

CPSC 425: Computer Vision

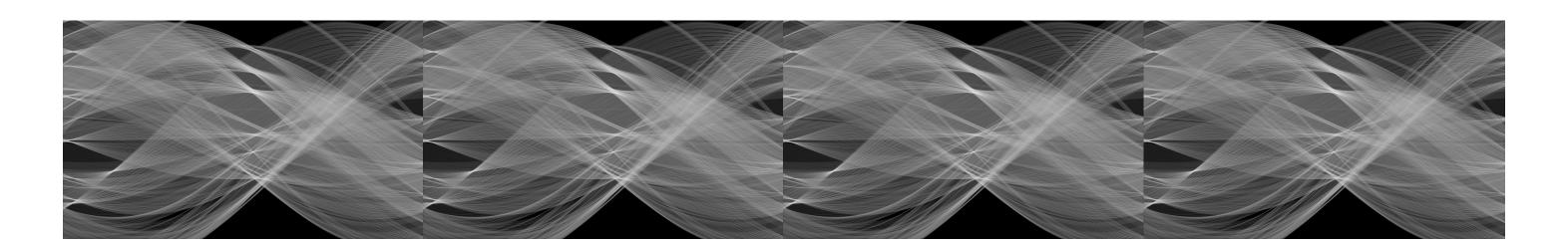


Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

Lecture 15: Hough Transform

Menu for Today

Topics:

- Hough Transform
- Transformation Space Voting

Line Detection

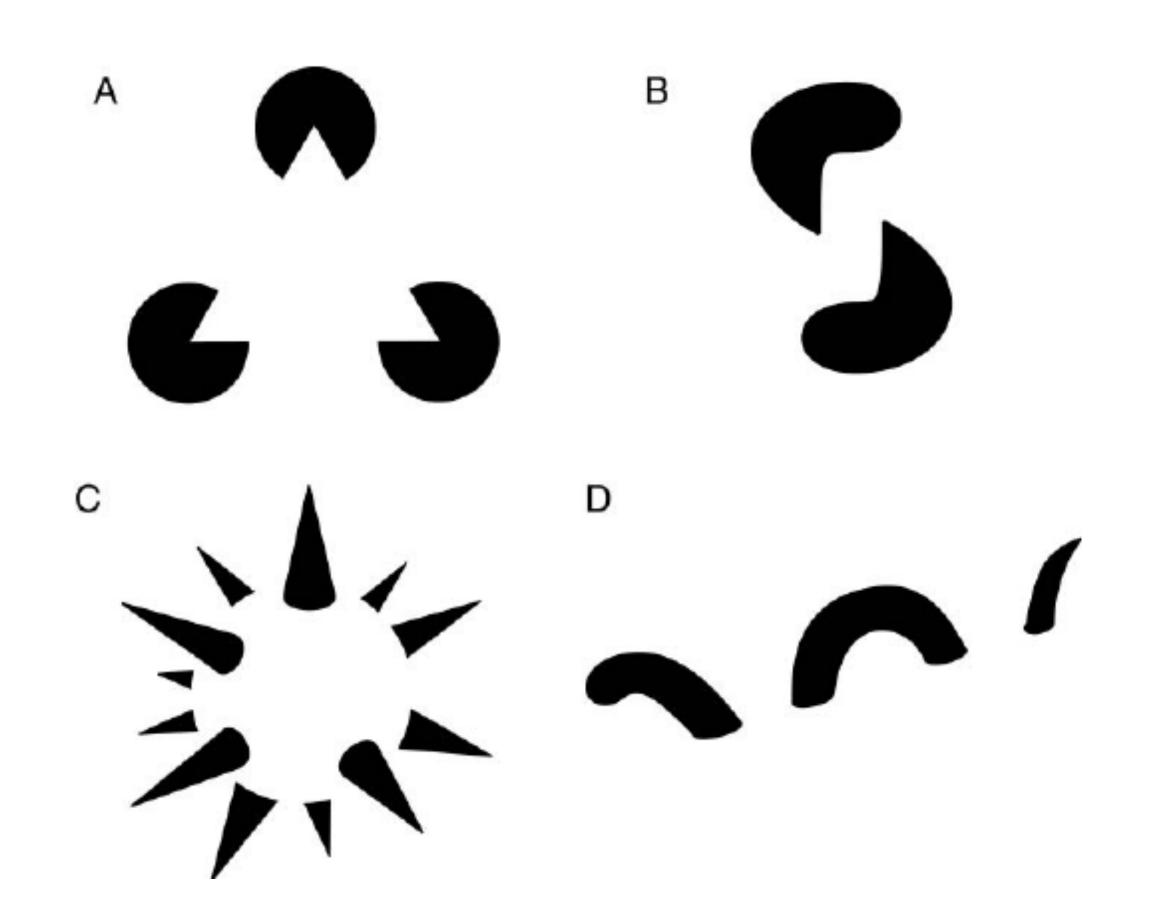
Readings:

- Today's Lecture: Szeliski 7.4, Forsyth & Ponce 10.1

Reminders:

Assignment 4: RANSAC and Panorama Stitching — now available

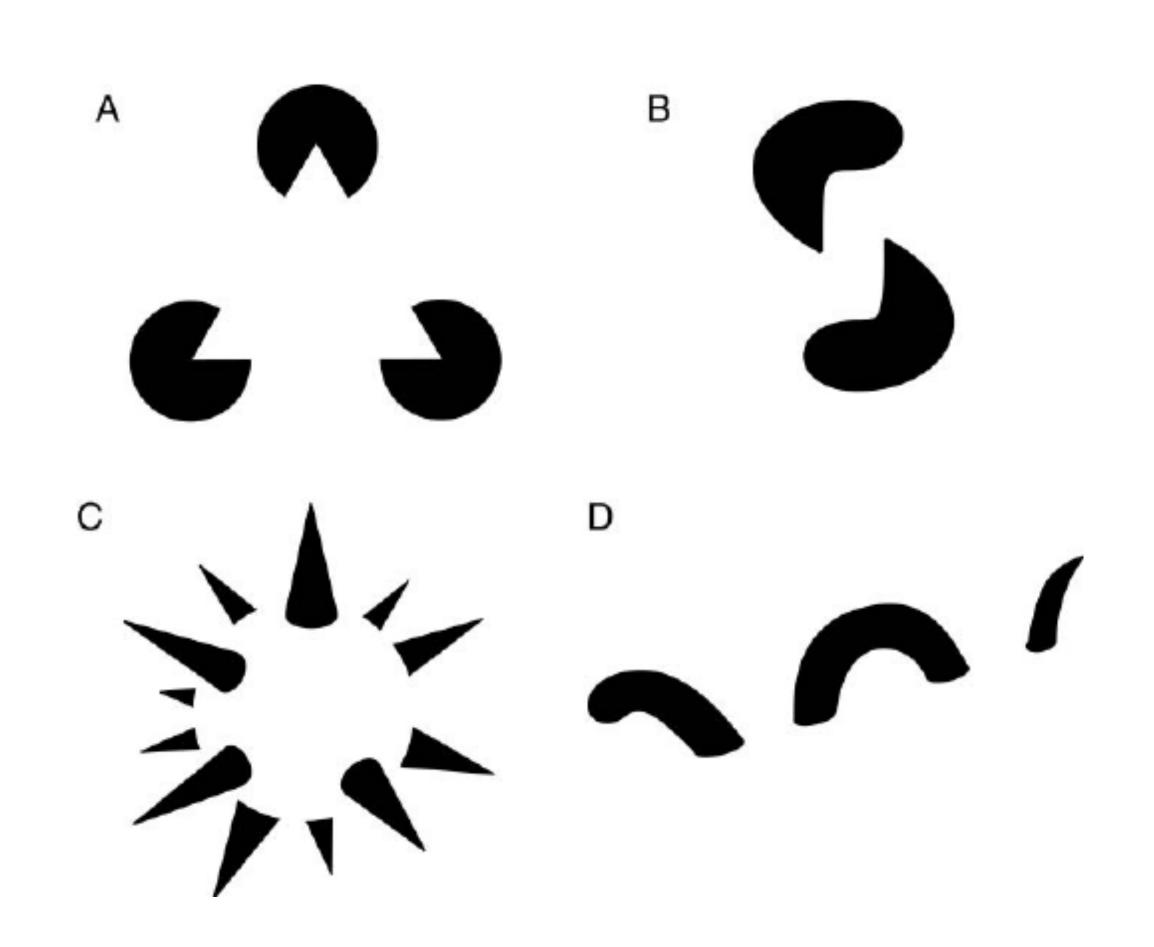
Today's "fun" Example: Tse's Volumetric Illusions

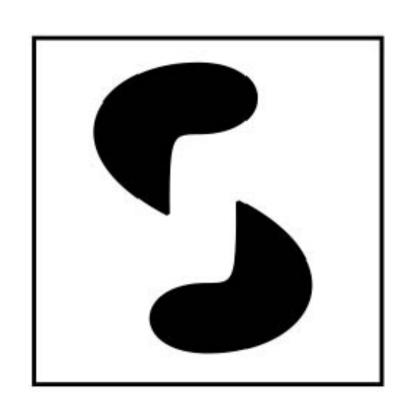


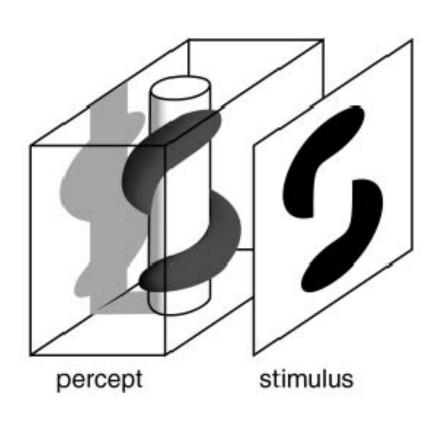
- A. Kanizsa triangle
- B. Tse's volumetric worm
- C. Idesawa's spiky sphere
- D. Tse's "sea monster"

Figure credit: Steve Lehar

Today's "fun" Example: Tse's Volumetric Illusions







Today's "fun" Example: FedEx



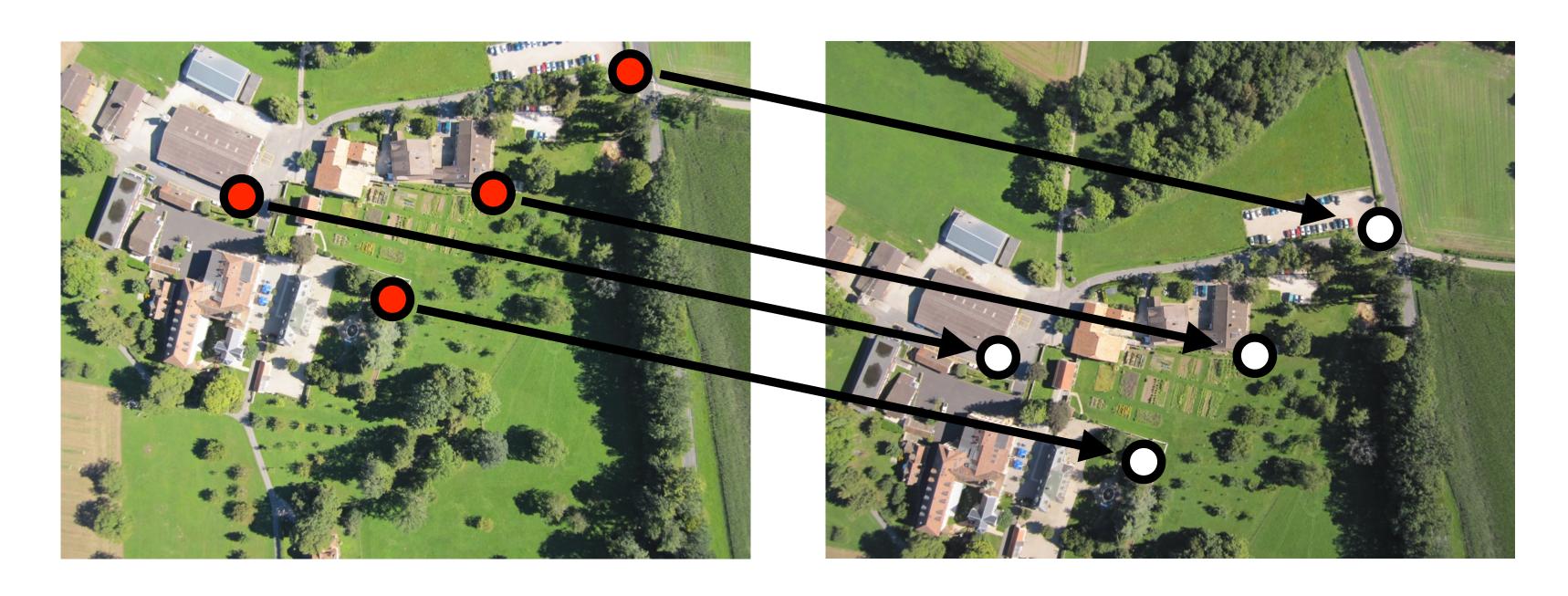
Image Alignment

Aim: Warp one image to align with another using a 2D transformation



Image Alignment

Step 1: Find correspondences (matching points) across two images



u = Hx

2 points for Similarity

3 for Affine

4 for Homography

Image Alignment

Step 2: Compute the transformation to align the two images



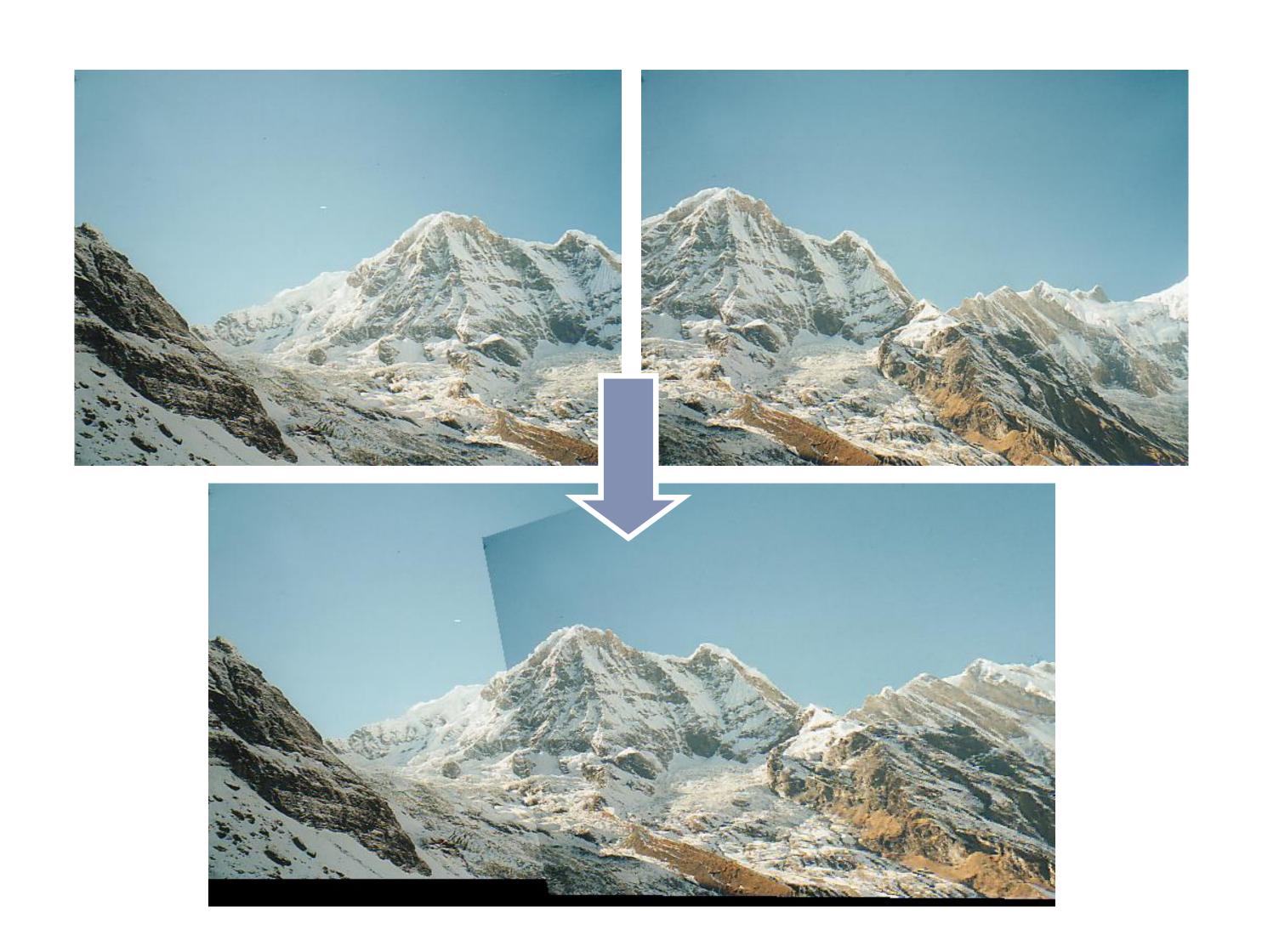
RANSAC (RANdom SAmple Consensus)

- 1. Randomly choose minimal subset of data points necessary to fit model (a **sample**)
- 2. Points within some distance threshold, t, of model are a **consensus set**. Size of consensus set is model's **support**
- 3. Repeat for N samples; model with biggest support is most robust fit
 - Points within distance t of best model are inliers
 - Fit final model to all inliers

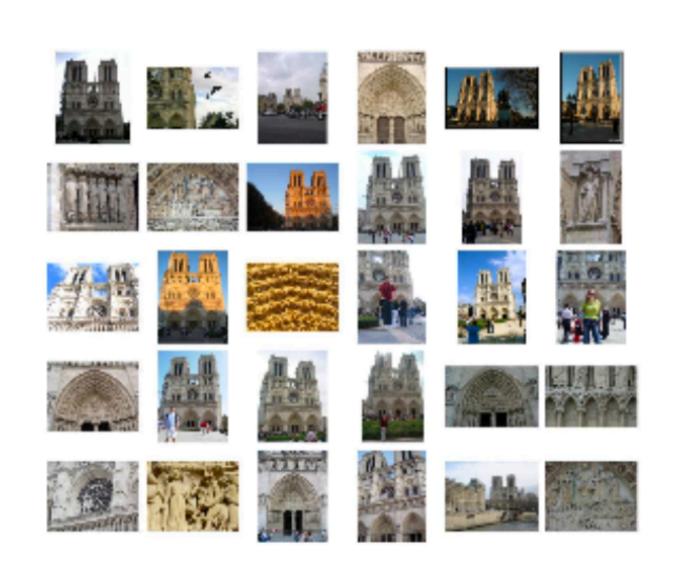
RANSAC is very useful for variety of applications

2-view Rotation Estimation

Final rotation estimation



Example: Photo Tourism



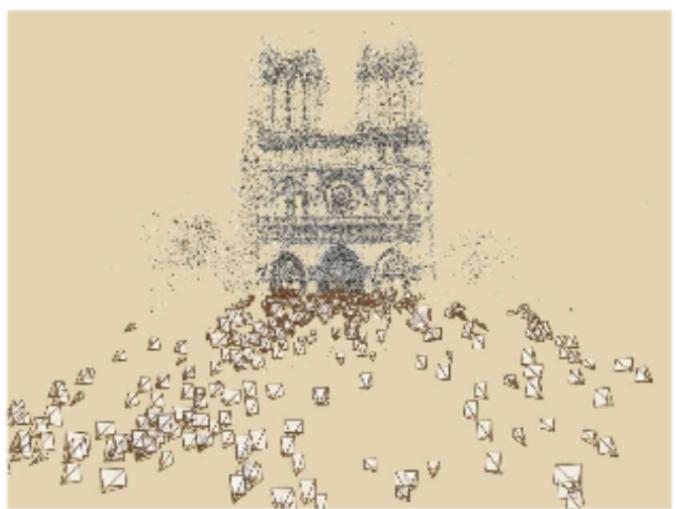




Figure credit: Snavely et al. 2006

Takes as input unstructured collections of photographs and reconstructs each photo's viewpoint and a sparse 3D model of the scene

Uses both SIFT and RANSAC

Example: Photo Tourism



[Agarwal, Furukawa, Snavely, Curless, Seitz, Szeliski, 2010]

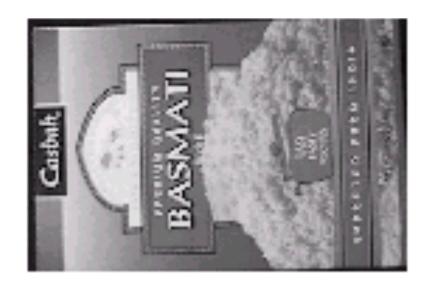
Example: Photo Tourism



[Agarwal, Furukawa, Snavely, Curless, Seitz, Szeliski, 2010]

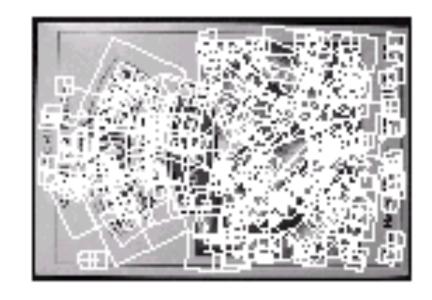
Object Instance Recognition

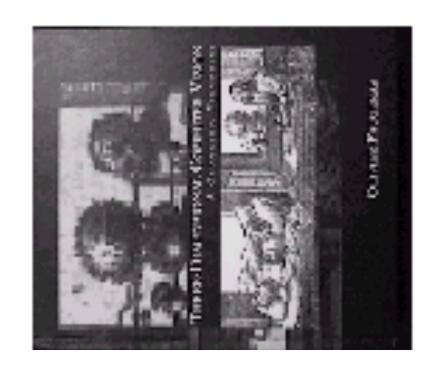
Database of planar objects

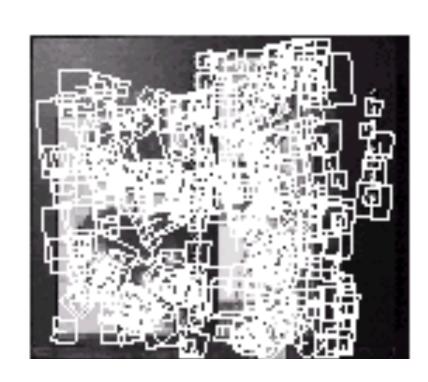












Instance recognition





Discussion of RANSAC

Advantages:

- General method suited for a wide range of model fitting problems
- Easy to implement and easy to calculate its failure rate

Disadvantages:

- Only handles a moderate percentage of outliers without cost blowing up
- Many real problems have high rate of outliers (but sometimes selective choice of random subsets can help)
- Hard to deal with multiple solutions (e.g., object detection with many objects)

The Hough transform can handle high percentage of outliers

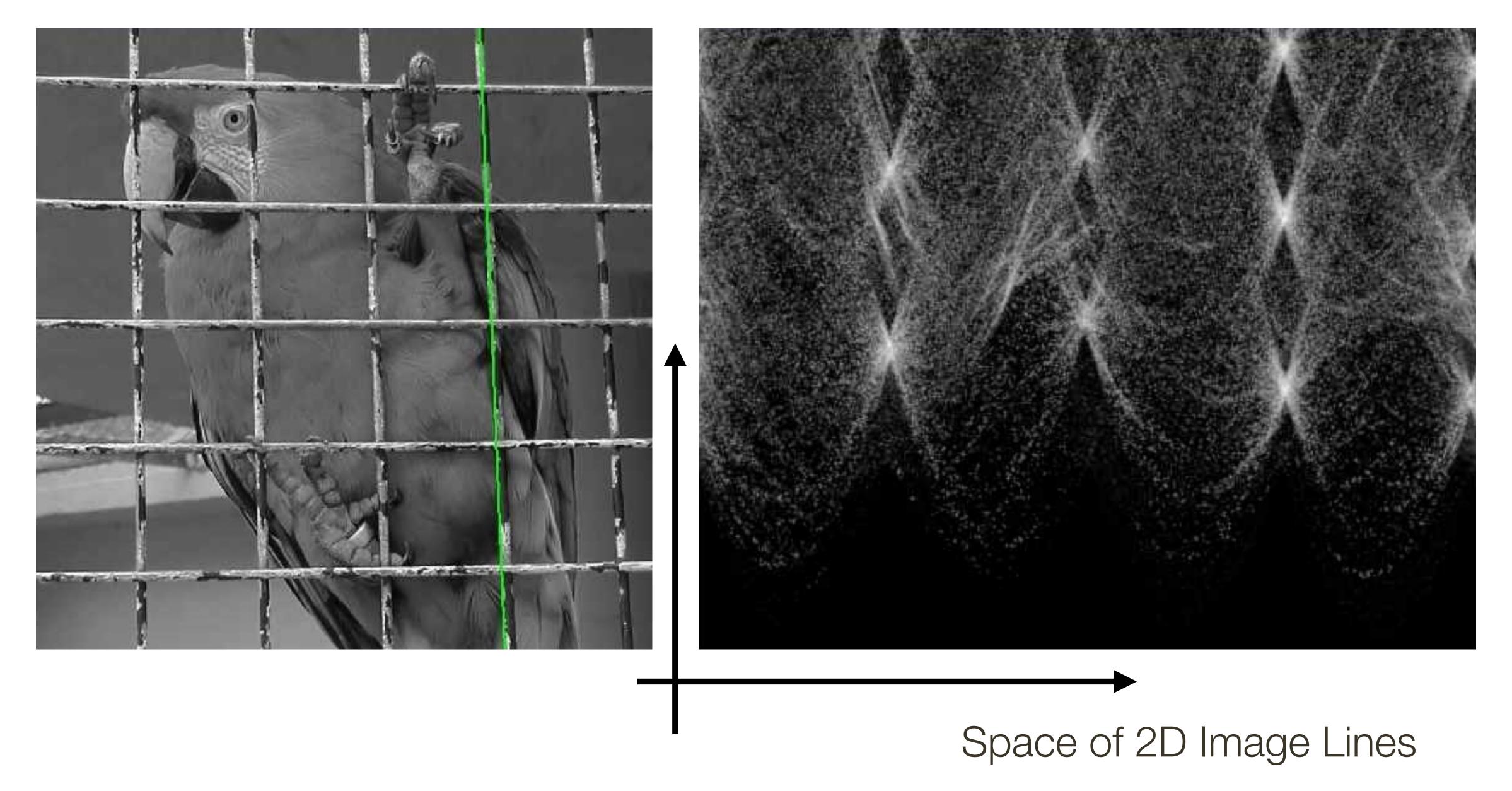
Hough Transform: Motivation



How to find lines in this image?

Hough Transform: Motivation

Votes / Probability Distribution



Hough Transform

Idea of Hough transform:

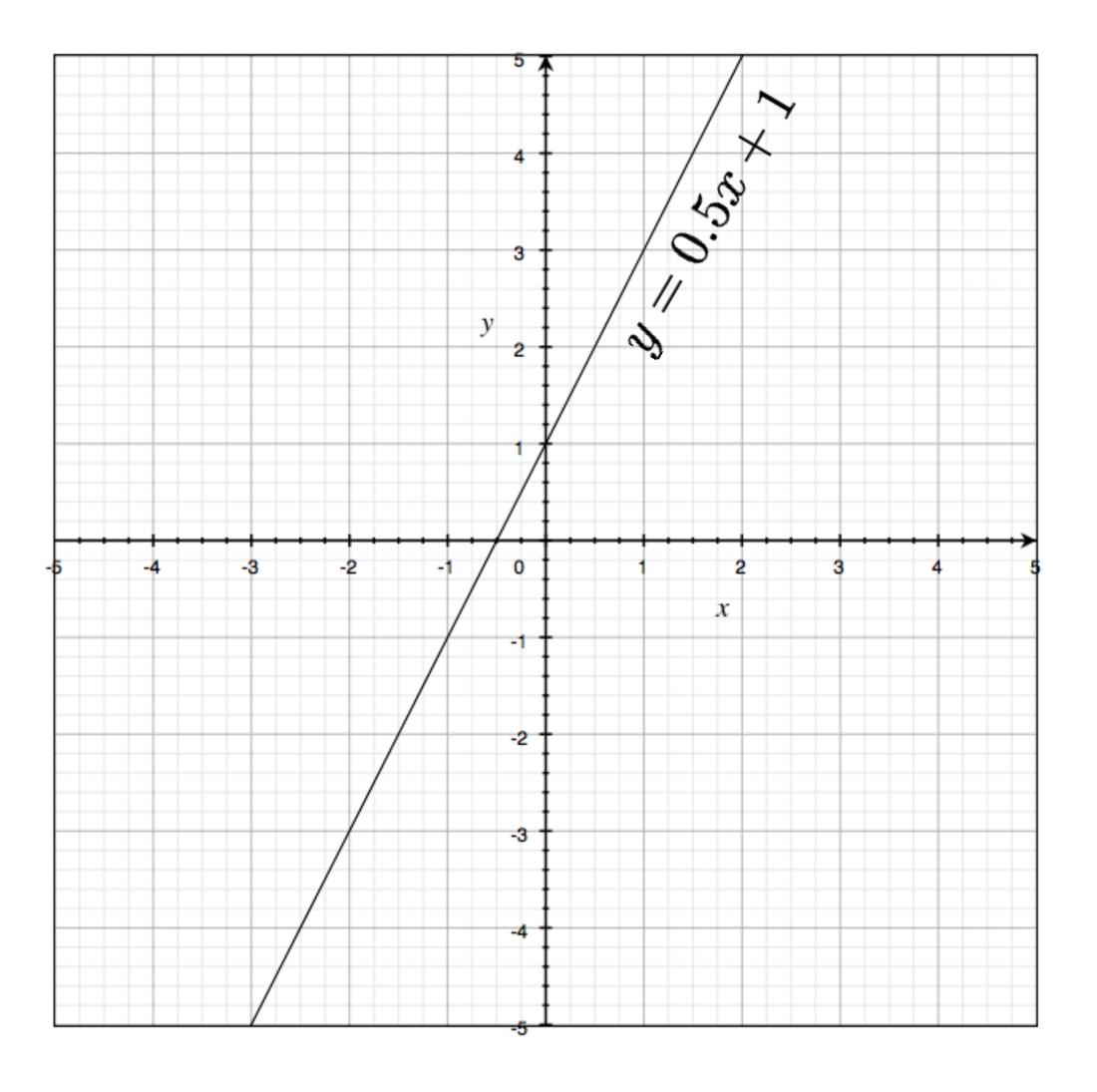
- For each token / data point vote for all models to which it could belong
- Return models that get many votes / distribution of possible models

Example: For each point, vote for all lines that could pass through it; the true lines will pass through many points and so receive many votes

c.f. **RANSAC** which optimizes a **single hypothesis** by maximizing the number of inliers (though modifications exist to find multiple instances of a model)

Lines: Slope intercept form

$$y=mx+b$$
slope y-intercept



Hough Transform: Image and Parameter Space

variables
$$y = mx + b$$

$$y = mx + b$$
parameters

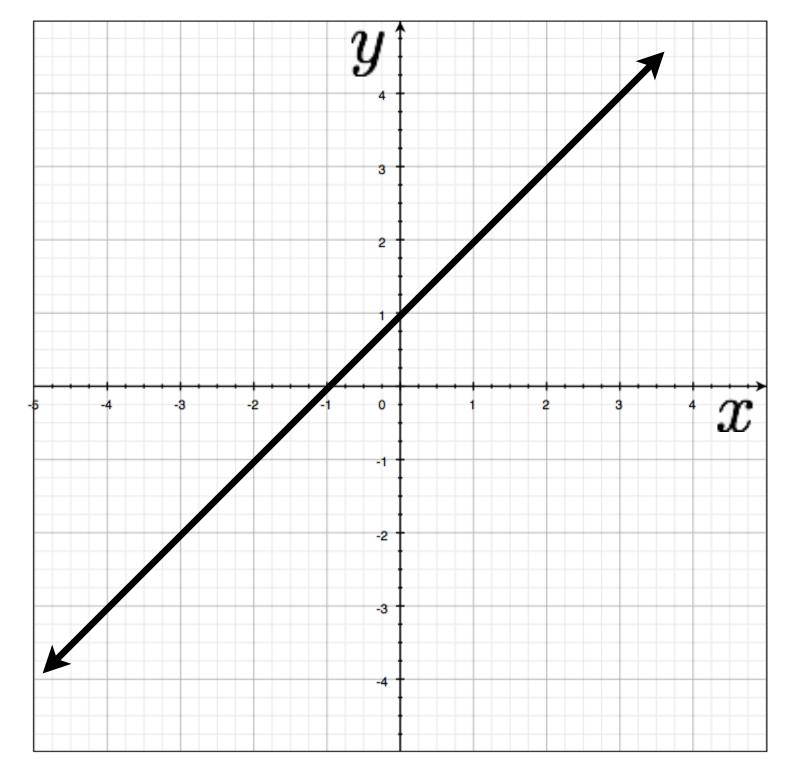


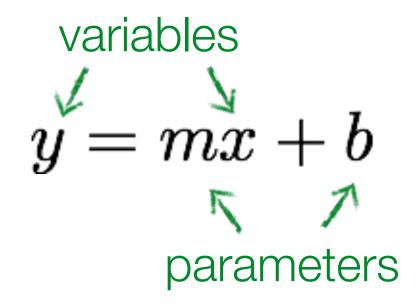
Image space

Hough Transform: Image and Parameter Space

a line

becomes a

point



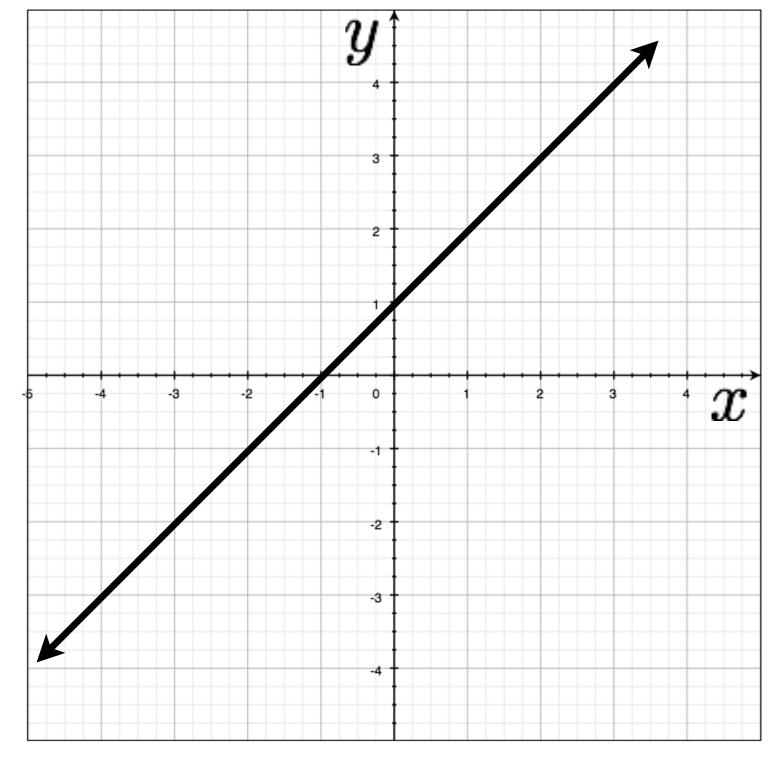
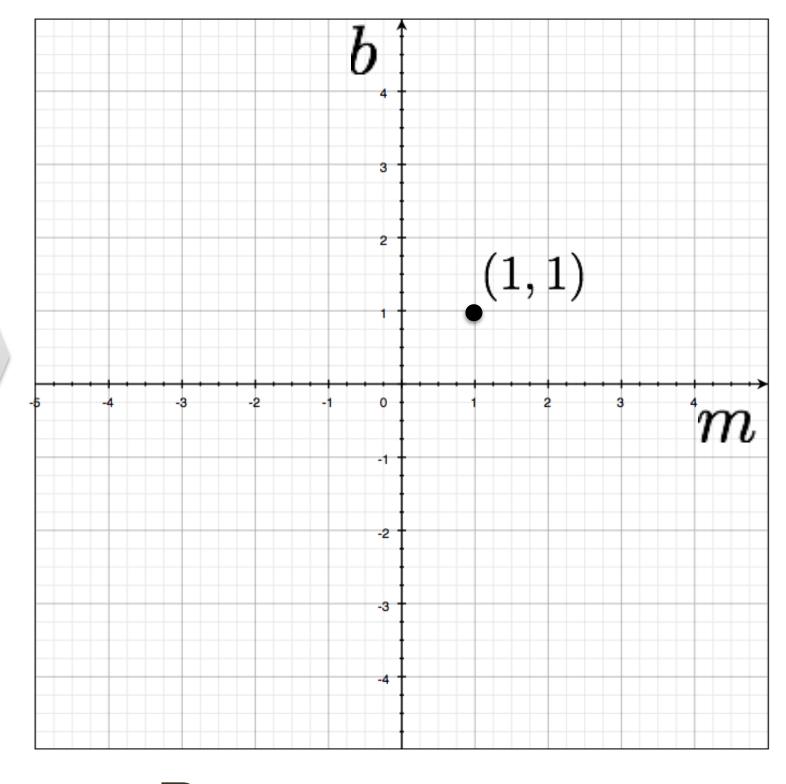


Image space

y - mx = b

variables

parameters



Parameter space

Hough Transform: Image and Parameter Space

variables
$$y = mx + b$$
 $y = mx + b$ parameters

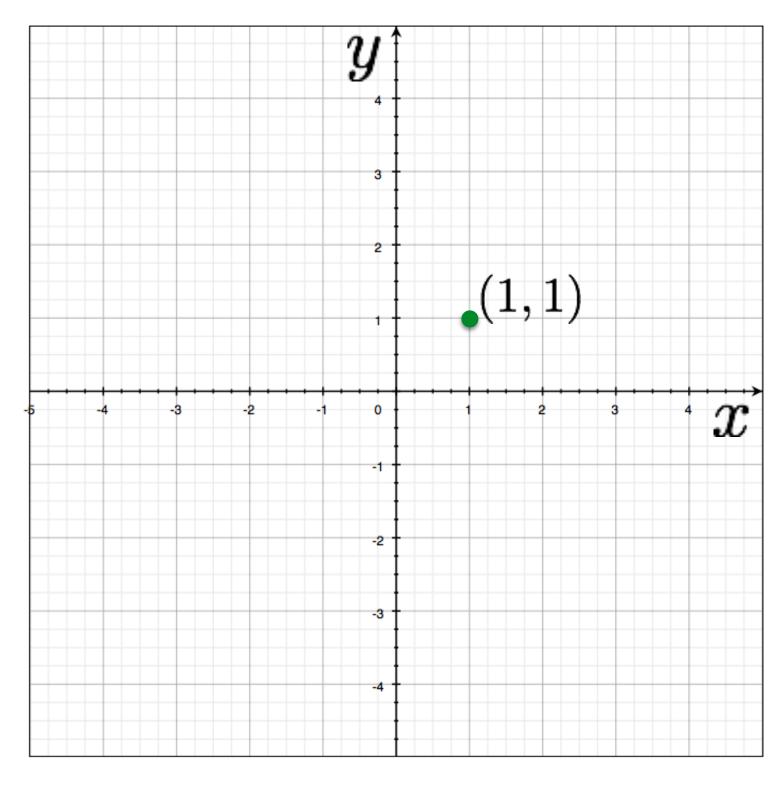


Image space

What would a **point** in image space become in **parameter space**?

variables
$$y = mx + b$$
 $y = mx + b$ parameters

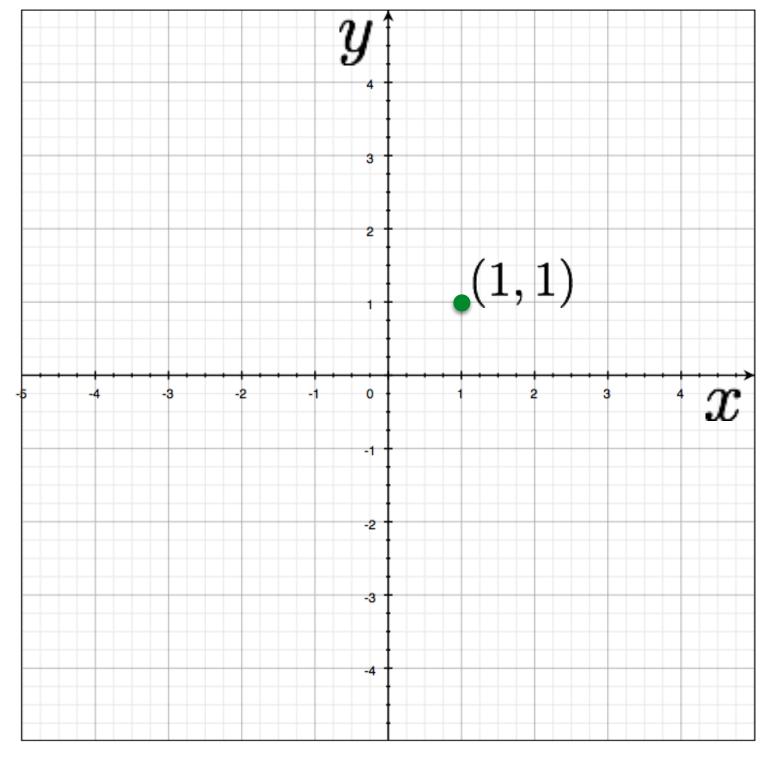
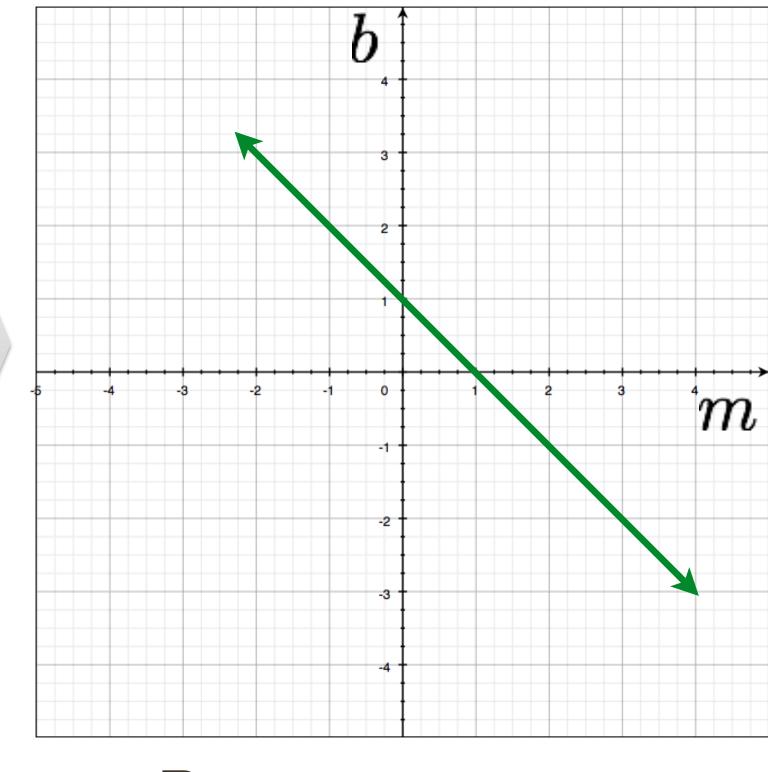


Image space

variables

$$y - mx = b$$

parameters



a point

becomes a

line

Parameter space

variables
$$y = mx + b$$
 $y = mx + b$ parameters

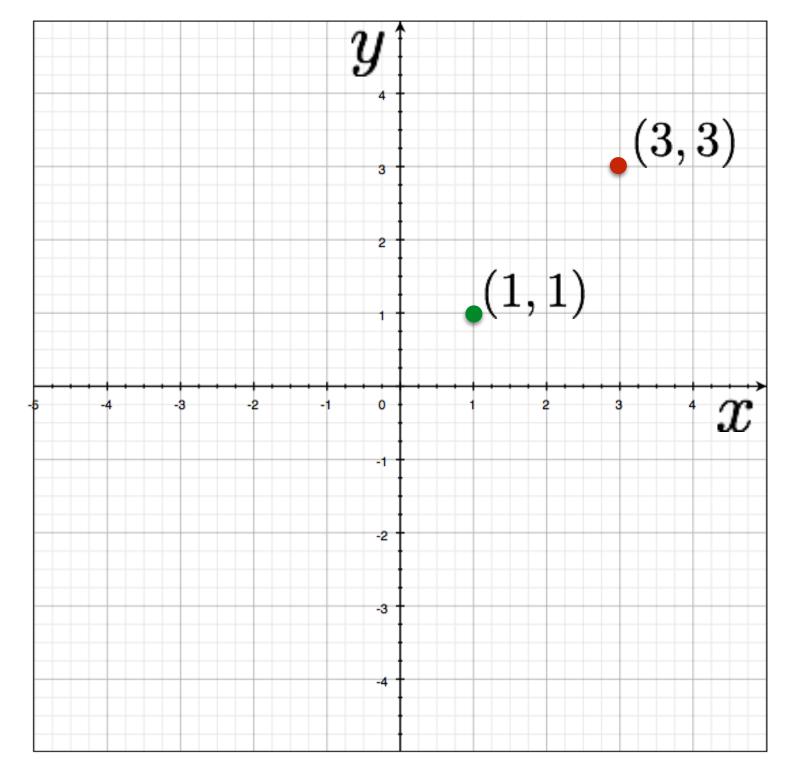
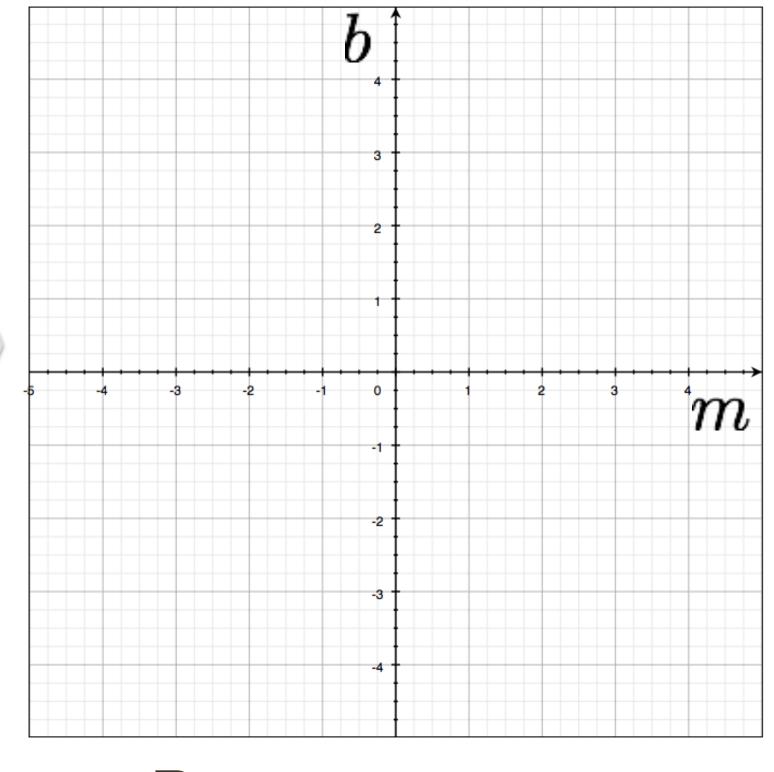


Image space

variables

$$y - mx = b$$

parameters



two

points?

Parameter space

variables
$$y = mx + b$$
 $y = mx + b$ parameters

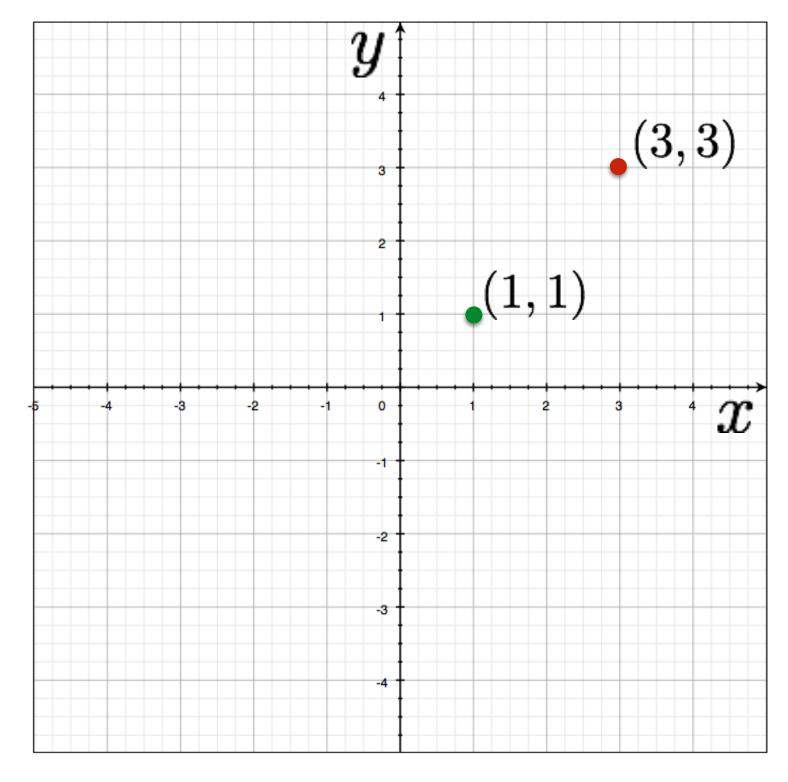
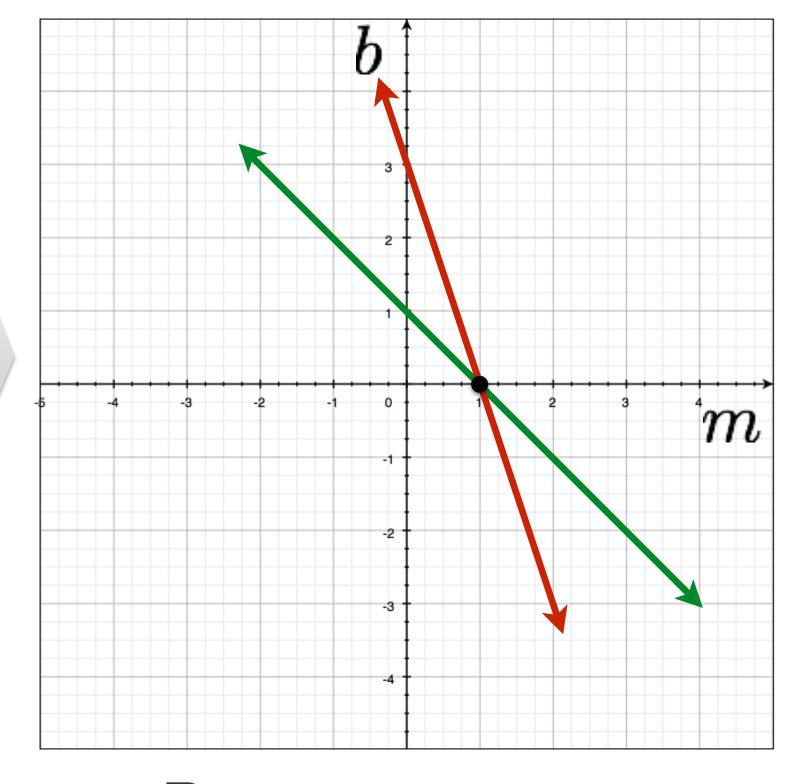


Image space

variables

$$y - mx = b$$

parameters



two

points?

Parameter space

variables
$$y = mx + b$$
 $y = mx + b$ parameters

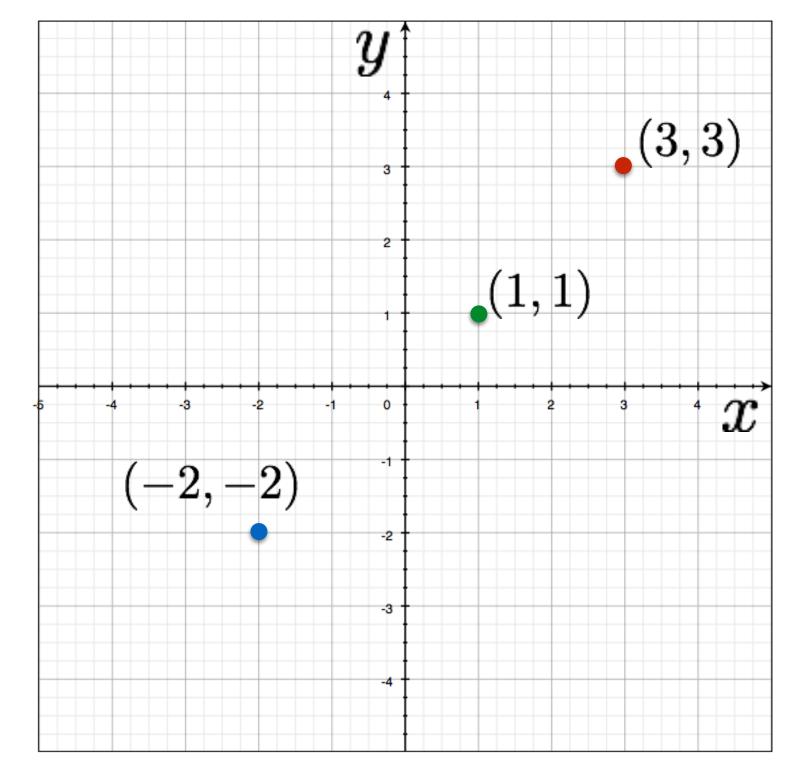
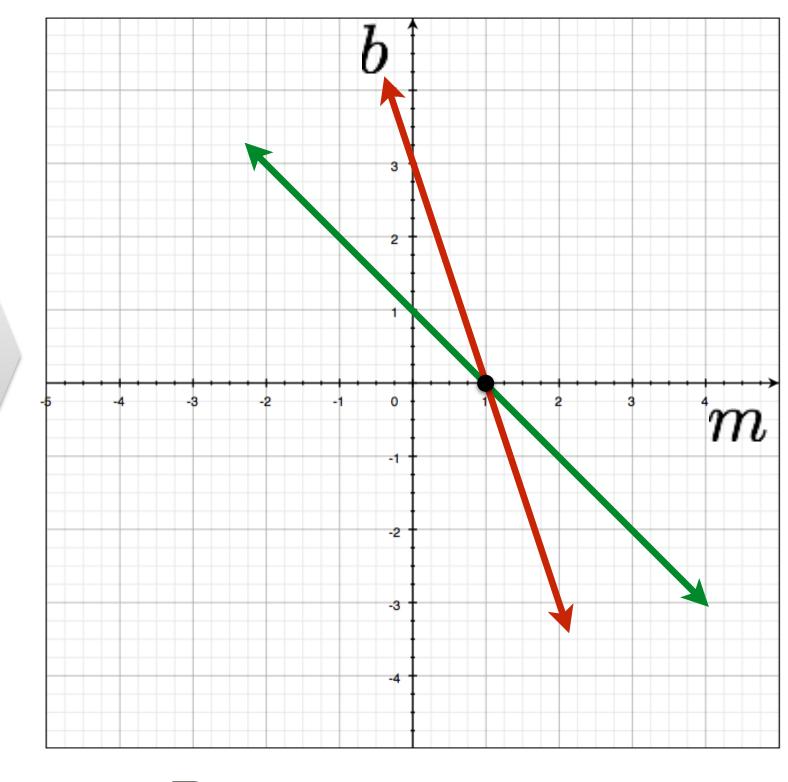


Image space

variables

$$y - mx = b$$
 \uparrow

parameters



three

points?

Parameter space

variables
$$y = mx + b$$

$$y = mx + b$$
parameters

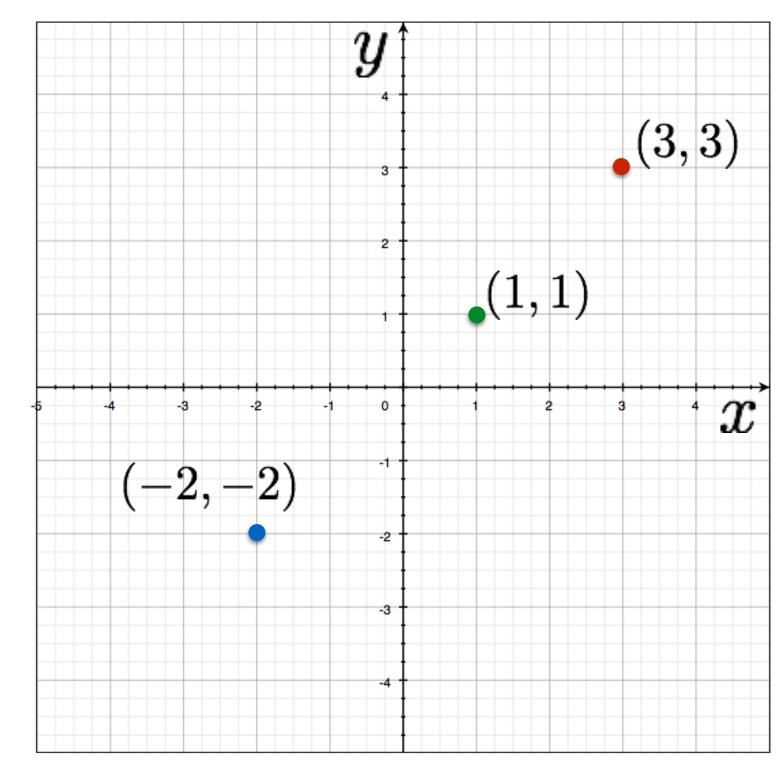
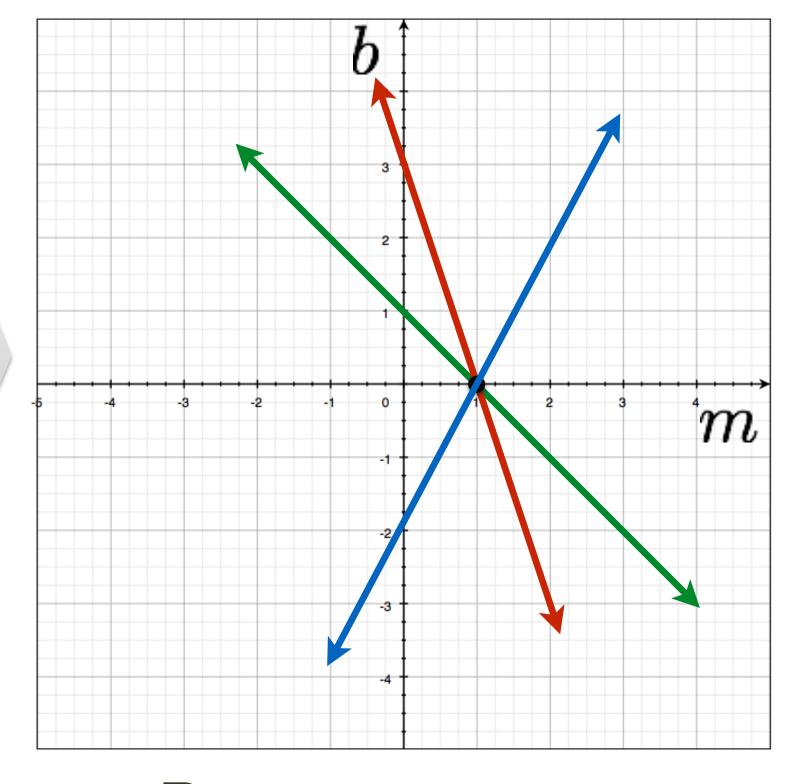


Image space

variables

$$y - mx = b$$
 \uparrow

parameters



three

points?

Parameter space

variables
$$y = mx + b$$
 $y = mx + b$ parameters

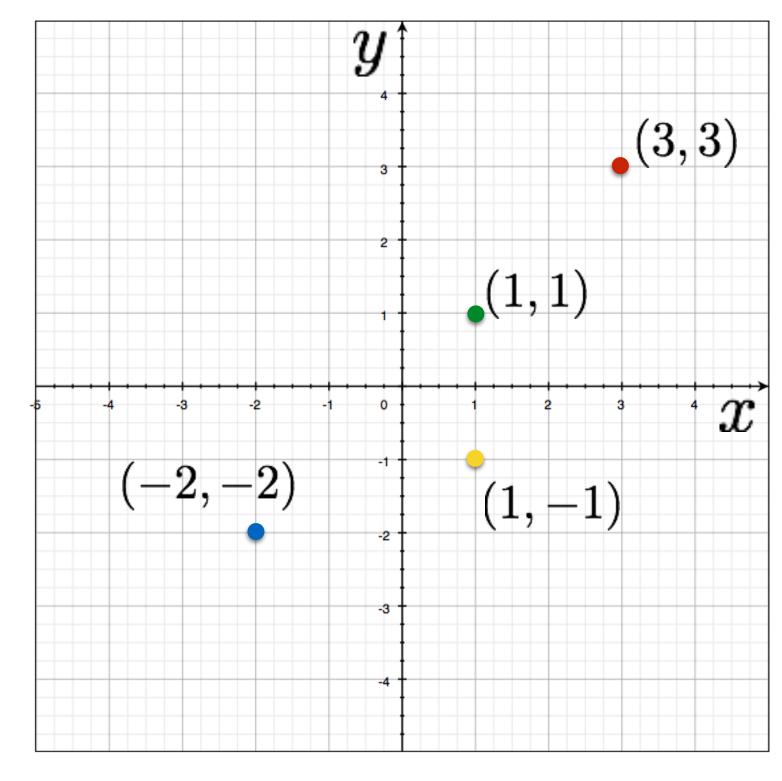
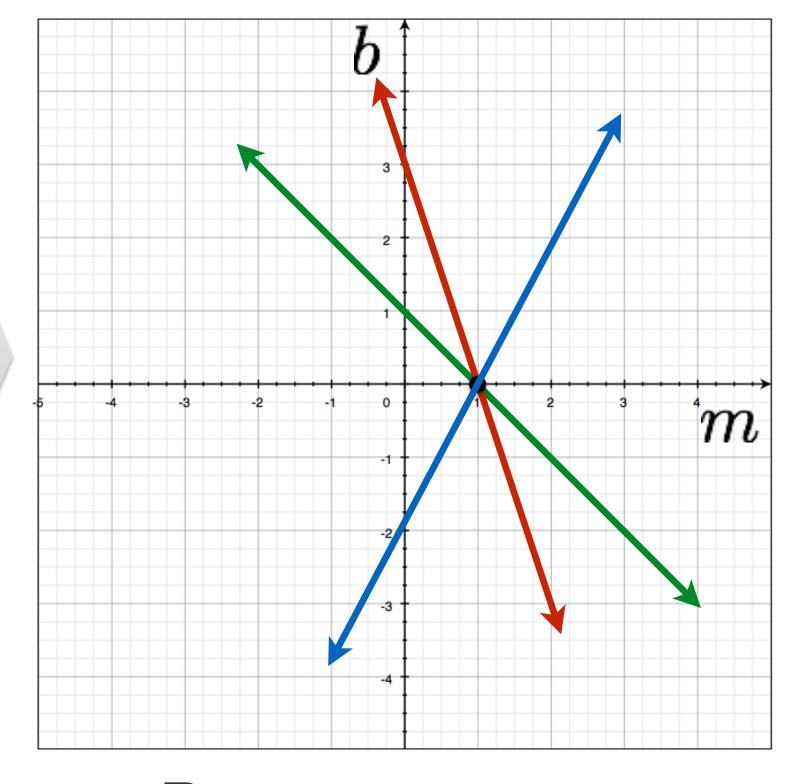


Image space

variables

$$y - mx = b$$
 \uparrow

parameters



four

points?

Parameter space

variables
$$y = mx + b$$
 $y = mx + b$ parameters

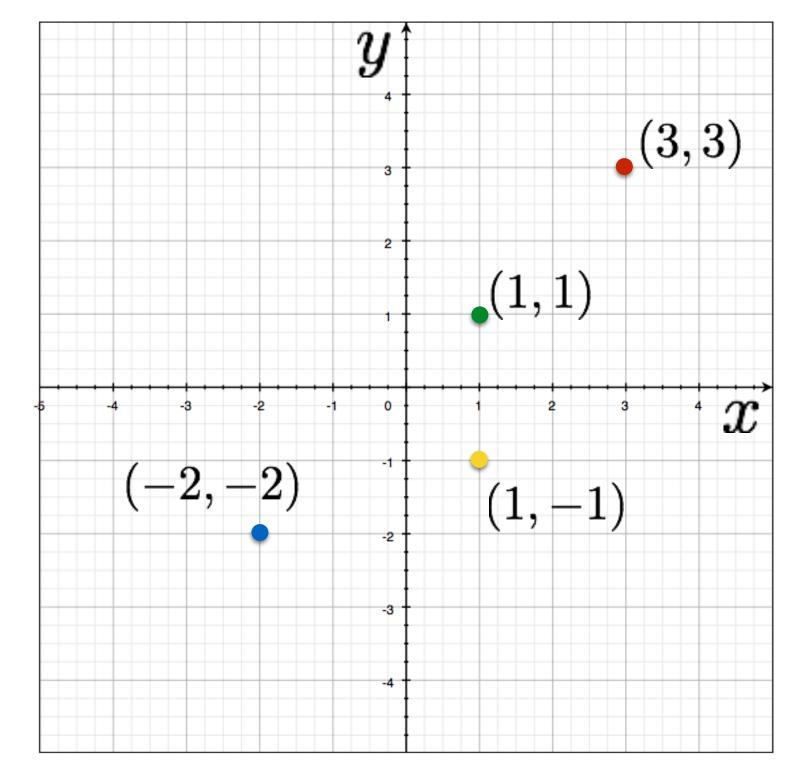
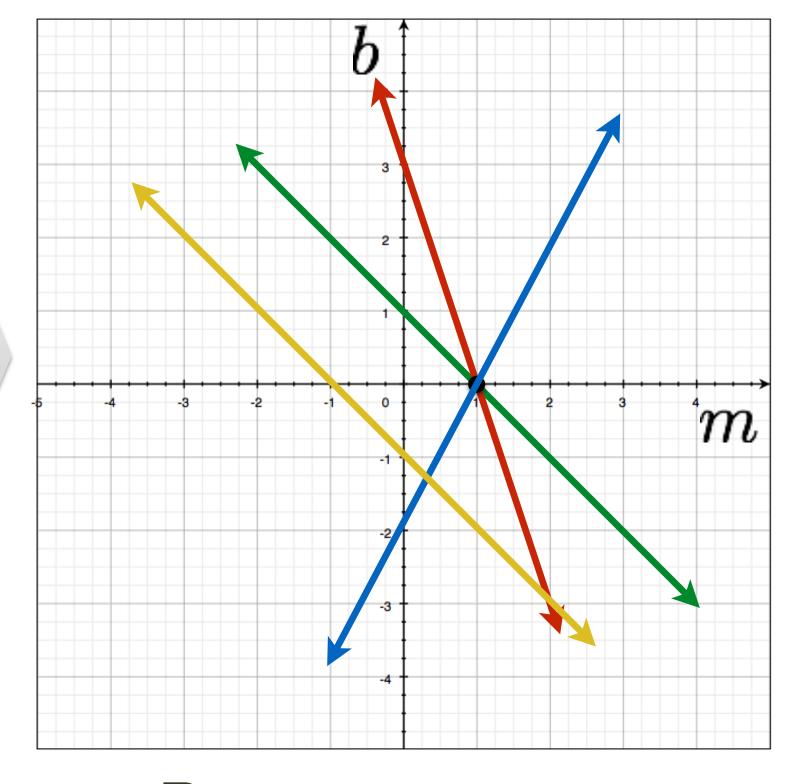


Image space

variables

$$y - mx = b$$
 \uparrow

parameters

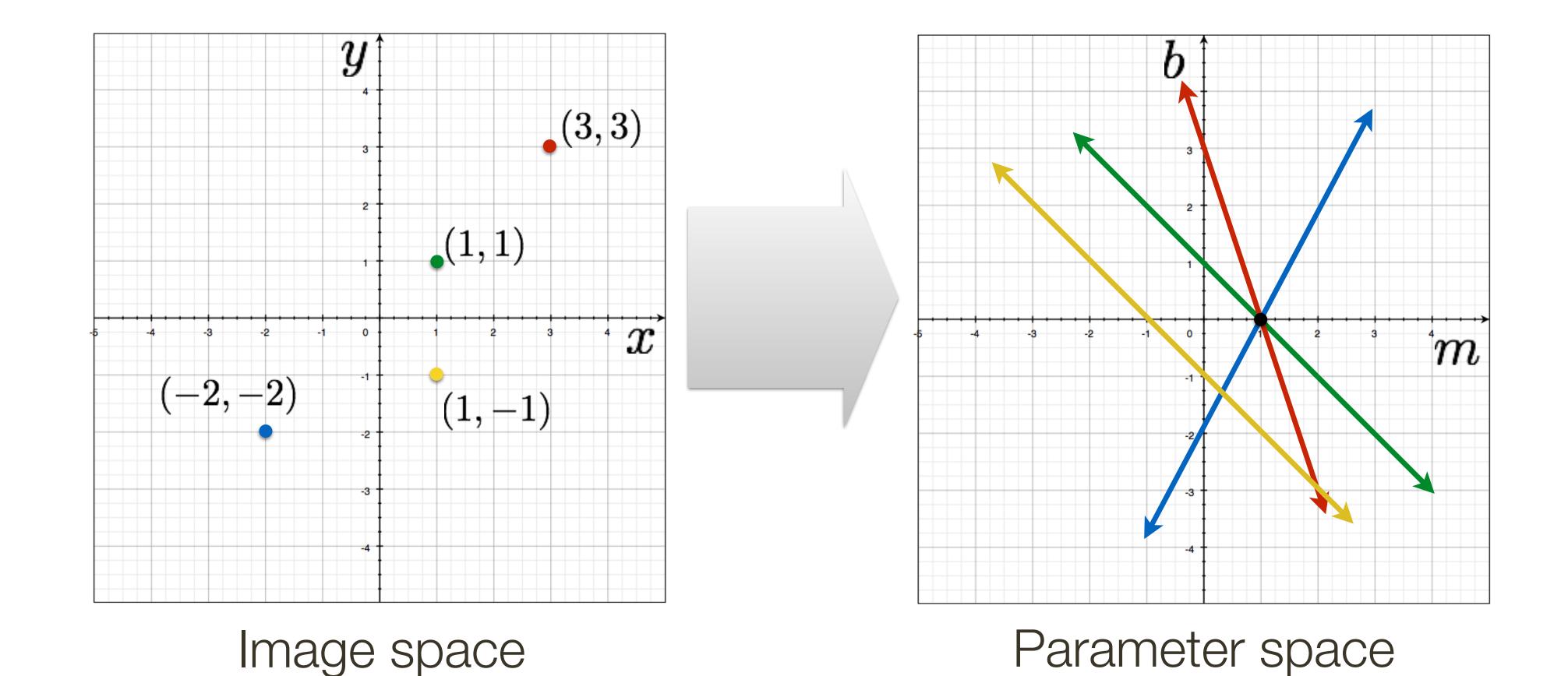


four

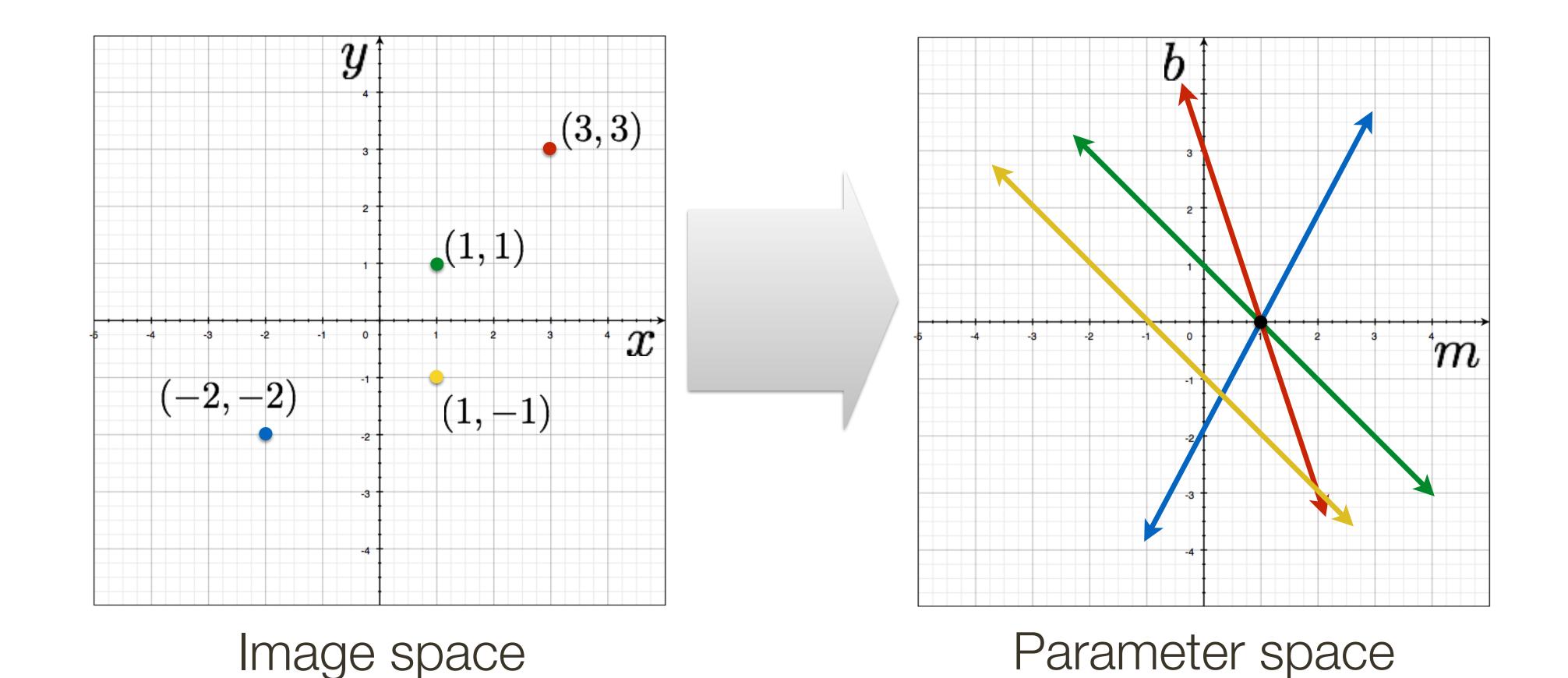
points?

Parameter space

How would you find the best fitting line?



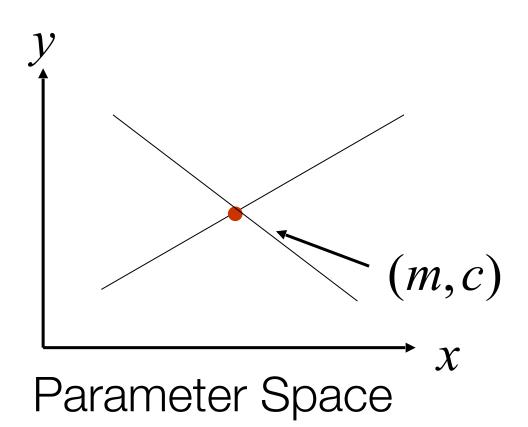
Is this method robust to measurement noise? clutter?



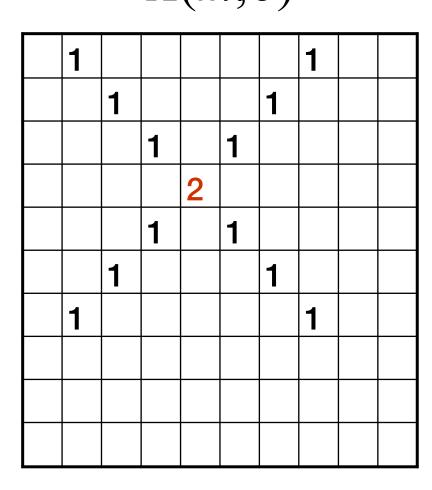
Line Detection by Hough Transform

Algorithm:

- 1. Quantize Parameter Space(m,c)
- 2. Create Accumulator Array A(m,c)
- 3.Set $A(m,c) = 0 \quad \forall m,c$
- 4. For each image edge (x_i, y_i) For each element in A(m,c)If (m,c) lies on the line: $c = -x_i m + y_i$ Increment A(m,c) = A(m,c) + 1
- 5. Find local maxima in A(m,c)

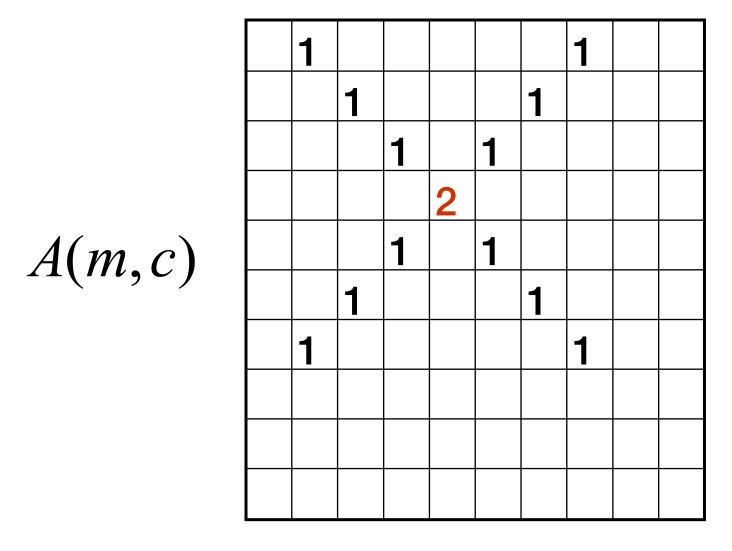






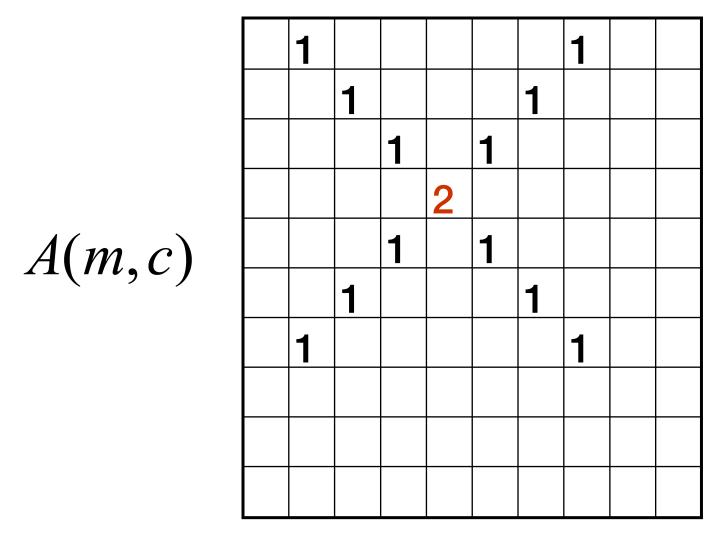
Problems with Parametrization

How big does the accumulator need to be for the parameterization (m,c)?



Problems with Parametrization

How big does the accumulator need to be for the parameterization (m,c)?



The space of m is huge!

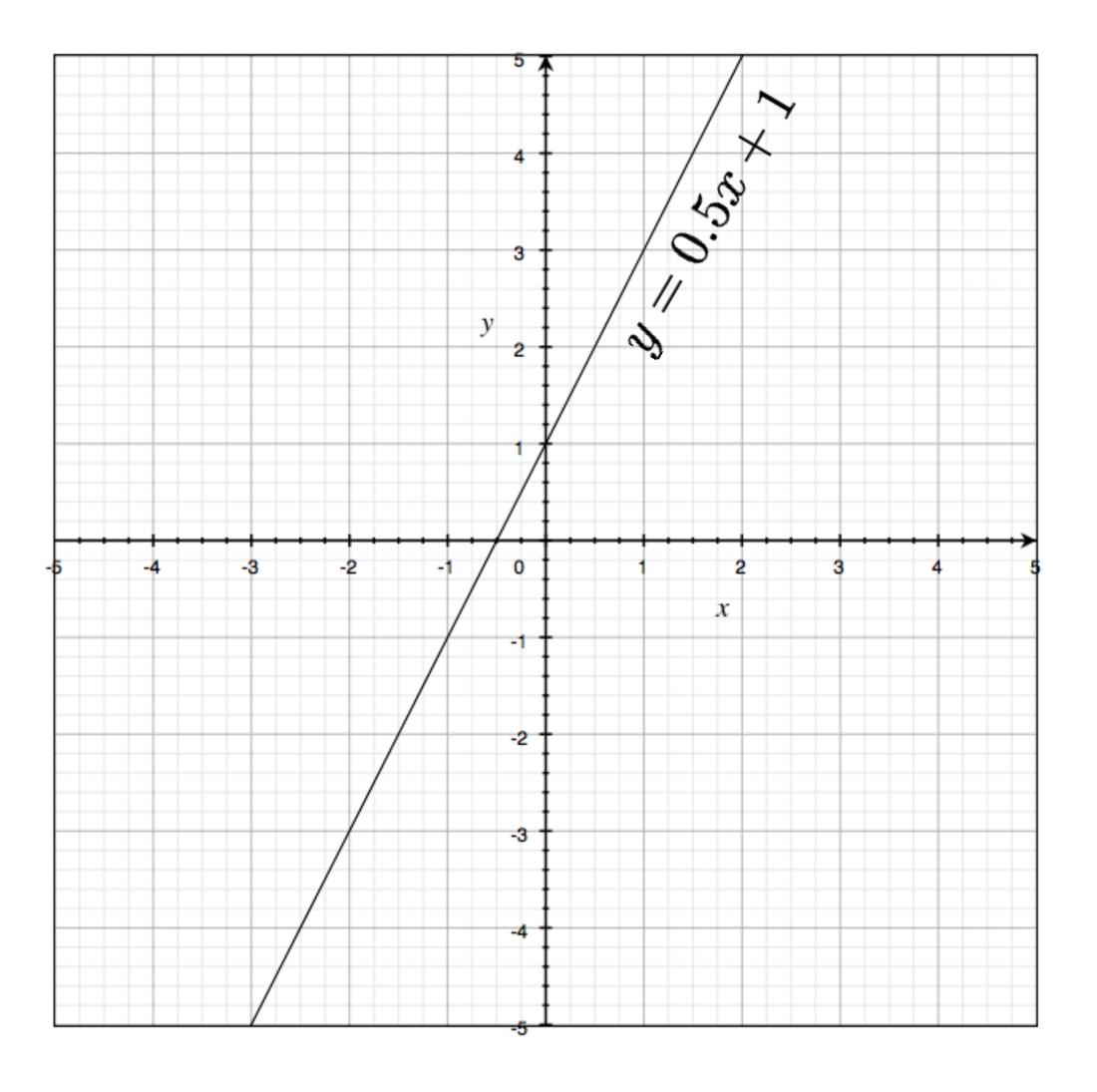
The space of c is huge!

$$-\infty \leq m \leq \infty$$

$$-\infty \leq c \leq \infty$$

Lines: Slope intercept form

$$y=mx+b$$
slope y-intercept



Lines: Normal form

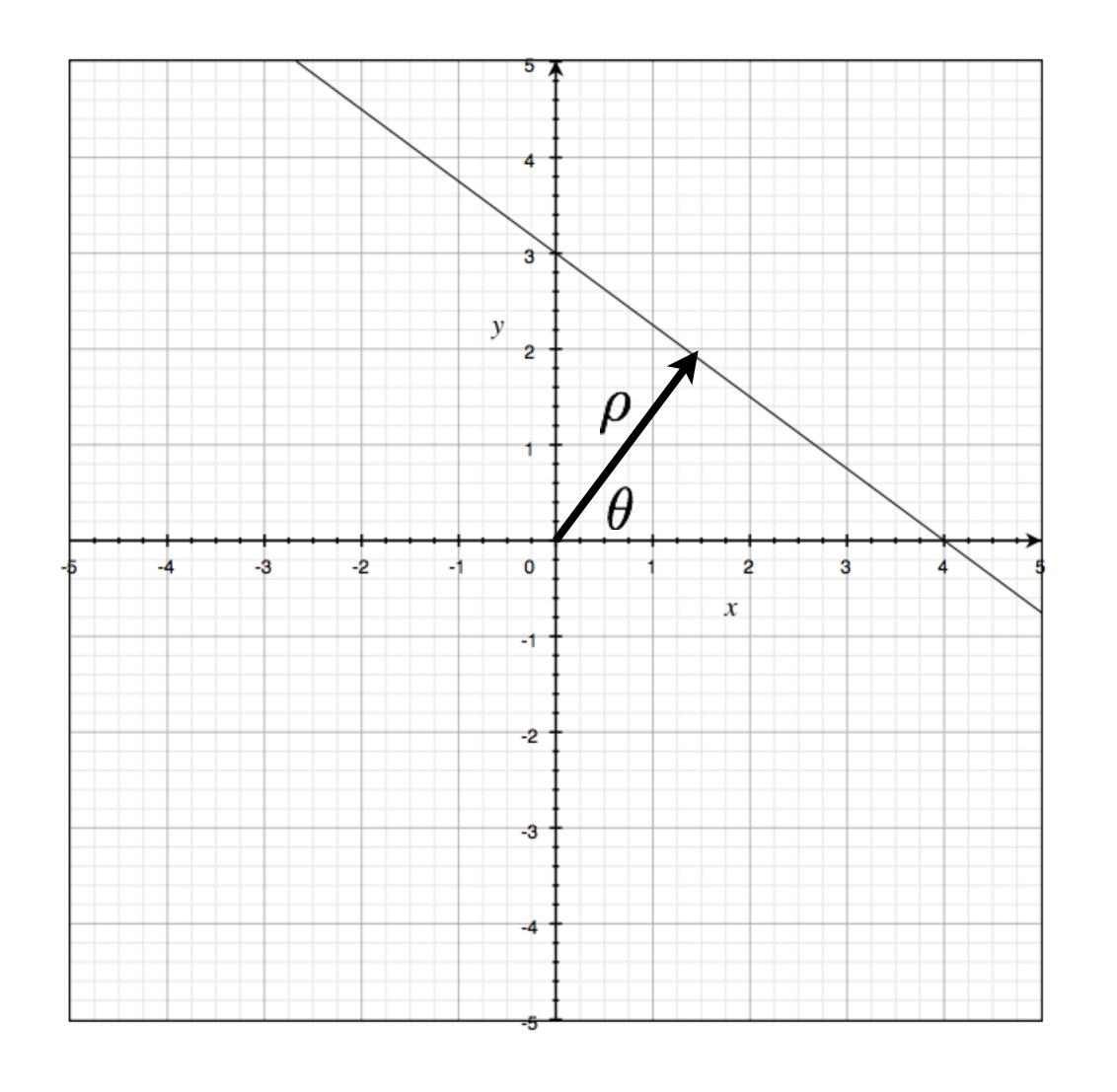
$$x \sin \theta + y \cos \theta = \rho$$

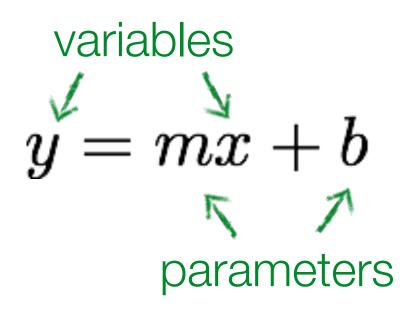
Forsyth/Ponce convention

$$x \sin \theta + y \cos \theta + r = 0$$

$$r \ge 0$$

$$0 < \theta < 2\pi$$





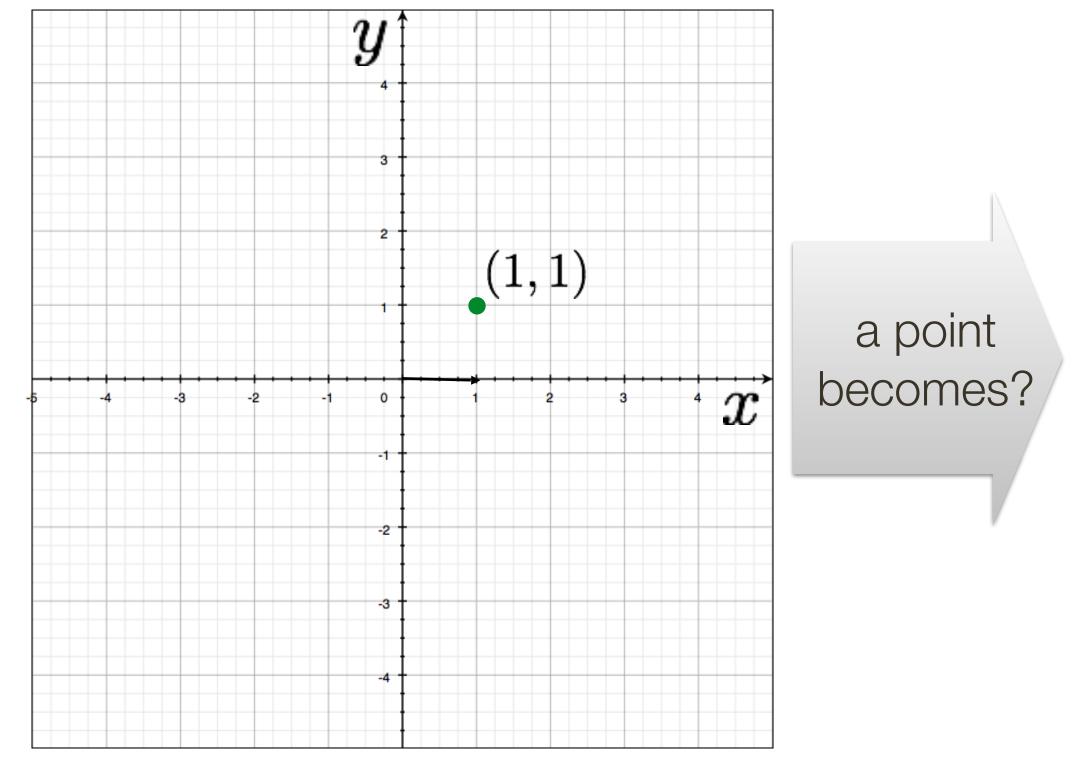
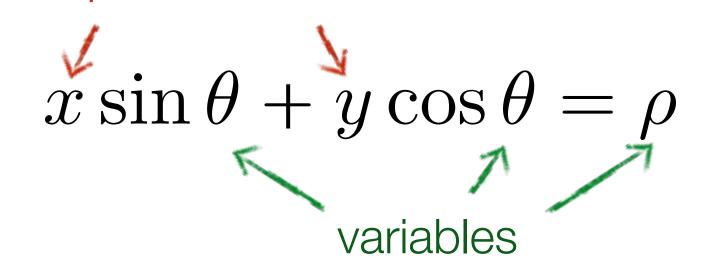
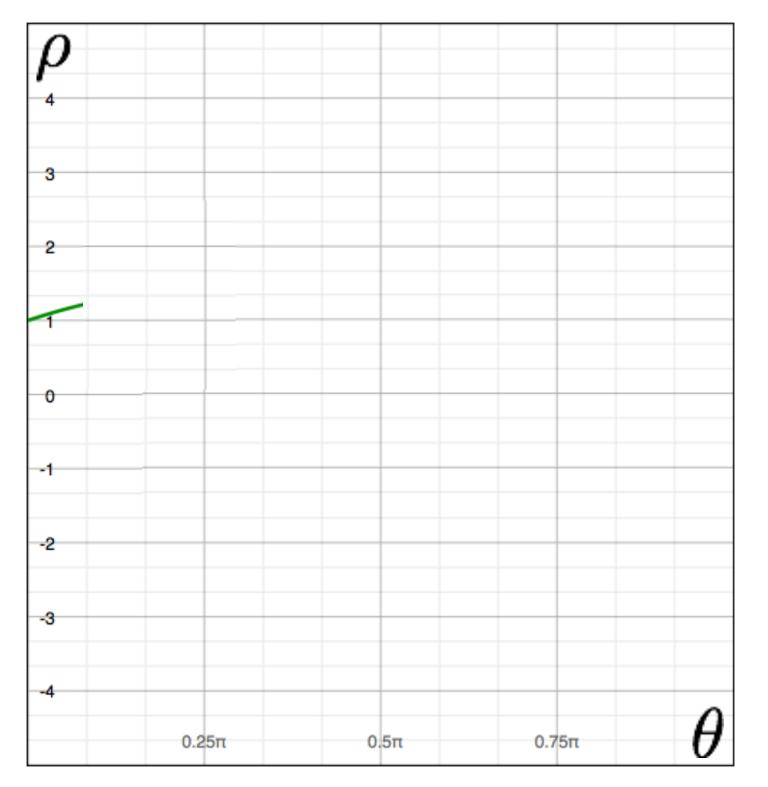


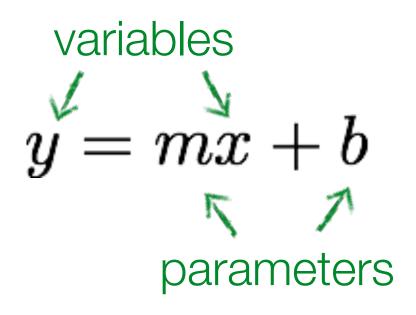
Image space







Parameter space



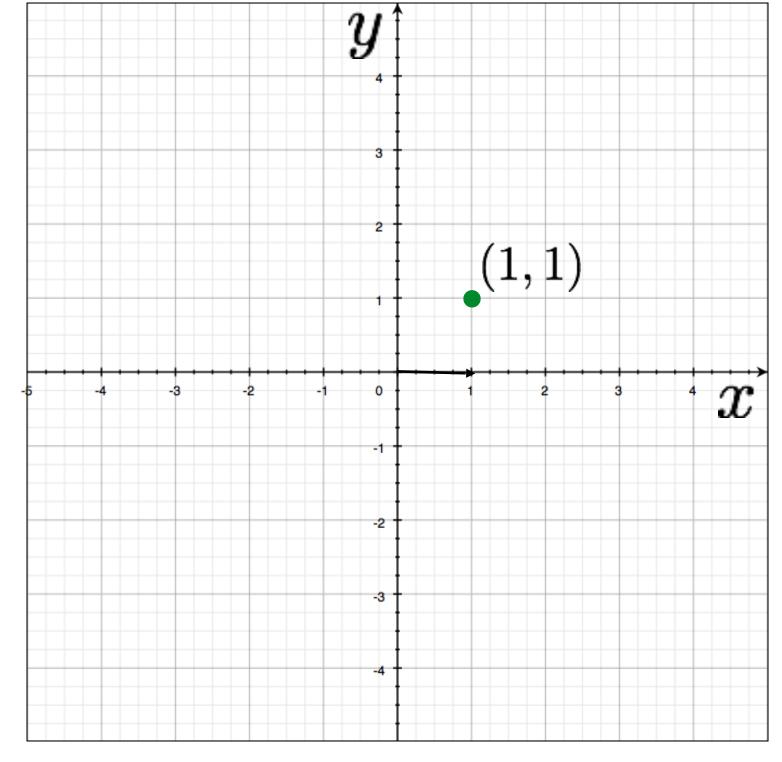


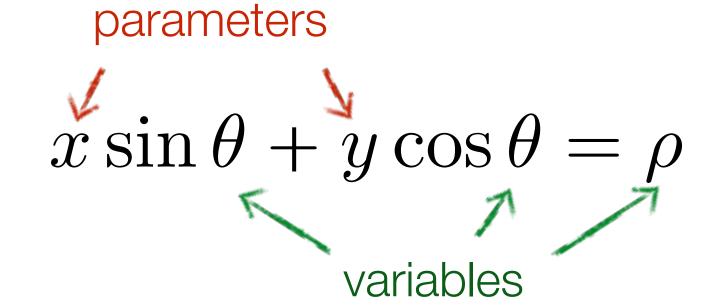
Image space

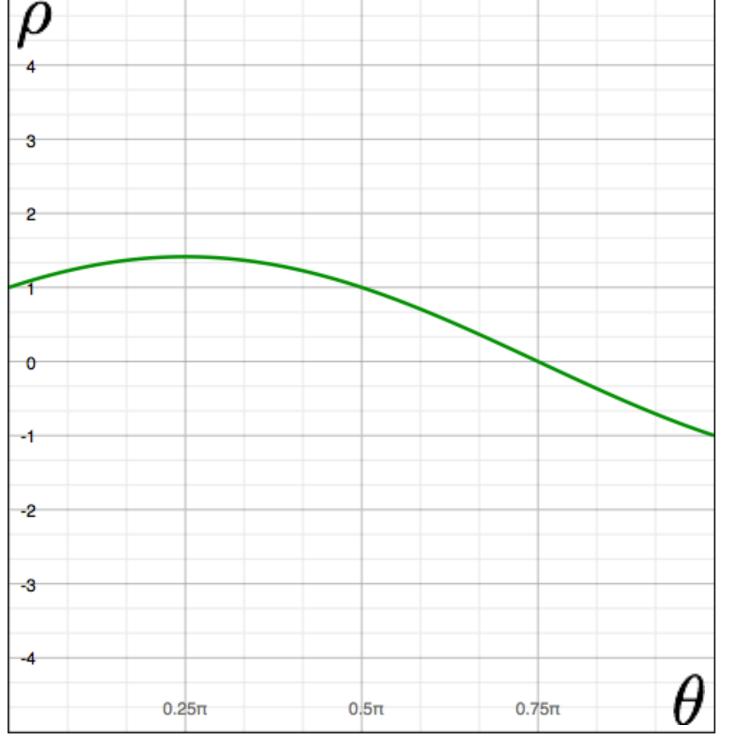


a point

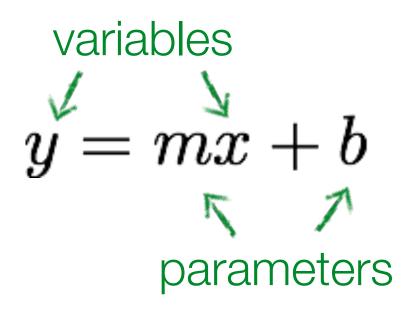
becomes

a wave





Parameter space



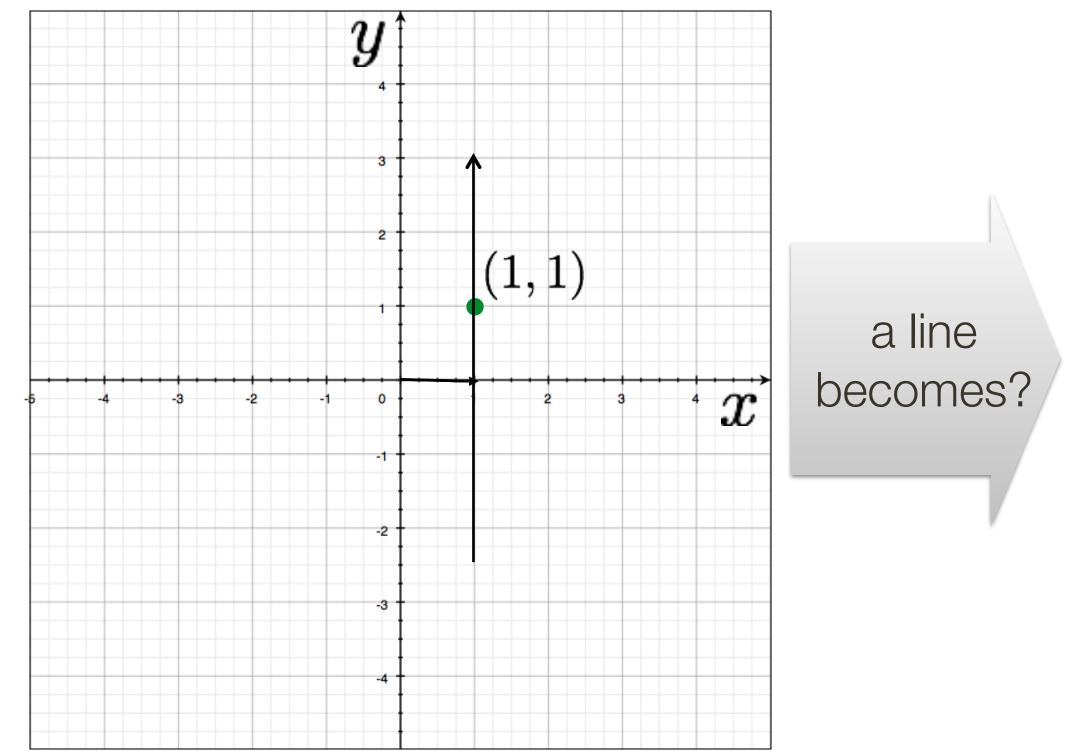
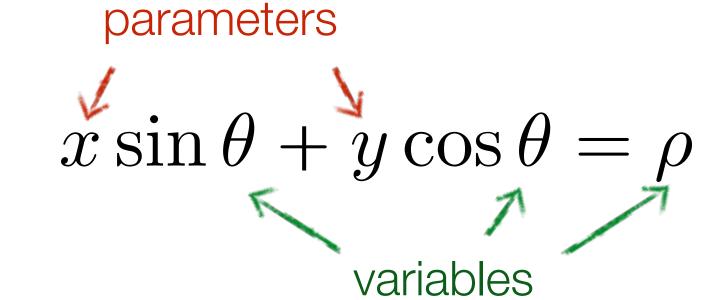
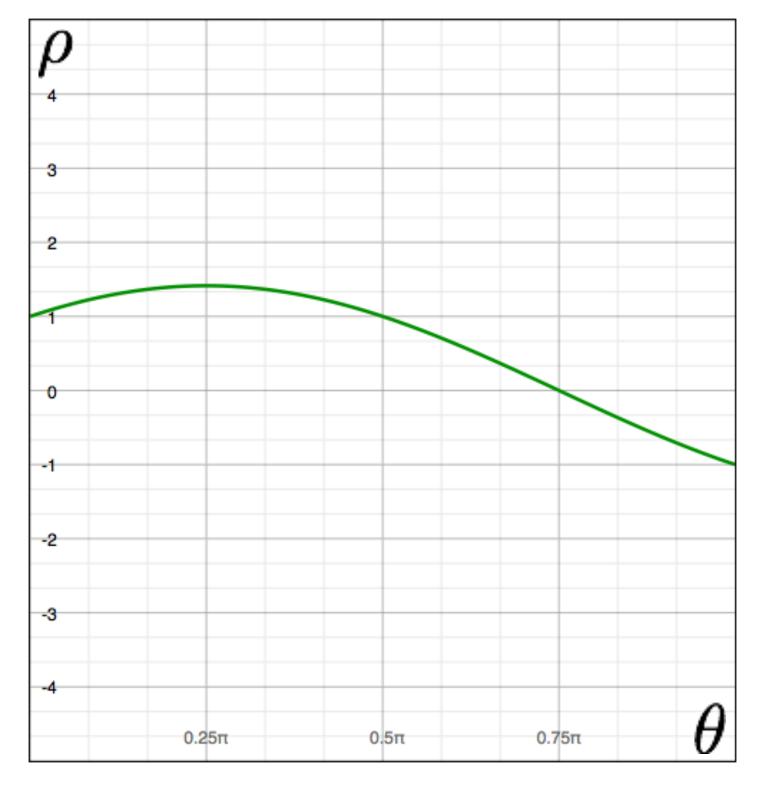
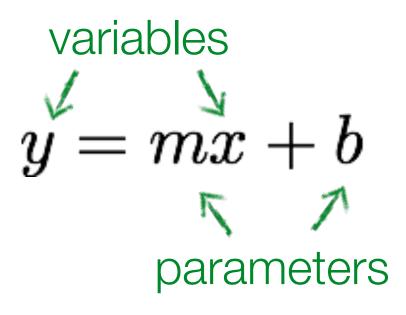


Image space





Parameter space



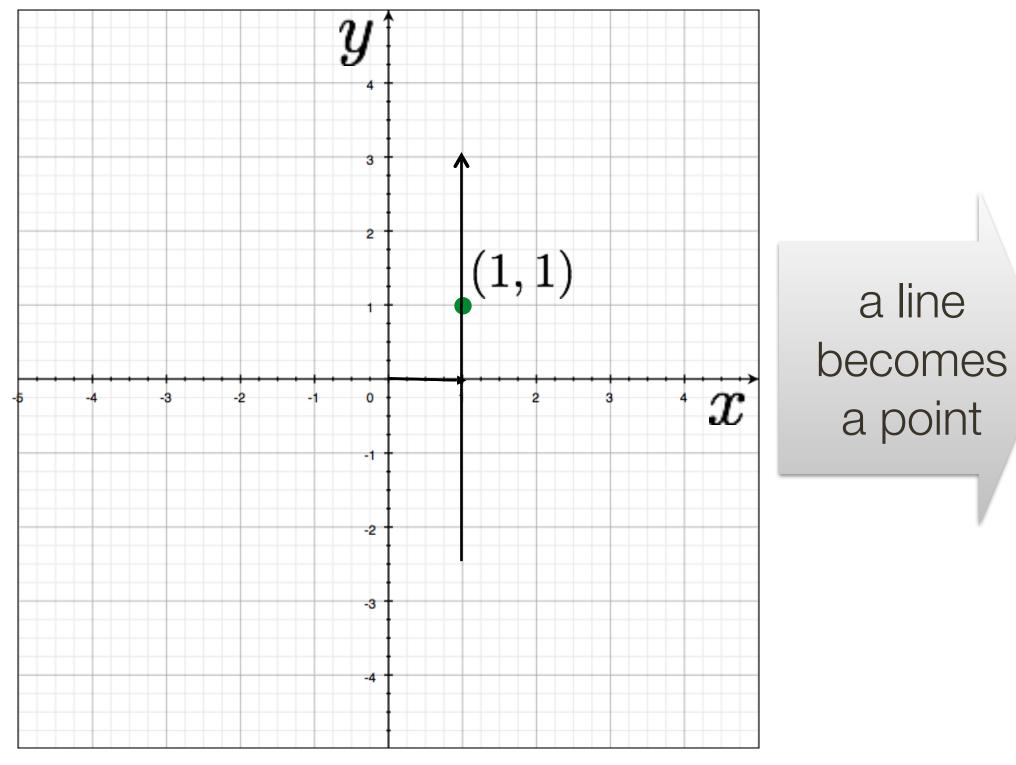
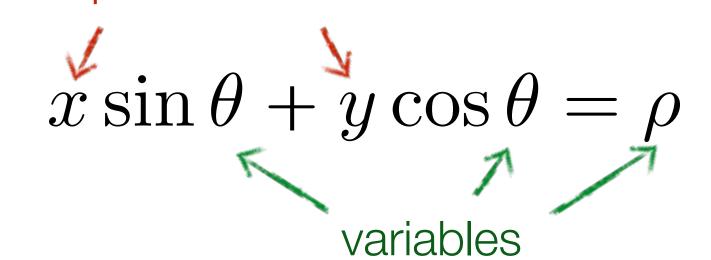
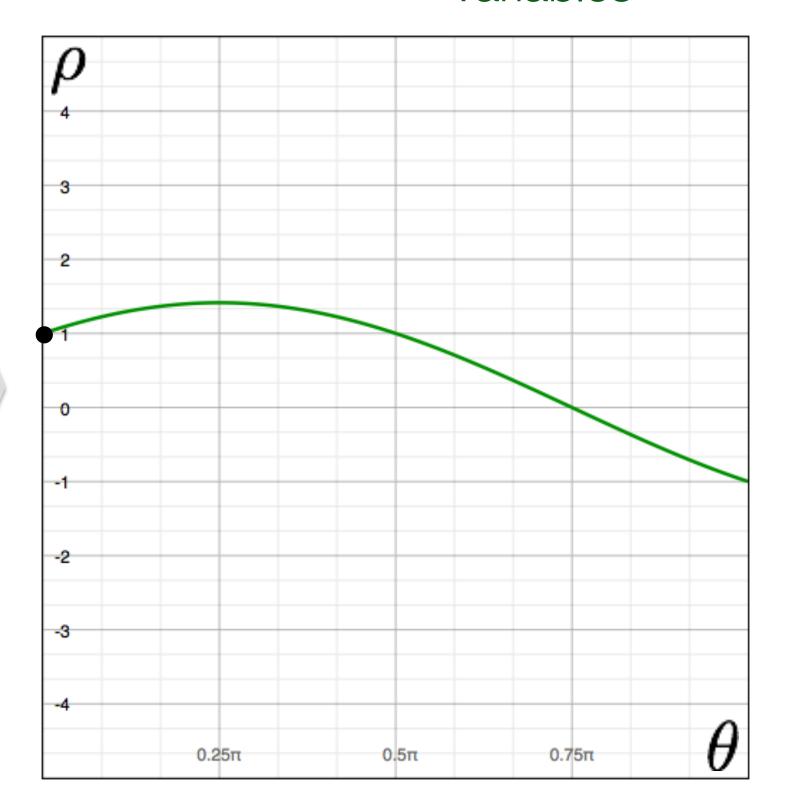


Image space

parameters





Parameter space

Hough Transform for Lines (switching to books notation)

Idea: Each point votes for the lines that pass through it

— A line is the set of points, (x, y), such that

$$x\sin\theta + y\cos\theta + r = 0$$

— Different choices of θ , r give different lines

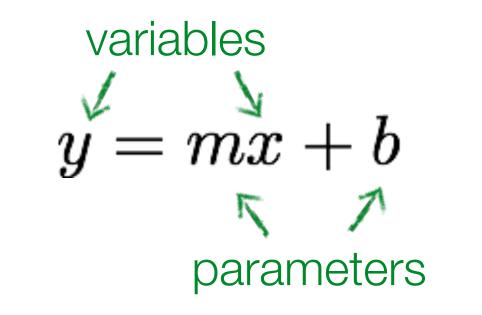
Hough Transform for Lines (switching to books notation)

Idea: Each point votes for the lines that pass through it

— A line is the set of points, (x, y), such that

$$x\sin\theta + y\cos\theta + r = 0$$

- Different choices of θ , r give different lines
- For any (x, y) there is a one parameter family of lines through this point. Just let (x, y) be constants and for each value of θ the value of r will be determined
- Each point enters votes for each line in the family
- If there is a line that has lots of votes, that will be the line passing near the points that voted for it



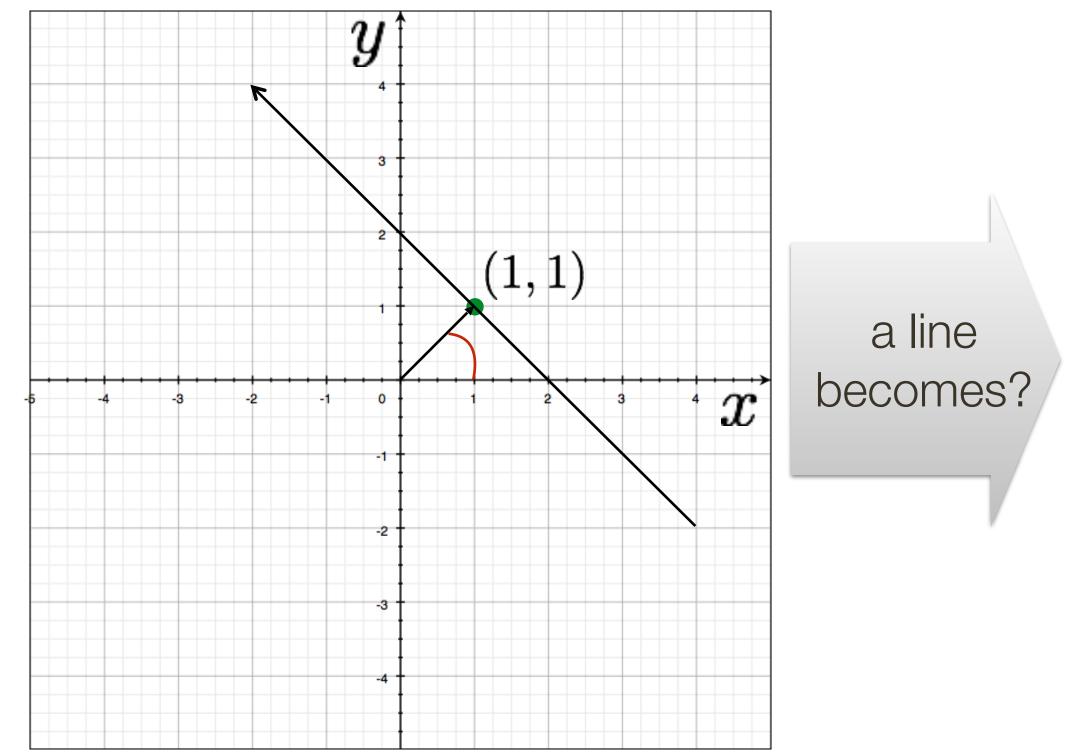
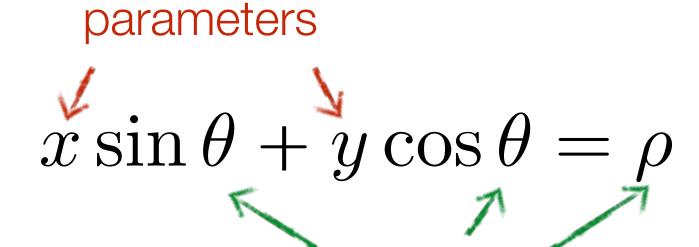
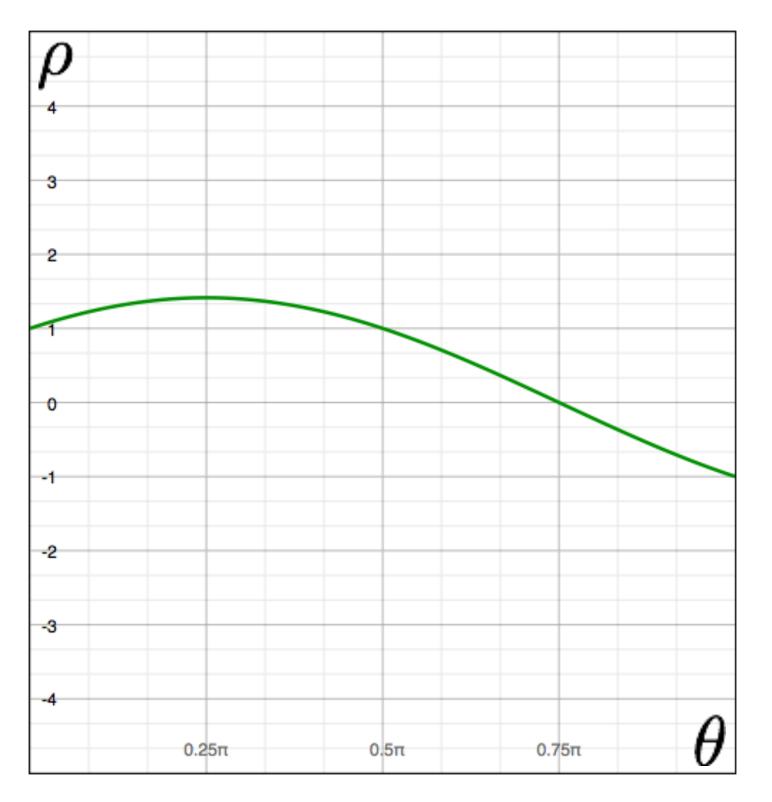


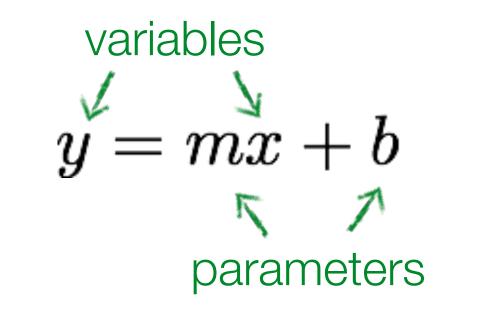
Image space

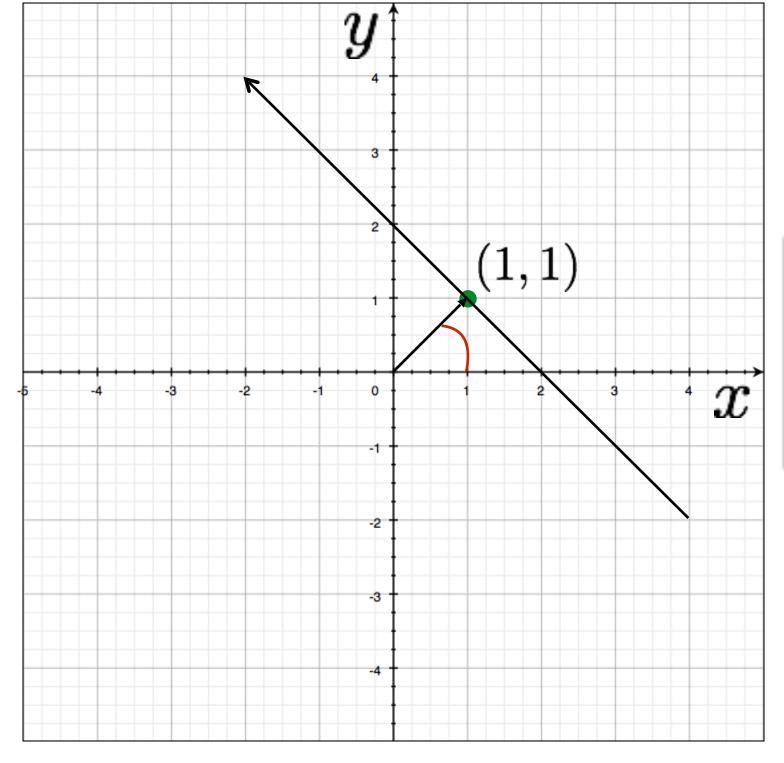


variables



Parameter space



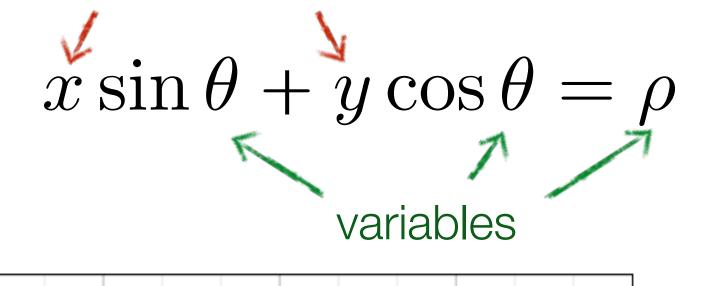


a line

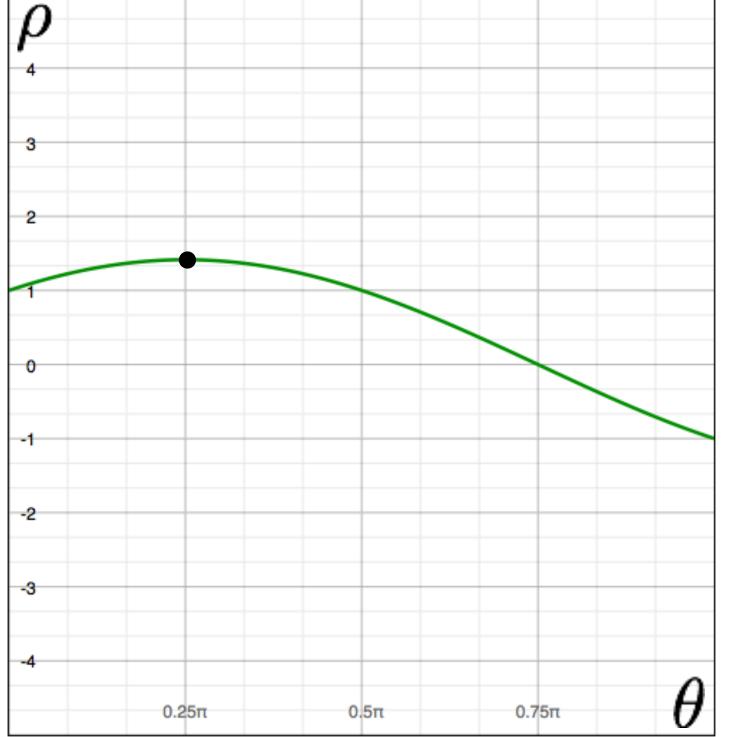
becomes

a point

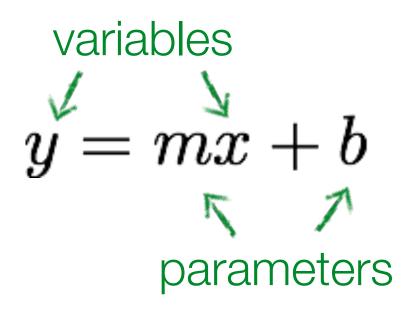
Image space

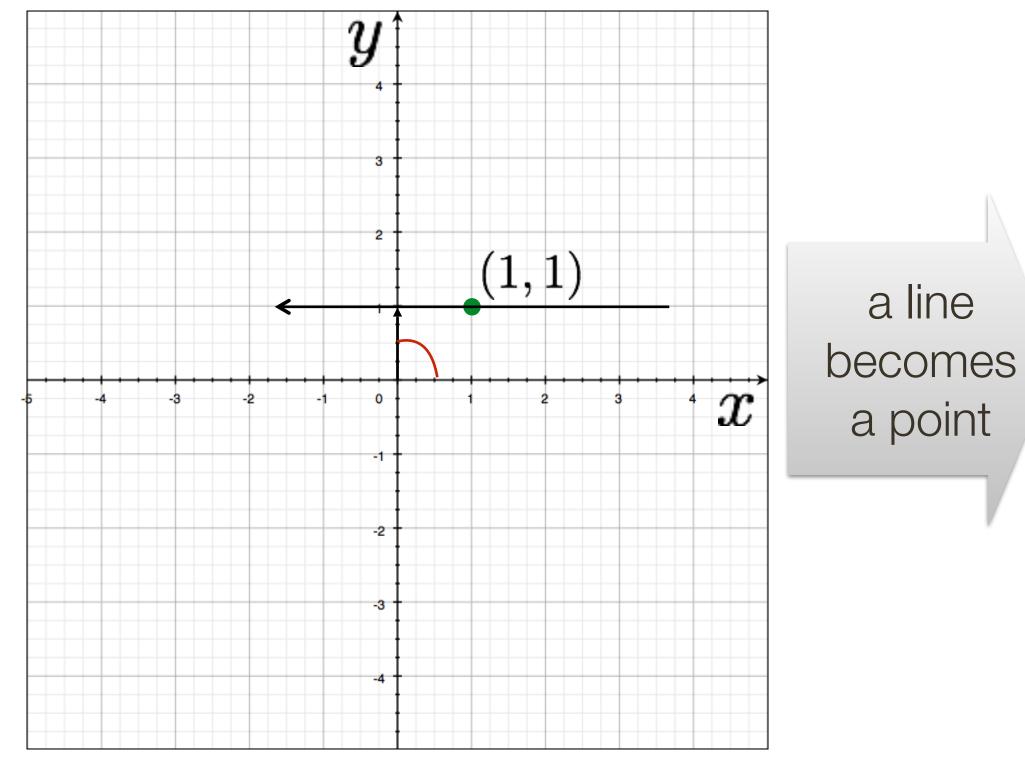


parameters



Parameter space



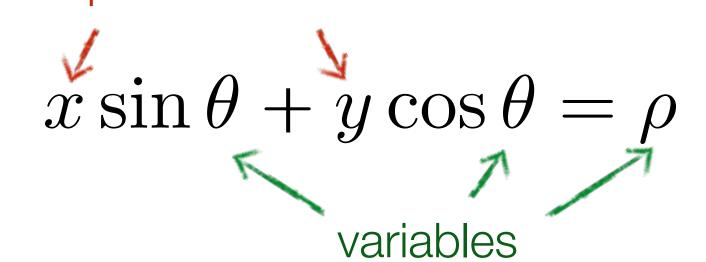


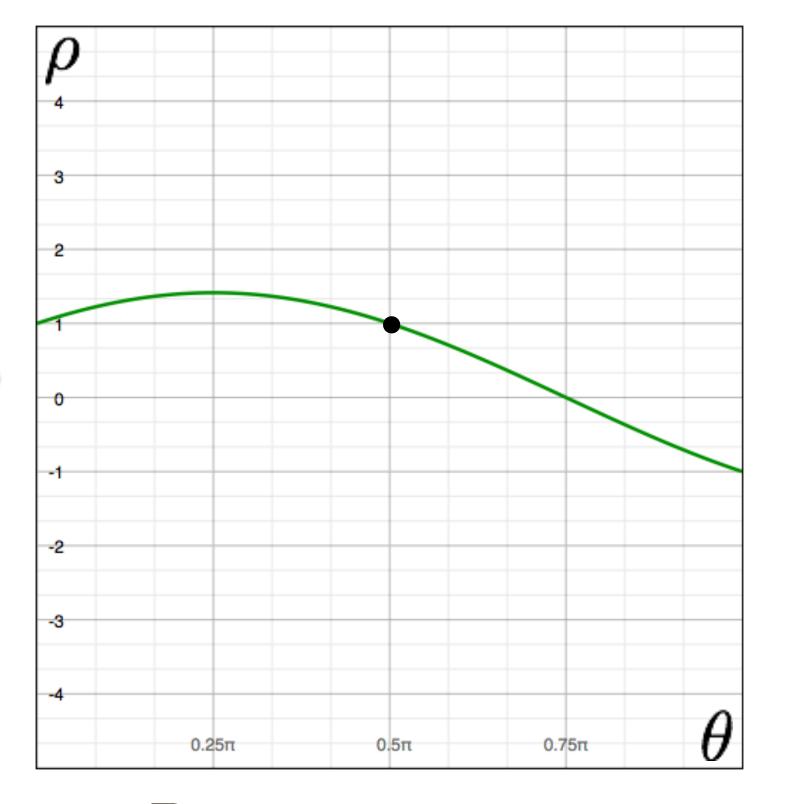
a line

a point

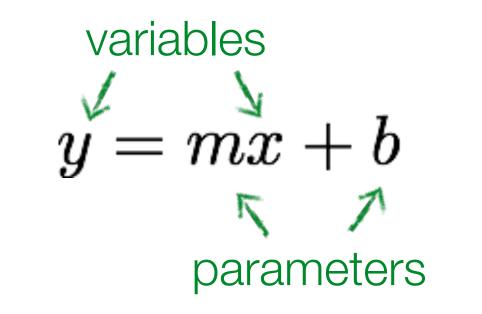
Image space

parameters





Parameter space



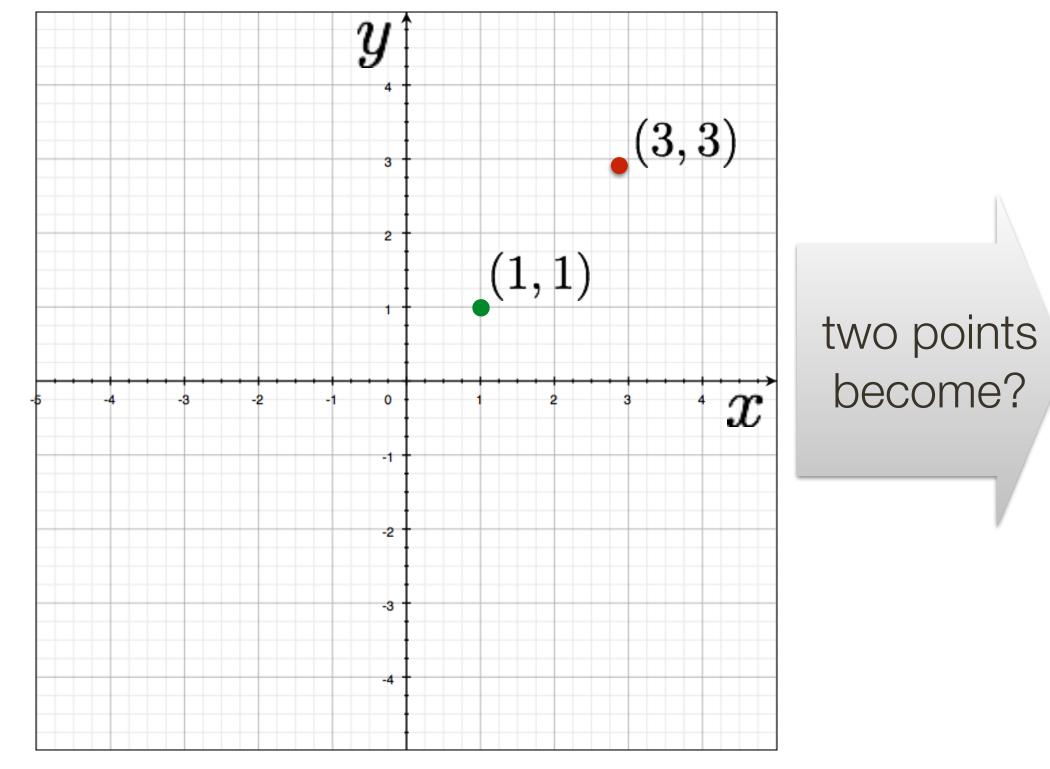
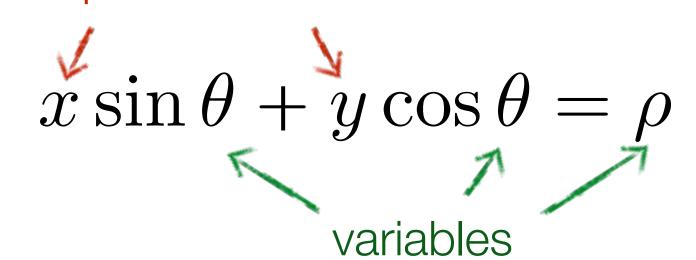
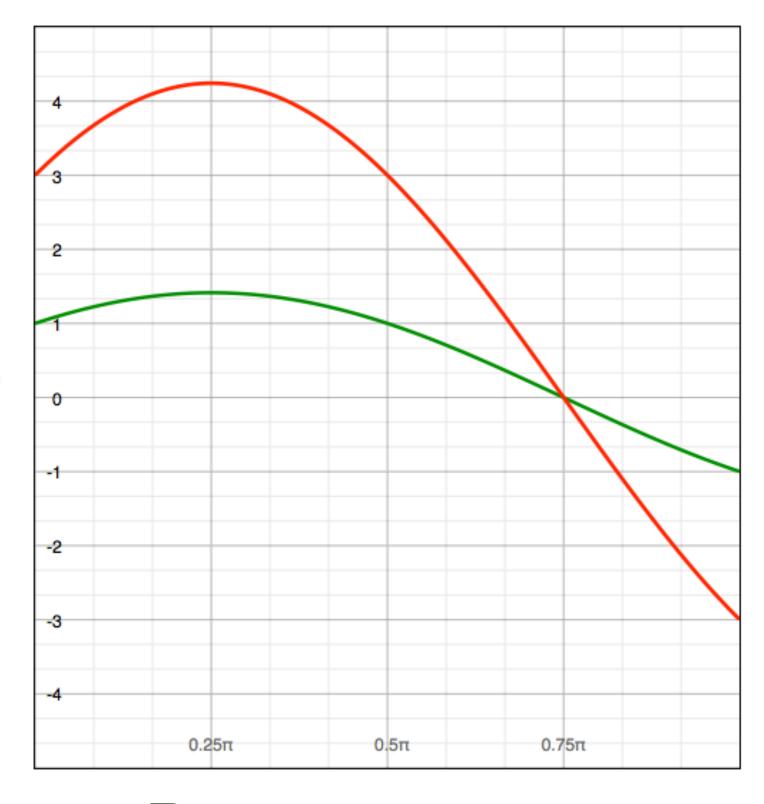


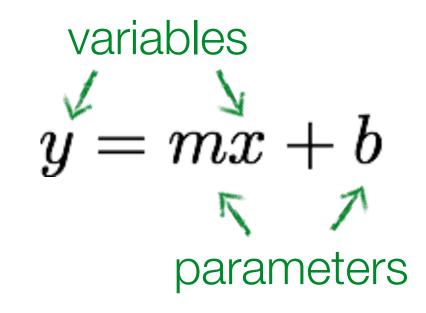
Image space







Parameter space



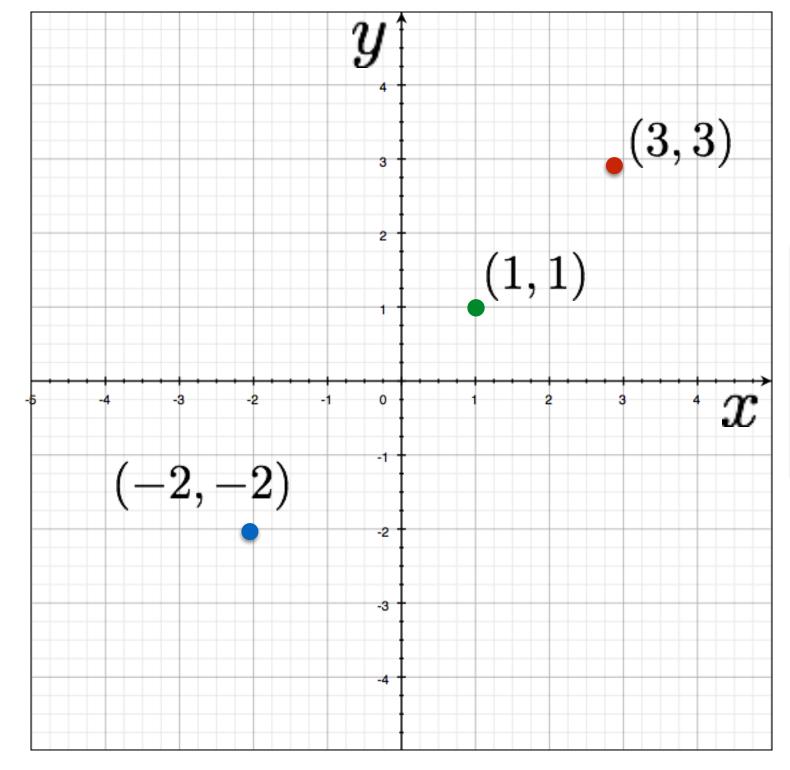
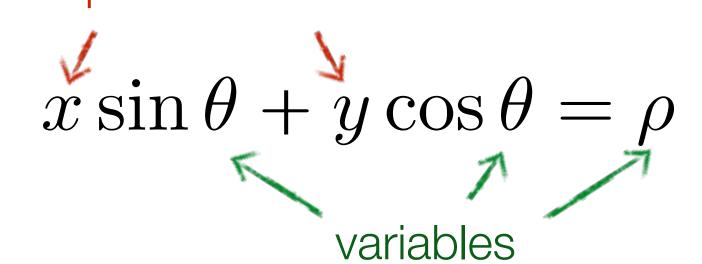
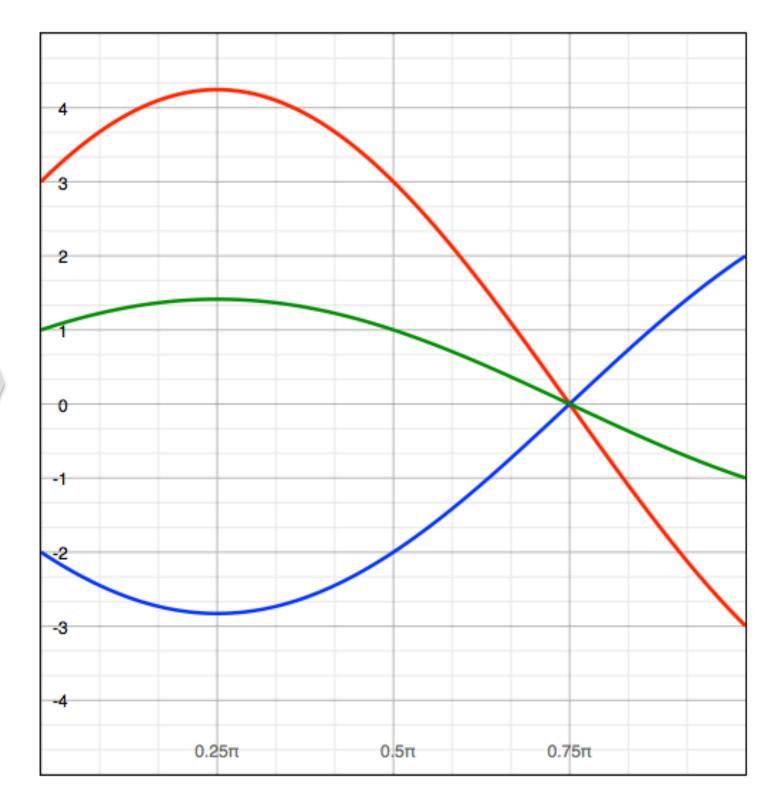


Image space



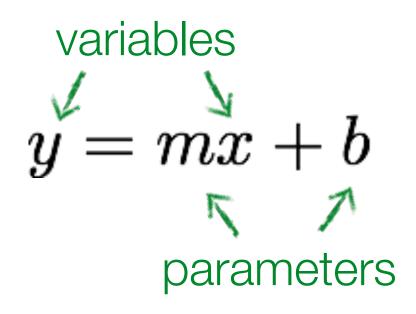




three points

become?

Parameter space



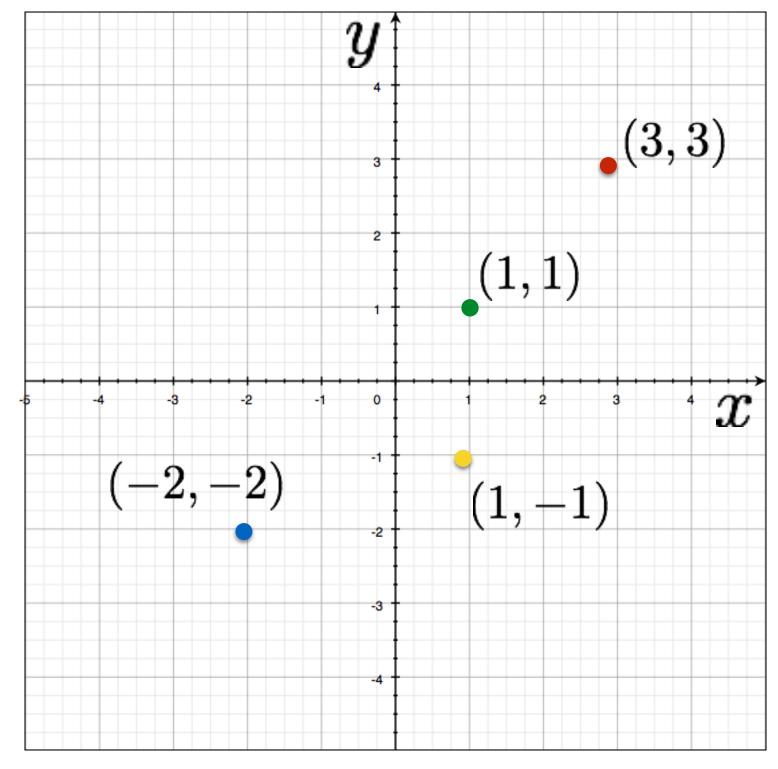
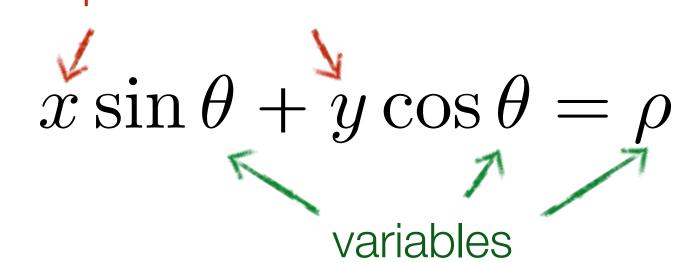
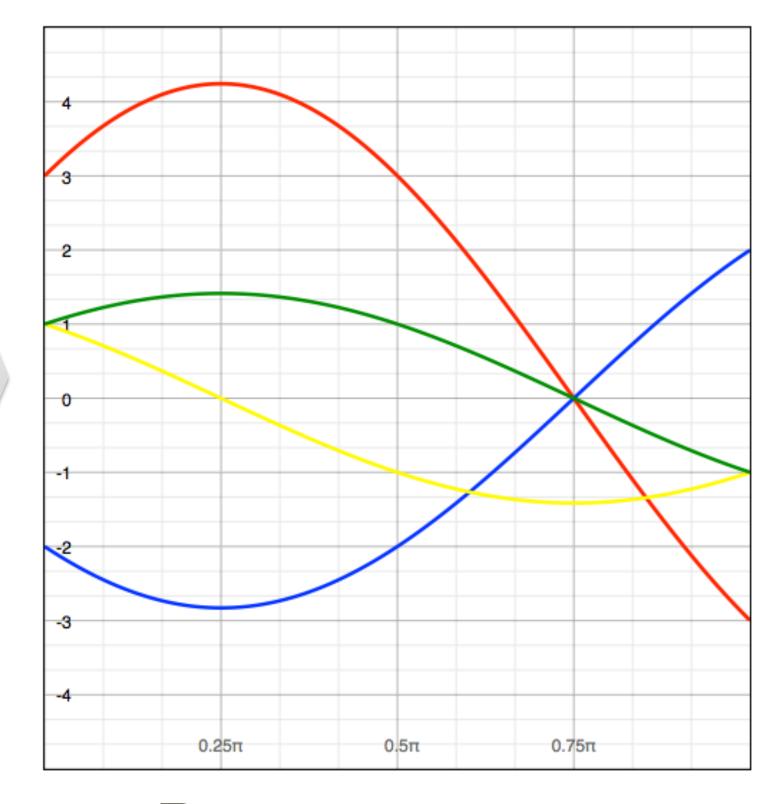


Image space



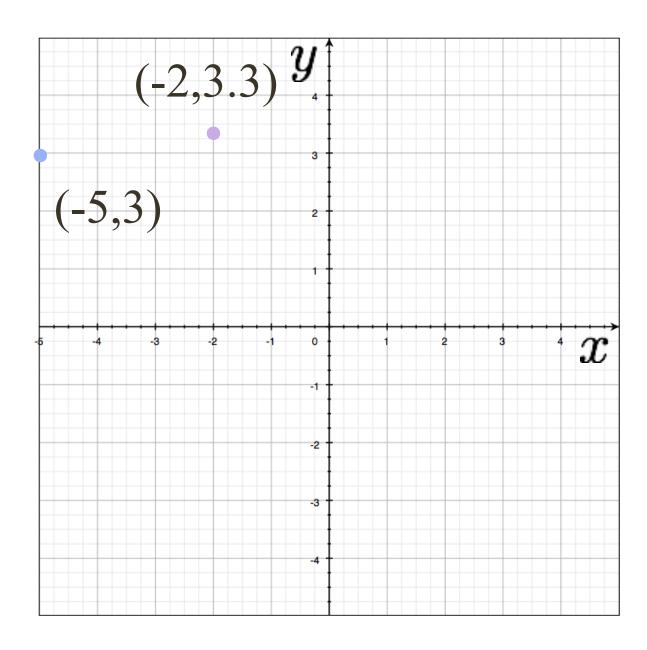


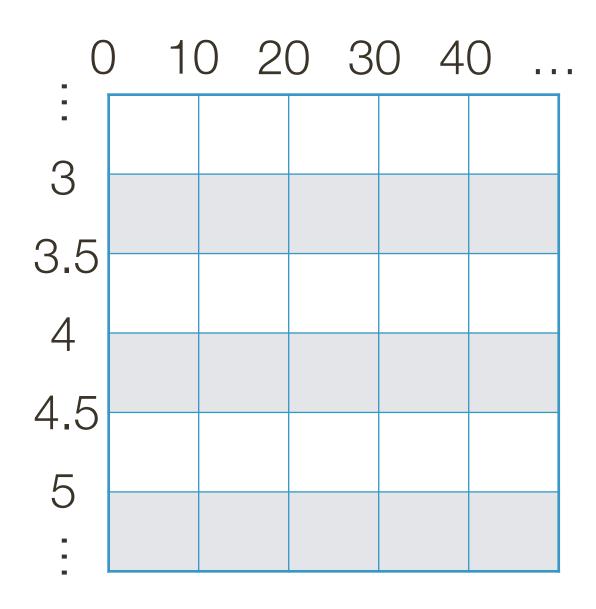


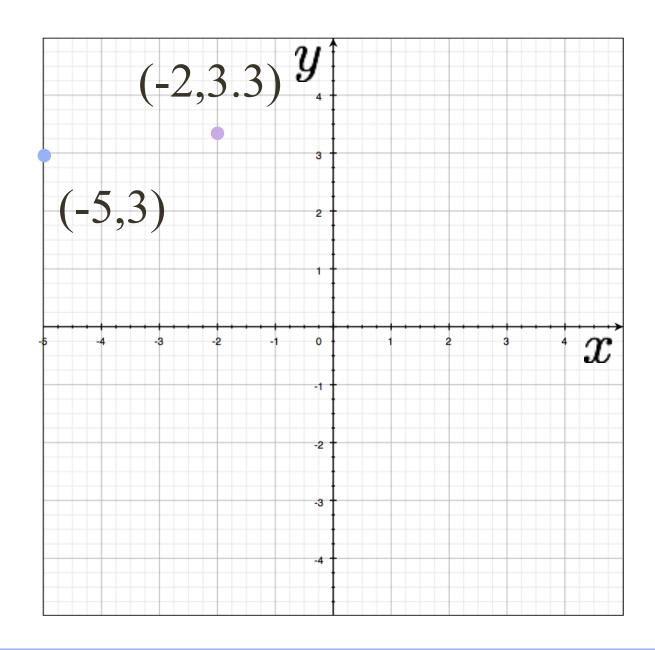
four points

become?

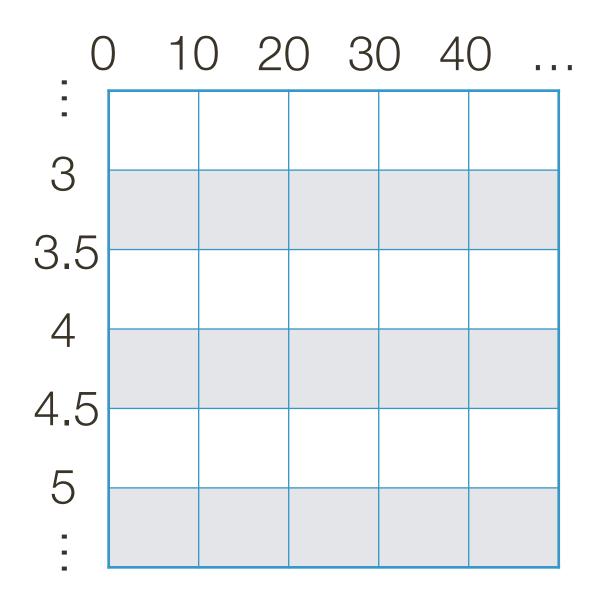
Parameter space

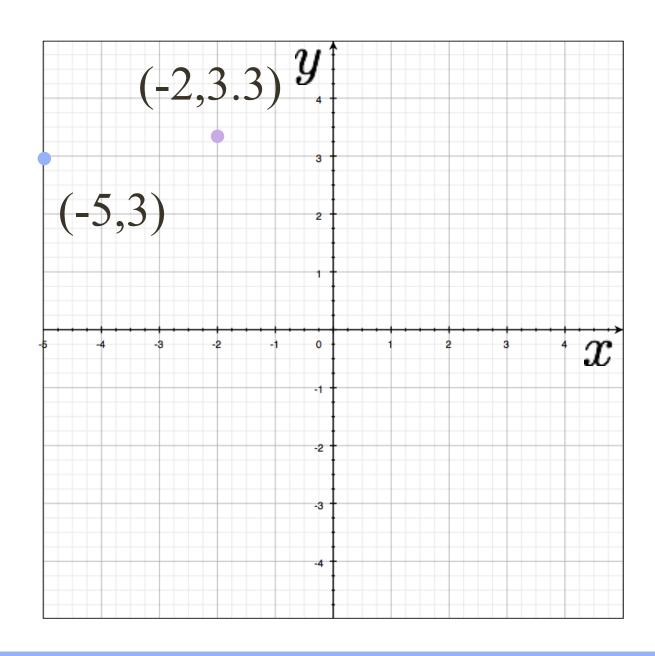


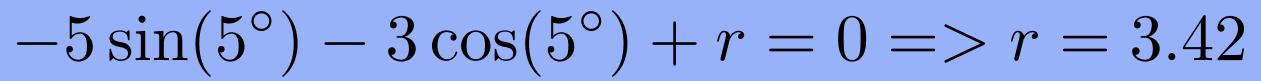


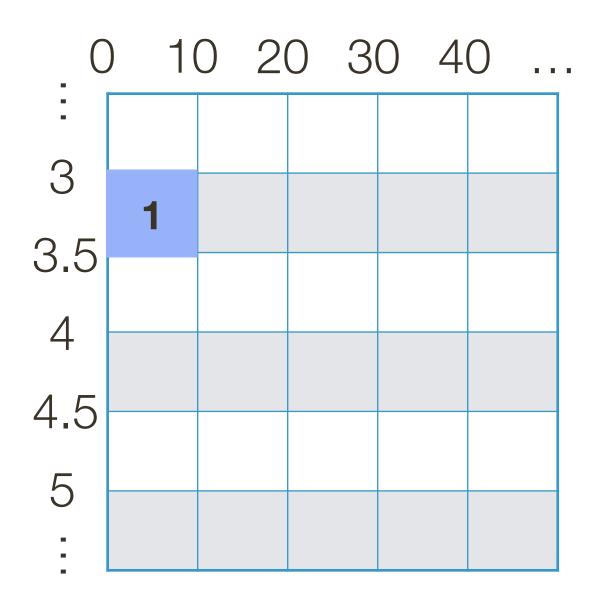


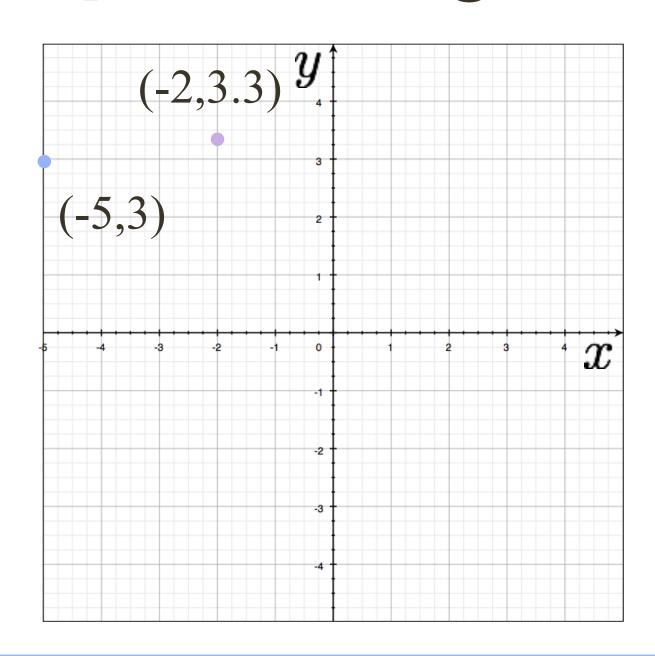
$$-5\sin(5^{\circ}) - 3\cos(5^{\circ}) + r = 0 => r = 3.42$$



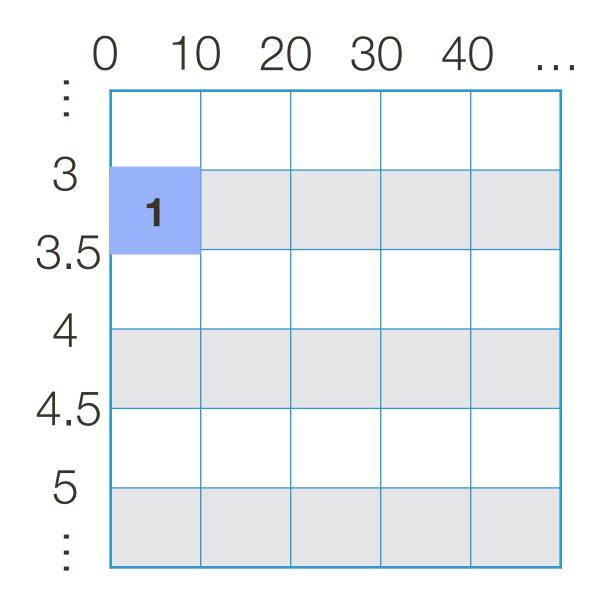


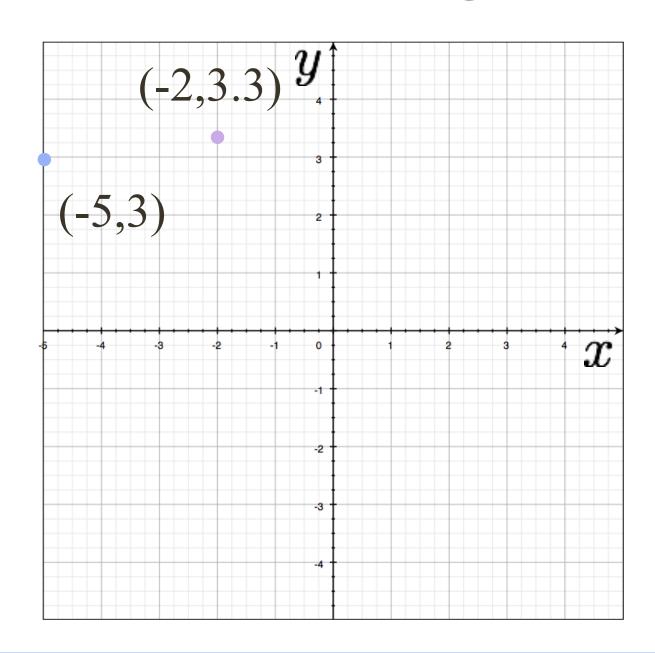




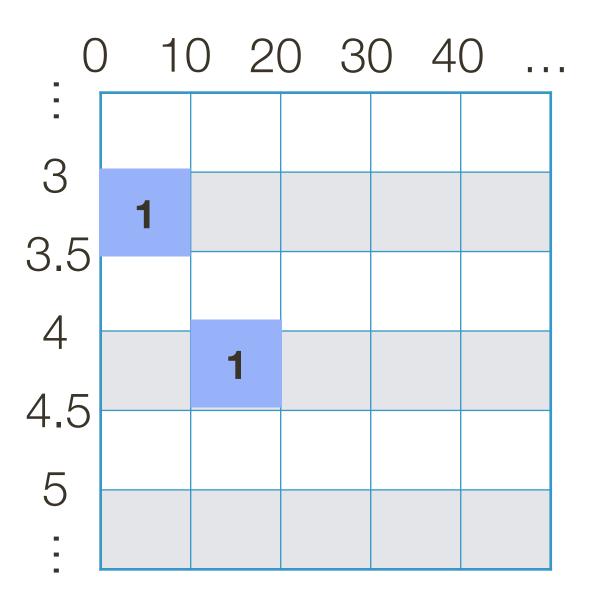


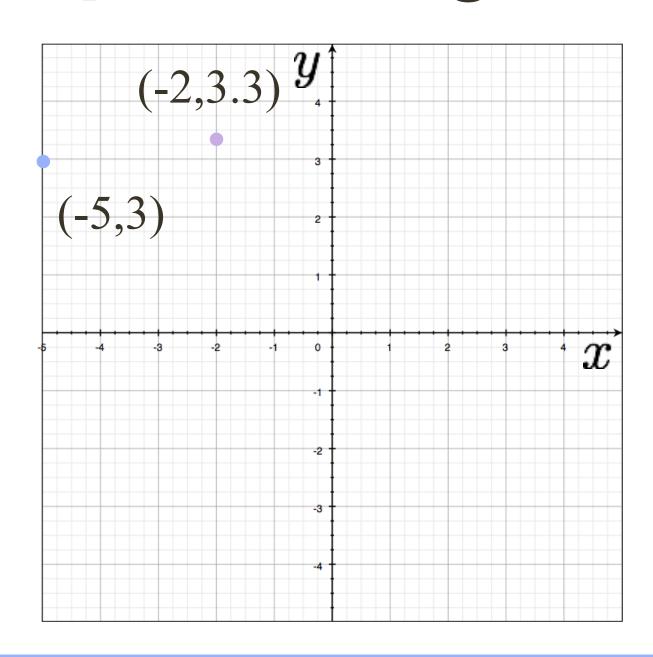
$$-5\sin(5^\circ) - 3\cos(5^\circ) + r = 0 => r = 3.42$$
$$-5\sin(15^\circ) - 3\cos(15^\circ) + r = 0 => r = 4.18$$





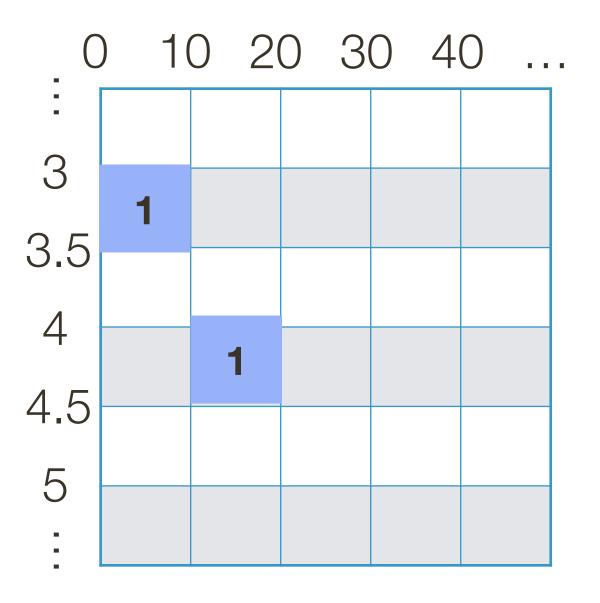
$$-5\sin(5^\circ) - 3\cos(5^\circ) + r = 0 => r = 3.42$$
$$-5\sin(15^\circ) - 3\cos(15^\circ) + r = 0 => r = 4.18$$

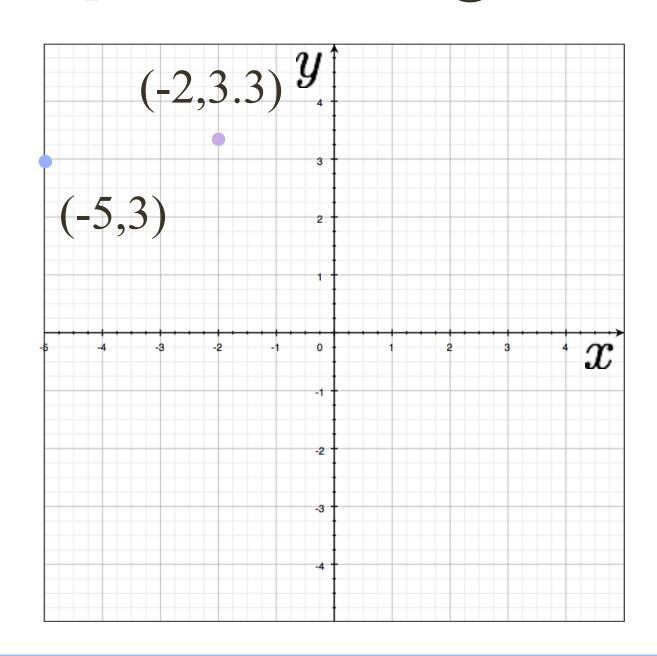




$$-5\sin(5^{\circ}) - 3\cos(5^{\circ}) + r = 0 => r = 3.42$$
$$-5\sin(15^{\circ}) - 3\cos(15^{\circ}) + r = 0 => r = 4.18$$

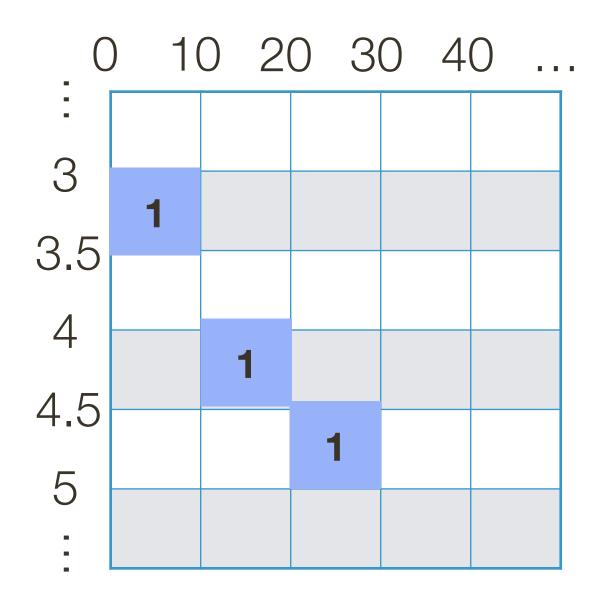
$$-5\sin(25^\circ) - 3\cos(25^\circ) + r = 0 => r = 4.83$$

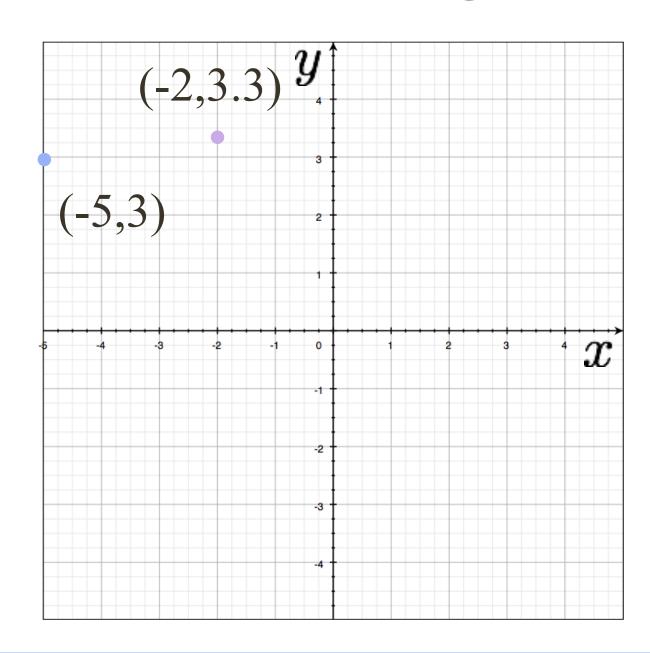


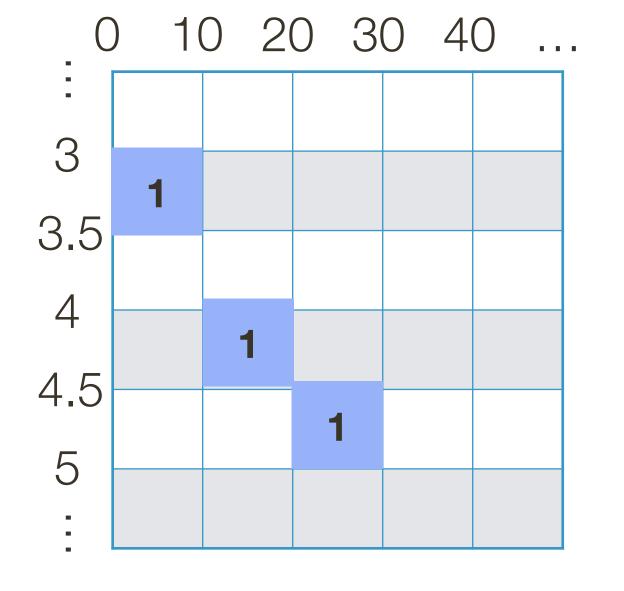


$$-5\sin(5^{\circ}) - 3\cos(5^{\circ}) + r = 0 => r = 3.42$$
$$-5\sin(15^{\circ}) - 3\cos(15^{\circ}) + r = 0 => r = 4.18$$

$$-5\sin(25^\circ) - 3\cos(25^\circ) + r = 0 => r = 4.83$$





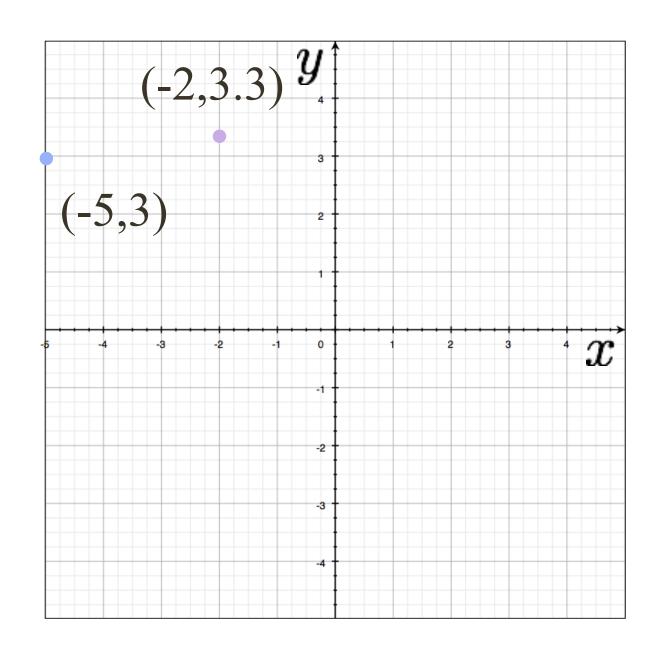


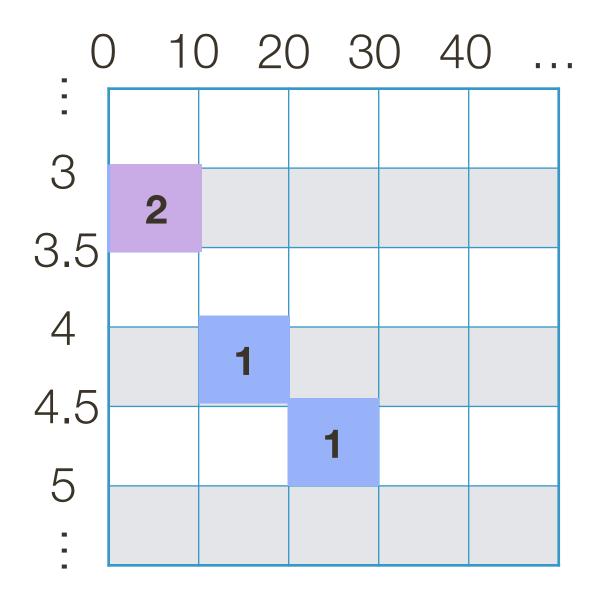
$$-5\sin(5^{\circ}) - 3\cos(5^{\circ}) + r = 0 => r = 3.42$$

$$-5\sin(15^{\circ}) - 3\cos(15^{\circ}) + r = 0 => r = 4.18$$

$$-5\sin(25^{\circ}) - 3\cos(25^{\circ}) + r = 0 => r = 4.83$$

$$-2\sin(5^\circ) - 3.3\cos(5^\circ) + r = 0 => r = 3.46$$



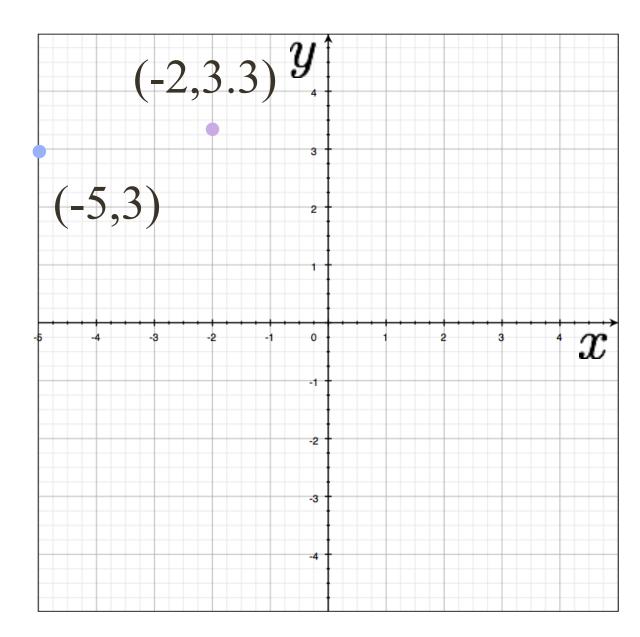


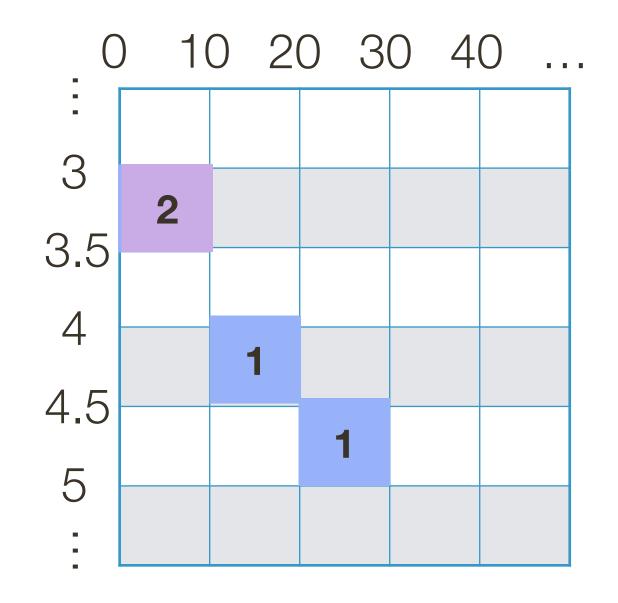
$$-5\sin(5^{\circ}) - 3\cos(5^{\circ}) + r = 0 => r = 3.42$$

$$-5\sin(15^{\circ}) - 3\cos(15^{\circ}) + r = 0 => r = 4.18$$

$$-5\sin(25^{\circ}) - 3\cos(25^{\circ}) + r = 0 => r = 4.83$$

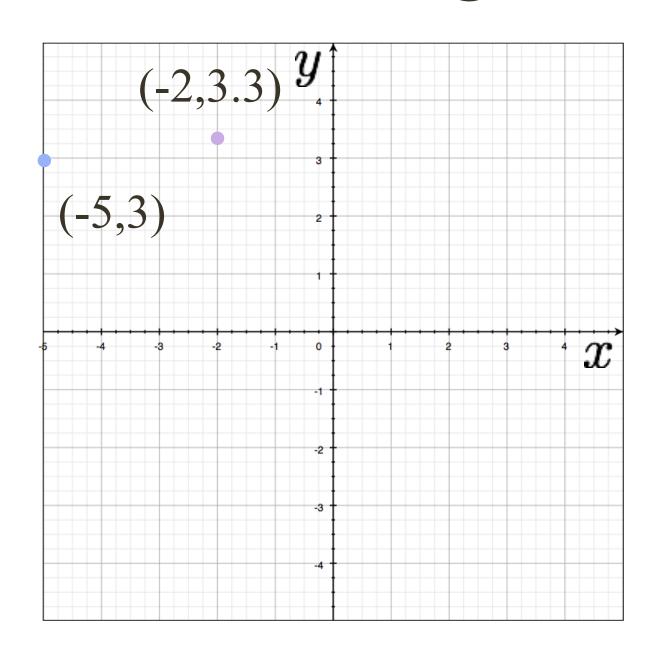
$$-2\sin(5^\circ) - 3.3\cos(5^\circ) + r = 0 => r = 3.46$$

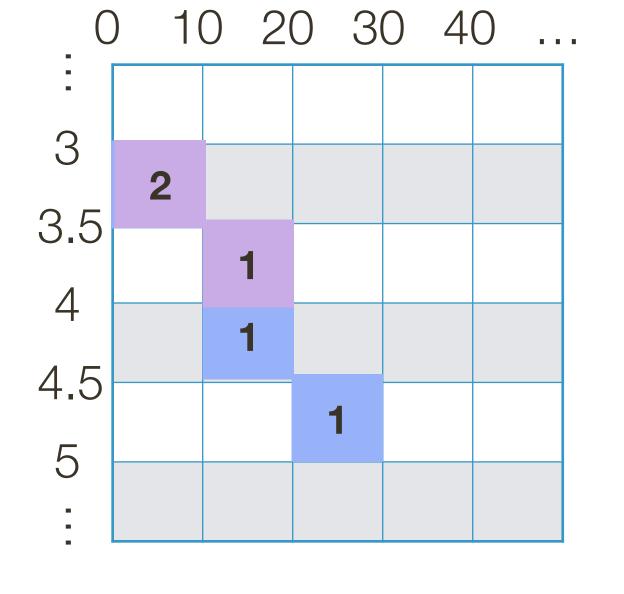




$$-5\sin(5^\circ) - 3\cos(5^\circ) + r = 0 => r = 3.42$$
$$-5\sin(15^\circ) - 3\cos(15^\circ) + r = 0 => r = 4.18$$
$$-5\sin(25^\circ) - 3\cos(25^\circ) + r = 0 => r = 4.83$$

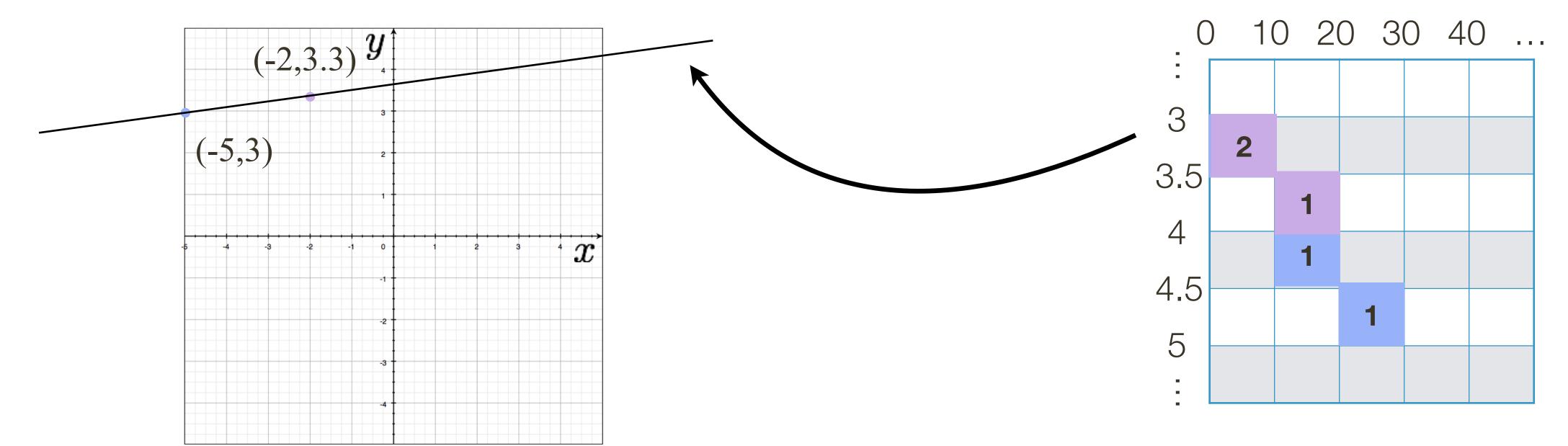
$$-2\sin(5^\circ) - 3.3\cos(5^\circ) + r = 0 => r = 3.46$$
$$-2\sin(15^\circ) - 3.3\cos(15^\circ) + r = 0 => r = 3.71$$





$$-5\sin(5^{\circ}) - 3\cos(5^{\circ}) + r = 0 => r = 3.42$$
$$-5\sin(15^{\circ}) - 3\cos(15^{\circ}) + r = 0 => r = 4.18$$
$$-5\sin(25^{\circ}) - 3\cos(25^{\circ}) + r = 0 => r = 4.83$$

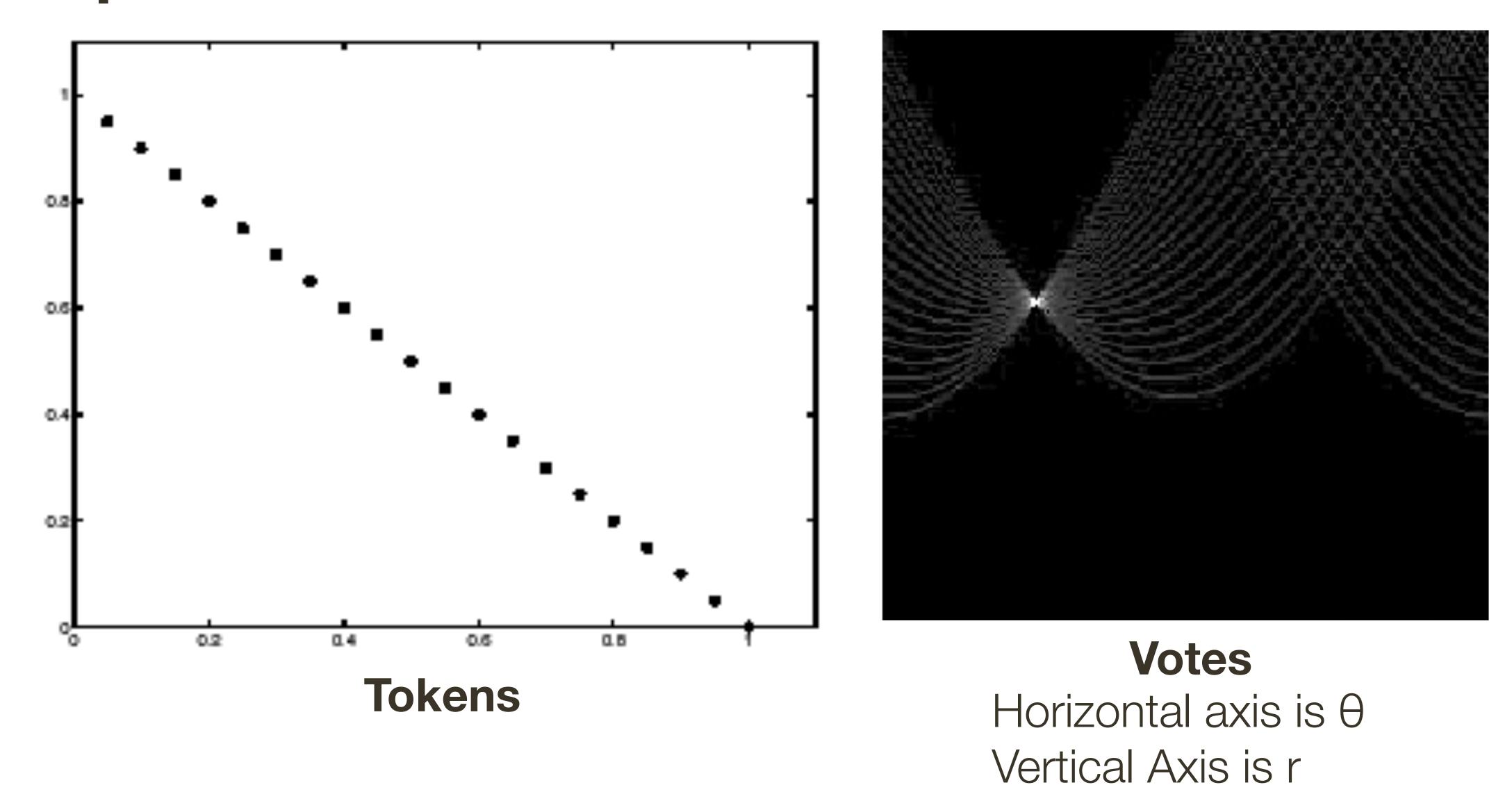
$$-2\sin(5^\circ) - 3.3\cos(5^\circ) + r = 0 => r = 3.46$$
$$-2\sin(15^\circ) - 3.3\cos(15^\circ) + r = 0 => r = 3.71$$



$$-5\sin(5^{\circ}) - 3\cos(5^{\circ}) + r = 0 => r = 3.42$$
$$-5\sin(15^{\circ}) - 3\cos(15^{\circ}) + r = 0 => r = 4.18$$
$$-5\sin(25^{\circ}) - 3\cos(25^{\circ}) + r = 0 => r = 4.83$$

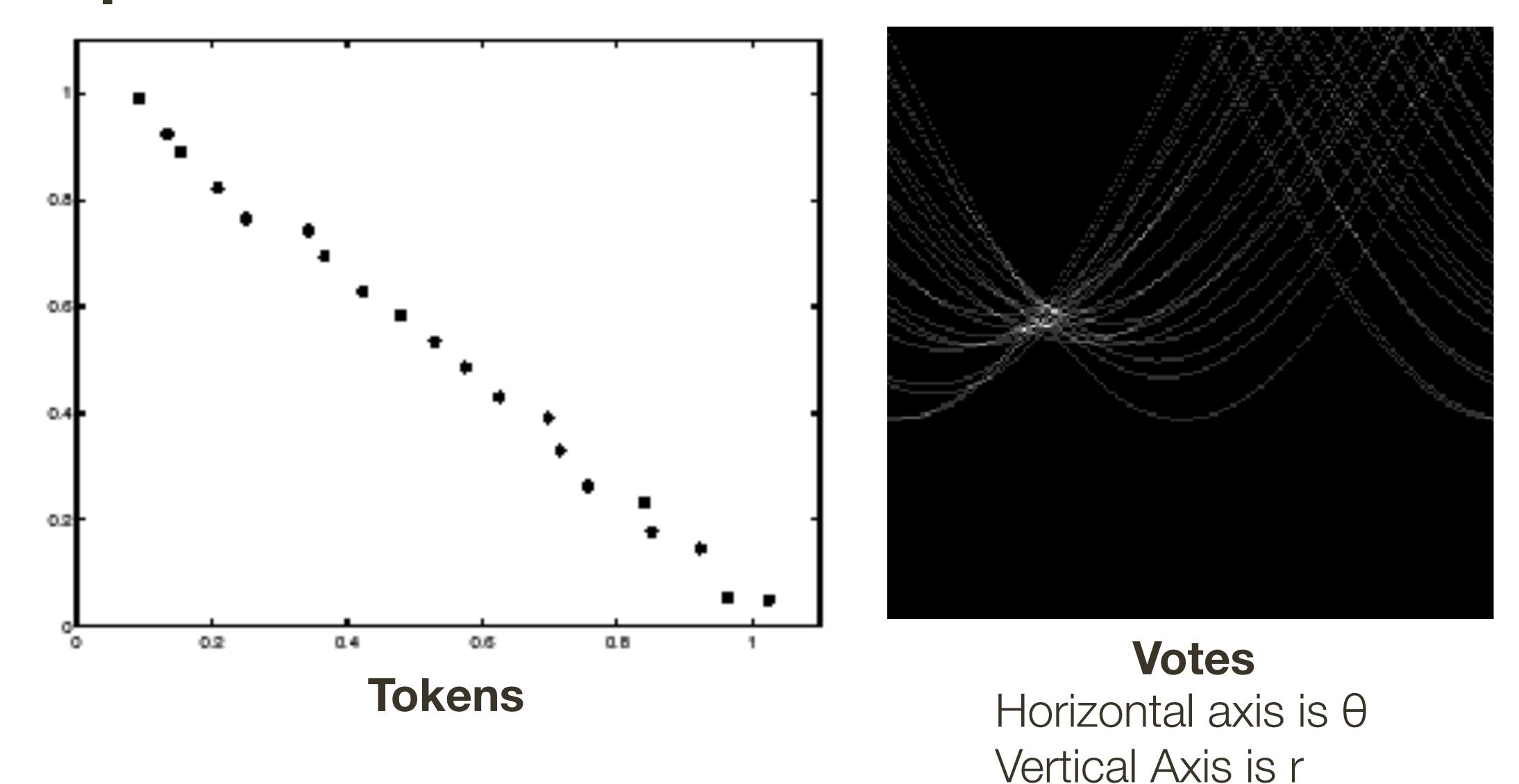
$$-2\sin(5^\circ) - 3.3\cos(5^\circ) + r = 0 => r = 3.46$$
$$-2\sin(15^\circ) - 3.3\cos(15^\circ) + r = 0 => r = 3.71$$

Example: Clean Data



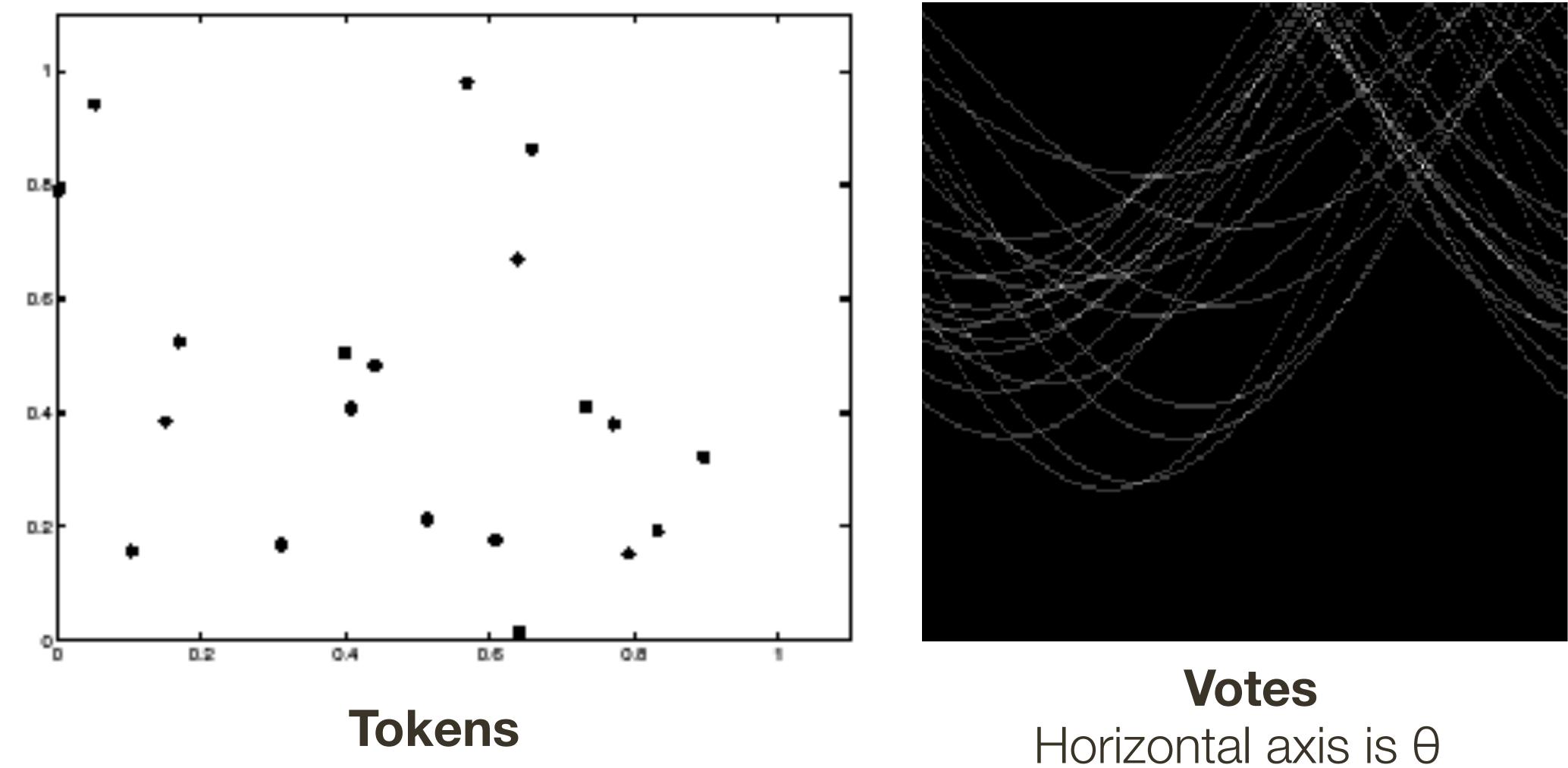
Forsyth & Ponce (2nd ed.) Figure 10.1 (Top)

Example: Some Noise



Forsyth & Ponce (2nd ed.) Figure 10.1 (Bottom)

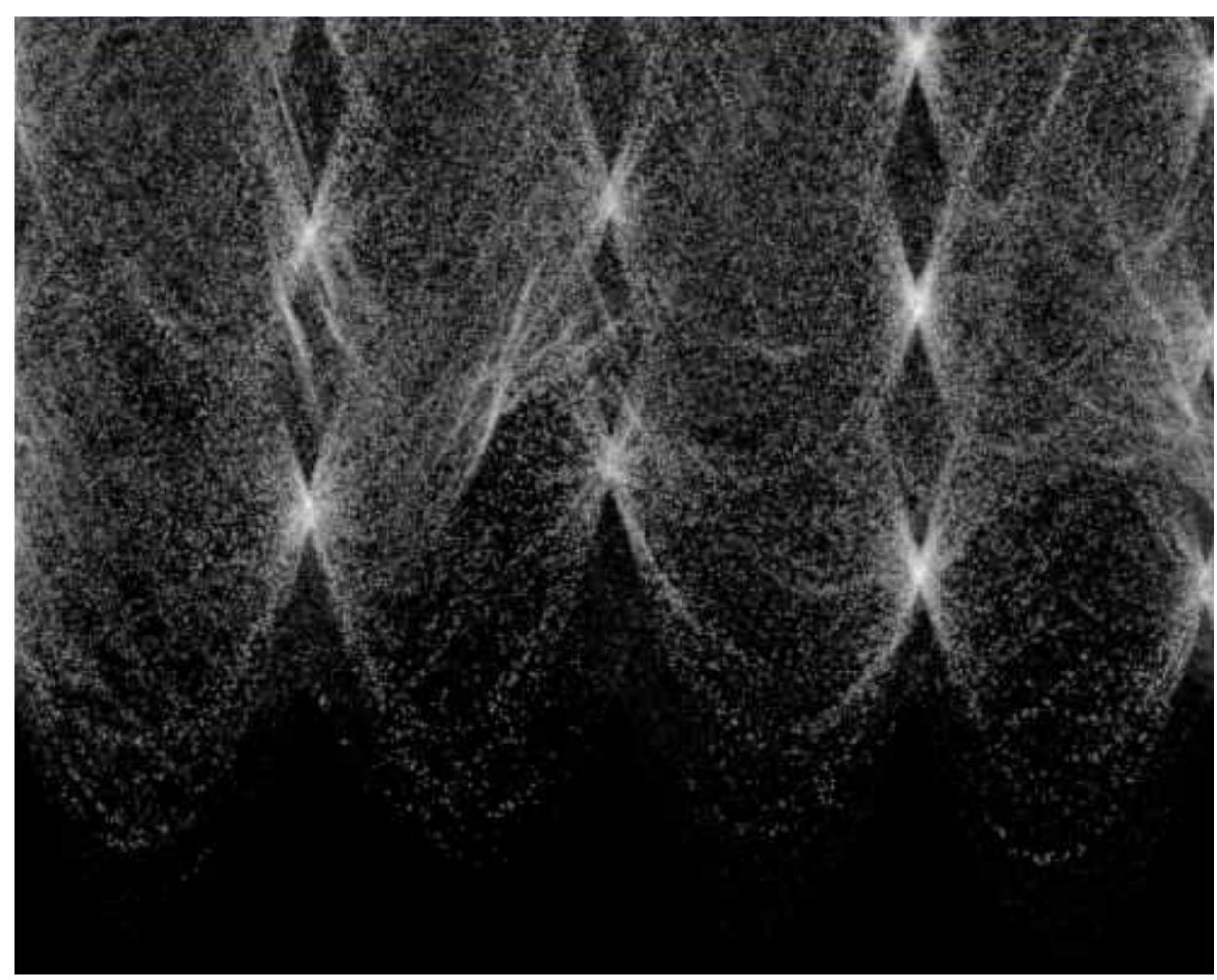
Example: Too Much Noise



Vertical Axis is r Forsyth & Ponce (2nd ed.) Figure 10.2

Real World Example

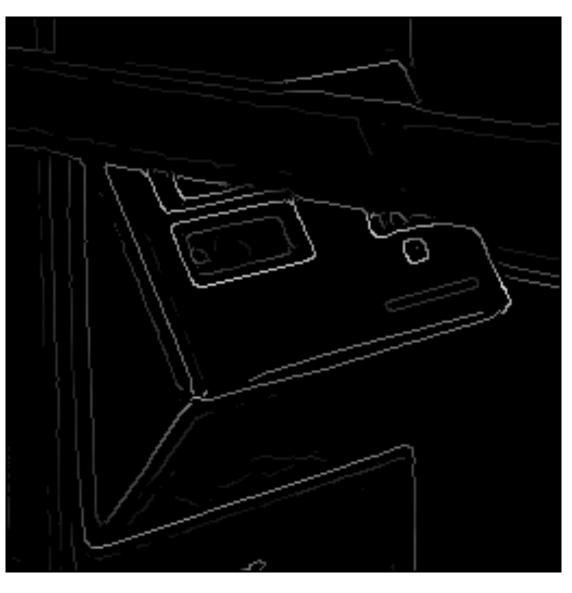




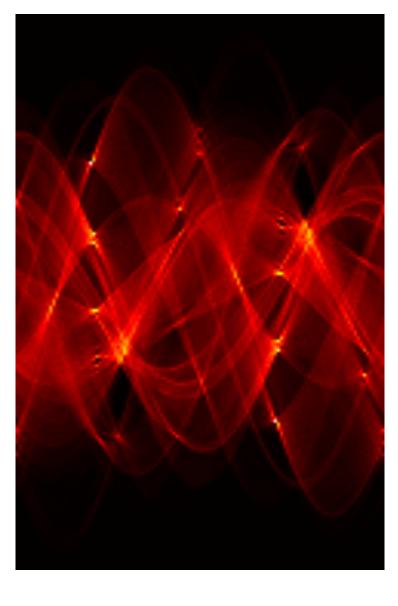
Real World Example



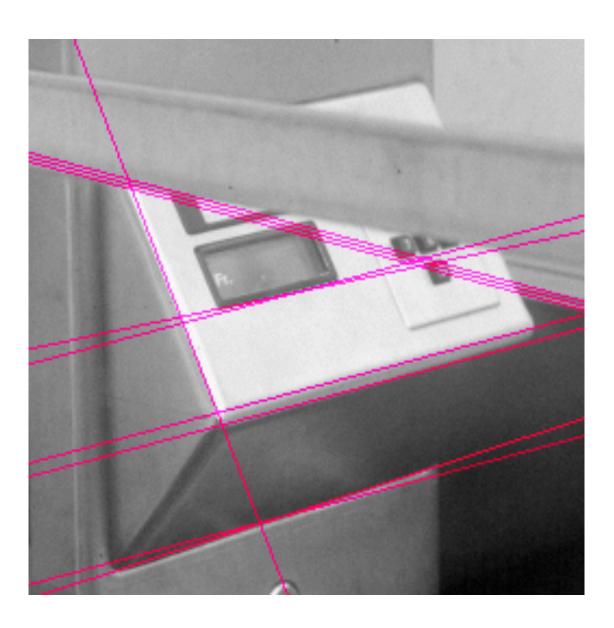
Original



Edges



Parameter space



Hough Lines

Mechanics of Hough Transform

- 1. Construct a quantized array to represent θ and r
- 2. For each point, render curve (θ , r) into this array adding one vote at each cell

Difficulties:

— How big should the cells be? (too big, and we merge quite different lines; too small, and noise causes lines to be missed)

How many lines?

- Count the peaks in the Hough array
- Treat adjacent peaks as a single peak

Some Practical Details of Hough Transform

It is best to **vote** for the two closest bins in each dimension, as the locations of the bin boundaries are arbitrary

— This means that peaks are "blurred" and noise will not cause similar votes to fall into separate bins

Can use a hash table rather than an array to store the votes

- This means that no effort is wasted on initializing and checking empty bins
- It avoids the need to predict the maximum size of the array, which can be non-rectangular

Hough Transform: Transformation Space Voting

Sometimes a single point / measurement can vote on the entire transformation

e.g., **SIFT** keypoint matches with **location, scale and orientation** vote on the 4 parameters of a **similarity transform** (x,y,s,theta)

In this case, the votes of each sample can be seen as a distribution in the parameter space of the transformation

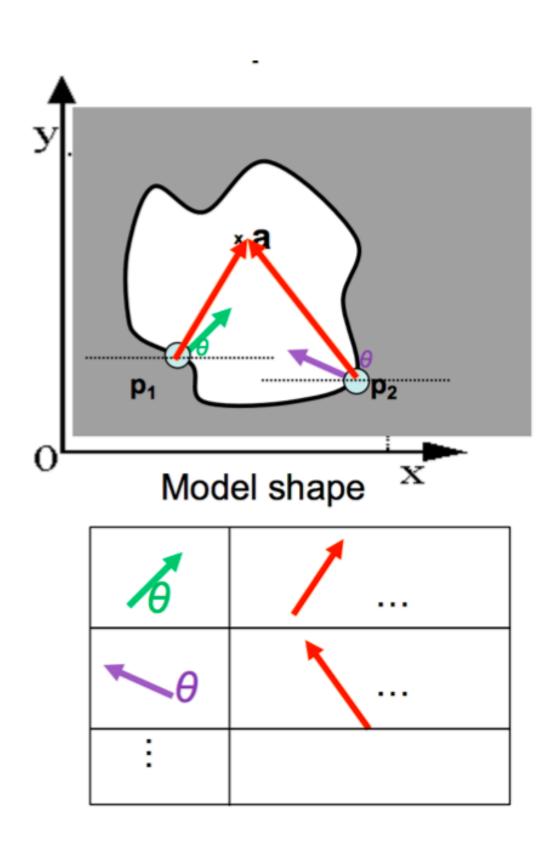
This can be effective in preventing noise in the distribution, e.g., edge detections with orientation can vote on **single lines** rather than **all lines** that pass through a point.

Generalized Hough Transform

What if we want to detect an arbitrary geometric shape?

Generalized Hough Transform

What if we want to detect an arbitrary geometric shape?



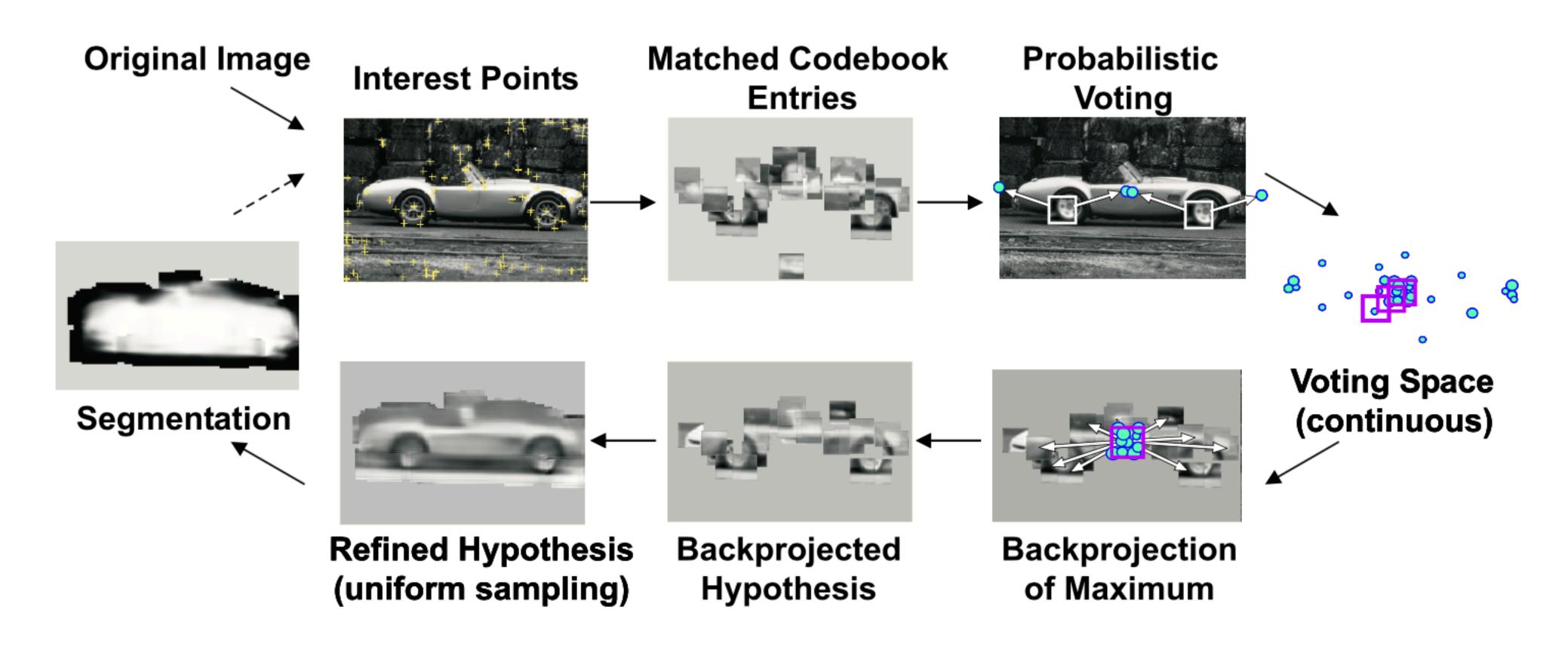
Offline procedure:

At each boundary point, compute displacement vector: $\mathbf{r} = \mathbf{a} - \mathbf{p_i}$.

Store these vectors in a table indexed by gradient orientation θ .

Example 1: Object Recognition — Implicit Shape Model

Combined object detection and segmentation using an implicit shape model. Image patches cast weighted votes for the object centroid.

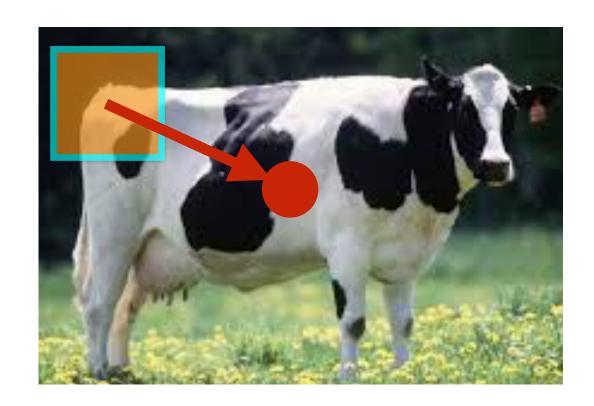


B. Leibe, A. Leonardis, and B. Schiele, Combined Object Categorization and Segmentation with an Implicit Shape Model, ECCV Workshop on Statistical Learning in Computer Vision 2004

Example 1: Object Recognition — Implicit Shape Model

Basic Idea:

- Find interest points/keypoints in an image (e.g., SIFT Keypoint detector or Corners)
- Match patch around each interest point to a training patch (e.g., SIFT Descriptor)
- Vote for object center given that training instances
- Find the patches that voted for the peaks (back-project)



lmage Index	Keypoint Index	Keypoint Detection (4D)	Keypoint Description (128D)	Offset to Centroid
Image 1	1	[x, y, s, Theta]	[]	[x,y]

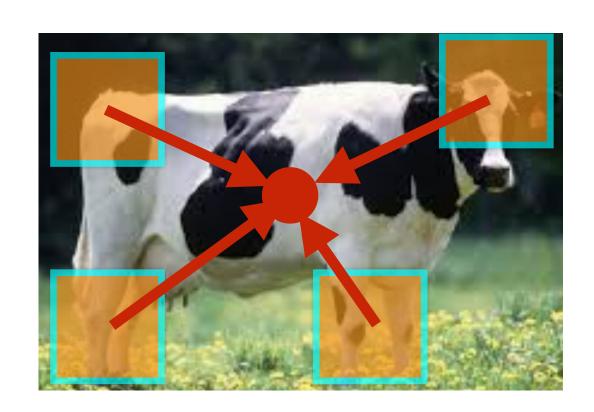
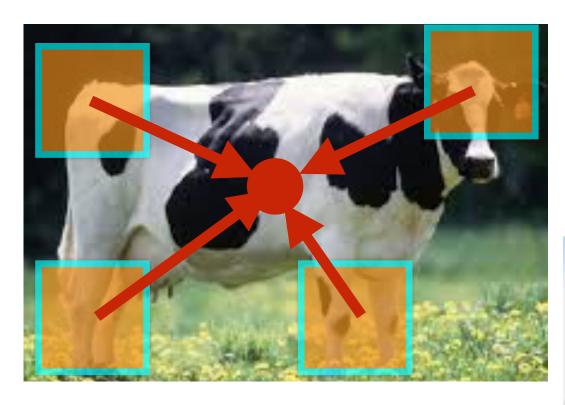
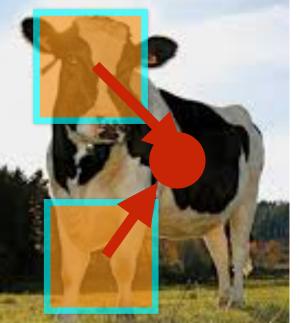


Image Index	Keypoint Index	Keypoint Detection (4D)	Keypoint Description (128D)	Offset to Centroid
lmage 1 lmage 1	1 2	[x, y, s, Theta] [x, y, s, Theta]	[] []	[x,y] [x,y]
 Image 1	265	[x, y, s, Theta]	 []	 [x,y]





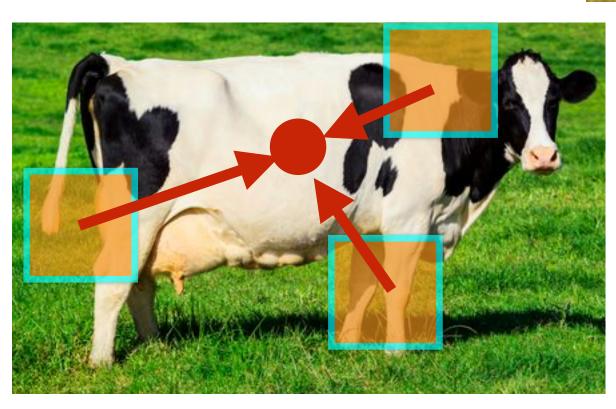


Image Index	Keypoint Index	Keypoint Detection (4D)	Keypoint Description (128D)	Offset to Centroid
Image 1 Image 1	1 2	[x, y, s, Theta] [x, y, s, Theta]	[] []	[x,y]
Image 1	265	[x, y, s, Theta]	[]	[x,y]
Image 2 Image 2	1 2	[x, y, s, Theta] [x, y, s, Theta]	[] []	[x,y] [x,y]
Image 2	645	[x, y, s, Theta]	 []	[x,y]
Image K Image K	1 2	[x, y, s, Theta] [x, y, s, Theta]	[] []	[x,y] [x,y]
Image K	134	[x, y, s, Theta]	[]	[x,y]









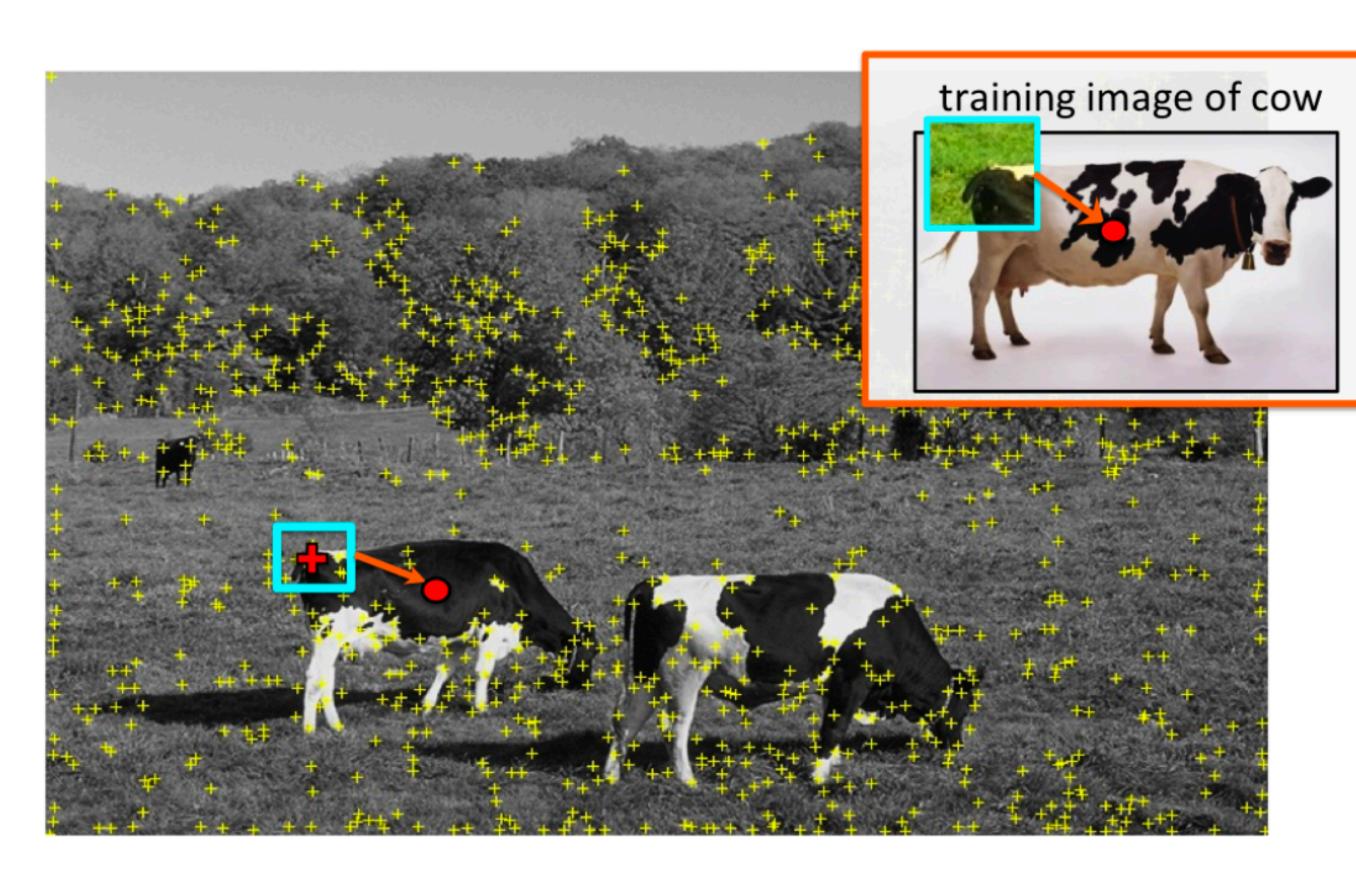


"Training" images of cows









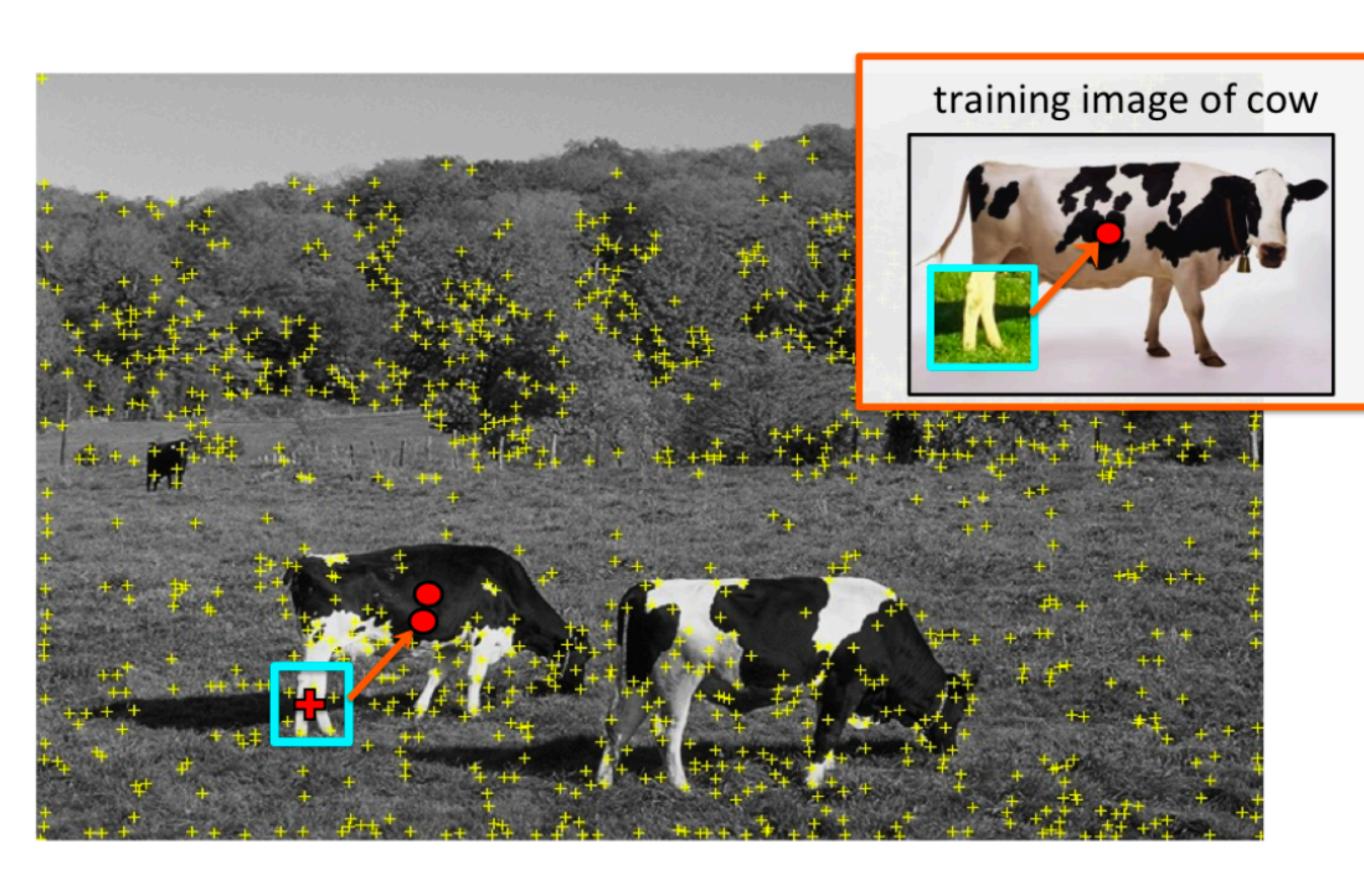
Vote for center of object

"Training" images of cows









Vote for center of object

"Training" images of cows









Vote for center of object

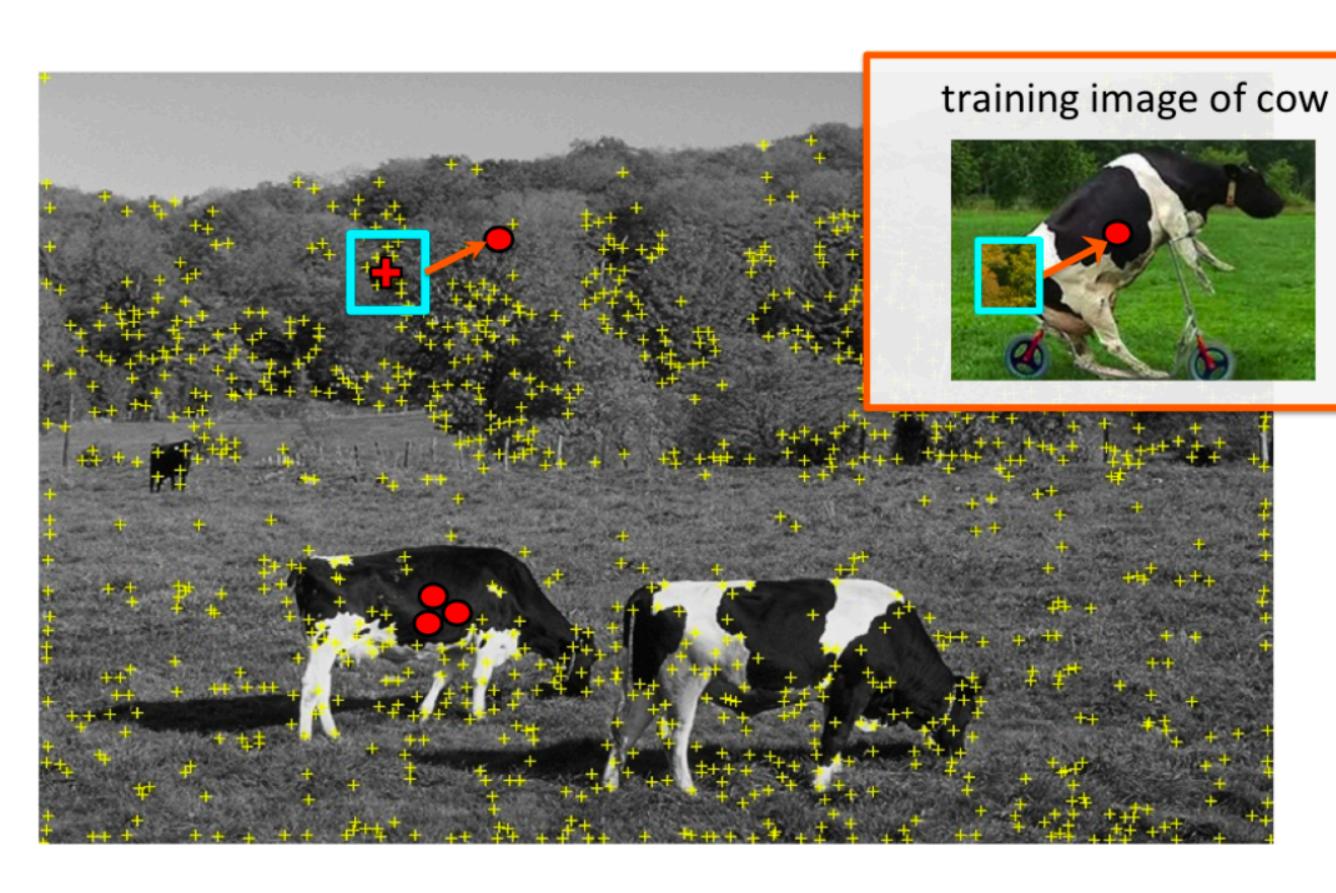
"Training" images of cows

"Testing" image









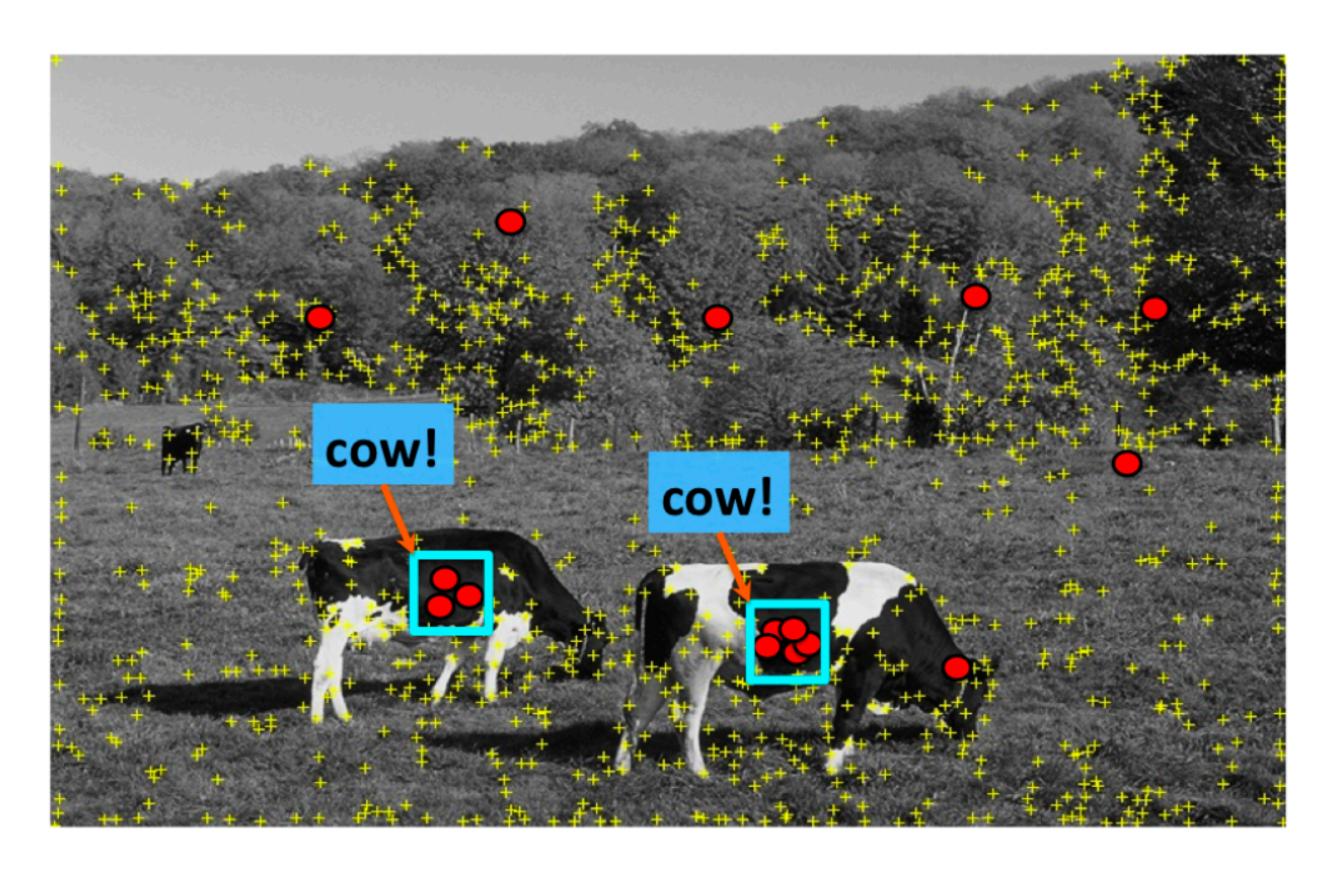
of course sometimes wrong votes are bound to happen

"Training" images of cows









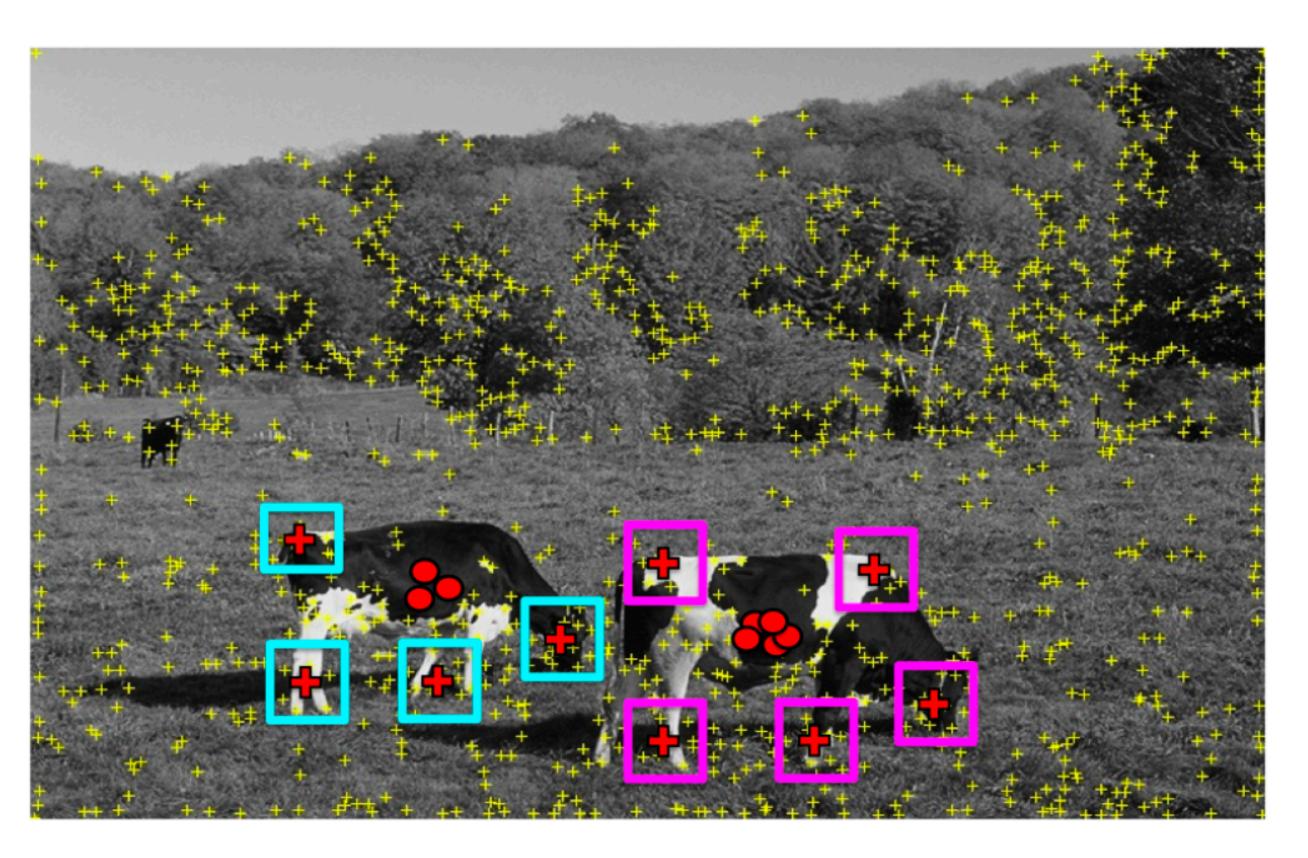
That's ok. We want only peaks in voting space.

"Training" images of cows









Find patches that voted for the peaks (back-project)

Image Index	Keypoint Index	Keypoint Detection (4D)	Keypoint Description (128D)	Offset to Centroid	
Image 1 Image 1	1 2	[x, y, s, Theta] [x, y, s, Theta]	[] []	[x,y] [x,y]	
Image 1	265	[x, y, s, Theta]	[]	[x,y]	
Image 2 Image 2	1 2	[x, y, s, Theta] [x, y, s, Theta]	[] []	[x,y] [x,y]	
Image 2	645	[x, y, s, Theta]	[]	[x,y]	
Image K Image K	1 2	[x, y, s, Theta] [x, y, s, Theta]	[] []	[x,y] [x,y]	
Image K	134	[x, y, s, Theta]	[]	[x,y]	

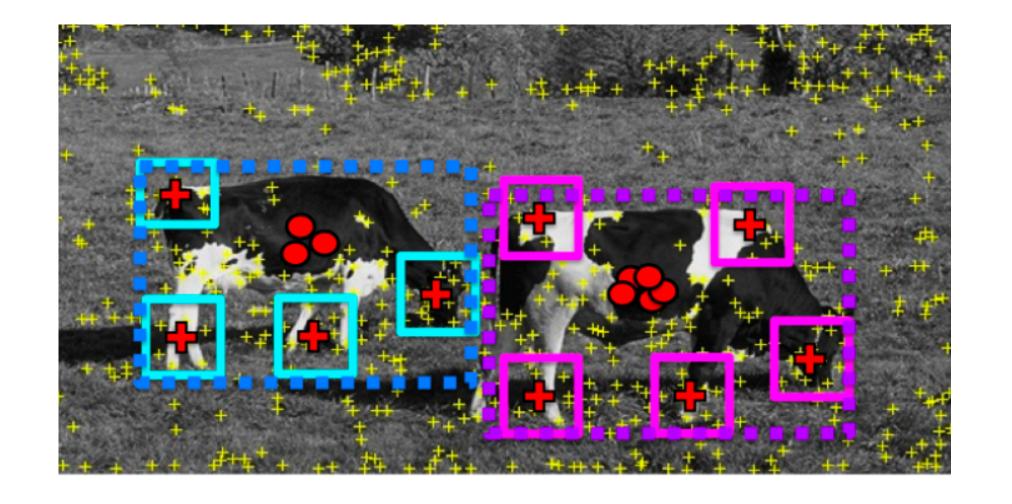
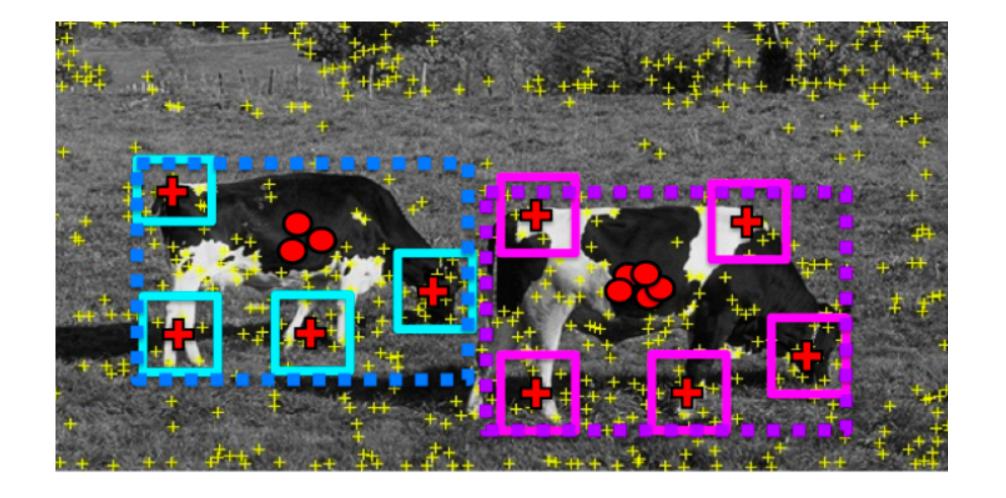


Image Index	Keypoint Index	Keypoint Detection (4D)	Keypoint Description (128D)	Offset to Centroid
Image 1	1 2	[x, y, s, Theta] [x, y, s, Theta]	[]	[x,y] [x,y]
Image 1	 265	[x, y, s, Theta]	 []	 [x,y]
Image 2 Image 2	1 2	[x, y, s, Theta] [x, y, s, Theta]	[]	[x,y] [x,y]
Image 2	645	[x, y, s, Theta]	[]	[x,y]
Image K Image K	1 2	[x, y, s, Theta] [x, y, s, Theta]	[] []	[x,y] [x,y]
Image K	134	[x, y, s, Theta]	[]	[x,y]



"Training" images of cows

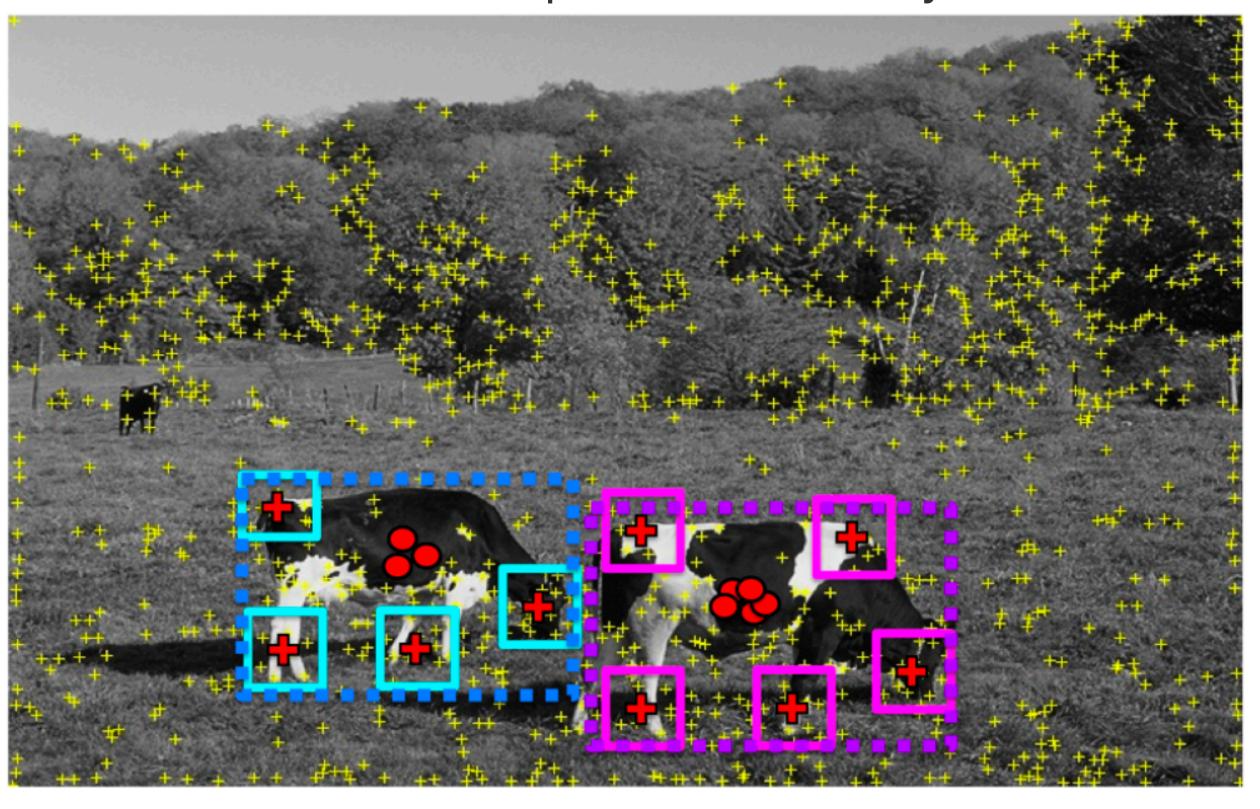






"Testing" image

box around patches = object



Find objects based on the back projected patches

"Training" images of cows



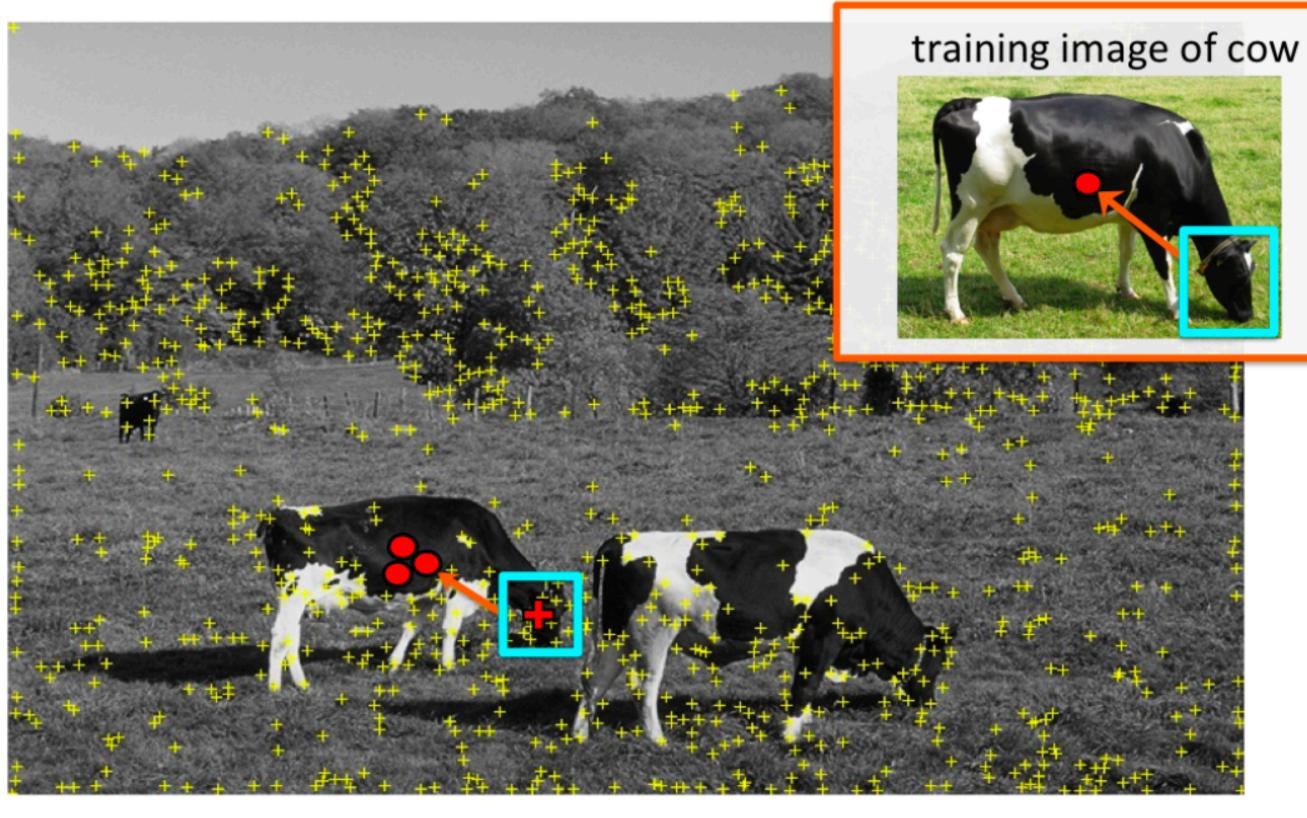
Really easy ... but slow ... how do we make it fast?







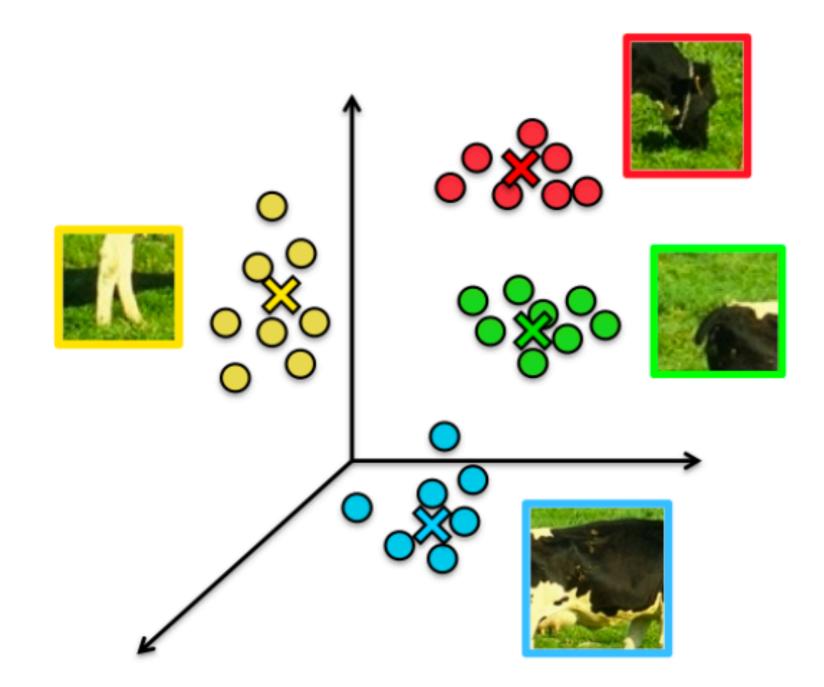




We need to match a patch around each yellow keypoint to all patches in all training images (slow)

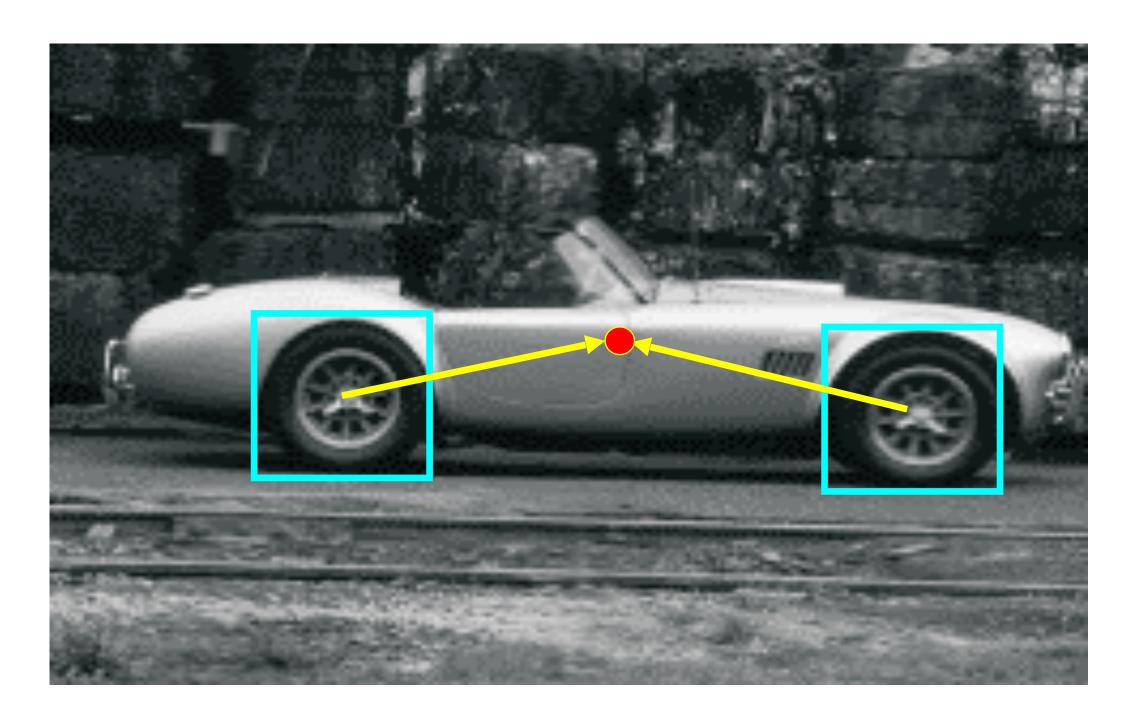
Visual Words

- Visual vocabulary (we saw this for retrieval)
- Compare each patch to a small set of visual words (clusters)

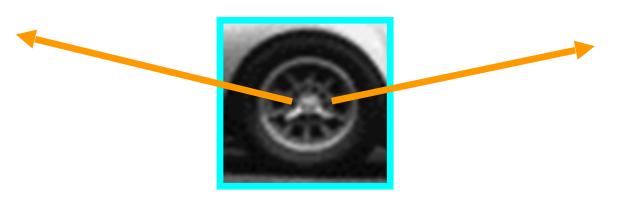


Visual words (visual codebook)!

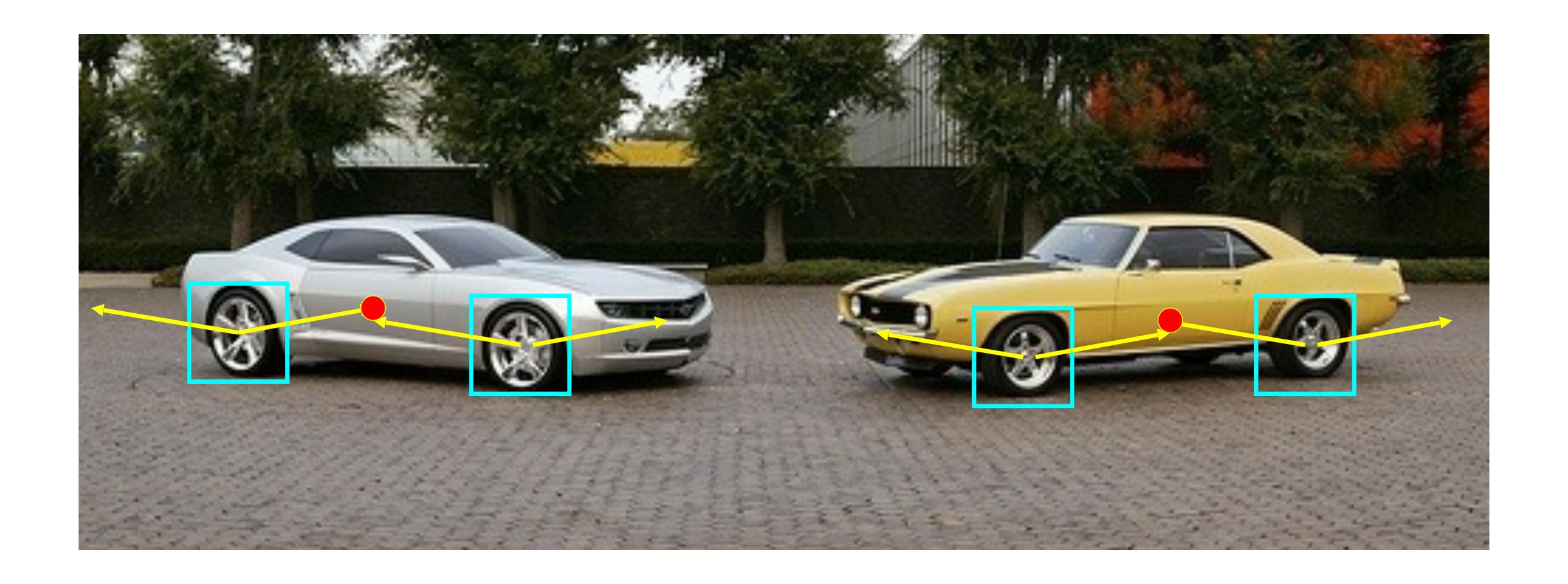
Index displacements by "visual codeword"



training image



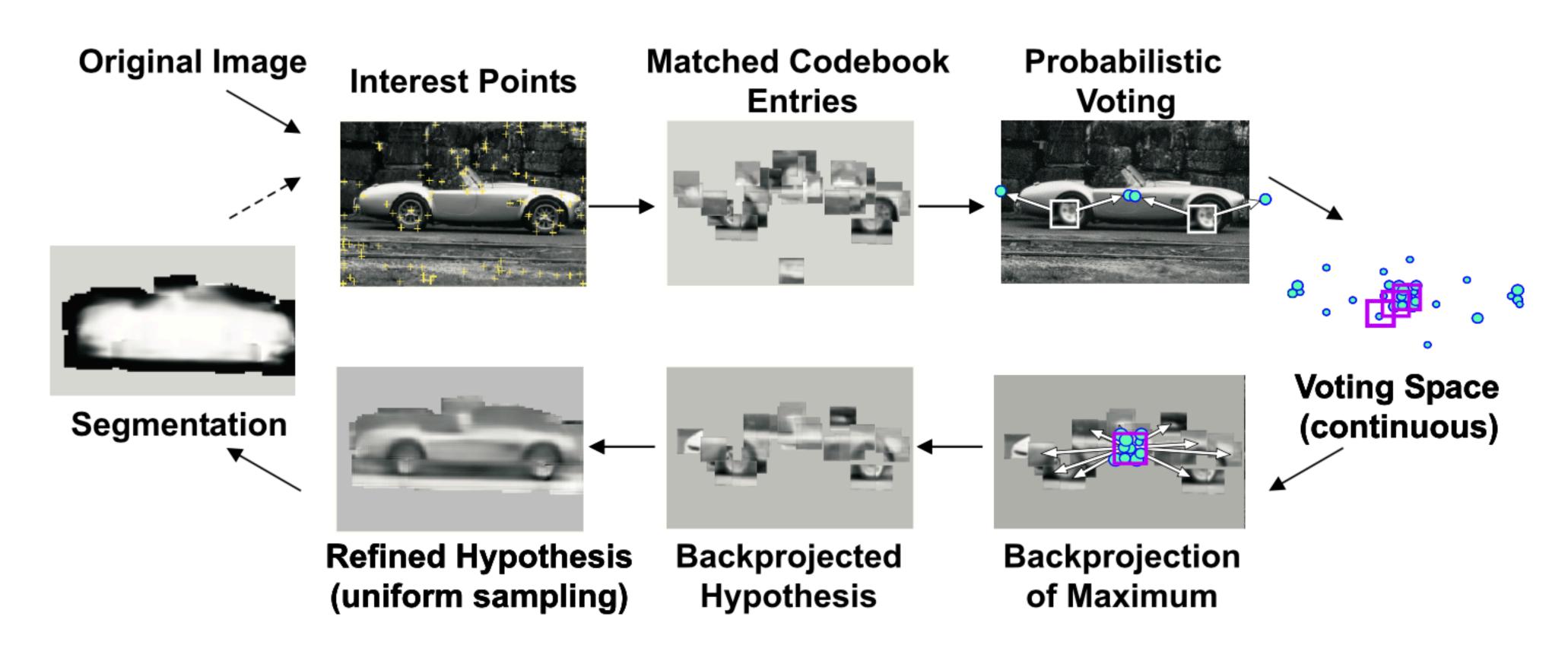
visual codeword with displacement vectors



B. Leibe, A. Leonardis, and B. Schiele, Combined Object Categorization and Segmentation with an Implicit Shape Model, ECCV Workshop on Statistical Learning in Computer Vision 2004

Inferring Other Information: Segmentation

Combined object detection and segmentation using an implicit shape model. Image patches cast weighted votes for the object centroid.



B. Leibe, A. Leonardis, and B. Schiele, Combined Object Categorization and Segmentation with an Implicit Shape Model, ECCV Workshop on Statistical Learning in Computer Vision 2004

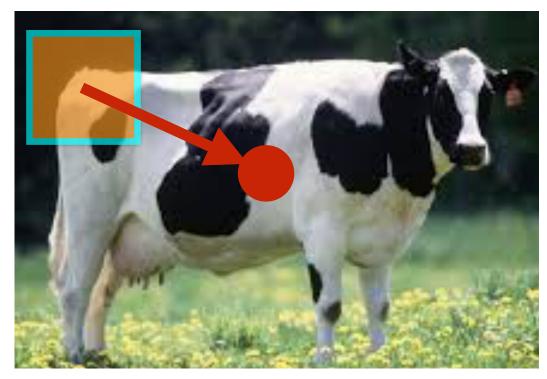


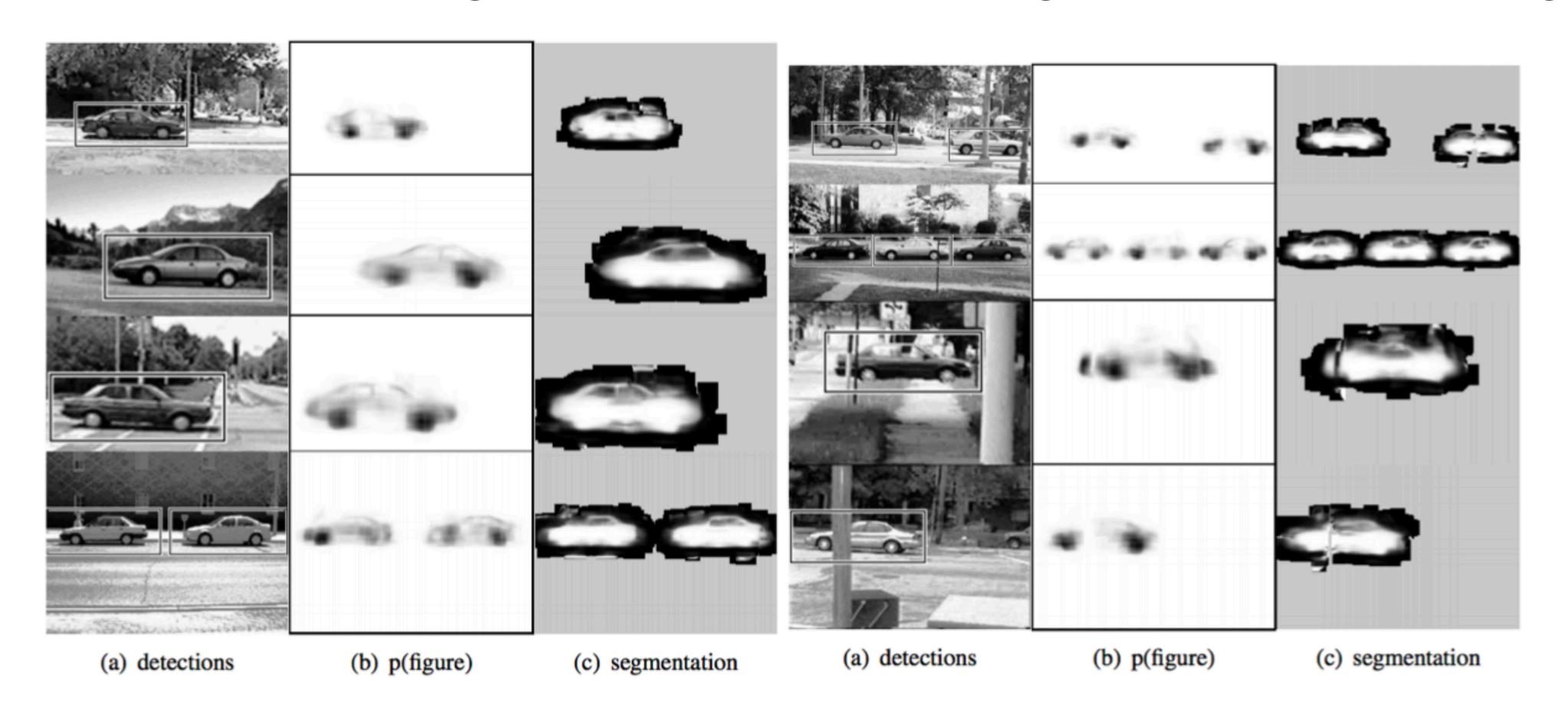




Image Index	Keypoint Index	Keypoint Detection (4D)	Keypoint Description (128D)	Offset to Centroid	Segm
Image 1 Image 1	1 2	[x, y, s, Theta] [x, y, s, Theta]		[x,y] [x,y]	
Image 1	265	[x, y, s, Theta]	[]	[x,y]	
Image 2 Image 2	1 2	[x, y, s, Theta] [x, y, s, Theta]		[x,y] [x,y]	
Image 2	645	[x, y, s, Theta]	[]	[x,y]	
Image K Image K	1 2	[x, y, s, Theta] [x, y, s, Theta]		[x,y] [x,y]	
Image K	134	[x, y, s, Theta]	 []	[x,y]	

Inferring Other Information: Segmentation

Idea: When back-projecting, back-project labeled segmentations per training patch



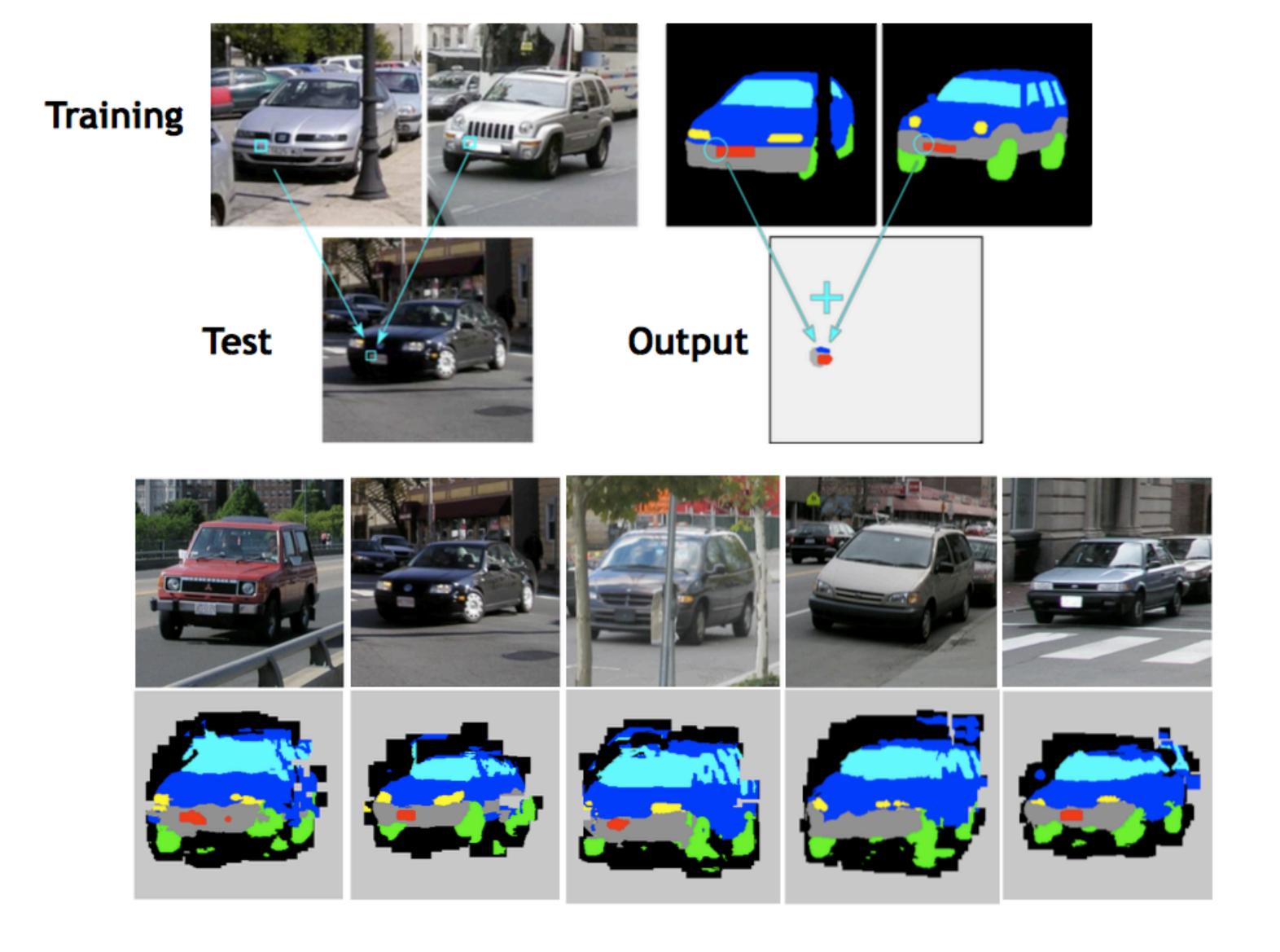
[Source: B. Leibe]

Inferring Other Information: Segmentation

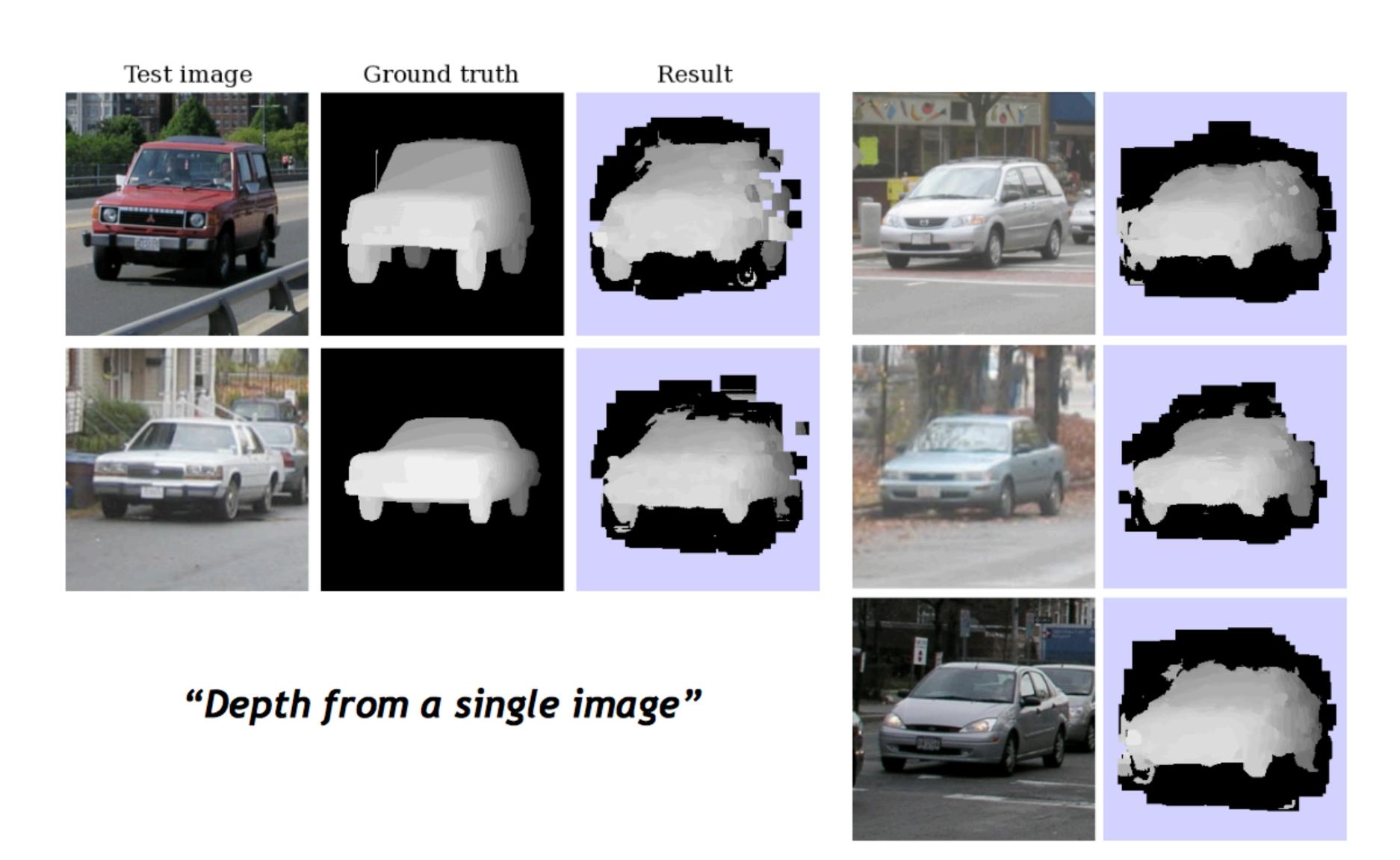


[Source: B. Leibe]

Inferring Other Information: Part Labels



Inferring Other Information: Depth



Example 2: Object Recognition — Boundary Fragments

Boundary fragments cast weighted votes for the object centroid. Also obtains an estimate of the object's contour.

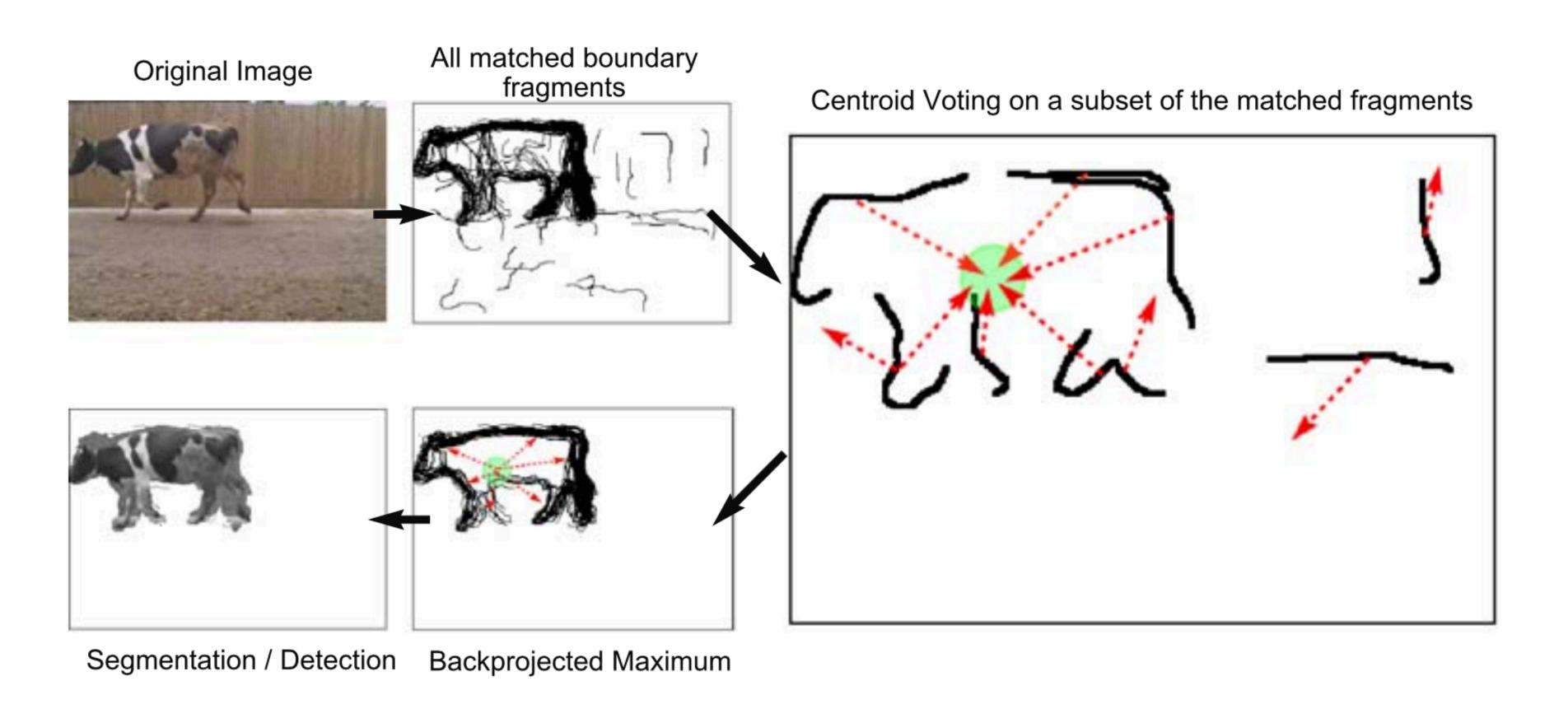


Image credit: Opelt et al., 2006

Example 2: Object Recognition — Boundary Fragments

Boundary fragments cast weighted votes for the object centroid. Also obtains an estimate of the object's contour.

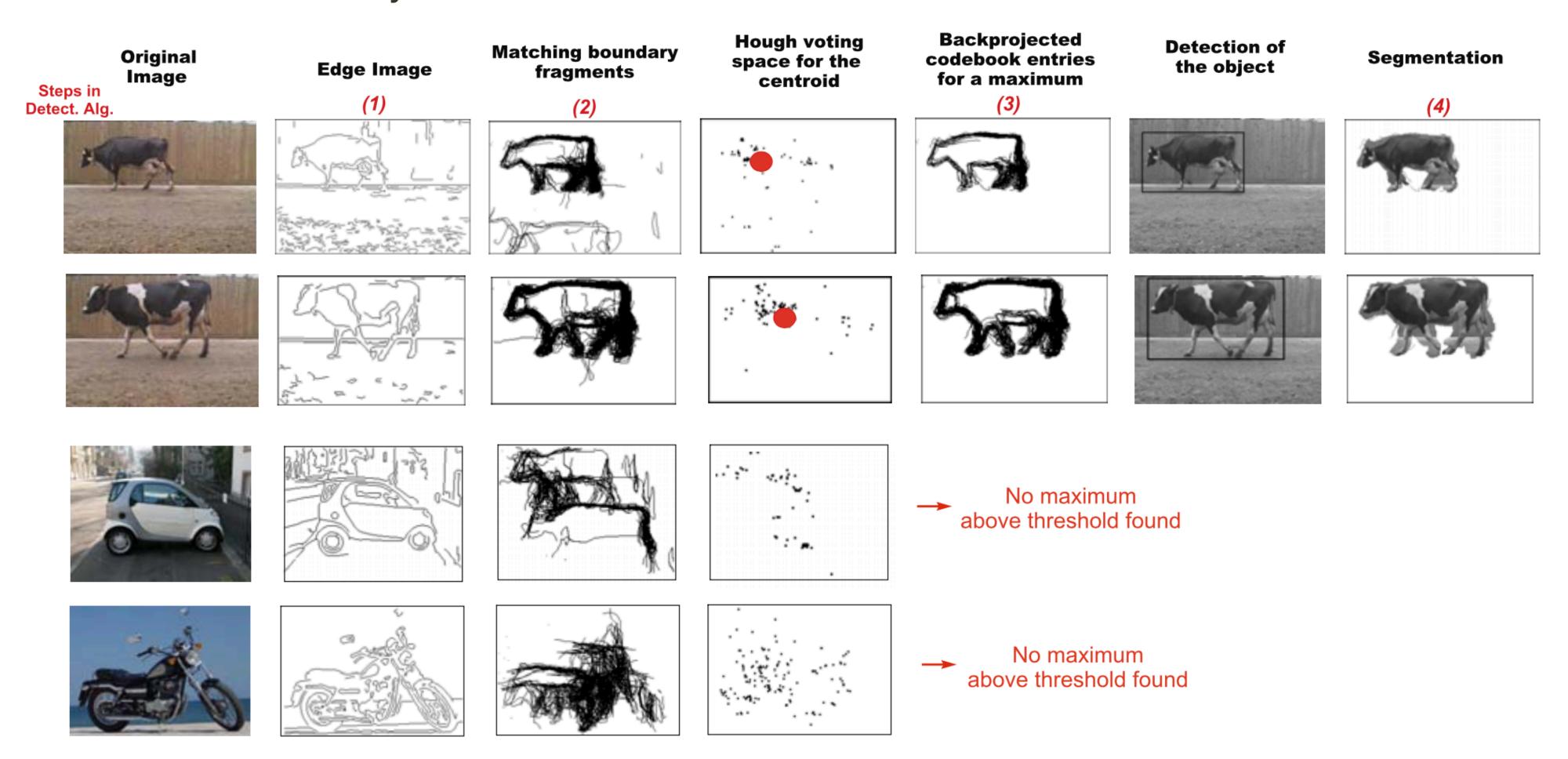


Image credit: Opelt et al., 2006

Example 3: Object Recognition — Poselets

Poselets are image patches that have distinctive appearance and can be used to infer some of the configuration of a parts-based object. Detected poselets

vote for the object configuration.

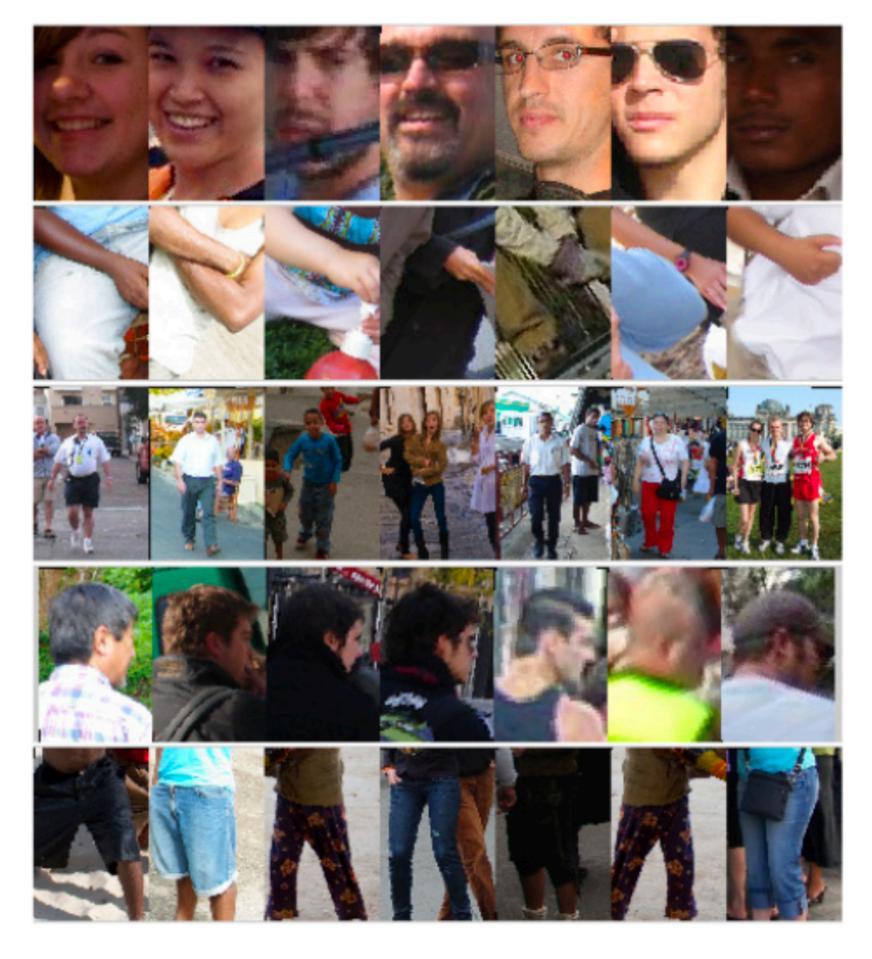
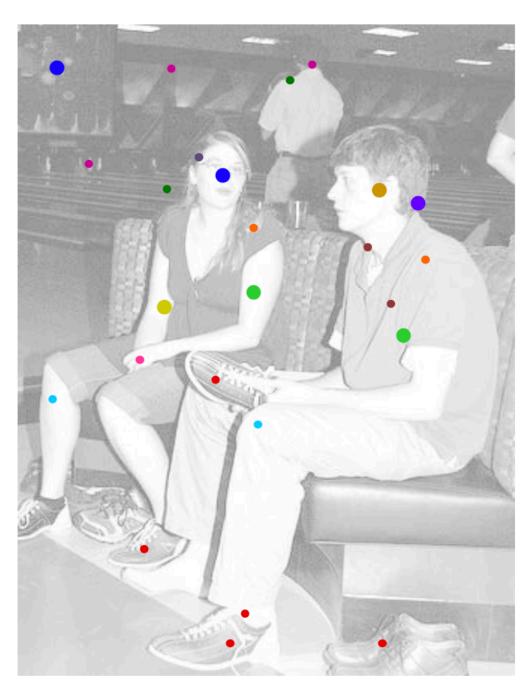


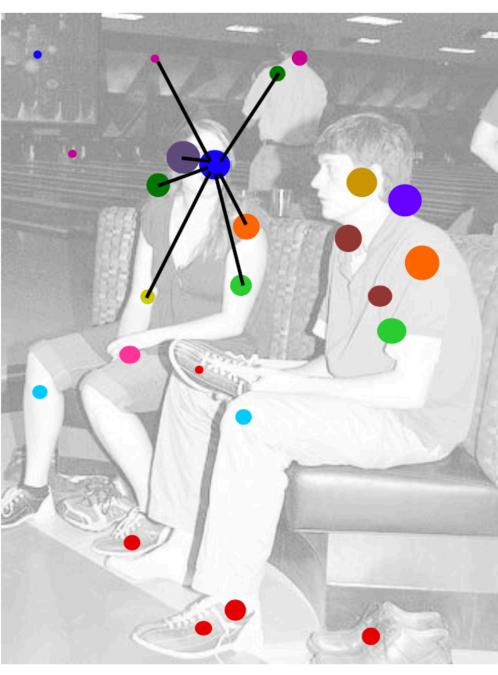
Image credit: Bourdev and Malik, 2009

Example 3: Object Recognition — Poselets

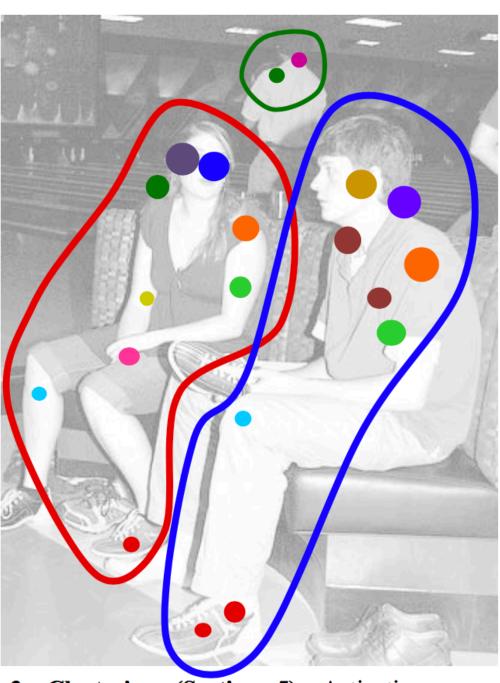
Poselets are image patches that have distinctive appearance and can be used to infer some of the configuration of a parts-based object. Detected poselets vote for the object configuration.



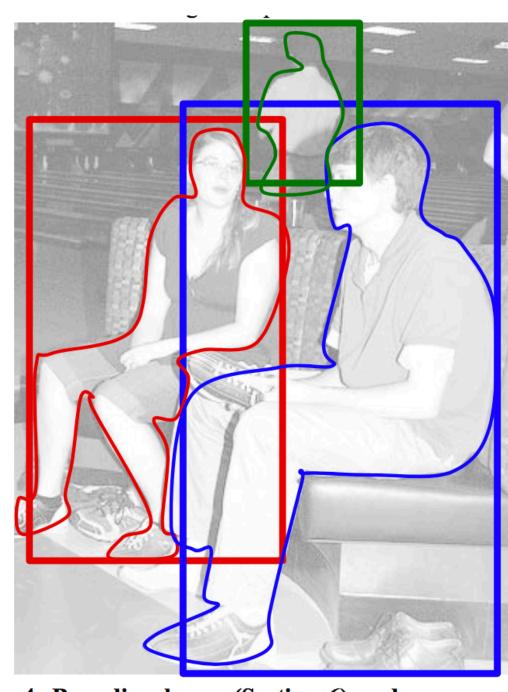
1. q-scores. Different colors illustrate different poselet detectors firing in the image. The blob size illustrates the score of the independent poselet classifier.



2. Q-scores (Section 4). Evidence from consistent poselet activations leads to a reranking based on mutual activation (Q-scores). Weaker activations consistent with others gain importance, whereas inconsistent ones get damped.



3. Clustering (Section 5). Activations are merged in a greedy manner starting with the strongest activation. Merging is based on pairwise consistency.



4. Bounding boxes (Section 6) and segmentations (Section 7). We predict the visible bounds and the contour of the person using the poselets within the cluster.

Image credit: Bourdev and Malik, 2009

Summary of Hough Transform

Idea of Hough transform:

- For each token vote for all models to which the token could belong
- Return models that get many votes

e.g., For each point, vote for all lines that could pass through it; the true lines will pass through many points and so receive many votes

Advantages:

- Can handle high percentage of outliers: each point votes separately
- Can detect multiple instances of a model in a single pass

Disadvantages:

- Search time increases exponentially with the number of model parameters
- Can be tricky to pick a good bin size