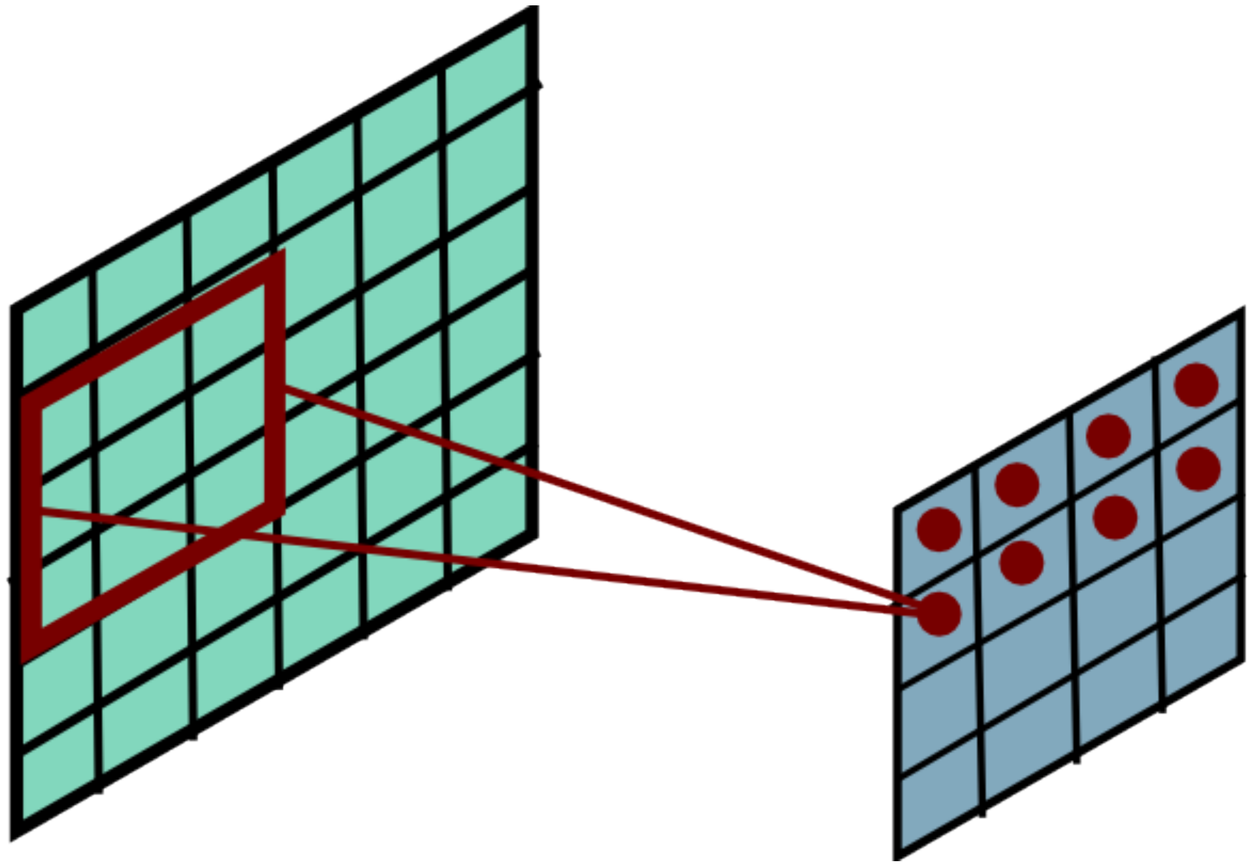
* slide from Marc'Aurelio Renzato

Convolutional Layer



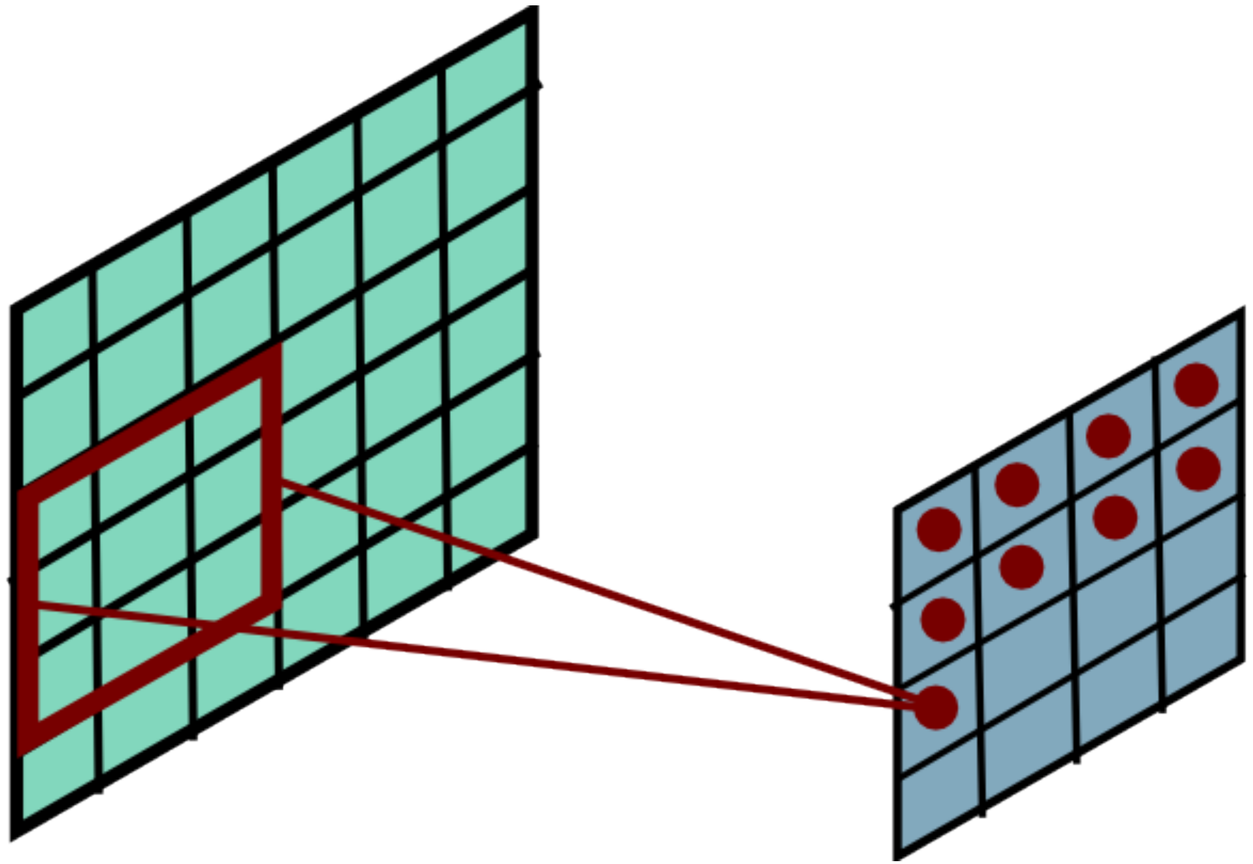
* slide from Marc'Aurelio Renzato

Convolutional Layer



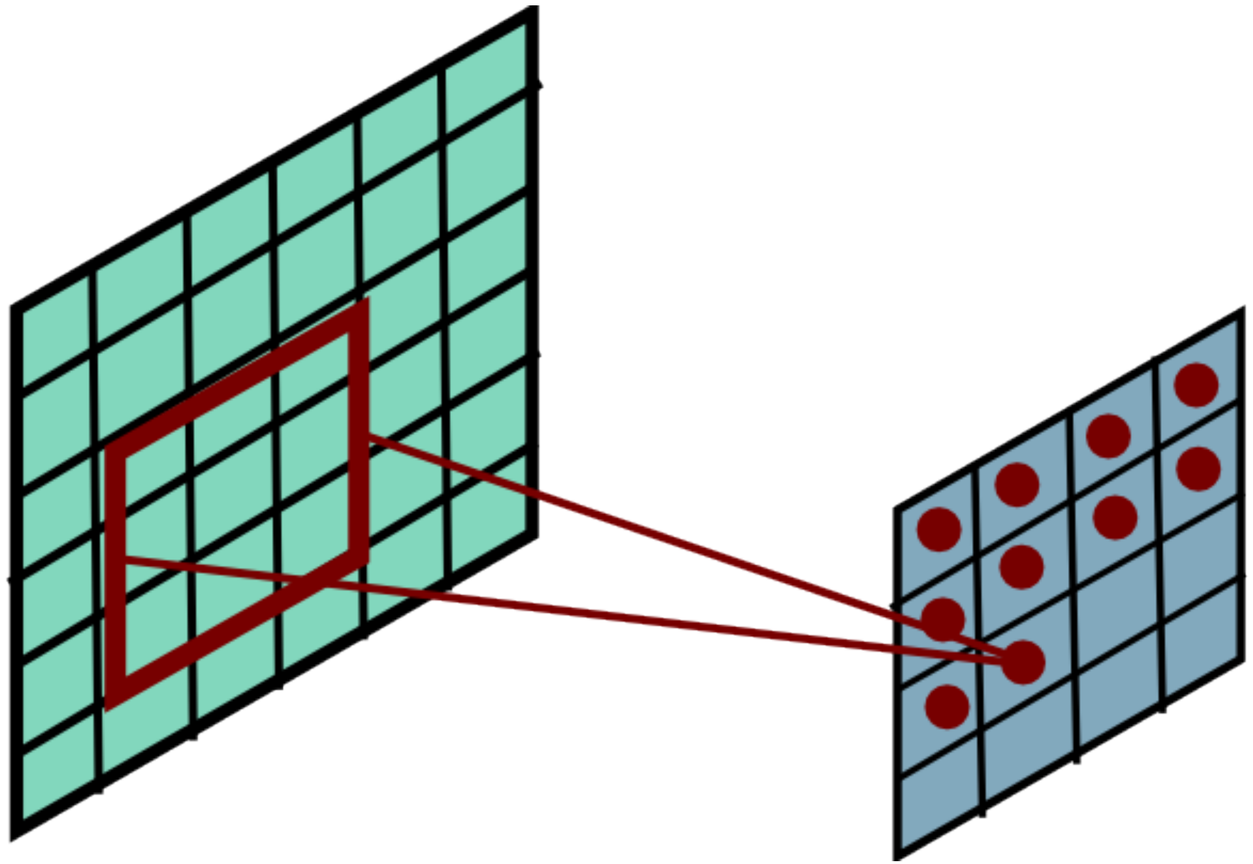
* slide from Marc'Aurelio Renzato

Convolutional Layer



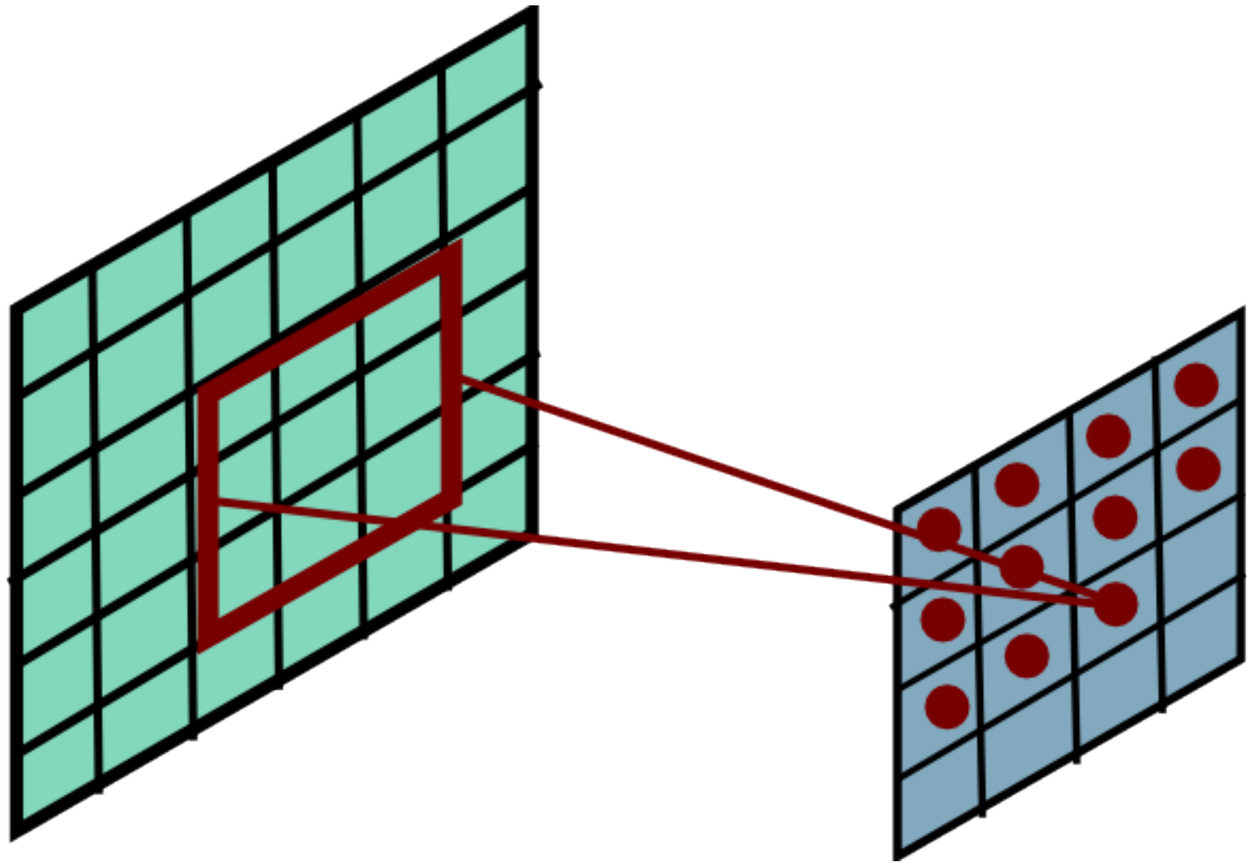
* slide from Marc'Aurelio Renzato

Convolutional Layer



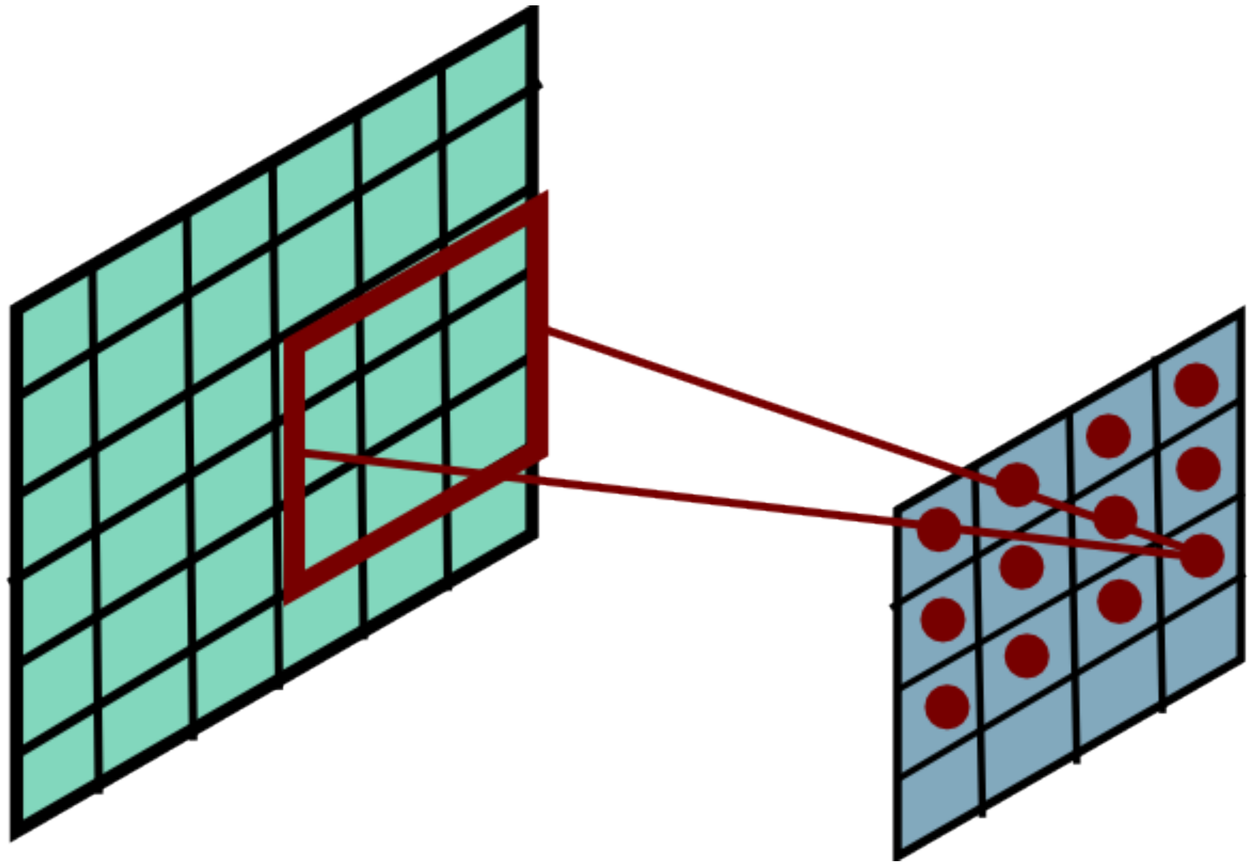
* slide from Marc'Aurelio Renzato

Convolutional Layer



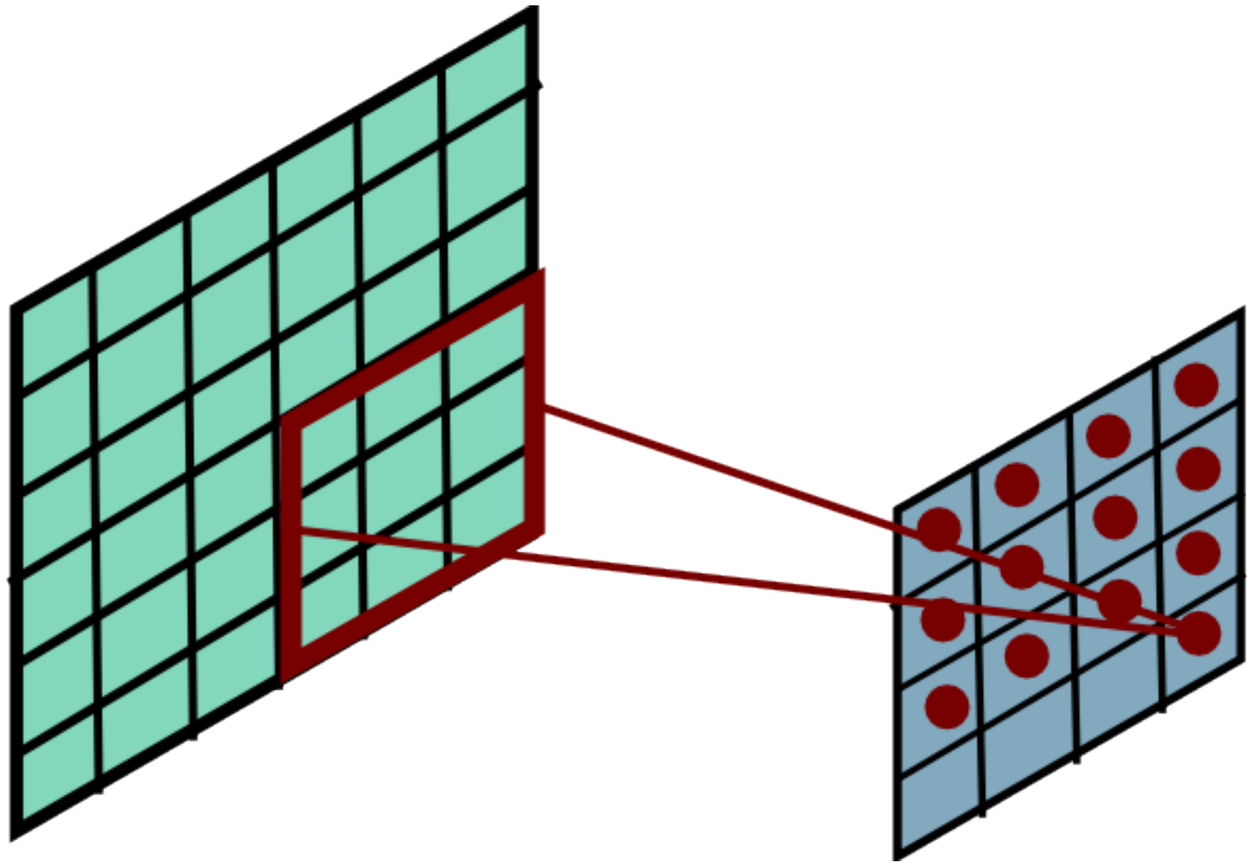
* slide from Marc'Aurelio Renzato

Convolutional Layer



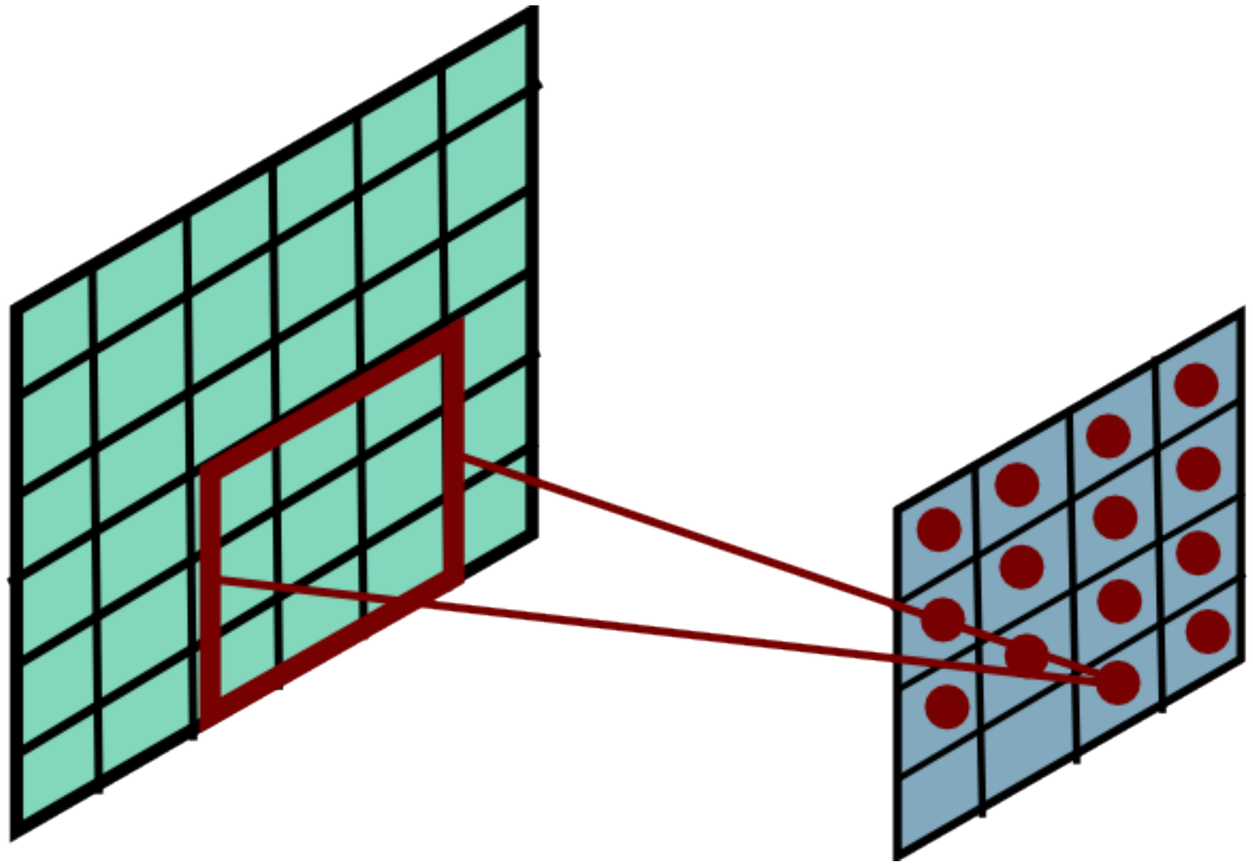
* slide from Marc'Aurelio Renzato

Convolutional Layer



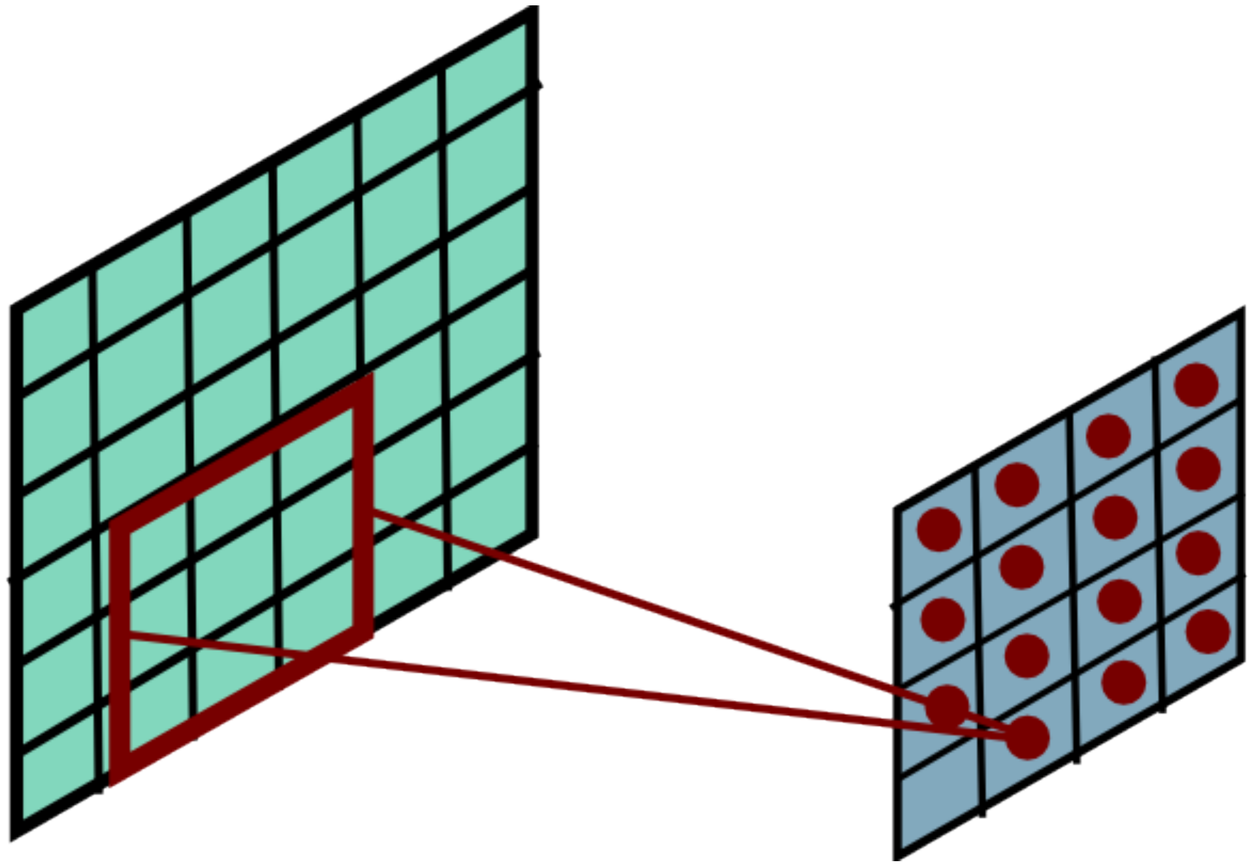
* slide from Marc'Aurelio Renzato

Convolutional Layer



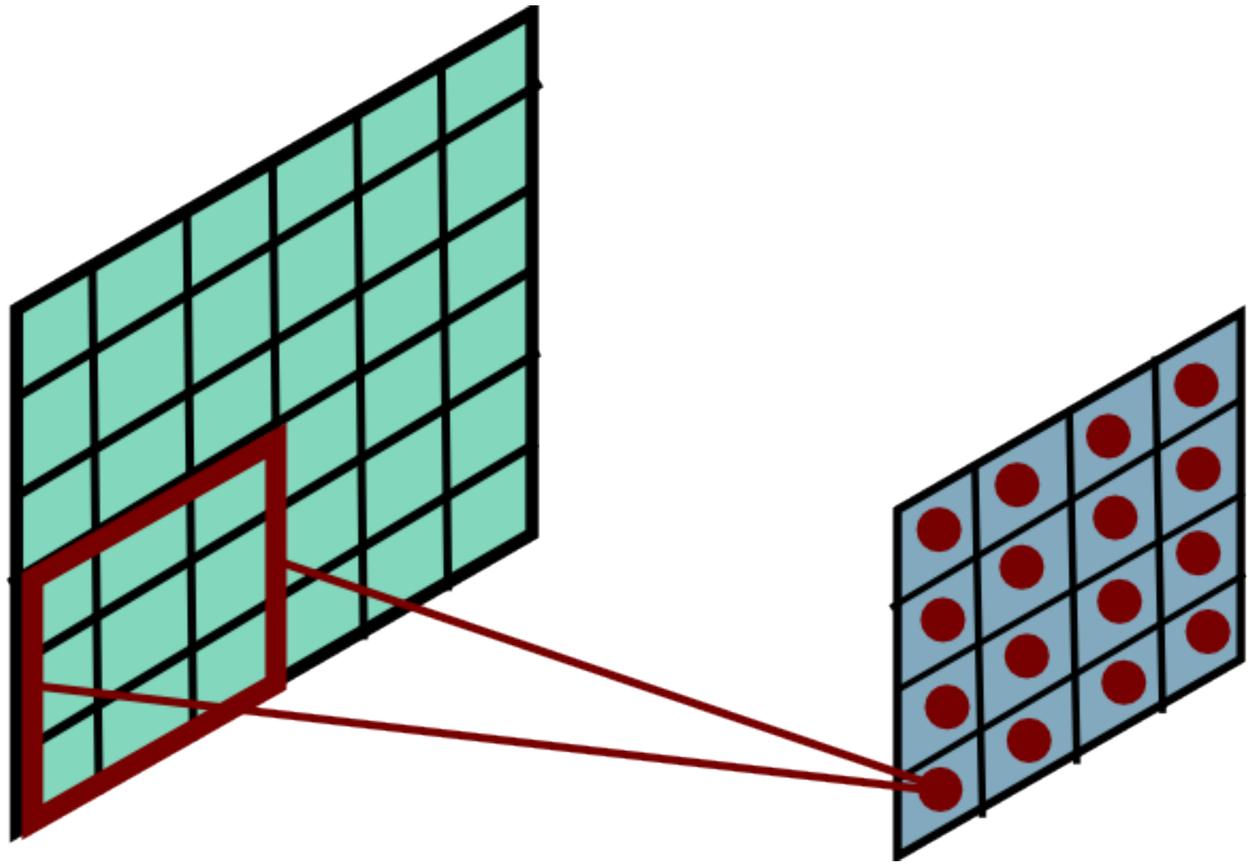
* slide from Marc'Aurelio Renzato

Convolutional Layer



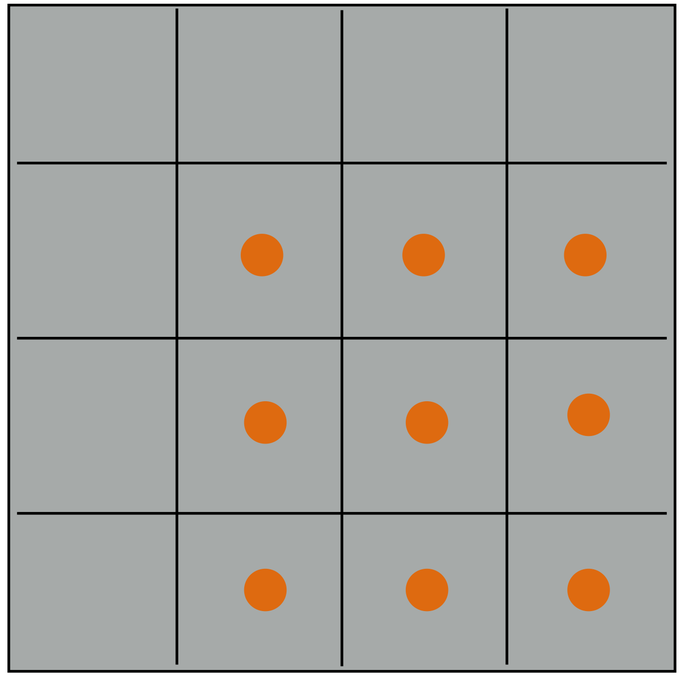
* slide from Marc'Aurelio Renzato

Convolutional Layer

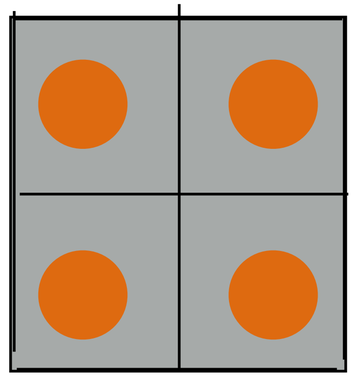


* slide from Marc'Aurelio Renzato

Convolutional Layer: Interpretation #2



One neuron applied as convolution (by shifting)



neurons

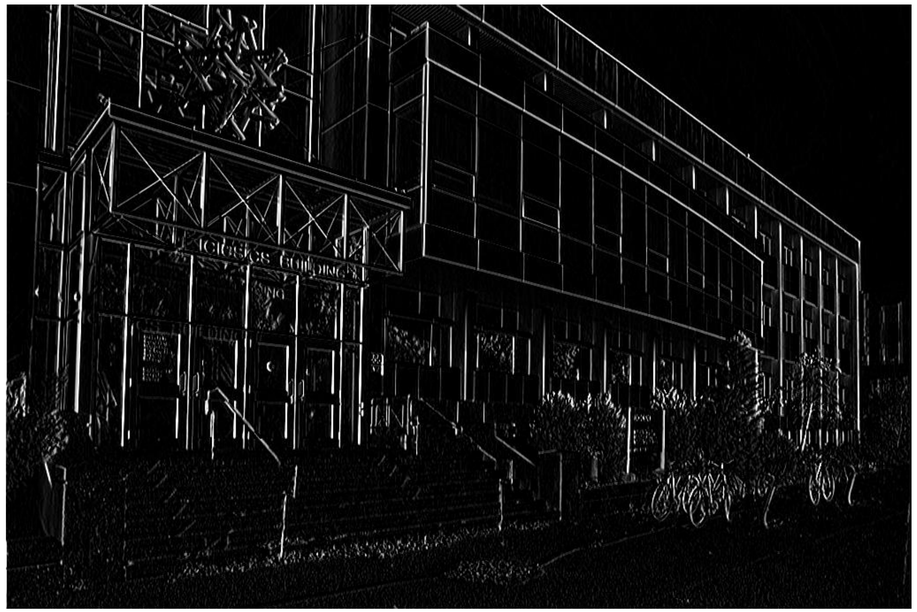
output

$$\sigma\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{W}_{i,j}\mathcal{I}(i,j)+b\right)$$

Similar to Filter in Normalized Correlation

Convolution Layer

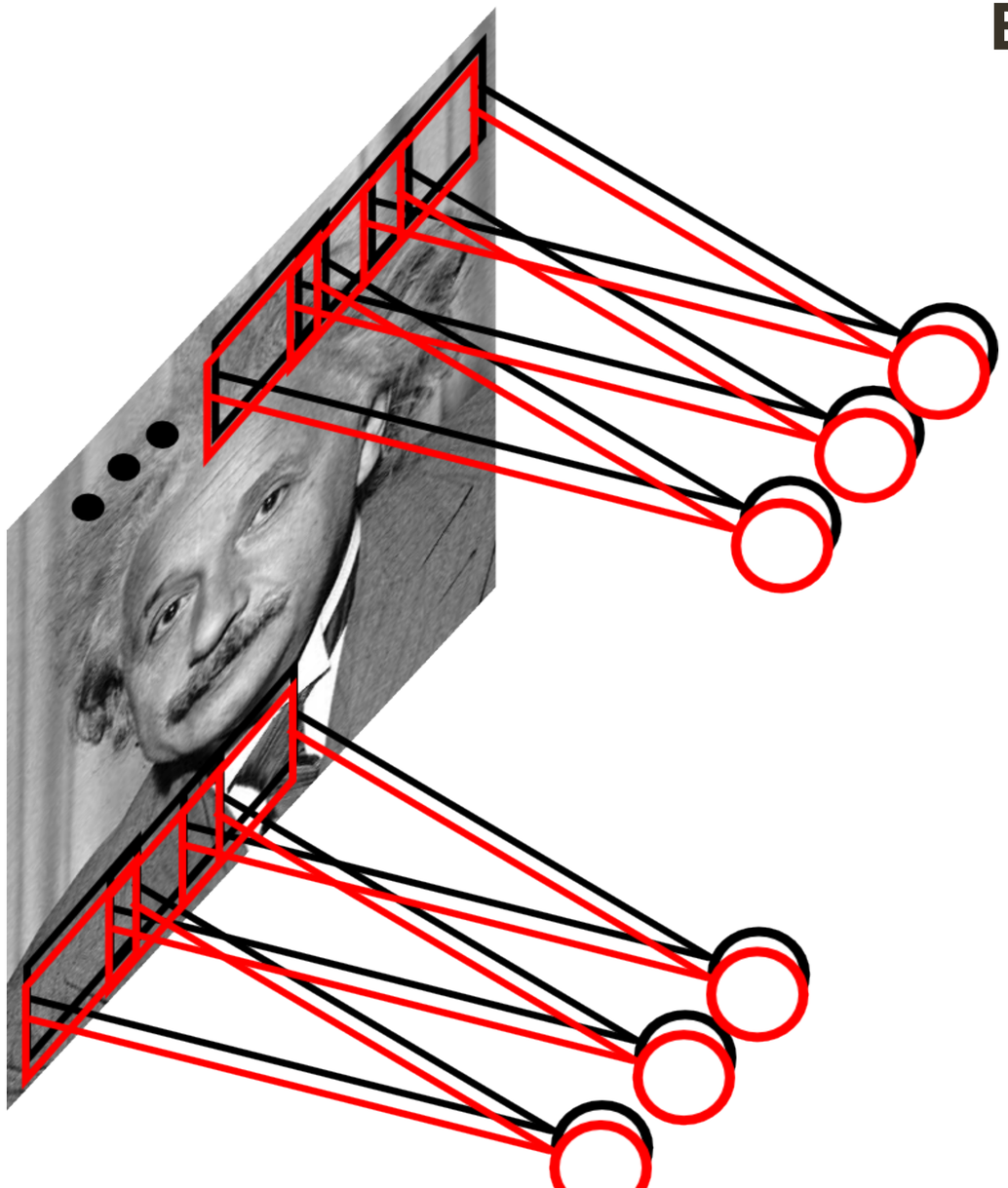
$$\star \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$



Convolution Layer

$$\star \begin{bmatrix} 0.11 & 0.11 & 0.11 \\ 0.11 & 0.11 & 0.11 \\ 0.11 & 0.11 & 0.11 \end{bmatrix} \rightarrow$$

Convolutional Layer



Example: 200 x 200 image (small)
x 40K hidden units

Filter size: 10 x 10

# of filters: 20

Learn **multiple filters**

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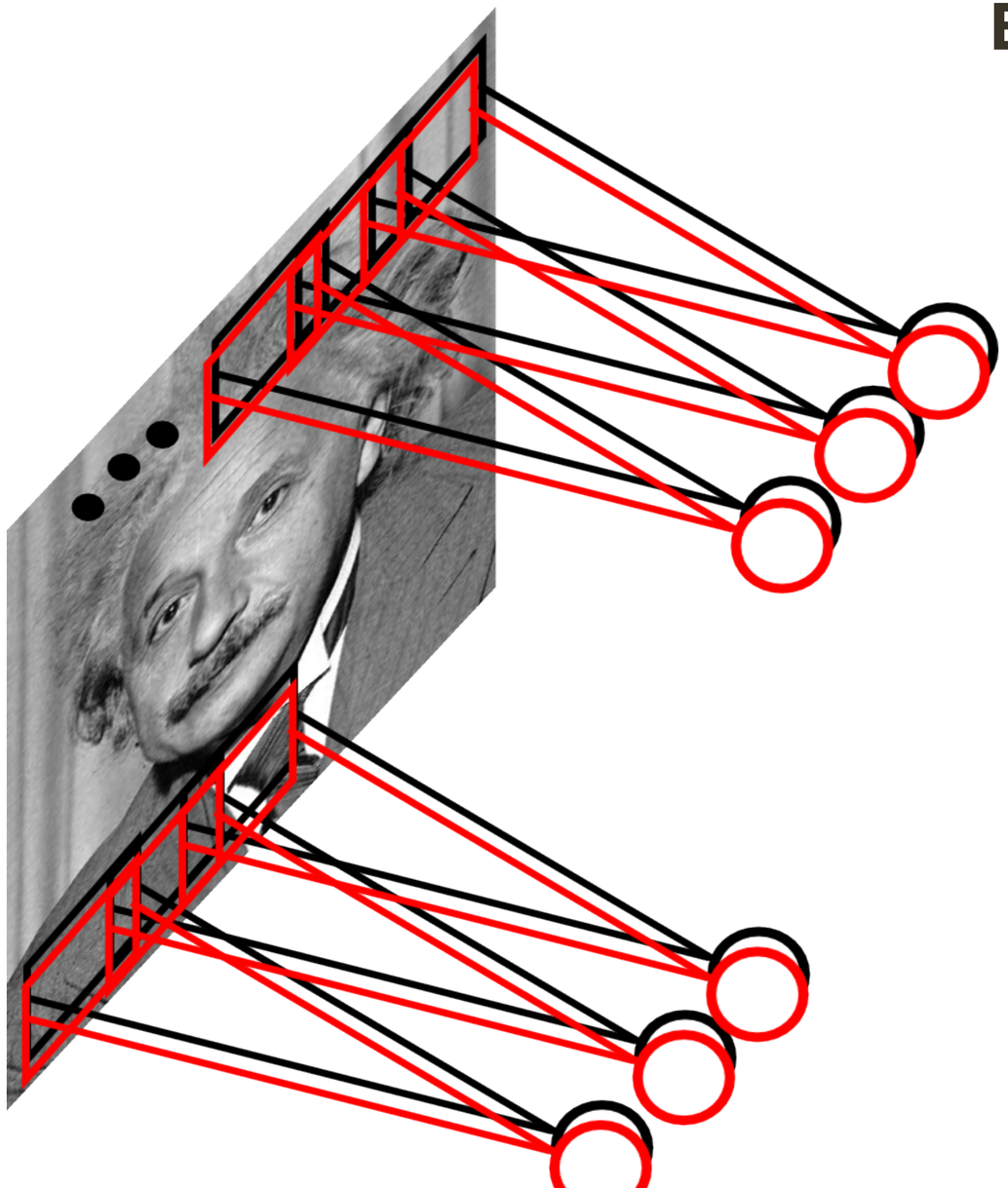
Convolutional Layer



Example: 200 x 200 image (small)
x 40K hidden units

Filter size: 10 x 10

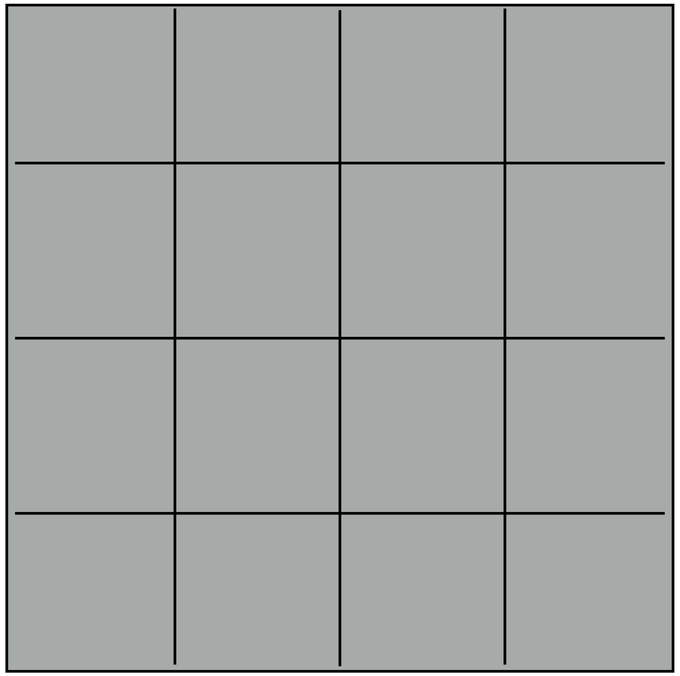
# of filters: 20

= 2000 parameters

Learn **multiple filters**

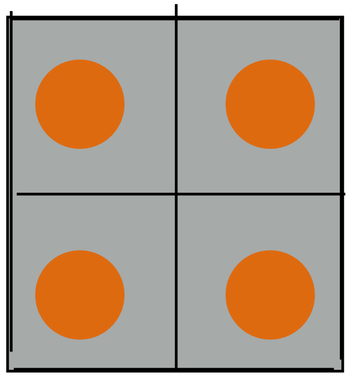
* slide from Marc'Aurelio Renzato

Convolutional Layer: Interpretation #2



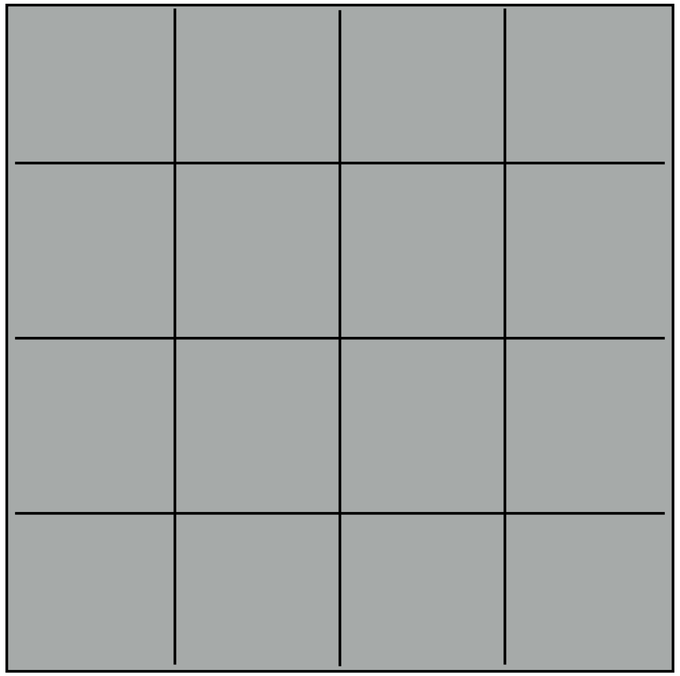
One neuron applied as convolution (by shifting)

neurons



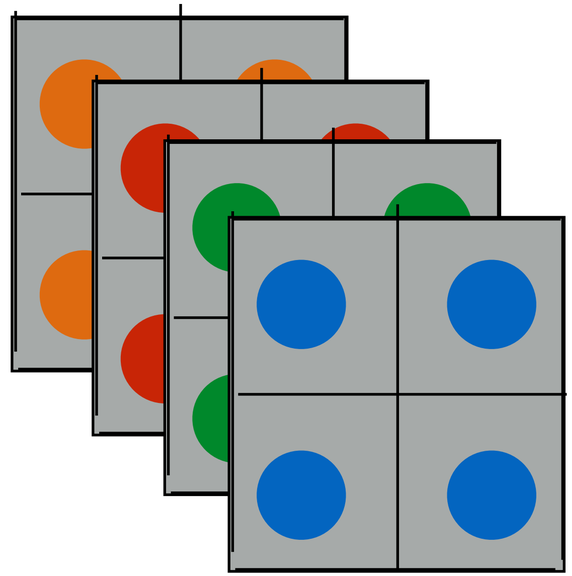
output

Convolutional Layer: Interpretation #2



One neuron applied as convolution (by shifting)

neurons



output