THE UNIVERSITY OF BRITISH COLUMBIA

CPSC 425: Computer Vision

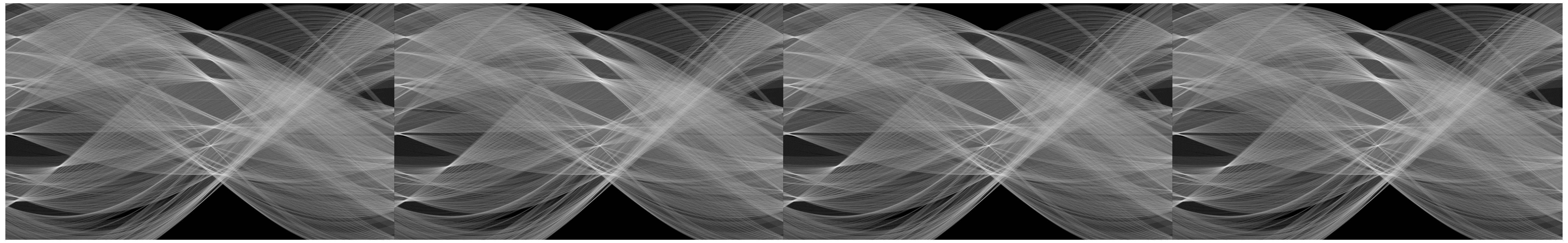


Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

Lecture 23: Stereo (cont)

Menu for Today (**November 4, 2020**)

Topics:

— Stereo Vision
— More Than 2 Cameras
— Structured Light
— Optical Flow

Redings:

— **Today's** Lecture: Forsyth & Ponce (2nd ed.) 10.6, 6.2.2, 9.3.1, 9.3.3, 9.4.2
— **Next** Lecture: None

Reminders:

— **Assignment 4**: RANSAC and Panoramas due **November 6th**

— **Quiz** next Monday, **November 9th**

Lecture 22: Re-cap Stereo Vision

With two eyes, we acquire images of the world from slightly different viewpoints

We perceive **depth** based on **differences in the relative position of points** in the left image and in the right image

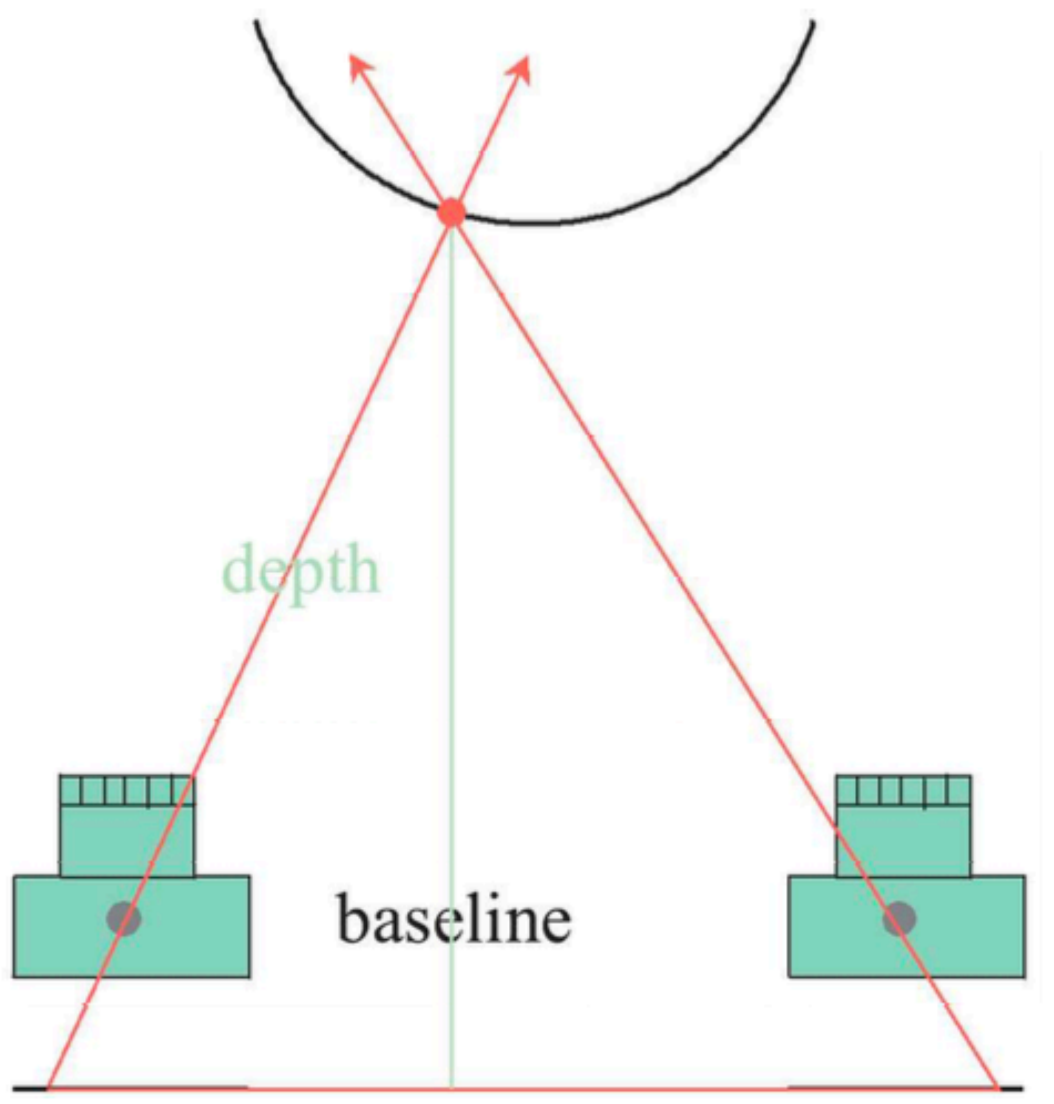
Lecture 22: Re-cap Stereo Vision

Task: Compute depth from two images acquired from (slightly) different viewpoints

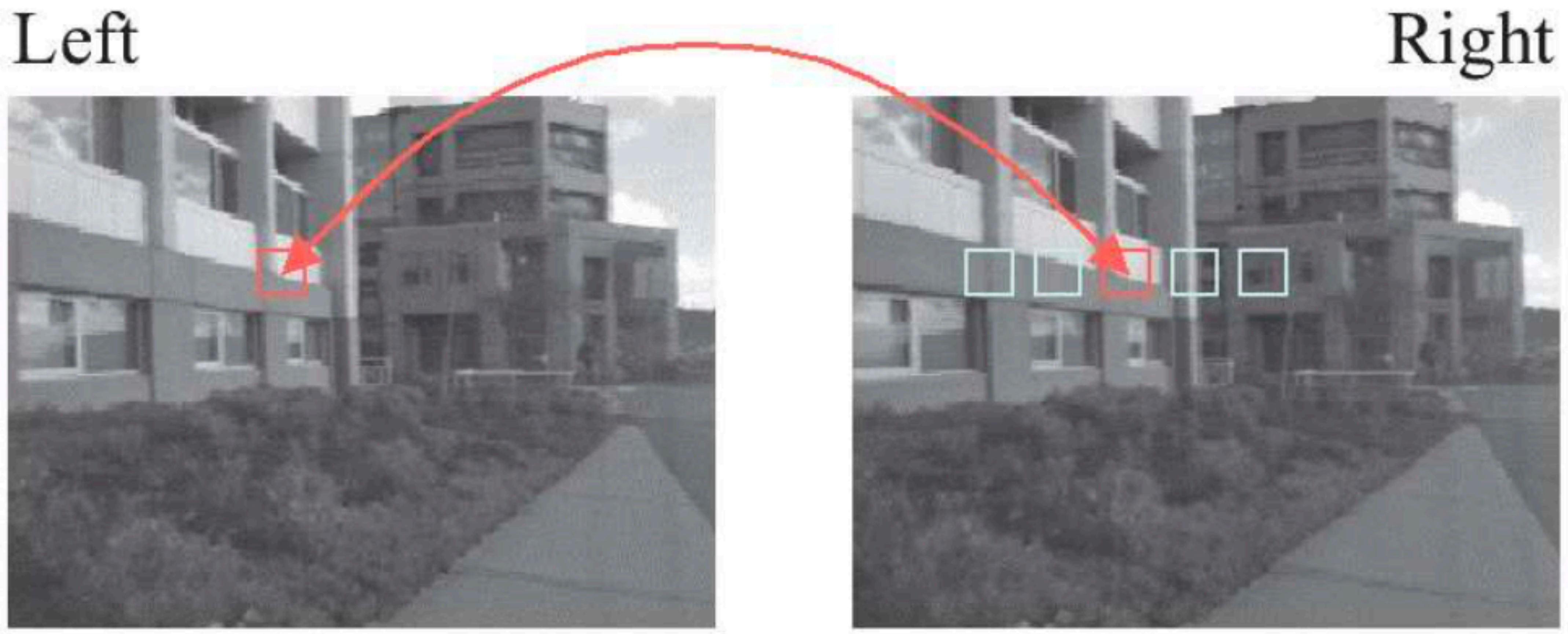
Approach: “Match” locations in one image to those in another

Sub-tasks:
— Calibrate cameras and camera positions
— Find all corresponding points (the hardest part)
— Compute depth and surfaces

Lecture 22: Re-cap Stereo Vision



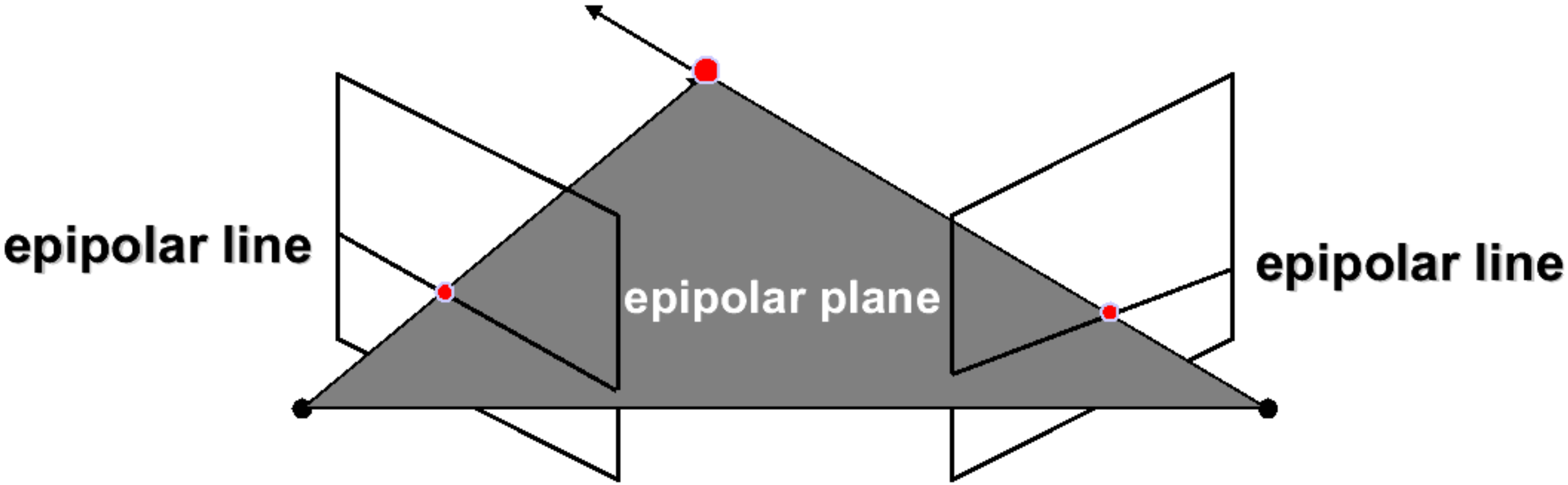
Triangulate on two images of the same point



Match correlation windows across scan lines

Image credit: Point Grey Research
Slide credit: Trevor Darrell

The **Epipolar** Constraint



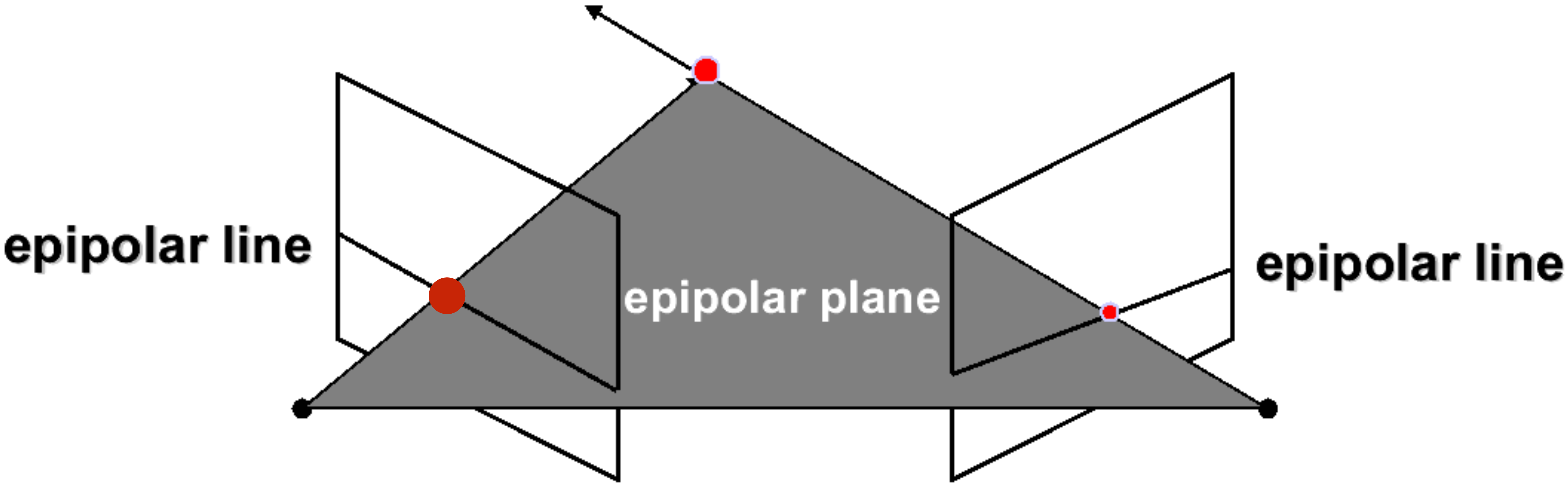
Matching points lie along corresponding epipolar lines

Reduces correspondence problem to 1D search along conjugate epipolar lines

Greatly reduces cost and ambiguity of matching

Slide credit: Steve Seitz

The **Epipolar** Constraint



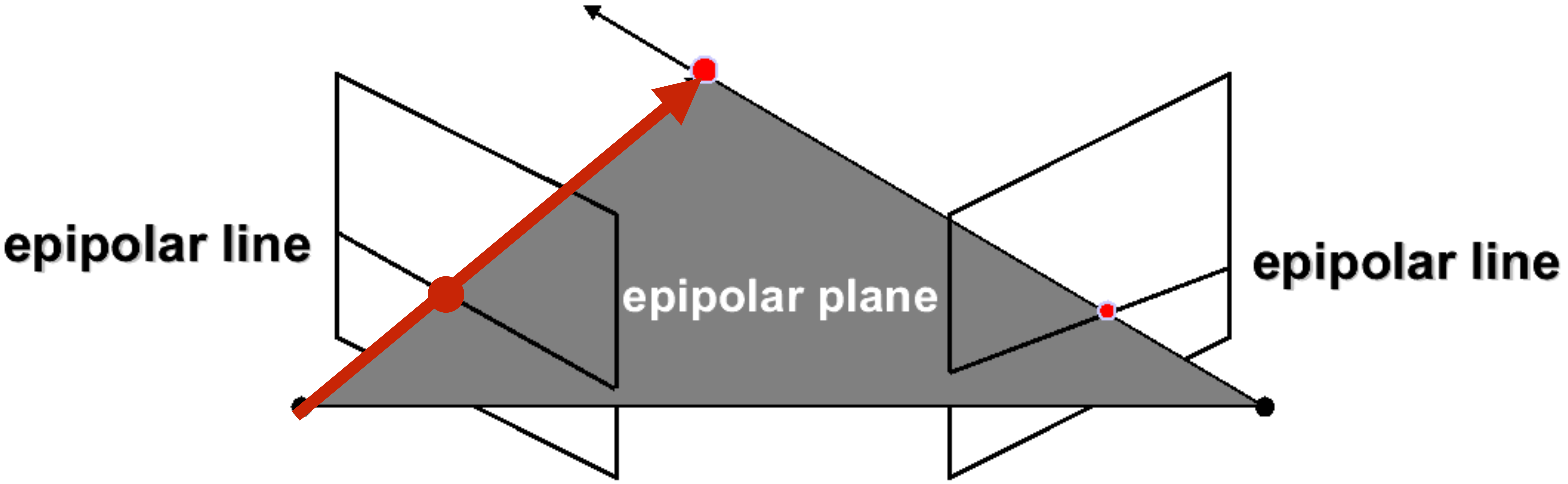
Matching points lie along corresponding epipolar lines

Reduces correspondence problem to 1D search along conjugate epipolar lines

Greatly reduces cost and ambiguity of matching

Slide credit: Steve Seitz

The **Epipolar** Constraint



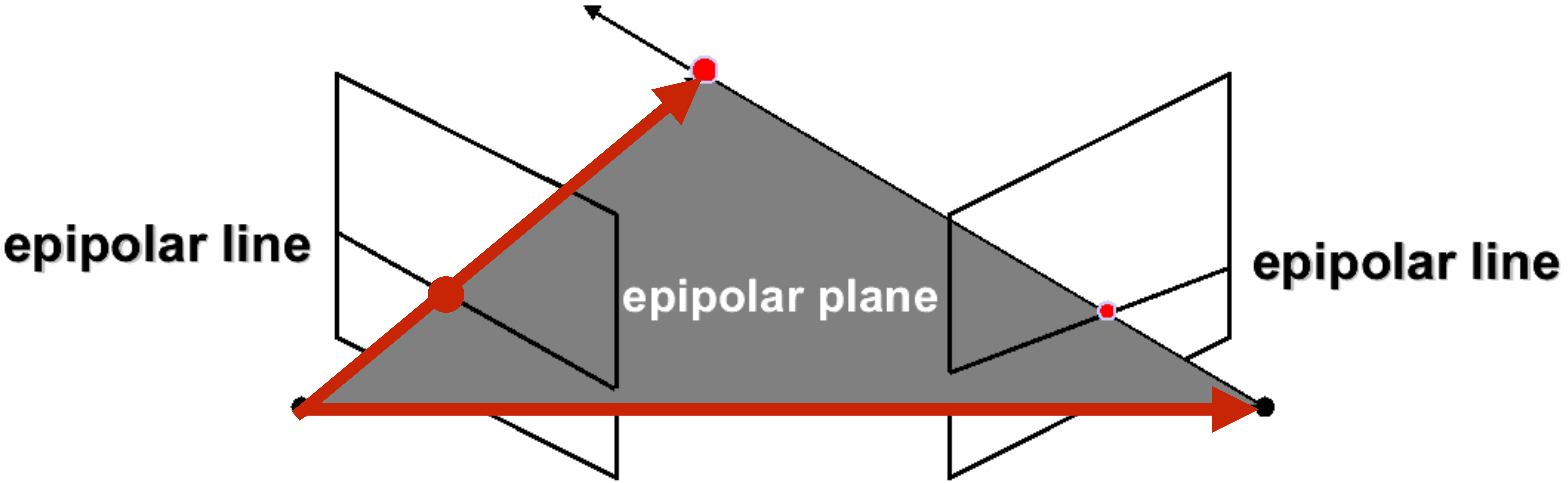
Matching points lie along corresponding epipolar lines

Reduces correspondence problem to 1D search along conjugate epipolar lines

Greatly reduces cost and ambiguity of matching

Slide credit: Steve Seitz

The **Epipolar** Constraint



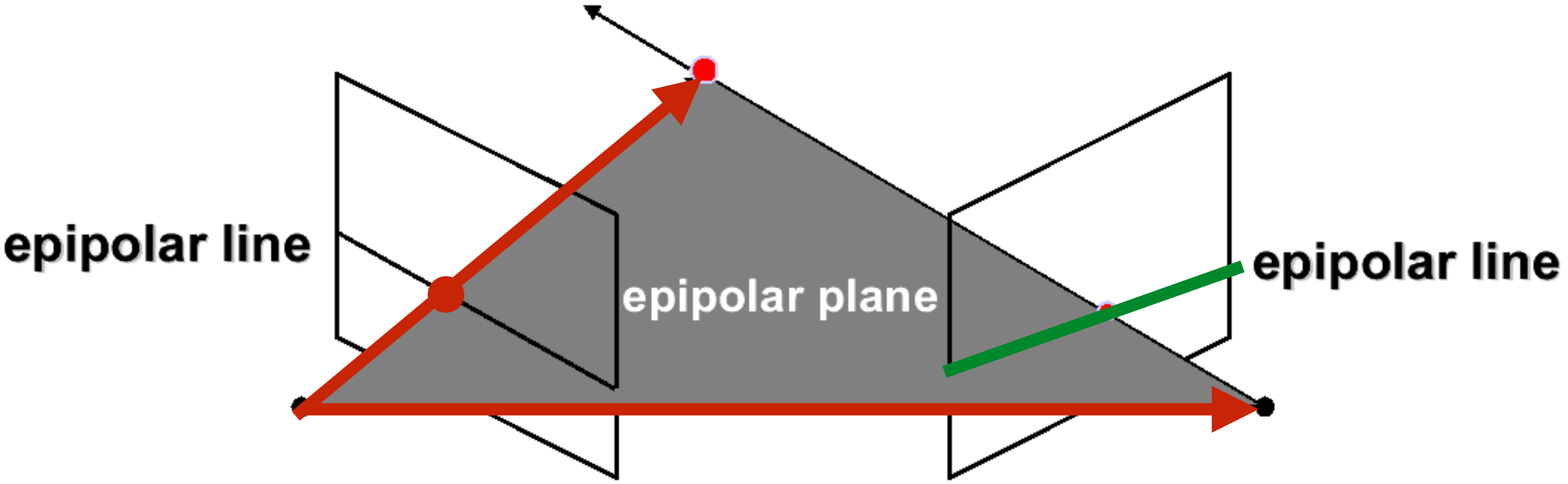
Matching points lie along corresponding epipolar lines

Reduces correspondence problem to 1D search along conjugate epipolar lines

Greatly reduces cost and ambiguity of matching

Slide credit: Steve Seitz

The **Epipolar** Constraint



Matching points lie along corresponding epipolar lines

Reduces correspondence problem to 1D search along conjugate epipolar lines

Greatly reduces cost and ambiguity of matching

Slide credit: Steve Seitz

Simplest Case: **Rectified** Images

Image planes of cameras are **parallel**

Focal **points** are at same height

Focal **lengths** same

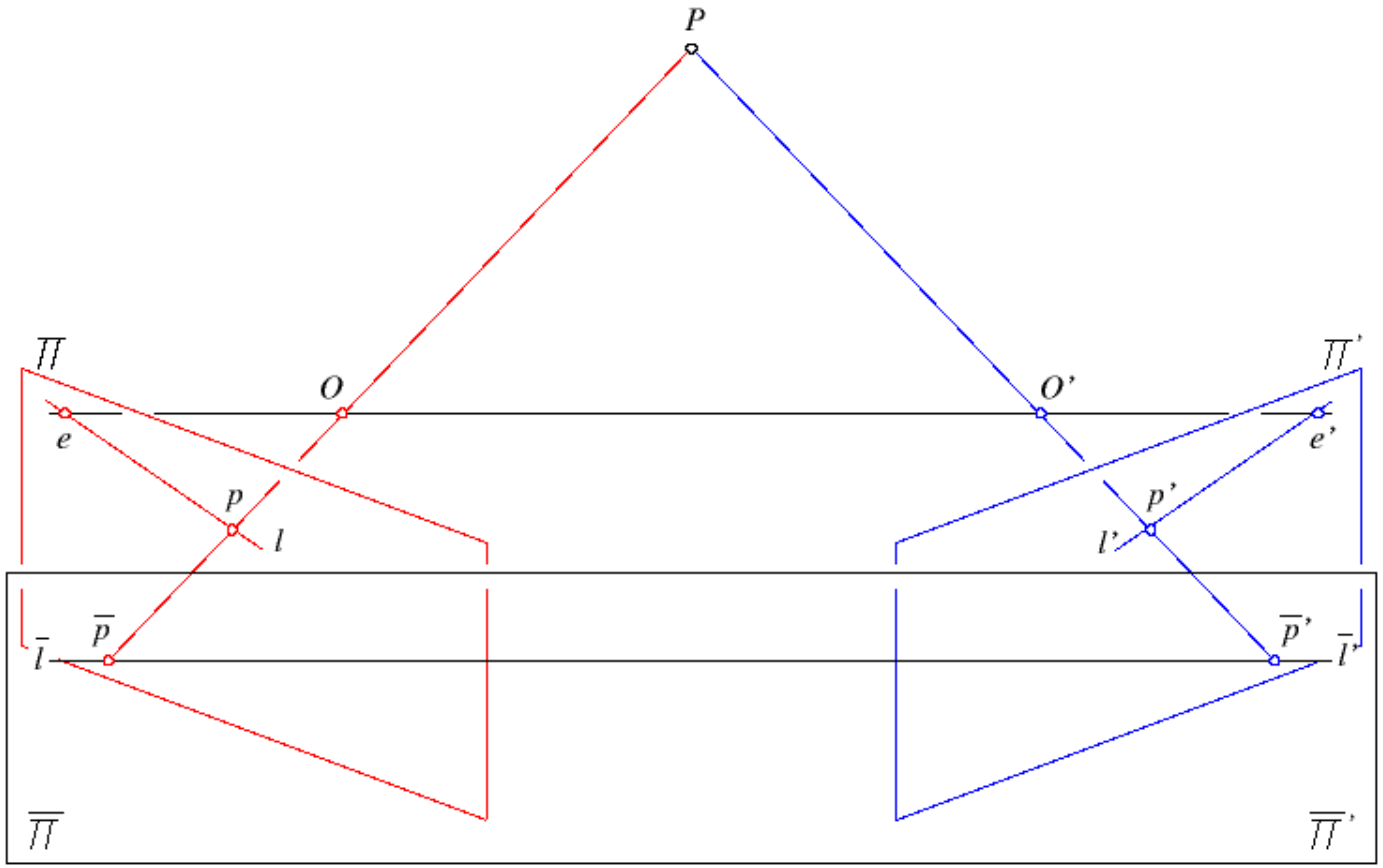
Then, **epipolar lines** fall along the **horizontal scan lines** of the images

We assume images have been **rectified** so that epipolar lines correspond to scan lines

— Simplifies algorithms

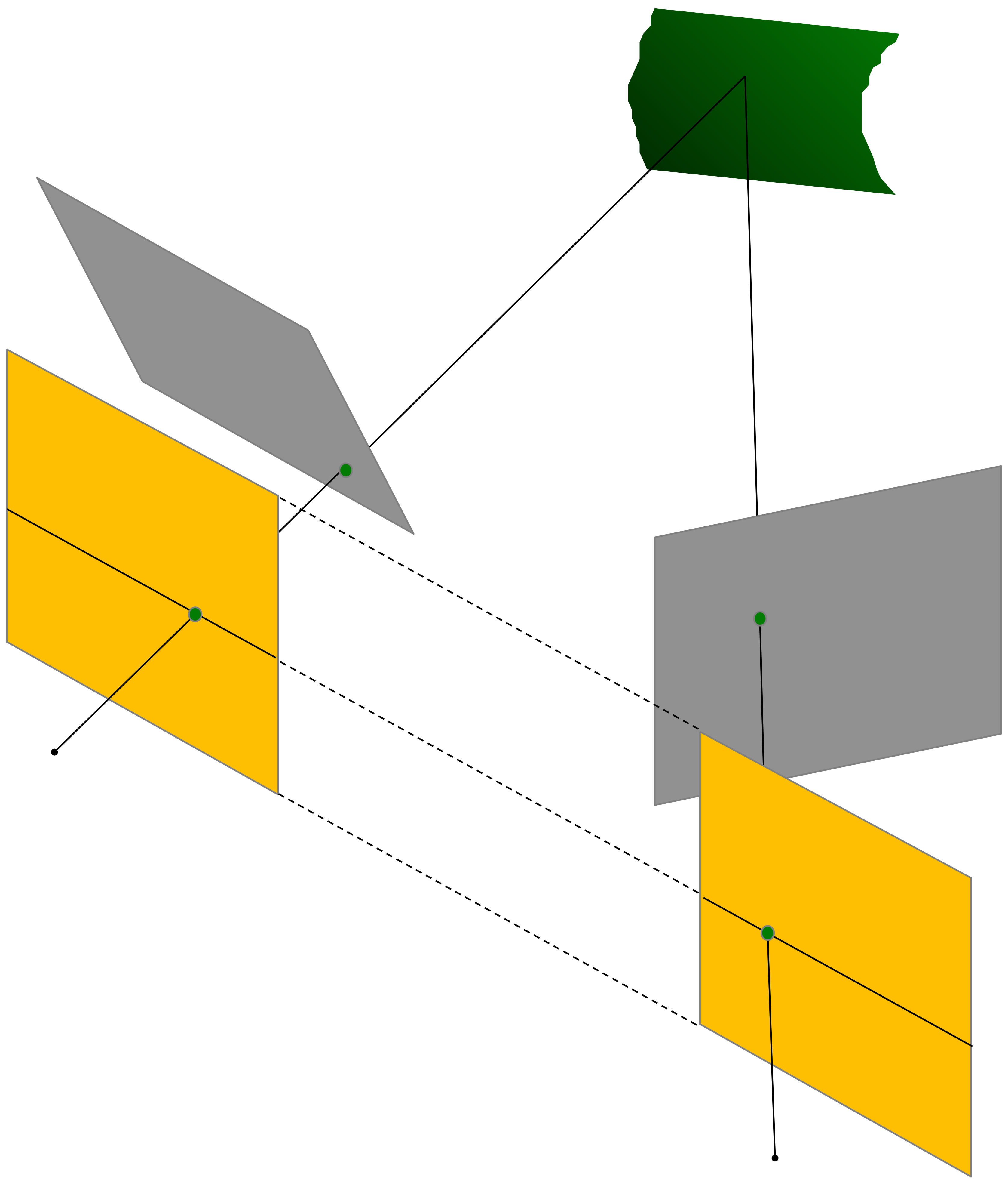
— Improves efficiency

Rectified Stereo Pair



Rectified Stereo Pair

Reproject image planes onto a common plane parallel to the line between camera centers



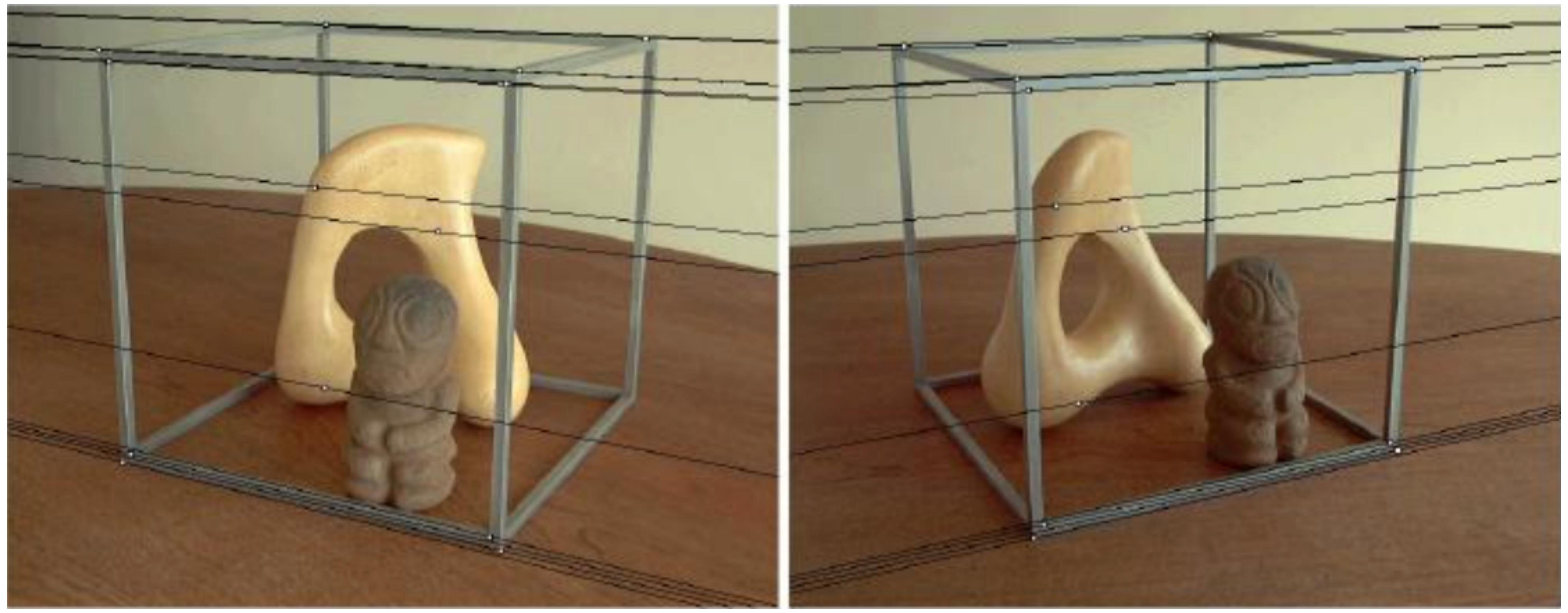
Need two homographies (3x3 transform), one for each input image reprojection

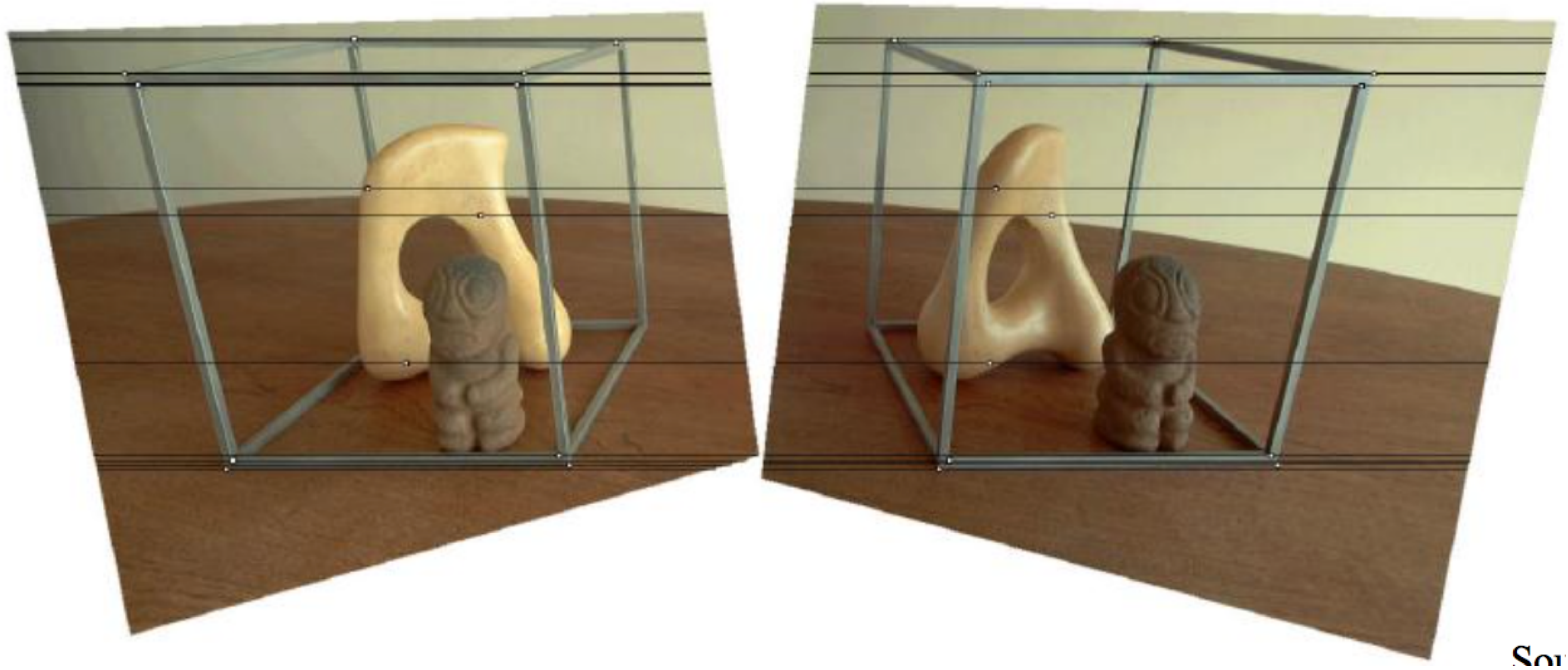
C. Loop and Z. Zhang. Computing Rectifying Homographies for Stereo Vision.Computer Vision and Pattern Recognition, 1999.

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Rectified Stereo Pair: Example

Before Rectification

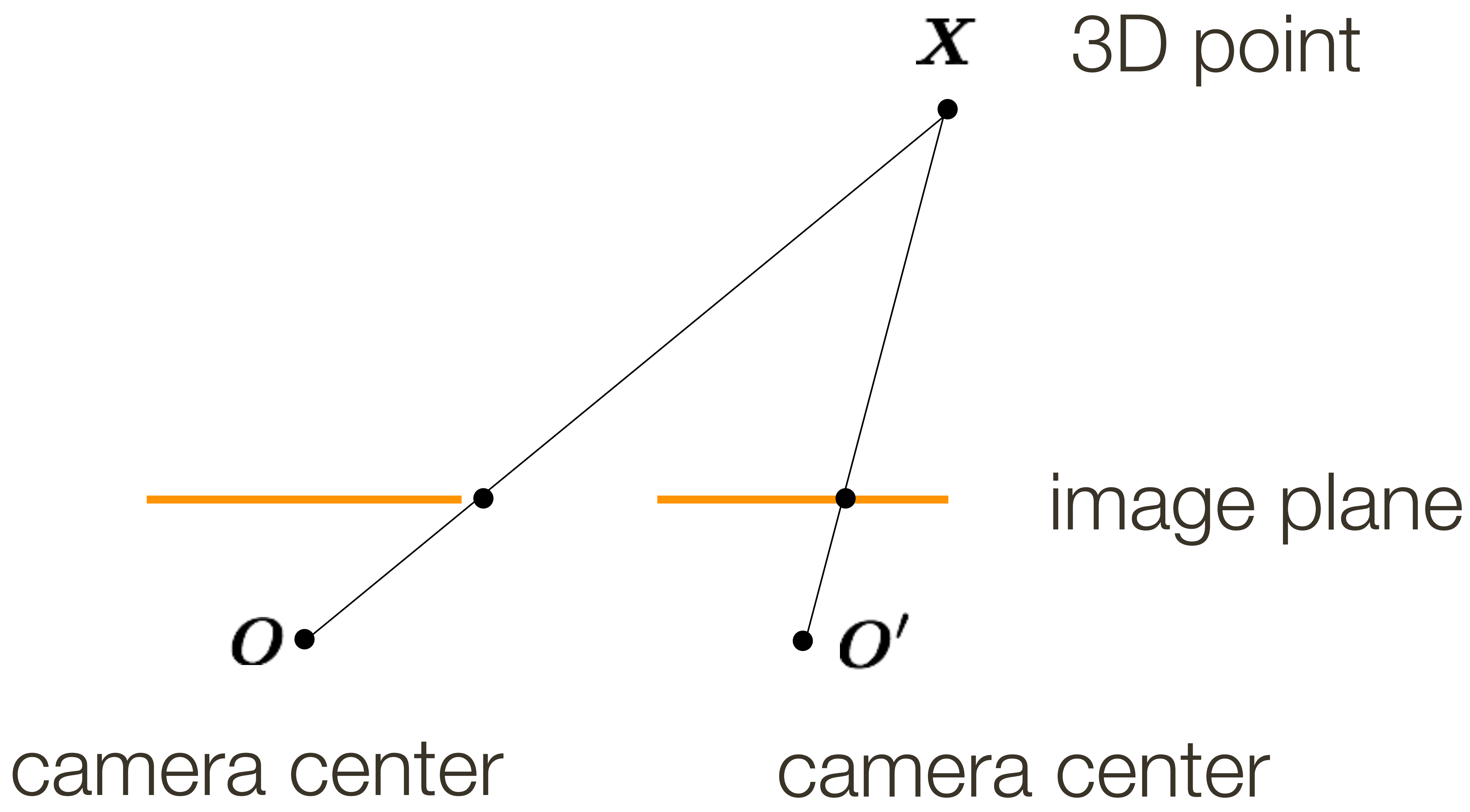




After Rectification

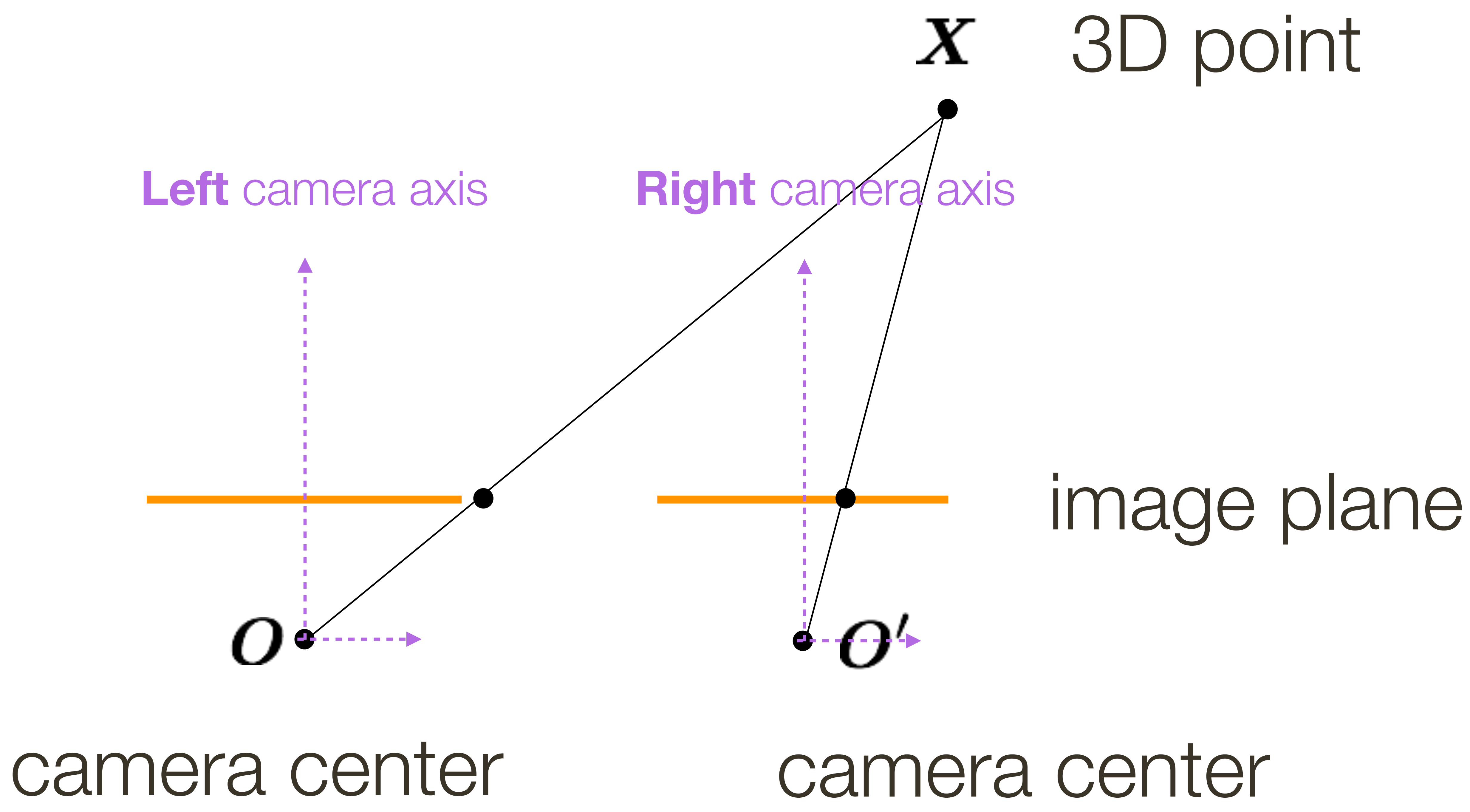
Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Rectified Stereo Pair: Depth Estimate



Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Rectified Stereo Pair: Depth Estimate



Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Rectified Stereo Pair: Depth Estimate

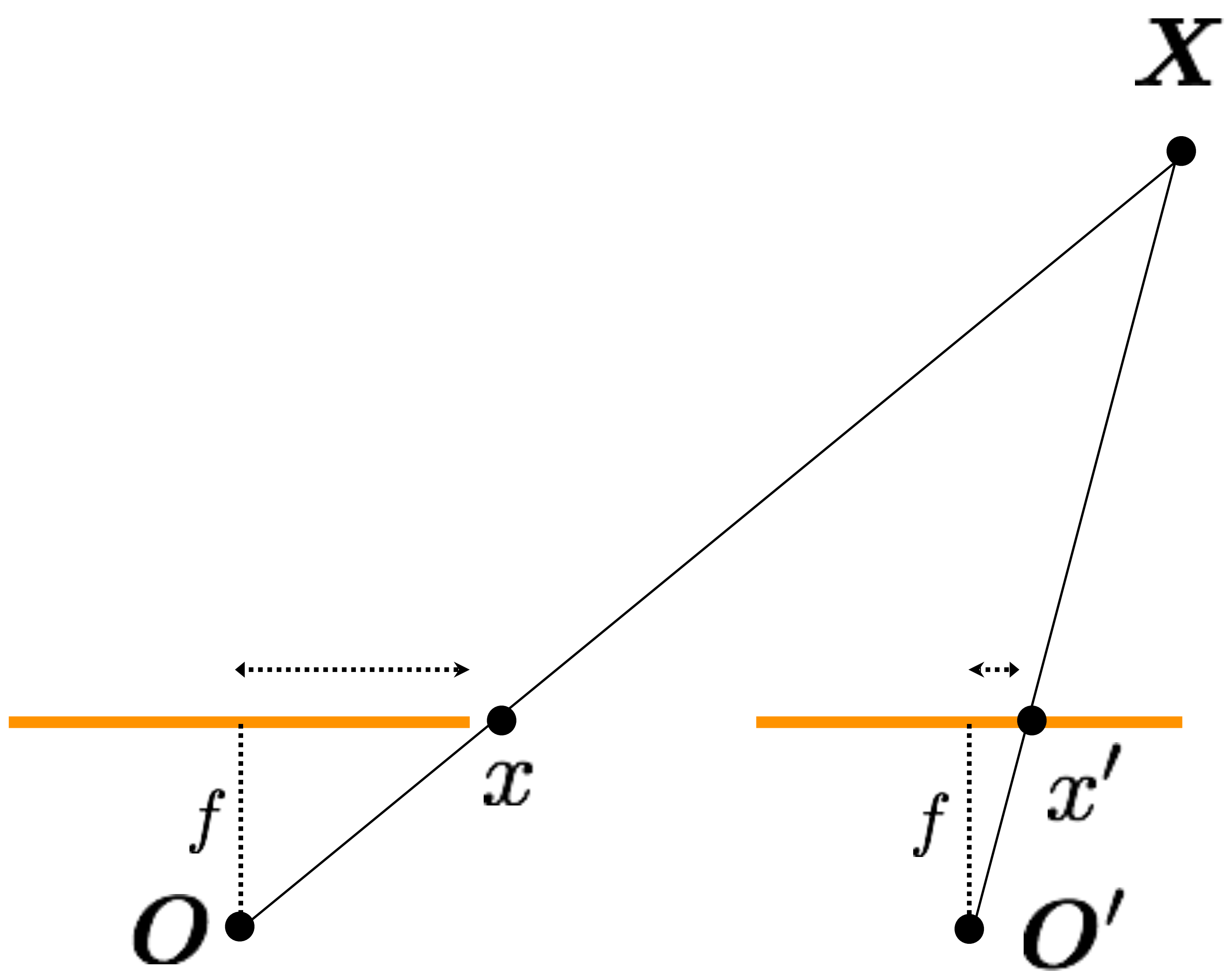
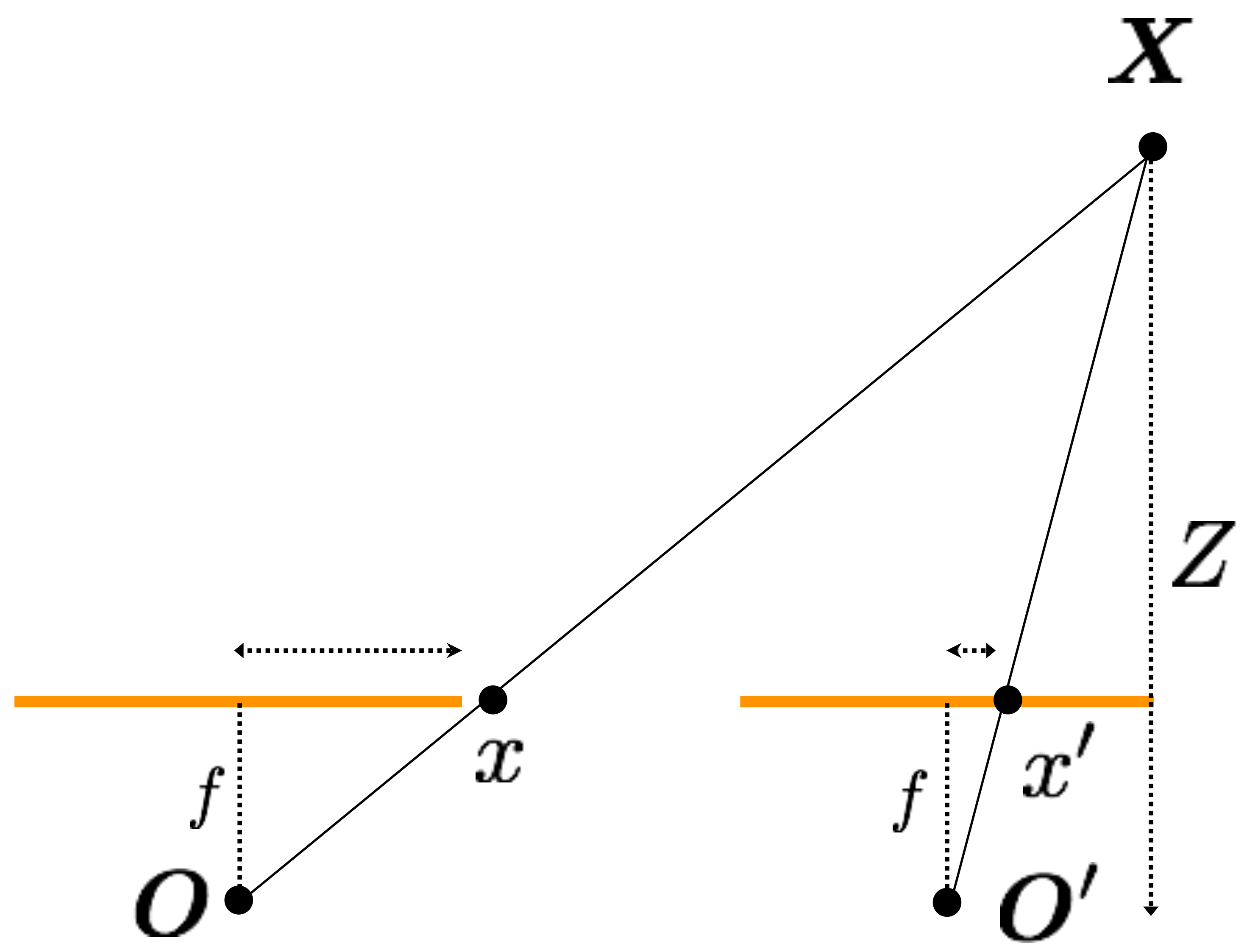


image plane

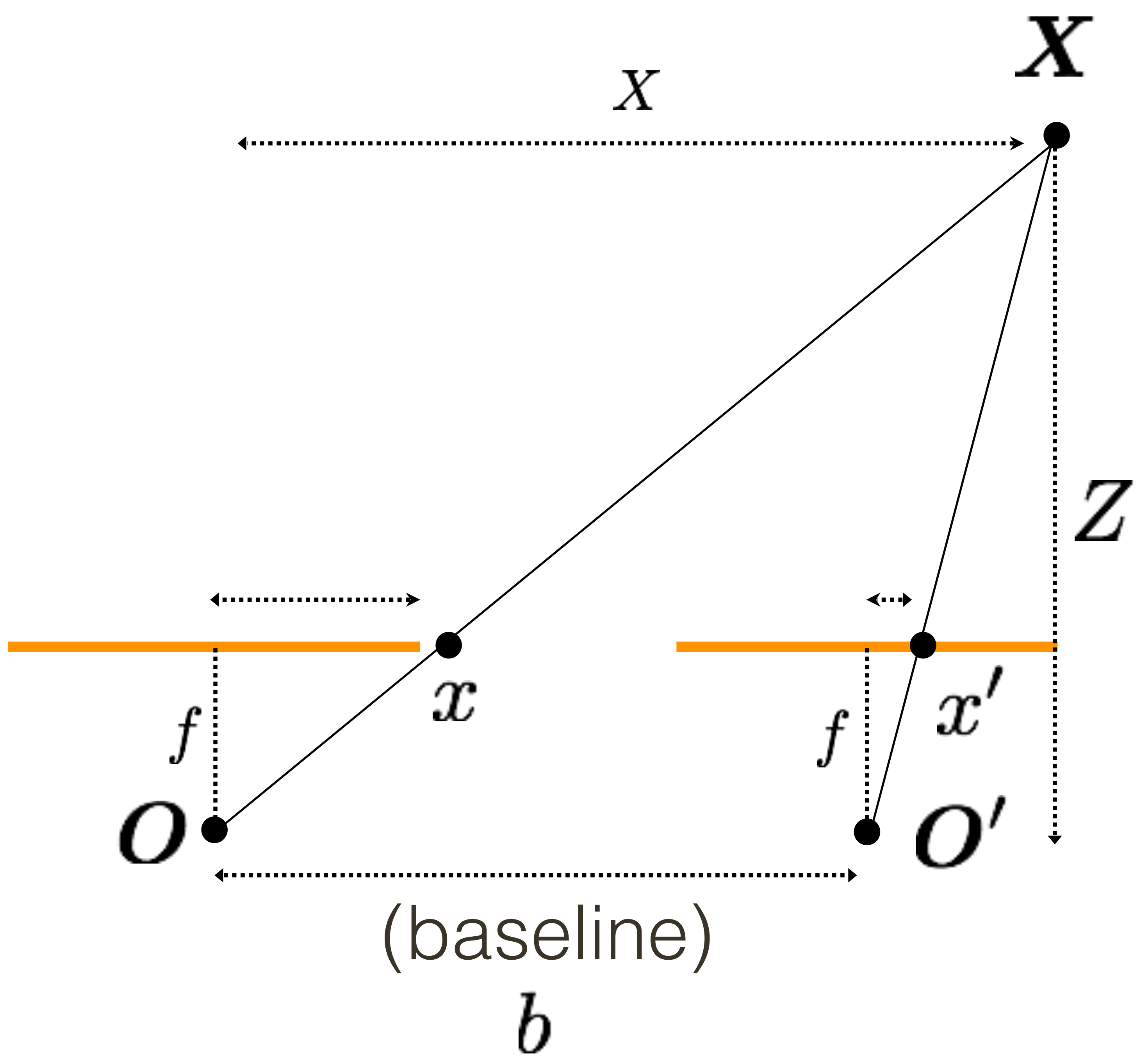
Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Rectified Stereo Pair: Depth Estimate



Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

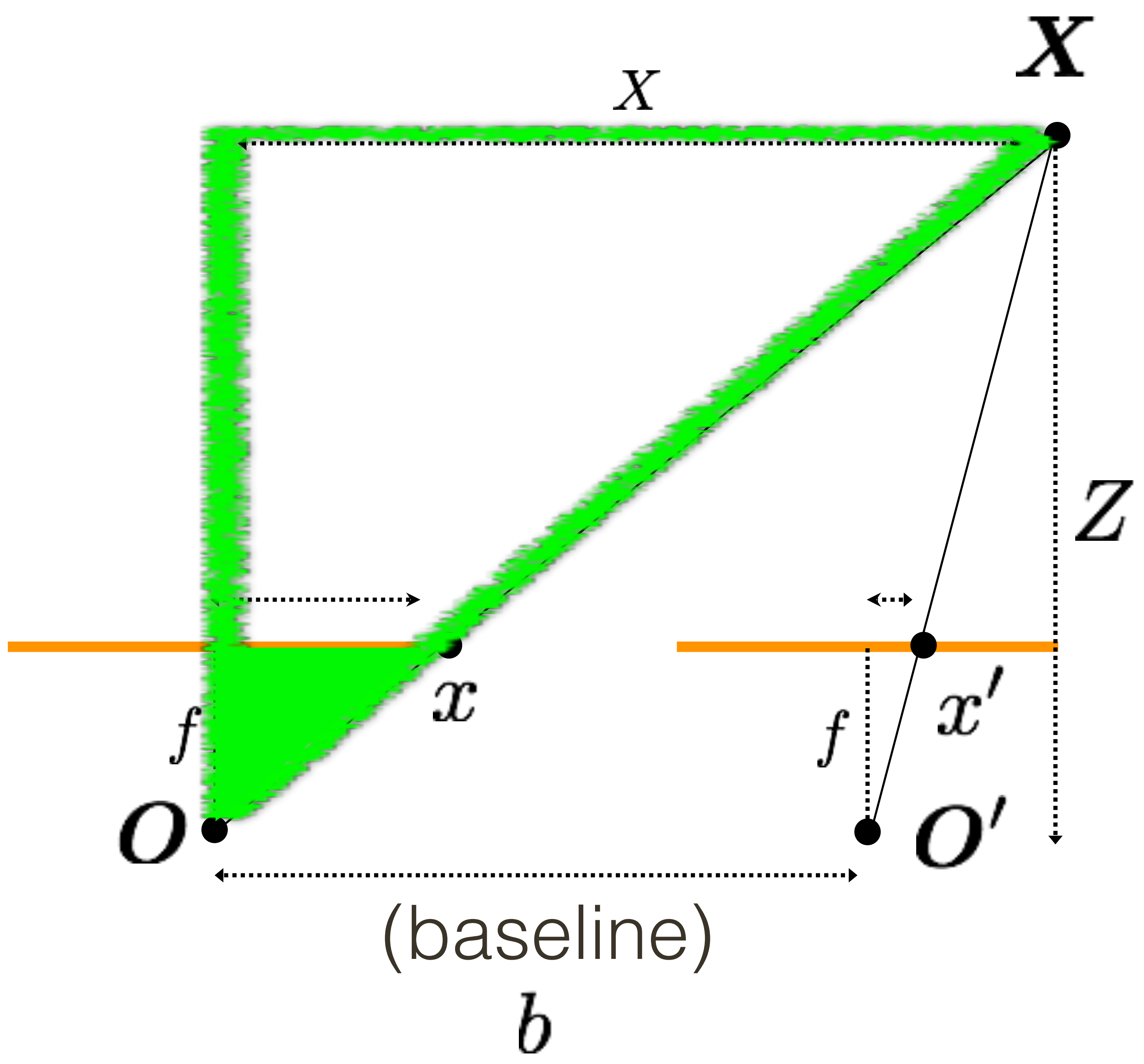
Rectified Stereo Pair: Depth Estimate



Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Rectified Stereo Pair: Depth Estimate

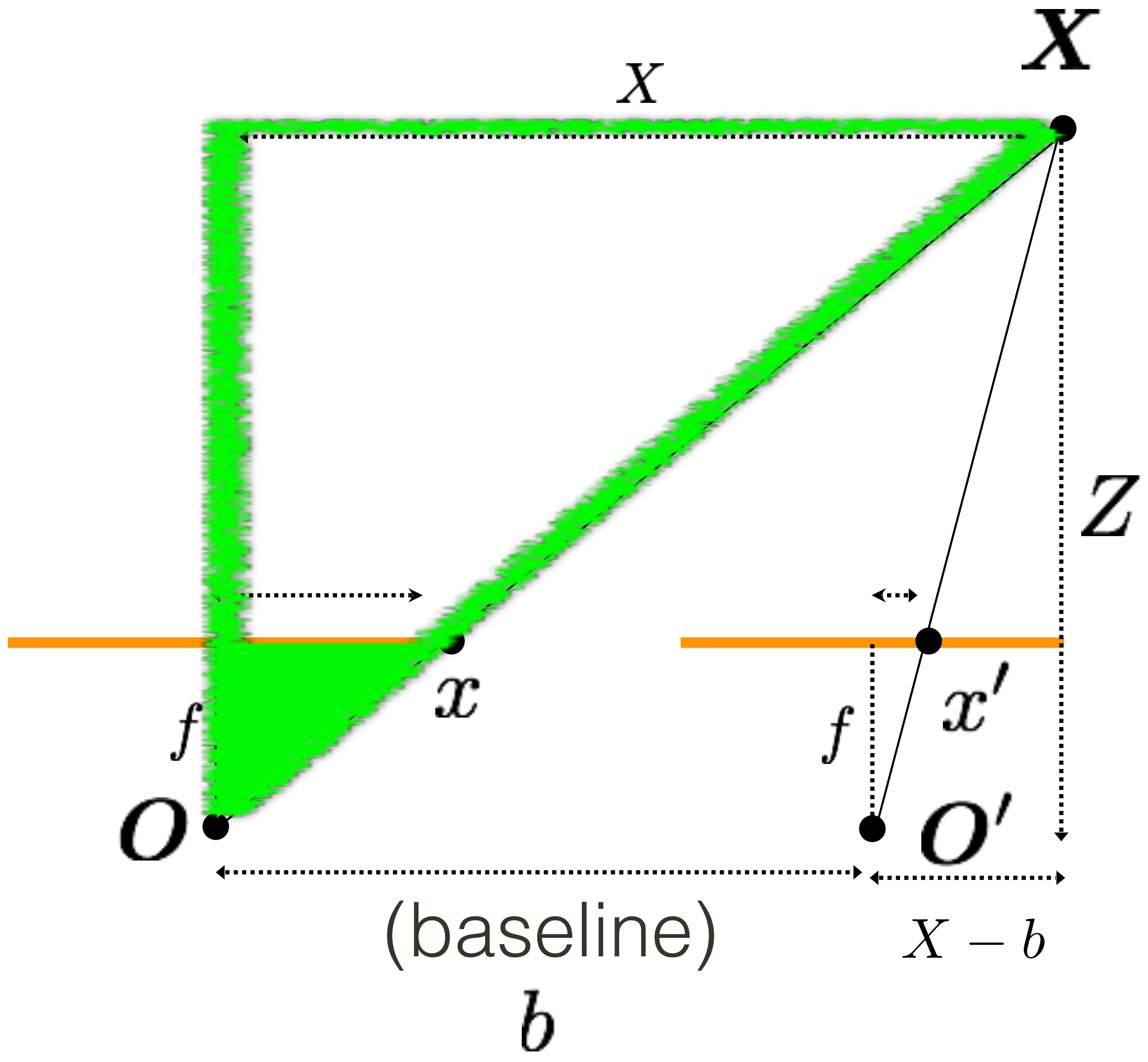
$$\frac{X}{Z} = \frac{x}{f}$$



Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Rectified Stereo Pair: Depth Estimate

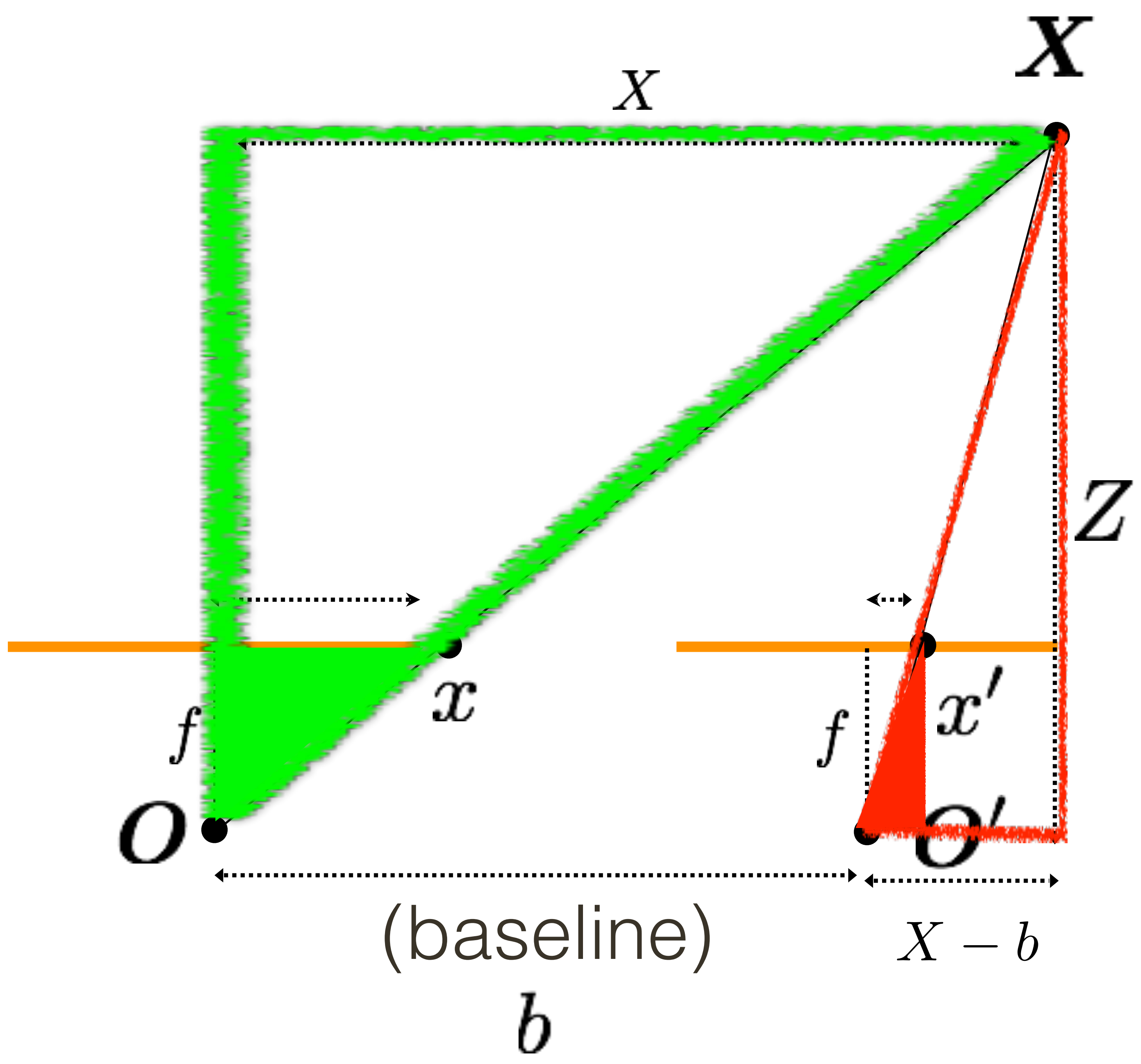
$$\frac{X}{Z} = \frac{x}{f}$$



Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Rectified Stereo Pair: Depth Estimate

$$\frac{X}{Z} = \frac{x}{f}$$

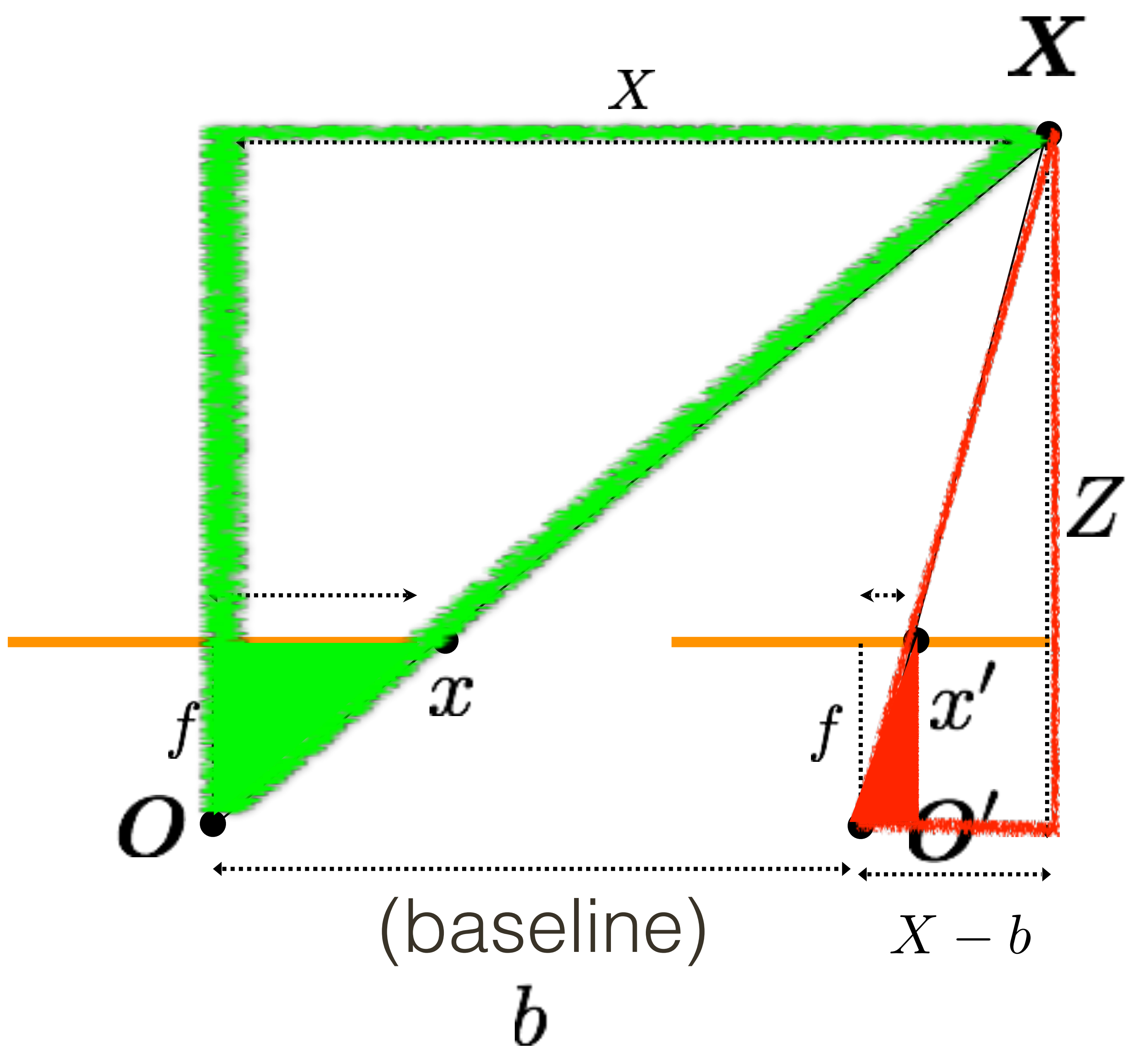


$$\frac{X-b}{Z} = \frac{x'}{f}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Rectified Stereo Pair: Depth Estimate

$$\frac{X}{Z} = \frac{x}{f}$$



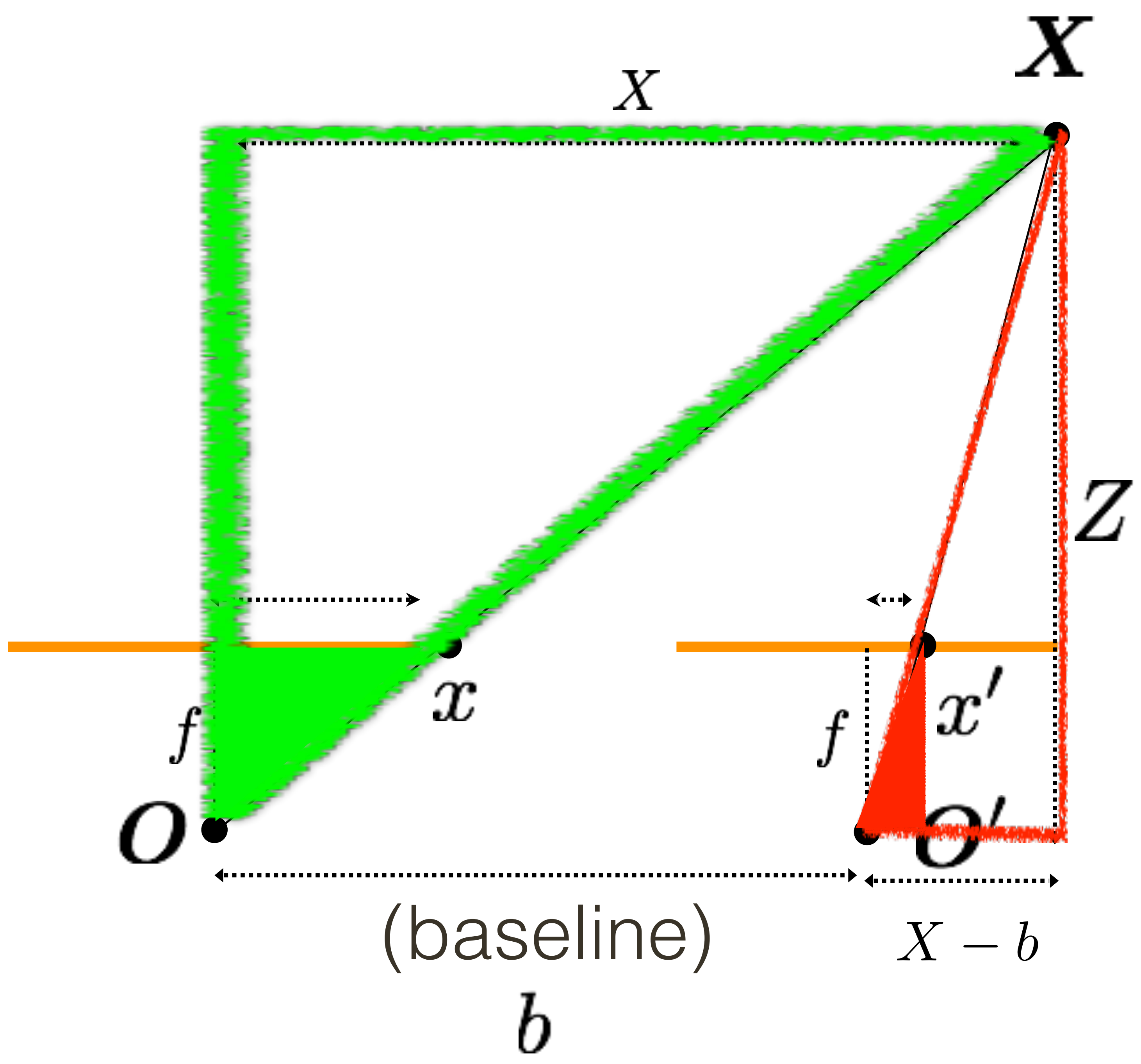
$$\frac{X - b}{Z} = \frac{x'}{f}$$

$$\frac{X}{Z} - \frac{b}{Z} = \frac{x'}{f}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Rectified Stereo Pair: Depth Estimate

$$\frac{X}{Z} = \frac{x}{f}$$



$$\frac{X - b}{Z} = \frac{x'}{f}$$

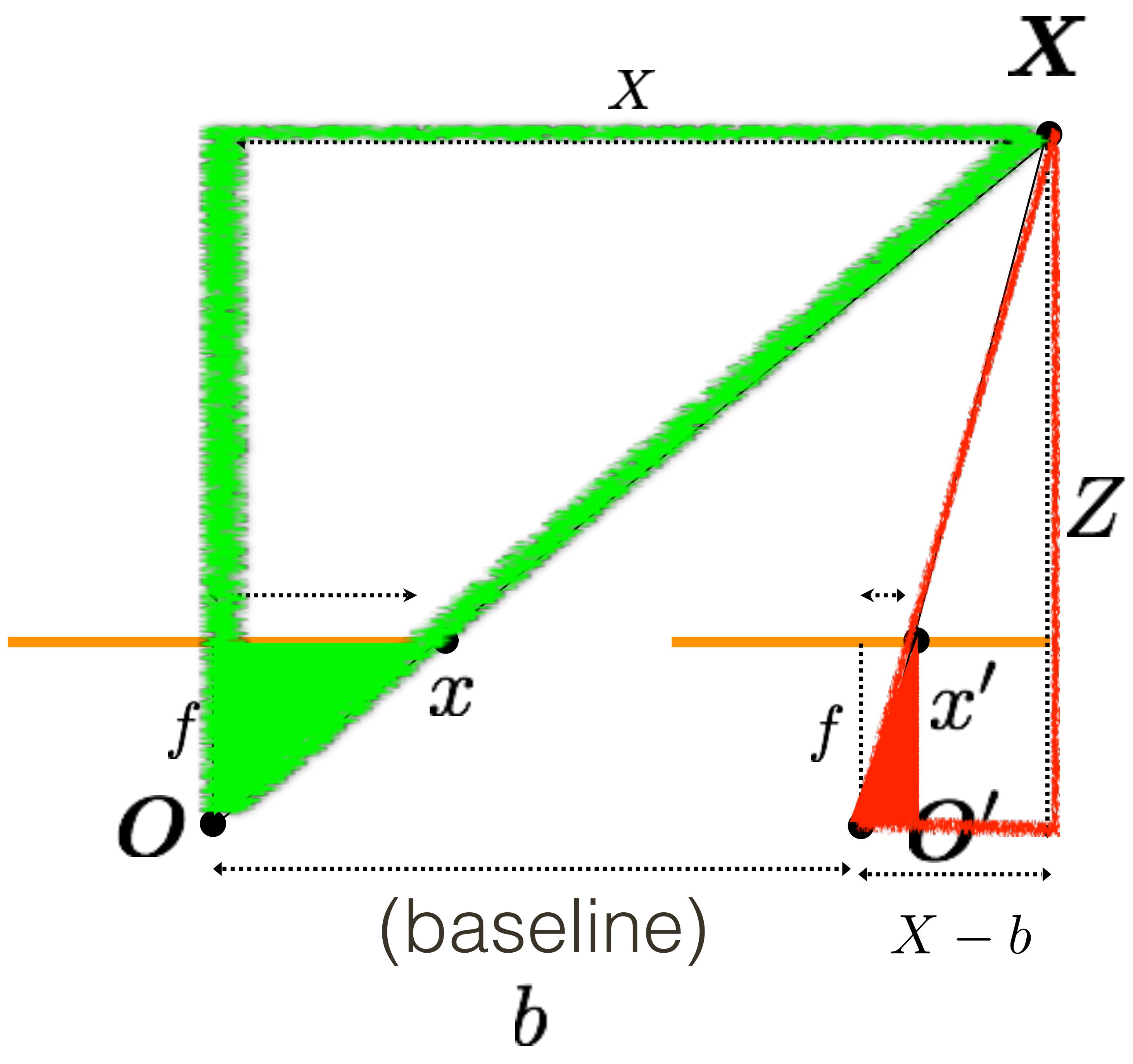
$$\frac{X}{Z} - \frac{b}{Z} = \frac{x'}{f}$$

$$\frac{x}{f} - \frac{b}{Z} = \frac{x'}{f}$$ (substitute)

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Rectified Stereo Pair: Depth Estimate

$$\frac{X}{Z} = \frac{x}{f}$$



$$\frac{X - b}{Z} = \frac{x'}{f}$$

$$\frac{X}{Z} - \frac{b}{Z} = \frac{x'}{f}$$

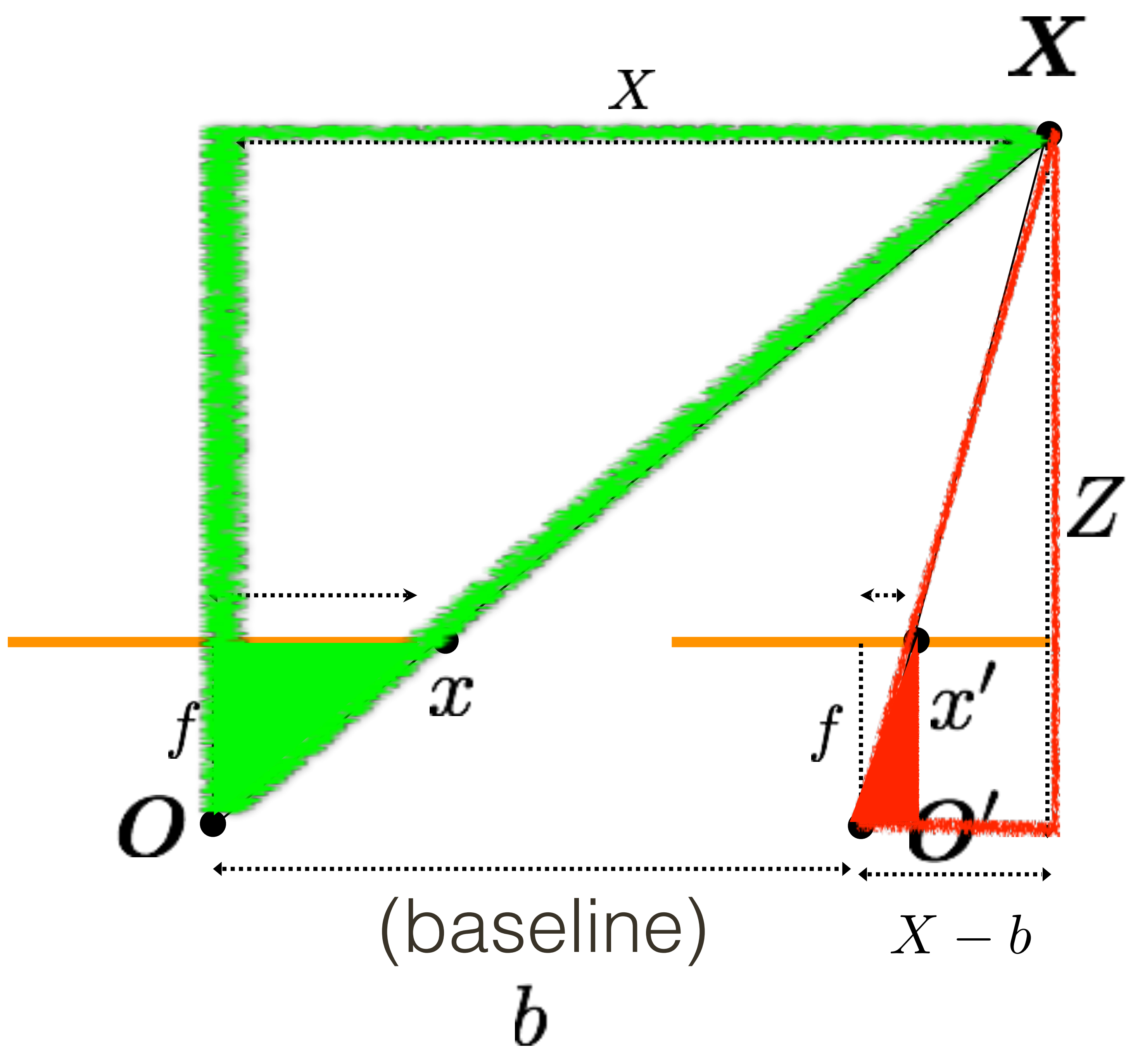
$$\frac{x}{f} - \frac{b}{Z} = \frac{x'}{f}$$

$$\frac{x - x'}{f} = \frac{b}{Z}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Rectified Stereo Pair: Depth Estimate

$$\frac{X}{Z} = \frac{x}{f}$$



$$\frac{X-b}{Z} = \frac{x'}{f}$$

$$\frac{X}{Z} - \frac{b}{Z} = \frac{x'}{f}$$

$$\frac{x}{f} - \frac{b}{Z} = \frac{x'}{f}$$

$$\frac{x - x'}{f} = \frac{b}{Z}$$

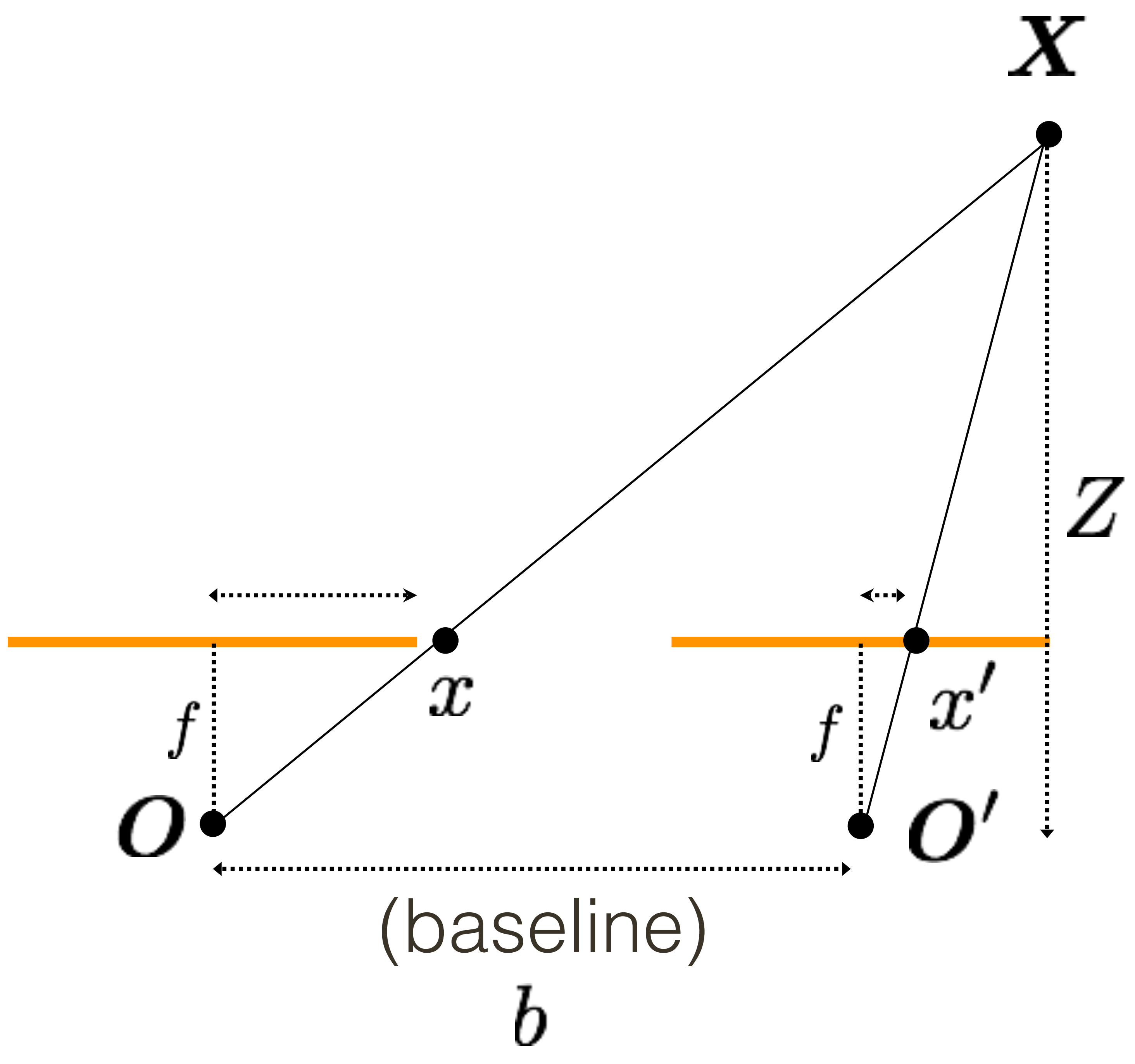
Disparity

(wrt to camera origin of image plane)

$$d = x - x'$$
$$= \frac{bf}{Z}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Rectified Stereo Pair: Depth Estimate



Disparity

(wrt to camera origin of image plane)

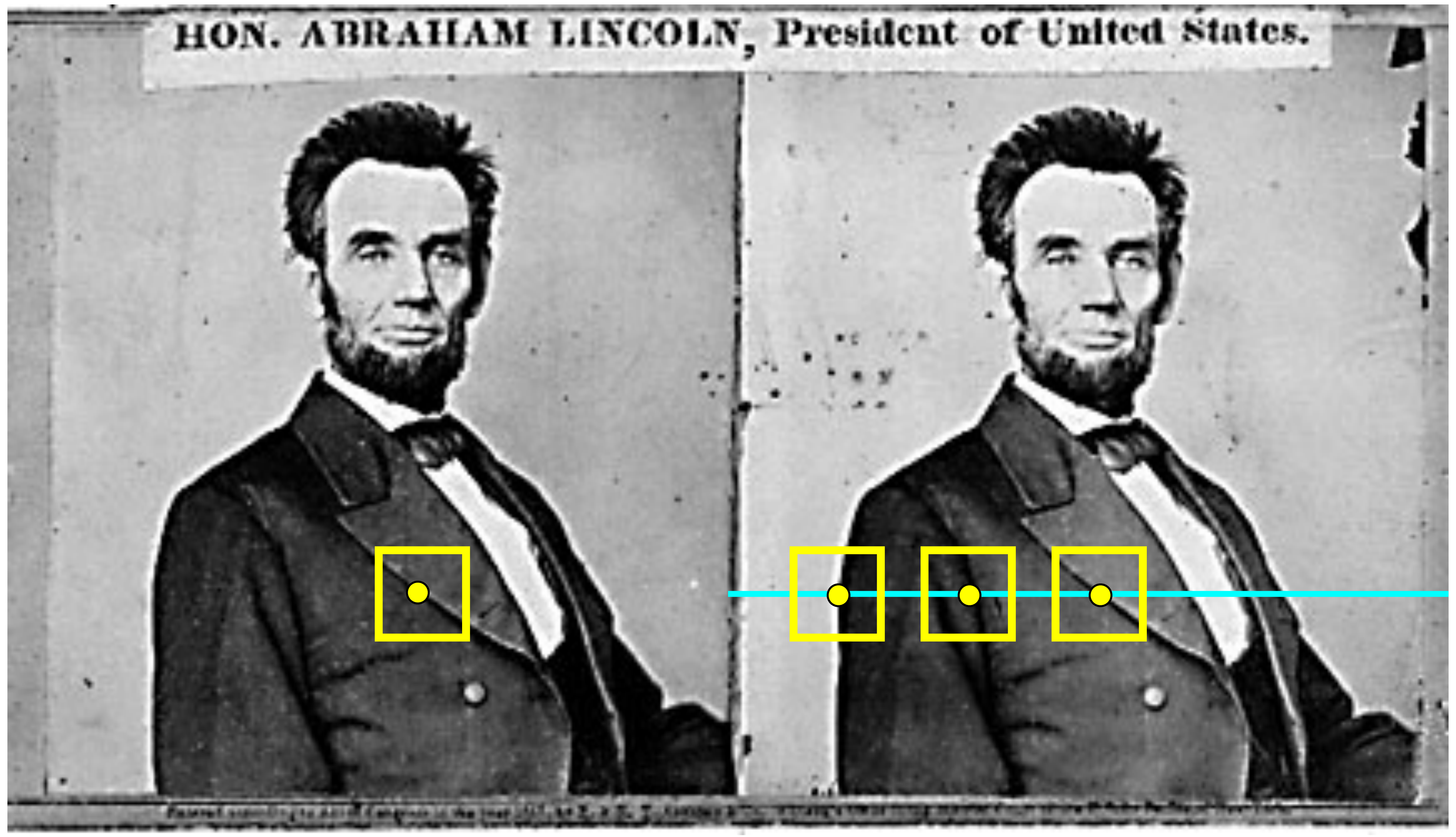
$$d = x - x'$$

$$= \frac{bf}{Z}$$

inversely proportional to depth

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

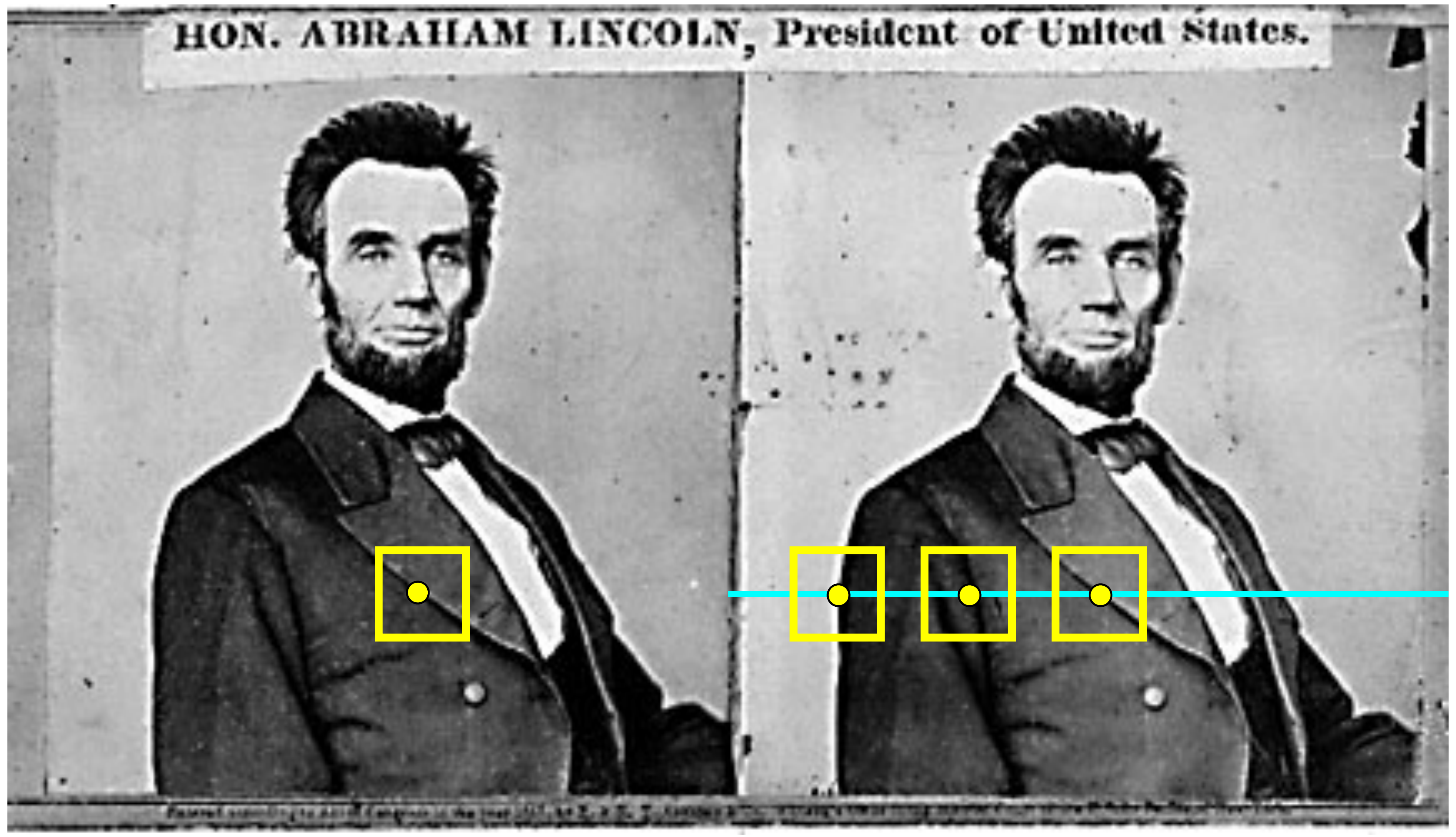
(simple) **Stereo** Algorithm



1. Rectify images
   (make epipolar lines horizontal)
2. For each pixel
   a. Find epipolar line
   b. Scan line for best match
   c. Compute depth from disparity $Z = \frac{bf}{d}$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

(simple) **Stereo** Algorithm



1. Rectify images
   (make epipolar lines horizontal)
2. For each pixel
   a. Find epipolar line
   b. Scan line for best match
   c. Compute depth from disparity $Z = \frac{bf}{d}$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Correspondence: What should we match?

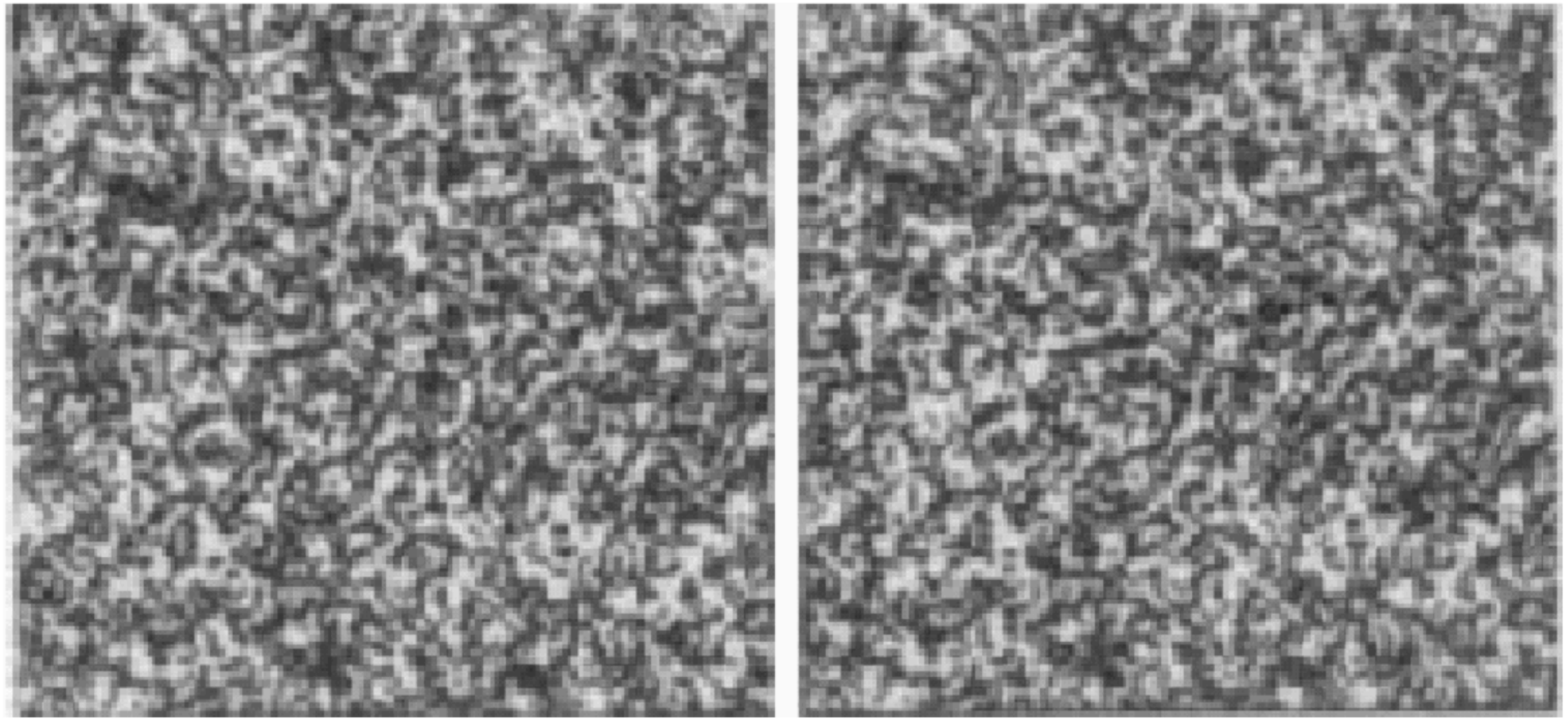
Objects?

Edges?

Pixels?

Collections of pixels?

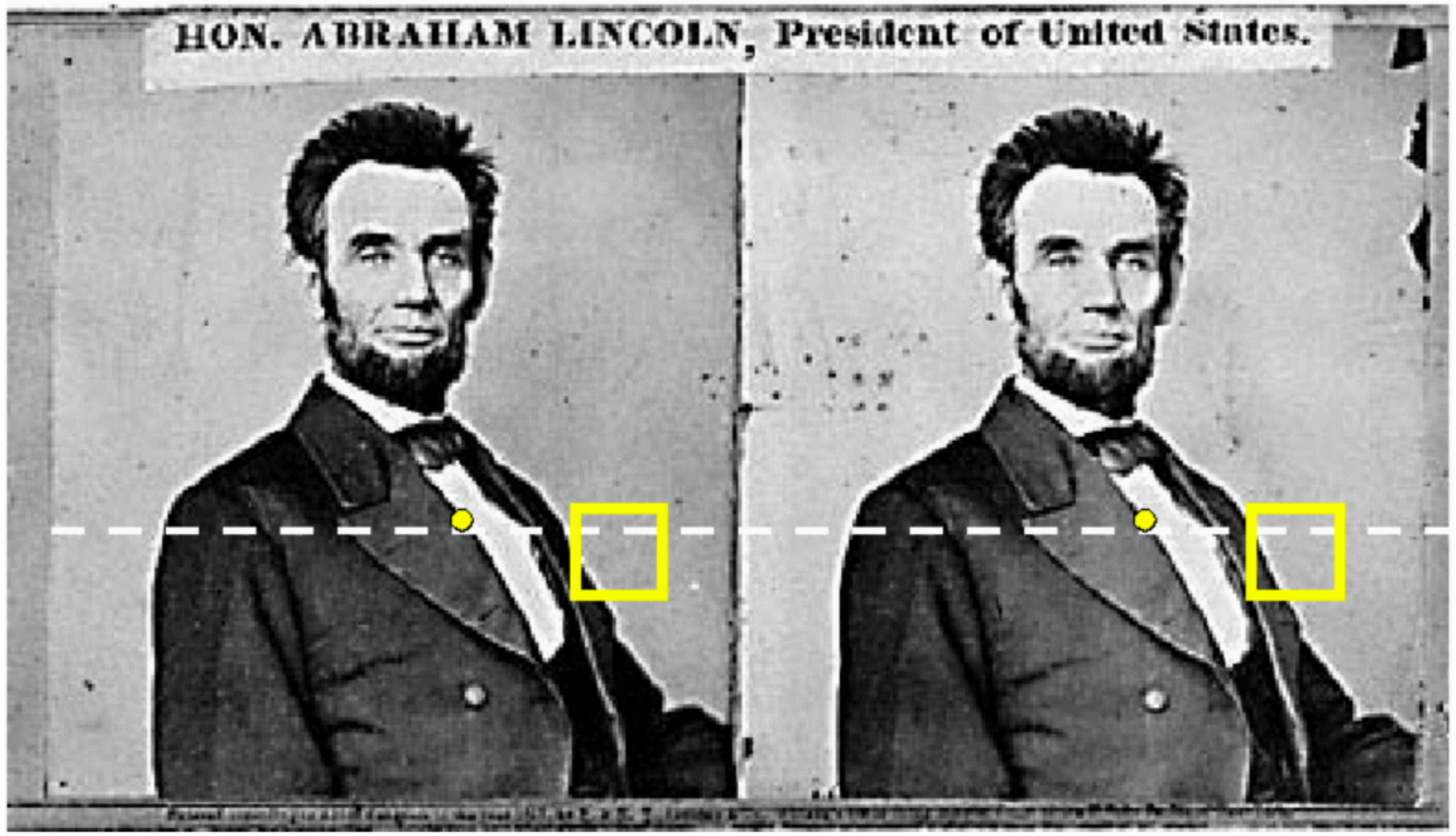
Random Dot Stereograms



Julesz (1960) showed that **recognition is not needed** for stereo

"When viewed monocularly, the images appear completely random. But when viewed stereoscopically, the image pair gives the impression of a square markedly in front of (or behind) the surround."

Method: Pixel Matching



For each **epipolar line**

For each **pixel** in the left image

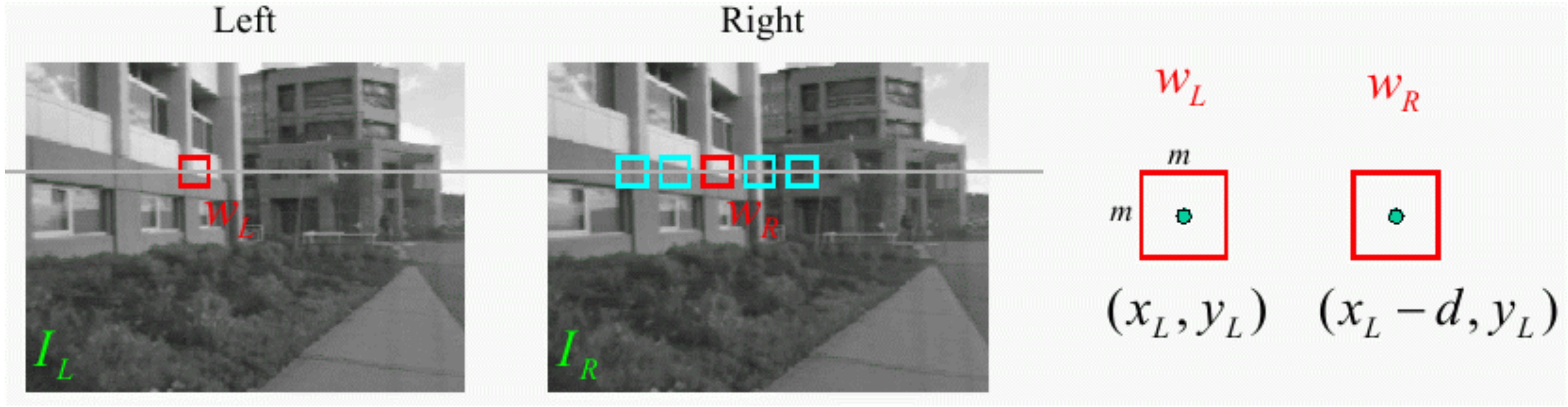
— compare with every pixel on same epipolar line in right image

— pick pixel with minimum match cost

This leaves too much ambiguity!

Slide credit: Steve Seitz

Sum of Squared (Pixel) Differences



$\mathbf{w}_L$ and $\mathbf{w}_R$ are corresponding $m \times m$ windows of pixels

Define the window function, $\mathbf{W}_m(x, y)$, by

$$\mathbf{W}_m(x,y) = \left\{(u,v) \mid x - \frac{m}{2} \leq u \leq x + \frac{m}{2}, y - \frac{m}{2} \leq v \leq y + \frac{m}{2}\right\}$$

SSD measures intensity difference as a function of disparity:

$$C_R(x,y,d) = \sum_{(u,v)\in\mathbf{W}_m(x,y)} [I_L(u,v) - I_R(u-d,v)]^2$$

Image Normalization

$$\bar{I} = \frac{1}{|\mathbf{W}_m(x,y)|} \sum_{(u,v)\in\mathbf{W}_m(x,y)} I(u,v)$$

Average Pixel

$$||I||_{\mathbf{W}_m(x,y)} = \sqrt{\sum_{(u,v)\in\mathbf{W}_m(x,y)} [I(u,v)]^2}$$

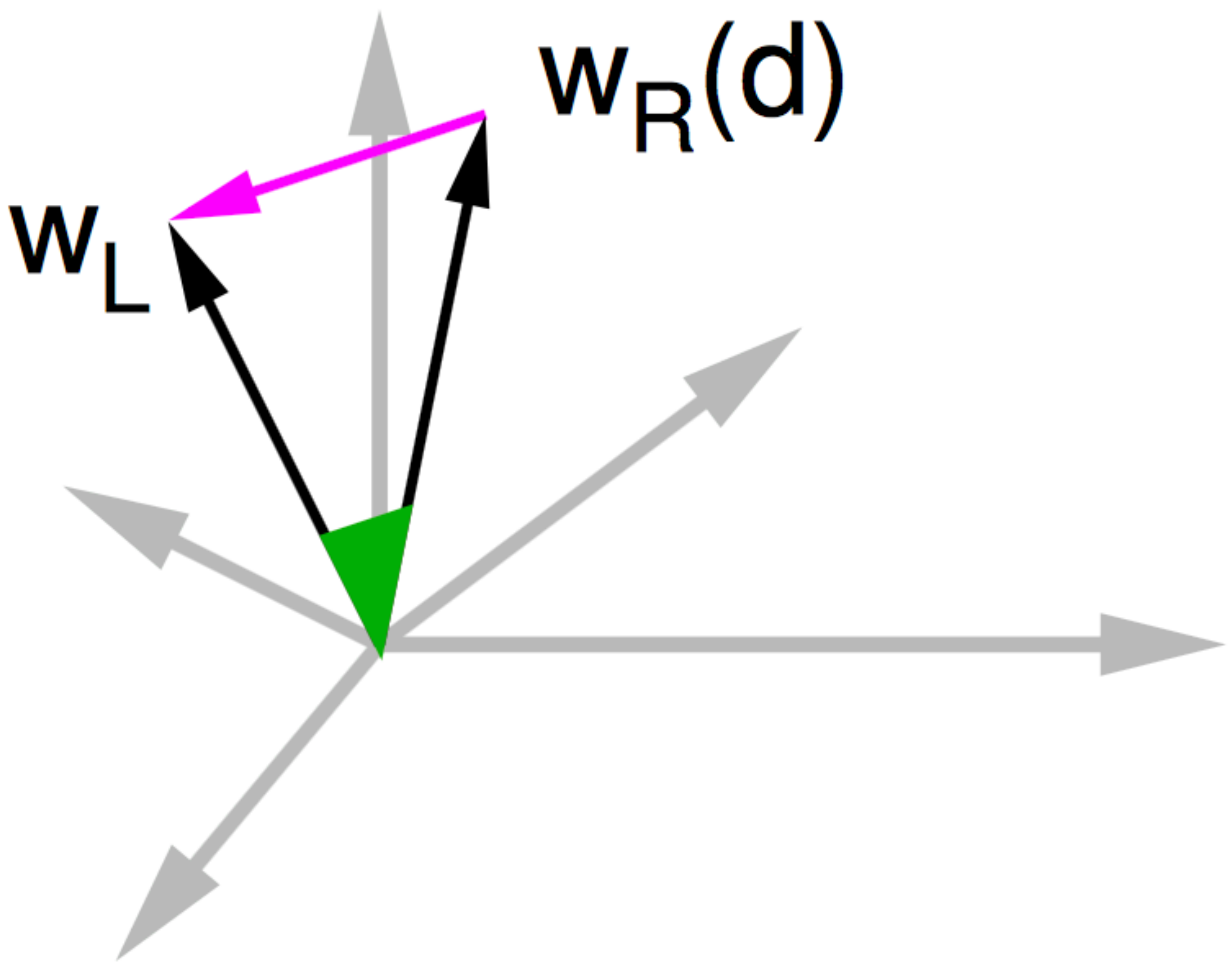
Window Magnitude

$$\hat{I}(x,y) = \frac{I(x,y) - \bar{I}}{||I - \bar{I}||_{\mathbf{W}_m(x,y)}}$$

Normalized Pixel: subtract the mean, normalize to unit length

Image **Metrics**

(Normalized) Sum of Squared Differences



(Normalized) Correlation

Image **Metrics**

Assume $\mathbf{w}_L$ and $\mathbf{w}_R(d)$ are normalized to unit length (Normalized)

Sum of Squared Differences:

$$C_{SSD}(d) = \sum_{(u,v)\in\mathbf{W}_m(x,y)} \left[\hat{I}_L(u,v) - \hat{I}_R(u-d,v)\right]^2$$

$$= ||\mathbf{w}_L - \mathbf{w}_R(d)||^2$$

(Normalized) **Correlation**:

$$C_{NC}(d) = \sum_{(u,v)\in\mathbf{W}_m(x,y)} \hat{I}_L(u,v)\hat{I}_R(u-d,v)$$

$$= \mathbf{w}_L \cdot \mathbf{w}_R(d) = \cos\theta$$

Image **Metrics**

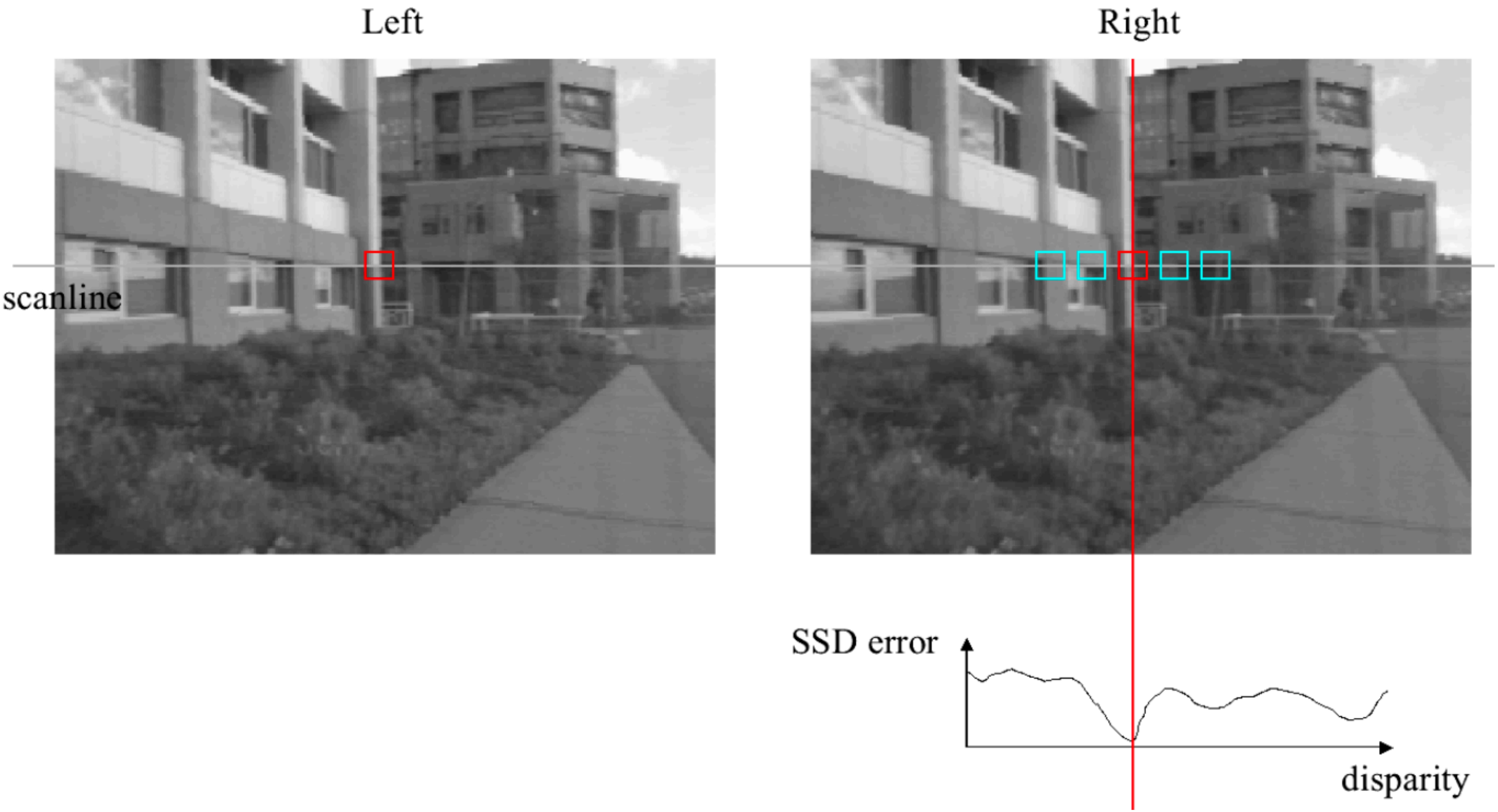
Let d^* be the value of d that minimizes C_{SSD}

Then d^* also is the value of d that maximizes C_{NC}

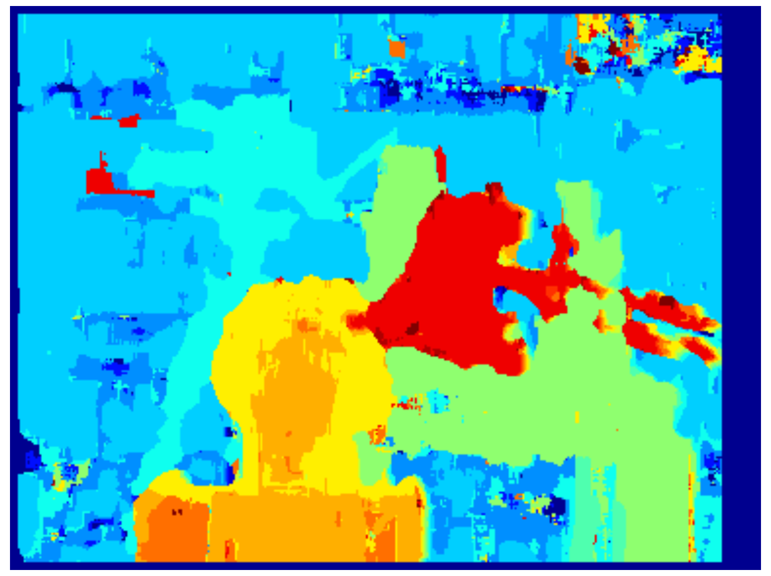
That is,

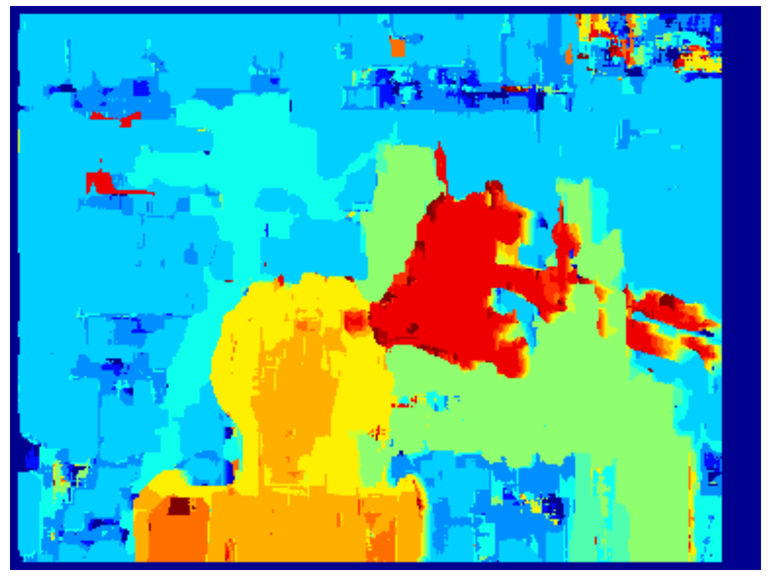
$$d^* = \arg\min_d ||\mathbf{w}_L - \mathbf{w}_R(d)||^2 = \arg\min_d \mathbf{w}_L \cdot \mathbf{w}_R(d)$$

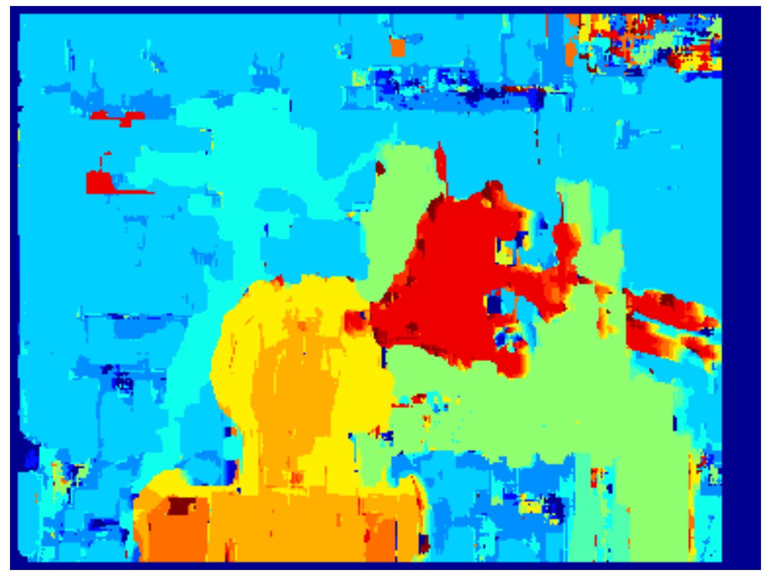
Method: Correlation



Similarity Measure	Formula
Sum of Absolute Differences (SAD)	$\sum_{(i,j)\in W} \lvert I_1(i,j) - I_2(x+i, y+j) \rvert$
Sum of Squared Differences (SSD)	$\sum_{(i,j)\in W} \left(I_1(i,j) - I_2(x+i, y+j)\right)^2$
Zero-mean SAD	$\sum_{(i,j)\in W} \lvert I_1(i,j) - \bar{I}_1(i,j) - I_2(x+i,y+j) + \bar{I}_2(x+i,y+j) \rvert$
Locally scaled SAD	$\sum_{(i,j)\in W} \lvert I_1(i,j) - \frac{\bar{I}_1(i,j)}{\bar{I}_2(x+i,y+j)} I_2(x+i,y+j) \rvert$
Normalized Cross Correlation (NCC)	$\frac{\sum_{(i,j)\in W} I_1(i,j) . I_2(x+i,y+j)}{\sqrt[2]{\sum_{(i,j)\in W} I_1^2(i,j) . \sum_{(i,j)\in W} I_2^2(x+i,y+j)}}$


SAD

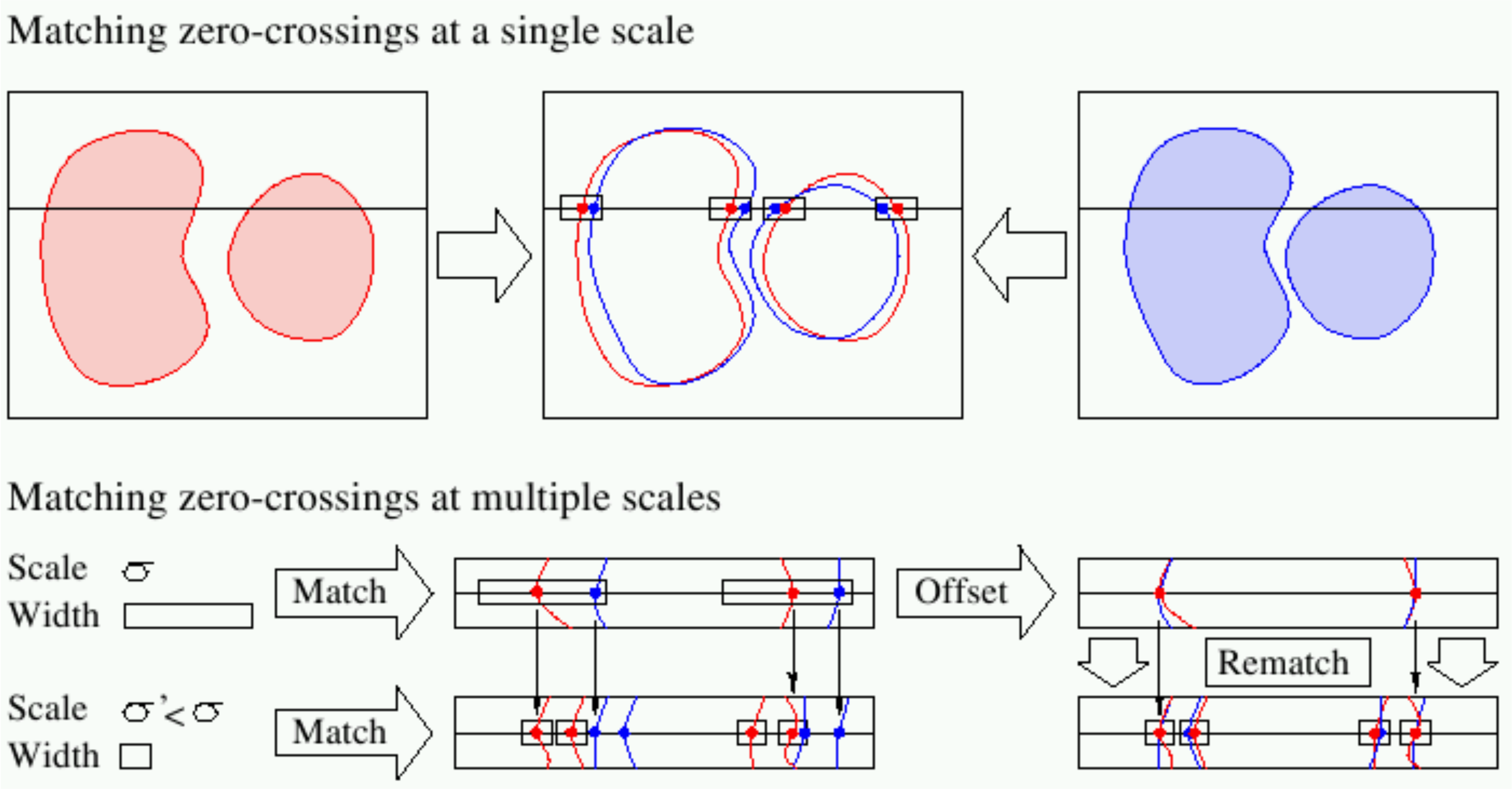

SSD


NCC


Ground truth

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Method: Edges



Forsyth & Ponce (2nd ed.) Figure 7.12 (Top & Middle)

Method: Edges (aside)

The **Marr/Poggio** (1979) multiscale stereo algorithm:

1. Convolve the two (rectified) images with $\nabla^2 G_\sigma$ filters of increasing $\sigma_1 < \sigma_2 < \sigma_3 < \sigma_4$

2. Find zero crossings along horizontal scanlines of the filtered images

3. For each filter scale σ, match zero crossings with the same parity and roughly equal orientations in a $[-\mathbf{w}_\sigma, +\mathbf{w}_\sigma]$ disparity range, with $\mathbf{w}_\sigma = 2\sqrt{2}\sigma$

4. Use the disparities found at larger scales to control eye vergence and cause unmatched regions at smaller scales to come into correspondence

Which Method is **Better**: Correlation or Edges?

Edges are more “meaningful” [Marr]. . . . . . but hard to find!

Edges tend to fail in dense texture (outdoors)

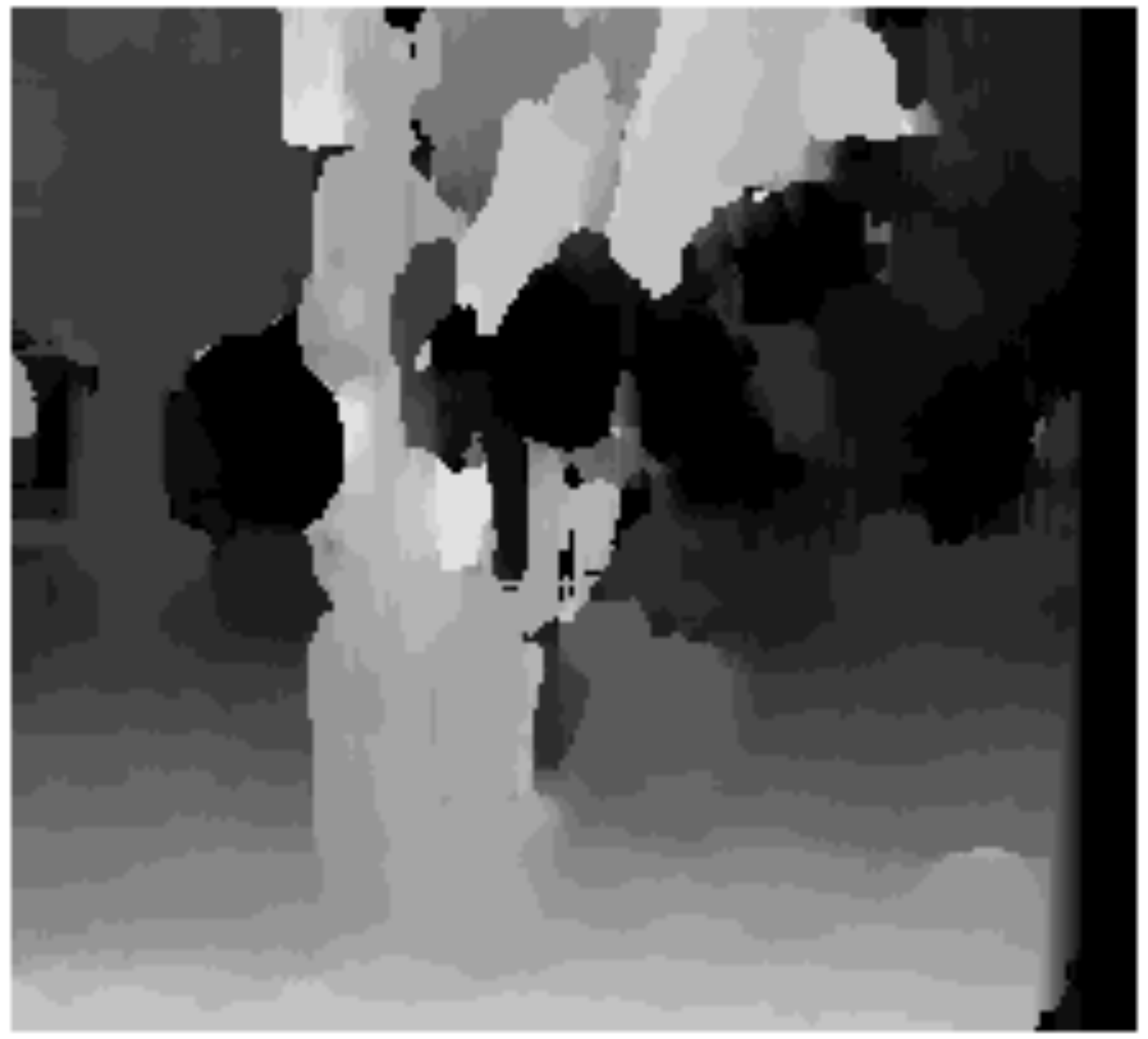
Correlation tends to fail in smooth, featureless regions

Note: Correlation-based methods are “dense.” Edge-based methods are “relatively sparse”

Effect of **Window Size**

W = 3

Smaller window
- \+ More detail
- \- More noise

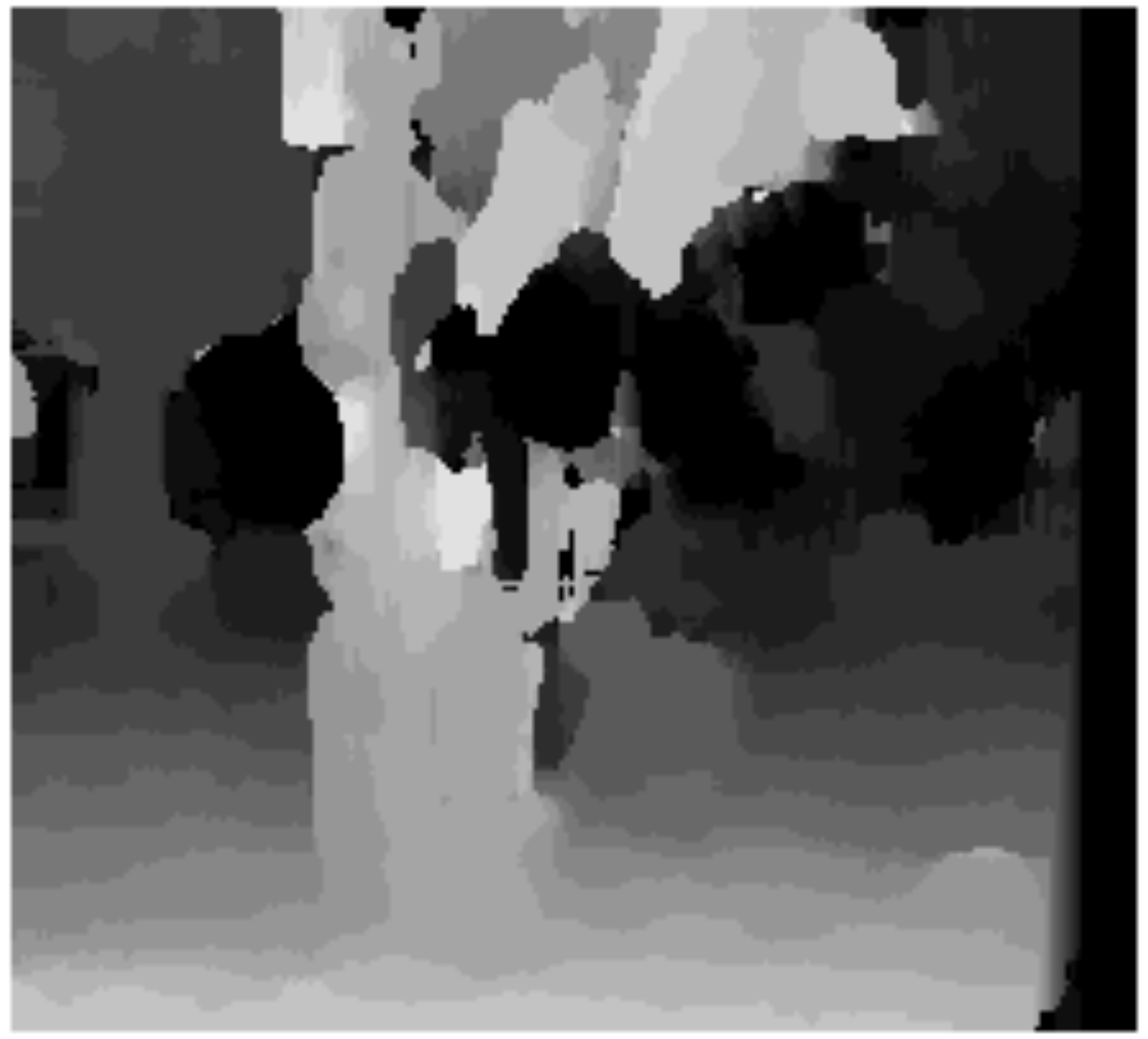


W = 20

Larger window
- \+ Smoother disparity maps
- \- Less detail
- \- Fails near boundaries

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Effect of **Window Size**

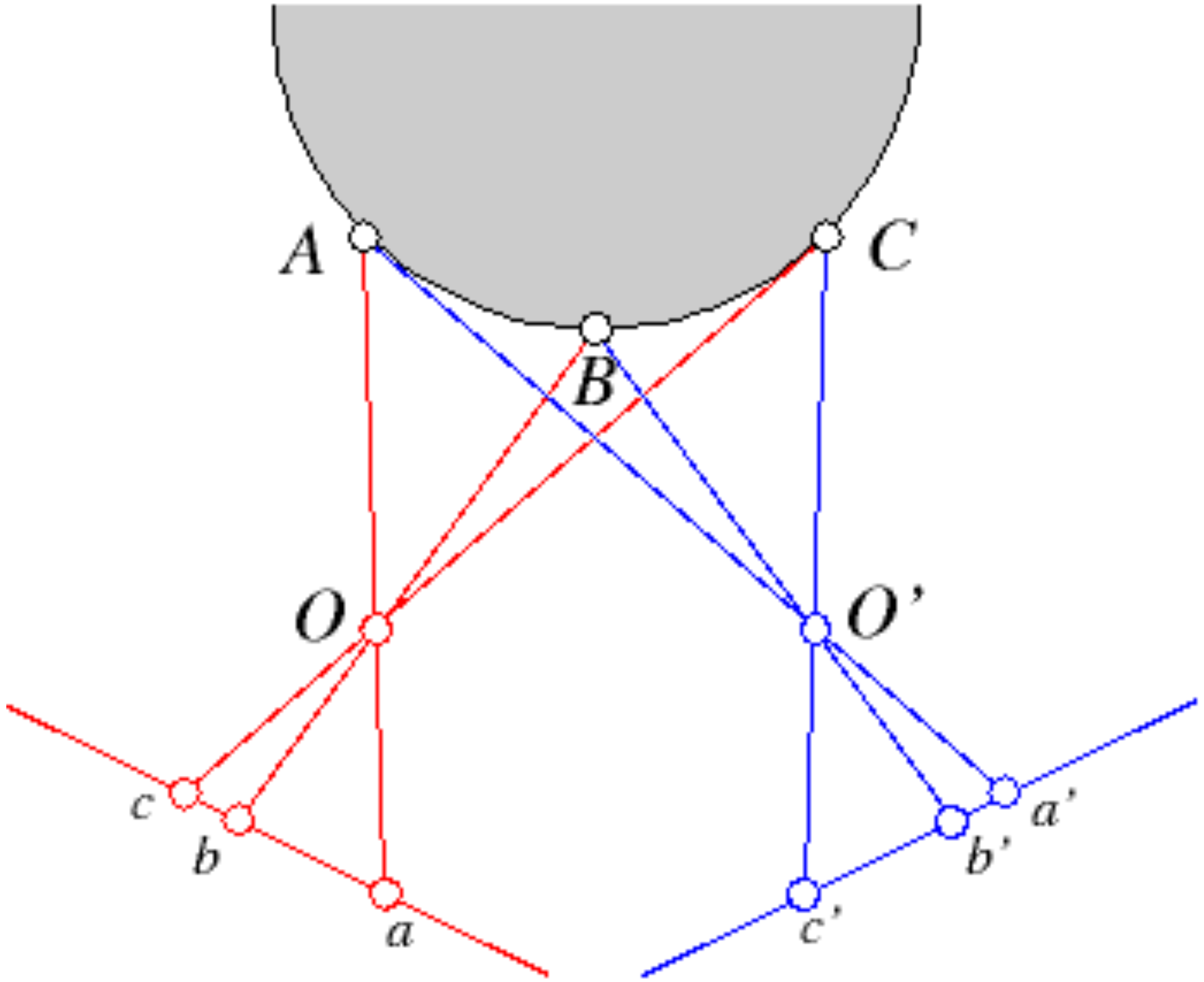
W = 3

W = 20

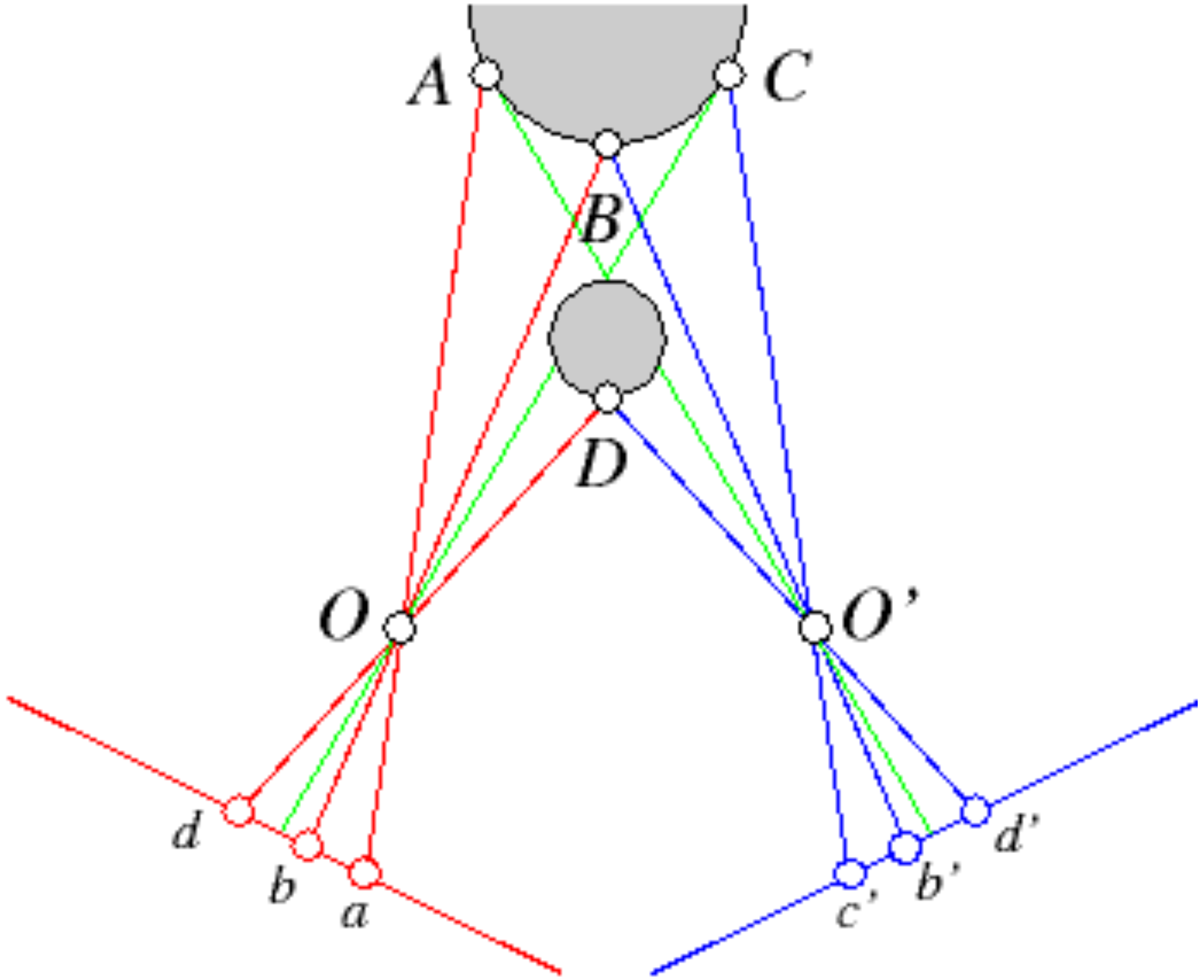
Note: Some approaches use an adaptive window size — try multiple sizes and select best match

Ordering Constraints

Ordering constraint …

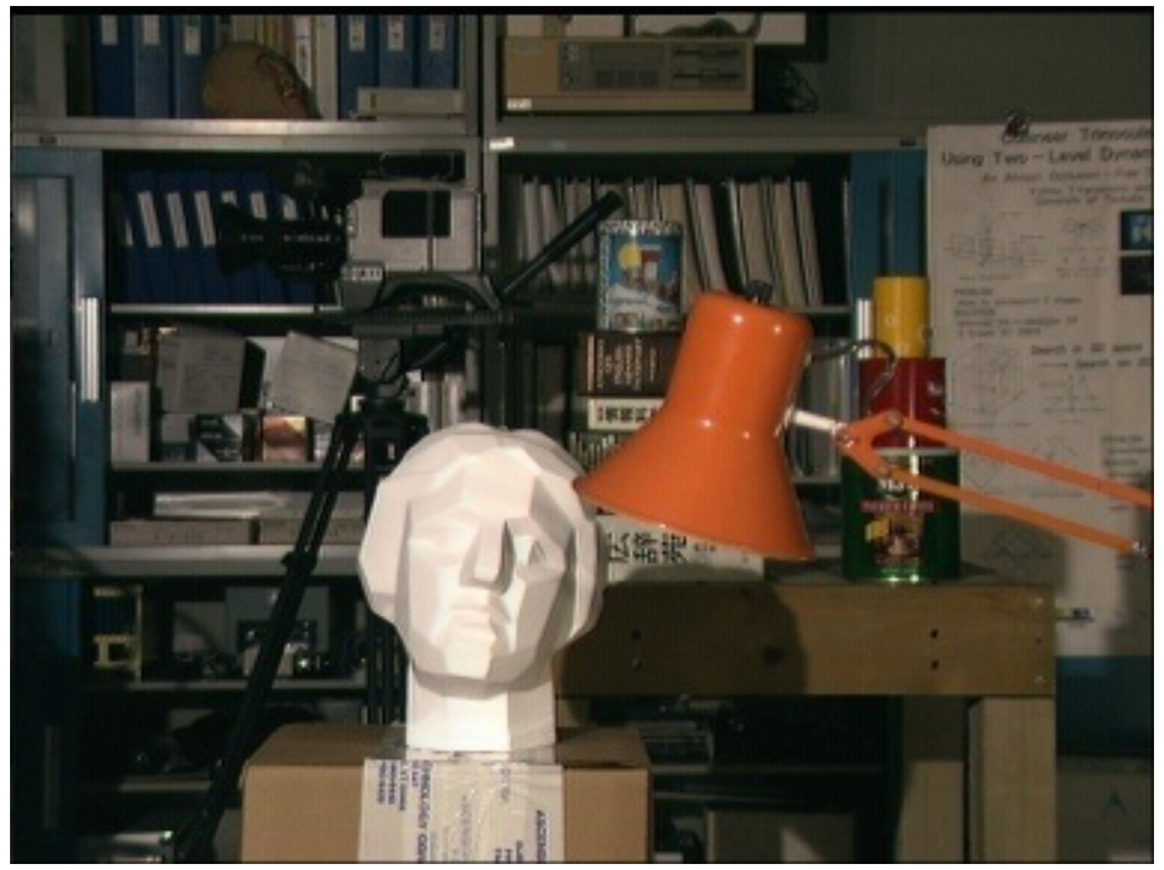


…. and a **failure** case

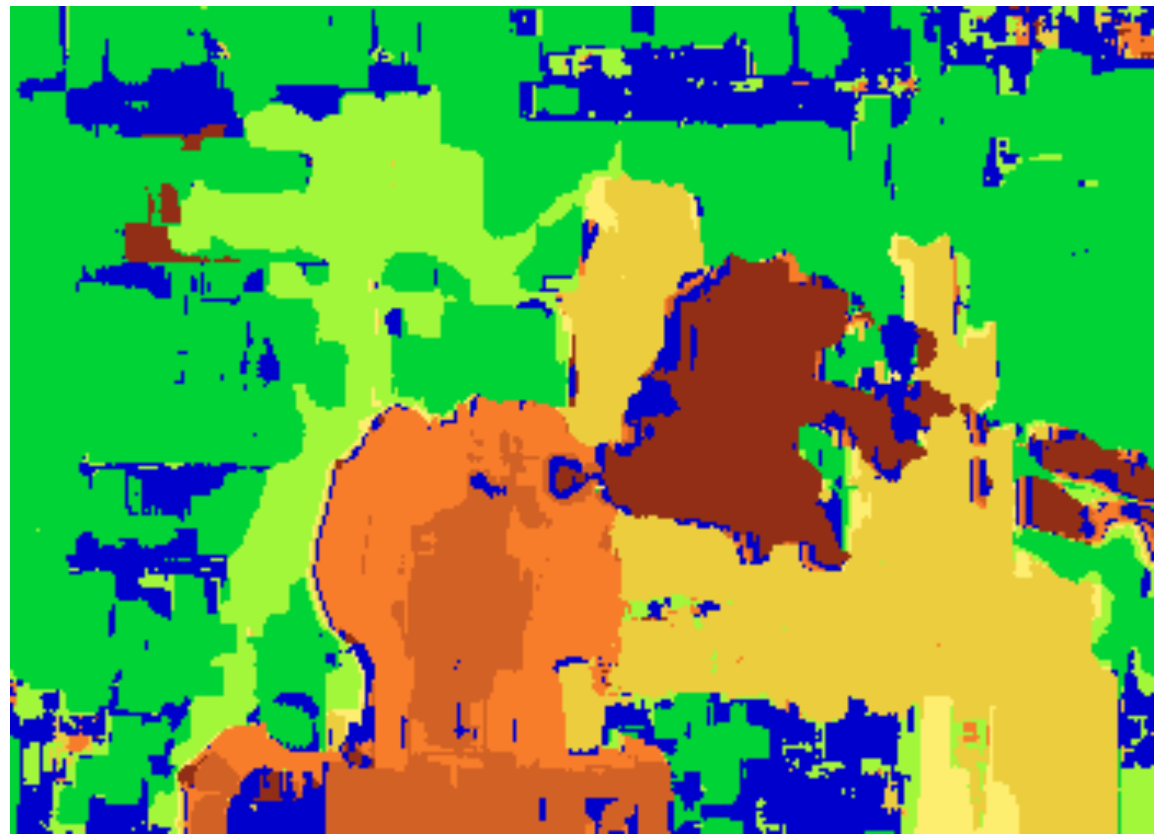


Forsyth & Ponce (2nd ed.) Figure 7.13

Block Matching Techniques: Result



Block matching



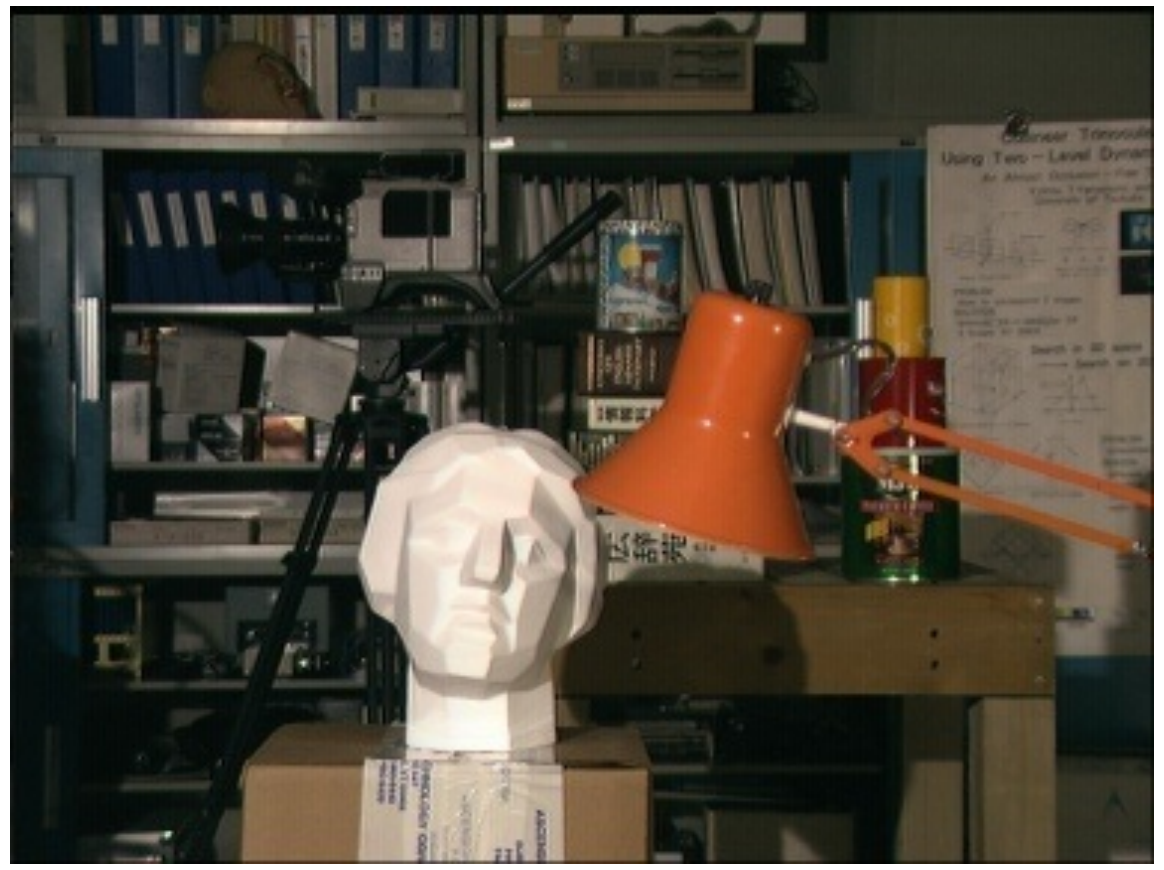
Ground truth

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

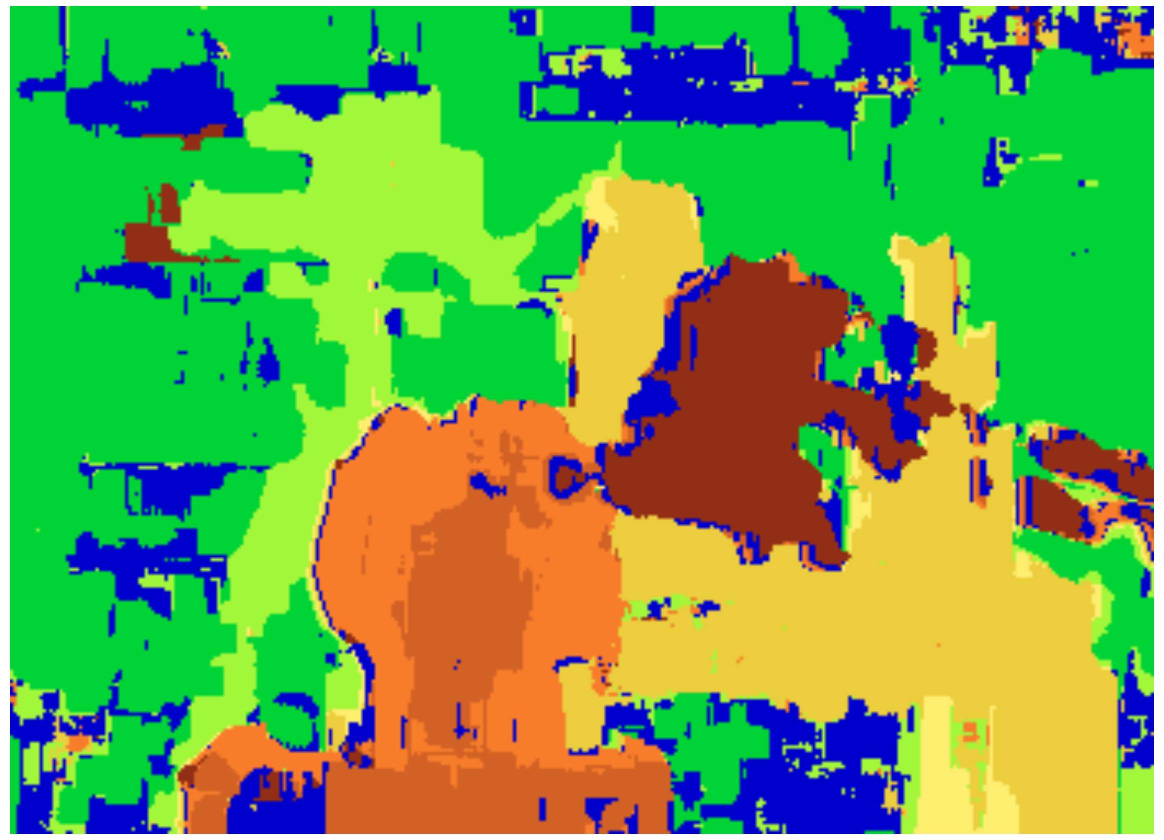
Block Matching Techniques: Result

Too many **discontinuities**.
We expect disparity values to change slowly.

Let's make an assumption:
depth should change smoothly



Block matching



Ground truth

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Stereo Matching as **Energy Minimization**

energy function
(for one pixel)

$$E(d) = \underbrace{E_d(d)}_{\text{data term}} + \lambda \underbrace{E_s(d)}_{\text{smoothness term}}$$

Want each pixel to find a good match in the other image

(block matching result)

Adjacent pixels should (usually) move about the same amount

(smoothness function)

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Stereo Matching as **Energy Minimization**

$$E(d) = E_d(d) + \lambda \boxed{E_s(d)}$$

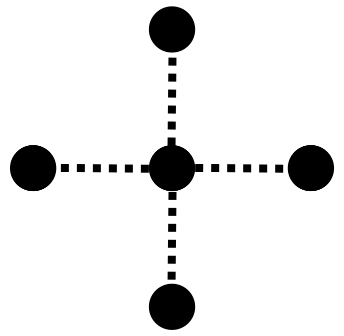
$$E_d(d) = \sum_{(x,y) \in I} C(x, y, d(x, y))$$

SSD distance between windows centered at I(x, y) and J(x+ d(x,y), y)

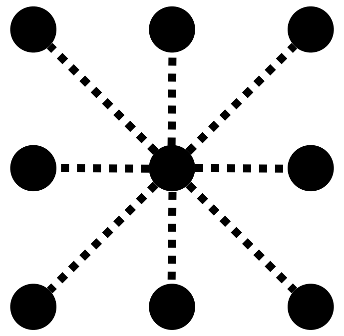
$$E_s(d) = \sum_{(p,q) \in \mathcal{E}} V(d_p, d_q)$$

smoothness term

$\mathcal{E}$: set of neighboring pixels



4-connected neighborhood



8-connected neighborhood

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

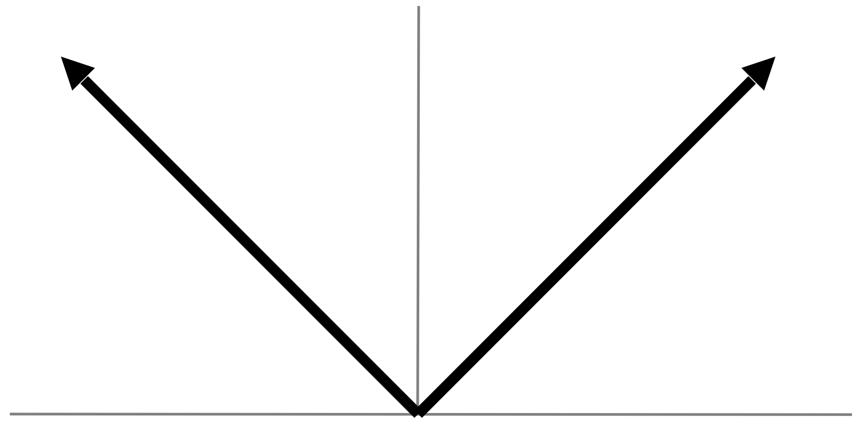
Stereo Matching as **Energy Minimization**

$$E_s(d) = \sum_{(p,q)\in\mathcal{E}} V(d_p, d_q)$$

smoothness term

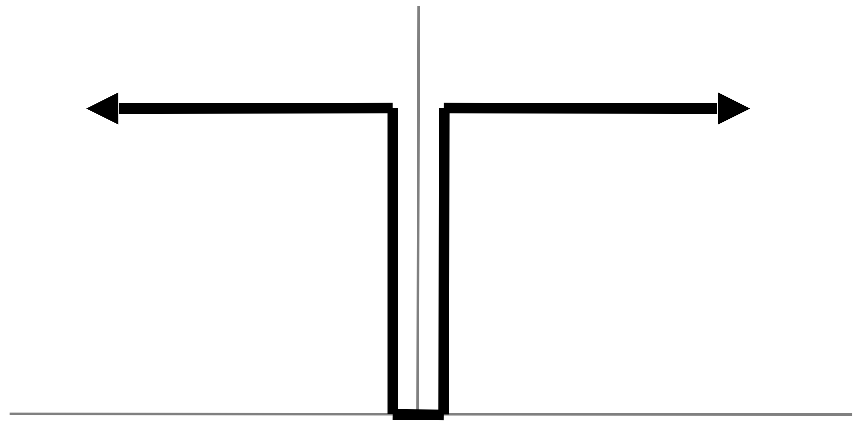
$$V(d_p, d_q) = |d_p - d_q|$$

L_1 distance



$$V(d_p, d_q) = \begin{cases} 0 & \text{if } d_p = d_q \\ 1 & \text{if } d_p \neq d_q \end{cases}$$

"Potts model"



Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

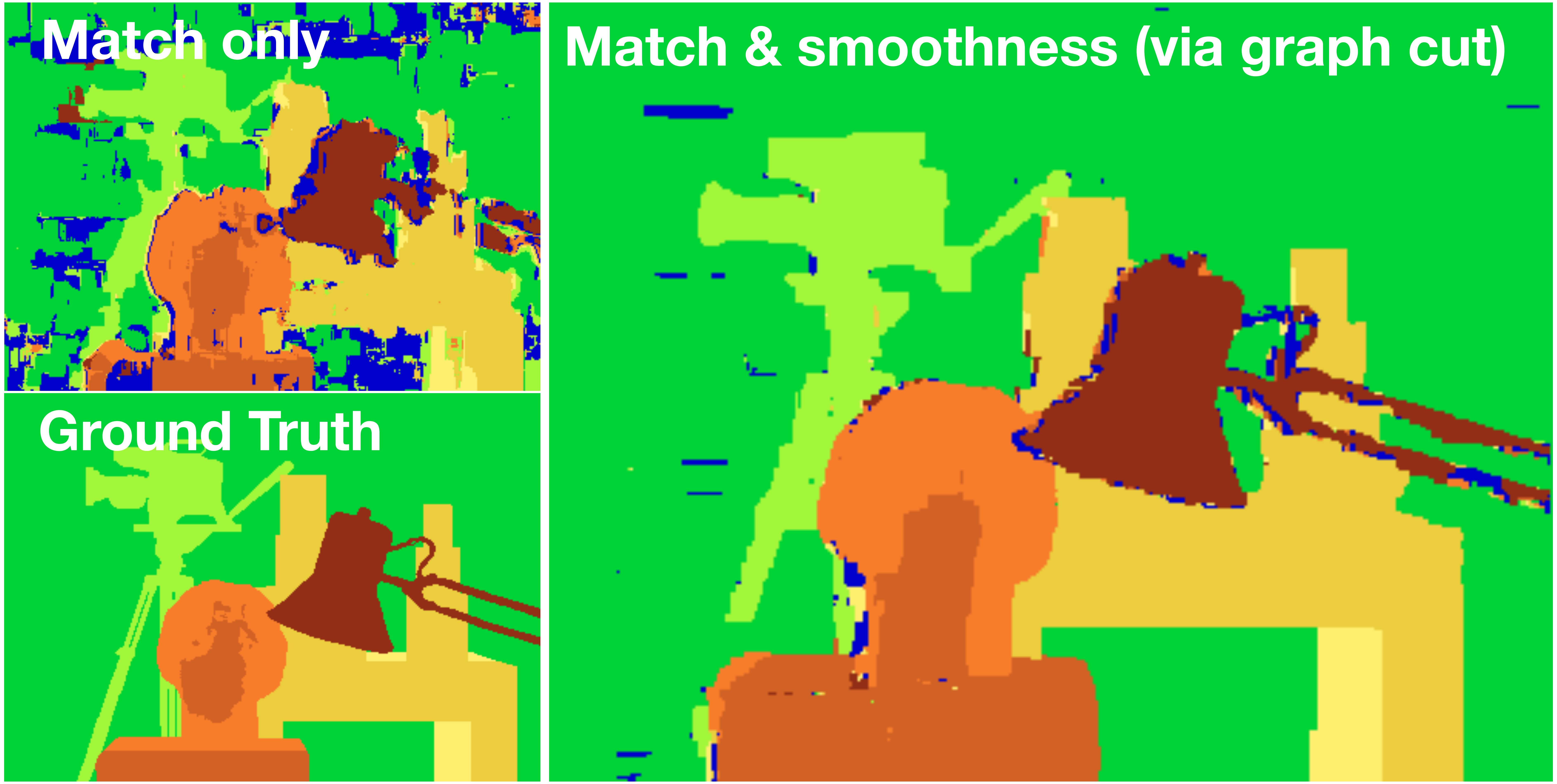
Stereo Matching as **Energy Minimization**: Solution

$$E(d) = E_d(d) + \lambda E_s(d)$$

Can minimize this independently per scanline
using **dynamic programming** (DP)

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Stereo Matching as **Energy Minimization**

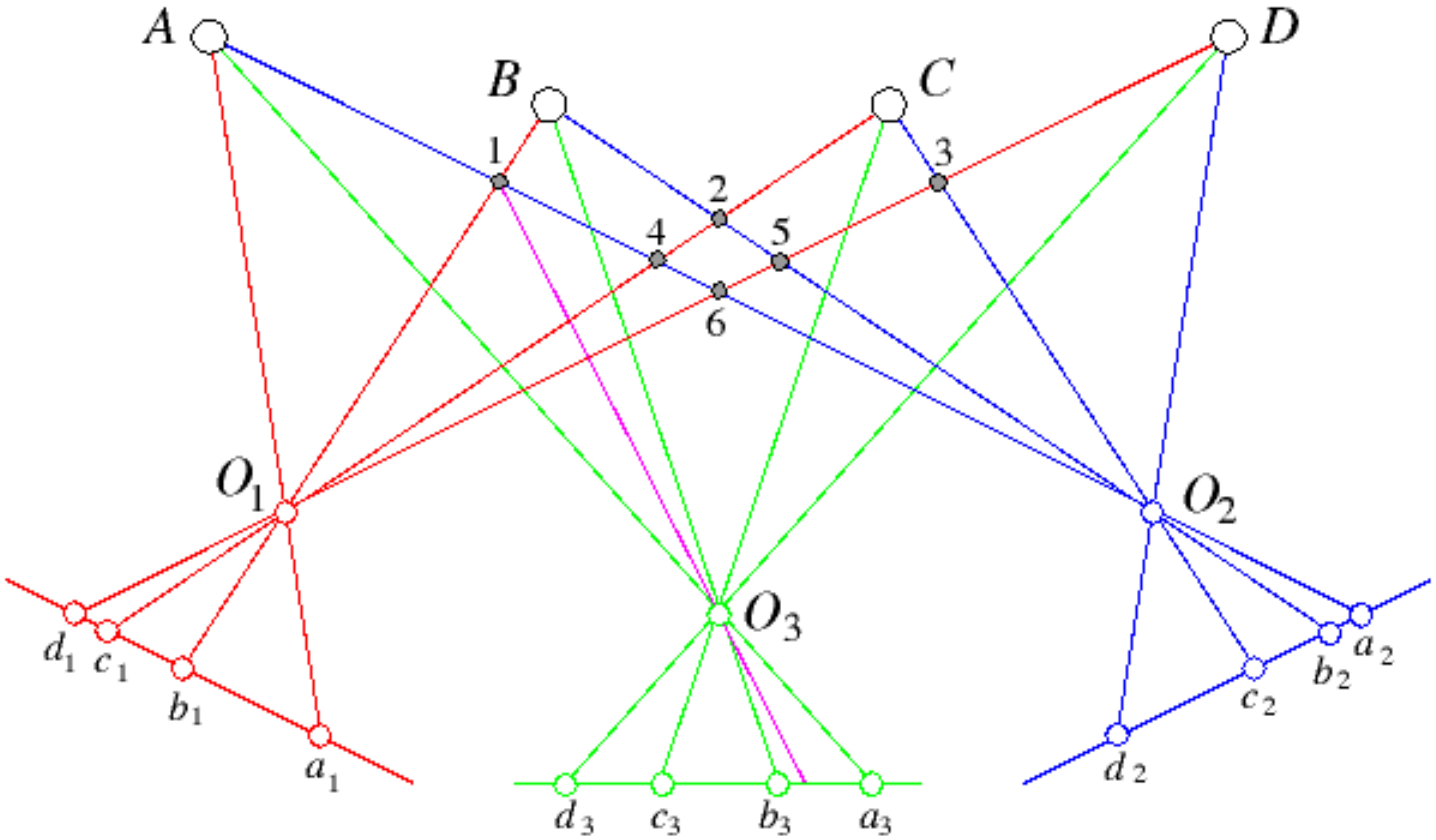


Y. Boykov, O. Veksler, and R. Zabih, Fast Approximate Energy Minimization via Graph Cuts, PAMI 2001

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Idea: Use More Cameras

Adding a third camera reduces ambiguity in stereo matching



Forsyth & Ponce (2nd ed.) Figure 7.17

Point Grey Research **Digiclops**

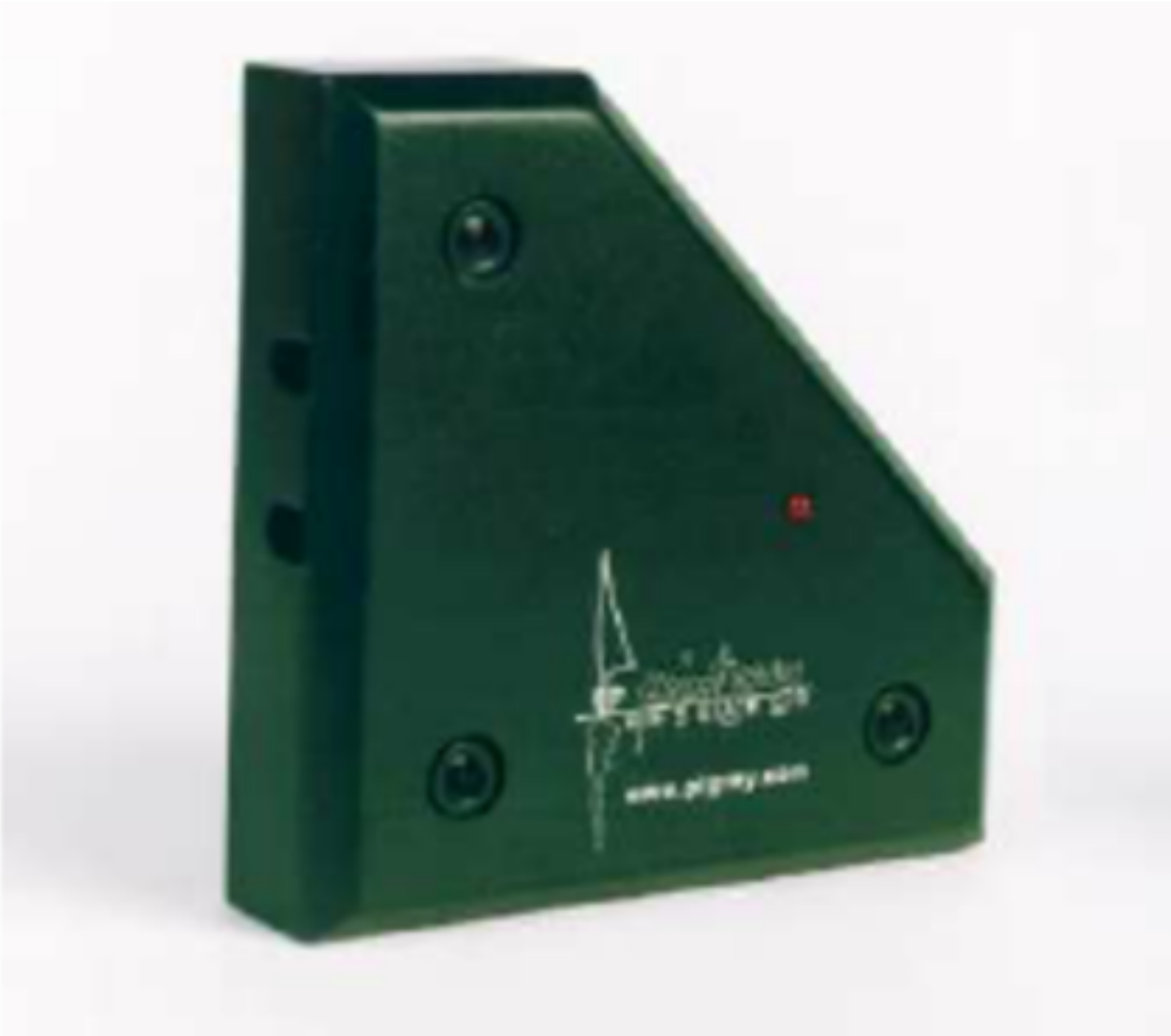
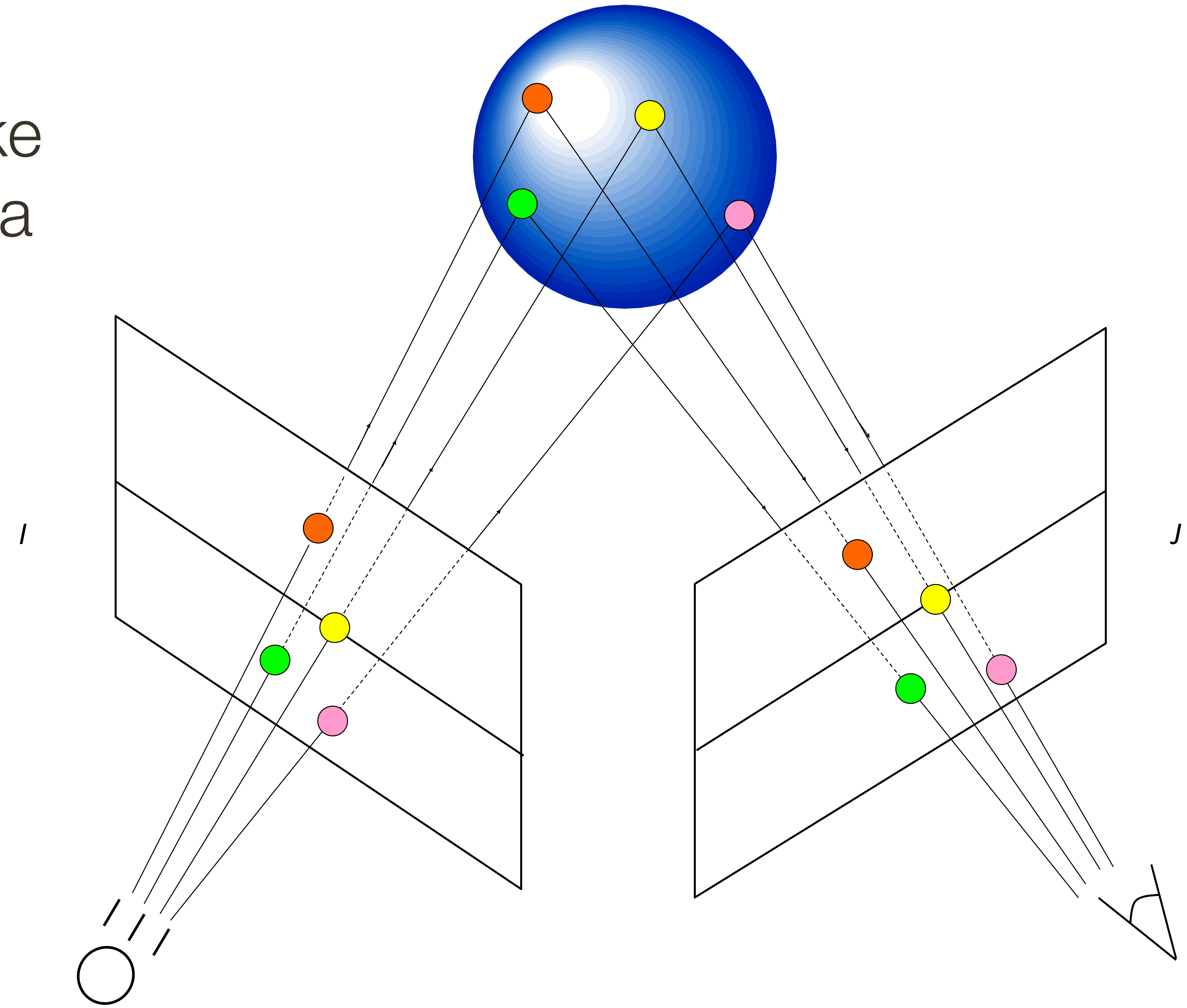


Image credit: Point Grey Research

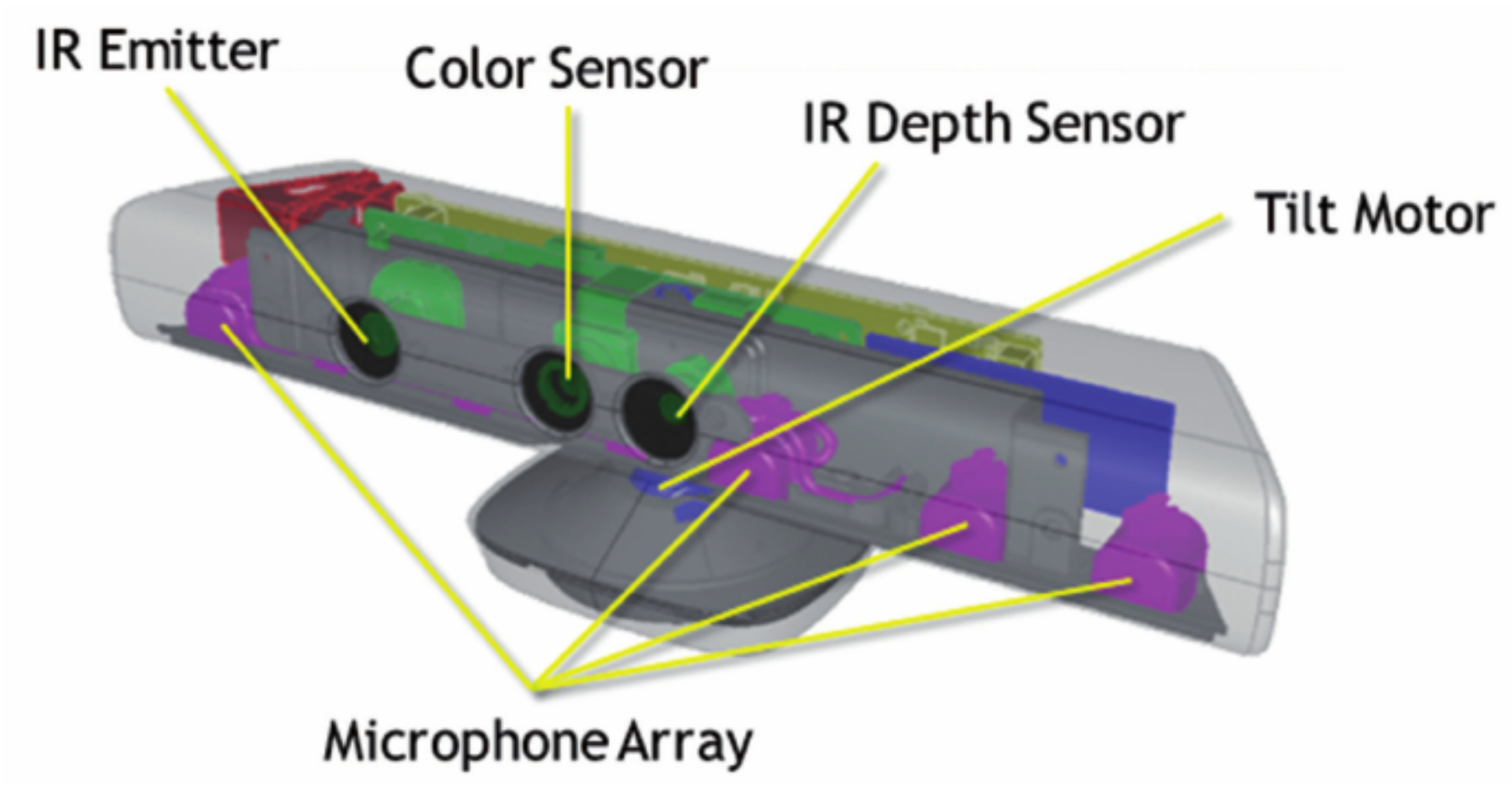
Structured Light Imaging: Structured Light and One Camera

Projector acts like
“reverse” camera

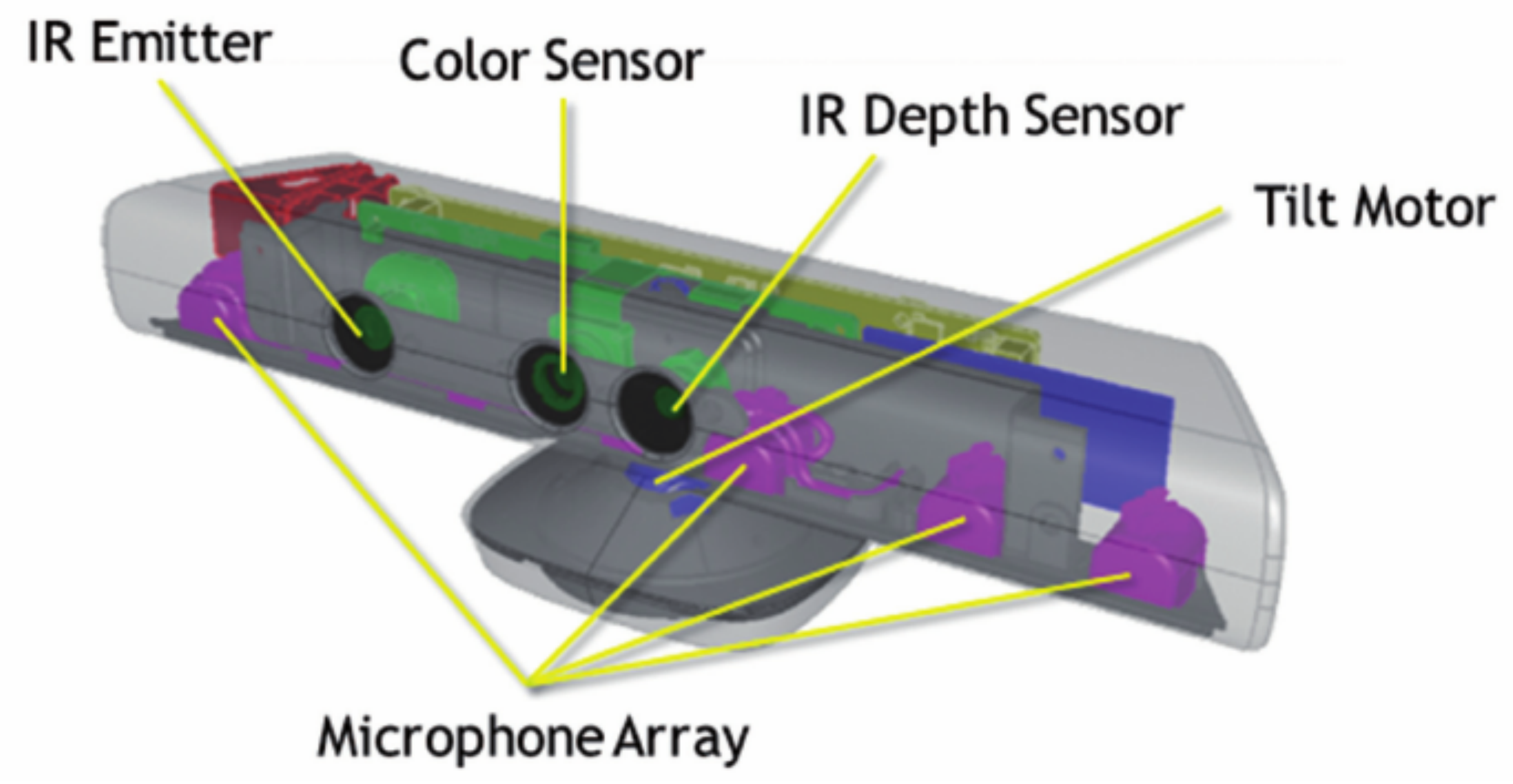


Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Microsoft **Kinect**



Microsoft **Kinect**



Summary

Stereo is formulated as a **correspondence** problem

— determine match between location of a scene point in one image and its location in another

If we assume calibrated cameras and image **rectification**, **epipolar lines** are horizontal scan lines

What do we match?

— Individual pixels?

— Patches?

— Edges?