THE UNIVERSITY OF BRITISH COLUMBIA

CPSC 425: Computer Vision

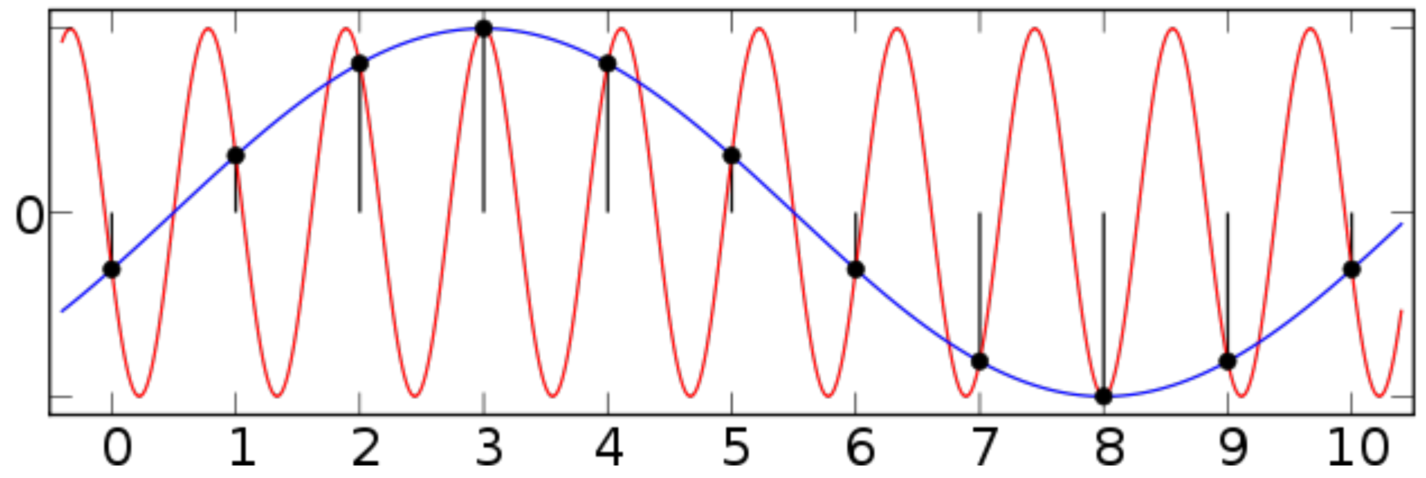


Image Credit: https://en.wikibooks.org/wiki/Analog_and_Digital_Conversion/Nyquist_Sampling_Rate

Lecture 6: Sampling (part 2)

(unless otherwise stated slides are taken or adopted from **Bob Woodham, Jim Little** and **Fred Tung**)

Menu for Today (**January 23, 2020**)

Topics:

- **Sampling** theory
- **Nyquist** rate
- Color **Filter Arrays**
- **Bayer** patterns

Redings:

- **Today's** Lecture: Forsyth & Ponce (2nd ed.) 4.5, 4.6
- **Next** Lecture: Forsyth & Ponce (2nd ed.) 4.6, 4.7

Reminders:

- **Assignment 1:** Image Filtering and Hybrid Images due **January 28**th
- Code for Piazza sign up is (**425S2**)

Today's "**fun**" Example: Optical Illusions

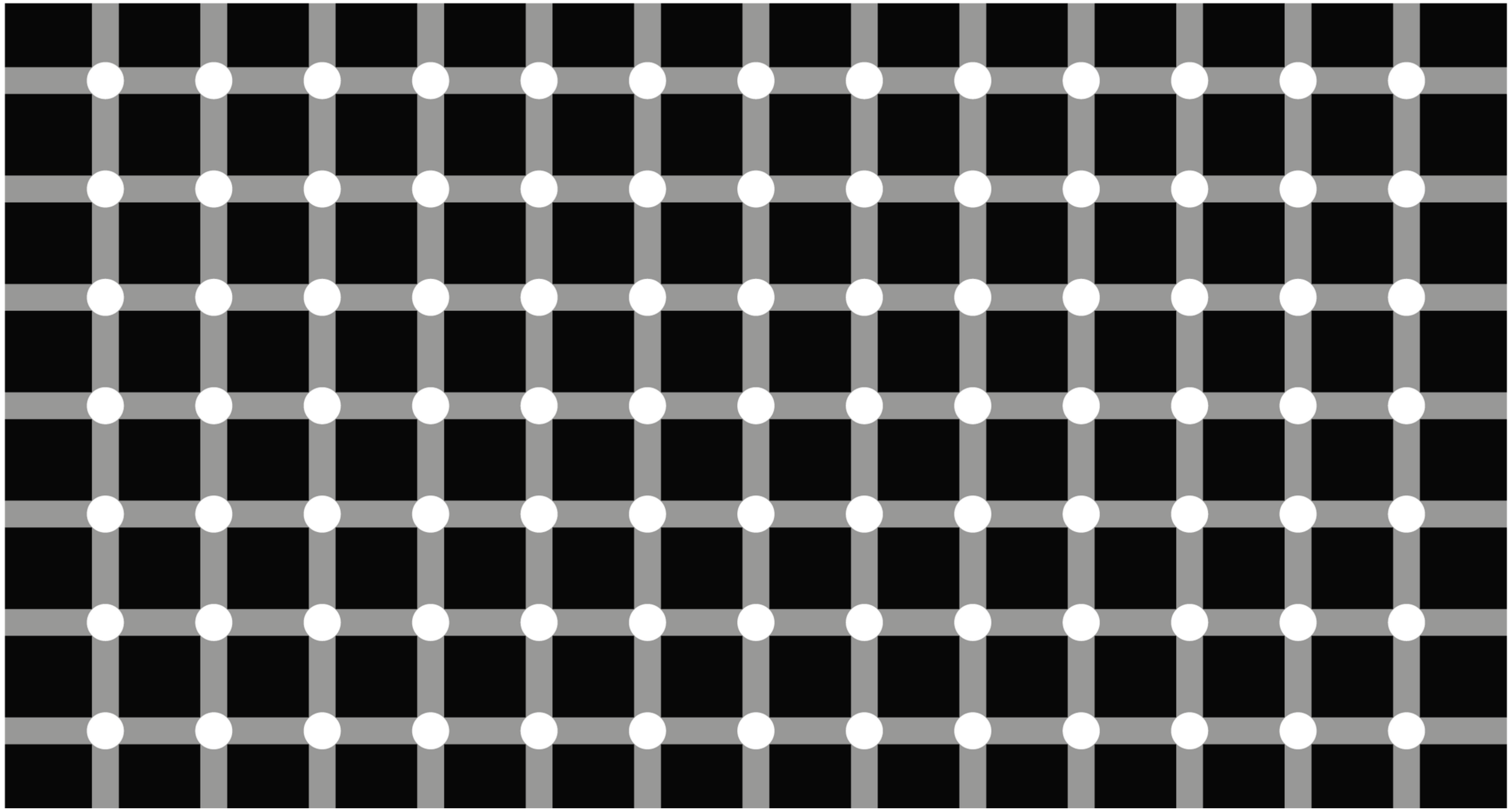


Image From: https://inudgeyou.com/en/nudging-traffic-safety-by-visual-illusions/

Today’s “**fun**” Example: Nudging

Aerial view of the white stripes at the lake shore drive in Chicago.

Today's "**fun**" Example: **Anchoring** and Ordering

Champagne, Sparkling, Rose, Sweet Wines

Champagne

CH18	NV	GREMILLET "Brut Selection" - Champagne	$65
CH31	NV	ERNEST RAPENEAU "Selection Brut" - Champagne	$65
CH12	NV	CHAMPAGNE ERNEST RAPENEAU - BRUT - Chardonnay/Pinot Noir/Pinot Meunier -	$75
CH05	NV	DRAPPIER "Carte d'Or" - Champagne	$78
CH30	2007	ERNEST RAPENEAU VINTAGE - Chardonnay/ Pinot Noir - Champagne	$80
CH32	NV	ERNEST RAPENEAU "Premier Cru Brut" - Champagne	$80
CH28	NV	DRAPPIER Brut Rose - Champagne	$85
CH29	2012	DRAPPIER "Millesime Exception" - Champagne	$98
CH11	2008	DRAPPIER " Cuvee Grande Sendree" - Champagne	$130
CH39	NV	ERNEST RAPENEAU "Grande Reserve"- Magnum - Champagne	$130

Sparkling Wines

CH06	NV	IL CORTIGIANO - Prosecco Extra Dry - Veneto	$30
CH17	NV	VALLFORMOSA "Clasic" Semi Seco - Cava	$30
CH24	NV	VEUVE MOISANS "Blanc de Blancs" - Loire Valley	$30
CH25	NV	VALDO - Prosecco Extra Dry - Treviso, Veneto	$30
CH33	NV	VALDO "Origine" Rose - Veneto	$30
CH03	2012	CHATEAU MONTGUERET Saumur Sec Rose - Cabernet Franc - Loire Valley	$32
CH04	NV	CAVA MASET RESERVA BRUT - Macabeo/Xarello/Parellada - Cava	$32
CH14	NV	TRIVENTO "Brut Nature" - Mendoza	$32
CH21	2015	CAMASELLA - Glera - Vaneto	$32
CH02	2013	BRUT D'ARGENT ICE - Chardonnay - France	$35
CH01	NV	VALDO "ORO PURO" Prosecco Superiore - Veneto	$36
CH40	NV	MAISON DARRAGON - AOC Vouvray Brut - Loire Valley	$38
CH09	NV	LOU MIRANDA ESTATE 'LEONE' - Sparkling Shiraz - Barossa Valley	$42

Rose Wines

PO03	2014	CASAL MENDES Rose - Baga - Portugal	$30
RH09	2014	LA VIE EN ROSE - Cinsault - Languedoc	$30
RH69	2015	LES EMBRUNS "La Croix des Saintes" - Sable de Camargue	$30
RH04	2015	LES MAITRES VIGNERONS DE ST TROPEZ - Cotes de Provence	$32
RH15	2015	MANON - COTES DE PROVENCE - Grenache/Cinsault/Syrah. - Provence	$34
RH04M	2015	LES MAITRES VIGNERONS DE LA PRESQU'ILE DE SAINT TROPEZ - Grenache/Mourve	$68

Sweet Wines

AR33	2015	TRIVENTO "Birds & Bees" White - Mendoza	$30
AR34	2016	TRIVENTO "Birds & Bees" Red - Mendoza	$30
AU05	2015	DEAKIN ESTATE - Moscato - Murray Darling	$30
AU12	2016	Chalk Hill - Moscato - McLaren Vale	$30
AU68	NV	WESTEND ESTATE "Richland" - Moscato - New South Wales	$30
AU107	NV	WESTEND ESTATE "Richland" - Pink Moscato - New South Wales	$30

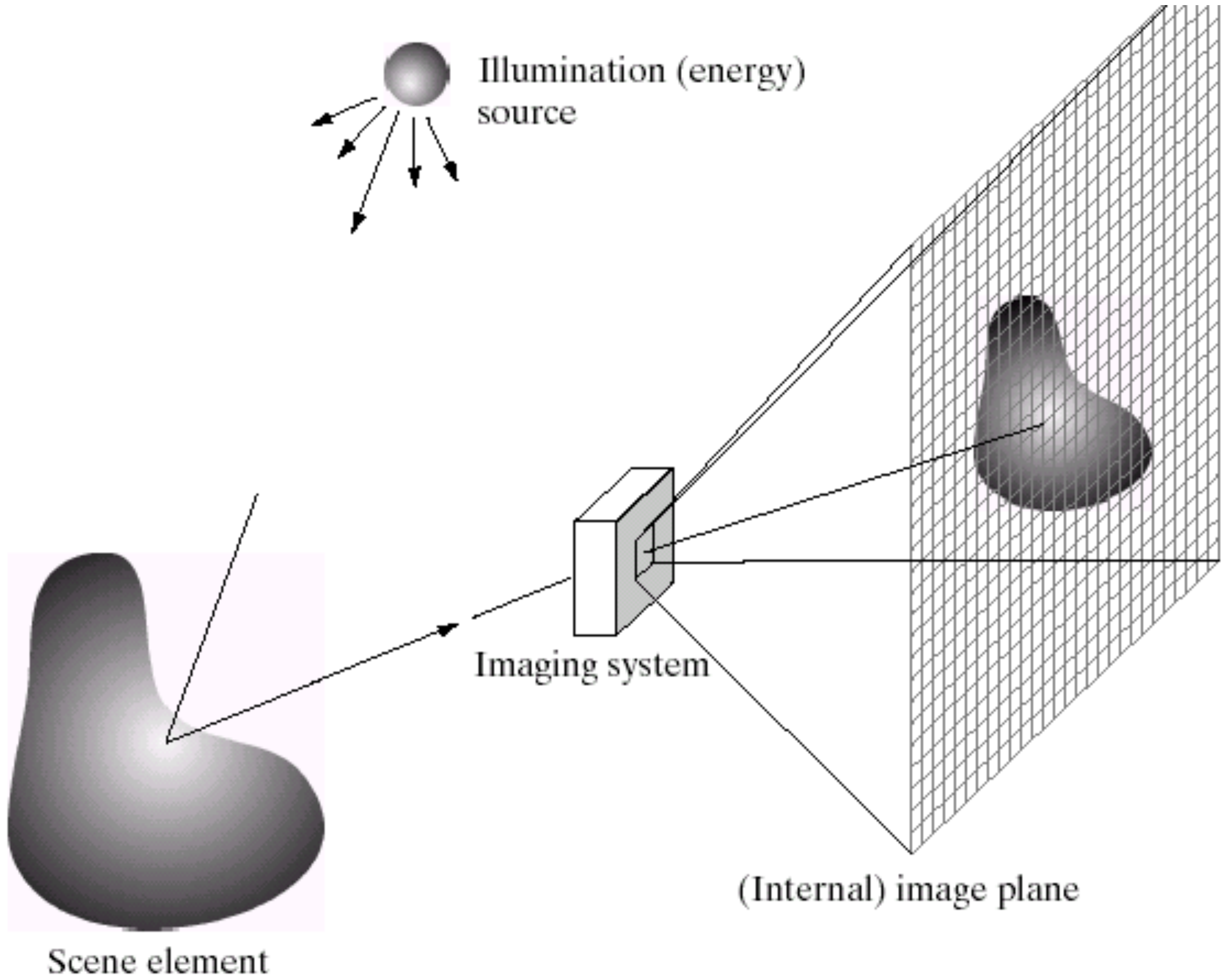
Framework for Today's Topic

Problem: How do we go from the optics of image formation to digital images as arrays of numbers?

Key Idea(s): Sampling and the notion of band limited functions

Theory: Sampling Theory

Reminder



Images are a **discrete**, or **sampled**, representation of a continuous world

What is an **Image**?

Up to now provided a **physical characterization**

— image formation as a problem in physics/optics

— we also talked about simple image processing algorithms on image arrays

Now provide a **mathematical characterization**

— to understand how to represent images digitally

— to understand how to compute with images

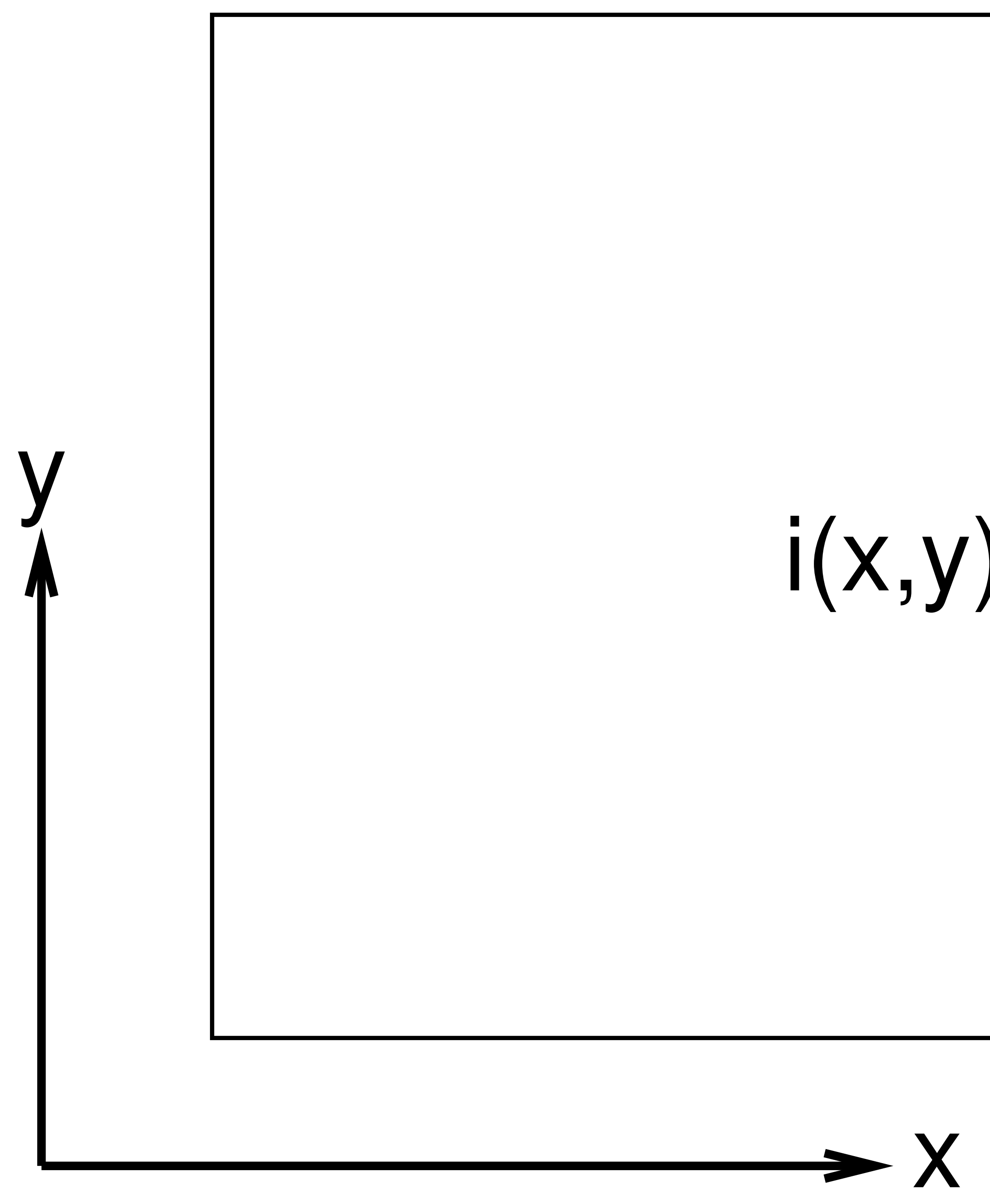
Continuous Case

“**Image**” suggests a 2D surface whose appearance varies from point–to–point — the surface typically is a plane (but might be curved, e.g., as is with an eye)

Appearance can be **Grayscale** (Black and White) or **Colour**

In **Grayscale**, variation in appearance can be described by a single parameter corresponding to the amount of light reaching the image at a given point in a given time

Continuous Case



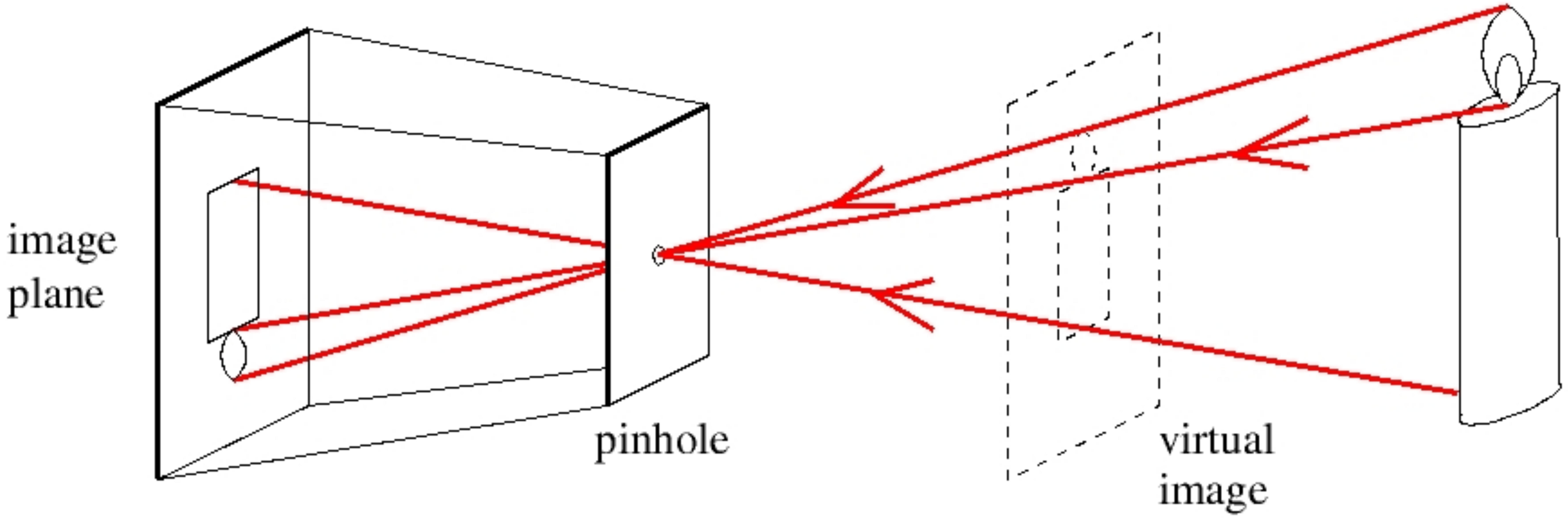
Denote the image as a function, $i(x, y)$, where x and y are spatial variables

Aside: The convention for this section is to use lower case letters for the continuous case and upper case letters for the discrete case

Continuous Case: Observations

– $i(x, y)$ is a **real-valued function** of **real spatial variables**, x and y

Recall: Pinhole Camera



Forsyth & Ponce (2nd ed.) Figure 1.2

Continuous Case: Observations

– $i(x, y)$ is a **real-valued function** of **real spatial variables**, x and y

– $i(x, y)$ is **bounded above and below**. That is

$$0 \leq i(x, y) \leq M$$

for some maximum brightness M

Continuous Case: Observations

- $i(x, y)$ is a **real-valued function** of **real spatial variables**, x and y

- $i(x, y)$ is **bounded above and below**. That is

$$0 \leq i(x, y) \leq M$$

  for some maximum brightness M

- $i(x, y)$ is **bounded in extent**. That is, $i(x, y)$ is non-zero (i.e., strictly positive) over, at most, a bounded region

Continuous Case

— Images also can be considered a function of time. Then, we write $i(x, y, t)$ where x and y are spatial variable and t is a **temporal variable**

— To make the dependence of brightness on wavelength explicit, we can instead write $i(x, y, t, \lambda)$ where x, y and t are as above and where λ is a **spectral variable**

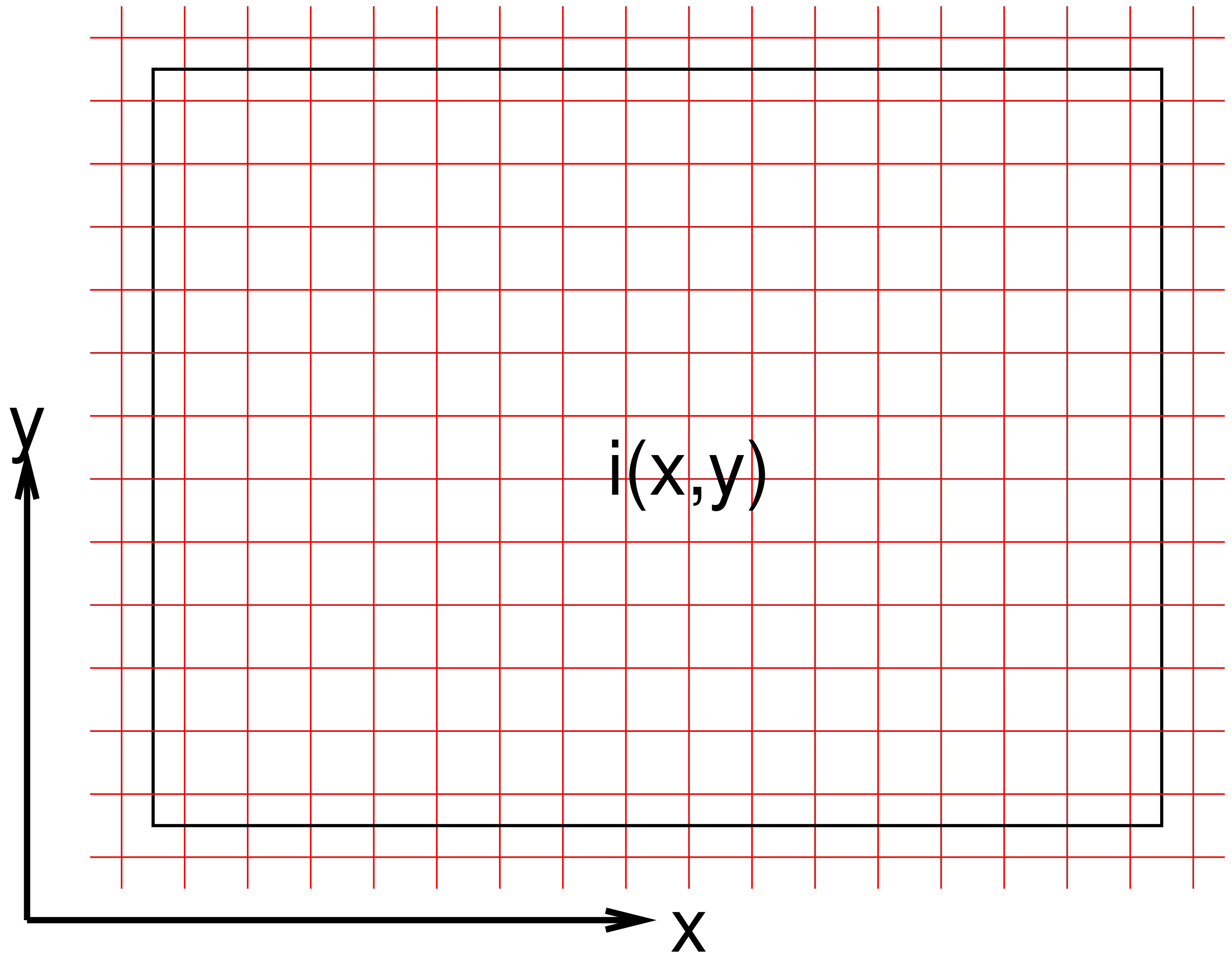
— More commonly, we think of "color" already as discrete and write

$$
\begin{gathered}
i_R(x, y) \\
i_G(x, y) \\
i_B(x, y)
\end{gathered}
$$

for specific colour channels, R, G and B

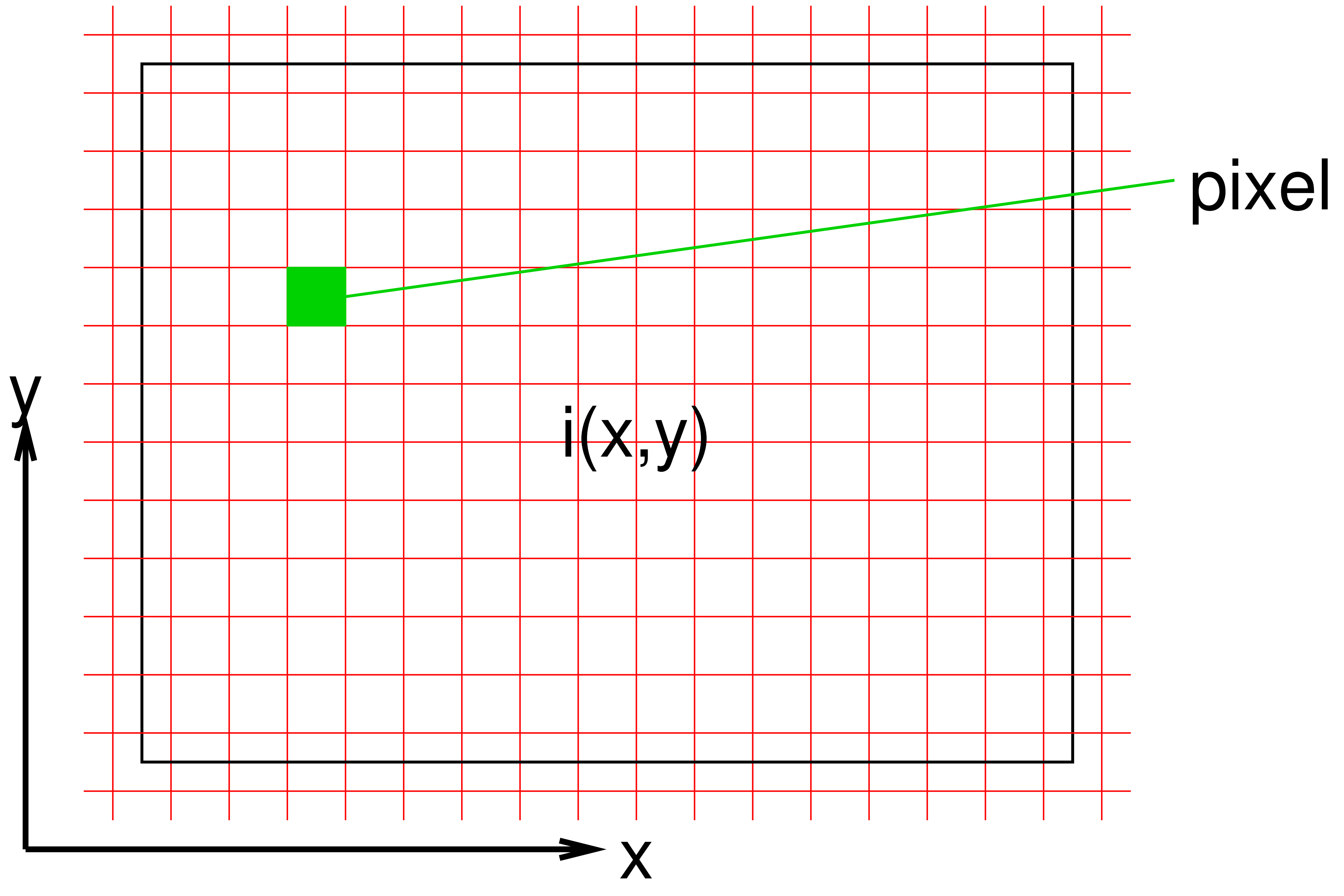
Discrete Case

Idea: Superimpose (regular) grid on continuous image



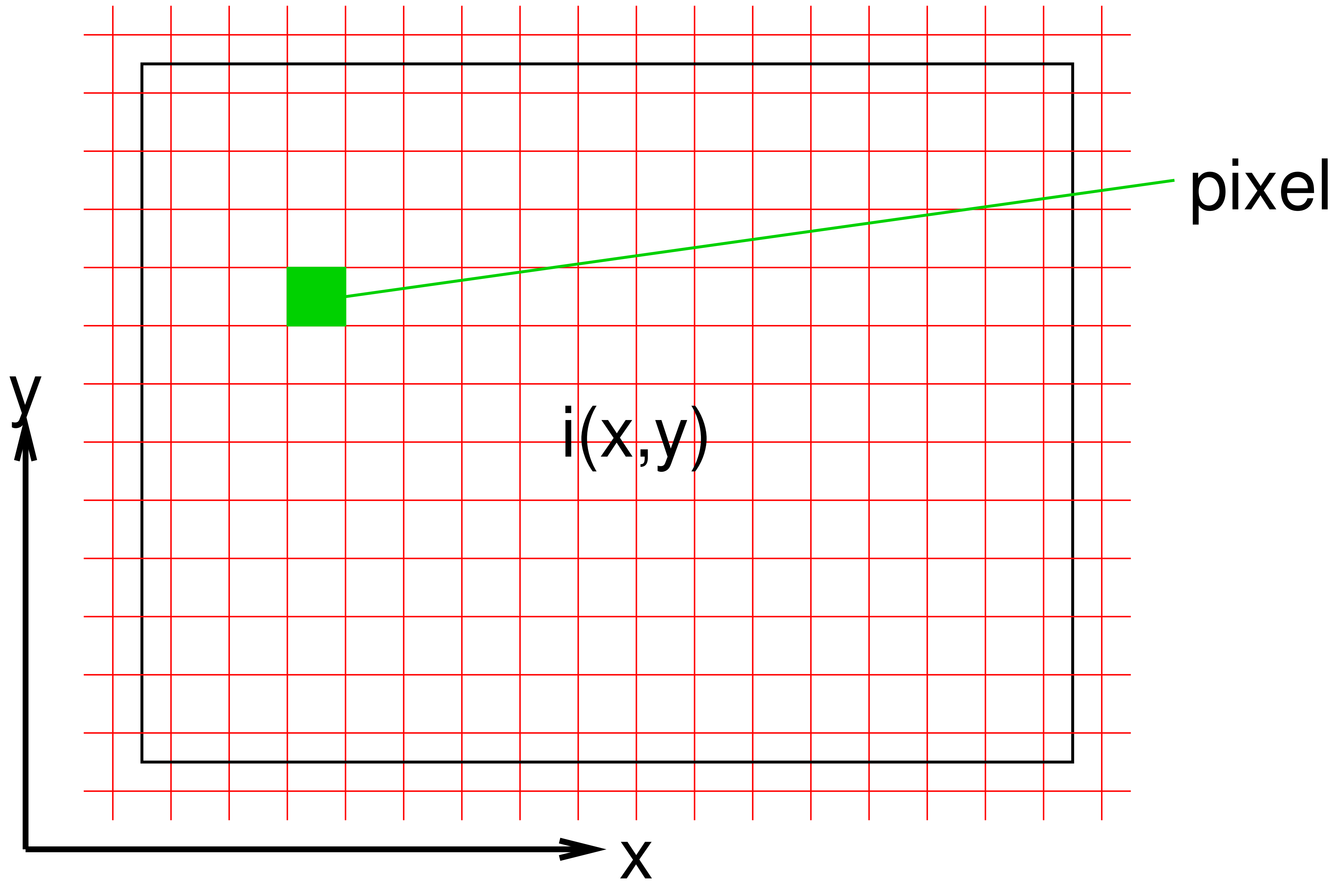
Sample the underlying continuous image according to the **tessellation** imposed by the grid

Discrete Case



Discrete Case

Each grid cell is called a picture element (**pixel**)



Denote the discrete image as $I(X, Y)$

We can store the pixels in a matrix or array

Discrete Case

Question: How to sample?

— Sample brightness at the point?

— "Average" brightness over entire pixel?

Answer:

— Point sampling is useful for theoretical development

— Area-based sampling occurs in practice

Discrete Case

Question: What about the brightness samples themselves?

Discrete Case

Question: What about the brightness samples themselves?

Answer: We make values of $I(X, Y)$ discrete as well

Recall: $0 \leq i(x, y) \leq M$

We divide the range $[0, M]$ into a finite number of equivalence classes. This is called **quantization**.

The values are called **grey-levels**.

Discrete Case

Quantization is a topic in its own right

For now, a simple linear scheme is sufficient

Suppose n bits-per-pixel are available. One can divide the range $[0, M]$ into evenly spaced intervals as follows:

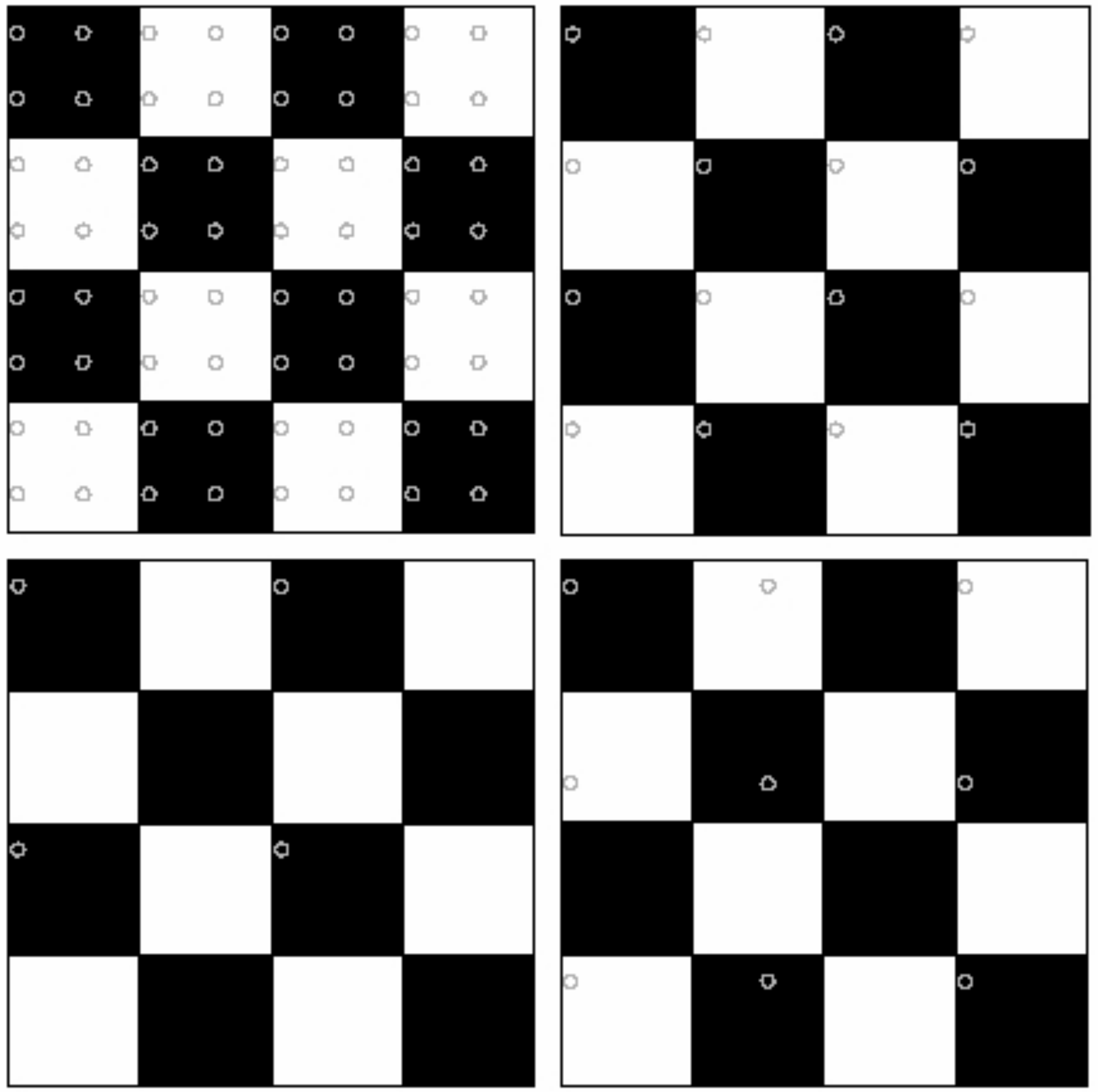
$$i(x, y) \to \left\lfloor \frac{i(x, y)}{M}(2^n - 1) + 0.5 \right\rfloor$$

where $\lfloor \quad \rfloor$ is floor (i.e., greatest integer less than or equal to)

Typically $n = 8$ resulting in grey-levels in the range $[0, 255]$

Sampling

It is clear that *some* information may be lost when we work on a discrete pixel grid.



Forsyth & Ponce (2nd ed.) Figure 4.7

Sampling

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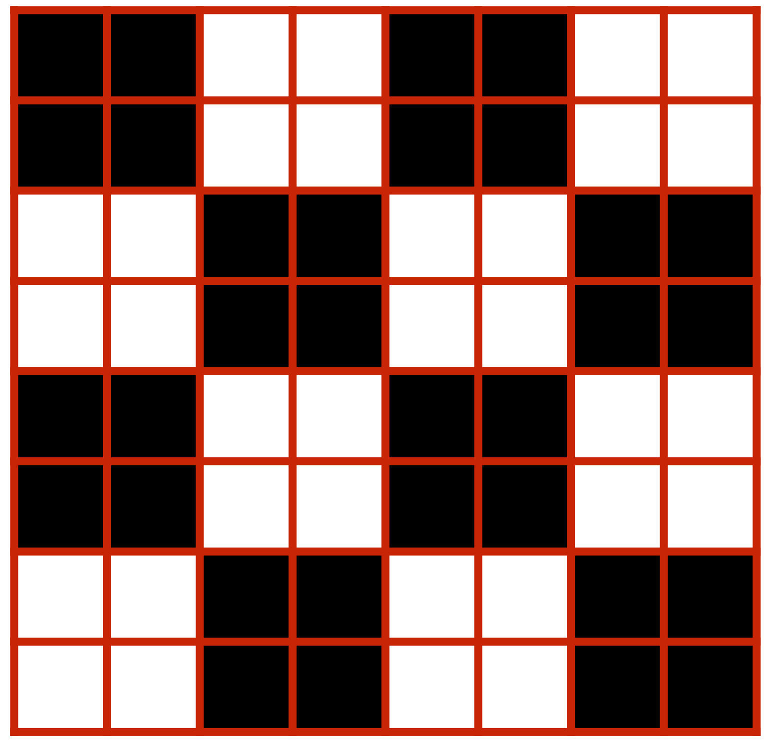

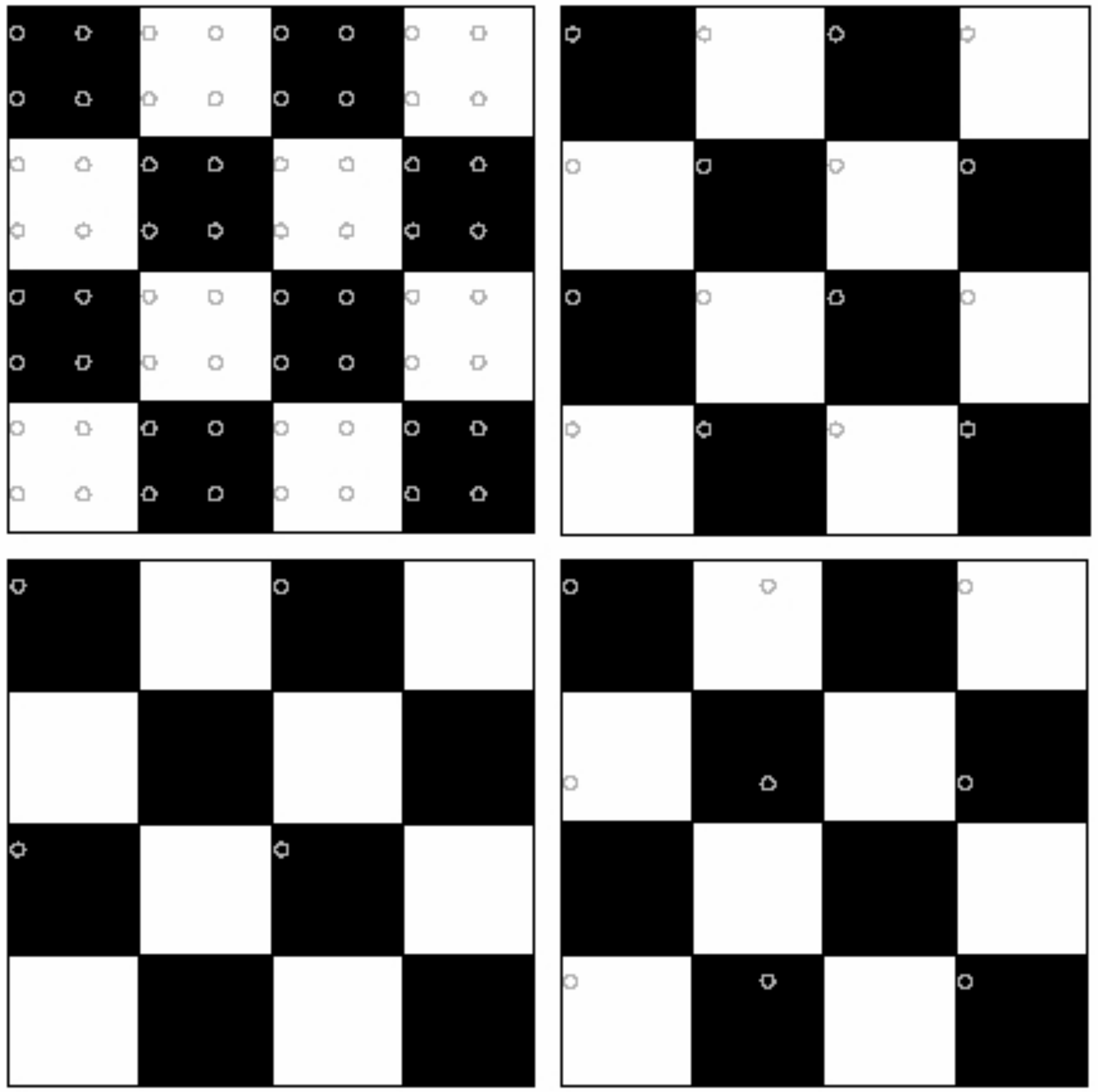

Forsyth & Ponce (2nd ed.) Figure 4.7

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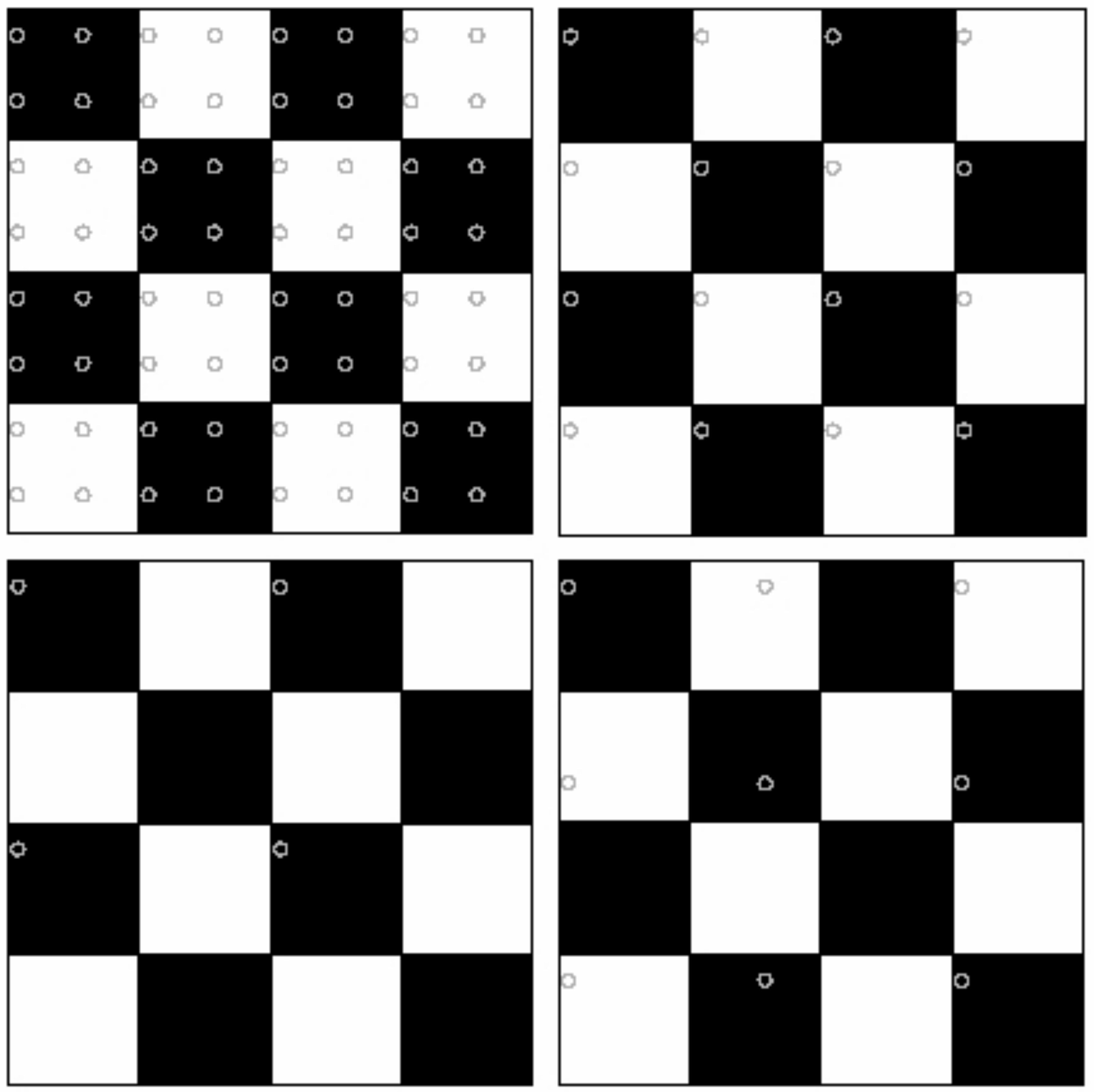
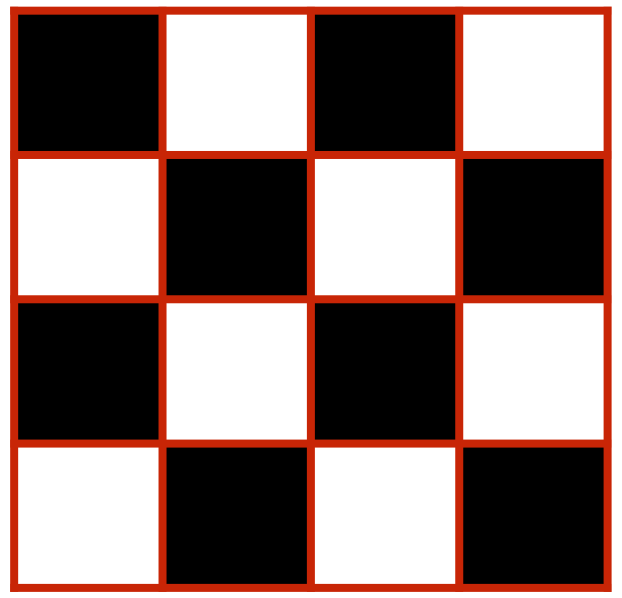


Forsyth & Ponce (2nd ed.) Figure 4.7



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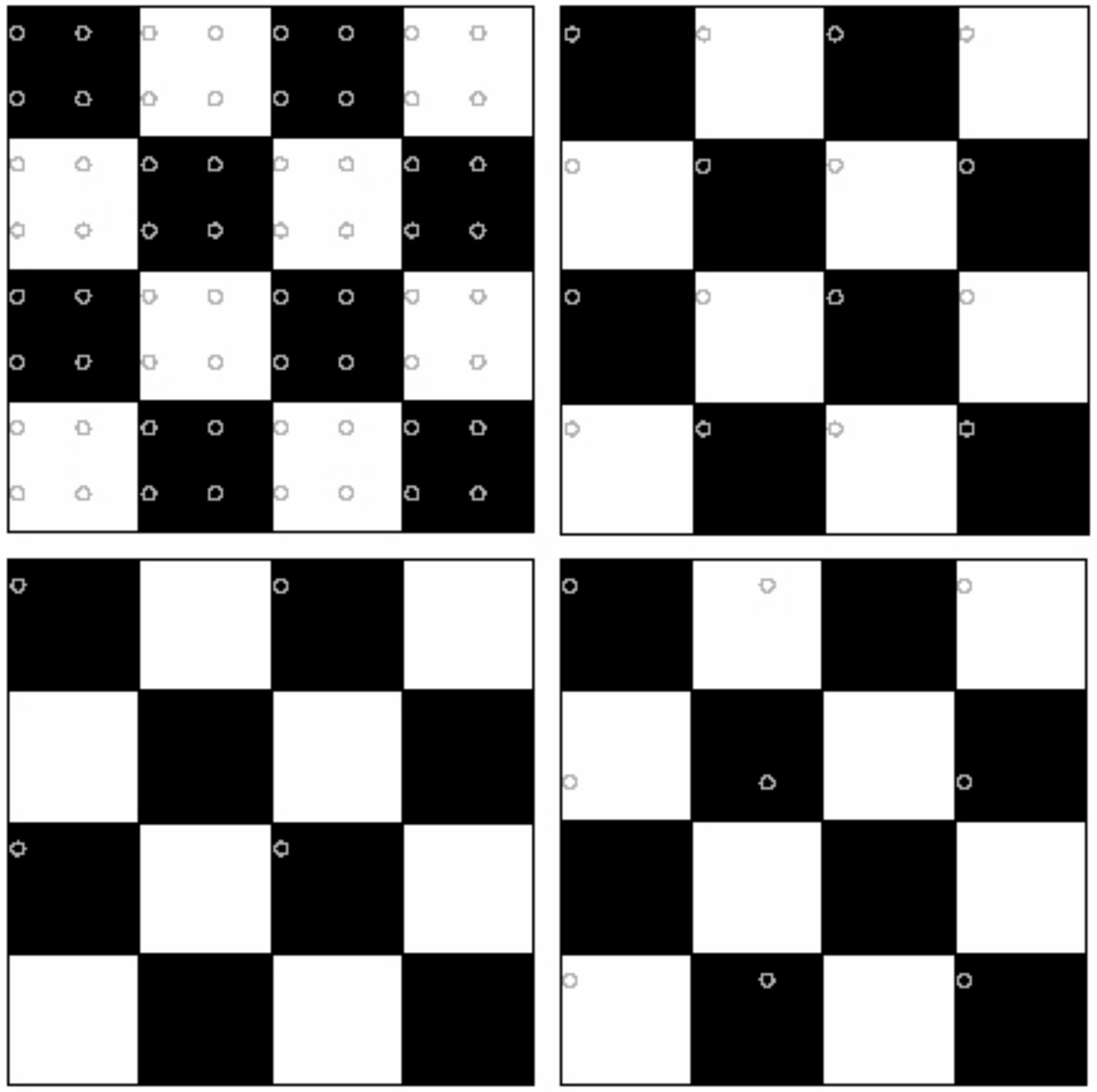
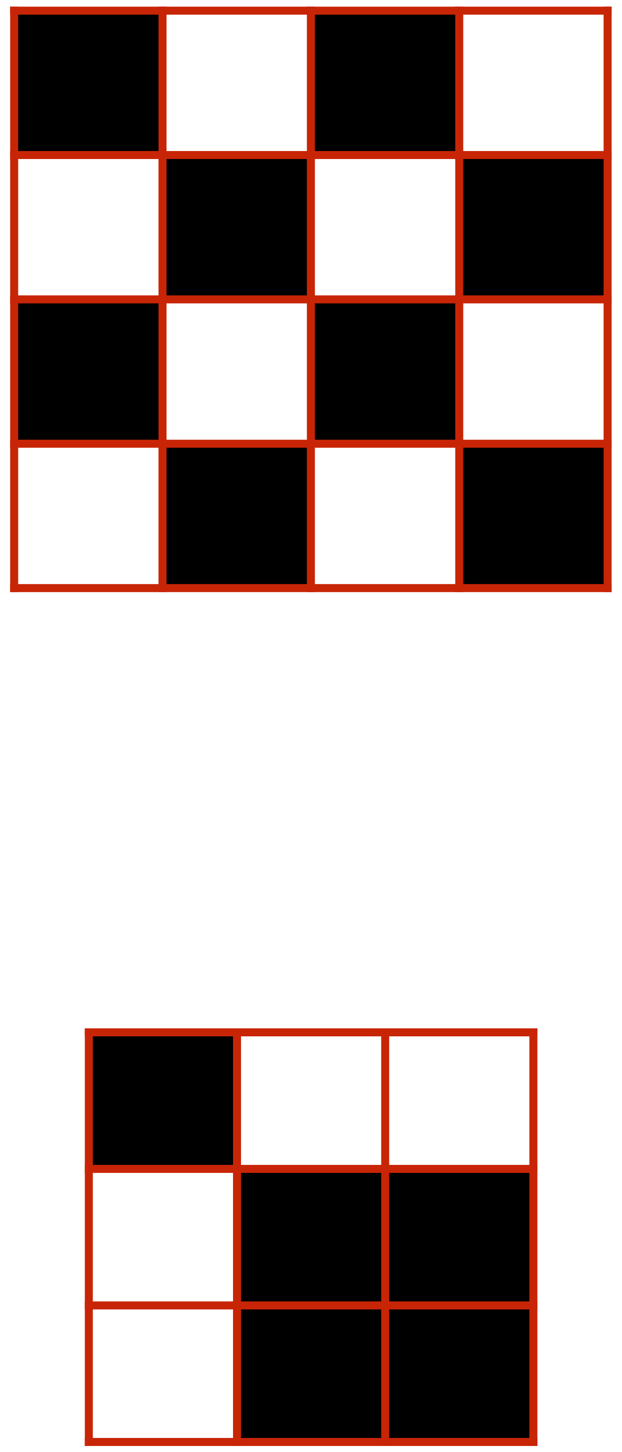


Forsyth & Ponce (2nd ed.) Figure 4.7



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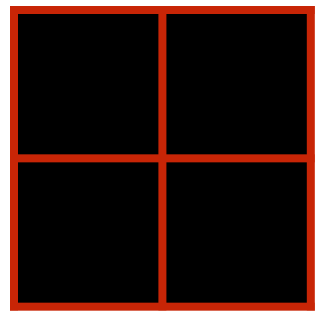

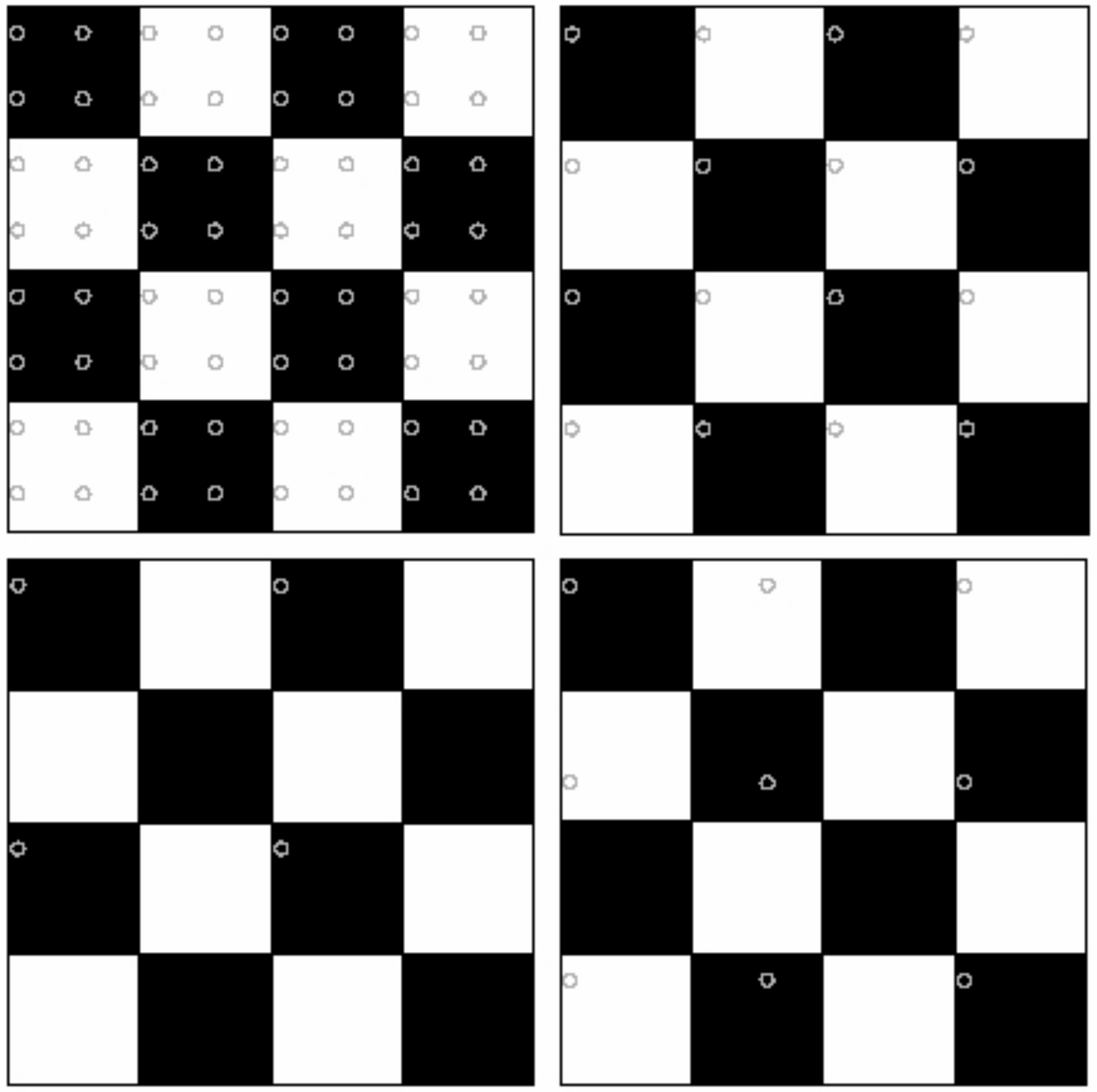

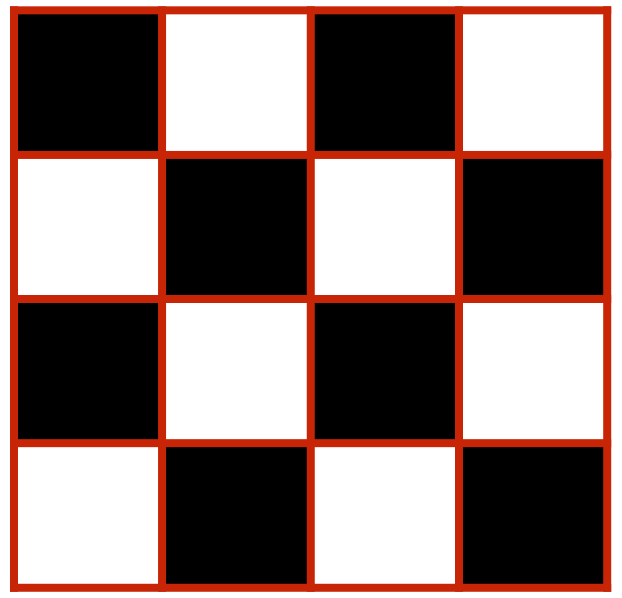

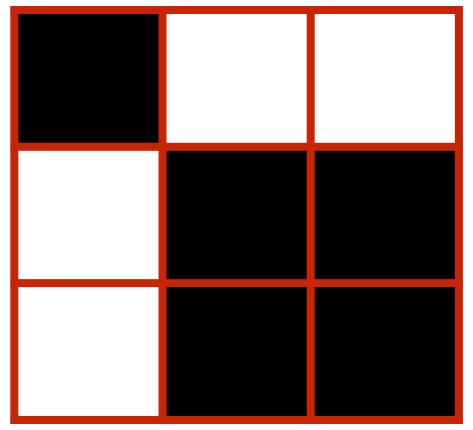

Forsyth & Ponce (2nd ed.) Figure 4.7

Sampling Theory (informal)

Question: When is $I(X,Y)$ an exact characterization of $i(x,y)$?

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Intuition: Reconstruction involves some kind of **interpolation**

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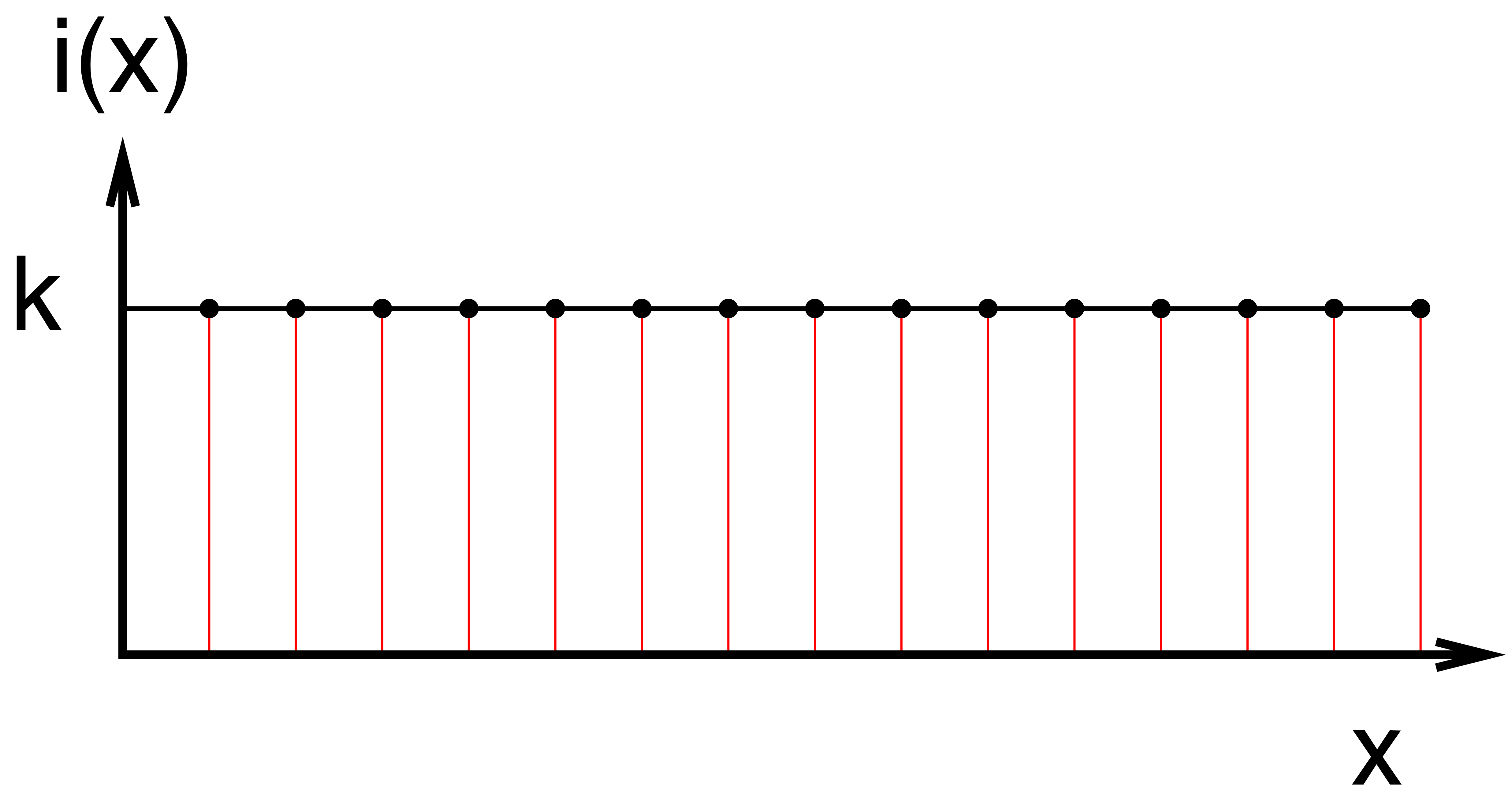
Question (modified): When can we reconstruct $i(x, y)$ exactly from $I(X, Y)$?

Intuition: Reconstruction involves some kind of **interpolation**

Heuristic: When in doubt, consider simple cases

Sampling Theory (informal)

Case 0: Suppose $i(x, y) = k$ (with k being one of our gray levels)



Note: we use equidistant sampling at integer values for convenience, in general, sampling doesn't need to be equidistant

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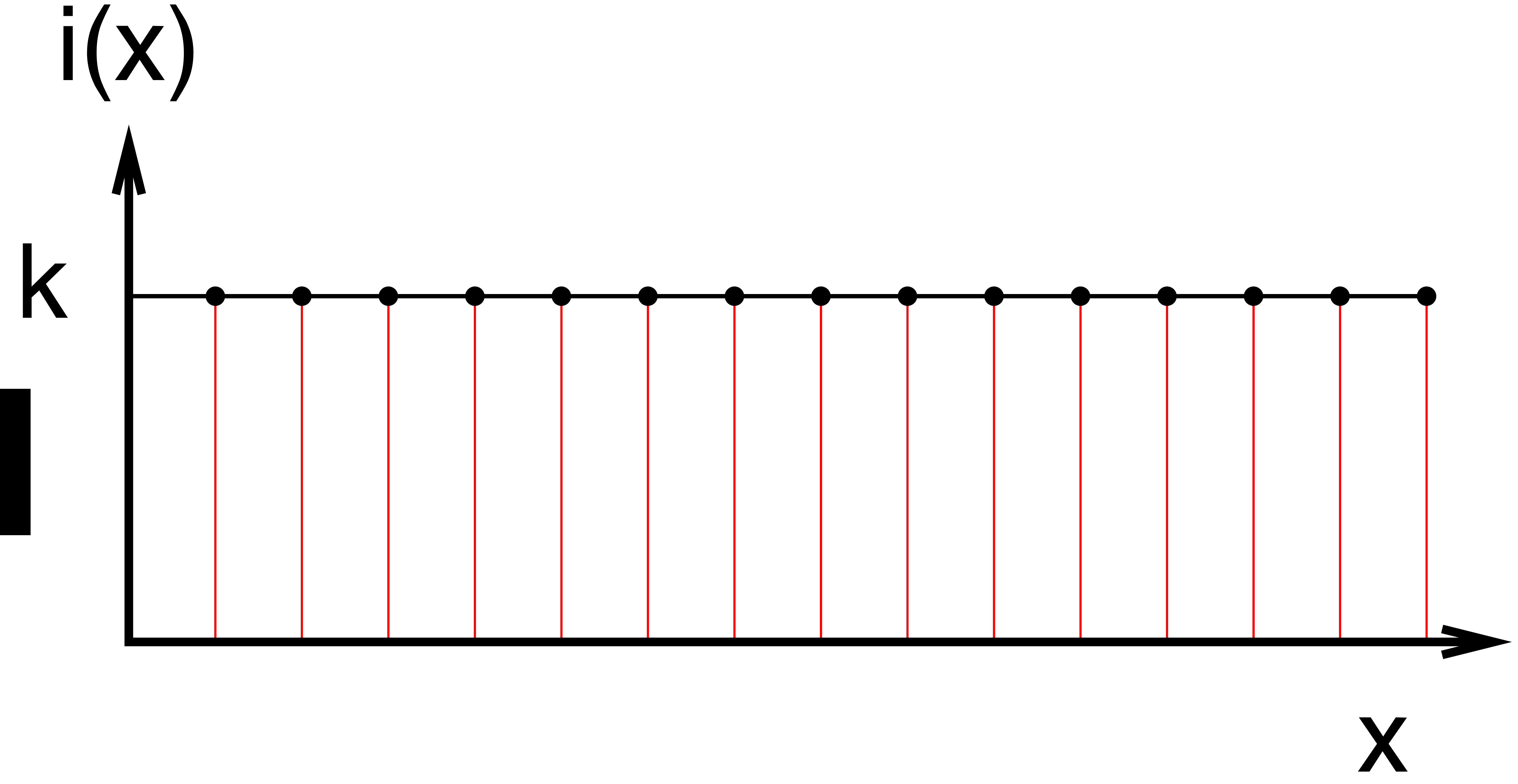

This is **easy**!

Sampling Theory (informal)

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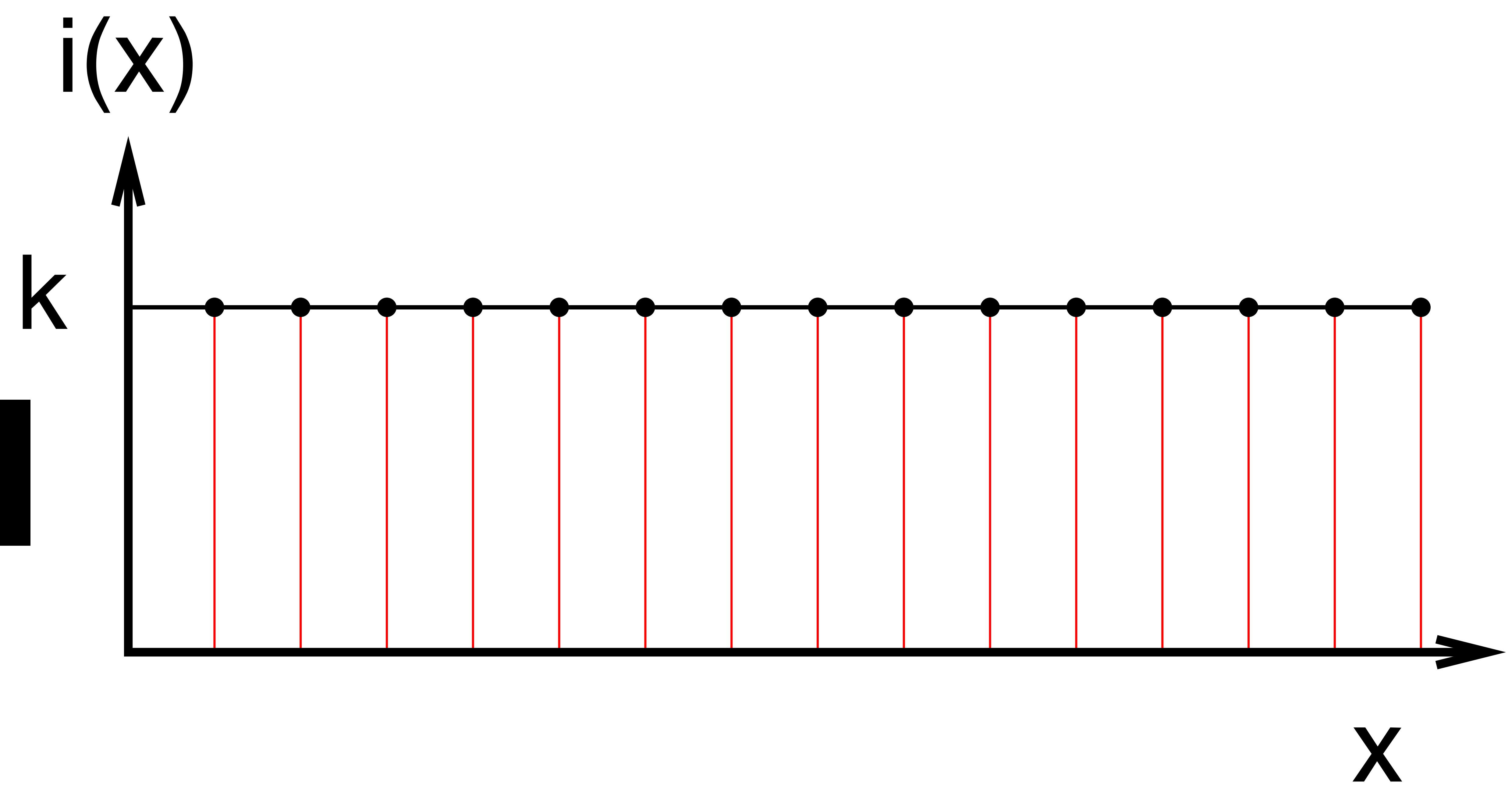
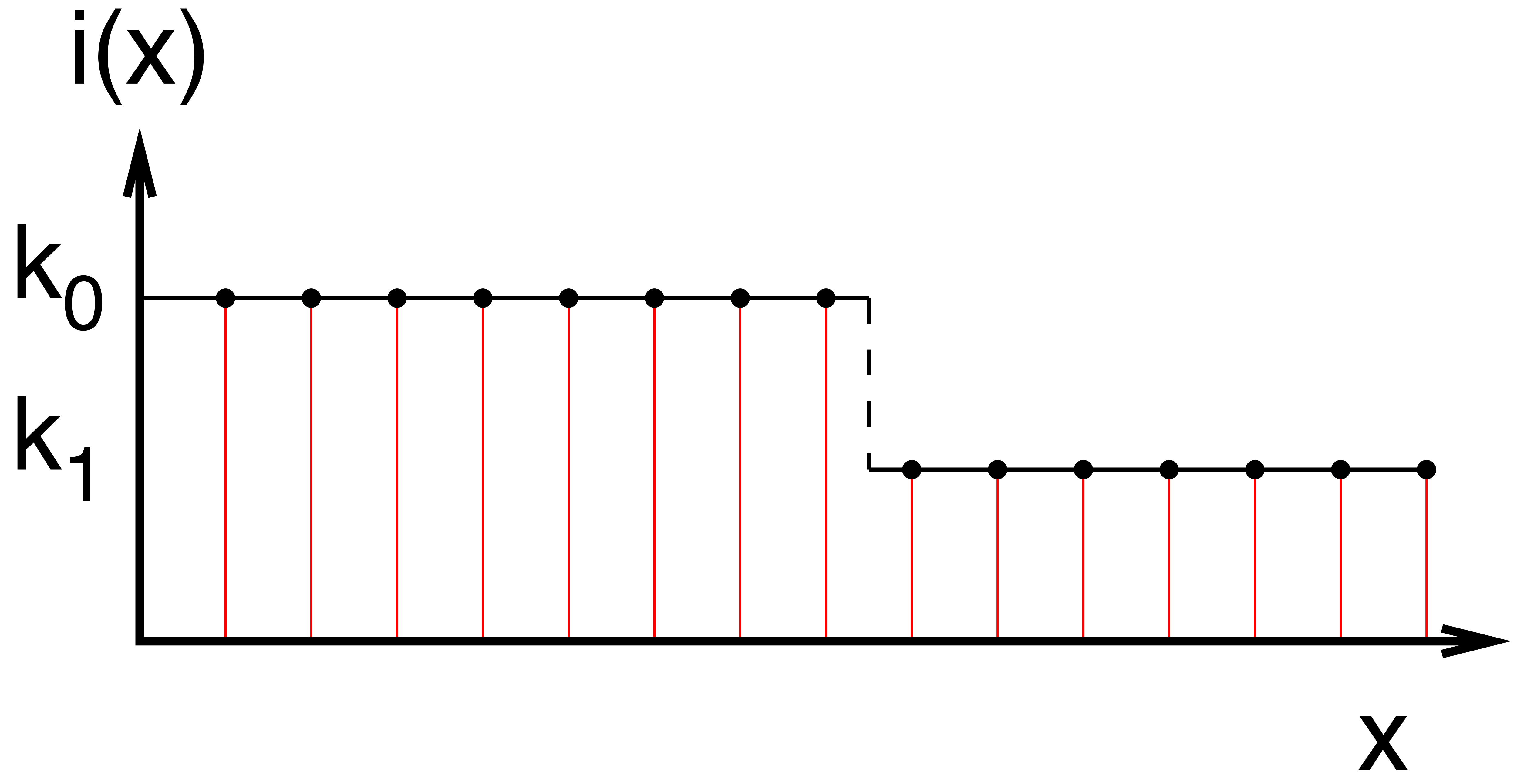


This is **easy**!

$I(X, Y) = k$. Any standard interpolation function would give $i(x, y) = k$ for non-integer x and y (irrespective oh how coarse the sampling is)

Sampling Theory (informal)

Case 0: Suppose $i(x, y)$ has a discontinuity not falling precisely at integer x, y



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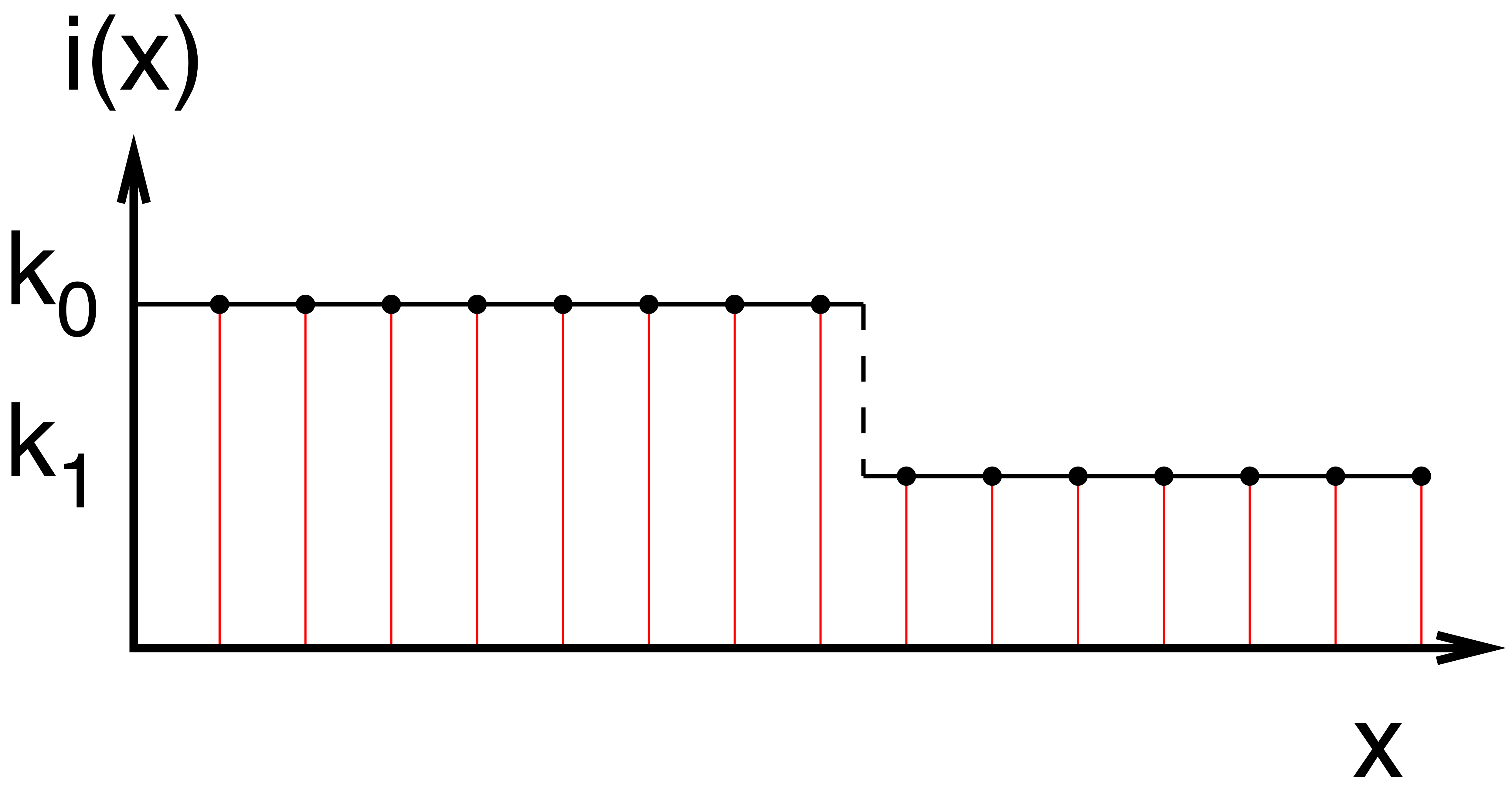

We cannot reconstruct $i(x, y)$ exactly because we can never know exactly where the discontinuity lies

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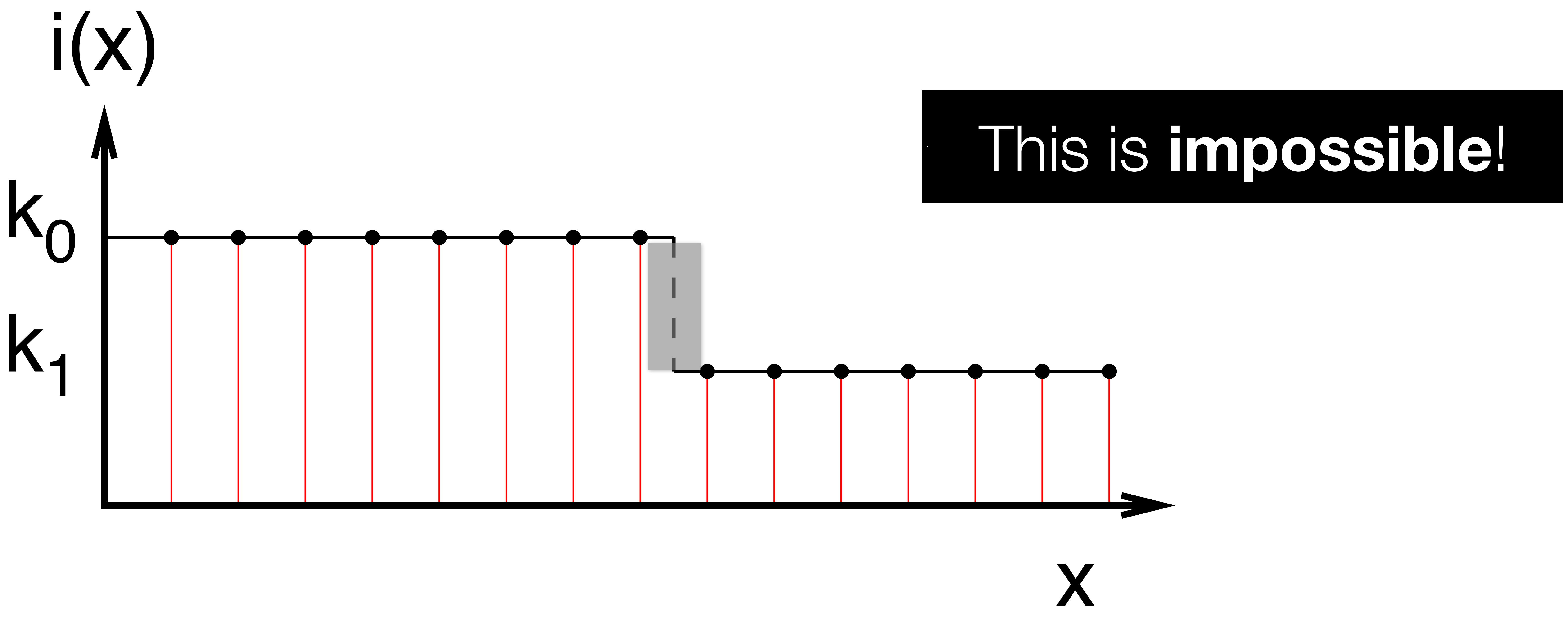


We cannot reconstruct $i(x, y)$ exactly because we can never know exactly where the discontinuity lies

Sampling Theory (informal)

Question: How do we close the gap between “**easy**” and “**impossible**?”

Next, we build intuition based on informal argument

Sampling Theory (informal)

Exact reconstruction requires constraint on the rate at which i(x,y) can change between samples

— “rate of change” means derivative

— the formal concept is **bandlimited signal**

— “bandlimit” and “constraint on derivative” are linked

Think of music

— bandlimited if it has some maximum **temporal frequency**

— the upper limit of human hearing is about 20 kHz

Think of imaging systems. Resolving power is measured in

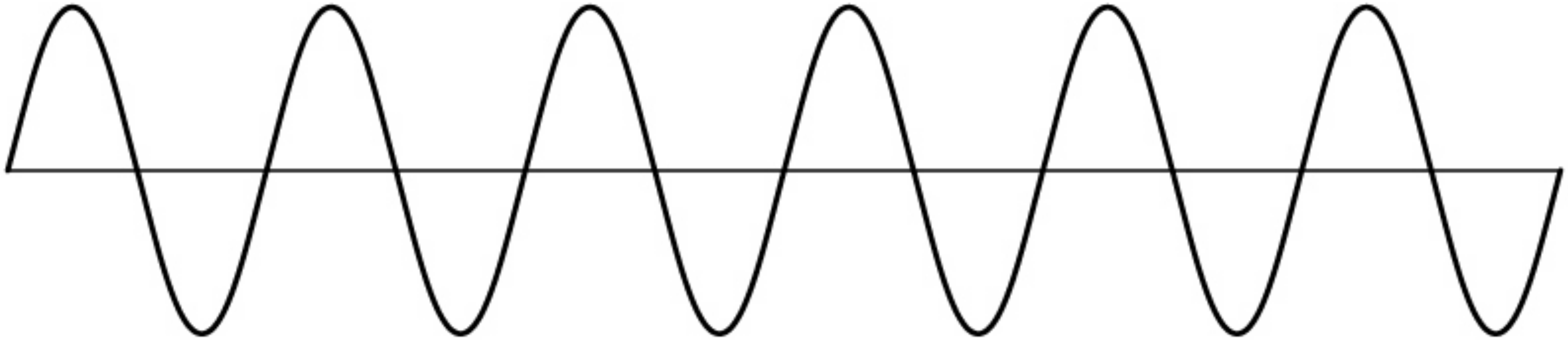
— “line pairs per mm” (for a bar test pattern)

— “cycles per mm” (for a sine wave test pattern)

An image is bandlimited if it has some maximum **spatial frequency**

Example: A Simple Sine Wave

How do we discretize the signal?



Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

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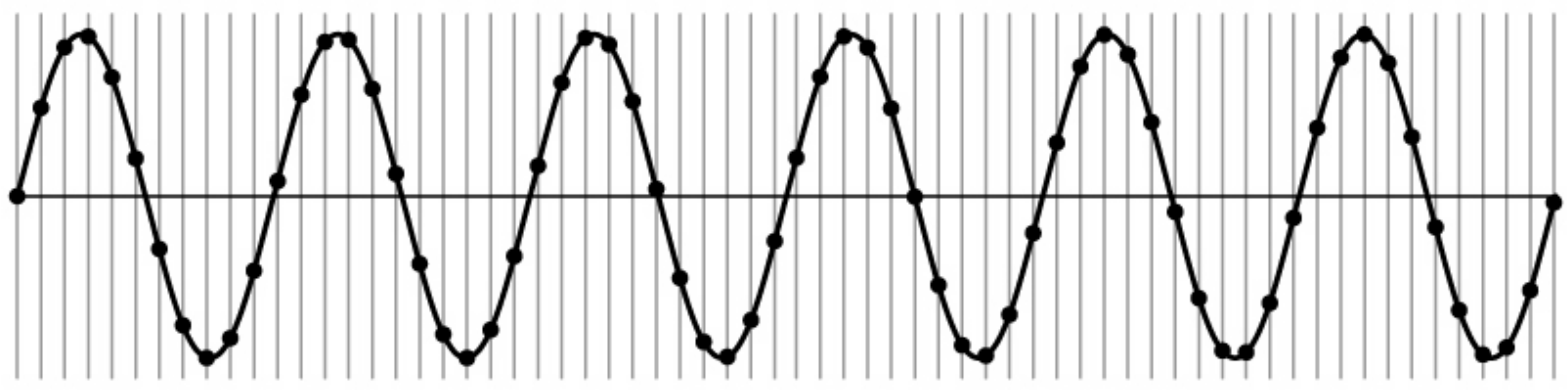

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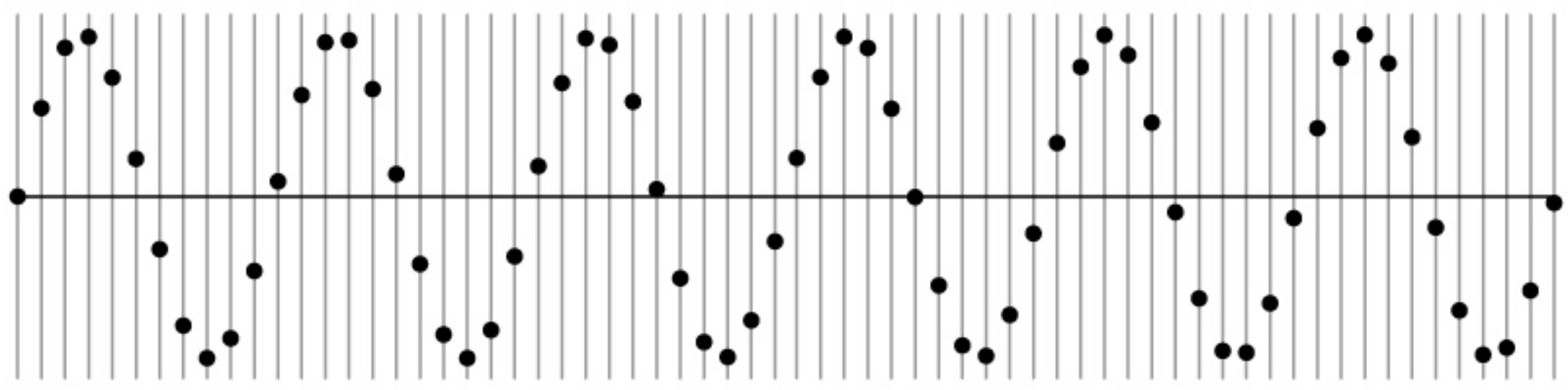

How many samples should I take?

Can I take as many samples as I want?

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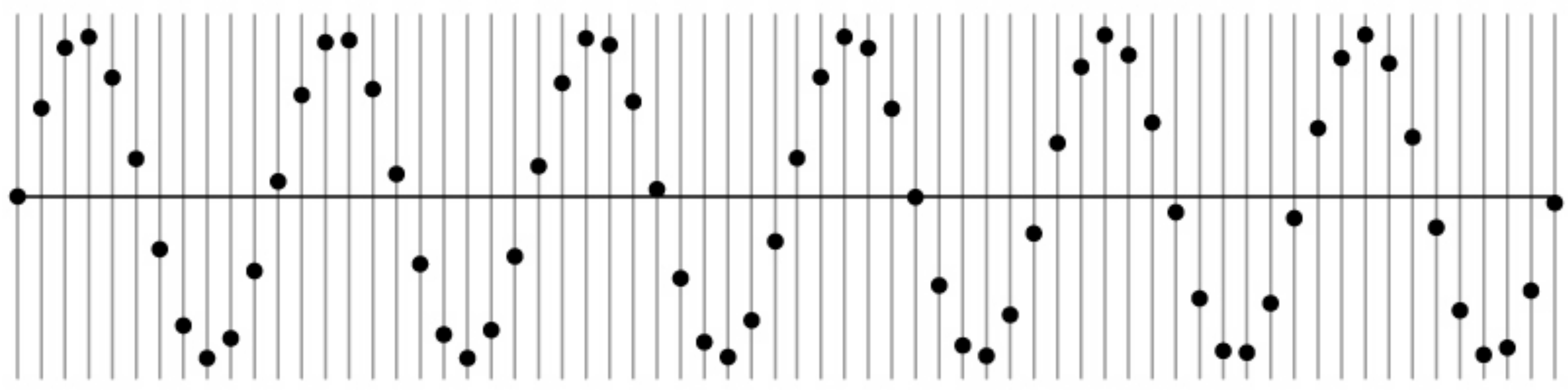

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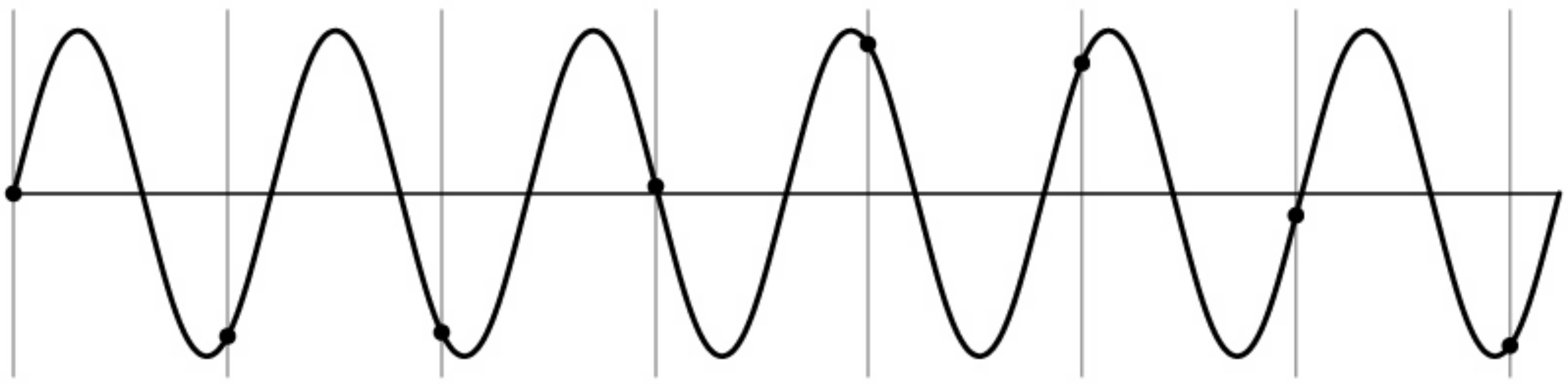

Signal can be confused with one at lower frequency

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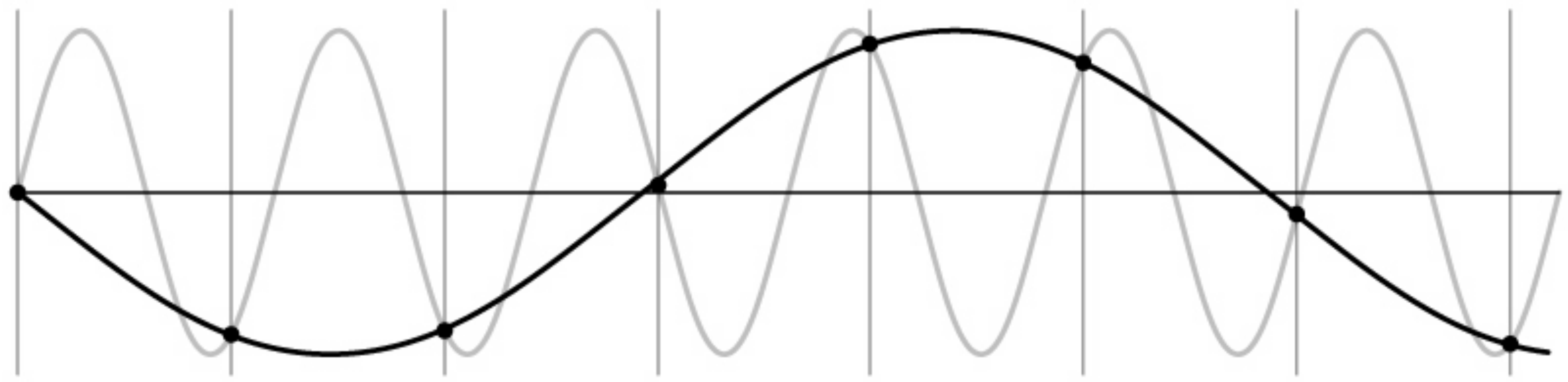

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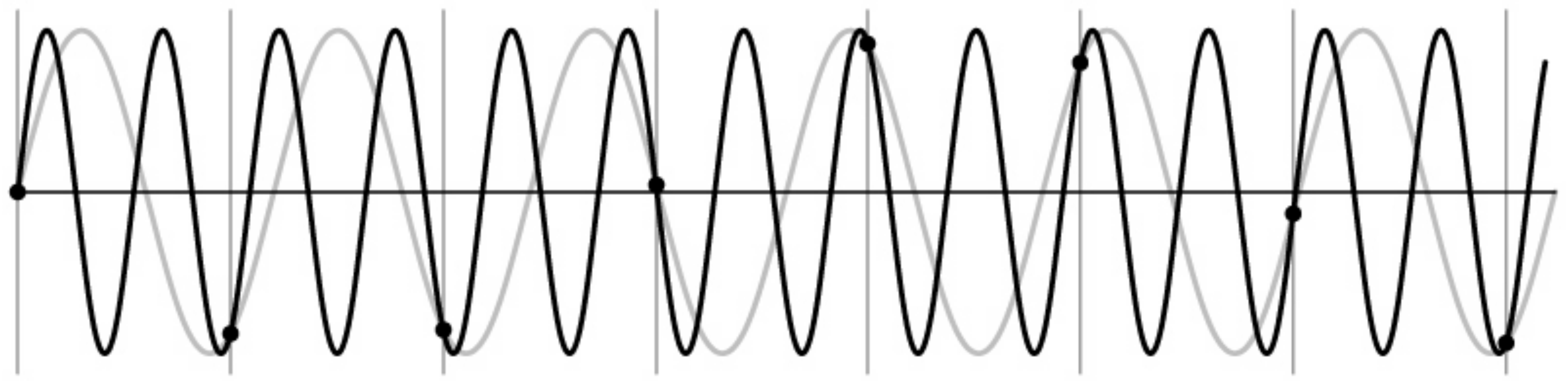
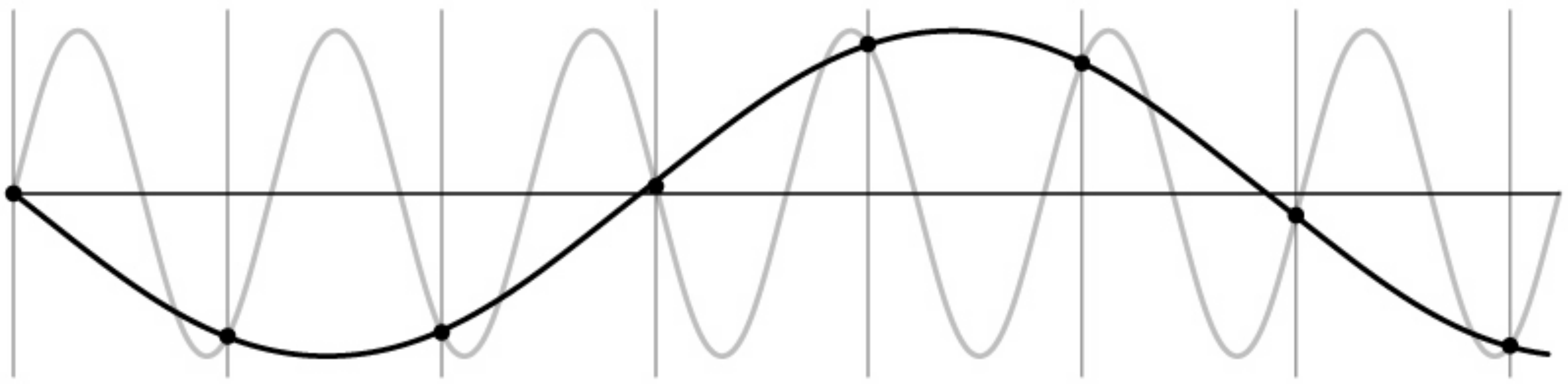


Signal can always be confused with one at higher frequency

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Undersampling = **Aliasing**



Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Sampling Theory (informal)

The challenge to intuition is the fact that music (in the 1D case) and images (in the 2D case) can be represented as linear combinations of individual sine waves of differing frequencies and phases (remember discussion on FFTs)

A fundamental result (**Sampling Theorem**) is:

> For bandlimited signals, if you sample regularly at or above twice the maximum frequency (called the **Nyquist rate**), then you can reconstruct the original signal exactly

Sampling Theory (informal)

Question: For a bandlimited signal, what if you **oversample** (i.e., sample at greater than the Nyquist rate)

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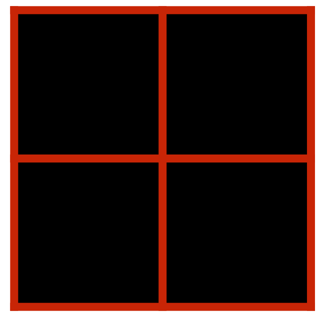
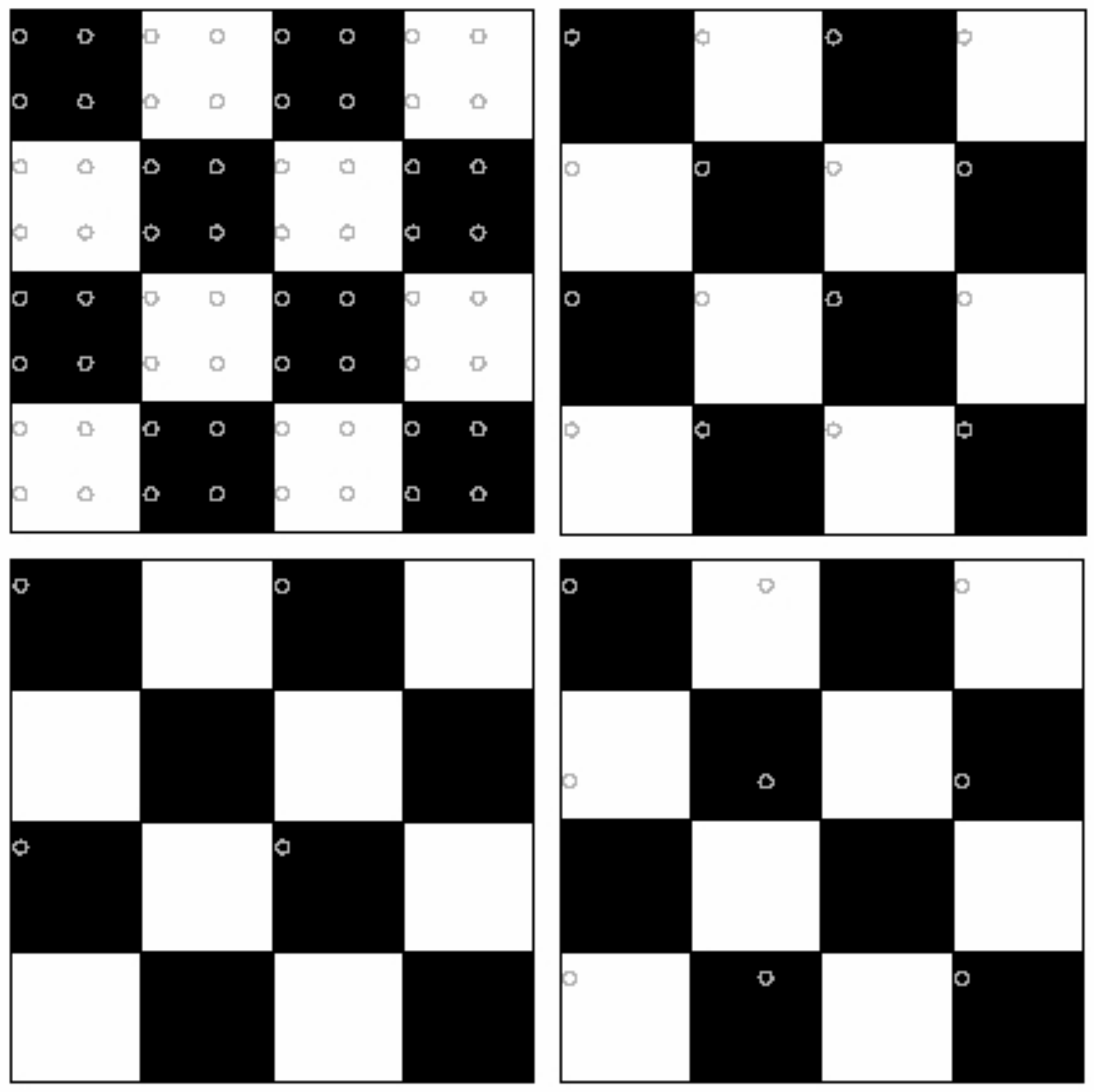
Answer: Nothing bad happens! Samples are redundant and there are wasted bits

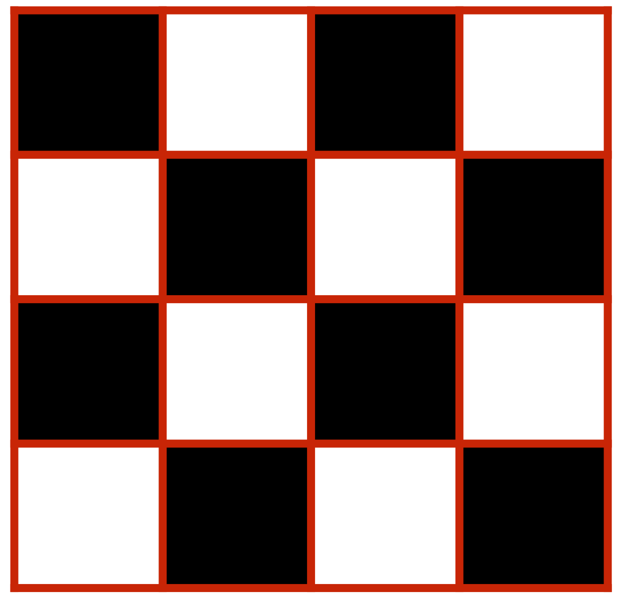
Question: For a bandlimited signal, what if you **undersample** (i.e., sample at less than the Nyquist rate)

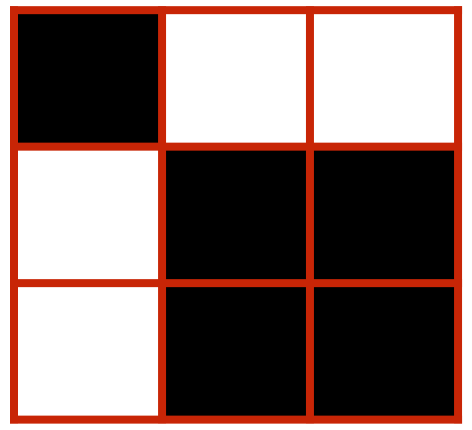
Answer: Two bad things happen! Things are missing (i.e., things that should be there aren't). There are artifacts (i.e., things that shouldn't be there are)

Sampling Theory (informal)



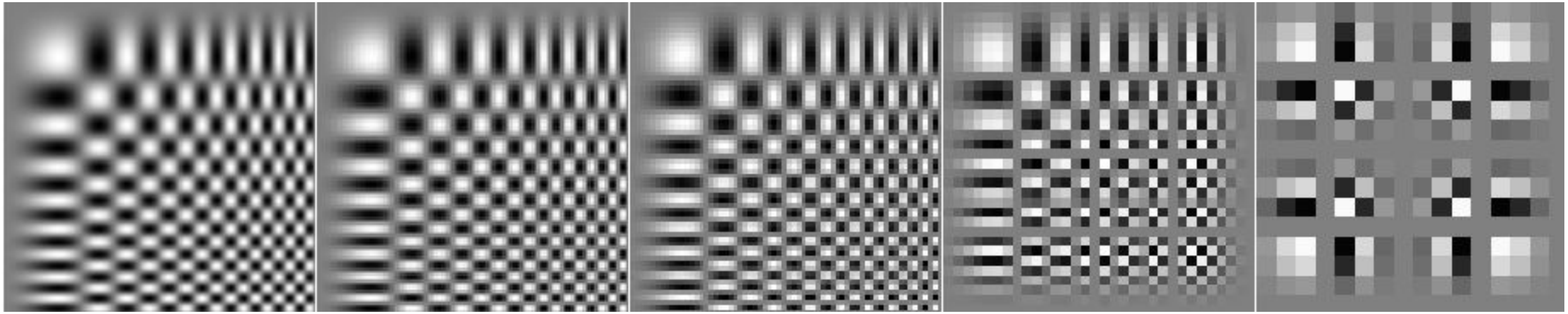






Forsyth & Ponce (2nd ed.) Figure 4.7

Sampling Theory (informal)

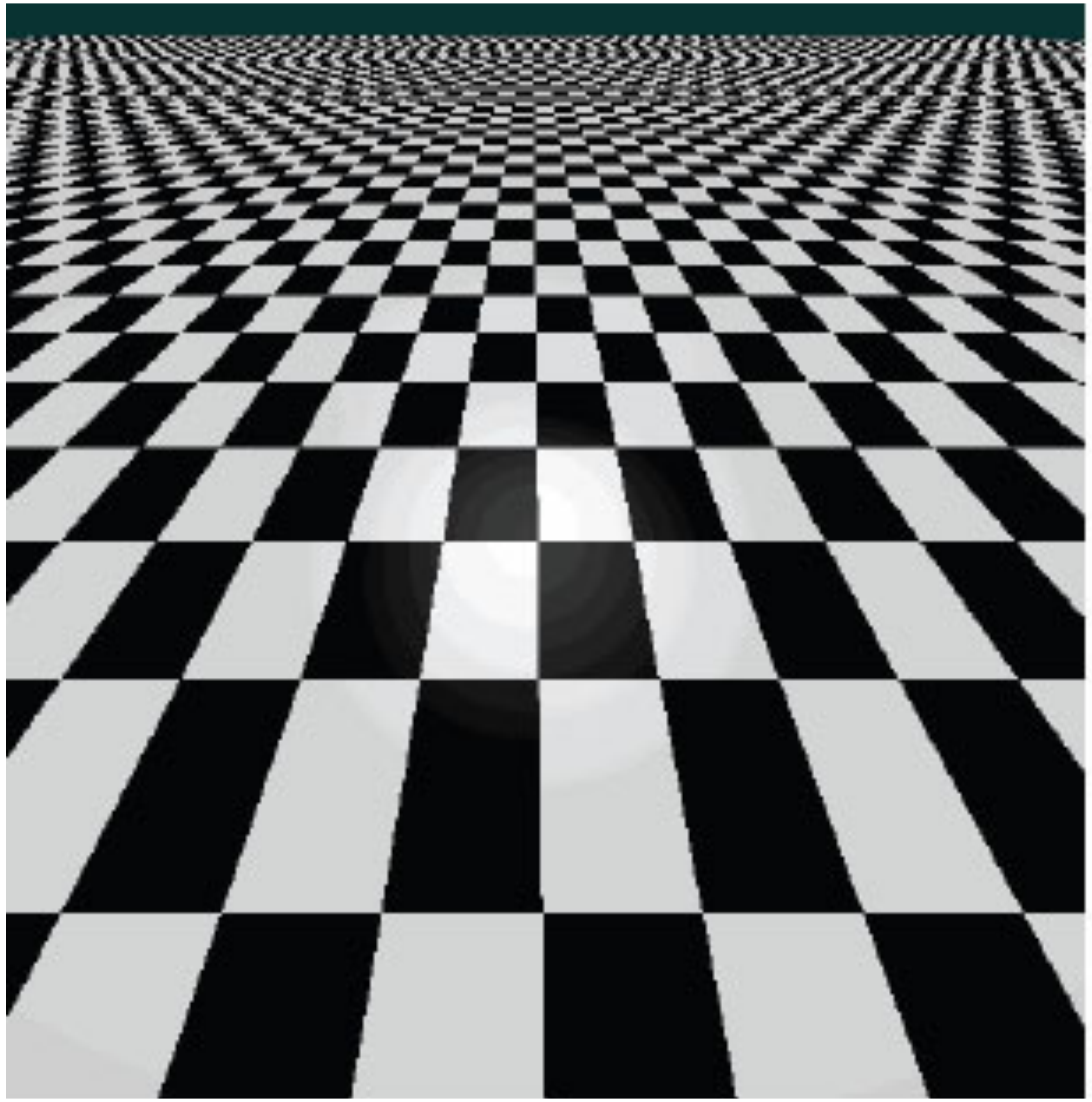


Forsyth & Ponce (2nd ed.) Figure 4.12

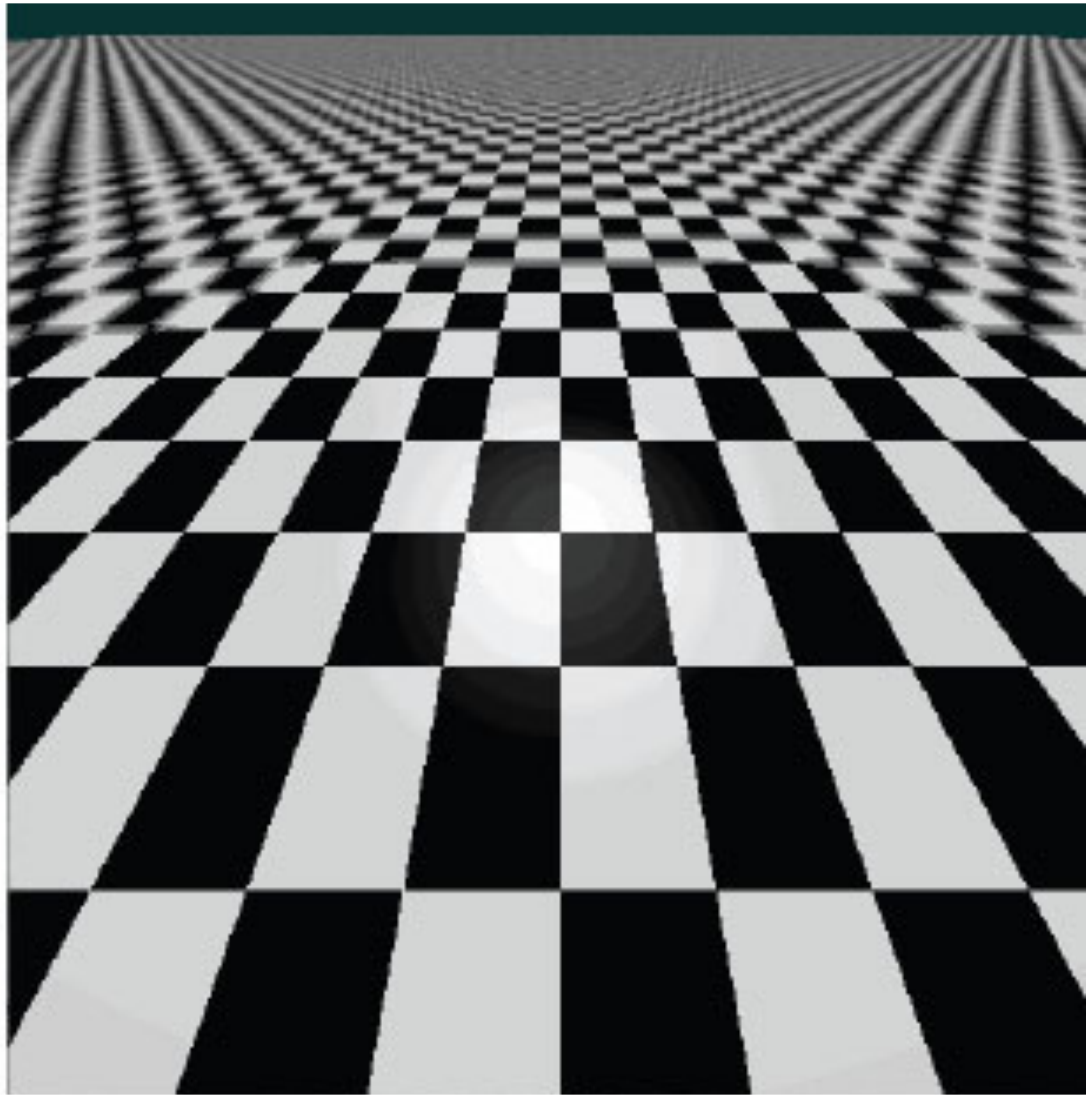
Reducing Aliasing Artifacts

1. **Oversampling** — sample more than you think you need and average (i.e., area sampling)

Aliasing



aliasing artifacts



anti-aliasing by oversampling

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Reducing Aliasing Artifacts

1. **Oversampling** — sample more than you think you need and average (i.e., area sampling)

2. **Smoothing** before sampling. Why?

Aliasing in Photographs

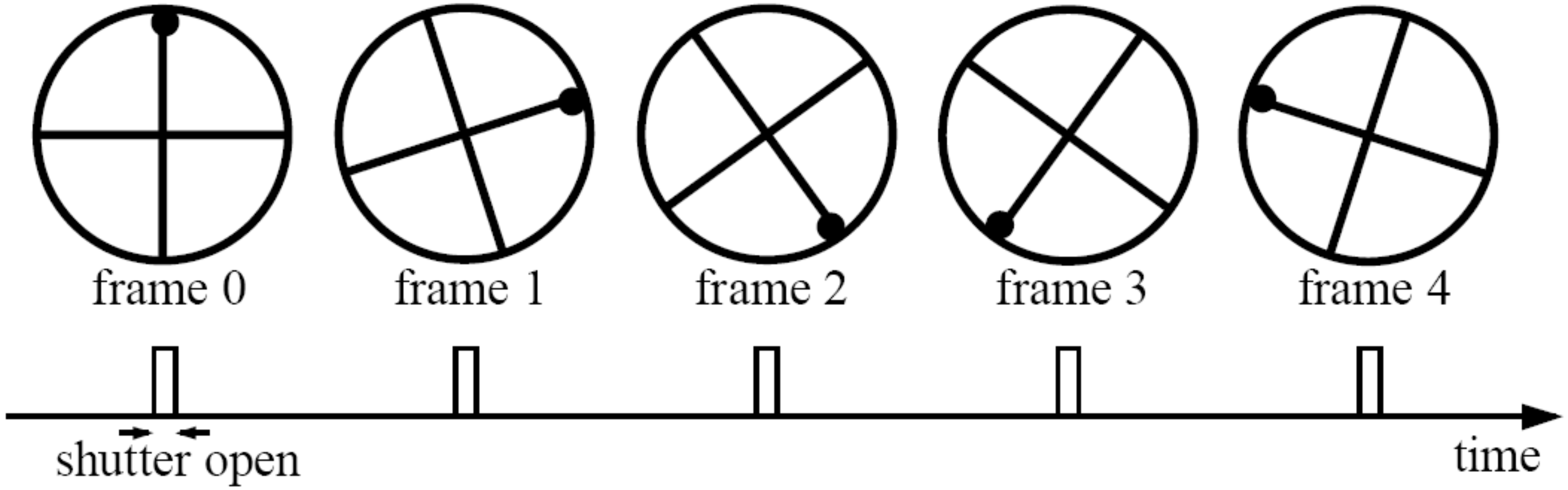
This is also known as “moire”

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Temporal Aliasing

Imagine a spoked wheel moving to the right (rotating clockwise). Mark wheel with dot so we can see what's happening.

If camera shutter is only open for a fraction of a frame time (frame time = 1/30 sec. for video, 1/24 sec. for film):



Without dot, wheel appears to be rotating slowly backwards! (counterclockwise)

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Temporal Aliasing

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Temporal Aliasing

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Temporal Aliasing

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Temporal Aliasing

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Sampling Theory (informal)

Sometimes **undersampling** is unavoidable, and there is a trade-off between "things missing" and "artifacts."

— **Medical imaging**: usually try to maximize information content, tolerate some artifacts

— **Computer graphics**: usually try to minimize artifacts, tolerate some information missing

Review: Continuous Case

— Images also can be considered a function of time. Then, we write $i(x, y, t)$ where x and y are spatial variable and t is a **temporal variable**

— To make the dependence of brightness on wavelength explicit, we can instead write $i(x, y, t, \lambda)$ where x, y and t are as above and where λ is a **spectral variable**

— More commonly, we think of "color" already as discrete and write

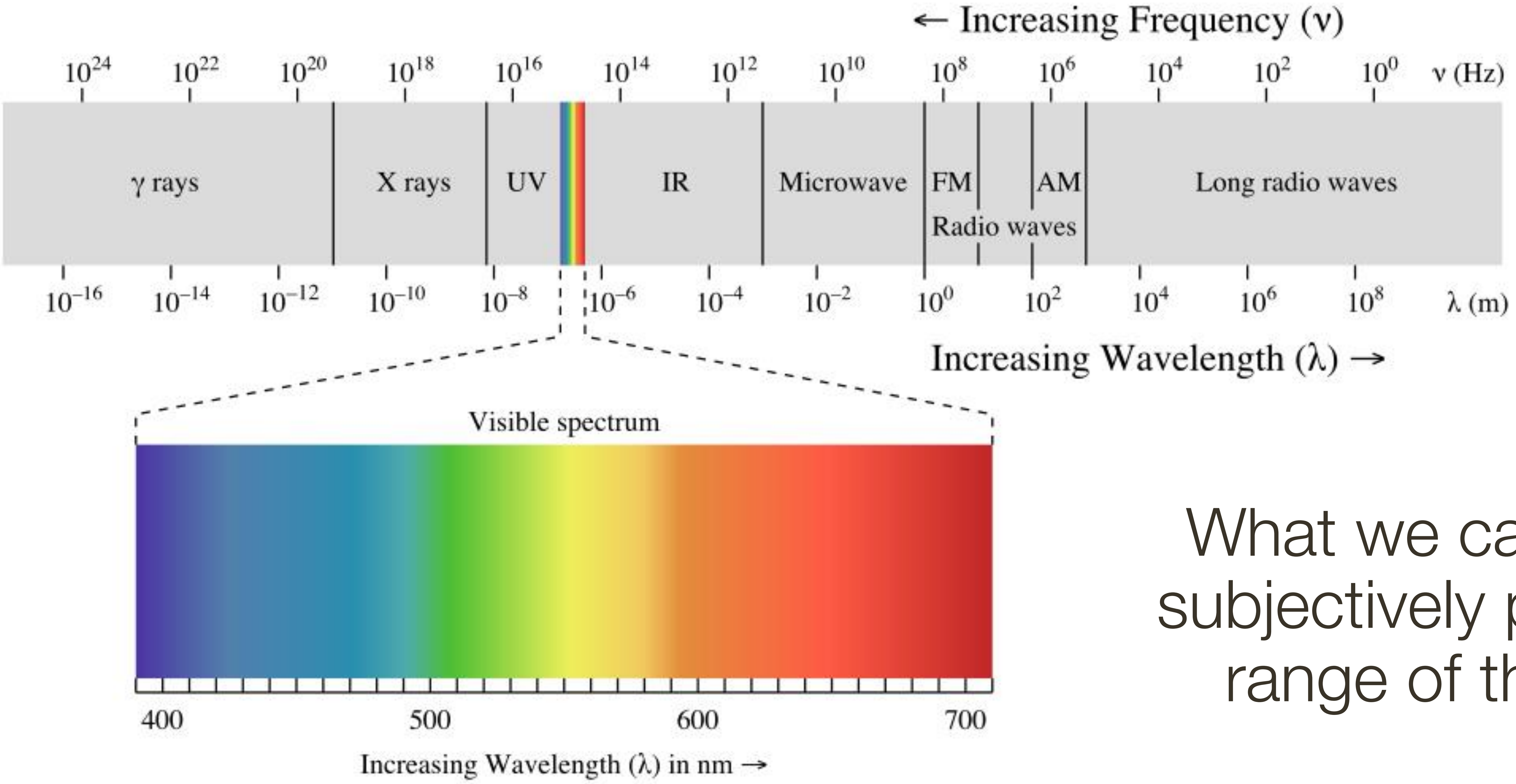
$$i_R(x, y)$$
$$i_G(x, y)$$
$$i_B(x, y)$$

for specific colour channels, R, G and B

Color is an Artifact of Human Perception

"Color" is **not** an objective physical property of light (electromagnetic radiation).

Instead, light is characterized by its wavelength.

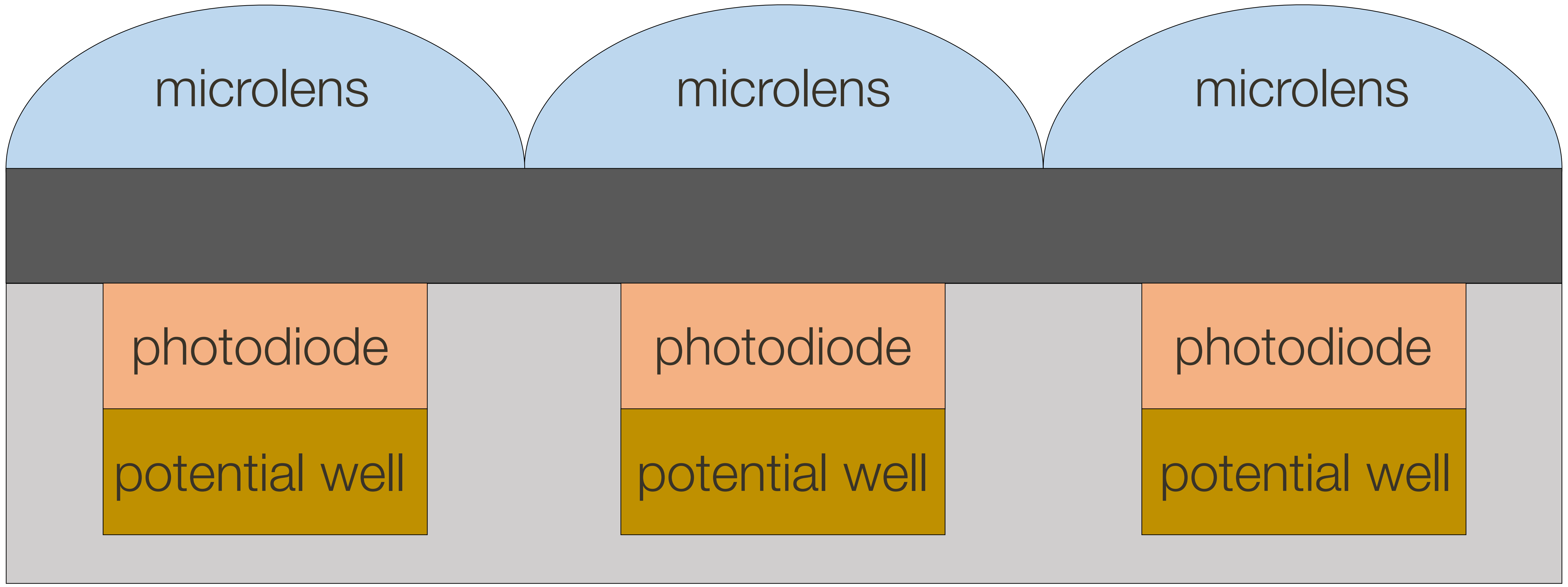


electromagnetic spectrum

What we call "color" is how we subjectively perceive a very small range of these wavelengths.

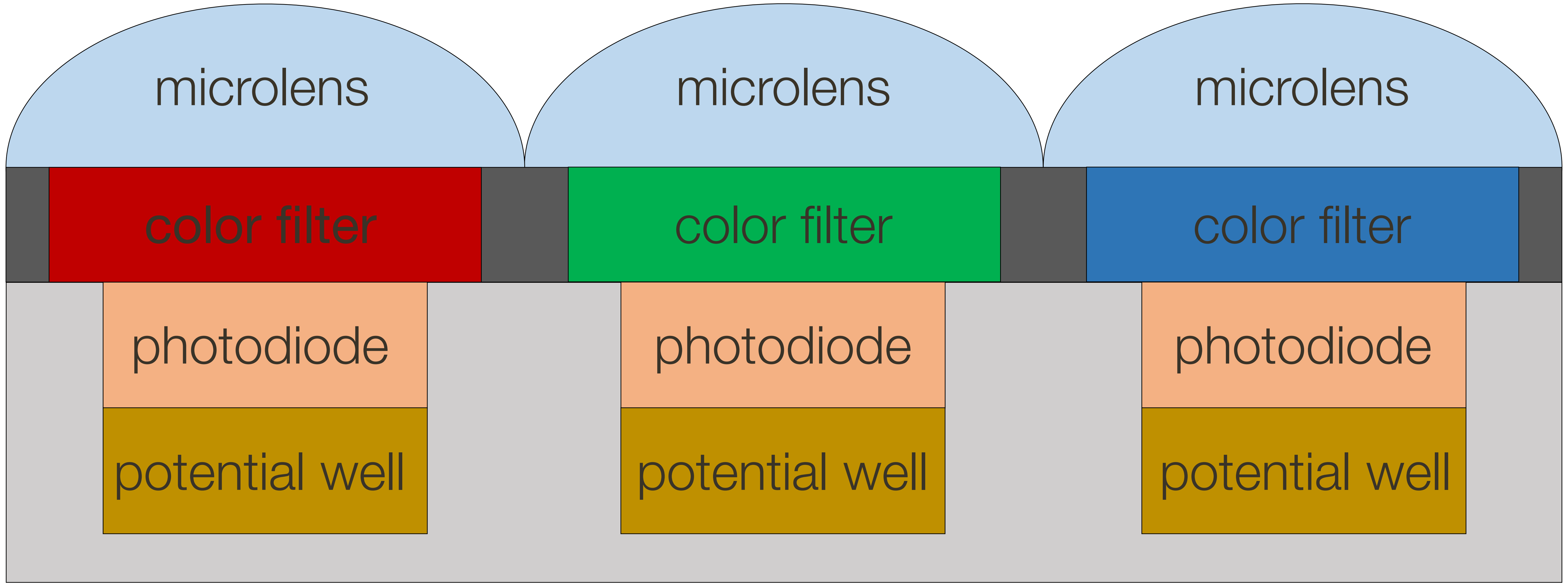
Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Color Filter Arrays (CFA)



Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Color Filter Arrays (CFA)



Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Color Filters

Two **design choices**:

— What spectral sensitivity functions $f(\lambda)$ to use for each color filter?

— How to spatially arrange ("**mosaic**") different color filters?

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Color Filters

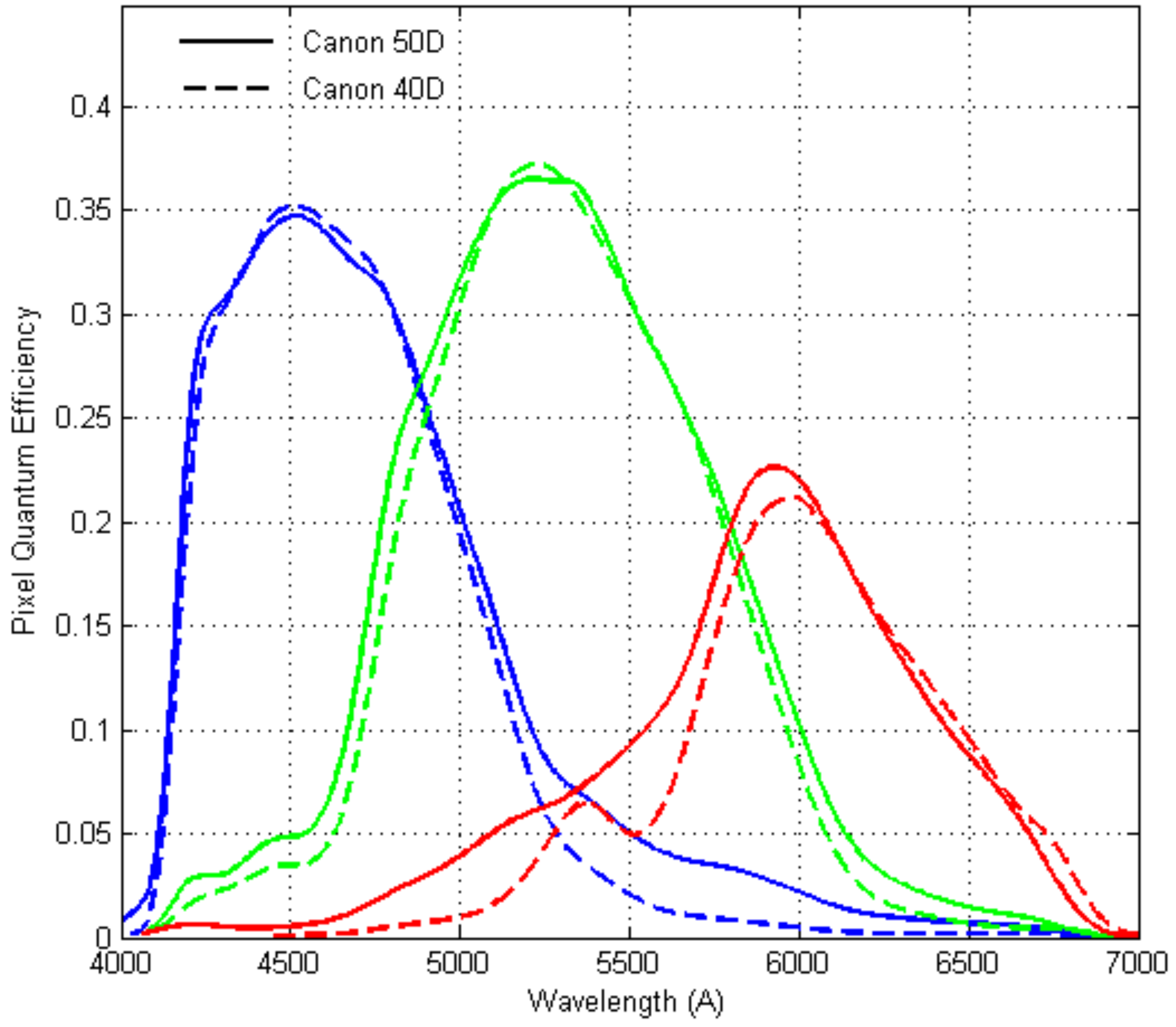
Two **design choices**:

— What spectral sensitivity functions $f(\lambda)$ to use for each color filter?

— How to spatially arrange (“**mosaic**”) different color filters?

Canon 50D

Generally do not match human sensitivity



$f(\lambda)$

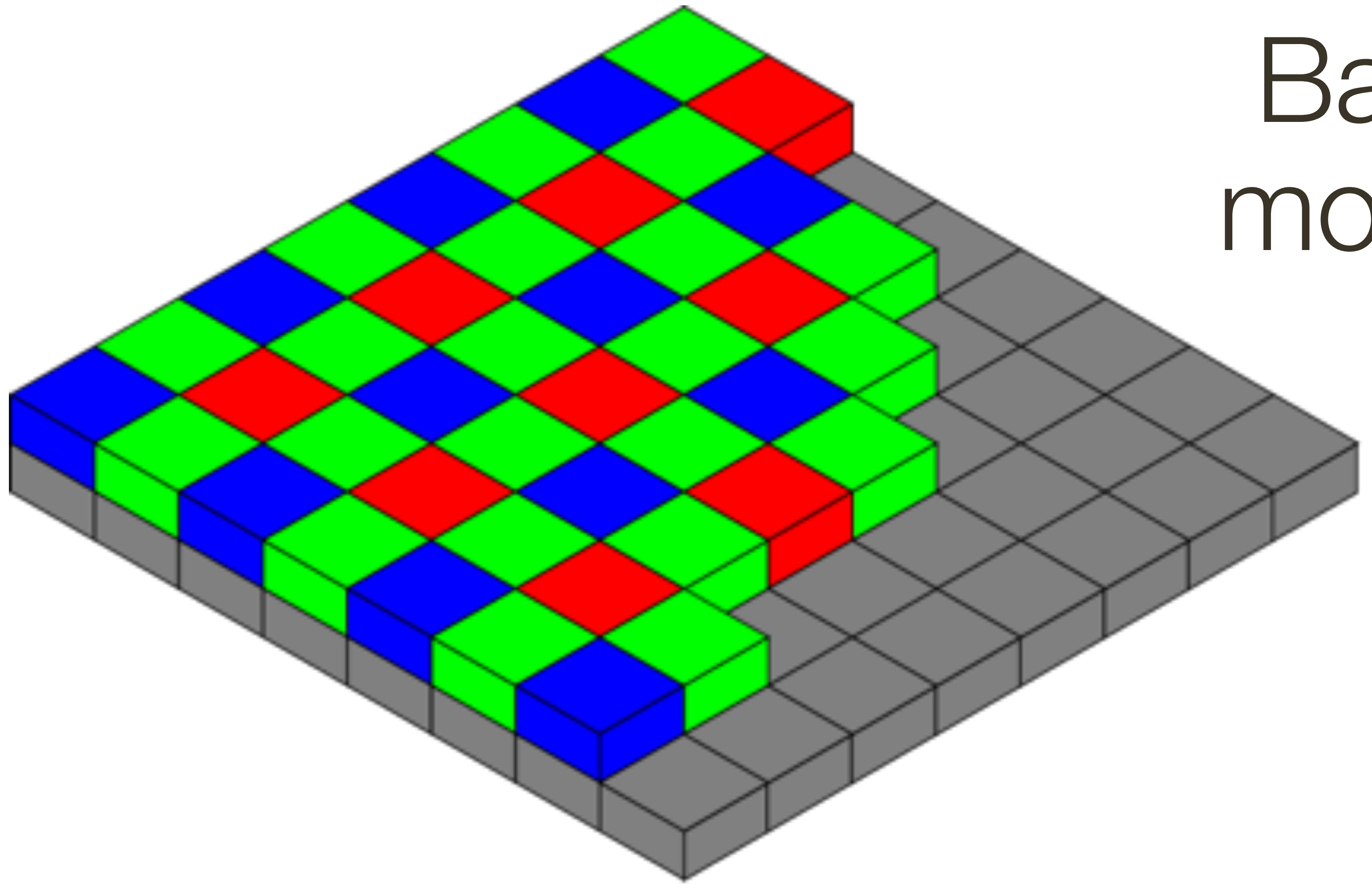
Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Color Filters

Two **design choices**:

— What spectral sensitivity functions $f(\lambda)$ to use for each color filter?

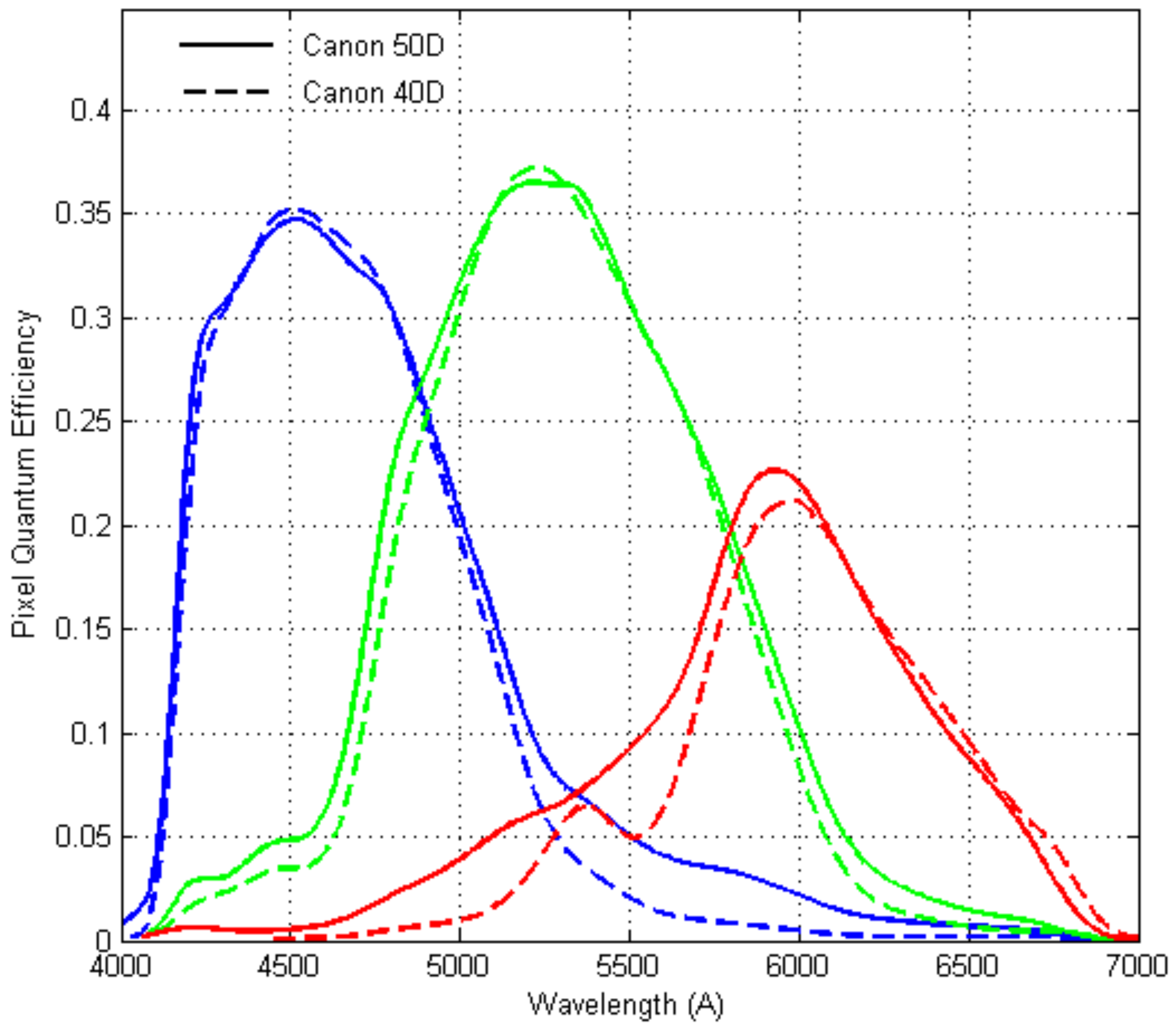
— How to spatially arrange ("**mosaic**") different color filters?



Bayer mosaic

Generally do not match human sensitivity

Canon 50D



$f(\lambda)$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Color Filters

Two **design choices**:

— What spectral sensitivity functions $f(\lambda)$ to use for each color filter?

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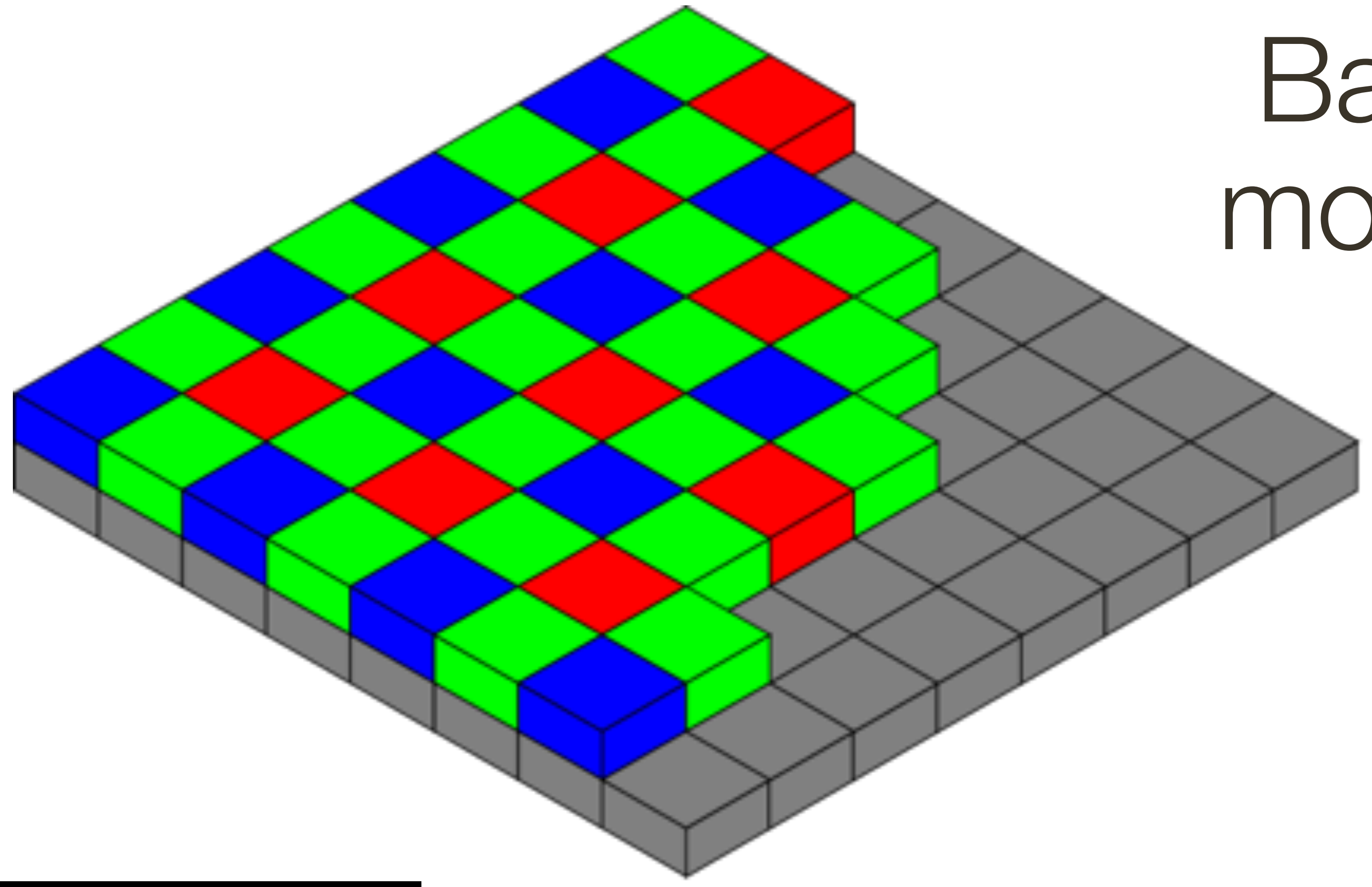
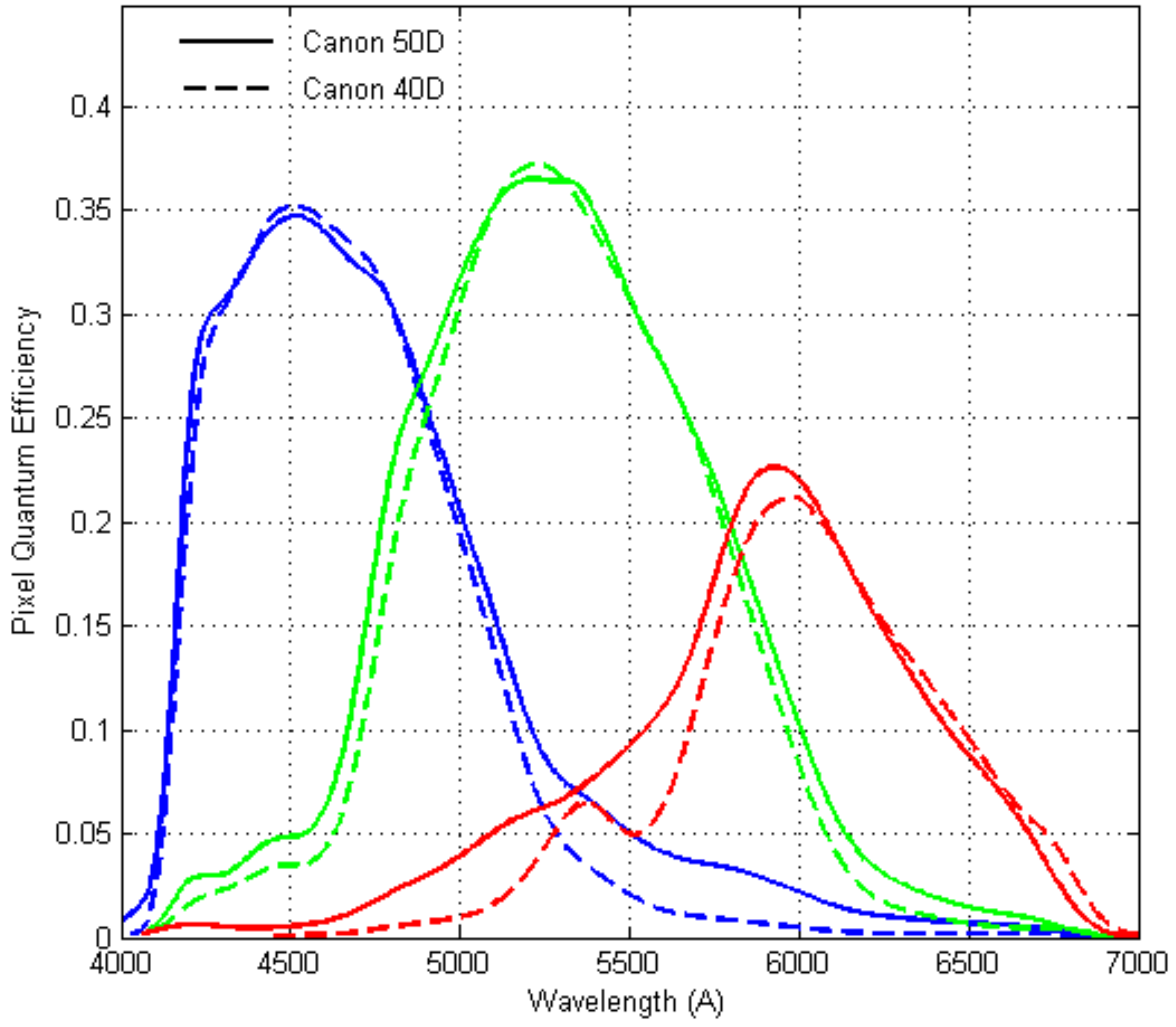


Bayer mosaic

Why more green pixels?

Canon 50D



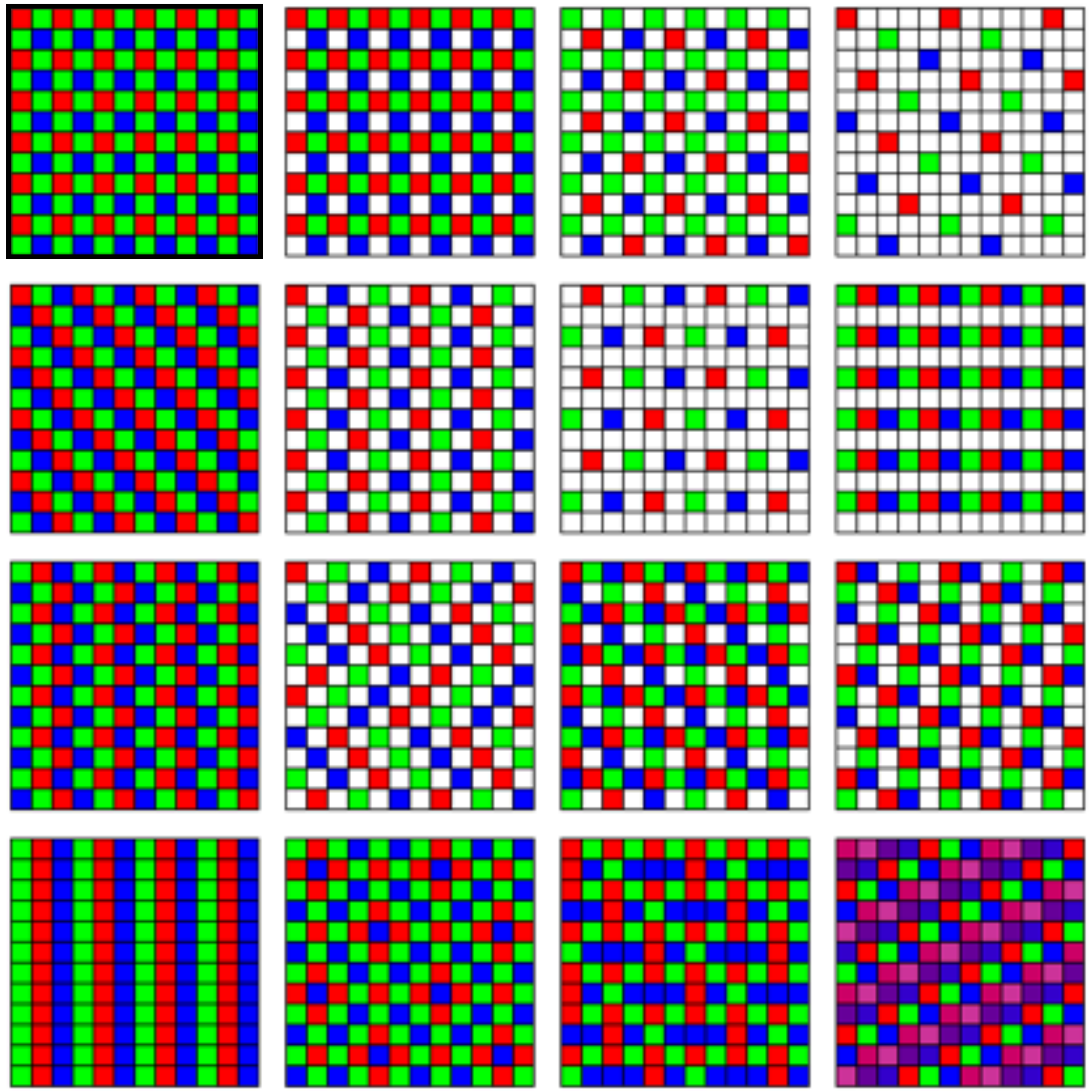
$f(\lambda)$

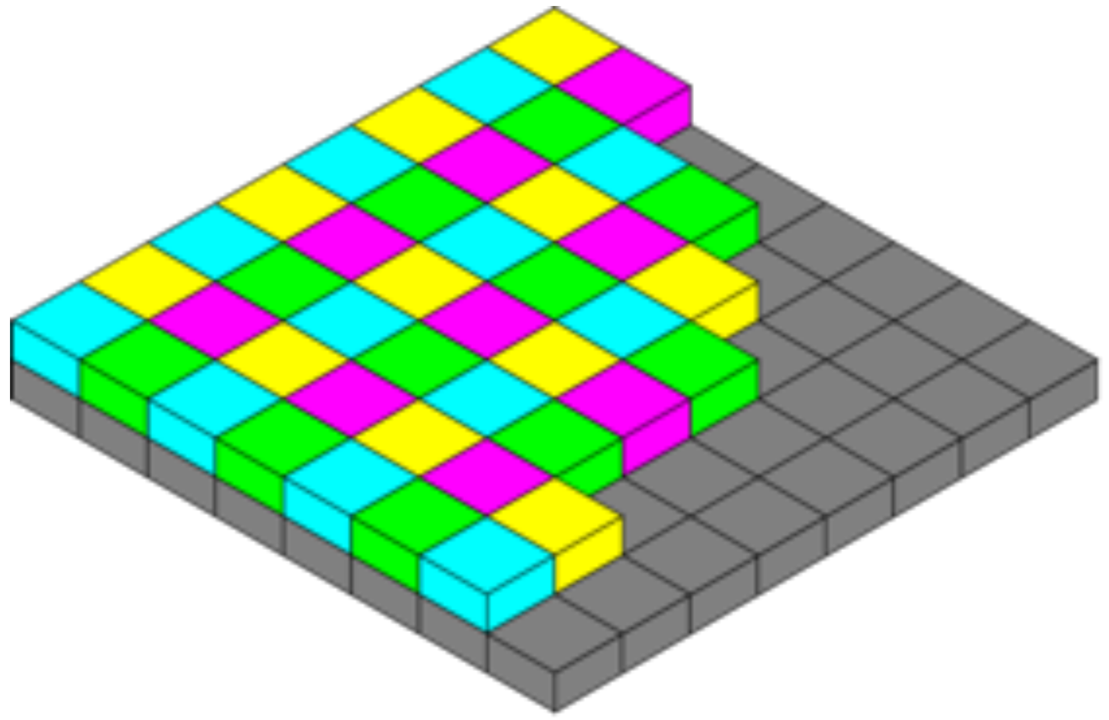
Generally do not match human sensitivity

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

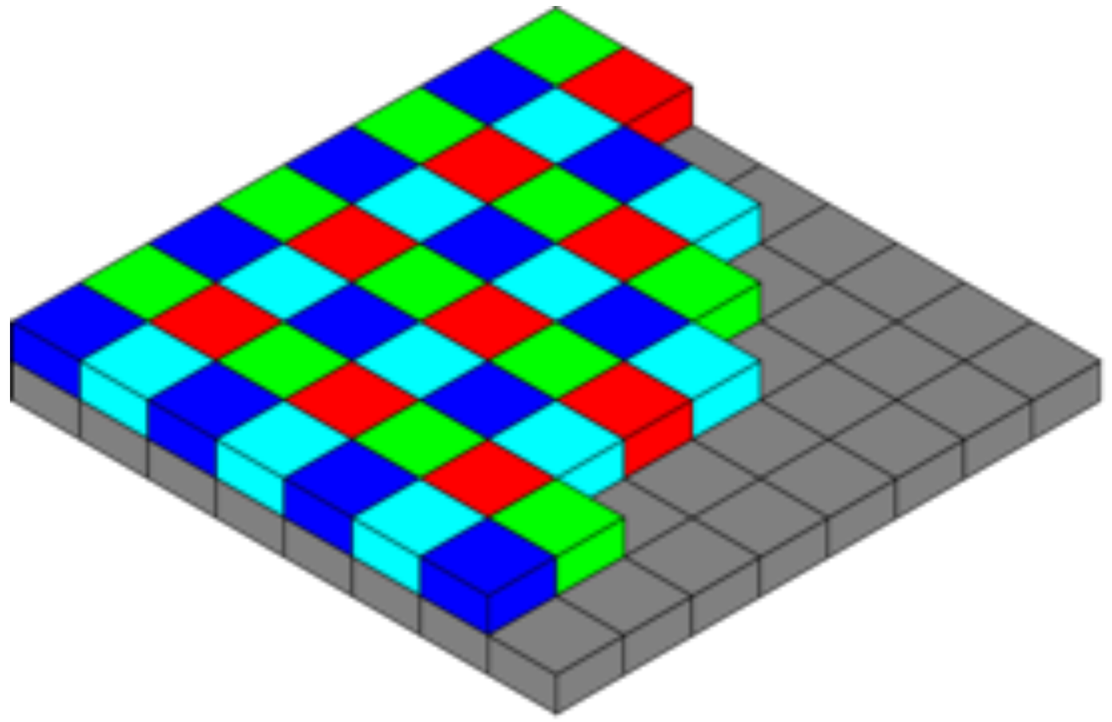
Different Color Filter Arrays (CFAs)

Finding the "**best**" CFA mosaic is an active research area.





CYGM
Canon IXUS, Powershot



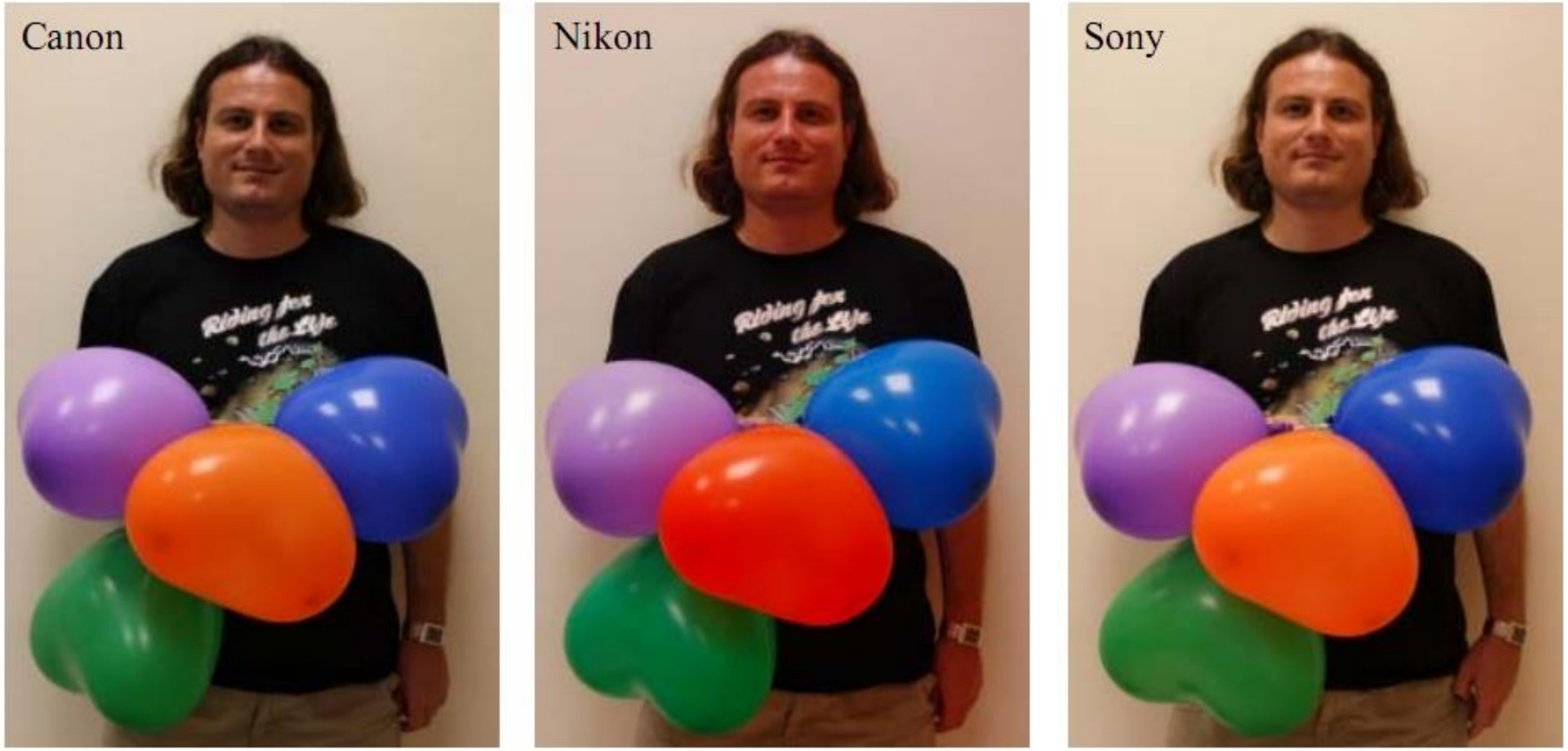
RGBE
Sony Cyber-shot

How would you go about designing your own CFA? What criteria would you consider?

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Many **Different Spectral Sensitivity** Functions

Each camera has its more or less unique, and most of the time secret, SSF

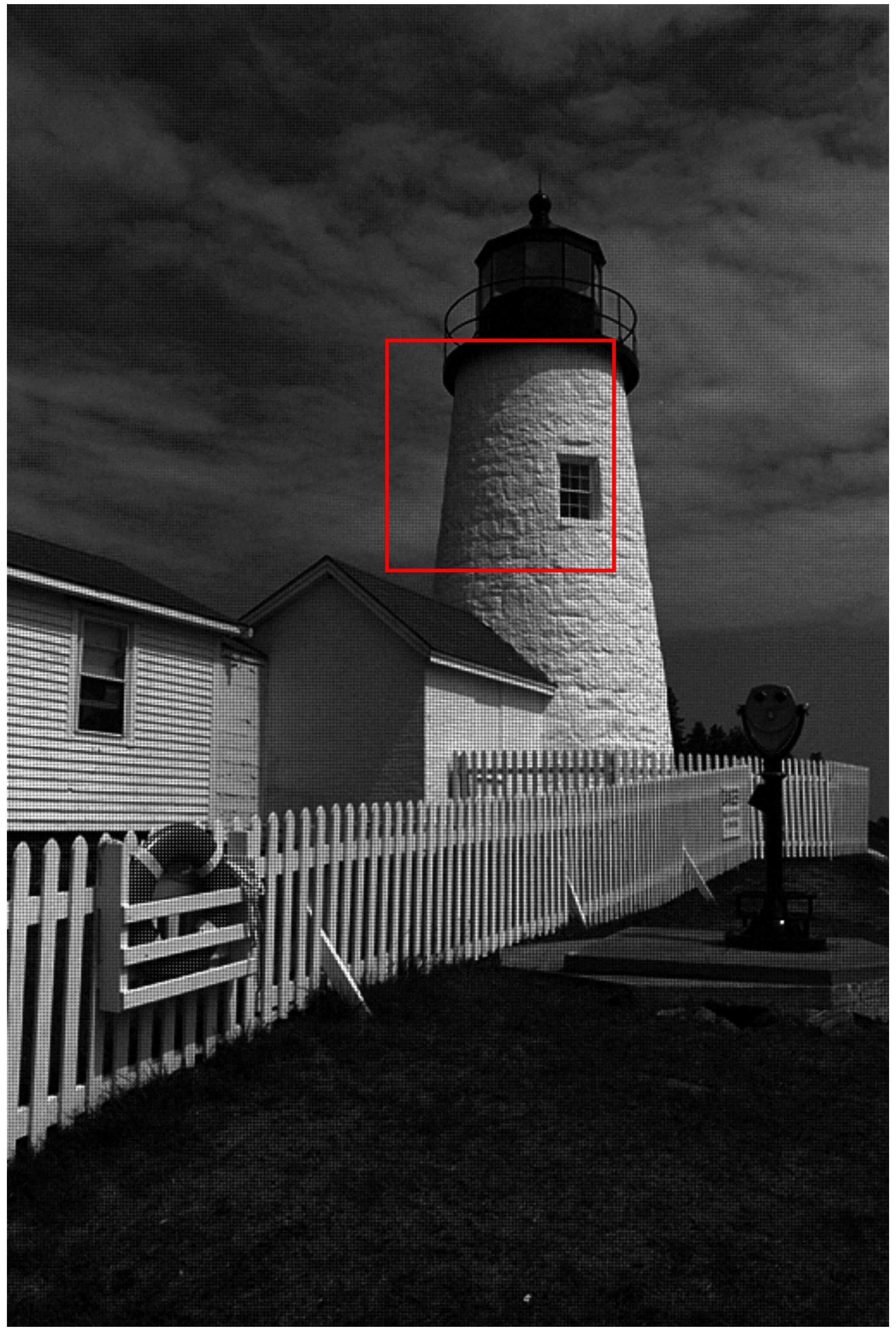


Same scene captured using 3 different cameras with identical settings

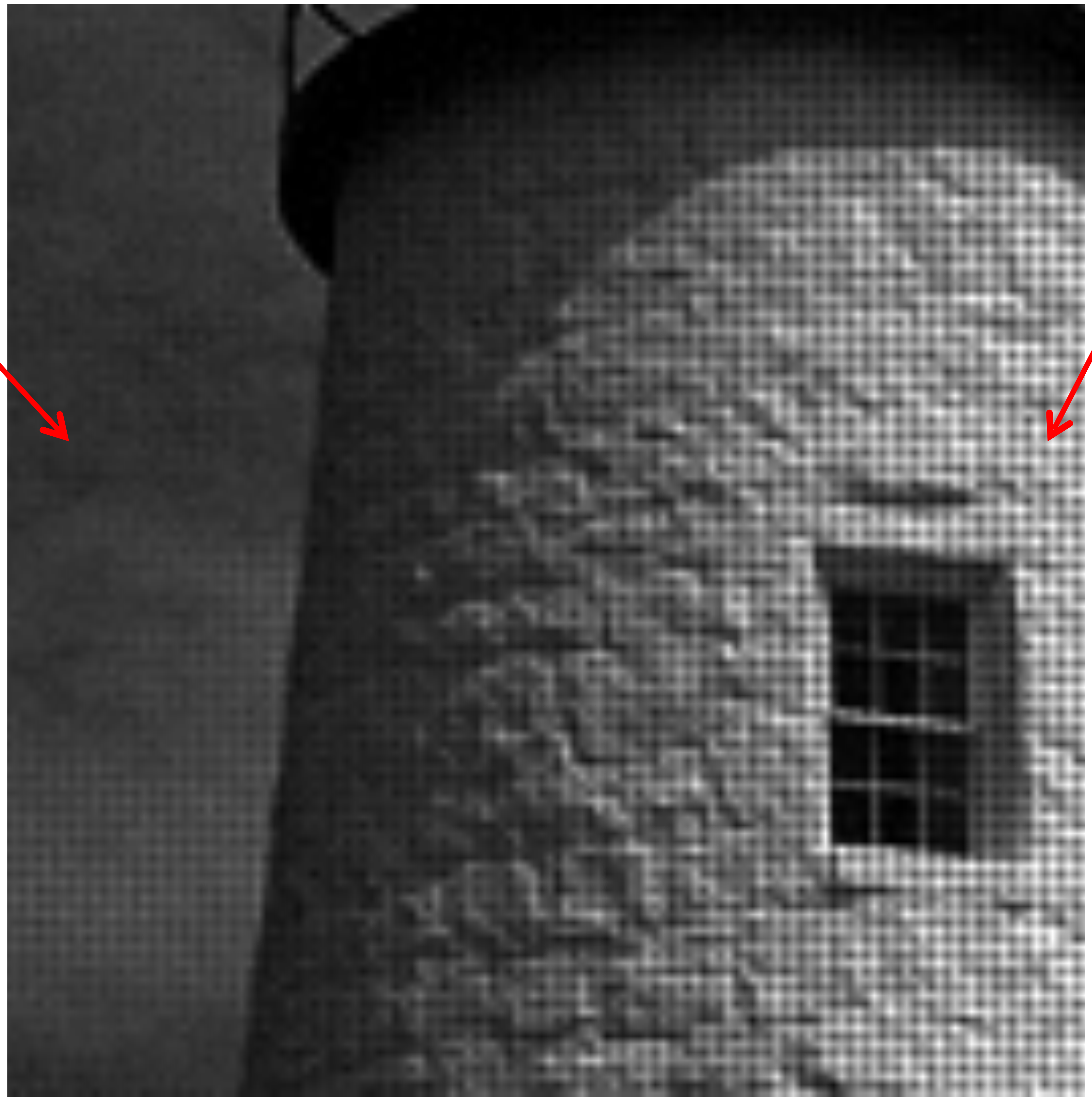
Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

RAW Bayer Image

After all of this, what does an image look like?



lots of noise



mosaicking artifacts

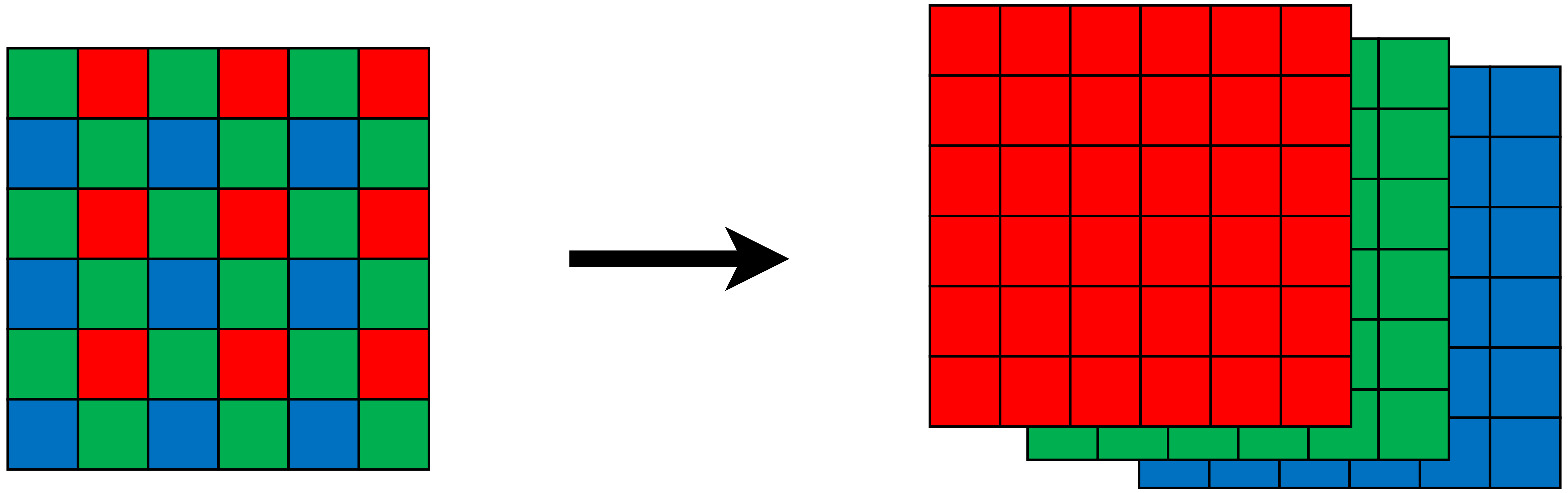
— Kind of disappointing

— We call this the RAW image

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

CFA **Demosicing**

Produce full RGB image from mosaiced sensor output

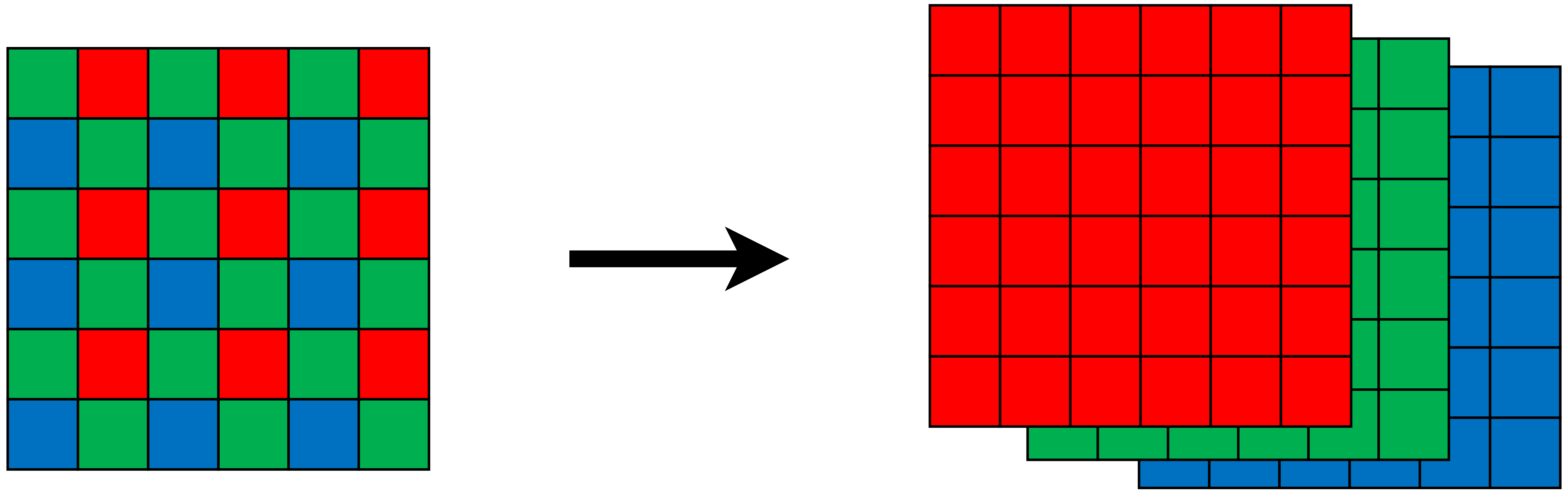


Any ideas on how to do this?

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

CFA **Demosicing**

Produce full RGB image from mosaiced sensor output



Interpolate from neighbors:

— Bilinear interpolation (needs 4 neighbors)

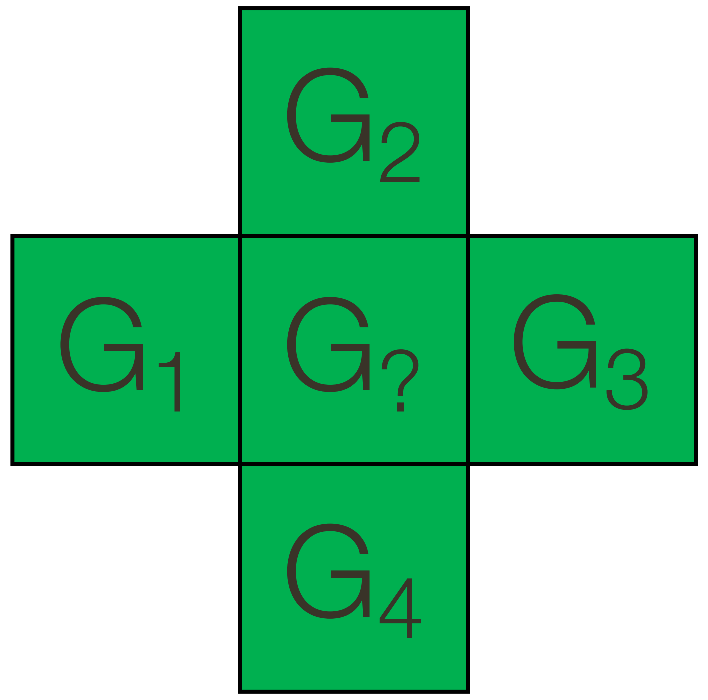
— Bicubic interpolation (needs more neighbors, may overblur)

— Edge-aware interpolation

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

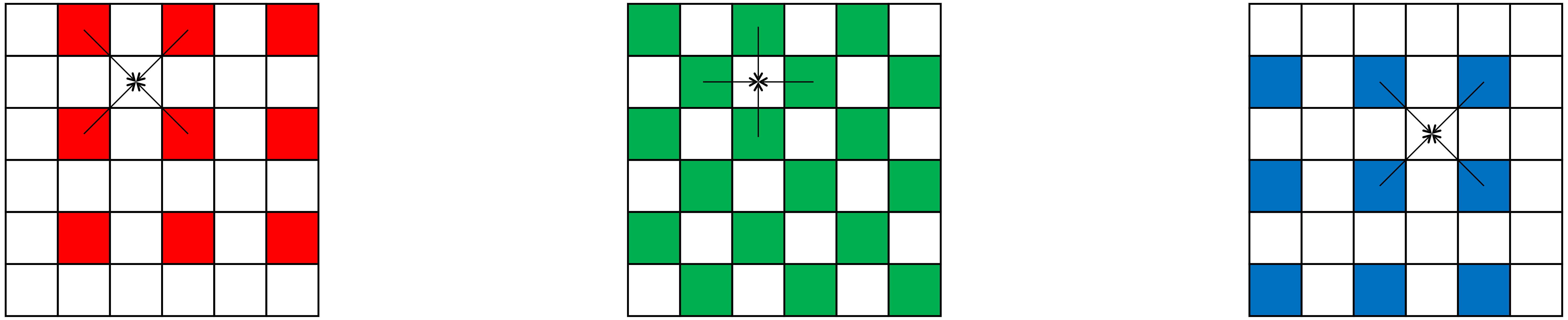
Demosaicing by Bilinear Interpolation

Bilinear interpolation: Simply average your 4 neighbors.



$$G_? = \frac{G_1 + G_2 + G_3 + G_4}{4}$$

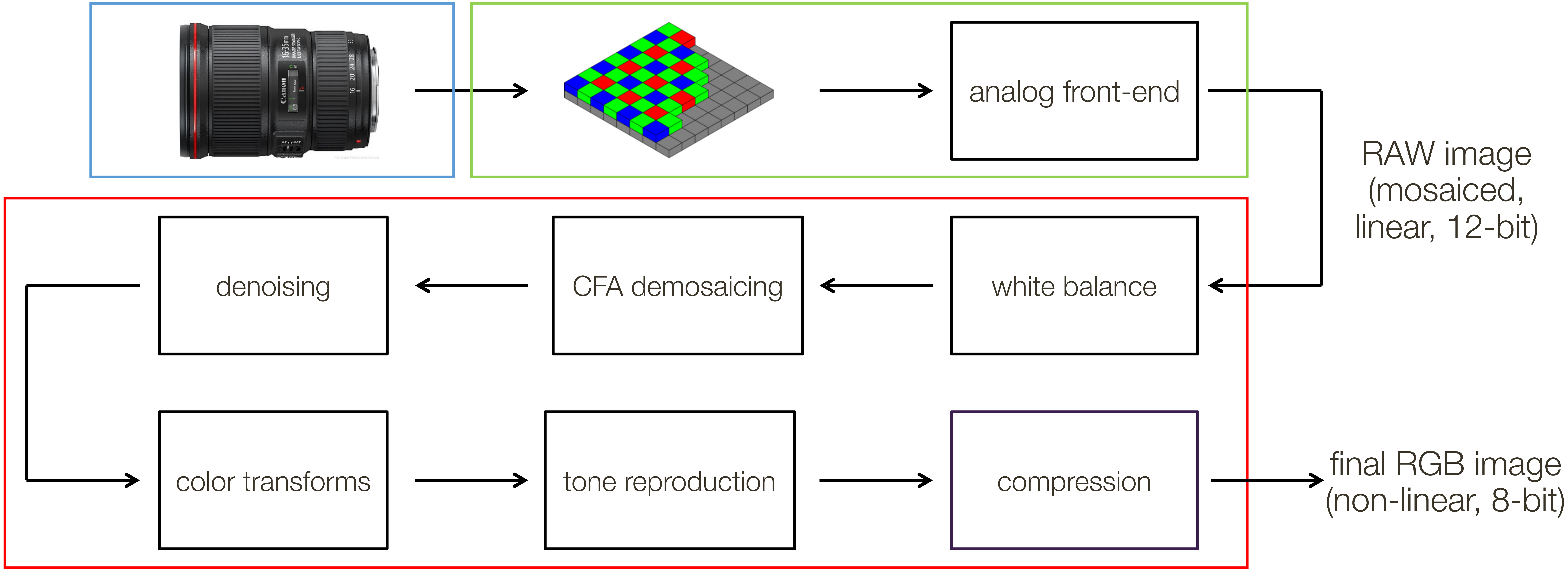
Neighborhood changes for different channels:



Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

(in camera) **Image** Processing Pipeline

The sequence of image processing operations applied by the camera's image signal processor (ISP) to convert a RAW image into a "conventional" image.



Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Summary

In the continuous case, images are functions of two spatial variables, x and y.

The discrete case is obtained from the continuous case via sampling (i.e. tessellation, quantization).

If a signal is **bandlimited** then it is possible to design a sampling strategy such that the sampled signal captures the underlying continuous signal exactly.

Adequate sampling may not always be practical. In such cases there is a trade-off between “things missing” and “artifacts”.

— Different applications make the trade-off differently