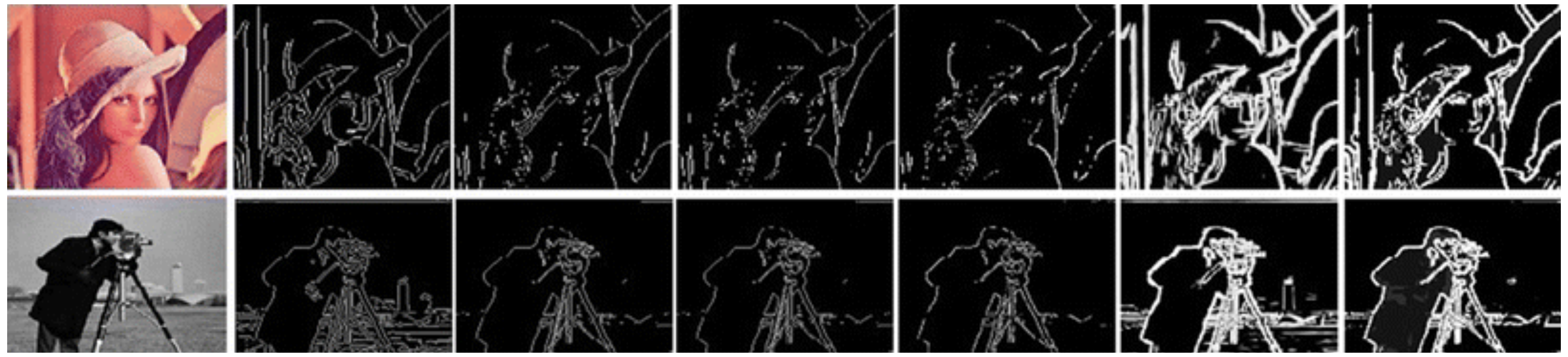
THE UNIVERSITY OF BRITISH COLUMBIA

CPSC 425: Computer Vision



Lecture 13: Laplacian Pyramids, Corner Detection

Menu for Today (**October 7, 2020**)

Topics:

— **Laplacian** Pyramids (revisited)
— **Corner** Detection
— **Autocorrelation**
— **Harris** Corner Detector

Redings:

— **Today's** Lecture: Forsyth & Ponce (2nd ed.) 5.3.0 - 5.3.1
— **Next** Lecture: Forsyth & Ponce (2nd ed.) 6.1, 6.3

Reminders:

— **Quiz 2**: due at the end of day **today**

— **Assignment 2**: Face Detection in a Scaled Representation is **October 14th**

Today's "**fun**" Example:

Wait for it! :)

Lecture 12: Re-cap

Physical properties of a 3D scene cause "**edges**" in an image:

— depth discontinuity

— surface orientation discontinuity

— reflectance discontinuity

— illumination boundaries

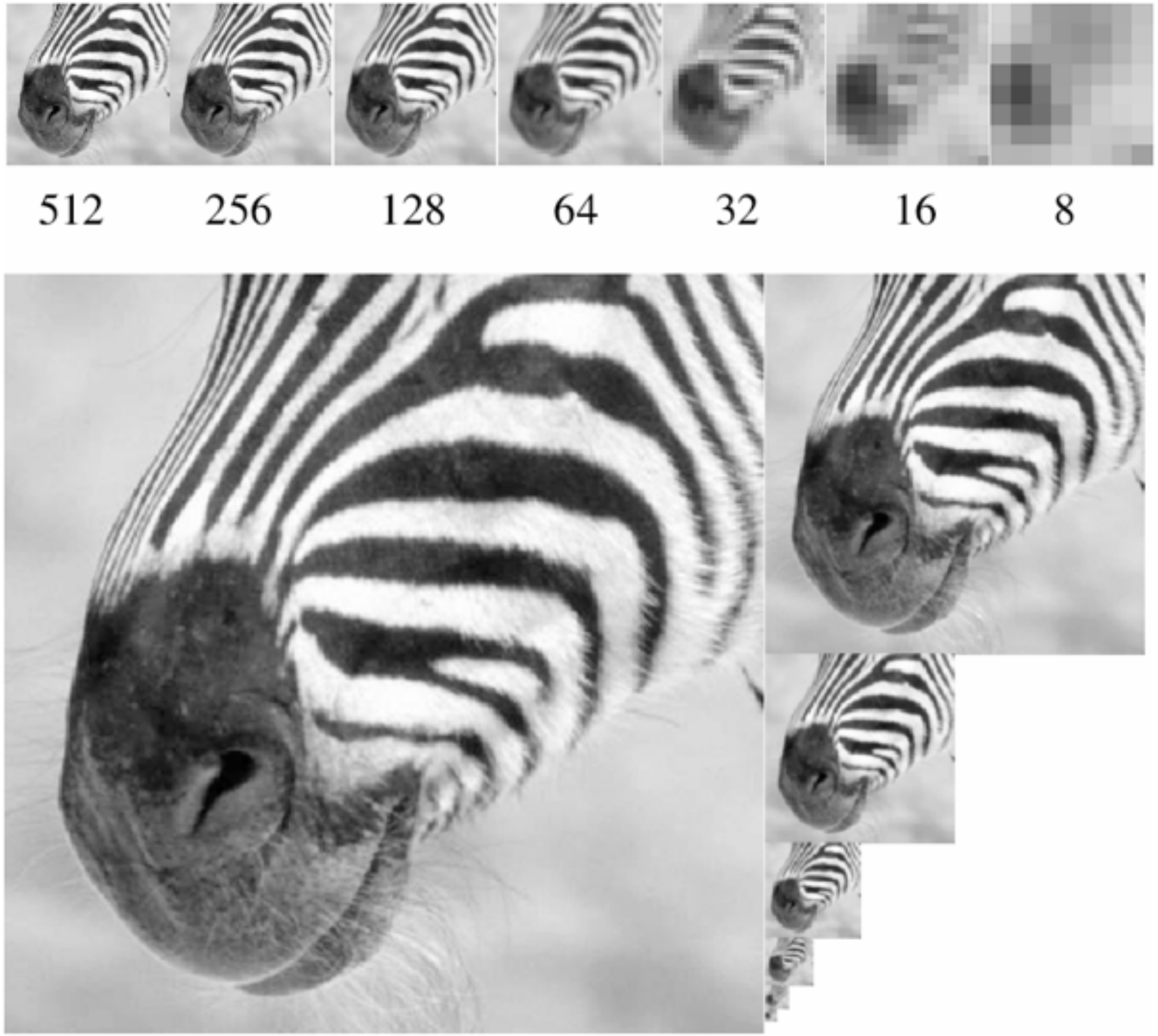
Two generic approaches to **edge detection**:

— local extrema of a first derivative operator → **Canny**

— zero crossings of a second derivative operator → **Marr/Hildreth**

Many algorithms consider "**boundary detection**" as a high-level recognition task and output a probability or confidence that a pixel is on a human-perceived boundary

Gaussian Pyramid



Forsyth & Ponce (2nd ed.) Figure 4.17

What happens to the details?

— They get smoothed out as we move to higher levels

What is preserved at the higher levels?

— Mostly large uniform regions in the original image

How would you reconstruct the original image from the image at the upper level?

— That's not possible

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Laplacian Pyramid

Building a **Laplacian** pyramid:

— Create a Gaussian pyramid

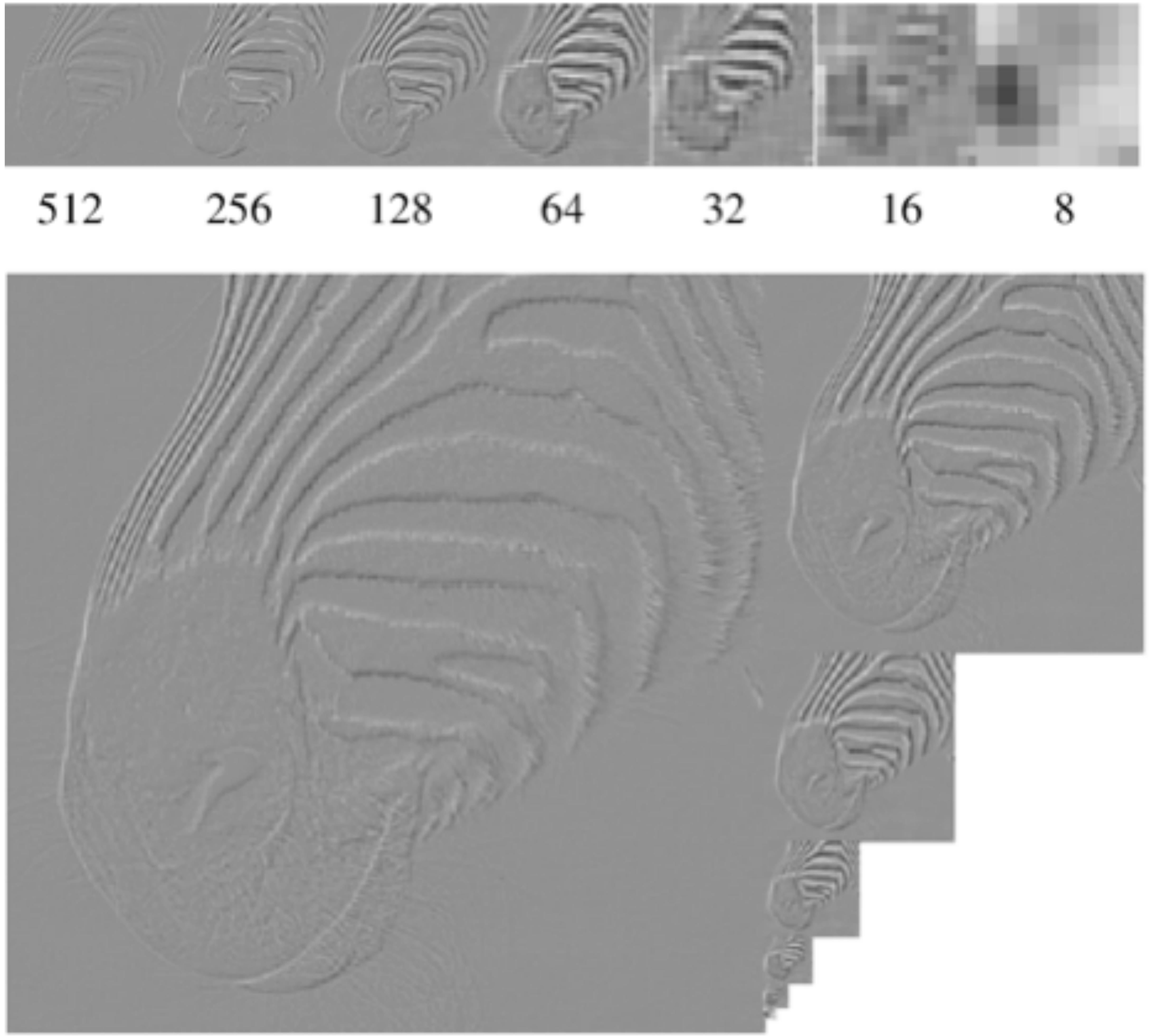
— Take the difference between one Gaussian pyramid level and the next (before subsampling)

Properties

— Also known as the difference-of-Gaussian (DOG) function, a close approximation to the Laplacian

— It is a band pass filter – each level represents a different band of spatial frequencies

Laplacian Pyramid

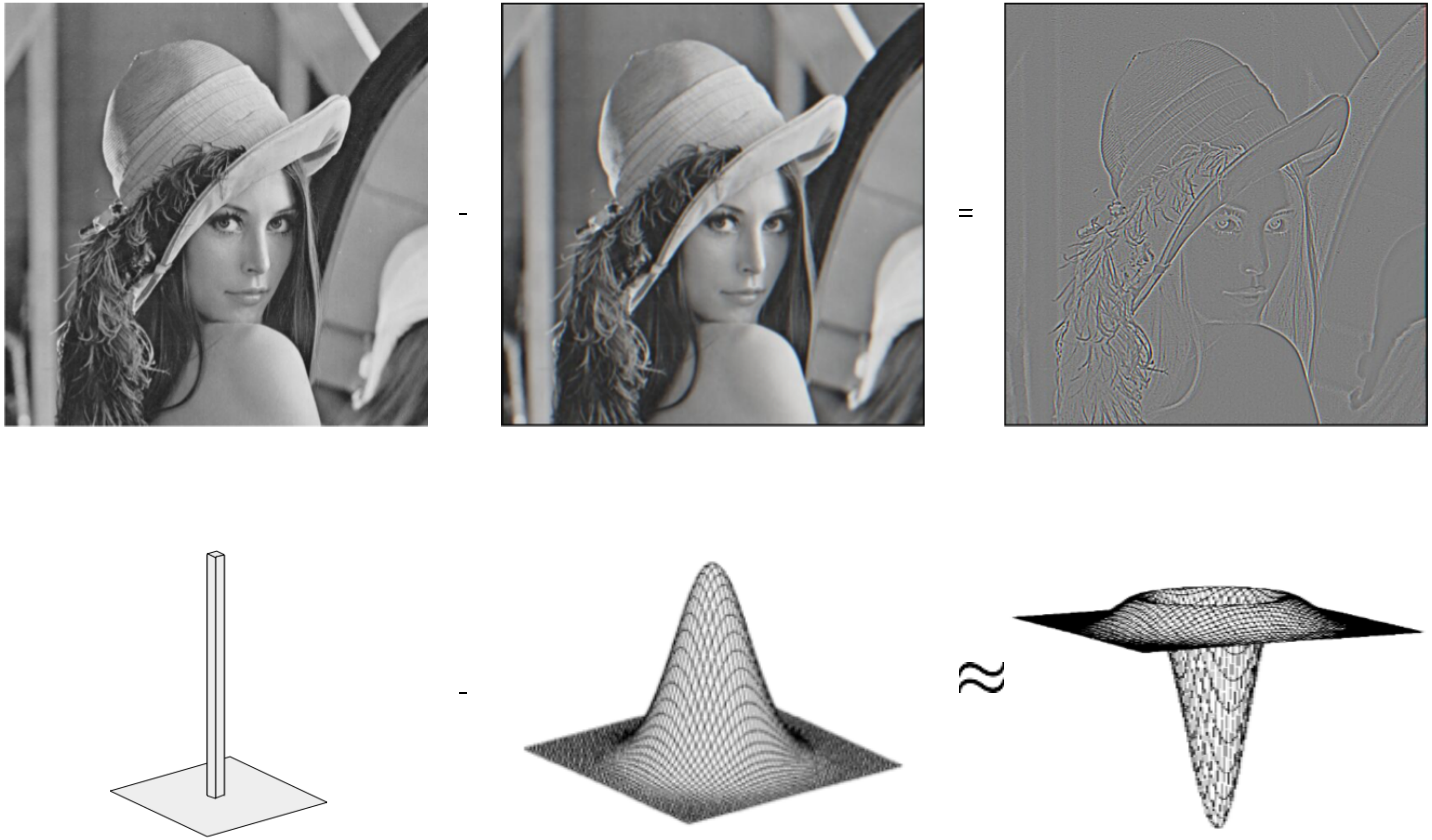


At each level, retain the residuals instead of the blurred images themselves.

Why is it called Laplacian Pyramid?

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

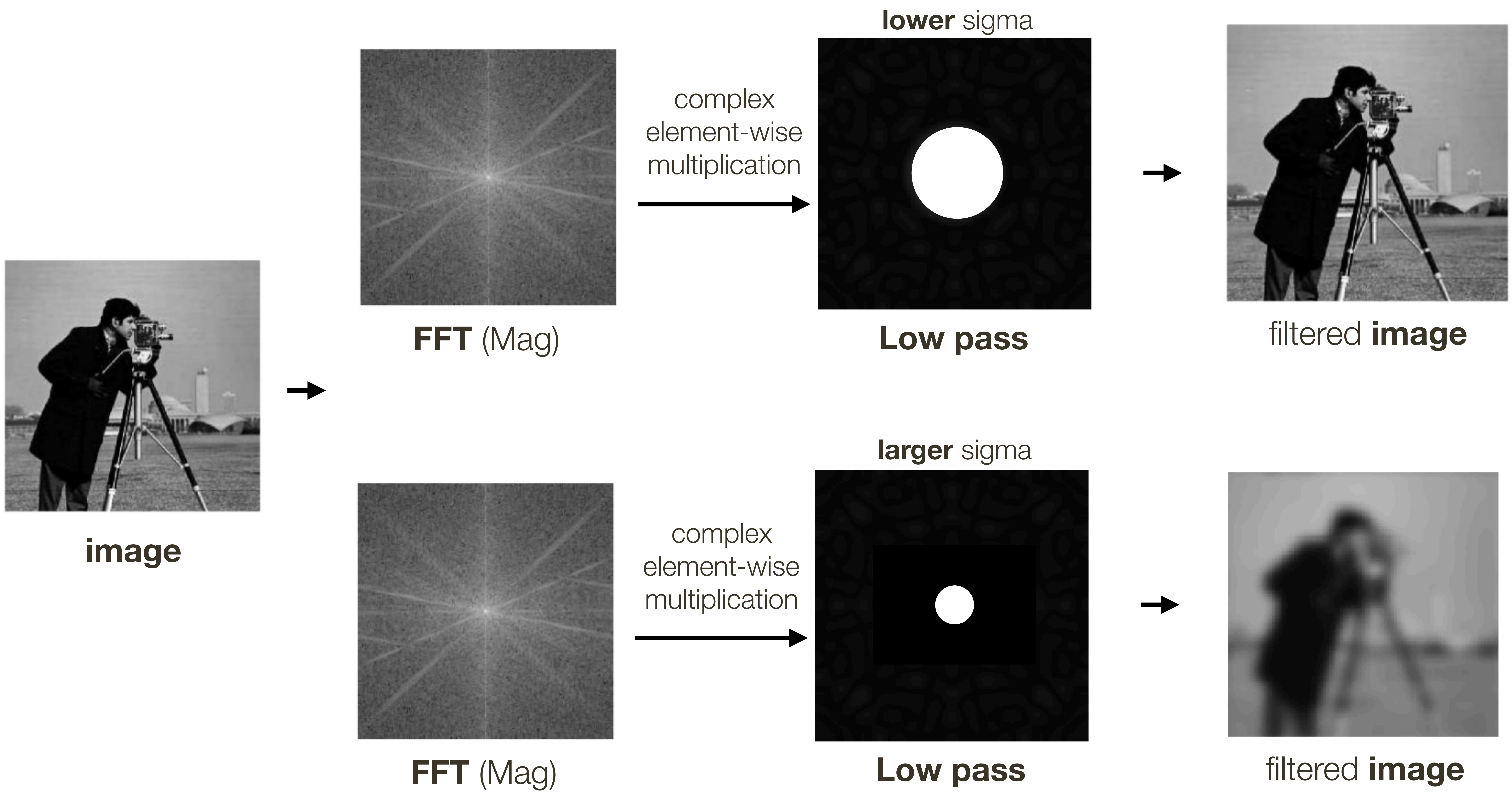
Why **Laplacian** Pyramid?



unit Gaussian Laplacian

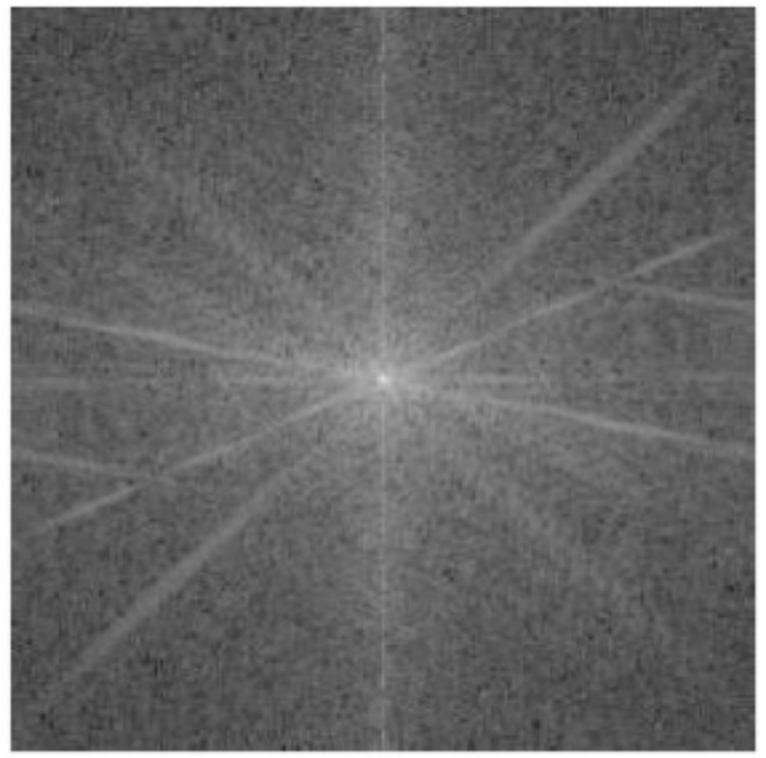
Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Laplacian is a Bandpass Filter



Laplacian is a Bandpass Filter

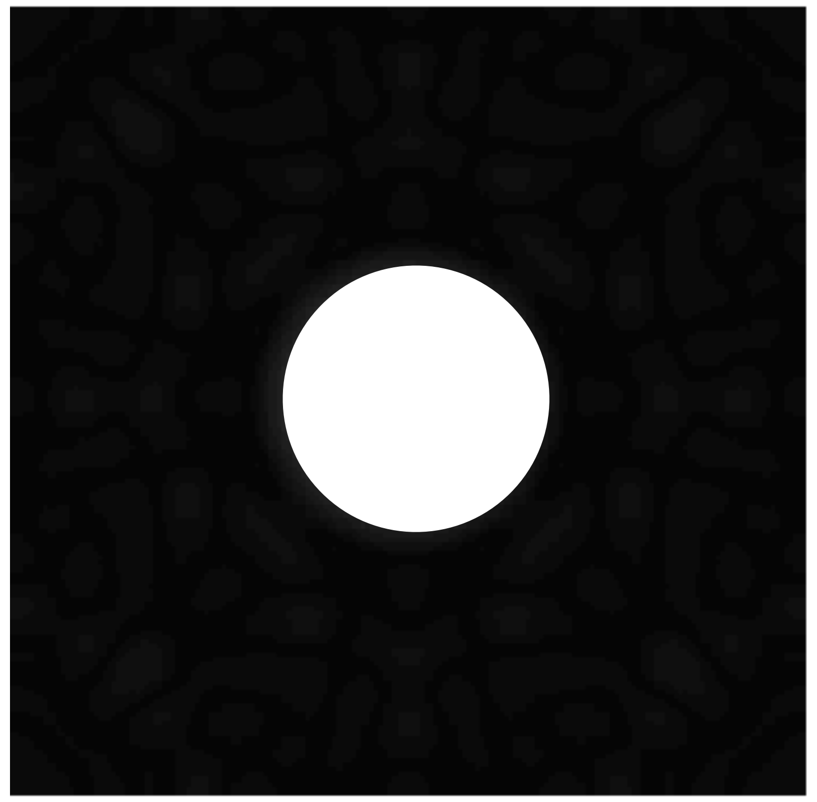
image



FFT (Mag)

complex
element-wise
multiplication

lower sigma

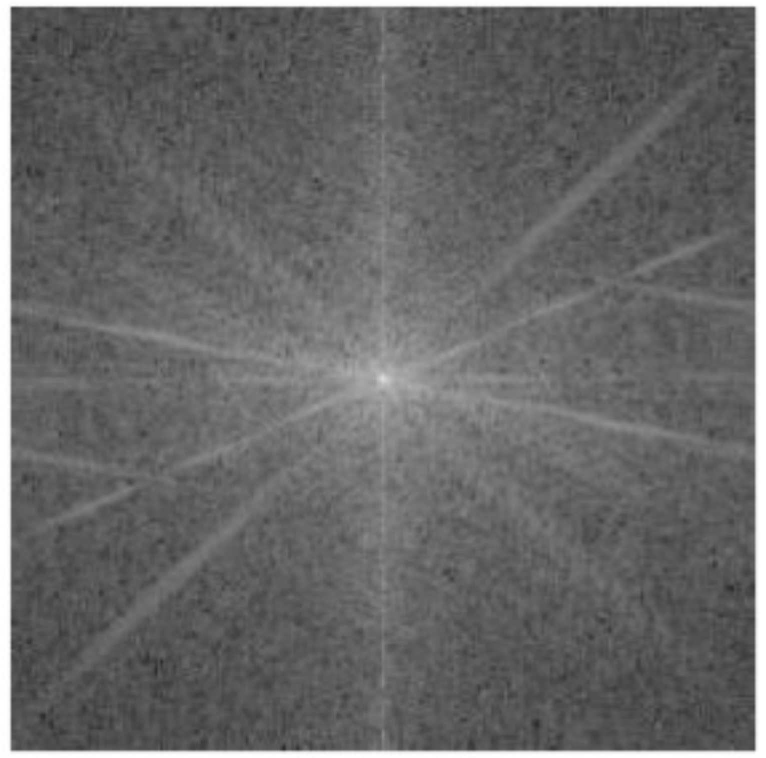


Low pass

filtered **image**

-

-



FFT (Mag)

complex
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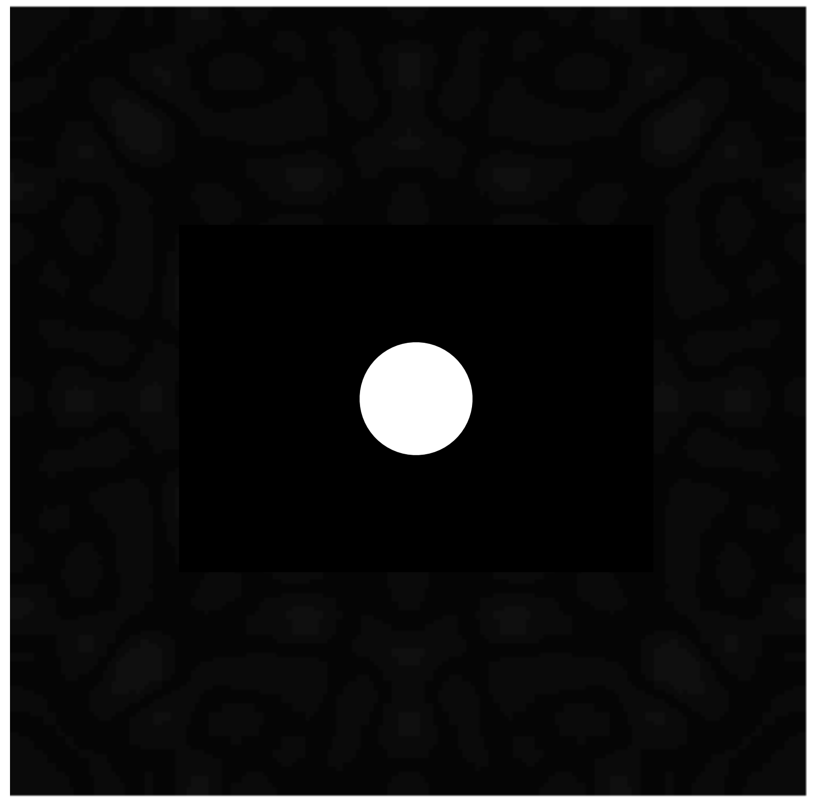
multiplication

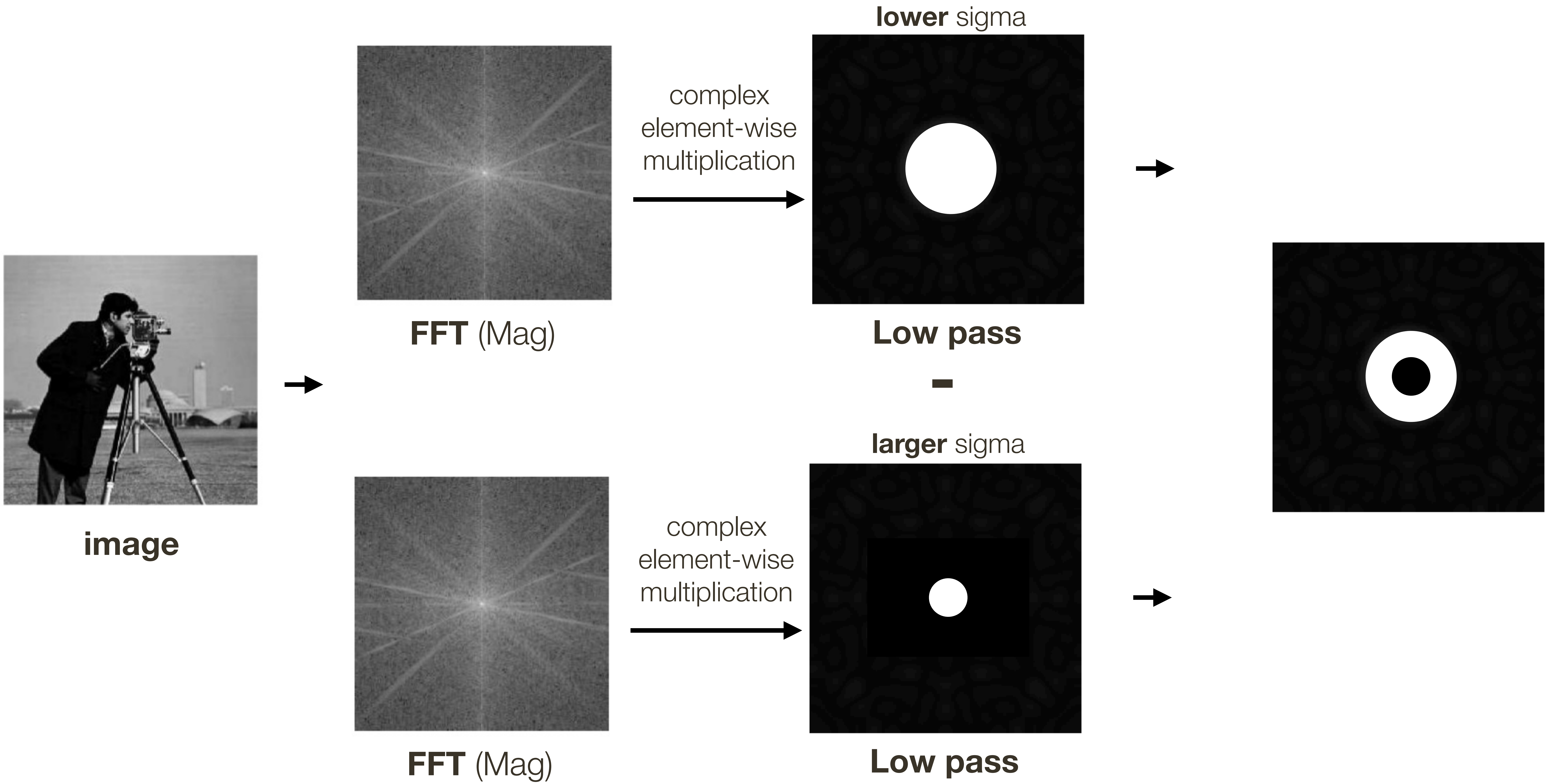
larger sigma



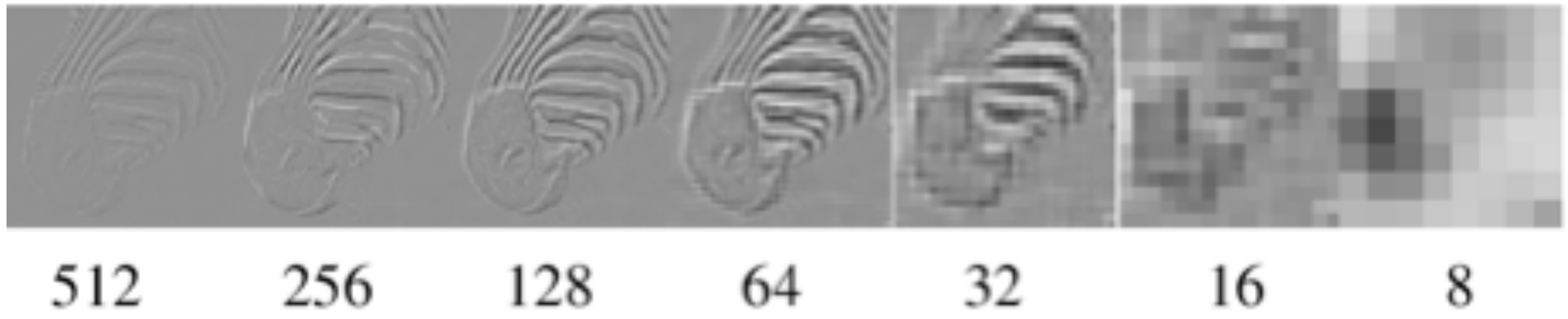
Low pass

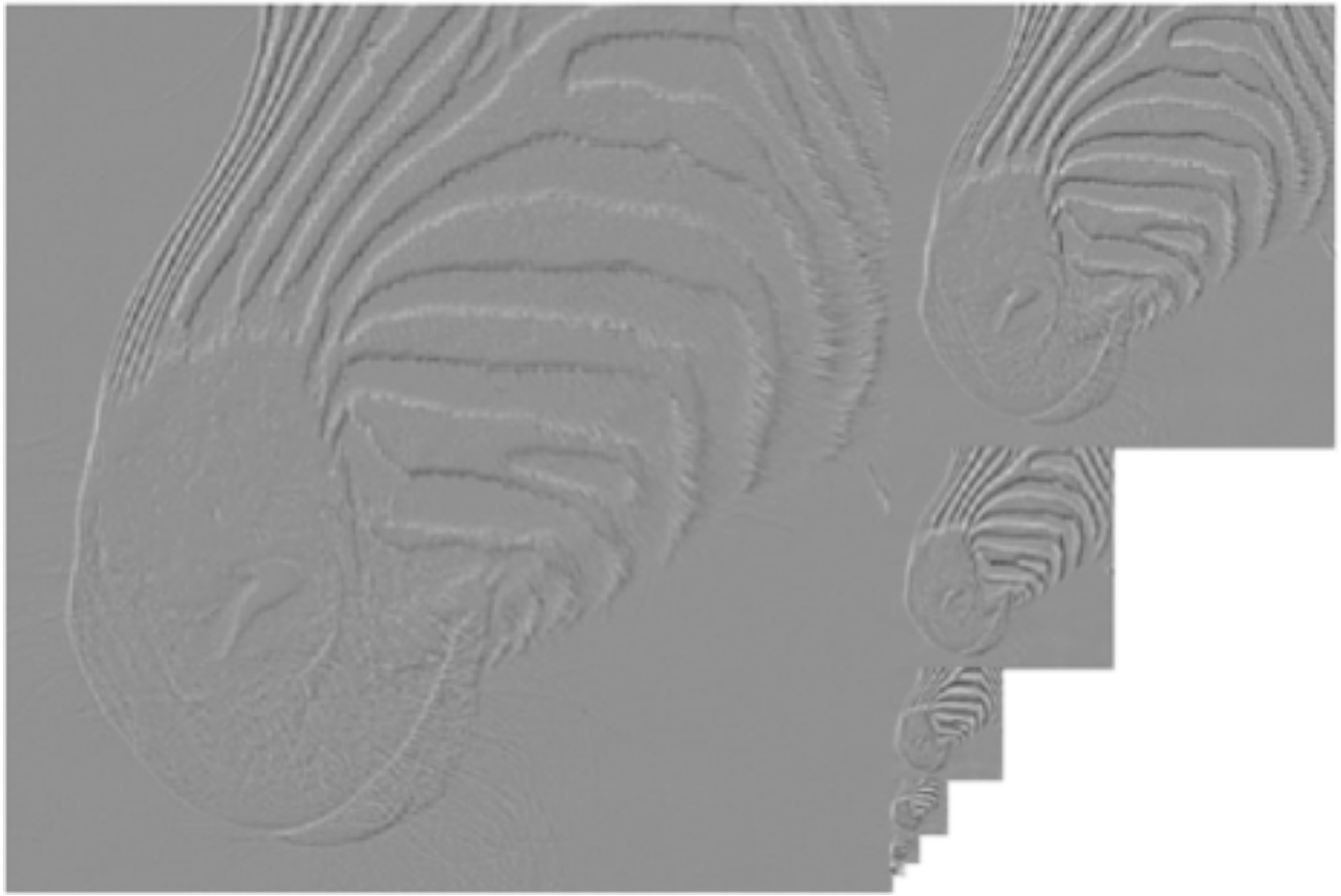
filtered **image**

Laplacian is a Bandpass Filter



Laplacian Pyramid





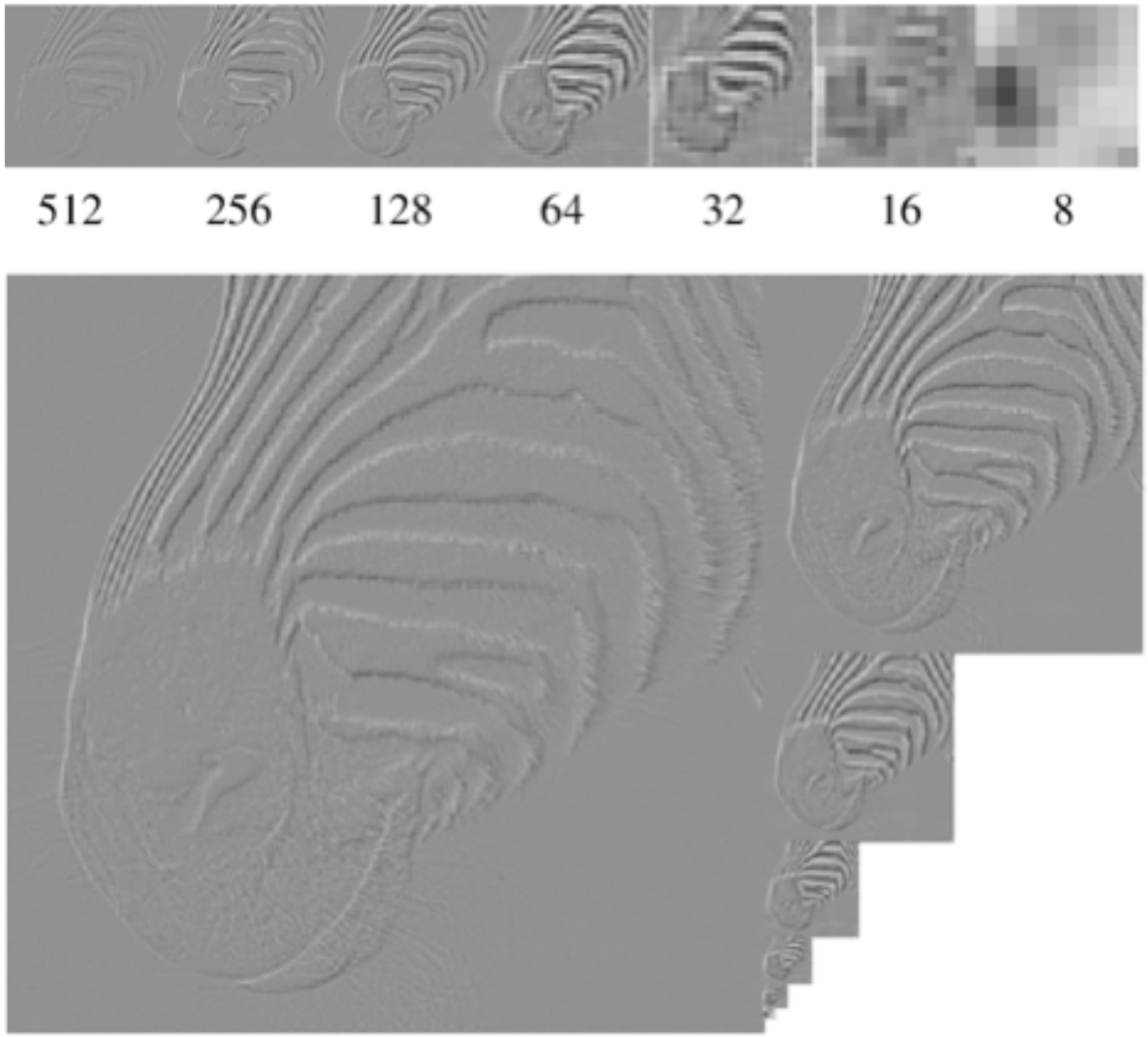
At each level, retain the residuals instead of the blurred images themselves.

Why is it called Laplacian Pyramid?

Can we reconstruct the original image using the pyramid?
— Yes we can!

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Laplacian Pyramid



At each level, retain the residuals instead of the blurred images themselves.

Why is it called Laplacian Pyramid?

Can we reconstruct the original image using the pyramid?
— Yes we can!

What do we need to store to be able to reconstruct the original image?

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Let's start by just looking at **one level**

level 0

=

level 1 (upsampled)

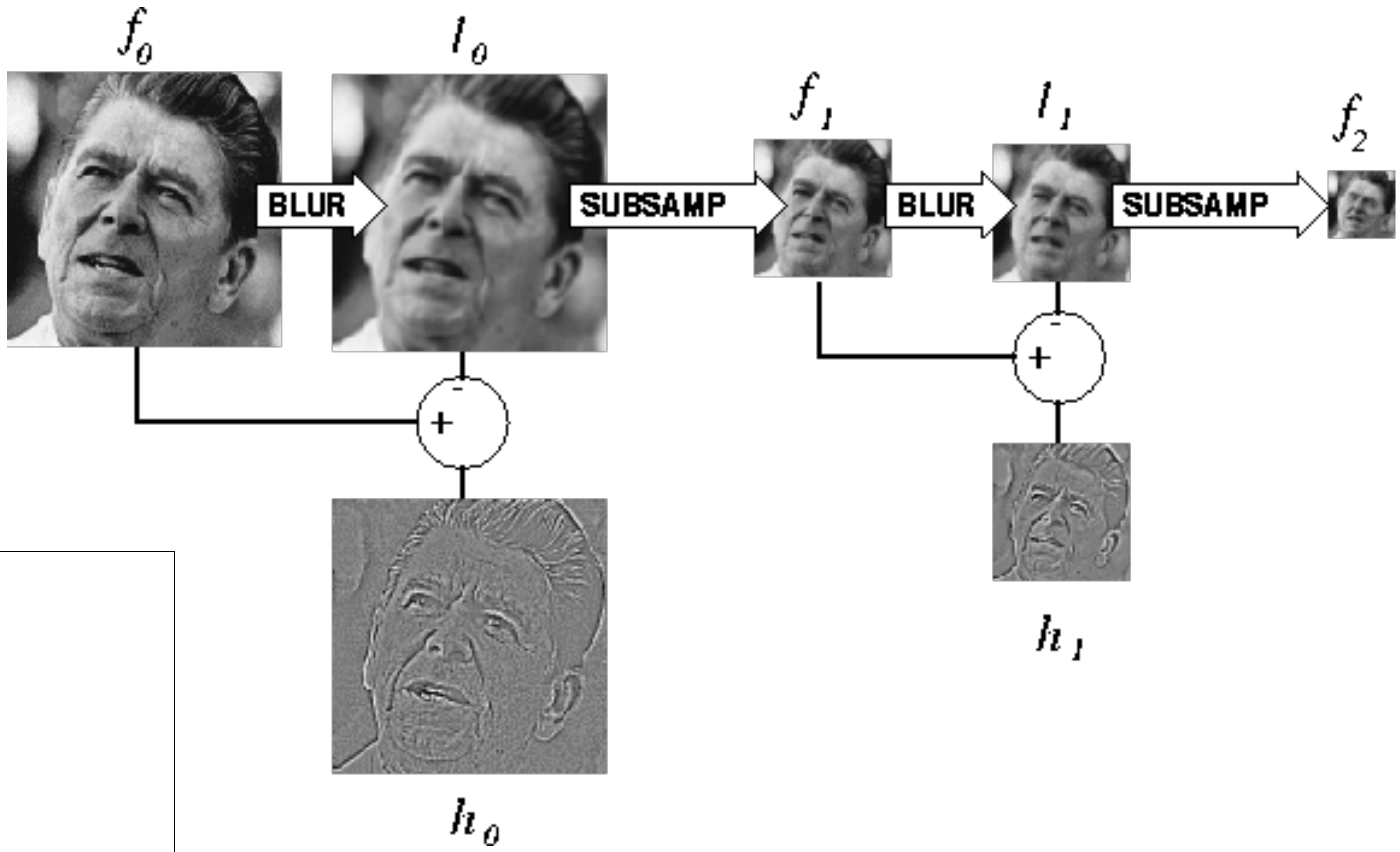
+

residual

Does this mean we need to store both residuals and the blurred copies of the original?

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Constructing a **Laplacian** Pyramid

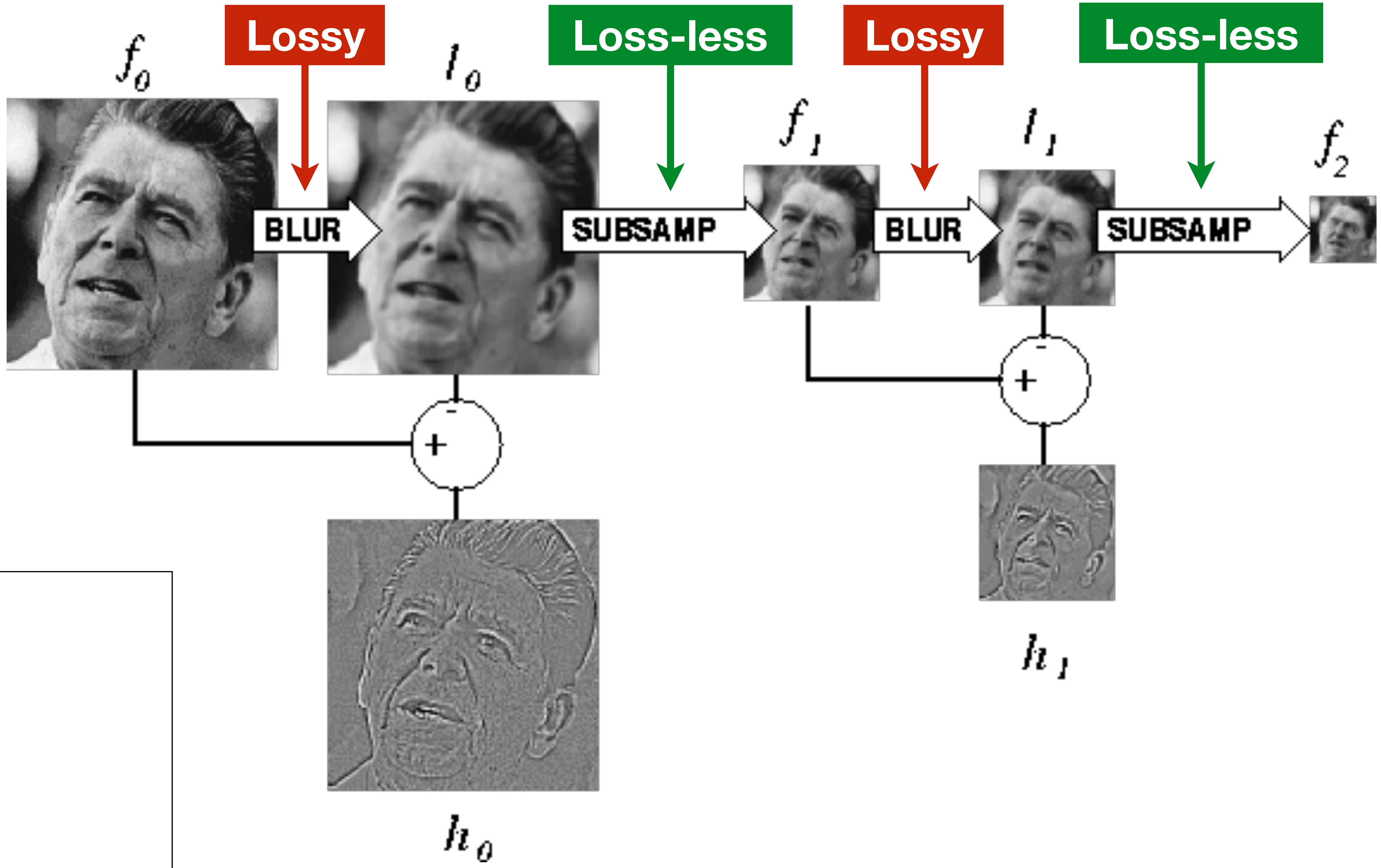


Algorithm

repeat:
  filter
  compute residual
  subsample
until min resolution reached

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Constructing a **Laplacian** Pyramid

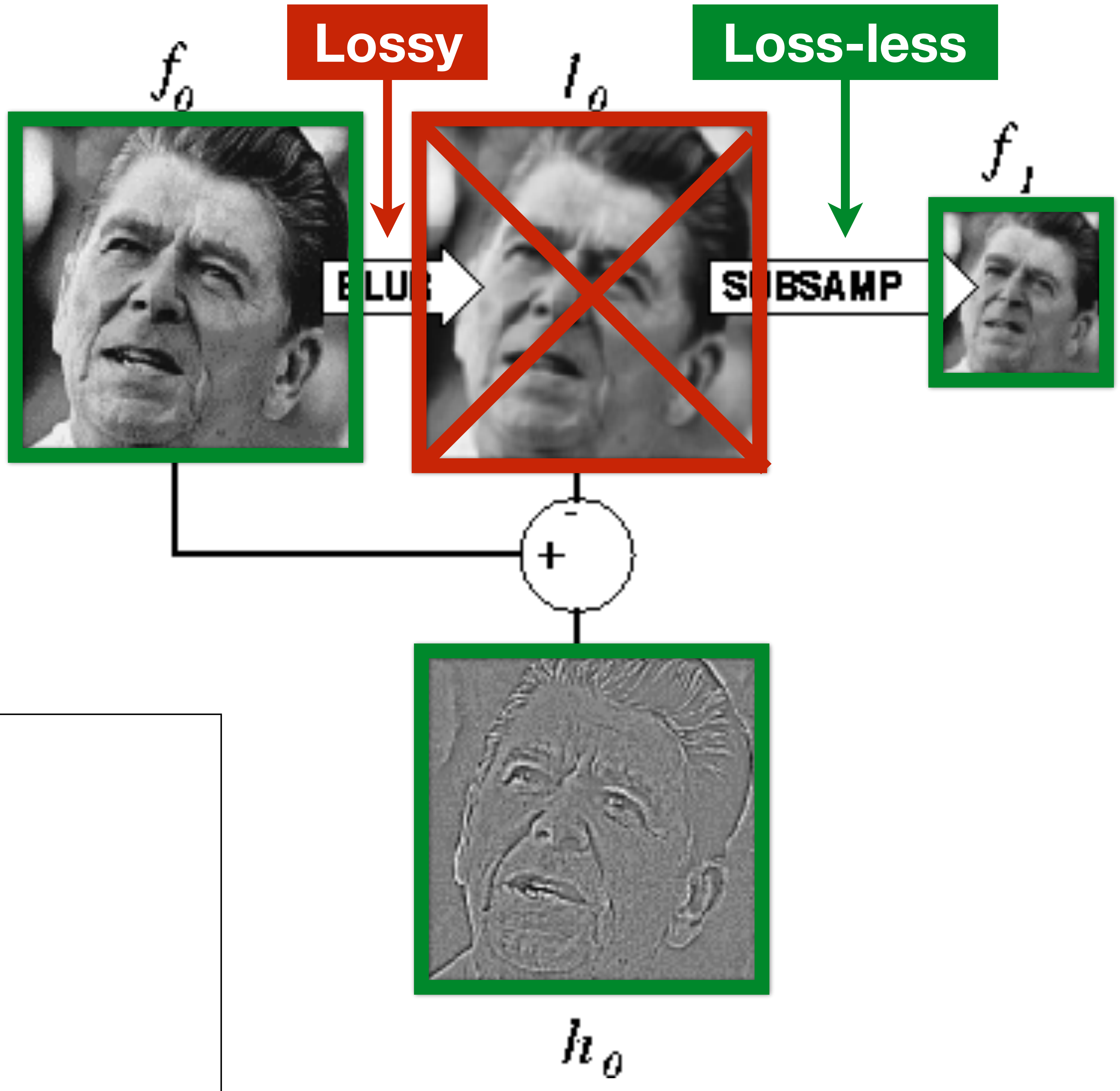


Algorithm

repeat: filter compute residual subsample until min resolution reached

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Constructing a **Laplacian** Pyramid



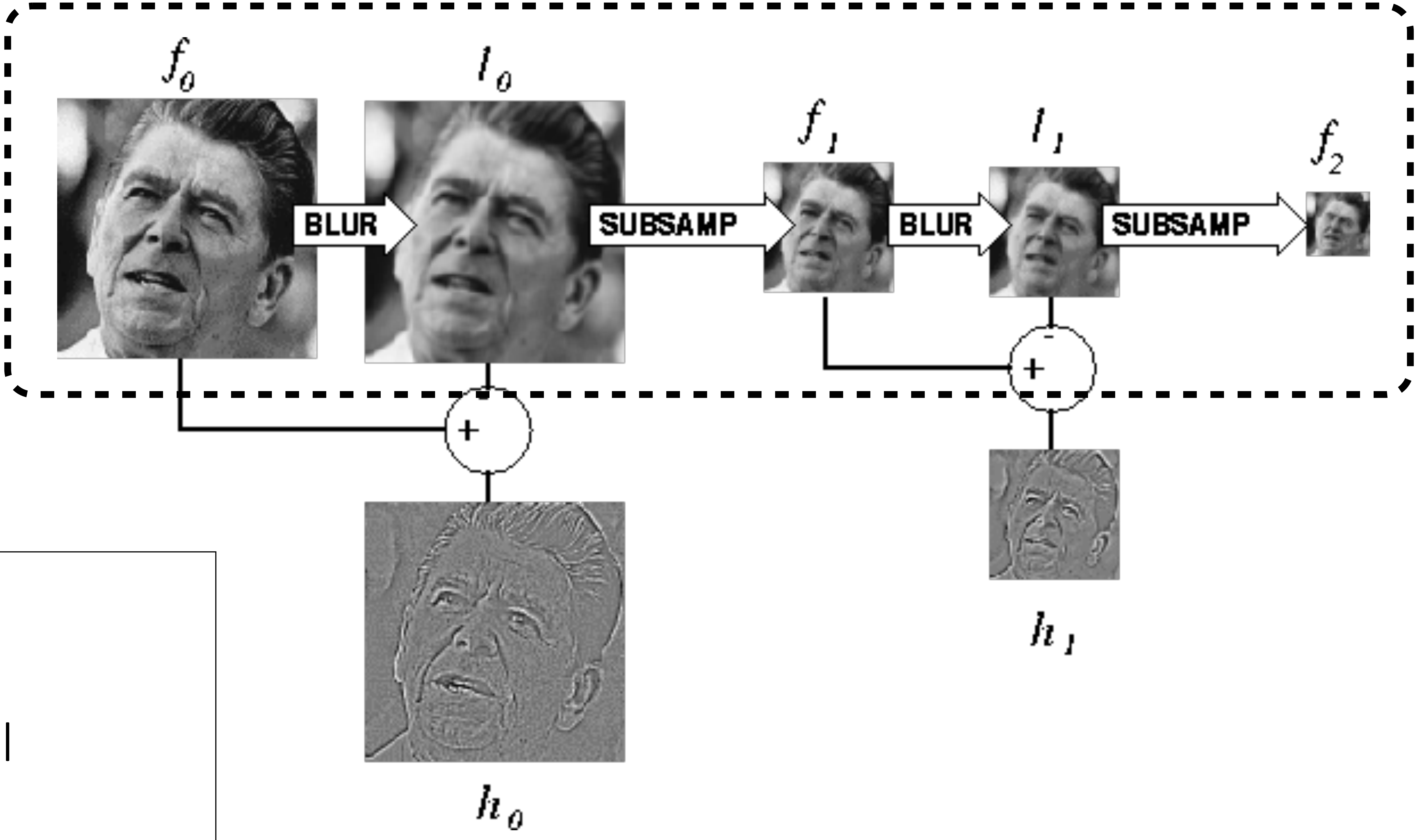
Algorithm

repeat:
filter
compute residual
subsample
until min resolution reached

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Constructing a **Laplacian** Pyramid

What is this part?



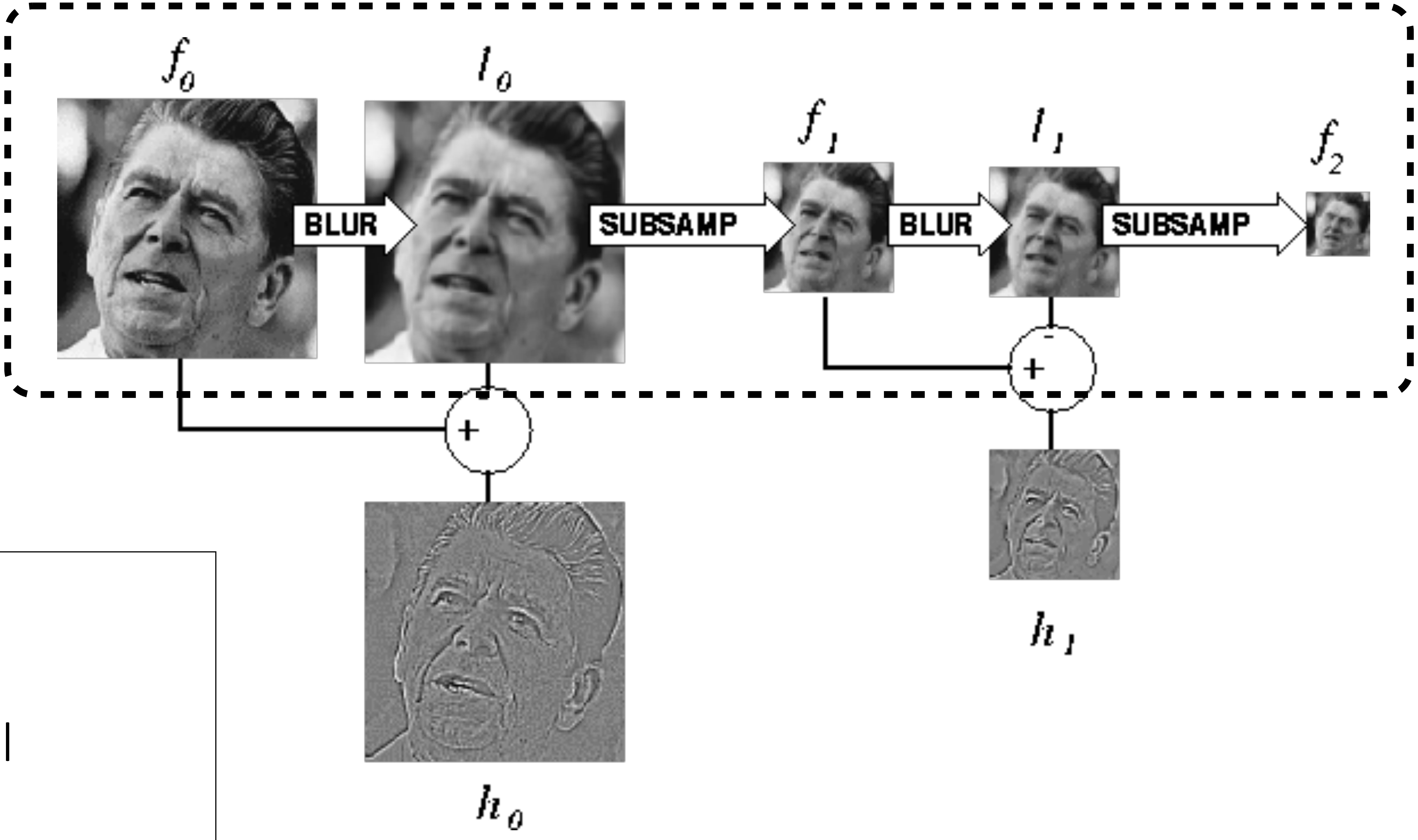
Algorithm

repeat:
    filter
    compute residual
    subsample
until min resolution reached

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Constructing a **Laplacian** Pyramid

It's a Gaussian Pyramid



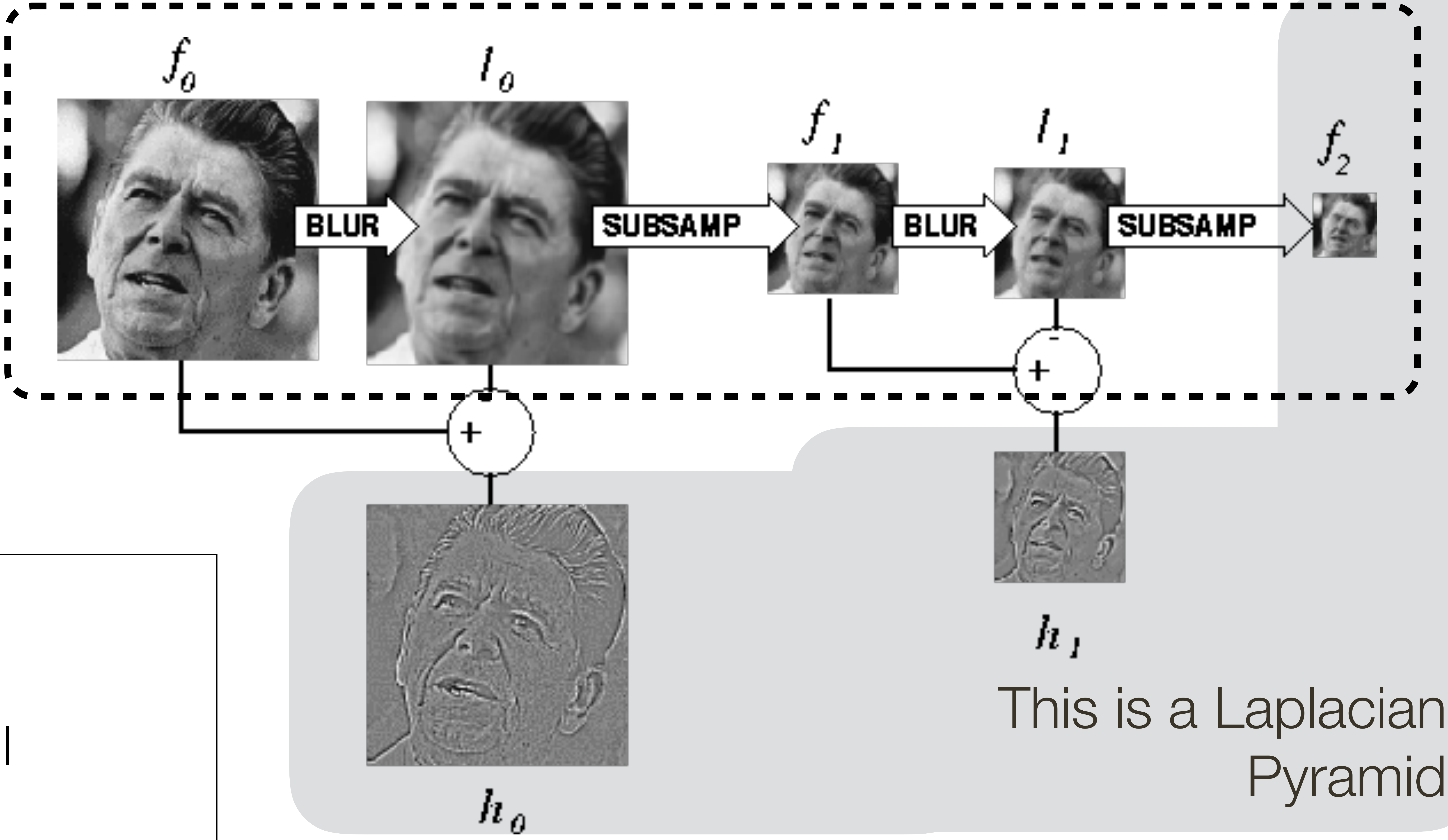
Algorithm

repeat:
    filter
    compute residual
    subsample
until min resolution reached

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Constructing a **Laplacian** Pyramid

It's a Gaussian Pyramid



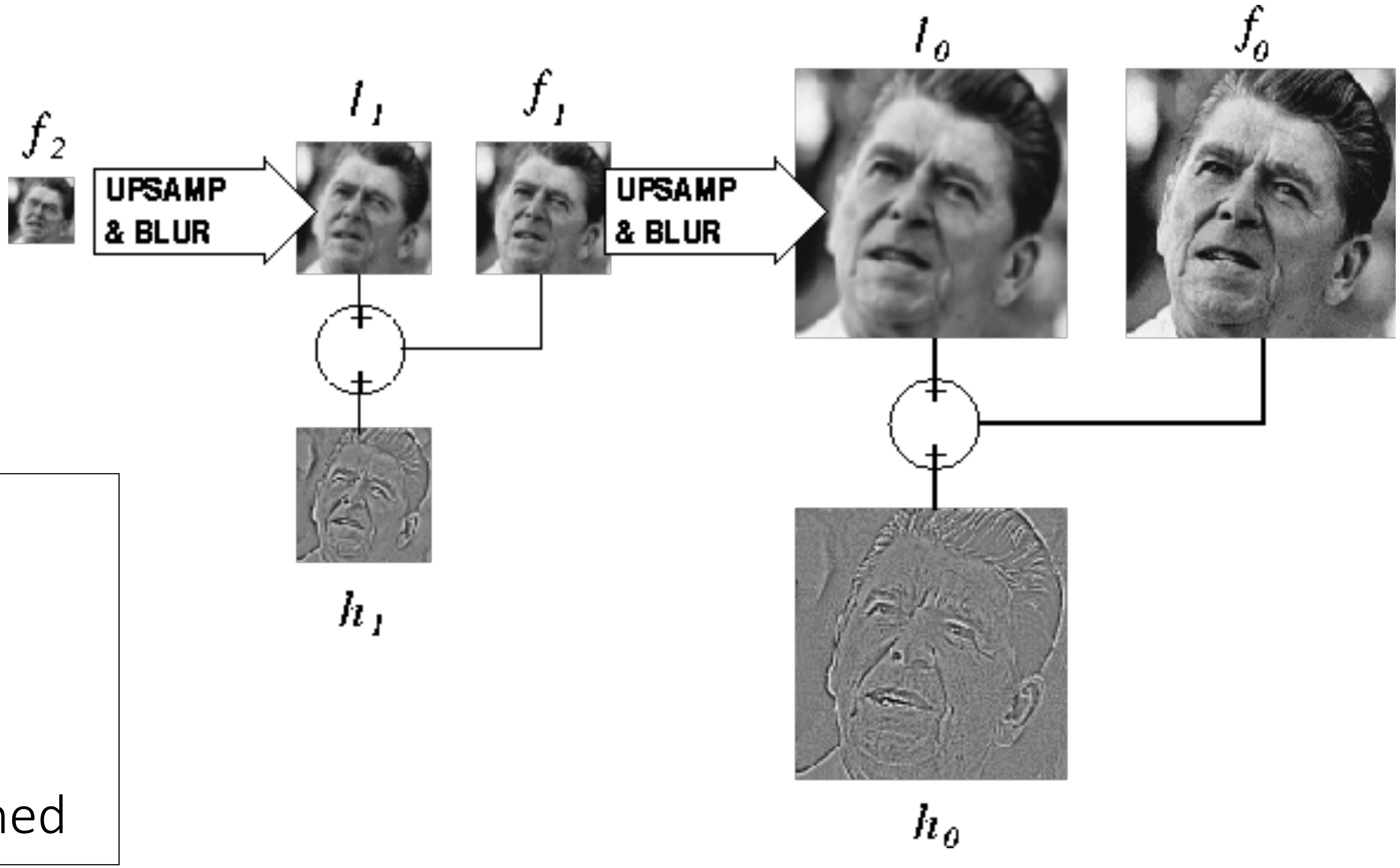
This is a Laplacian Pyramid

Algorithm

repeat:
  filter
  compute residual
  subsample
until min resolution reached

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Reconstructing the Original Image



Algorithm

repeat:

upsample

sum with residual

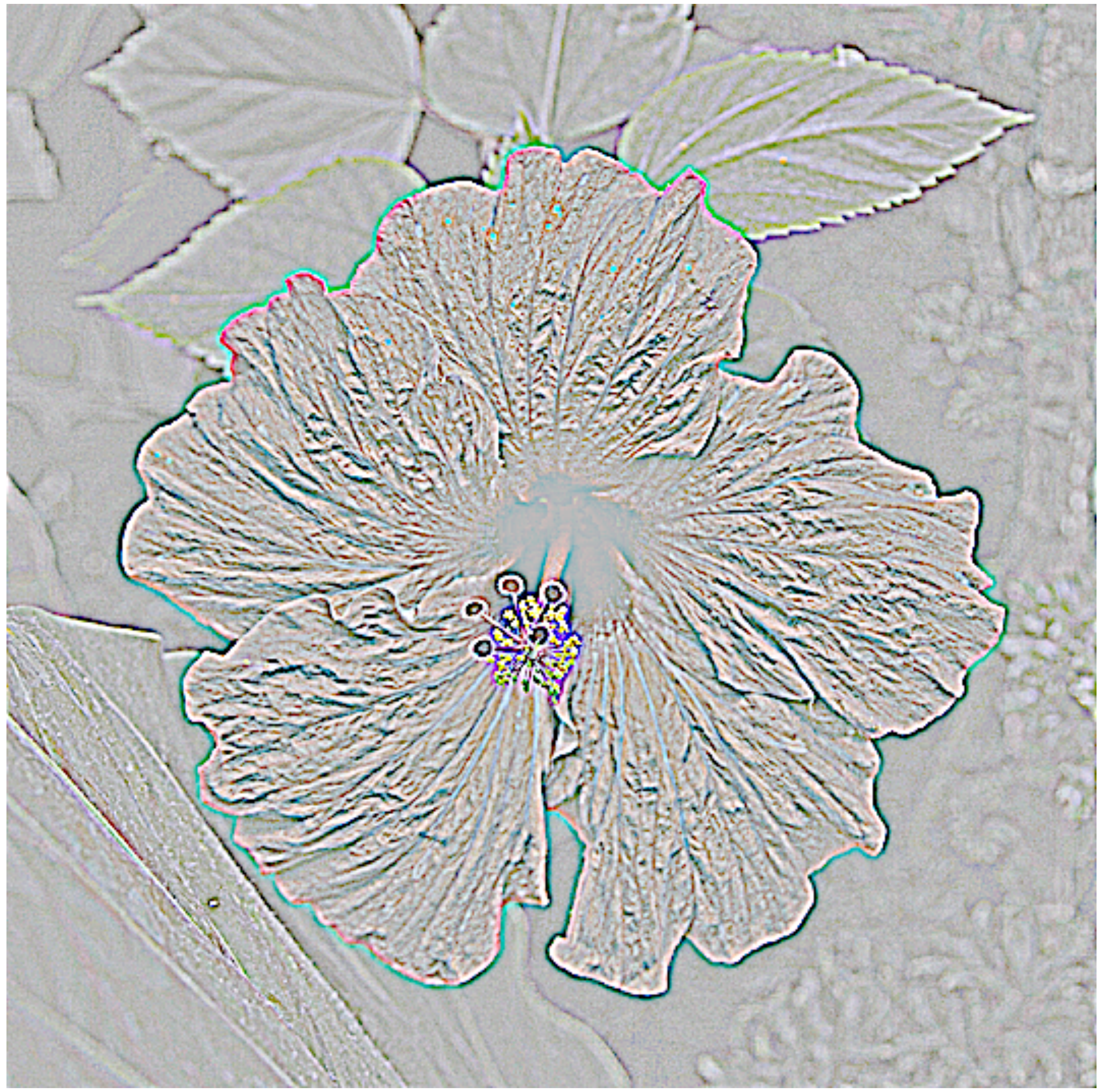
until orig resolution reached

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Gaussian vs **Laplacian** Pyramid

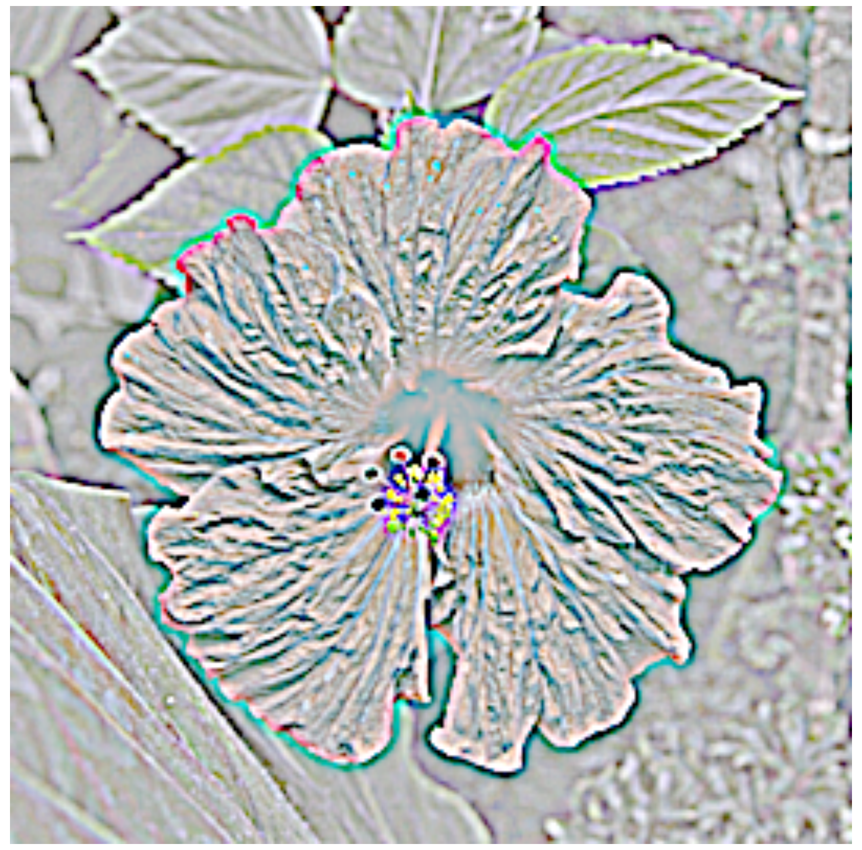



Shown in opposite order for space



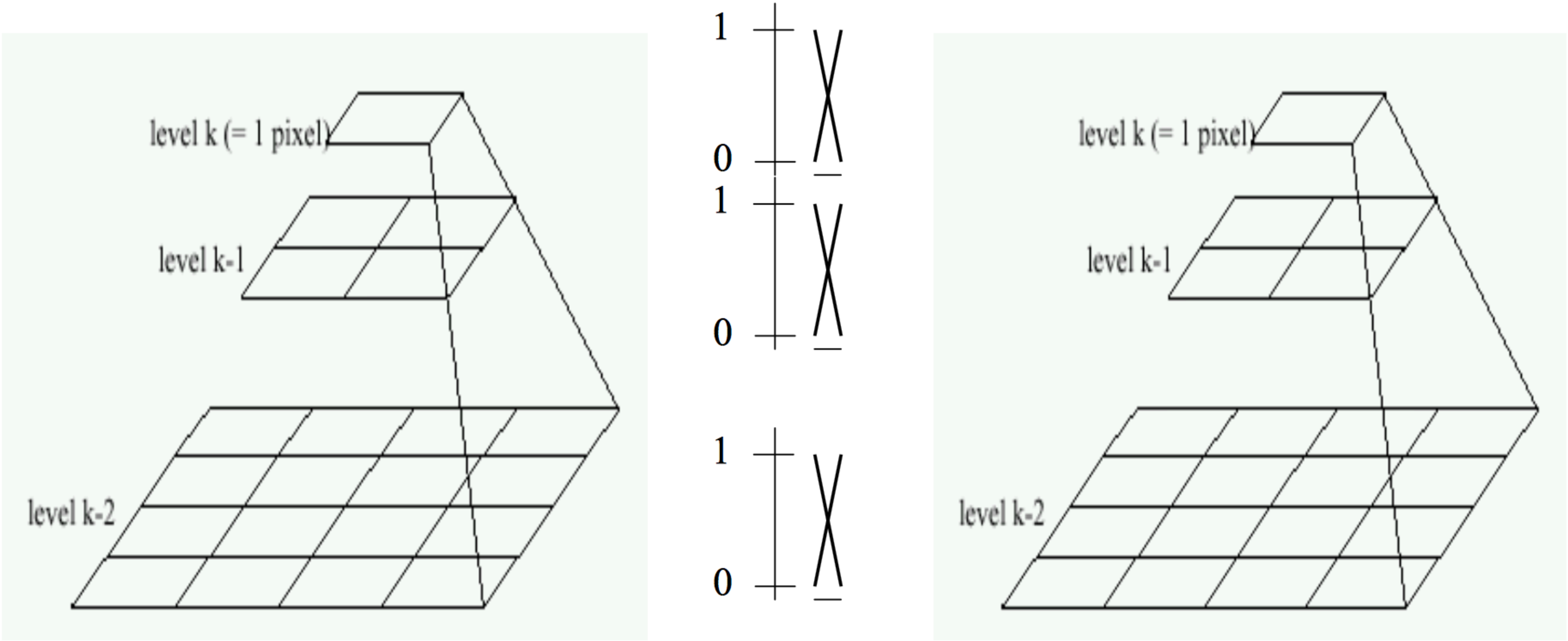
Which one takes more space to store?





Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Aside: Image Blending



Left pyramid blend Right pyramid

Burt and Adelson, “A multiresolution spline with application to image mosaics,”ACM Transactions on Graphics, 1983, Vol.2, pp.217-236.

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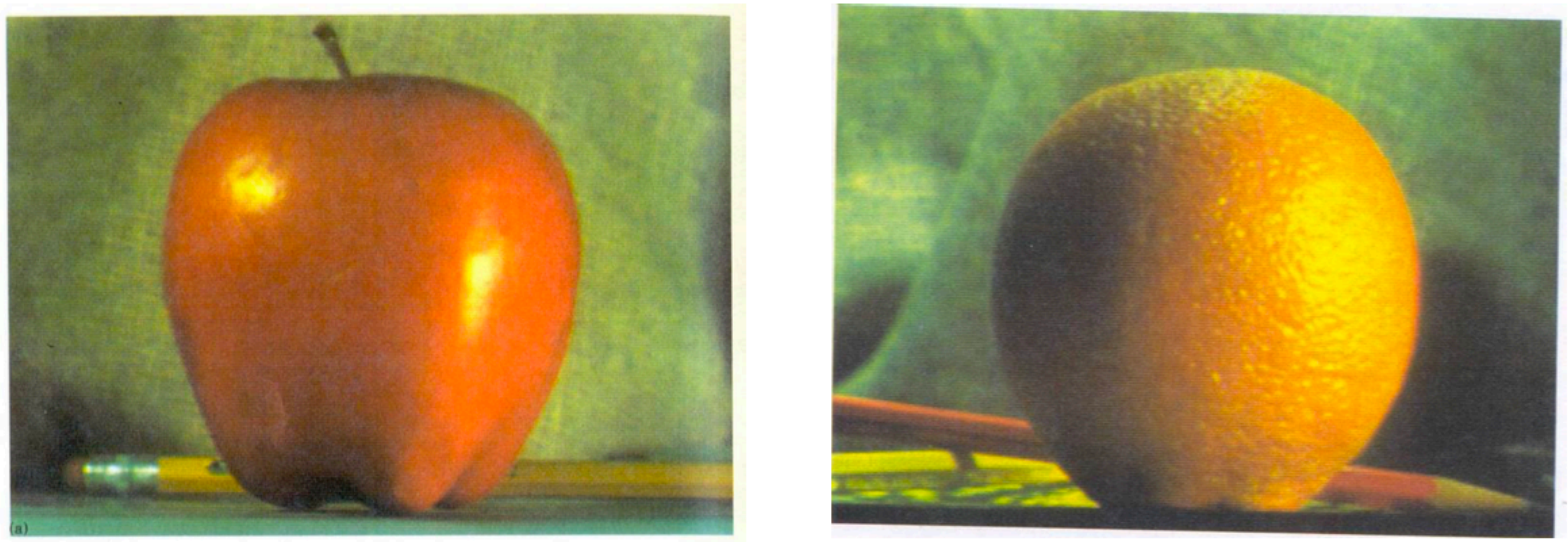

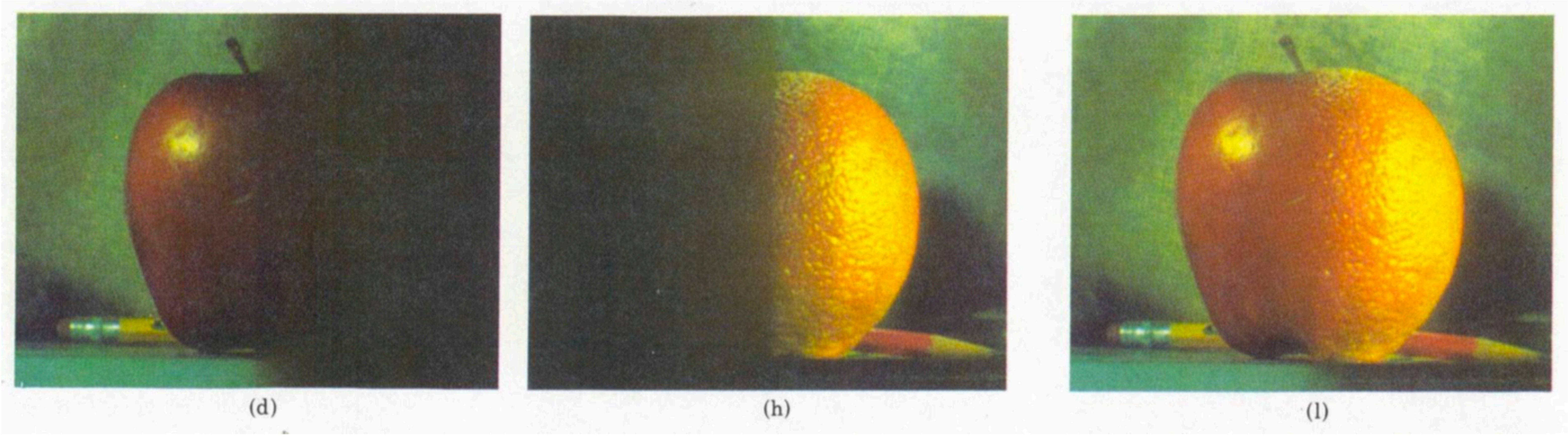

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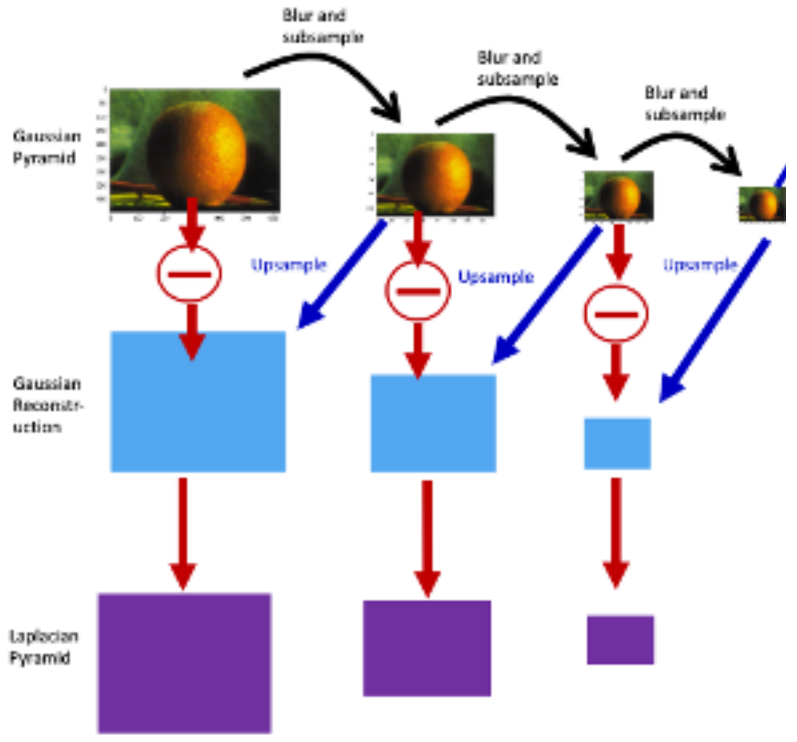

https://becominghuman.ai/image-blending-using-laplacian-pyramids-2f8e9982077f

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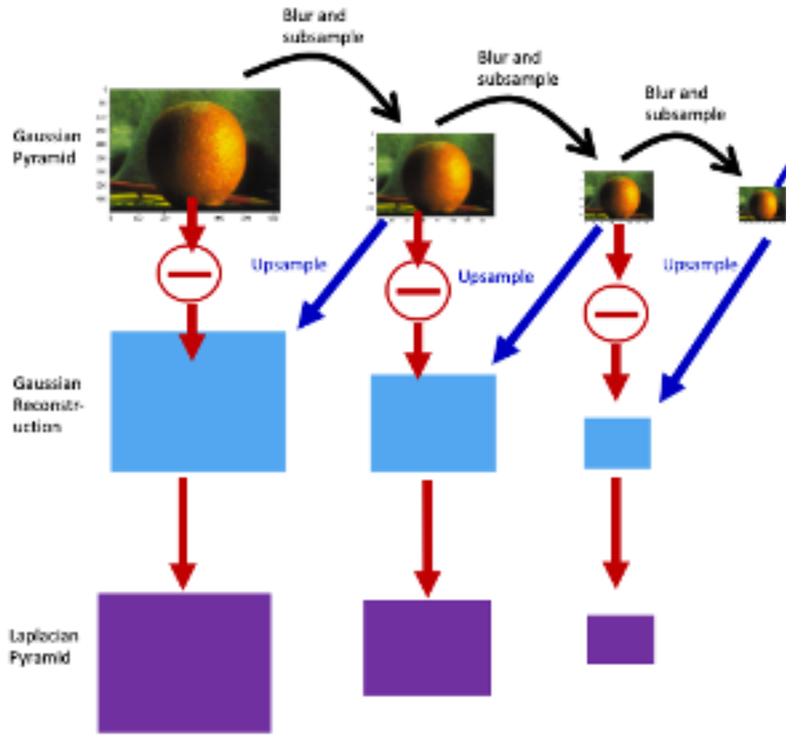

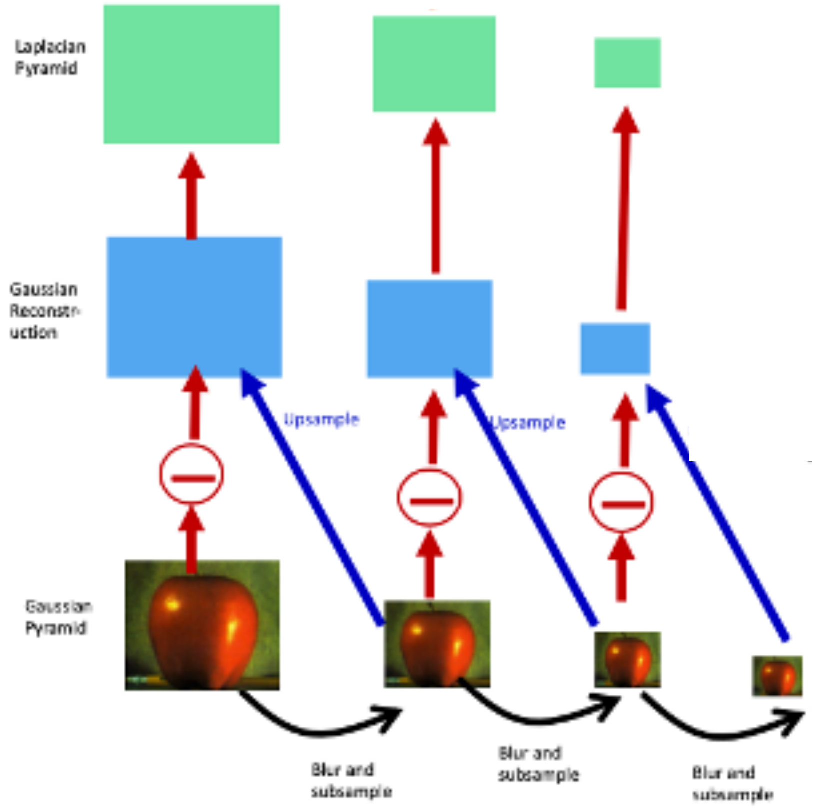

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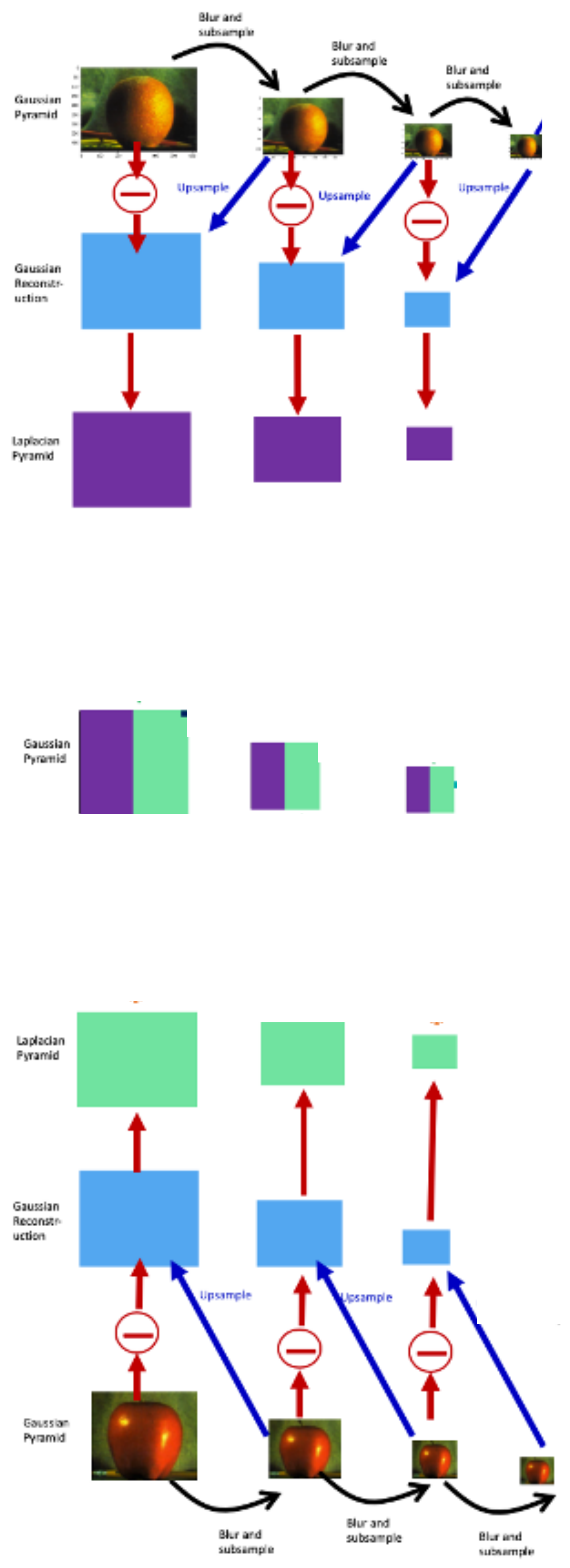

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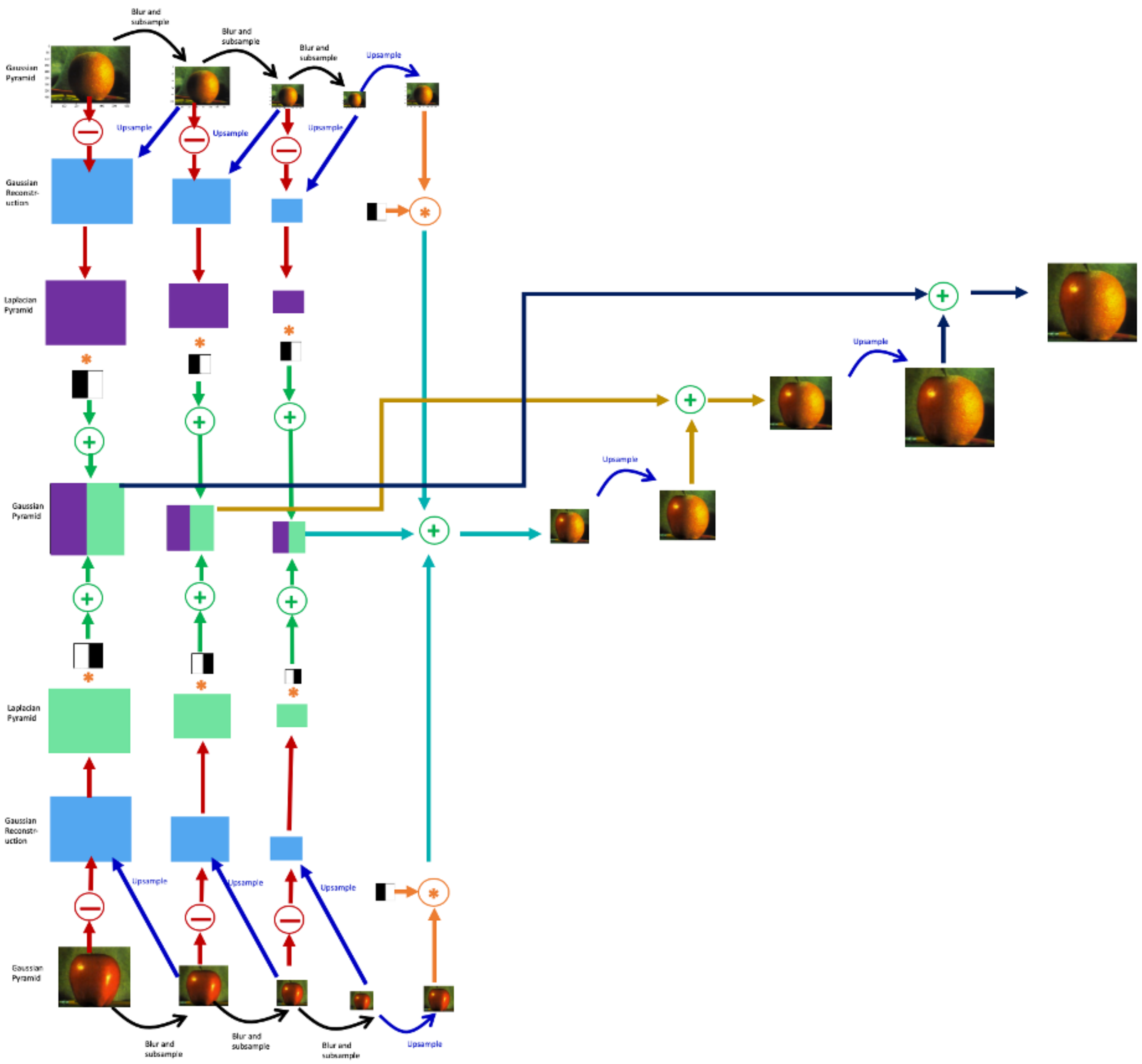

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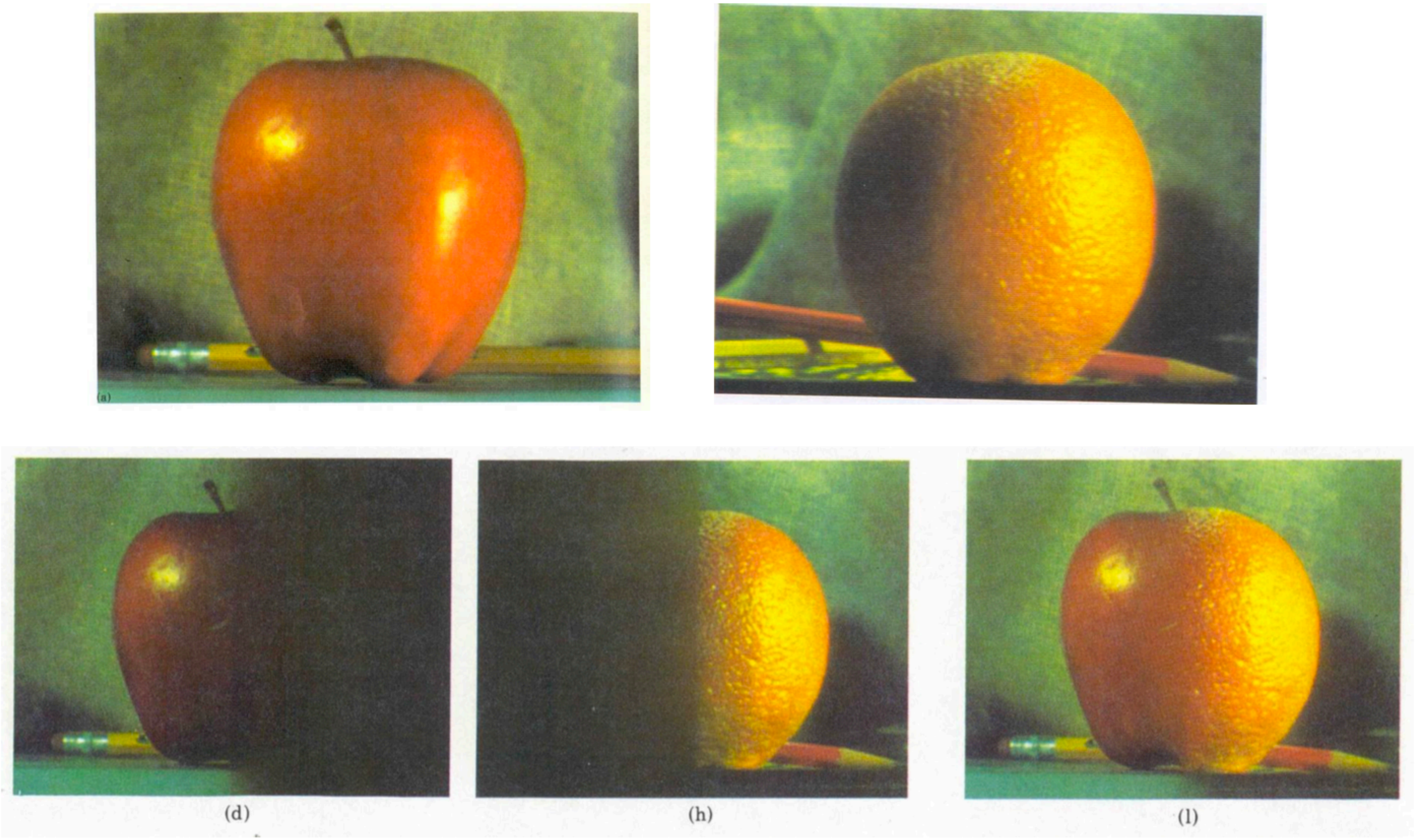
Aside: Image Blending



High-level Intuition: Smoother blending of flatter regions, sharper blending of more detailed regions

Aside: Image Blending

Algorithm:

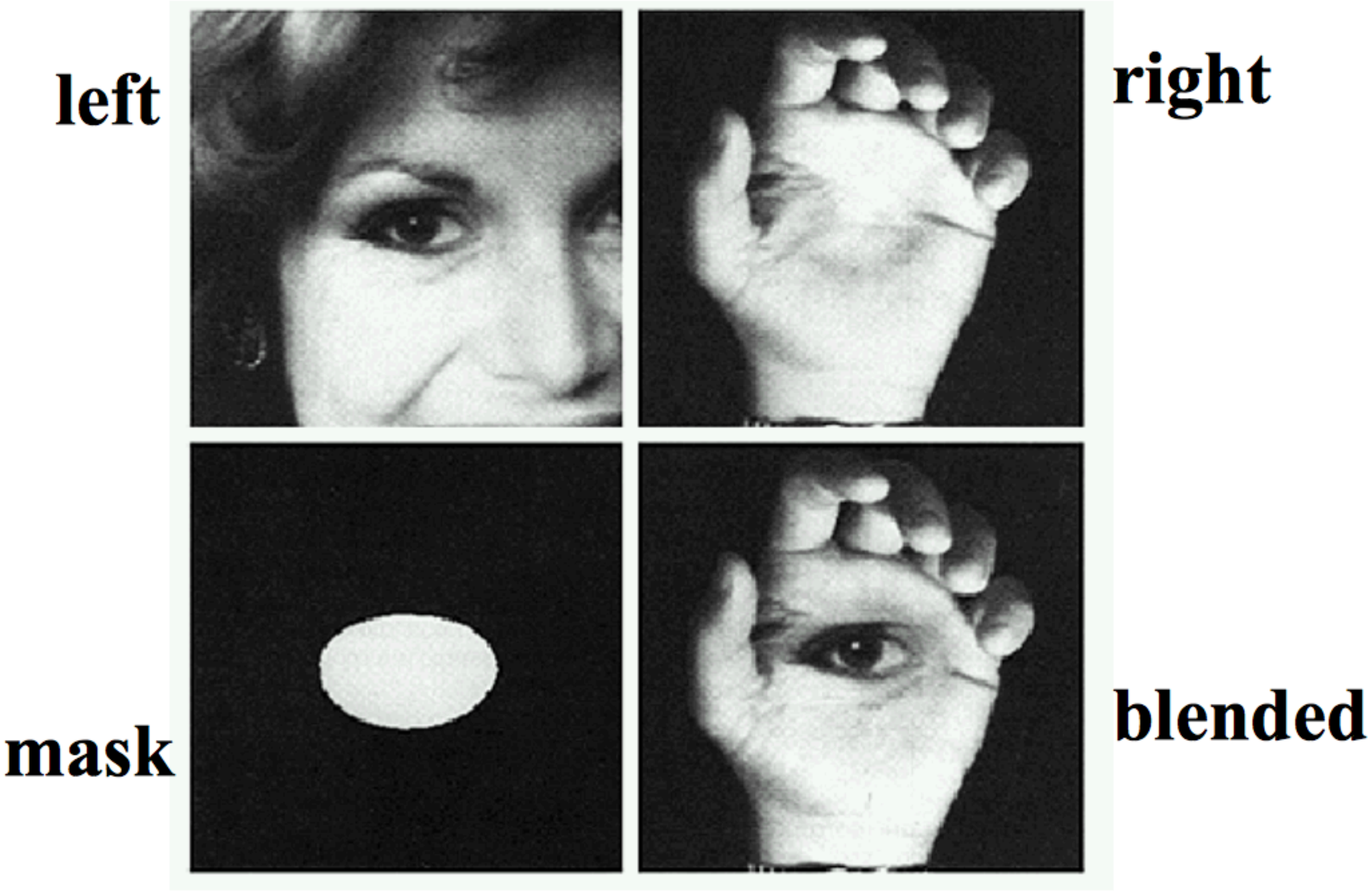
1. Build Laplacian pyramid LA and LB from images A and B

2. Build a Gaussian pyramid GR from mask image R (the mask defines which image pixels should be coming from A or B)

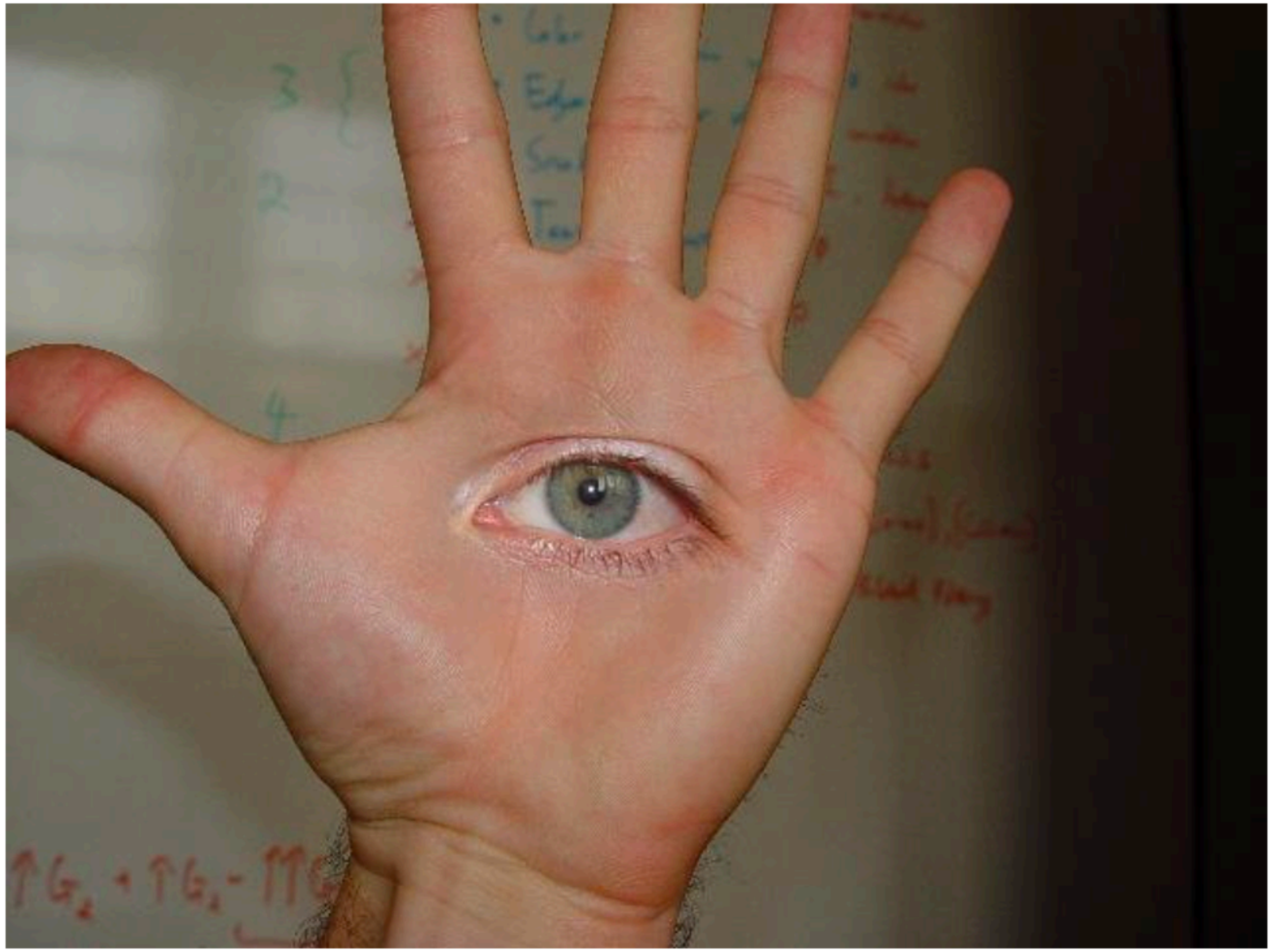
3. From a combined (blended) Laplacian pyramid LS, using nodes of GR as weights: $LS(i,j) = GR(i,j) * LA(i,j) + (1-GR(i,j)) * LB(i,j)$

4. Reconstruct the final blended image from LS

Aside: Image Blending



Aside: Image Blending

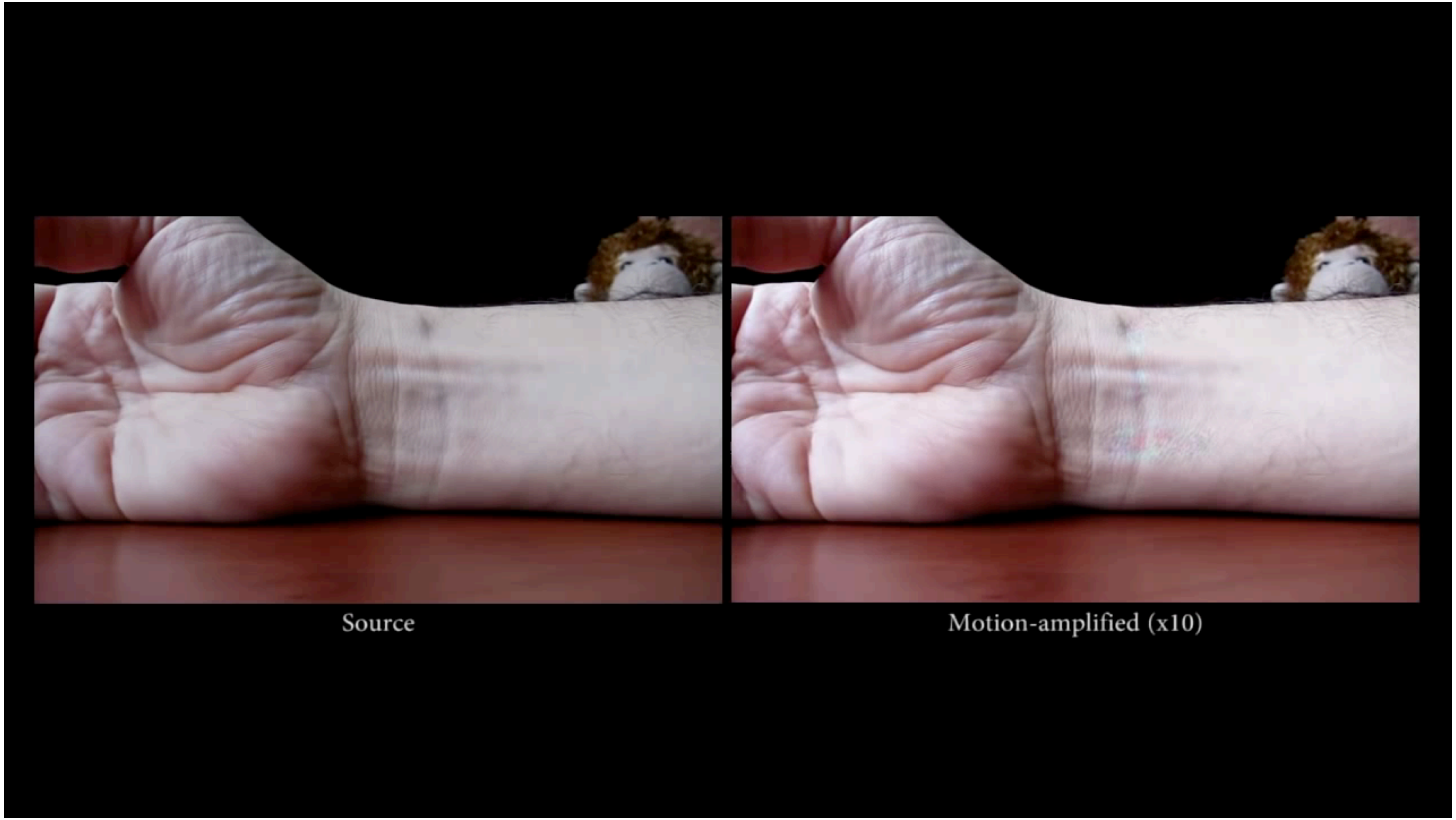


© david dmartin (Boston College)

Aside: Image Blending

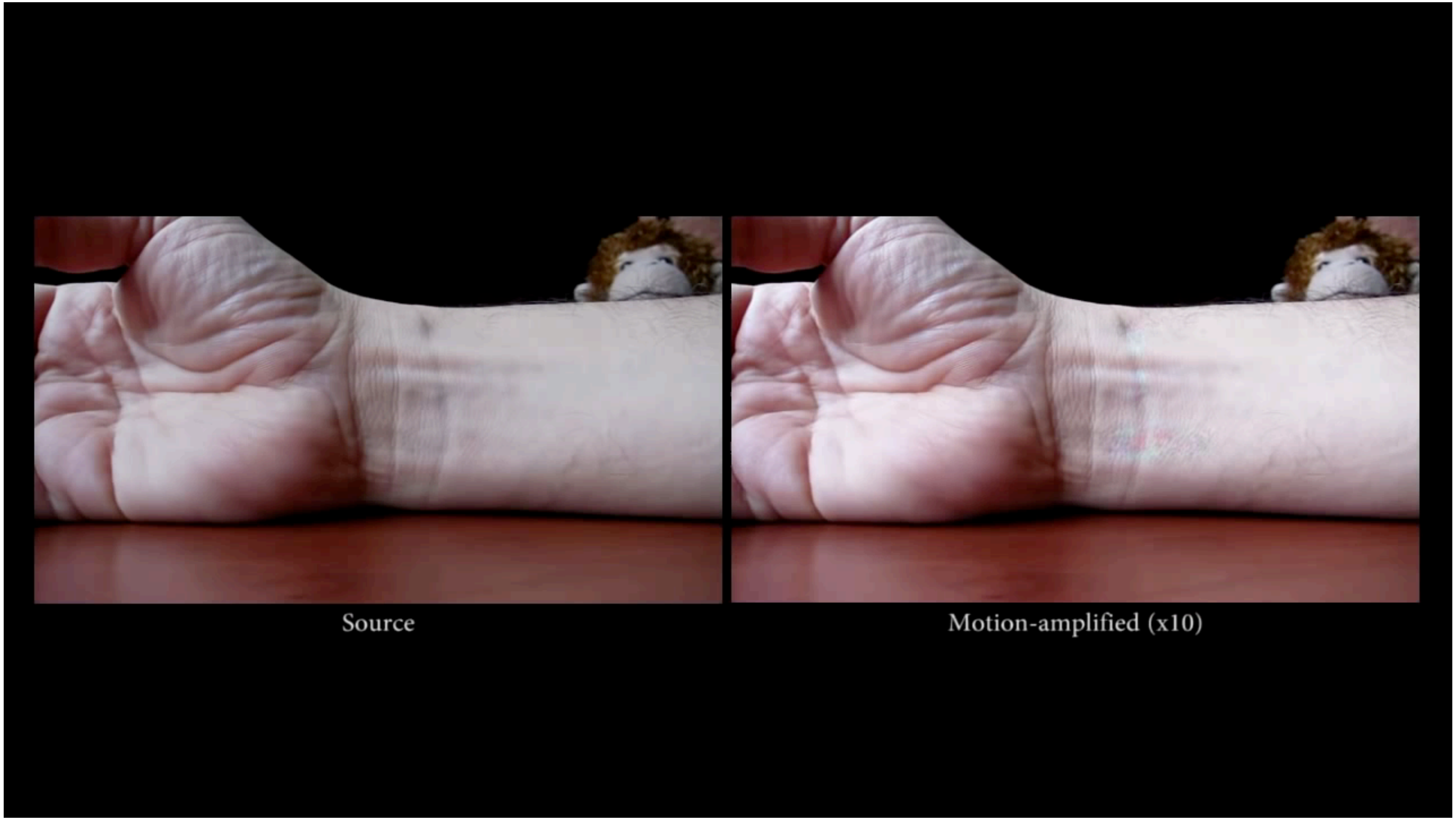
© Chris Cameron

Today's "**fun**" Example: Eulerian Video Magnification



Video From: Wu at al., Siggraph 2012

Today’s “**fun**” Example: Eulerian Video Magnification



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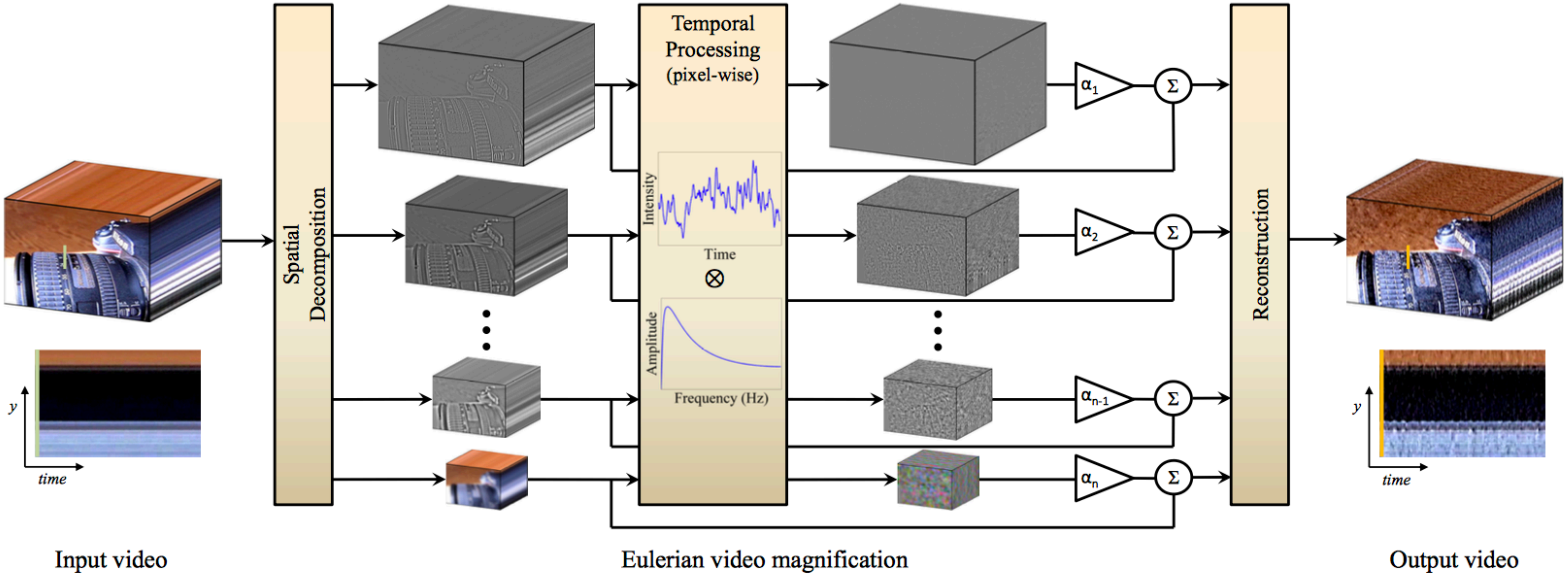

Figure From: Wu at al., Siggraph 2012

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CPSC 425: Computer Vision

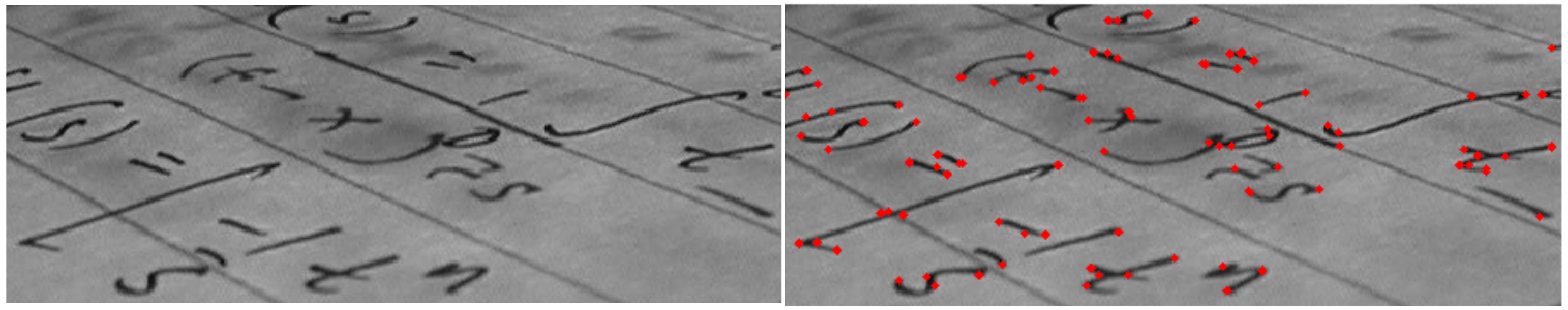


Image Credit: https://en.wikipedia.org/wiki/Corner_detection

Lecture 13: Corner Detection

(unless otherwise stated slides are taken or adopted from **Bob Woodham, Jim Little** and **Fred Tung**)

Motivation: Template Matching

When might **template matching fail**?

— Different scales  

— Different orientation 

— Lighting conditions 

— Left vs. Right hand  

— Partial Occlusions 

— Different Perspective

— Motion / blur

Motivation: Template Matching in Scaled Representation

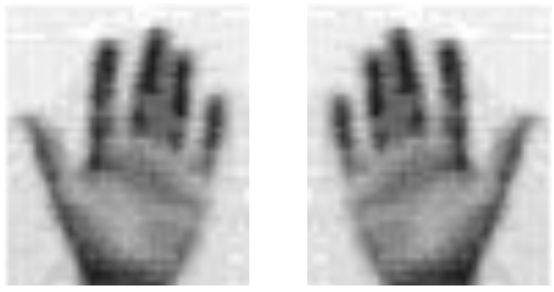
When might **template matching** in scaled representation **fail**?

— ~~Different scales~~

— Different orientation

— Lighting conditions

— Left vs. Right hand



— Partial Occlusions

— Different Perspective

— Motion / blur

Motivation: Edge Matching in Scaled Representation

When might **edge matching** in scaled representation **fail**?

— ~~Different scales~~  

— Different orientation 

— ~~Lighting conditions~~ 

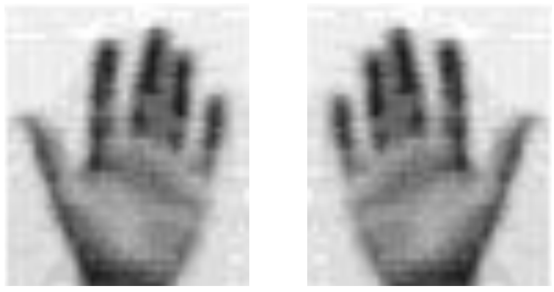
— Left vs. Right hand  

— Partial Occlusions 

— Different Perspective

— Motion / blur

Motivation: Edge Matching in Scaled Representation

- ~~Different scales~~ 
- ~~Different orientation~~ 
- ~~Lighting conditions~~ 
- Left vs. Right hand 

- ~~Partial Occlusions~~ 
- ~~Different Perspective~~
- Motion / blur

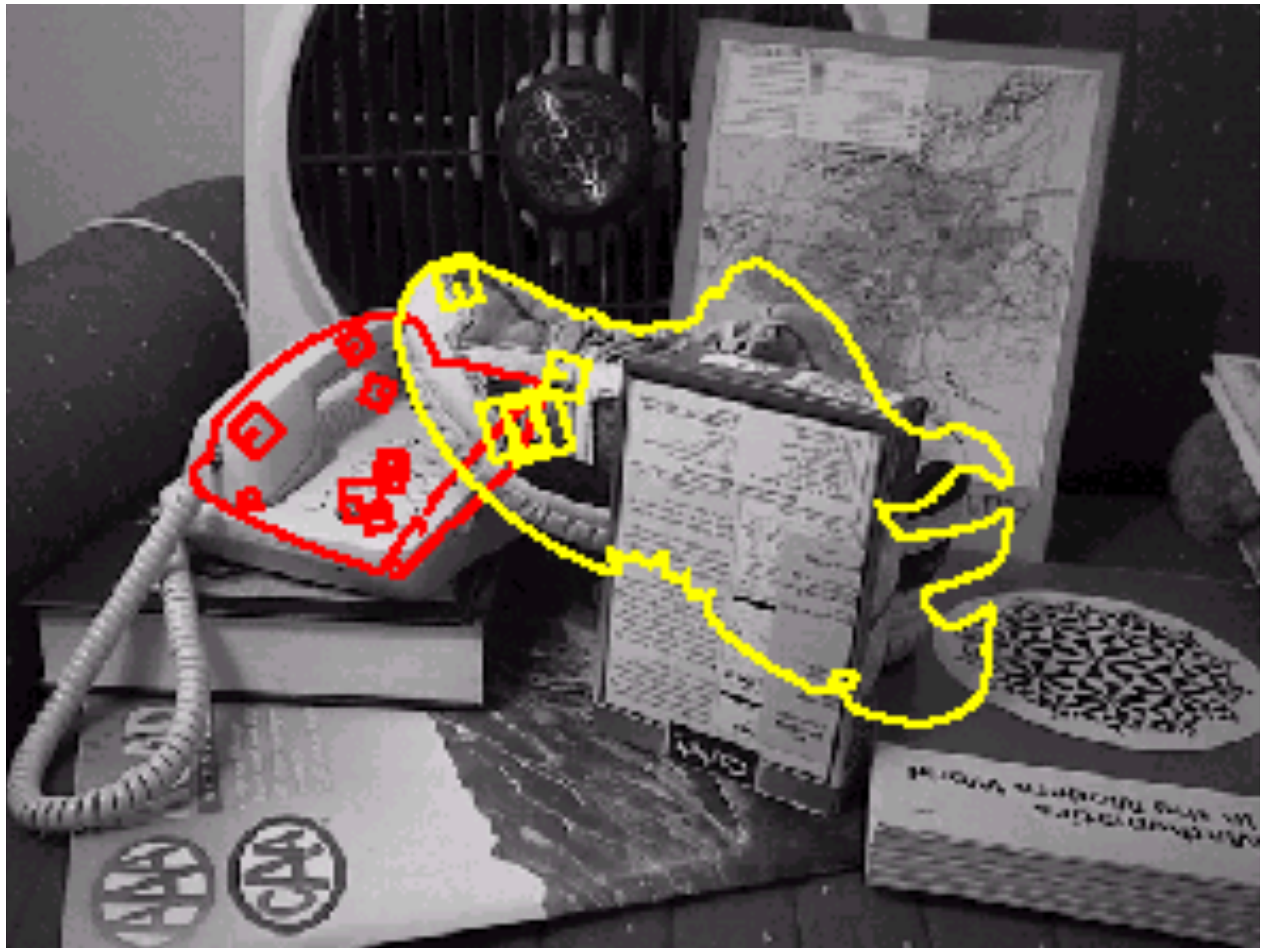
Planar Object **Instance Recognition**

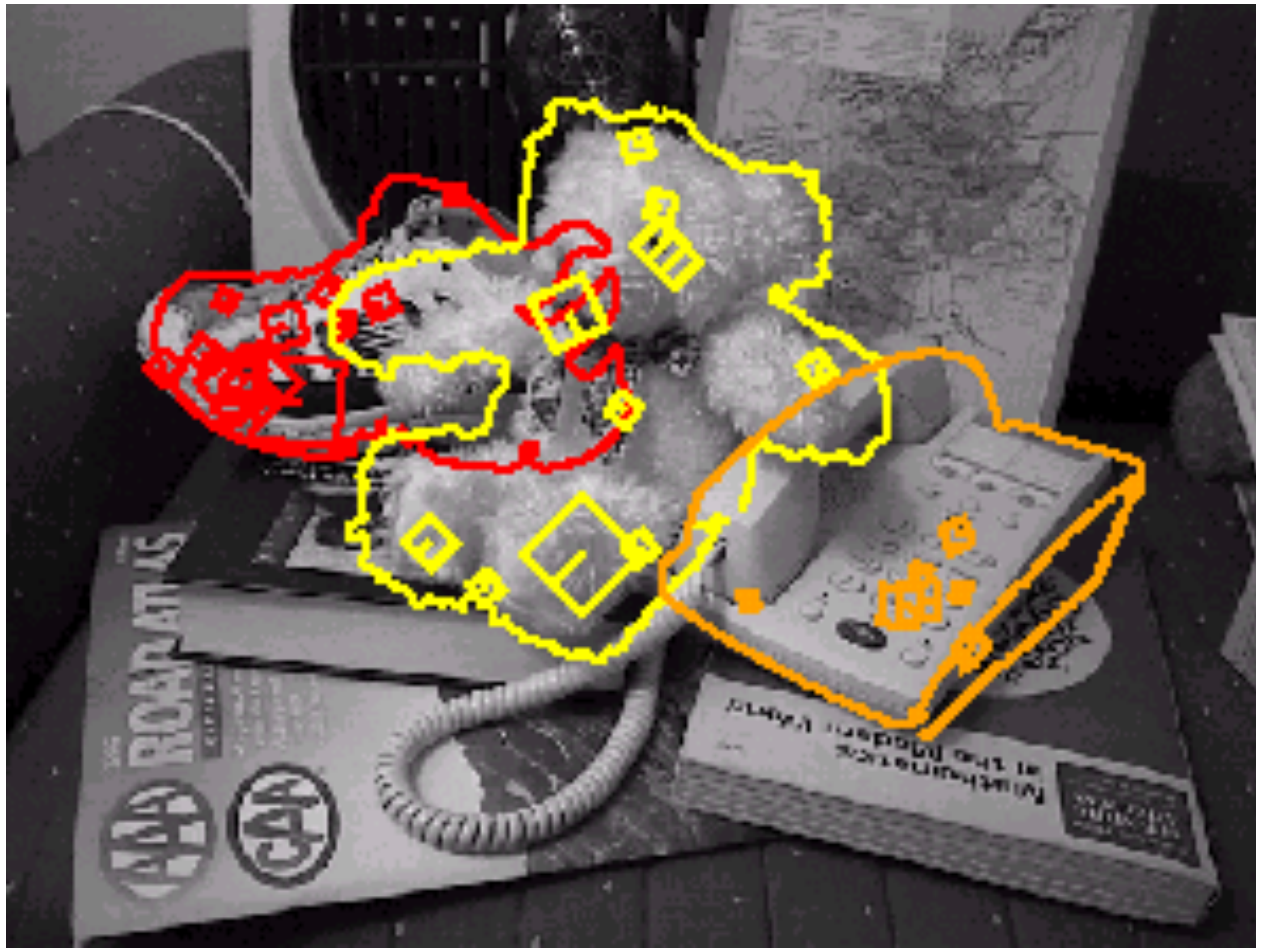
Database of planar objects

Instance recognition

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Recognition under **Occlusion**





Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

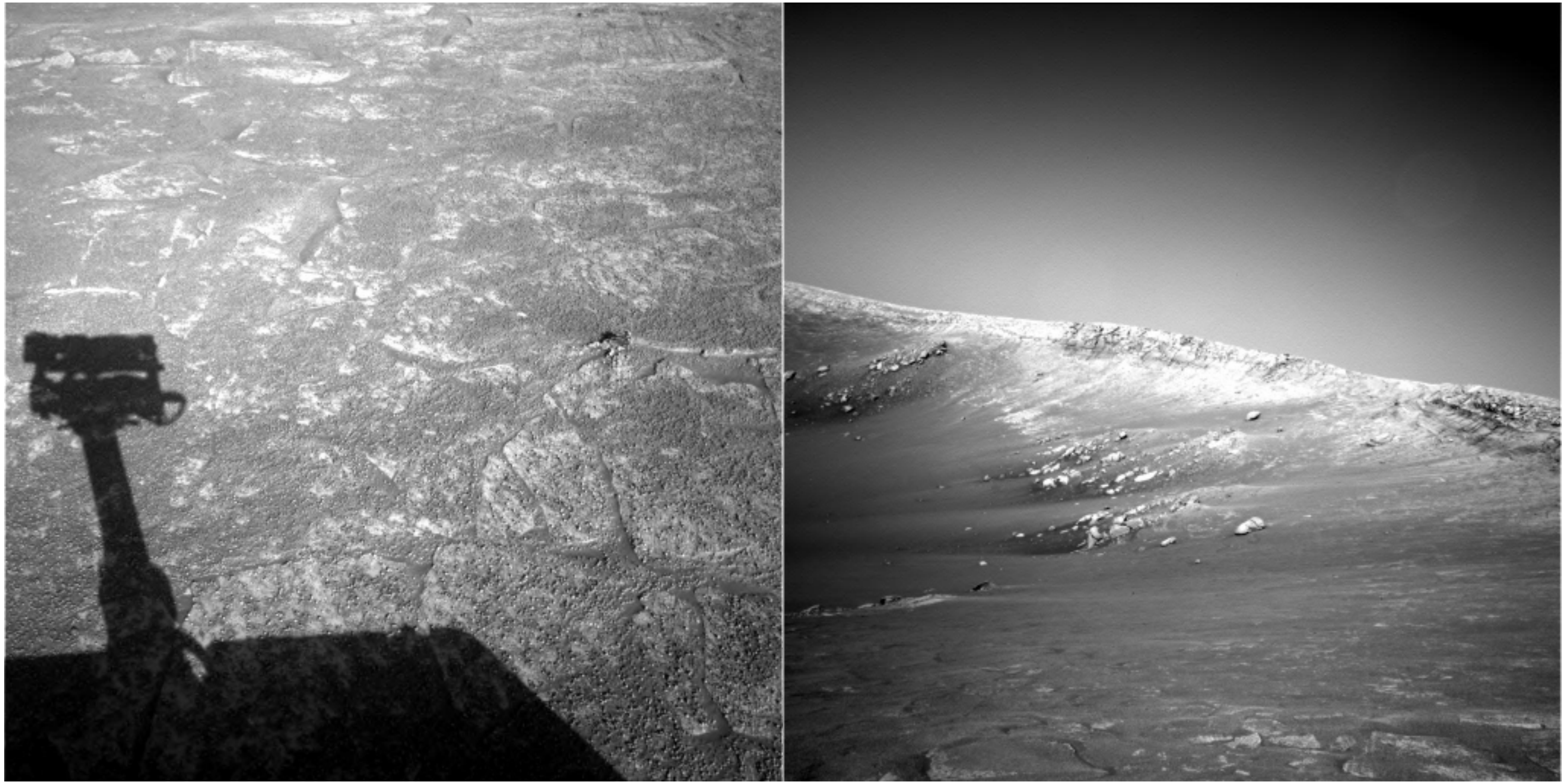
Image **Matching**

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Image **Matching**

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

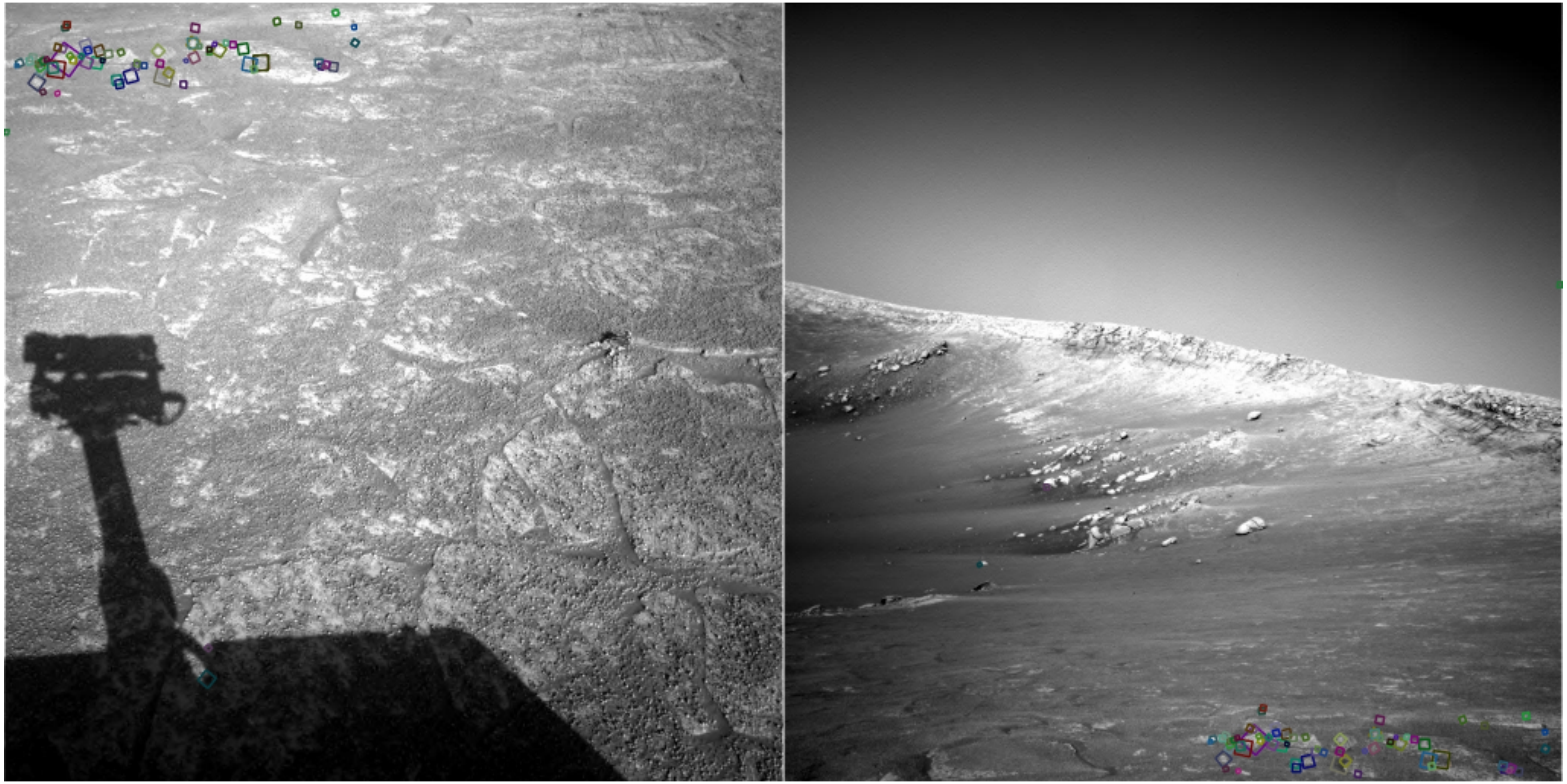
Finding **Correspondences**



NASA Mars Rover images

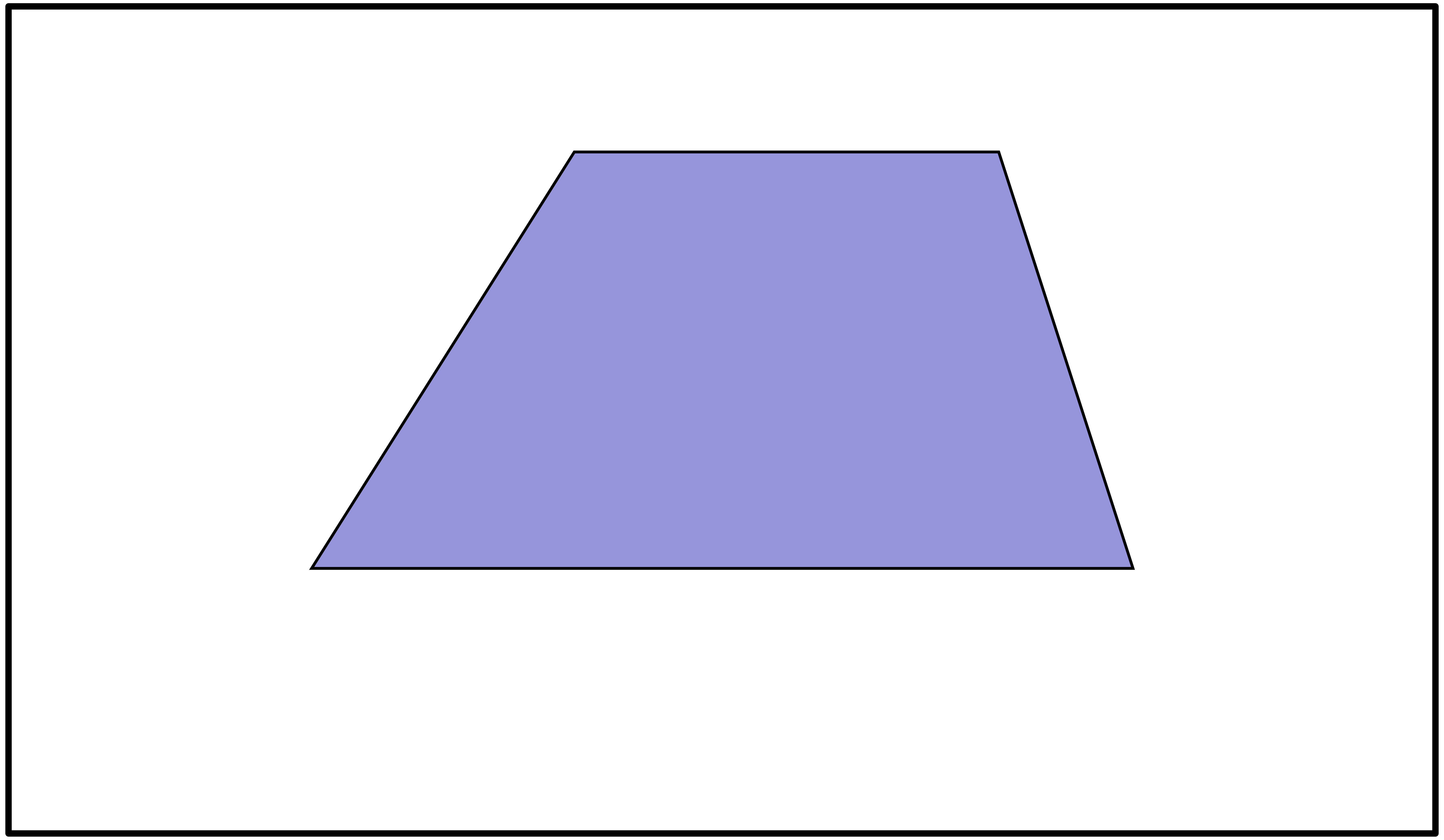
Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Finding **Correspondences**



Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

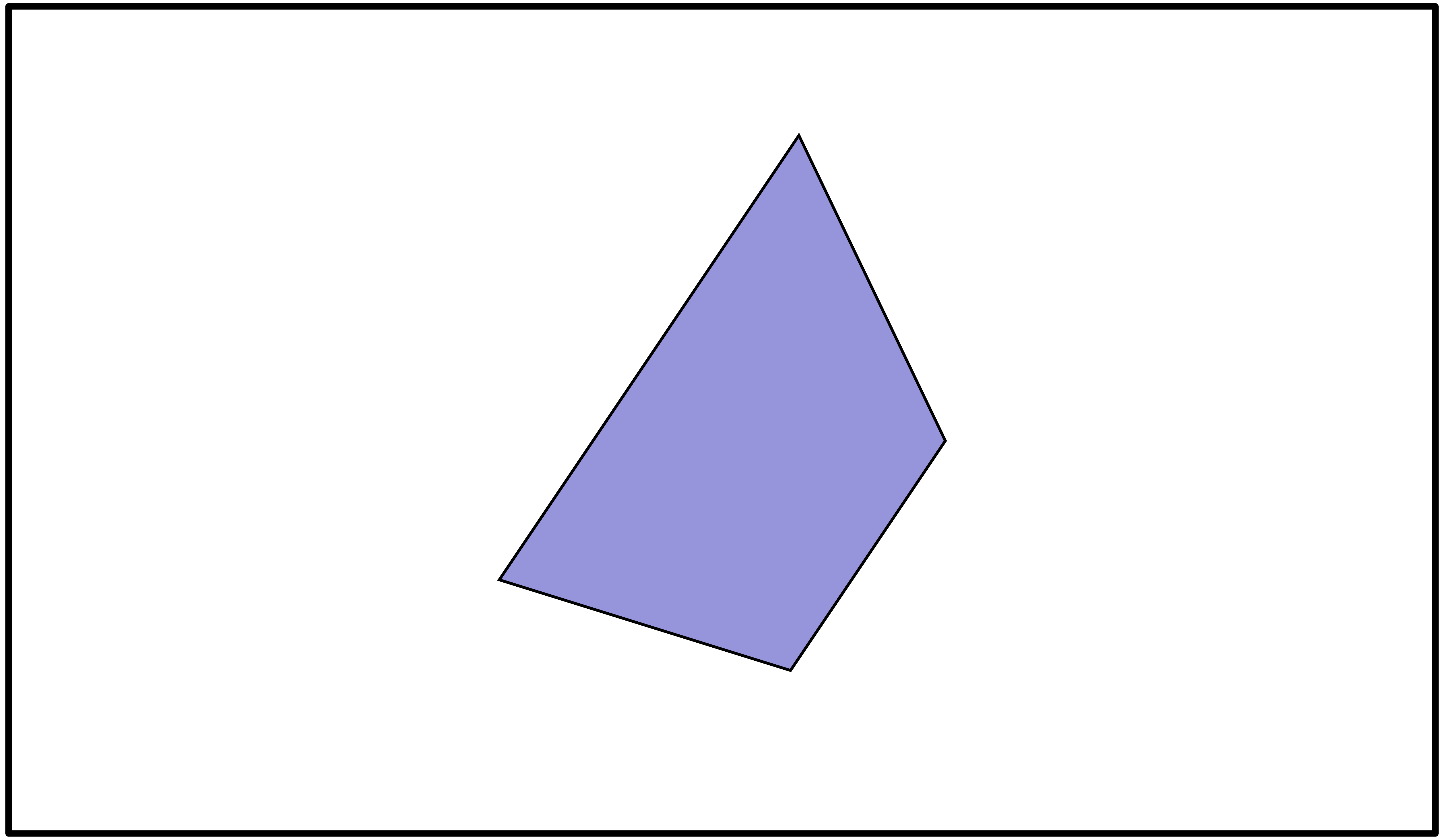
What is a **Good Feature**?



Pick a point in the image.
Find it again in the next image.

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

What is a **Good Feature**?



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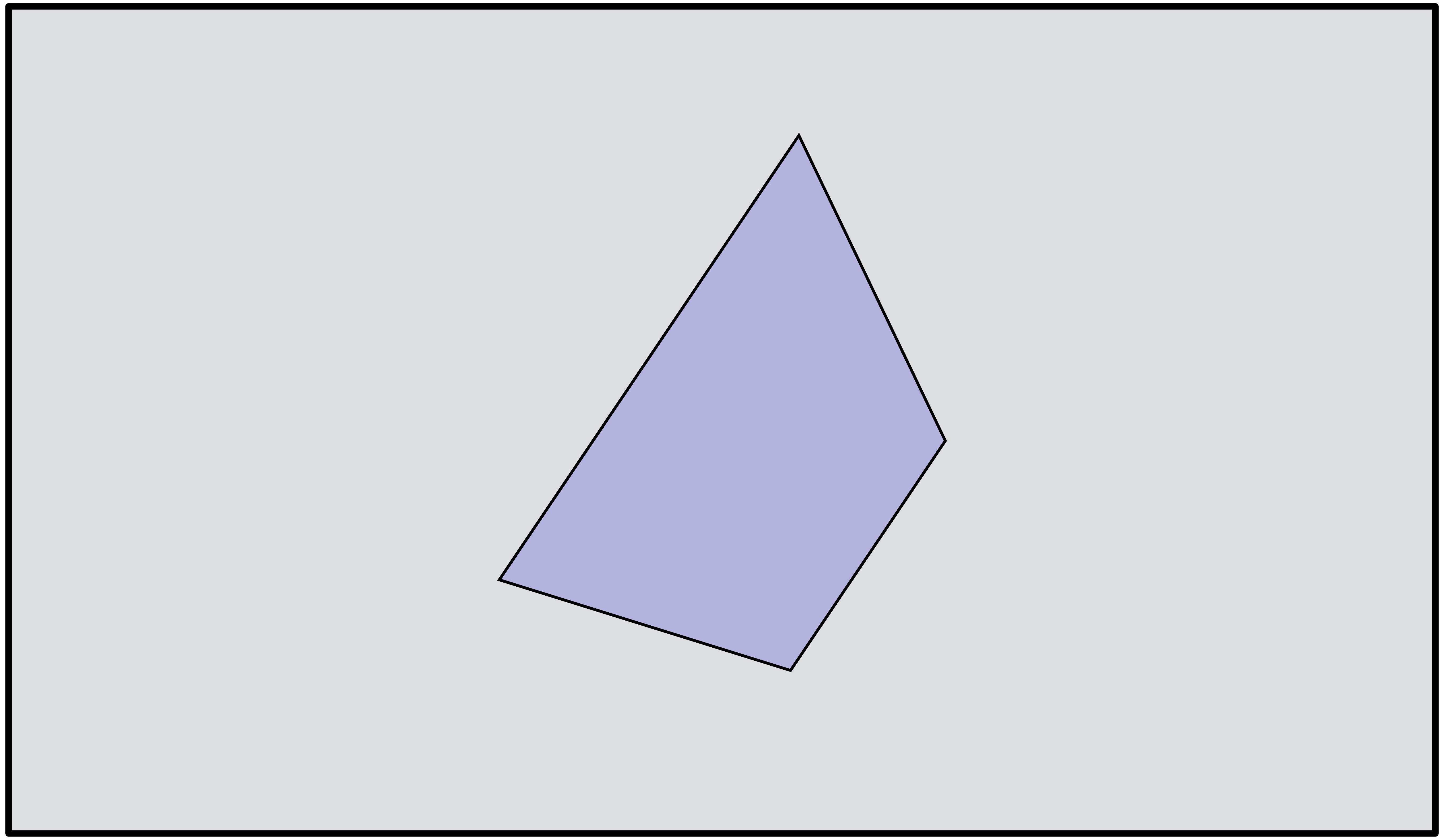


Pick a point in the image.
Find it again in the next image.

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

What is a **Good Feature**?

Local: features are local, robust to occlusion and clutter

Accurate: precise localization

Robust: noise, blur, compression, etc. do not have a big impact on the feature.

Distinctive: individual features can be easily matched

Efficient: close to real-time performance

What is a **corner**?

Image Credit: John Shakespeare, Sydney Morning Herald

We can think of a corner as any **locally distinct** 2D image feature that (hopefully) corresponds to a distinct position on an 3D object of interest in the scene.

What is a **corner**?

Corner

Interest Point

Image Credit: John Shakespeare, Sydney Morning Herald

We can think of a corner as any **locally distinct** 2D image feature that (hopefully) corresponds to a distinct position on an 3D object of interest in the scene.

Why are corners **distinct**?

A corner can be **localized reliably**.

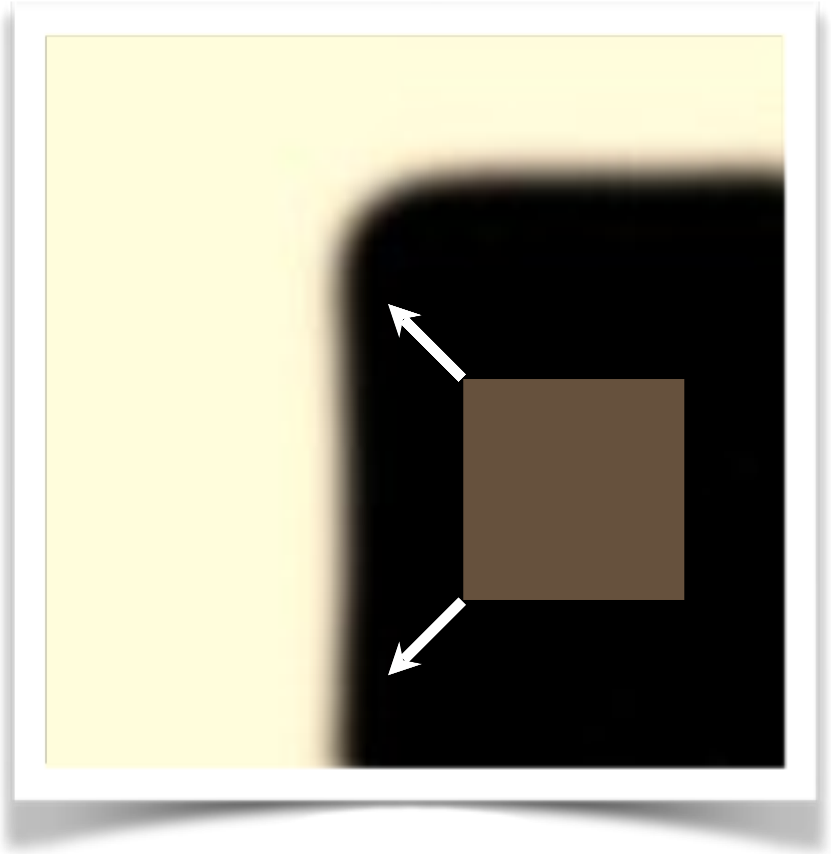
Thought experiment:

Why are corners **distinct**?

A corner can be **localized reliably**.

Thought experiment:

— Place a small window over a patch of constant image value.



"**flat**" region:

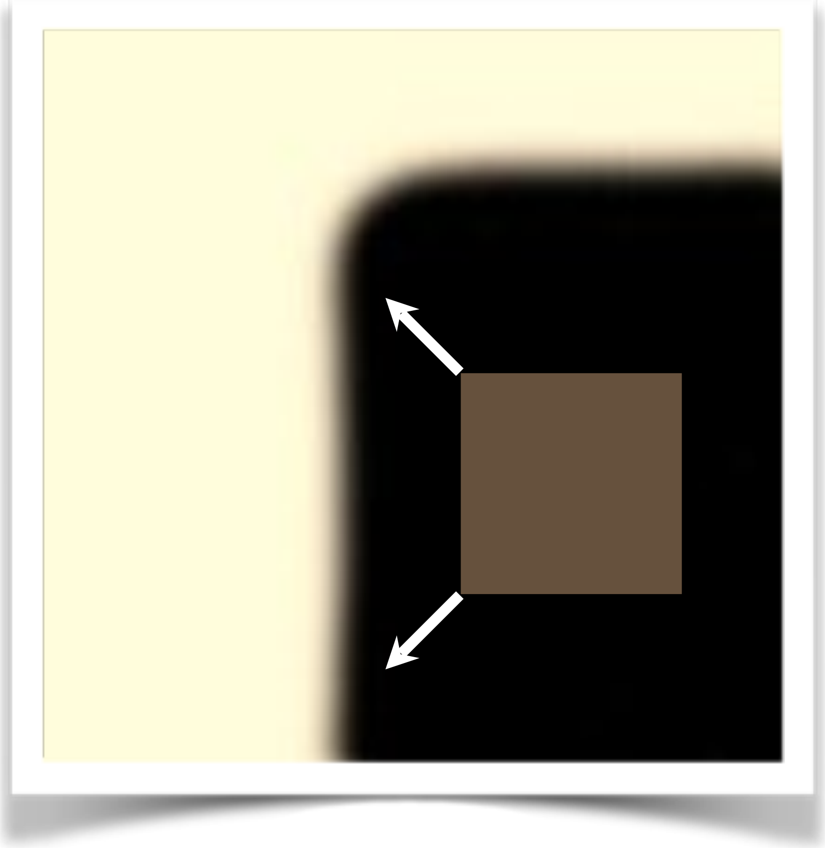
Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

Why are corners **distinct**?

A corner can be **localized reliably**.

Thought experiment:

— Place a small window over a patch of constant image value. If you slide the window in any direction, the image in the window will not change.



“**flat**” region:
no change in all
directions

Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

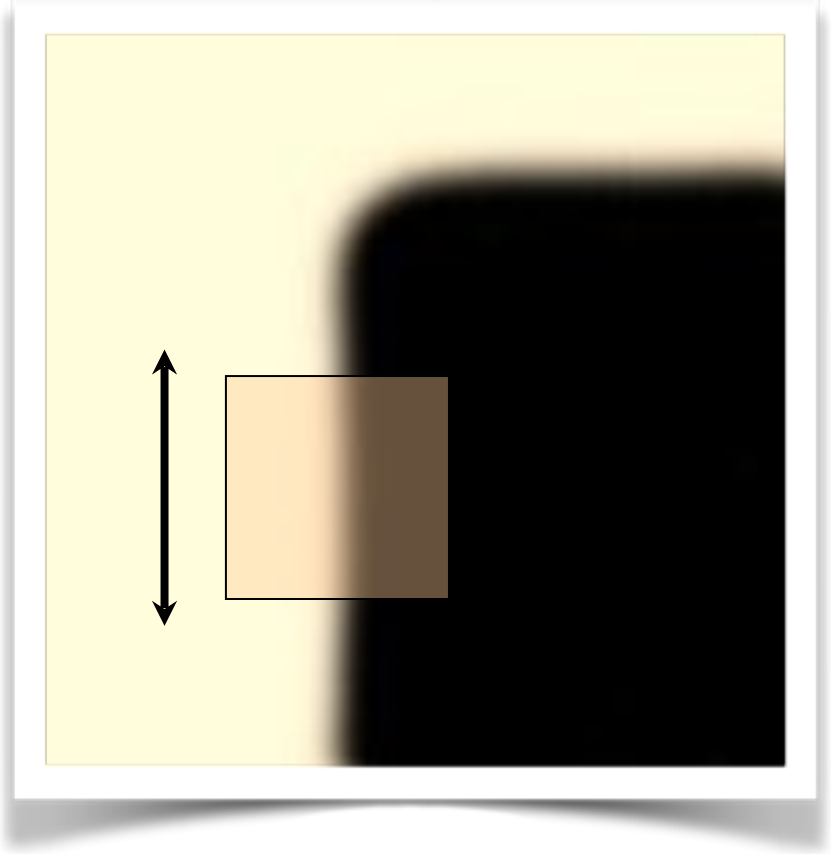
Why are corners **distinct**?

A corner can be **localized reliably**.

Thought experiment:

— Place a small window over a patch of constant image value. If you slide the window in any direction, the image in the window will not change.

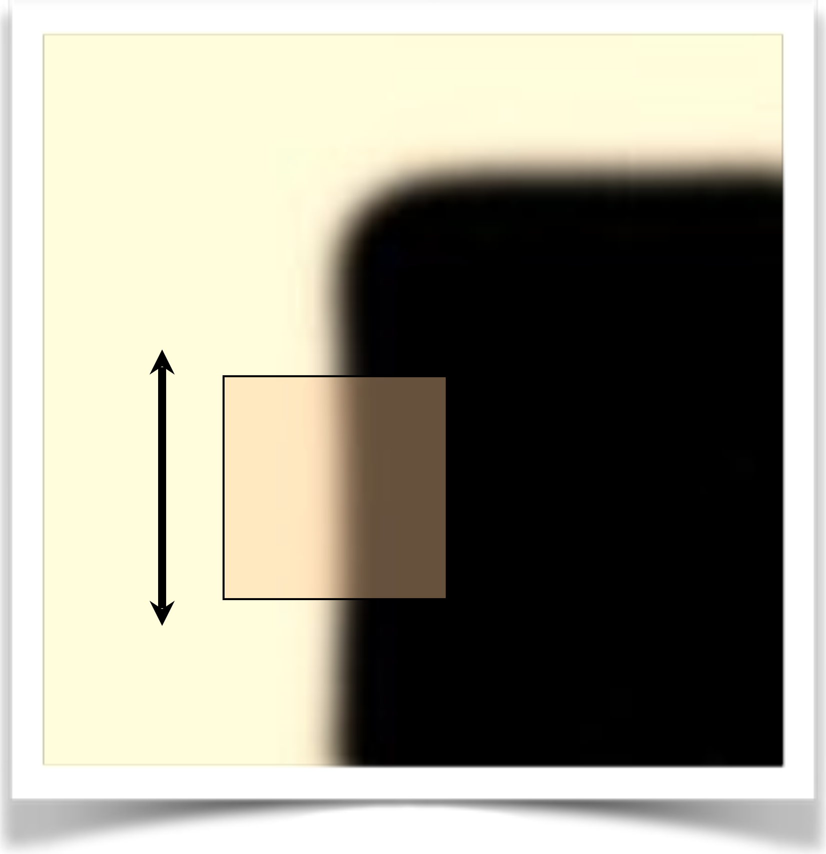
— Place a small window over an edge.



“**edge**”:

Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

Why are corners **distinct**?



"**edge**":
no change along
the edge direction

A corner can be **localized reliably**.

Thought experiment:

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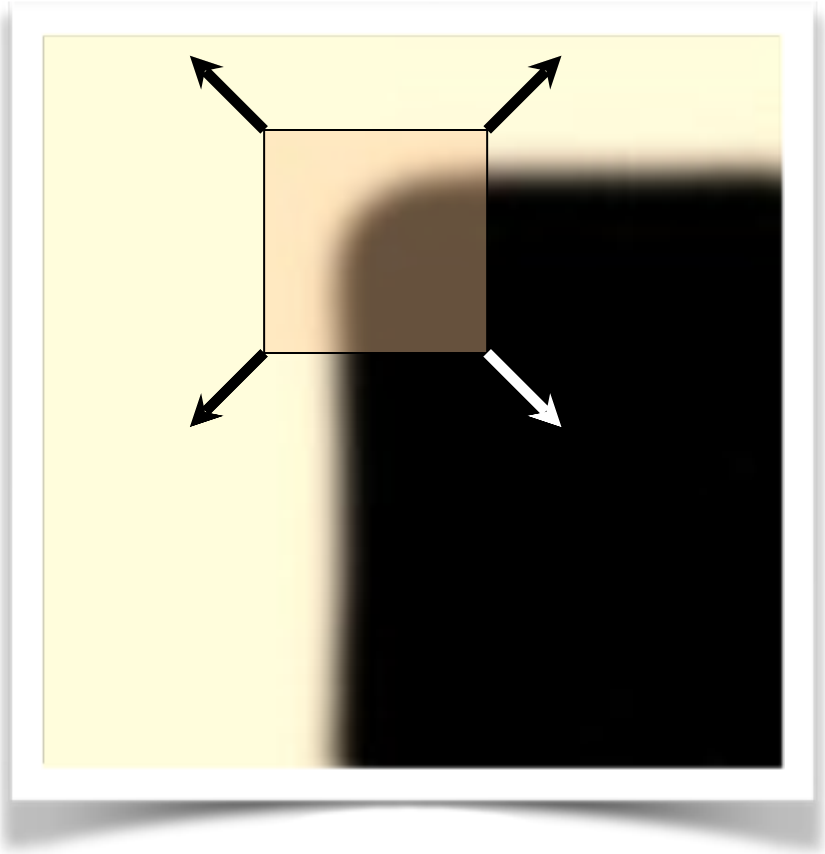
— Place a small window over an edge. If you slide the window in the direction of the edge, the image in the window will not change

→ Cannot estimate location along an edge (a.k.a., **aperture** problem)

Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

Why are corners **distinct**?



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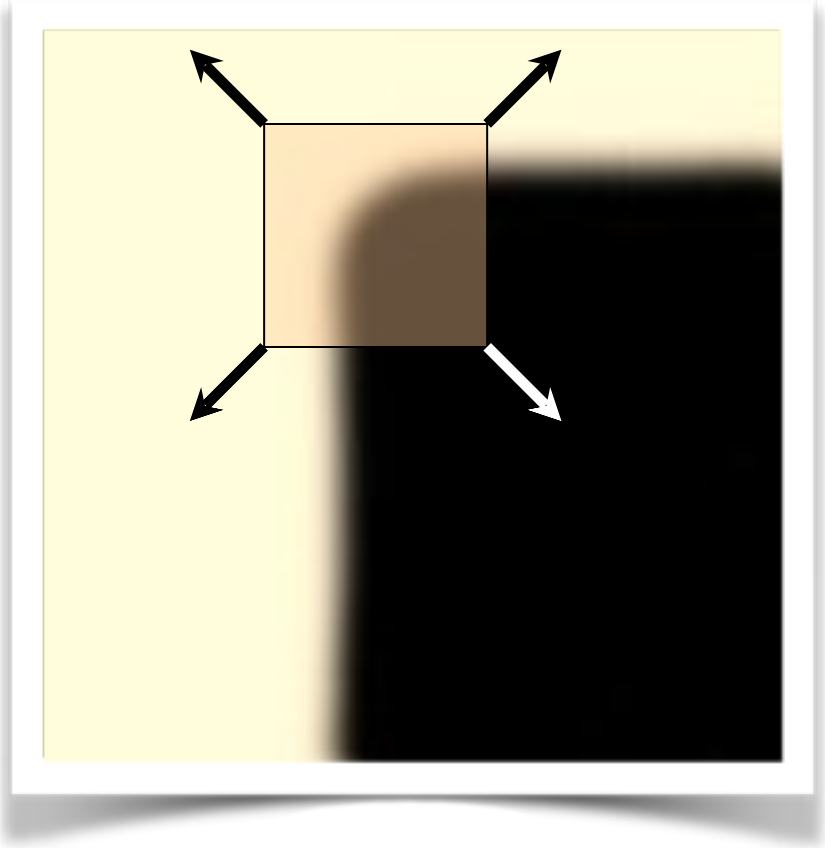
— Place a small window over a corner. If you slide the window in any direction, the image in the window changes.



"**corner**": significant change in all directions

Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

Corner Detection

Edge detectors perform poorly at corners

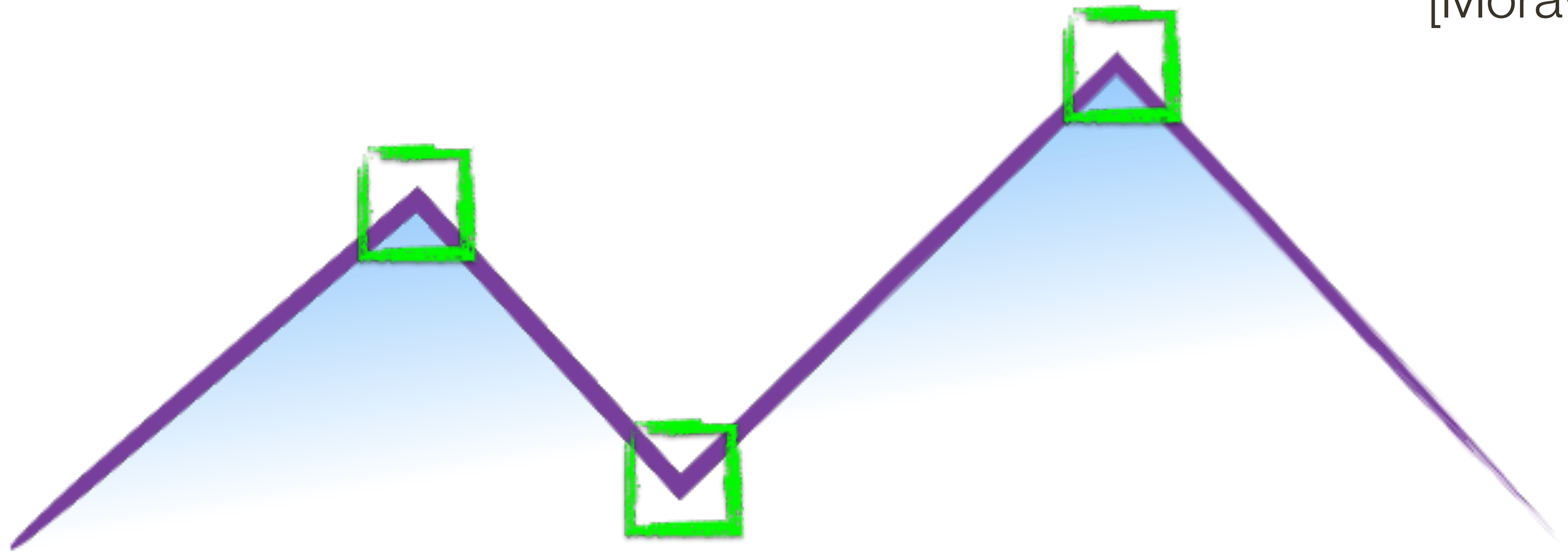
Observations:

— The gradient is ill defined exactly at a corner

— Near a corner, the gradient has two (or more) distinct values

How do you find a **corner**?

[Moravec 1980]



Easily recognized by looking through a small window

Shifting the window should give large change in intensity

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Autocorrelation

Autocorrelation is the correlation of the image with itself.

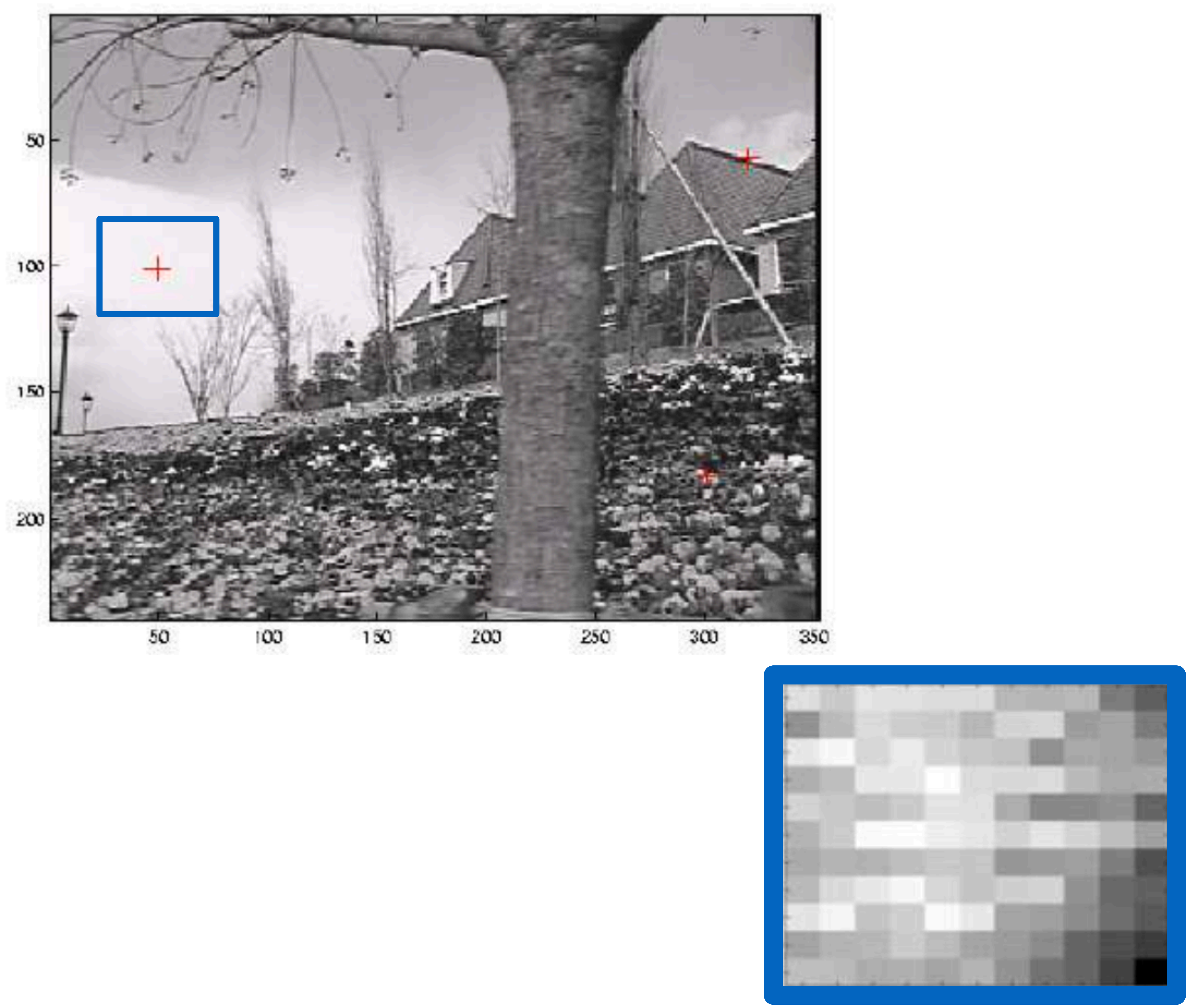
— Windows centered on an edge point will have autocorrelation that falls off slowly in the direction along the edge and rapidly in the direction across (perpendicular to) the edge.

— Windows centered on a corner point will have autocorrelation that falls of rapidly in all directions.

Autocorrelation

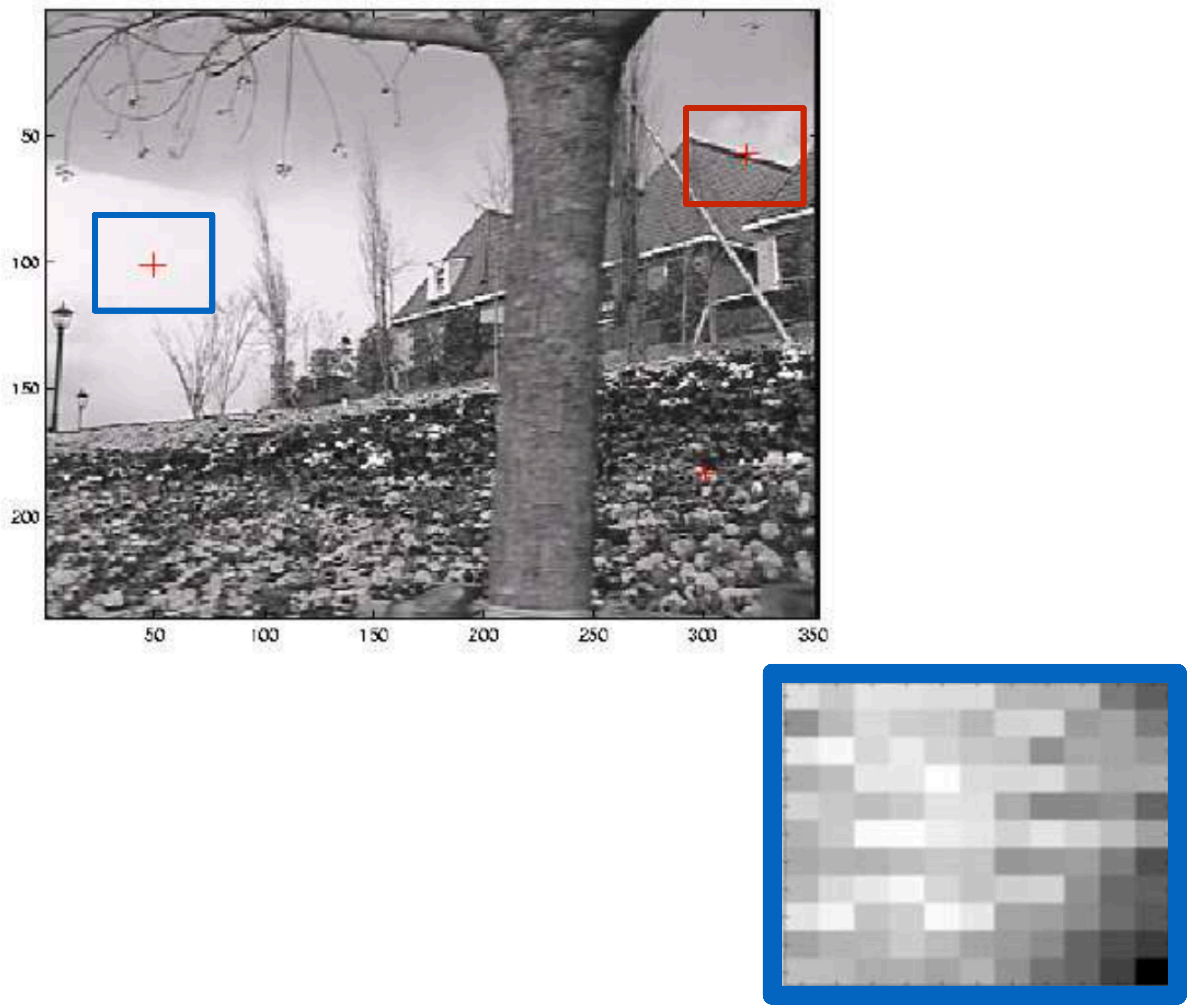
Szeliski, Figure 4.5

Autocorrelation



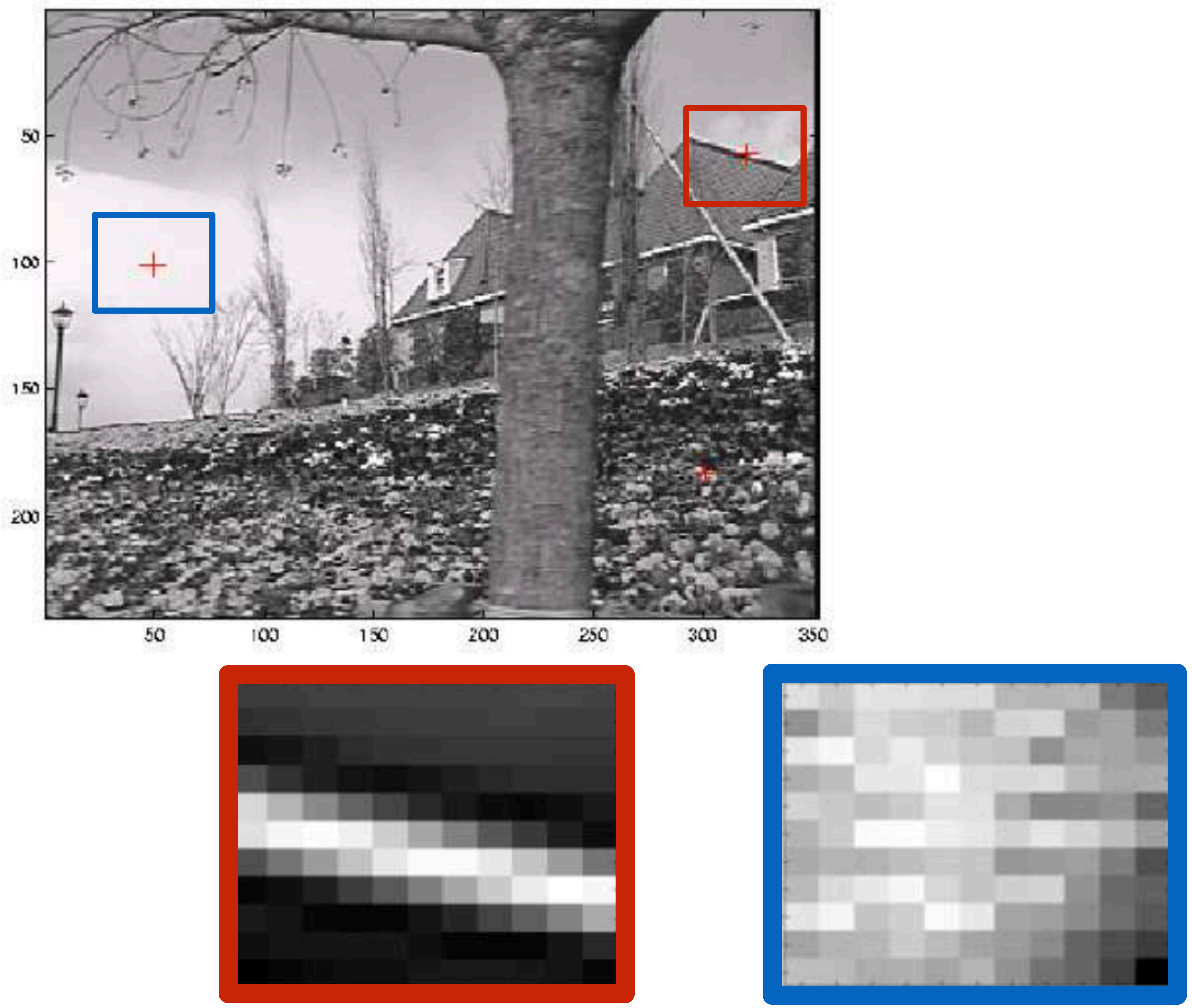
Szeliski, Figure 4.5

Autocorrelation



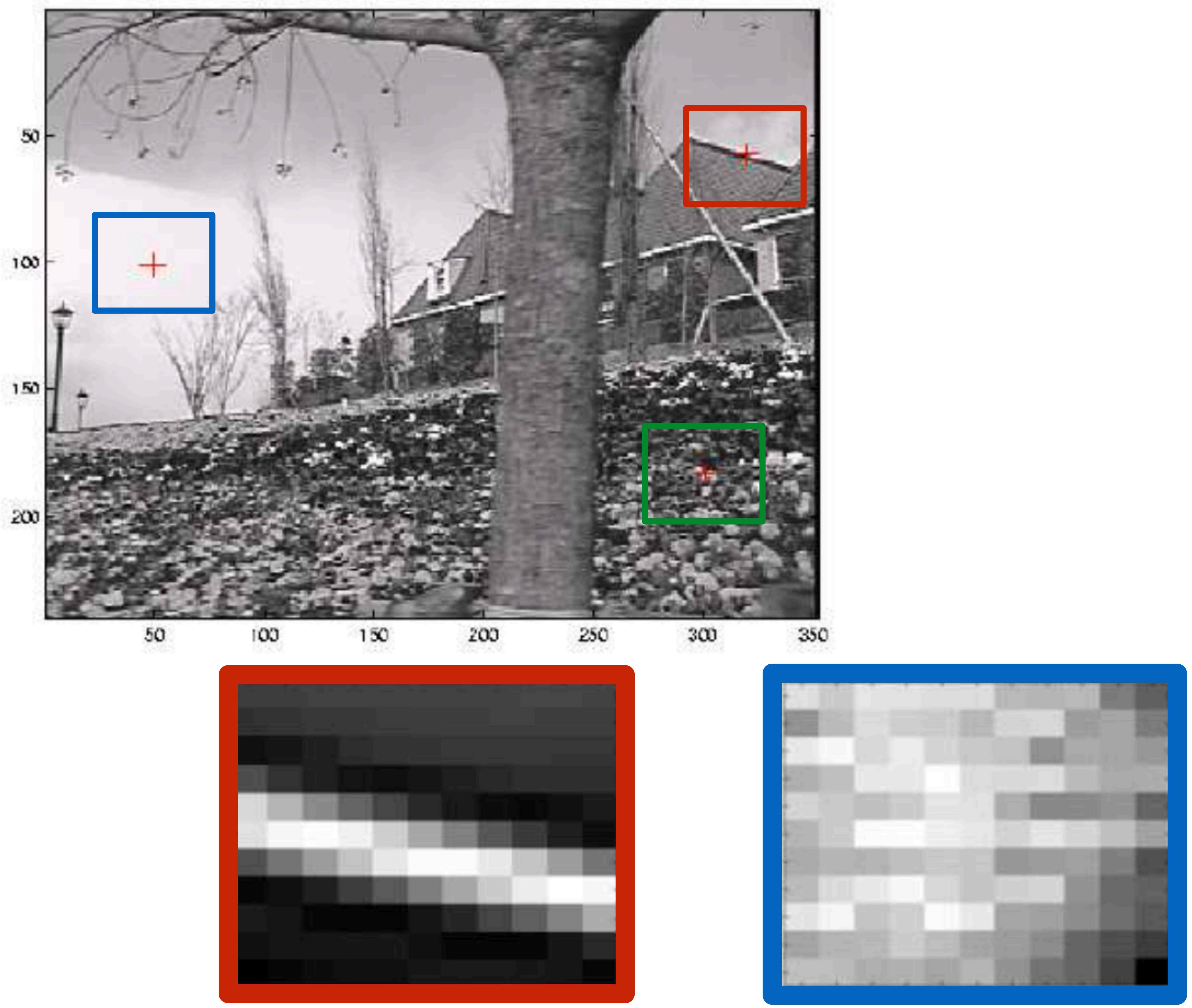
Szeliski, Figure 4.5

Autocorrelation



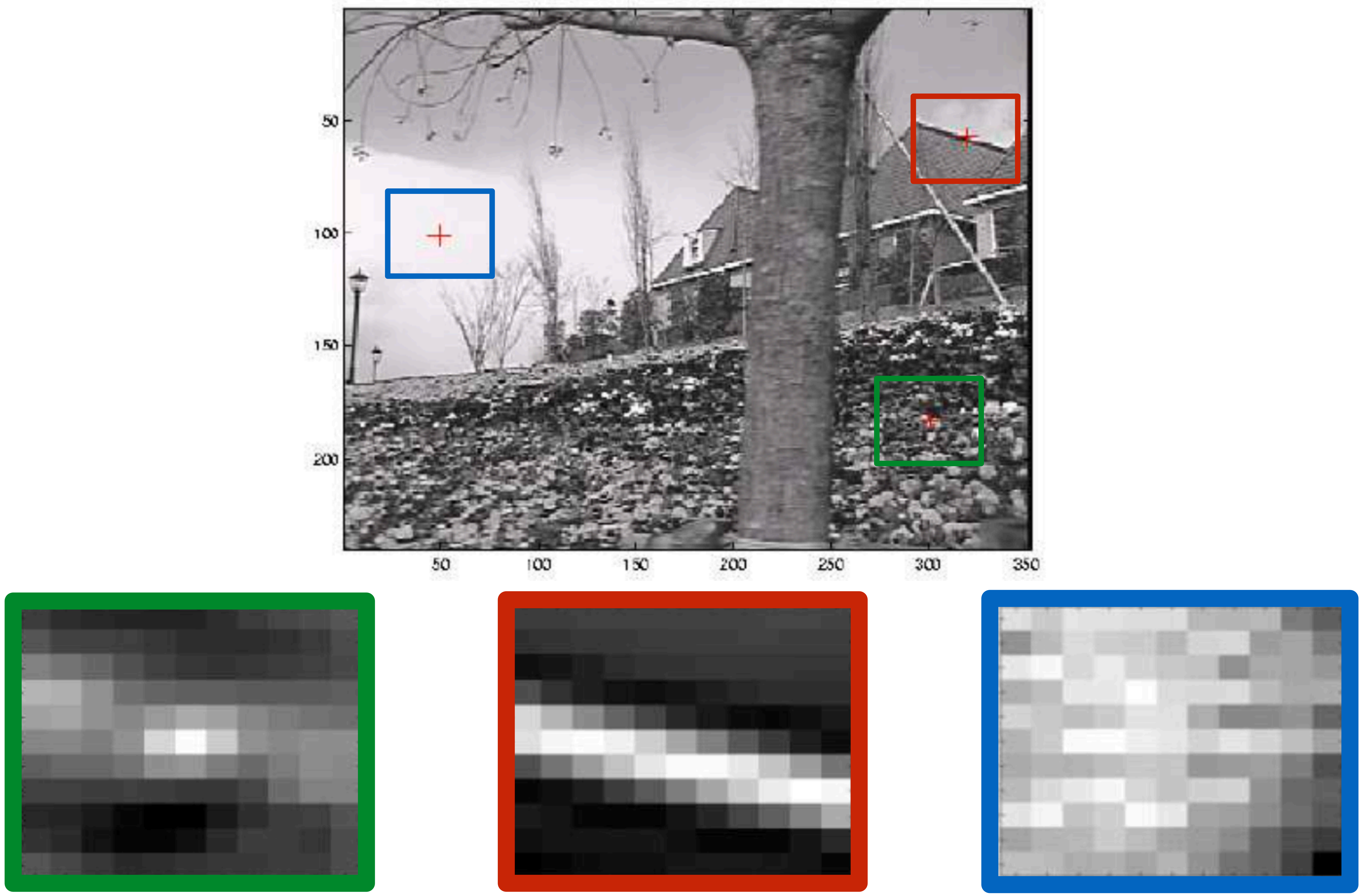
Szeliski, Figure 4.5

Autocorrelation



Szeliski, Figure 4.5

Autocorrelation



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— Windows centered on a corner point will have autocorrelation that falls of rapidly in all directions.

Harris Corner Detection

1. Compute image gradients over small region

2. Compute the covariance matrix

3. Compute eigenvectors and eigenvalues

4. Use threshold on eigenvalues to detect corners

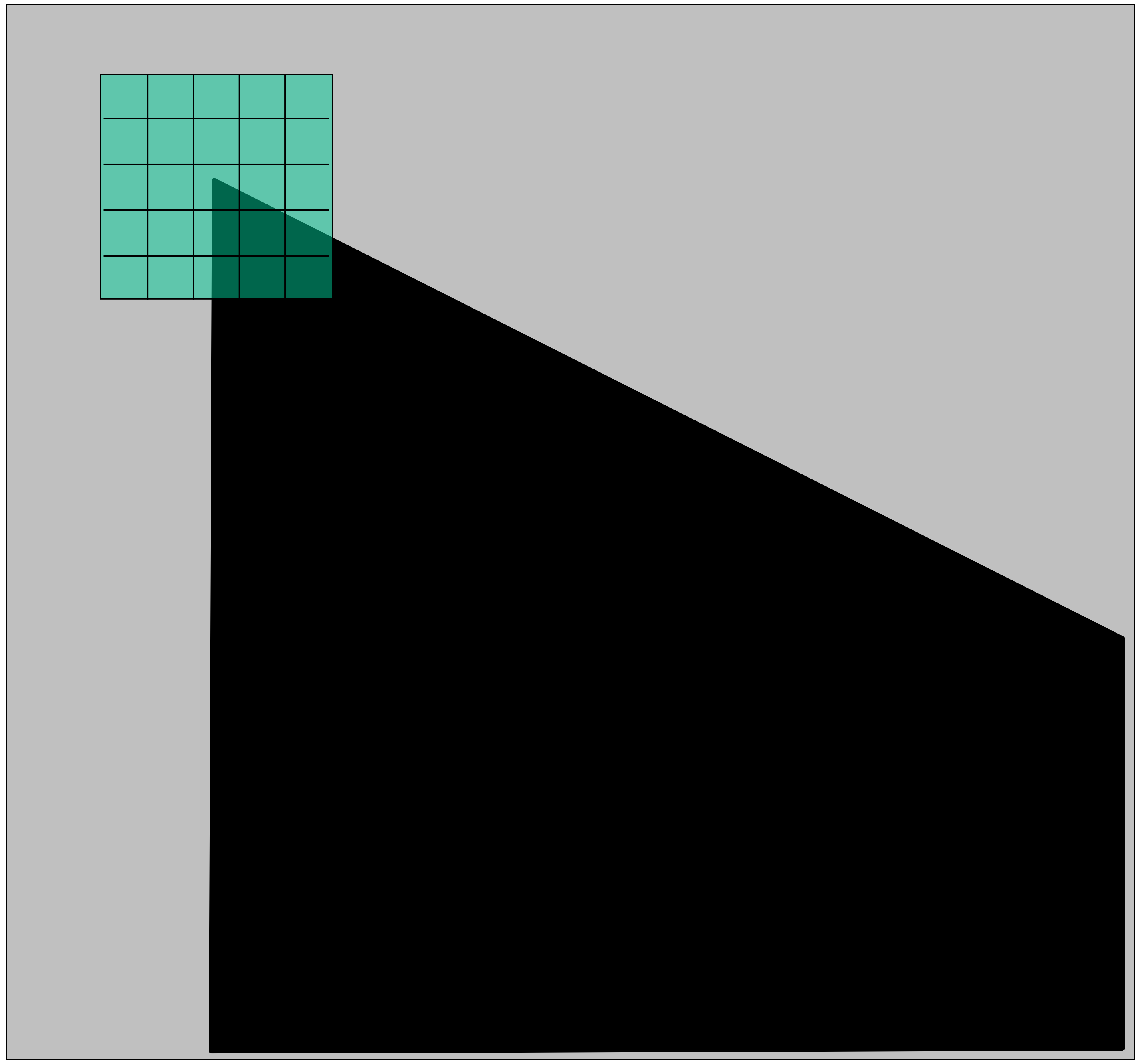
$$I_x = \frac{\partial I}{\partial x} \qquad I_y = \frac{\partial I}{\partial y}$$

$$\begin{bmatrix} \sum\limits_{p \in P} I_x I_x & \sum\limits_{p \in P} I_x I_y \\ \sum\limits_{p \in P} I_y I_x & \sum\limits_{p \in P} I_y I_y \end{bmatrix}$$

Slide Adopted: Ioannis (Yannis) Gkioulekas (CMU)

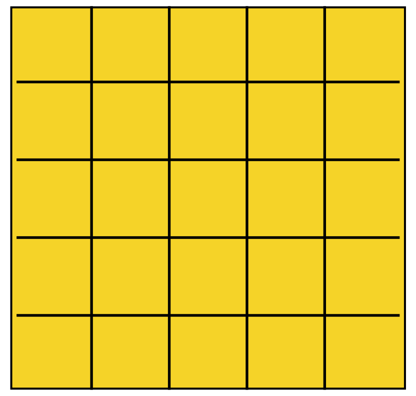
1. Compute **image gradients** over a small region

(not just a single pixel)



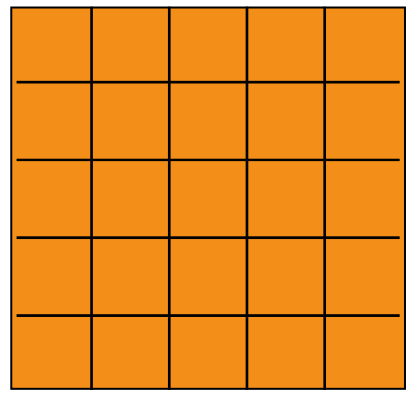
array of x gradients

$$I_x = \frac{\partial I}{\partial x}$$



array of y gradients

$$I_y = \frac{\partial I}{\partial y}$$



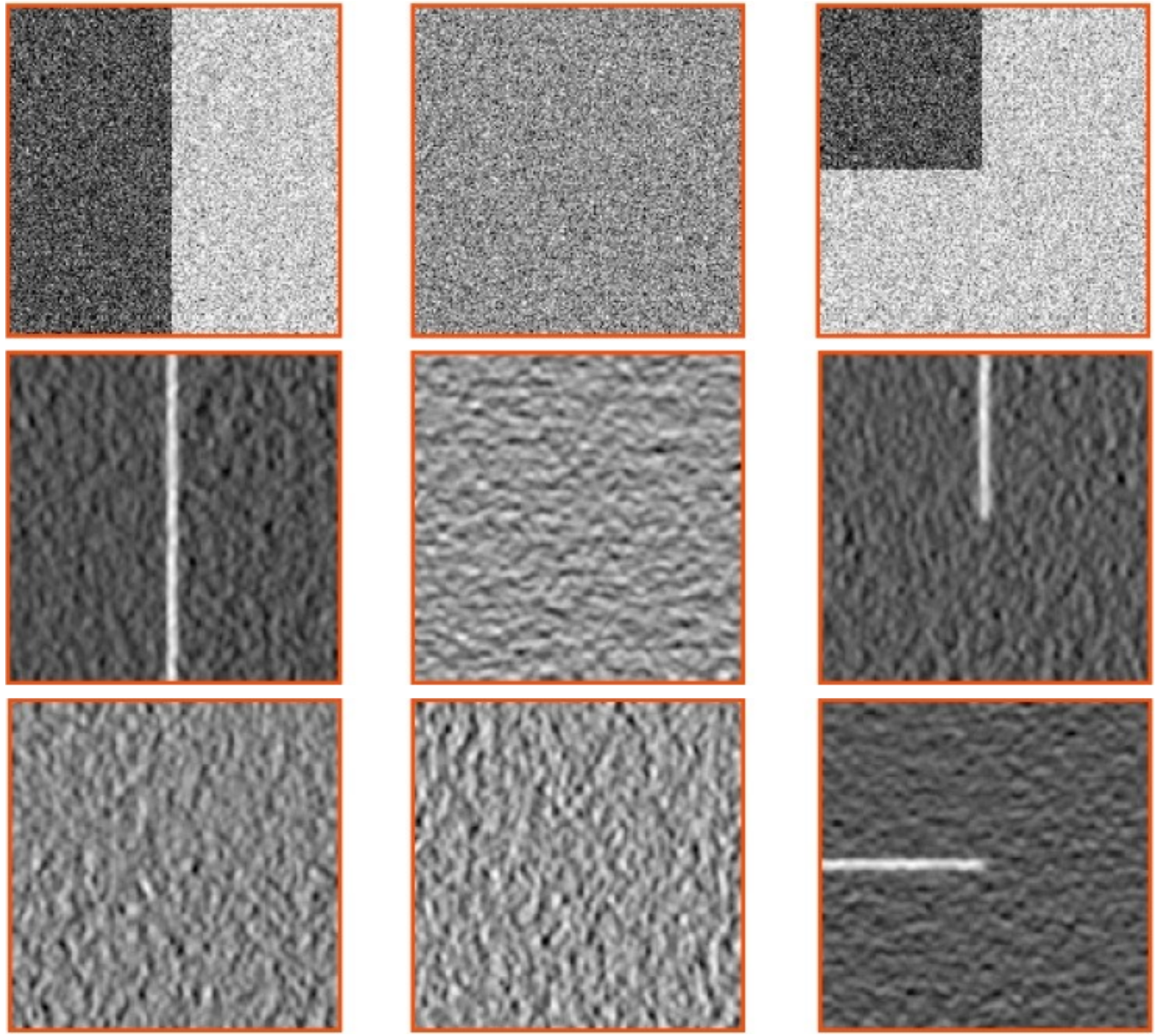
Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Visualization of Gradients

image

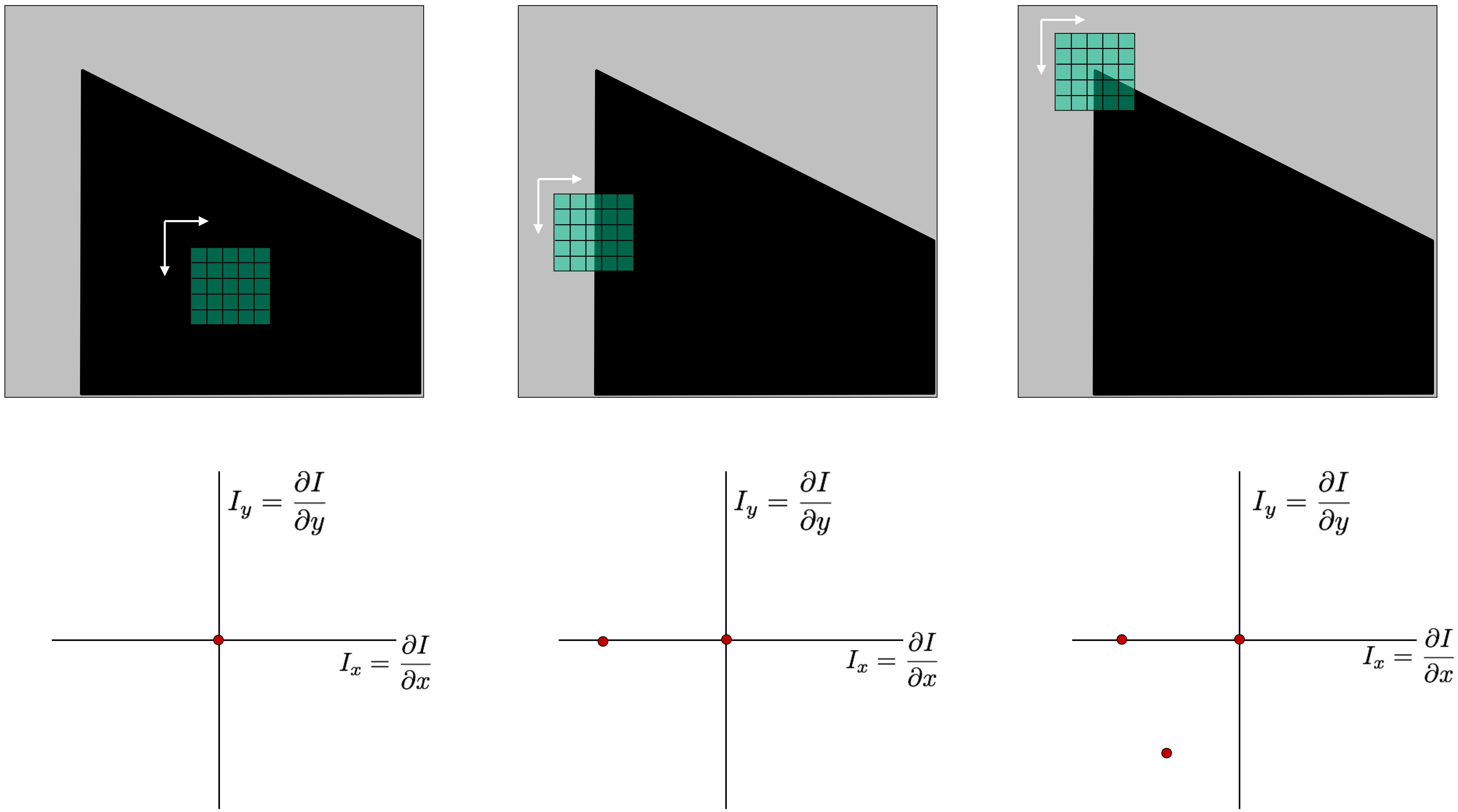
X derivative

Y derivative



Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

What Does a **Distribution** Tells You About the **Region**?



Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

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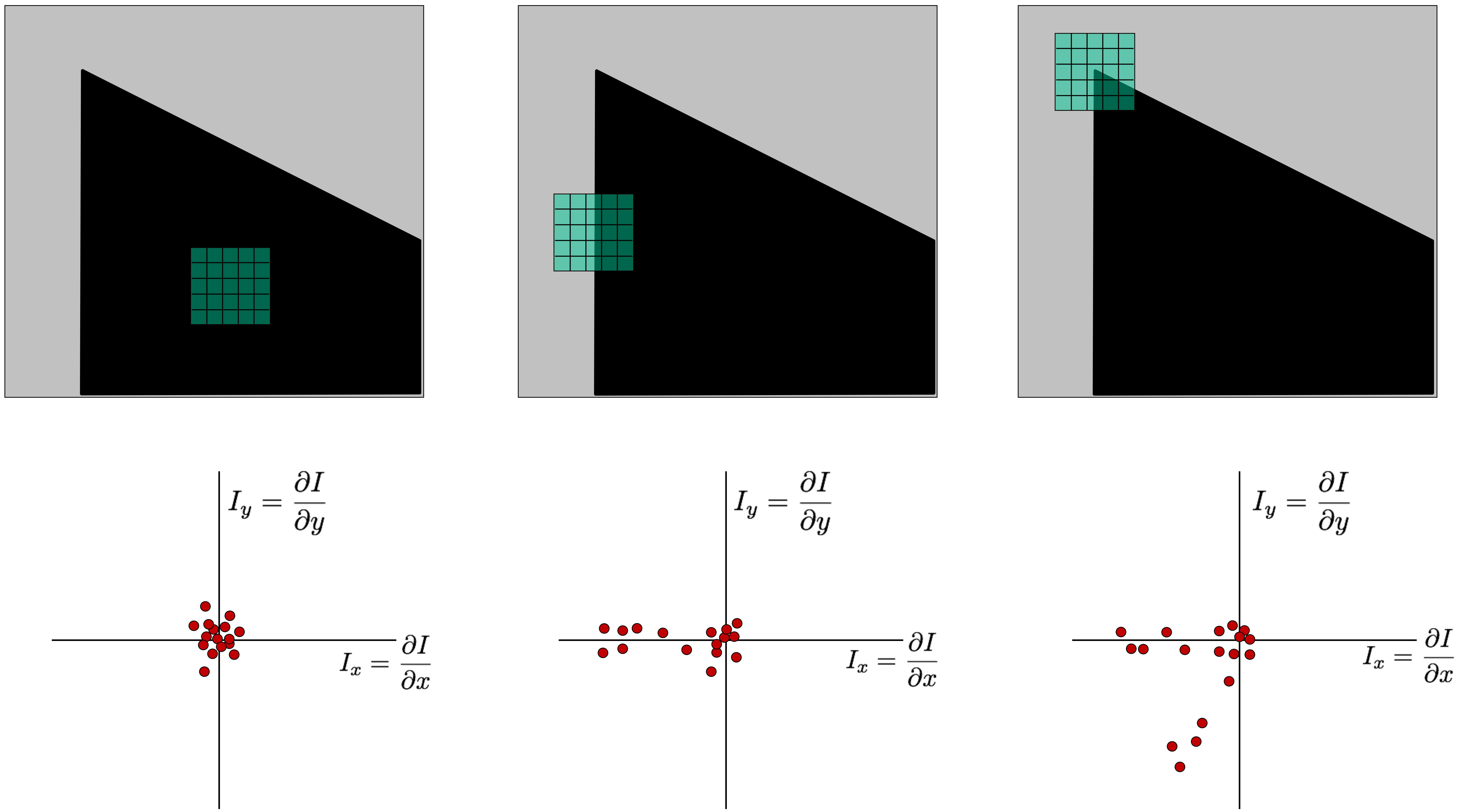

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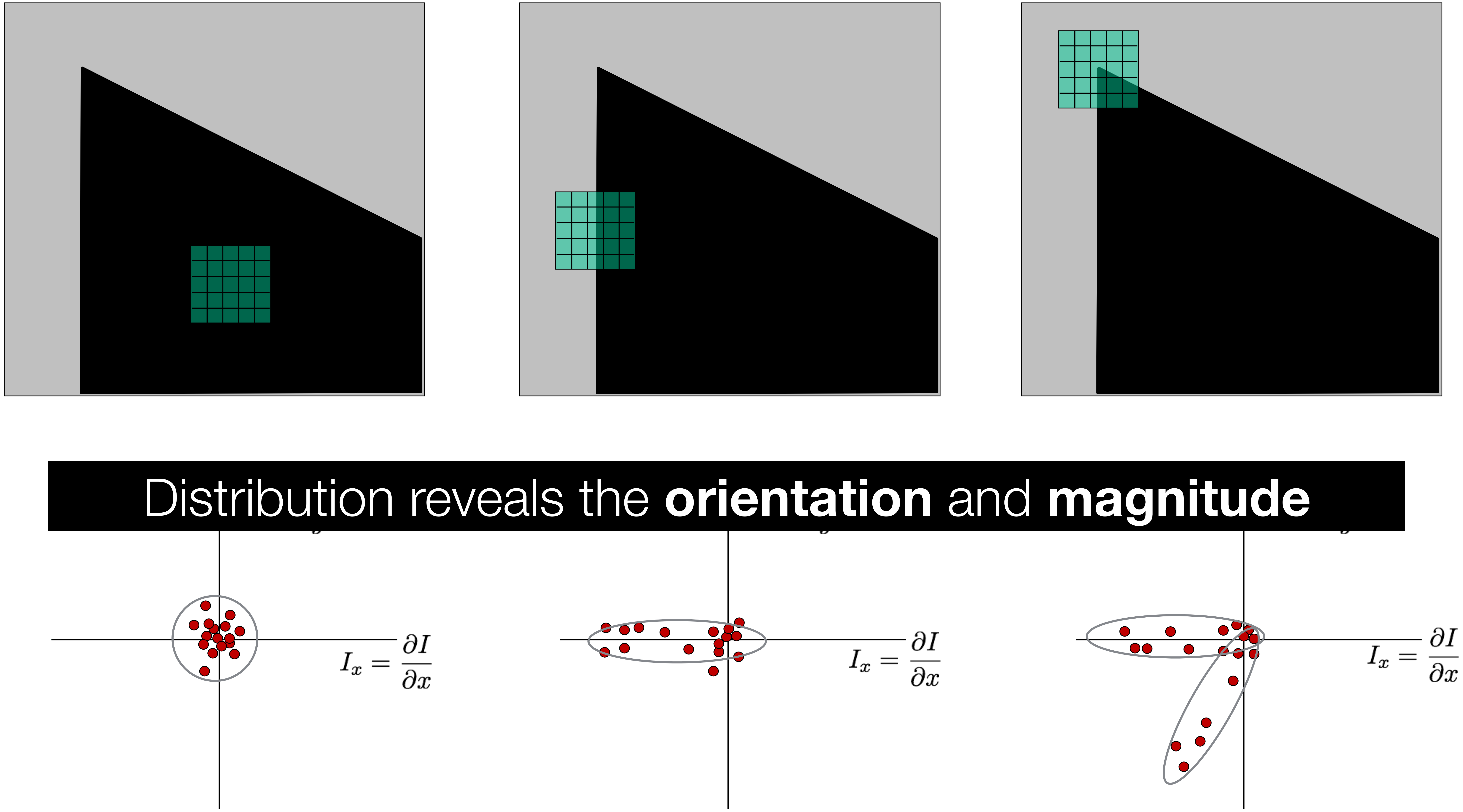
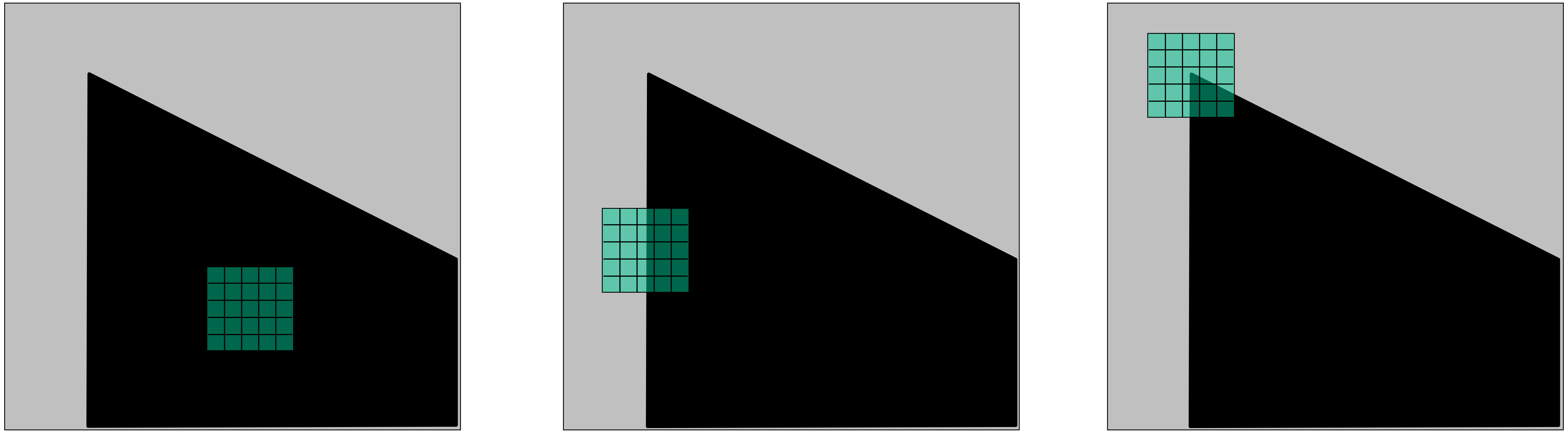


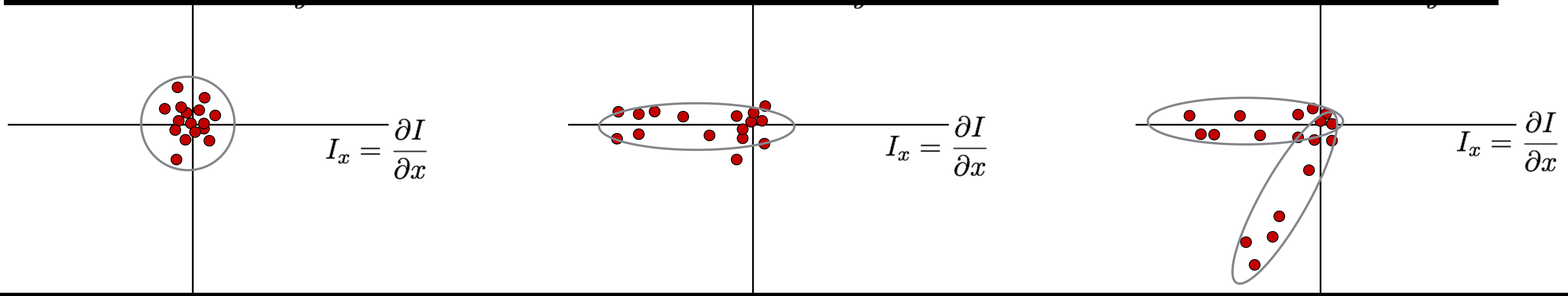
Distribution reveals the **orientation** and **magnitude**

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

What Does a **Distribution** Tells You About the **Region**?



Distribution reveals the orientation and magnitude



How do we quantify the orientation and magnitude?

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

2. Compute the **covariance matrix** (a.k.a. 2nd moment matrix)

$$C = \begin{bmatrix} \sum\limits_{p \in P} I_x I_x & \sum\limits_{p \in P} I_x I_y \\ \sum\limits_{p \in P} I_y I_x & \sum\limits_{p \in P} I_y I_y \end{bmatrix}$$

2. Compute the **covariance matrix** (a.k.a. 2nd moment matrix)

Sum over small region
around the corner

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Sum over small region around the corner

Gradient with respect to x, times gradient with respect to y

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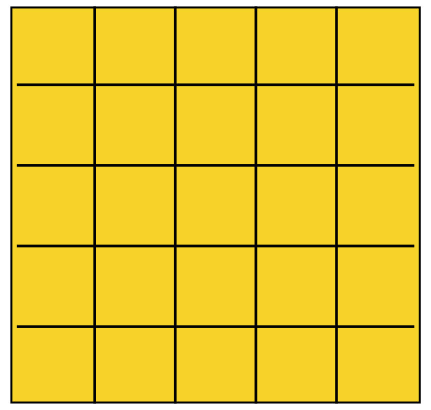
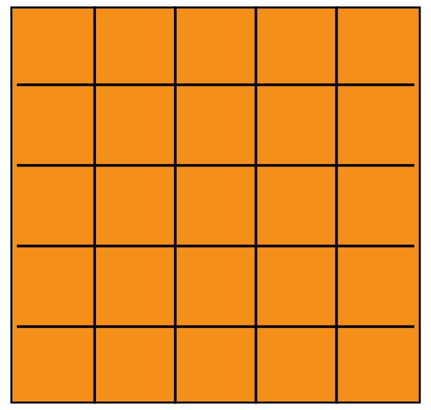
$$I_x = \frac{\partial I}{\partial x} \qquad I_y = \frac{\partial I}{\partial y}$$

$\sum_{p \in P} I_x I_y$ =sum( .* )

array of x gradients

array of y gradients

2. Compute the **covariance matrix** (a.k.a. 2nd moment matrix)

Sum over small region around the corner

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Matrix is **symmetric**

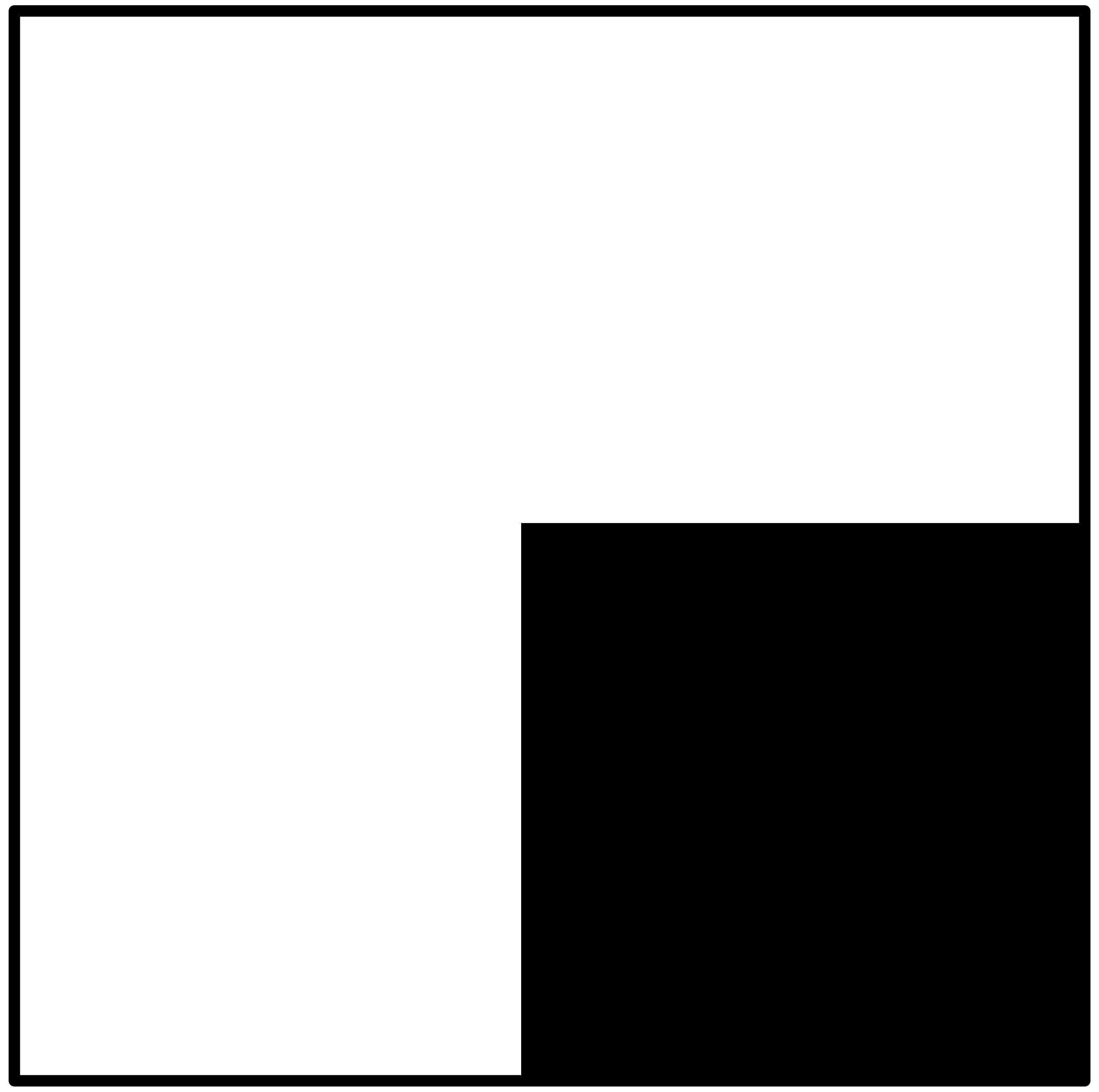
2. Compute the **covariance matrix** (a.k.a. 2nd moment matrix)

By computing the **gradient covariance matrix** …

$$C = \begin{bmatrix} \sum\limits_{p \in P} I_x I_x & \sum\limits_{p \in P} I_x I_y \\ \sum\limits_{p \in P} I_y I_x & \sum\limits_{p \in P} I_y I_y \end{bmatrix}$$

we are fitting a **quadratic** to the gradients over a small image region

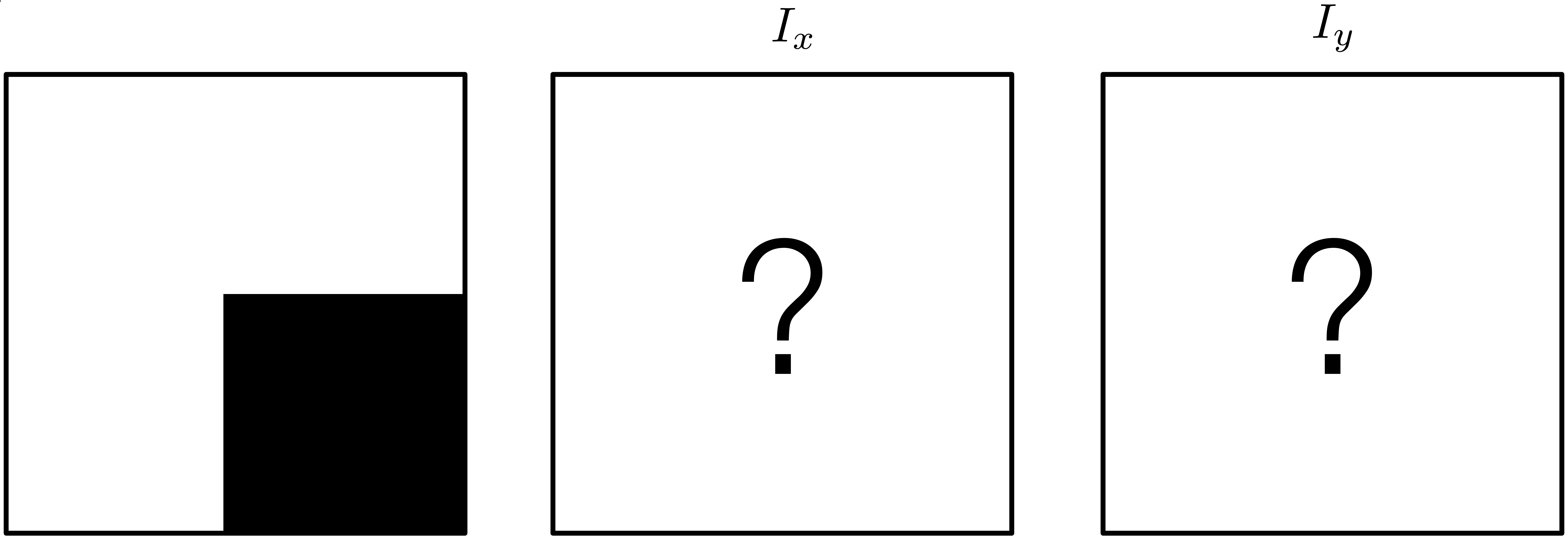
Simple Case



Local Image Patch

$$C = \begin{bmatrix} \sum\limits_{p \in P} I_x I_x & \sum\limits_{p \in P} I_x I_y \\ \sum\limits_{p \in P} I_y I_x & \sum\limits_{p \in P} I_y I_y \end{bmatrix} = ?$$

Simple Case



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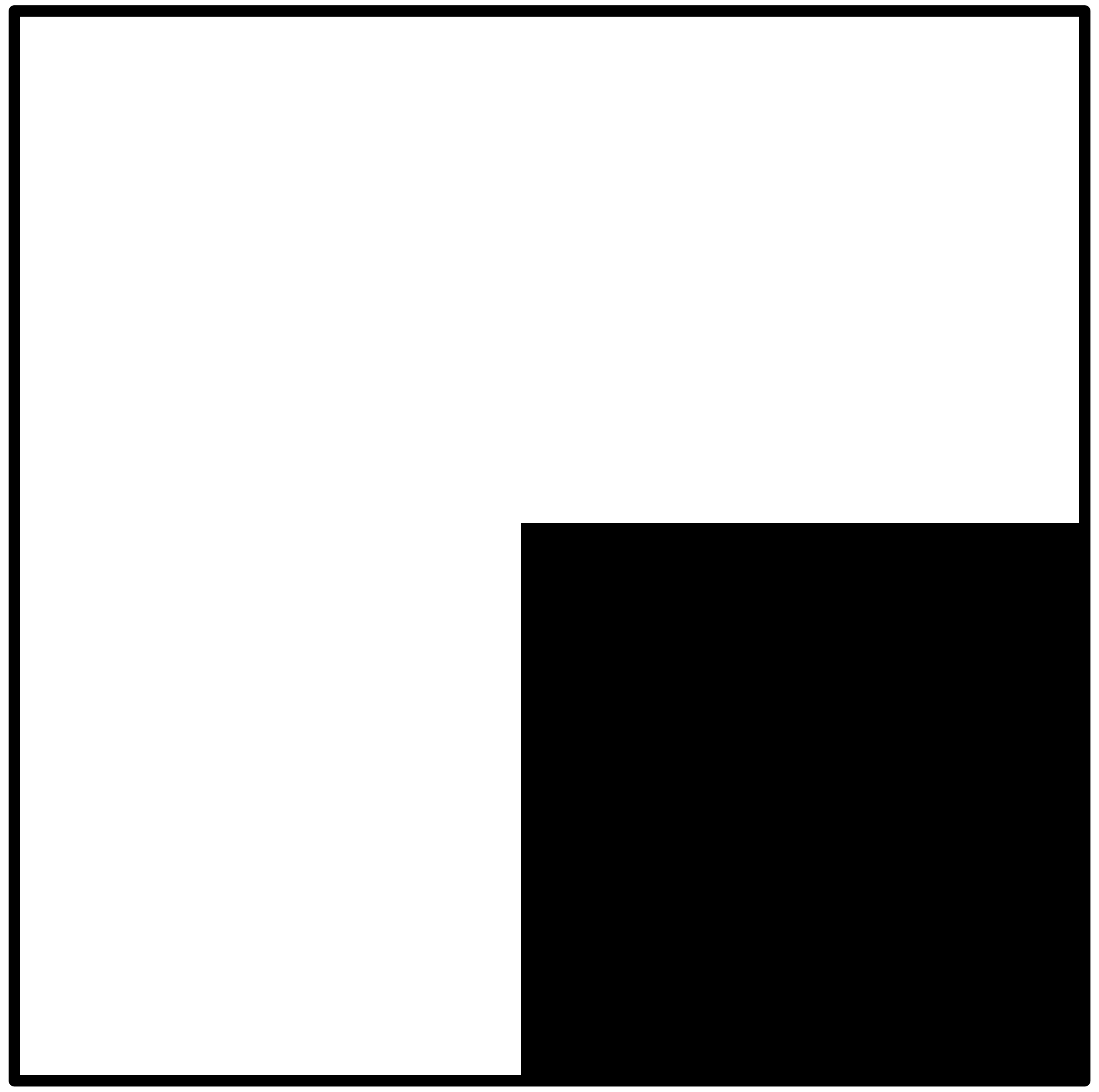
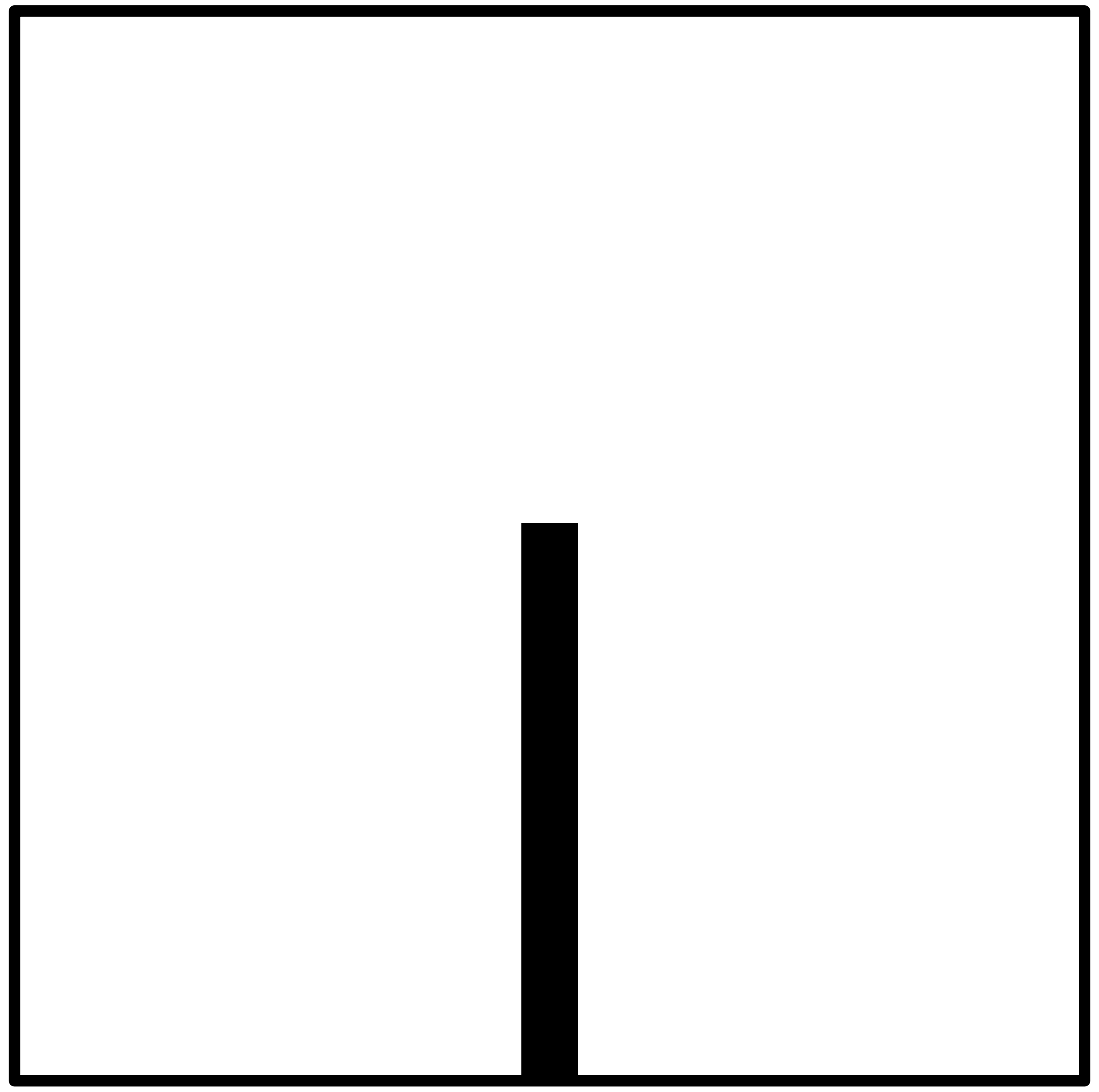


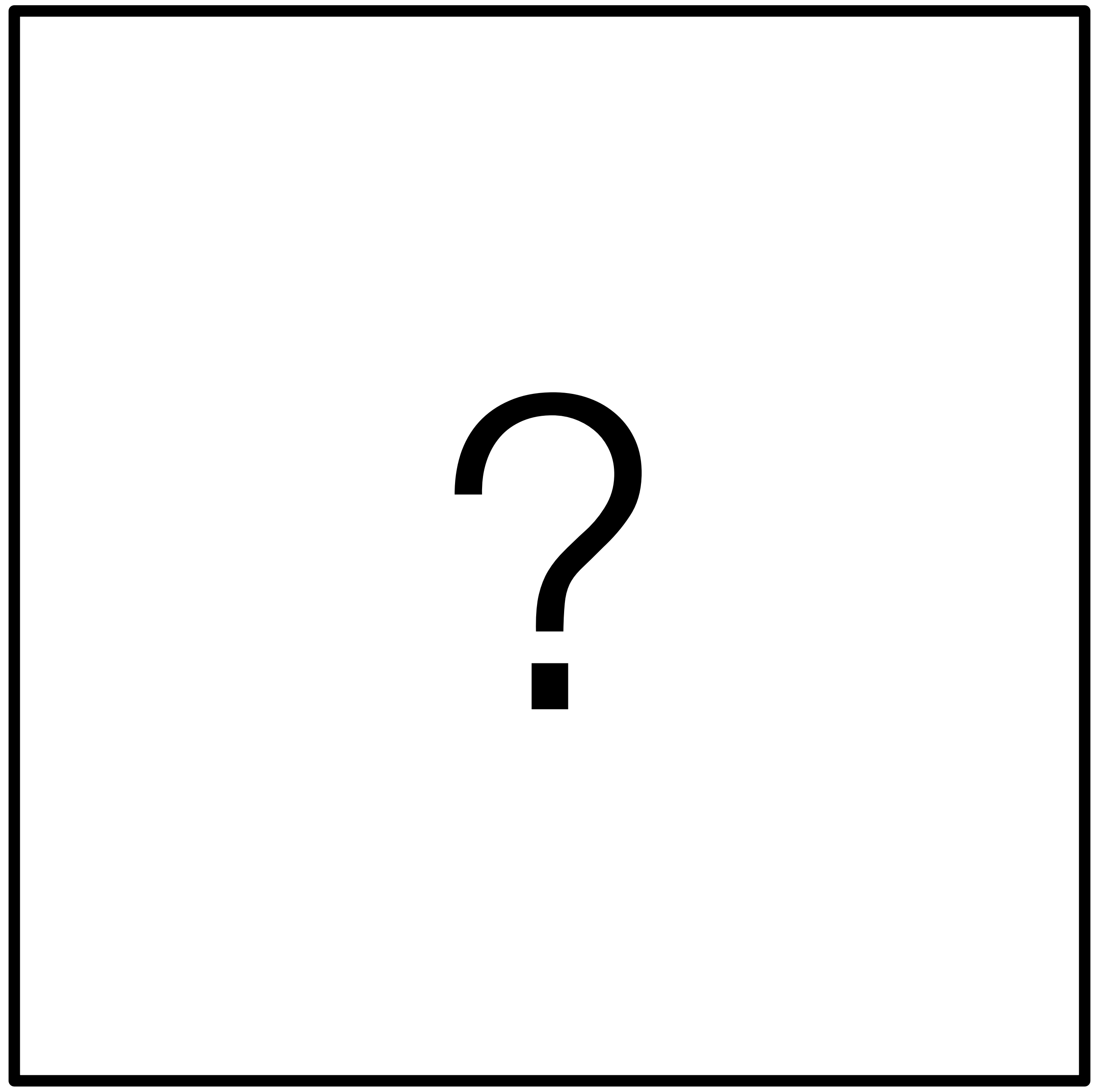
Local Image Patch

I_x



high value along vertical
strip of pixels and 0 elsewhere

I_y



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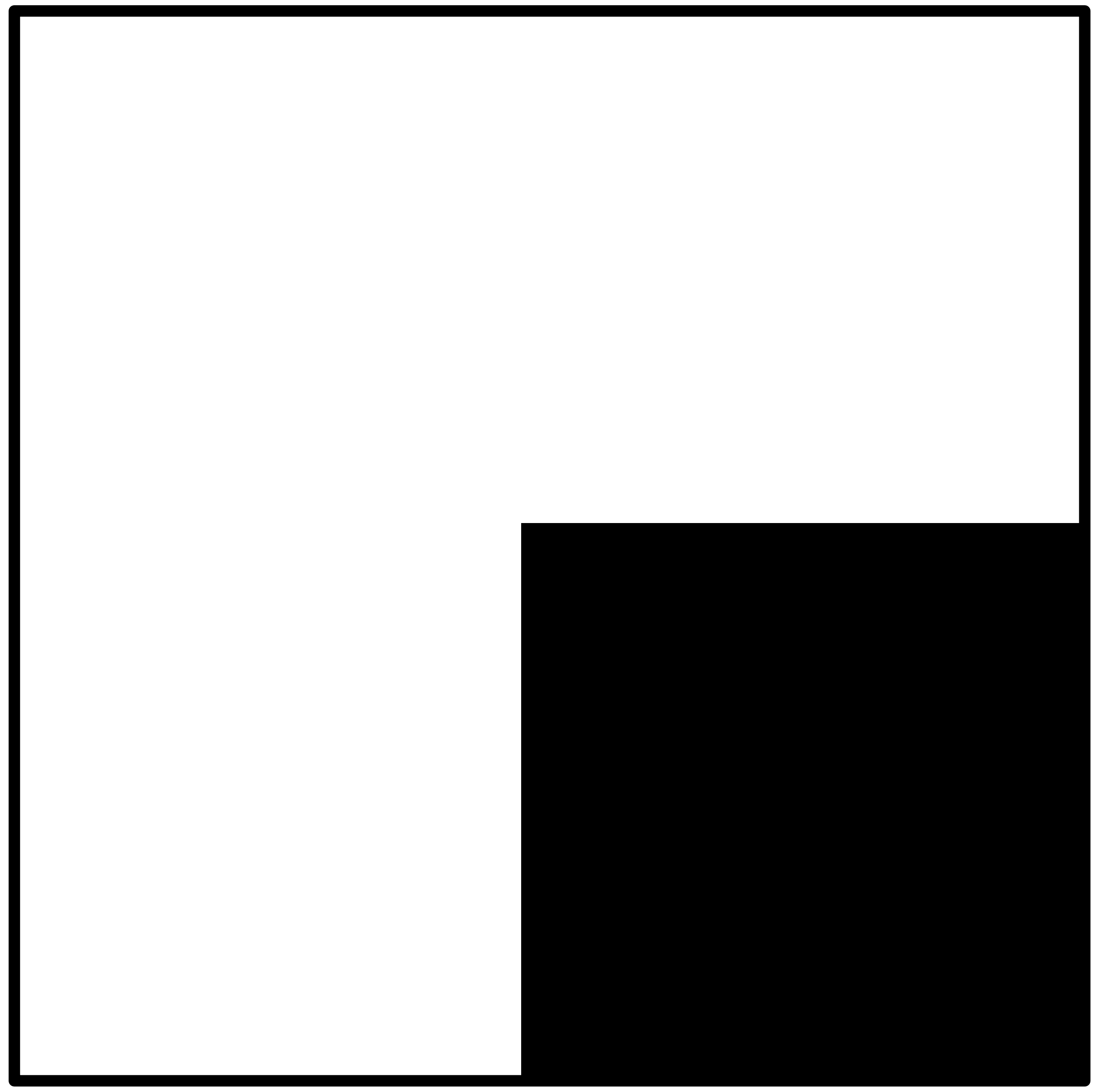

Local Image Patch

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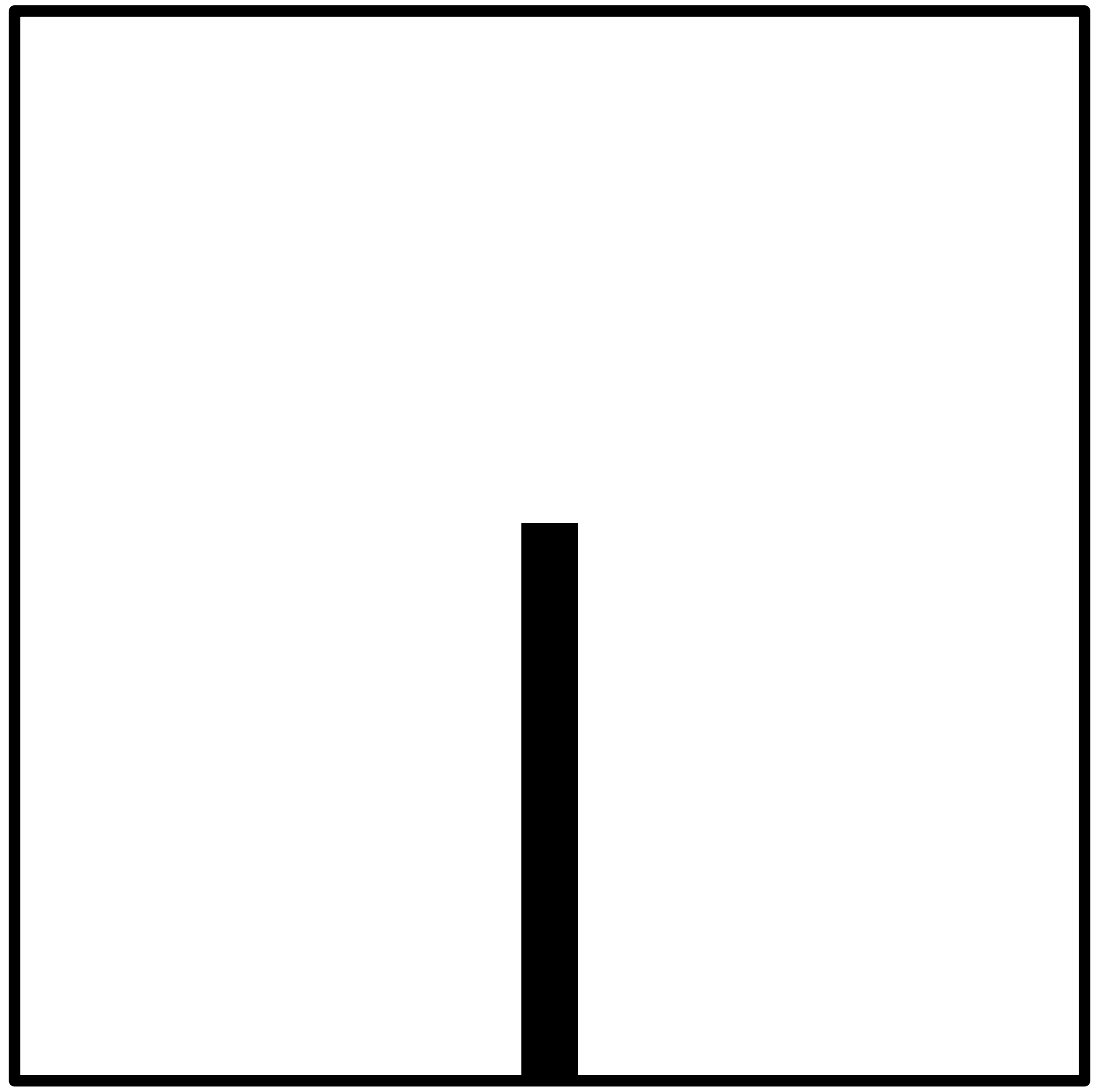
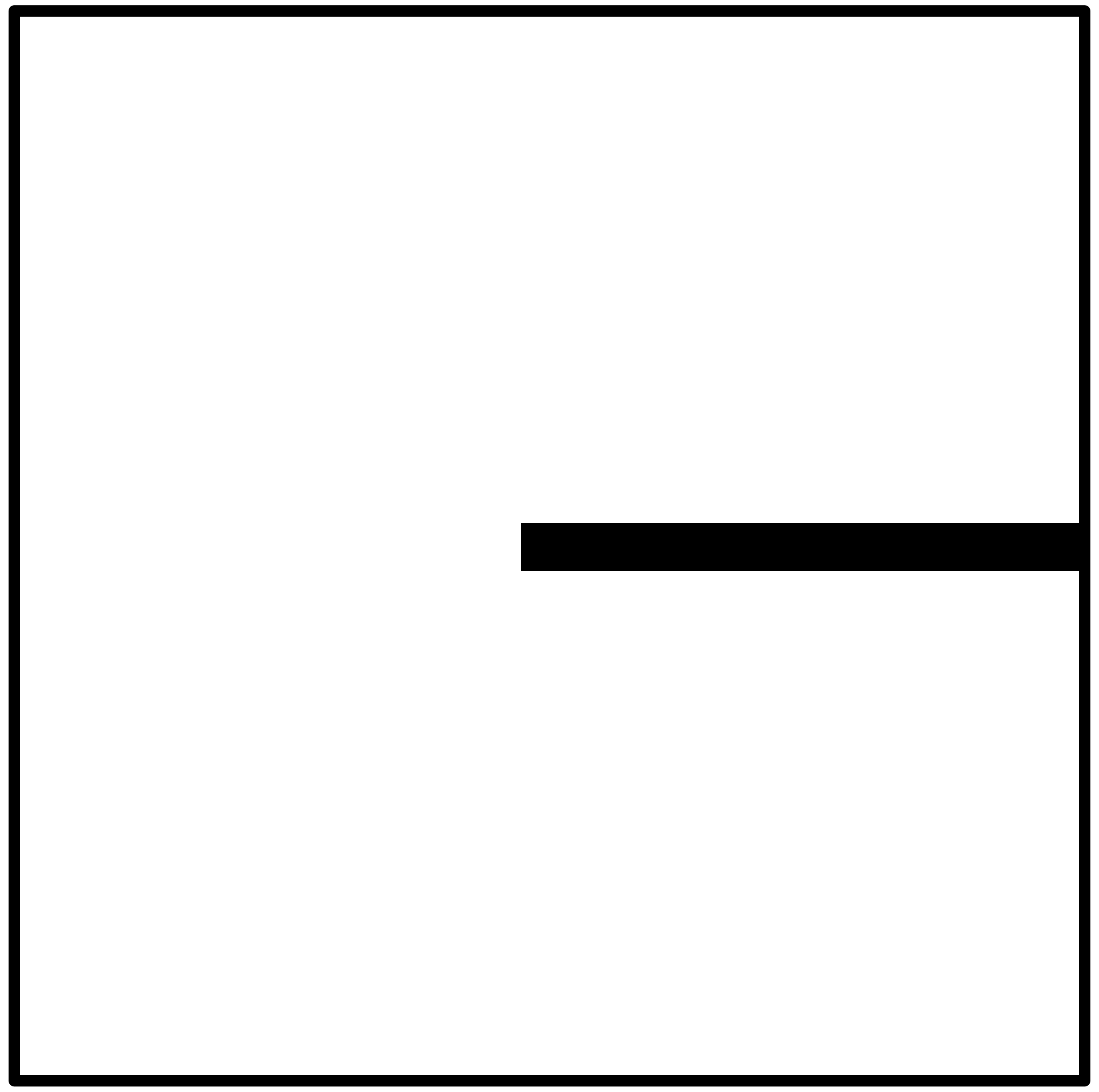


high value along vertical strip of pixels and 0 elsewhere

I_y



high value along horizontal strip of pixels and 0 elsewhere

$$C = \begin{bmatrix} \sum\limits_{p \in P} I_x I_x & \sum\limits_{p \in P} I_x I_y \\ \sum\limits_{p \in P} I_y I_x & \sum\limits_{p \in P} I_y I_y \end{bmatrix} = ?$$

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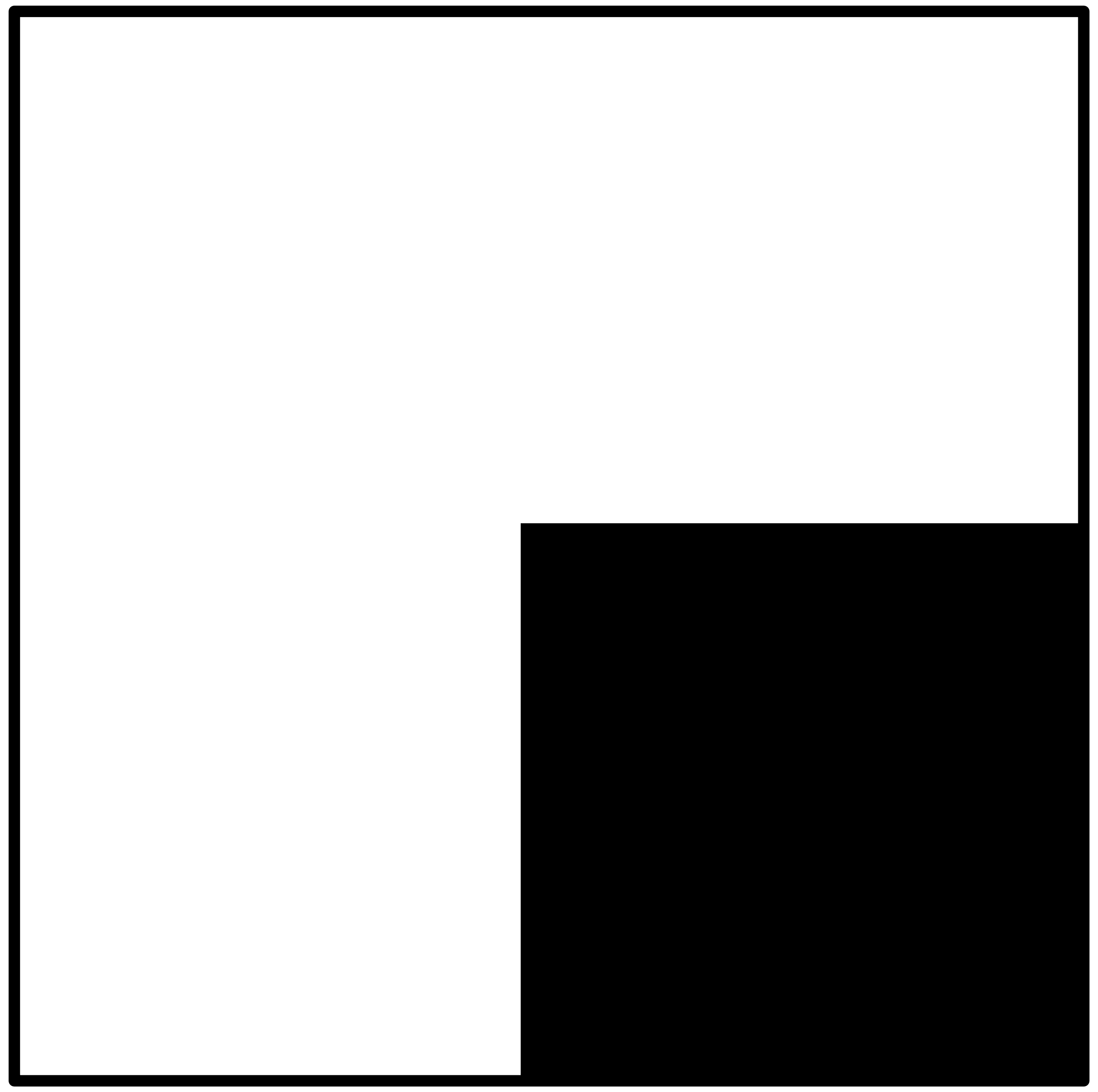
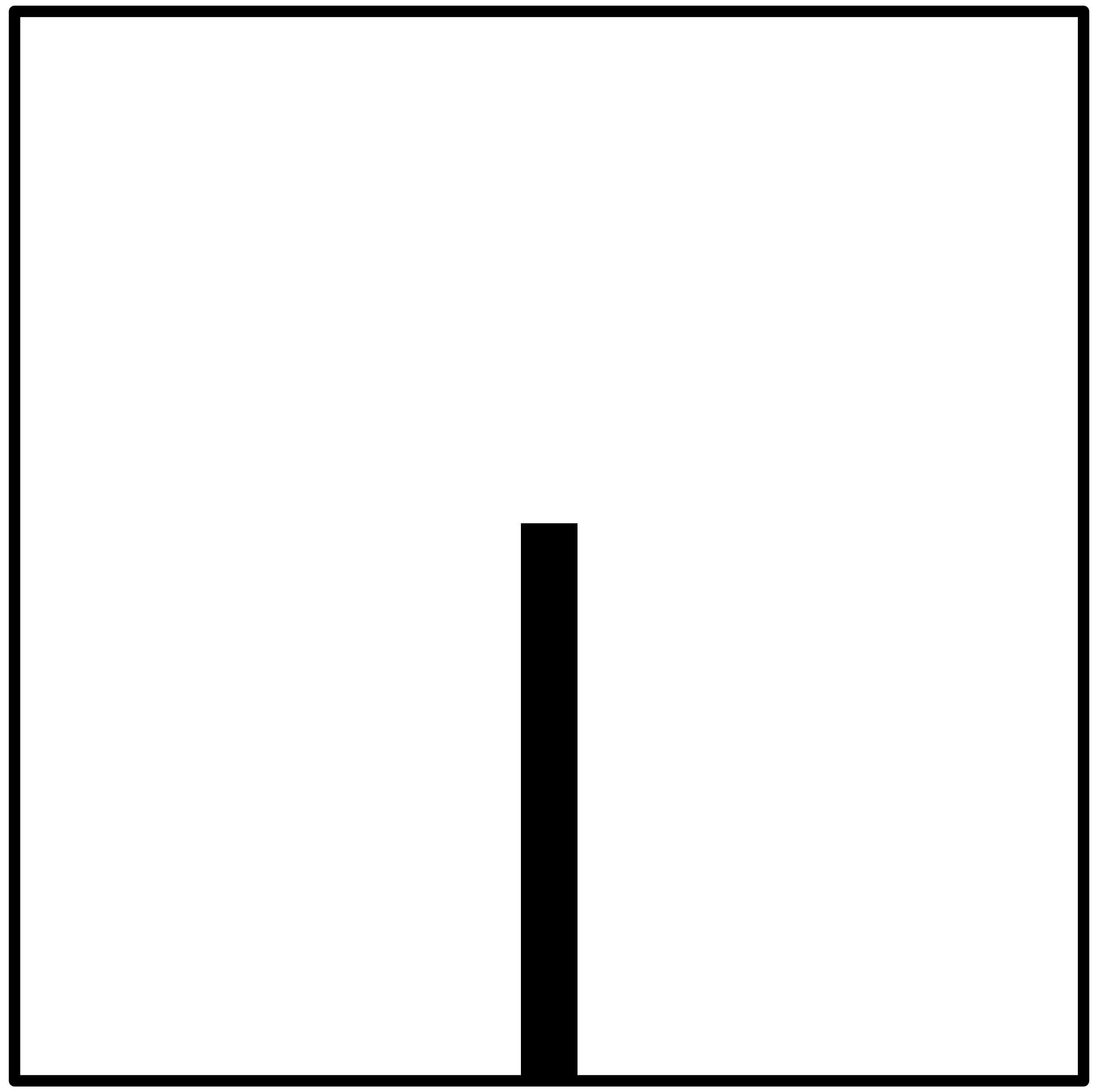


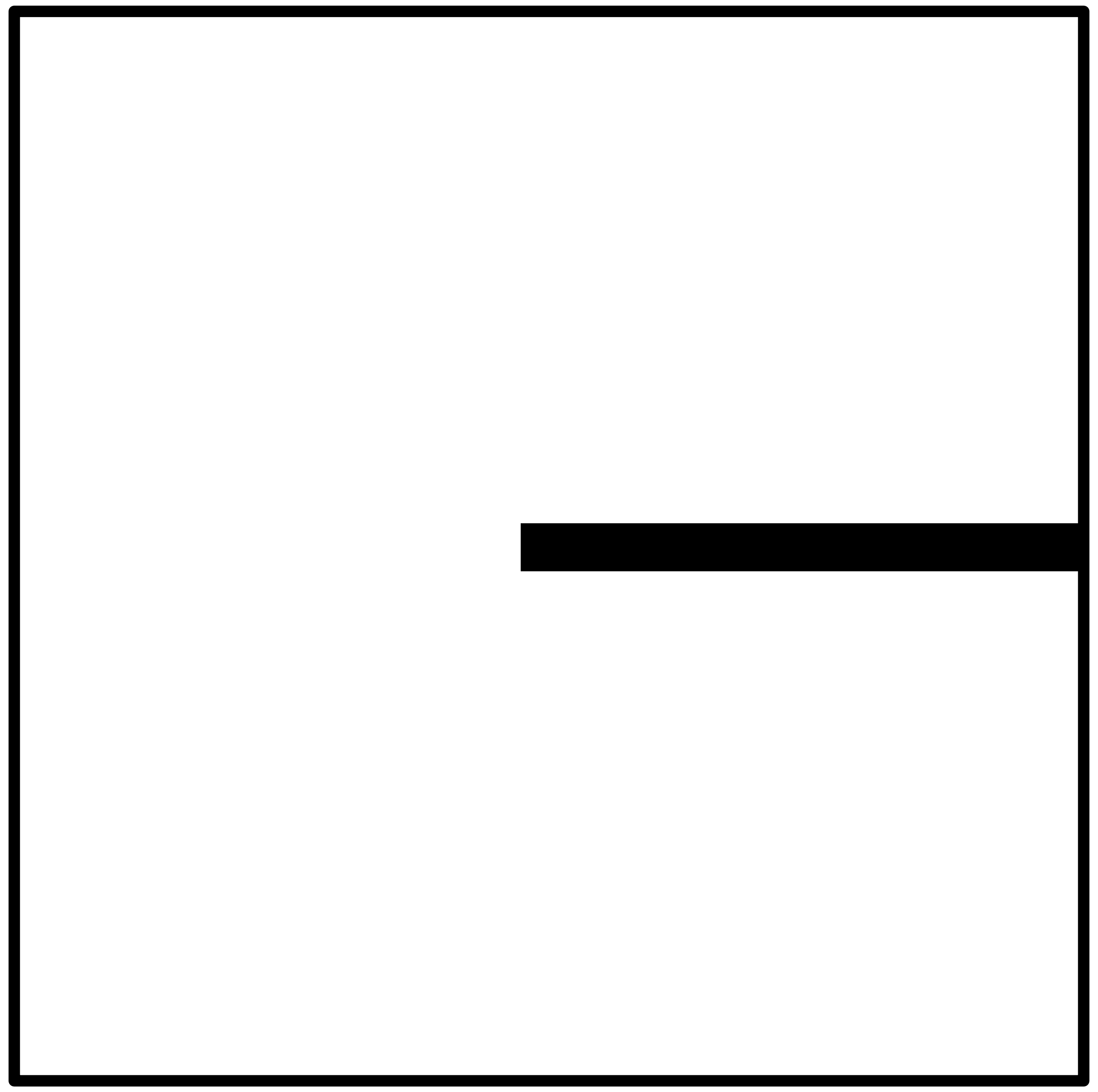
Local Image Patch

I_x



high value along vertical
strip of pixels and 0 elsewhere

I_y



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General Case

It can be shown that since every C is symmetric:

$$C = \begin{bmatrix} \sum\limits_{p \in P} I_x I_x & \sum\limits_{p \in P} I_x I_y \\ \sum\limits_{p \in P} I_y I_x & \sum\limits_{p \in P} I_y I_y \end{bmatrix} = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

… so general case is like a **rotated** version of the simple one

3. Computing **Eigenvalues** and **Eigenvectors**

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Quick **Eigenvalue/Eigenvector** Review

Given a square matrix $\mathbf{A}$, a scalar λ is called an **eigenvalue** of $\mathbf{A}$ if there exists a nonzero vector $\mathbf{v}$ that satisfies

$$\mathbf{A}\mathbf{v} = \lambda\mathbf{v}$$

The vector $\mathbf{v}$ is called an **eigenvector** for $\mathbf{A}$ corresponding to the eigenvalue λ.

The eigenvalues of $\mathbf{A}$ are obtained by solving

$$\det(\mathbf{A} - \lambda I) = 0$$

3. Computing **Eigenvalues** and **Eigenvectors**

eigenvalue

$$Ce = \lambda e$$

eigenvector

$$(C - \lambda I)e = 0$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

3. Computing **Eigenvalues** and **Eigenvectors**

eigenvalue

$$Ce = \lambda e$$

eigenvector

$$(C - \lambda I)e = 0$$

1. Compute the determinant of $C - \lambda I$
(returns a polynomial)

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

3. Computing **Eigenvalues** and **Eigenvectors**

eigenvalue

$$Ce = \lambda e$$

eigenvector

$$(C - \lambda I)e = 0$$

1. Compute the determinant of (returns a polynomial)	$C - \lambda I$
2. Find the roots of polynomial (returns eigenvalues)	$\det(C - \lambda I) = 0$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

3. Computing **Eigenvalues** and **Eigenvectors**

eigenvalue

$$Ce = \lambda e$$

eigenvector

$$(C - \lambda I)e = 0$$

1. Compute the determinant of (returns a polynomial)	$C - \lambda I$
2. Find the roots of polynomial (returns eigenvalues)	$\det(C - \lambda I) = 0$
3. For each eigenvalue, solve (returns eigenvectors)	$(C - \lambda I)e = 0$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Example

$$C = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

1. Compute the determinant of
(returns a polynomial)
$$C - \lambda I$$

2. Find the roots of polynomial
(returns eigenvalues)
$$\det(C - \lambda I) = 0$$

3. For each eigenvalue, solve
(returns eigenvectors)
$$(C - \lambda I)e = 0$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Example

$$C = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\det\left(\begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix}\right)$$

1. Compute the determinant of (returns a polynomial) $\qquad C - \lambda I$

2. Find the roots of polynomial (returns eigenvalues) $\qquad \det(C - \lambda I) = 0$

3. For each eigenvalue, solve (returns eigenvectors) $\qquad (C - \lambda I)e = 0$

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Example

$$C = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\det\left(\begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix}\right)$$

$$(2-\lambda)(2-\lambda)-(1)(1)$$

1. Compute the determinant of (returns a polynomial) $$C - \lambda I$$

2. Find the roots of polynomial (returns eigenvalues) $$\det(C - \lambda I) = 0$$

3. For each eigenvalue, solve (returns eigenvectors) $$(C - \lambda I)e = 0$$

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$$C = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\det\left(\begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix}\right)$$

$$(2-\lambda)(2-\lambda)-(1)(1)$$

$$(2-\lambda)(2-\lambda)-(1)(1)=0$$
$$\lambda^2-4\lambda+3=0$$
$$(\lambda-3)(\lambda-1)=0$$
$$\lambda_1=1, \lambda_2=3$$

1. Compute the determinant of (returns a polynomial) $$C-\lambda I$$

2. Find the roots of polynomial (returns eigenvalues) $$\det(C-\lambda I)=0$$

3. For each eigenvalue, solve (returns eigenvectors) $$(C-\lambda I)e=0$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)