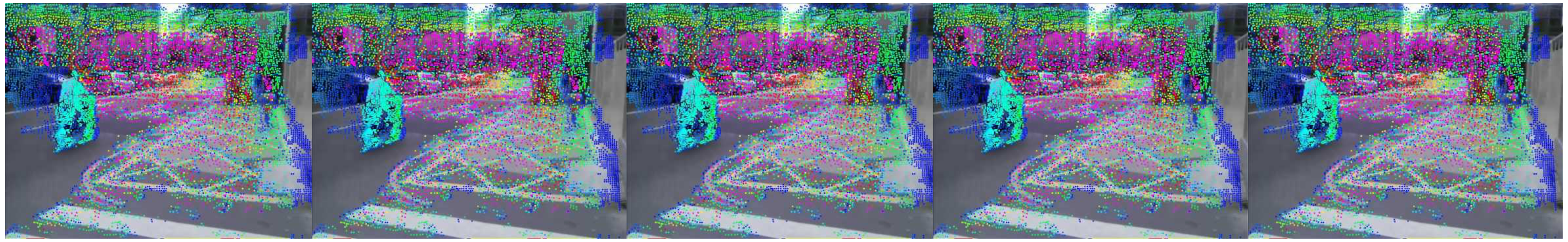
THE UNIVERSITY OF BRITISH COLUMBIA

CPSC 425: Computer Vision



Lecture 17: Optical Flow

Menu for Today

Topics:

- **Optical Flow**
- **Quiz 4**

Readings:

- **Today's** Lecture: Szeliski 15.1, 15.2

Reminders:

- **Midterm results** are now posted on Canvas (we will review some questions)
- **Assignment 4**: RANSAC and Panoramas due **March 20th**

Midterm - Q8 (Projection)

A rectangle in the Y-plane in the world is defined by the following four points:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ a \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ a \end{bmatrix},$$

where a is a variable.

(a) [2 marks] Compute the perspective projection of the rectangle in the image plane (i.e. give numerical expression for the projected points in terms of a and focal length f where needed).

Midterm - Q8 (Projection)

Perspective Projection: $x' = \frac{fX}{Z}$ $y' = \frac{fY}{Z}$

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where a is a variable.

(a) [2 marks] Compute the perspective projection of the rectangle in the image plane (i.e. give numerical expression for the projected points in terms of a and focal length f where needed).

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \frac{f}{2} \\ \frac{f}{2} \end{bmatrix}, \begin{bmatrix} \frac{f}{a} \\ \frac{f}{a} \end{bmatrix}, \begin{bmatrix} \frac{-f}{2} \\ \frac{f}{2} \end{bmatrix}, \begin{bmatrix} \frac{-f}{a} \\ \frac{f}{a} \end{bmatrix}$$

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(b) [2 marks] Sketch the projection in the imaging plane for $f = 2$ and $a = 4$.

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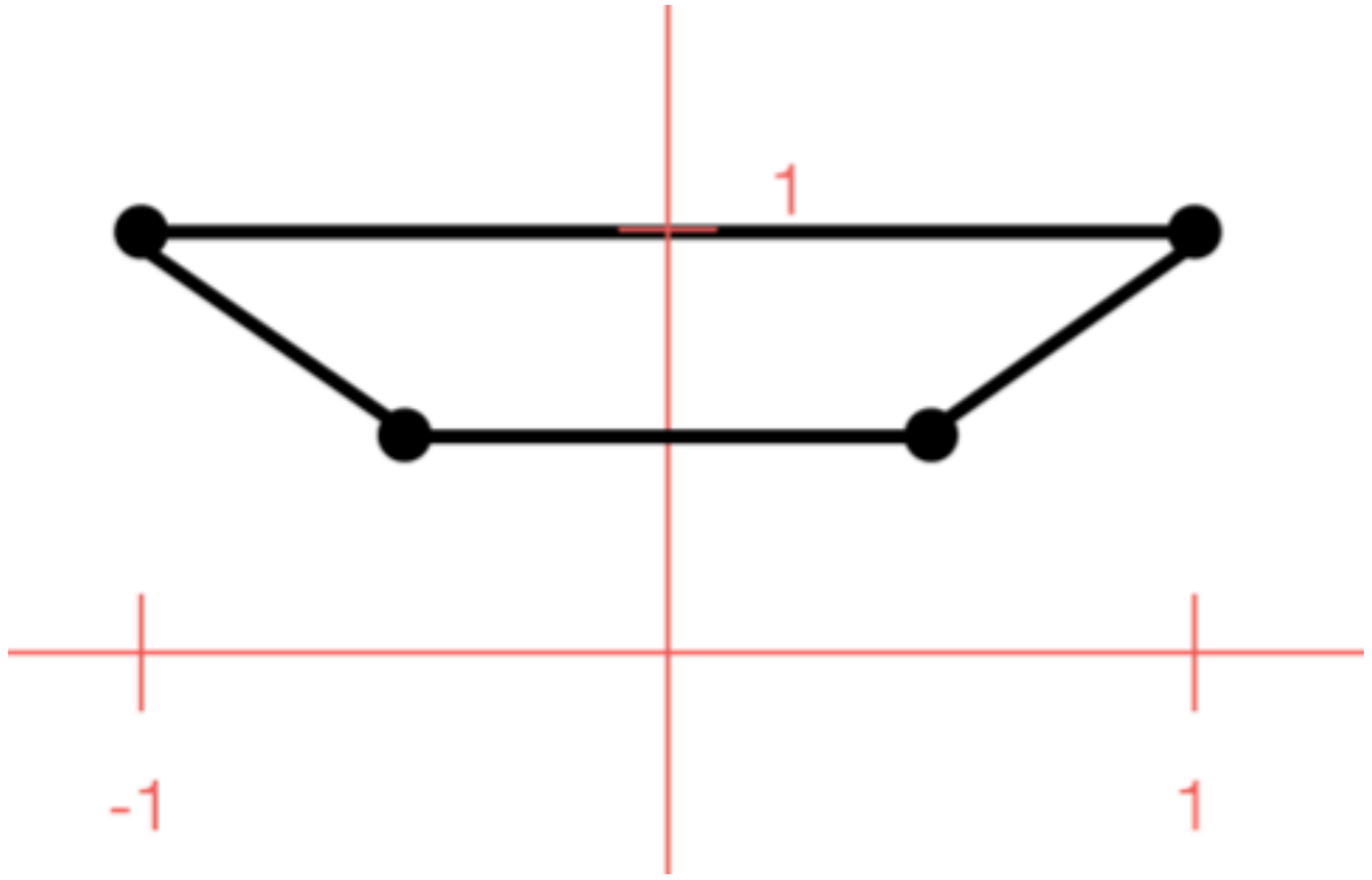

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(c) [2 marks] Describe both numerically and in terms of concepts we learned about in projection, what happens as $a \to \infty$. Sketch what a projection will look like at that point.

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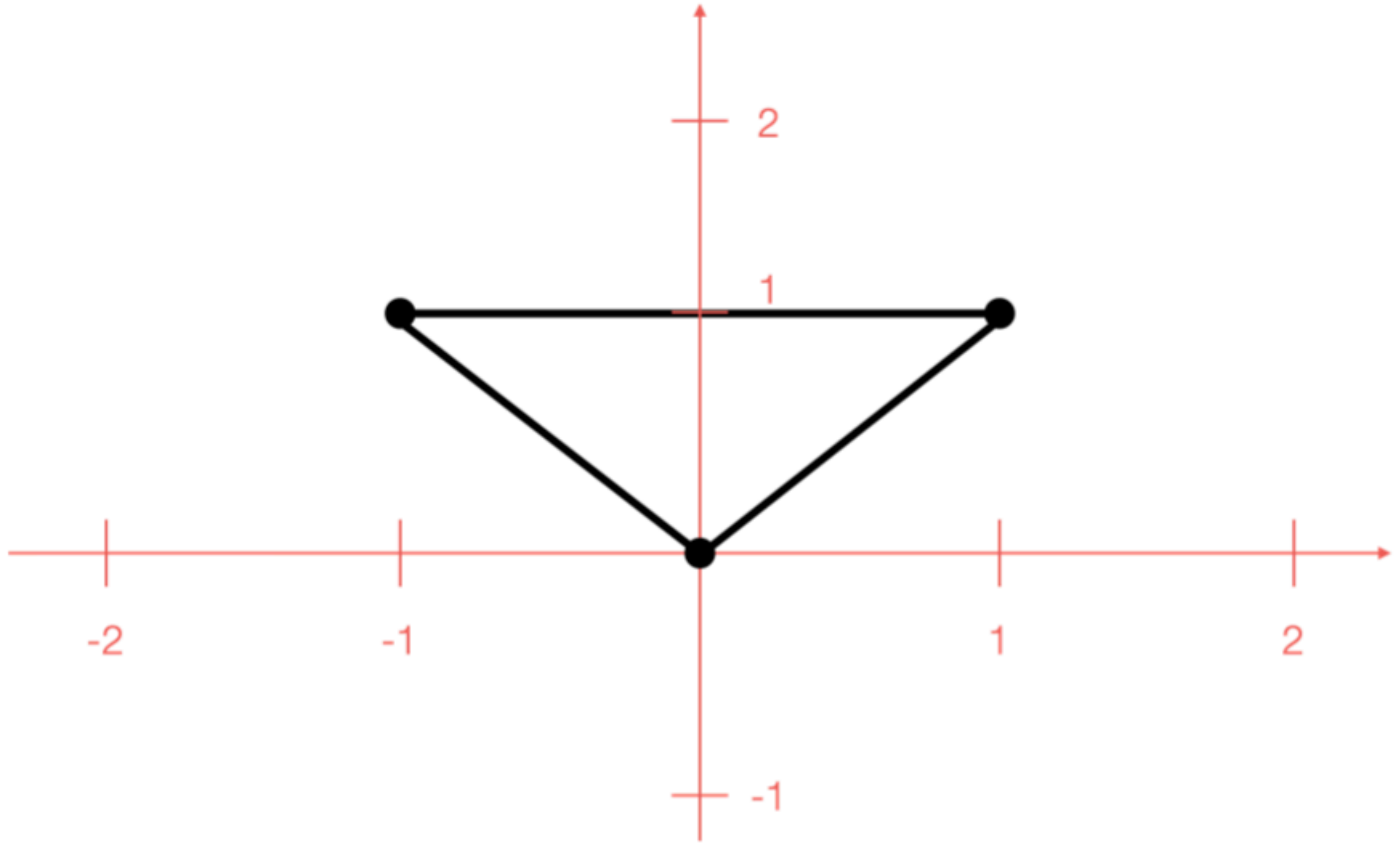

Midterm - Q8 (Projection)

(d) [3 marks] Consider what happens if the projection is not perspective, but rather weak perspective which is governed by a scaling parameter m, i.e.,

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = m \begin{bmatrix} X \\ Y \end{bmatrix}. \tag{1}$$

Compute an appropriate value for m in that case in terms of focal length f and a. Describe what must be true of a and/or f for this to be a good (accurate) approximation.

Midterm - Q8 (Projection)

Weak perspective: $m = \frac{f}{z_o}$

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$$m = \frac{f}{1 + \frac{1}{2}a}.$$

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Midterm - Q10 (Smoothing)

Consider making a smoothing filter out of the upside down cone defined by the following function:

$$P_r(x, y) = 1 - \sqrt{\frac{x^2 + y^2}{r^2}} \qquad (2)$$

which is only defined on it's positive domain of $-r \le x \le r$ and $-r \le y \le r$, where parameter $r > 0$ is a radius of the cone base, which, similar to σ in a Gaussian, controls the amount of smoothing.

(b) [4 marks] For a particular $r = 5$ we obtain the following 2D smoothing parabolic filter. Briefly describe two things that are wrong with the filter and how they could be fixed.

0.43	0.55	0.60	0.55	0.43
0.55	0.72	0.80	0.72	0.55
0.60	0.80	1.00	0.80	0.60
0.55	0.72	0.80	0.72	0.55
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1. Values do not sum to 1

2. Extent of the filter does not capture the full function

Midterm - Q10 (Smoothing)

(c) [4 marks] Consider the **central pixel** in the following image patches. State whether the the value of this center pixel will increase, decrease or stay the same in the case of smoothing with standard filters:

$$\text{Image Patch 1} \quad \begin{bmatrix} 220 & 10 & 10 \\ 10 & 10 & 200 \\ 10 & 240 & 10 \end{bmatrix} \qquad \text{Image Patch 2} \quad \begin{bmatrix} 5 & 5 & 18 \\ 5 & 17 & 5 \\ 18 & 5 & 5 \end{bmatrix}$$

Box filter

Median filter

Gaussian filter (with $\sigma = 1$)

Bilateral filter (with $\sigma_r = \sigma_d = 1$)

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	Image Patch 1	Image Patch 2
Box filter	Increase	Decrease
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Median filter	Same	Decrease
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Median filter	Same	Decrease
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Bilateral filter (with $\sigma_r = \sigma_d = 1$)	Same	Increase

Midterm - Q13 (Texture)

Consider texture synthesis approach of Efros and Leung for filling in a pixel marked (q) in the texture below. Assume we are using the rest of the image as the source of texture for copying.

230	230	100	230	230	230	230	100	230	100
230	100	230	100	230	230	230	230	100	230
100	230	100	230	230	100	230	230	230	230
230	230	230	230	230	230	230	230	230	230
230	230	230	100	230	230	230	230	230	100
230	230	100	230	230	100	230	230	100	230
230	230	230	230	***q***	230	230	100	230	230
100	230	230	100	230	230	230	230	230	230
230	230	100	230	230	100	230	230	230	230
230	230	230	230	230	100	230	230	230	100

(a) [3 marks] Assuming we only consider exact matches and a 3×3 neighborhood, compute the probability of the pixel q being each color:

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230	230	230	230	230	230	230	230	230	230
230	230	230	100	230	230	230	230	230	100
230	230	100	230	230	100	230	230	100	230
230	230	230	230	***q***	230	230	100	230	230
100	230	230	100	230	230	230	230	230	230
230	230	100	230	230	100	230	230	230	230
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100	230	100	230	230	100	230	230	230	230
230	230	230	230	230	230	230	230	230	230
230	230	230	100	230	230	230	230	230	100
230	230	100	230	230	100	230	230	100	230
230	230	230	230	***q***	230	230	100	230	230
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100	230	100	230	230	100	230	230	230	230
230	230	230	230	230	230	230	230	230	230
230	230	230	100	230	230	230	230	230	100
230	230	100	230	230	100	230	230	100	230
230	230	230	230	***q***	230	230	100	230	230
100	230	230	100	230	230	230	230	230	230
230	230	100	230	230	100	230	230	230	230
230	230	230	230	230	100	230	230	230	100

$$p(q = 230) = \frac{3}{5}$$

$$p(q = 100) = \frac{2}{5}$$

$$p(q = 0) = 0$$

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100	230	100	230	230	100	230	230	230	230
230	230	230	230	230	230	230	230	230	230
230	230	230	100	230	230	230	230	230	100
230	230	100	230	230	100	230	230	100	230
230	230	230	230	***q***	230	230	100	230	230
100	230	230	100	230	230	230	230	230	230
230	230	100	230	230	100	230	230	230	230
230	230	230	230	230	100	230	230	230	100

(b) [3 marks] Now consider a 5×5 neighborhood. Compute the probability of the pixel q being each color now:

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230	100	230	100	230	230	230	230	100	230
100	230	100	230	230	100	230	230	230	230
230	230	230	230	230	230	230	230	230	230
230	230	230	100	230	230	230	230	230	100
230	230	100	230	230	100	230	230	100	230
230	230	230	230	***q***	230	230	100	230	230
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100	230	100	230	230	100	230	230	230	230
230	230	230	230	230	230	230	230	230	230
230	230	230	100	230	230	230	230	230	100
230	230	100	230	230	100	230	230	100	230
230	230	230	230	***q***	230	230	100	230	230
100	230	230	100	230	230	230	230	230	230
230	230	100	230	230	100	230	230	230	230
230	230	230	230	230	100	230	230	230	100

$$p(q = 230) = 1$$

$$p(q = 100) = 1$$

$$p(q = 0) = 0$$

(b) [3 marks] Now consider a 5×5 neighborhood. Compute the probability of the pixel q being each color now:

Optical Flow

Problem:

Determine how objects (and/or the camera itself) move in the 3D world

Key Idea(s):

Images acquired as a (continuous) function of time provide additional constraint. Formulate motion analysis as finding (dense) point correspondences over time.

What is **Optical Flow**?

[vision.middlebury.edu/flow]

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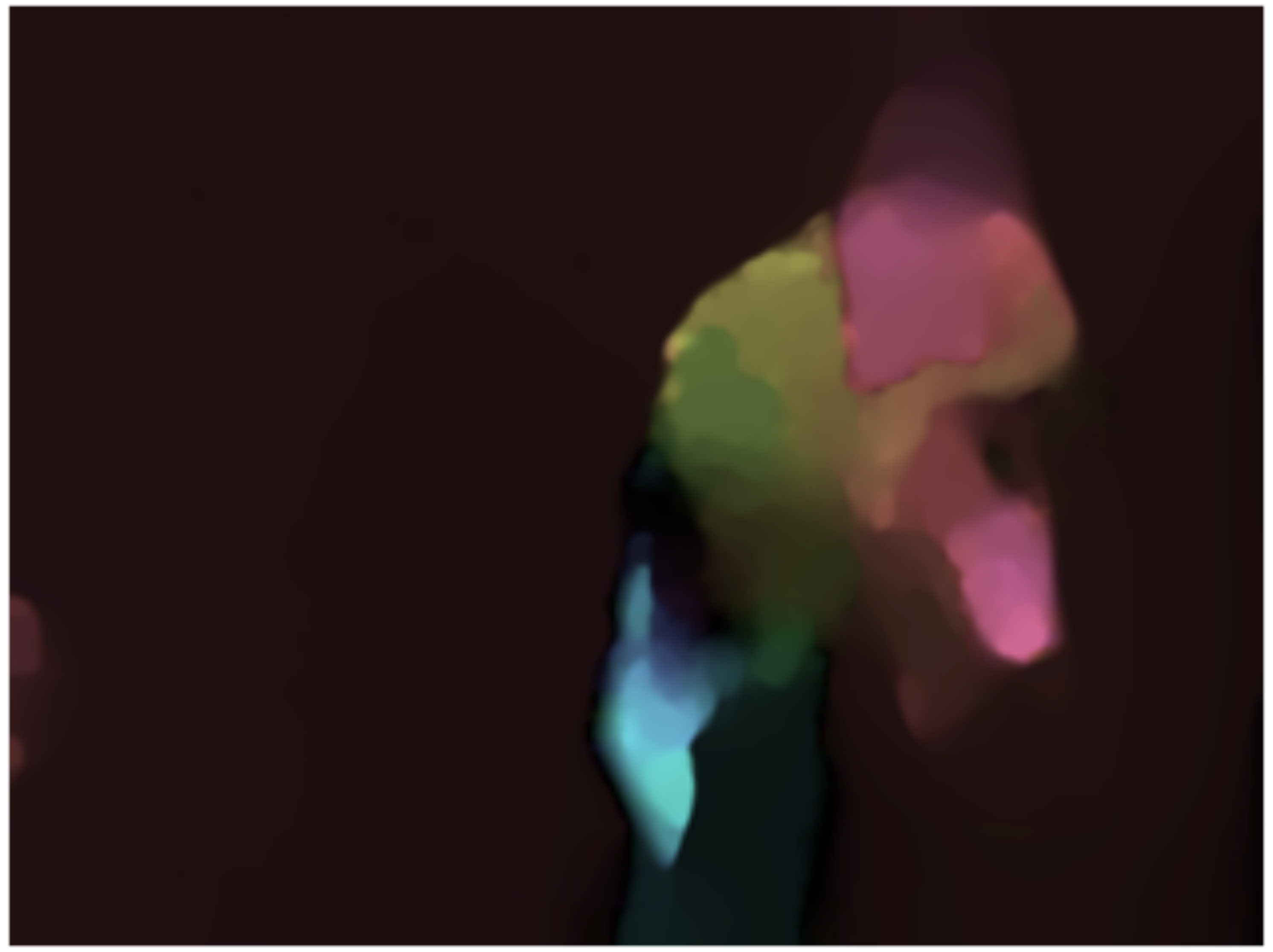

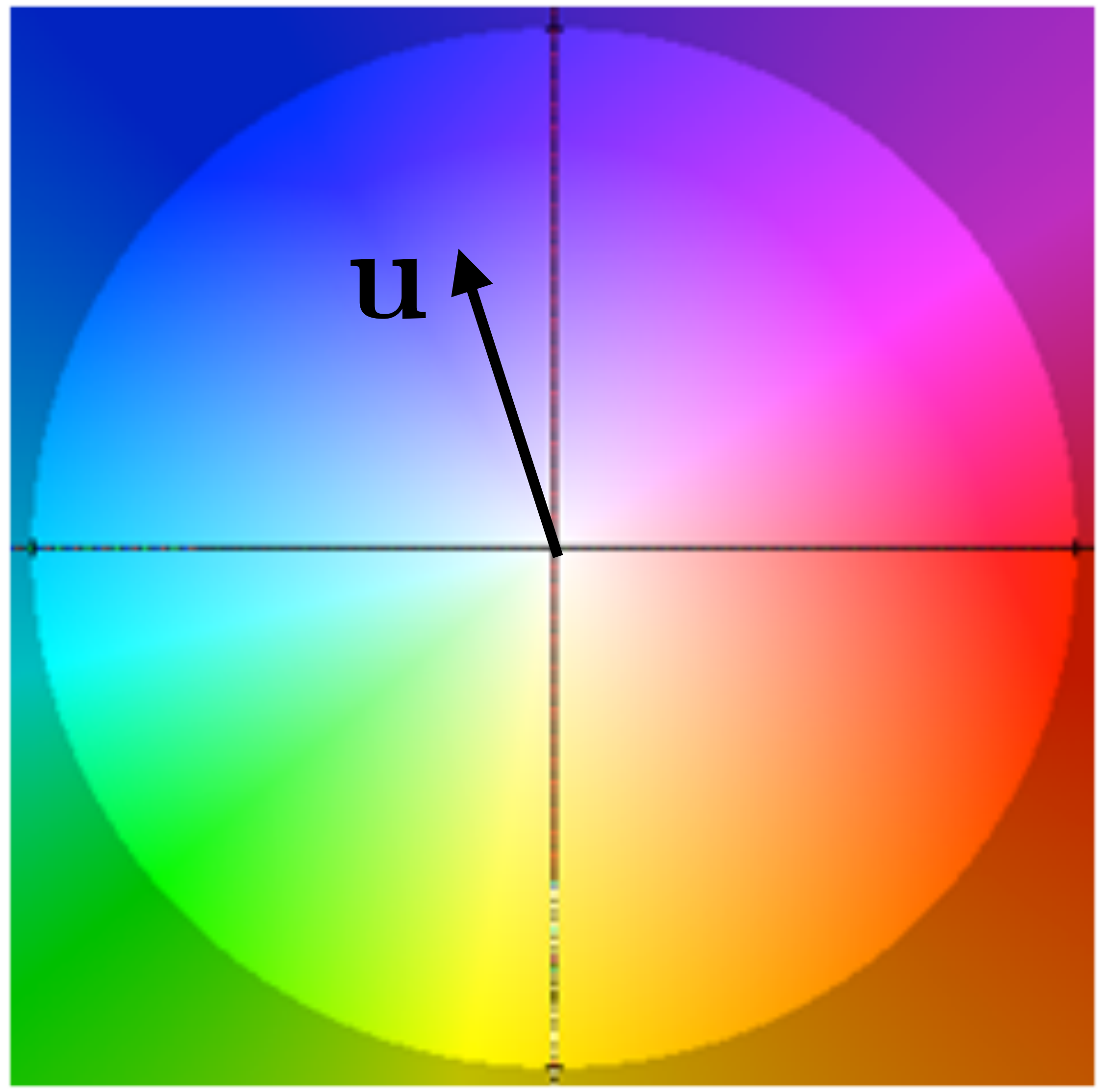

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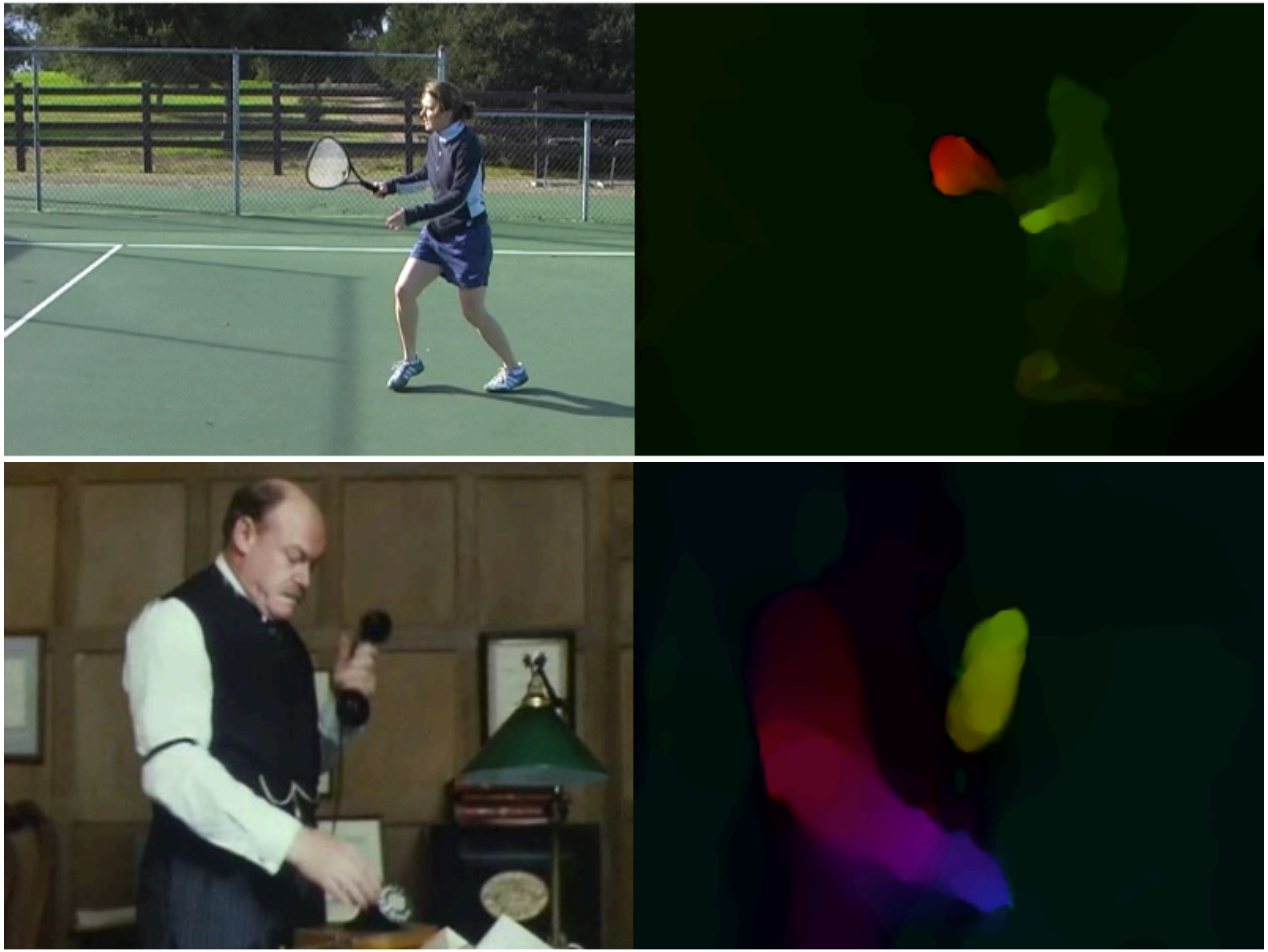


[Brox Malik 2011]

Optical Flow and 2D Motion

Optical flow is the apparent motion of brightness patterns in the image

Applications

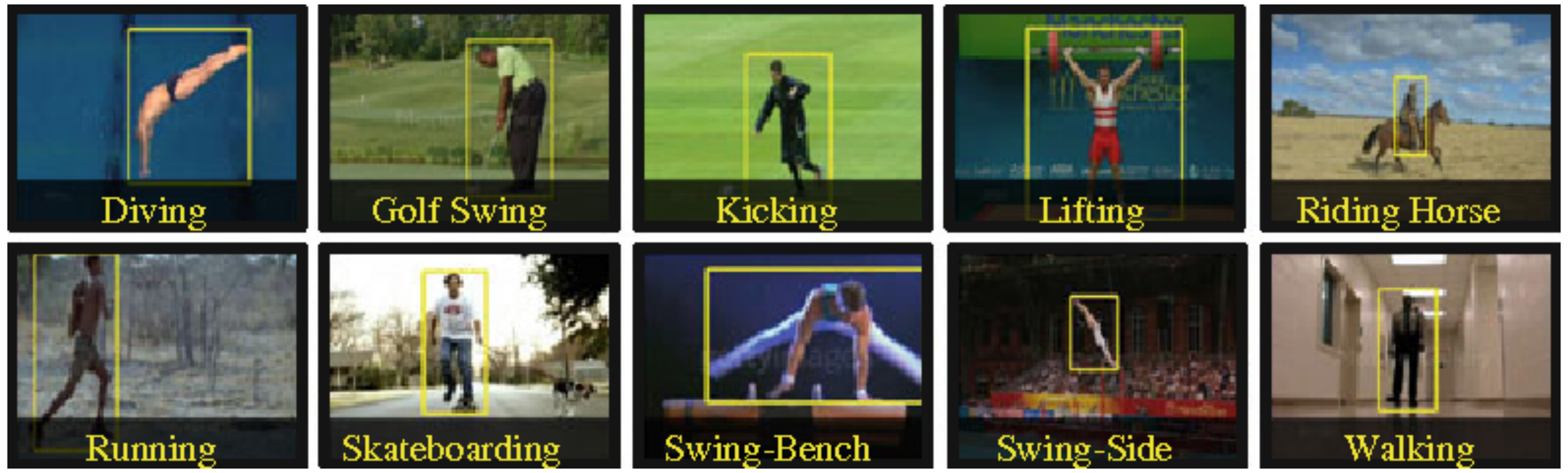
— image and video stabilization in digital cameras, camcorders

— motion-compensated video compression schemes such as MPEG

— image registration for medical imaging, remote sensing

— action recognition

— motion segmentation



Optical Flow and 2D Motion

Motion is geometric

Optical flow is radiometric

Usually we assume that optical flow and 2-D motion coincide ... but this is not always the case!

Optical Flow and 2D Motion

Optical flow but **no motion** . . .

Optical Flow and 2D Motion

Optical flow but **no motion** . . .

. . . moving light source(s), lights going on/off, inter-reflection, shadows

Optical Flow and 2D Motion

Optical flow but **no motion** . . .

. . . moving light source(s), lights going on/off, inter-reflection, shadows

Motion but **no optical flow** . . .

Optical Flow and 2D Motion

Optical flow but **no motion** . . .

. . . moving light source(s), lights going on/off, inter-reflection, shadows

Motion but **no optical flow** . . .

. . . spinning sphere.

Optical Flow and 2D Motion

Here's a video example of a very skilled Japanese contact juggler working with a clear acrylic ball

Source: http://youtu.be/CtztrcGkCBw?t=1m20s

A key element to the illusion is motion without corresponding optical flow

Optical Flow and 2D Motion

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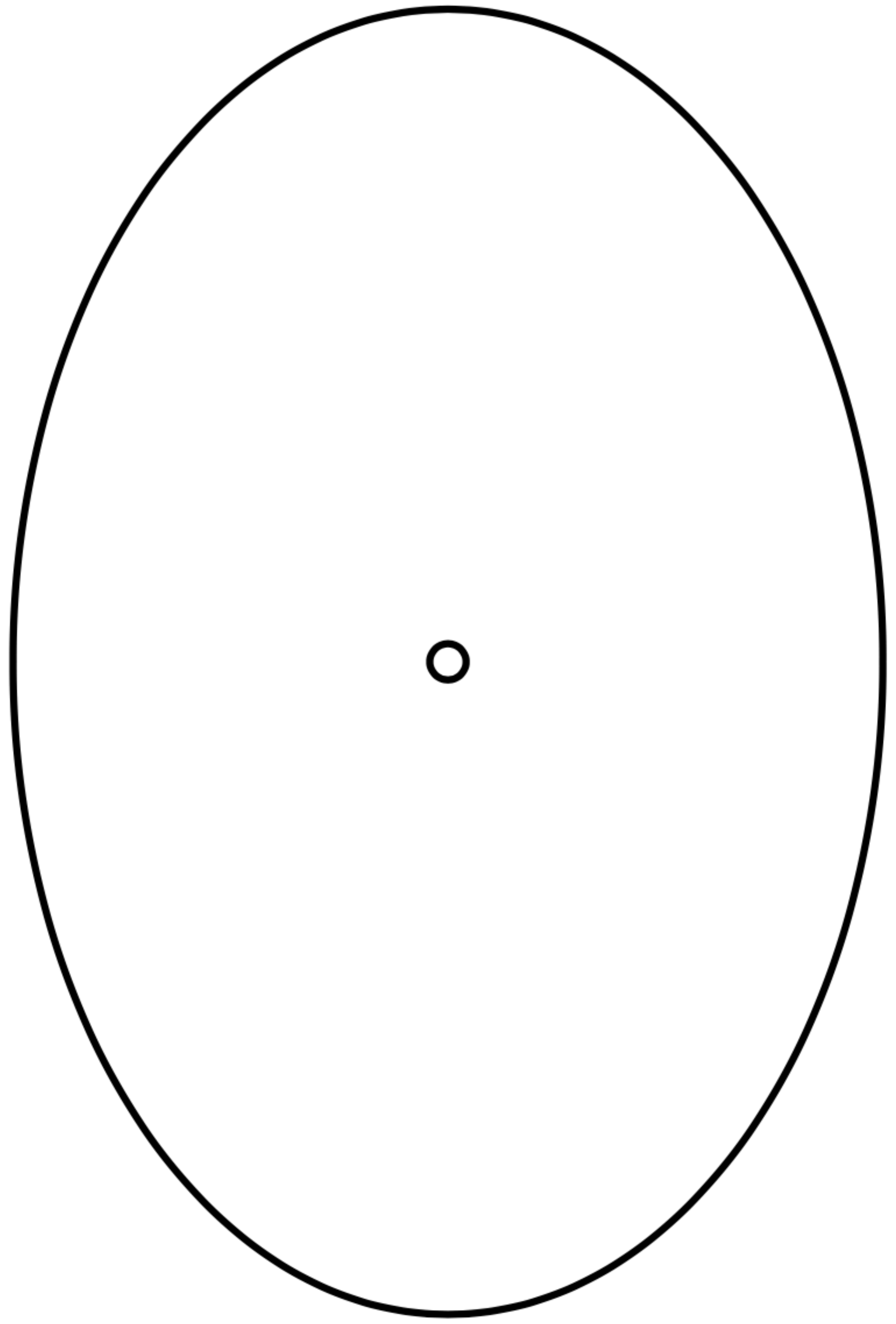
Example 1: Rotating Ellipse



Example 1: Three "Percepts"

1. **Veridical**:

— a 2-D rigid, flat, rotating ellipse

2. **Amoeboid**:

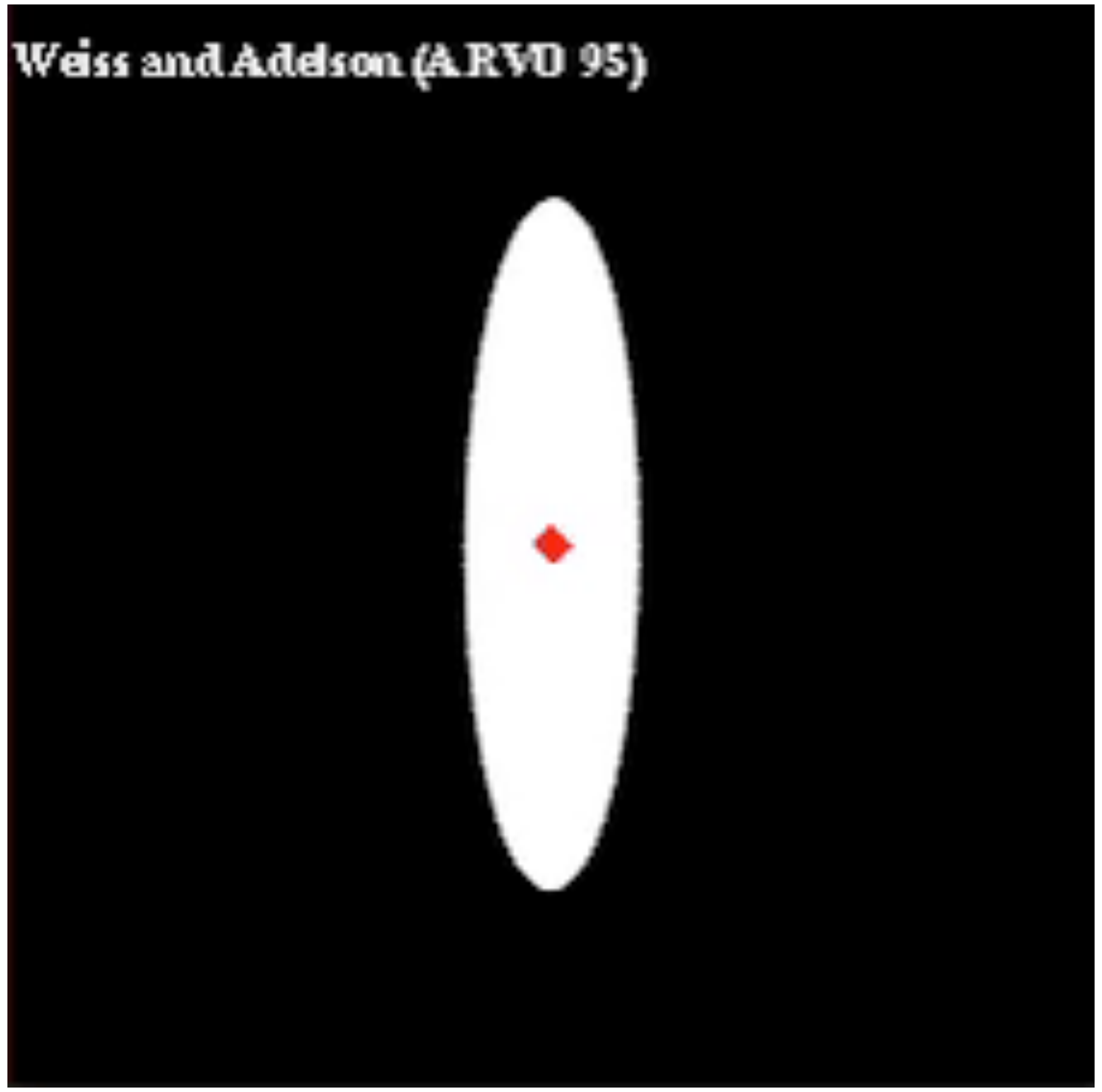
— a 2-D, non-rigid "gelatinous" smoothly deforming shape

3. **Stereokinetic**:

— a circular, rigid disk rolling in 3-D

Example 1: Rotating Ellipse

A narrow ellipse oscillating rigidly about its center appears **rigid**



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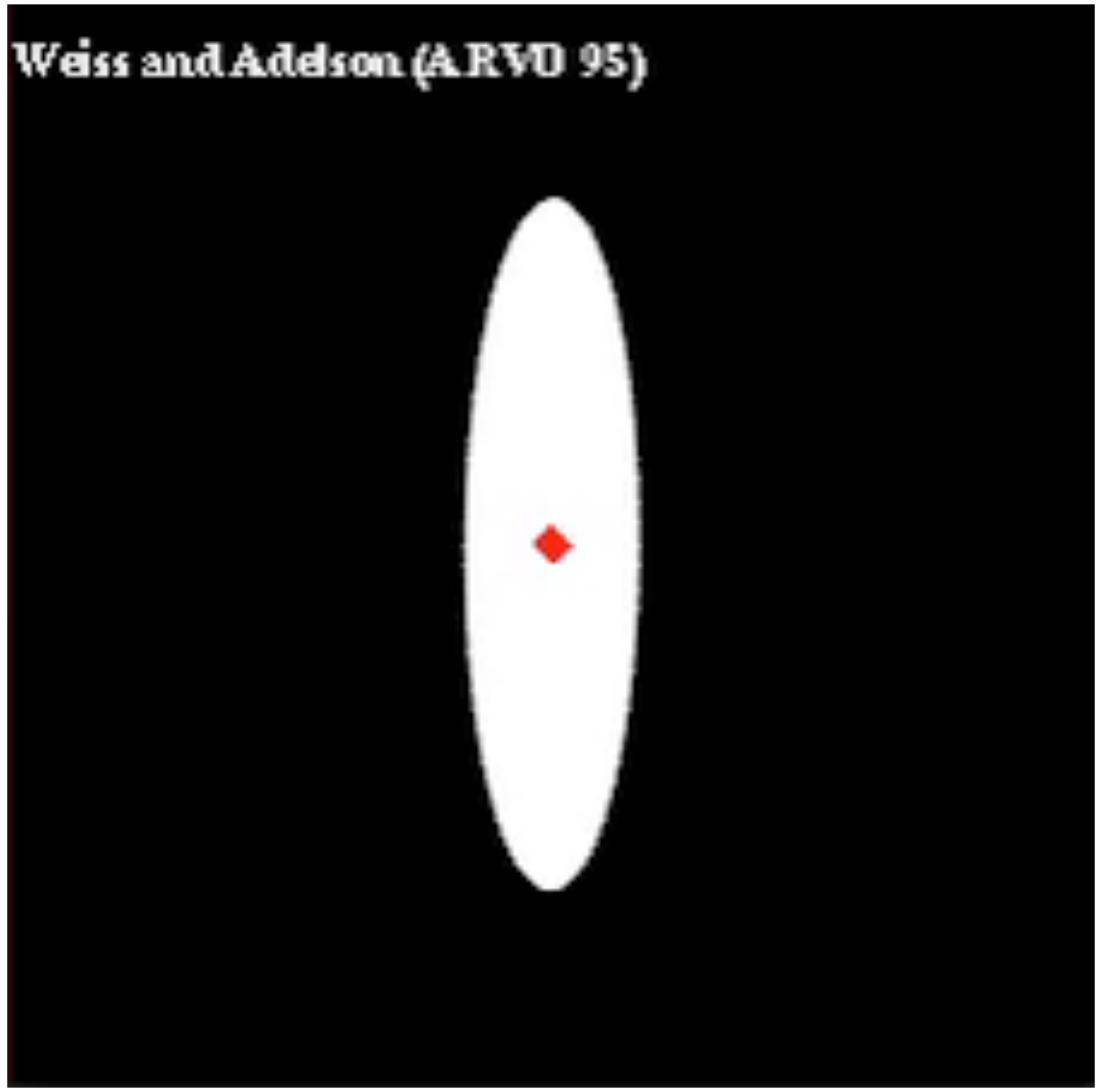

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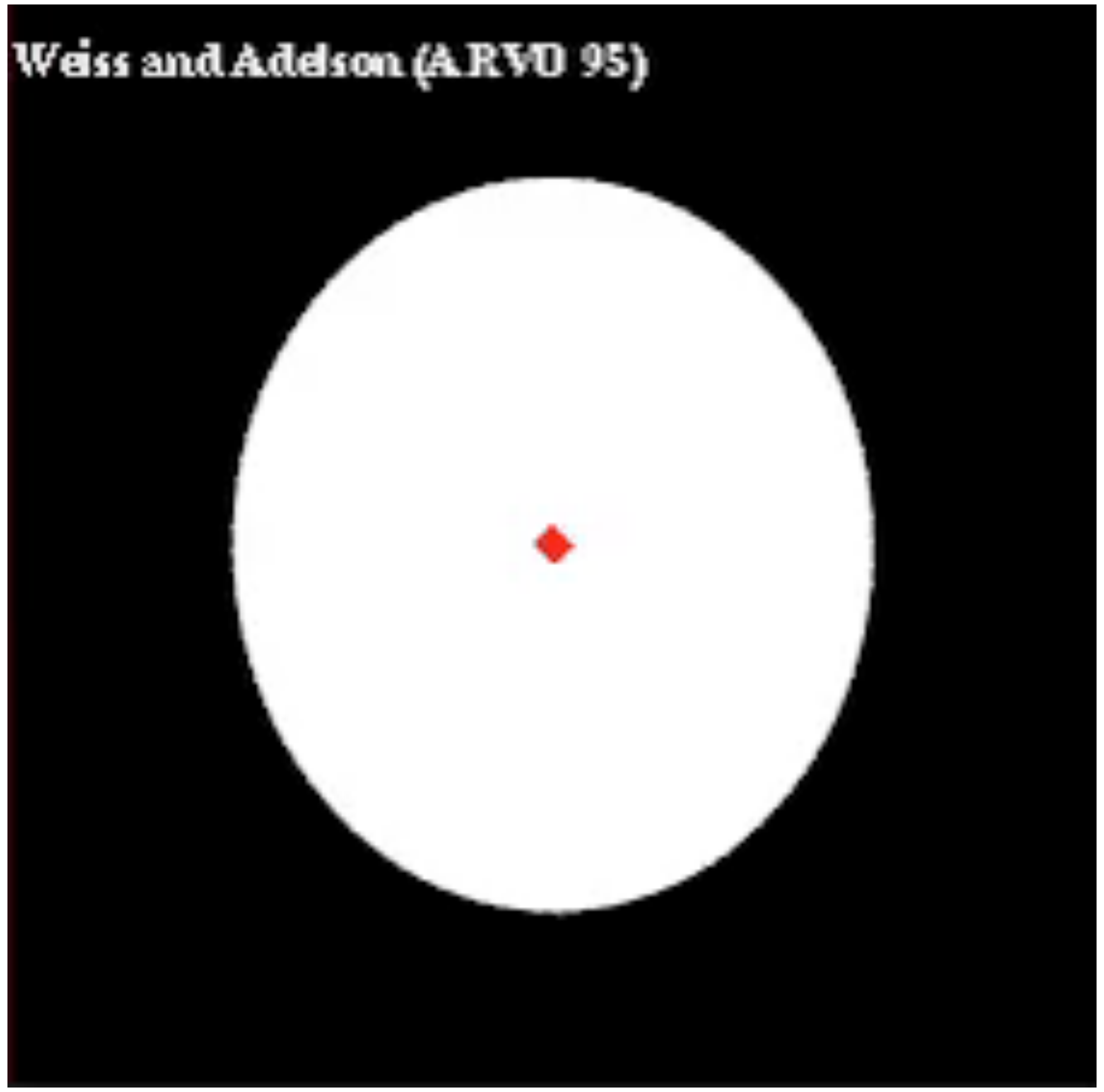
However, a fat ellipse undergoing the same motion appears **nonrigid**



Video credits: Yair Weiss

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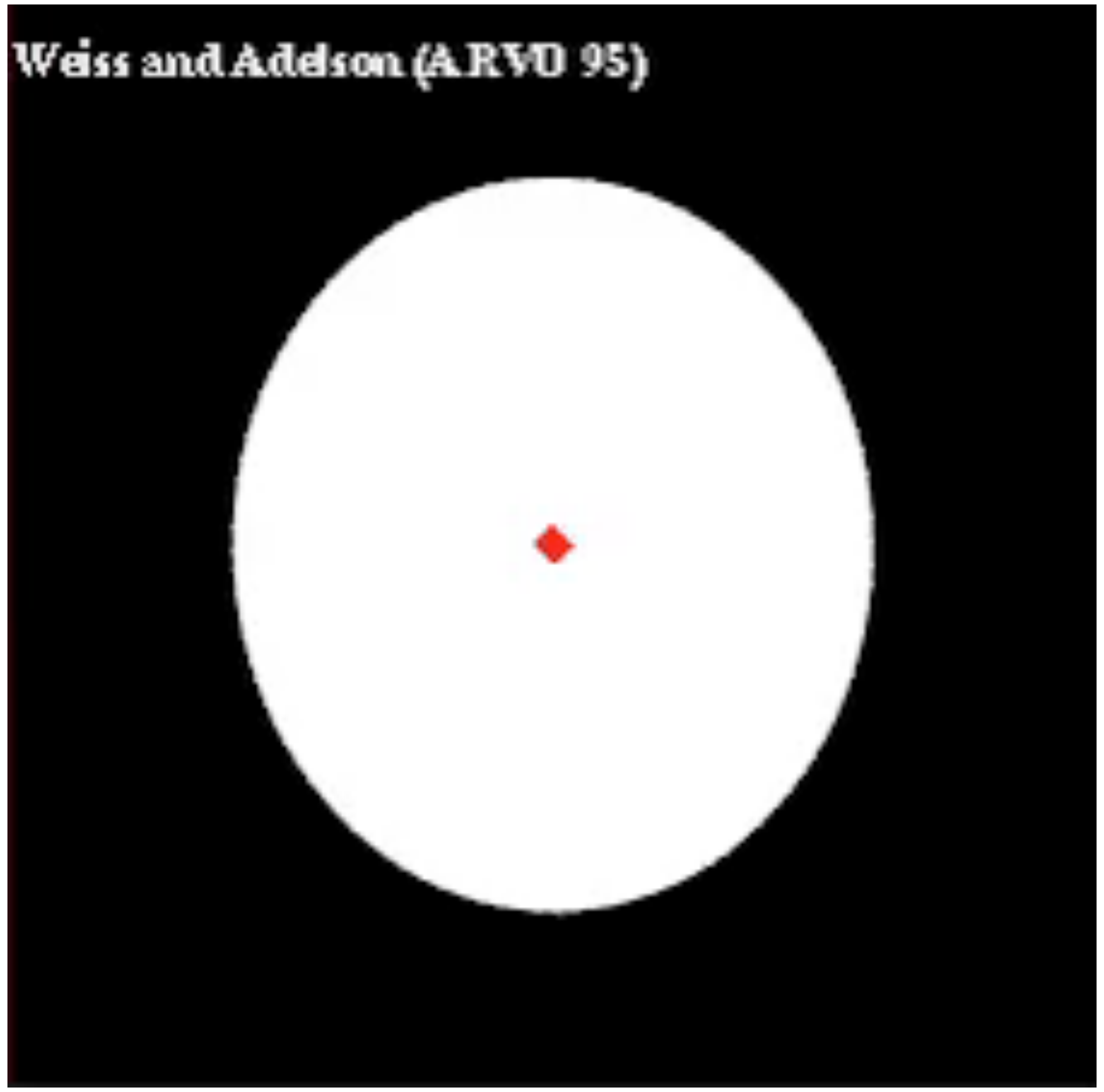

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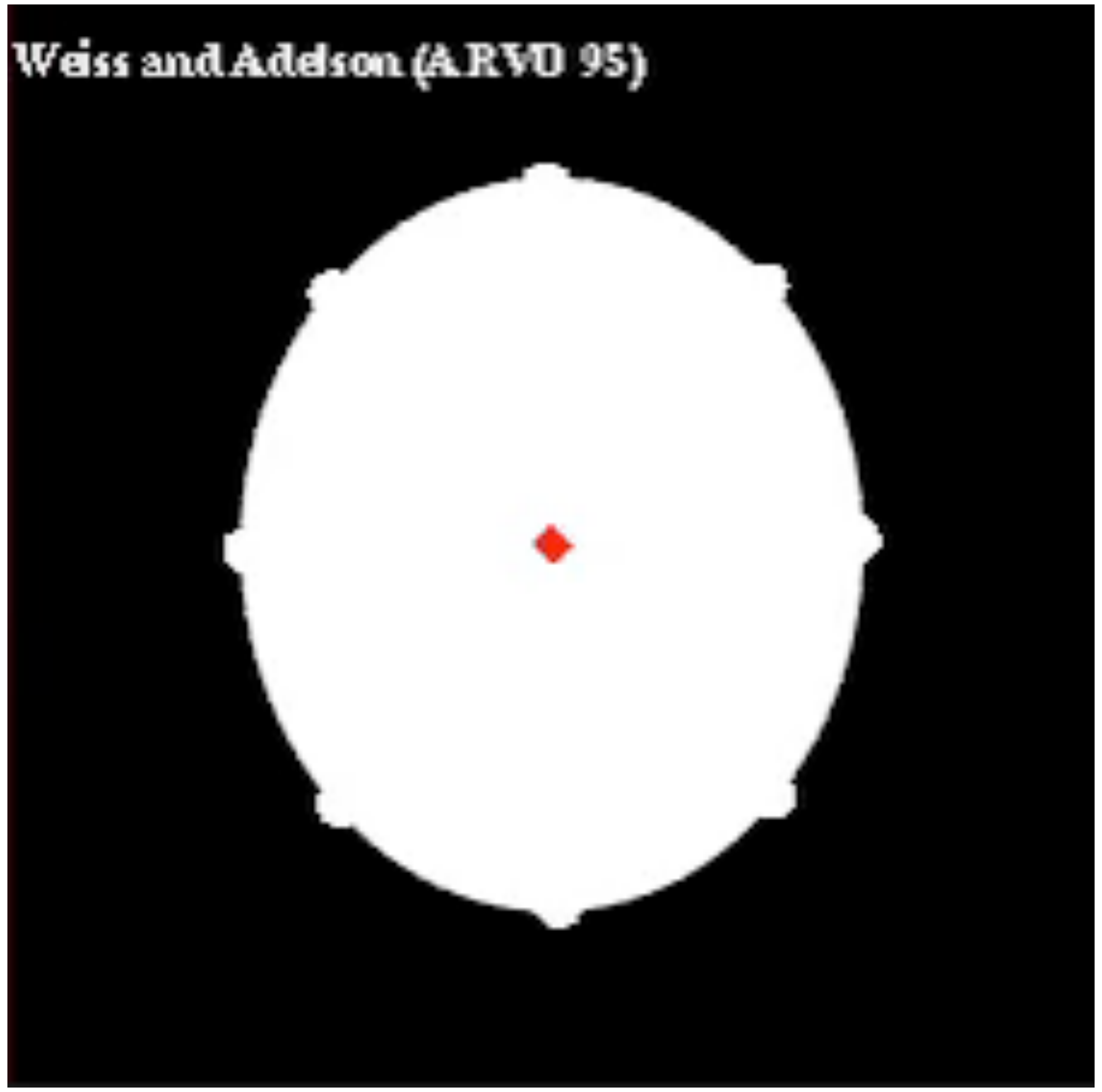
The apparent nonrigidity of a fat ellipse is not really a "visual illusion". A rotating ellipse or a nonrigid pulsating ellipse can cause the exact same stimulation on our retinas. In this sequence the ellipse contour is always doing the same thing, only the markers' motion changes.



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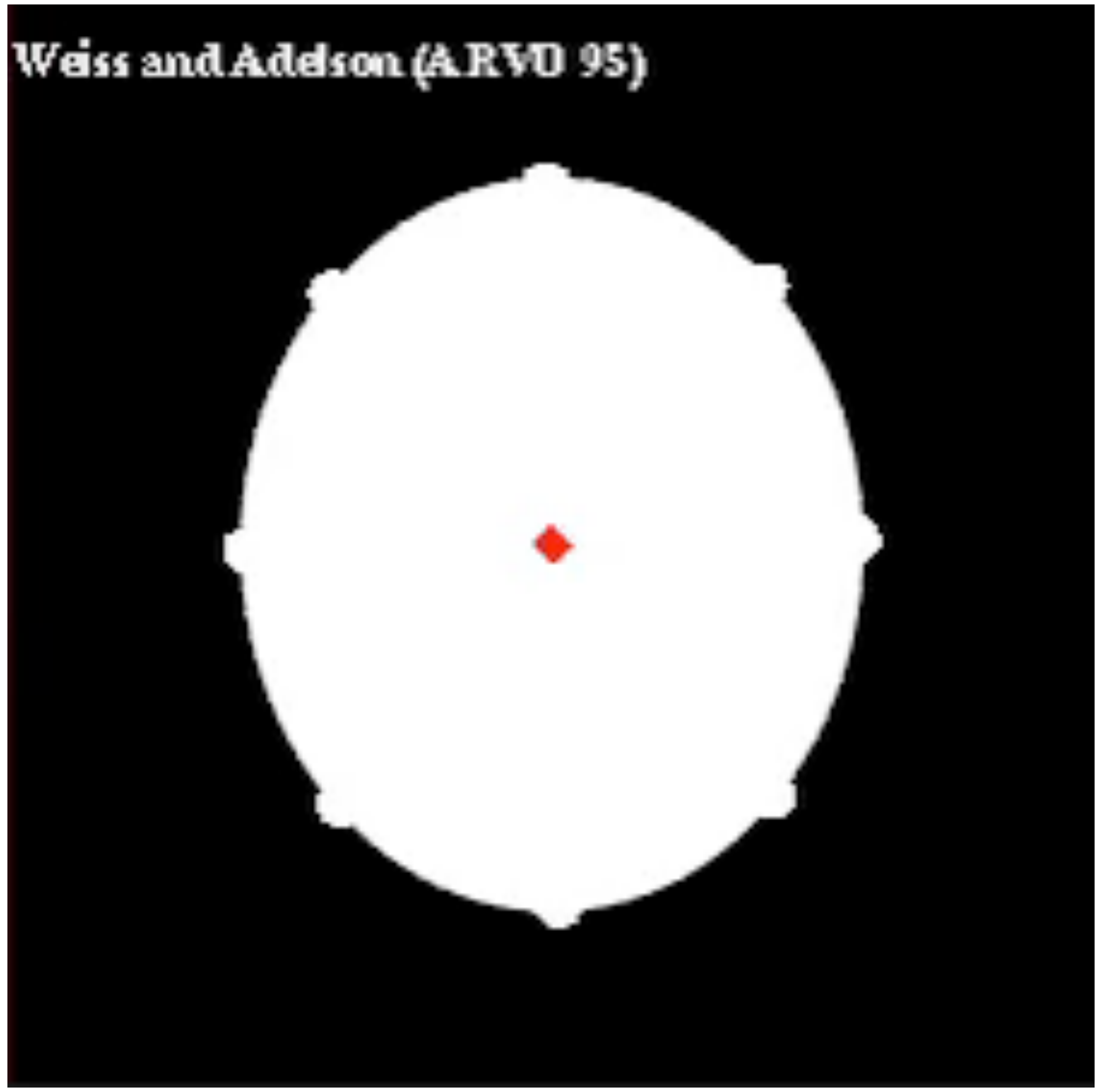

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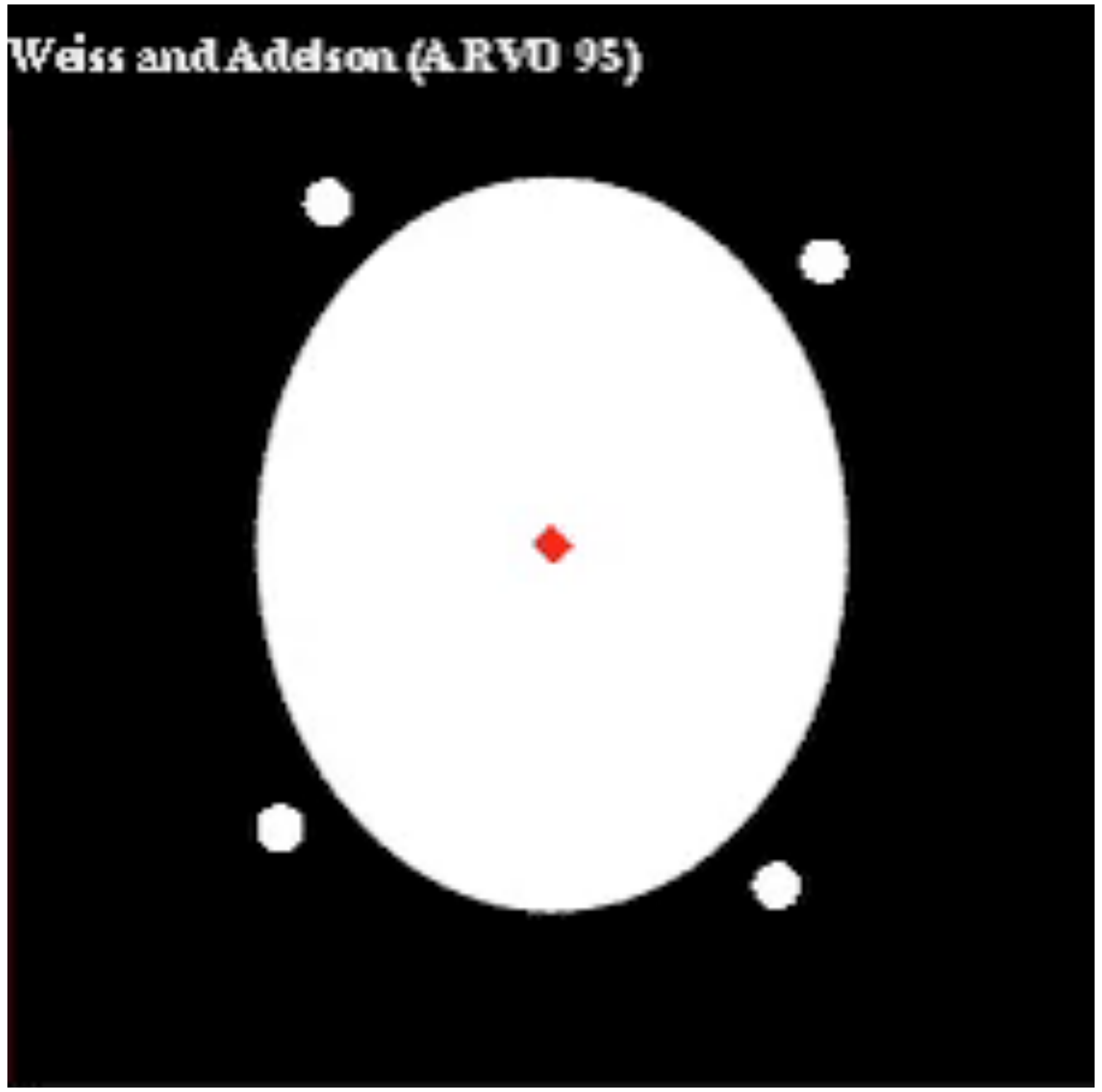
Example 1: Rotating Ellipse

The ellipse's motion can be influenced by features not physically connected to the ellipse. In this sequence the ellipse is always doing the same thing, only the dots' motion changes.



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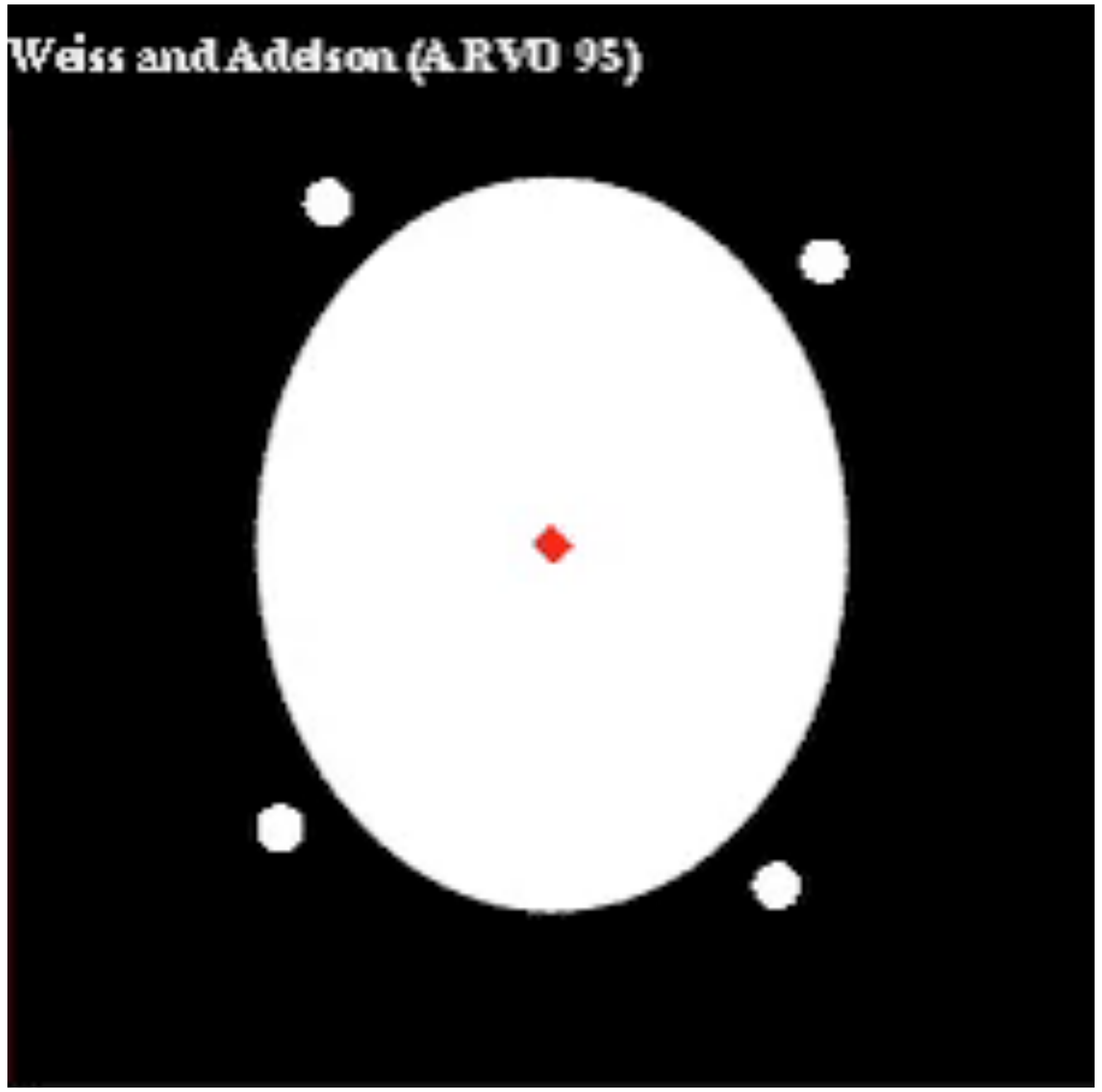

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Example: Flying Insects and Birds

Bees have very limited stereo perception. How do they fly safely through narrow passages?

Example: Flying Insects and Birds

Bees have very limited stereo perception. How do they fly safely through narrow passages?

A simple strategy would be to balance the speeds of motion of the images of the two walls. If wall A is moving faster than wall B, what should you (as a bee) do?

Example: Flying Insects and Birds

Bee strategy: Balance the optical flow experienced by the two eyes

Figure credit: M. Srinivasan

Example: Flying Insects and Birds

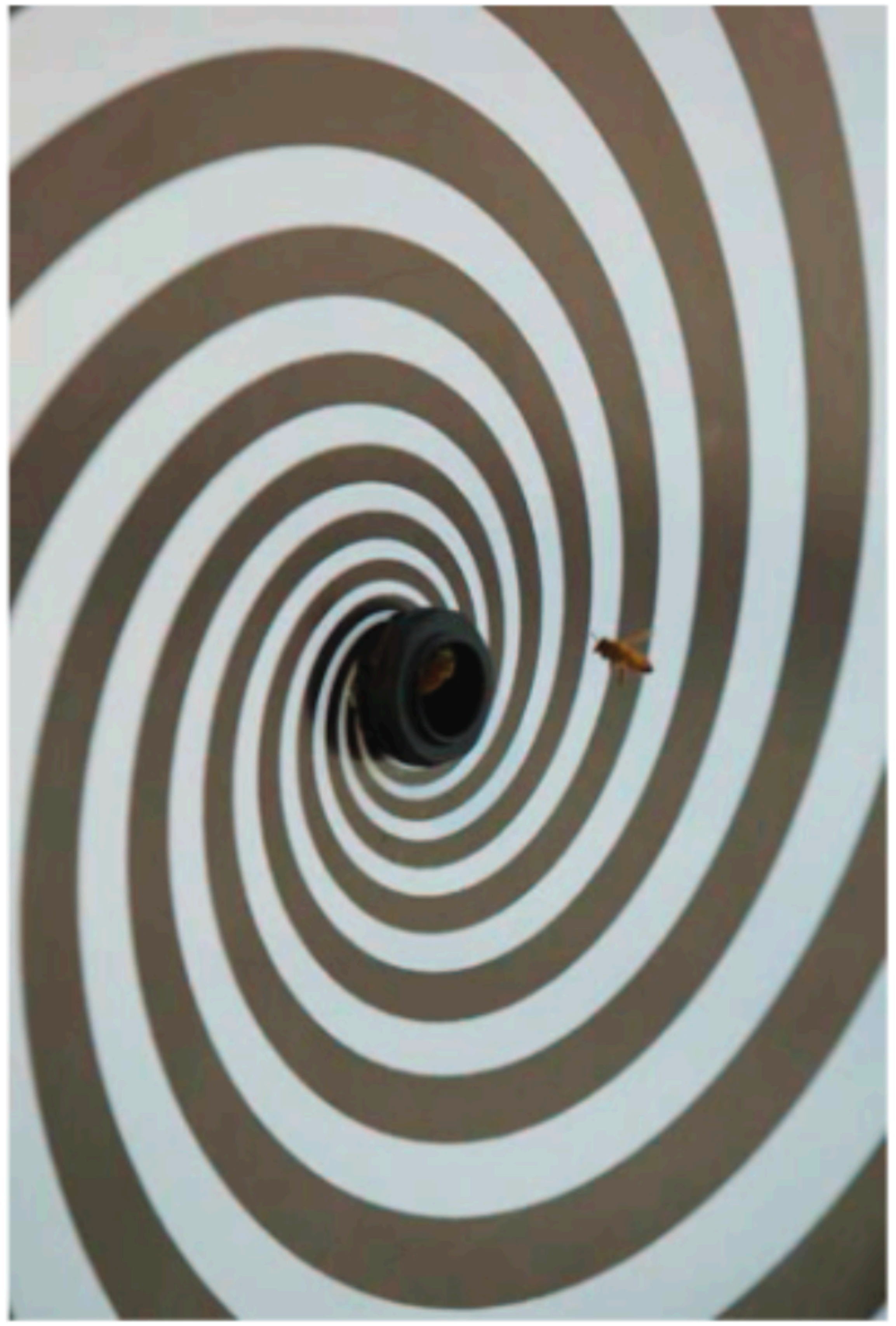
How do bees land safely on surfaces?

During their approach, bees continually adjust their speed to hold constant the optical flow in the vicinity of the target

— approach speed decreases as the target is approached and reduces to zero at the point of touchdown

— no need to estimate the distance to the target at any time

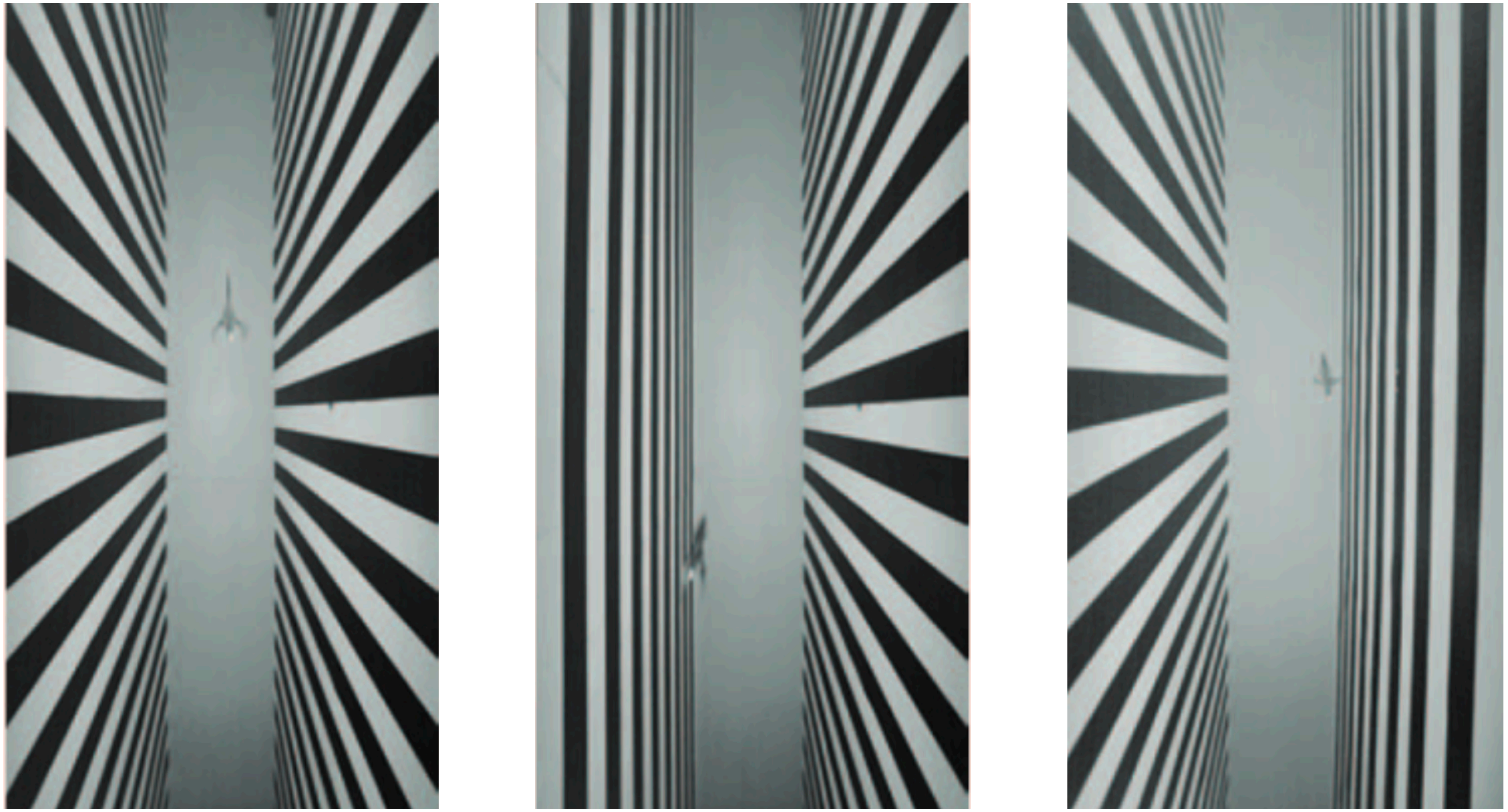
Example: Flying Insects and Birds



Bees approach the surface more slowly if the spiral is rotated to augment the rate of expansion, and more quickly if the spiral is rotated in the opposite direction

Figure credit: M. Srinivasan

Example: Flying Insects and Birds



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Optical Flow **Constraint Equation**

Consider image intensity also to be a function of time, t. We write

$$I(x, y, t)$$

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Scene point moving through image sequence

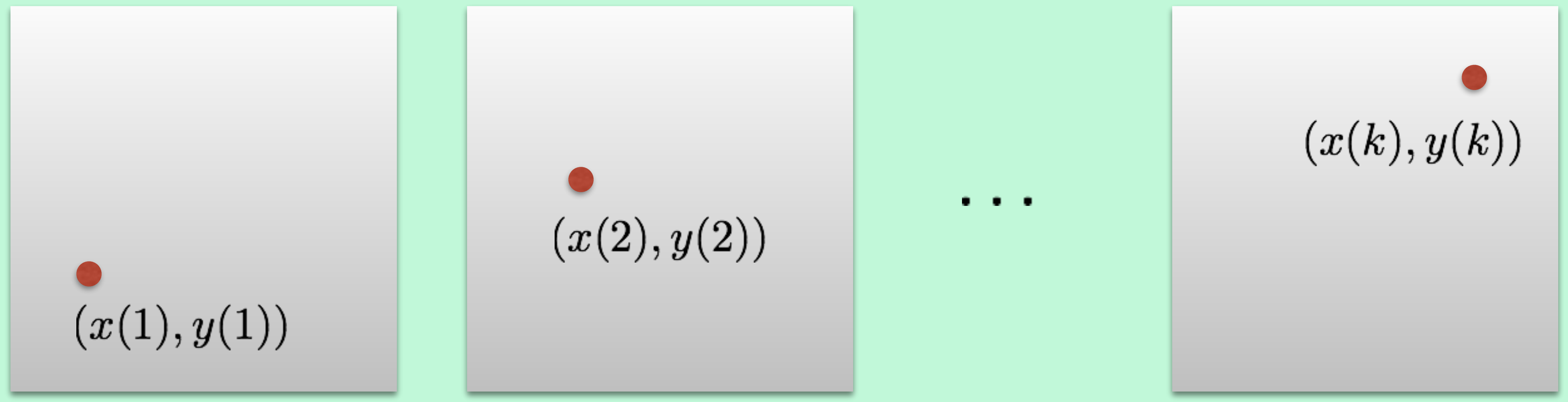

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Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

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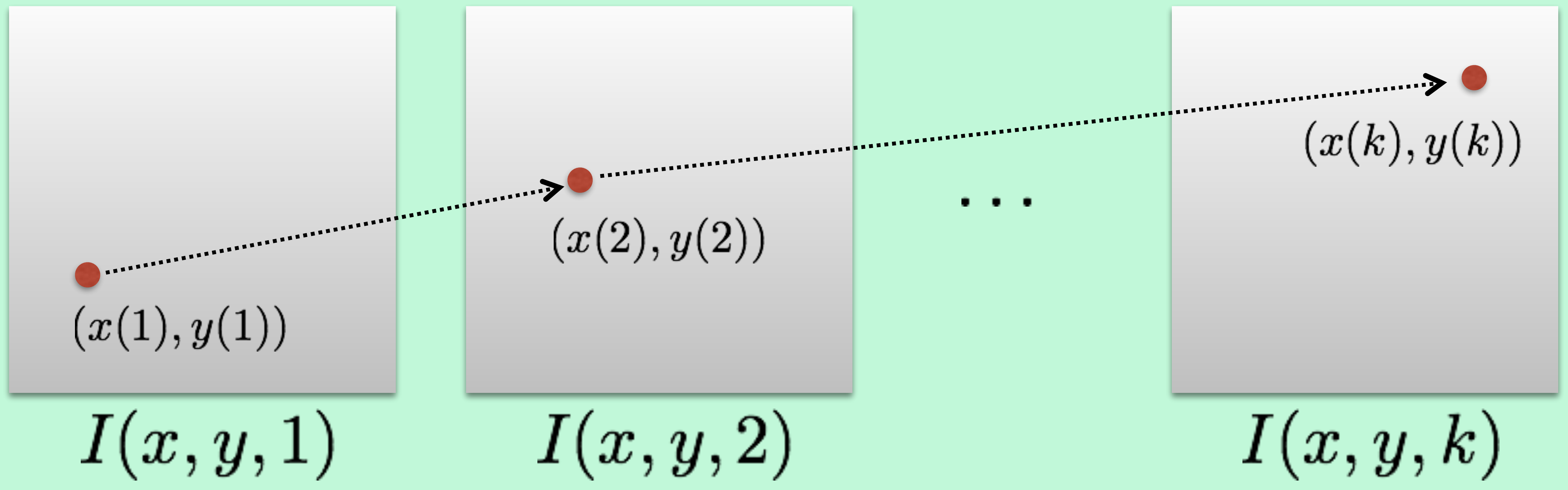

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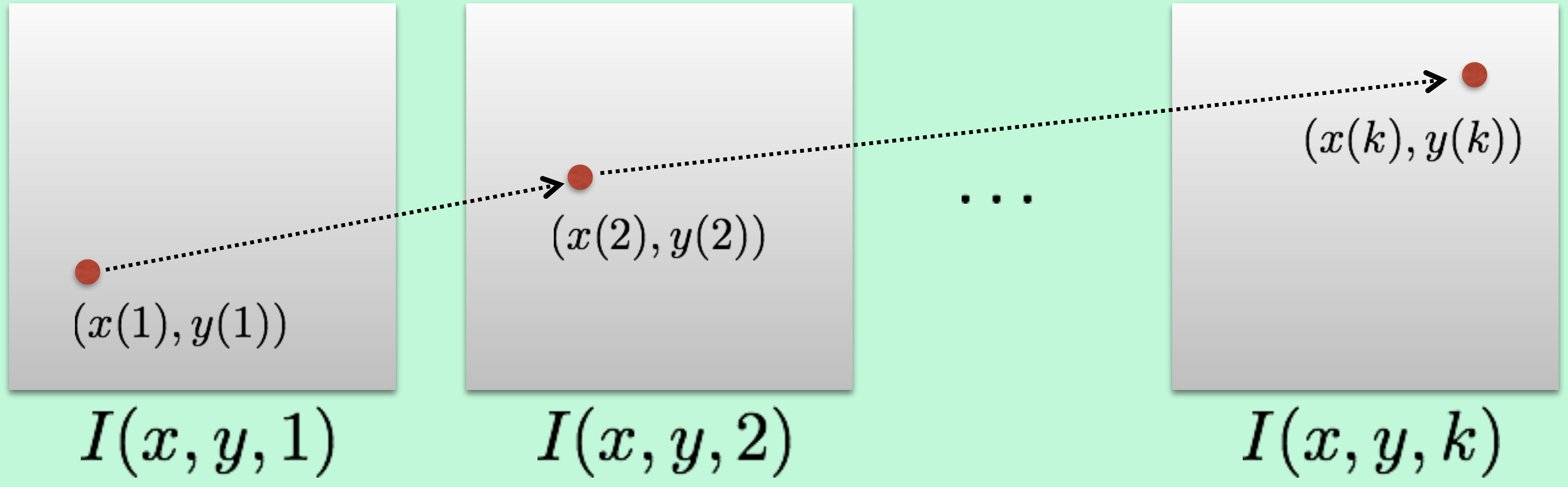
Optical Flow **Constraint Equation**

Brightness Constancy Assumption: Brightness of the point remains the same



$$I(x(t), y(t), t) = C$$

constant

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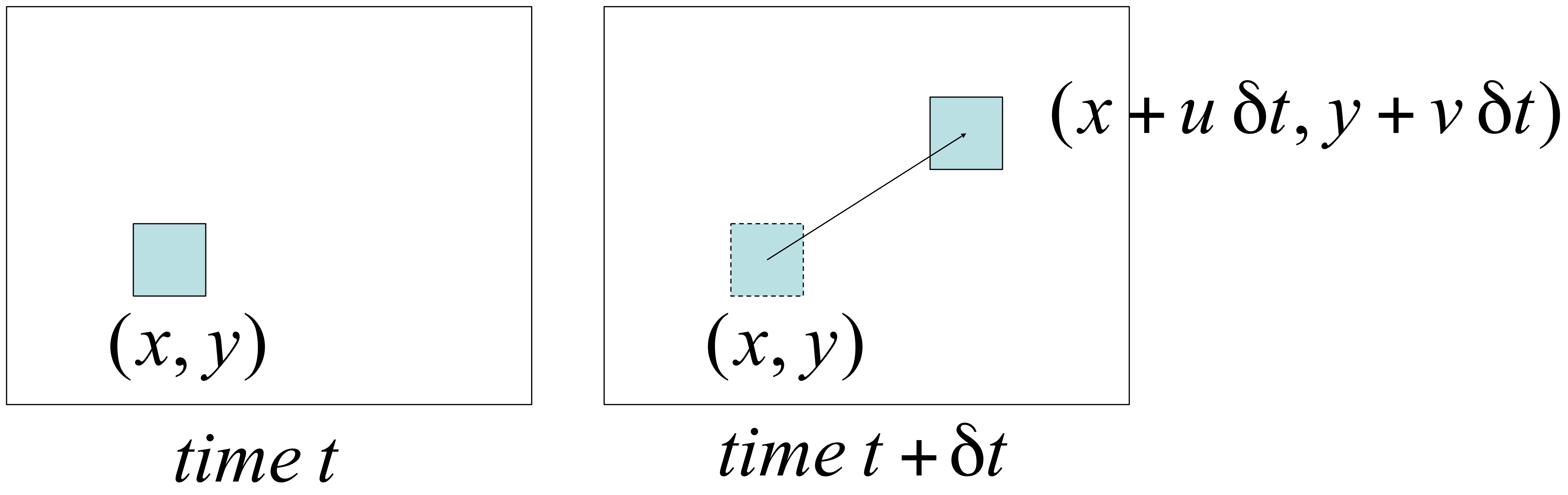
Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

Aside: Derivation of Optical Flow Constraint

$$I(x + u\delta t, y + v\delta t, t + \delta t) = I(x, y, t)$$

For small space-time step, brightness of a point is the same



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Insight:
If the time step is really small,
we can *linearize* the intensity function
(and motion is really-small … think less than a pixel)

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

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Multivariable Taylor Series Expansion

(First order approximation, two variables)

$$f(x,y) \approx f(a,b) + f_x(a,b)(x-a) - f_y(a,b)(y-b)$$

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partial derivative

$$I(x,y,t) + \frac{\partial I}{\partial x}\delta x + \frac{\partial I}{\partial y}\delta y + \frac{\partial I}{\partial t}\delta t = I(x,y,t)$$

fixed point

assuming small motion

cancel terms

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take limit $\delta t \rightarrow 0$

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Brightness Constancy Equation

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

How do we **compute** …

$$I_x u + I_y v + I_t = 0$$

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How do we **compute** …

$$I_x u + I_y v + I_t = 0$$

$$I_x = \frac{\partial I}{\partial x} \quad I_y = \frac{\partial I}{\partial y}$$

spatial derivative

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Forward difference
Sobel filter
Scharr filter
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Frame differencing

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Frame Differencing: Example

$t+1$

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	10	10	10
1	1	10	10	10
1	1	10	10	10

-

t

1	1	1	1	1
1	1	1	1	1
1	10	10	10	10
1	10	10	10	10
1	10	10	10	10
1	10	10	10	10

=

$$I_t = \frac{\partial I}{\partial t}$$

0	0	0	0	0
0	0	0	0	0
0	-9	-9	-9	-9
0	-9	0	0	0
0	-9	0	0	0
0	-9	0	0	0

(example of a forward temporal difference)

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t

1	1	1	1	1
1	1	1	1	1
1	10	10	10	10
1	10	10	10	10
1	10	10	10	10
1	10	10	10	10

$t+1$

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	10	10	10
1	1	10	10	10
1	1	10	10	10

$$I_x = \frac{\partial I}{\partial x}$$

-	0	0	0	-
-	0	0	0	-
-	9	0	0	-
-	9	0	0	-
-	9	0	0	-
-	9	0	0	-

-1 0 1

$$I_y = \frac{\partial I}{\partial y}$$

-	-	-	-	-
0	0	0	0	0
0	9	9	9	9
0	0	0	0	0
0	0	0	0	0
-	-	-	-	-

-1
0
1

$$I_t = \frac{\partial I}{\partial t}$$

0	0	0	0	0
0	0	0	0	0
0	-9	-9	-9	-9
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Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

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spatial derivative

$$u = \frac{dx}{dt} \quad v = \frac{dy}{dt}$$

optical flow

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temporal derivative

Forward difference
Sobel filter
Scharr filter
…

How do you compute this?

Frame differencing

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We need to solve for this!
(this is the unknown in the optical flow problem)

Frame differencing

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Forward difference
Sobel filter
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…

(u, v)
Solution lies on a line

Cannot be found uniquely
with a single constraint

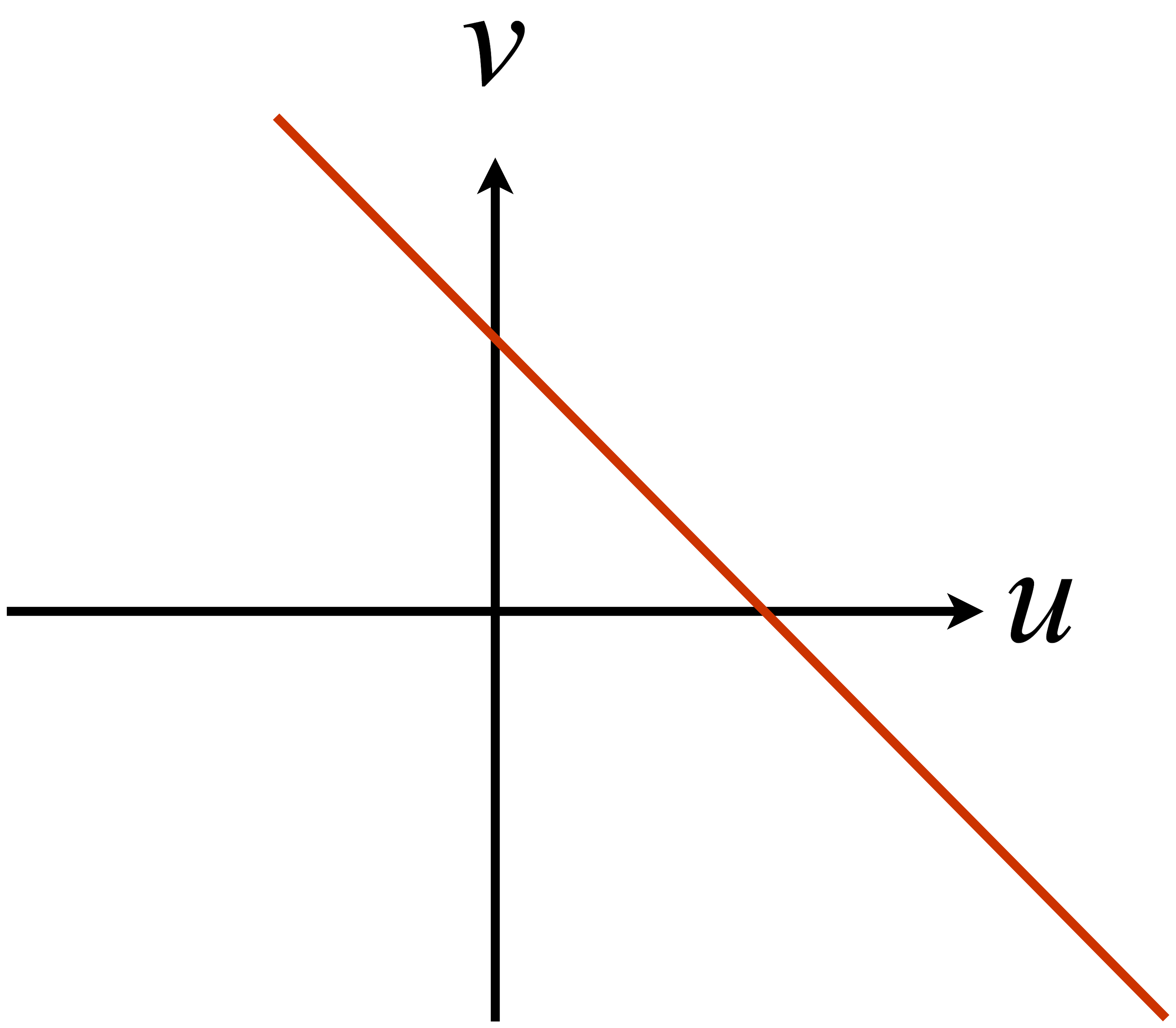
Frame differencing

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Optical Flow **Constraint Equation**

$$I_x u + I_y v + I_t = 0$$

many combinations of u and v will satisfy the equality



Equation determines a **straight line** in velocity space

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Flow **Ambiguity**

- The stripes can be interpreted as moving vertically, horizontally (rotation), or somewhere in between!
- The component of velocity parallel to the edge is unknown

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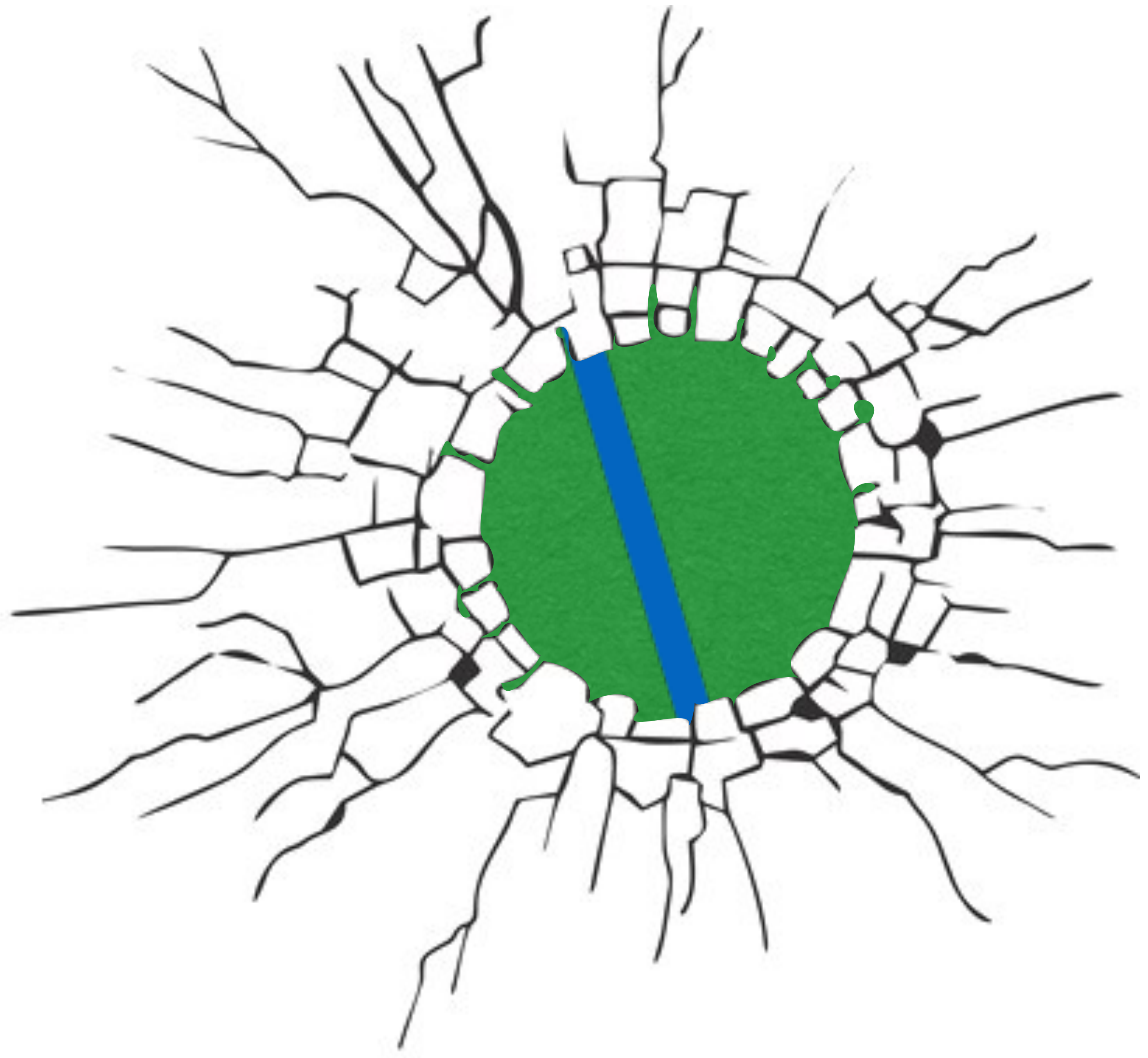
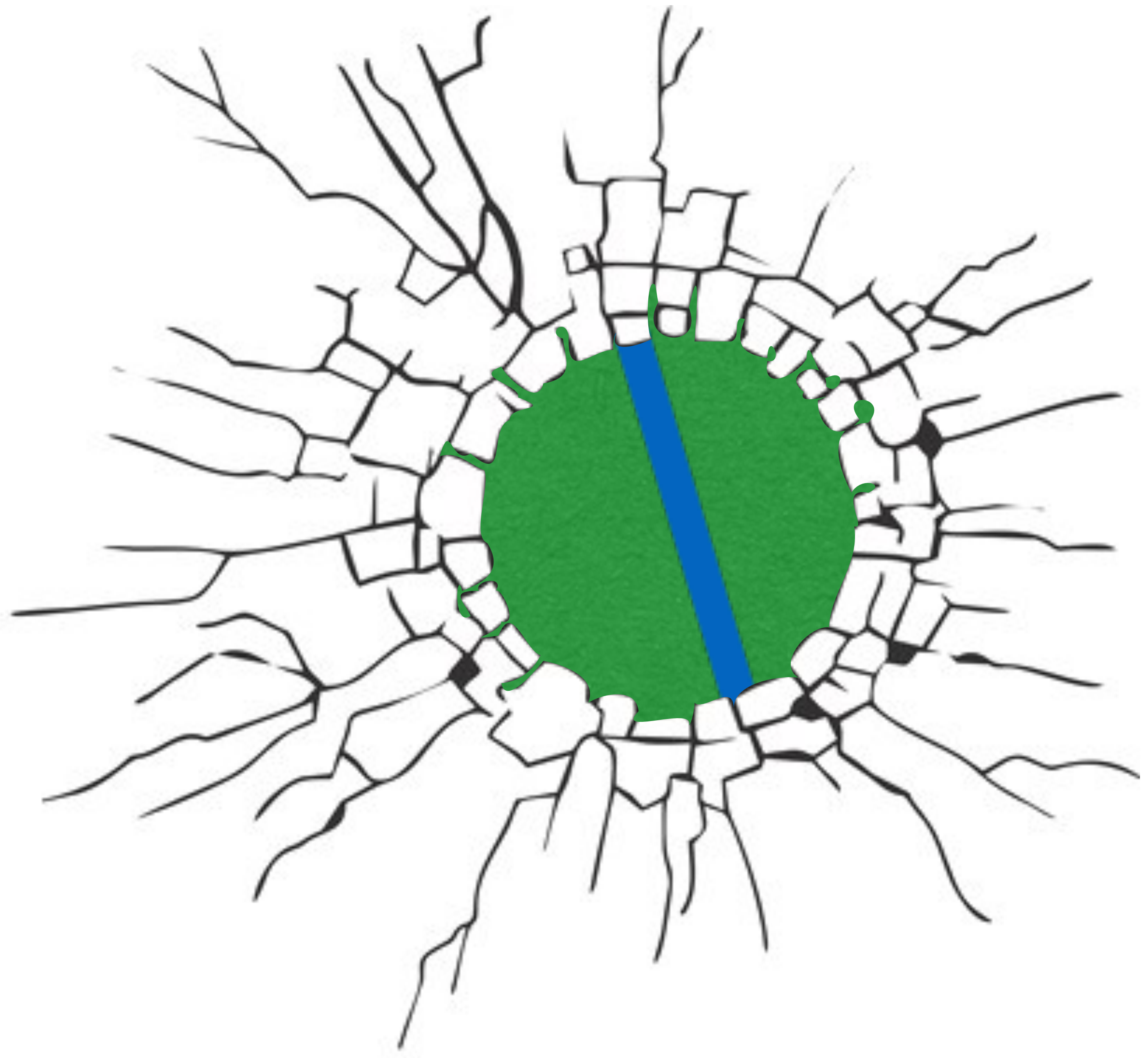
Aperture Problem



In which direction is the line moving?

Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

Aperture Problem



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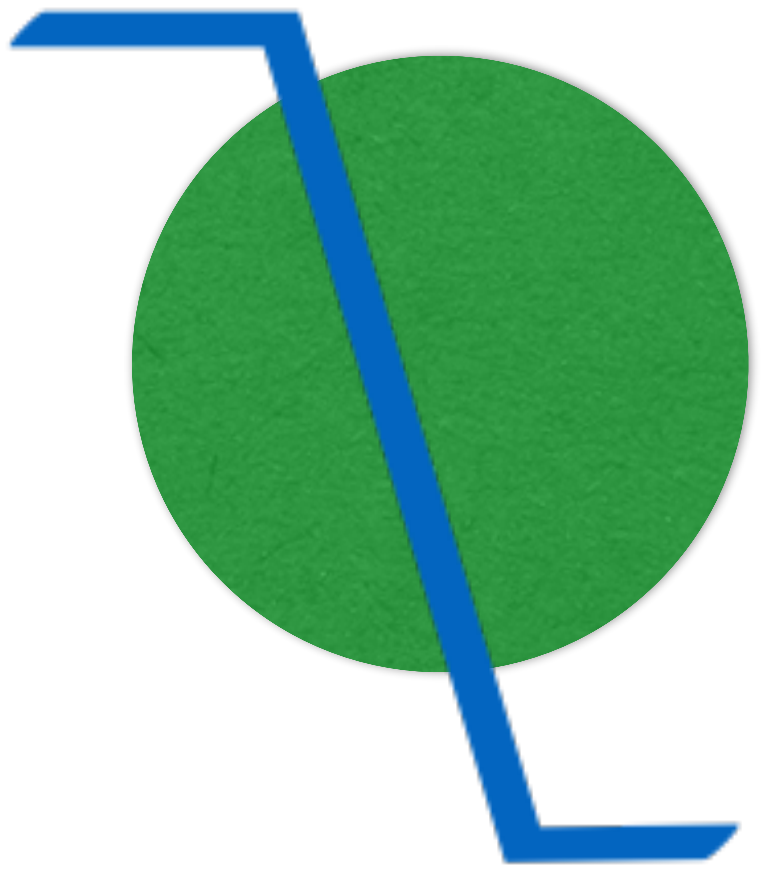

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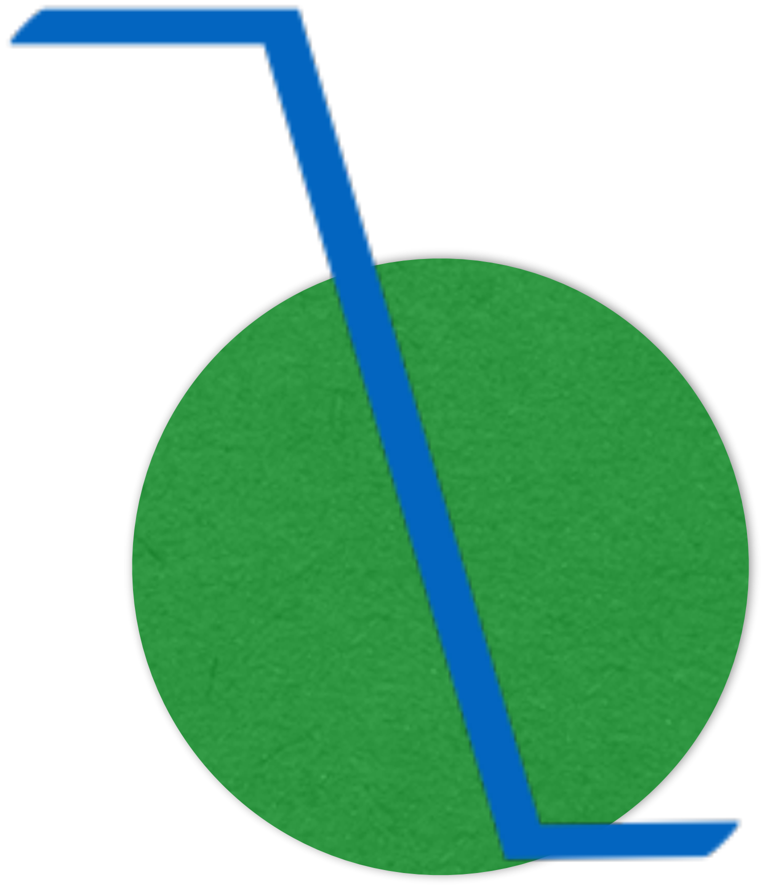

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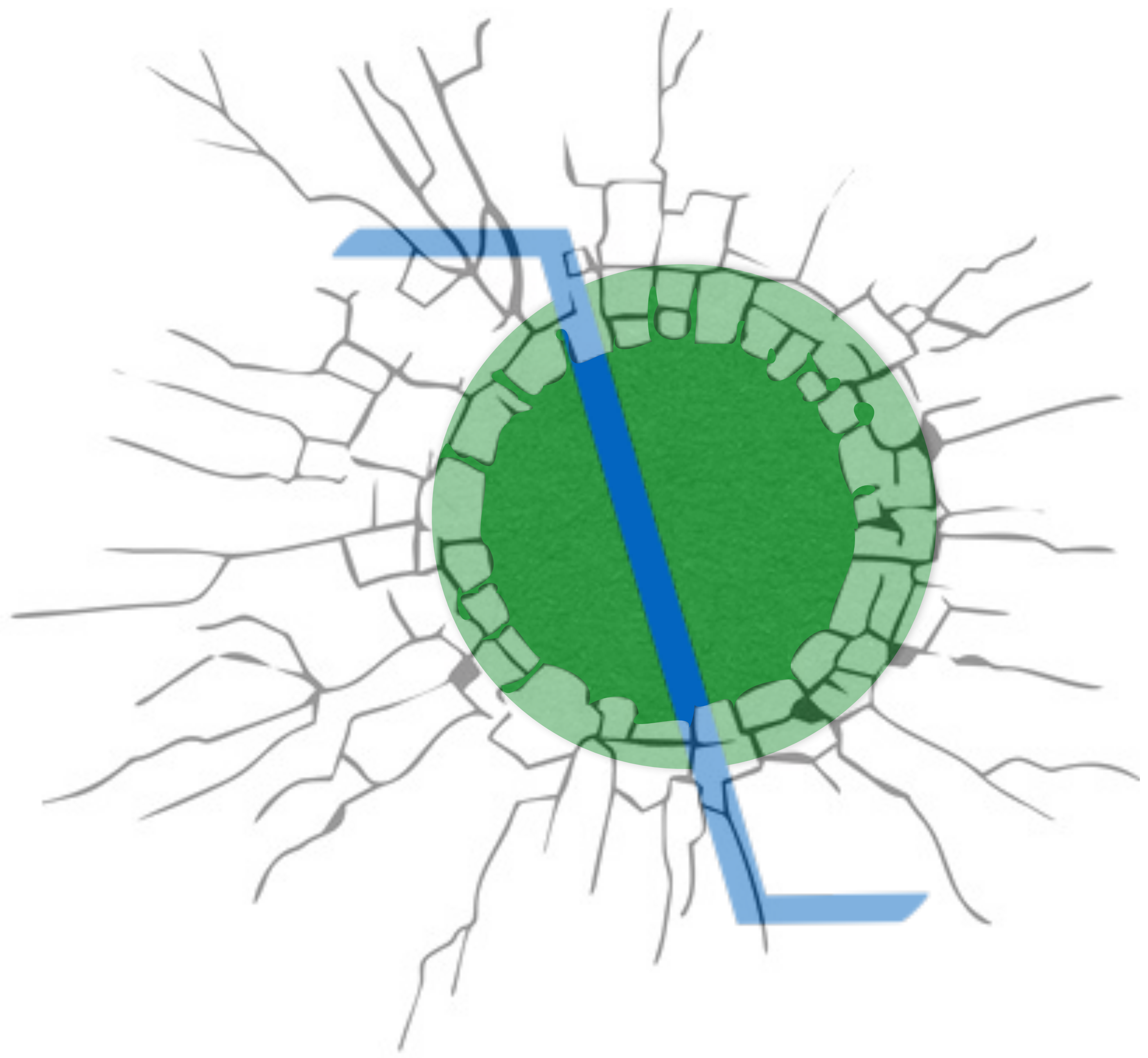

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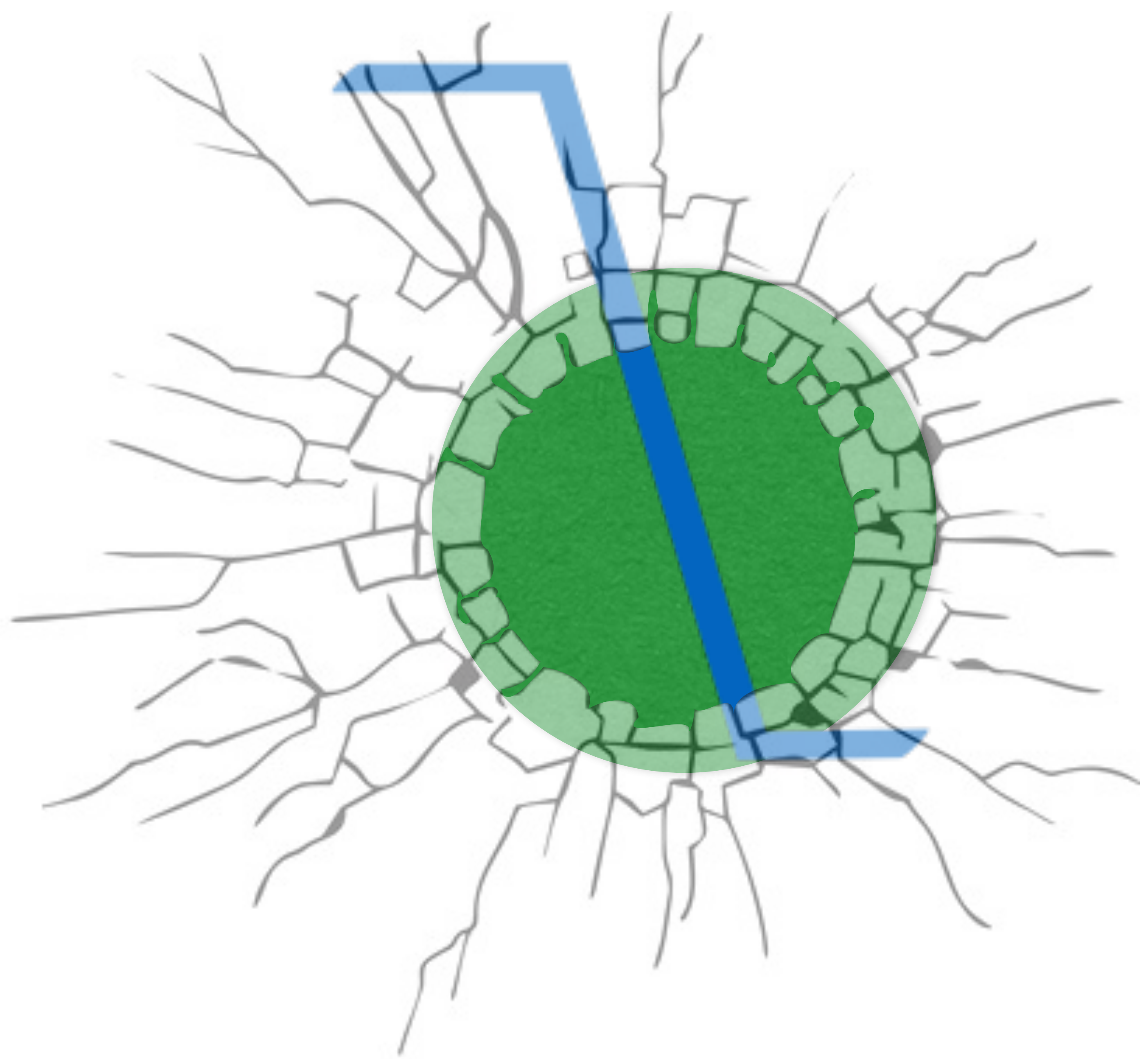

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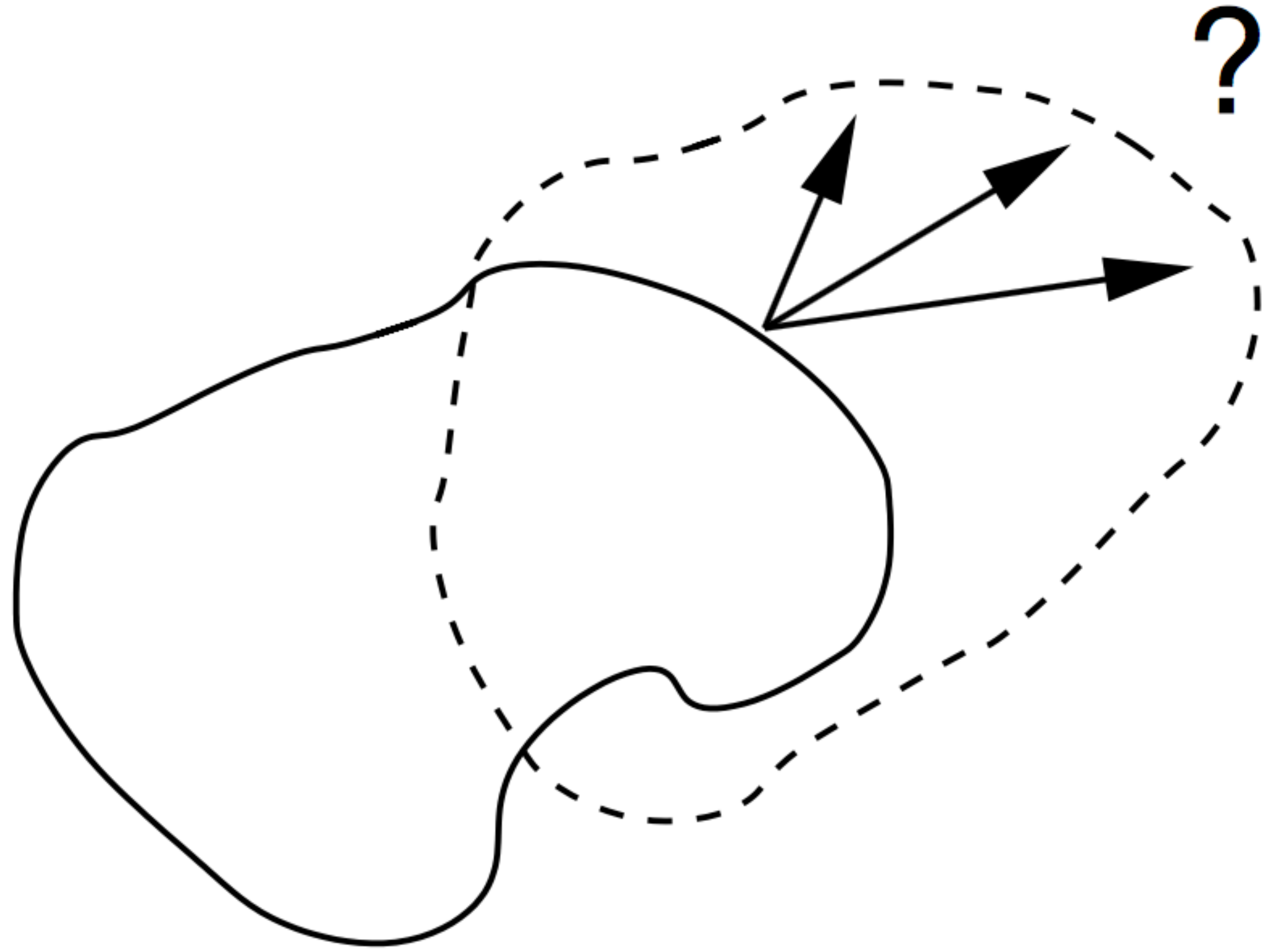

— Without distinct features to track, the true visual motion is ambiguous

— Locally, one can compute only the component of the visual motion in the direction perpendicular to the contour

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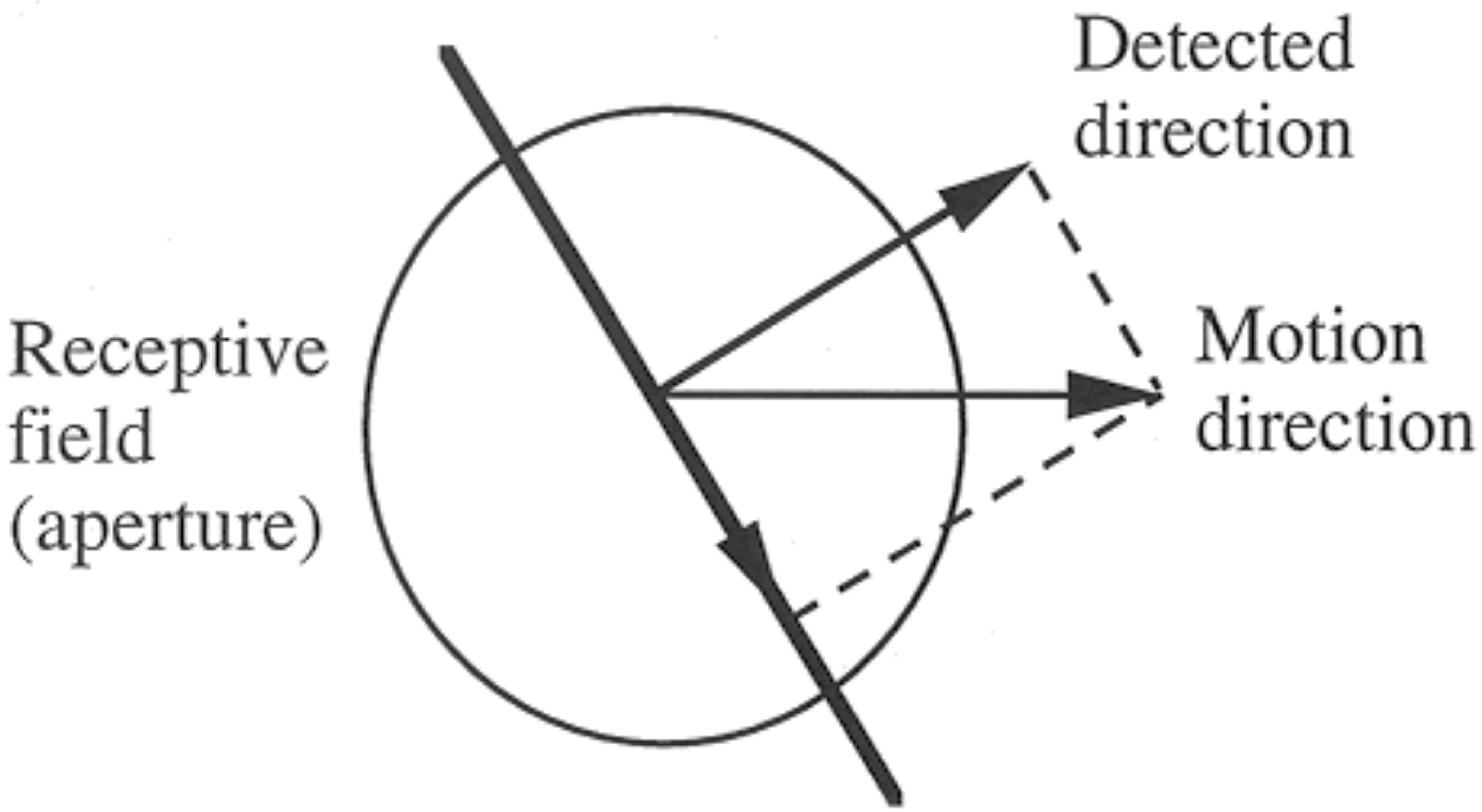

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Lucas-Kanade

Observations:

1. The 2-D motion, $[u, v]$, at a given point, $[x, y]$, has two degrees-of-freedom

2. The partial derivatives, I_x, I_y, I_t, provide one constraint

3. The 2-D motion, $[u, v]$, cannot be determined locally from I_x, I_y, I_t alone

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3. The 2-D motion, $[u, v]$, cannot be determined locally from I_x, I_y, I_t alone

Lucas–Kanade Idea:

Obtain additional local constraint by computing the partial derivatives, I_x, I_y, I_t, in a window centered at the given $[x, y]$

Lucas-Kanade

Observations:

1. The 2-D motion, $[u, v]$, at a given point, $[x, y]$, has two degrees-of-freedom

2. The partial derivatives, I_x, I_y, I_t, provide one constraint

3. The 2-D motion, $[u, v]$, cannot be determined locally from I_x, I_y, I_t alone

Lucas–Kanade Idea:

Obtain additional local constraint by computing the partial derivatives, I_x, I_y, I_t, in a window centered at the given $[x, y]$

Constant Flow Assumption: nearby pixels will likely have same optical flow

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Optical Flow Constraint Equation: $I_x u + I_y v + I_t = 0$

Suppose $[x_1, y_1] = [x, y]$ is the (original) center point in the window. Let $[x_2, y_2]$ be any other point in the window. This gives us two equations that we can write

$$I_{x_1} u + I_{y_1} v = -I_{t_1}$$
$$I_{x_2} u + I_{y_2} v = -I_{t_2}$$

and that can be solved locally for u and v as

$$\begin{bmatrix} u \\ v \end{bmatrix} = -\begin{bmatrix} I_{x_1} & I_{y_1} \\ I_{x_2} & I_{y_2} \end{bmatrix}^{-1} \begin{bmatrix} I_{t_1} \\ I_{t_2} \end{bmatrix}$$

provided that u and v are the same in both equations and provided that the required matrix inverse exists.

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Optical Flow Constraint Equation: $I_x u + I_y v + I_t = 0$

Considering all n points in the window, one obtains

$$\begin{aligned} I_{x_1} u + I_{y_1} v &= -I_{t_1} \\ I_{x_2} u + I_{y_2} v &= -I_{t_2} \\ &\vdots \\ I_{x_n} u + I_{y_n} v &= -I_{t_n} \end{aligned}$$

which can be written as the matrix equation

$$\mathbf{A}\mathbf{v} = \mathbf{b}$$

where $\mathbf{v} = [u, v]^T$, $\mathbf{A} = \begin{bmatrix} I_{x_1} & I_{y_1} \\ I_{x_2} & I_{y_2} \\ \vdots & \vdots \\ I_{x_n} & I_{y_n} \end{bmatrix}$ and $\mathbf{b} = -\begin{bmatrix} I_{t_1} \\ I_{t_2} \\ \vdots \\ I_{t_n} \end{bmatrix}$

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The standard least squares solution, $\bar{\mathbf{v}}$, to is

$$\bar{\mathbf{v}} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b}$$

again provided that u and v are the same in all equations and provided that the rank of $\mathbf{A}^T\mathbf{A}$ is 2 (so that the required inverse exists)

Lucas-Kanade

The standard least squares solution, $\bar{\mathbf{v}}$, to is

$$\begin{bmatrix} I_{x_1} & I_{x_2} & \cdots & I_{x_n} \\ I_{y_1} & I_{y_2} & \cdots & I_{y_n} \end{bmatrix} \quad \begin{bmatrix} I_{x_1} & I_{y_1} \\ I_{x_2} & I_{y_2} \\ \vdots & \vdots \\ I_{x_n} & I_{y_n} \end{bmatrix}$$

$$\bar{\mathbf{v}} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b}$$

again provided that u and v are the same in all equations and provided that the rank of $\mathbf{A}^T\mathbf{A}$ is 2 (so that the required inverse exists)

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Note that we can explicitly write down an expression for $\mathbf{A}^T\mathbf{A}$ as

$$\mathbf{A}^T\mathbf{A} = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & I_y^2 \end{bmatrix}$$

which is identical to the matrix $\mathbf{C}$ that we saw in the context of Harris corner detection

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What does that mean?

Lucas-Kanade **Summary**

A dense method to compute motion, $[u, v]$ at every location in an image

Key Assumptions:

1. Motion is slow enough and smooth enough that differential methods apply (i.e., that the partial derivatives, I_x, I_y, I_t, are well-defined)

2. The optical flow constraint equation holds (i.e., $\frac{dI(x,y,t)}{dt} = 0$)

3. A window size is chosen so that motion, $[u, v]$, is constant in the window

4. A window size is chosen so that the rank of $\mathbf{A}^T\mathbf{A}$ is 2 for the window

Aside: Optical Flow Smoothness Constraint

Many methods trade off a ‘departure from the optical flow constraint’ cost with a ‘departure from smoothness’ cost.

The optimization objective to minimize becomes

$$E = \int \int (I_x u + I_y v + I_t)^2 + \lambda(|| \bigtriangledown u||^2 + || \bigtriangledown v||^2)$$

where λ is a weighing parameter.

Horn-**Schunck** Optical Flow

smoothness brightness constancy

$$\min_{\boldsymbol{u},\boldsymbol{v}} \sum_{i,j} \left\{ E_s(i,j) + \lambda E_d(i,j) \right\}$$

weight

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

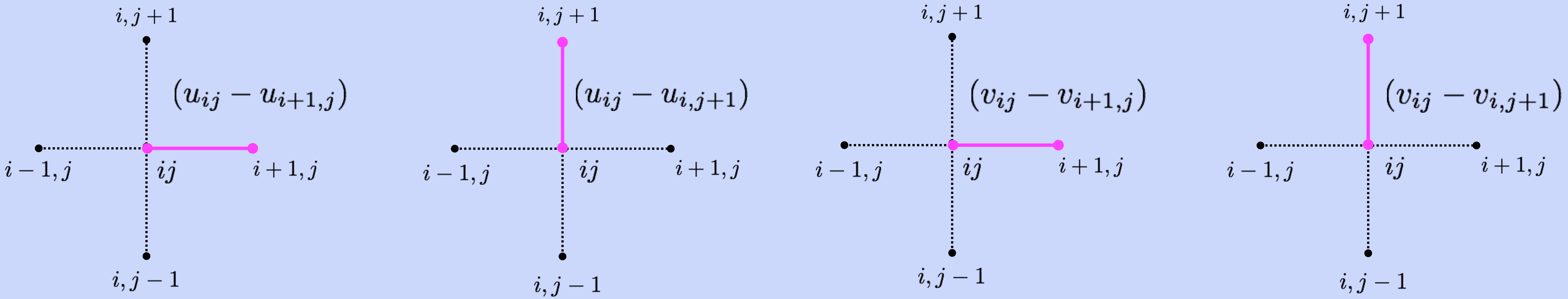
Horn-Schunck Optical Flow

Brightness constancy

$$E_d(i,j) = \left[I_x u_{ij} + I_y v_{ij} + I_t \right]^2$$

Smoothness

$$E_s(i,j) = \frac{1}{4}\left[(u_{ij} - u_{i+1,j})^2 + (u_{ij} - u_{i,j+1})^2 + (v_{ij} - v_{i+1,j})^2 + (v_{ij} - v_{i,j+1})^2 \right]$$



Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Optical Flow and 2D Motion

Motion is geometric, **Optical flow** is radiometric

Usually we assume that optical flow and 2-D motion coincide ... but this is not always the case!

Optical flow with **no motion:**

. . . moving light source(s), lights going on/off, inter-reflection, shadows

Motion with **no optical flow:**

. . . spinning cylinder, sphere.

Optical Flow **Summary**

Motion, like binocular stereo, can be formulated as a matching problem. That is, given a scene point located at (x_0, y_0) in an image acquired at time t_0, what is its position, (x_1, y_1), in an image acquired at time t_1?

Assuming image intensity does not change as a consequence of motion, we obtain the (classic) **optical flow constraint equation**

$$I_x u + I_y v + I_t = 0$$

where $[u, v]$, is the 2-D motion at a given point, $[x, y]$, and I_x, I_y, I_t are the partial derivatives of intensity with respect to x, y, and t

Lucas–Kanade is a dense method to compute the motion, $[u, v]$, at every location in an image