

### THE UNIVERSITY OF BRITISH COLUMBIA

# **CPSC 425: Computer Vision**



Lecture 25: Classification

## Menu for Today (November 9, 2020)

### **Topics:**

- Classification
- kNN, SVMs

### **Redings:**

- **Today's** Lecture: Forsyth & Ponce (2nd ed.) 15
- **Next** Lecture:

### **Reminders:**

- Assignment 5: Scene Recognition with Bag of Words is out
- Quiz 4 is out and due at the end of day today
- No class on Wednsday



### Bag of Words Representation Scene Classification

# Forsyth & Ponce (2nd ed.) 16.1.3, 16.1.4, 16.1.9



### Today's "fun" Example:

### Audio-Visual Scene Analysis with Self-Supervised Multisensory Features

Andrew Owens Alexei A. Efros UC Berkeley



### Today's "fun" Example:

### Audio-Visual Scene Analysis with Self-Supervised Multisensory Features

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### **Problem**:

Assign new observations into one of a fixed set of categories (classes)

### Key Idea(s):

Build a model of data in a given category based on observations of instances in that category



(assume given set of discrete labels) {dog, cat, truck, plane, ...}

cat



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A classifier is a procedure that accepts as input a set of features and outputs a class **label** (probability over class labels)

Classifiers can be **binary** (face vs. not-face) or **multi-class** (cat, dog, horse, ...).

We build a classifier using a **training set** of labelled examples  $\{(\mathbf{x}_i, y_i)\}$ , where each  $\mathbf{x}_i$  is a feature vector and each  $y_i$  is a class label.

Given a previously unseen observation, we use the classifier to predict its class label.

**Binary**: [0]/[1]

**Multi-class**: [1, 0, 0, 0, ...] (one-hot)





Collect a database of images with labels

- Use ML to train an image classifier
- Evaluate the classifier on test images



### Example training set

### **Example 1**: A Classification Problem

Categorize images of fish - "Atlantic salmon" vs "Pacific salmon"

Use **features** such as length, width, lightness, fin shape & number, mouth position, etc.

Given a previously unobserved image of a salmon, use the learned classifier to guess whether it is an Atlantic or Pacific salmon



### Figure credit: Duda & Hart

## **Example 2**: Real Classification Problem

### **SUN Dataset**

- 131K images
- 908 scene categories

indoor	shopping and dining	auto showroom
outdoor natural	workplace (office building, factory, lab, etc.)	bakery kitchen
outdoor man-made	home or hotel	bakery shop
	transportation (vehicle interiors, stations, etc.)	bank indoor
	sports and leisure	bank vault
	cultural (art, education, religion, millitary, law, politics, etc.)	banquet hall
		bar

## **Example 3**: Real Classification Problem



An object occurring naturally; not made by man

Numbers in Numbers synsets in the

Image

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### ImageNet Dataset

- 14 Million images
- 21K object categories

### Natural object





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water table, water level, ground

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An object occurring naturally; not made by man

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water table, water level, ground

## **Closed-world** problem

**Issue:** Classification assumes that incoming image belongs to one of k classes. However, in practice it is impossible to enumerate all relevant classes in the world, nor would doing so be useful. So how do we deal with images which don't belong?

Solution: Create an "unknown" or "irrelevant" class.

1

Let c be the **class label** and let x be the **measurement** (i.e., evidence)

### posterior probability



 $P(c|x) = \frac{P(x|c)p(c)}{P(x)}$ 

Let c be the **class label** and let x be the **measurement** (i.e., evidence)

class-conditional probability (a.k.a. likelihood)



### posterior probability

Let c be the **class label** and let x be the **measurement** (i.e., evidence)

### Simple case:

- binary classification; i.e.,  $c \in \{1, 2\}$
- features are 1D; i.e.,  $x \in \mathbb{R}$

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Classify **x** as

1 if  $p(1|\mathbf{x}) > p(2|\mathbf{x})$ 

2 if  $p(1|\mathbf{x}) < p(2|\mathbf{x})$ 

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### **General** case:

- multi-class; i.e.,  $c \in \{1, ..., 1000\}$
- features are high-dimensional; i.e.,  $x \in \mathbb{R}^{2,000+}$

Assume we have two classes:  $c_1 = male$ 

### $c_2 = \mathbf{female}$

We have a person who's gender we don't know, who's name is *drew* 



### Assume we have two classes: $c_1 = male$ $c_2 = \mathbf{female}$ We have a person who's gender we don't know, who's name is *drew*



Drew Carey



Drew Barrymore



Assume we have two classes:

We have a person who's gender we don't know, who's name is *drew* 

Classifying drew as being male or female is equivalent to asking is it more probable that *drew* is male or female, i.e. which is greater p(male|drew) $p(\mathbf{female}|drew)$ 



Drew Carey

### $c_1 =$ male $c_2 = \mathbf{female}$



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Assume we have two classes:

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Classifying drew as being male or female is equivalent to asking is it more probable that *drew* is male or female, i.e. which is greater p(male|drew) $p(\mathbf{female}|drew)$ 

### $c_1 = \mathbf{male}$ $c_2 = \mathbf{female}$

 $p(\mathbf{male}|drew) = \frac{p(drew|\mathbf{male})p(\mathbf{male})}{p(drew)}$ 



Name	Gend
Drew	Male
Claudia	Female
Drew	Female
Drew	Female
Alberto	Male
Karin	Female
Nina	Female
Sergio	Male

 $p(\mathbf{male}|drew) = \frac{p(drew|\mathbf{male})p(\mathbf{male})}{p(drew)}$ 





p(male) =

 $p(drew|\mathbf{male}) =$ 

p(drew) =

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 $p(\text{male}) = \frac{3}{8}$  $p(drew|\mathbf{male}) =$ 

p(drew) =

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 $p(\mathbf{male}|drew) = rac{p(drew|\mathbf{male})p(\mathbf{male})}{p(drew)} = 0.125$ 

$$e^{3} = \frac{5}{8}$$

$$e^{3} = \frac{2}{5}$$

Name	Gend
Drew	Male
Claudia	Female
Drew	Female
Drew	Female
Alberto	Male
Karin	Female
Nina	Female
Sergio	Male

 $p(\mathbf{female}|drew) = \frac{p(drew|\mathbf{female})p(\mathbf{female})}{p(\mathbf{female})}$ = 0.25





- 17 samples
- 15 samples

0

30



Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

- 17 samples
- 15 samples 0

Given a (g,b) pixel value from a new patch is it more likely to be be grass or sky?

These could be (g,b) pixel value of an image patch with sky

0

### These could be (g,b) pixel value of an image patch with grass





• 17 samples • 15 samples

$$p(blue) = \frac{17}{17 + 15}$$

$$p(green) = \frac{15}{17 + 15}$$

0



Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

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• 17 samples • 15 samples

$$p(blue) = \frac{17}{17 + 15}$$

$$p(green) = \frac{15}{17 + 15}$$



# **Example**: 2D Bayes Classifier • 17 samples

0

0

0

• 15 samples

$$p(blue) = \frac{17}{17 + 15}$$

$$p(green) = \frac{15}{17 + 15}$$



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### Simple case:

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## Bayes' Risk

# Some errors may be inevitable: the minimum risk (shaded area) is called the **Bayes' risk**





## **Discriminative** vs. Generative

Finding a decision boundary is not the same as modeling a conditional density — while a normal density here is a poor fit to P(1|x), the quality of the classifier depends only on how well the boundary is positioned





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## Bayes' Risk

# Some errors may be inevitable: the minimum risk (shaded area) is called the **Bayes' risk**





### **Loss Functions** and Classifiers

### Loss

- Some errors may be more expensive than others **Example:** A fatal disease that is easily cured by a cheap medicine with no side-effects. Here, false positives in diagnosis are better than false negatives
- We discuss two class classification:  $L(1 \rightarrow 2)$  is the loss caused by calling 1 a 2
- **Total risk** of using classifier **s** is

### $R(s) = Pr\{1 \rightarrow 2 \mid \text{using } \mathbf{s}\} \ L(1 \rightarrow 2) + Pr\{2 \rightarrow 1 \mid \text{using } \mathbf{s}\} \ L(2 \rightarrow 1)$

### **Two Class** Classification

than for 2

Classify **x** as

**Decision boundary:** points where the loss is the same for either class.

### Generally, we should classify as 1 if the expected loss of classifying as 1 is less

### 1 if $p(1|\mathbf{x}) L(1 \rightarrow 2) > p(2|\mathbf{x}) L(2 \rightarrow 1)$

2 if  $p(1|\mathbf{x}) L(1 \rightarrow 2) < p(2|\mathbf{x}) L(2 \rightarrow 1)$ 

**Training error** is the error a classifier makes on the training set

unseen testing set

error

called **overfitting** 

- We want to minimize the **testing error** the error the classifier makes on an

Classifiers that have small training error may not necessarily have small testing

The phenomenon that causes testing error to be worse than training error is

**Underfitting**: model is too simple to represent all the relevant class characteristics



**Underfitting**: model is too simple to represent all the relevant class characteristics

**Overfitting**: model is too complex and fits irrelevant characteristics (noise) in the data



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the classifier on the rest of the data and evaluate on the validation set

Try out what hyperparameters work best on test set.



### We cannot reliably estimate the error rate of the classifier using the training set

# An alternative is to split some training data to form a validation set, then train

the classifier on the rest of the data and evaluate on the validation set

Trying out what hyperparameters work best on test set: Very bad idea. The test set is a proxy for the generalization performance! Use only VERY SPARINGLY, at the end.

train data

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# An alternative is to split some training data to form a validation set, then train

test data

the classifier on the rest of the data and evaluate on the validation set

		train da
fold 1	fold 2	fold 3

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# **Cross-validation** involves performing multiple splits and averaging the error over all splits



## **Confusion** Matrix

When evaluating a multi-class classifier, it may be useful to know how often certain classes are often misclassified as others.

### A confusion matrix is a table whose (i,j)th entry is the frequency (or proportion) an item of true class i was labelled as j by the classifier.



Forsyth & Ponce (2nd ed.) Figure 15.3. Original credit: H. Zhang et al., 2006. 50

## **Receiver Operating Characteristics (ROC)**

### **ROC curves** plot trade-off between false positives and false negatives

Figure from M. J. Jones and J. Rehg, "Statistical color models with application to skin detection," Proc. CVPR, 1999, IEEE



## **Receiver Operating Characteristics (ROC)**

What is a ROC curve for a perfect classifier?

**ROC curves** plot trade-off between false positives and false negatives

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Parametric classifiers are **model driven**. The parameters of the model are learned from training examples. New data points are classified by the learned model.

- fast, compact
- flexibility and accuracy depend on model assumptions

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Parametric classifiers are **model driven**. The parameters of the model are learned from training examples. New data points are classified by the learned model.

- fast, compact
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Non-parametric classifiers are **data driven**. New data points are classified by comparing to the training examples directly. "The data is the model". - slow

highly flexible decision boundaries

## Nearest Neighbor Classifier

space.

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### Given a new data point, assign the label of nearest training example in feature



## Nearest Neighbor Classifier

space.

### Given a new data point, assign the label of nearest training example in feature



Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

## k-Nearest Neighbor (kNN) Classifier

by **majority vote**.

various dimensions

minimizing probability of error

- We can gain some robustness to noise by voting over **multiple** neighbours.
- Given a **new** data point, find the k nearest training examples. Assign the label

Simple method that works well if the **distance measure** correctly weights the

For **large data sets**, as k increases kNN approaches optimality in terms of

## k-Nearest Neighbor (kNN) Classifier

1-Nearest Neighbor Classifier



15-Nearest Neighbor Classifier

kNN decision boundaries respond to local clusters where one class dominates

Figure credit: Hastie, Tibshirani & Friedman (2nd ed.)