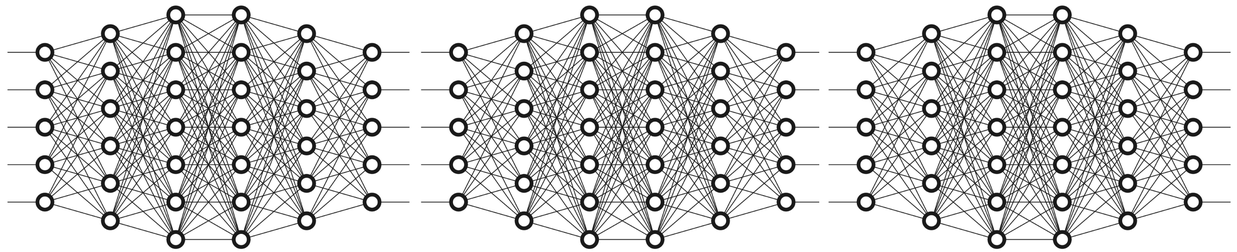
THE UNIVERSITY OF BRITISH COLUMBIA

CPSC 425: Computer Vision



Lecture 30: Neural Networks

Menu for Today (**November 23, 2020**)

Topics:

- Backpropagation
- Convolutional Layers
- Pooling Layer

Redings:

- **Today's** Lecture: N/A
- **Next** Lecture: N/A

Reminders:

- **Assignment 6**: Deep Learning due **Wednsday, November 2nd**

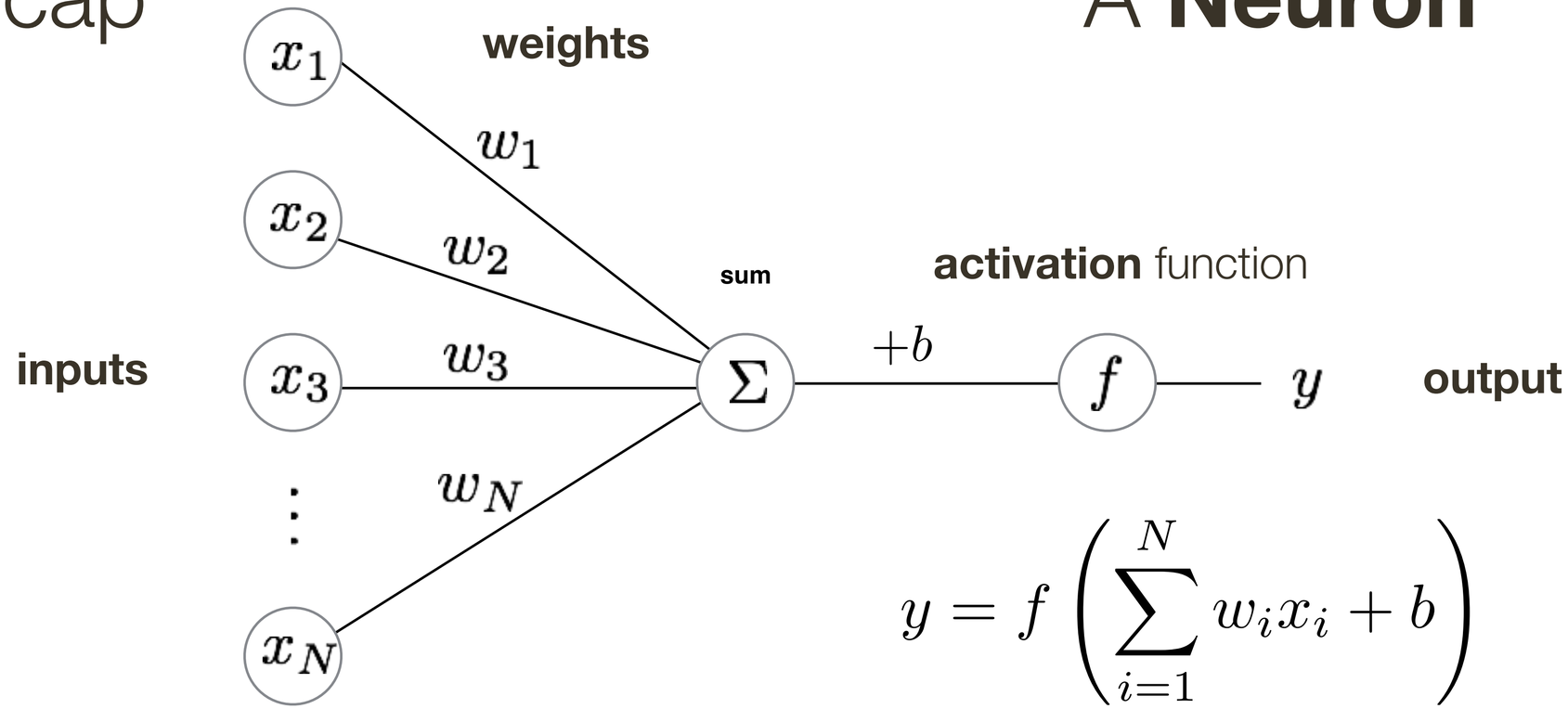
Please fill out

Student Evaluations

(on Canvas)

Lecture 29: Re-cap

A **Neuron**



$$y = f\left(\sum_{i=1}^{N} w_i x_i + b\right)$$

— The basic unit of computation in a neural network is a neuron.

— A neuron accepts some number of input signals, computes their weighted sum, and applies an **activation function** (or **non-linearity**) to the sum.

— Common activation functions include sigmoid and rectified linear unit (ReLU)

Lecture 29: Re-cap **Neural** Network

A neural network comprises neurons connected in an acyclic graph

The outputs of neurons can become inputs to other neurons

Neural networks typically contain multiple layers of neurons

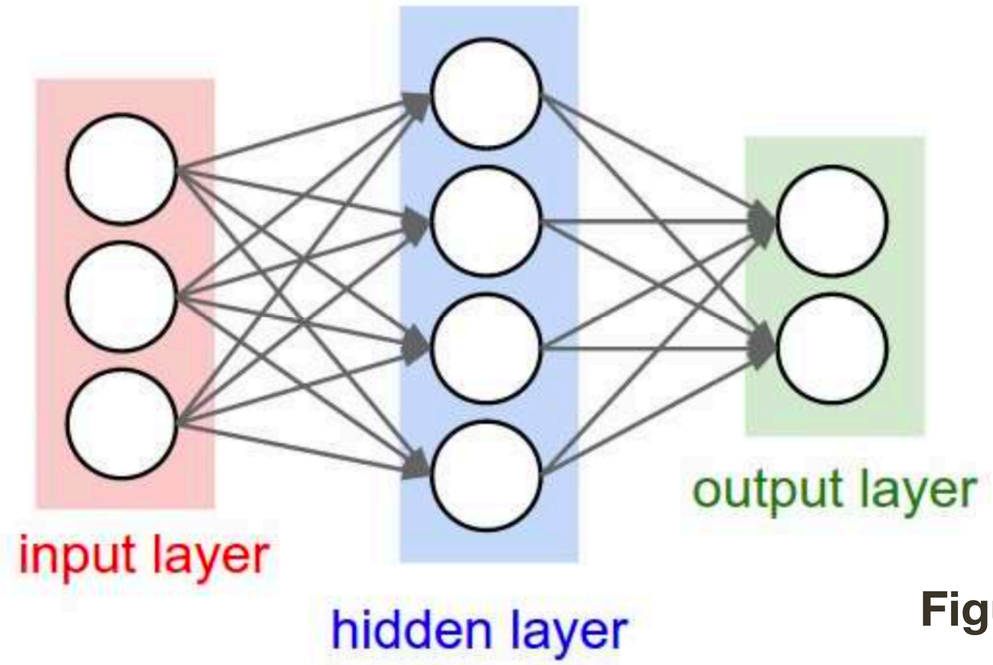
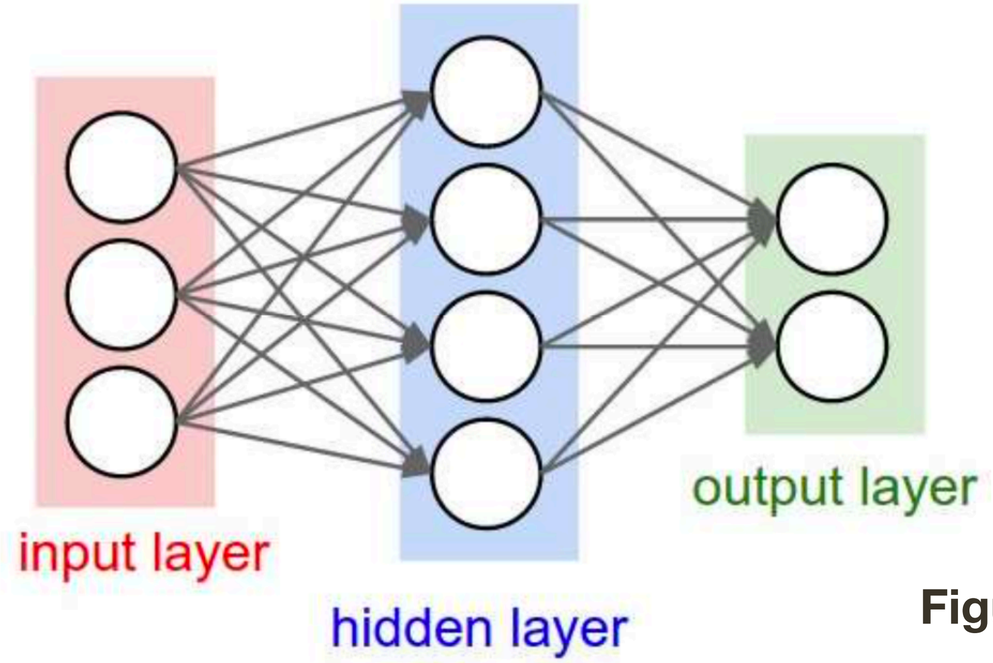


Figure credit: Fei-Fei and Karpathy

Example of a neural network with three inputs, a single hidden layer of four neurons, and an output layer of two neurons

Lecture 29: Re-cap **Neural** Network

Note: each neuron will have its own vector of weights and a bias, its easier to think of all neurons in a layer as a single entity with a matrix of weights (size = number of inputs x number of neurons) and a vector of biases (size = number of neurons)



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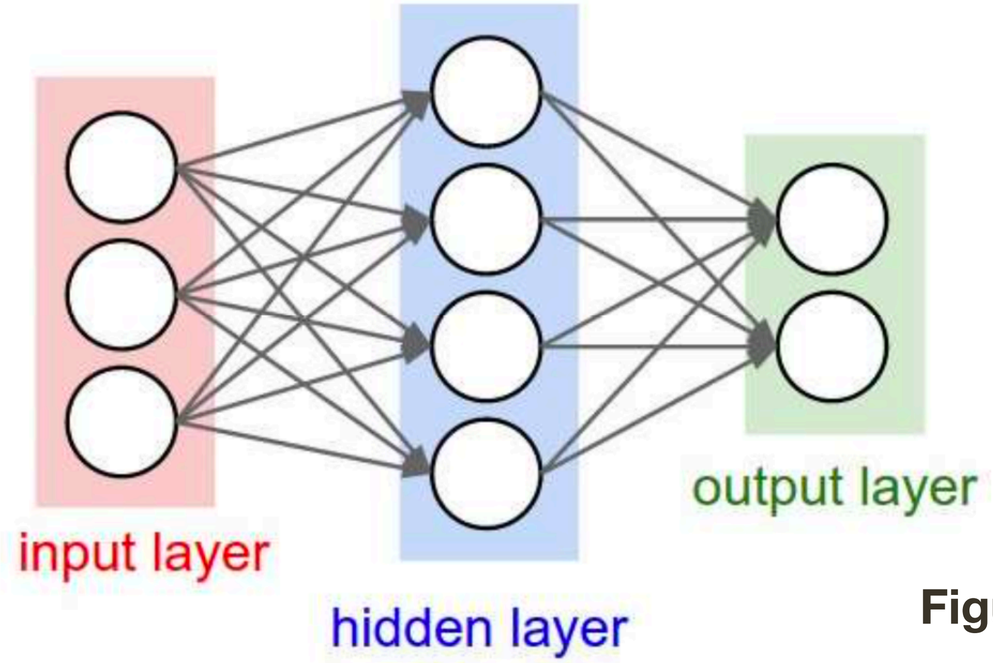

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$$\hat{\mathbf{y}} = f(\mathbf{x}, \mathbf{W}_1, \mathbf{W}_2, \mathbf{b}_1, \mathbf{b}_2) = \sigma\left(\mathbf{W}_2^{(2\times4)}\sigma\left(\mathbf{W}_1^{(4\times3)}\mathbf{x} + \mathbf{b}_1^{(4)}\right) + \mathbf{b}_2^{(2)}\right)$$

Backpropagation

When training a neural network, the final output will be some loss (error) function

— e.g. cross-entropy loss: $$L_i = -\log\left(\frac{e^{f_{y_i}}}{\sum_j e^{f_{y_j}}}\right)$$

which defines loss for i-th training example with true class index y_i; and f_j is the j-th element of the vector of class scores coming from neural net.

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Consider neural net which takes input vector $\mathbf{x}_i$ and predicts scores for 3 classes, with true class being class 3:

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$$f$$
$$c_1 = -2.85$$
$$c_2 = 0.86$$
$$c_3 = 0.28$$

Backpropagation

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$$\begin{array}{c} f \\ c_1 = -2.85 \\ c_2 = 0.86 \\ c_3 = 0.28 \end{array} \xrightarrow{\exp} \begin{array}{c} \\ 0.058 \\ 2.36 \\ 1.32 \end{array}$$

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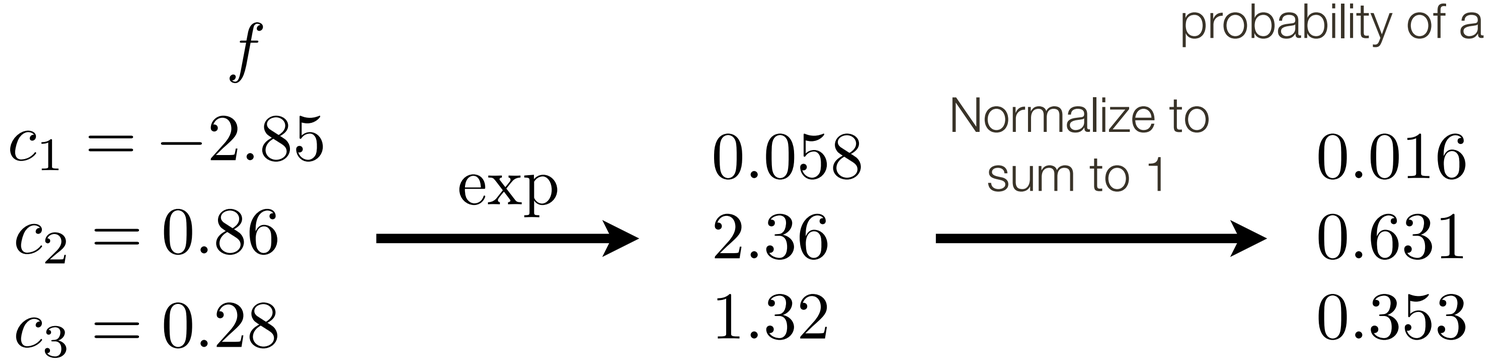

Backpropagation

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— e.g. cross-entropy loss: $L_i = -\log\left(\frac{e^{f_{y_i}}}{\sum_j e^{f_{y_j}}}\right)$

softmax function
multi-class classifier

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Consider neural net which takes input vector $\mathbf{x}_i$ and predicts scores for 3 classes, with true class being class 3:

probability of a class

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probability of a class

$$L_i = -\log(0.353) = 1.04$$

Backpropagation

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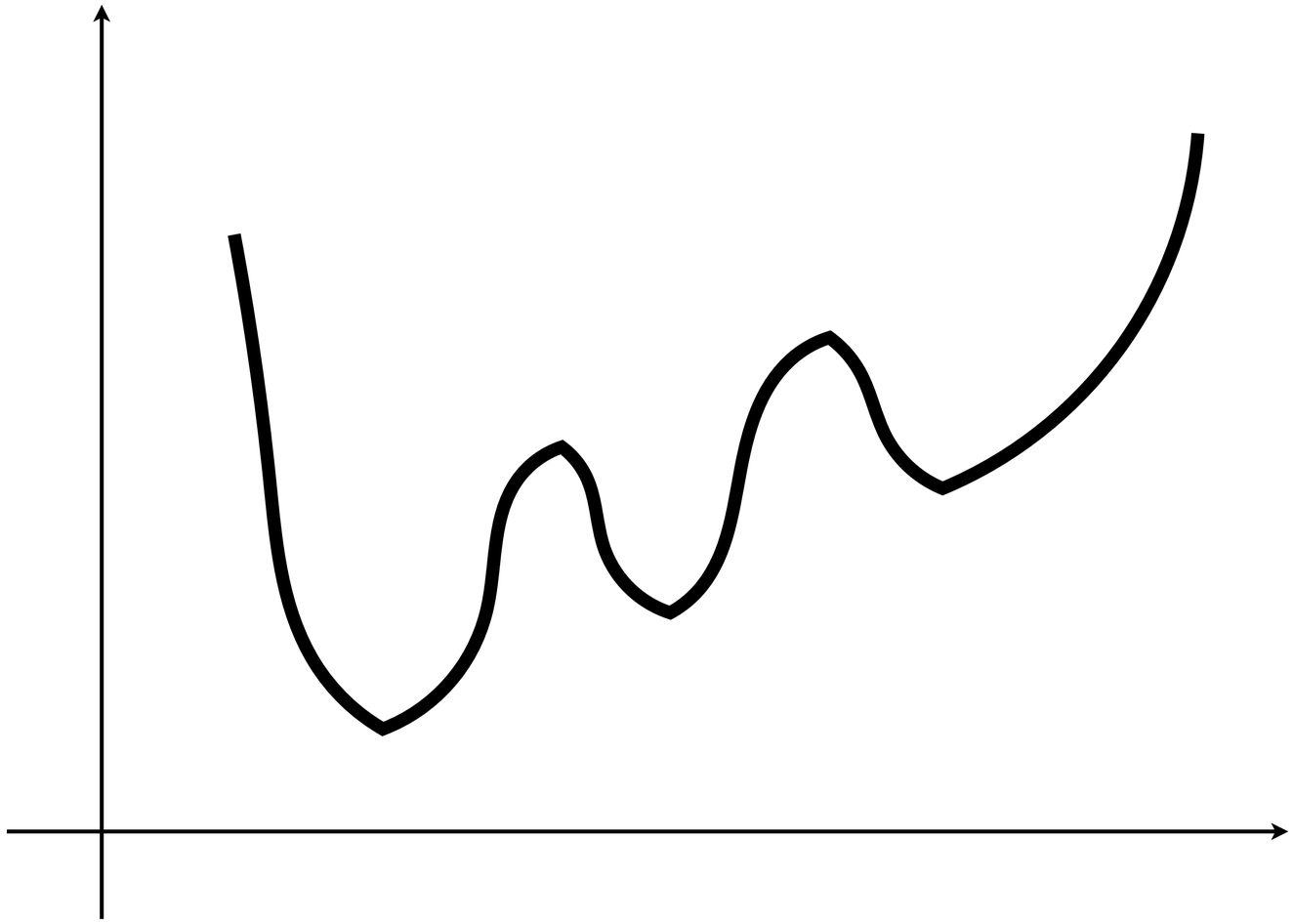
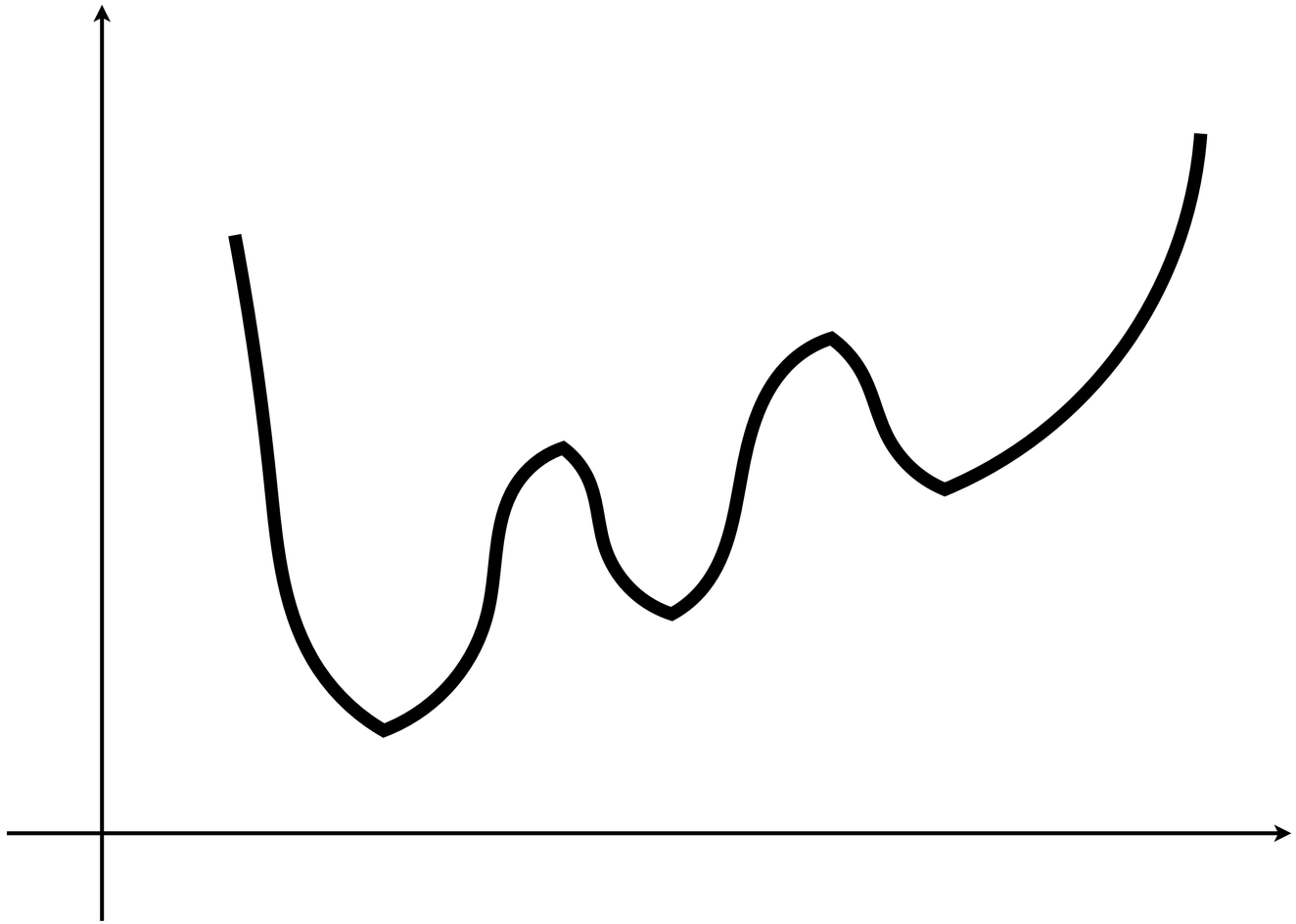
We want to compute the **gradient** of the loss with respect to the network parameters so that we can incrementally adjust the network parameters

Gradient Descent



*slide adopted from V. Ordonex

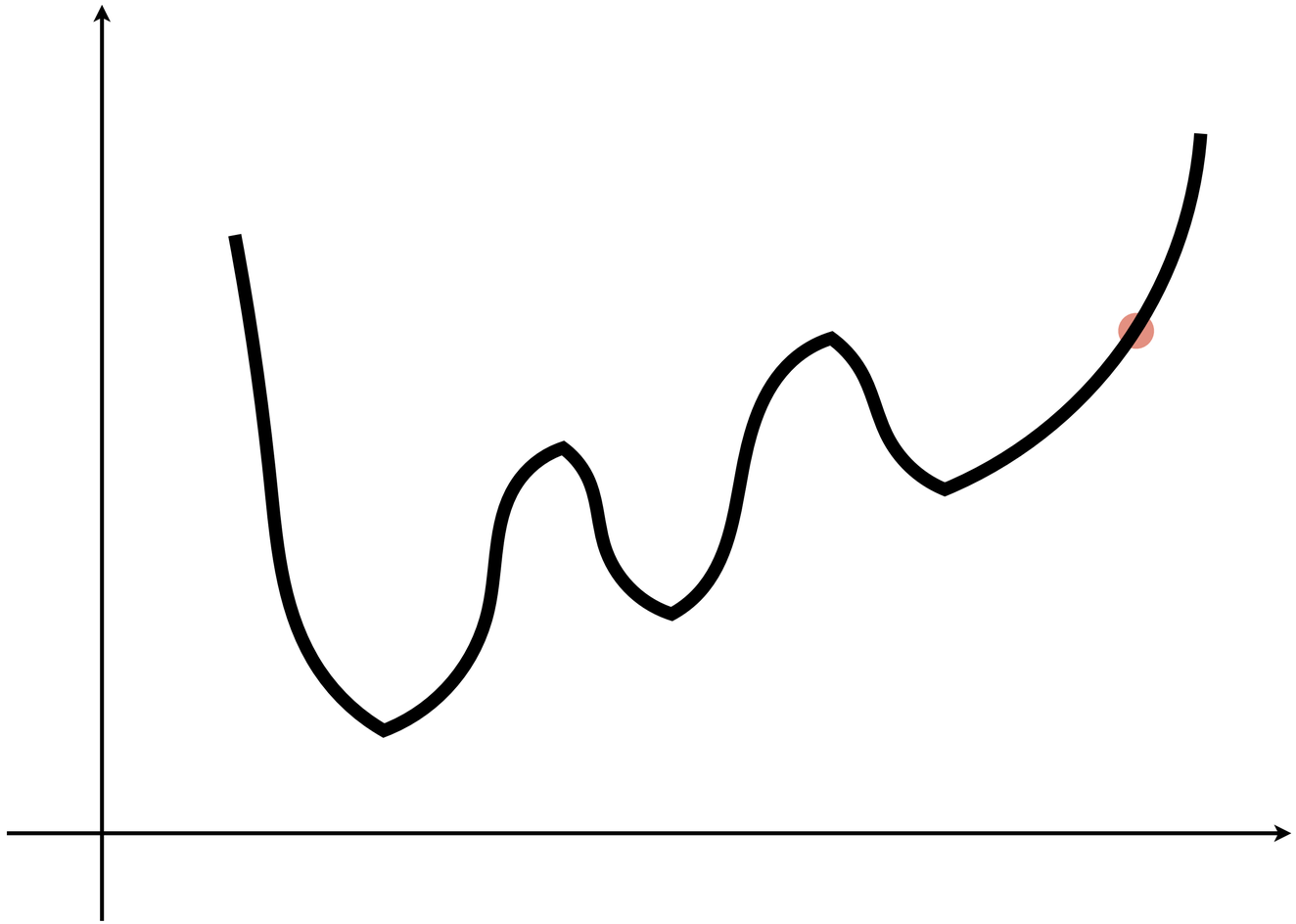
Gradient Descent



1. Start from random value of $\mathbf{W}_0, \mathbf{b}_0$

*slide adopted from V. Ordonex

Gradient Descent



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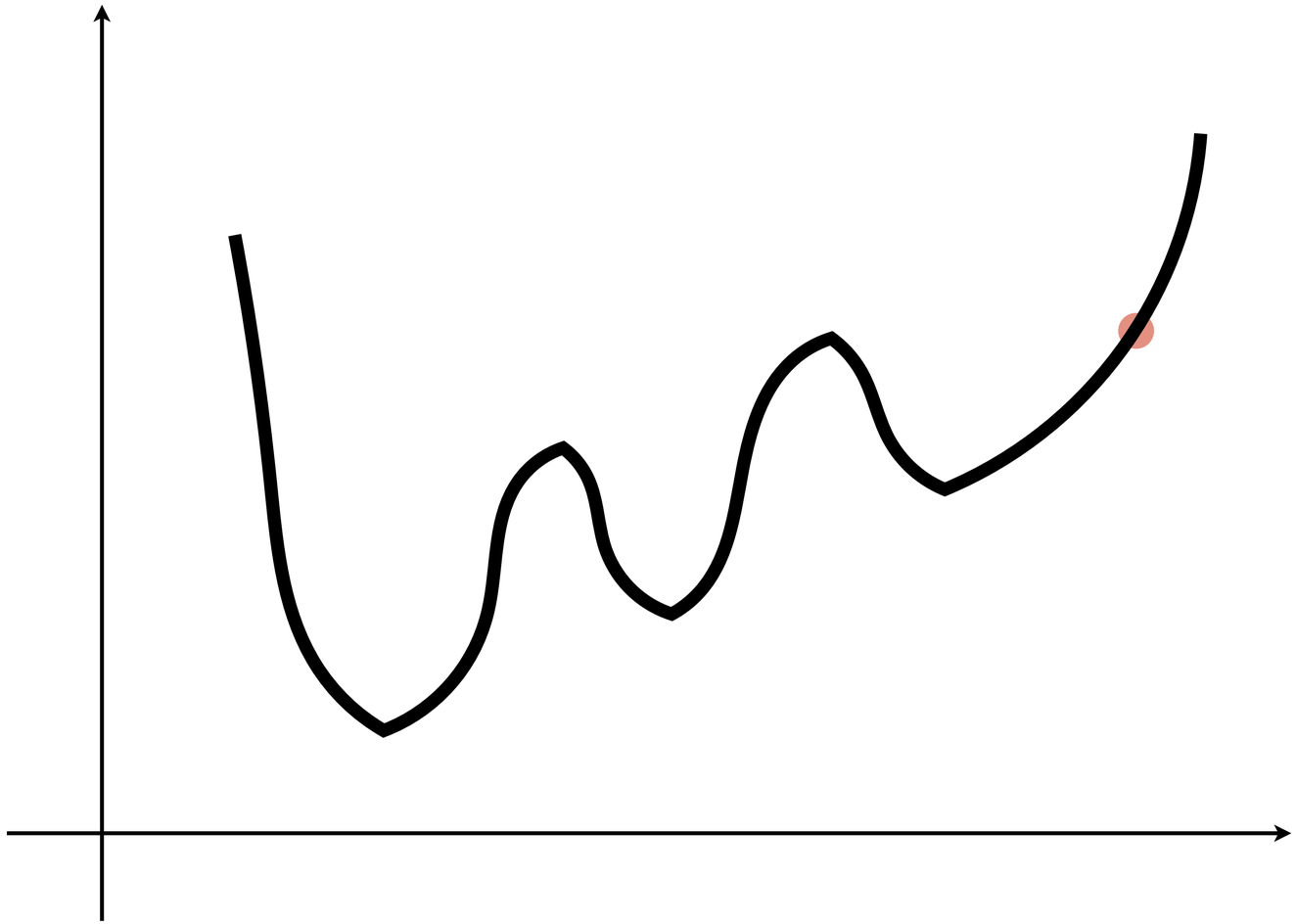
Gradient Descent



1. Start from random value of $\mathbf{W}_0, \mathbf{b}_0$

For $k = 0$ to max number of iterations

2. Compute gradient of the loss with respect to previous (initial) parameters:

$$\nabla \mathcal{L}(\mathbf{W}, \mathbf{b})|_{\mathbf{W}=\mathbf{W}_k, \mathbf{b}=\mathbf{b}_k}$$

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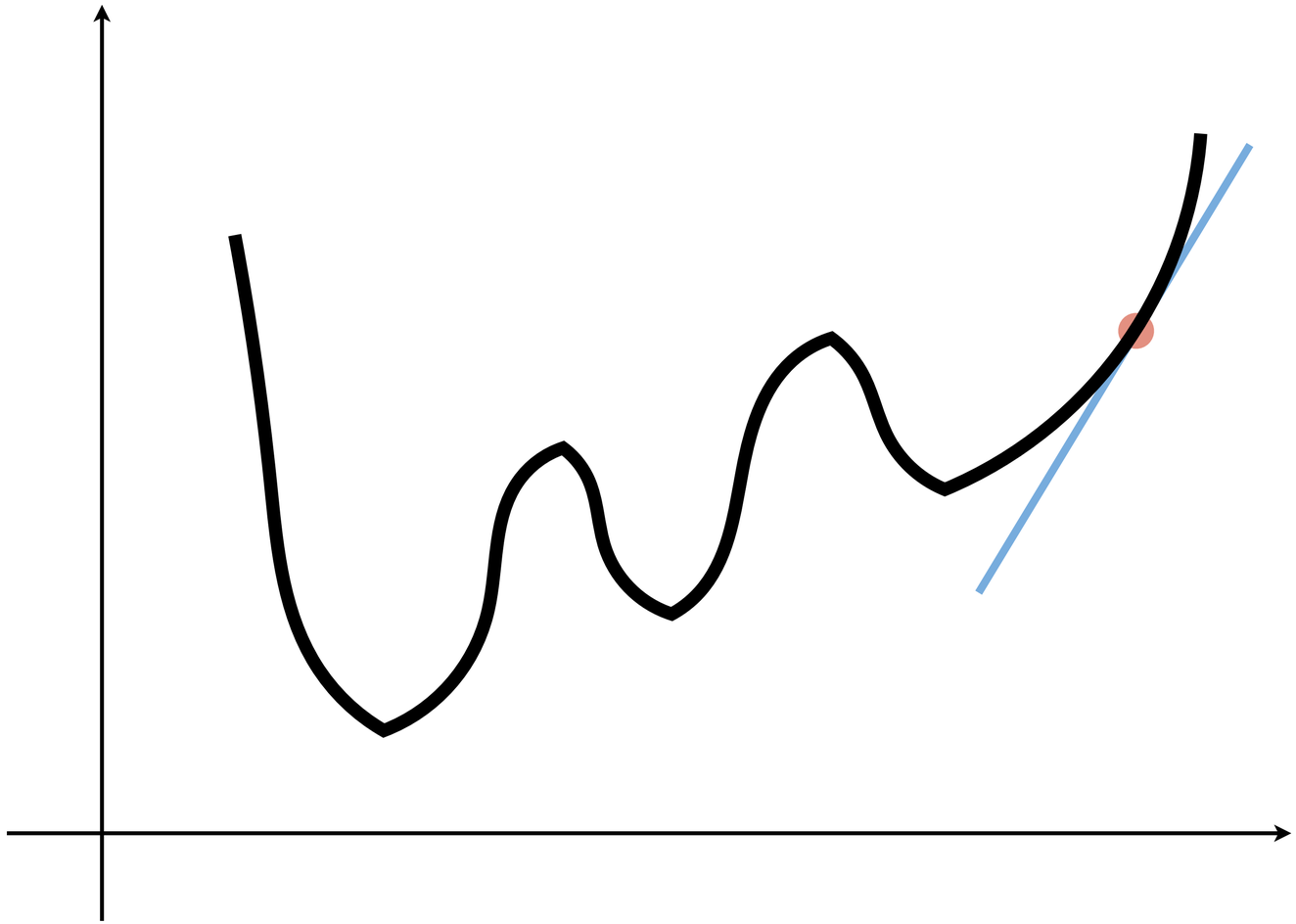

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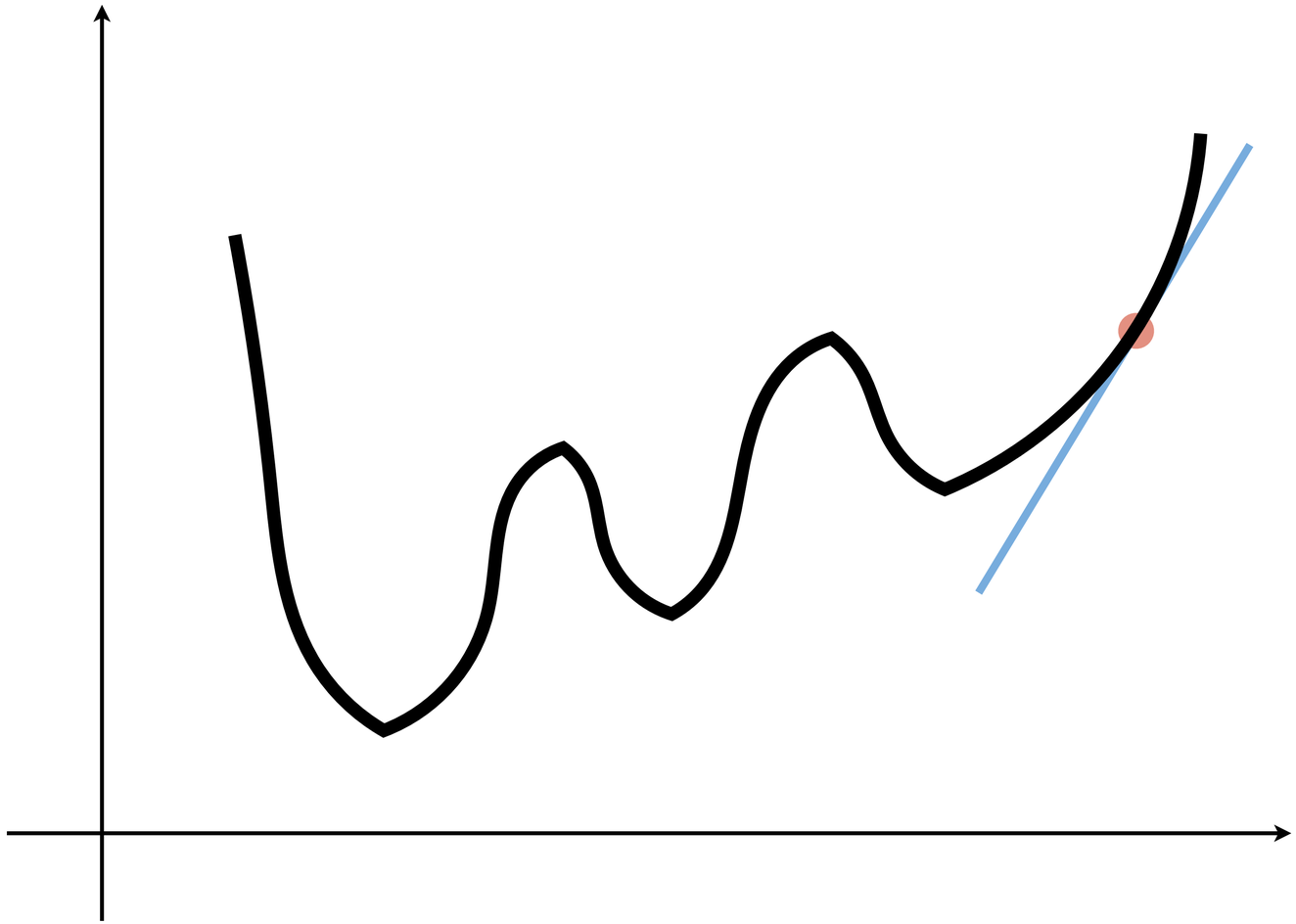

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3. Re-estimate the parameters

$$\mathbf{W}_{k+1} = \mathbf{W}_k - \lambda \left. \frac{\partial \mathcal{L}(\mathbf{W}, \mathbf{b})}{\partial \mathbf{W}} \right|_{\mathbf{W}=\mathbf{W}_k, \mathbf{b}=\mathbf{b}_k}$$

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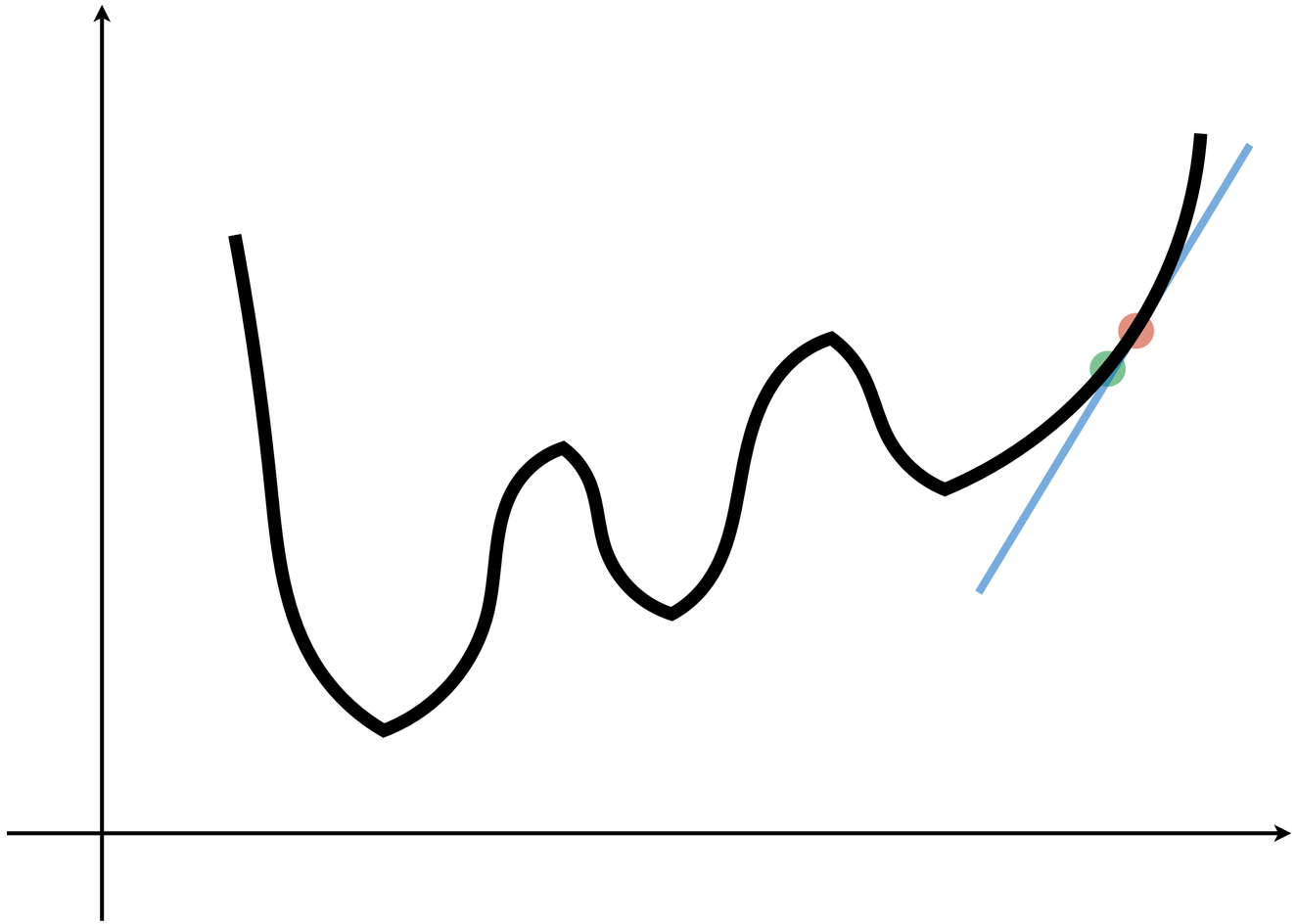

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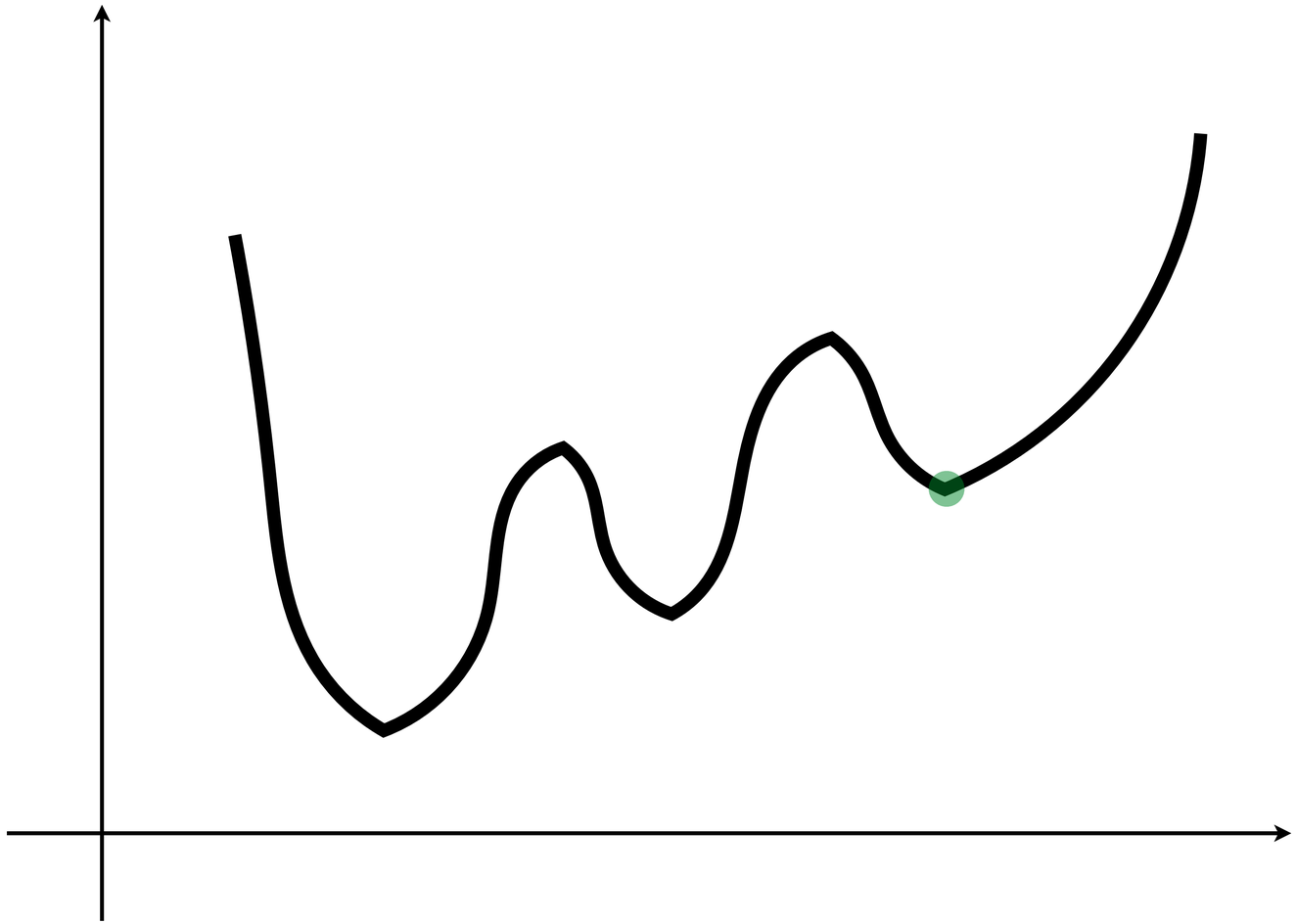

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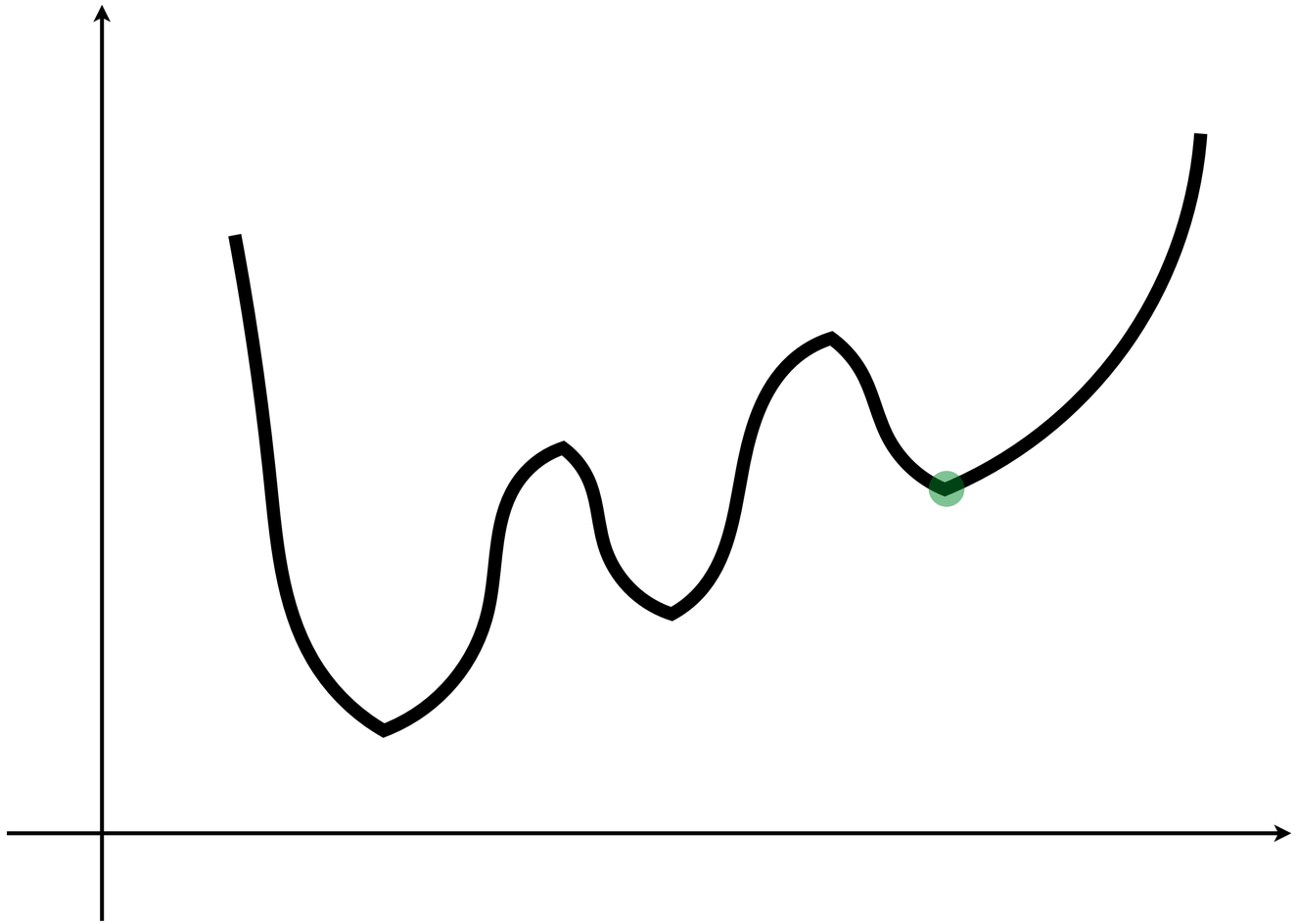

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λ - is the learning rate

*slide adopted from V. Ordonex

Gradient Descent

Loss:

$$\mathcal{L}(\mathbf{y}, \hat{\mathbf{y}}) = ||\mathbf{y} - \hat{\mathbf{y}}|| = ||\mathbf{y} - f(\mathbf{x}, \mathbf{W}_1, \mathbf{W}_2, \mathbf{b}_1, \mathbf{b}_2)||$$

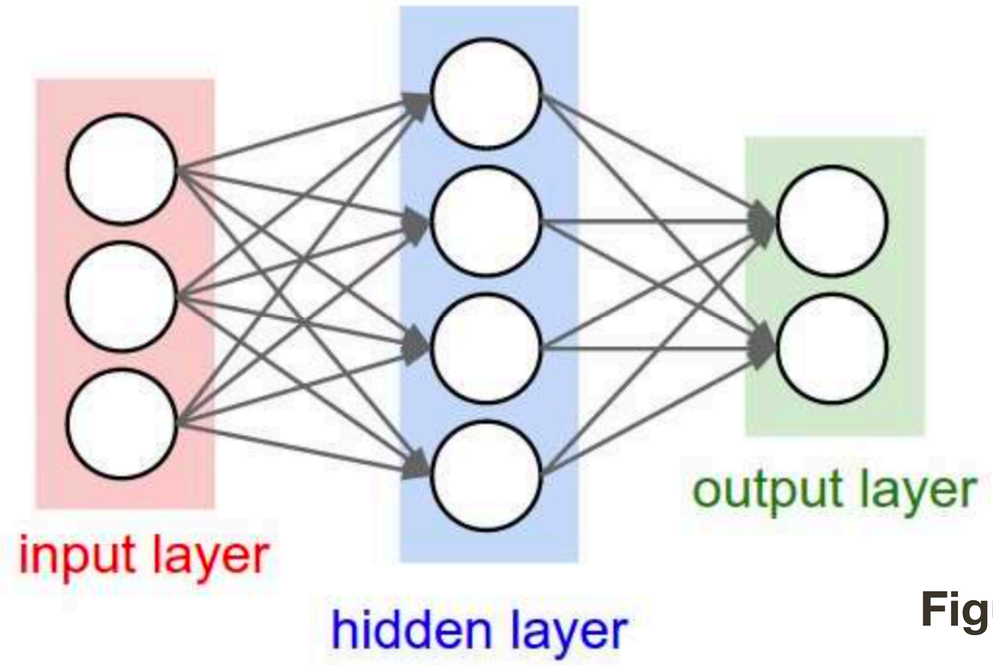

Figure credit: Fei-Fei and Karpathy

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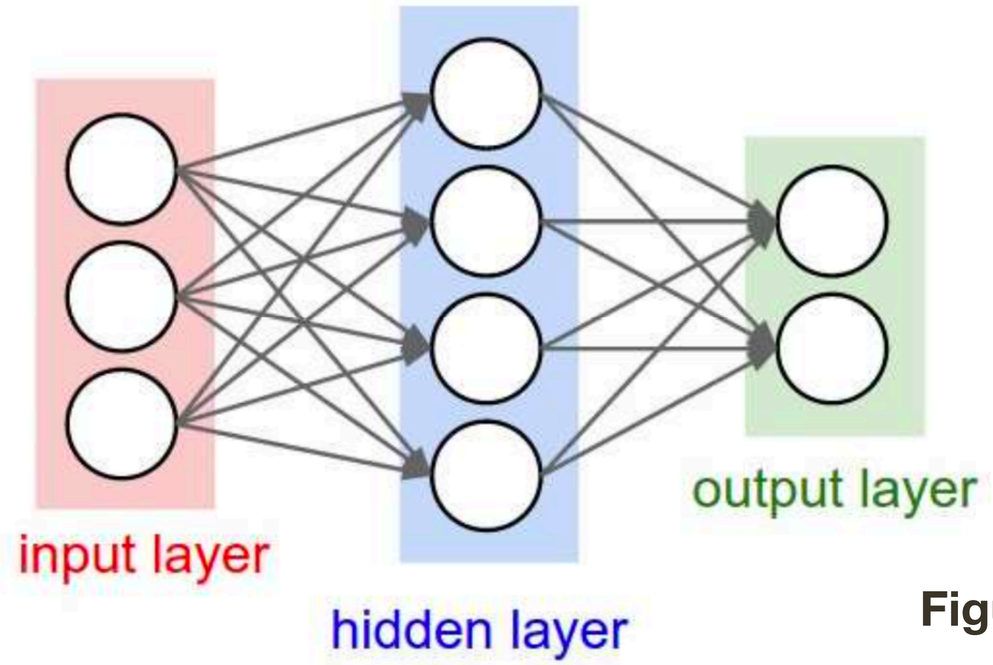
Gradient Descent

$$\mathbf{W}_{1,i,j} = \mathbf{W}_{1,i,j} - \lambda \frac{\partial \mathcal{L}(\mathbf{y}, \hat{\mathbf{y}})}{\partial \mathbf{W}_{1,i,j}}$$

$$\mathbf{b}_{1,i} = \mathbf{b}_{1,i} - \lambda \frac{\partial \mathcal{L}(\mathbf{y}, \hat{\mathbf{y}})}{\partial \mathbf{b}_{1,i}}$$



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Stochastic Gradient Descent

$$\frac{\partial \mathcal{L}(\mathbf{W}, \mathbf{b})}{\partial w_{ji}} = \frac{\partial}{\partial w_{ji}} \sum_{d=1}^{|D_{train}|} \left(\mathbf{sigmoid}\left(\mathbf{W}^T \mathbf{x}^{(d)} + \mathbf{b} \right) - \mathbf{y}^{(d)} \right)^2$$

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Problem: For large datasets computing sum is expensive

Stochastic Gradient Descent

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Solution: Compute approximate gradient with mini-batches of much smaller size (as little as 1-example sometimes)

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Problem: For large datasets computing sum is expensive

Solution: Compute approximate gradient with mini-batches of much smaller size (as little as 1-example sometimes)

Problem: How do we compute the actual gradient?

Numerical Differentiation

$\mathbf{1}_i$ - Vector of all zeros, except for one 1 in i-th location

We can approximate the gradient numerically, using:

$$\frac{\partial f(\mathbf{x})}{\partial x_i} \approx \lim_{h \to 0} \frac{f(\mathbf{x} + h\mathbf{1}_i) - f(\mathbf{x})}{h}$$

*slide adopted from T. Chen, H. Shen, A. Krishnamurthy CSE 599G1 lecture at UWashington

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However, both of theses suffer from rounding errors and are not good enough for learning (they are very good tools for checking the correctness of implementation though, e.g., use $h = 0.000001$).

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Numerical Differentiation

$\mathbf{1}_i$ - Vector of all zeros, except for one 1 in i-th location

$\mathbf{1}_{ij}$ - Matrix of all zeros, except for one 1 in (i,j)-th location

We can approximate the gradient numerically, using:

$$\frac{\partial \mathcal{L}(\mathbf{W}, \mathbf{b})}{\partial w_{ij}} \approx \lim_{h \to 0} \frac{\mathcal{L}(\mathbf{W} + h\mathbf{1}_{ij}, \mathbf{b}) - \mathcal{L}(\mathbf{W}, \mathbf{b})}{h} \qquad \frac{\partial \mathcal{L}(\mathbf{W}, \mathbf{b})}{\partial b_j} \approx \lim_{h \to 0} \frac{\mathcal{L}(\mathbf{W}, \mathbf{b} + h\mathbf{1}_j) - \mathcal{L}(\mathbf{W}, \mathbf{b})}{h}$$

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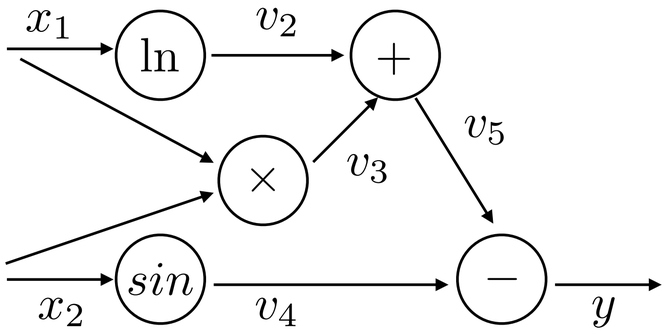
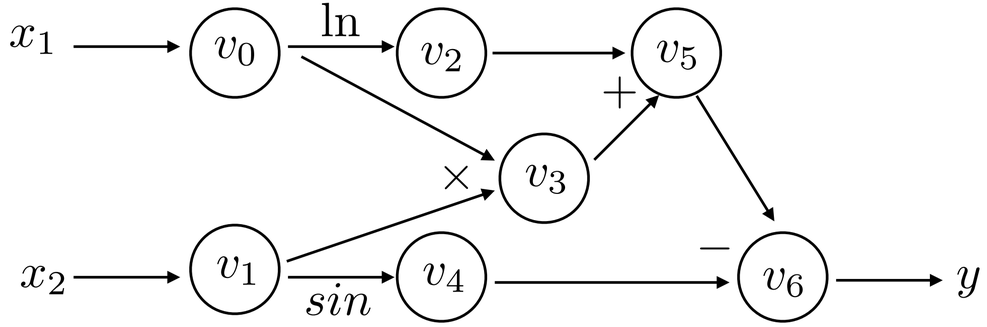
Symbolic Differentiation

$$y = f(x_1, x_2) = \ln(x_1) + x_1 x_2 - sin(x_2)$$

Input function is represented as **computational graph** (a symbolic tree)





Implements differentiation rules for composite functions:

Sum Rule

$$\frac{\mathrm{d}\left(f(x) + g(x)\right)}{\mathrm{d}x} = \frac{\mathrm{d}f(x)}{\mathrm{d}x} + \frac{\mathrm{d}g(x)}{\mathrm{d}x}$$

Product Rule

$$\frac{\mathrm{d}\left(f(x) \cdot g(x)\right)}{\mathrm{d}x} = \frac{\mathrm{d}f(x)}{\mathrm{d}x} g(x) + f(x) \frac{\mathrm{d}g(x)}{\mathrm{d}x}$$

Chain Rule

$$\frac{\mathrm{d}(f(g(x)))}{\mathrm{d}x} = \frac{\mathrm{d}f(g(x))}{\mathrm{d}x} \cdot \frac{\mathrm{d}g(x)}{\mathrm{d}x}$$

*slide adopted from T. Chen, H. Shen, A. Krishnamurthy CSE 599G1 lecture at UWashington

Symbolic Differentiation

$$y = f(x_1, x_2) = \ln(x_1) + x_1 x_2 - sin(x_2)$$

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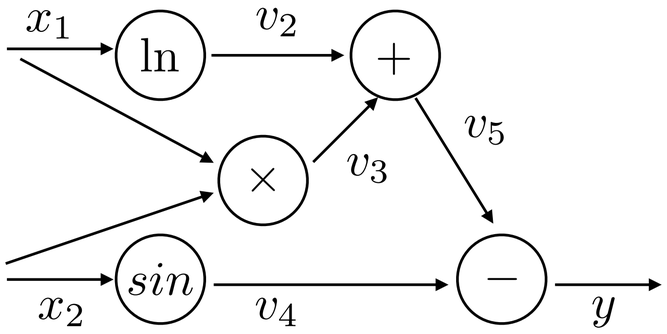

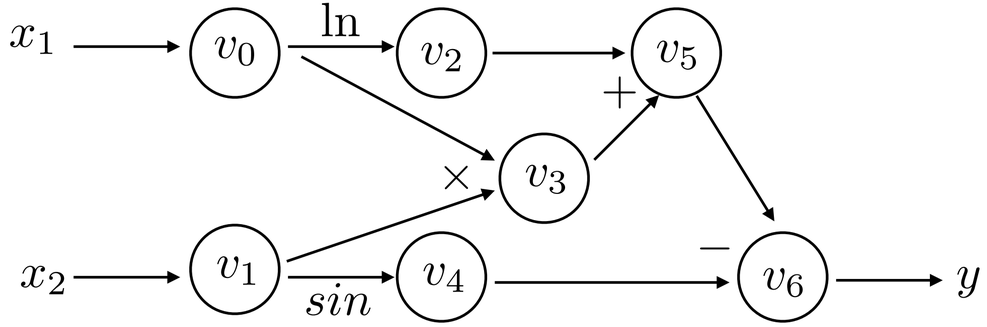

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Problem: For complex functions, expressions can be exponentially large; also difficult to deal with piece-wise functions (creates many symbolic cases)

*slide adopted from T. Chen, H. Shen, A. Krishnamurthy CSE 599G1 lecture at UWashington

Automatic Differentiation (AutoDiff)

$$y = f(x_1, x_2) = \ln(x_1) + x_1 x_2 - sin(x_2)$$

Intuition: Interleave symbolic differentiation and simplification

Key Idea: apply symbolic differentiation at the elementary operation level, evaluate and keep intermediate results

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Success of **deep learning** owes A LOT to success of AutoDiff algorithms
(also to advances in parallel architectures, and large datasets, …)

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Backpropagation

The parameters of a neural network are learned using **backpropagation**, which computes gradients via recursive application of the **chain rule** from calculus

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Suppose $f(x, y) = xy$. What is the partial derivative of f with respect to x? What is the partial derivative of f with respect to y?

$$\frac{\partial f}{\partial x} = y \qquad \frac{\partial f}{\partial y} = x$$

Backpropagation

Suppose $f(x, y) = x + y$. What is the partial derivative of f with respect to x? What is the partial derivative of f with respect to y?

Backpropagation

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$$\frac{\partial f}{\partial x} = 1 \qquad \frac{\partial f}{\partial y} = 1$$

Backpropagation

A trickier example: $f(x, y) = \max(x, y)$

Backpropagation

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$$\frac{\partial f}{\partial x} = \mathbf{1}(x \geq y) \qquad \frac{\partial f}{\partial y} = \mathbf{1}(y \geq x)$$

That is, the (sub)gradient is 1 on the input that is larger, and 0 on the other input

— For example, say x = 4, y = 2. Increasing y by a tiny amount does not change the value of f (f will still be 4), hence the gradient on y is zero.

Backpropagation

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By the chain rule

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q}\frac{\partial q}{\partial x} = z \cdot 1 = z$$

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Backpropagation

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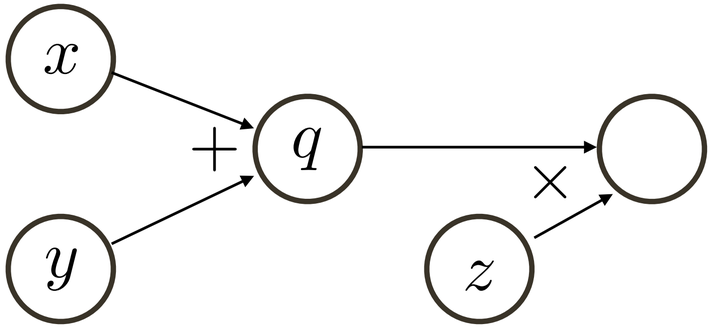

Computational graph (a DAG) with variable ordering from topological sort, where each **node** is an input, intermediate, or output variable

Backpropagation

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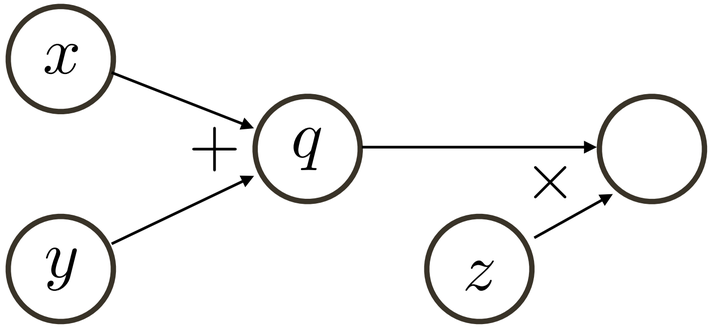

Computational graph (a DAG) with variable ordering from topological sort, where each **node** is an input, intermediate, or output variable

Suppose the network input is: $(x, y, z) = (-2, 5, -4)$

Then: $q = x + y = 3$ $f = qz = -12$ (**forward** pass)

Backpropagation

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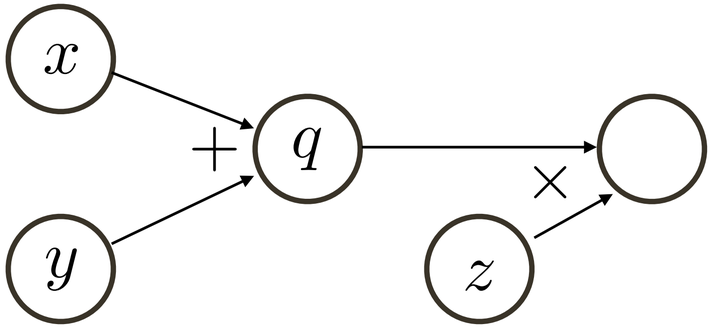

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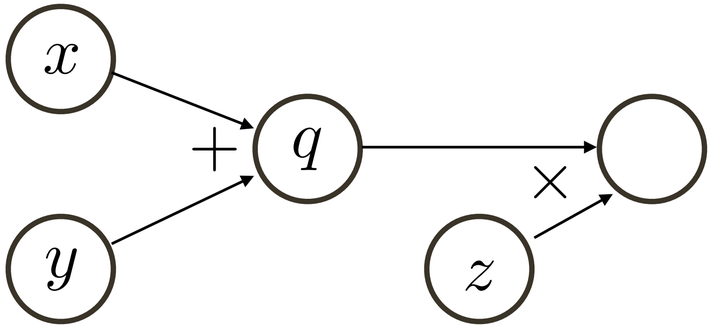

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q}\frac{\partial q}{\partial x} = \frac{\partial f}{\partial q} \cdot 1$$

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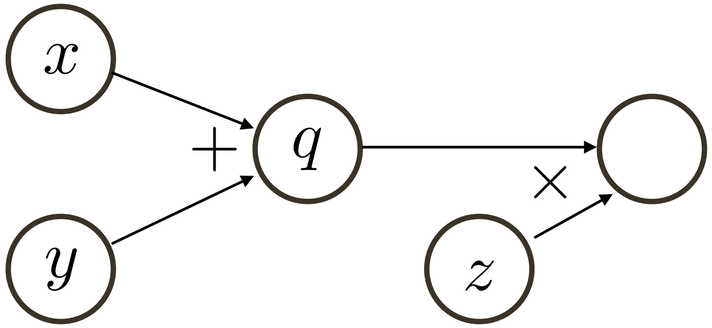

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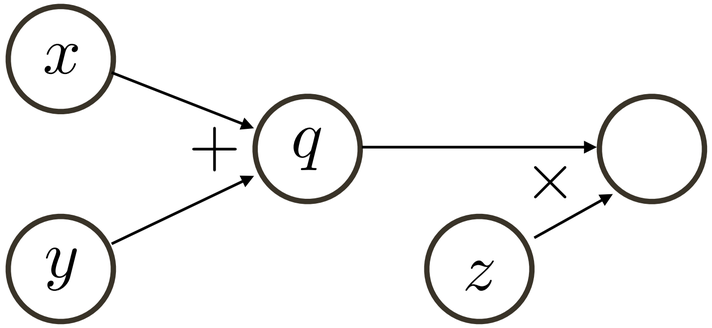

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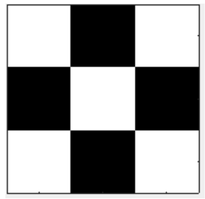
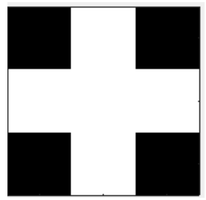
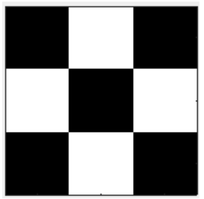
Example: Let's Build (world smallest) Neural Network

Lets create a neural network that will be able to differentiate (classify) these patterns of simple 3x3 pixel images







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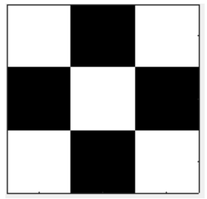
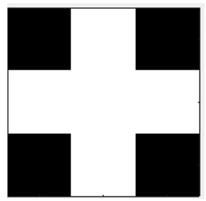
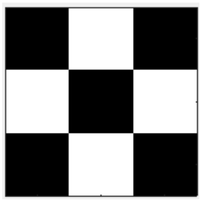
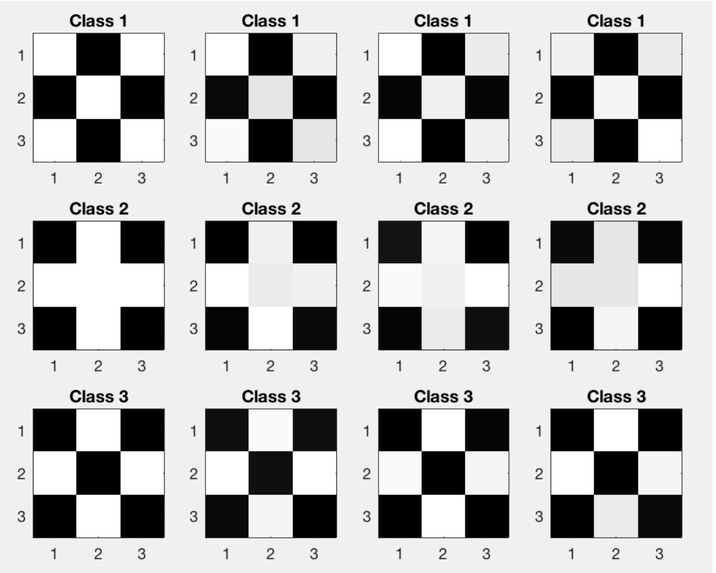






We will need some labeled data



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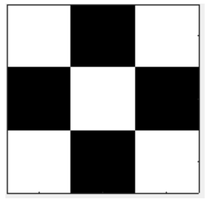

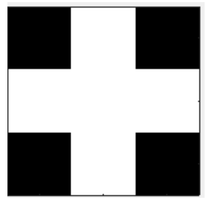

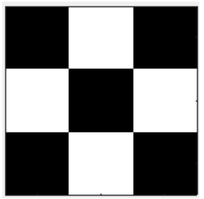

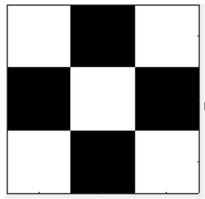

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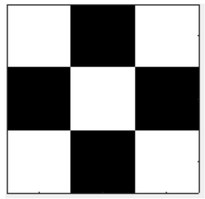

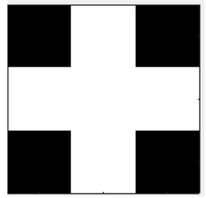

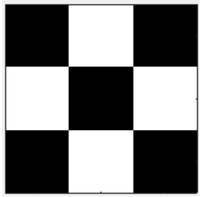

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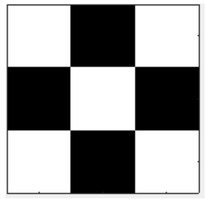

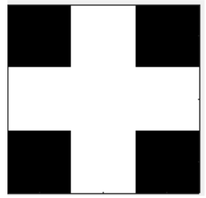

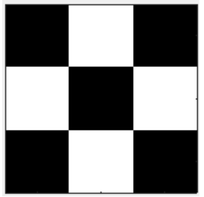

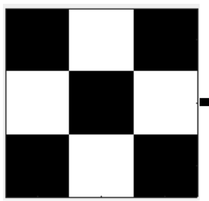

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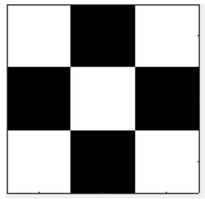
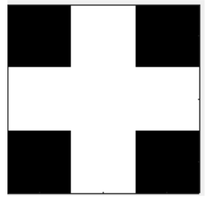
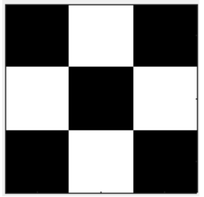
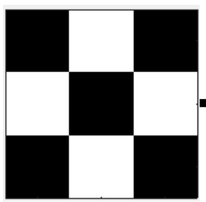




What do we need to do?



First, lets re-formulate the problem

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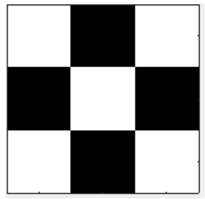
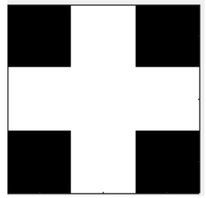
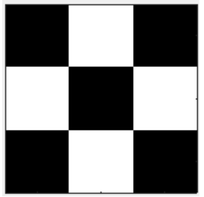

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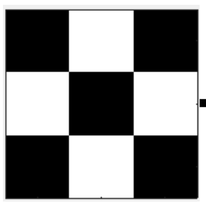

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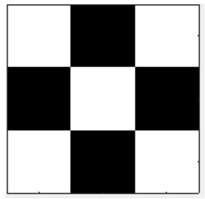
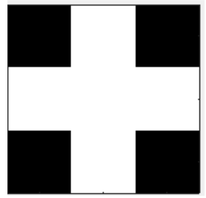
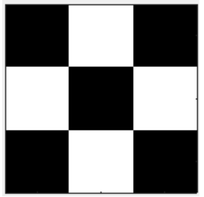
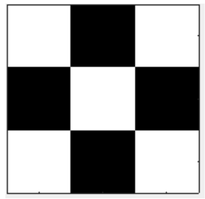




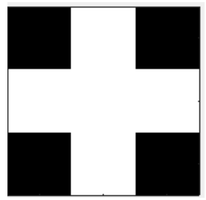
Now, lets build a **network**!

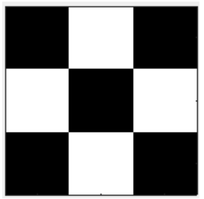
How many inputs should the network have? How neuron outputs?

Example: Let's Build (world smallest) Neural Network

Lets create a neural network that will be able to differentiate (classify) these patterns of simple 3x3 pixel images

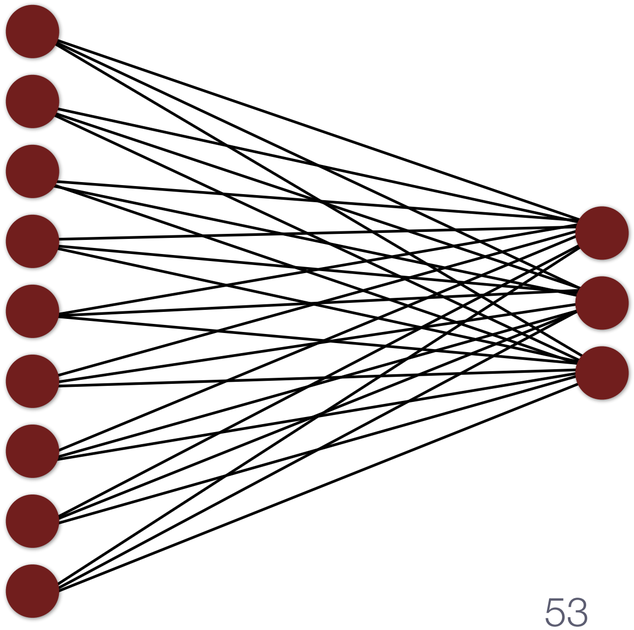






Input Layer

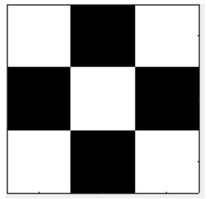
Output Layer



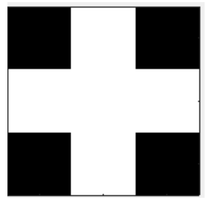
What else is missing for us to train it?

Example: Let's Build (world smallest) Neural Network

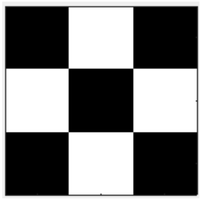
Lets create a neural network that will be able to differentiate (classify) these patterns of simple 3x3 pixel images



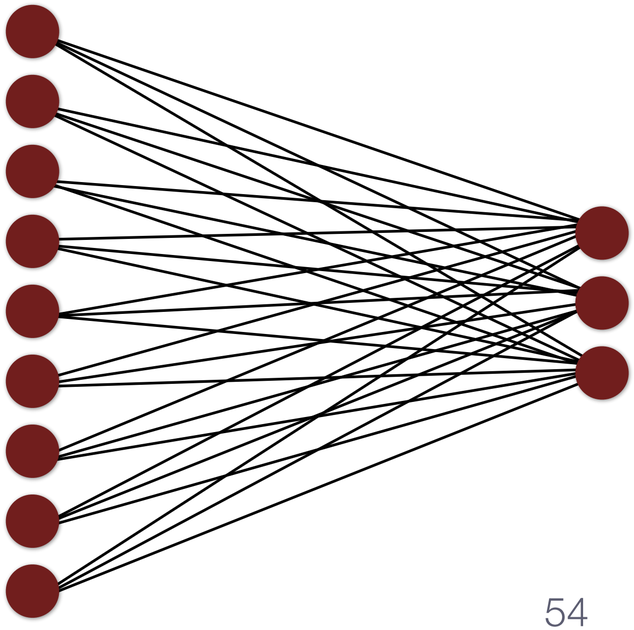
Input Layer



Output Layer



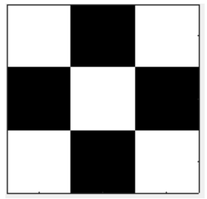
Loss

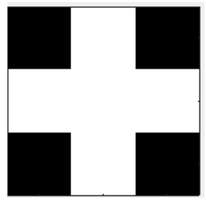


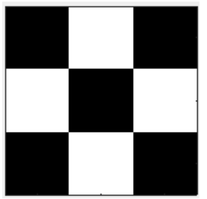
$$L_i = -\log\left(\frac{e^{f_{y_i}}}{\sum_j e^{f_{y_j}}}\right)$$

Example: Let's Build (world smallest) Neural Network

Lets create a neural network that will be able to differentiate (classify) these patterns of simple 3x3 pixel images



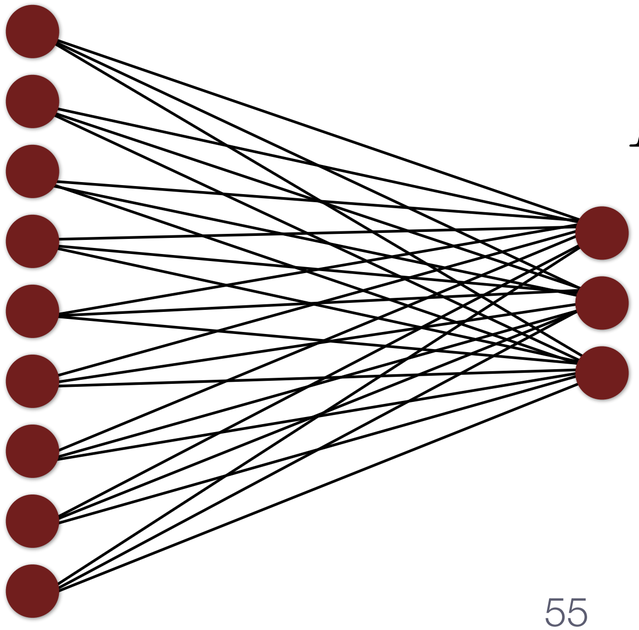




Input Layer

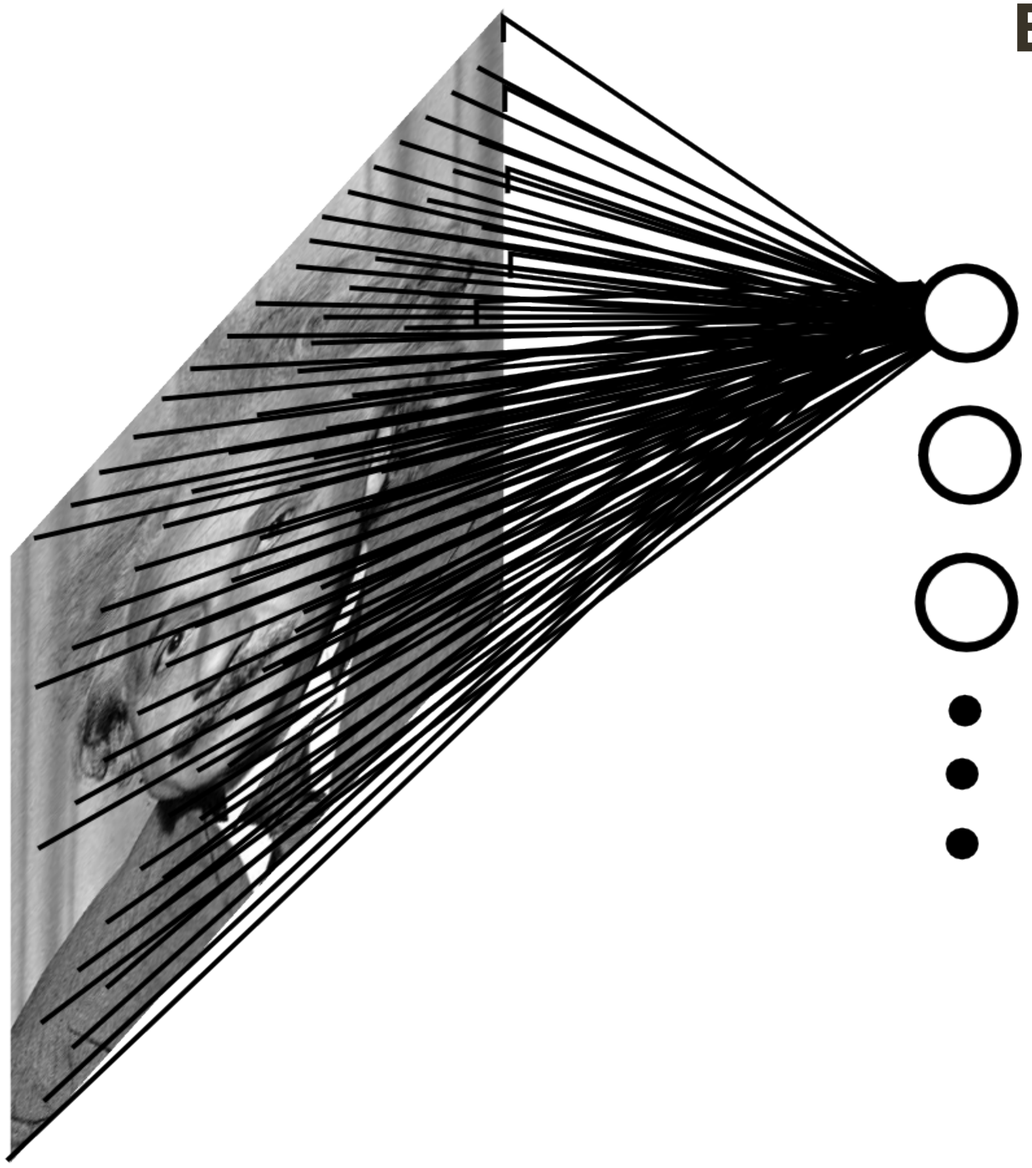
Output Layer

Loss



$$L_1 = -log\left(\frac{e^{\sum_{i=1}^{9}\sigma(w_{1,i}x_i+b_1)}}{\sum_{j=1}^{3}e^{\sum_{i=1}^{9}\sigma(w_{1,i}x_i+b_1)}}\right)$$

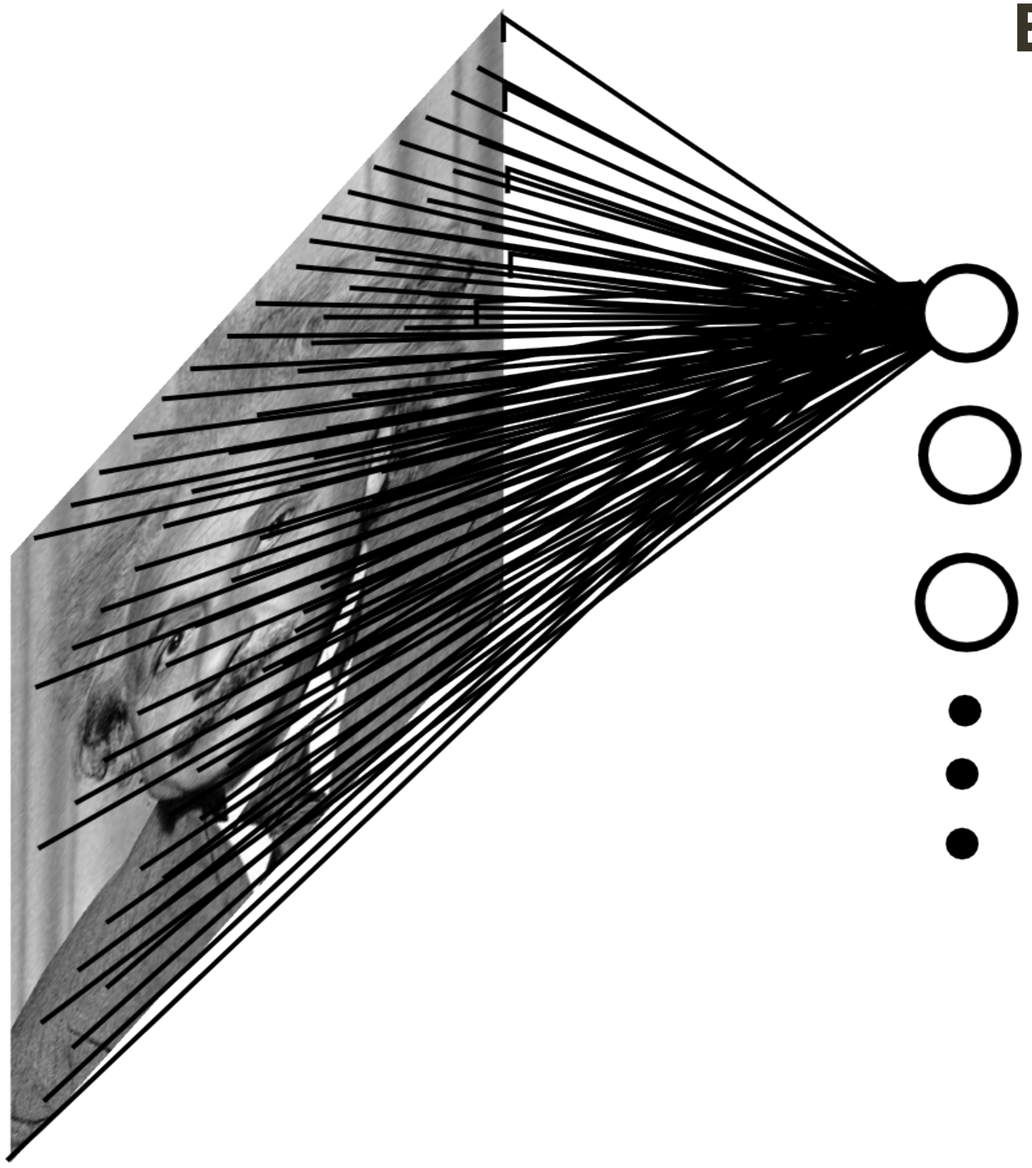
Fully Connected Layer



Example: 200 x 200 image (small)
x 40K hidden units

* slide from Marc'Aurelio Renzato

Fully Connected Layer

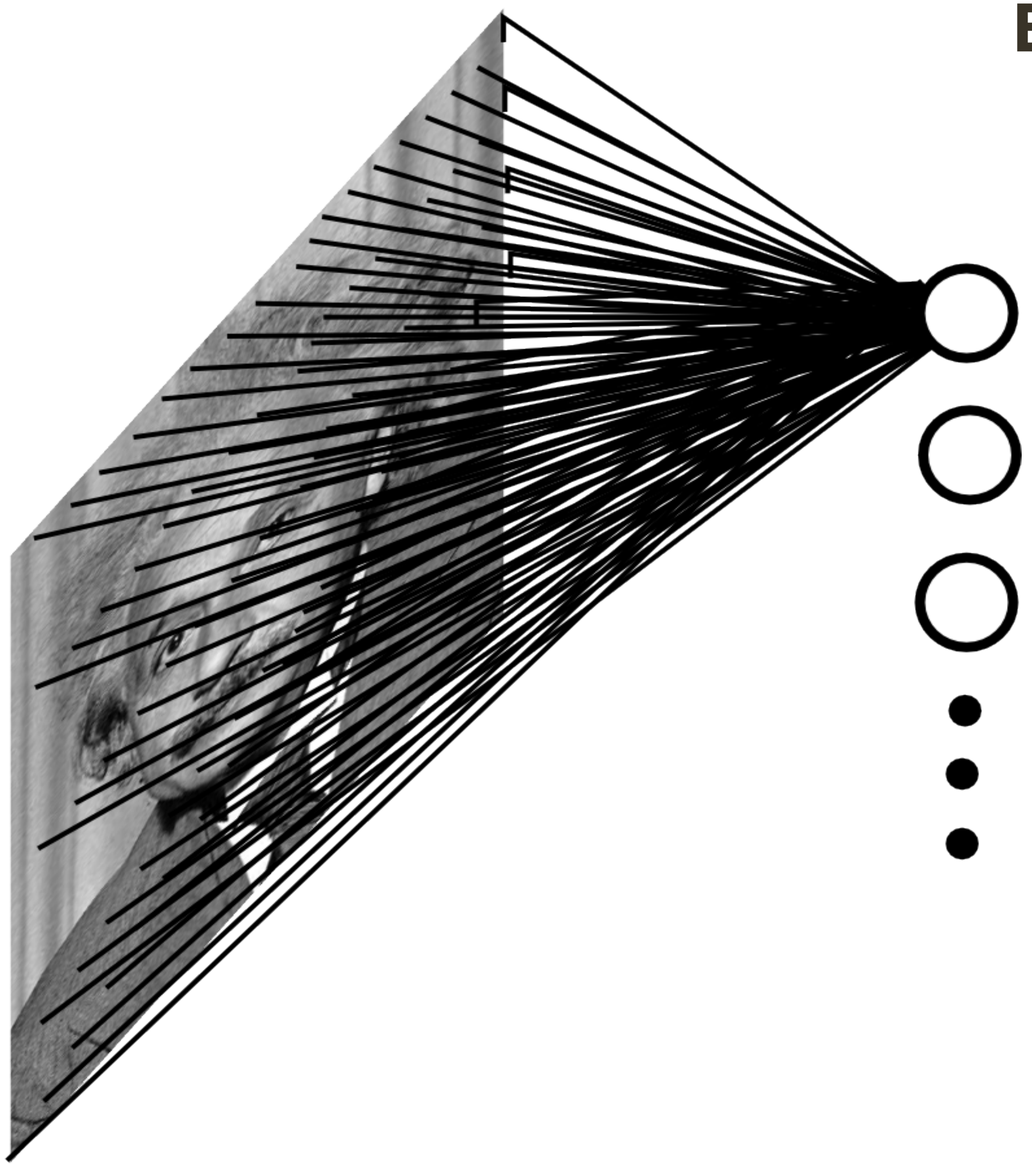


Example: 200 x 200 image (small)
x 40K hidden units

= ~ **2 Billion** parameters (for one layer!)

* slide from Marc'Aurelio Renzato

Fully Connected Layer



Example: 200 x 200 image (small)
x 40K hidden units

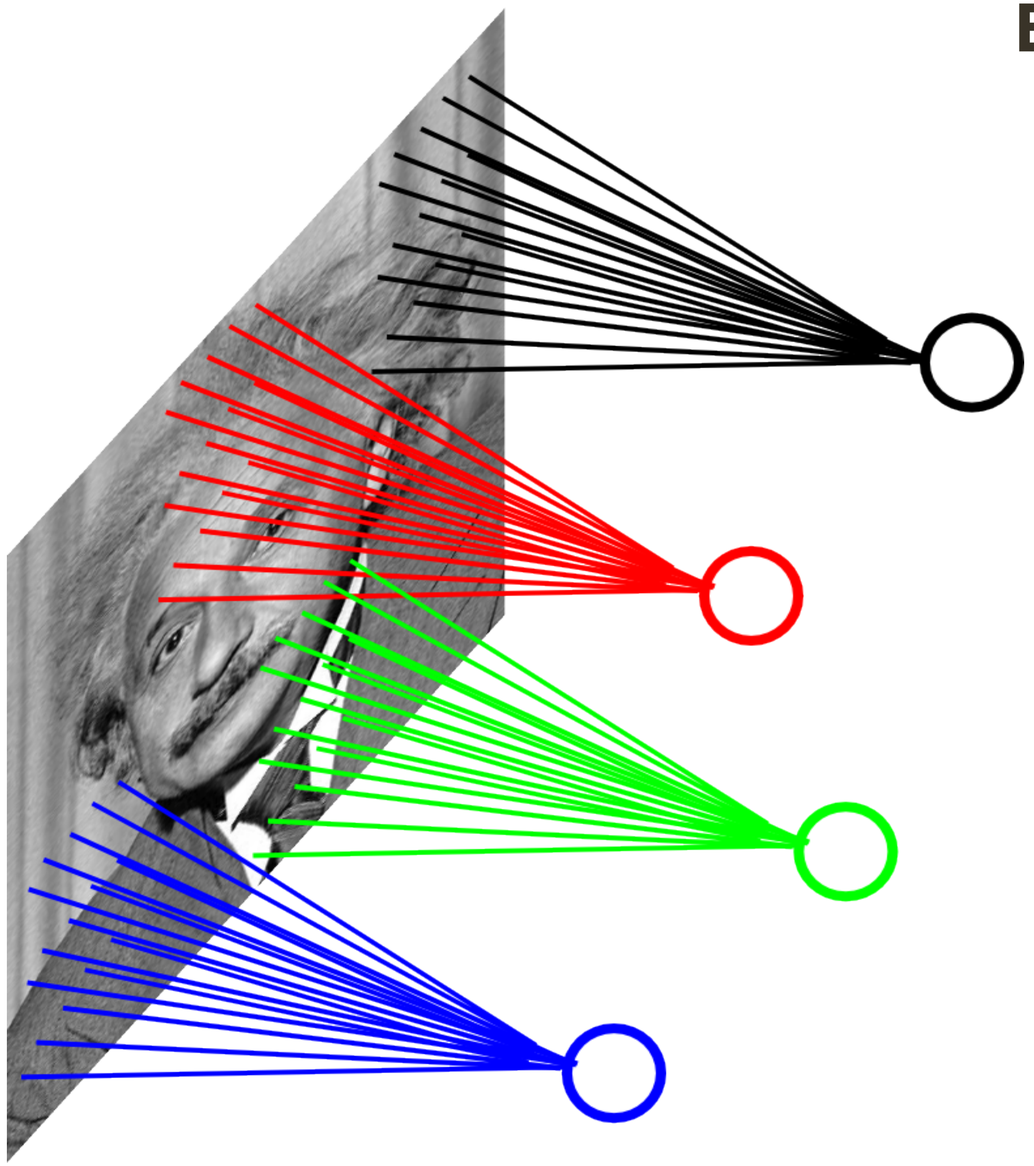
= ~ **2 Billion** parameters (for one layer!)

Spatial correlations are generally local

Waste of resources + we don't have enough data to train networks this large

* slide from Marc'Aurelio Renzato

Locally Connected Layer



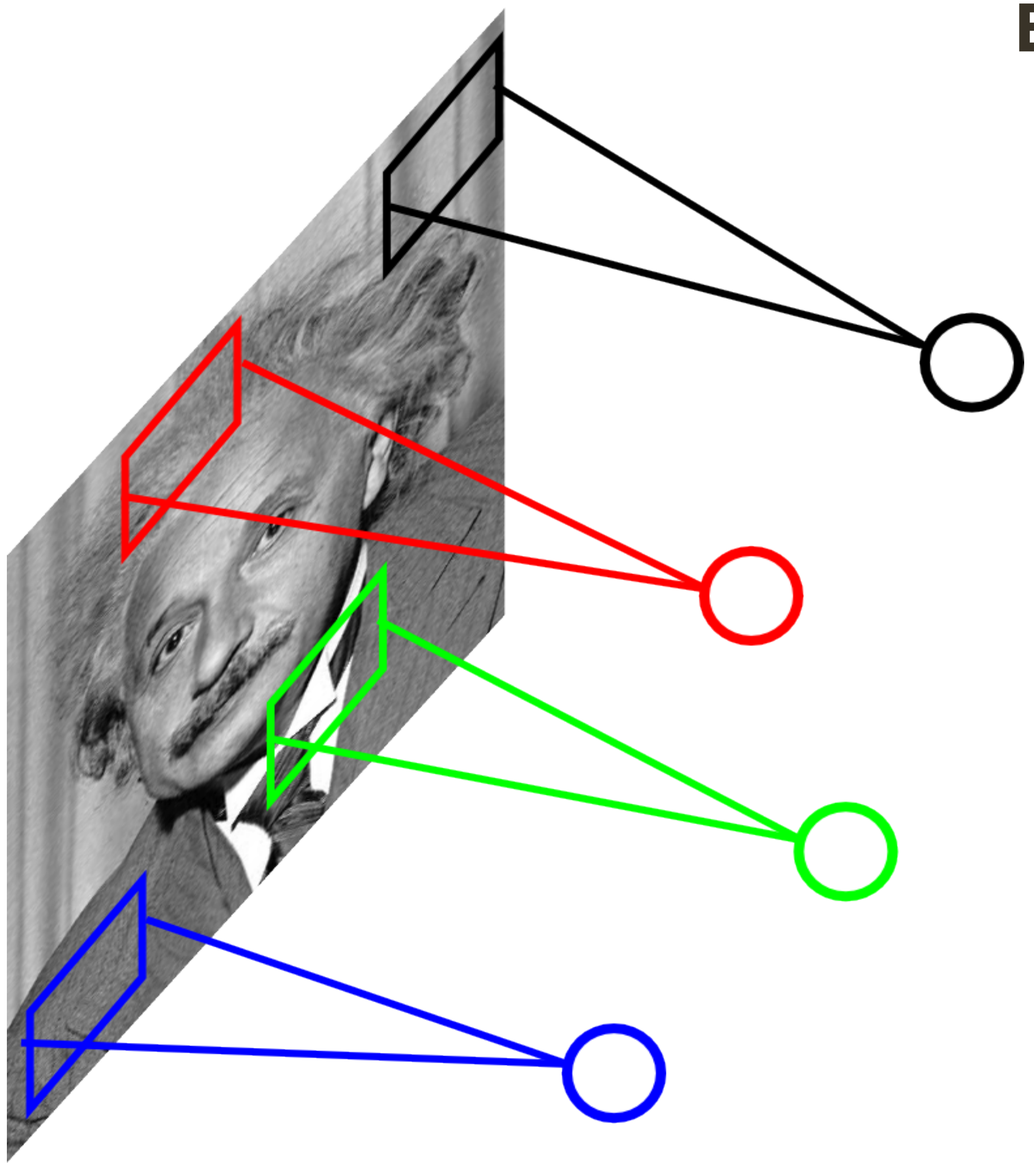
Example: 200 x 200 image (small)
x 40K hidden units

Filter size: 10 x 10

= ~ **4 Million** parameters

* slide from Marc'Aurelio Renzato

Locally Connected Layer



Example: 200 x 200 image (small)
x 40K hidden units

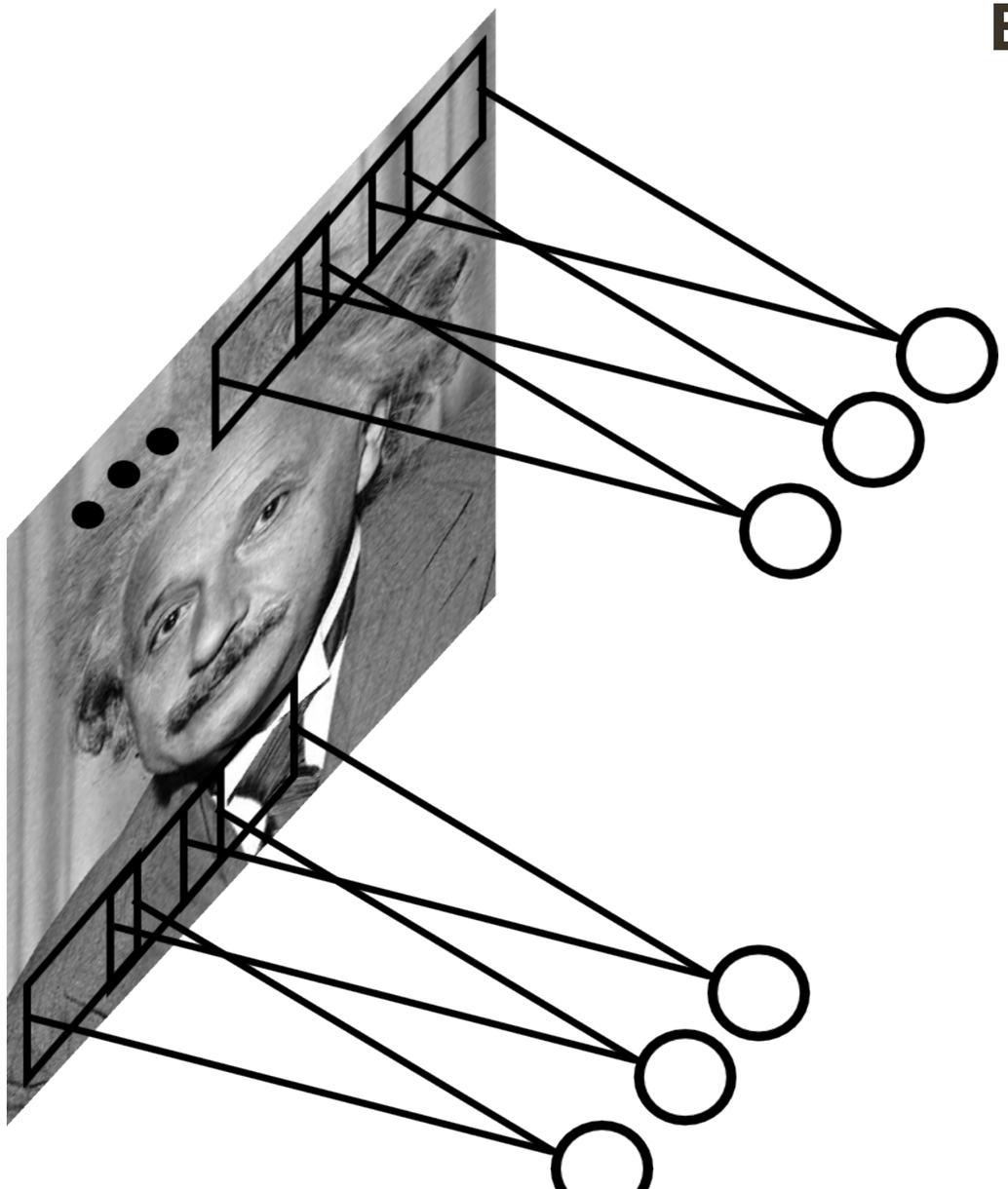
Filter size: 10 x 10

= ~ **4 Million** parameters

Stationarity — statistics is similar at different locations

* slide from Marc'Aurelio Renzato

Convolutional Layer



Example: 200 x 200 image (small)
x 40K hidden units

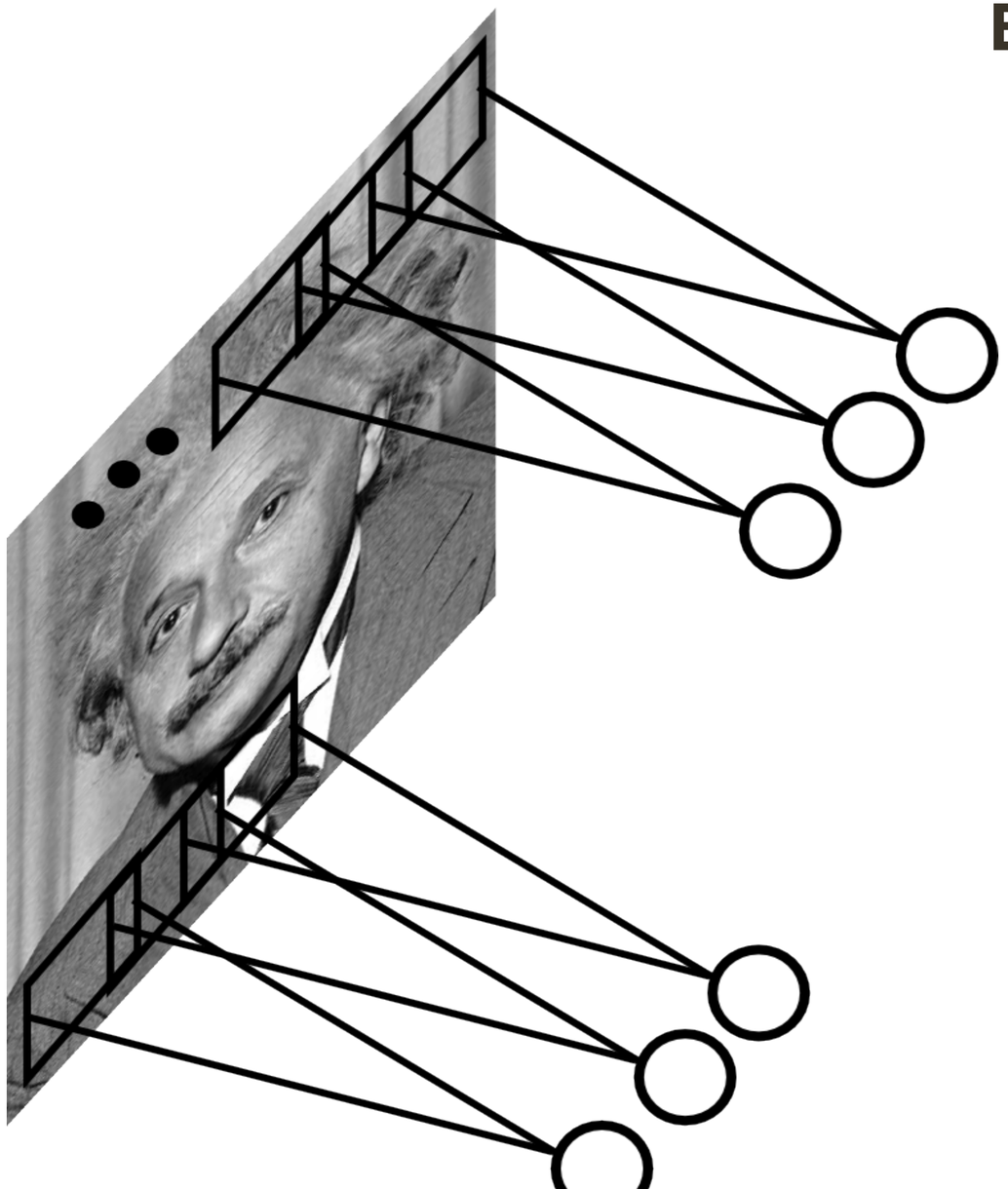
Filter size: 10 x 10

= ~ **4 Million** parameters

Share the same parameters across the locations (assuming input is stationary)

* slide adopted from Marc'Aurelio Renzato

Convolutional Layer



Example: 200 x 200 image (small)
x 40K hidden units

Filter size: 10 x 10

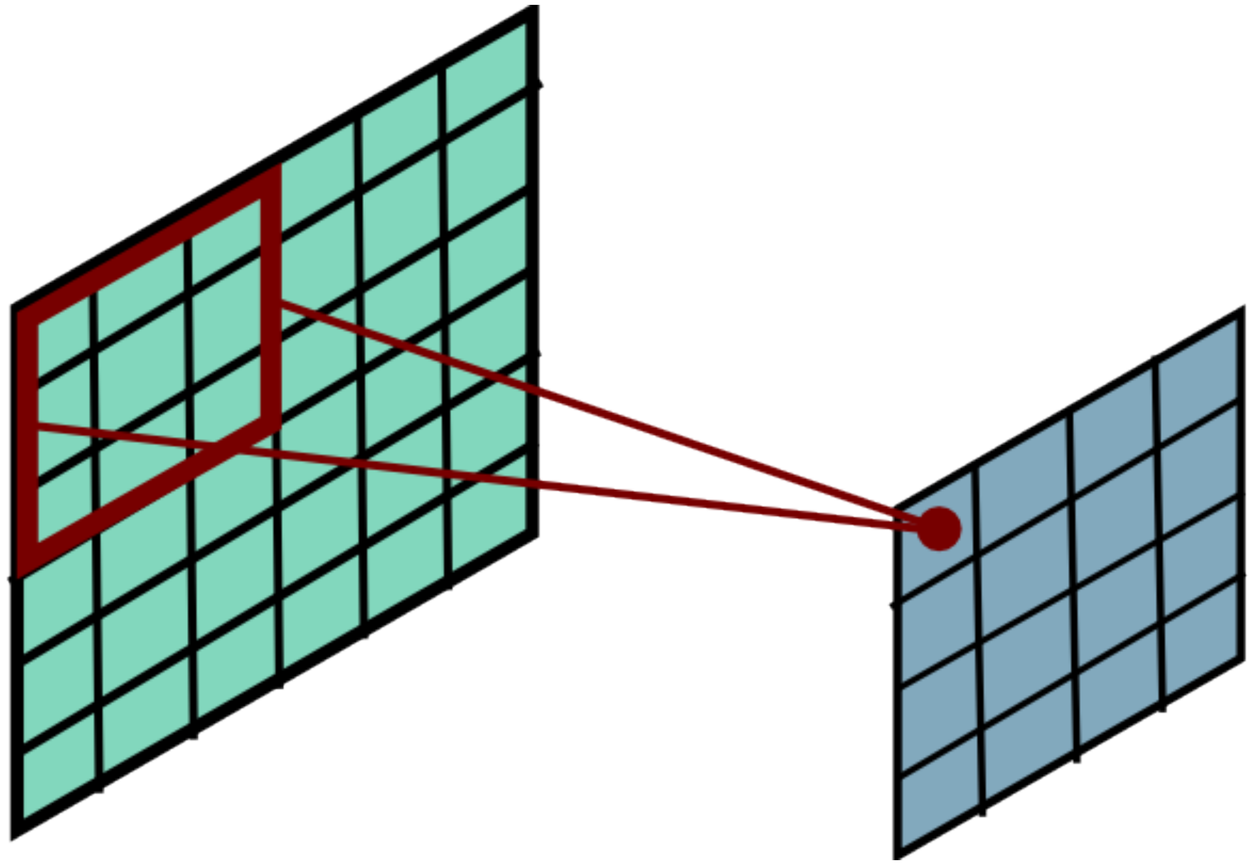
= ~ **4 Million** parameters

= 100 parameters

Share the same parameters across the locations (assuming input is stationary)

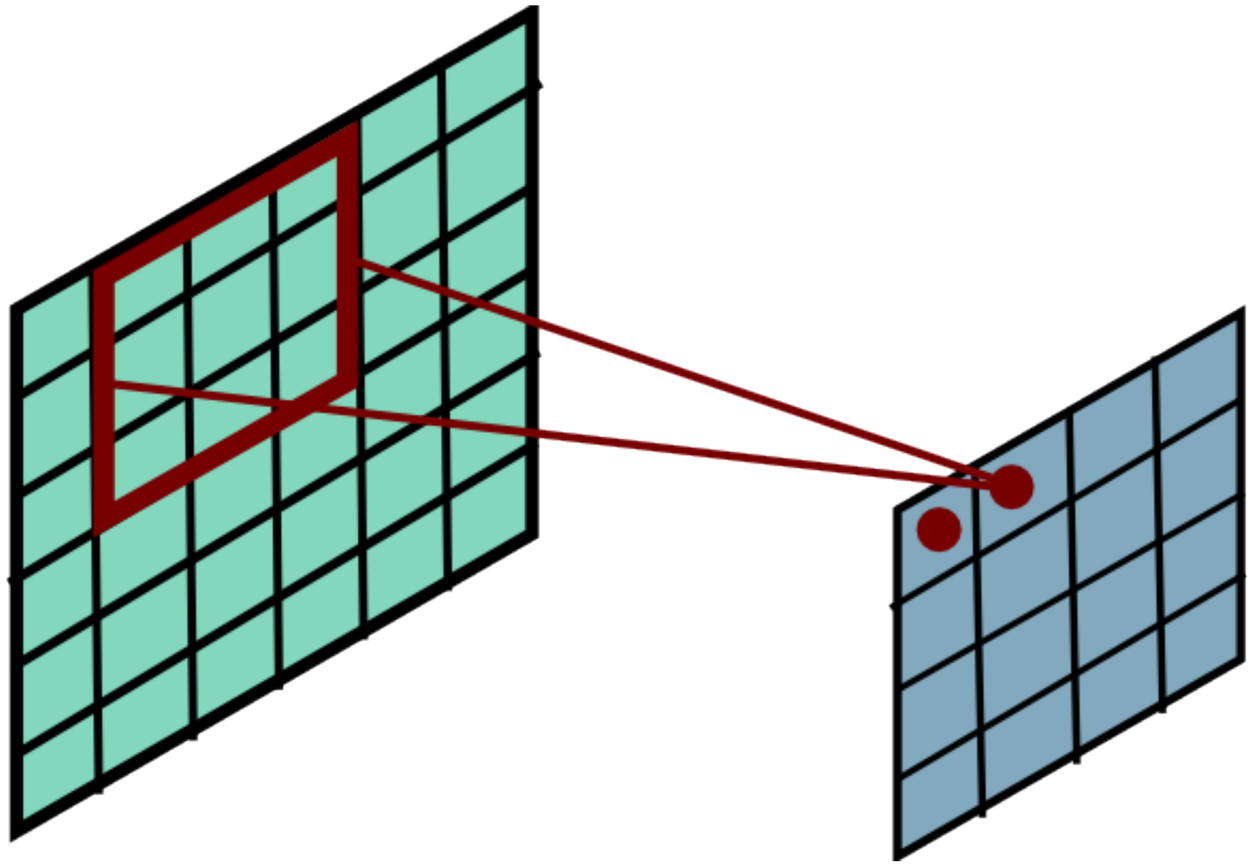
* slide adopted from Marc'Aurelio Renzato

Convolutional Layer



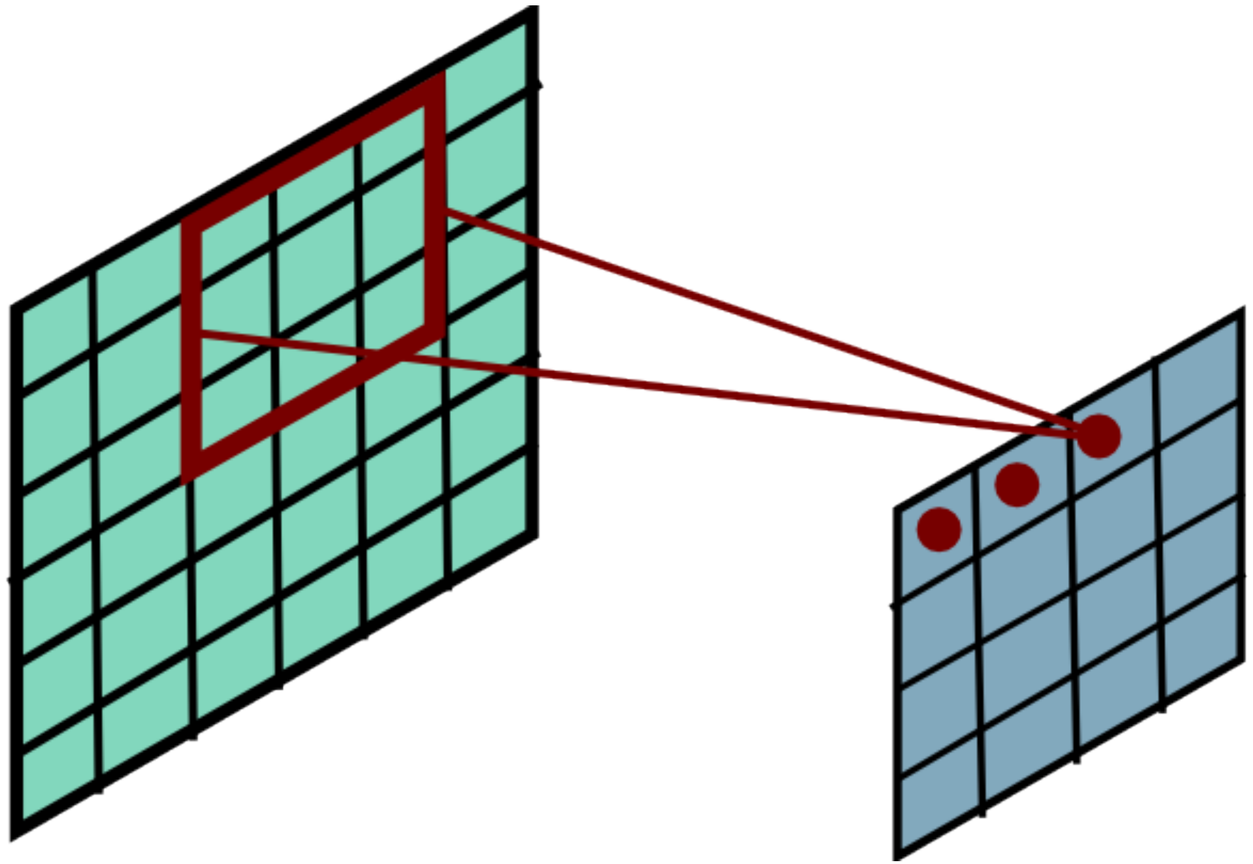
* slide from Marc'Aurelio Renzato

Convolutional Layer



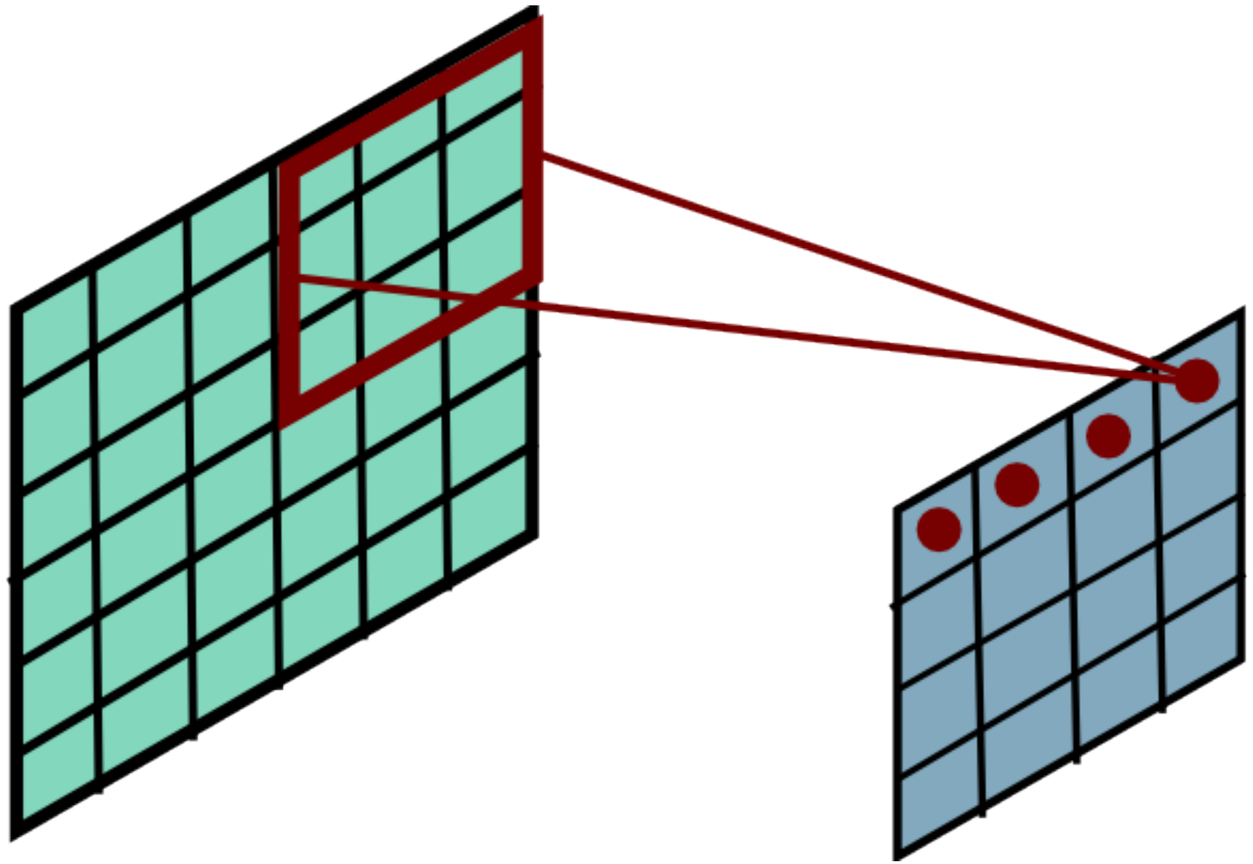
* slide from Marc'Aurelio Renzato

Convolutional Layer



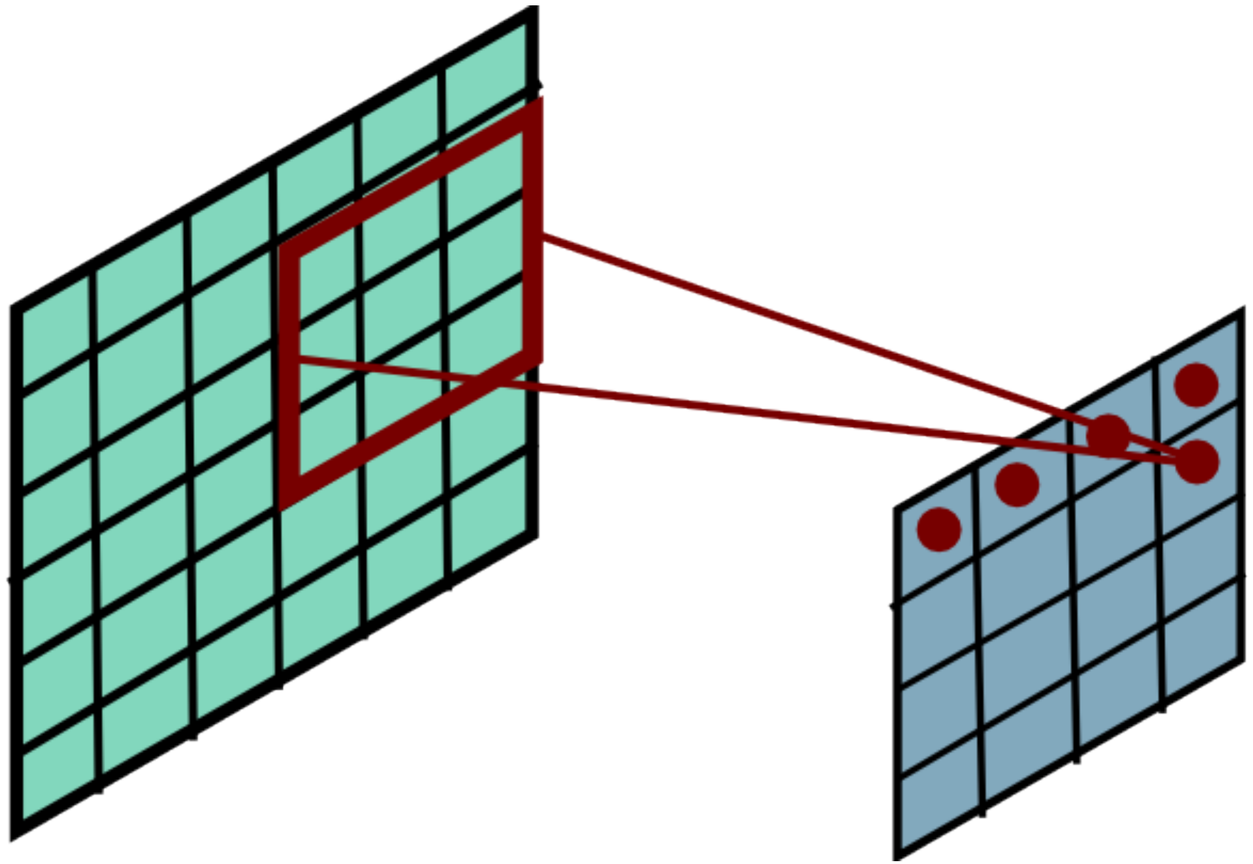
* slide from Marc'Aurelio Renzato

Convolutional Layer



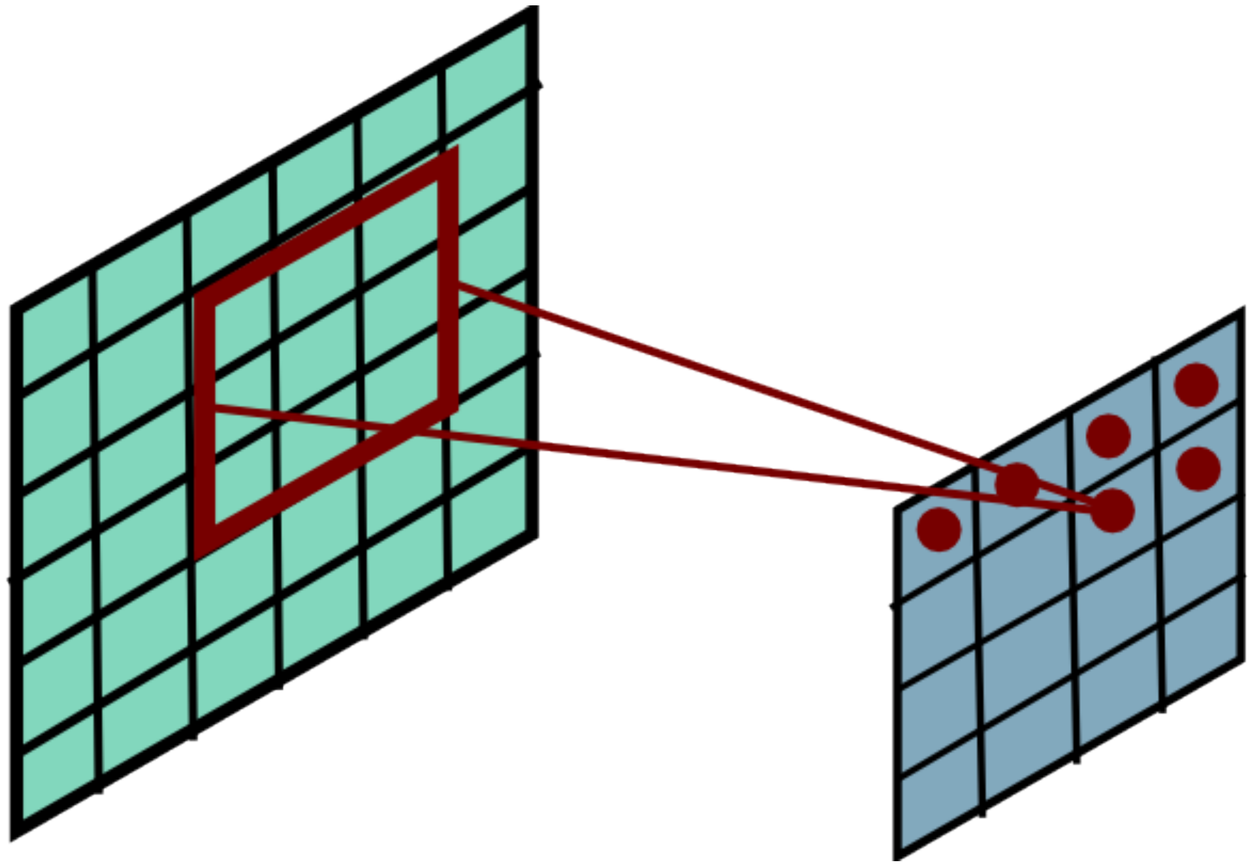
* slide from Marc'Aurelio Renzato

Convolutional Layer



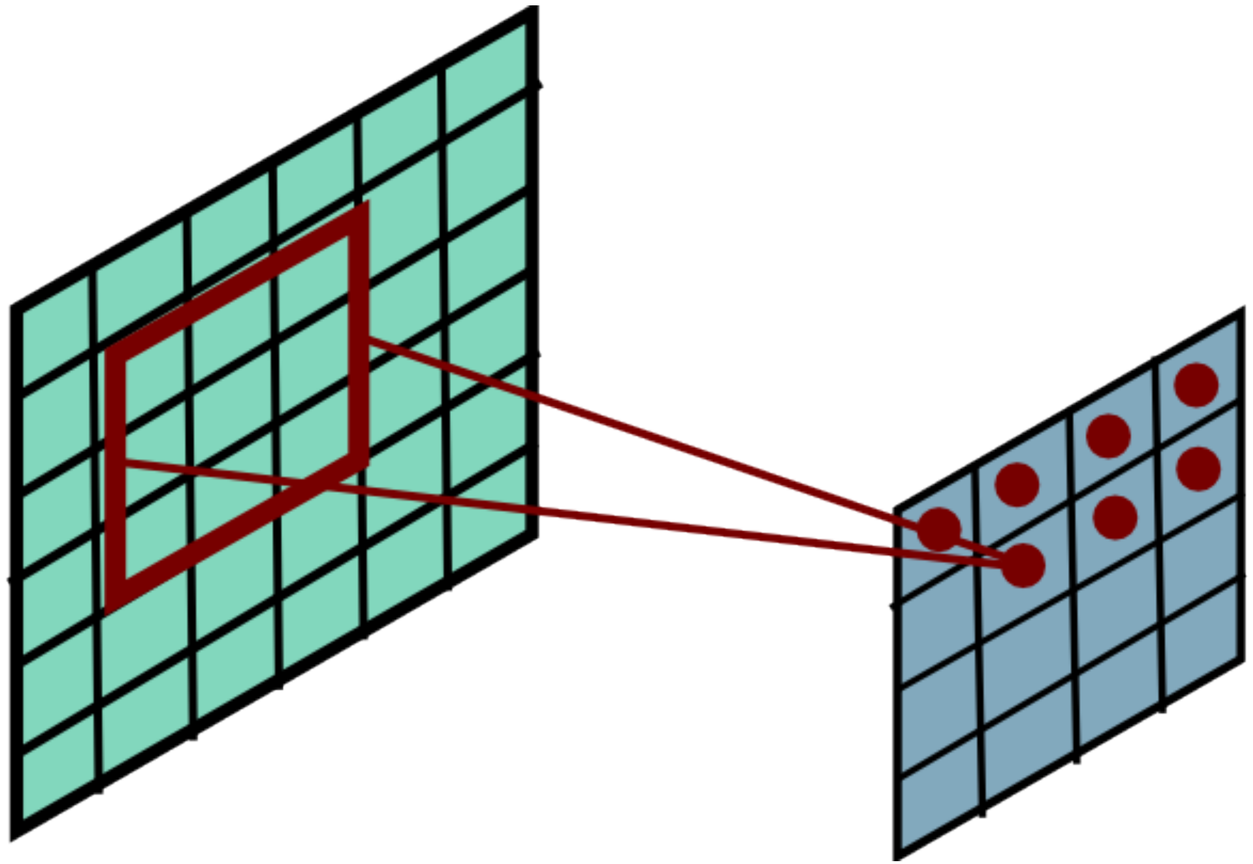
* slide from Marc'Aurelio Renzato

Convolutional Layer



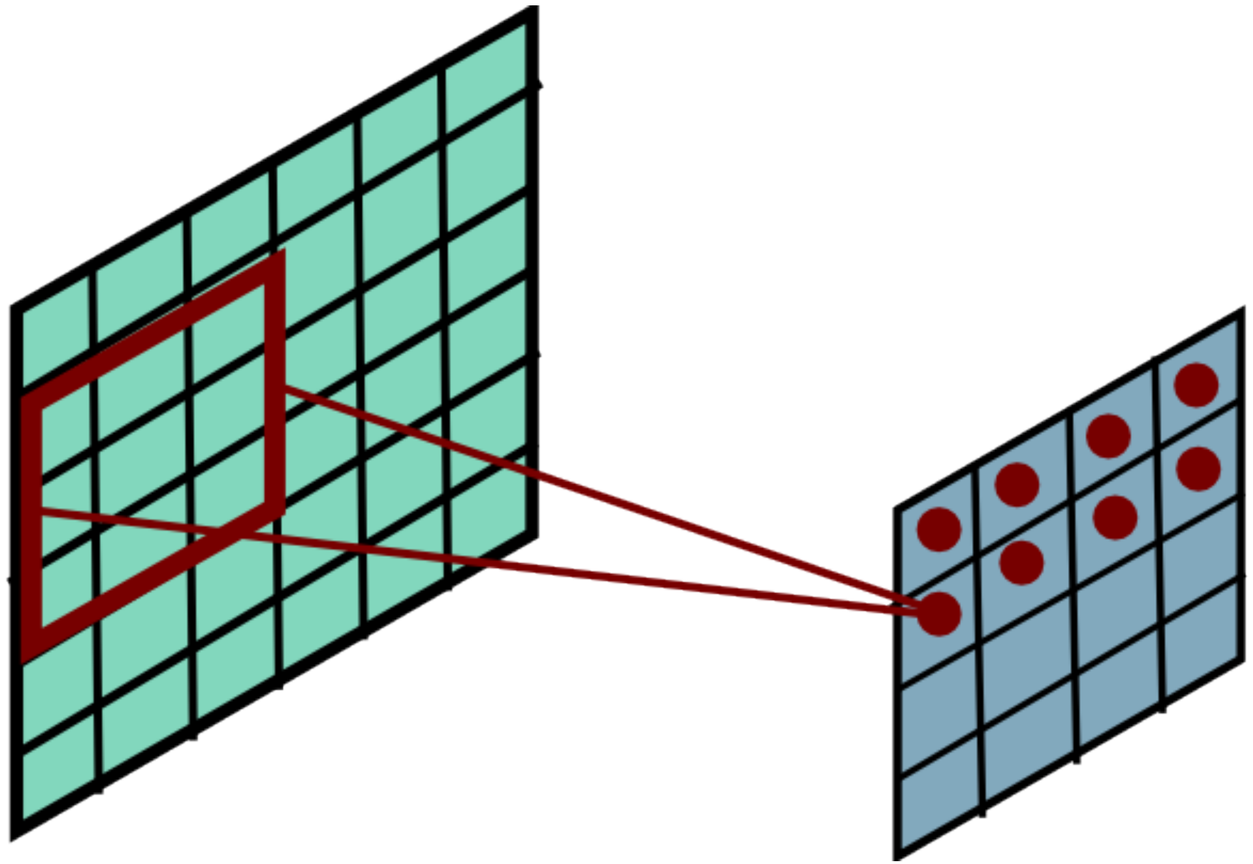
* slide from Marc'Aurelio Renzato

Convolutional Layer



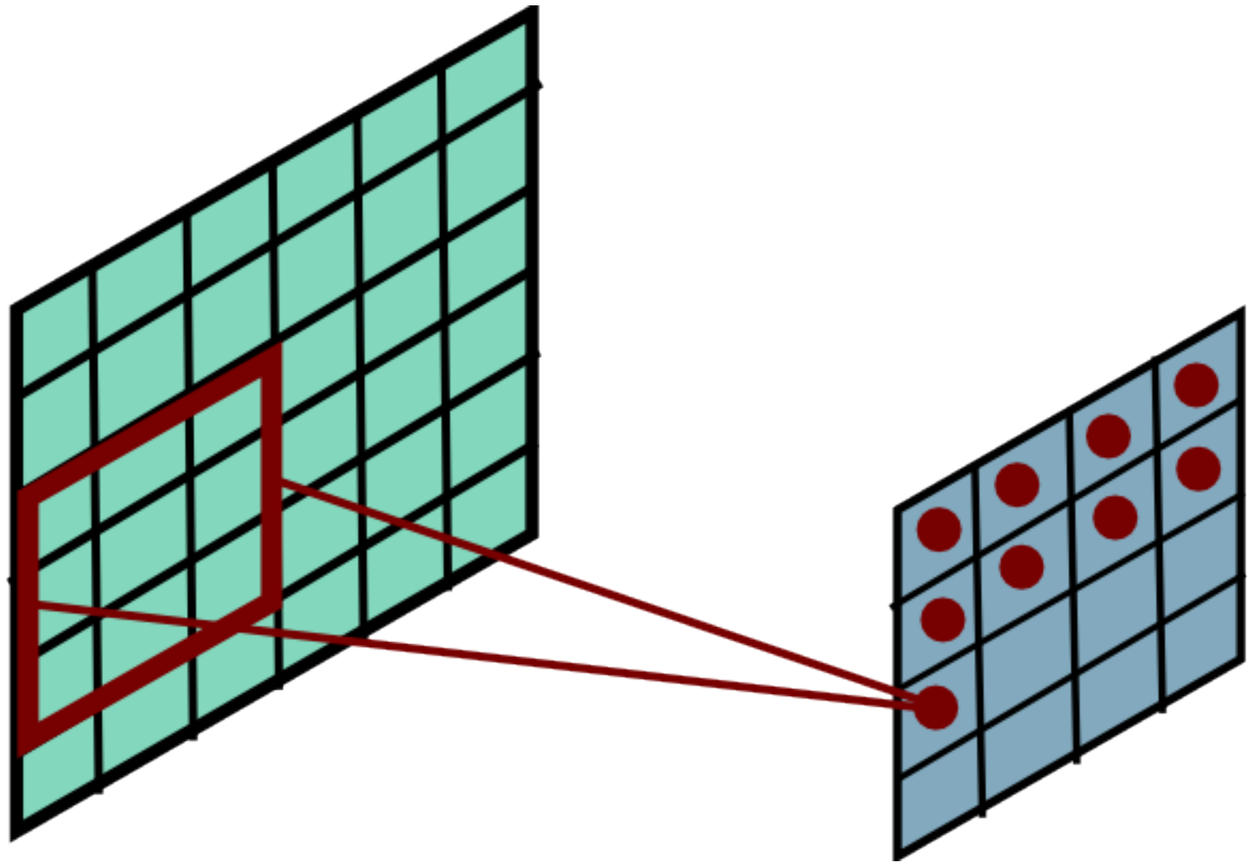
* slide from Marc'Aurelio Renzato

Convolutional Layer



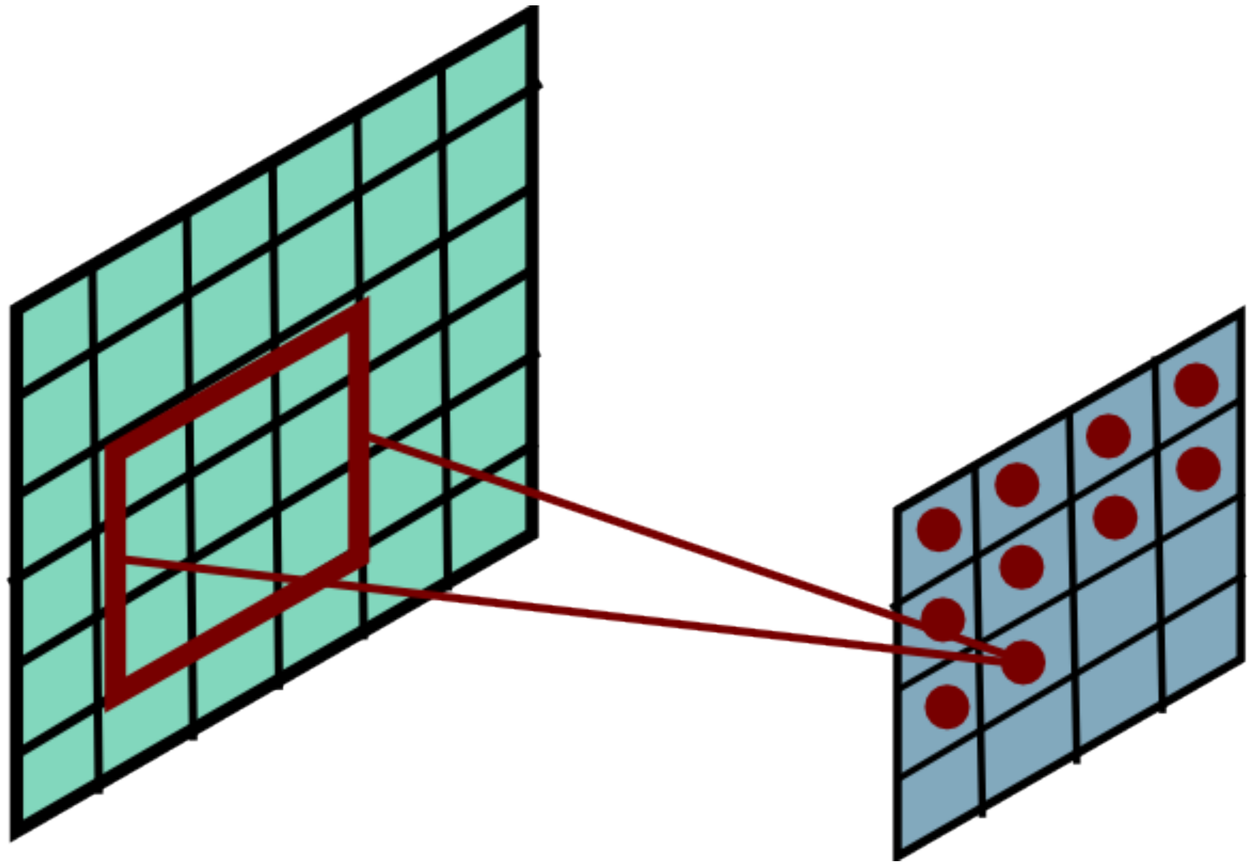
* slide from Marc'Aurelio Renzato

Convolutional Layer



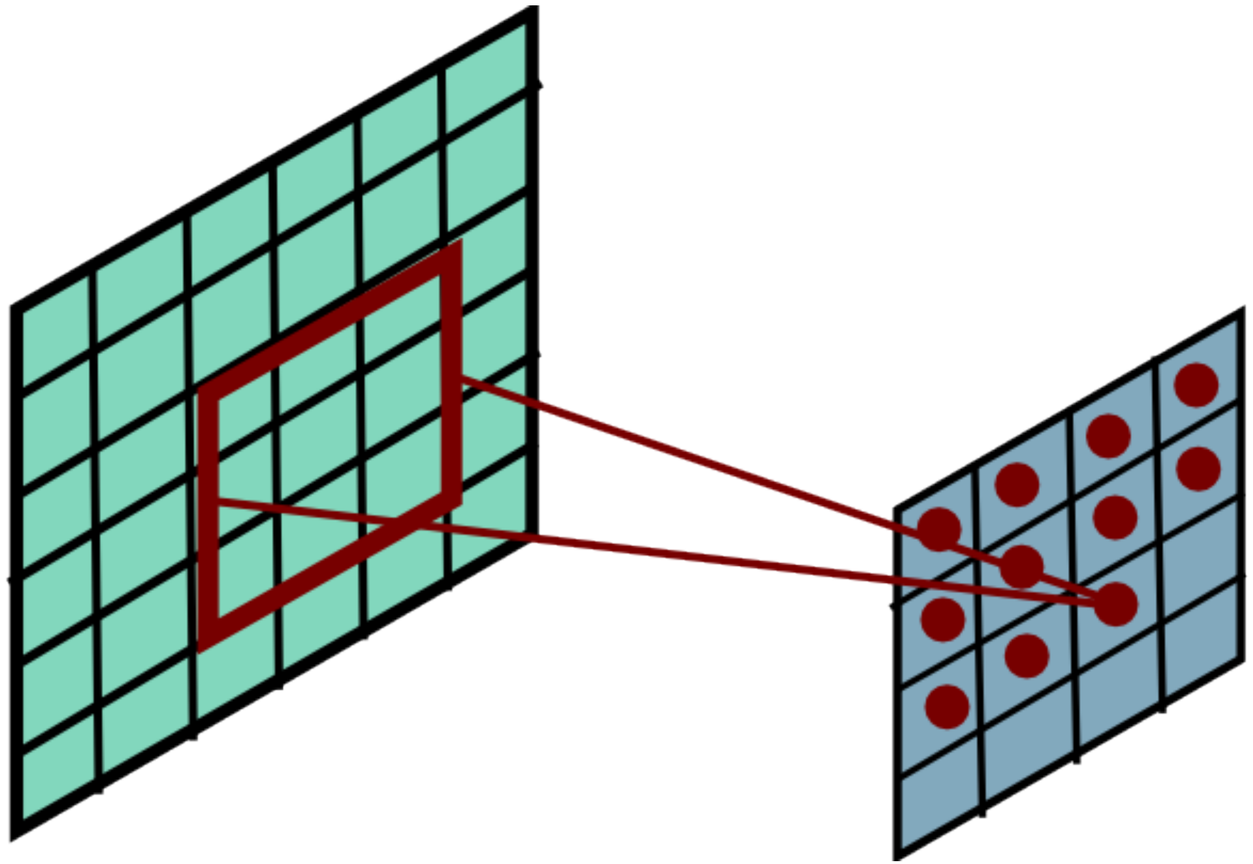
* slide from Marc'Aurelio Renzato

Convolutional Layer



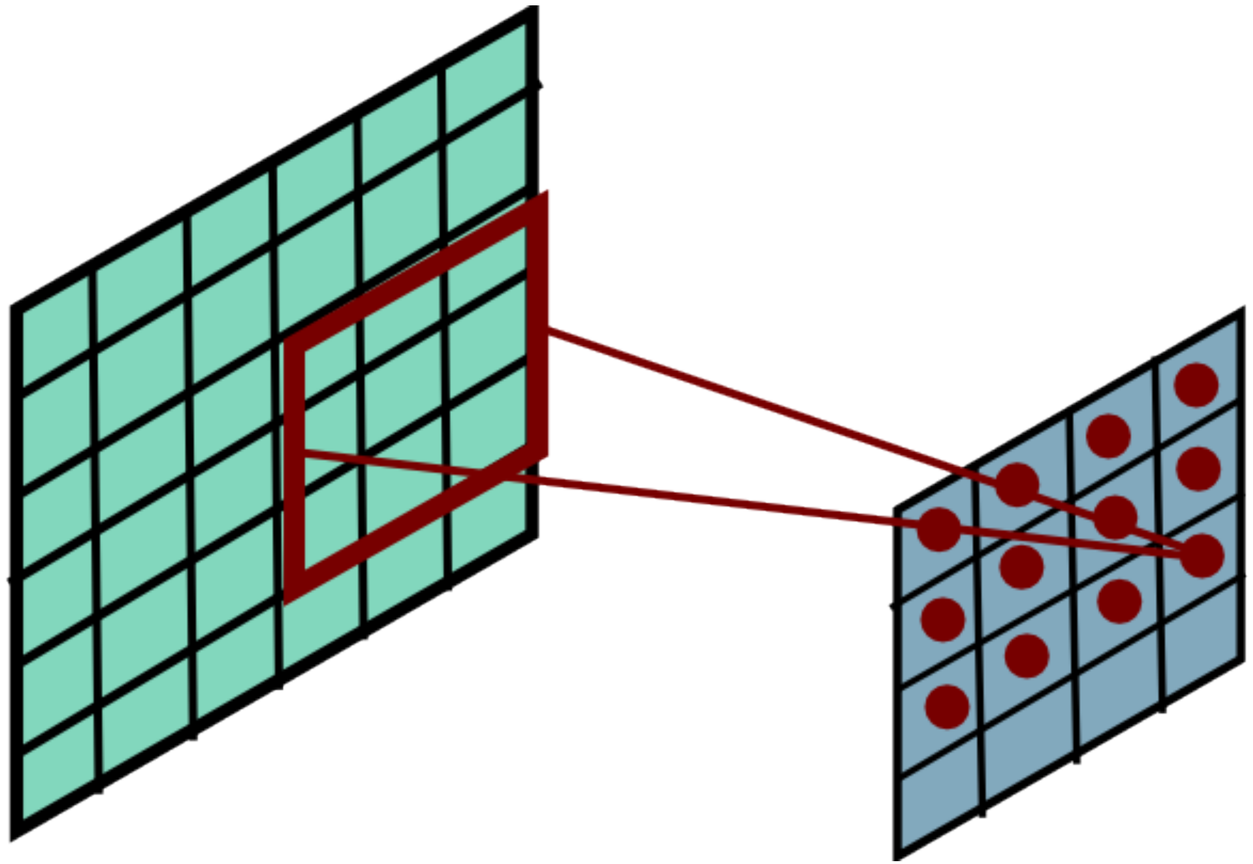
* slide from Marc'Aurelio Renzato

Convolutional Layer



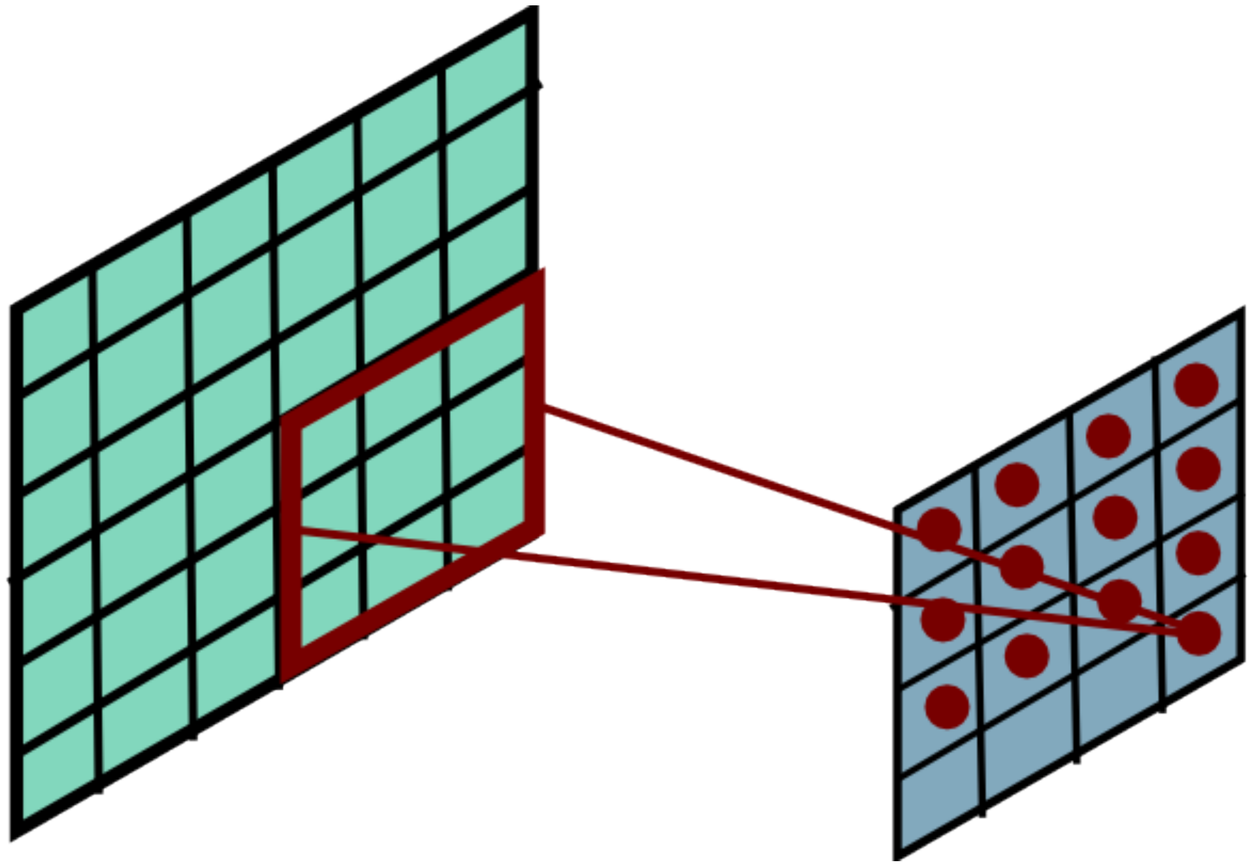
* slide from Marc'Aurelio Renzato

Convolutional Layer



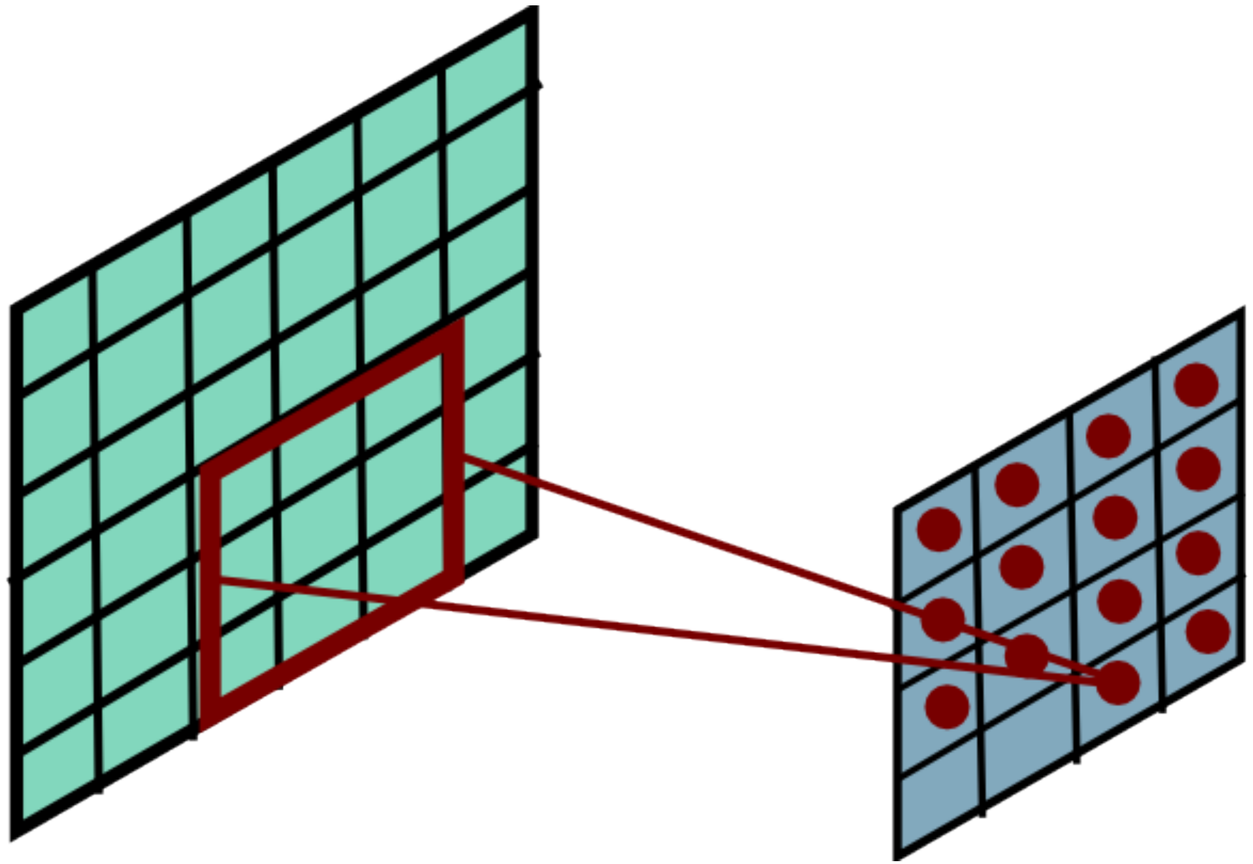
* slide from Marc'Aurelio Renzato

Convolutional Layer



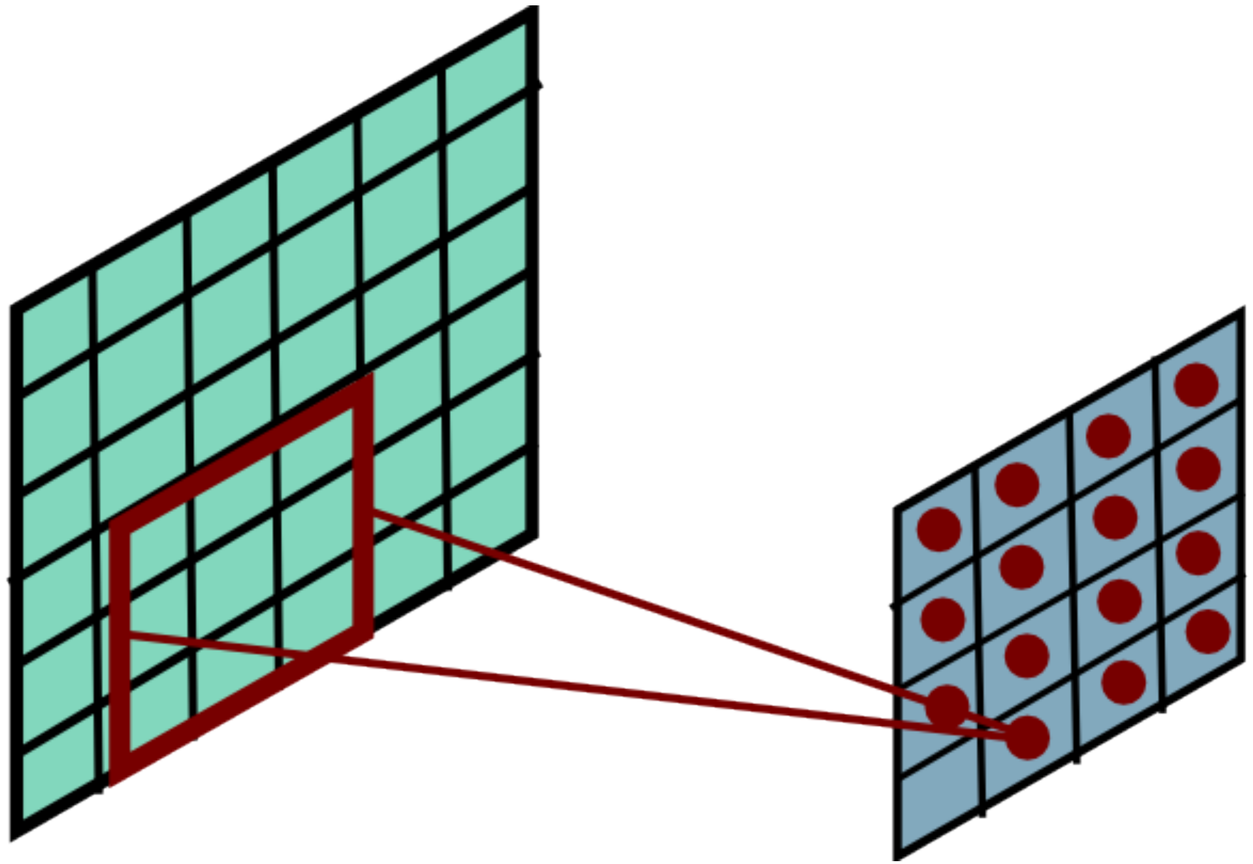
* slide from Marc'Aurelio Renzato

Convolutional Layer



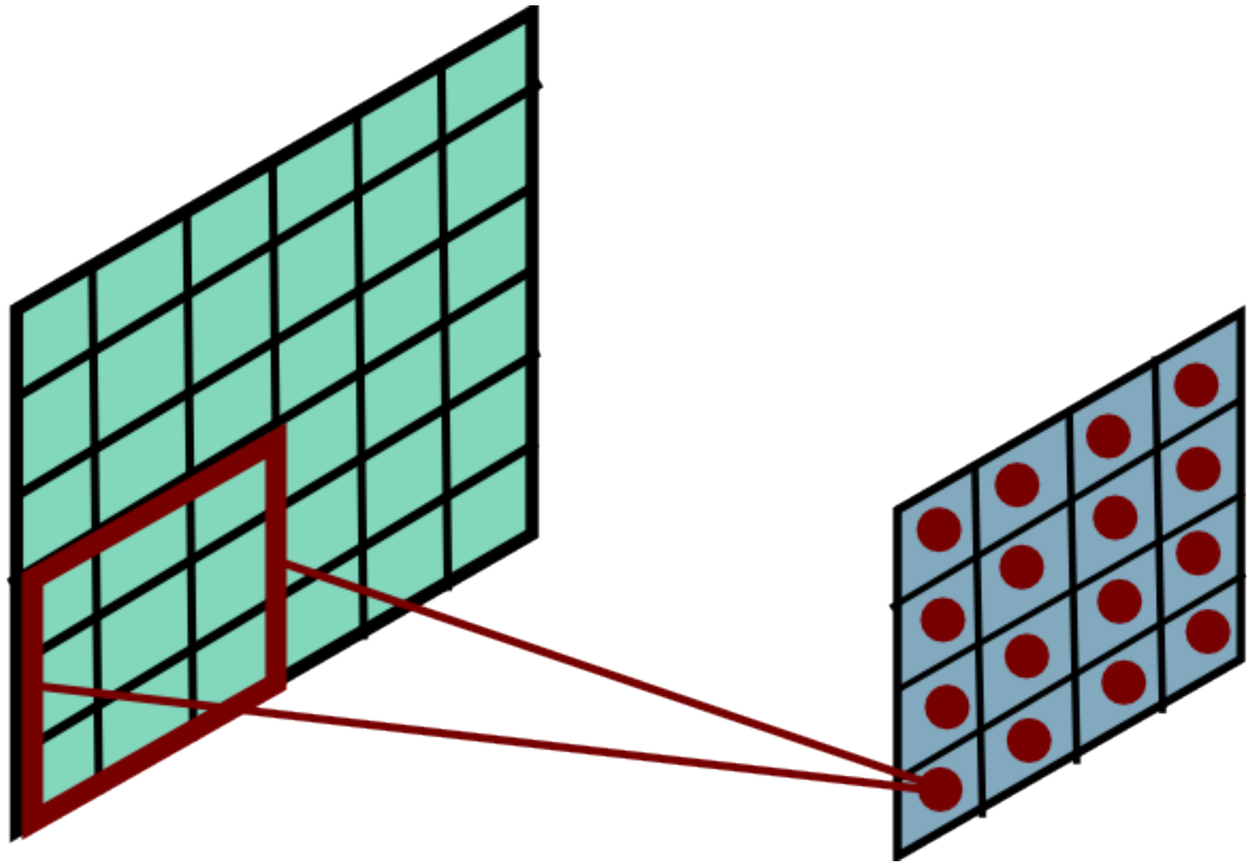
* slide from Marc'Aurelio Renzato

Convolutional Layer



* slide from Marc'Aurelio Renzato

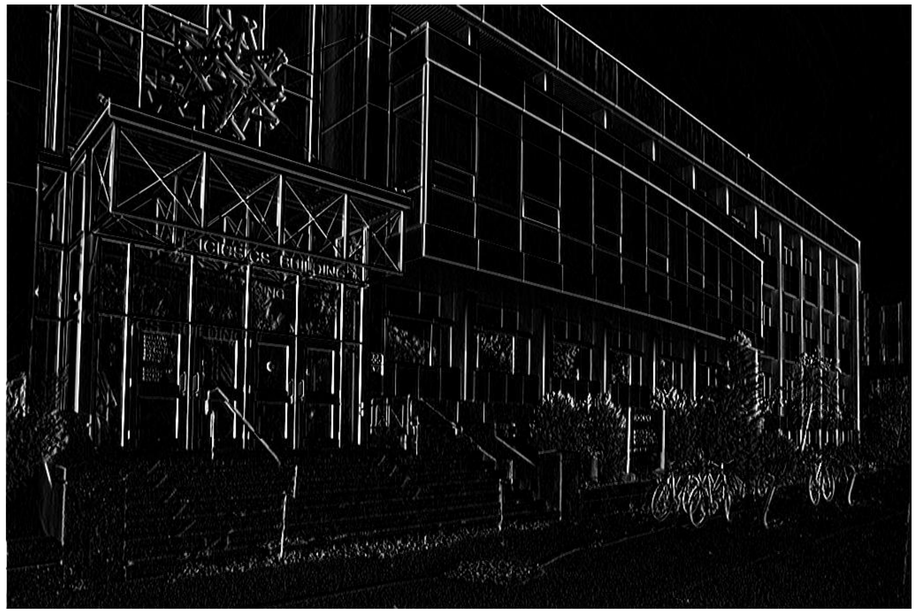
Convolutional Layer



* slide from Marc'Aurelio Renzato

Convolution Layer

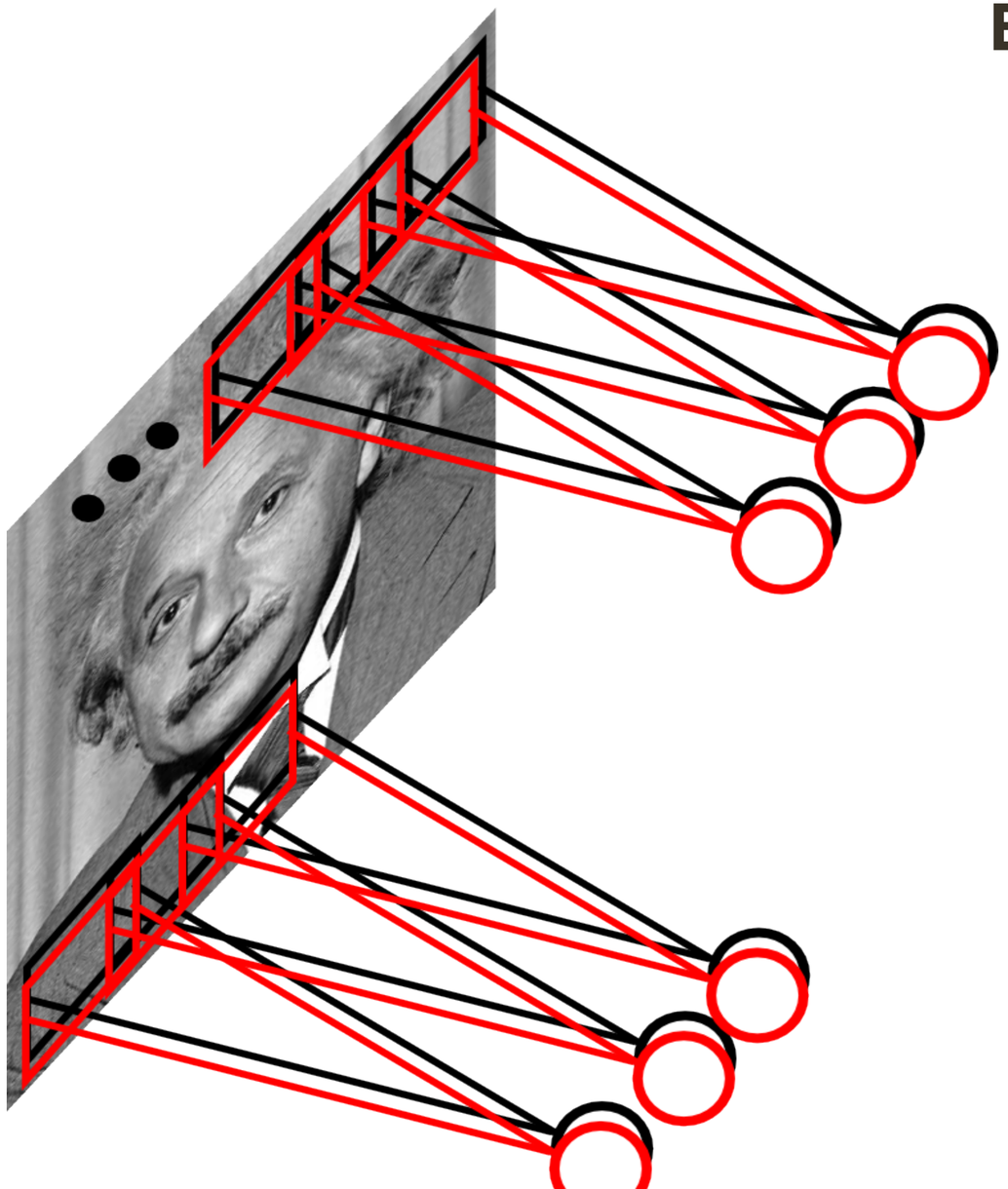
$$\star \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$



Convolution Layer

$$\star \begin{bmatrix} 0.11 & 0.11 & 0.11 \\ 0.11 & 0.11 & 0.11 \\ 0.11 & 0.11 & 0.11 \end{bmatrix} \rightarrow$$

Convolutional Layer



Example: 200 x 200 image (small)
x 40K hidden units

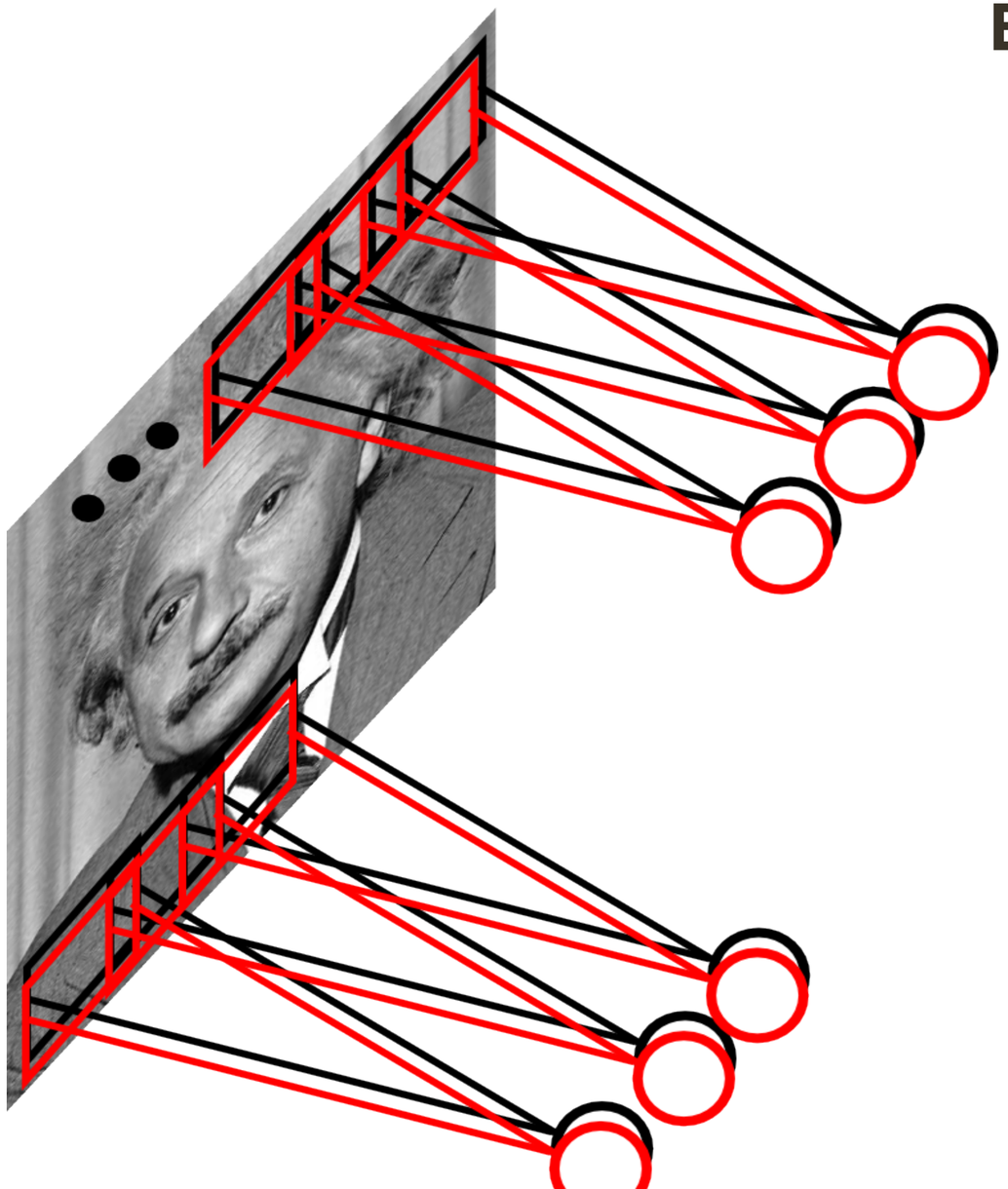
Filter size: 10 x 10

# of filters: 20

Learn **multiple filters**

* slide from Marc'Aurelio Renzato

Convolutional Layer



Example: 200 x 200 image (small)
x 40K hidden units

Filter size: 10 x 10

# of filters: 20

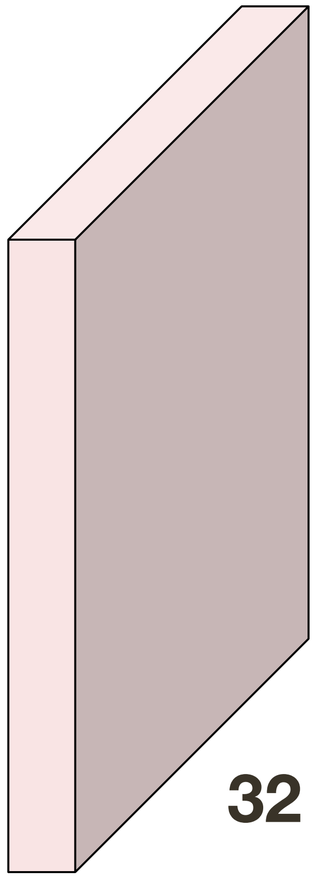
= 2000 parameters

Learn **multiple filters**

* slide from Marc'Aurelio Renzato

Convolutional Layer

32 x 32 x 3 **image** (note the image preserves spatial structure)



32 height

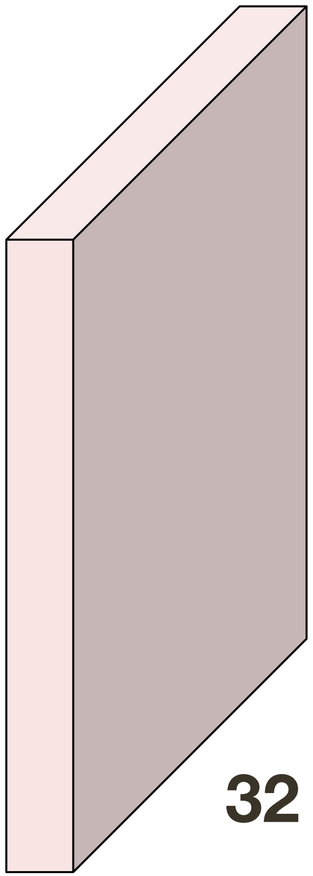
32 width

3 depth

* slide from Fei-Dei Li, Justin Johnson, Serena Yeung, **cs231n Stanford**

Convolutional Layer

32 x 32 x 3 **image**

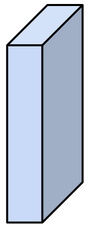


32 height

32 width

3 depth

5 x 5 x 3 **filter**

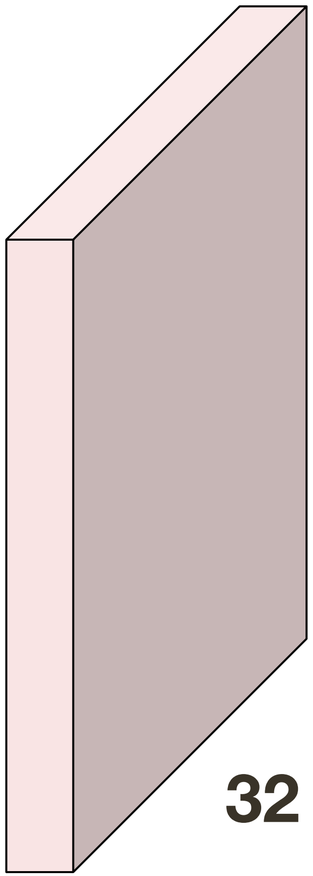


Convolve the filter with the image (i.e., “slide over the image spatially, computing dot products”)

* slide from Fei-Dei Li, Justin Johnson, Serena Yeung, **cs231n Stanford**

Convolutional Layer

32 x 32 x 3 **image**



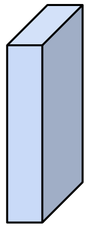
32 height

32 width

3 depth

Filters always extend the full depth of the input volume

5 x 5 x 3 **filter**

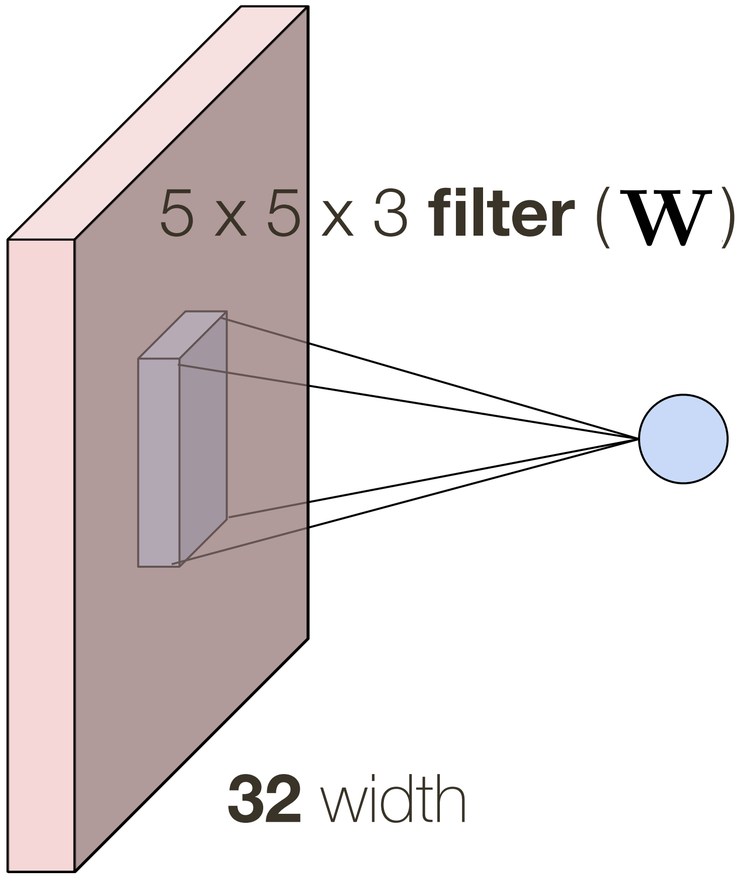


Convolve the filter with the image (i.e., “slide over the image spatially, computing dot products”

* slide from Fei-Dei Li, Justin Johnson, Serena Yeung, **cs231n Stanford**

Convolutional Layer

32 x 32 x 3 **image**



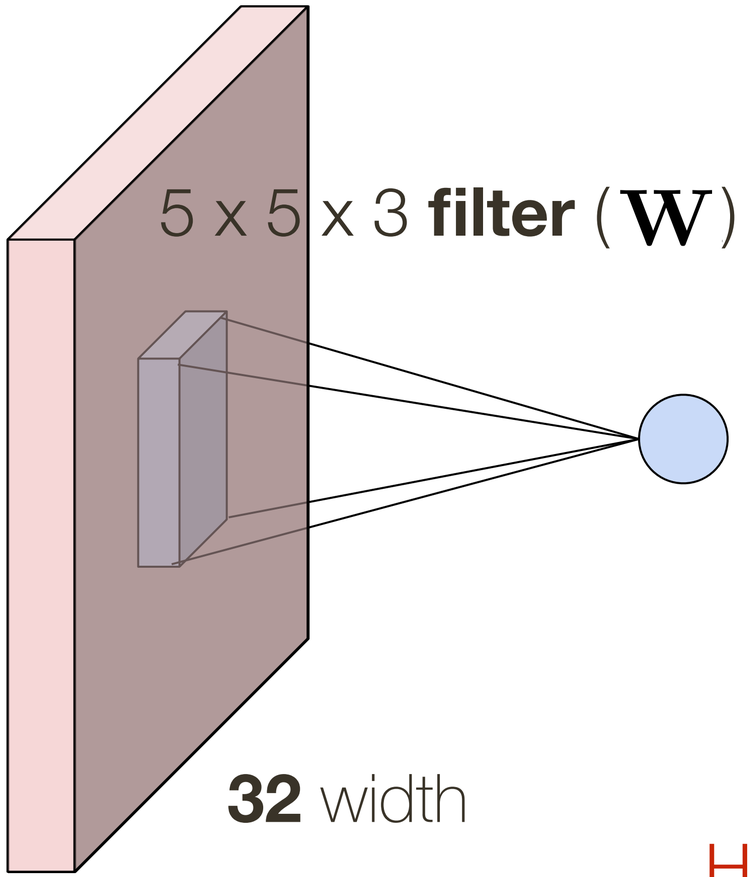
1 number: the result of taking a dot product between the filter and a small 5 x 5 x 3 part of the image

$$\mathbf{W}^T\mathbf{x} + b, \text{where } \mathbf{W}, \mathbf{x} \in \mathbb{R}^{75}$$

* slide from Fei-Dei Li, Justin Johnson, Serena Yeung, **cs231n Stanford**

Convolutional Layer

32 x 32 x 3 **image**



1 number: the result of taking a dot product between the filter and a small 5 x 5 x 3 part of the image

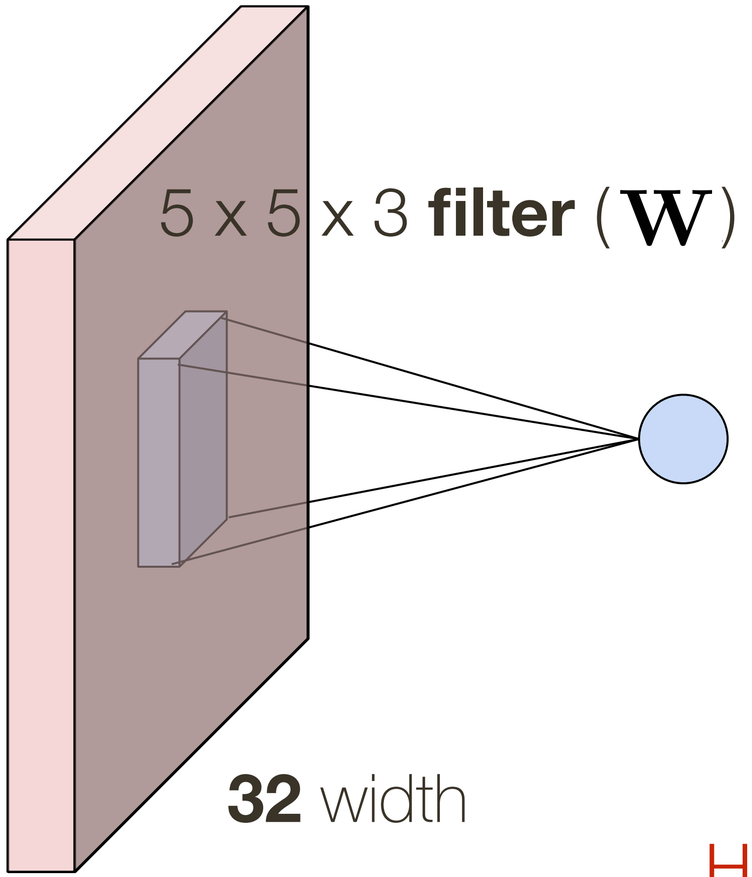
$$\mathbf{W}^T\mathbf{x} + b, \text{where } \mathbf{W}, \mathbf{x} \in \mathbb{R}^{75}$$

How many **parameters** does the layer have?

* slide from Fei-Dei Li, Justin Johnson, Serena Yeung, **cs231n Stanford**

Convolutional Layer

32 x 32 x 3 **image**



1 number: the result of taking a dot product between the filter and a small 5 x 5 x 3 part of the image

$$\mathbf{W}^T\mathbf{x} + b, \text{where } \mathbf{W}, \mathbf{x} \in \mathbb{R}^{75}$$

How many **parameters** does the layer have? **76**

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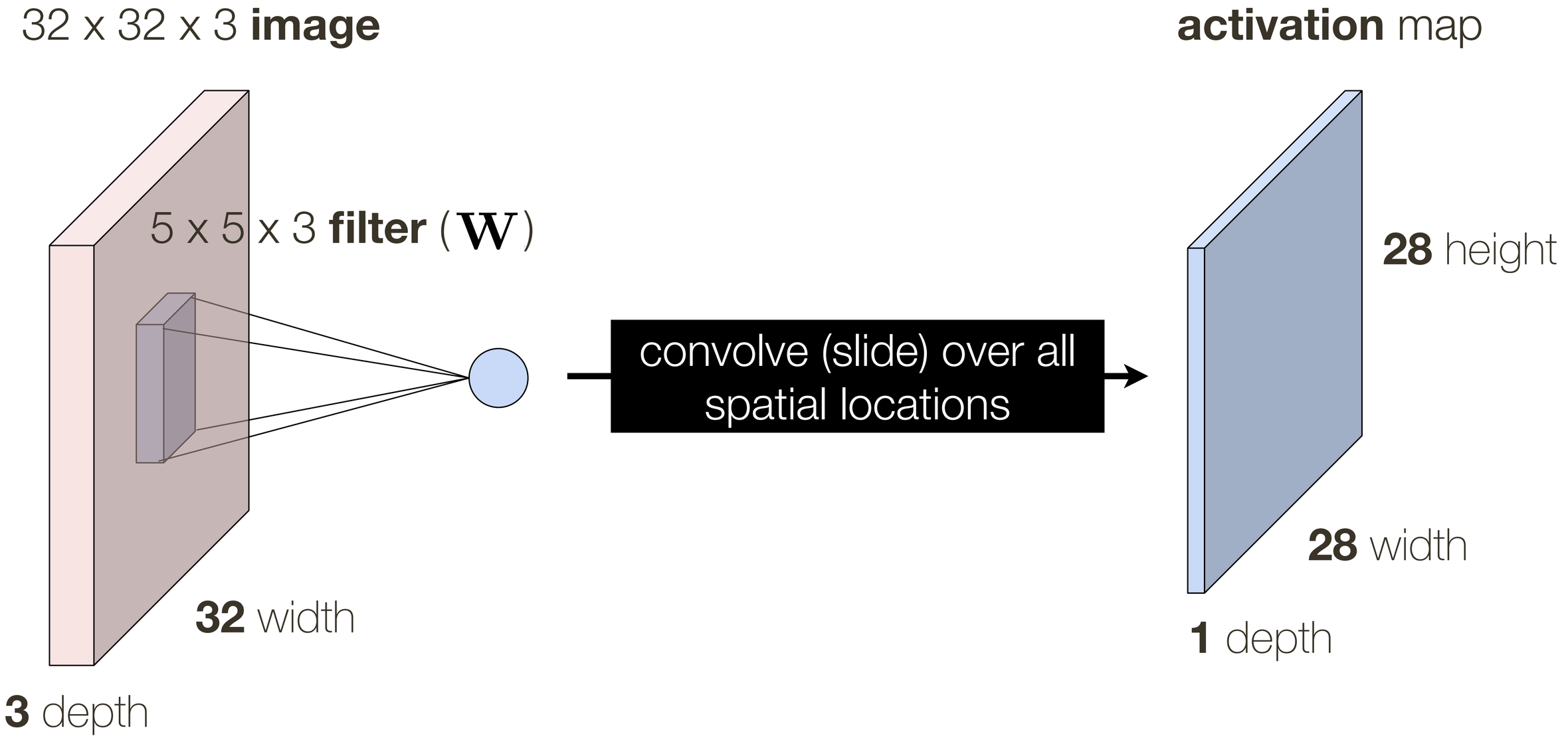
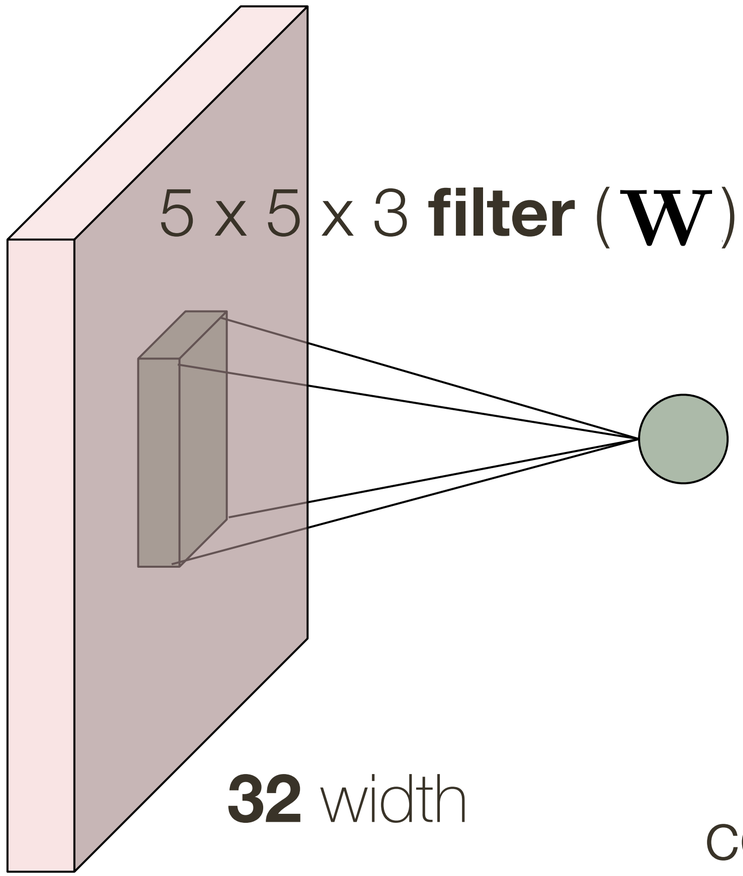
Convolutional Layer



* slide from Fei-Dei Li, Justin Johnson, Serena Yeung, **cs231n Stanford**

Convolutional Layer

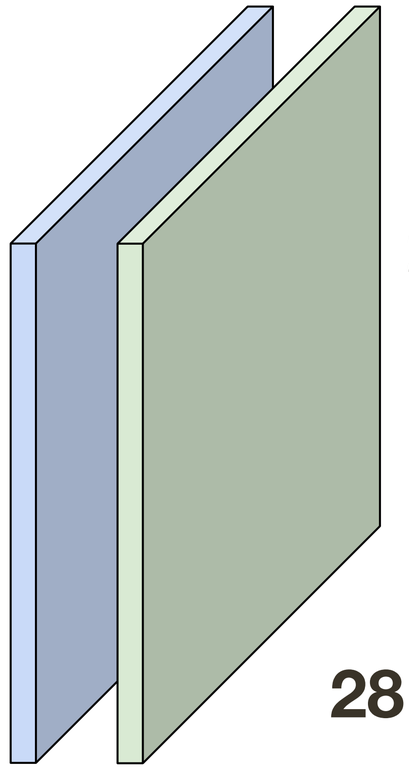
32 x 32 x 3 **image**



convolve (slide) over all spatial locations

consider another **green** filter

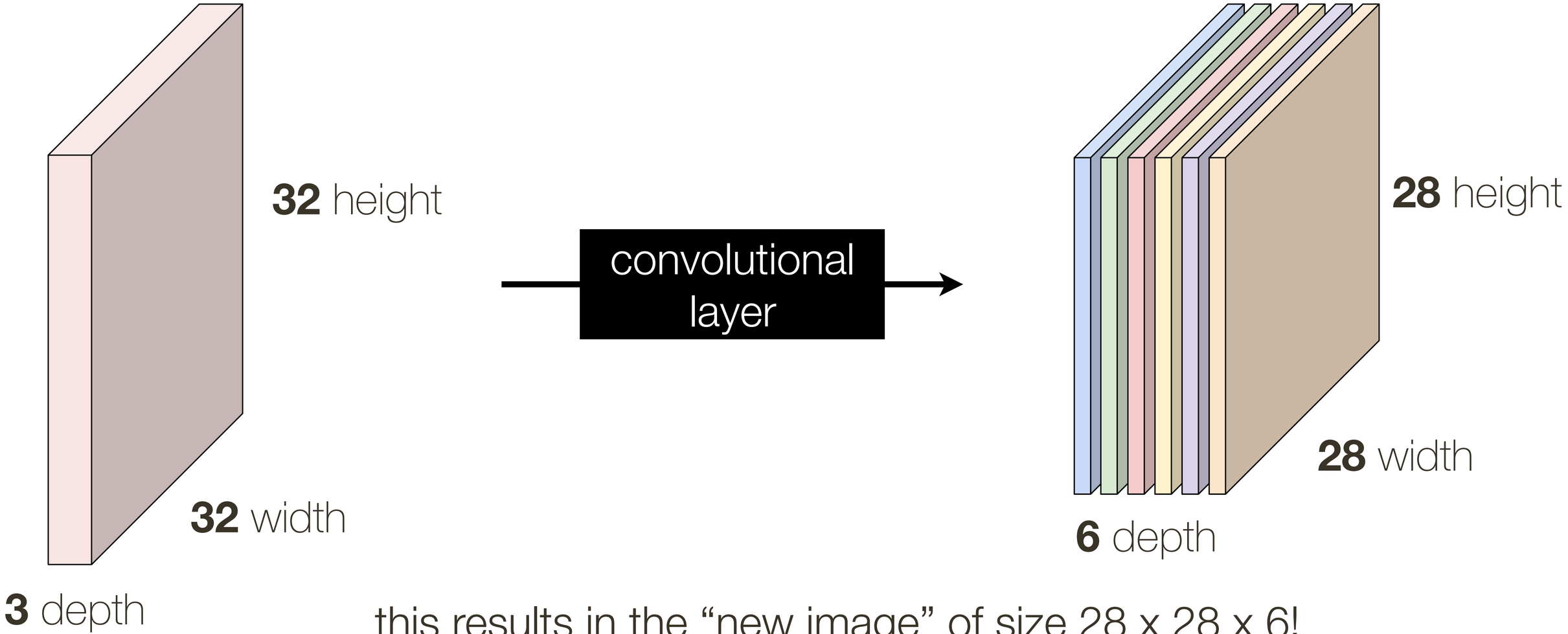
activation map



* slide from Fei-Dei Li, Justin Johnson, Serena Yeung, **cs231n Stanford**

Convolutional Layer

If we have 6 5x5 filter, we'll get 6 separate activation maps: **activation** map



this results in the "new image" of size 28 x 28 x 6!

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Convolutional Layer

The number of neurons in a layer is determined by depth and stride parameter

— also affected by zero-padding

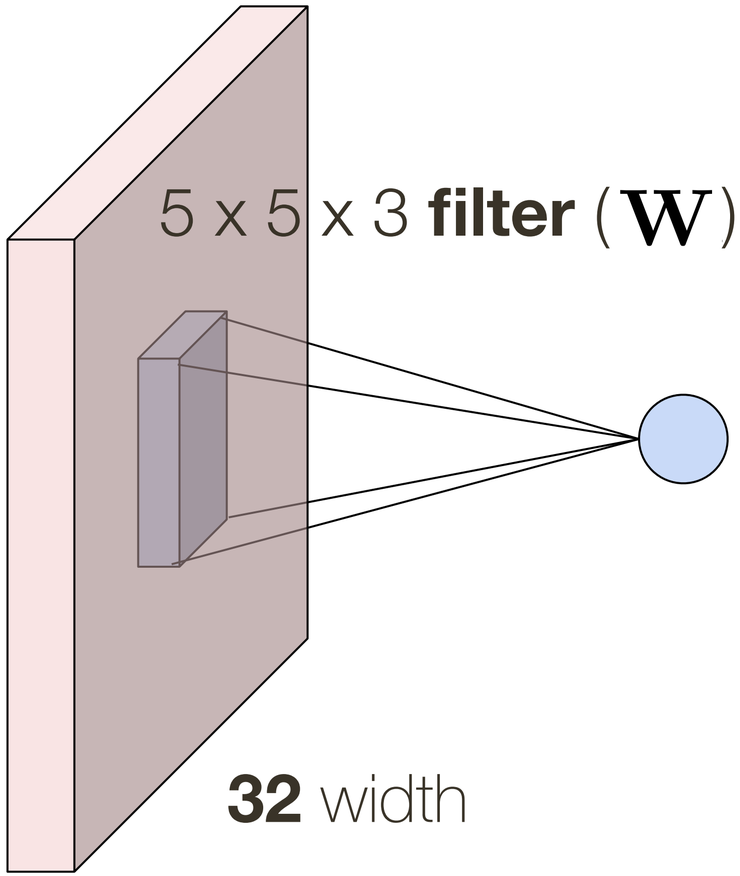
Depth: Controls number of neurons that connect to the same region of the input layer

— a set of neurons connected to the same region is called a **depth column**

Stride: Controls spatial density. How far apart are depth columns?

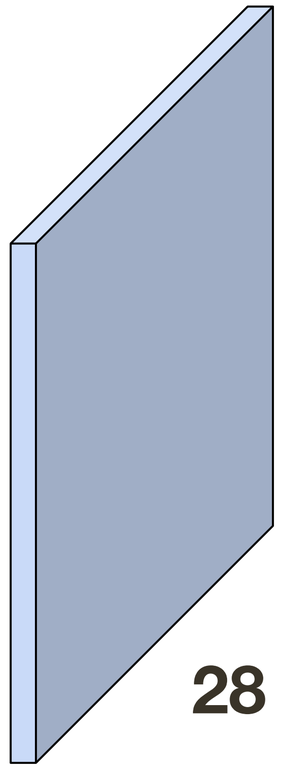
Convolutional Layer: Closer Look at **Spatial Dimensions**

32 x 32 x 3 **image**



convolve (slide) over all spatial locations

activation map



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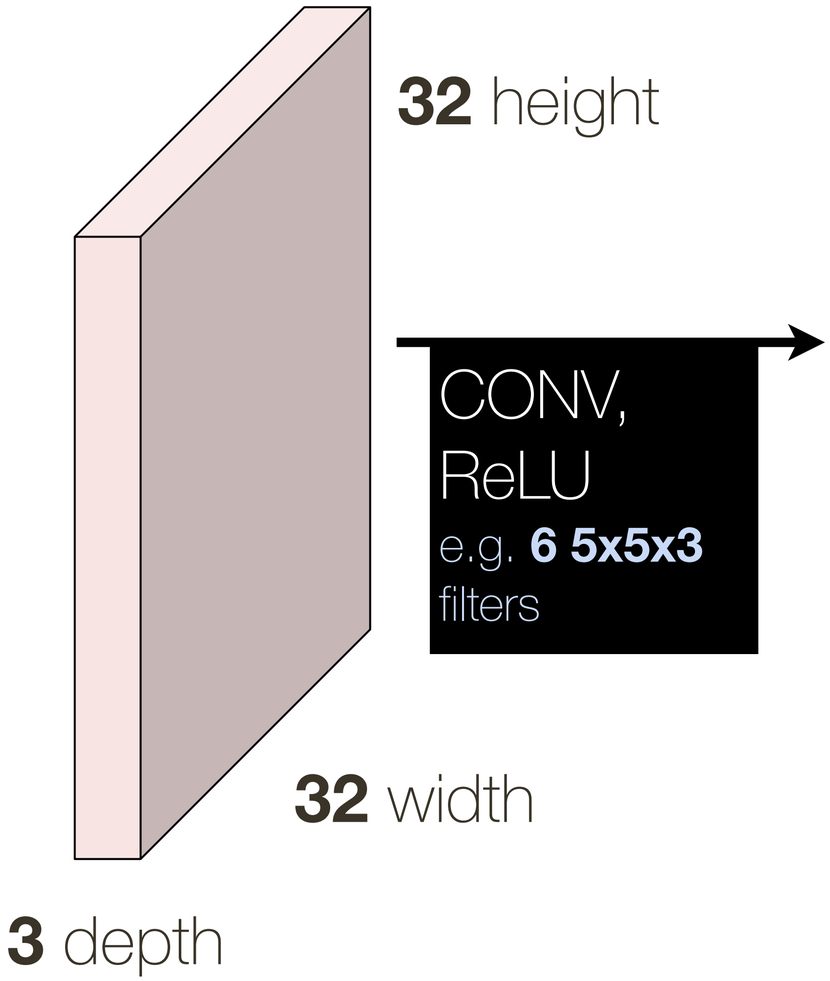
Convolutional Neural Network (ConvNet)



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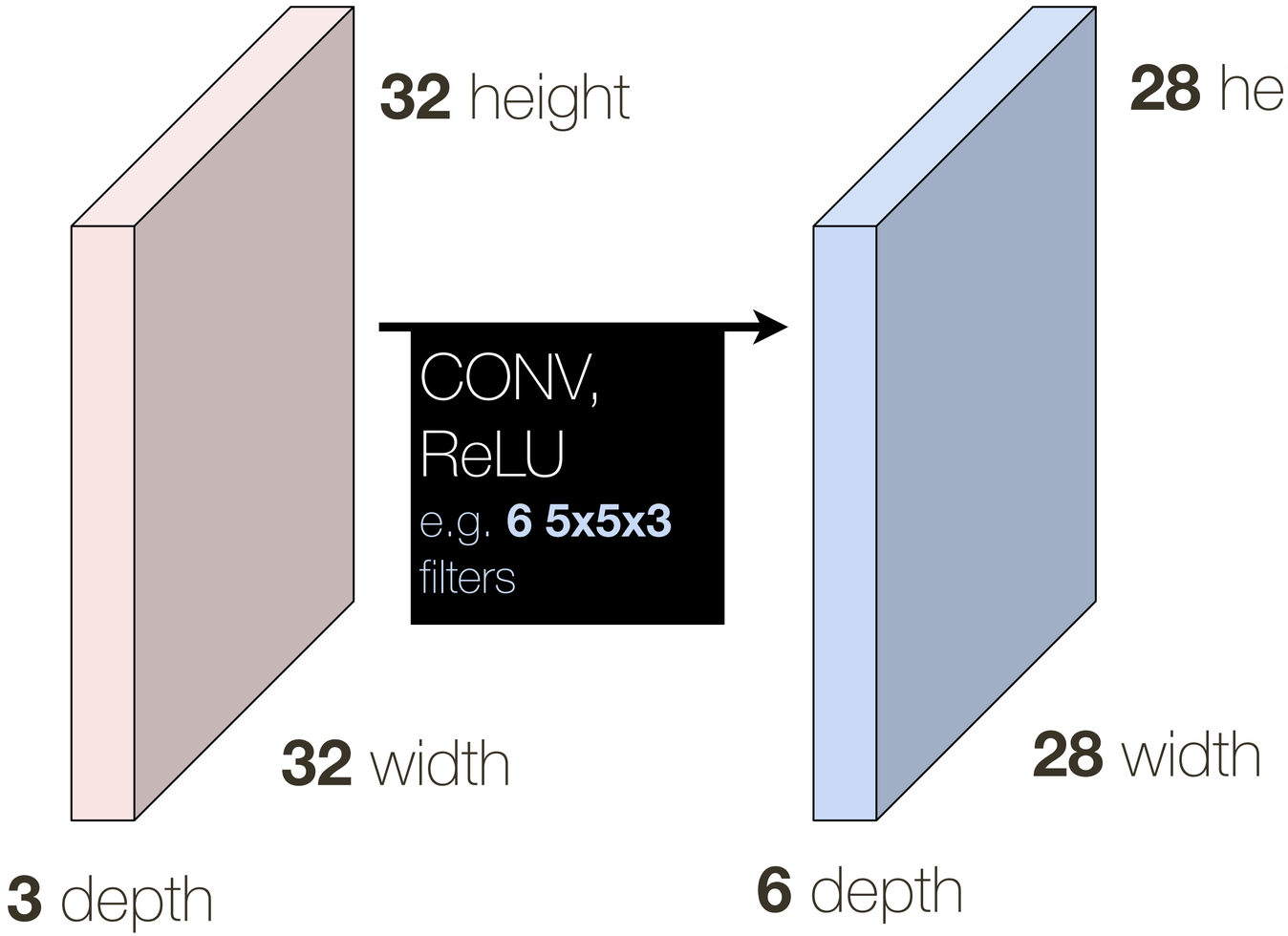
Convolutional Neural Network (ConvNet)



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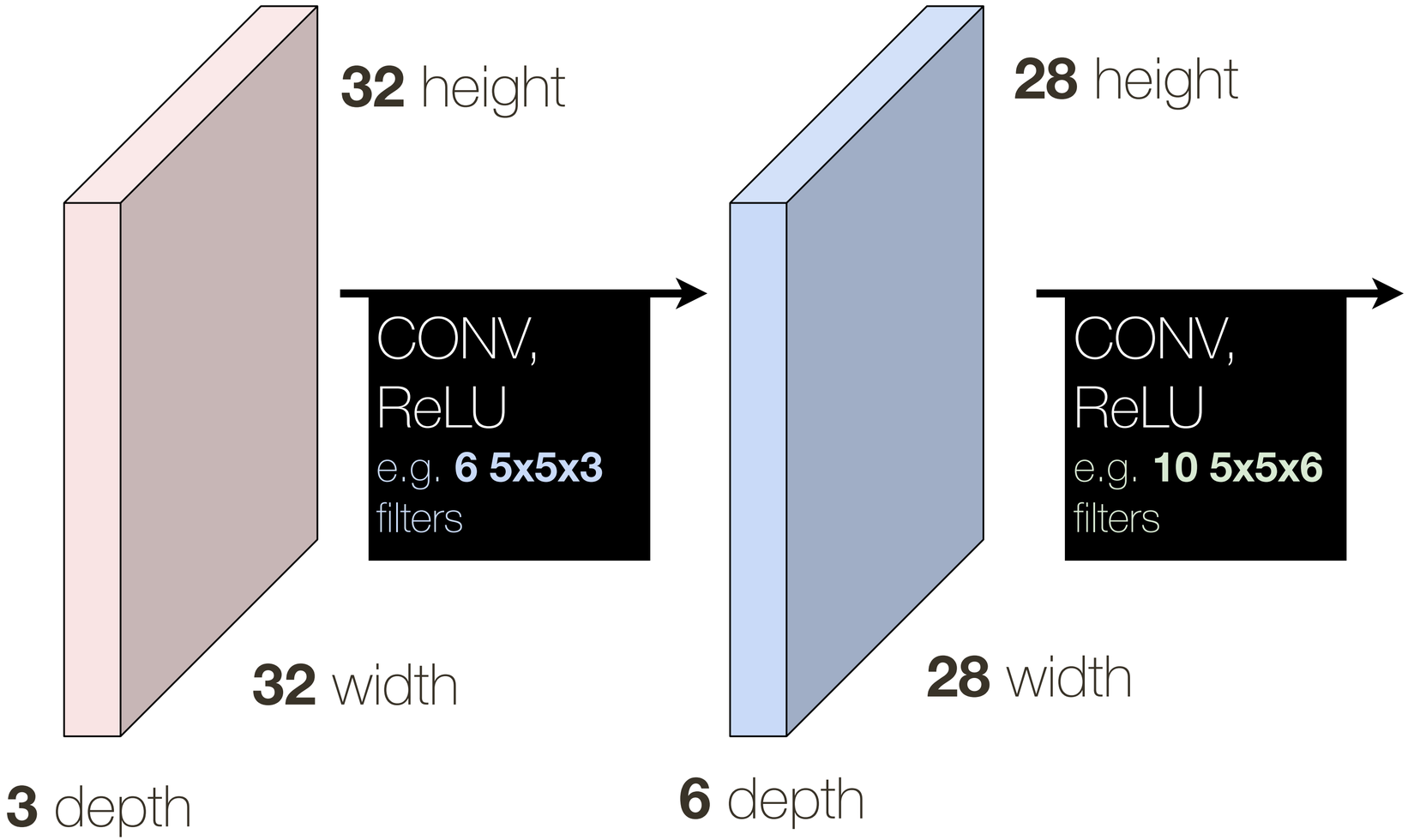
Convolutional Neural Network (ConvNet)



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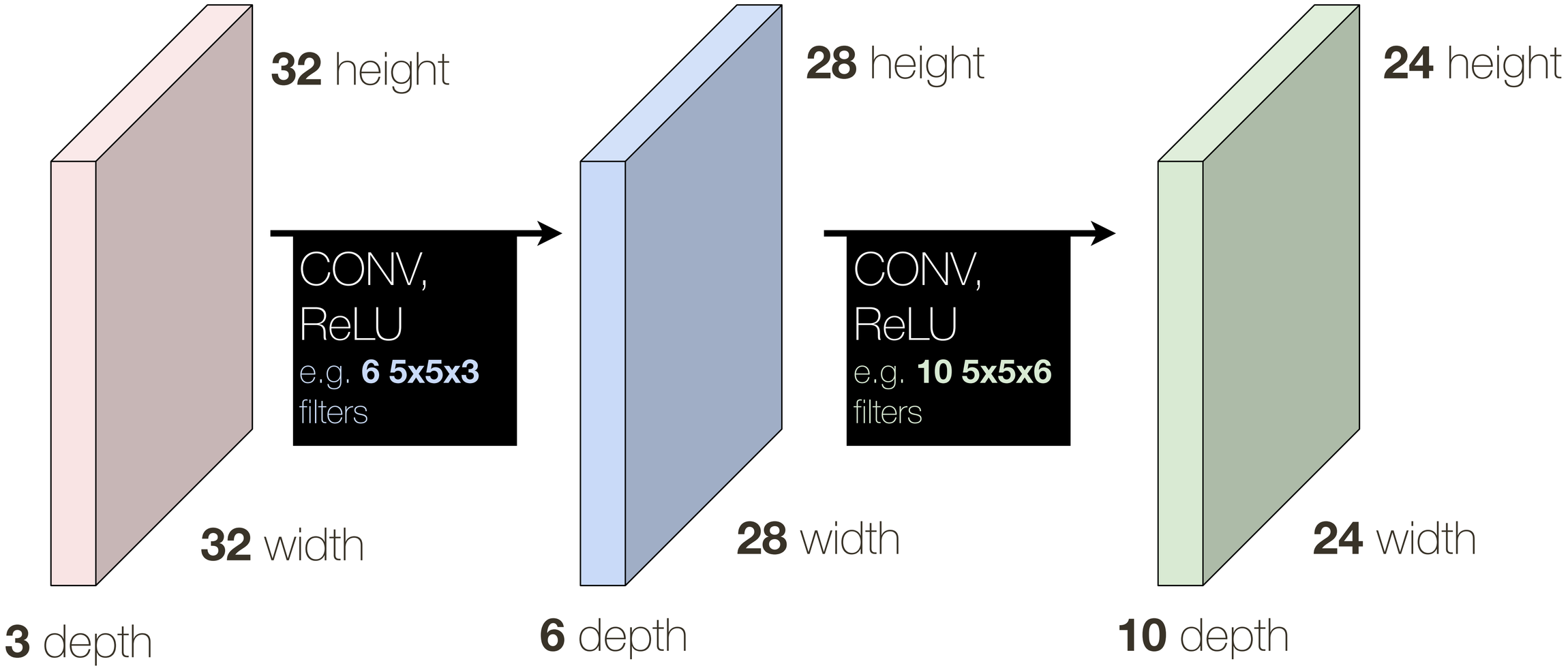

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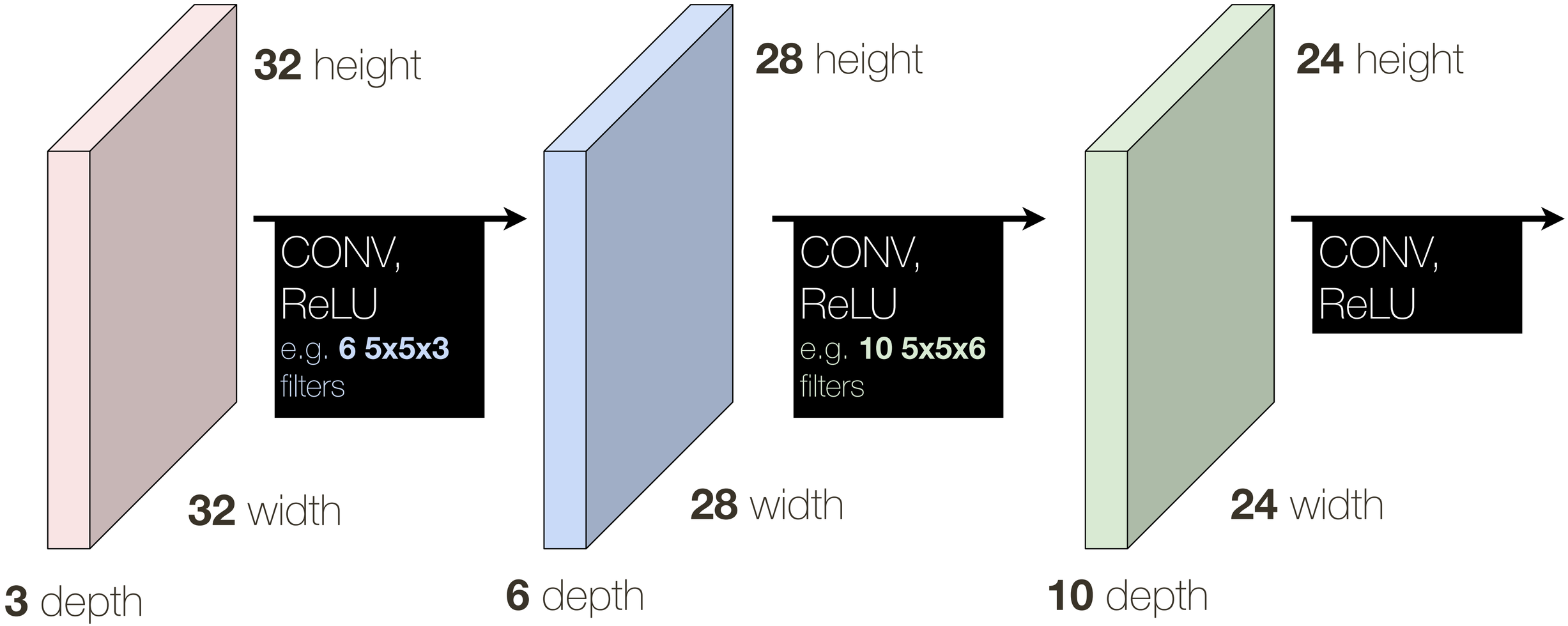
Convolutional Neural Network (ConvNet)



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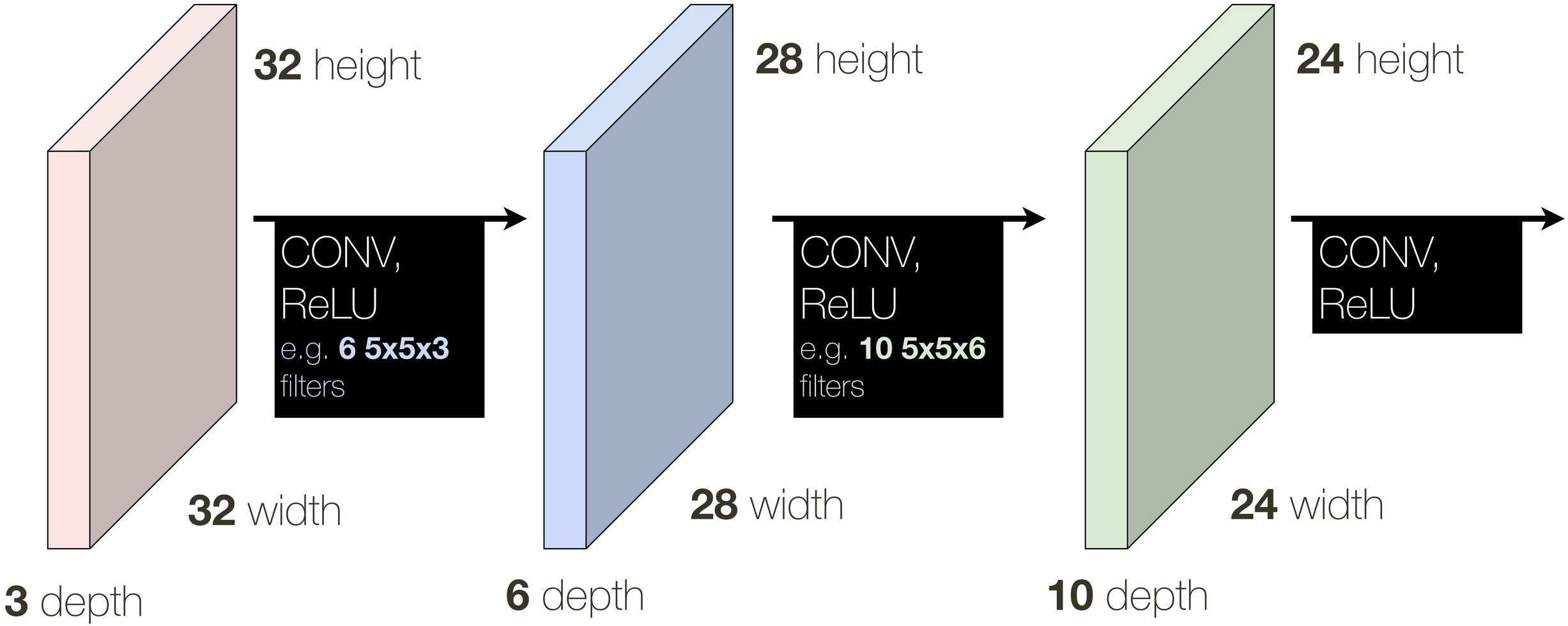
Convolutional Neural Network (ConvNet)

With padding we can achieve no shrinking (32 -> 28 -> 24); shrinking quickly (which happens with larger filters) doesn't work well in practice



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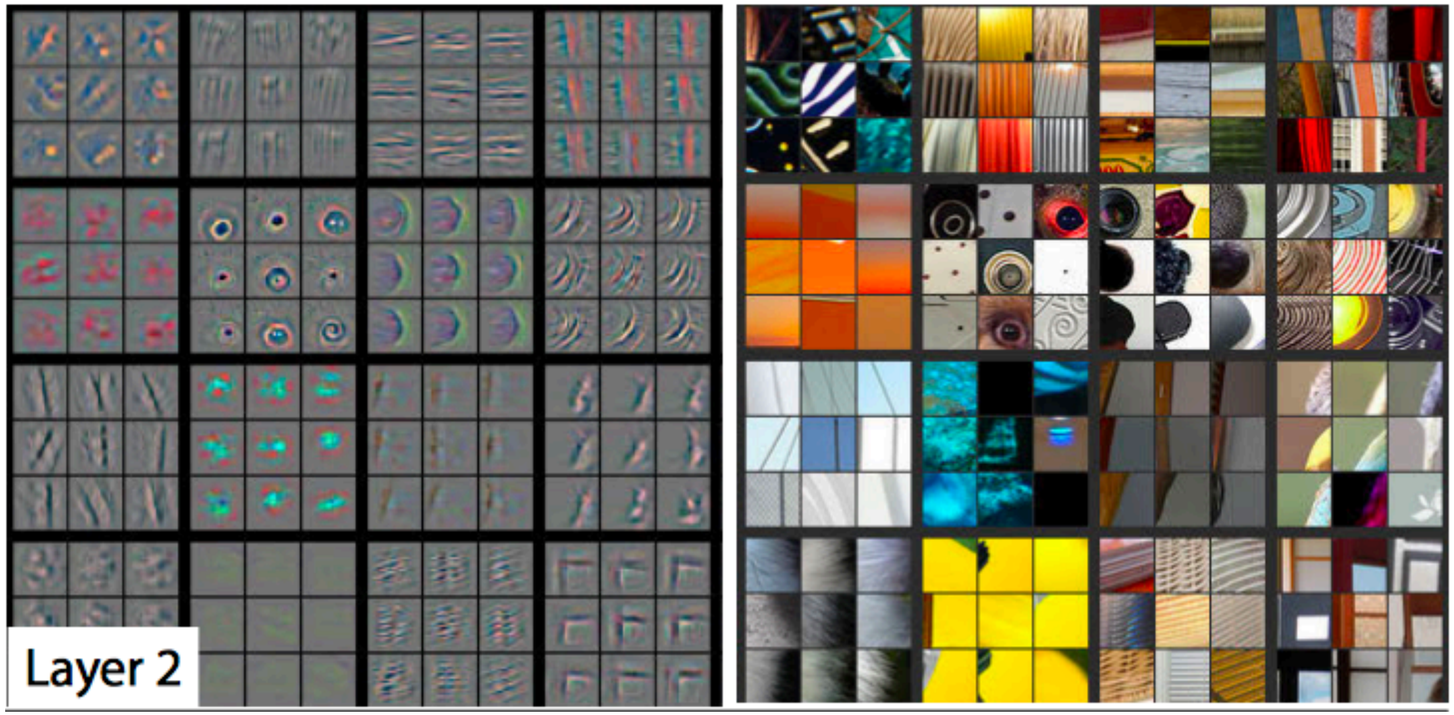
Convolutional Neural Network (ConvNet)

Convolutional neural networks can be seen as learning a hierarchy of filters.

As we go deeper in the network, filters learn and respond to increasingly specialized structures
— The first layers may contain simple orientation filters, middle layers may respond to common substructures, and final layers may respond to entire objects

What **filters** do networks learn?

Layer 1



[Zeiler and Fergus, 2013]

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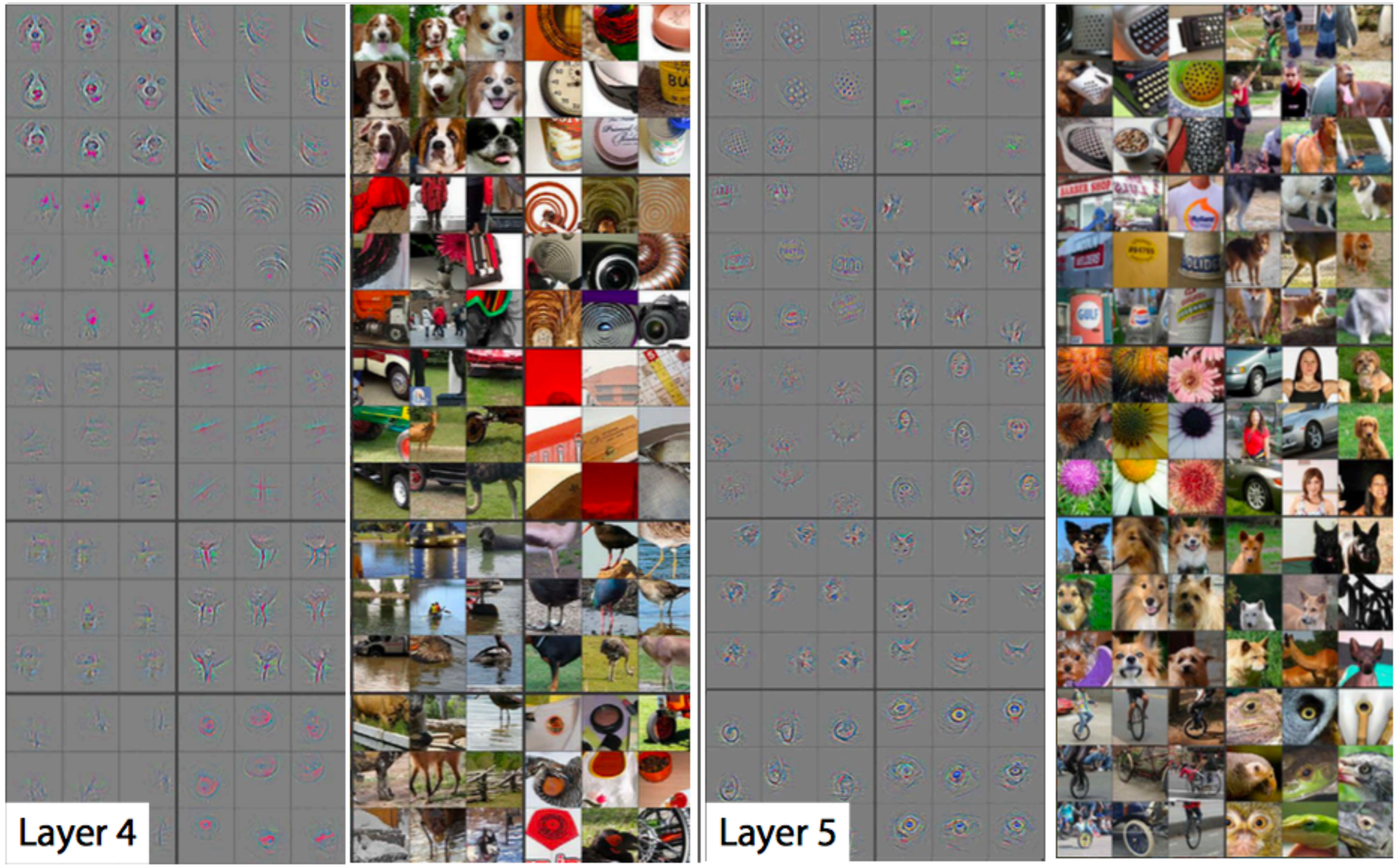
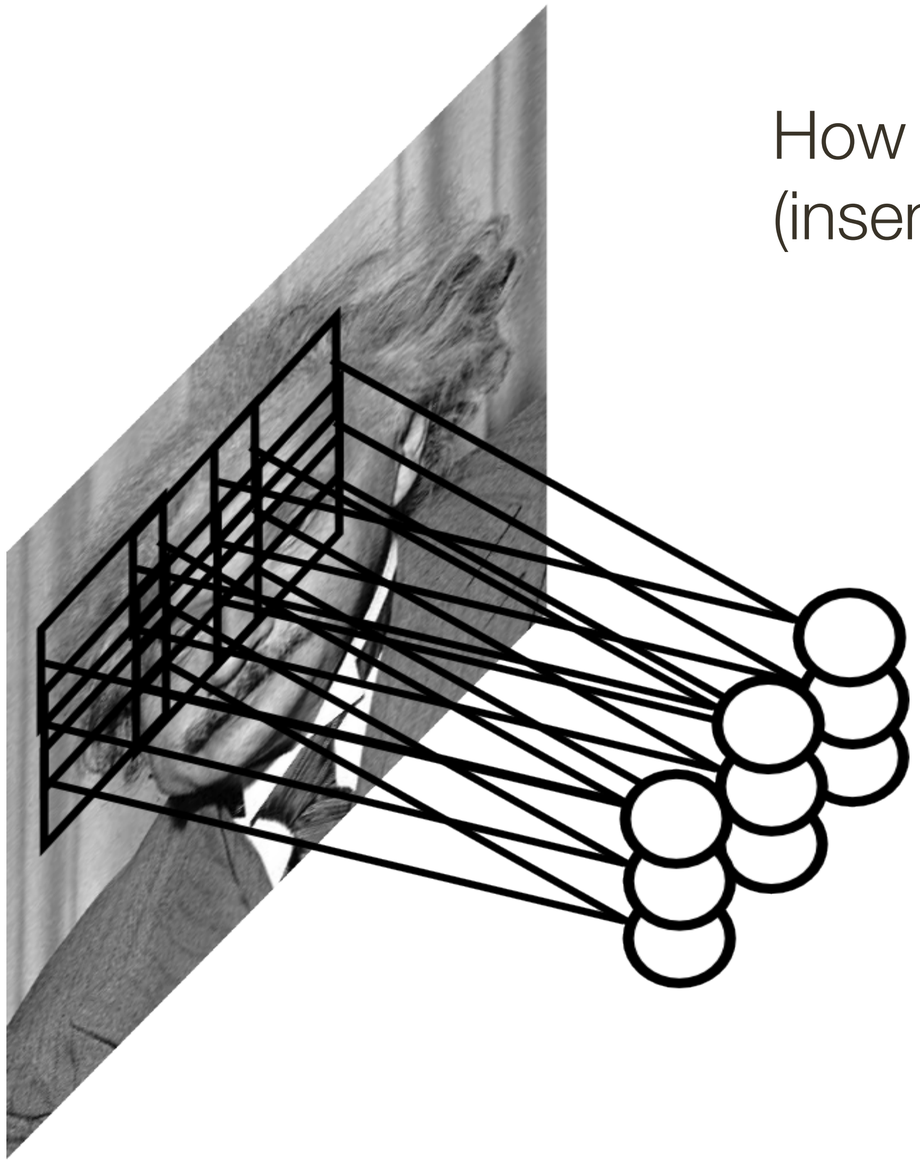


[Zeiler and Fergus, 2013]

Pooling Layer



Let us assume the filter is an “eye” detector

How can we make detection spatially invariant (insensitive to position of the eye in the image)

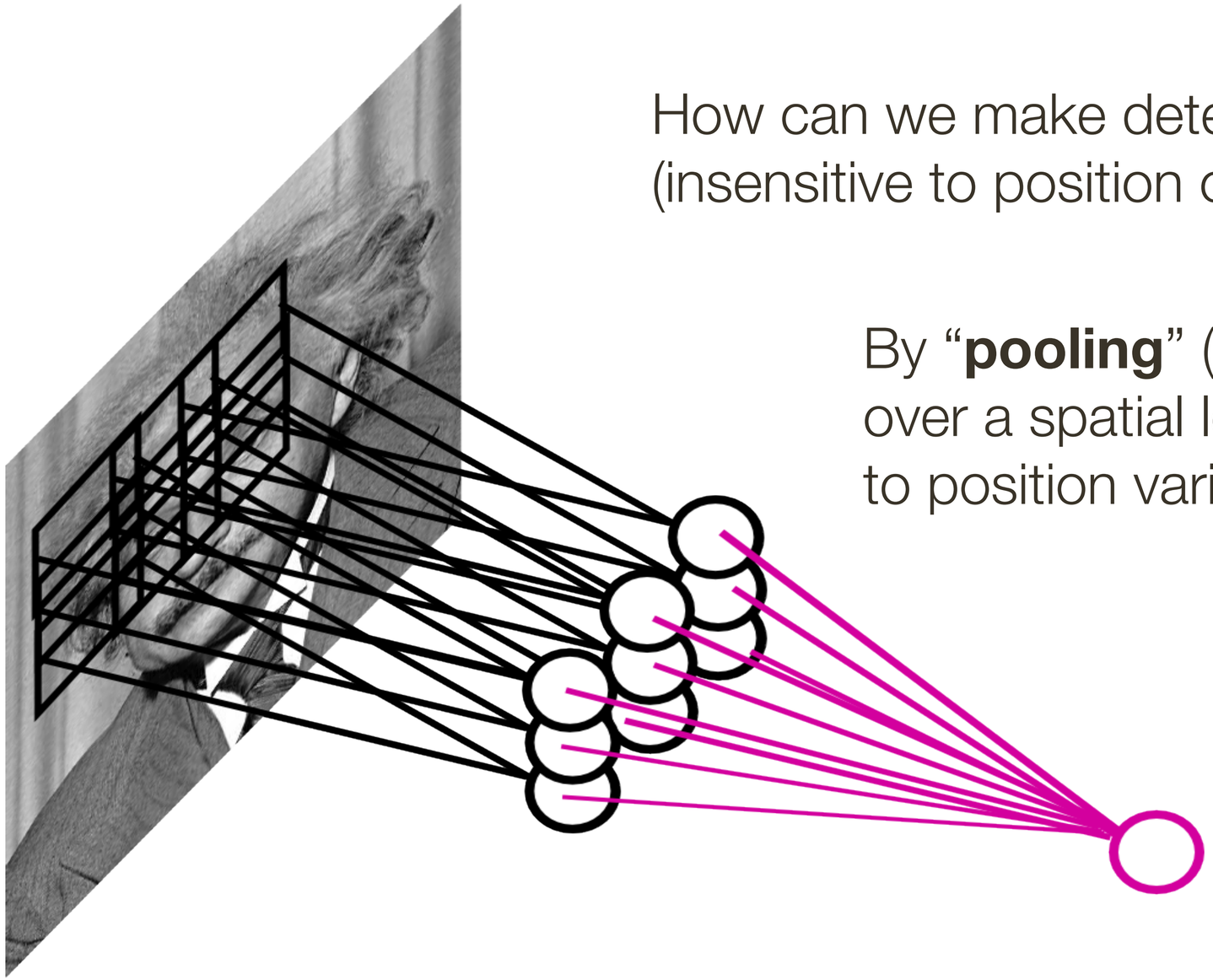
* slide from Marc’Aurelio Renzato

Pooling Layer

Let us assume the filter is an “eye” detector

How can we make detection spatially invariant (insensitive to position of the eye in the image)

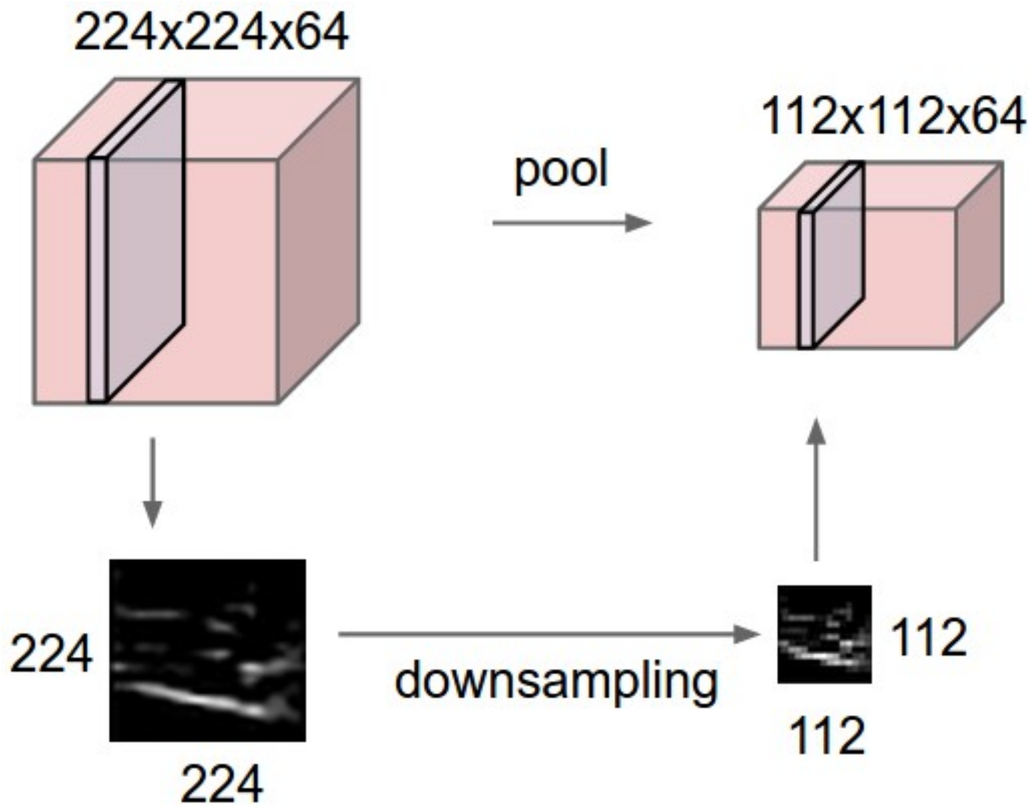
By “**pooling**” (e.g., taking a max) response over a spatial locations we gain robustness to position variations



* slide from Marc’Aurelio Renzato

Pooling Layer

- Makes representation smaller, more manageable and spatially invariant
- Operates over each activation map independently



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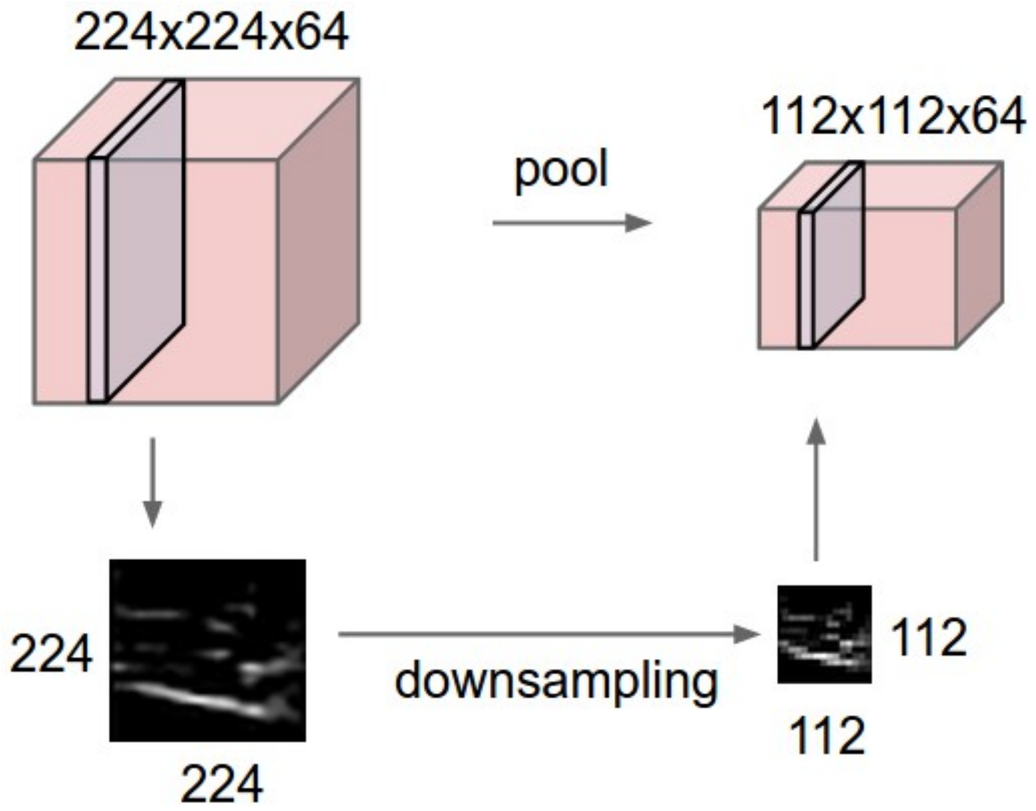

How many **parameters**?

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Pooling Layer

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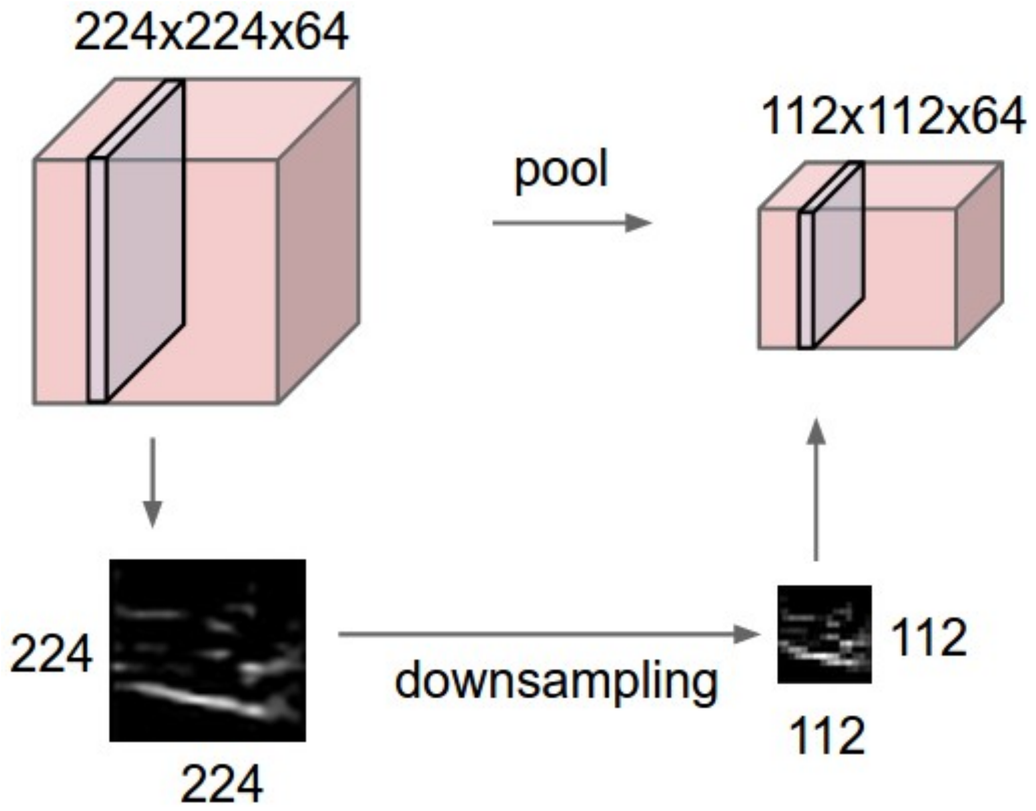

How many **parameters**?

None!

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Max **Pooling**

activation map

1	1	2	4
5	6	7	8
3	2	1	0
1	2	3	4

max pool with 2 x 2 filter
and stride of 2

6	8
3	4

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Average **Pooling**

activation map

1	1	2	4
5	6	7	8
3	2	1	0
1	2	3	4

3.25	5.25
2	2