THE UNIVERSITY OF BRITISH COLUMBIA

CPSC 425: Computer Vision

Image Credit: https://docs.adaptive-vision.com/4.7/studio/machine_vision_guide/TemplateMatching.html

Lecture 7: Template Matching (cont.) and Scaled Representations

(unless otherwise stated slides are taken or adopted from **Bob Woodham, Jim Little** and **Fred Tung**)

Menu for Today (**January 28, 2020**)

Topics:

- Template **Matching**
- **Normalized** Correlation
- **Scaled** Representations
- **Image Derivatives**

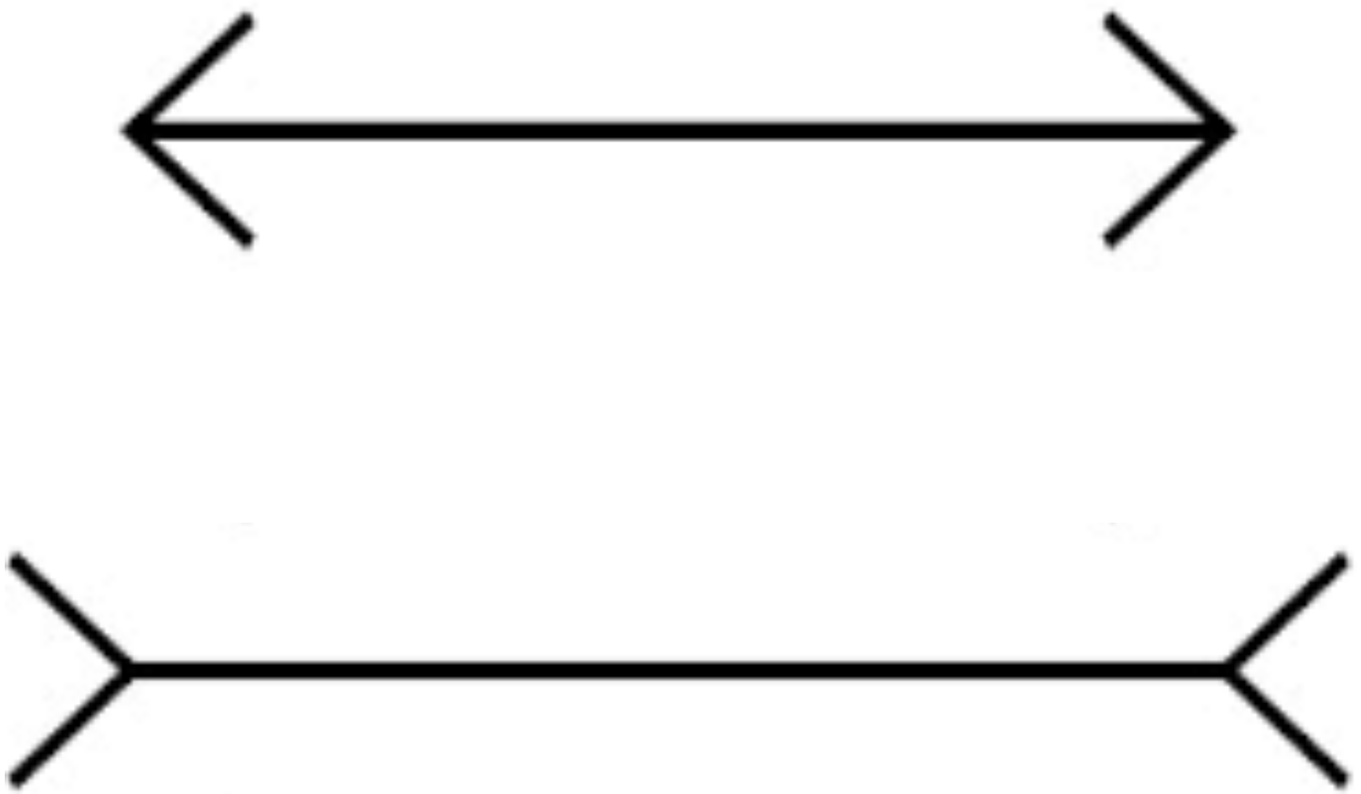
Redings:

- **Today's** Lecture: Forsyth & Ponce (2nd ed.) 4.5 - 4.7
- **Next** Lecture: Forsyth & Ponce (2nd ed.) 5.1 - 5.2

Reminders:

- **Assignment 1:** Image Filtering and Hybrid Images is due **today**
- **Assignment 2**: Scaled Representations, Face Detection and Image Blending

Today's "**fun**" Example: Müller-Lyer Illusion



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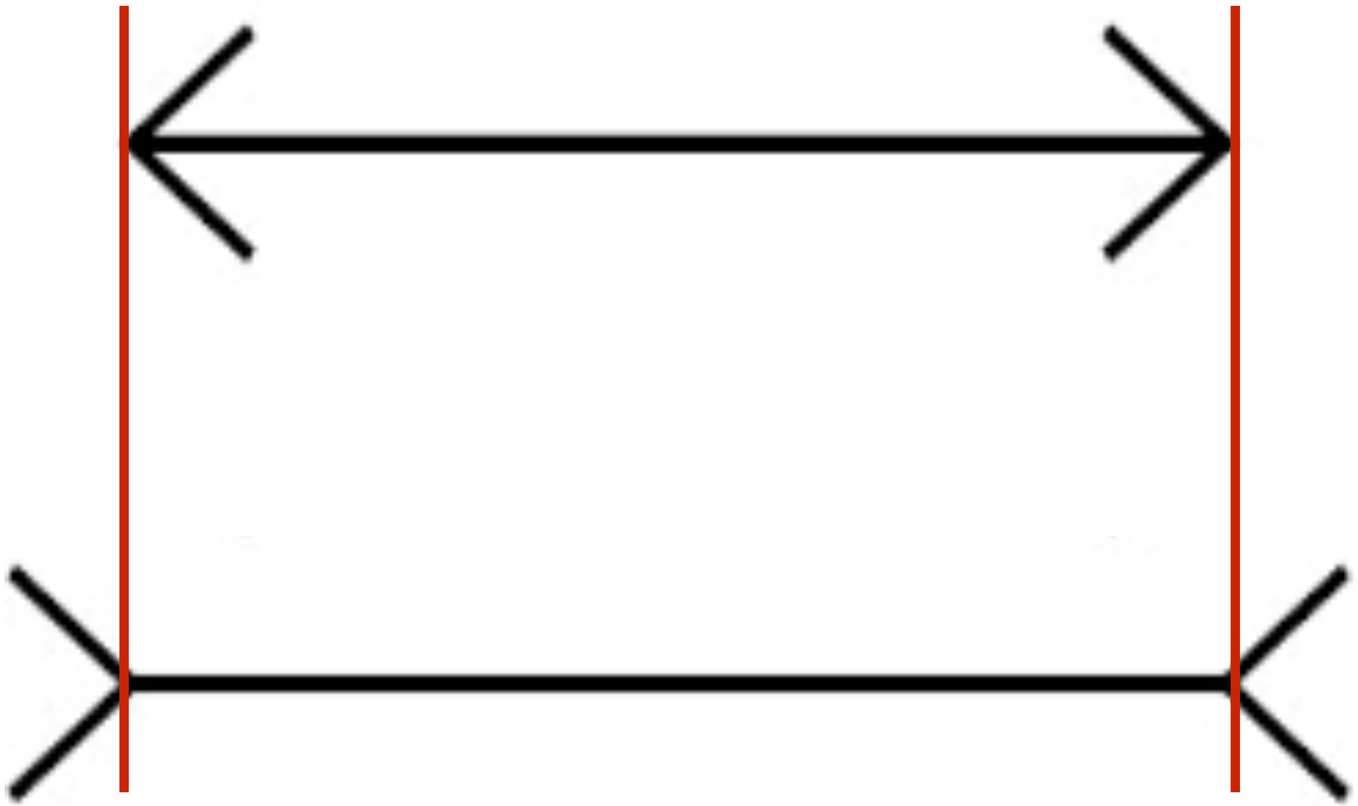

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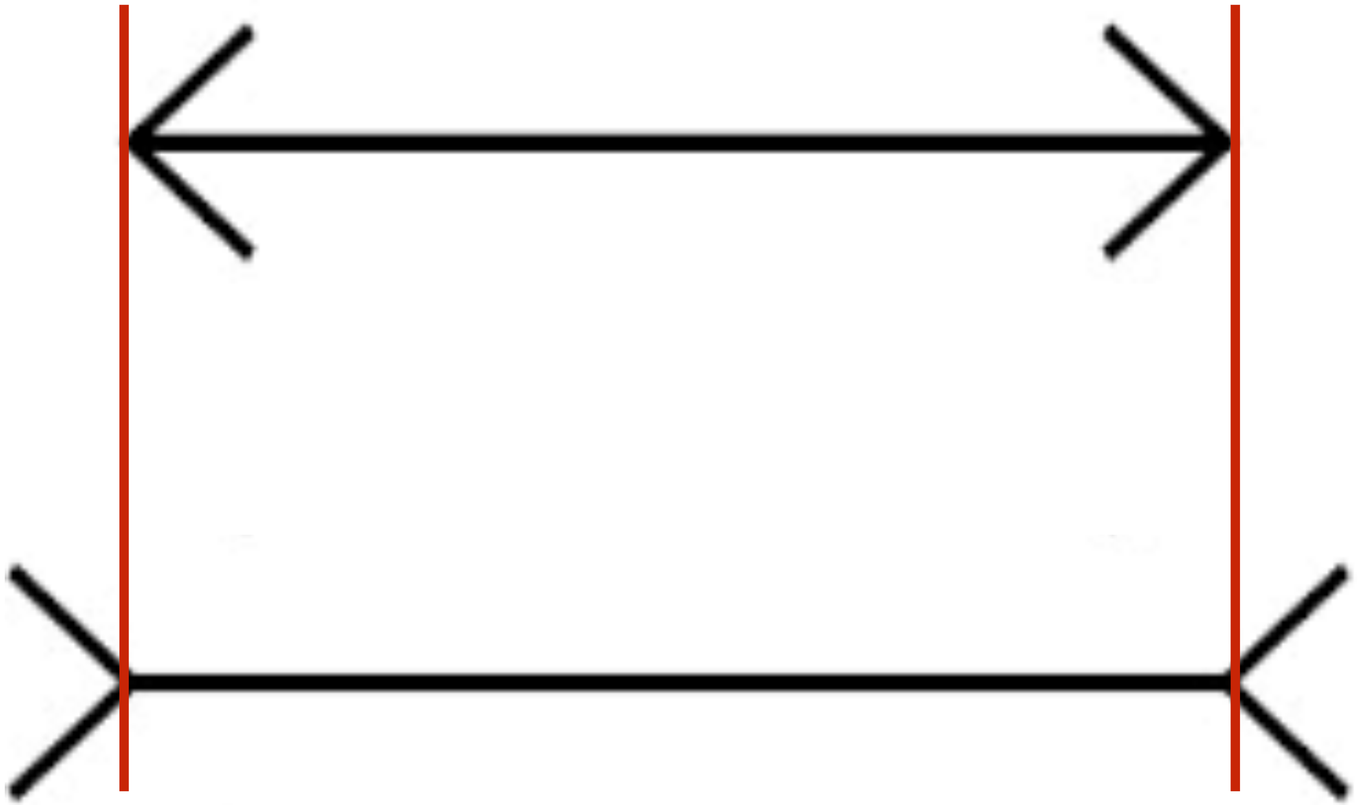
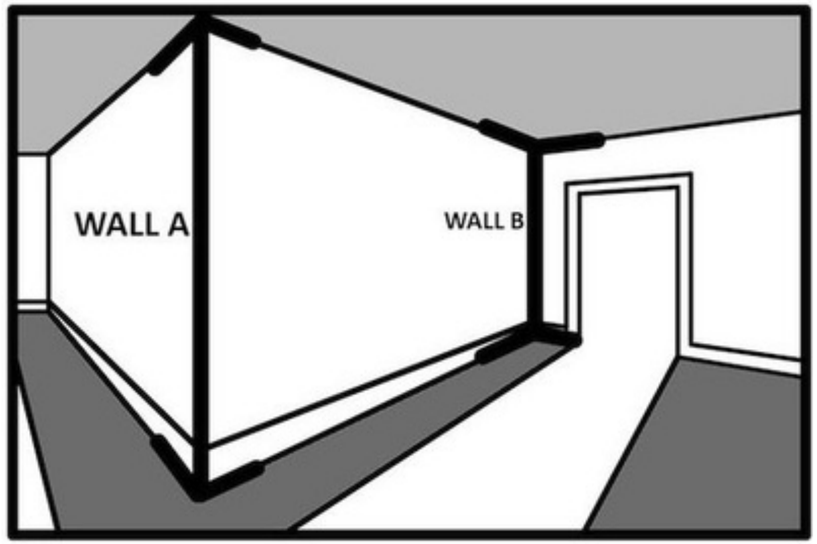




Lecture 6: Re-cap

In the continuous case, images are functions of two spatial variables, x and y.

The discrete case is obtained from the continuous case via sampling (i.e. tessellation, quantization).

If a signal is **bandlimited** then it is possible to design a sampling strategy such that the sampled signal captures the underlying continuous signal exactly.

Adequate sampling may not always be practical. In such cases there is a trade-off between “things missing” and “artifacts”.

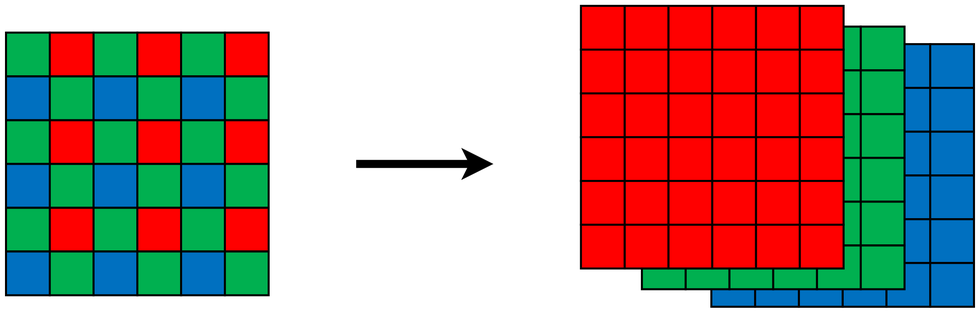
— Different applications make the trade-off differently

Lecture 6: Re-cap

"Color" is **not** an objective physical property of light (electromagnetic radiation). Instead, light is characterized by its wavelength.

Color Filter Arrays (CFAs) allow capturing of mosaiced color information; the layout of the mosaic is called **Bayer** pattern.

Demosaicing is the process of taking the RAW image and interpolating missing color pixels per channel



Lecture 6: Re-cap

How can we find a part of one image that matches another?

or,

How can we find instances of a pattern in an image?

Lecture 6: Re-cap

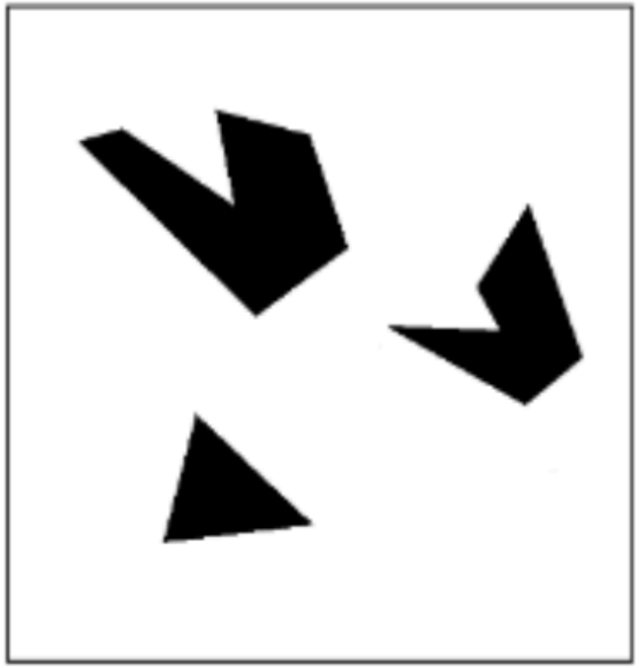
How can we find a part of one image that matches another?

or,

How can we find instances of a pattern in an image?

Key Idea: Use the pattern as a **template**

Template Matching



Scene

Template (mask)

A toy example

Slide Credit: Kristen Grauman

Template Matching

We can think of convolution/**correlation** as comparing a template (the filter) with each local image patch.

— Consider the filter and image patch as vectors.

— Applying a filter at an image location can be interpreted as computing the dot product between the filter and the local image patch.

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Template

0	0	0
0	1	0
0	1	1

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Image Patch 1

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0	1	0
0	1	1

Image Patch 2

1	0	1
0	1	0
0	0	0

Template

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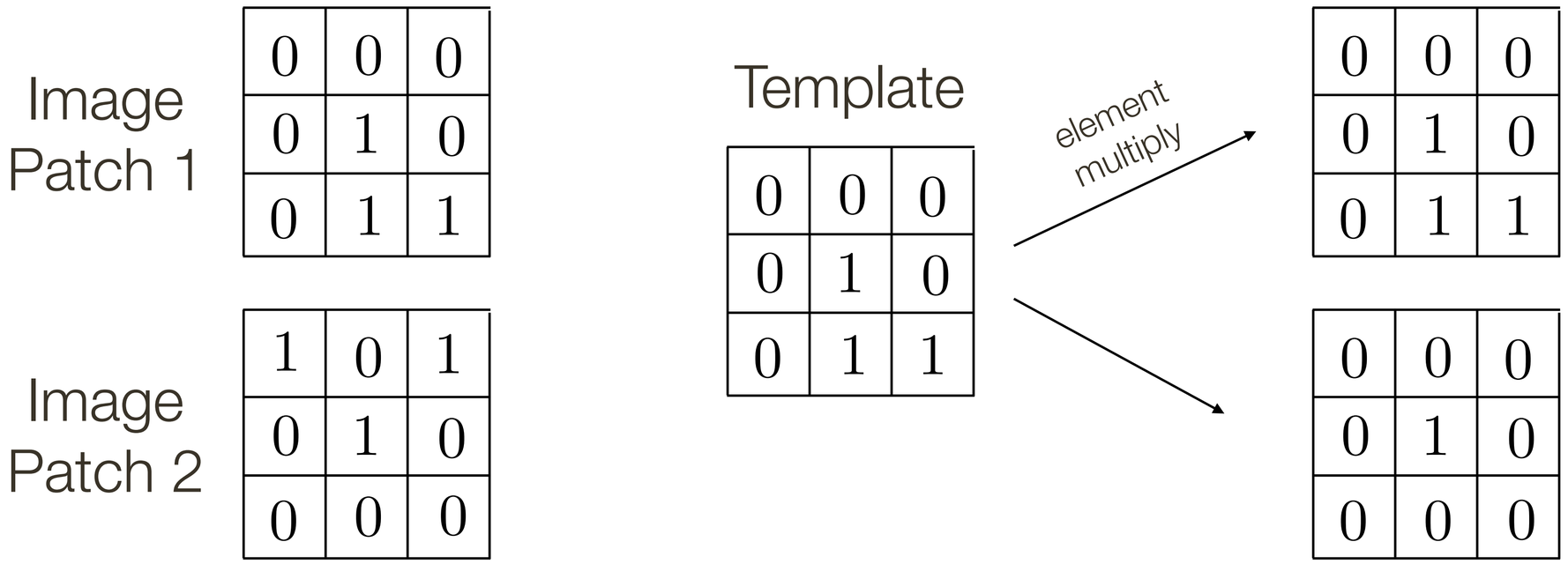

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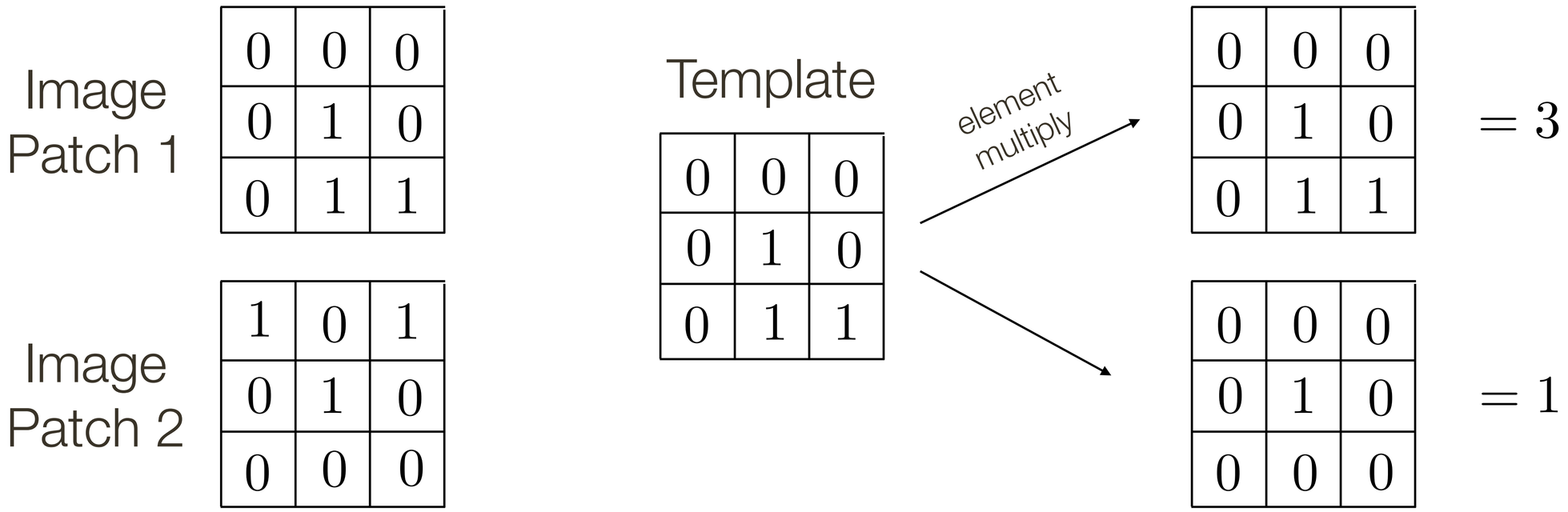

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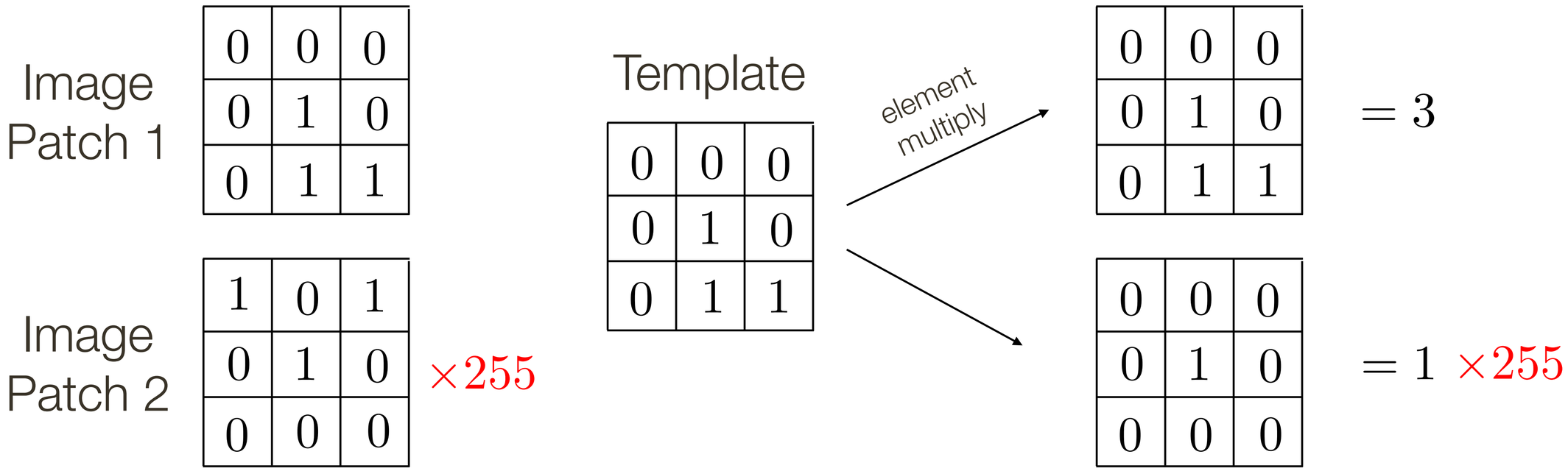

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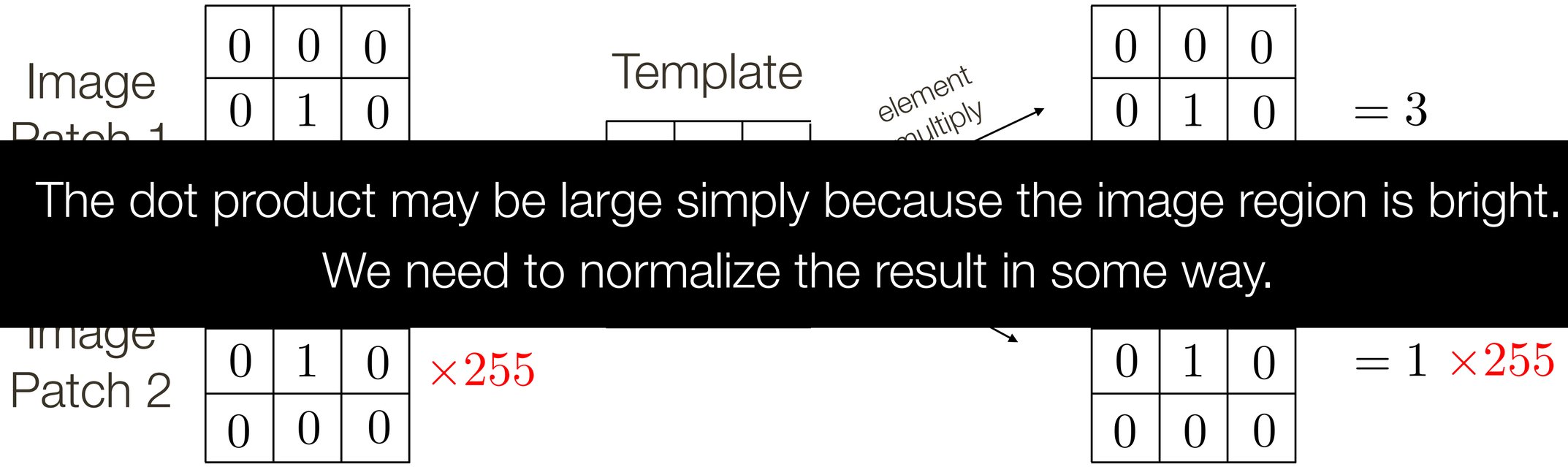


The dot product may be large simply because the image region is bright.

We need to normalize the result in some way.

Template Matching

Let a and b be vectors. Let θ be the angle between them. We know

$$\cos\theta = \frac{a \cdot b}{|a||b|} = \frac{a \cdot b}{\sqrt{(a \cdot a)(b \cdot b)}} = \frac{a}{|a|}\frac{b}{|b|}$$

where · is dot product and | | is vector magnitude

Correlation is a dot product

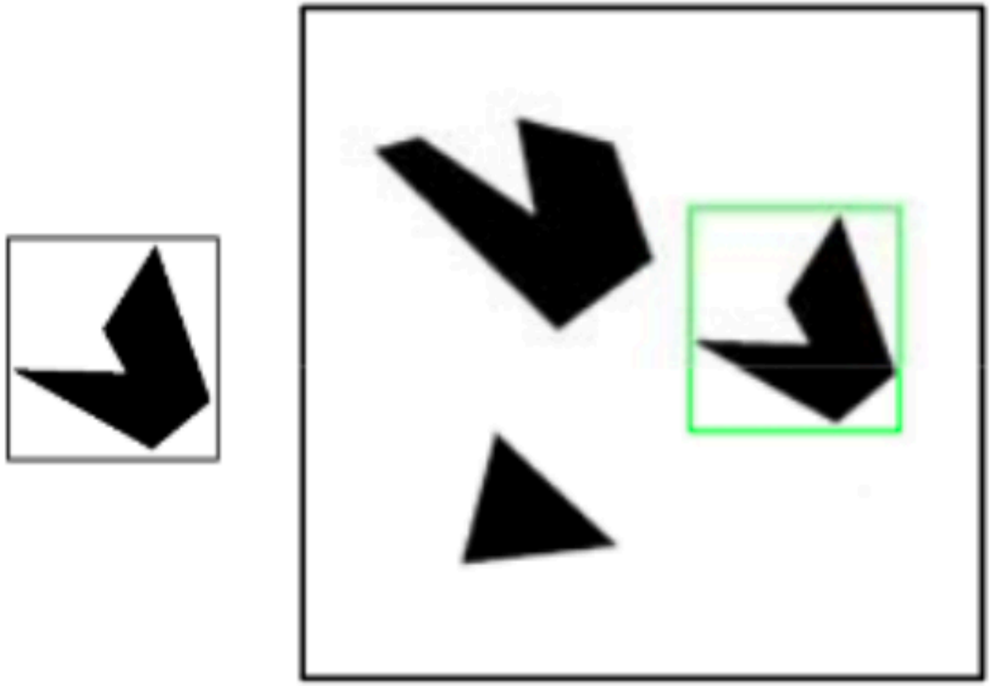
Correlation measures similarity between the filter and each local image region

Normalized correlation varies between –1 and 1

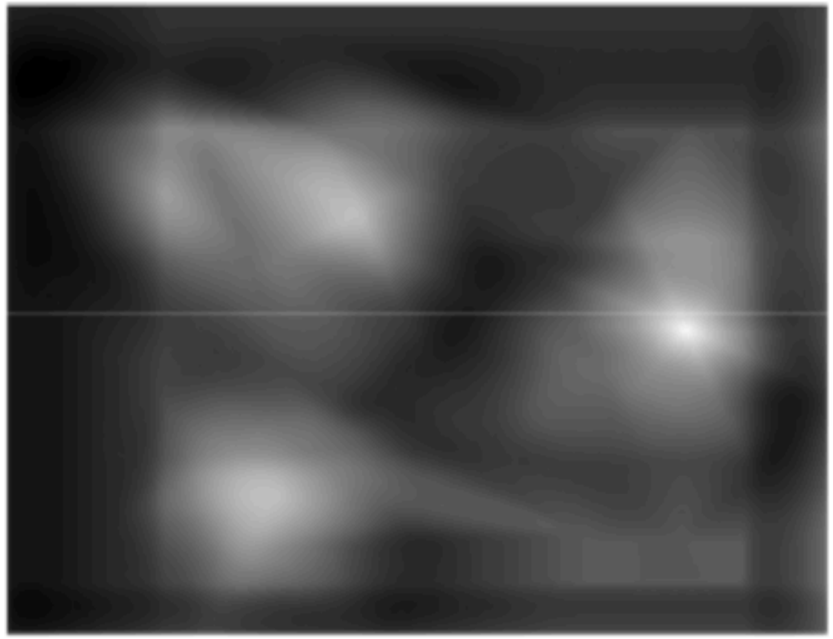
Normalized correlation attains the value 1 when the filter and image region are identical (up to a scale factor)

Template Matching

Assuming template is all positive, what does this tell us about correlation map?



Detected template



Correlation map

Slide Credit: Kristen Grauman

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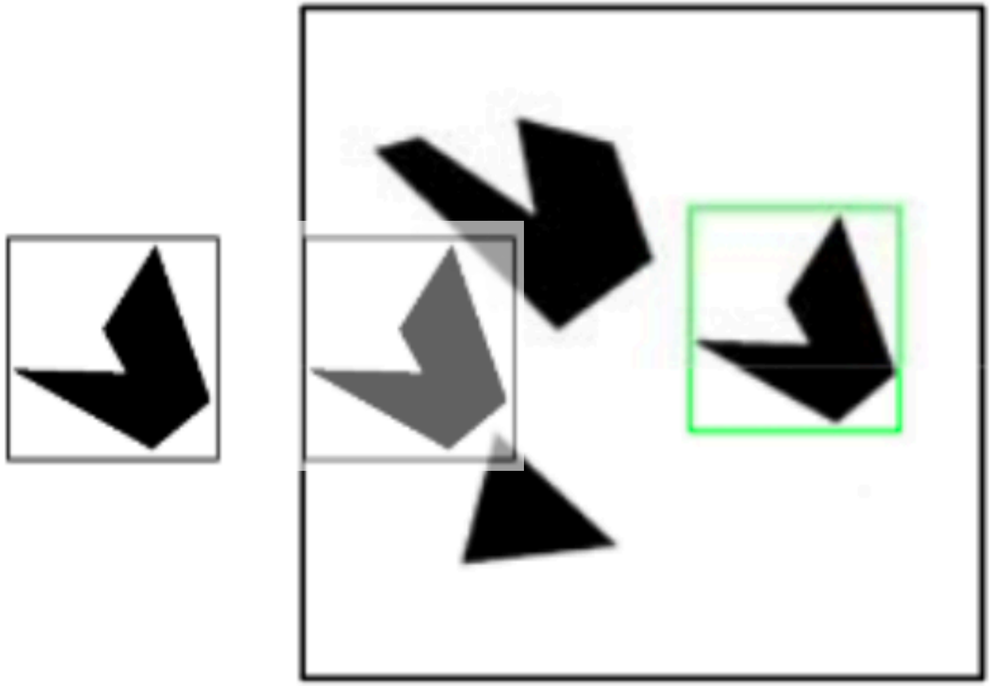

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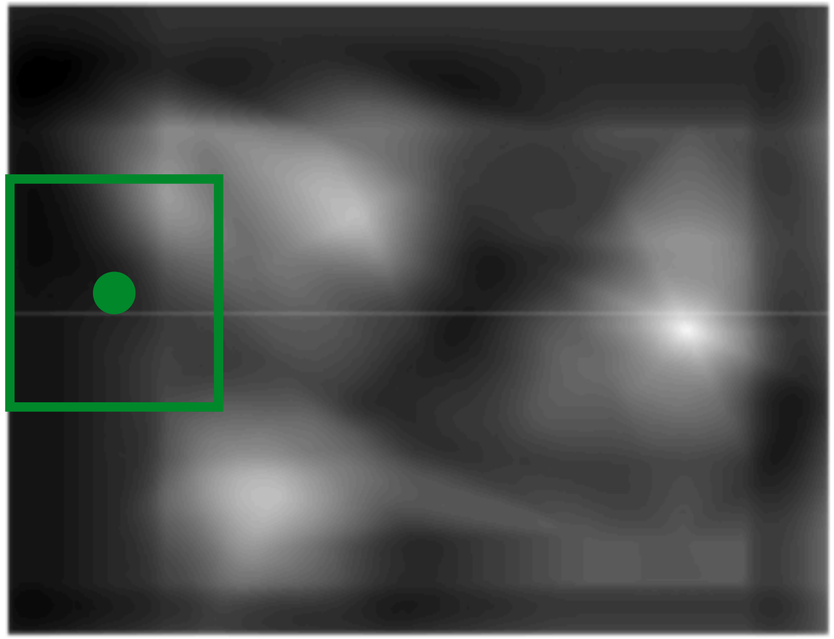

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$$\frac{a}{|a|}\frac{b}{|b|} = ?$$

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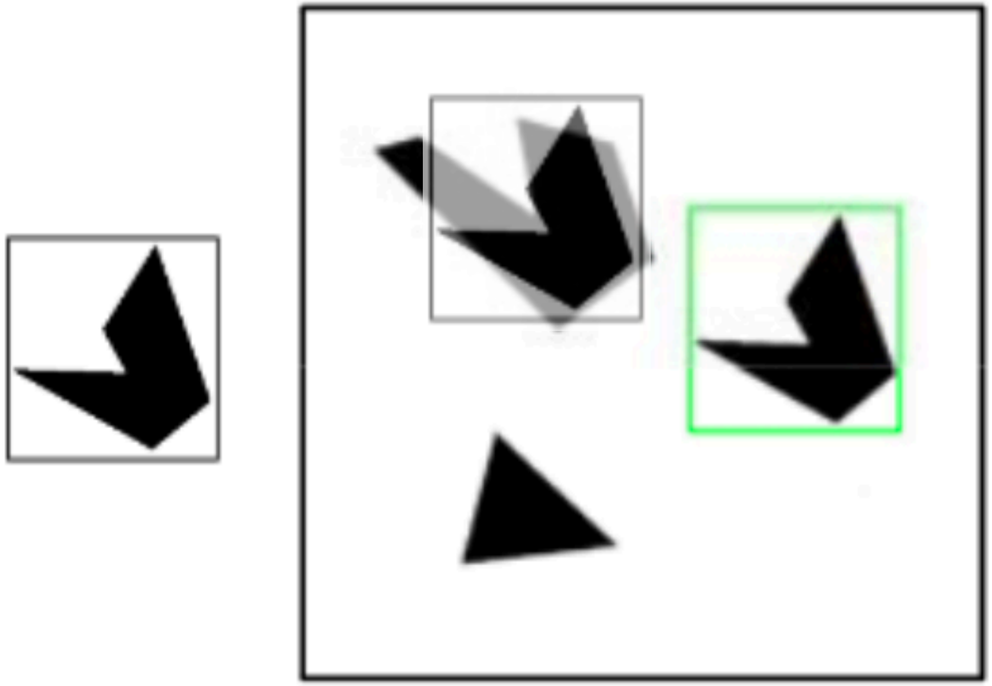

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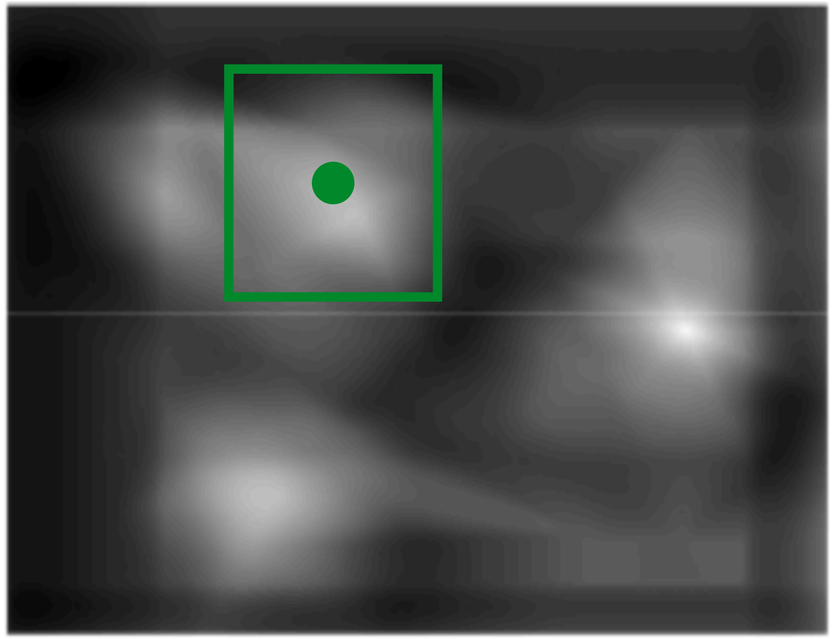

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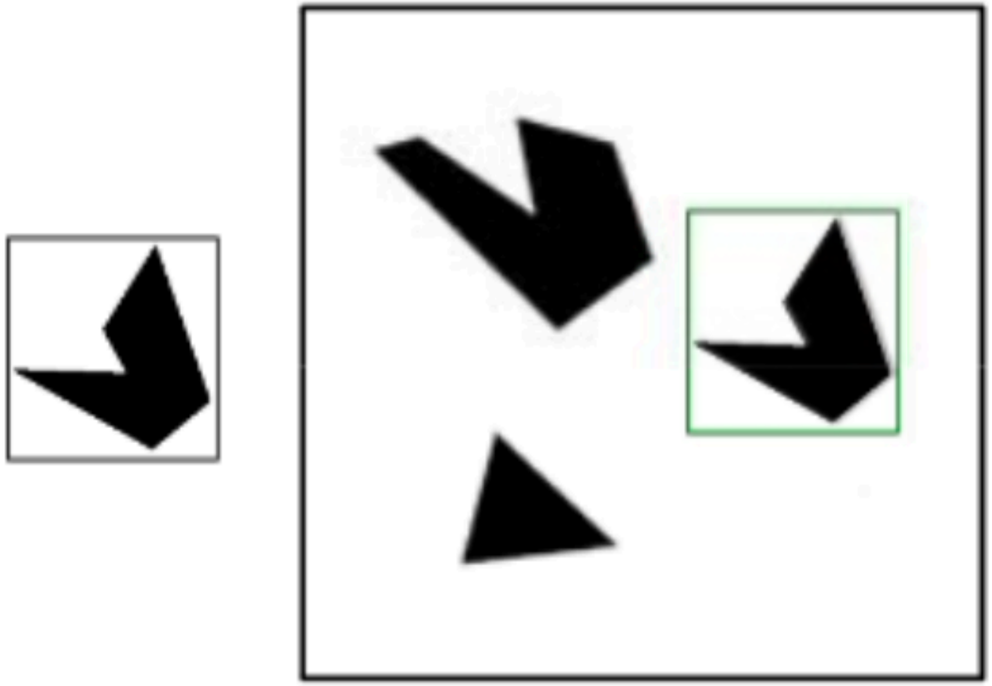

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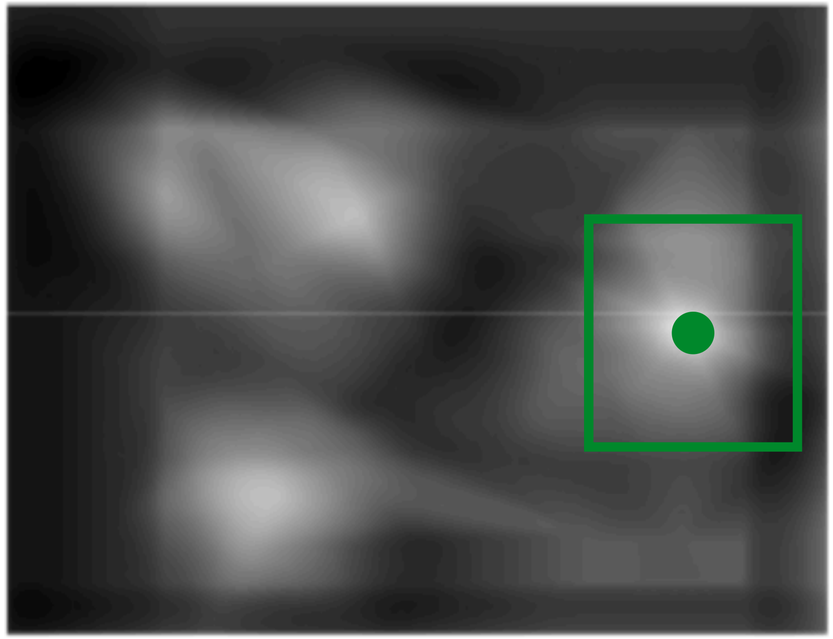

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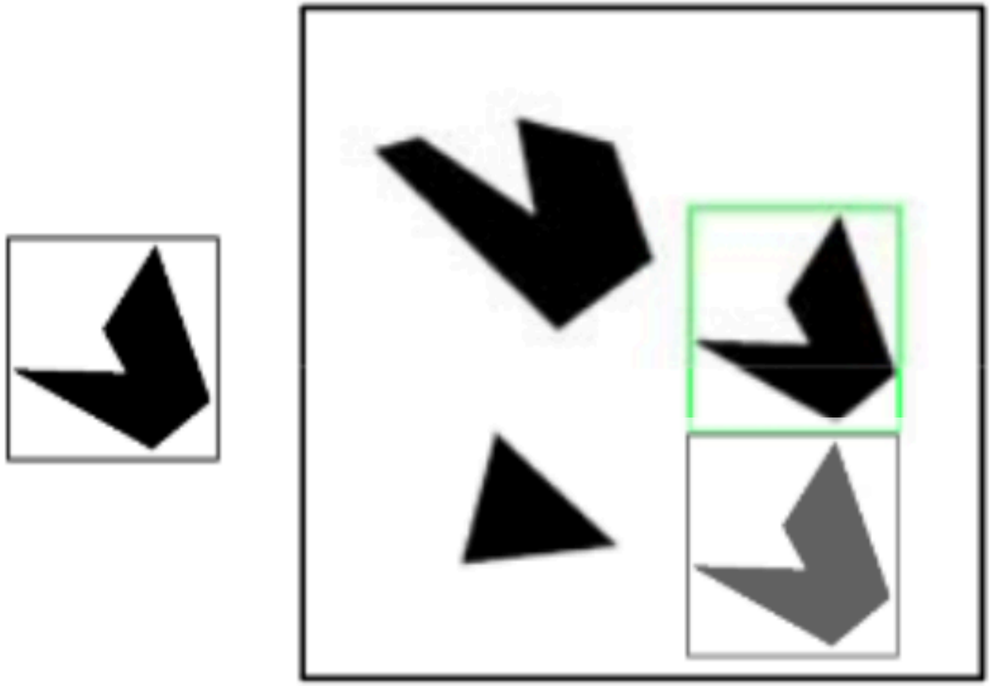

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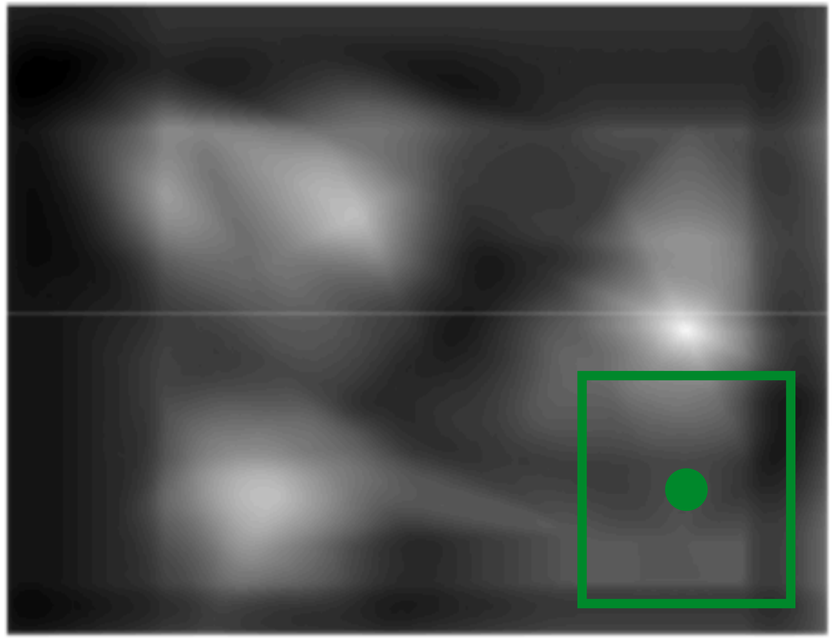

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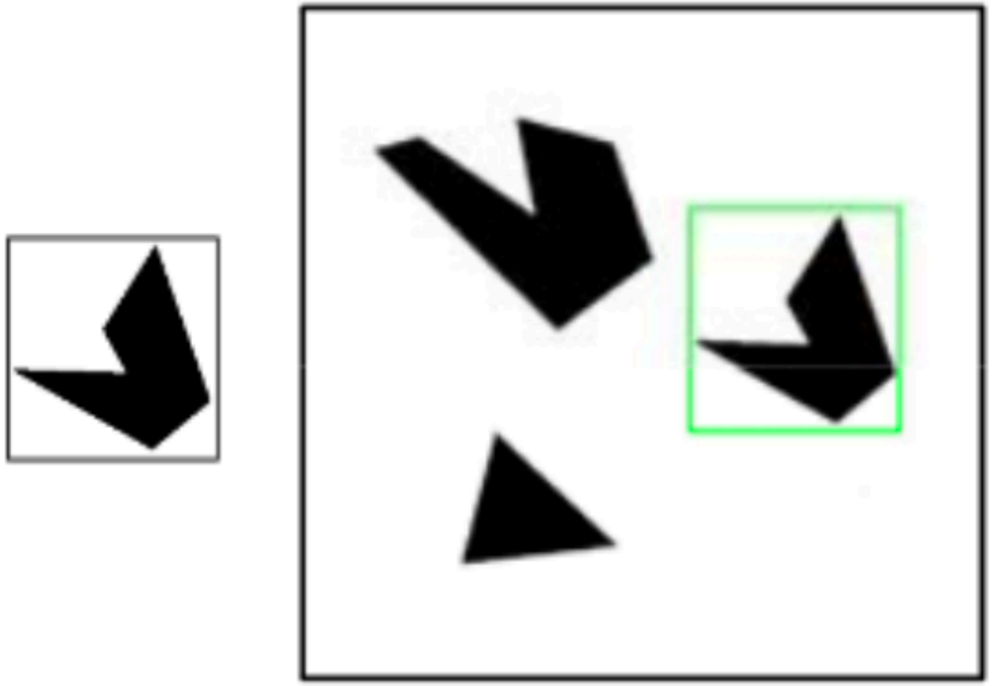
Template Matching

Detection can be done by comparing correlation map score to a threshold



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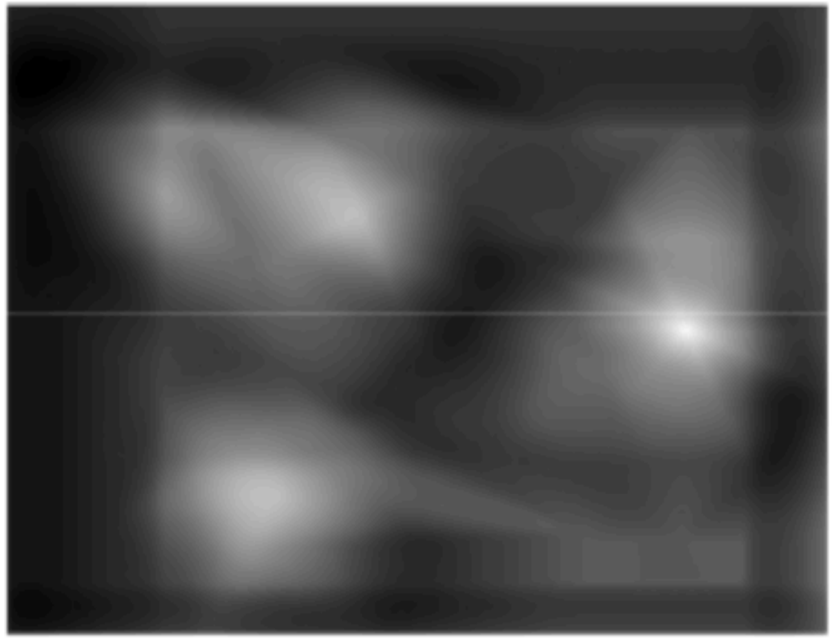

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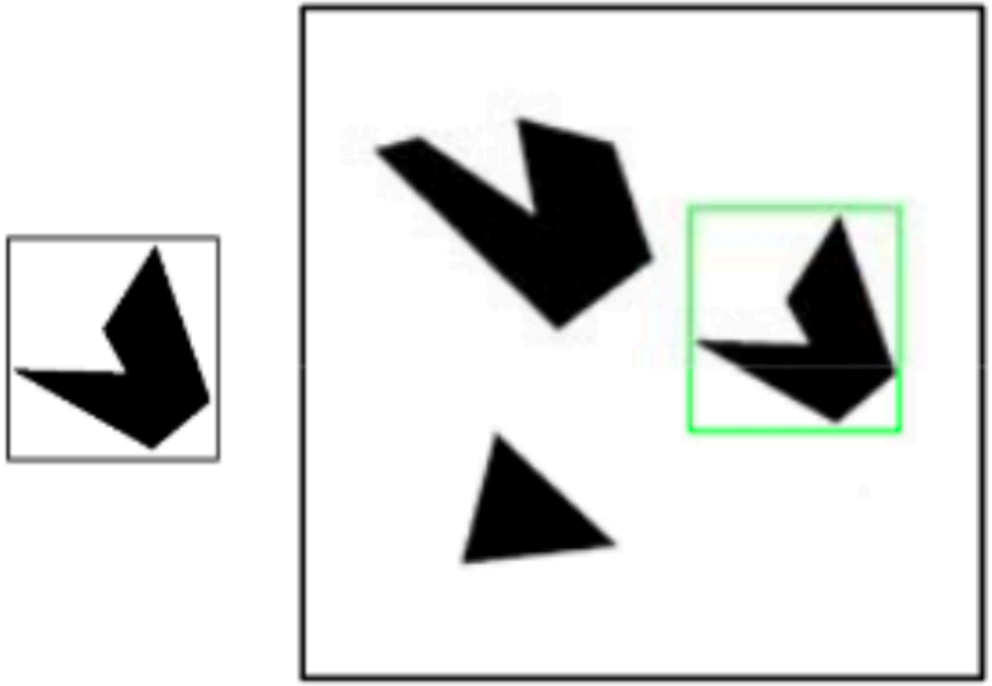

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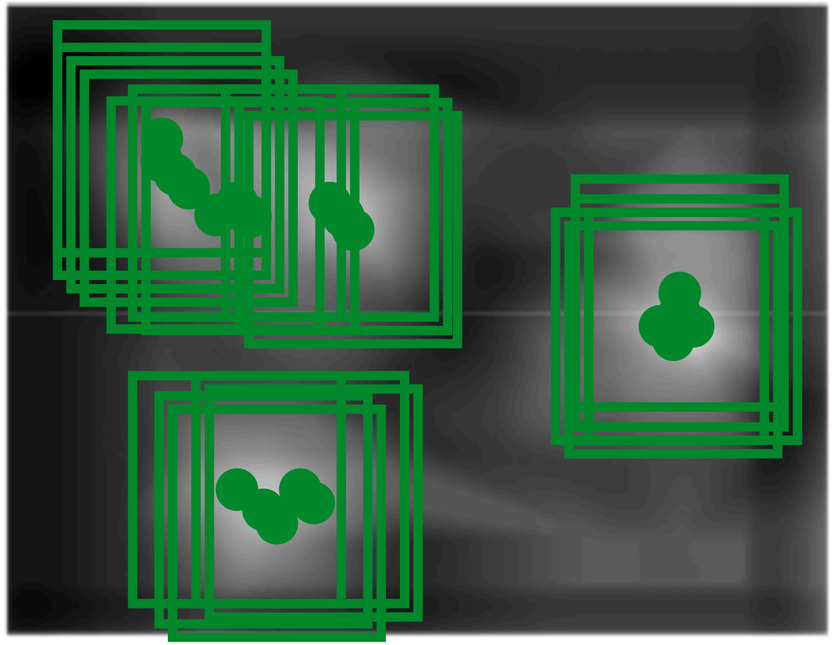

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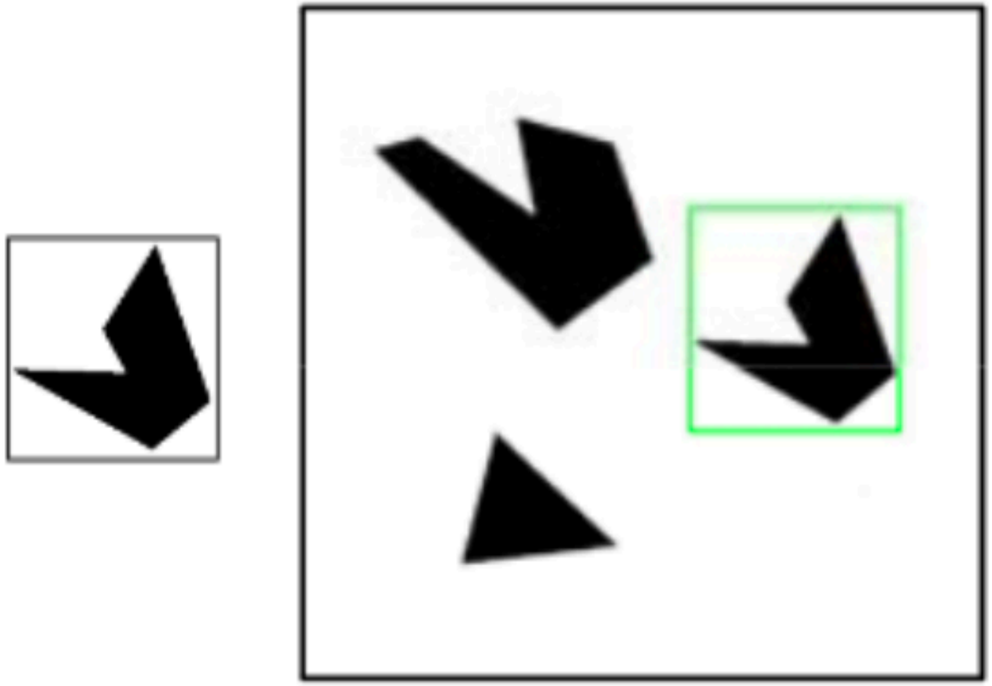

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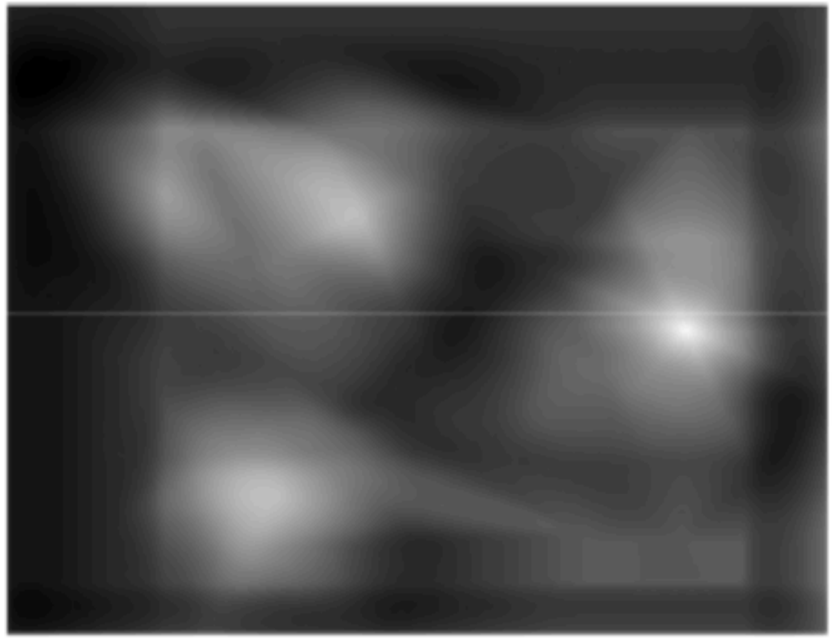

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What happens if the threshold is very high (e.g., 0.99)?

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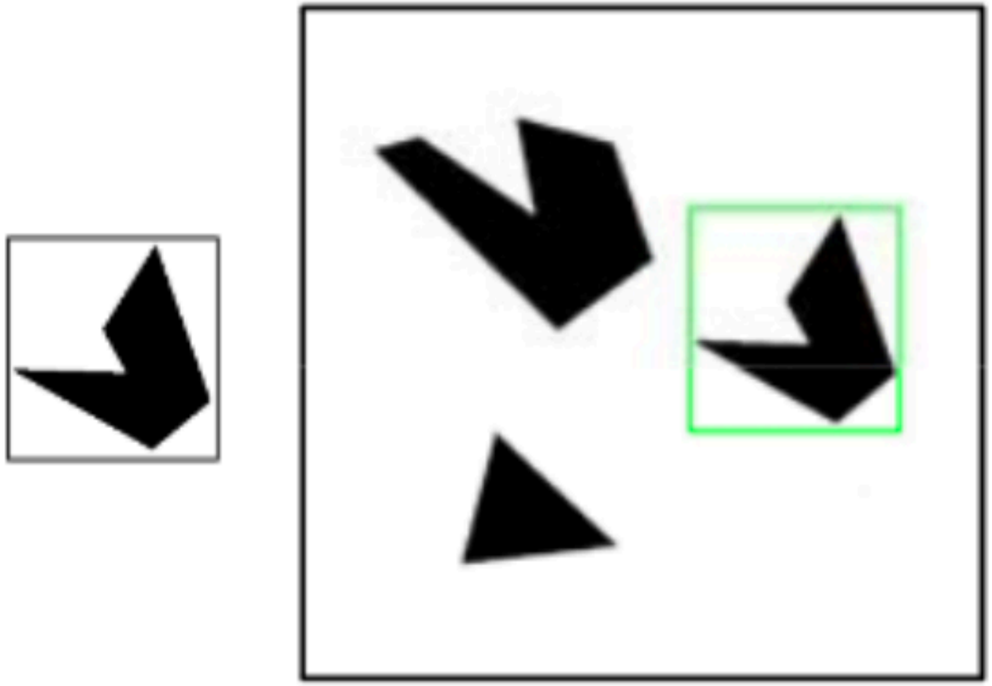

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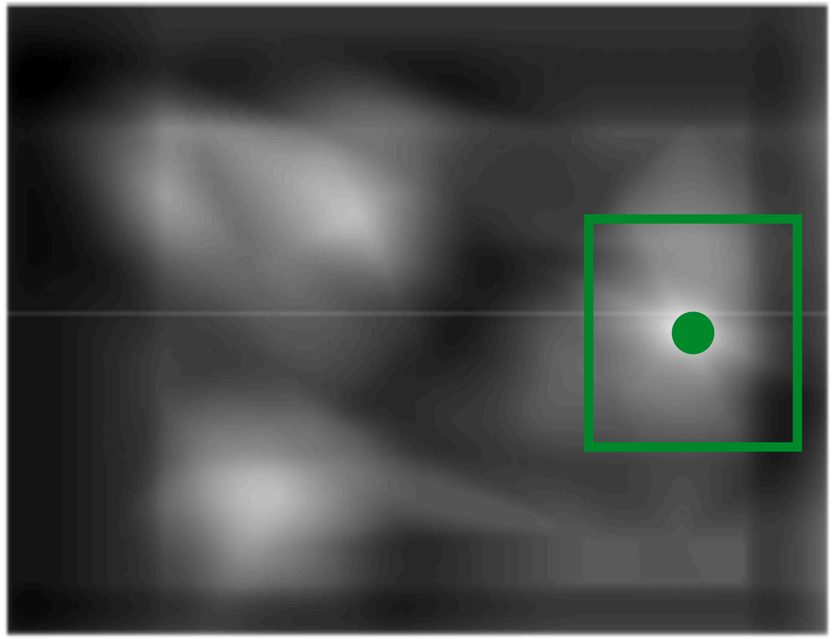

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Template Matching

Linear filtering the entire image computes the entire set of dot products, one for each possible alignment of filter and image

Important **Insight**:

— filters look like the pattern they are intended to find

— filters find patterns they look like

Linear filtering is sometimes referred to as **template matching**

Template Matching

Let a and b be vectors. Let θ be the angle between them. We know

$$\cos\theta = \frac{a \cdot b}{|a||b|} = \frac{a \cdot b}{\sqrt{(a \cdot a)(b \cdot b)}} = \frac{a}{|a|}\frac{b}{|b|}$$

where $\cdot$ is dot product and $|\,|$ is vector magnitude

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1. Normalize the template / filter (b) in the beginning
2. Compute norm of $|a|$ by convolving squared image with a filter of all 1's of equal size to the the template and squarooting the response
3. We can compute the dot product by convolution of image (a) with normalized filter (b)
4. We can finally compute the normalized correlation by dividing element-wise result in Step 3 by result ins Step 2

Example 1:

Credit: W. Freeman et al., “Computer Vision for Interactive Computer Graphics,” IEEE Computer Graphics and Applications, 1998

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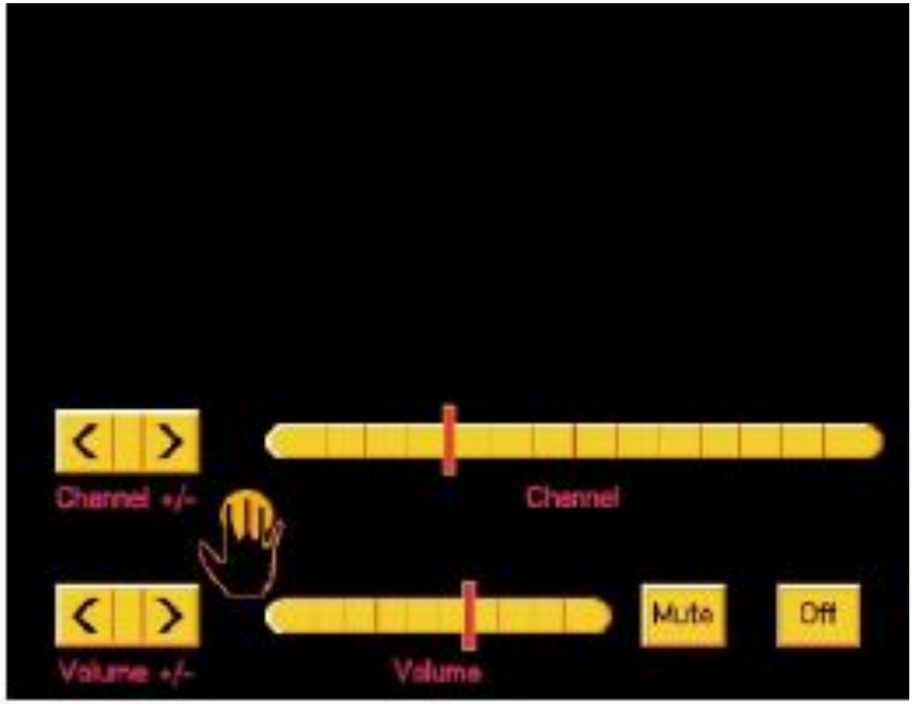

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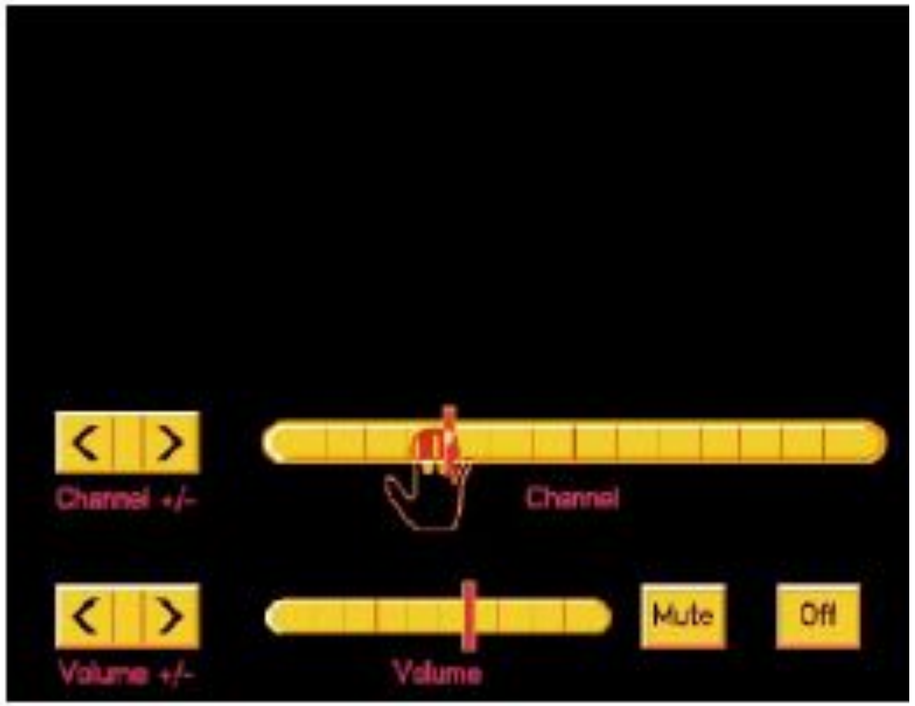

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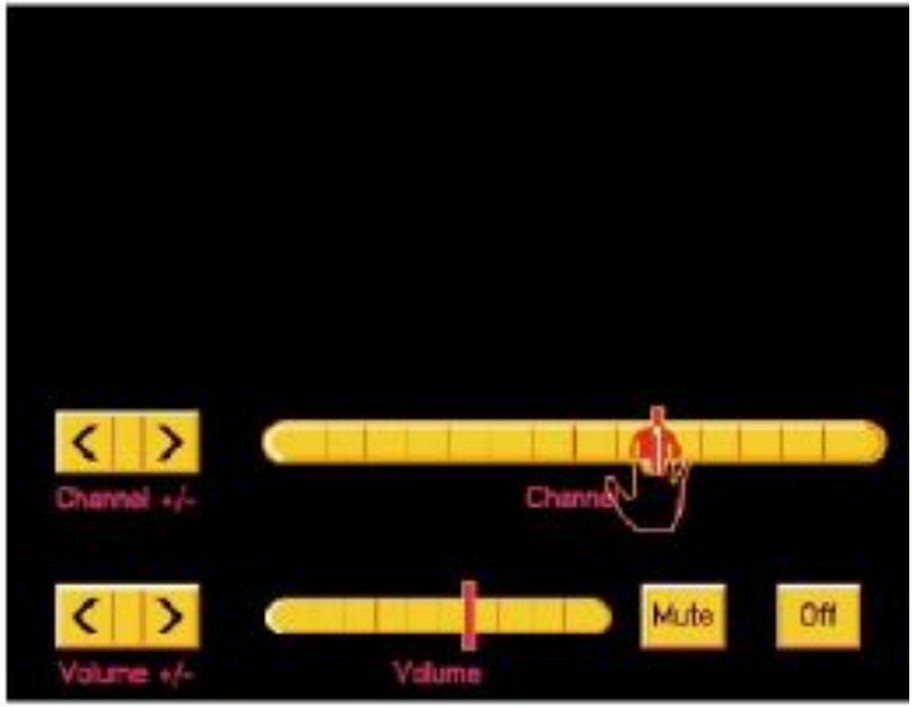

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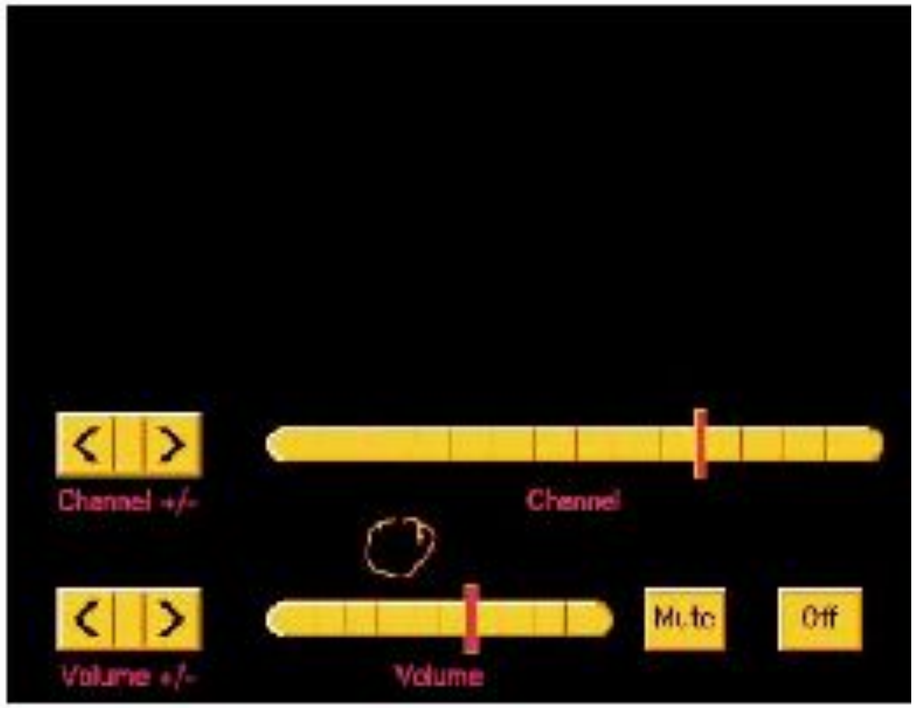

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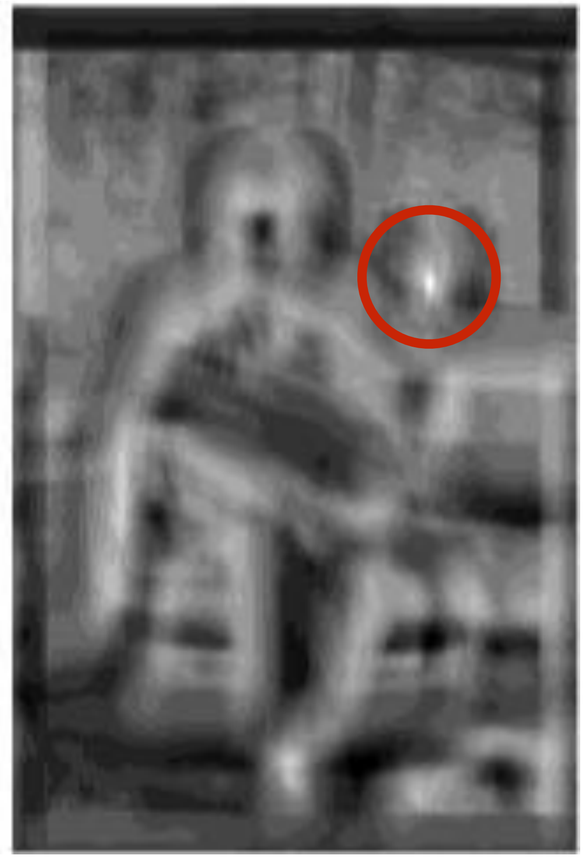
Example 1:

Template (left), image (middle), normalized correlation (right)

Note peak value at the true position of the hand



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Template Matching

When might **template matching fail**?

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— Different scales  

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— Different orientation 

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— Lighting conditions 

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— Left vs. Right hand 

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— Partial Occlusions 

Template Matching

When might **template matching fail**?

- Different scales 
- Different orientation 
- Lighting conditions 
- Left vs. Right hand 

- Partial Occlusions 
- Different Perspective
- Motion / blur

Template Matching Summary

Good News:

— works well in presence of noise

— relatively easy to compute

Bad News:

— sensitive to (spatial) scale change

— sensitive to 2D rotation

More Bad News:

When imaging 3D worlds:

— sensitive to viewing direction and pose

— sensitive to conditions of illumination

Scaled Representations

Problem: Make template matching robust to changes in 2D (spatial) scale.

Key Idea(s): Build a scaled representation: the Gaussian image pyramid

Alternatives:

— use multiple sizes for each given template

— ignore the issue of 2D (spatial) scale

Theory: Sampling theory allows us to build image pyramids in a principled way

“**Gotchas**:”

— template matching remains sensitive to 2D orientation, 3D pose and illumination

Scaled Representations

Why build a scaled representation of the **image** instead of scaled representation of the **template**?

Scaled Representations: Goals

to find **template matches** at all scales

— template size constant, image scale varies

— finding hands or faces when we don’t know what size they are in the image

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efficient search for image–to–image correspondences

— look first at coarse scales, refine at finer scales

— much less cost (but may miss best match)

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efficient search for image–to–image correspondences

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to examine all **levels of detail**

— find edges with different amounts of blur

— find textures with different spatial frequencies (i.e., different levels of detail)

Shrinking the Image

We can’t shrink an image simply by taking every second pixel

Shrinking the Image

We can’t shrink an image simply by taking every second pixel

Why?

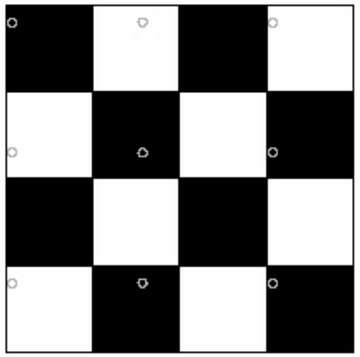
Shrinking the Image

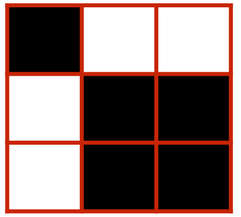
We can’t shrink an image simply by taking every second pixel

If we do, characteristic **artifacts** appear:

— small phenomena can look bigger

— fast phenomena can look slower





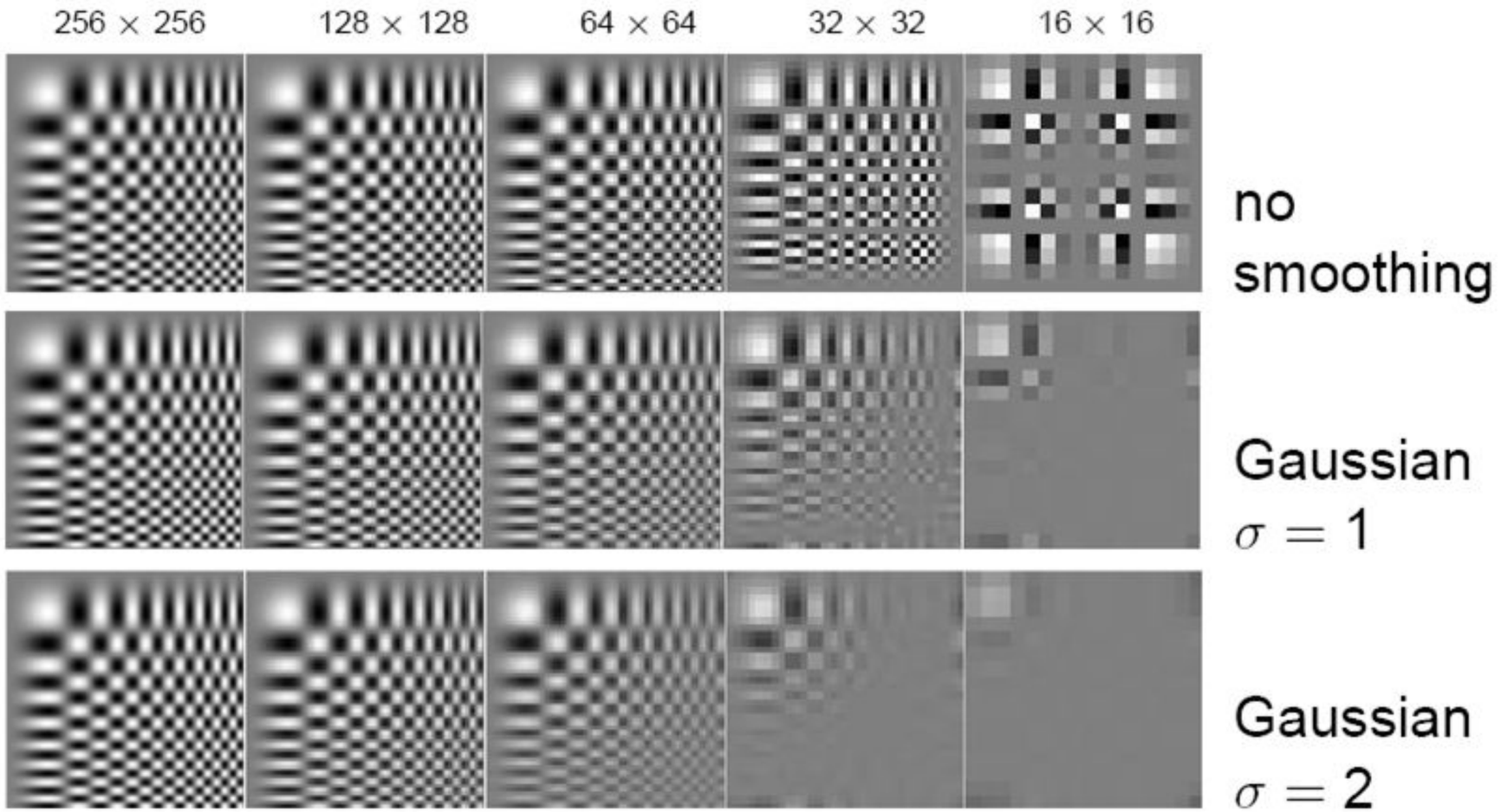
Common **examples** include:

— checkerboard patterns misrepresented in video games

— striped shirts look funny on colour television

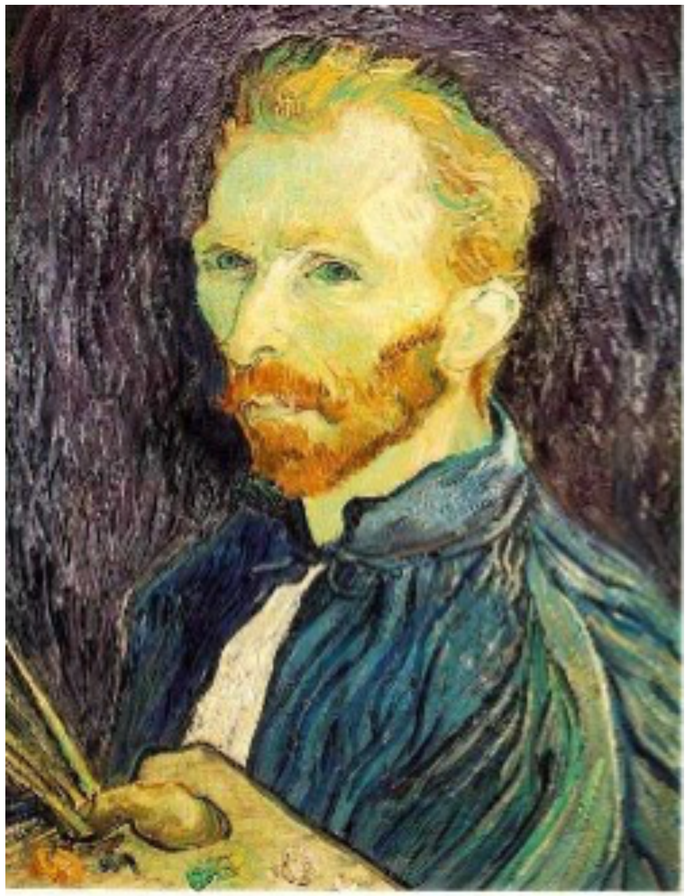
— wagon wheels roll the wrong way in movies

Shrinking the Image



Forsyth & Ponce (2nd ed.) Figure 4.12-4.14 (top rows)

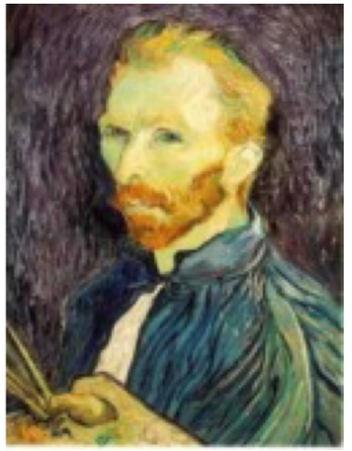
Template Matching: Sub-sample with Gaussian Pre-filtering



1/2

Apply a smoothing filter first, then throw away half the rows and columns

Gaussian filter delete even rows delete even columns



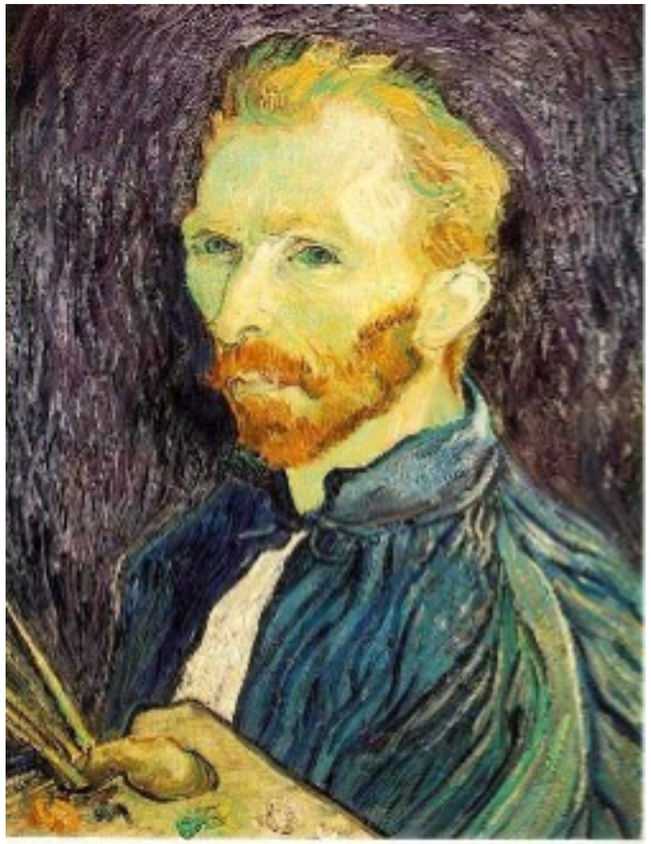
1/4

Gaussian filter delete even rows delete even columns

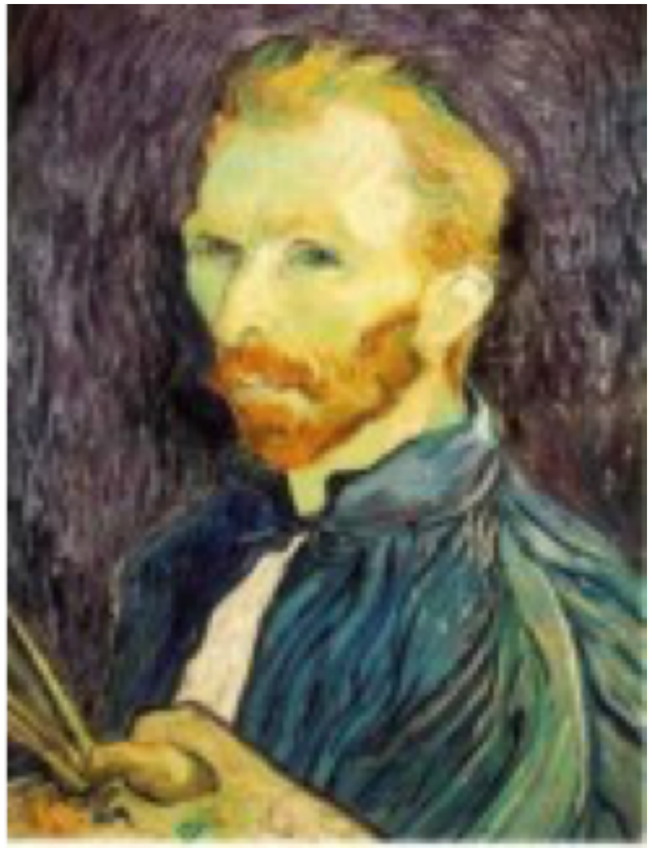
1/8

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Template Matching: Sub-sample with Gaussian Pre-filtering



1/2

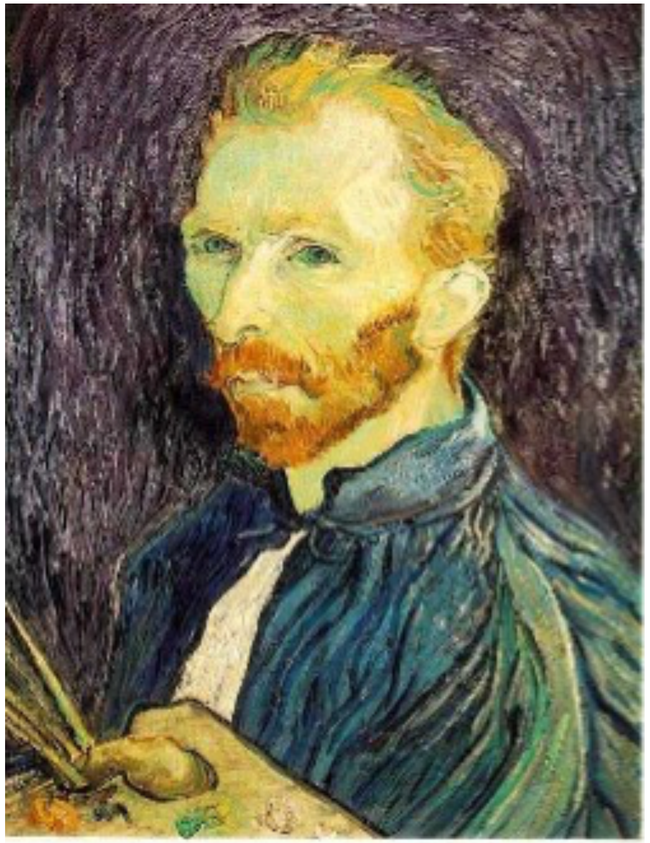


1/4 (2x zoom)

1/8 (4x zoom)

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Template Matching: Sub-sample with NO Pre-filtering



1/2

1/4 (2x zoom)

1/8 (4x zoom)

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Gaussian Pre-filtering

Question: How much smoothing is needed to avoid aliasing?

Gaussian Pre-filtering

Question: How much smoothing is needed to avoid aliasing?

Answer: Smoothing should be sufficient to ensure that the resulting image is band limited “enough” to ensure we can sample every other pixel.

Practically: For every image reduction of 0.5, smooth by $\sigma = 1$

Image **Pyramid**

An **image pyramid** is a collection of representations of an image. Typically, each layer of the pyramid is half the width and half the height

of the previous layer.

In a **Gaussian pyramid**, each layer is smoothed by a Gaussian filter and resampled to get the next layer

Gaussian Pyramid

Again, let $\otimes$ denote convolution

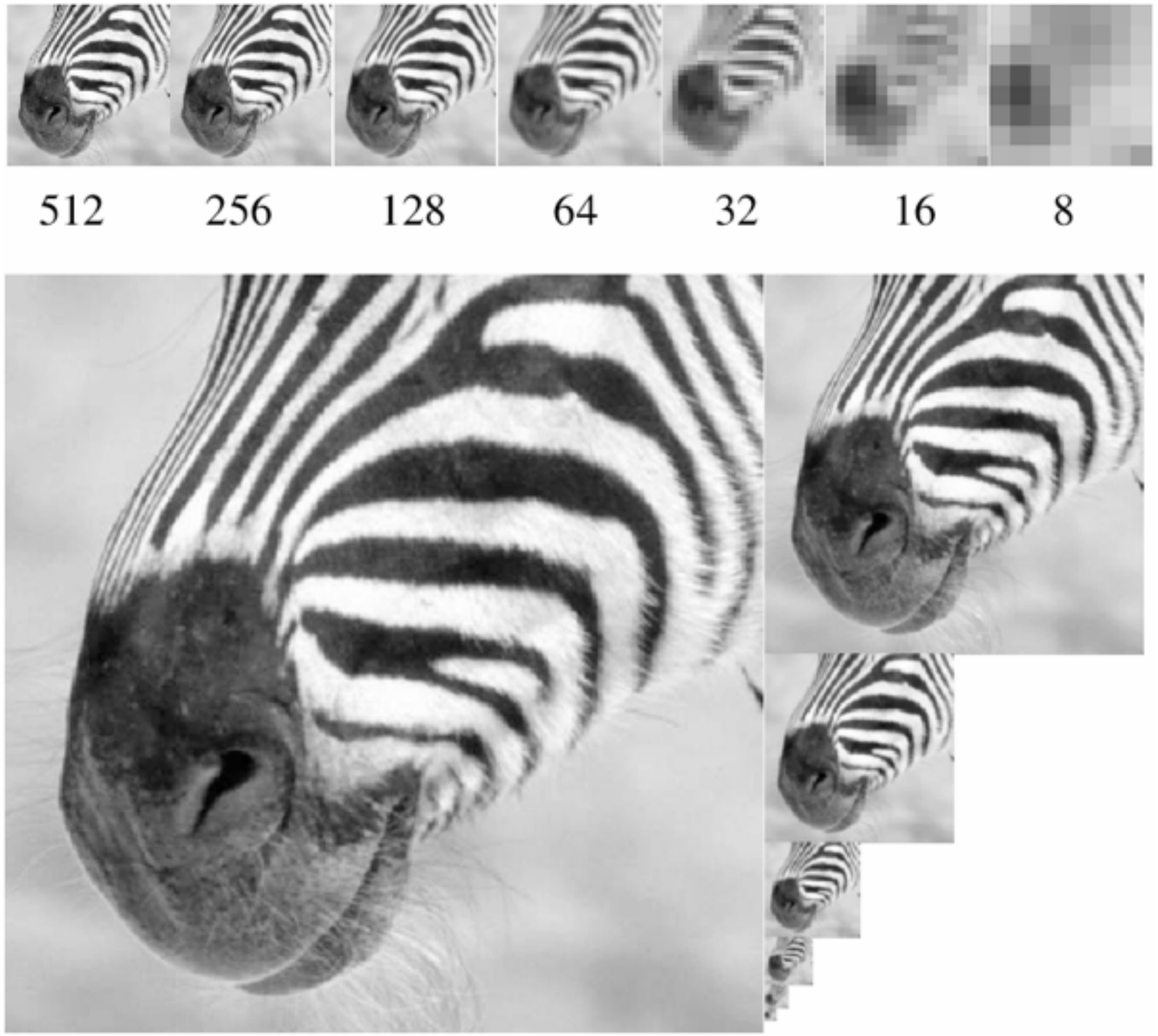
Create each level from previous one

— smooth and (re)sample

Smooth with Gaussian, taking advantage of the fact that

$$G_{\sigma_1}(x) \otimes G_{\sigma_2}(x) = G_{\sqrt{\sigma_1^2+\sigma_2^2}}(x)$$

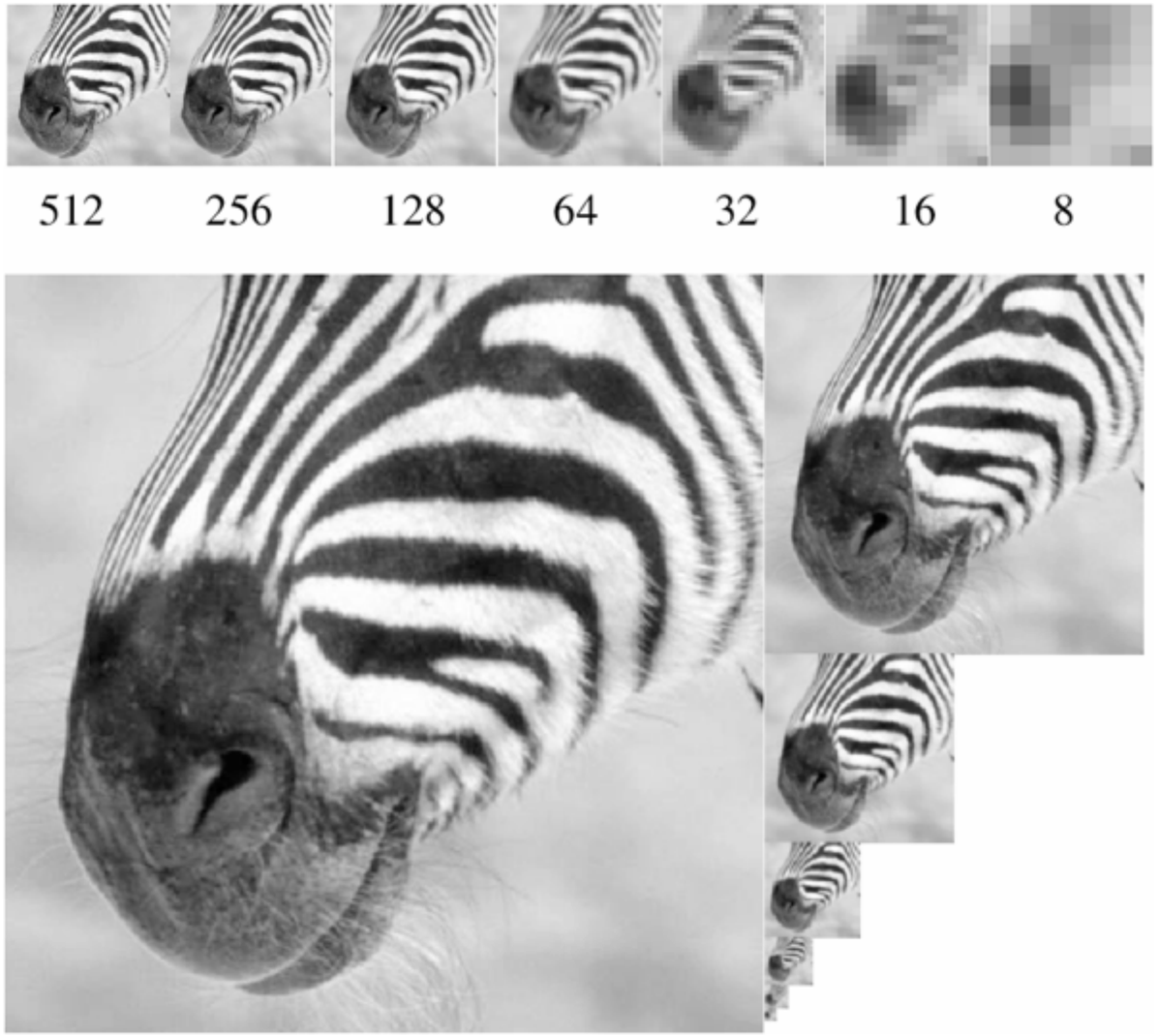
Example 2: Gaussian Pyramid



Forsyth & Ponce (2nd ed.) Figure 4.17

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Example 2: Gaussian Pyramid



What happens to the details?

Forsyth & Ponce (2nd ed.) Figure 4.17

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

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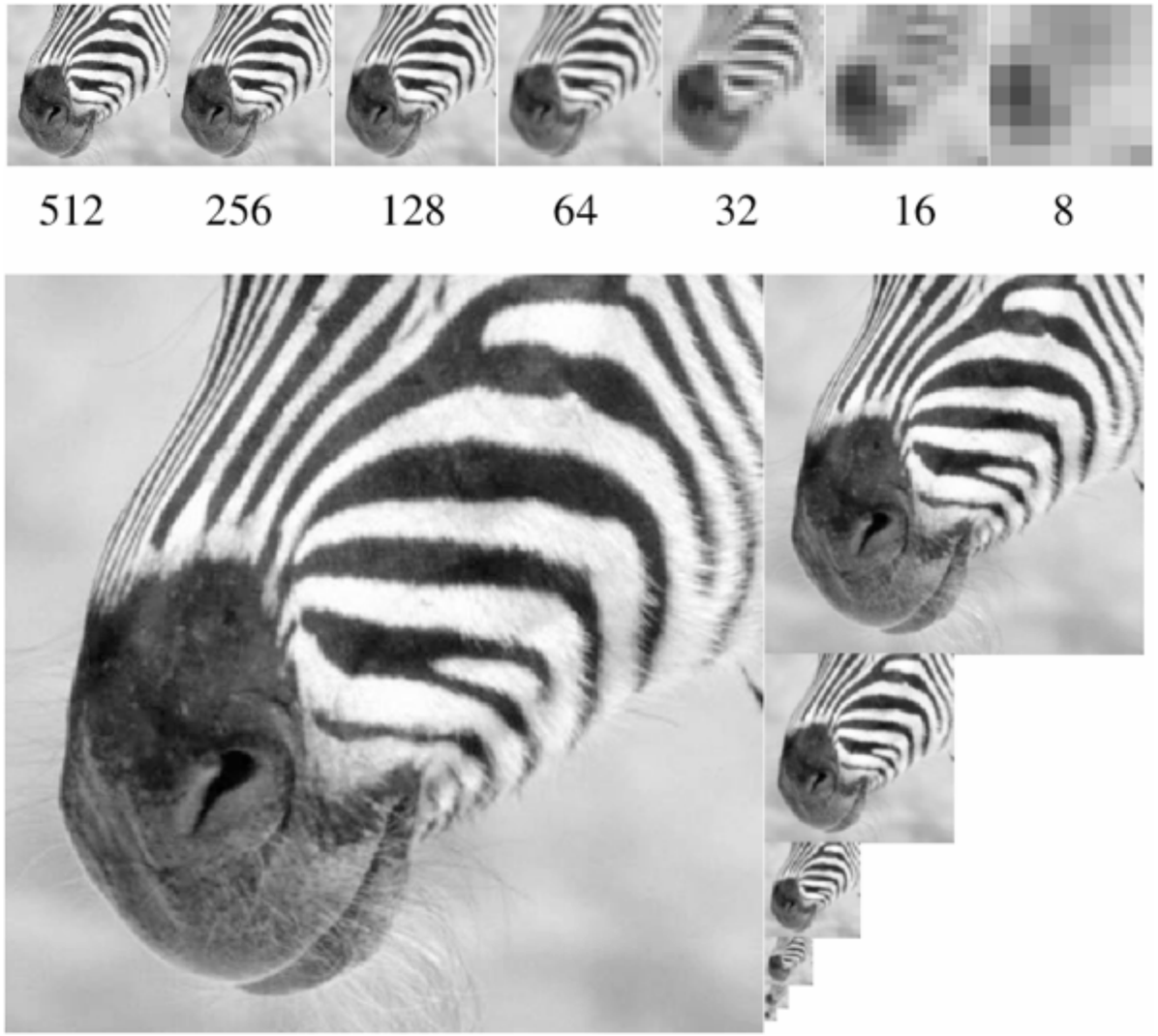


Forsyth & Ponce (2nd ed.) Figure 4.17

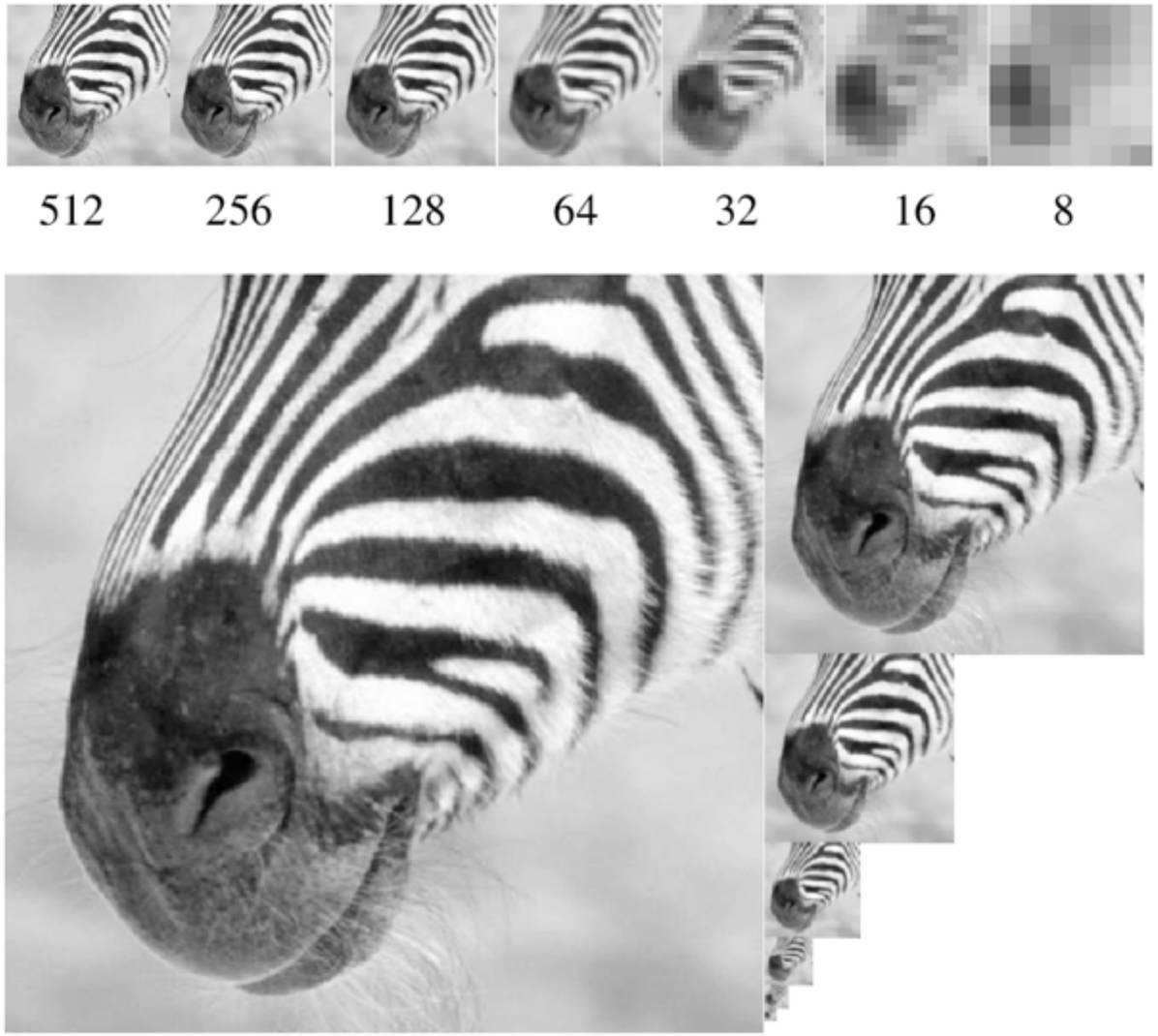
What happens to the details?

— They get smoothed out as we move to higher levels

What is preserved at the higher levels?

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Example 2: Gaussian Pyramid



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What happens to the details?

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What is preserved at the higher levels?

— Mostly large uniform regions in the original image

How would you reconstruct the original image from the image at the upper level?

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

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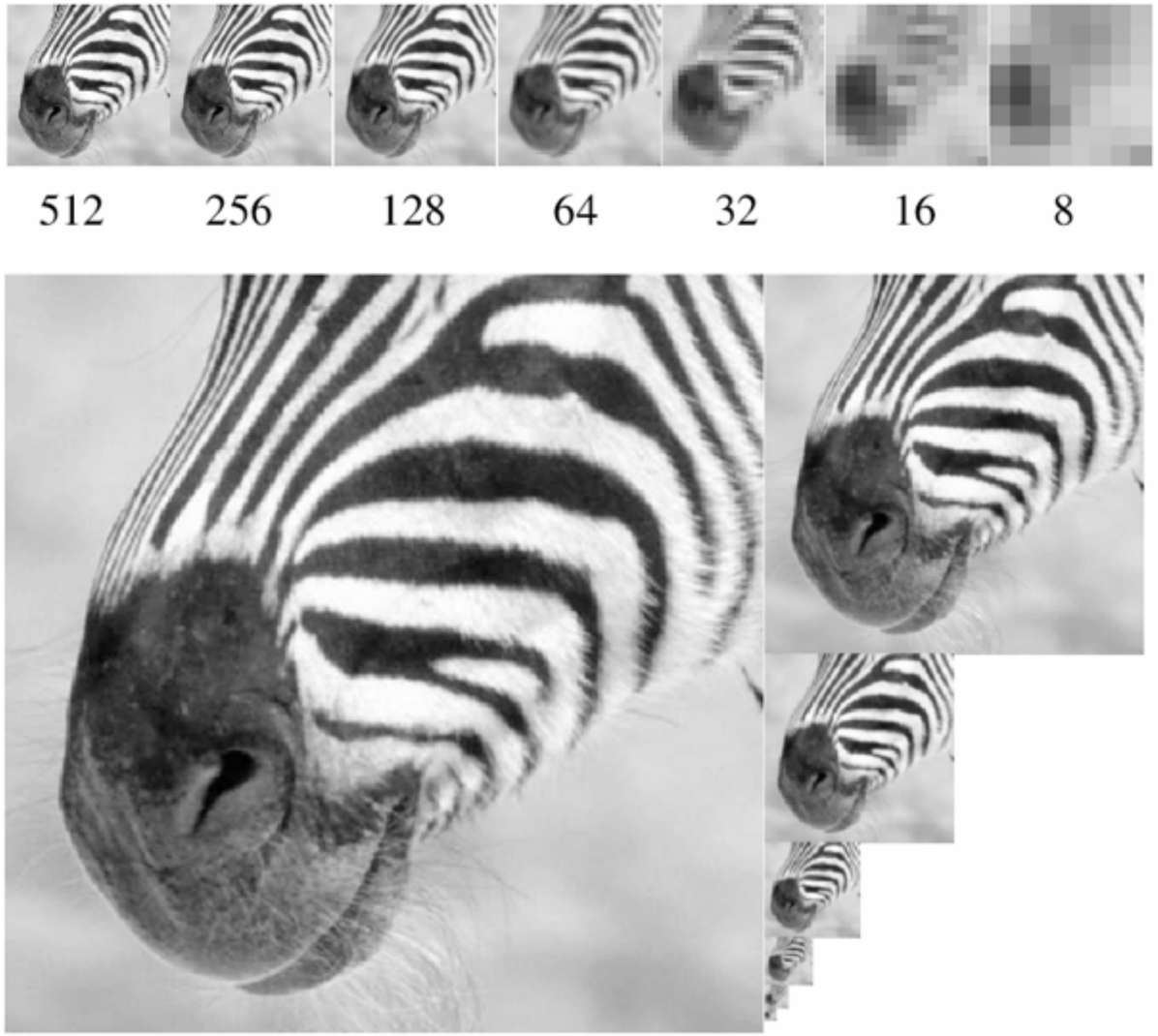

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What happens to the details?

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How would you reconstruct the original image from the image at the upper level?

— That's not possible

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

From Template Matching to **Local Feature Detection**

We'll now shift from global template matching to **local feature detection**

Consider the problem of finding images of an elephant using a template

From Template Matching to **Local Feature Detection**

We'll now shift from global template matching to **local feature detection**

Consider the problem of finding images of an elephant using a template

An elephant looks different from different viewpoints

— from above (as in an aerial photograph or satellite image)

— head on

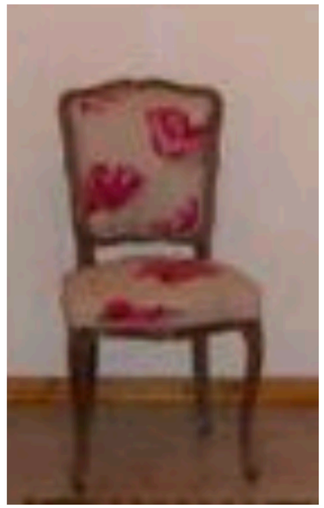
— sideways (i.e., in profile)

— rear on

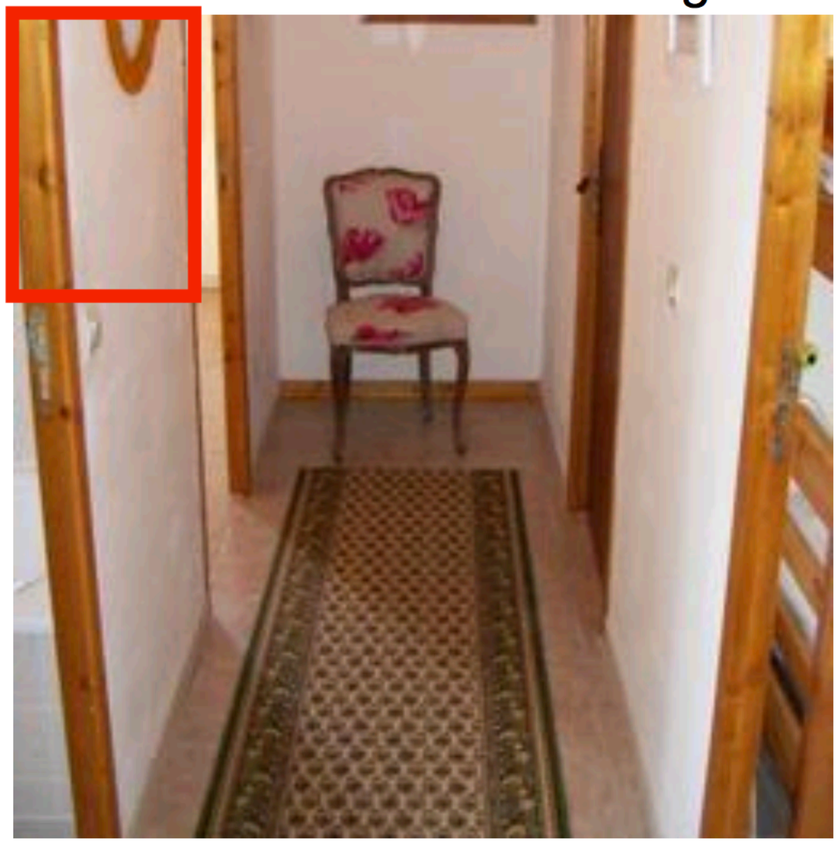
What happens if parts of an elephant are obscured from view by trees, rocks, other elephants?

From Template Matching to **Local Feature Detection**

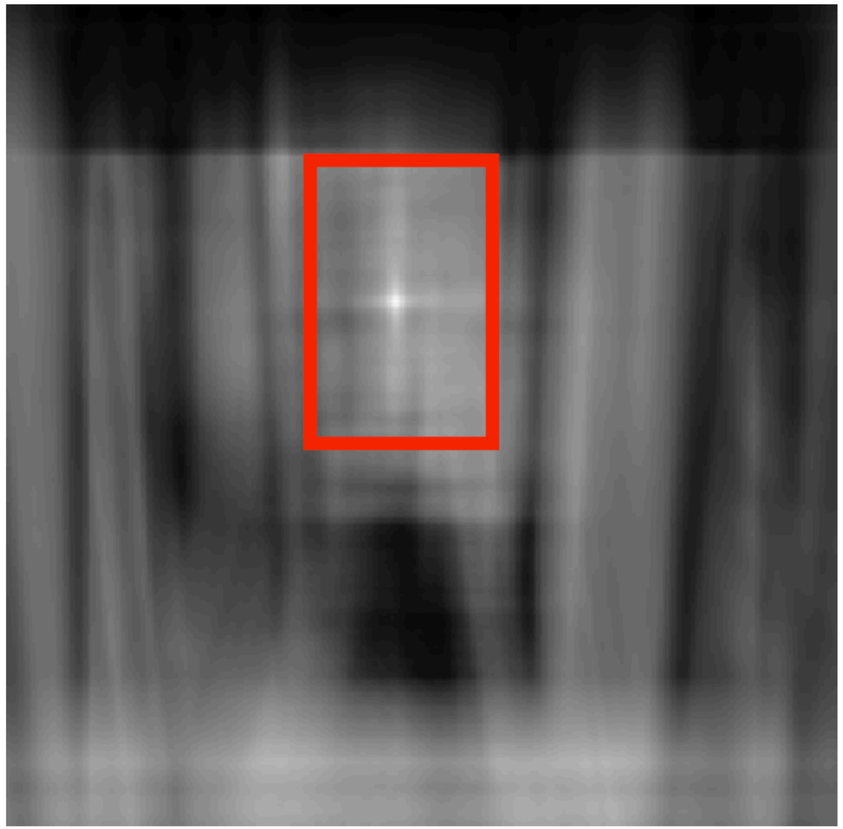
This is a chair



Find the chair in this image

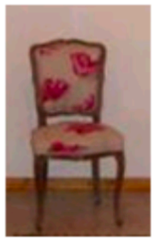


Output of normalized correlation

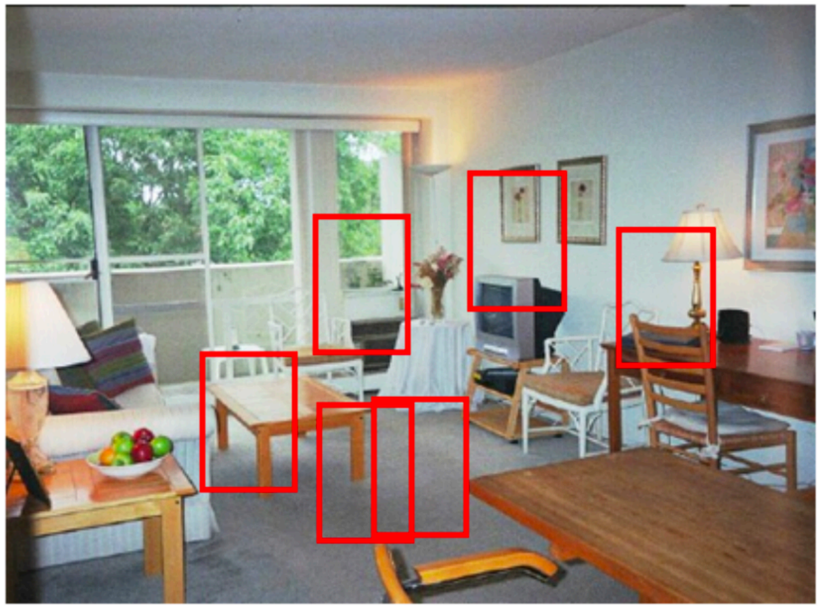


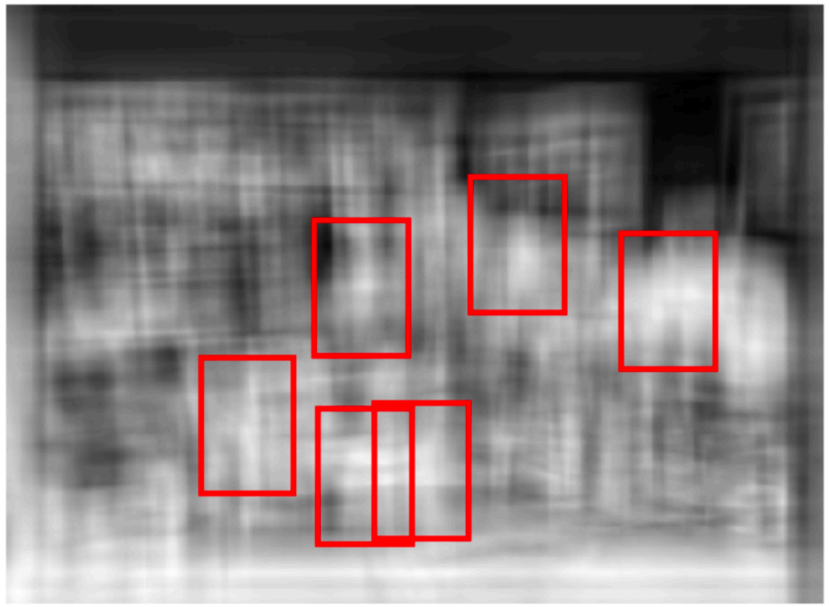
Slide Credit: Li Fei-Fei, Rob Fergus, and Antonio Torralba

From Template Matching to **Local Feature Detection**



Find the chair in this image





Pretty much garbage
Simple template matching is not going to make it

Slide Credit: Li Fei-Fei, Rob Fergus, and Antonio Torralba

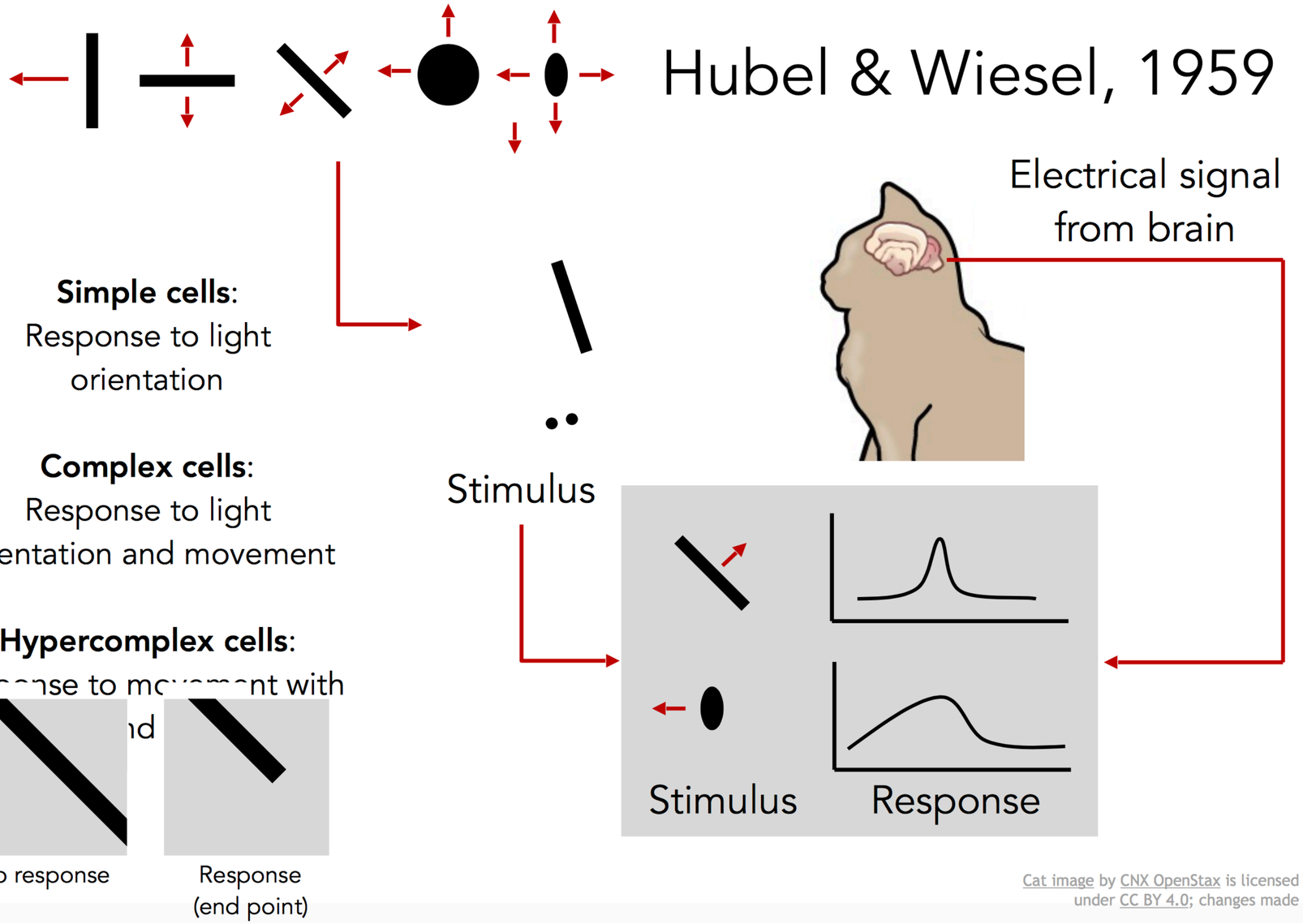
From Template Matching to **Local Feature Detection**

— Move from global template matching to **local template matching**

— Local template matching also called local **feature detection**

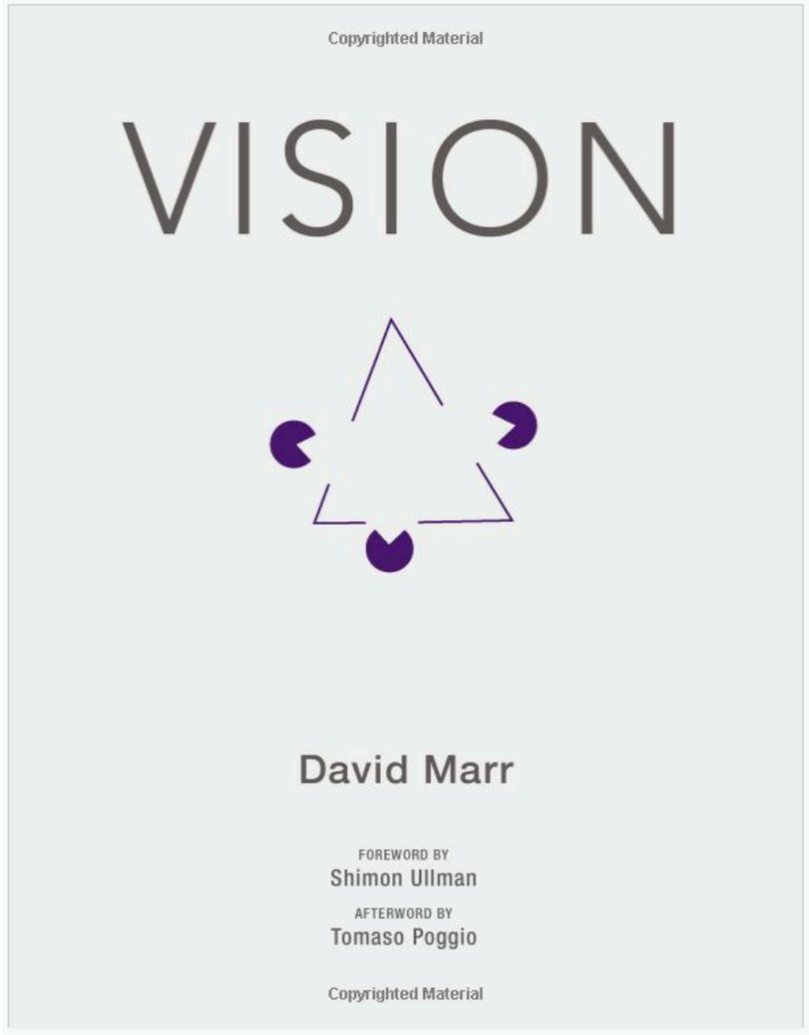
— Obvious local features to detect are **edges** and **corners**

Human vision …



* slide from Fei-Dei Li, Justin Johnson, Serena Yeung, **cs231n Stanford**

David **Marr**, 1970s



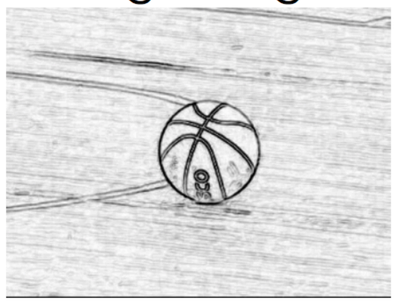
* slide from Fei-Dei Li, Justin Johnson, Serena Yeung, **cs231n Stanford**

David **Marr**, 1970s

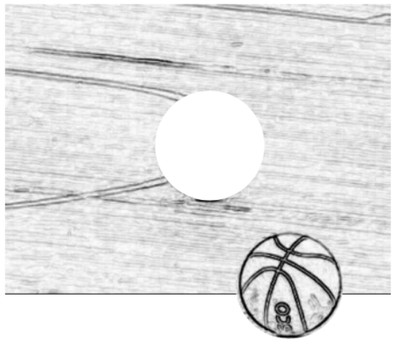
Input image

This image is CC0 1.0 public domain

Edge image

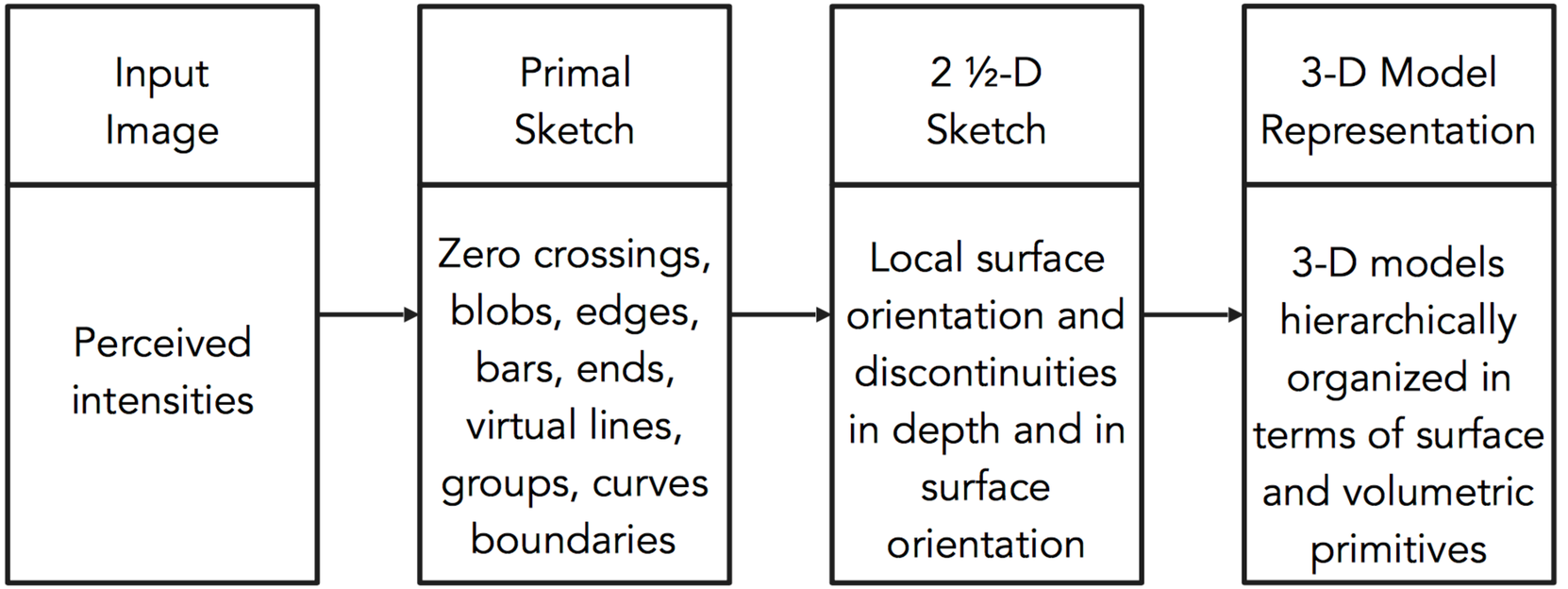


2 ½-D sketch



3-D model

This image is CC0 1.0 public domain



[Stages of Visual Representation, **David Marr**]

* slide from Fei-Dei Li, Justin Johnson, Serena Yeung, **cs231n Stanford**

From Template Matching to **Local Feature Detection**

- Move from global template matching to **local template matching**
- Local template matching also called local **feature detection**
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Estimating **Derivatives**

Recall, for a 2D (continuous) function, f(x,y)

$$\frac{\partial f}{\partial x} = \lim_{\epsilon \to 0} \frac{f(x+\epsilon, y) - f(x, y)}{\epsilon}$$

Differentiation is linear and shift invariant, and therefore can be implemented as a convolution

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$$\frac{\partial f}{\partial x} \approx \frac{F(X+1, y) - F(x,y)}{\Delta x}$$

-1	1

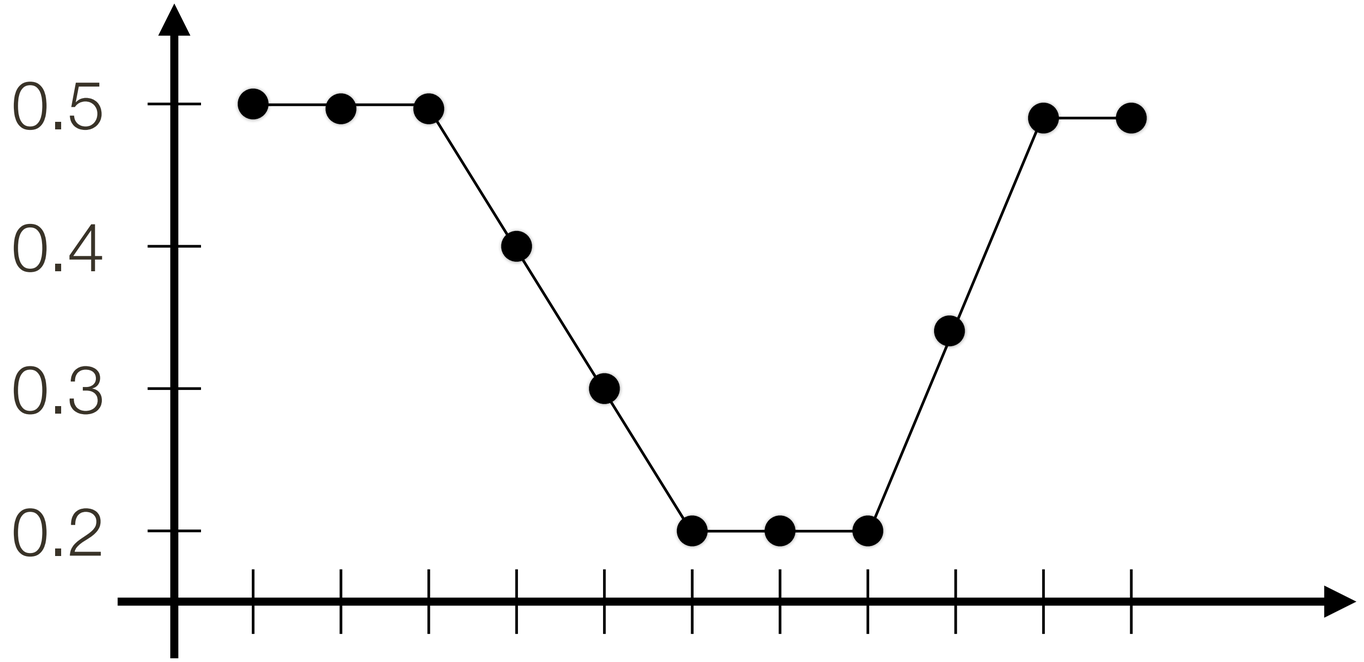
Estimating **Derivatives**

A similar definition (and approximation) holds for $\frac{\partial f}{\partial y}$

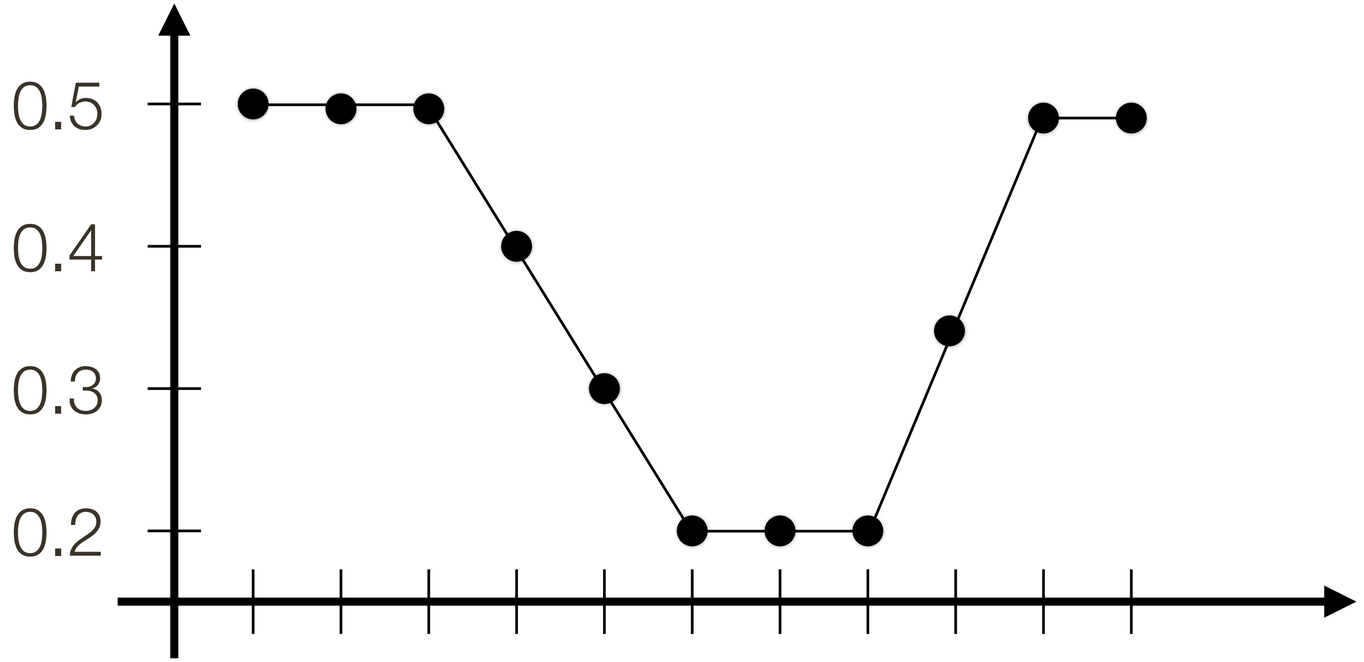
Image **noise** tends to result in pixels not looking exactly like their neighbours, so simple “finite differences” are sensitive to noise.

The usual way to deal with this problem is to **smooth** the image prior to derivative estimation.

Example 1D

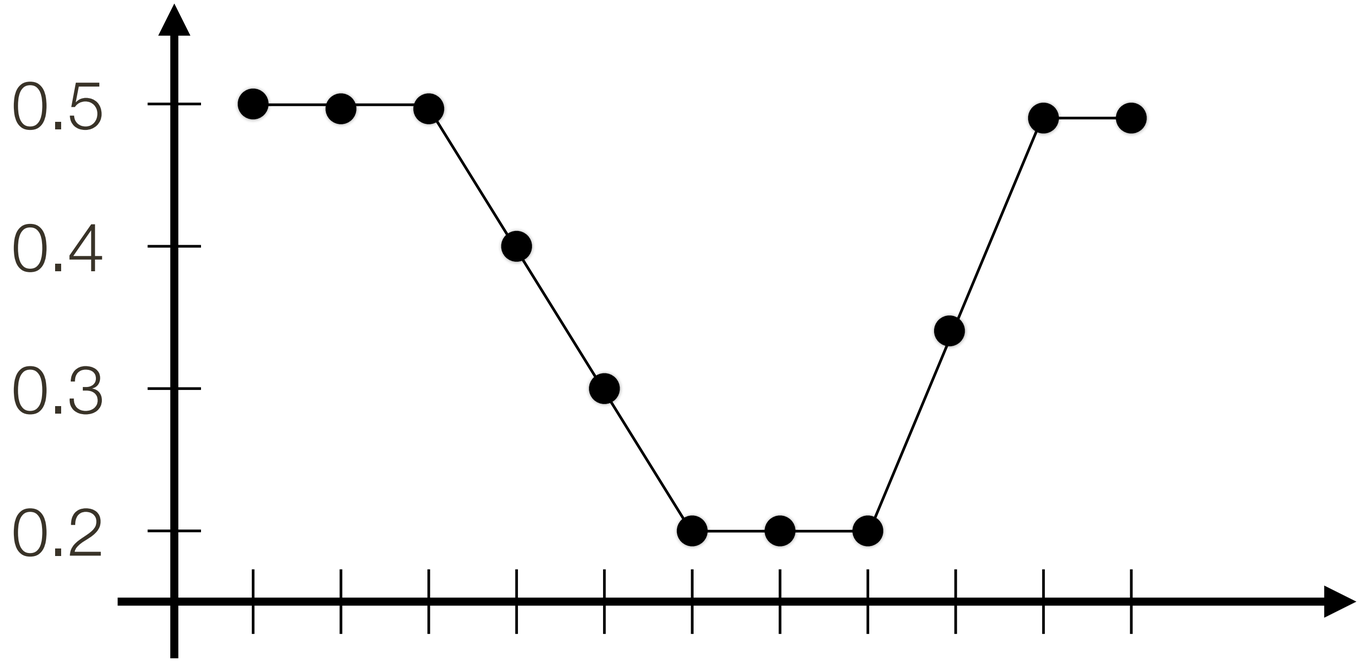


Example 1D



Signal 0.5 0.5 0.5 0.4 0.3 0.2 0.2 0.2 0.35 0.5 0.5

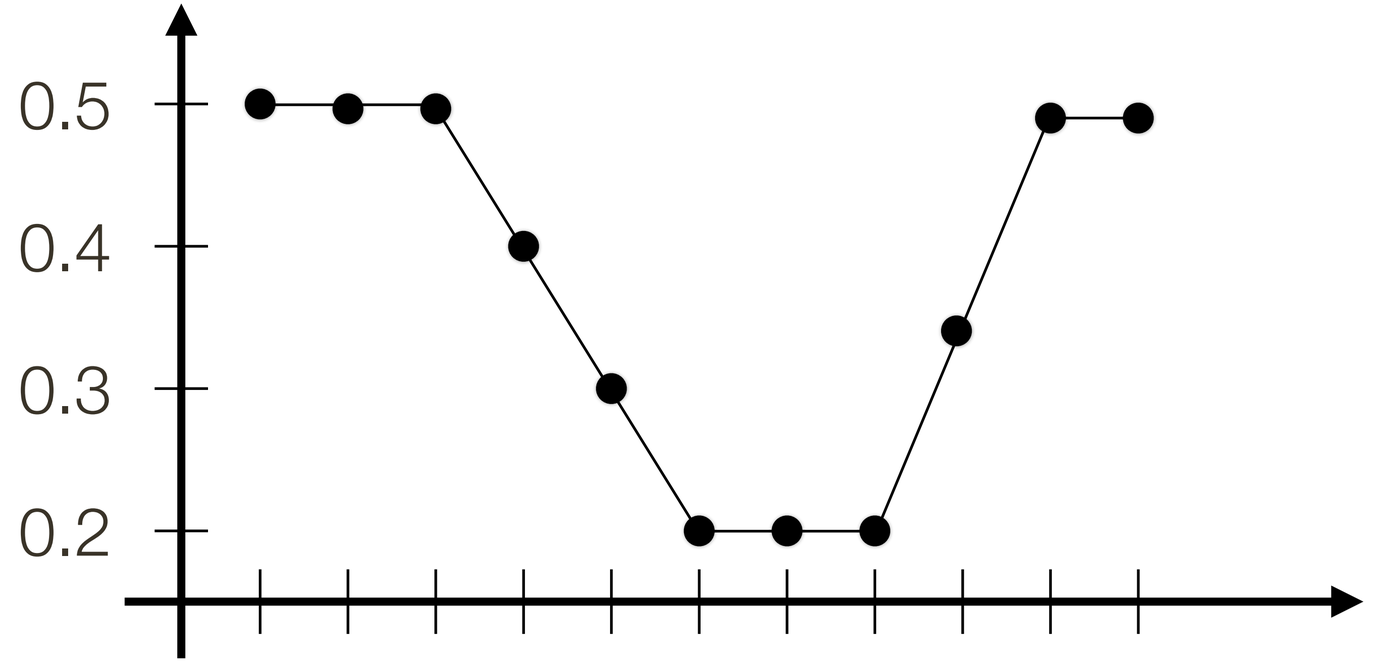
Example 1D



Signal	0.5	0.5	0.5	0.4	0.3	0.2	0.2	0.2	0.35	0.5	0.5

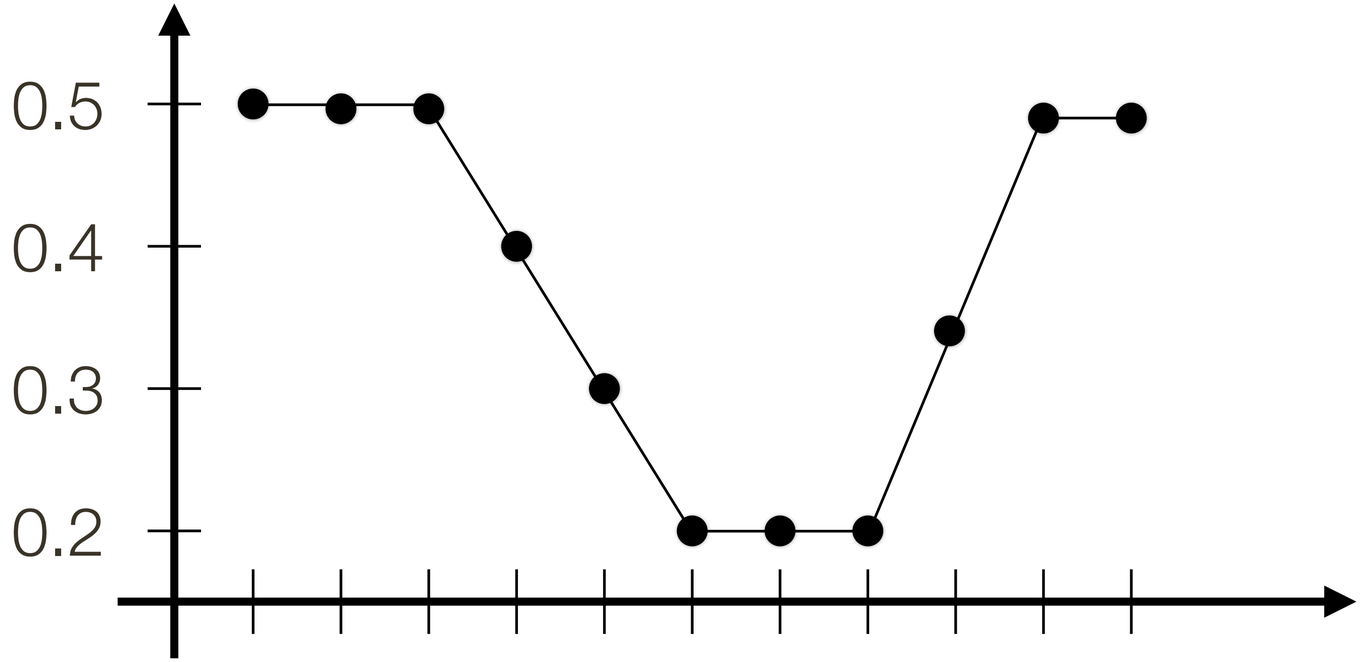
Derivative

Example 1D



Signal	0.5	0.5	0.5	0.4	0.3	0.2	0.2	0.2	0.35	0.5	0.5
Derivative	0.0										

Example 1D



Signal	0.5	0.5	0.5	0.4	0.3	0.2	0.2	0.2	0.35	0.5	0.5
Derivative	0.0										

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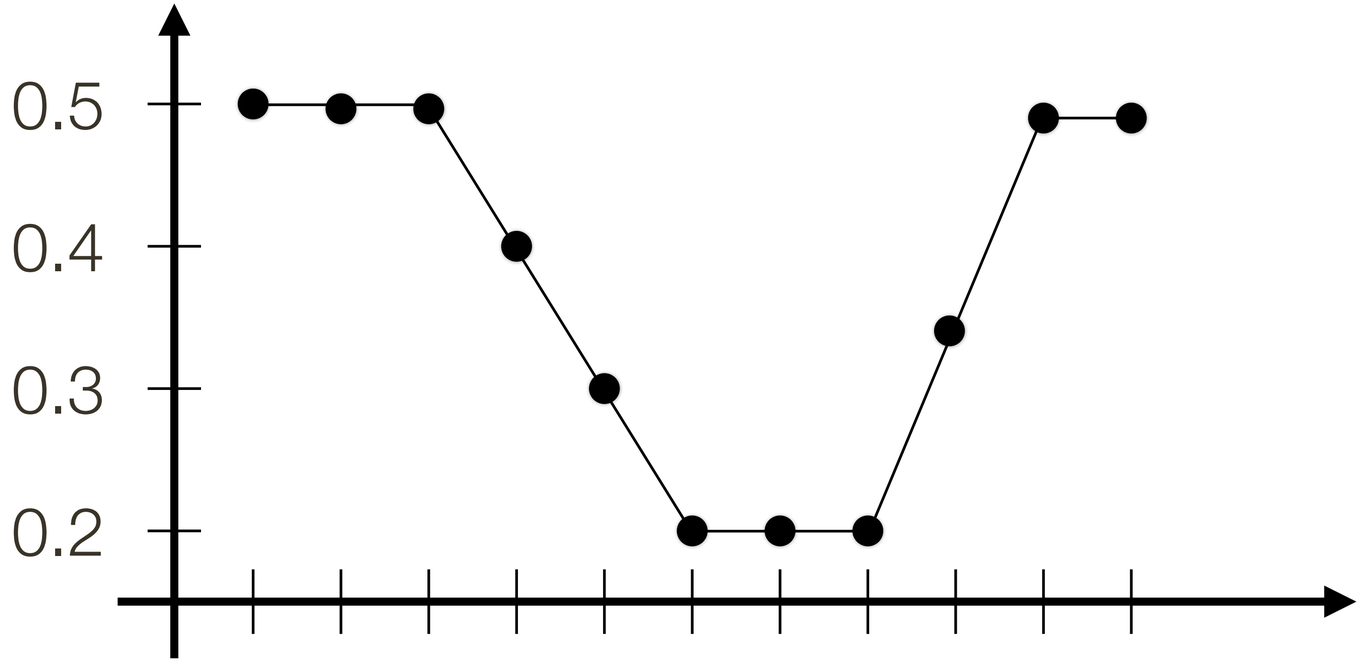

Signal	0.5	0.5	0.5	0.4	0.3	0.2	0.2	0.2	0.35	0.5	0.5
Derivative	0.0	0.0									

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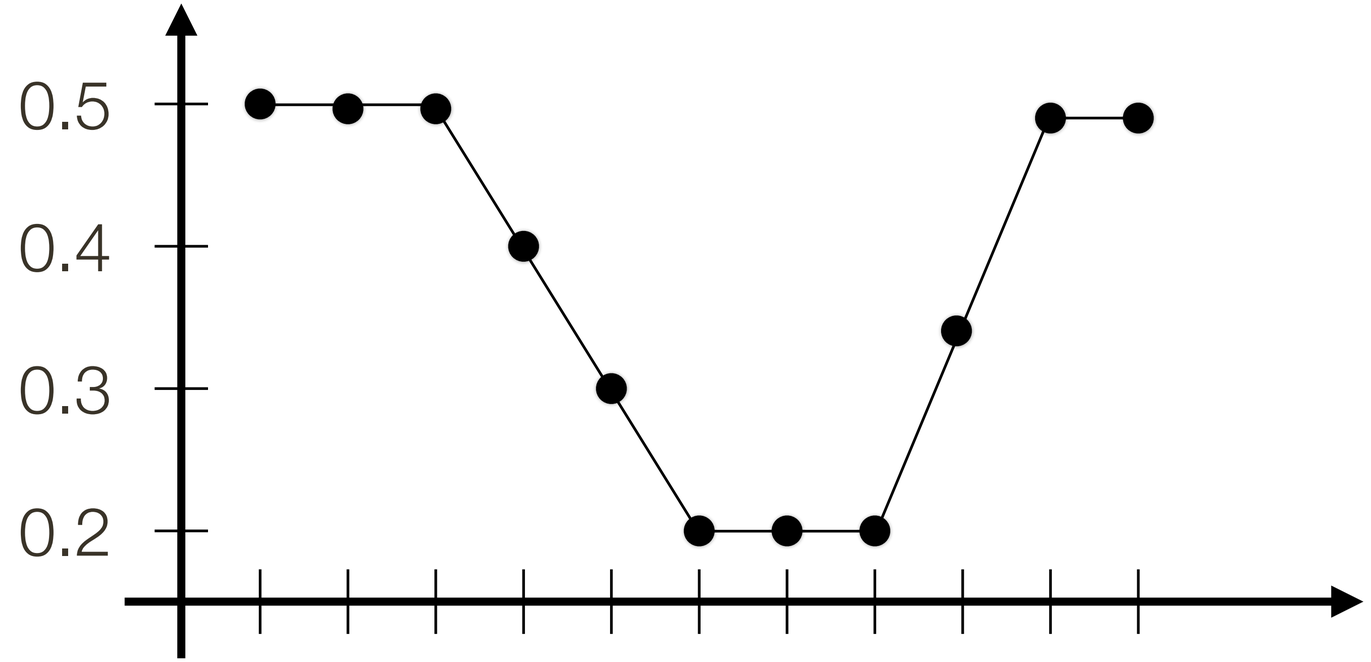
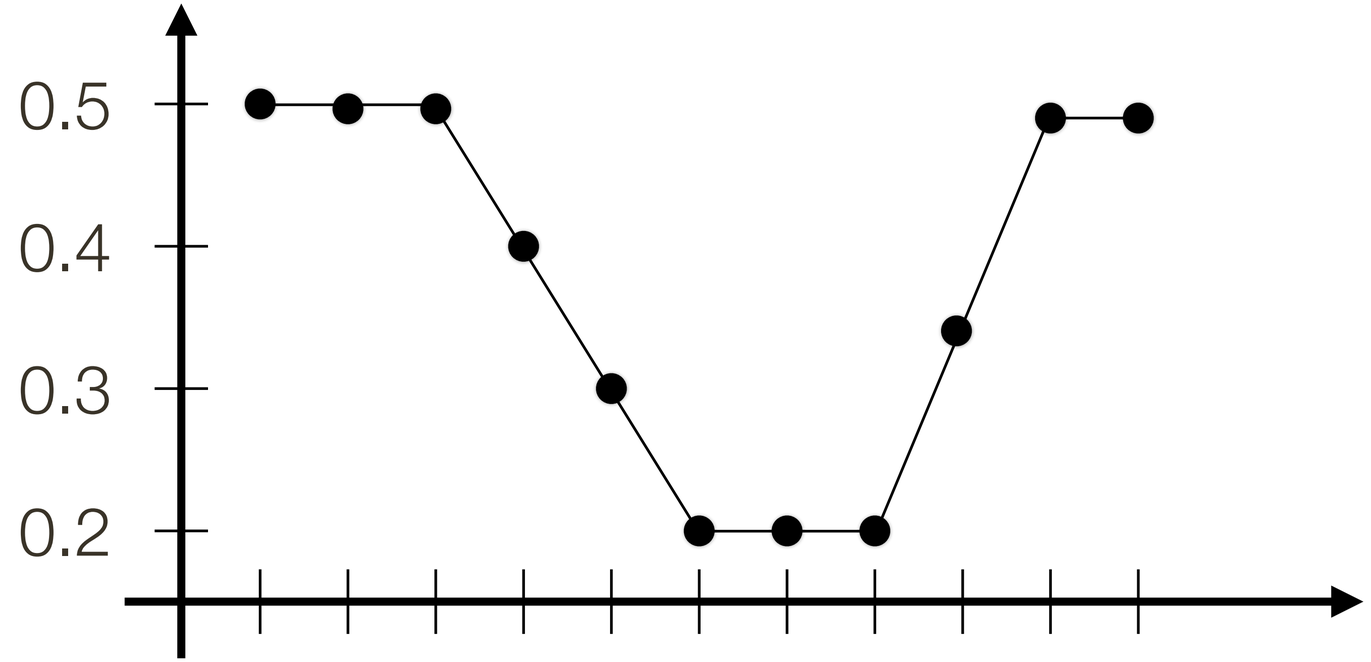


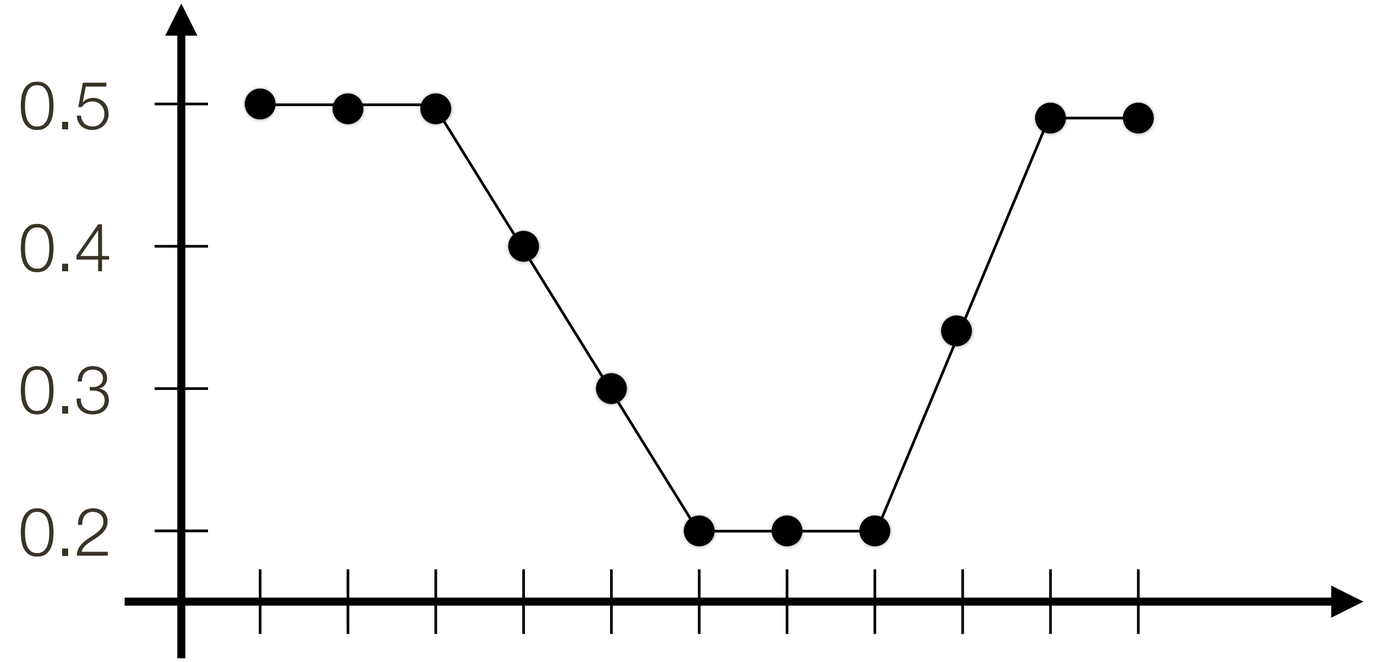
Signal	0.5	0.5	0.5	0.4	0.3	0.2	0.2	0.2	0.35	0.5	0.5
Derivative	0.0	0.0									

Example 1D



Signal	0.5	0.5	0.5	0.4	0.3	0.2	0.2	0.2	0.35	0.5	0.5
Derivative	0.0	0.0	-0.1								

Example 1D



Signal	0.5	0.5	0.5	0.4	0.3	0.2	0.2	0.2	0.35	0.5	0.5
Derivative	0.0	0.0	-0.1	-0.1	-0.1	0.0	0.0	0.15	0.15	0.0	X

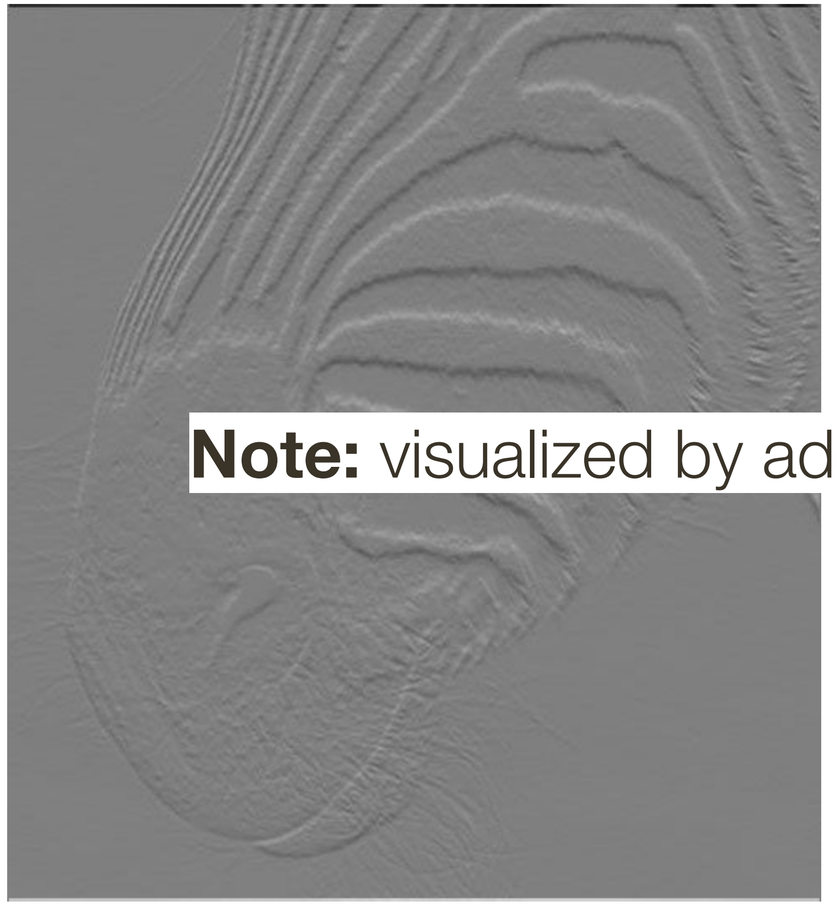
Estimating **Derivatives**

Derivative in Y (i.e., vertical) direction

Forsyth & Ponce (1st ed.) Figure 7.4 (top left & top middle)

Estimating **Derivatives**

Derivative in Y (i.e., vertical) direction

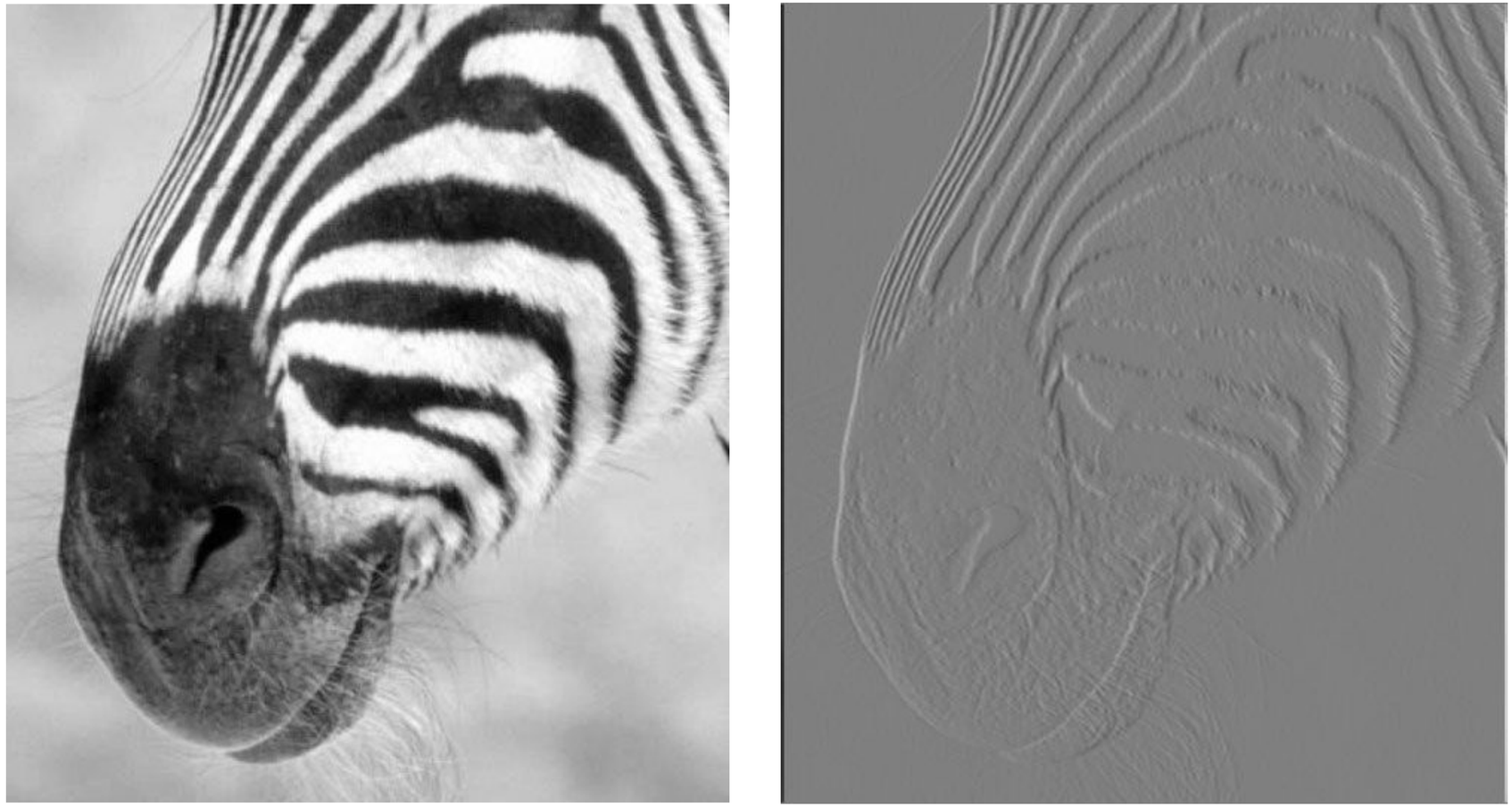


Note: visualized by adding 0.5/128

Forsyth & Ponce (1st ed.) Figure 7.4 (top left & top middle)

Estimating **Derivatives**

Derivative in X (i.e., horizontal) direction



Forsyth & Ponce (1st ed.) Figure 7.4 (top left & top right)

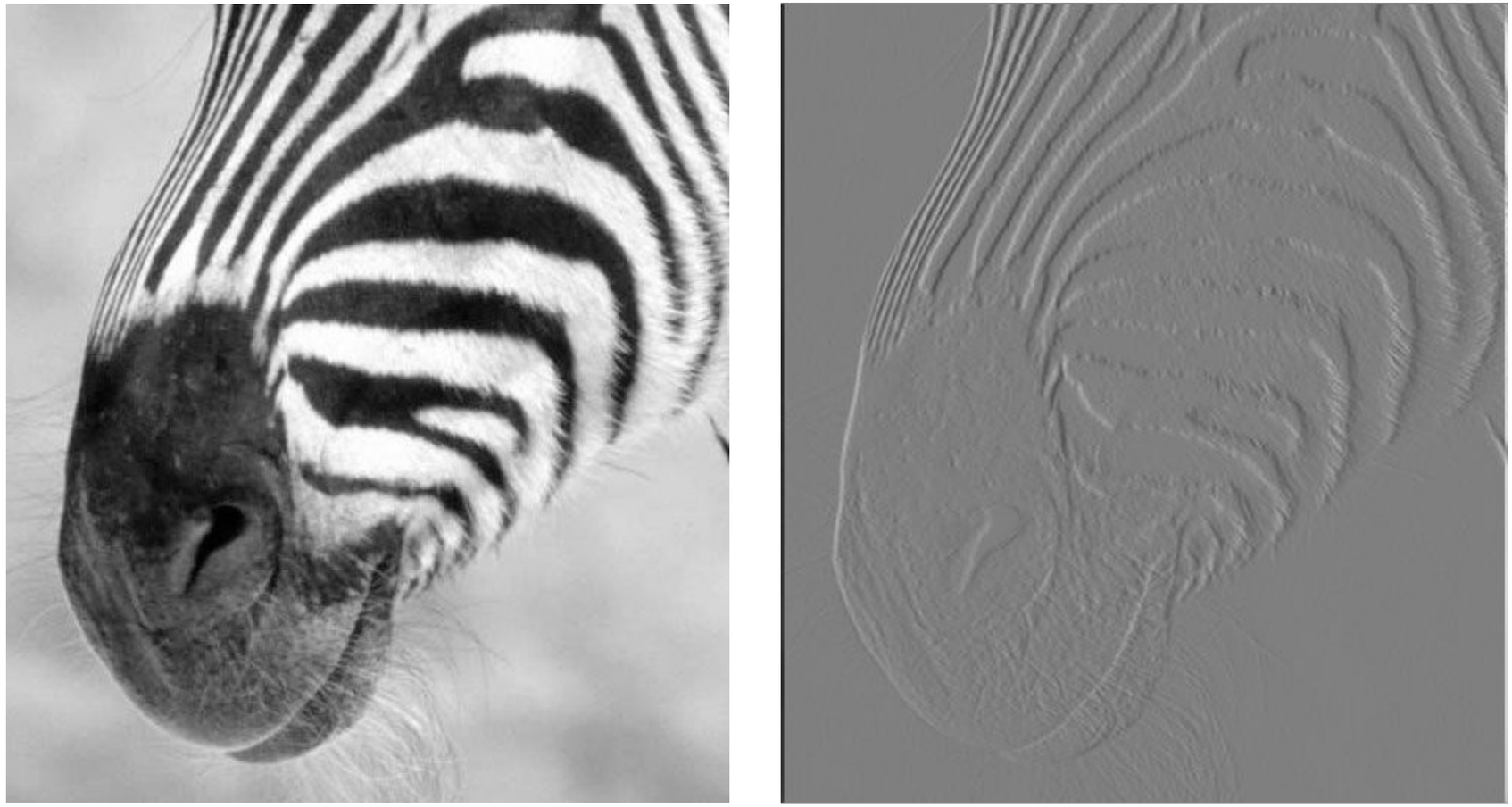
Estimating **Derivatives**

Derivative in Y (i.e., vertical) direction

Forsyth & Ponce (1st ed.) Figure 7.4 (top left & top middle)

Estimating **Derivatives**

Derivative in X (i.e., horizontal) direction



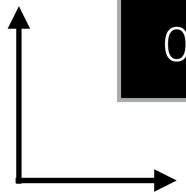
Forsyth & Ponce (1st ed.) Figure 7.4 (top left & top right)

A Sort **Exercise**

Use the “first forward difference" to compute the image derivatives in X and Y directions.

(Compute two arrays, one of $\frac{\partial f}{\partial x}$ values and one of $\frac{\partial f}{\partial y}$ values.)

1	1	0.6	0.3	0	0
1	1	0.6	0.3	0	0
0	0	0	0	0	0
0	0	0	0	0	0



A Sort **Exercise**: Derivative in X Direction

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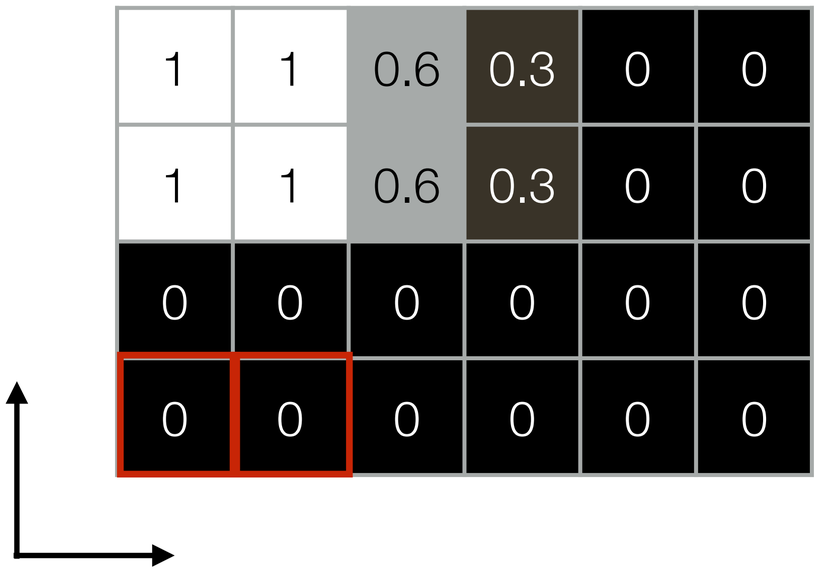

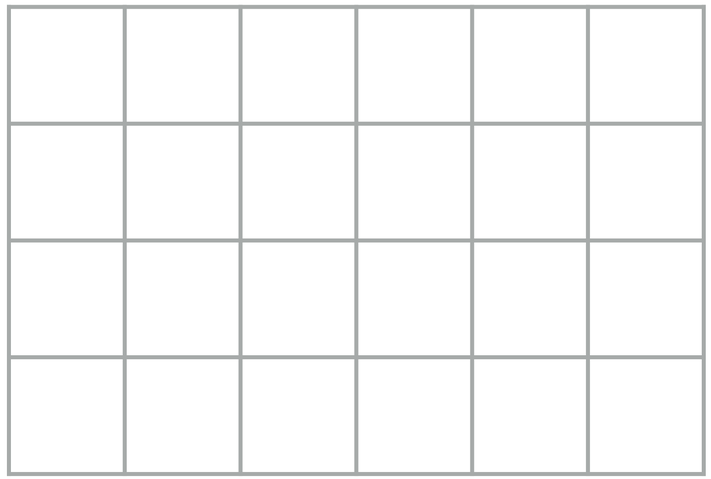

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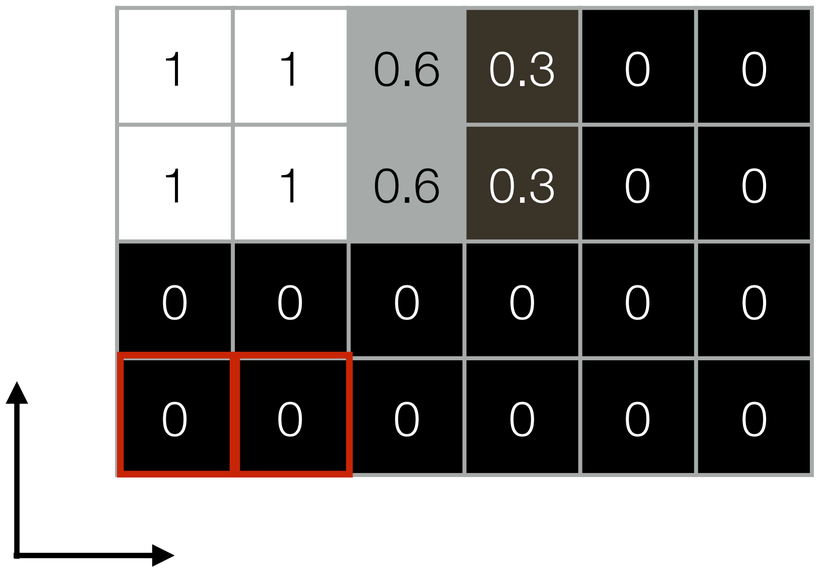

0					

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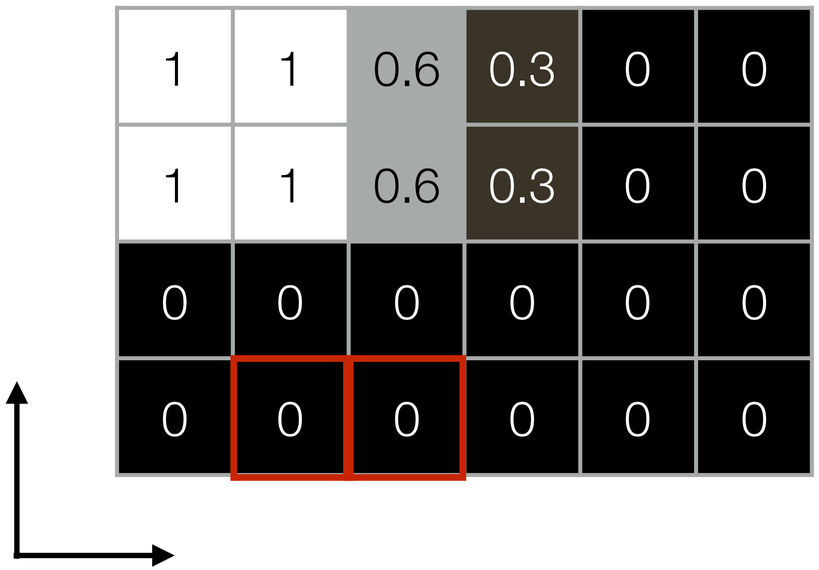

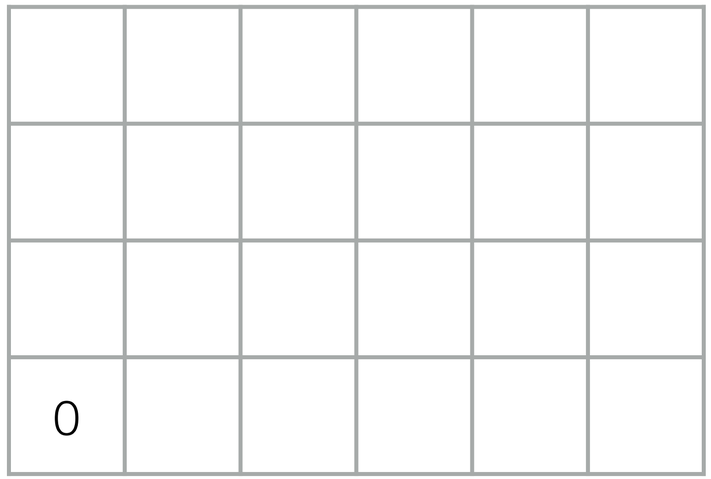

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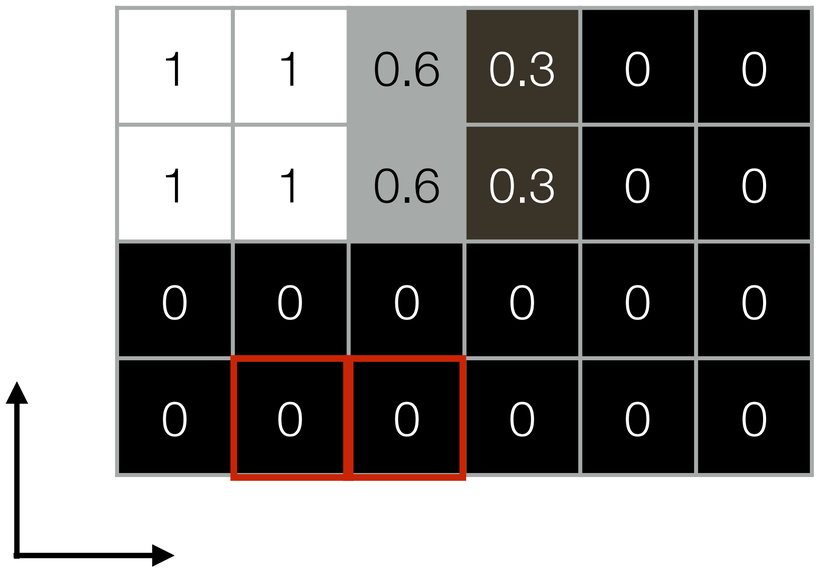
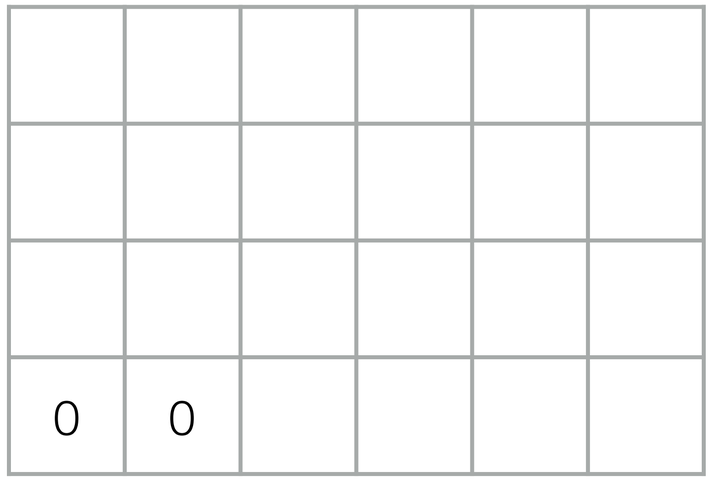


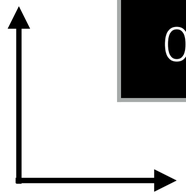


A Sort **Exercise**: Derivative in X Direction

Use the “first forward difference" to compute the image derivatives in X and Y directions.

(Compute two arrays, one of $\frac{\partial f}{\partial x}$ values and one of $\frac{\partial f}{\partial y}$ values.)

1	1	0.6	0.3	0	0
1	1	0.6	0.3	0	0
0	0	0	0	0	0
0	0	0	0	0	0



0					
0	0	0	0	0	
0	0	0	0	0	

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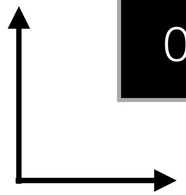

0	-0.4				
0	0	0	0	0	
0	0	0	0	0	

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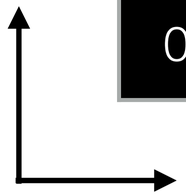
0	0	0	0	0	0
0	0	0	0	0	0



0	-0.4	-0.3	-0.3	0	
0	-0.4	-0.3	-0.3	0	
0	0	0	0	0	
0	0	0	0	0	

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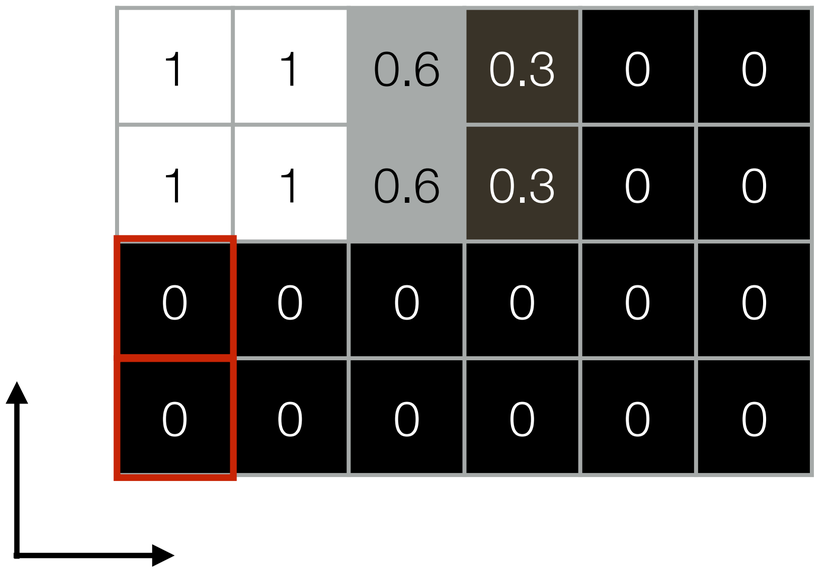

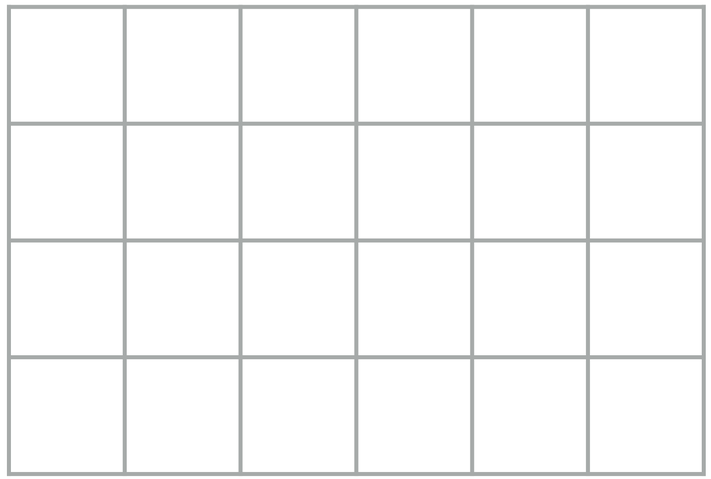

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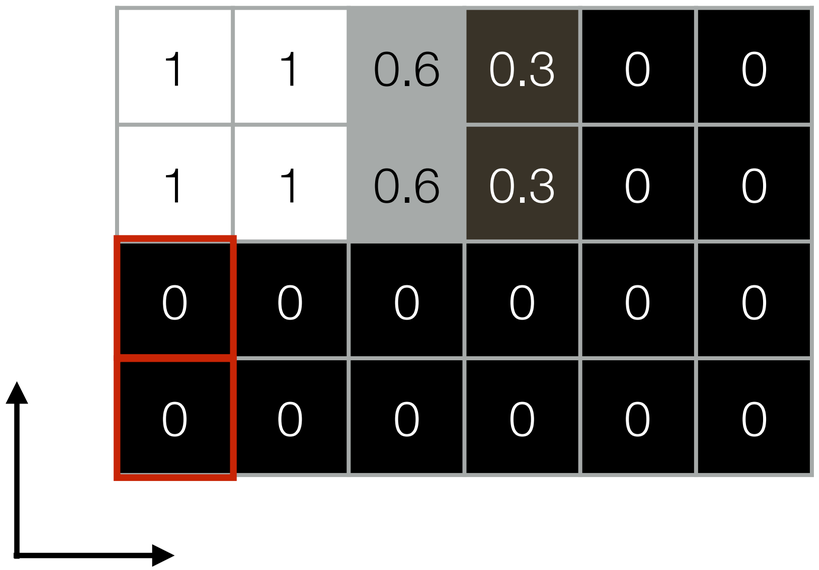

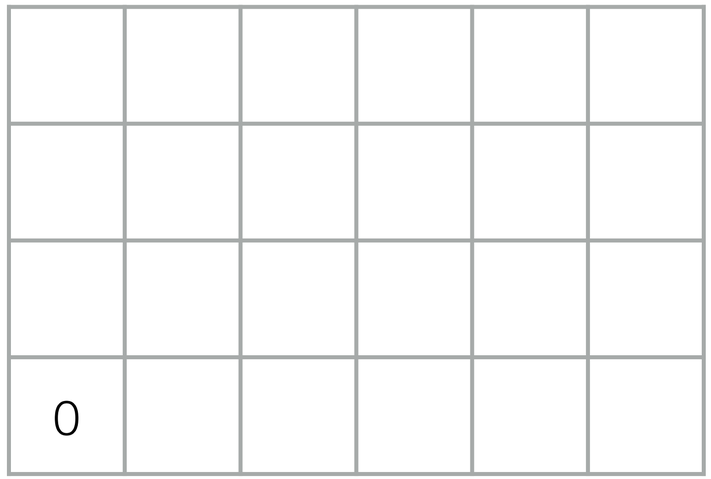

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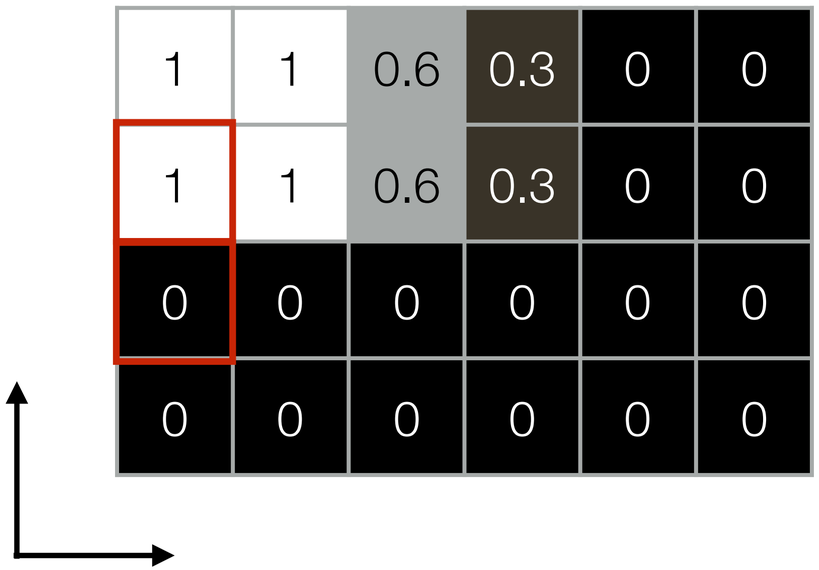

0	0	0	0	0	0

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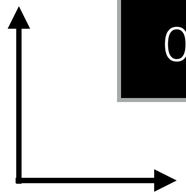
0	0	0	0	0	0
0	0	0	0	0	0



0	0	0	0	0	0
1	1	0.6	0.3	0	0
0	0	0	0	0	0

Estimating **Derivatives**

Question: Why, in general, should the weights of a filter used for differentiation sum to 0?

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$$\sum_{i=1}^{N} f_i \cdot k = k \sum_{i=1}^{N} f_i = 0 \implies \sum_{i=1}^{N} f_i = 0$$

Summary

Template matching as (normalized) correlation. Template matching is not robust to changes in:

— 2D spatial scale and 2D orientation

— 3D pose and viewing direction

— illumination

Scaled representations facilitate

— template matching at multiple scales

— efficient search for image–to–image correspondences

— image analysis at multiple levels of detail

A **Gaussian pyramid** reduces artifacts introduced when sub-sampling to coarser scales