CPSC 425: Computer Vision

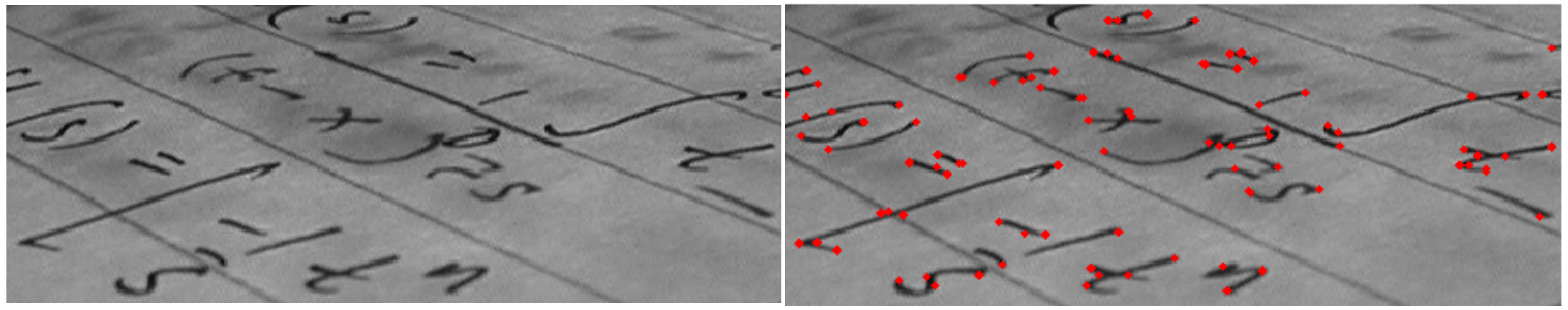


Image Credit: https://en.wikipedia.org/wiki/Corner_detection

Lecture 11: Corner Detection (cont.)

(unless otherwise stated slides are taken or adopted from **Bob Woodham, Jim Little** and **Fred Tung**)

Menu for Today

Topics:

- Harris **Corner** Detector (review)
- **Blob** Detection
- Searching over **Scale**
- **Texture** Synthesis & Analysis

Readings:

- **Today's** Lecture: Forsyth & Ponce (2nd ed.) 5.3, 6.1, 6.3, 3.1-3.3

Reminders:

- **Assignment 2**: Face Detection in a Scaled Representation is due **today**
- **Assignment 3:** Texture Synthesis is out **today**
- Study questions for **Midterm** are on Canvas (answers on Friday)
- (practice) **Quiz 1** is on Canvas, Quiz 2 & 3 coming

Today's "**fun**" Example: Texture Camouflage

https://en.wikipedia.org/wiki/File:Camouflage.jpg

Today’s “**fun**” Example: Texture Camouflage

Cuttlefish on gravel seabed

Seconds later…

http://www.marinet.org.uk/campaign-article/an-illustrated-guide-to-uk-marine-animals

Lecture 10: Re-cap (Harris Corner Detection)

1. Compute image gradients over small region

2. Compute the covariance matrix

3. Compute eigenvectors and eigenvalues

4. Use threshold on eigenvalues to detect corners

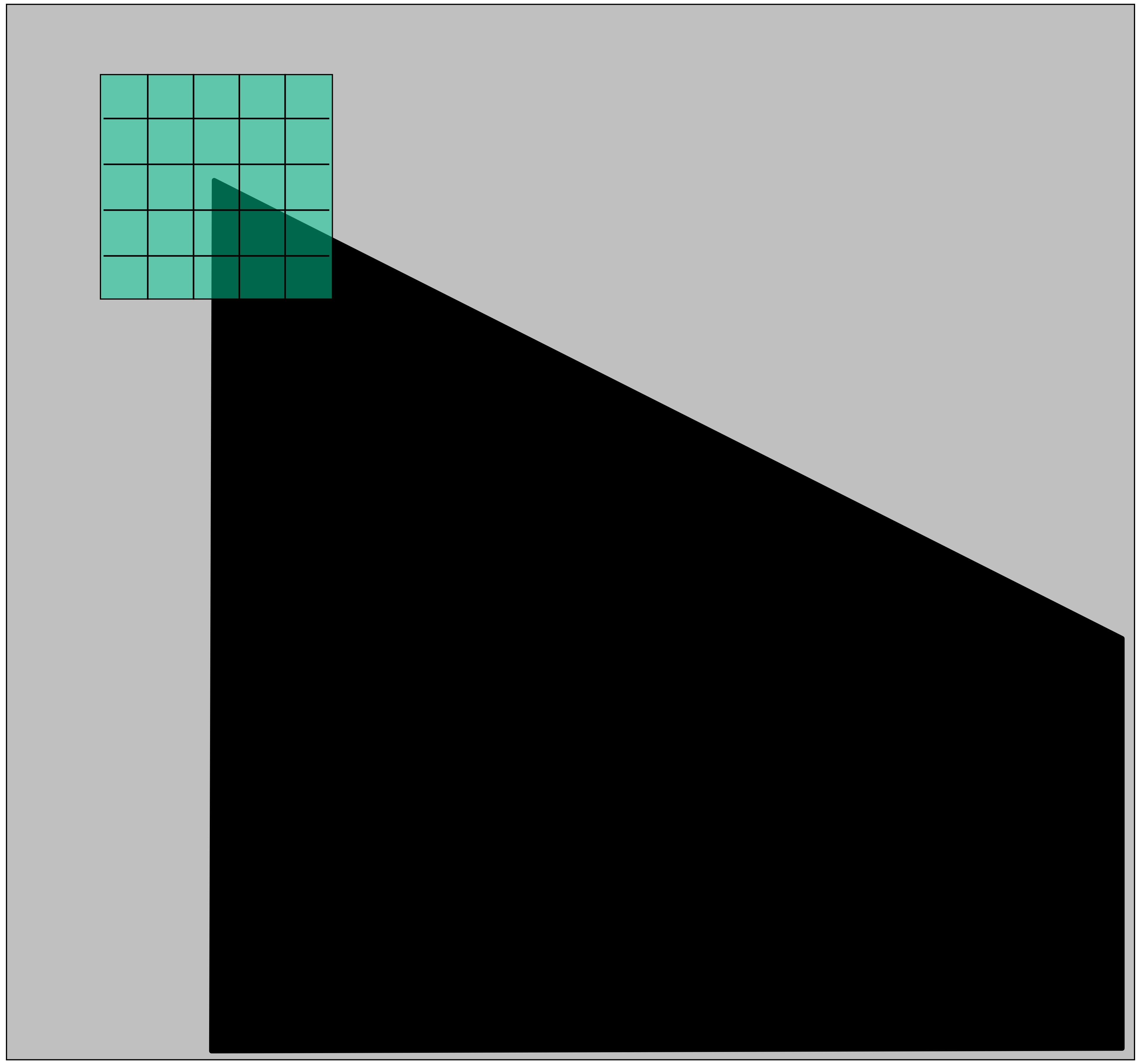
$$I_x = \frac{\partial I}{\partial x} \qquad I_y = \frac{\partial I}{\partial y}$$

$$\begin{bmatrix} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{bmatrix}$$

Slide Adopted: Ioannis (Yannis) Gkioulekas (CMU)

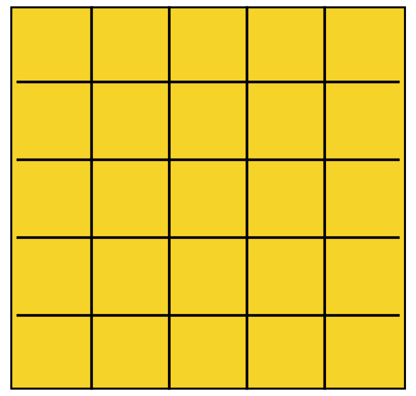
Lecture 10: Re-cap (compute image gradients at patch)

(not just a single pixel)



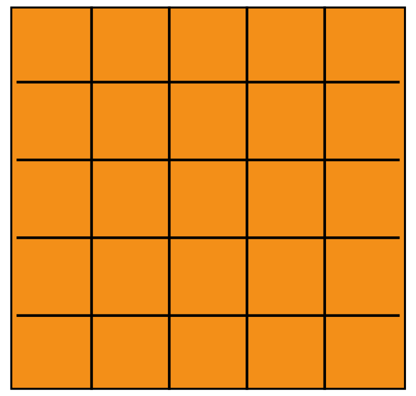
array of x gradients

$$I_x = \frac{\partial I}{\partial x}$$



array of y gradients

$$I_y = \frac{\partial I}{\partial y}$$



Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Lecture 10: Re-cap (compute the covariance matrix)

Sum over small region around the corner

Gradient with respect to x, times gradient with respect to y

$$C = \begin{bmatrix} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{bmatrix}$$

Matrix is **symmetric**

Lecture 10: Re-cap

It can be shown that since every C is symmetric:

$$C = \begin{bmatrix} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{bmatrix} = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

Lecture 10: Re-cap (computing eigenvalues and eigenvectors)

eigenvalue

$$Ce = \lambda e$$

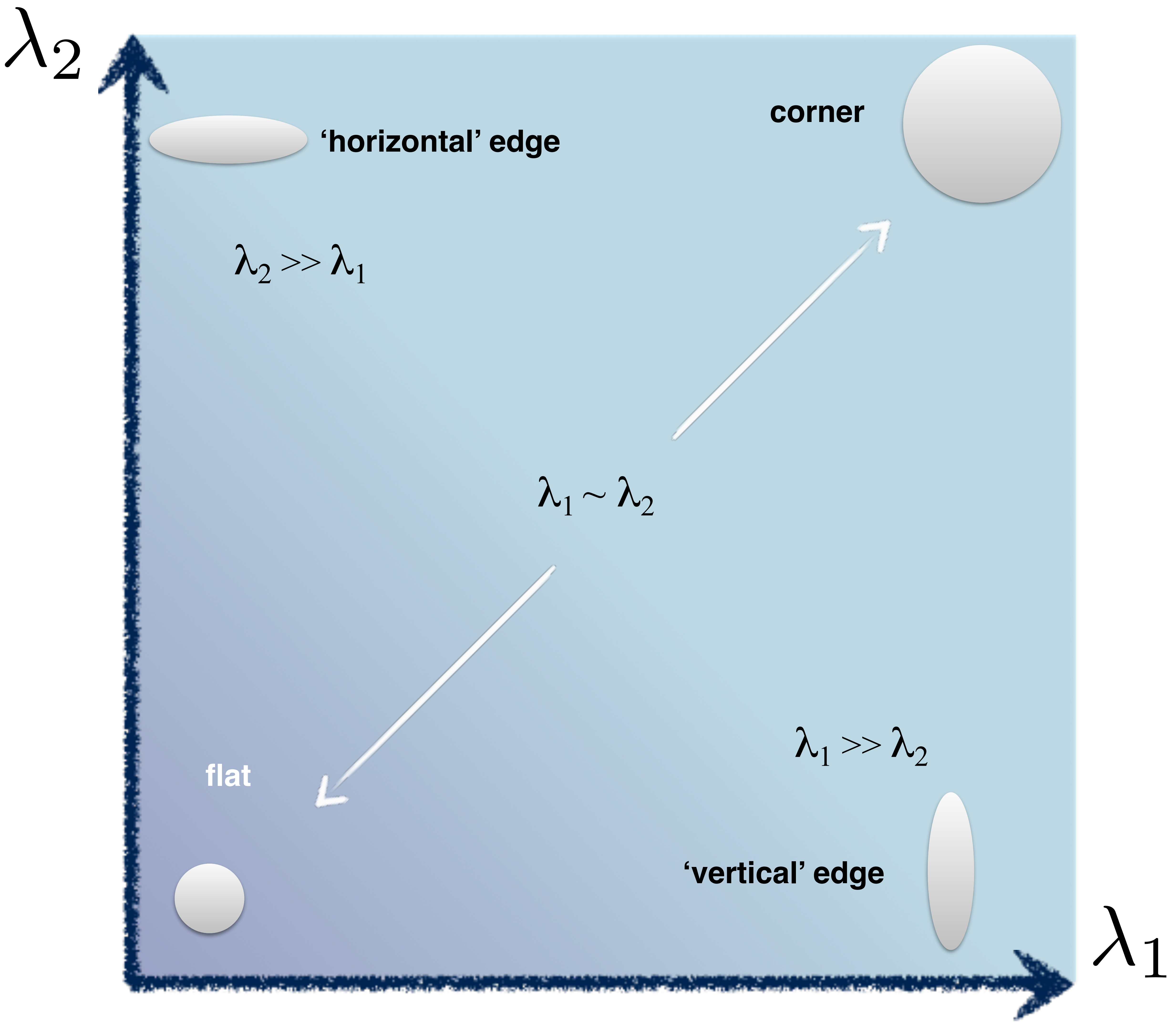
eigenvector

$$(C - \lambda I)e = 0$$

1. Compute the determinant of (returns a polynomial)	$C - \lambda I$
2. Find the roots of polynomial (returns eigenvalues)	$\det(C - \lambda I) = 0$
3. For each eigenvalue, solve (returns eigenvectors)	$(C - \lambda I)e = 0$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Lecture 10: Re-cap (interpreting eigenvalues)



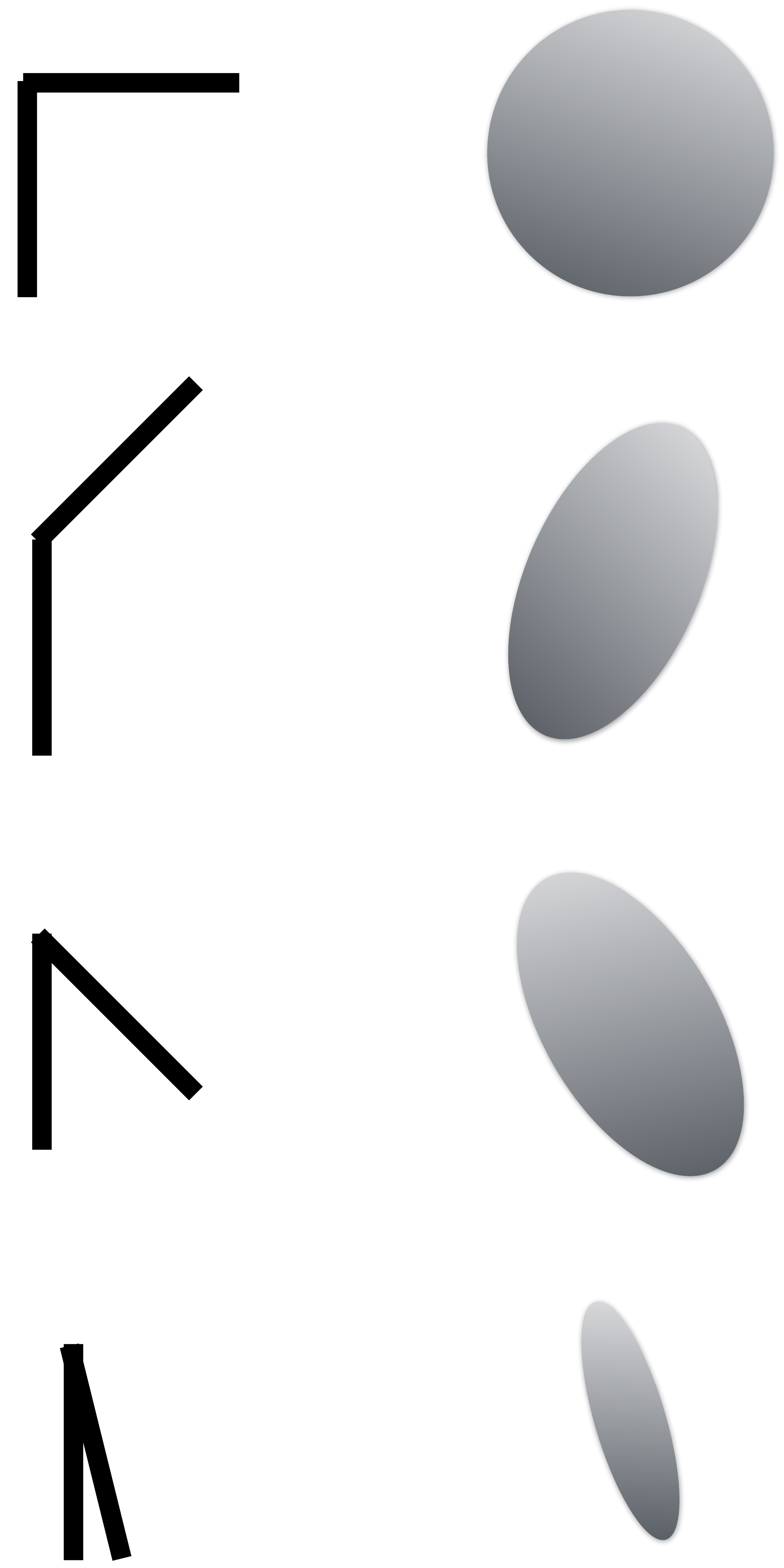
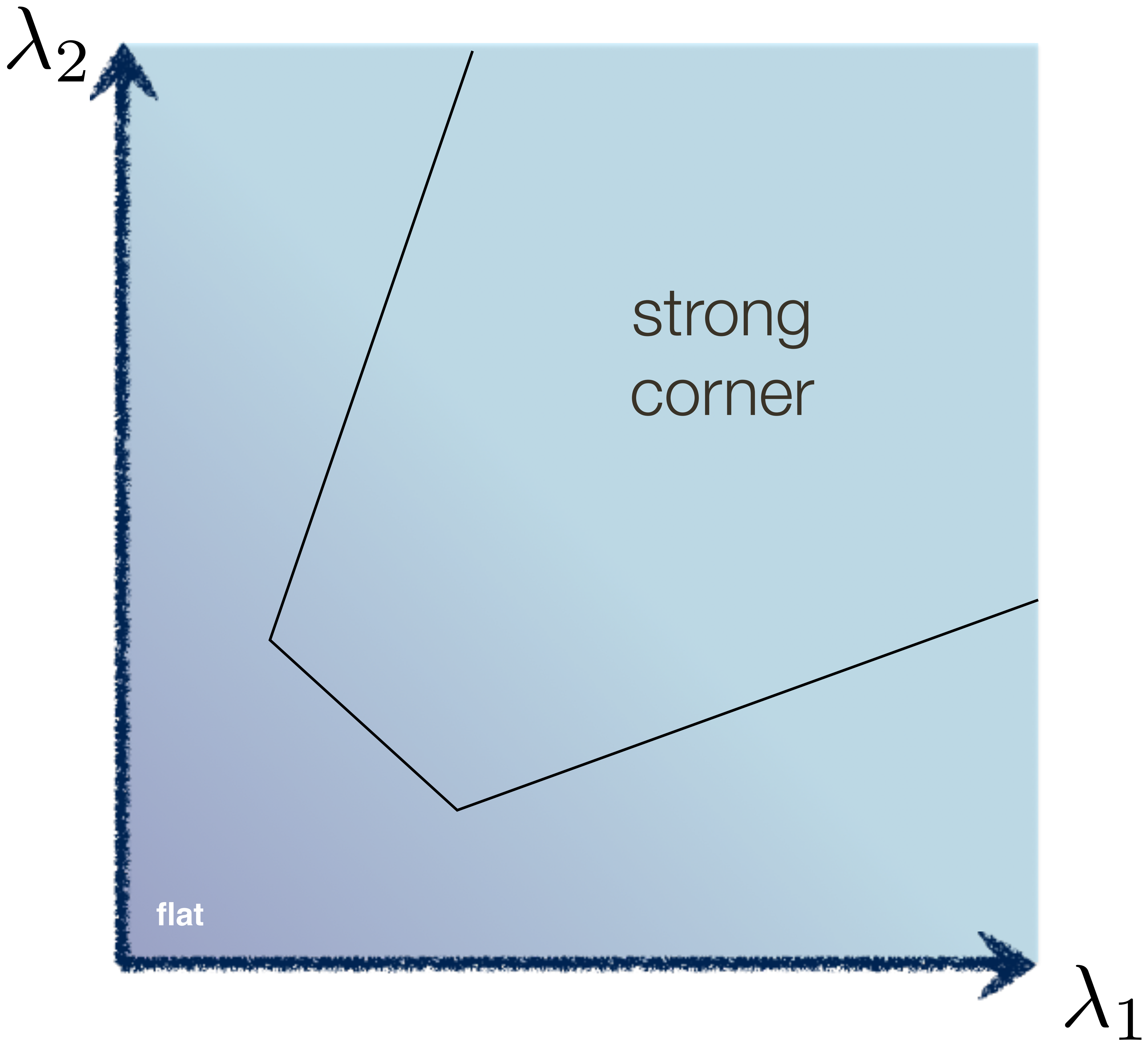


Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

Lecture 10: Re-cap (**Threshold** on Eigenvalues to **Detect Corners)**



Think of a function to score 'cornerness'

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Lecture 10: Re-cap (**Threshold** on Eigenvalues to **Detect Corners)**

Harris & Stephens (1988)

$$\det(C) - \kappa \operatorname{trace}^2(C)$$

Kanade & Tomasi (1994)

$$\min(\lambda_1, \lambda_2)$$

Nobel (1998)

$$\frac{\det(C)}{\operatorname{trace}(C) + \epsilon}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Example: Harris Corner Detection

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

Example: Harris Corner Detection

Lets compute a measure of “corner-ness” for the green pixel:

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

Example: Harris Corner Detection

Lets compute a measure of “corner-ness” for the green pixel:

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

Example: Harris Corner Detection

Lets compute a measure of "corner-ness" for the green pixel:

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

$$I_x = \frac{\partial I}{\partial x}$$

0	0	0	0	0	0	
-1	1	0	0	-1	1	
-1	0	0	0	1	0	
-1	0	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	

Example: Harris Corner Detection

Lets compute a measure of “corner-ness” for the green pixel:

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

$$I_x = \frac{\partial I}{\partial x}$$

0	0	0	0	0	0	
-1	1	0	0	-1	1	
-1	0	0	0	1	0	
-1	0	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	

$$I_y = \frac{\partial I}{\partial y}$$

0	-1	0	0	0	-1	0
0	0	-1	-1	-1	1	0
0	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Example: Harris Corner Detection

Lets compute a measure of "corner-ness" for the green pixel:

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

$$\sum \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix} = 3$$

$$I_x = \frac{\partial I}{\partial x}$$

0	0	0	0	0	0	
-1	1	0	0	-1	1	
-1	0	0	0	1	0	
-1	0	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	

$$I_y = \frac{\partial I}{\partial y}$$

0	-1	0	0	0	-1	0
0	0	-1	-1	-1	1	0
0	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Example: Harris Corner Detection

Lets compute a measure of “corner-ness” for the green pixel:

$$\mathbf{C} = \begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}$$

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

$$I_x = \frac{\partial I}{\partial x}$$

0	0	0	0	0	0	
-1	1	0	0	-1	1	
-1	0	0	0	1	0	
-1	0	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	

$$I_y = \frac{\partial I}{\partial y}$$

0	-1	0	0	0	-1	0
0	0	-1	-1	-1	1	0
0	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Example: Harris Corner Detection

Lets compute a measure of “corner-ness” for the green pixel:

$$\mathbf{C} = \begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix} => \lambda_1 = 1.4384; \lambda_2 = 5.5616$$

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

$$I_x = \frac{\partial I}{\partial x}$$

0	0	0	0	0	0	
-1	1	0	0	-1	1	
-1	0	0	0	1	0	
-1	0	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	

$$I_y = \frac{\partial I}{\partial y}$$

0	-1	0	0	0	-1	0
0	0	-1	-1	-1	1	0
0	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Example: Harris Corner Detection

Lets compute a measure of “corner-ness” for the green pixel:

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

$$\mathbf{C} = \begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix} => \lambda_1 = 1.4384; \lambda_2 = 5.5616$$

$$\det(\mathbf{C}) - 0.04\text{trace}^2(\mathbf{C}) = 6.04$$

$$I_x = \frac{\partial I}{\partial x}$$

0	0	0	0	0	0	
-1	1	0	0	-1	1	
-1	0	0	0	1	0	
-1	0	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	

$$I_y = \frac{\partial I}{\partial y}$$

0	-1	0	0	0	-1	0
0	0	-1	-1	-1	1	0
0	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Example: Harris Corner Detection

Lets compute a measure of “corner-ness” for the green pixel:

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

$$\mathbf{C} = \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix} => \lambda_1 = 3; \lambda_2 = 0$$

$$\det(\mathbf{C}) - 0.04\text{trace}^2(\mathbf{C}) = -0.36$$

$I_x = \frac{\partial I}{\partial x}$

0	0	0	0	0	0	
-1	1	0	0	-1	1	
-1	0	0	0	1	0	
-1	0	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	

$I_y = \frac{\partial I}{\partial y}$

0	-1	0	0	0	-1	0
0	0	-1	-1	-1	1	0
0	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Example: Harris Corner Detection

Lets compute a measure of "corner-ness" for the green pixel:

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

$$\mathbf{C} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} => \lambda_1 = 3; \lambda_2 = 2$$

$$\det(\mathbf{C}) - 0.04\text{trace}^2(\mathbf{C}) = 5$$

$I_x = \frac{\partial I}{\partial x}$

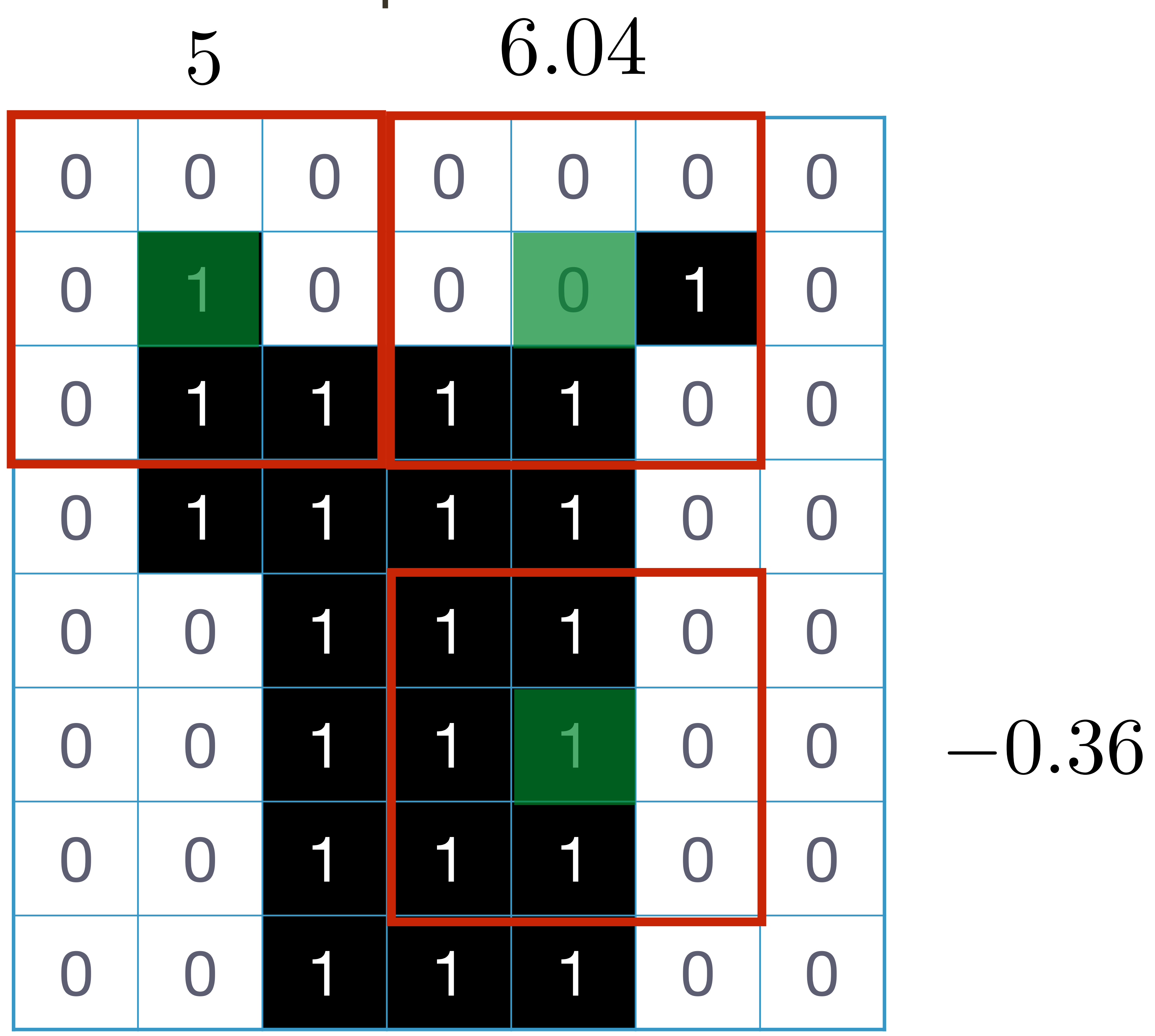
0	0	0	0	0	0	
-1	1	0	0	-1	1	
-1	0	0	0	1	0	
-1	0	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	

$I_y = \frac{\partial I}{\partial y}$

0	-1	0	0	0	-1	0
0	0	-1	-1	-1	1	0
0	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Example: Harris Corner Detection

Lets compute a measure of “corner-ness” for the green pixel:



Harris Corner Detection Review

— Filter image with **Gaussian**

— Compute magnitude of the x and y **gradients** at each pixel

— Construct C in a window around each pixel

— Harris uses a **Gaussian window**

— Solve for product of the λ's

— If λ's both are big (product reaches local maximum above threshold) then we have a corner

— Harris also checks that ratio of λs is not too high

Harris & Stephens (1988)

$$\det(C) - \kappa \operatorname{trace}^2(C)$$

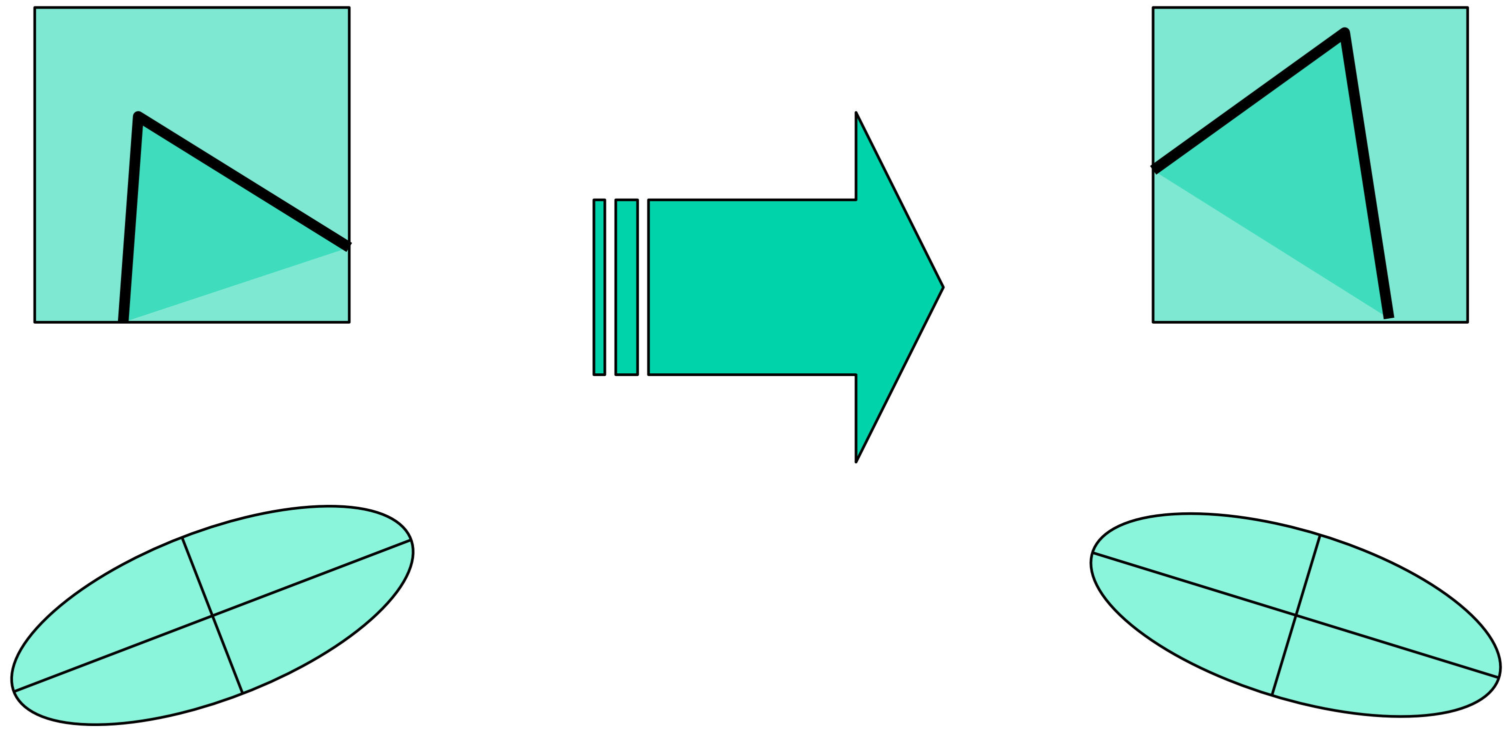
Compute the **Covariance Matrix**

Sum can be implemented as an (unnormalized) box filter with

$$C = \begin{bmatrix} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{bmatrix}$$

Harris uses a **Gaussian** weighting instead

Properties: Rotational Invariance



Ellipse rotates but its shape (**eigenvalues**) remains the same

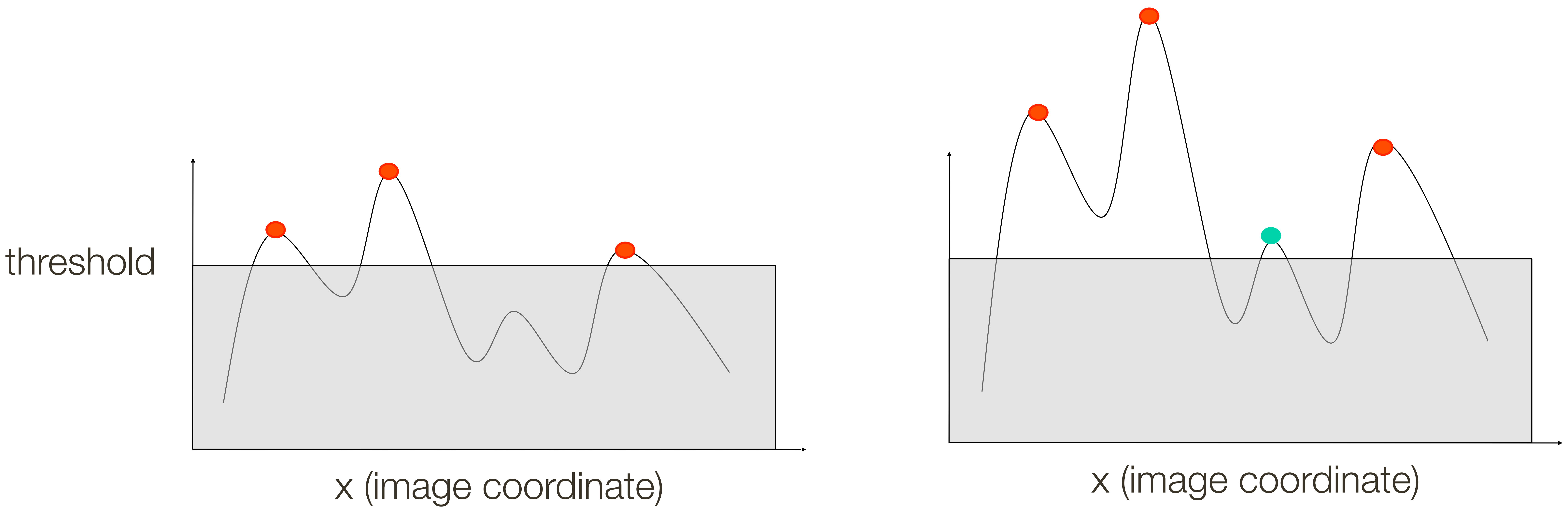
Corner response is **invariant** to image rotation

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Properties: (partial) Invariance to Intensity Shifts and Scaling

Only derivatives are used -> Invariance to intensity shifts

Intensity scale could effect performance



Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Properties: NOT Invariant to Scale Changes

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Example 2: Wagon Wheel (Harris Results)

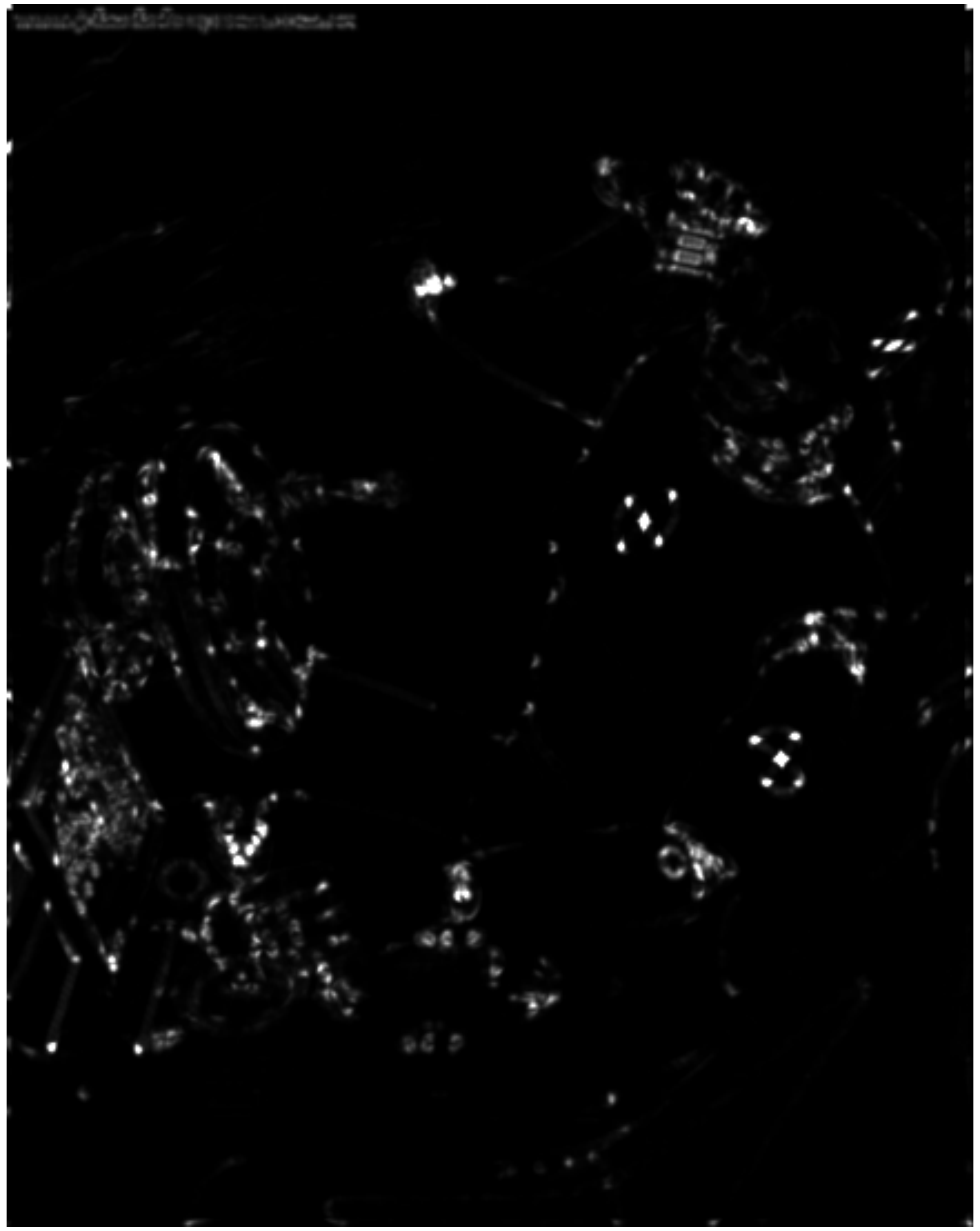
$\sigma = 1$ (219 points)

$\sigma = 2$ (155 points)

$\sigma = 3$ (110 points)

$\sigma = 4$ (87 points)

Example 3: Crash Test Dummy (Harris Result)



corner response image

$\sigma = 1$ (175 points)

Original Image Credit: John Shakespeare, Sydney Morning Herald

Example 2: Wagon Wheel (Harris Results)

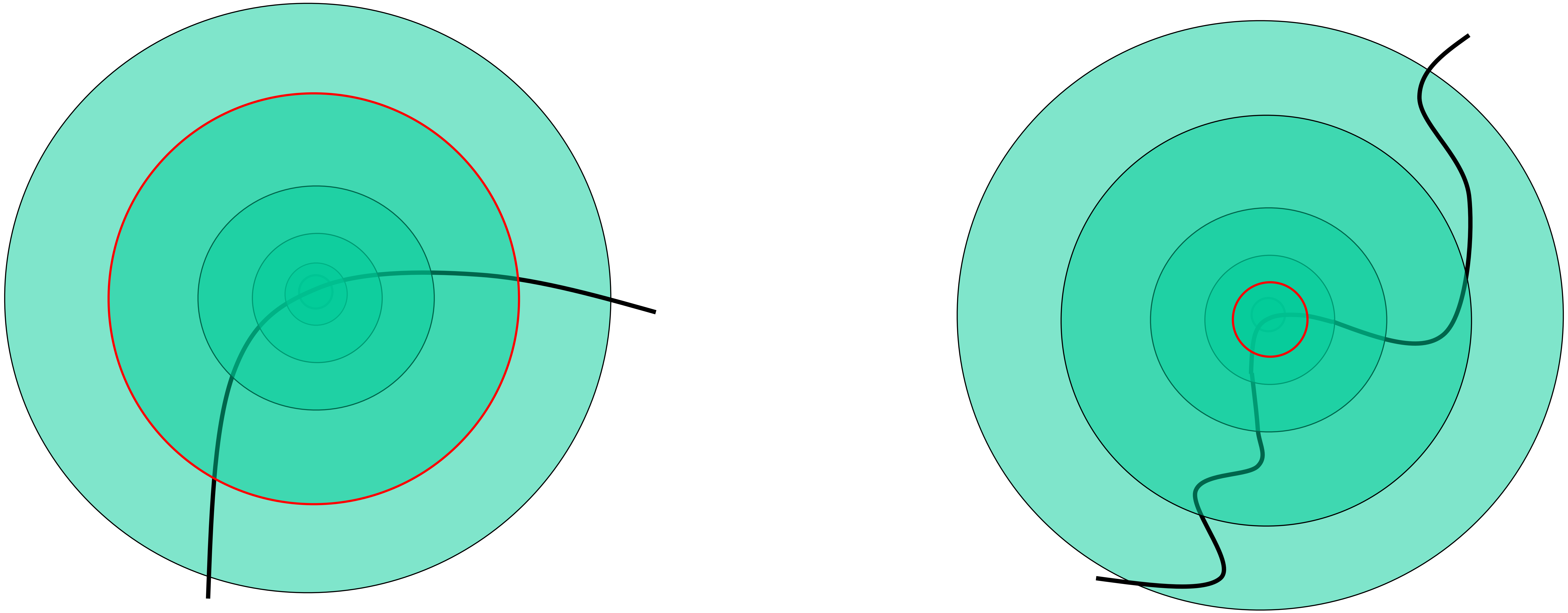
$\sigma = 1$ (219 points)

$\sigma = 2$ (155 points)

$\sigma = 3$ (110 points)

$\sigma = 4$ (87 points)

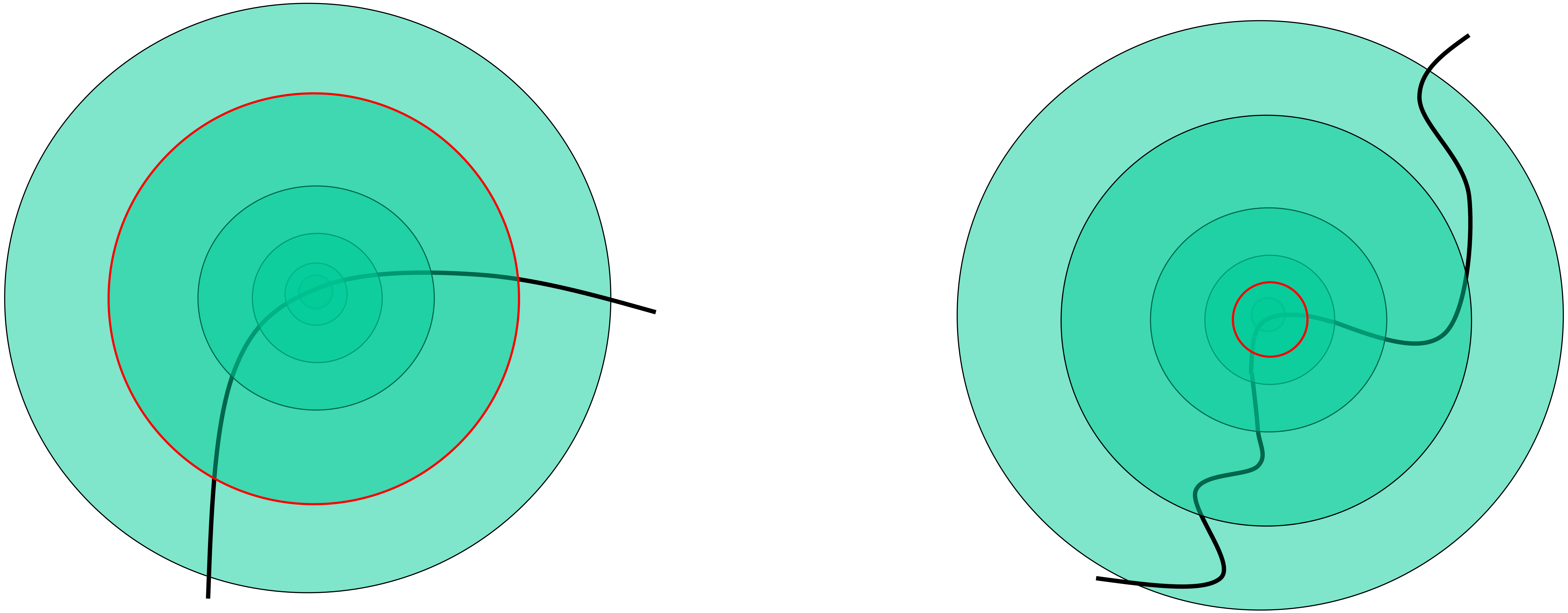
Intuitively ...



Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Intuitively …

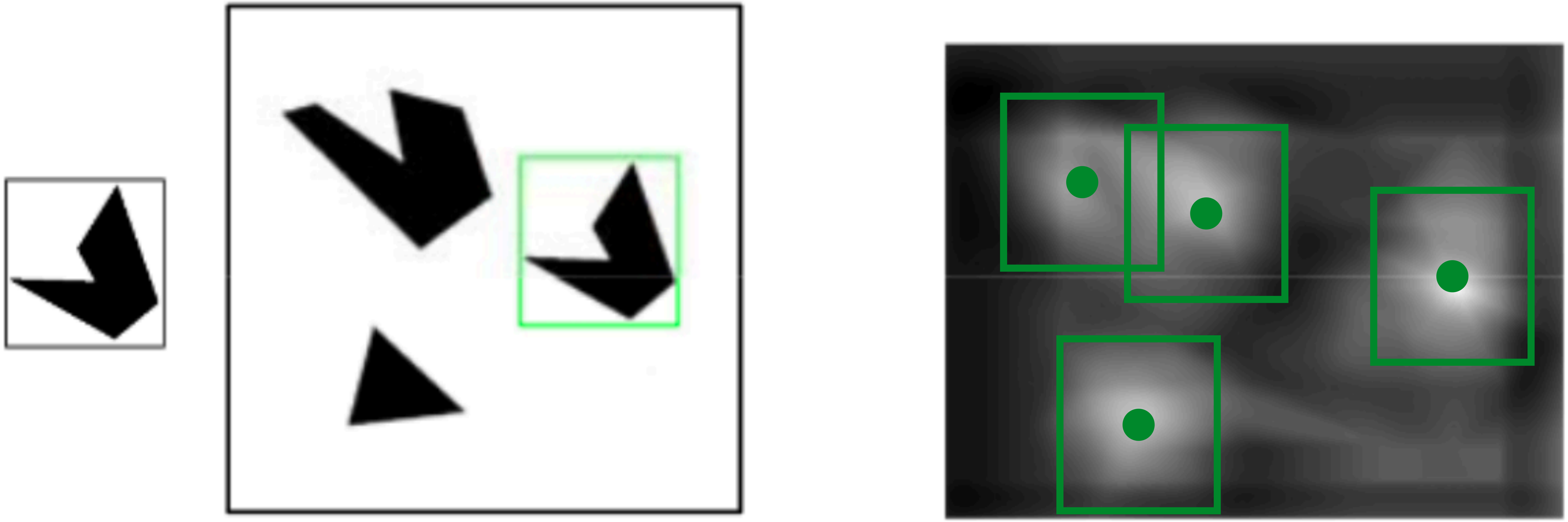
Find local maxima in both **position** and **scale**



Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Non-maxima Suppression in Template Matching

Idea: suppress near-by similar detections to obtain one “true” result

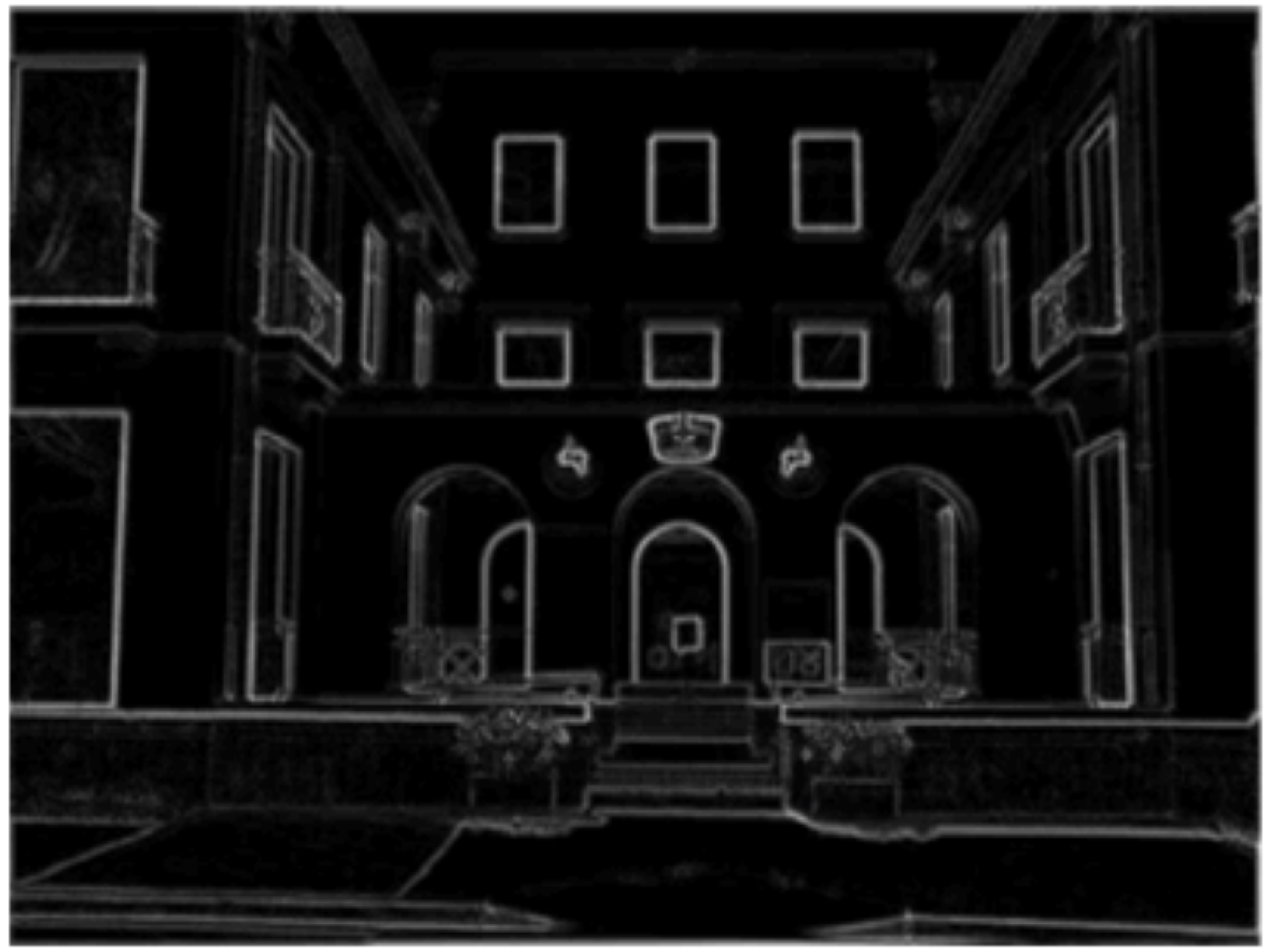


Detected template **Correlation map**

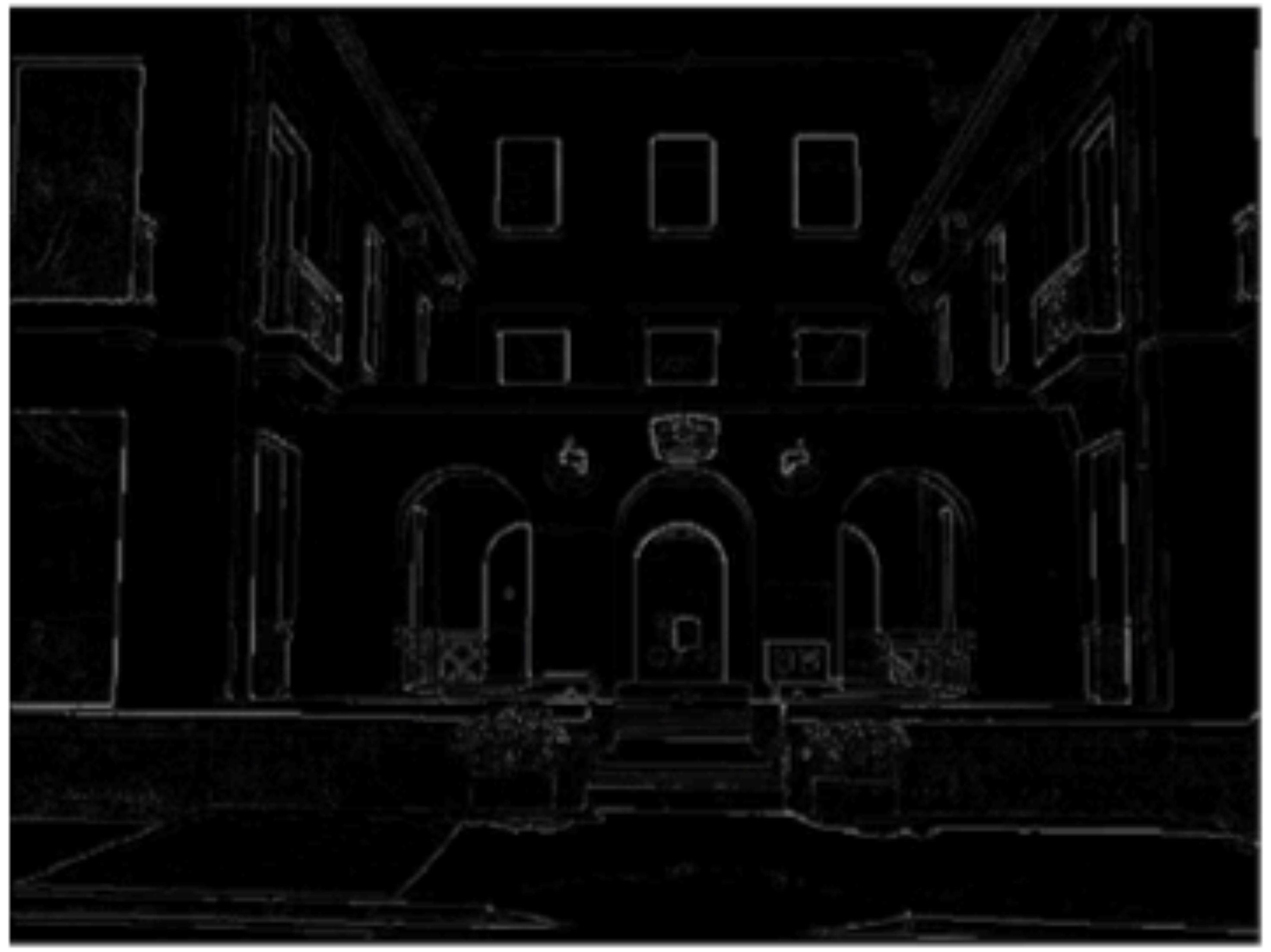
Slide Credit: Kristen Grauman

Non-maxima Suppression in Edge Detection (Canny)

Original Image



Gradient Magnitude

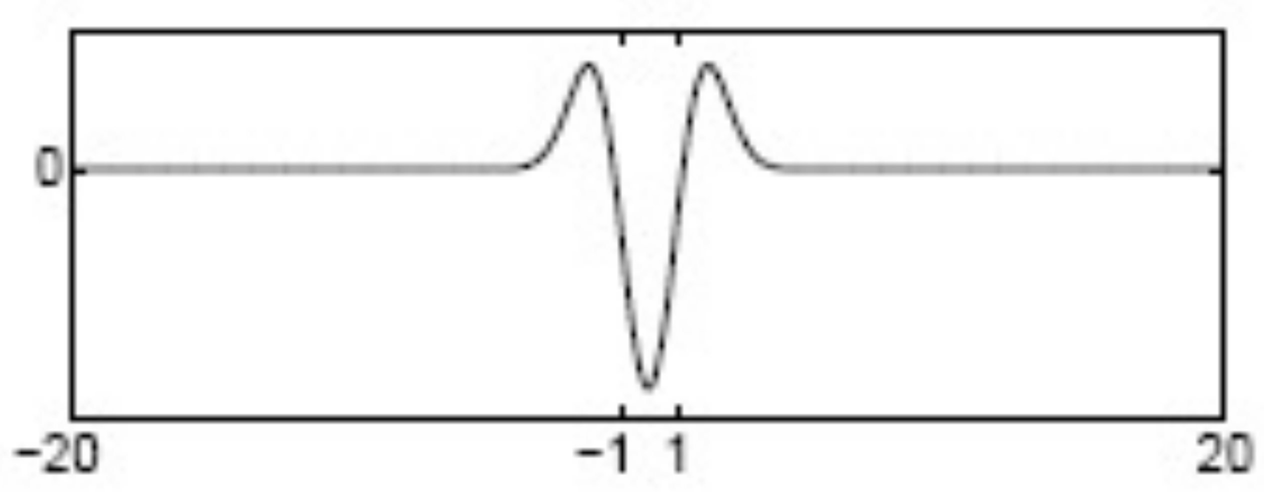


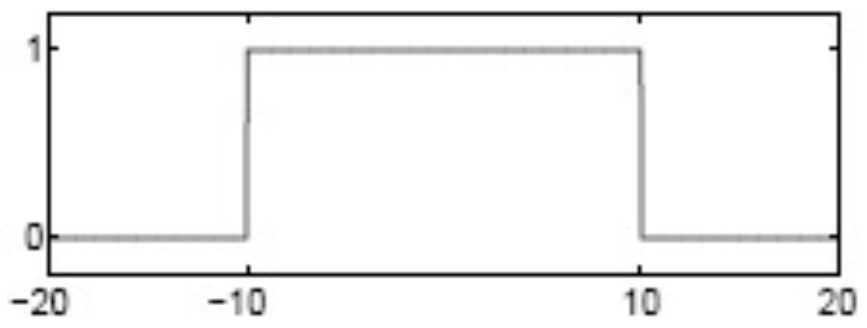
Non-maxima
Suppression

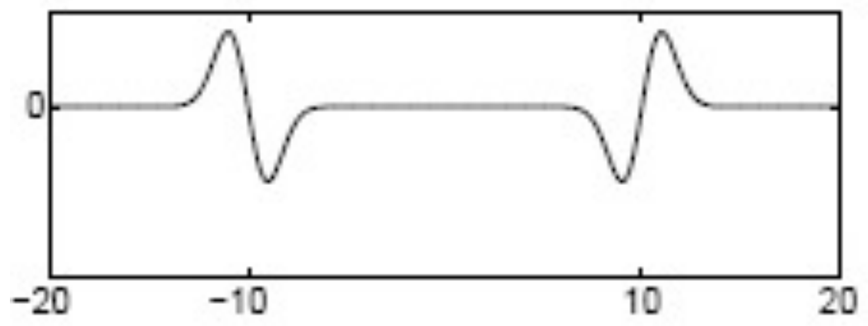
Slide Credit: Christopher Rasmussen

Formally …

Laplacian filter



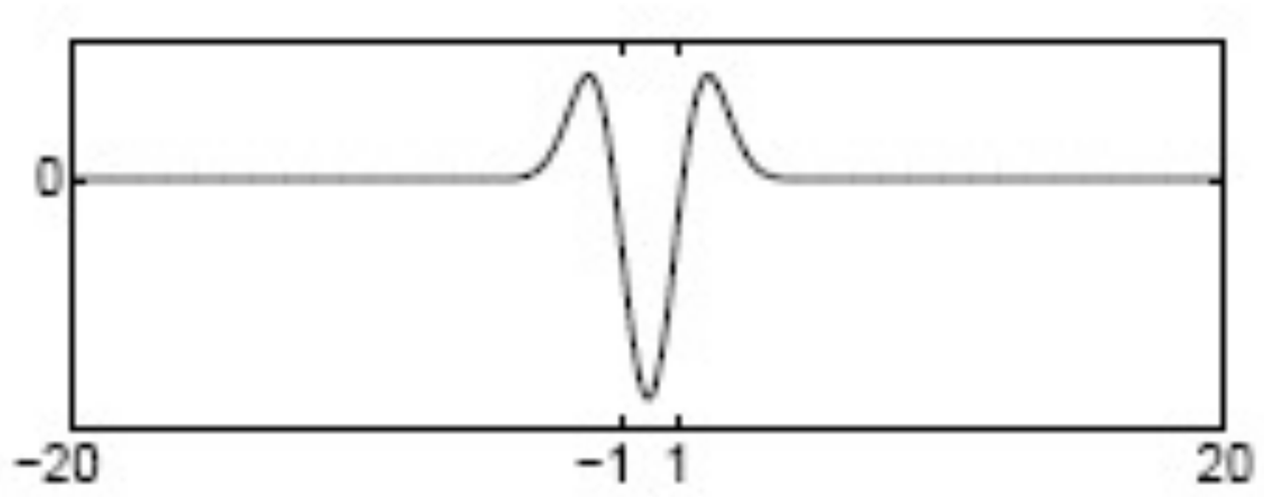


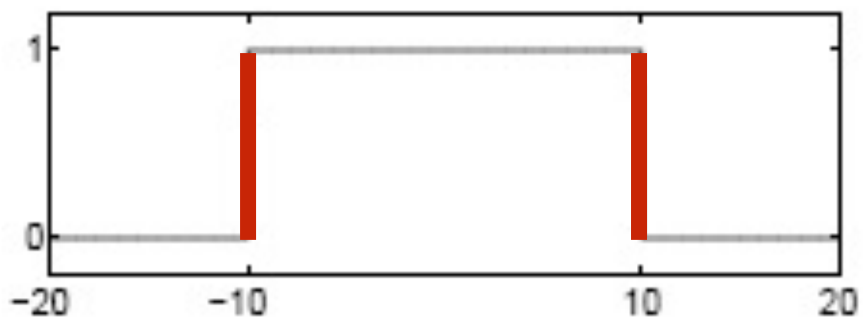


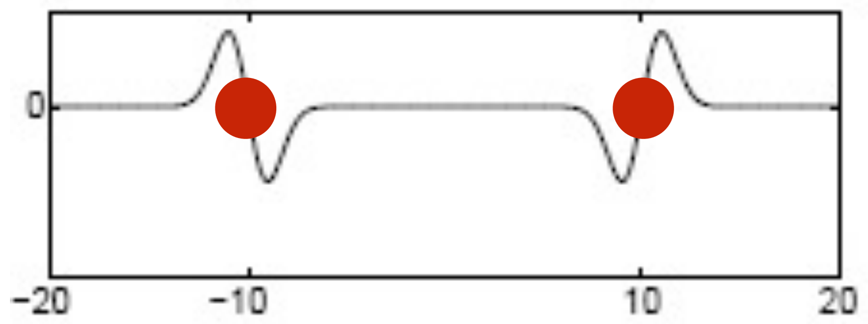
Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Formally …

Laplacian filter

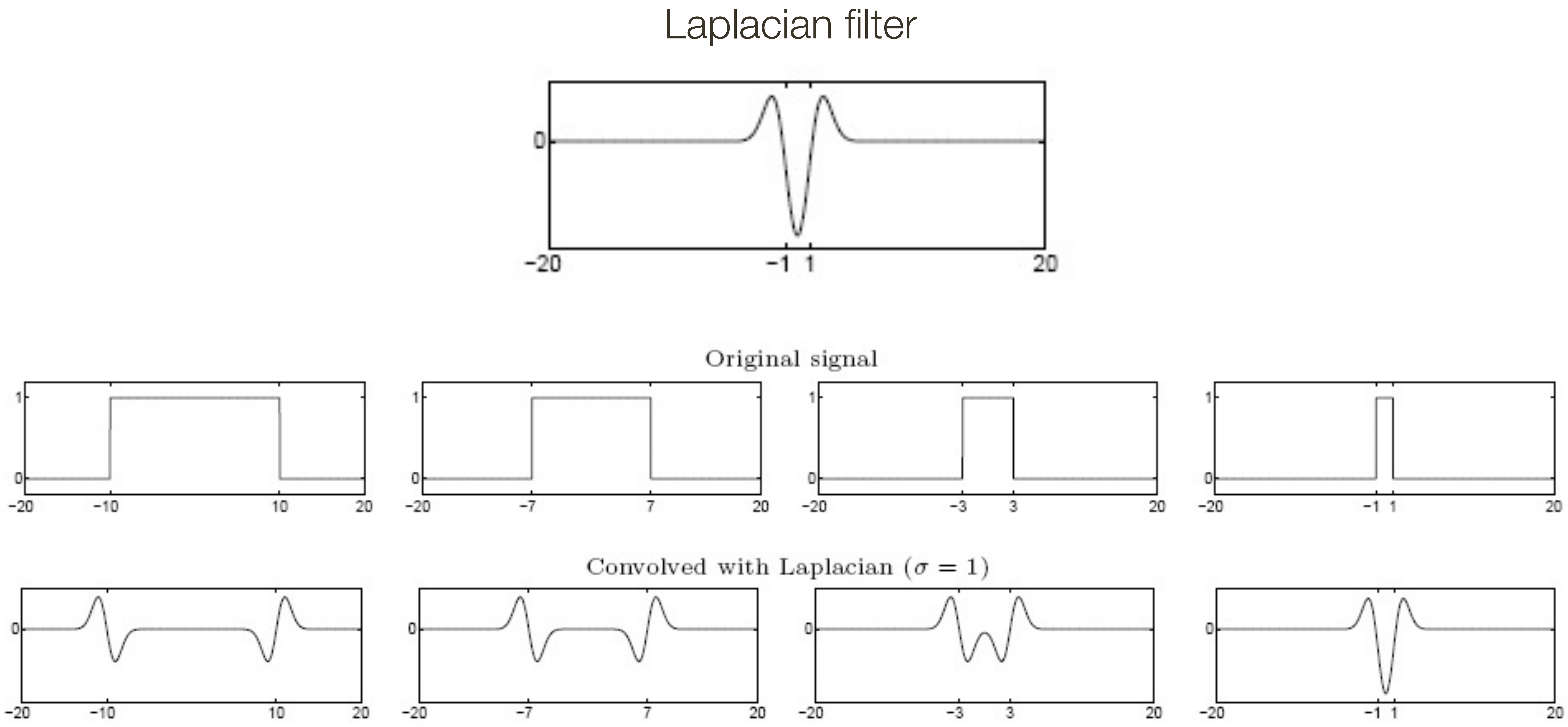






Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Formally …

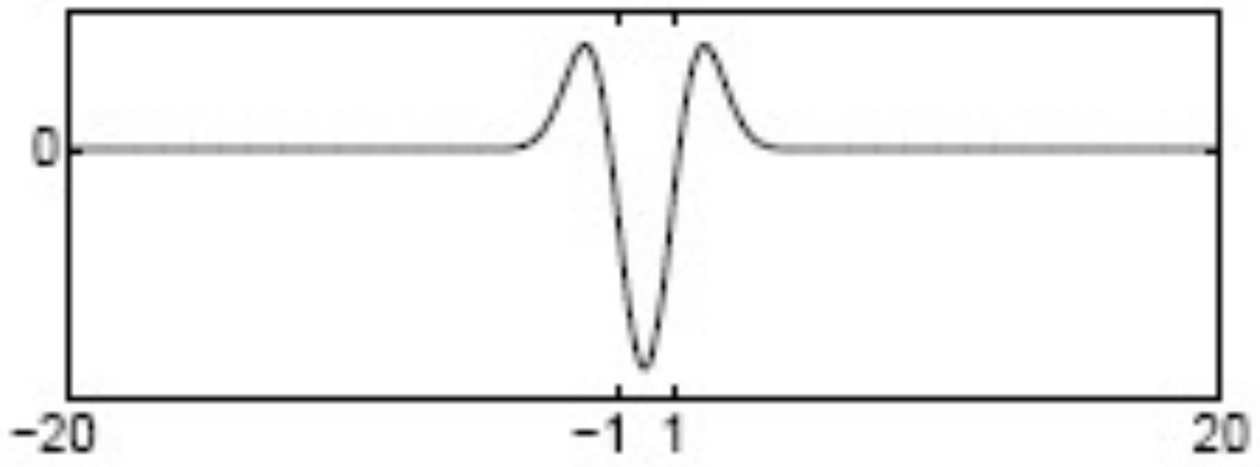


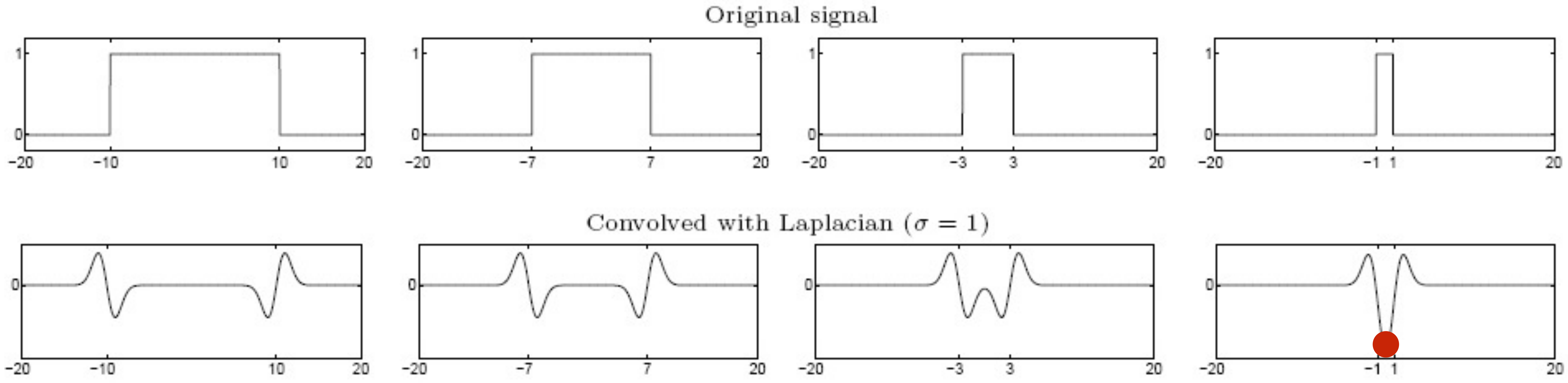
Highest response when the signal has the same **characteristic scale** as the filter

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Formally …

Laplacian filter





Highest response when the signal has the same **characteristic scale** as the filter

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

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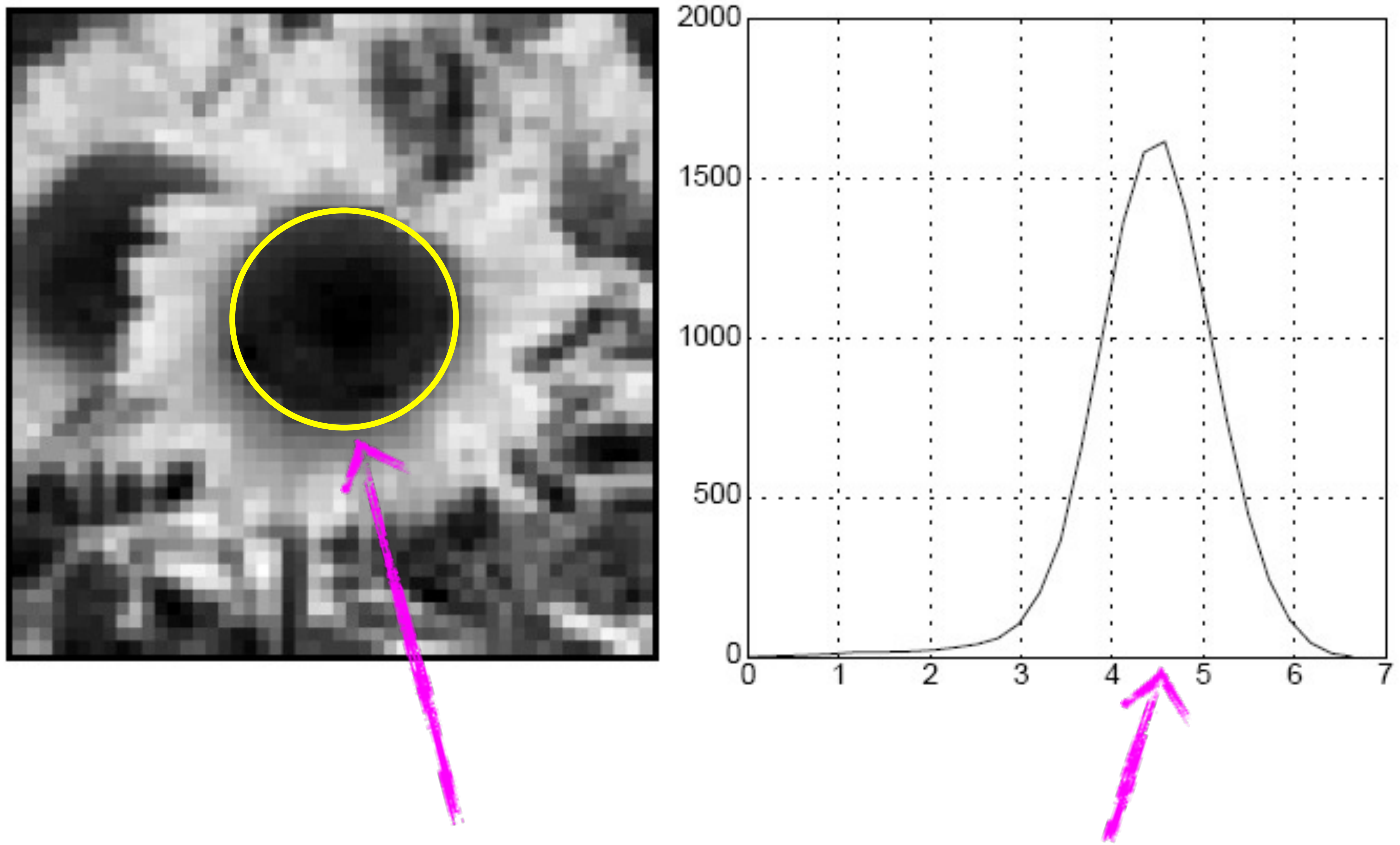
Characteristic Scale

characteristic scale - the scale that produces peak filter response

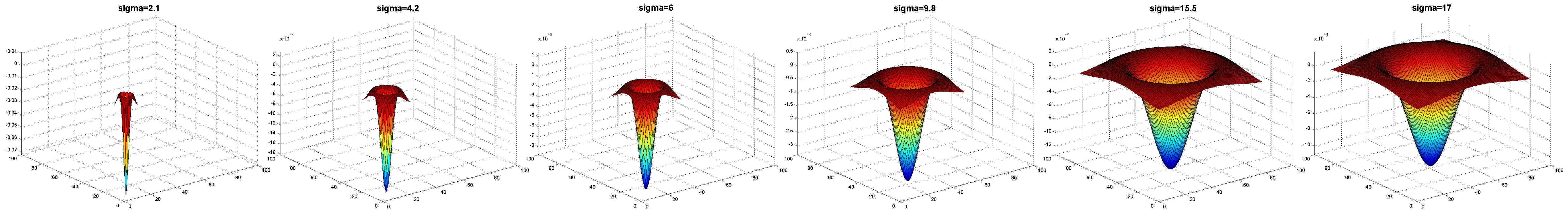


characteristic scale

we need to search over characteristic scales

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Applying **Laplacian** Filter at Different **Scales**



Full size

3/4 size

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

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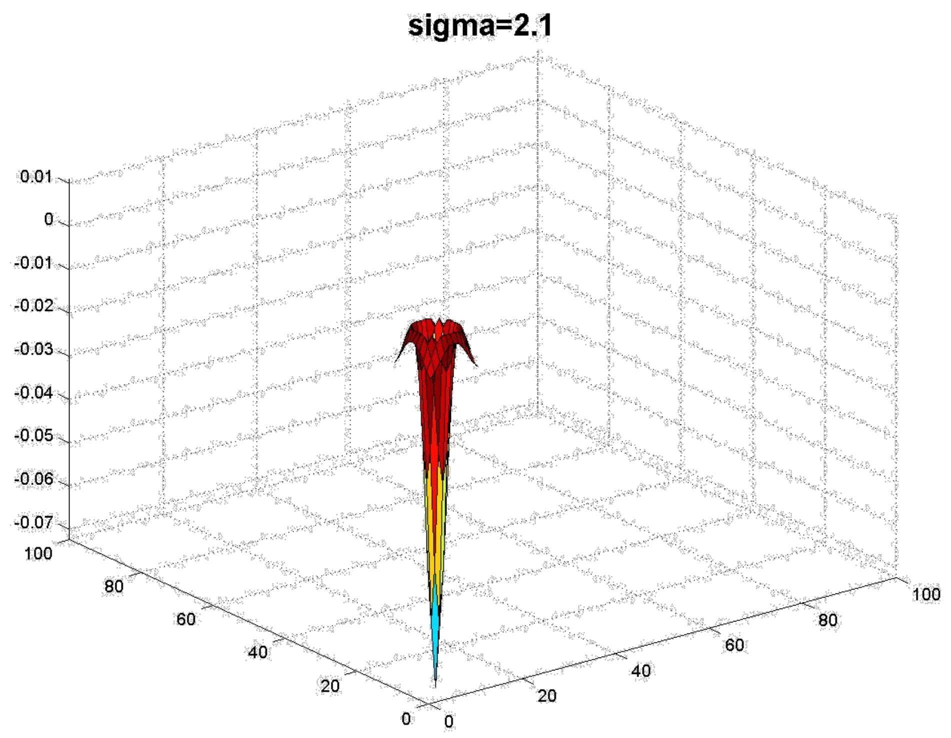

jet color scale
blue: low, red: high

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

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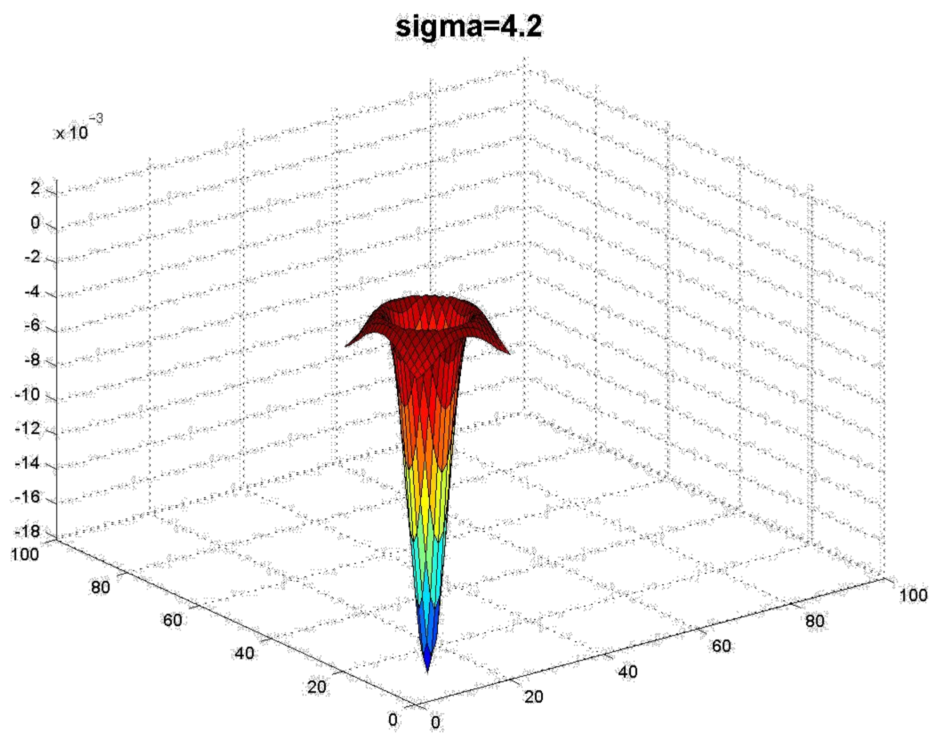

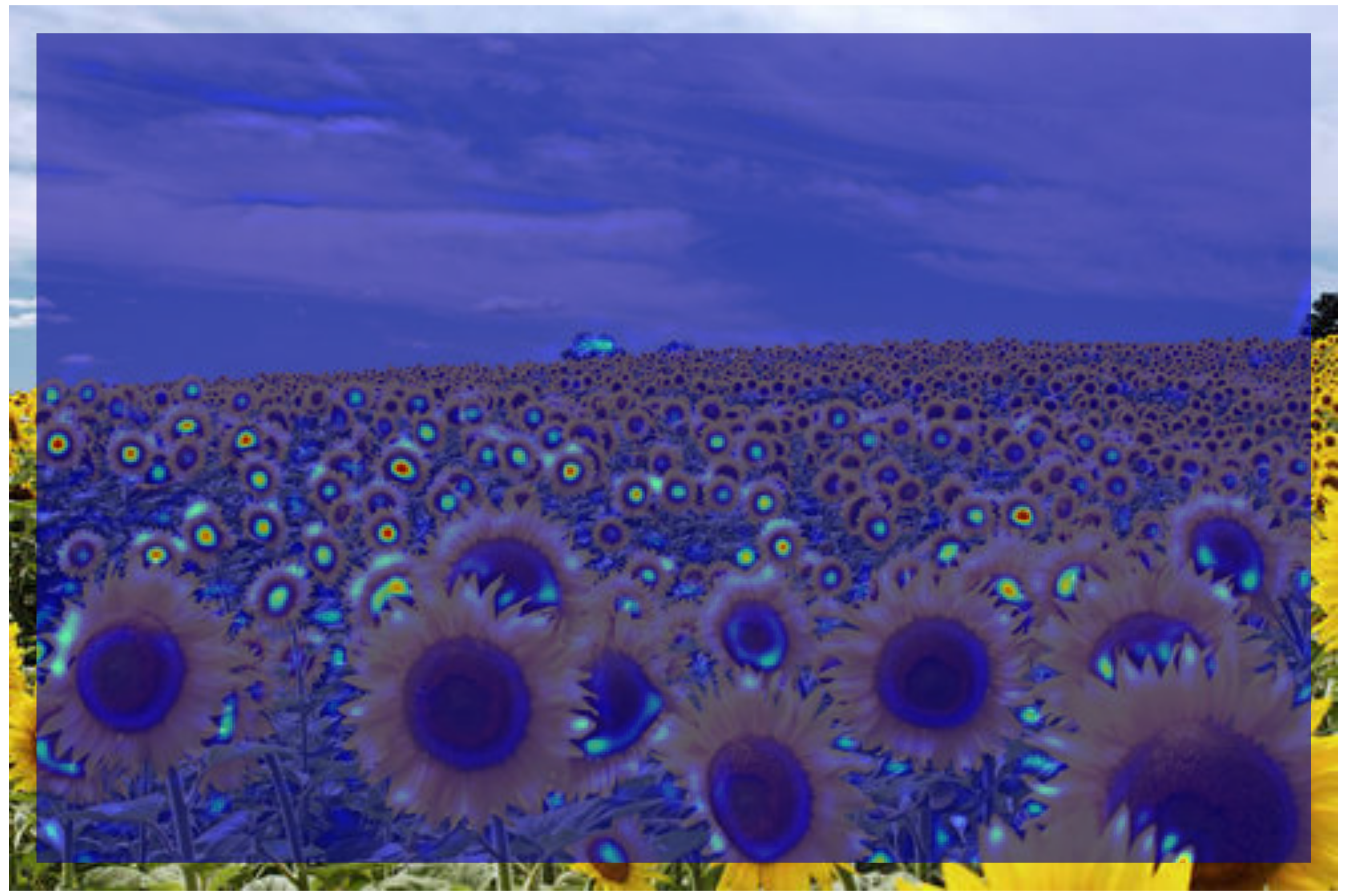

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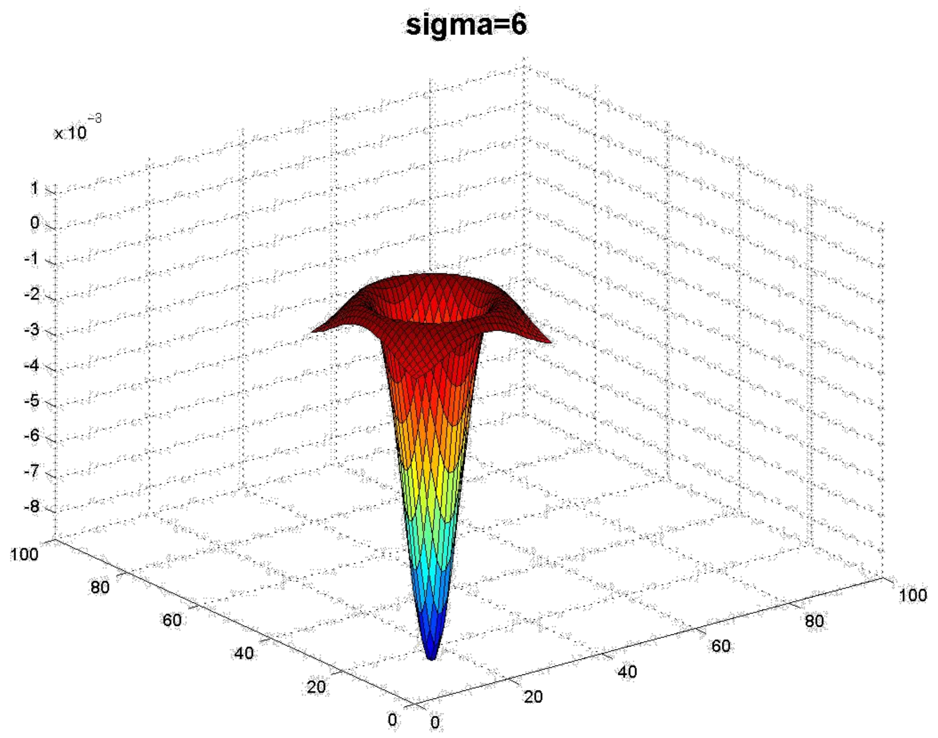

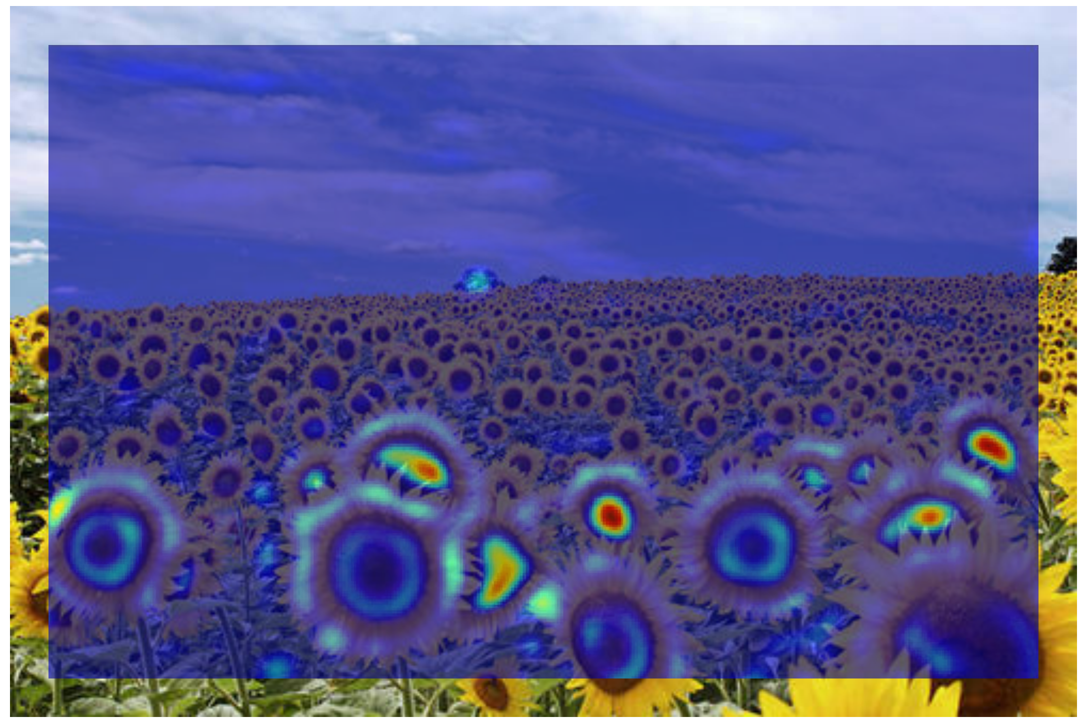

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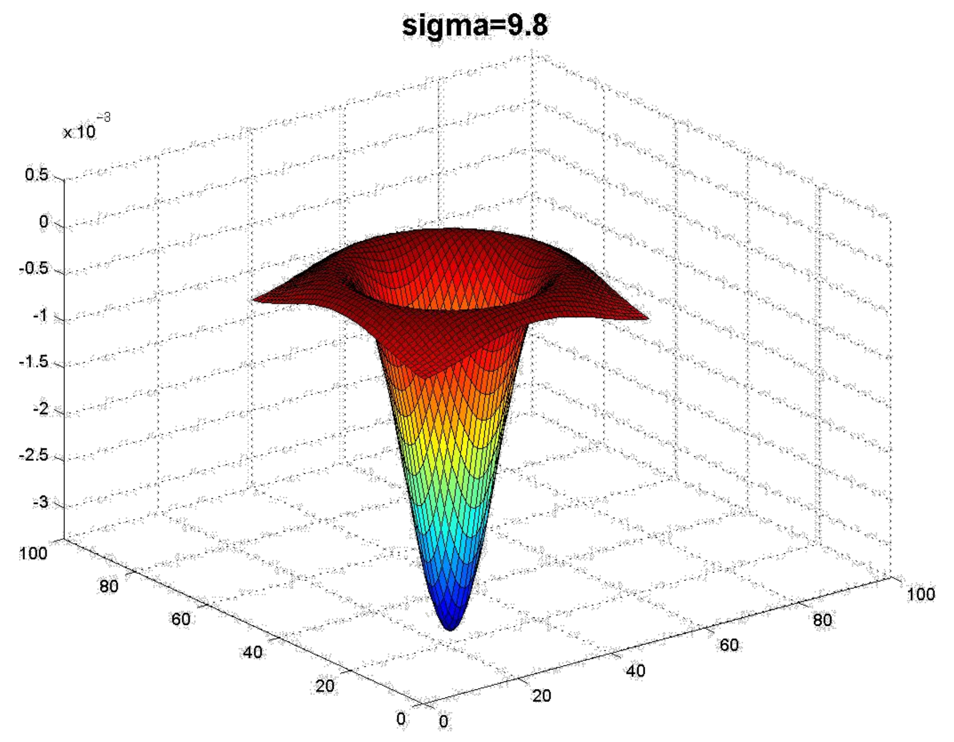

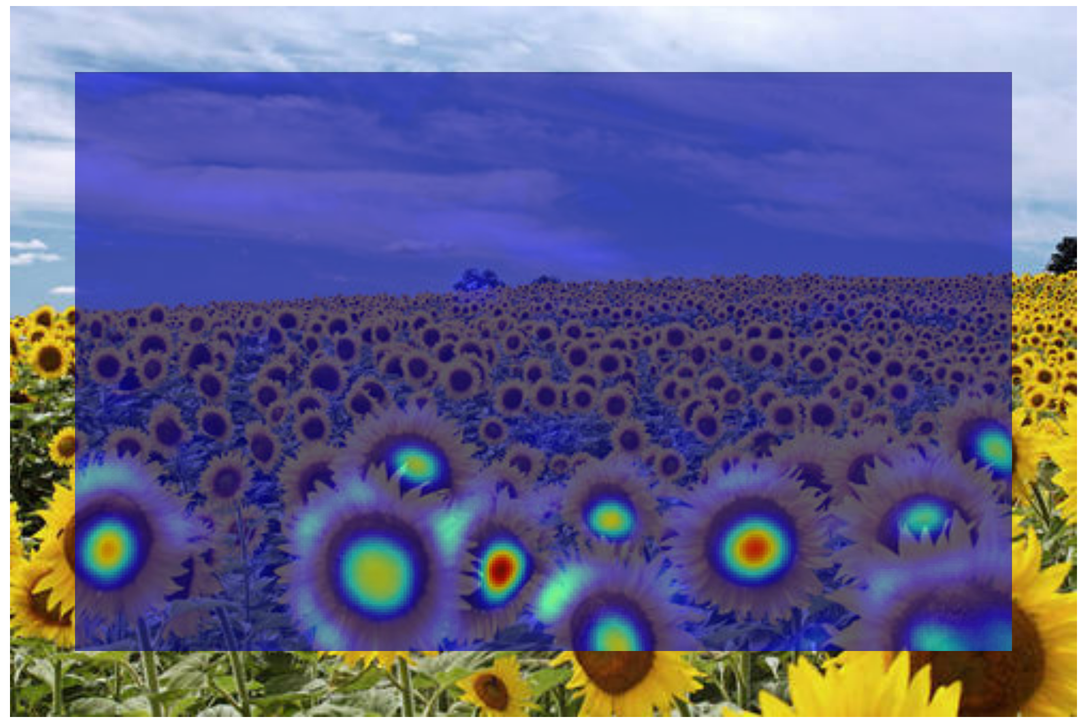

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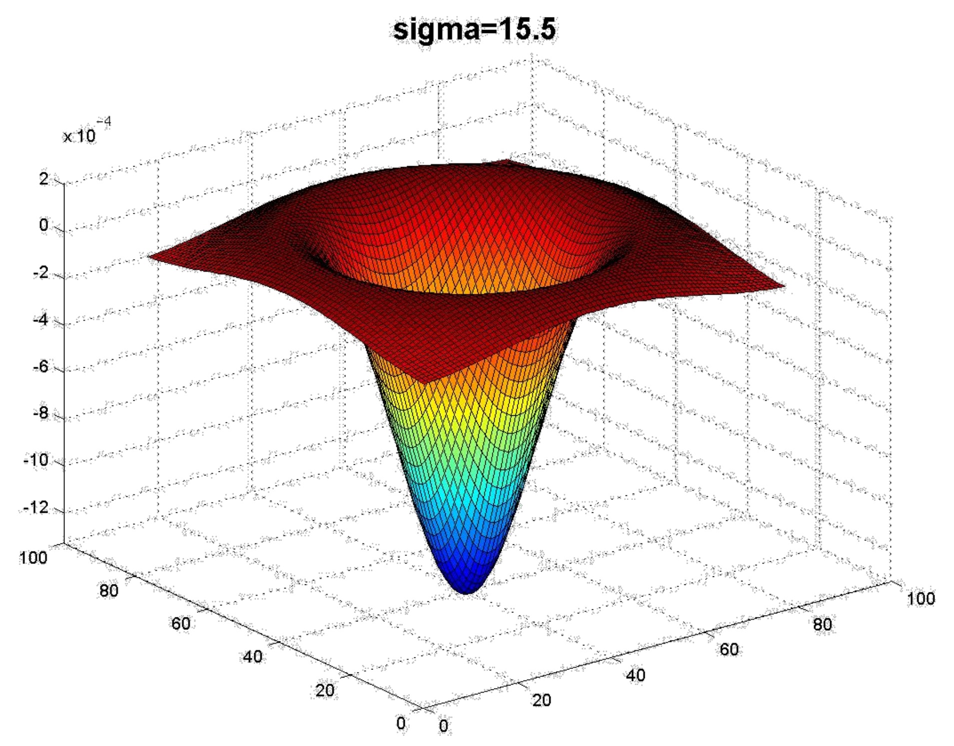

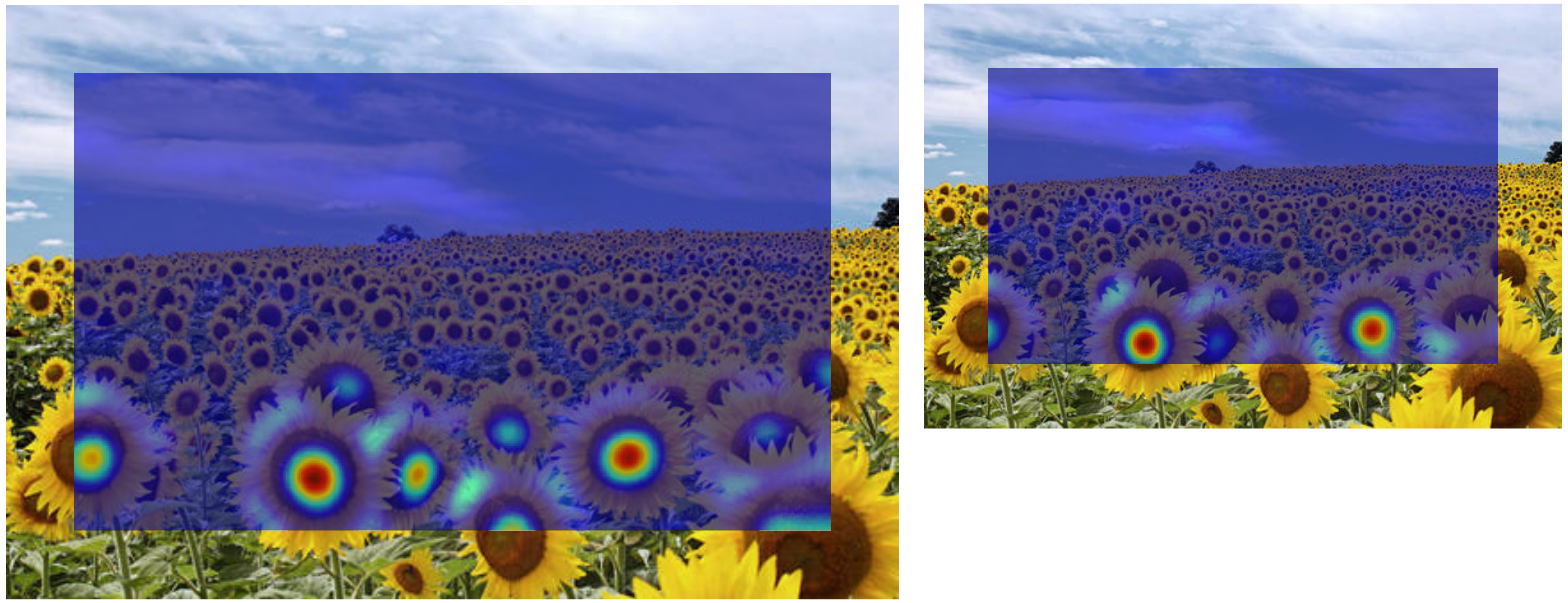

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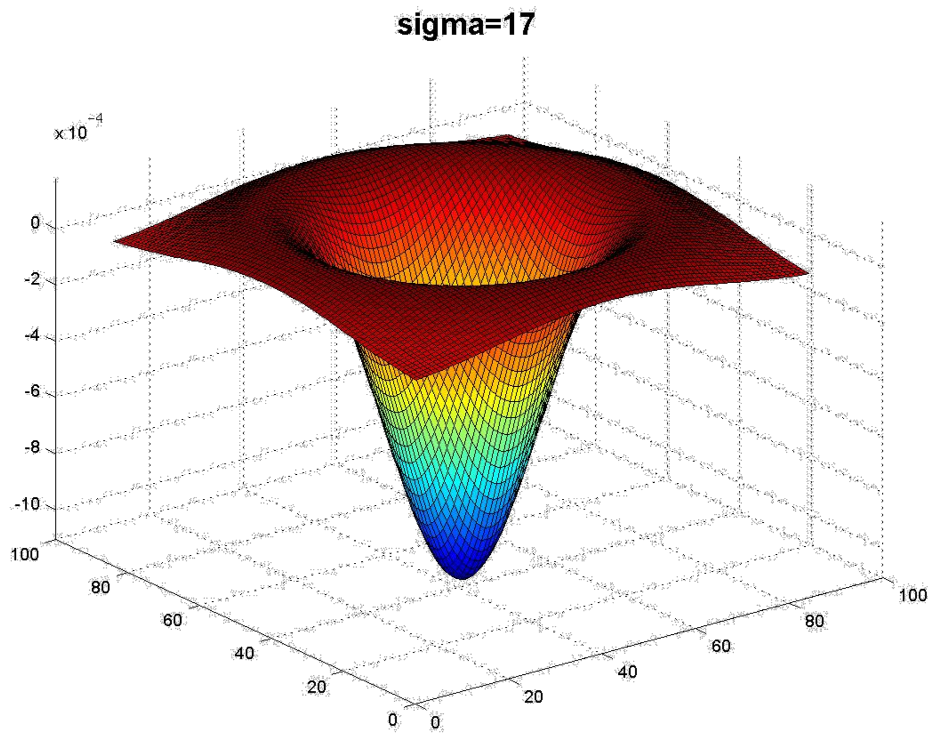

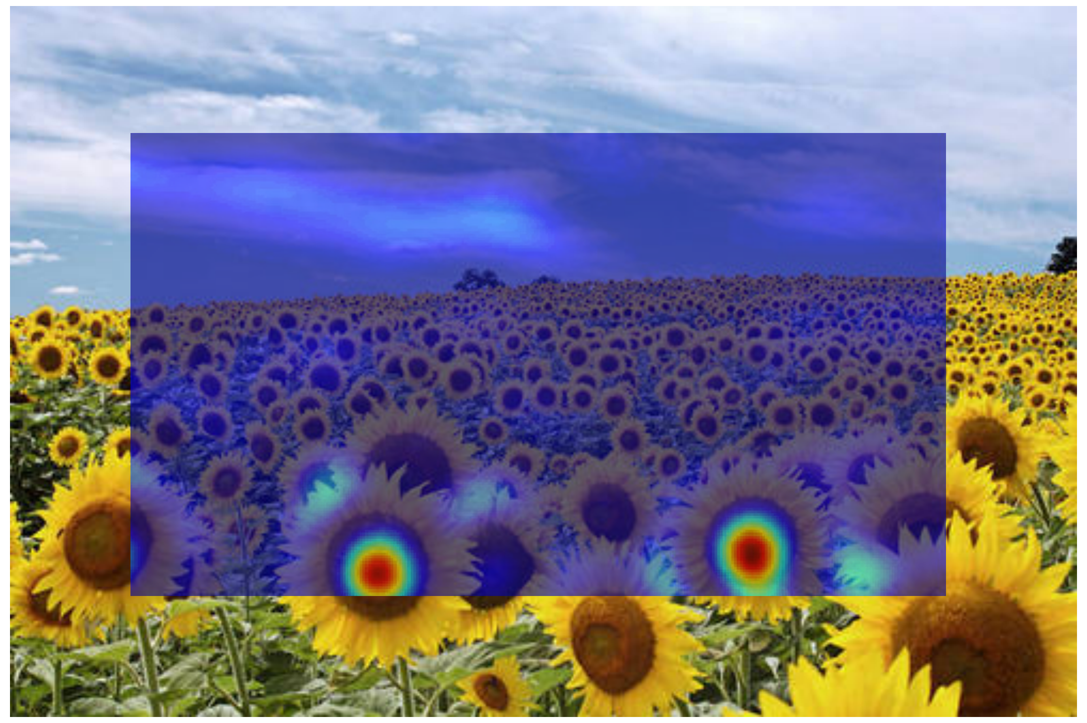

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Applying **Laplacian** Filter at Different **Scales**

Full size

3/4 size

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

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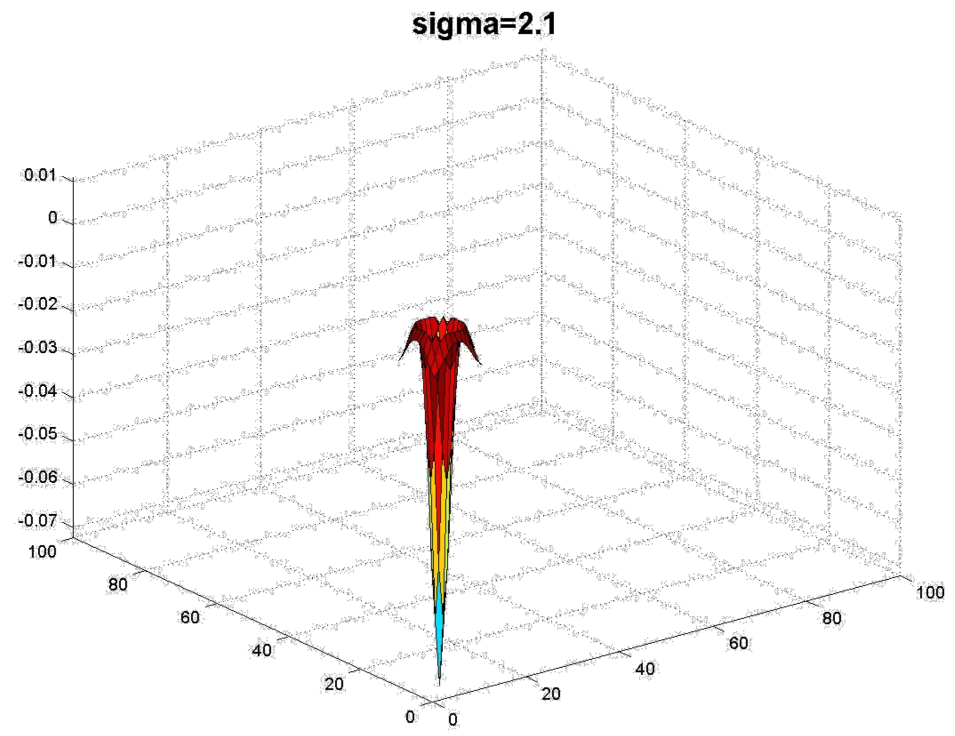

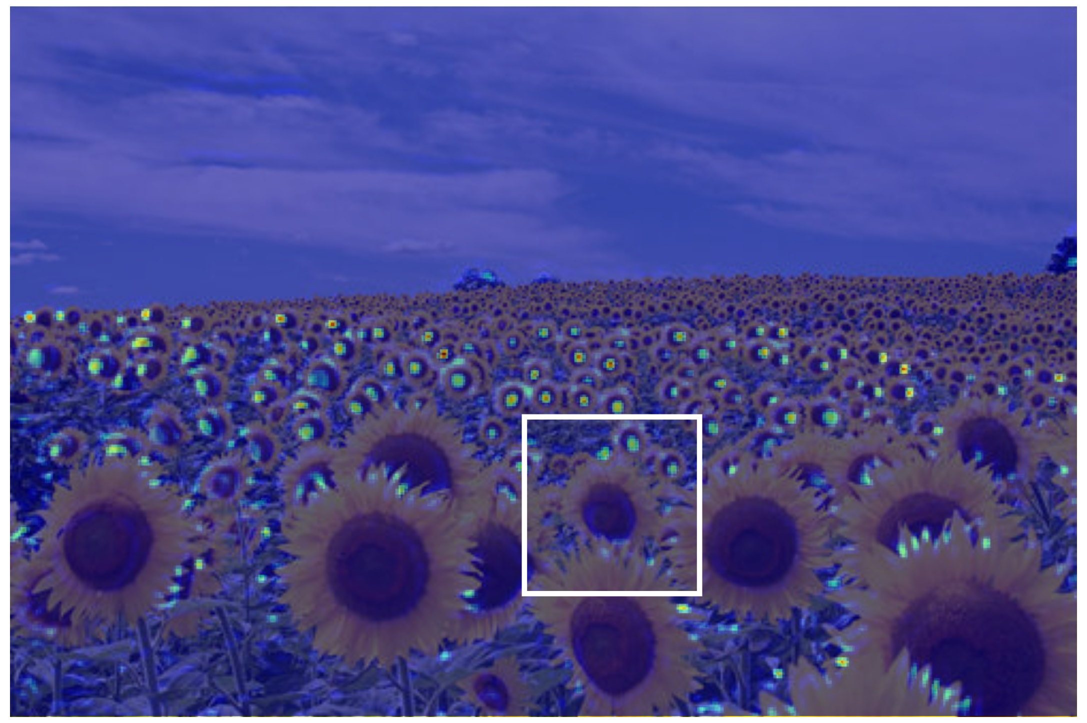

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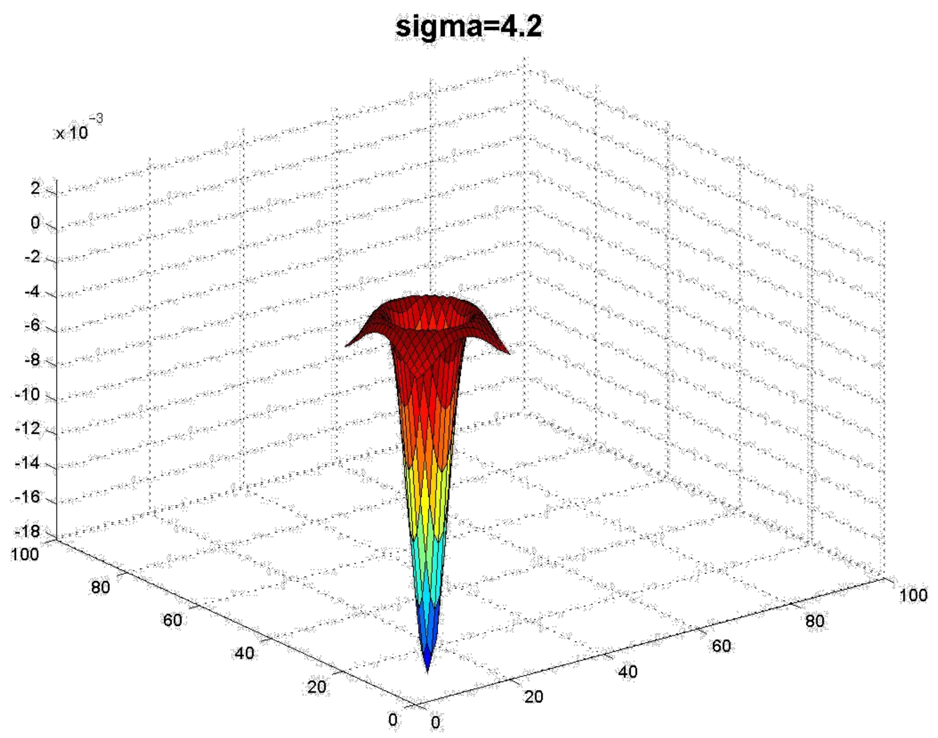

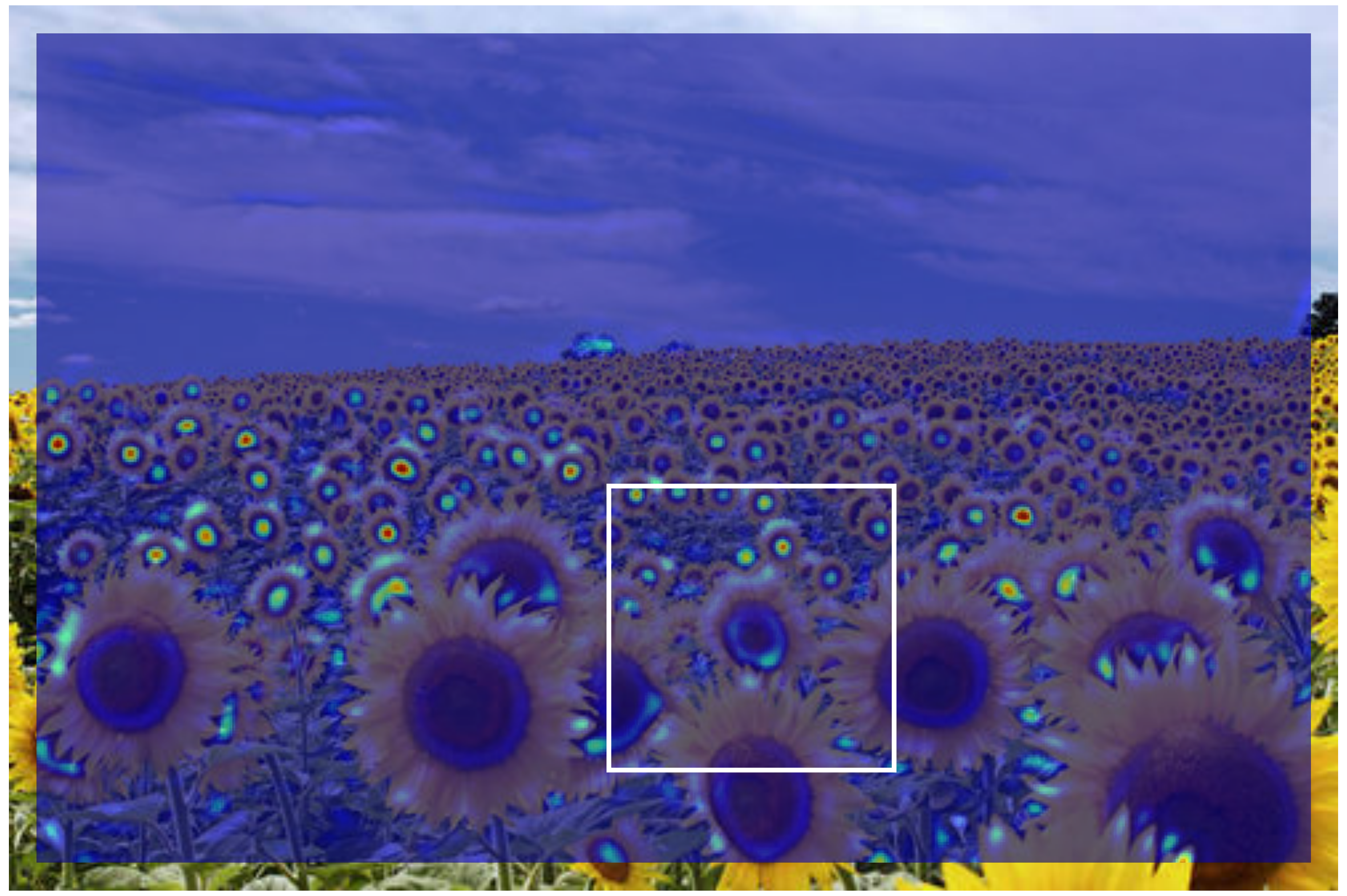

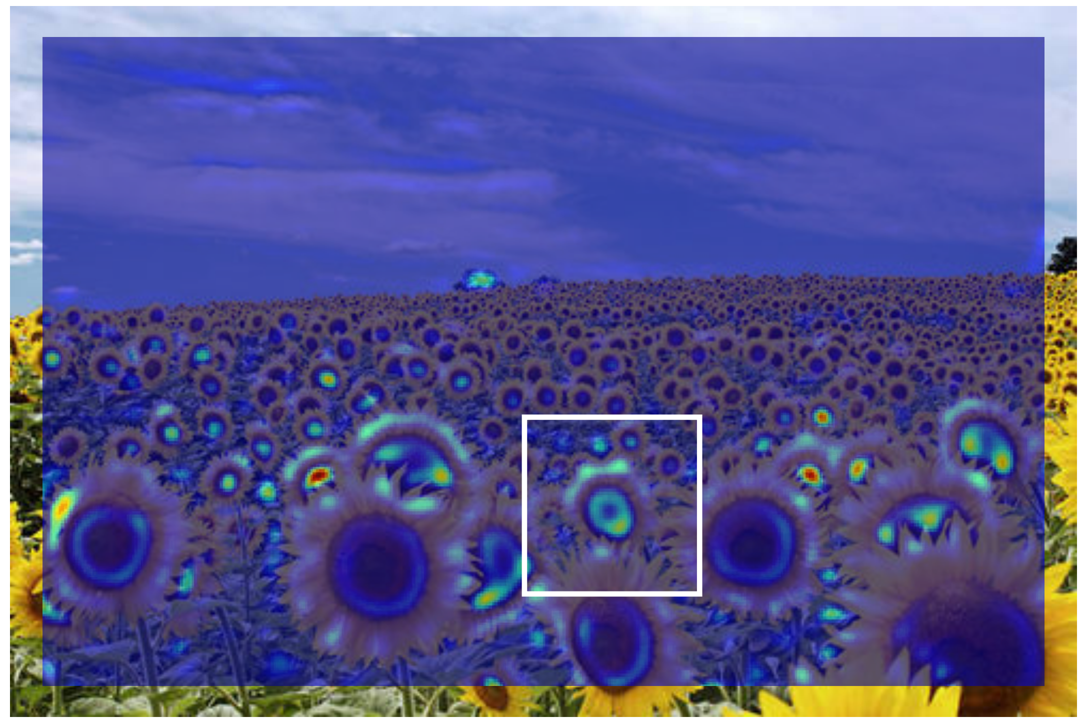

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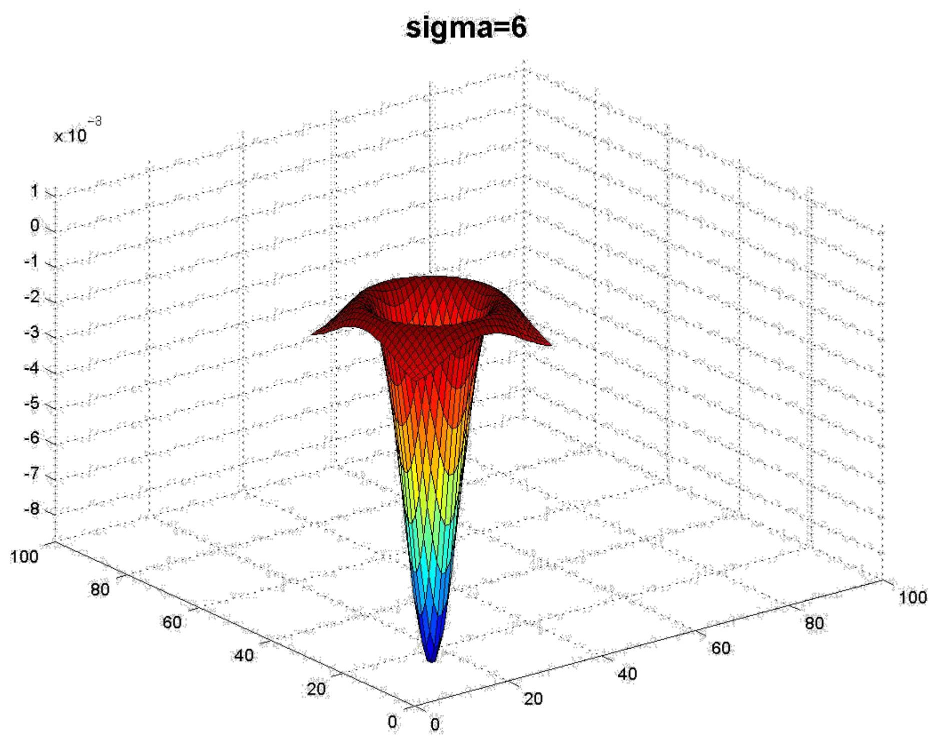

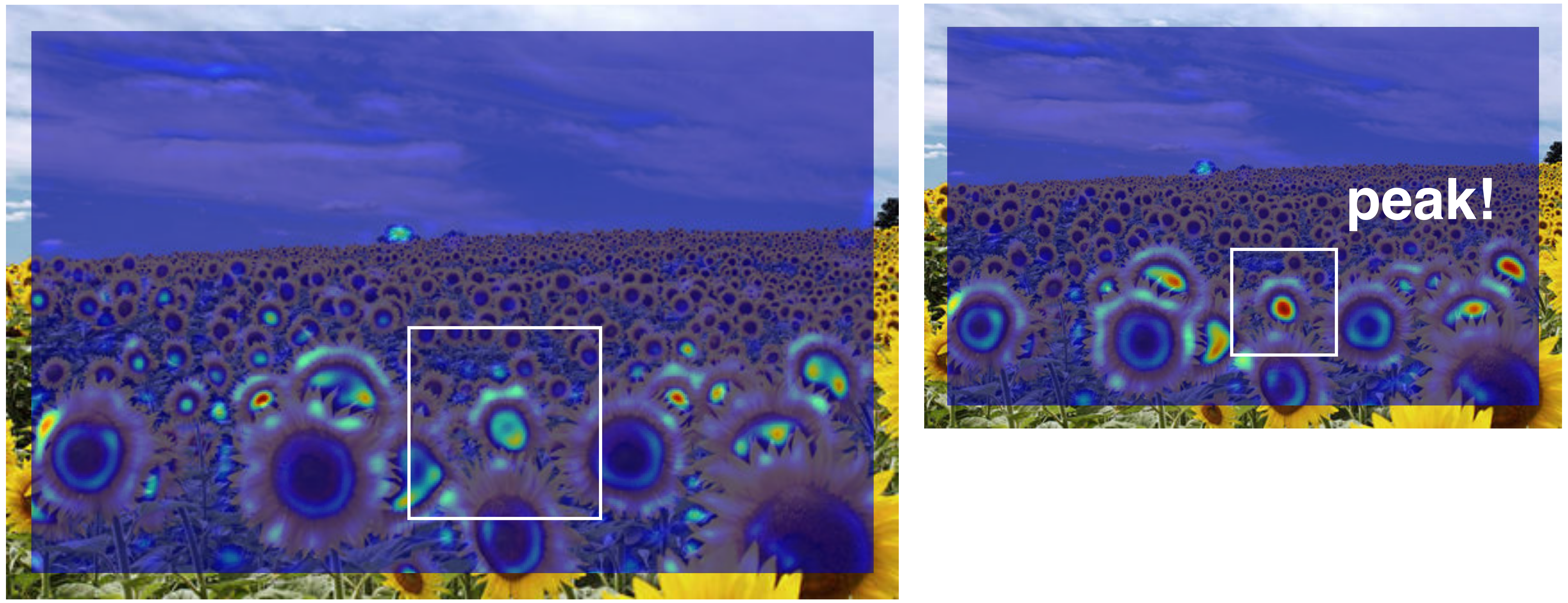

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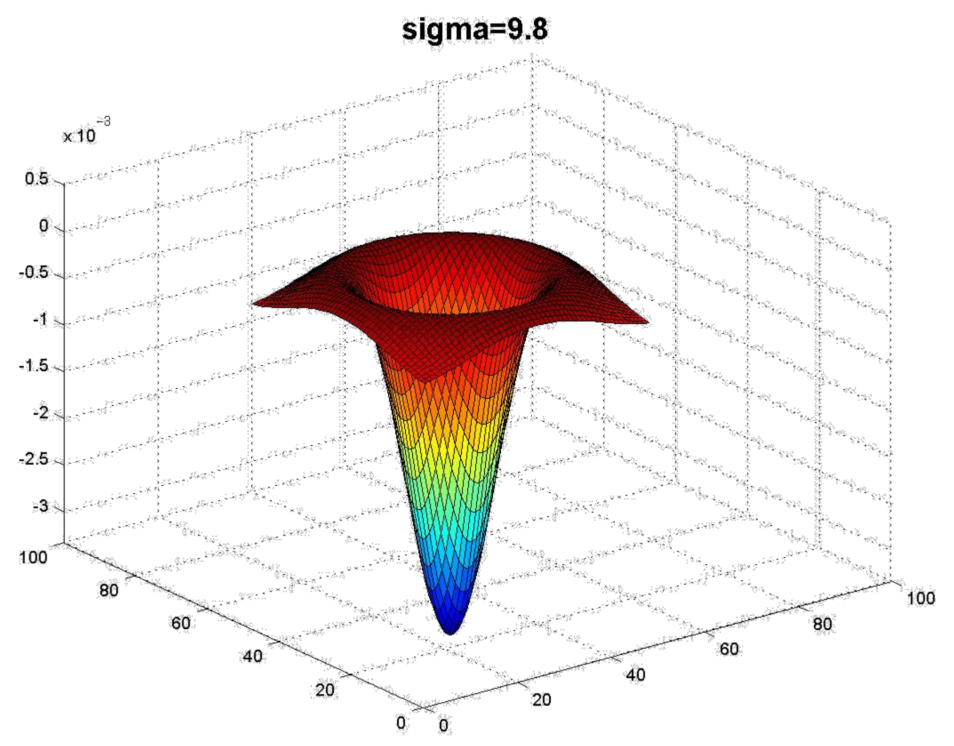

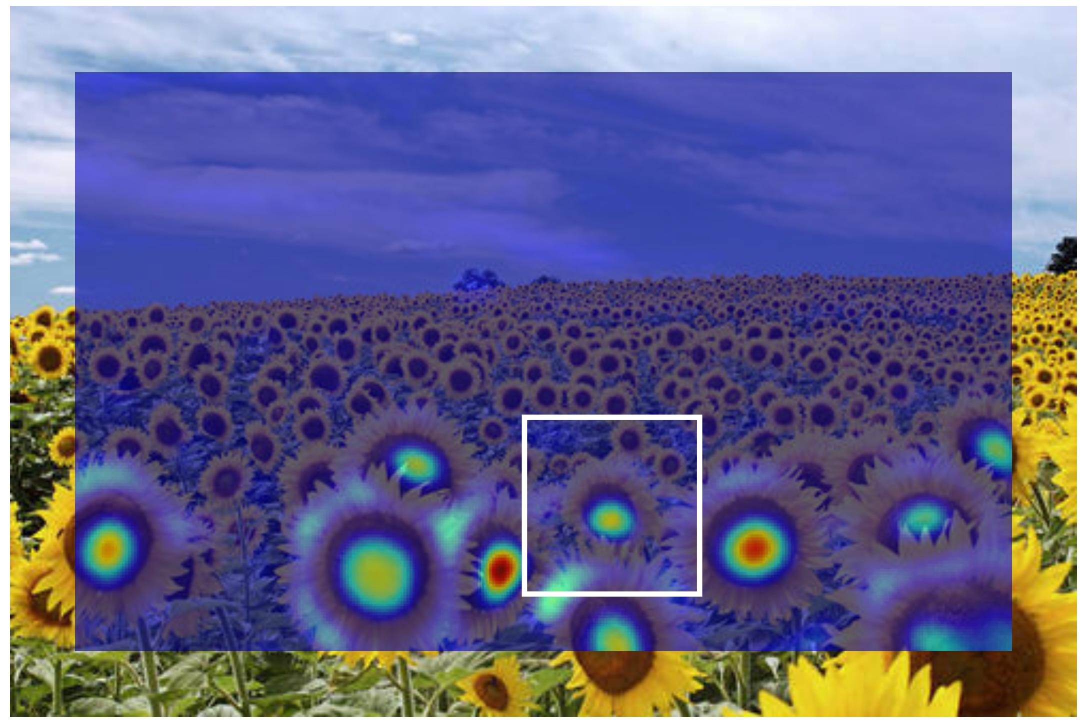

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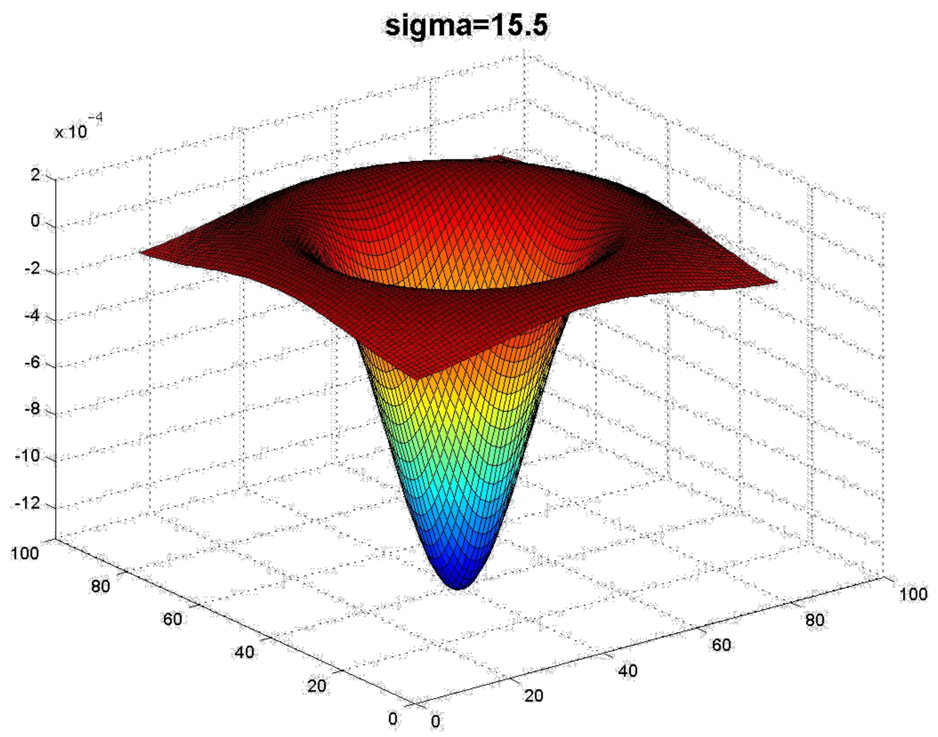

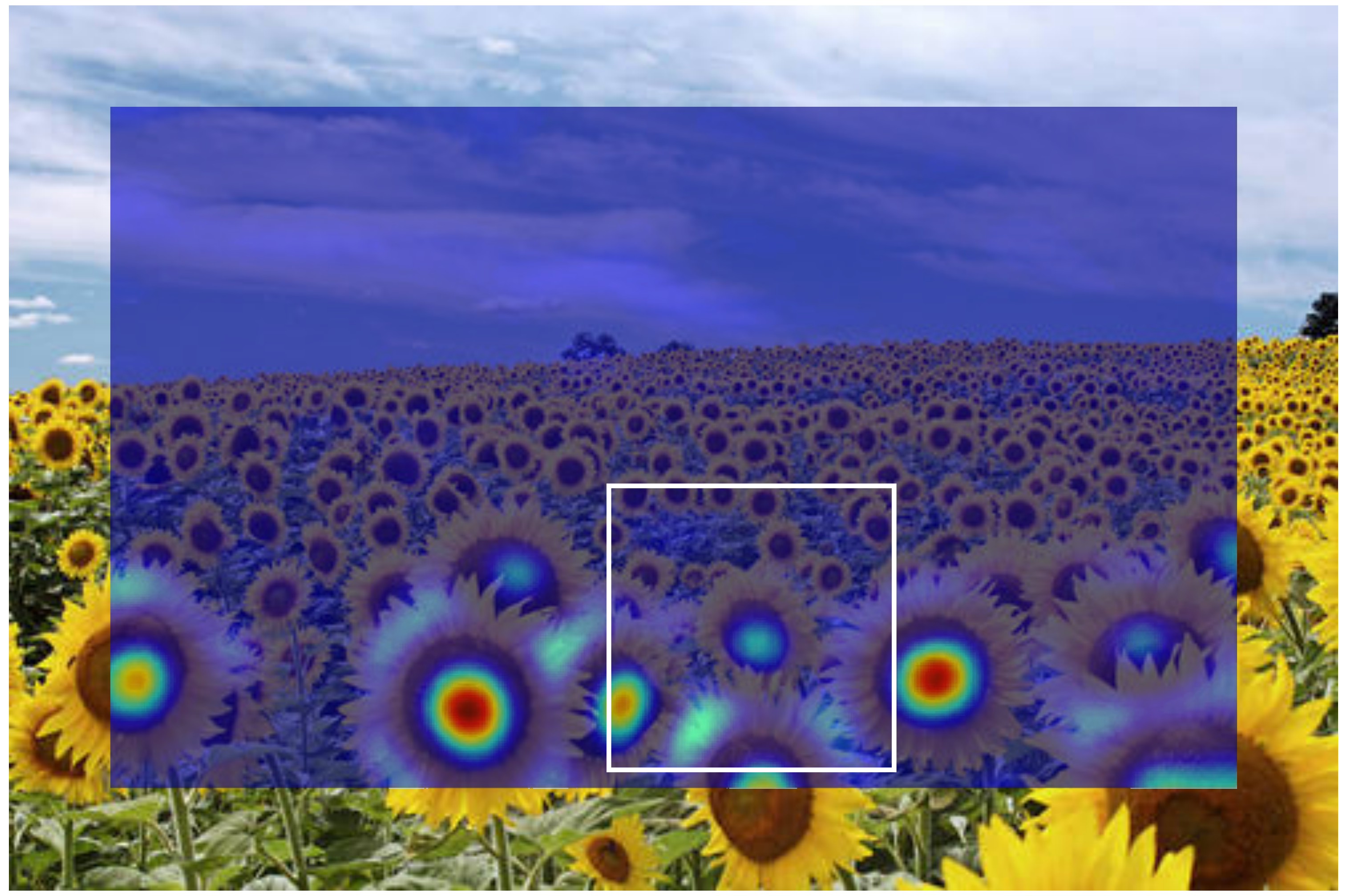

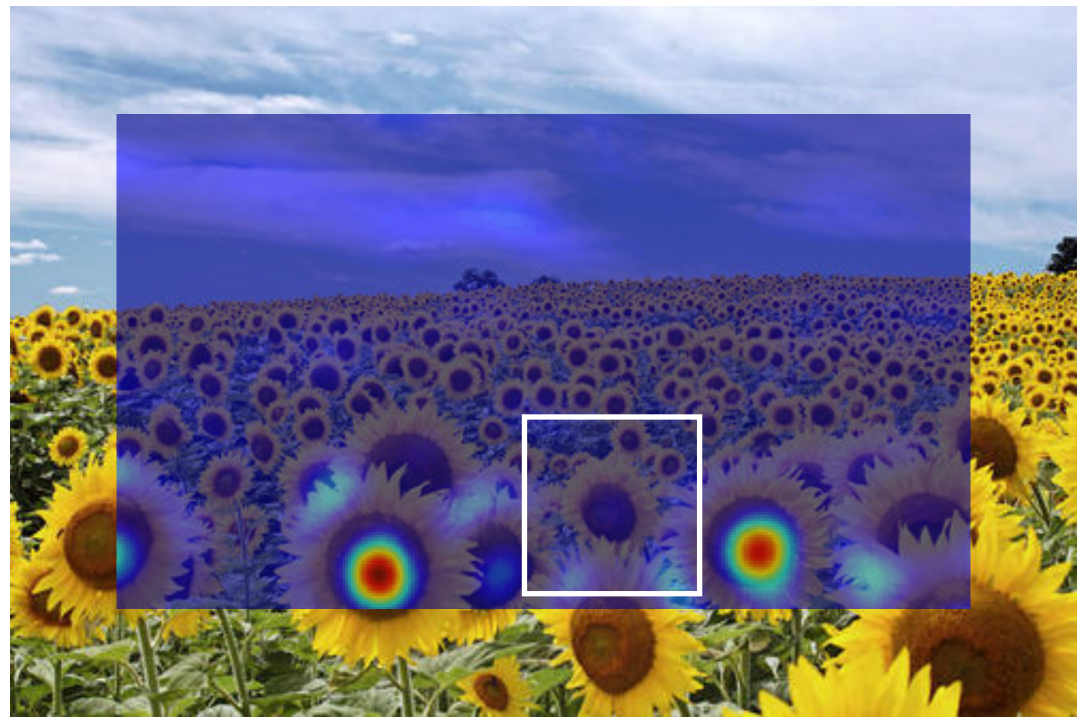

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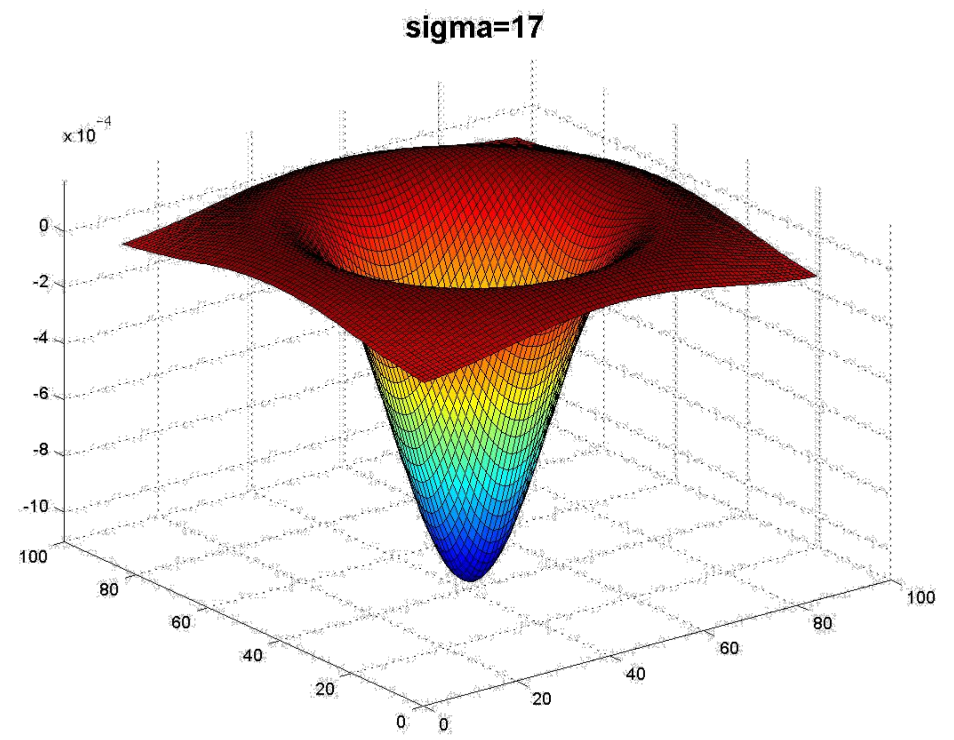

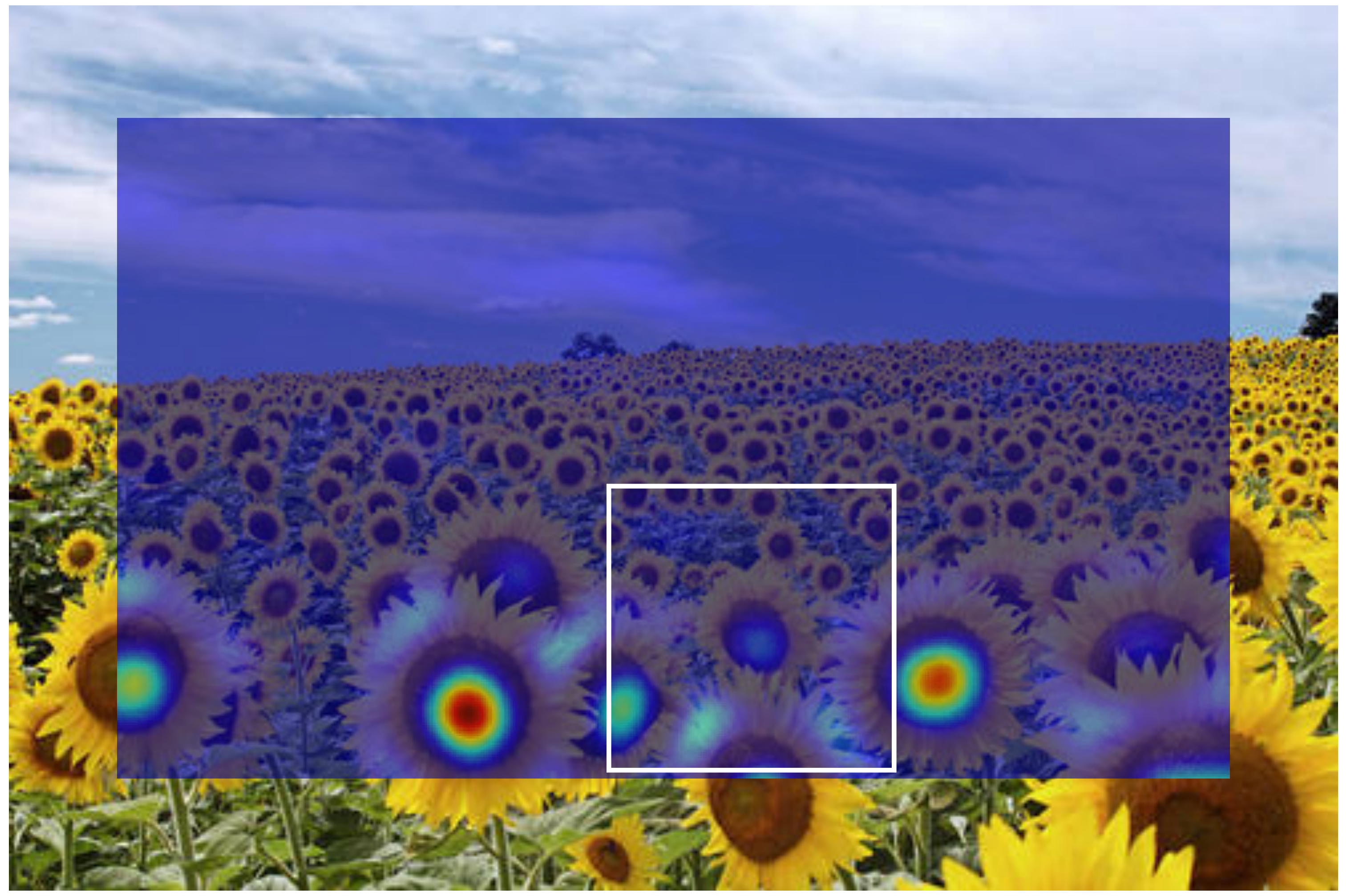

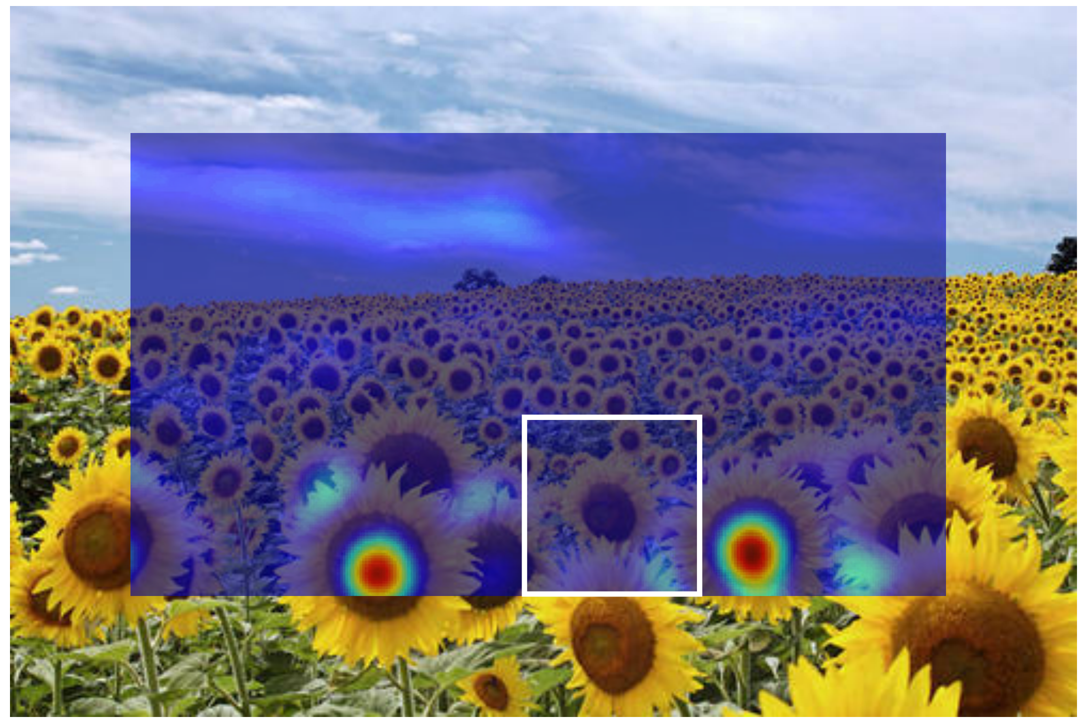

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Applying **Laplacian** Filter at Different **Scales**

Full size

3/4 size

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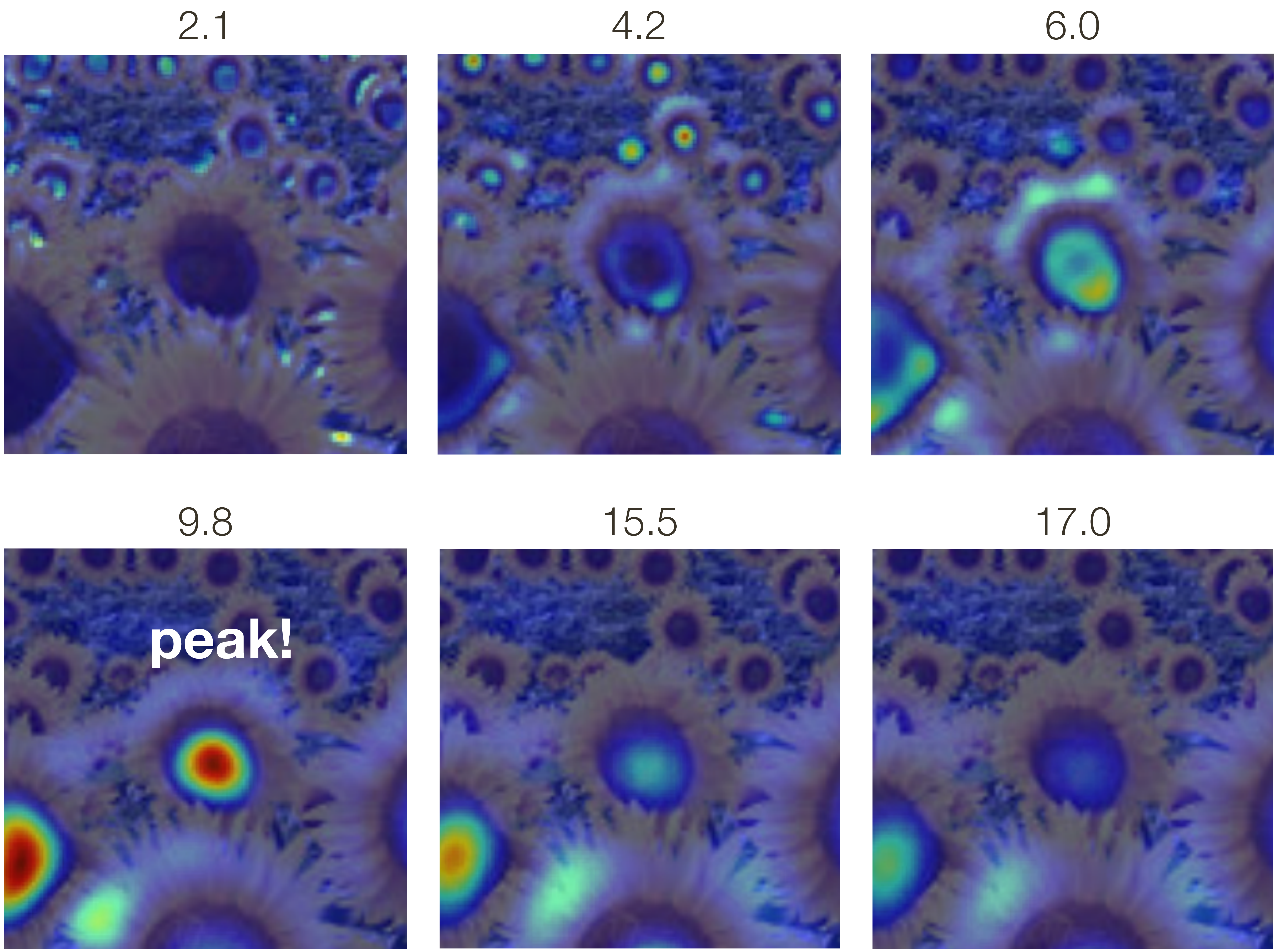

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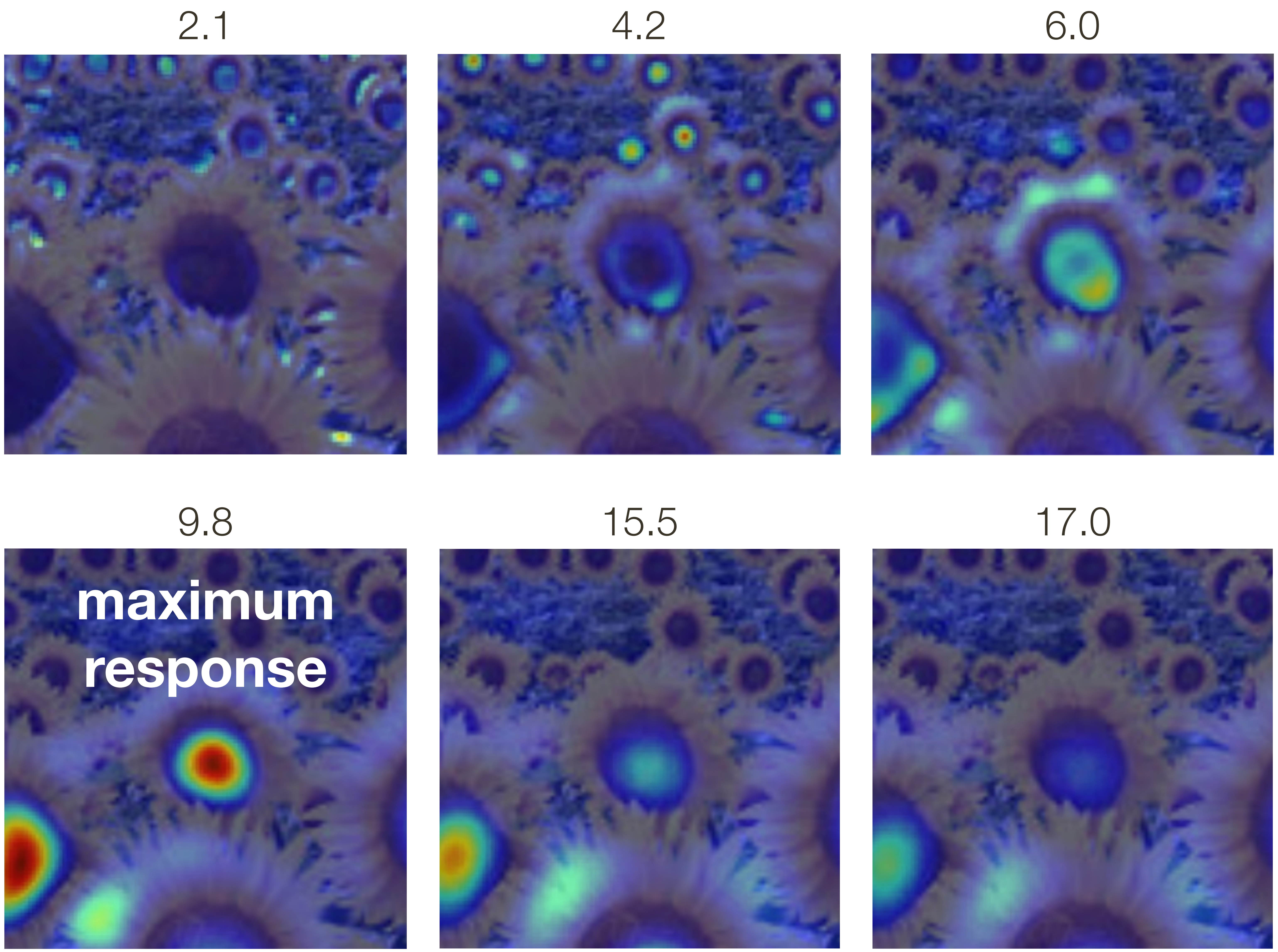
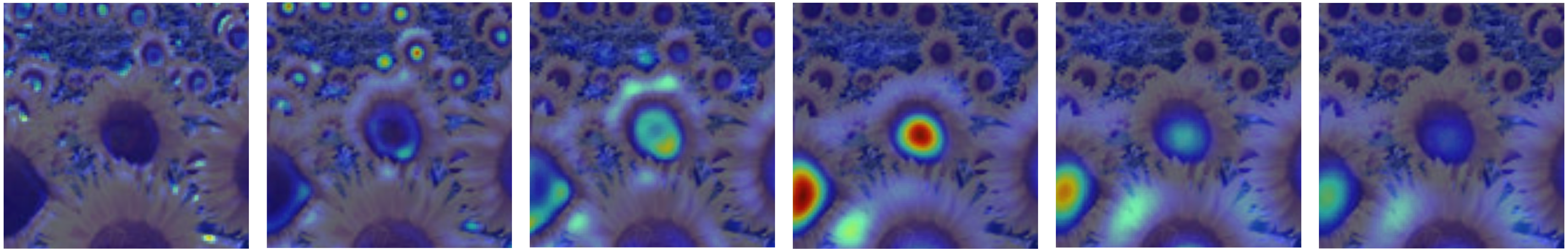


Optimal **Scale**

2.1 4.2 6.0 9.8 15.5 17.0



Full size image

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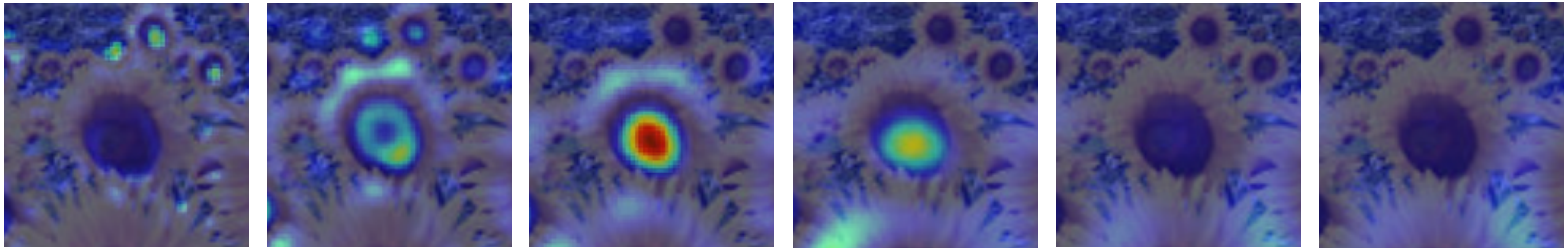

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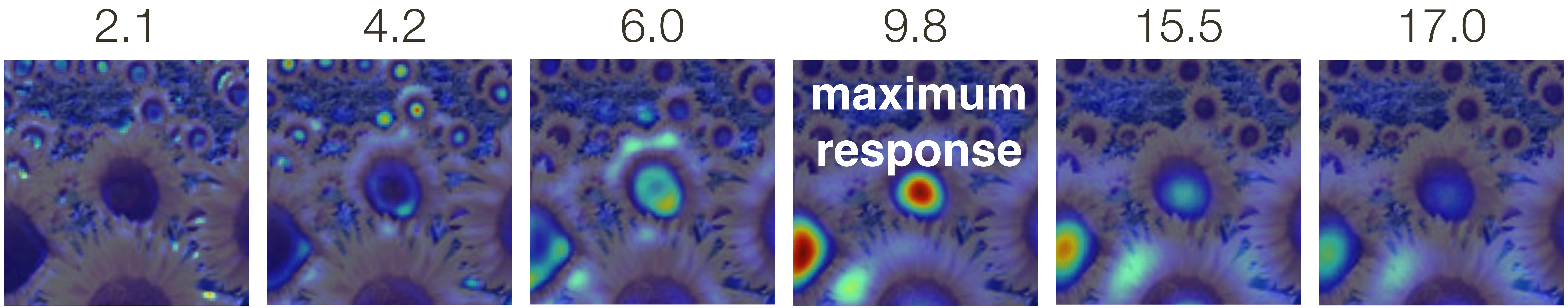
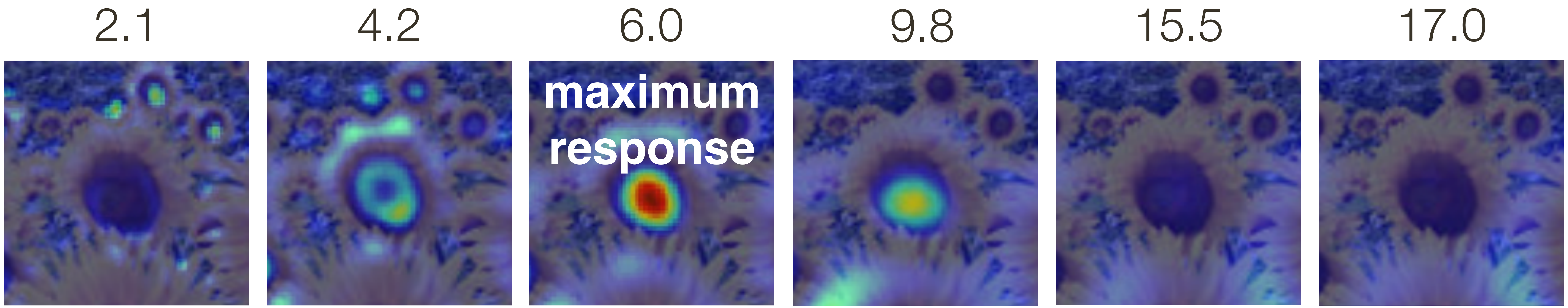


Full size image



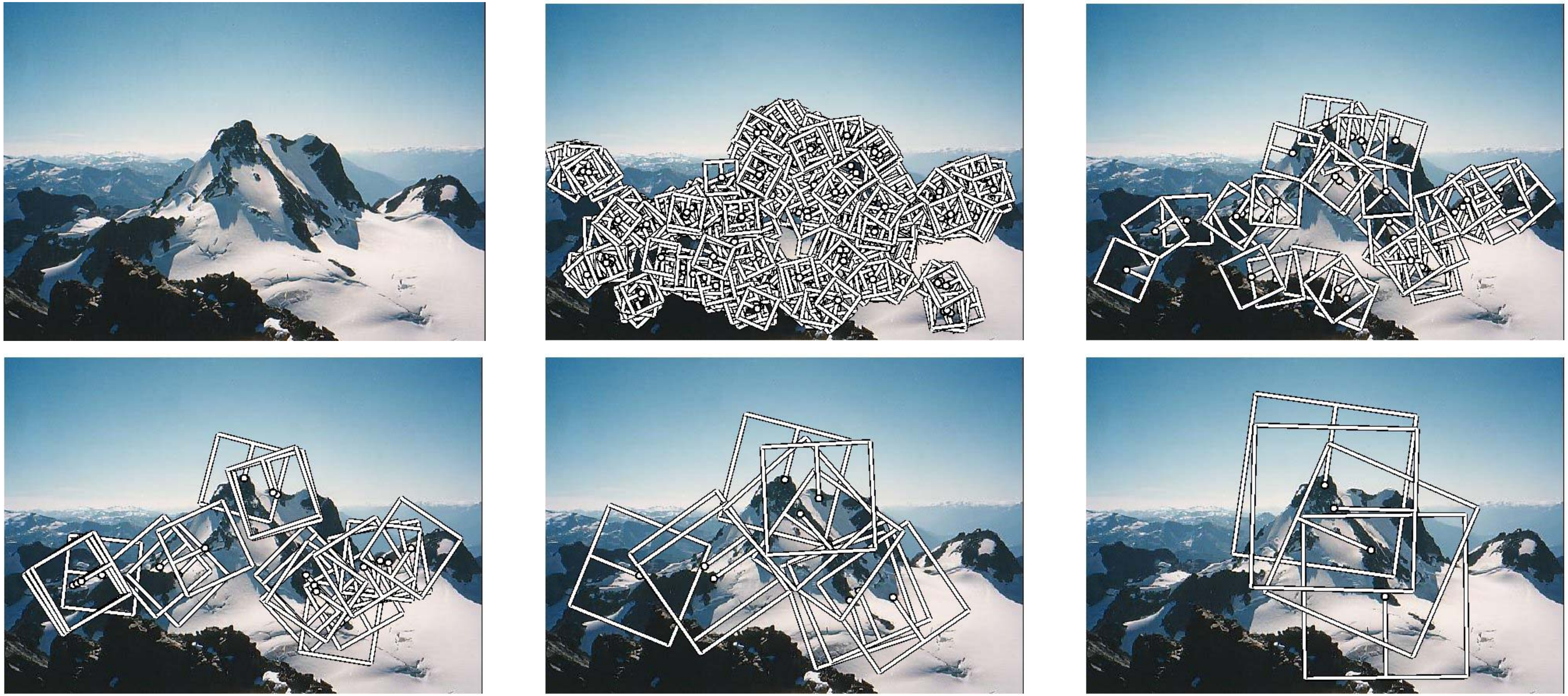
3/4 size image

Implementation

```
For each level of the Gaussian pyramid

compute feature response (e.g. Harris, Laplacian)

For each level of the Gaussian pyramid

    if local maximum and cross-scale

    save scale and location of feature
```

(x, y, s)

Multi-Scale Harris Corners



Re-cap

Summary of what we have seen so far:

Representation	Results in	Approach	Technique
intensity	dense	template matching	(normalized) correlation
edge	relatively sparse	derivatives	Sobel, LoG, Canny
corner	sparse	locally distinct features	Harris (and variants)
blob	sparse	locally distinct features	LoG

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Summary

Edges are useful image features for many applications, but suffer from the aperture problem

Canny Edge detector combines edge filtering with linking and hysteresis steps

Corners / Interest Points have 2D structure and are useful for correspondence

Harris corners are minima of a local SSD function

DoG maxima can be reliably located in scale-space and are useful as interest points