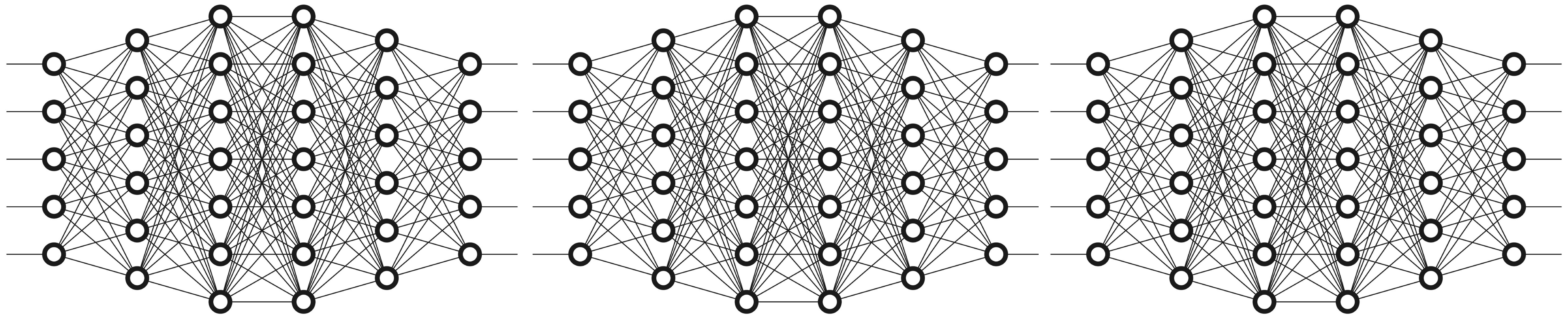
THE UNIVERSITY OF BRITISH COLUMBIA

CPSC 425: Computer Vision



Lecture 23: Neural Networks (cont), CNNs

Menu for Today (**April 2, 2020**)

Topics:

- Backpropagation
- Convolutional Layers
- Pooling Layer
- R-CNN

Redings:

- **Today's** Lecture: N/A
- **Next** Lecture: N/A

Reminders:

- **Assignment 6**: Deep Learning due **Tuesday, April 7th**

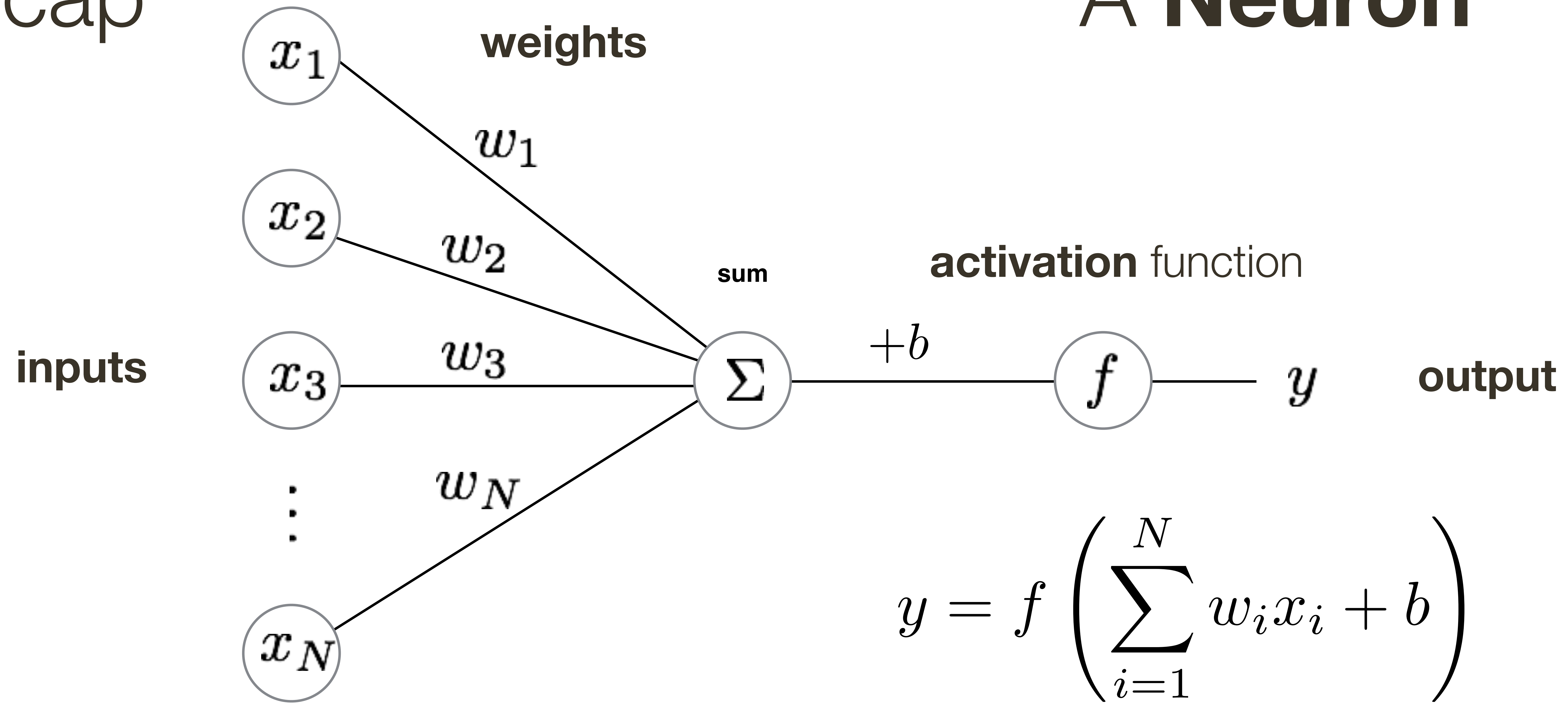
Please fill out

Student Evaluations

(on Canvas)

Lecture 22: Re-cap

A **Neuron**



$$y = f\left(\sum_{i=1}^{N} w_i x_i + b\right)$$

— The basic unit of computation in a neural network is a neuron.

— A neuron accepts some number of input signals, computes their weighted sum, and applies an **activation function** (or **non-linearity**) to the sum.

— Common activation functions include sigmoid and rectified linear unit (ReLU)

Lecture 22: Re-cap

Neural Network

A neural network comprises neurons connected in an acyclic graph

The outputs of neurons can become inputs to other neurons

Neural networks typically contain multiple layers of neurons

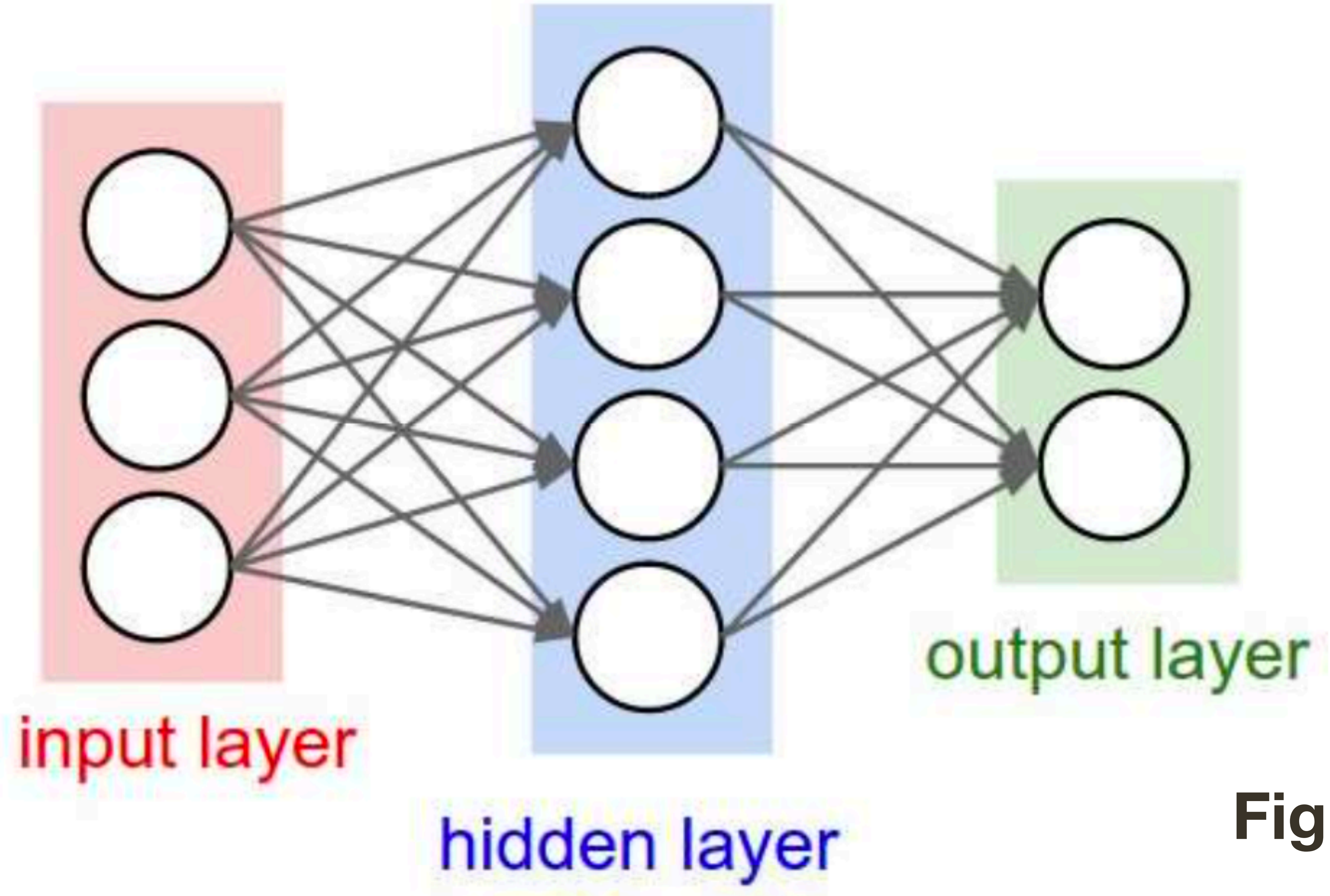
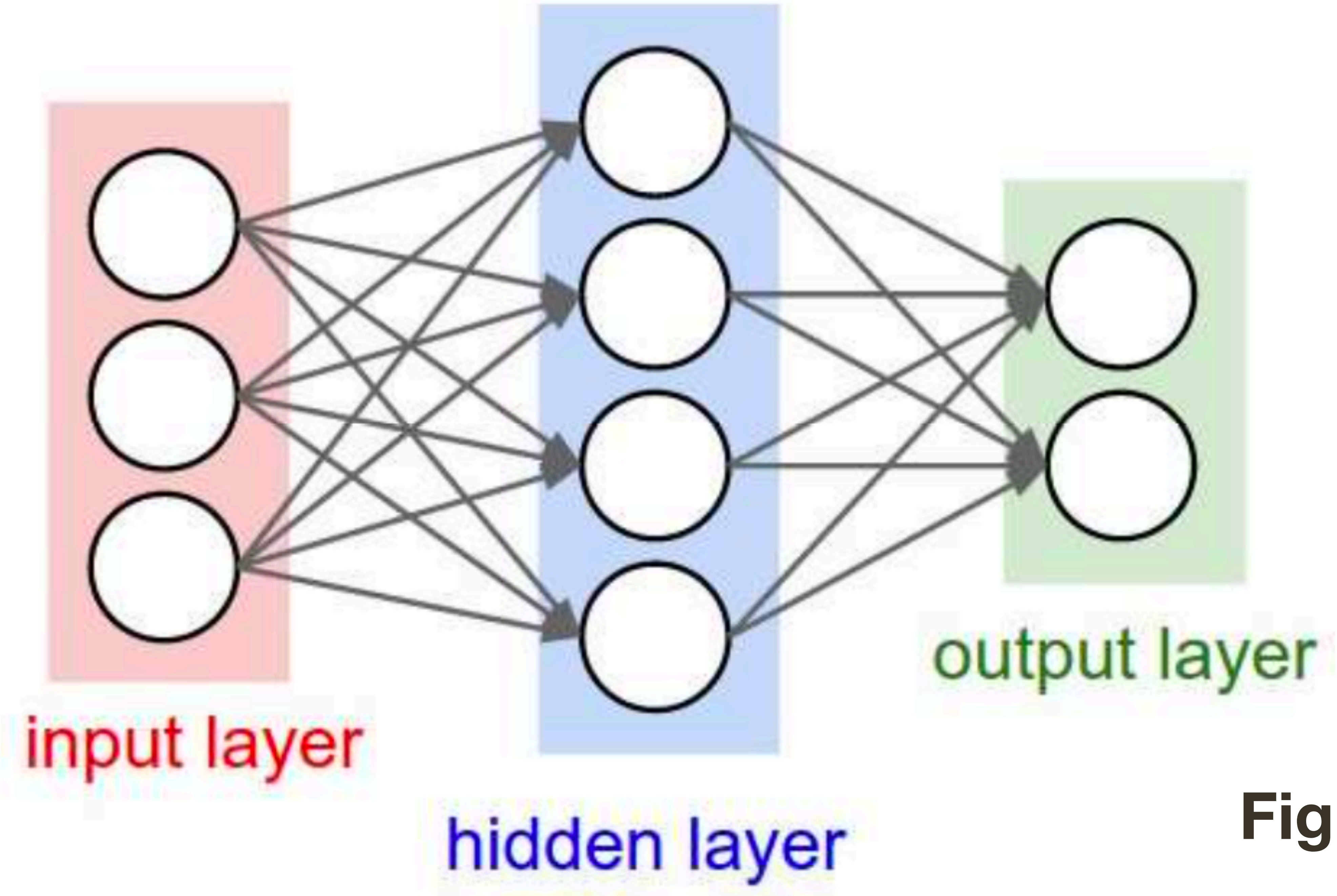


Figure credit: Fei-Fei and Karpathy

Example of a neural network with three inputs, a single hidden layer of four neurons, and an output layer of two neurons

Lecture 22: Re-cap **Neural** Network

Note: each neuron will have its own vector of weights and a bias, its easier to think of all neurons in a layer as a single entity with a matrix of weights (size = number of inputs x number of neurons) and a vector of biases (size = number of neurons)



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Lecture 22: Re-cap

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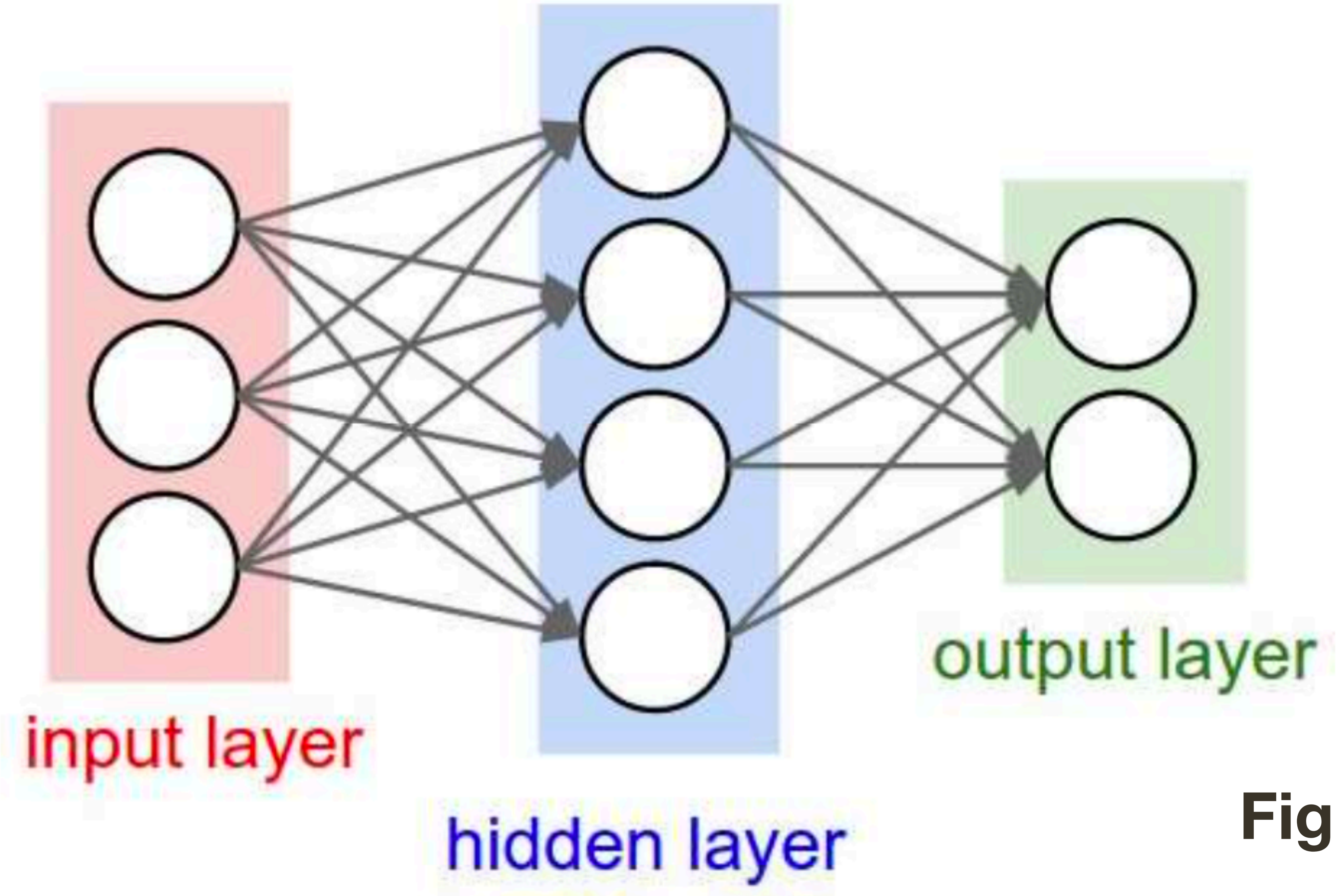
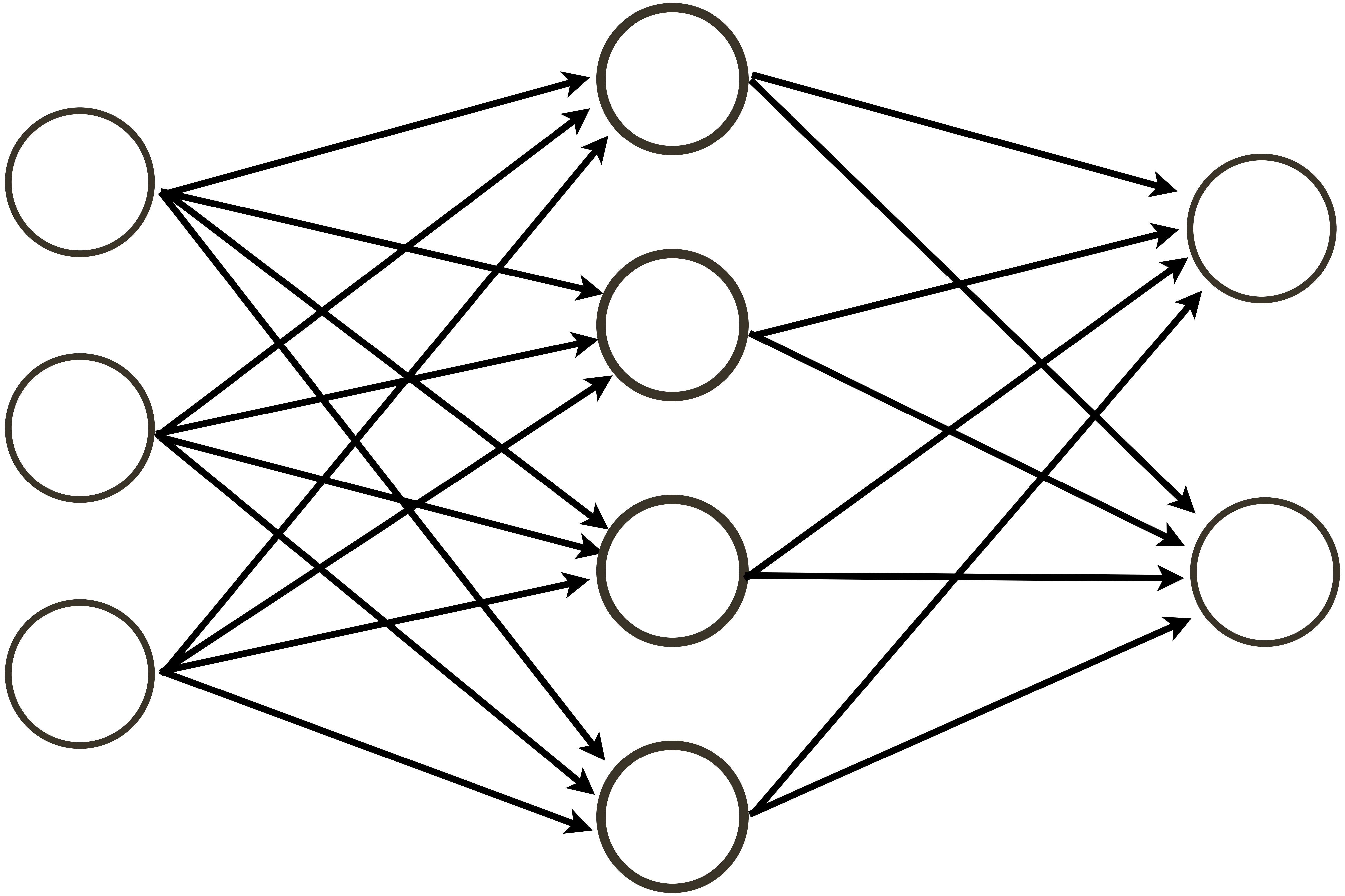


Figure credit: Fei-Fei and Karpathy

$$\hat{\mathbf{y}} = f(\mathbf{x}, \mathbf{W}_1, \mathbf{W}_2, \mathbf{b}_1, \mathbf{b}_2) = \sigma\left(\mathbf{W}_2^{(2\times 4)} \sigma\left(\mathbf{W}_1^{(4\times 3)}\mathbf{x} + \mathbf{b}_1^{(4)}\right) + \mathbf{b}_2^{(2)}\right)$$

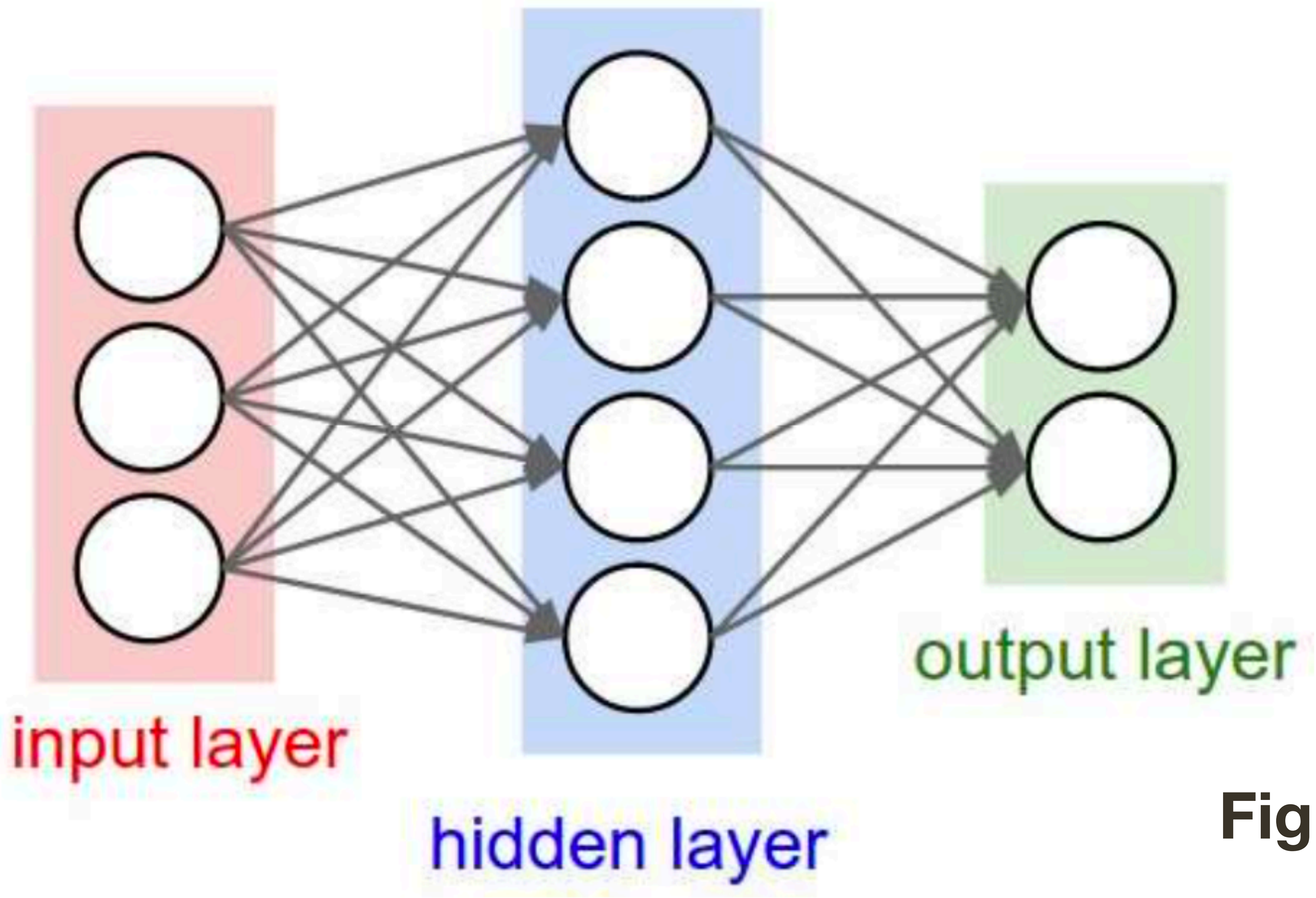
Activation Function

Why can't we have **linear** activation functions? Why have non-linear activations?



Activation Function

$$\hat{\mathbf{y}} = f(\mathbf{x}, \mathbf{W}_1, \mathbf{W}_2, \mathbf{b}_1, \mathbf{b}_2) = \sigma\left(\mathbf{W}_2^{(2\times 4)}\sigma\left(\mathbf{W}_1^{(4\times 3)}\mathbf{x} + \mathbf{b}_1^{(4)}\right) + \mathbf{b}_2^{(2)}\right)$$



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Activation Function

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$$= \mathbf{W}_2^{(2\times4)}\left(\mathbf{W}_1^{(4\times3)}\mathbf{x} + \mathbf{b}_1^{(4)}\right) + \mathbf{b}_2^{(2)}$$

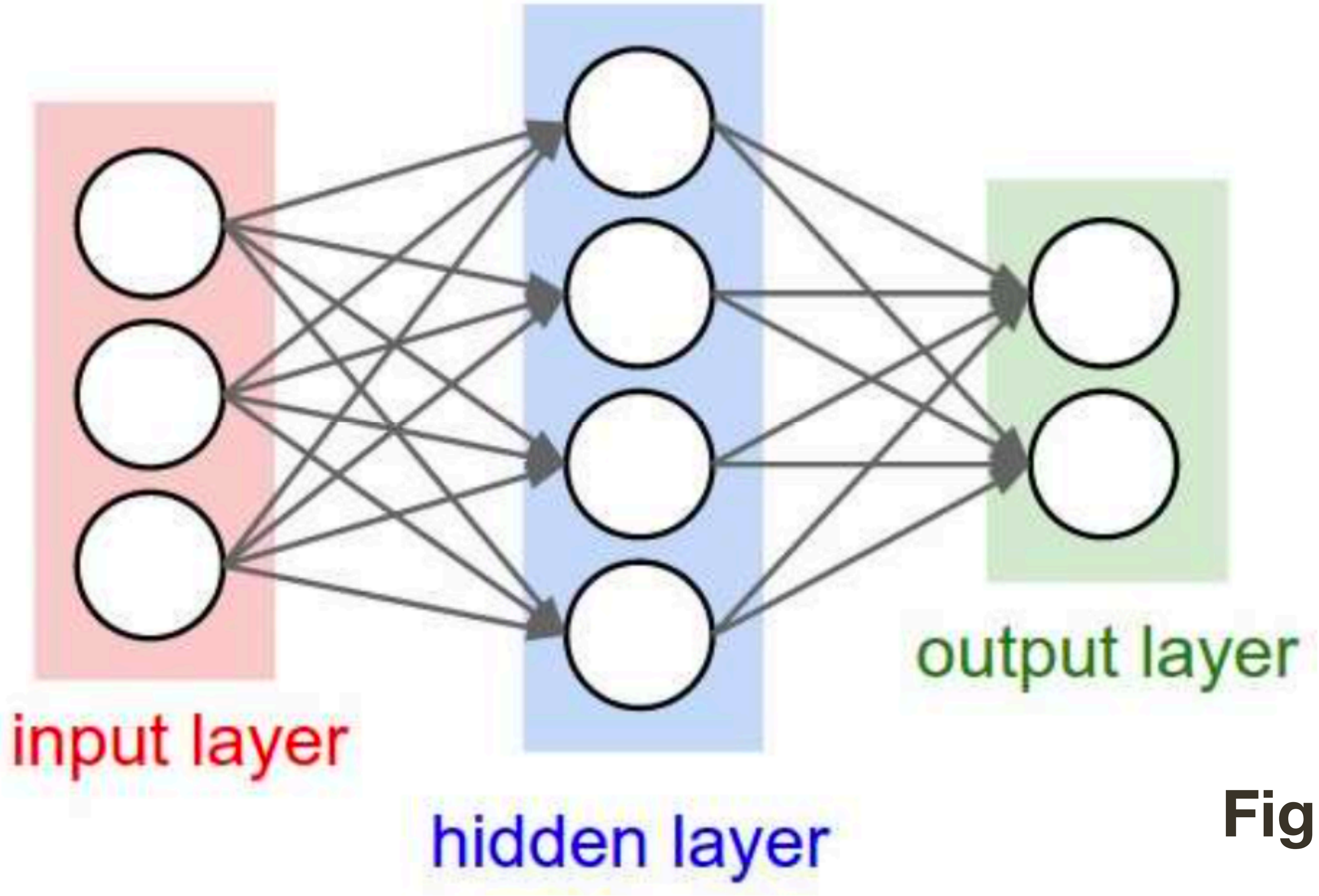
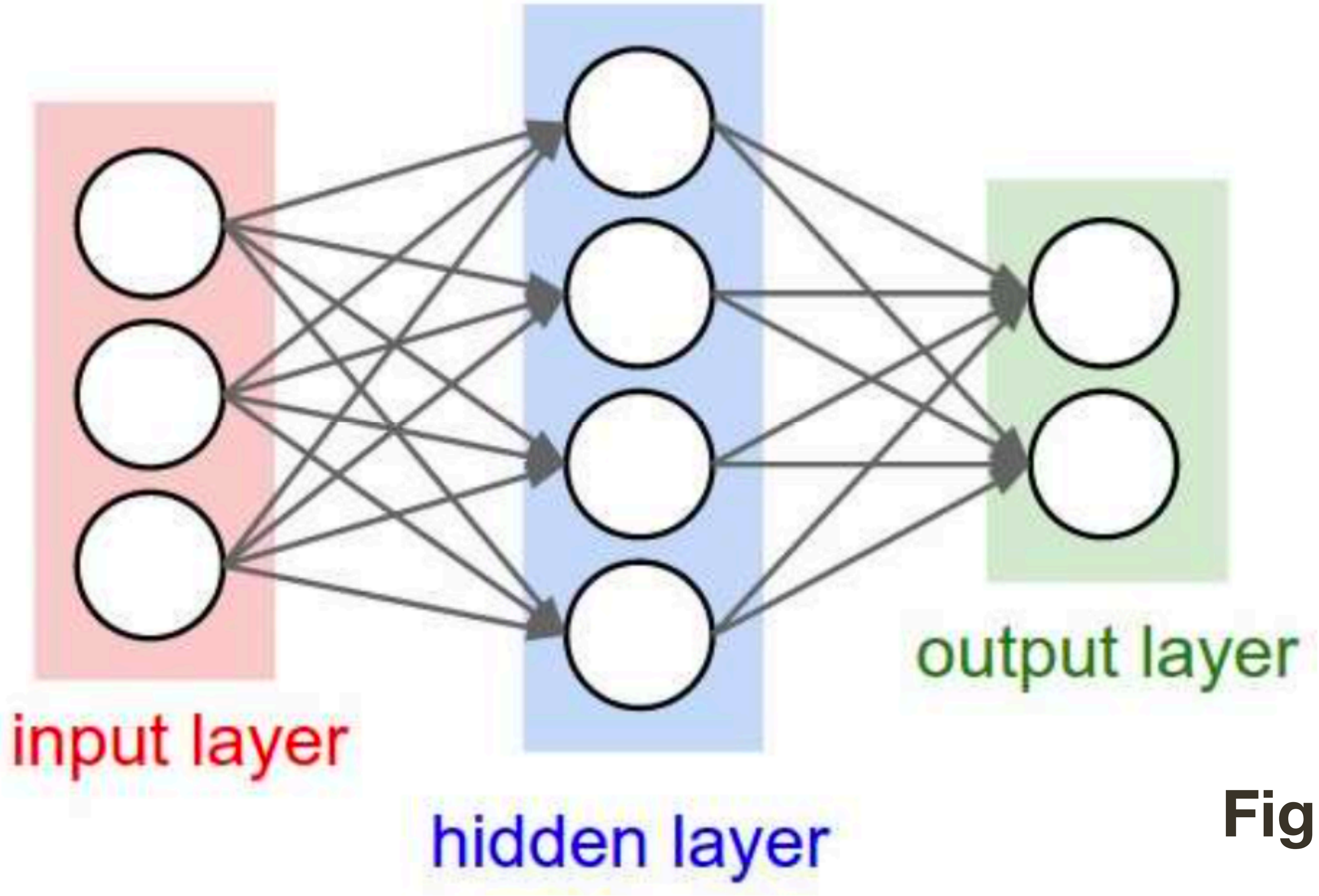


Figure credit: Fei-Fei and Karpathy

Activation Function

$$
\begin{aligned}
\hat{\mathbf{y}} = f(\mathbf{x}, \mathbf{W}_1, \mathbf{W}_2, \mathbf{b}_1, \mathbf{b}_2) &= \sigma\left(\mathbf{W}_2^{(2\times 4)} \sigma\left(\mathbf{W}_1^{(4\times 3)}\mathbf{x} + \mathbf{b}_1^{(4)}\right) + \mathbf{b}_2^{(2)}\right) \\
&= \mathbf{W}_2^{(2\times 4)} \left(\mathbf{W}_1^{(4\times 3)}\mathbf{x} + \mathbf{b}_1^{(4)}\right) + \mathbf{b}_2^{(2)} \\
&= \mathbf{W}_2^{(2\times 4)}\mathbf{W}_1^{(4\times 3)}\mathbf{x} + \mathbf{W}_2^{(2\times 4)}\mathbf{b}_1^{(4)} + \mathbf{b}_2^{(2)}
\end{aligned}
$$



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Activation Function

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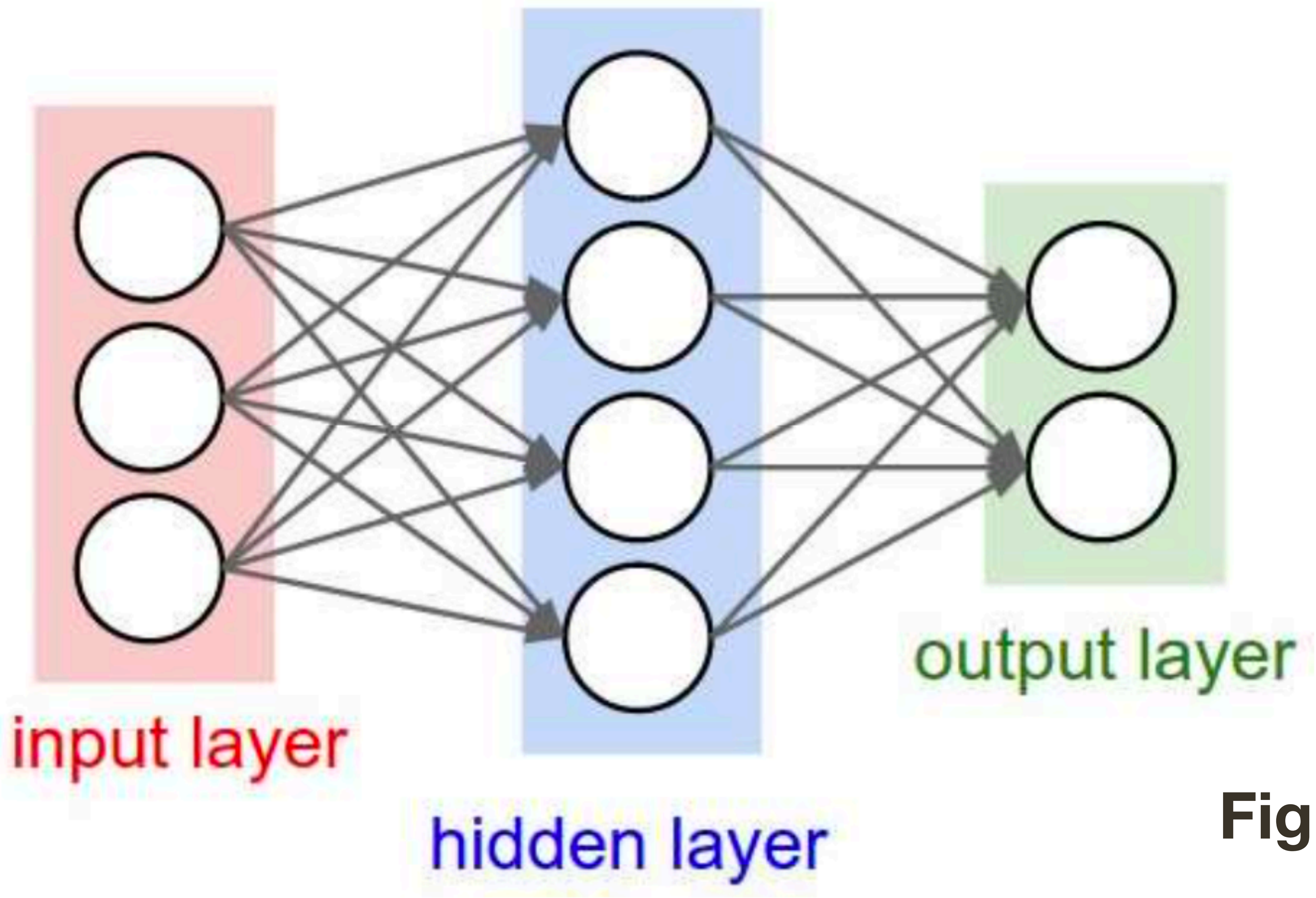
$$= \underbrace{\mathbf{W}_2^{(2\times4)}\mathbf{W}_1^{(4\times3)}}_{\mathbf{W}_*^{(2\times3)}}\mathbf{x} + \underbrace{\mathbf{W}_2^{(2\times4)}\mathbf{b}_1^{(4)} + \mathbf{b}_2^{(2)}}_{\mathbf{b}^{(2)}}$$



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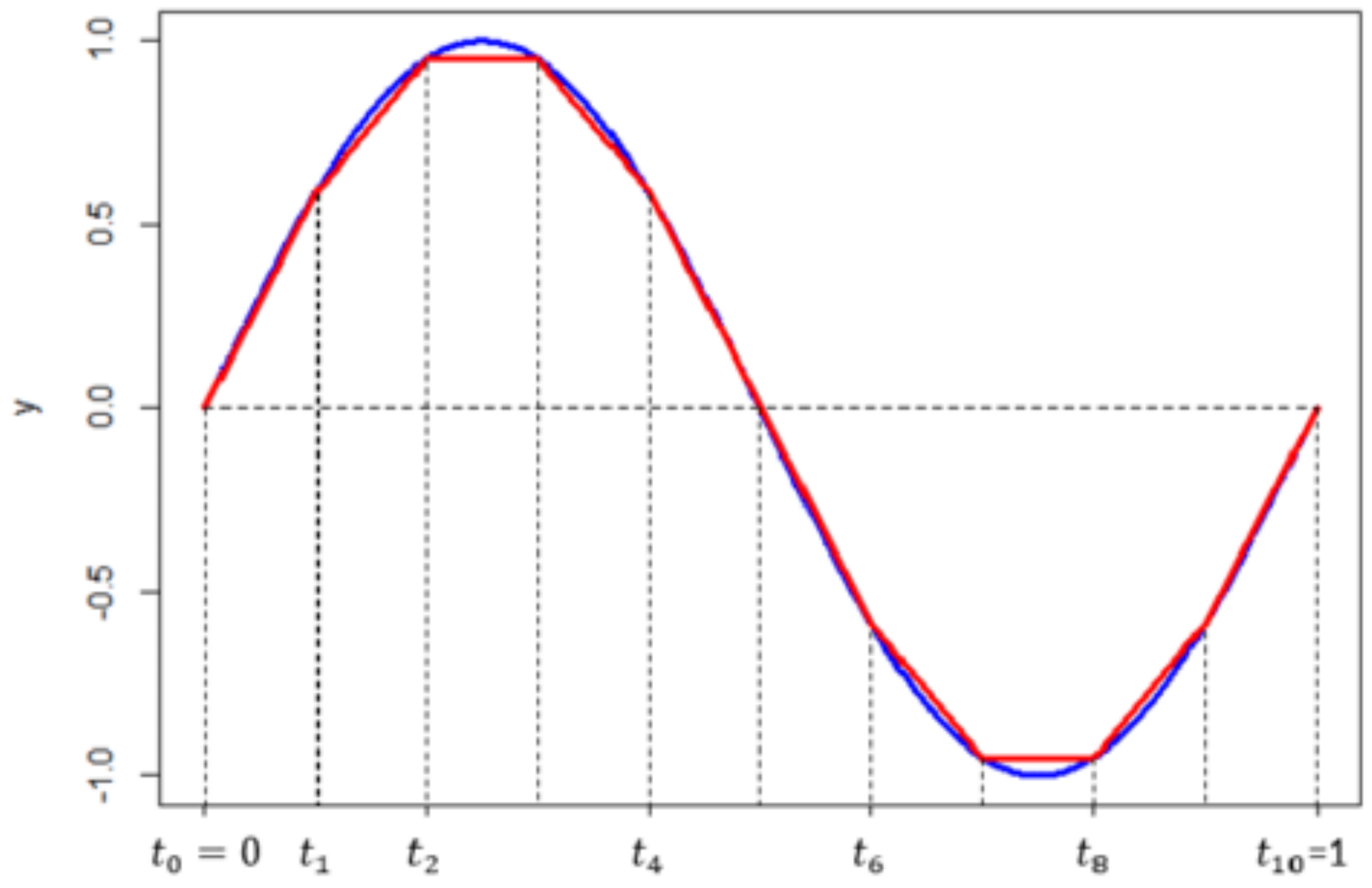
Activation Function

Non-linear activation is required to provably make the Neural Net a **universal function approximator**

Intuition: with ReLU activation, we effectively get a linear spline approximation to any function.

Optimization of neural net parameters = finding slops and transitions of linear pieces

The quality of approximation depends on the number of linear segments



Number of linear segments for large input dimension: $\Omega(2^{\frac{2}{3}Ln})$

Light Theory: Neural Network as Universal Approximator

Universal Approximation Theorem: Single hidden layer can approximate any continuous function with compact support to arbitrary accuracy, when the width goes to infinity.

[Hornik *et al*., 1989]

Universal Approximation Theorem (revised): A network of infinite depth with a hidden layer of size $d+1$ neurons, where d is the dimension of the input space, can approximate any continuous function.

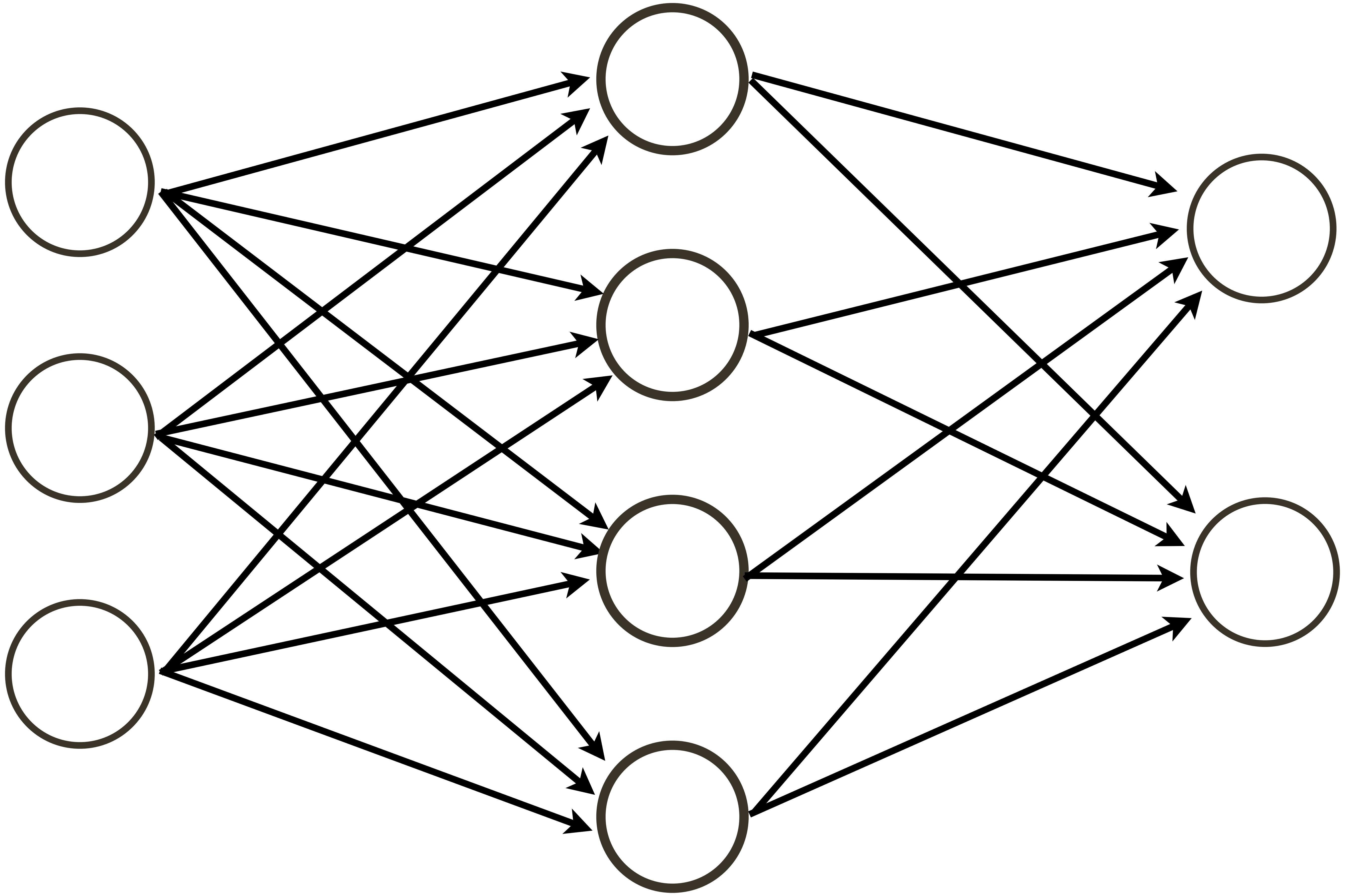
[Lu *et al*., NIPS 2017]

Universal Approximation Theorem (further revised): ResNet with a single hidden unit and infinite depth can approximate any continuous function.

[Lin and Jegelka, NIPS 2018]

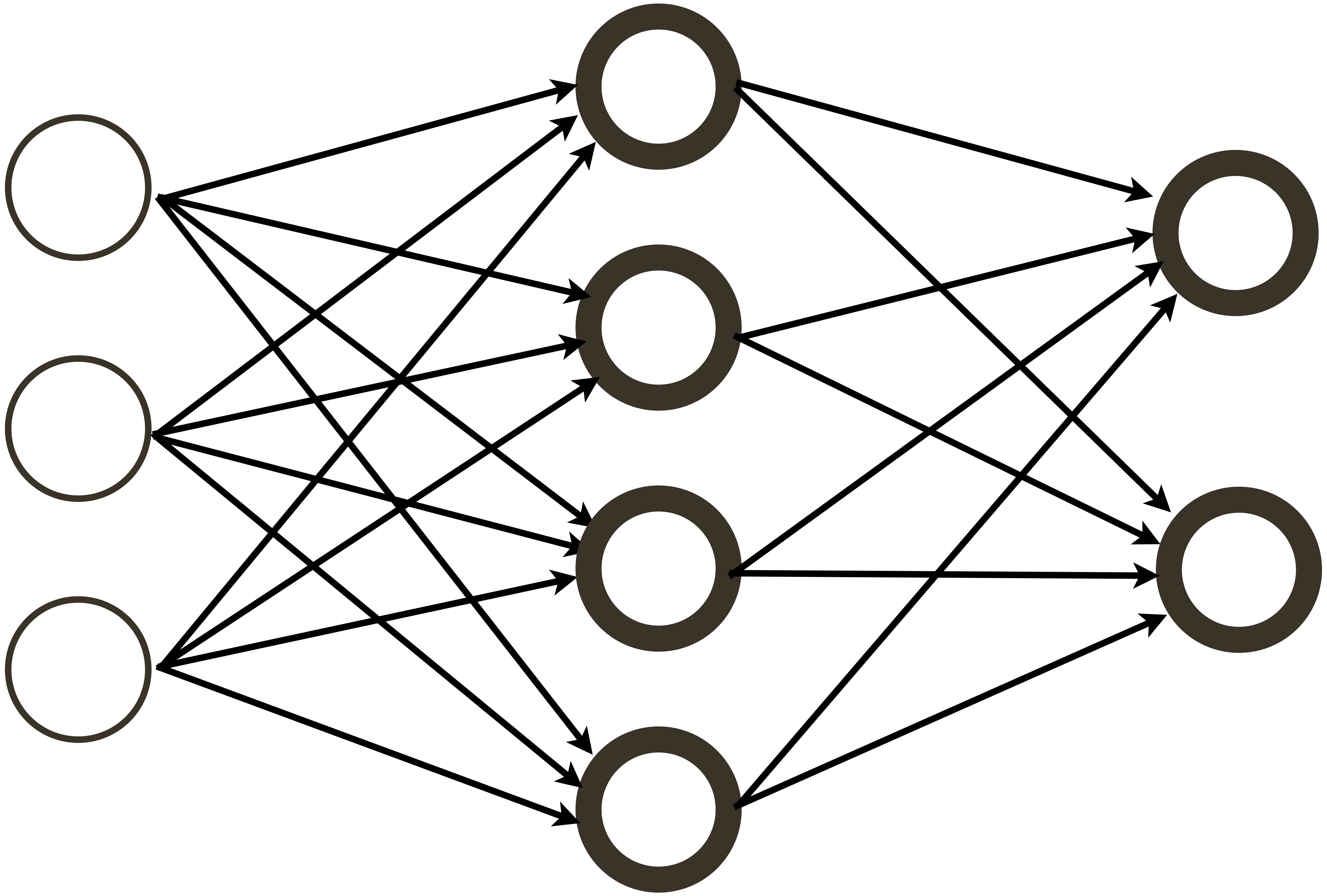
Neural Network

How many neurons?



Neural Network

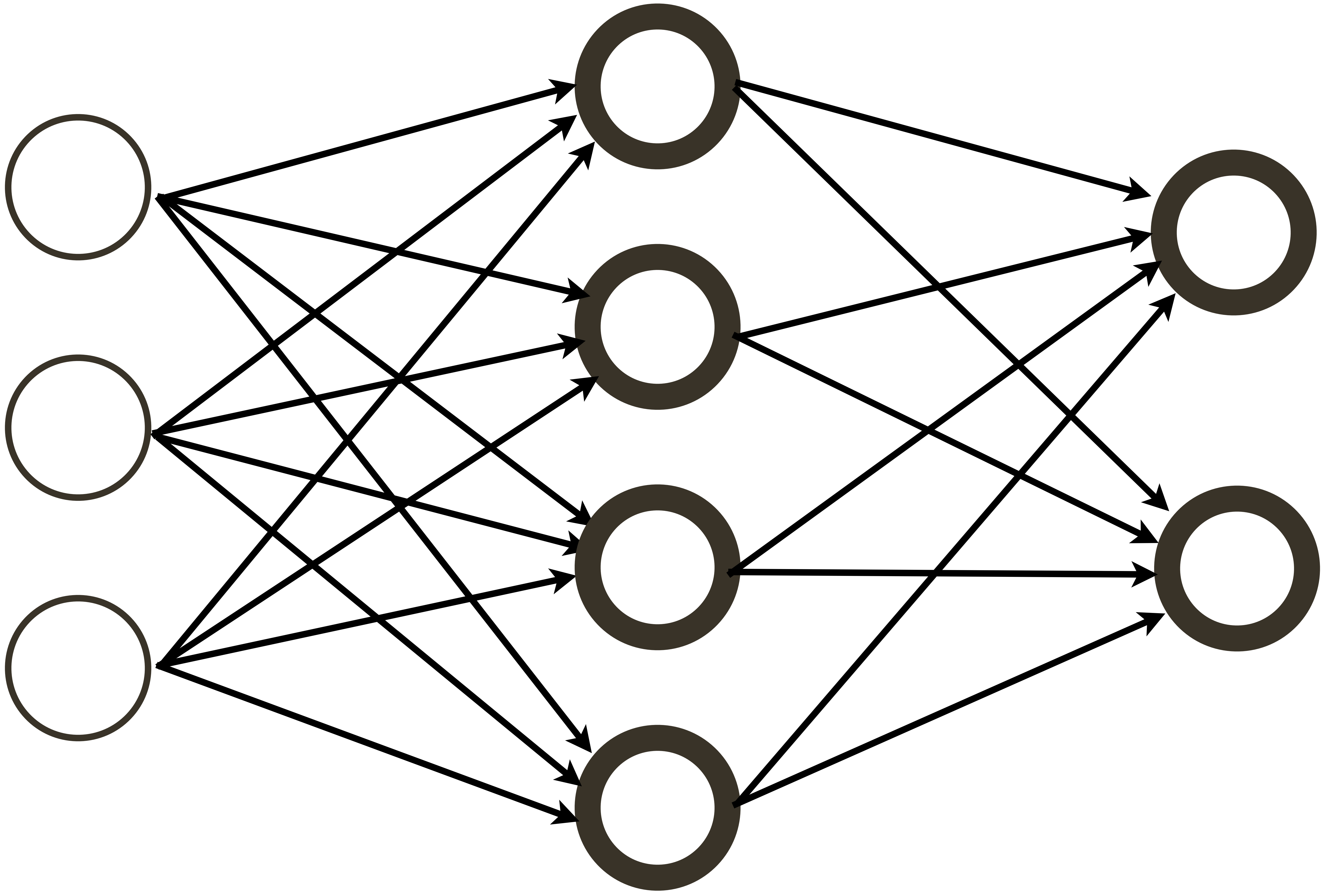
How many neurons?   4+2 = 6



Neural Network

How many neurons? 4+2 = 6

How many weights?

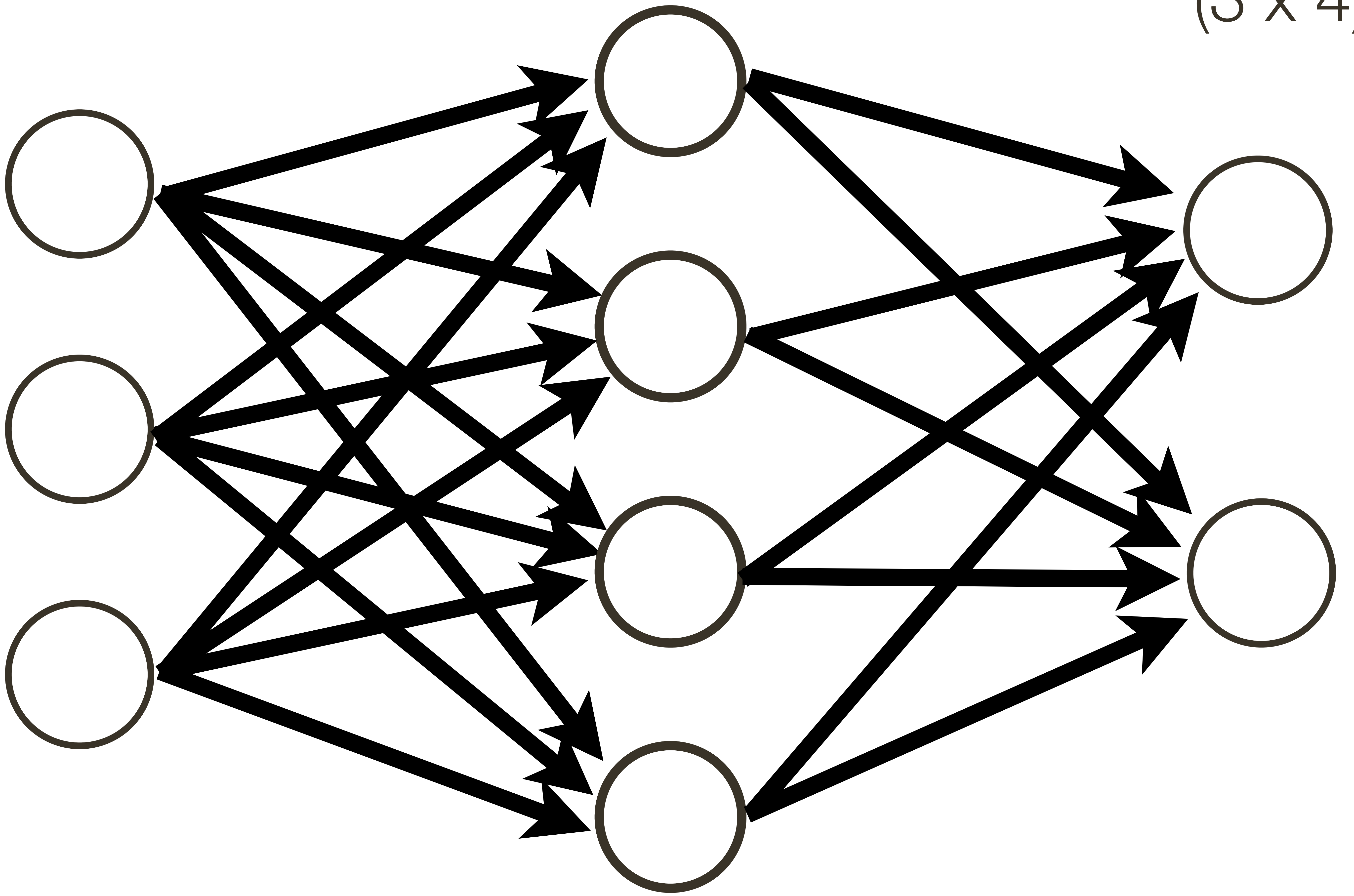


Neural Network

How many neurons? 4+2 = 6

How many weights?

(3 x 4) + (4 x 2) = 20



Neural Network

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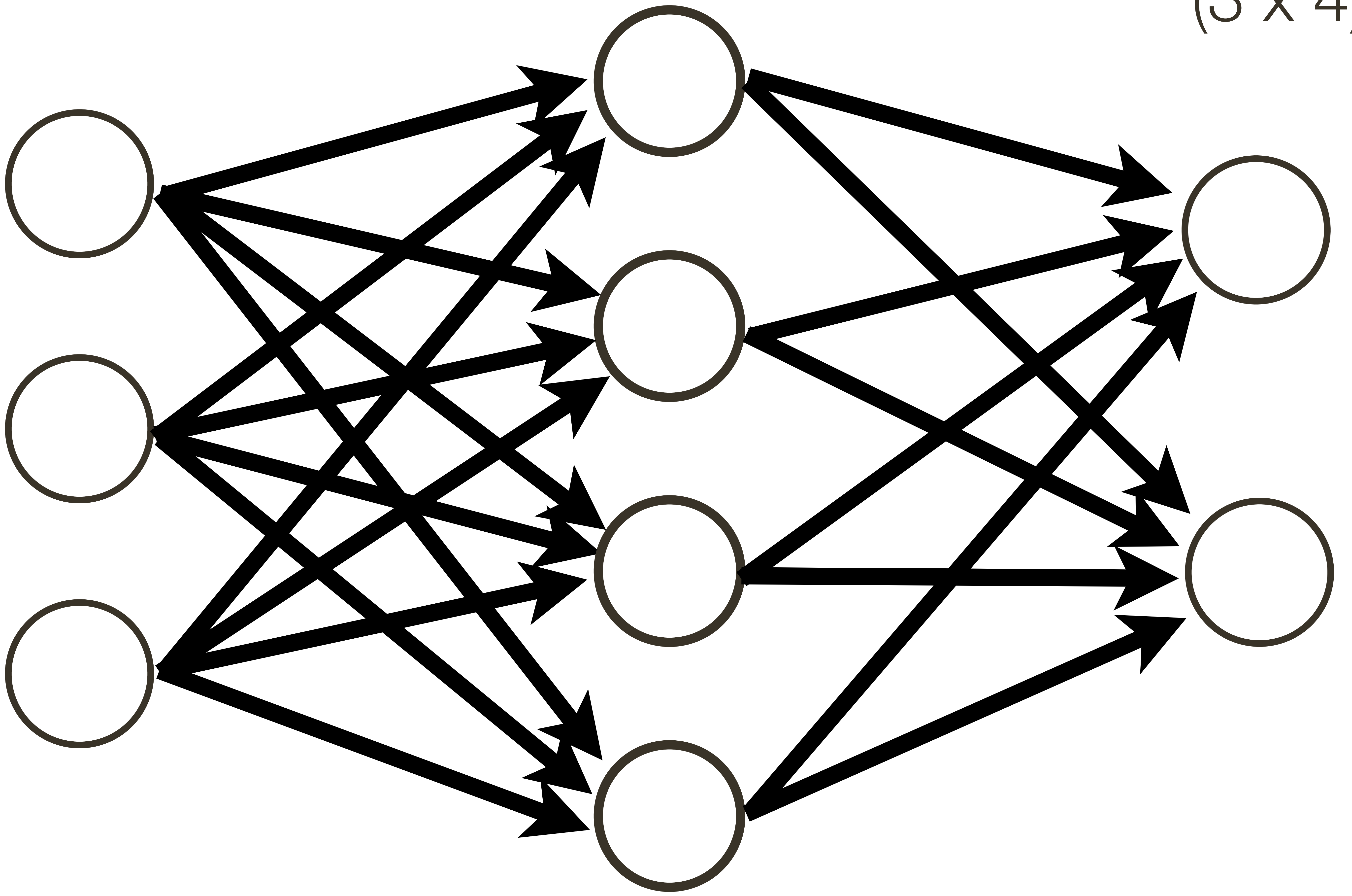


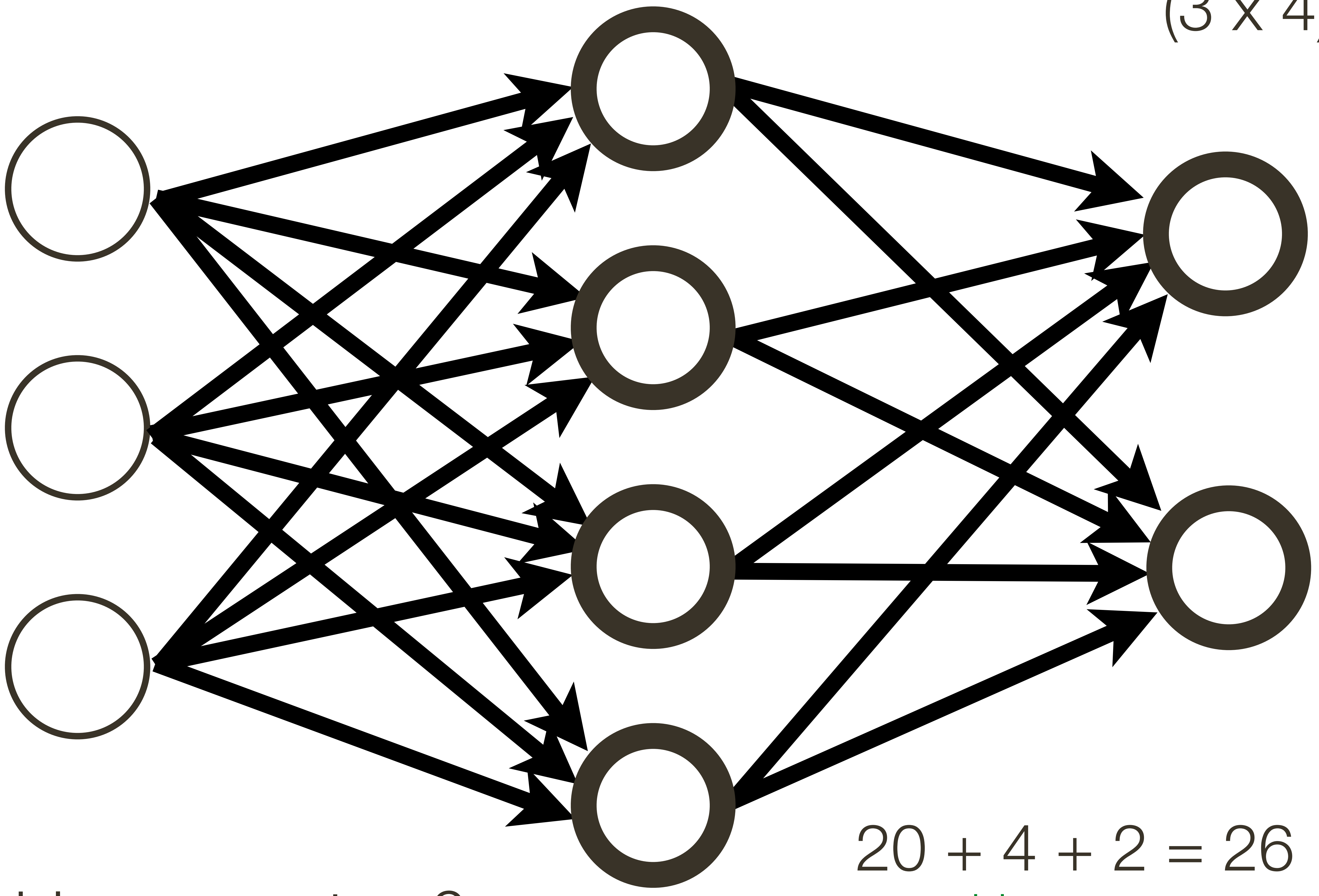
How many learnable parameters?

Neural Network

How many neurons? 4+2 = 6

How many weights?

(3 x 4) + (4 x 2) = 20



How many learnable parameters?

20 + 4 + 2 = 26

bias terms

Neural Networks

Modern **convolutional neural networks** contain 10-20 layers and on the order of 100 million parameters

Training a neural network requires estimating a large number of parameters

Backpropagation

When training a neural network, the final output will be some loss (error) function

— e.g. cross-entropy loss: $$L_i = -\log\left(\frac{e^{f_{y_i}}}{\sum_j e^{f_{y_j}}}\right)$$

which defines loss for i-th training example with true class index y_i; and f_j is the j-th element of the vector of class scores coming from neural net.

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Consider neural net which takes input vector $\mathbf{x}_i$ and predicts scores for 3 classes, with true class being class 3:

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Consider neural net which takes input vector $\mathbf{x}_i$ and predicts scores for 3 classes, with true class being class 3:

$$f$$

$$c_1 = -2.85$$

$$c_2 = 0.86$$

$$c_3 = 0.28$$

Backpropagation

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Consider neural net which takes input vector $\mathbf{x}_i$ and predicts scores for 3 classes, with true class being class 3:

f		
$c_1 = -2.85$	$\xrightarrow{\exp}$	0.058
$c_2 = 0.86$		2.36
$c_3 = 0.28$		1.32

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Consider neural net which takes input vector $\mathbf{x}_i$ and predicts scores for 3 classes, with true class being class 3:

$$f$$

f			Normalize to sum to 1	
$c_1 = -2.85$	$\xrightarrow{\exp}$	0.058	$\longrightarrow$	0.016
$c_2 = 0.86$		2.36		0.631
$c_3 = 0.28$		1.32		0.353

Backpropagation

When training a neural network, the final output will be some loss (error) function

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Consider neural net which takes input vector $\mathbf{x}_i$ and predicts scores for 3 classes, with true class being class 3:

f				probability of a class
$c_1 = -2.85$	exp →	0.058	Normalize to sum to 1 →	0.016
$c_2 = 0.86$		2.36		0.631
$c_3 = 0.28$		1.32		0.353

Backpropagation

When training a neural network, the final output will be some loss (error) function

— e.g. cross-entropy loss: $$L_i = -\log\left(\frac{e^{f_{y_i}}}{\sum_j e^{f_{y_j}}}\right)$$

softmax function
multi-class classifier

which defines loss for i-th training example with true class index y_i; and f_j is the j-th element of the vector of class scores coming from neural net.

Consider neural net which takes input vector $\mathbf{x}_i$ and predicts scores for 3 classes, with true class being class 3:

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$c_1 = -2.85$	exp →	0.058	→	0.016
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Consider neural net which takes input vector $\mathbf{x}_i$ and predicts scores for 3 classes, with true class being class 3:

f			Normalize to sum to 1	probability of a class	
$c_1 = -2.85$	exp →	0.058	→	0.016	
$c_2 = 0.86$		2.36		0.631	$L_i = -\log(0.353) = 1.04$
$c_3 = 0.28$		1.32		0.353	

Backpropagation

When training a neural network, the final output will be some loss (error) function

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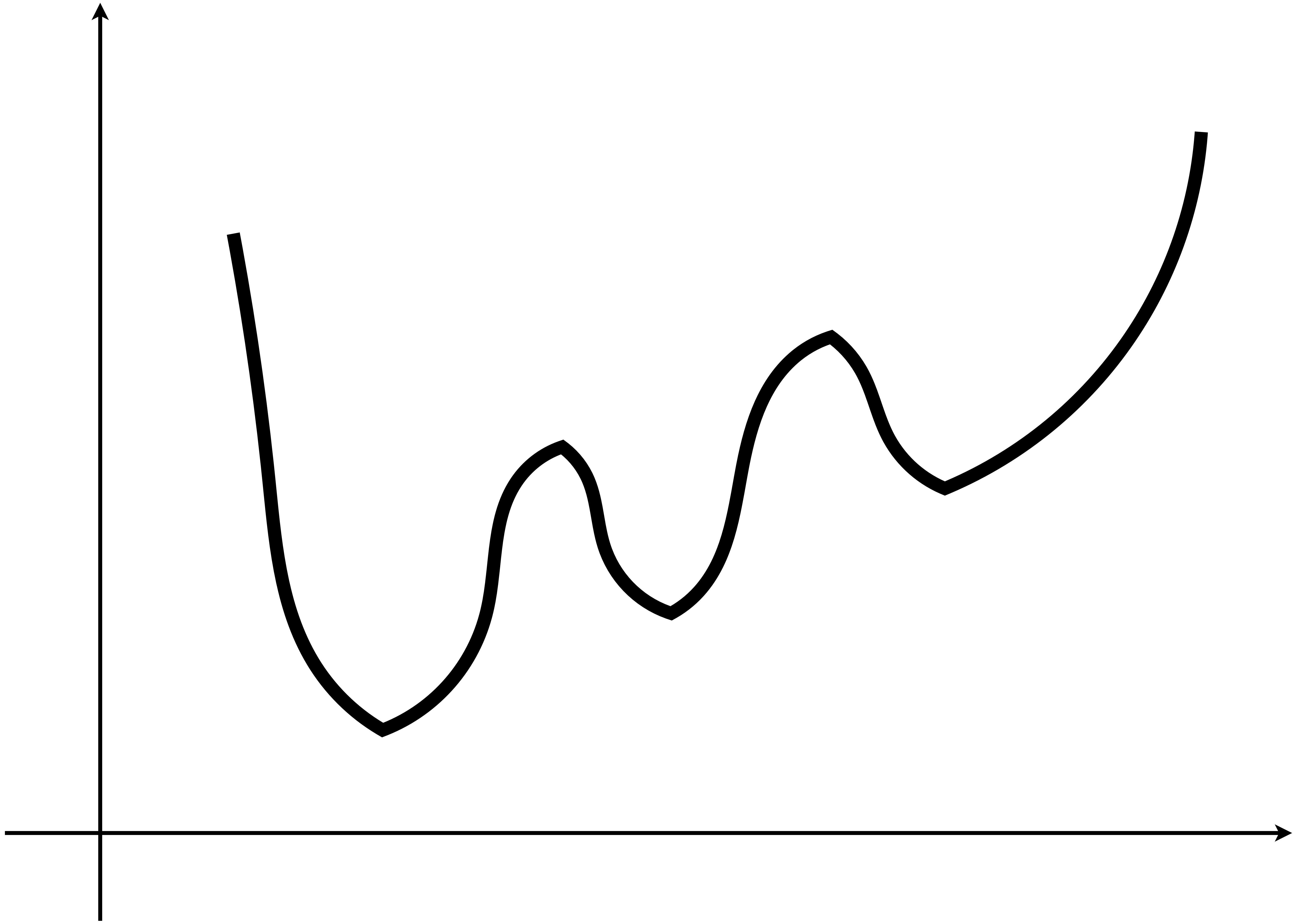
which defines loss for i-th training example with true class index y_i; and f_j is the j-th element of the vector of class scores coming from neural net.

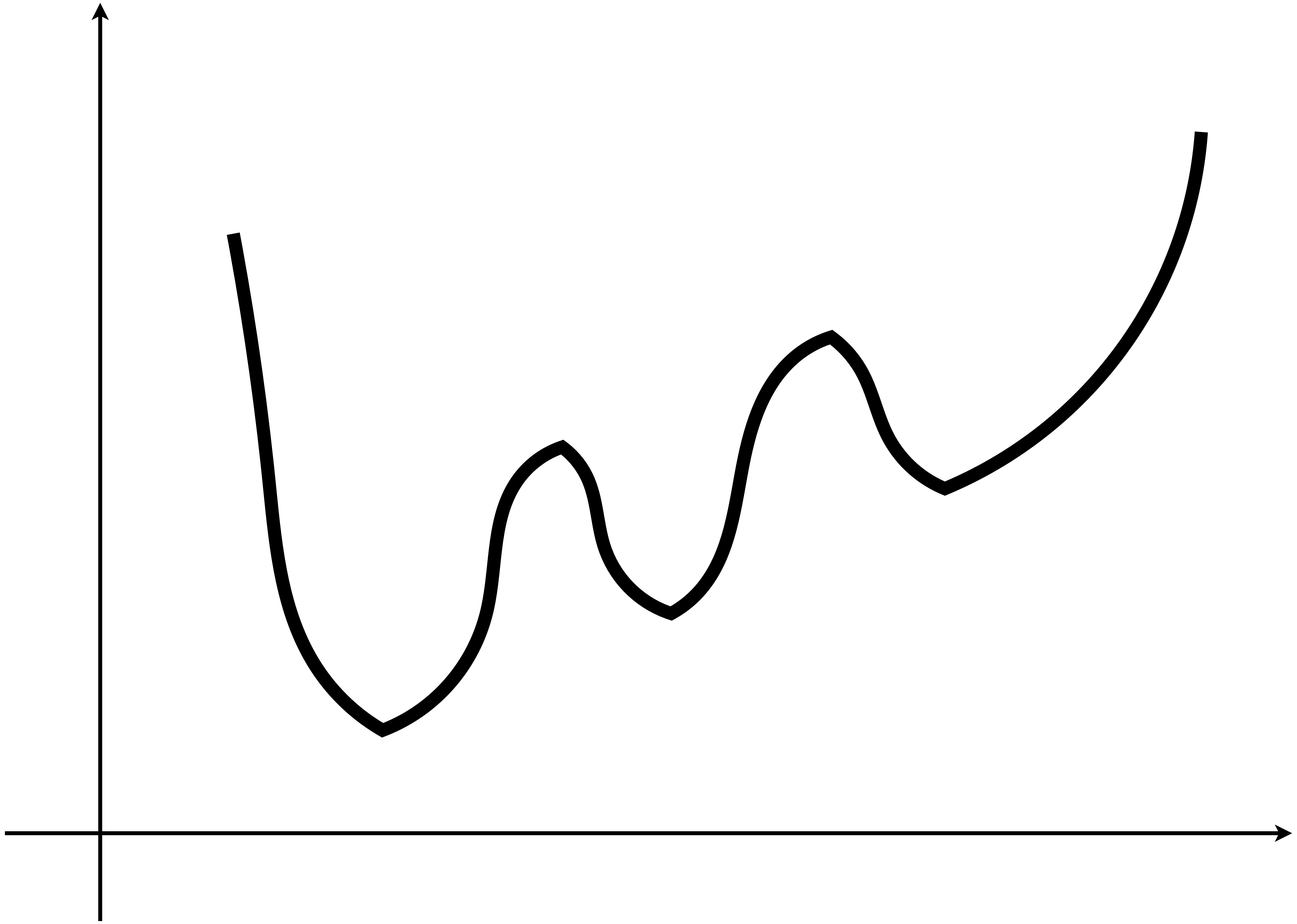
We want to compute the **gradient** of the loss with respect to the network parameters so that we can incrementally adjust the network parameters

Gradient Descent



*slide adopted from V. Ordonex

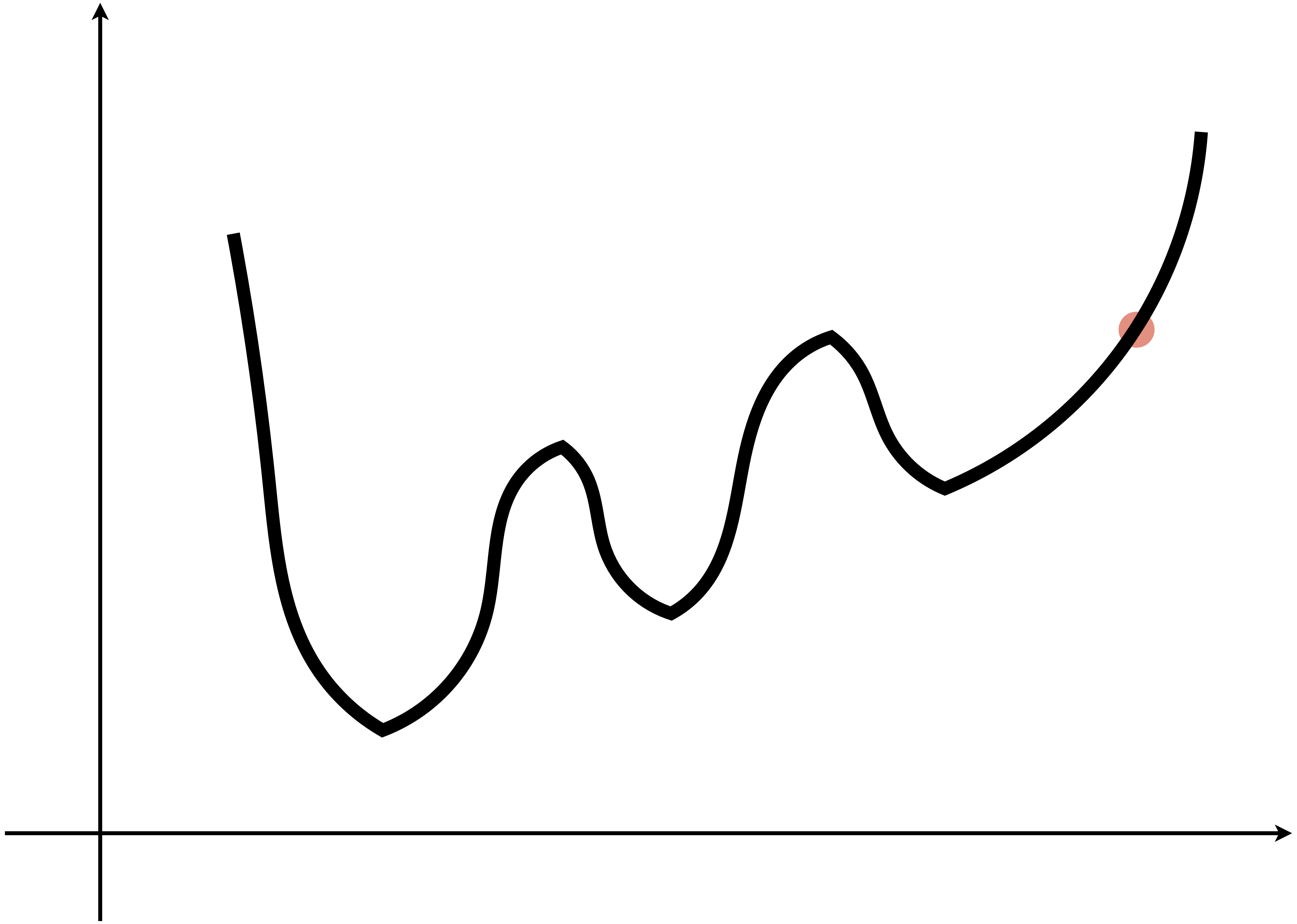
Gradient Descent



1. Start from random value of $\mathbf{W}_0, \mathbf{b}_0$

*slide adopted from V. Ordonex

Gradient Descent



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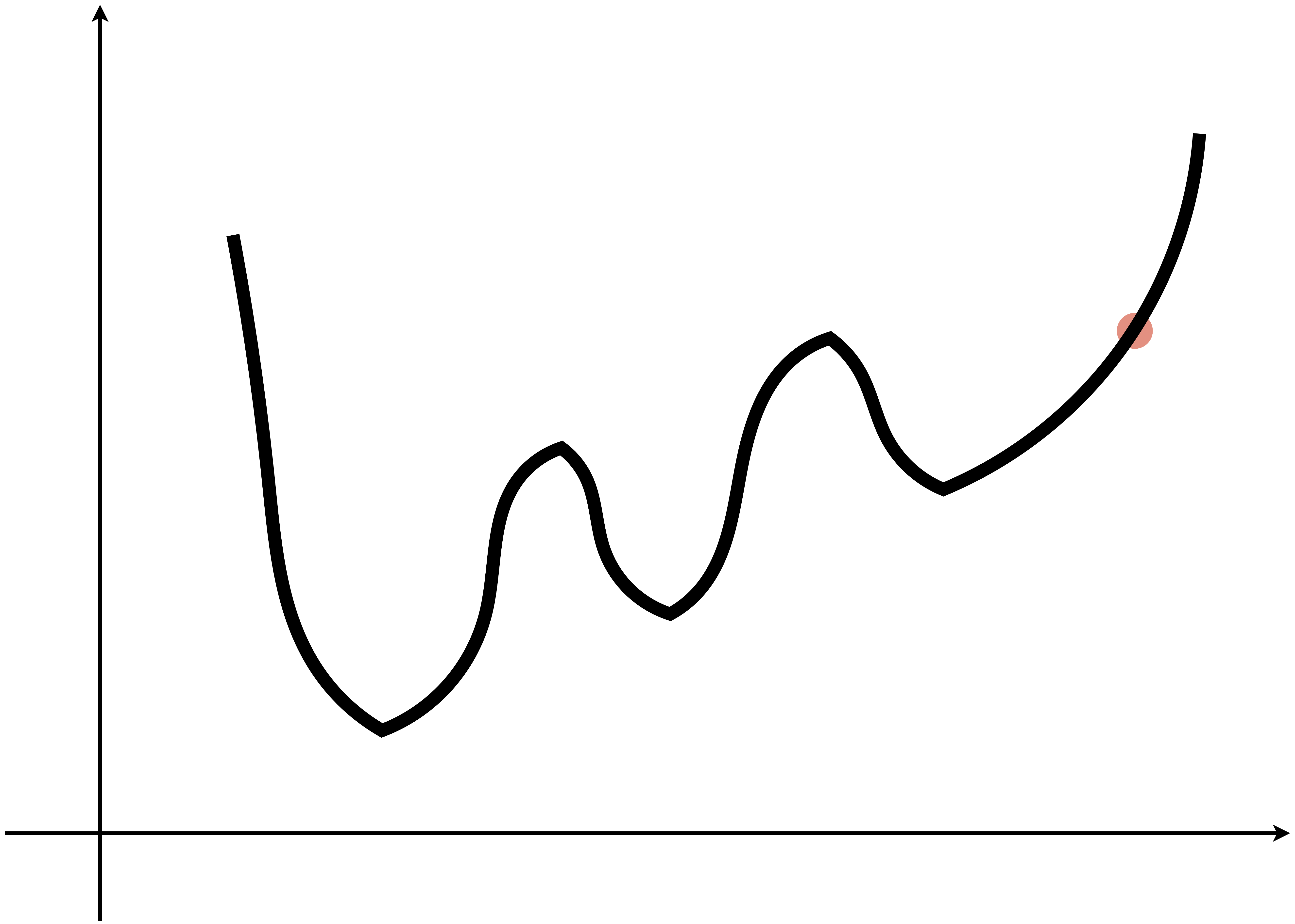


1. Start from random value of $\mathbf{W}_0, \mathbf{b}_0$

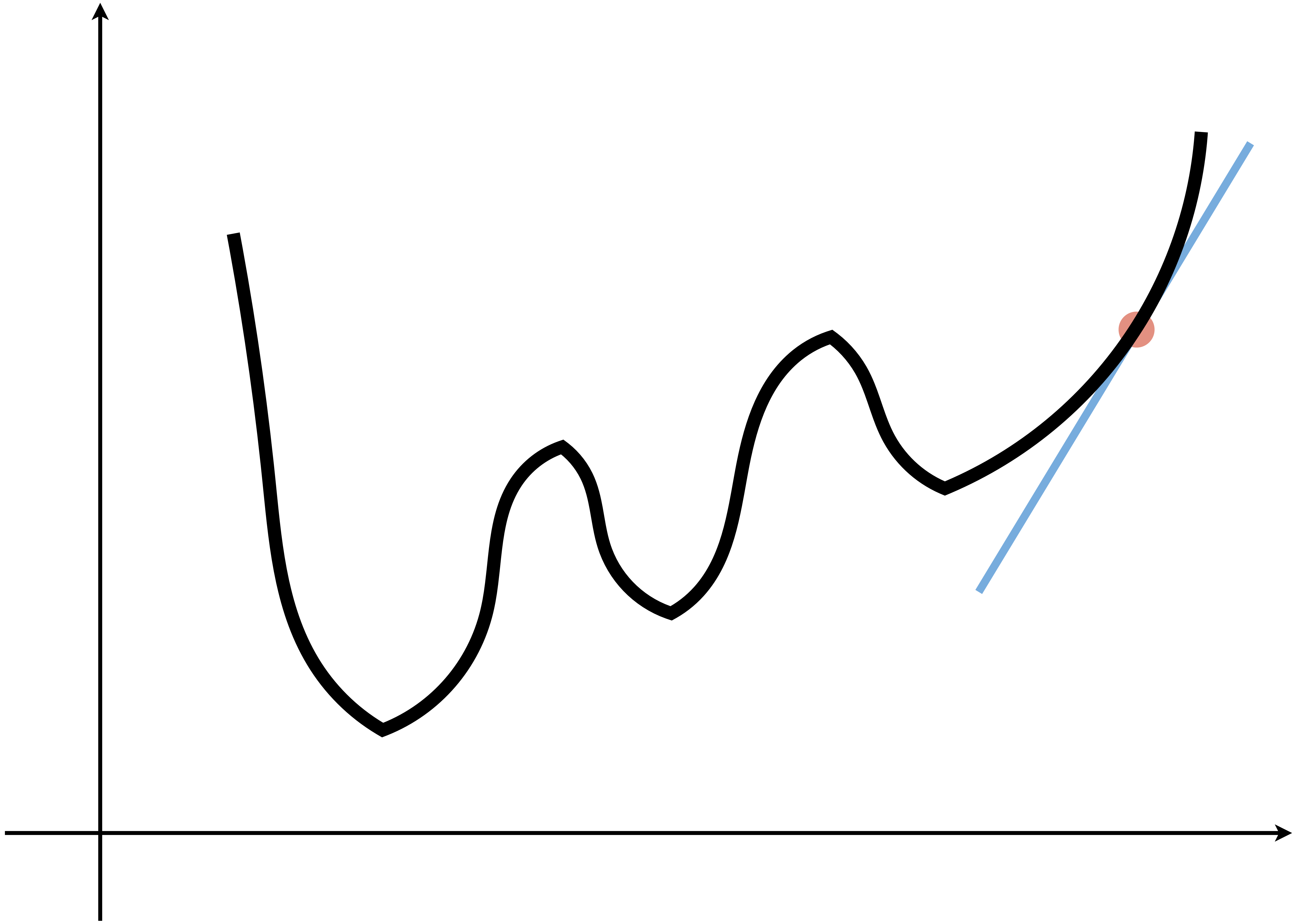
For $k = 0$ to max number of iterations

2. Compute gradient of the loss with respect to previous (initial) parameters:

$$\nabla \mathcal{L}(\mathbf{W}, \mathbf{b})|_{\mathbf{W}=\mathbf{W}_k, \mathbf{b}=\mathbf{b}_k}$$

*slide adopted from V. Ordonex

Gradient Descent



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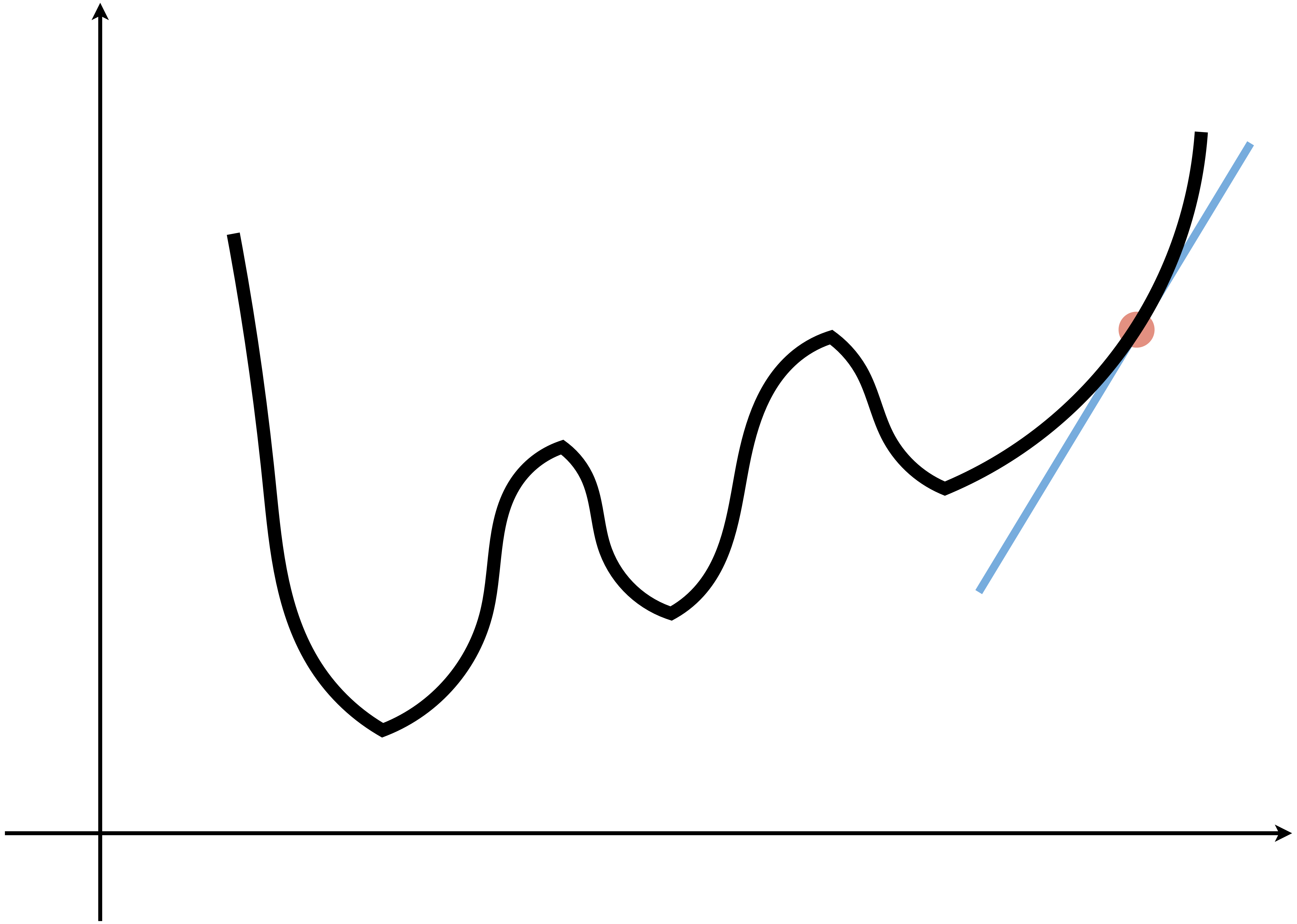

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3. Re-estimate the parameters

$$\mathbf{W}_{k+1} = \mathbf{W}_k - \lambda \left.\frac{\partial \mathcal{L}(\mathbf{W}, \mathbf{b})}{\partial \mathbf{W}}\right|_{\mathbf{W}=\mathbf{W}_k, \mathbf{b}=\mathbf{b}_k}$$

$$\mathbf{b}_{k+1} = \mathbf{b}_k - \lambda \left.\frac{\partial \mathcal{L}(\mathbf{W}, \mathbf{b})}{\partial \mathbf{b}}\right|_{\mathbf{W}=\mathbf{W}_k, \mathbf{b}=\mathbf{b}_k}$$

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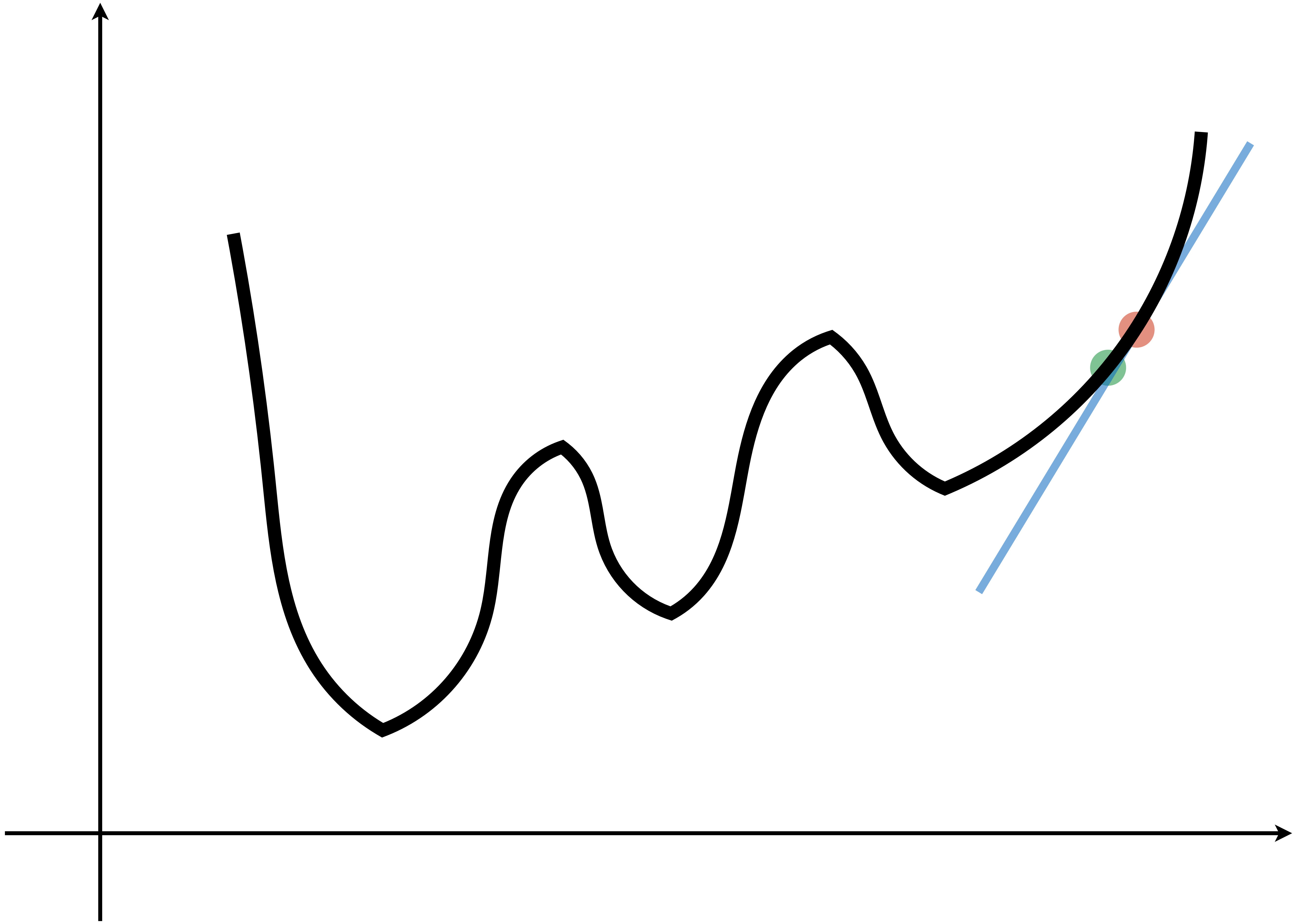
Gradient Descent



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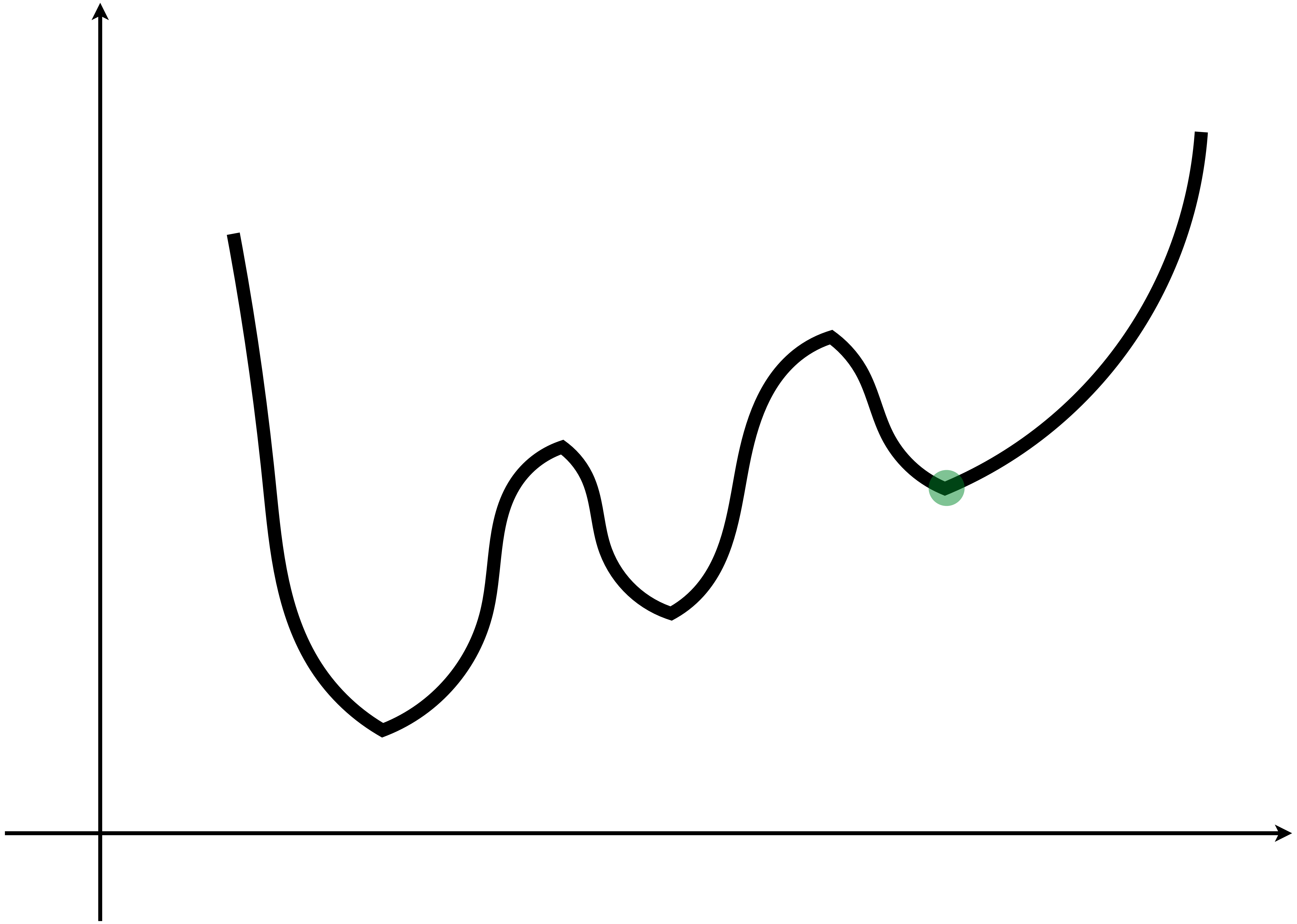
Gradient Descent



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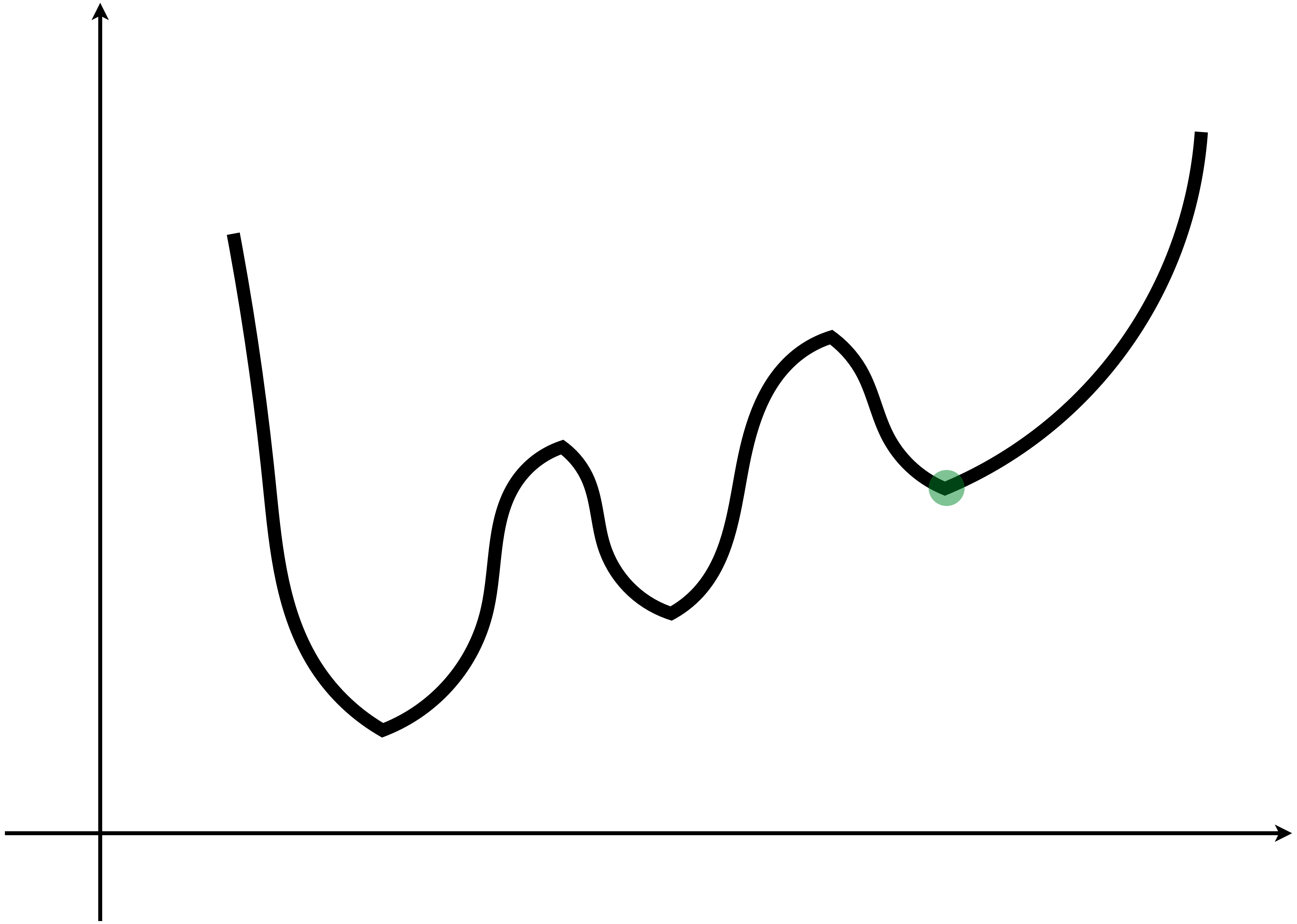
Gradient Descent



1. Start from random value of $\mathbf{W}_0, \mathbf{b}_0$

For $k = 0$ to max number of iterations

2. Compute gradient of the loss with respect to previous (initial) parameters:

$$\nabla \mathcal{L}(\mathbf{W}, \mathbf{b})\big|_{\mathbf{W}=\mathbf{W}_k, \mathbf{b}=\mathbf{b}_k}$$

3. Re-estimate the parameters

$$\mathbf{W}_{k+1} = \mathbf{W}_k - \underline{\lambda} \left. \frac{\partial \mathcal{L}(\mathbf{W}, \mathbf{b})}{\partial \mathbf{W}} \right|_{\mathbf{W}=\mathbf{W}_k, \mathbf{b}=\mathbf{b}_k}$$

$$\mathbf{b}_{k+1} = \mathbf{b}_k - \underline{\lambda} \left. \frac{\partial \mathcal{L}(\mathbf{W}, \mathbf{b})}{\partial \mathbf{b}} \right|_{\mathbf{W}=\mathbf{W}_k, \mathbf{b}=\mathbf{b}_k}$$

λ - is the learning rate

*slide adopted from V. Ordonex

Loss:

$$\mathcal{L}(\mathbf{y}, \hat{\mathbf{y}}) = ||\mathbf{y} - \hat{\mathbf{y}}|| = ||\mathbf{y} - f(\mathbf{x}, \mathbf{W}_1, \mathbf{W}_2, \mathbf{b}_1, \mathbf{b}_2)||$$

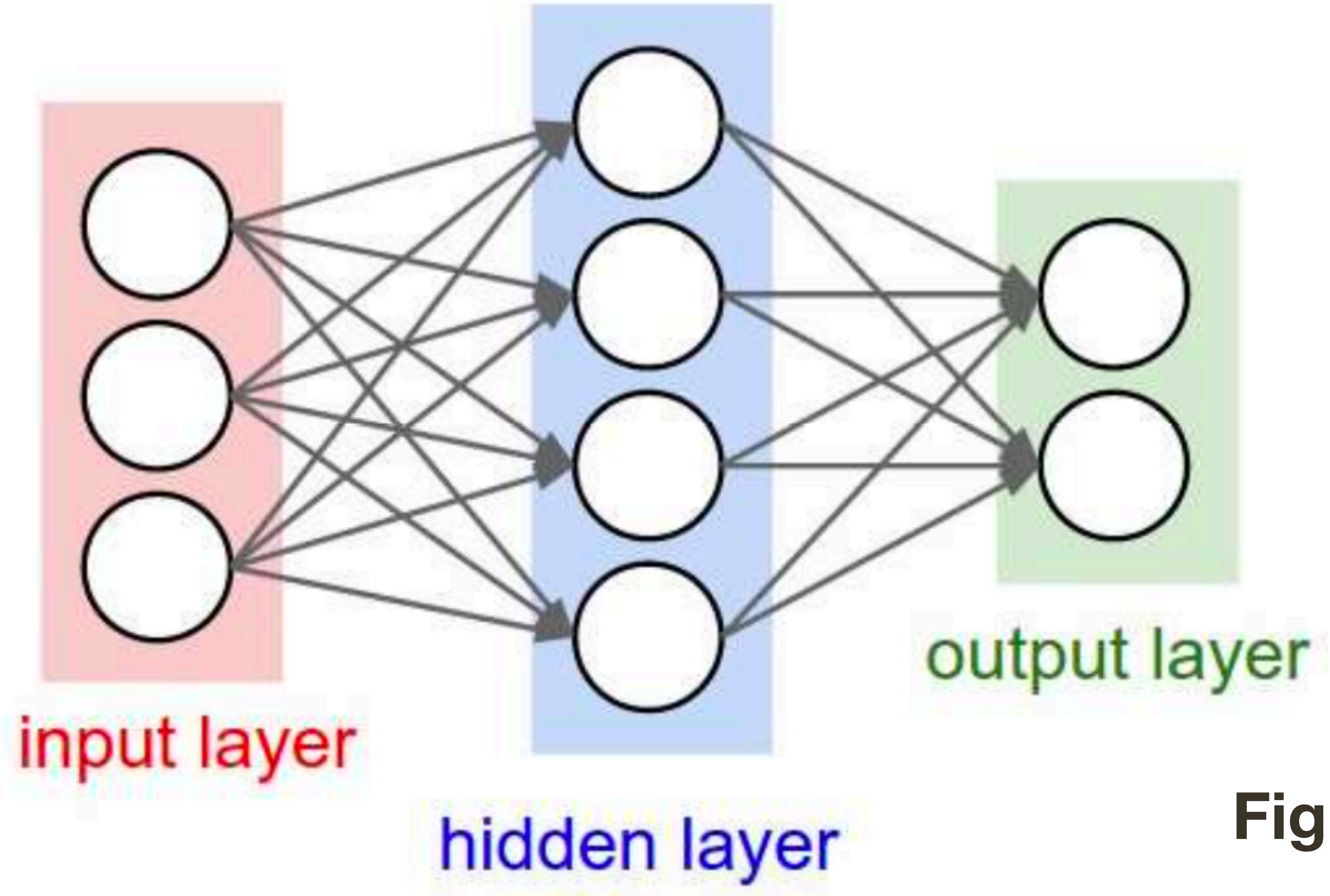


Figure credit: Fei-Fei and Karpathy

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Re-cap

Neural Network

Loss:

$$\mathcal{L}(\mathbf{y}, \hat{\mathbf{y}}) = ||\mathbf{y} - \hat{\mathbf{y}}|| = ||\mathbf{y} - f(\mathbf{x}, \mathbf{W}_1, \mathbf{W}_2, \mathbf{b}_1, \mathbf{b}_2)||$$

Gradient Descent

$$\mathbf{W}_{1,i,j} = \mathbf{W}_{1,i,j} - \lambda \frac{\partial \mathcal{L}(\mathbf{y}, \hat{\mathbf{y}})}{\partial \mathbf{W}_{1,i,j}}$$

$$\mathbf{b}_{1,i} = \mathbf{b}_{1,i} - \lambda \frac{\partial \mathcal{L}(\mathbf{y}, \hat{\mathbf{y}})}{\partial \mathbf{b}_{1,i}}$$

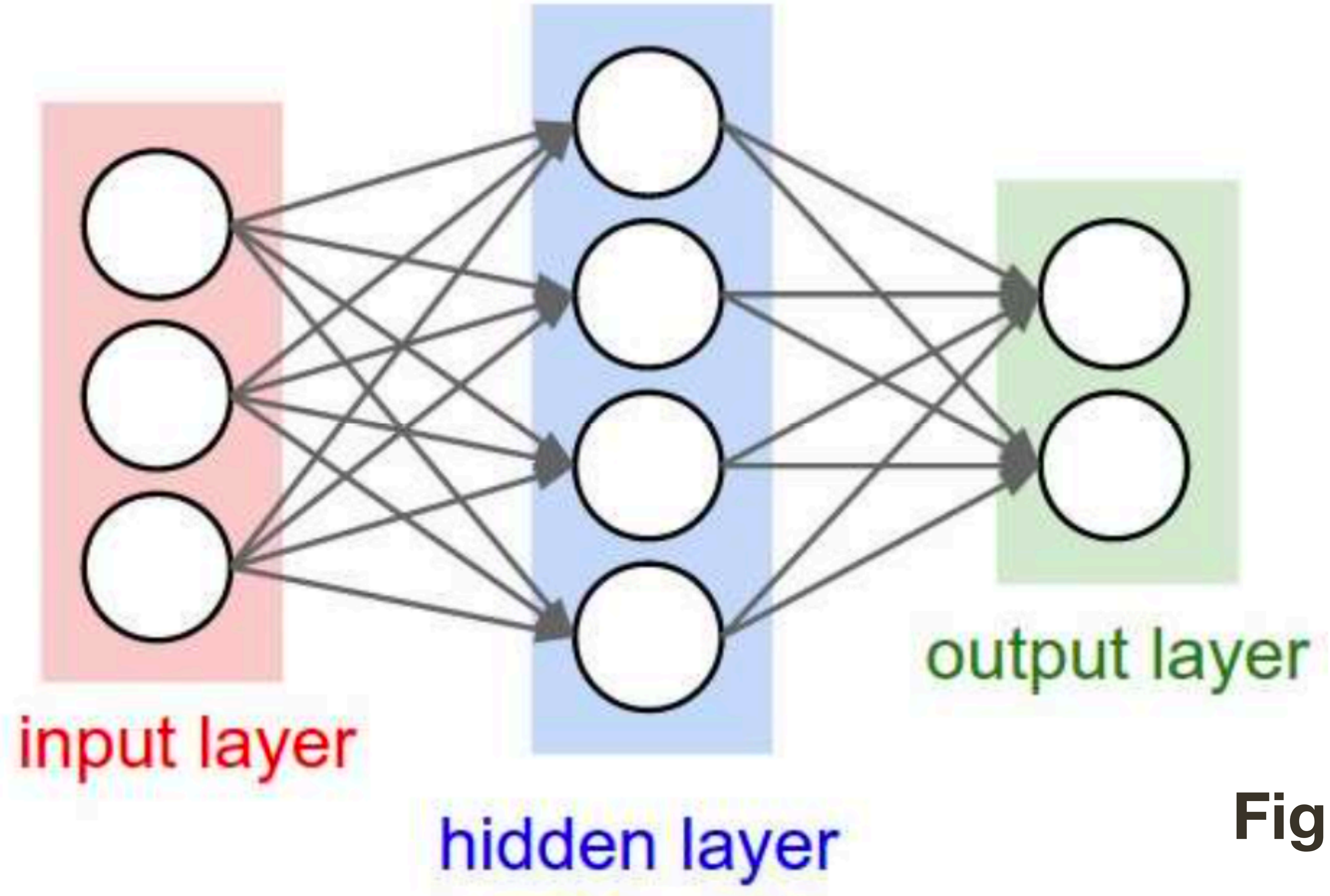

Figure credit: Fei-Fei and Karpathy

$$\hat{\mathbf{y}} = f(\mathbf{x}, \mathbf{W}_1, \mathbf{W}_2, \mathbf{b}_1, \mathbf{b}_2) = \sigma\left(\mathbf{W}_2^{(2\times 4)} \sigma\left(\mathbf{W}_1^{(4\times 3)}\mathbf{x} + \mathbf{b}_1^{(4)}\right) + \mathbf{b}_2^{(2)}\right)$$

Backpropagation

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$$\frac{\partial f}{\partial x} = y \qquad\qquad \frac{\partial f}{\partial y} = x$$

Backpropagation

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Backpropagation

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$$\frac{\partial f}{\partial x} = 1 \qquad \frac{\partial f}{\partial y} = 1$$

Backpropagation

A trickier example: $f(x, y) = \max(x, y)$

Backpropagation

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$$\frac{\partial f}{\partial x} = \mathbf{1}(x \geq y) \qquad \frac{\partial f}{\partial y} = \mathbf{1}(y \geq x)$$

That is, the (sub)gradient is 1 on the input that is larger, and 0 on the other input

— For example, say x = 4, y = 2. Increasing y by a tiny amount does not change the value of f (f will still be 4), hence the gradient on y is zero.

Backpropagation

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For illustration we break this expression into $q = x + y$ and $f = qz$. This is a sum and a product, and we have just seen how to compute partial derivatives for these.

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$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q}\frac{\partial q}{\partial x} = z \cdot 1 = z$$

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Backpropagation

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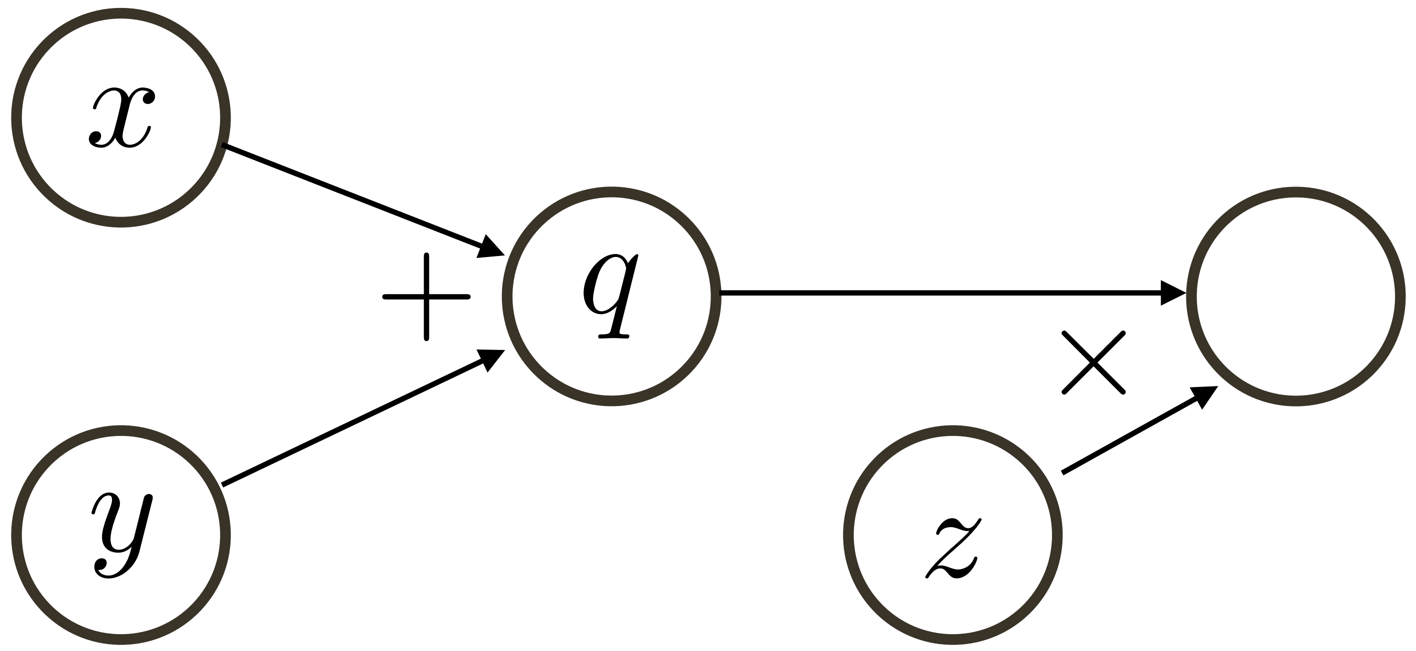

Computational graph (a DAG) with variable ordering from topological sort, where each **node** is an input, intermediate, or output variable

Backpropagation

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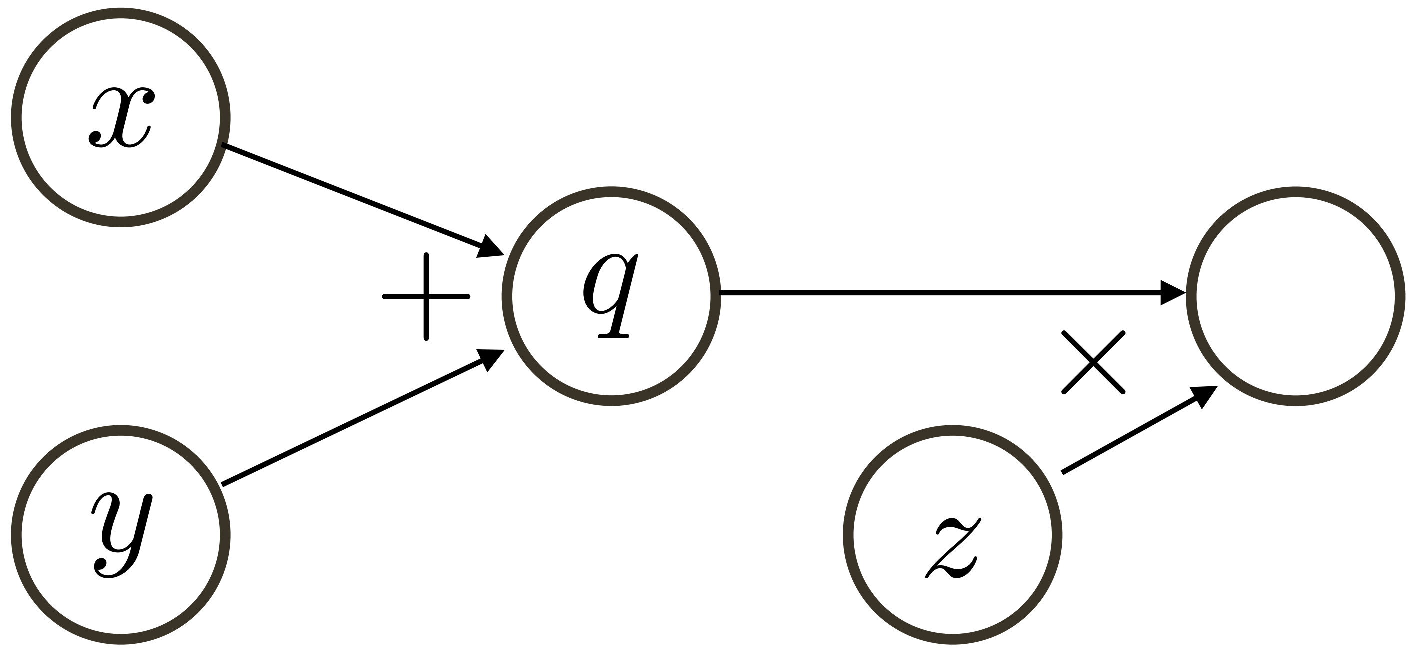

Computational graph (a DAG) with variable ordering from topological sort, where each **node** is an input, intermediate, or output variable

Suppose the network input is: $(x, y, z) = (-2, 5, -4)$

Then: $q = x + y = 3$ $\quad f = qz = -12$ (**forward** pass)

Backpropagation

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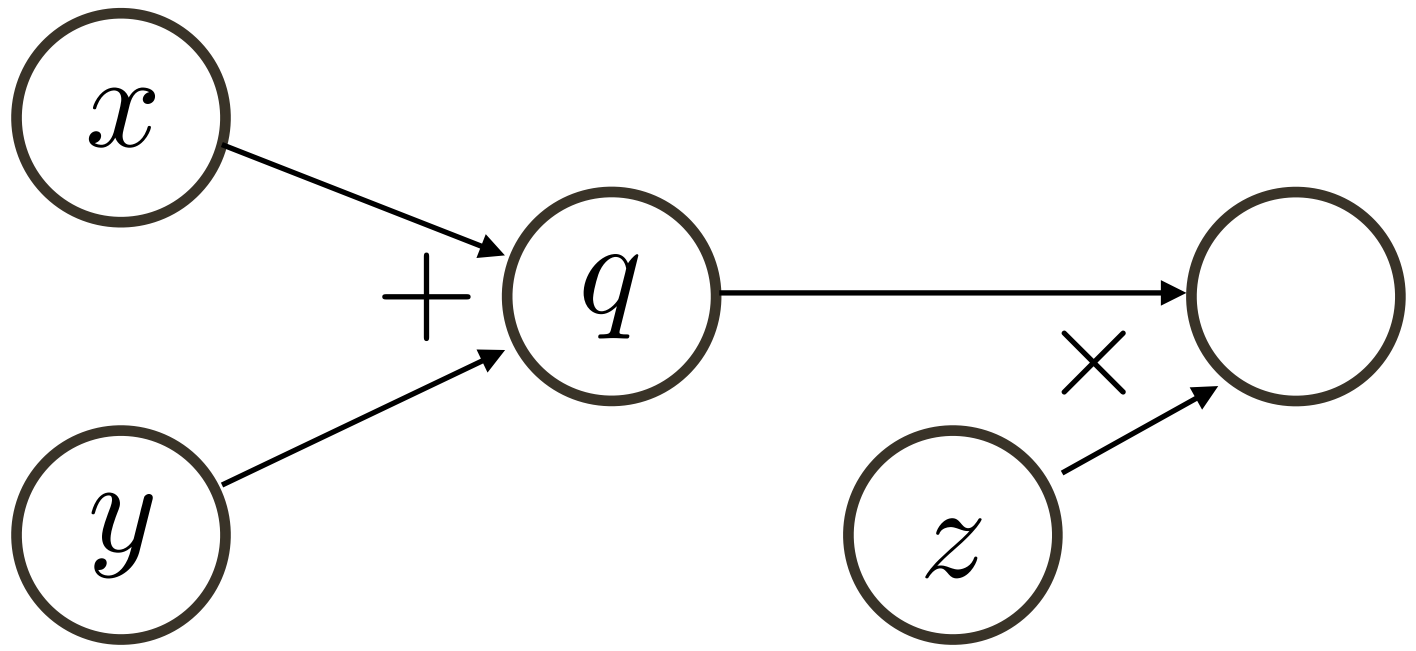

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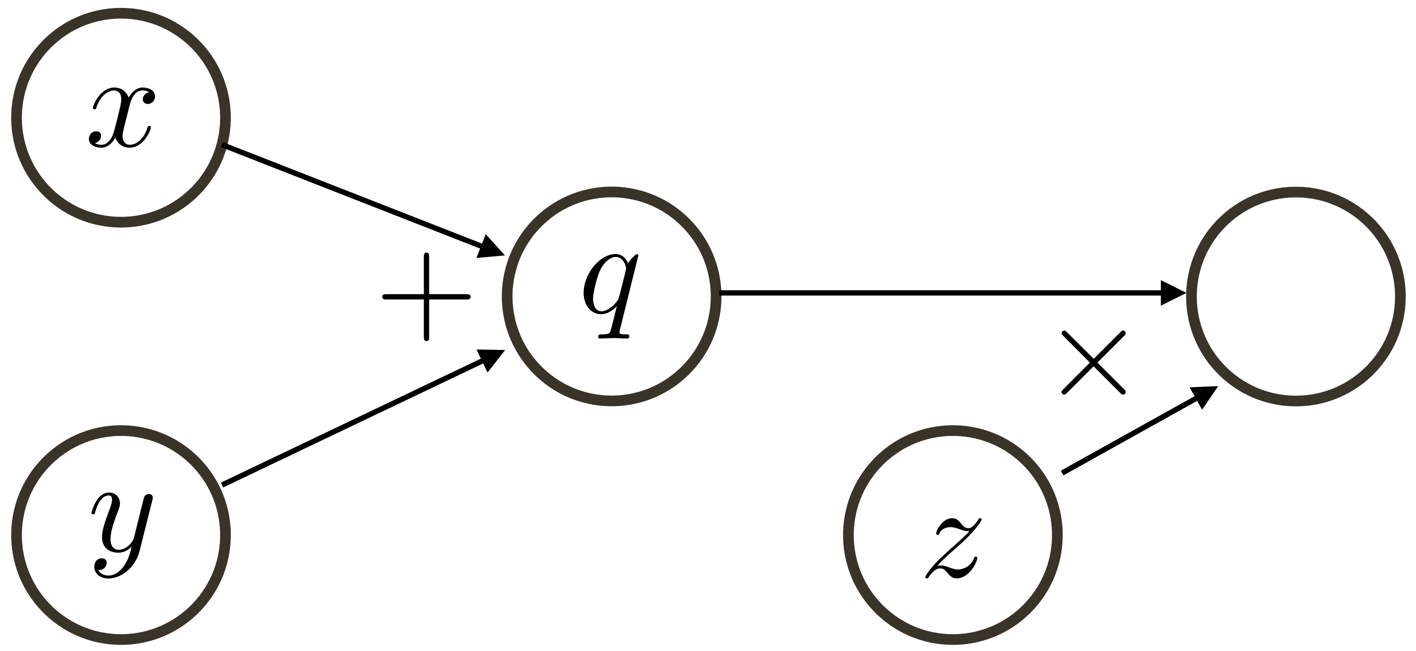

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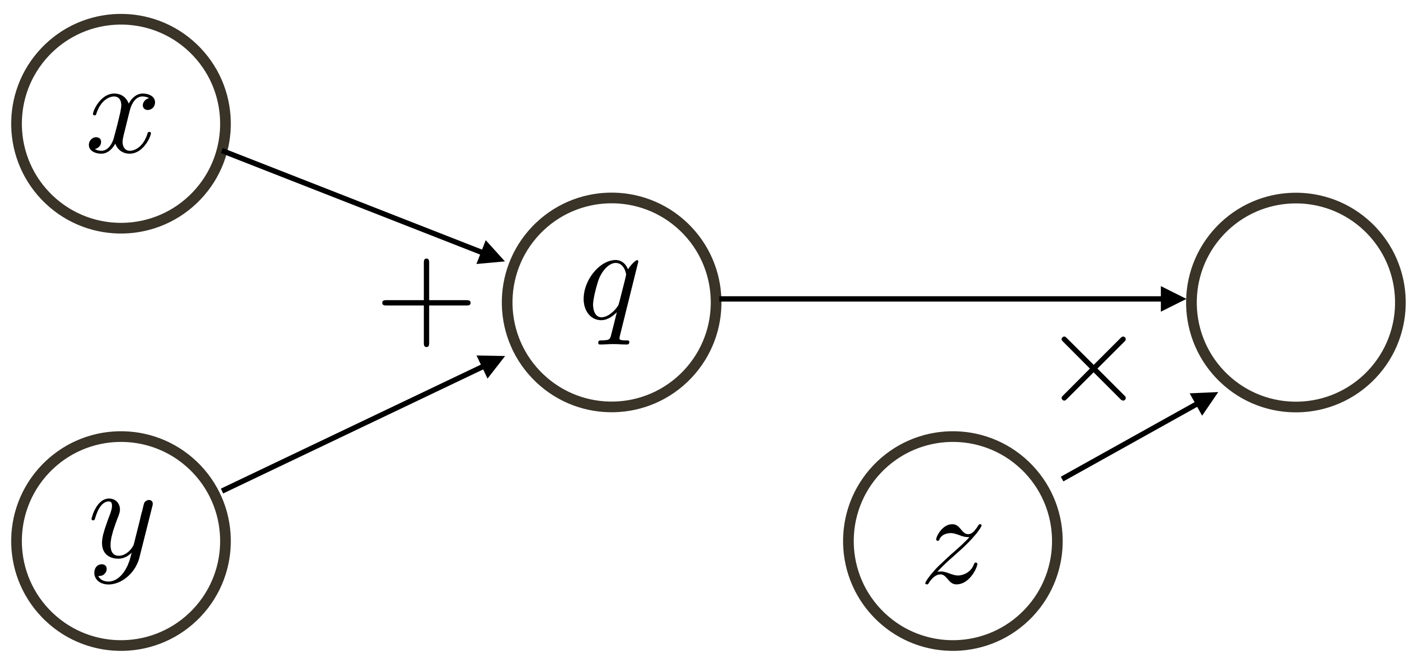

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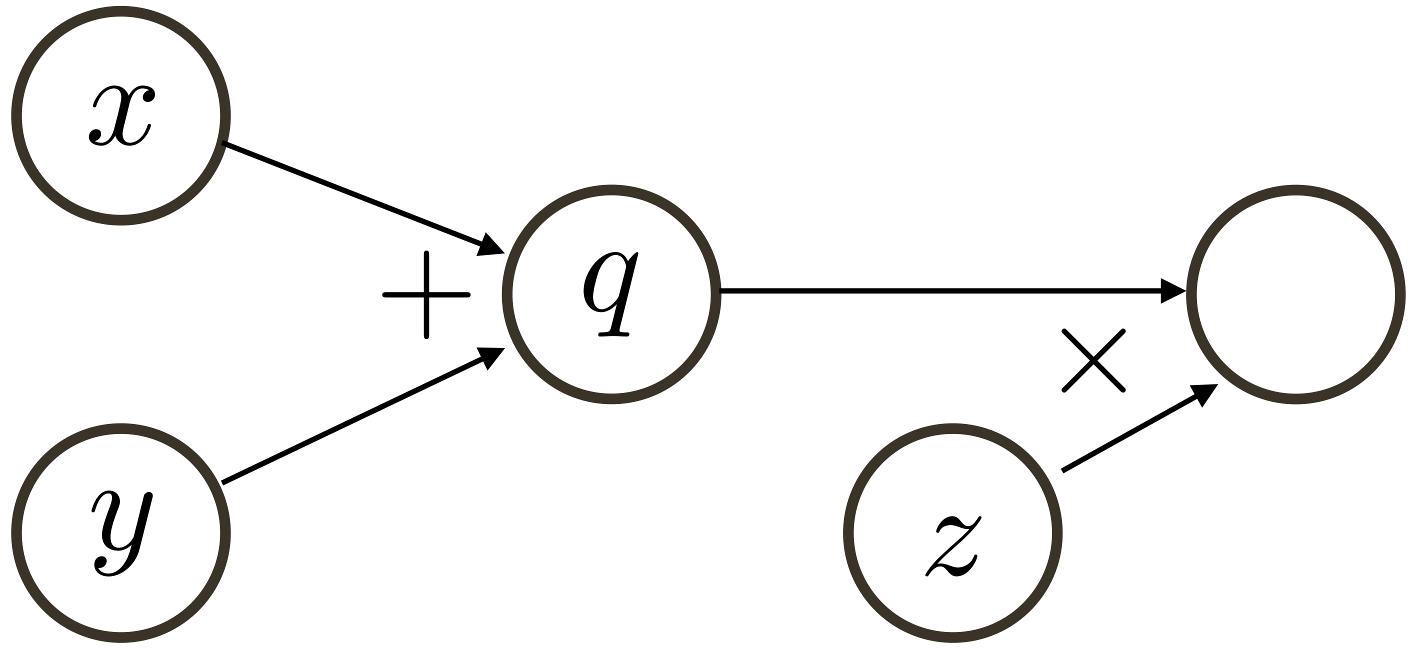
(**backward** pass)

Backpropagation



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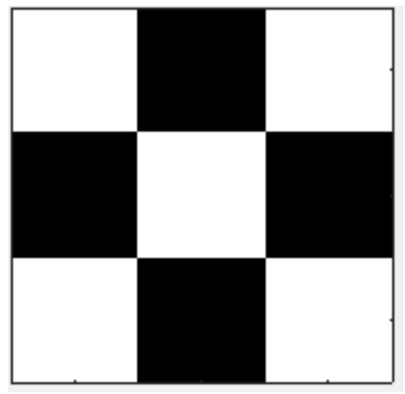
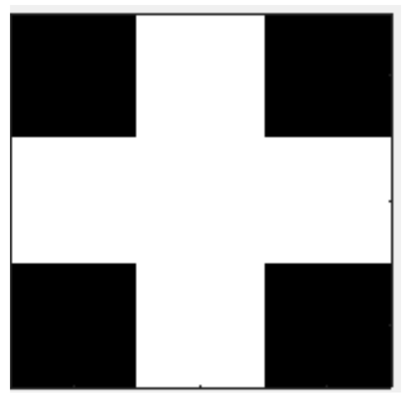
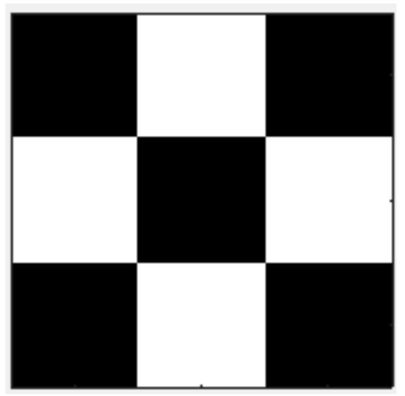
$$\frac{\partial f}{\partial q} = z = -4 \qquad \frac{\partial f}{\partial x} = -4 \qquad \frac{\partial f}{\partial y} = -4 \qquad \frac{\partial f}{\partial z} = 3$$

(**backward** pass)

Example: Let’s Build (world smallest) Neural Network

Lets create a neural network that will be able to differentiate (classify) these patterns of simple 3x3 pixel images

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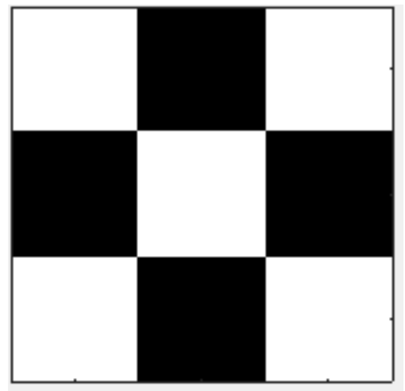
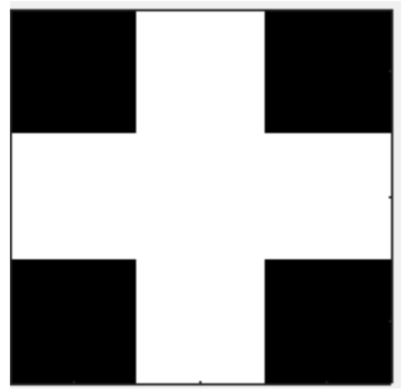
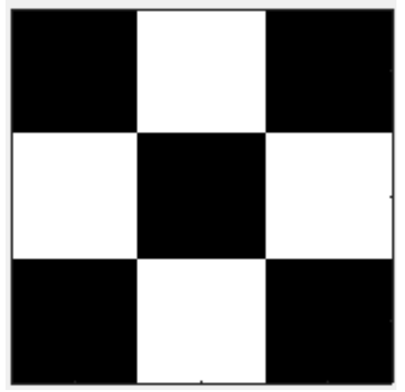
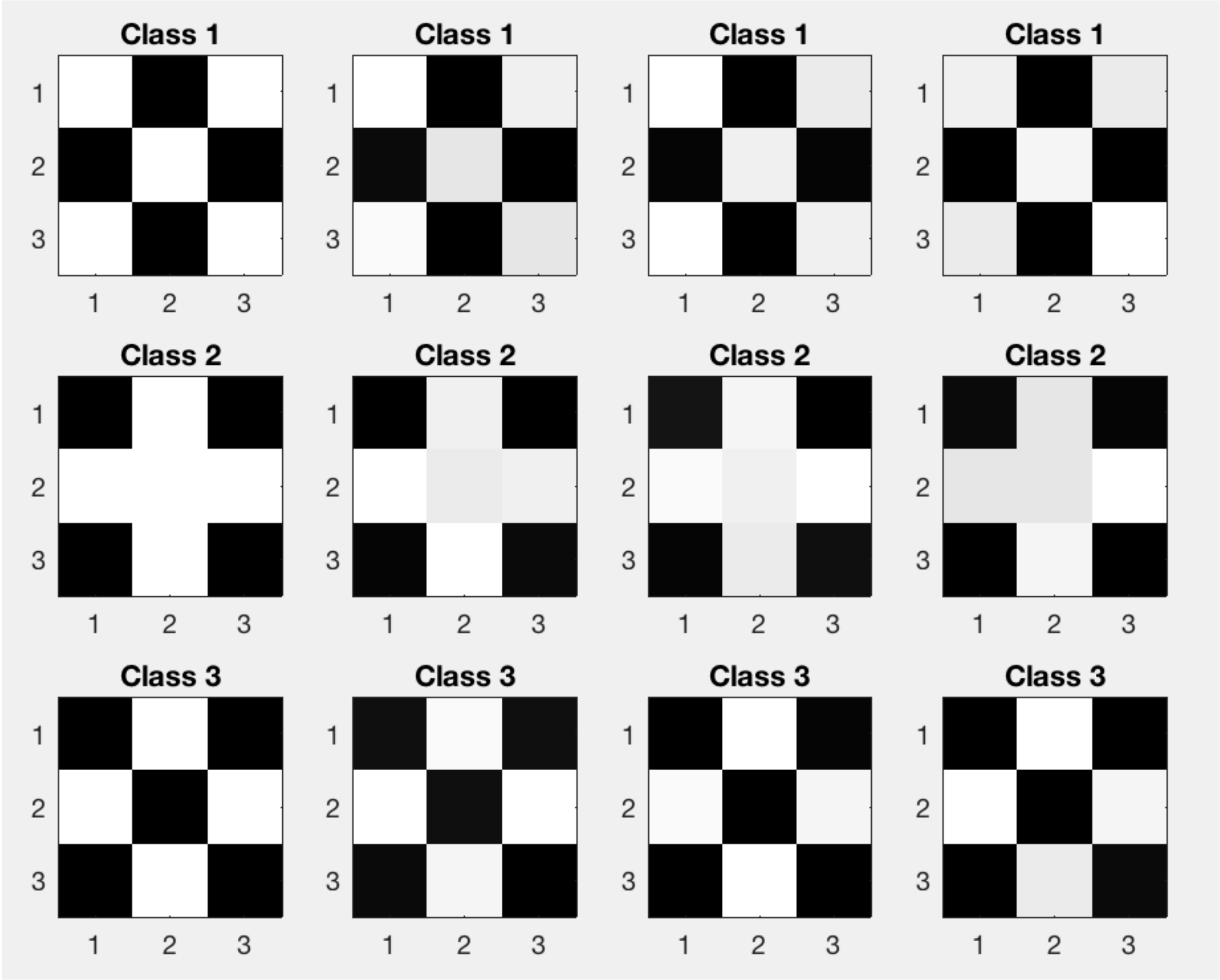




We will need some labeled data



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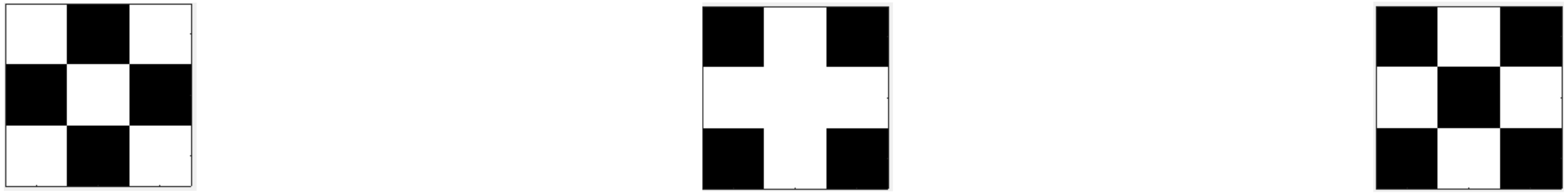

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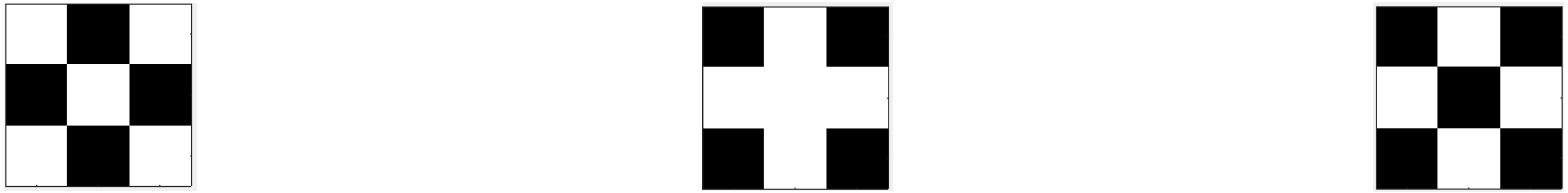

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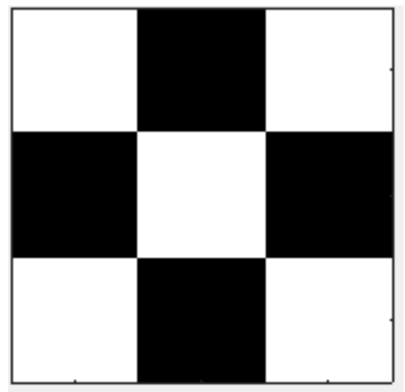

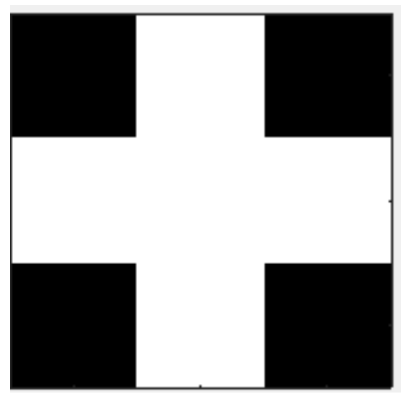

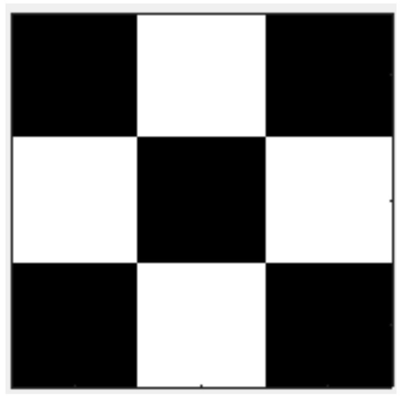

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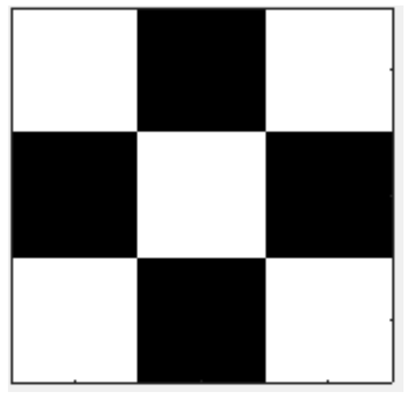
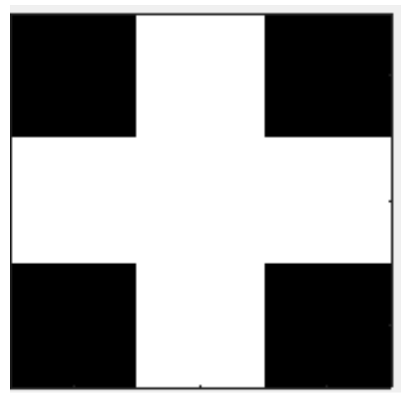
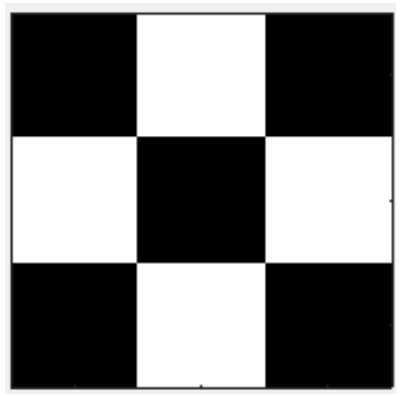

What do we need to do?

First, lets re-formulate the problem

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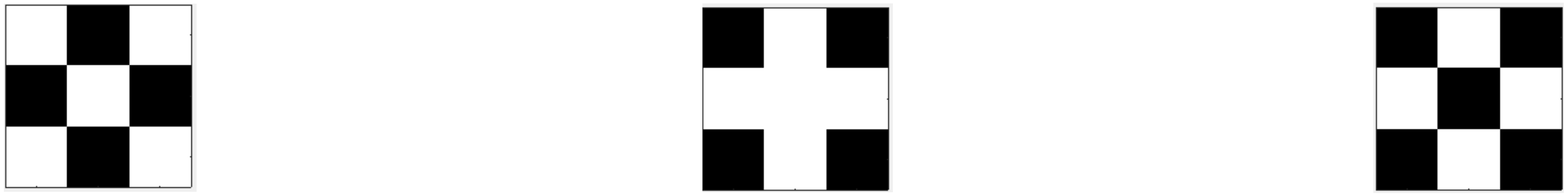

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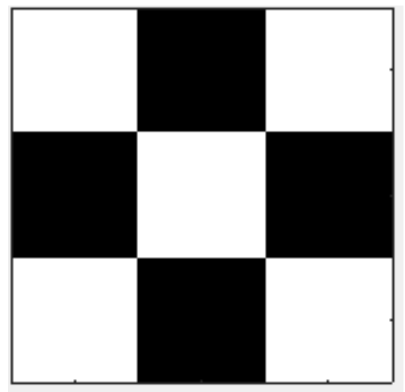

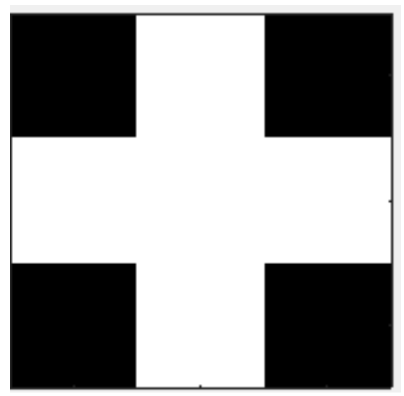

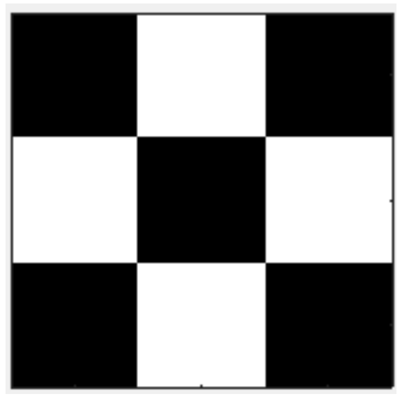

Now, lets build a **network**!

How many inputs should the network have? How neuron outputs?

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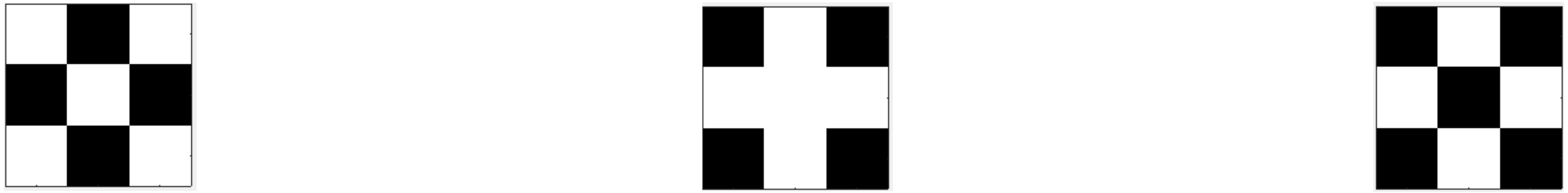
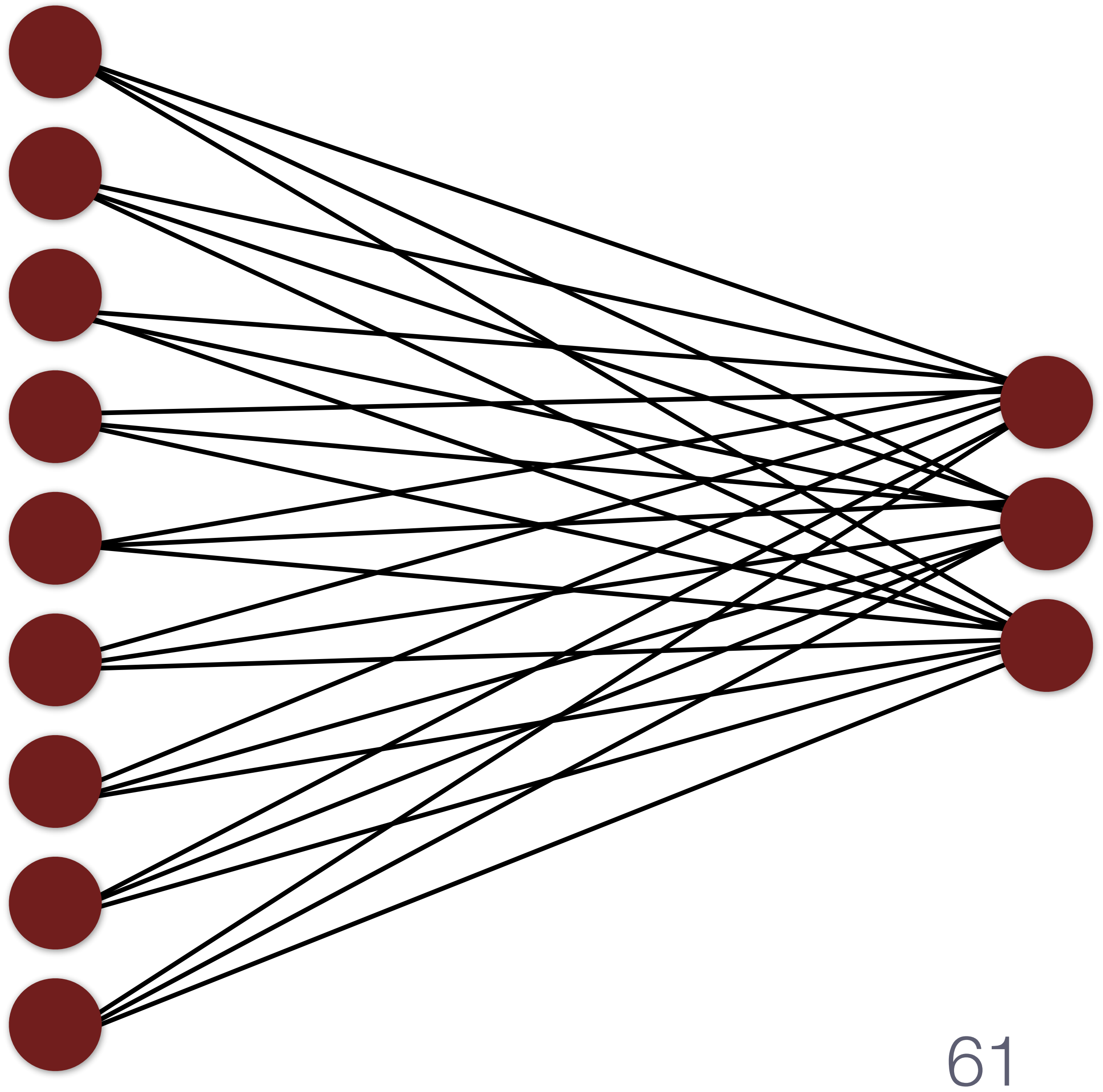
Lets create a neural network that will be able to differentiate (classify) these patterns of simple 3x3 pixel images



Input Layer **Output** Layer



What else is missing for us to train it?

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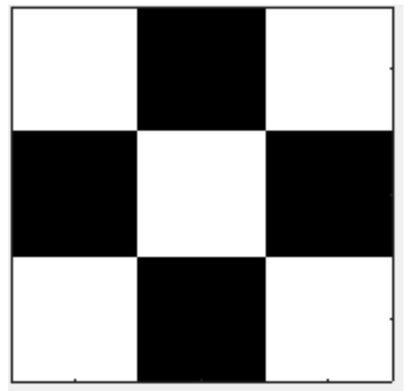
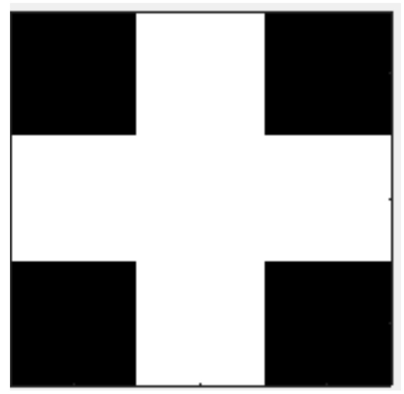
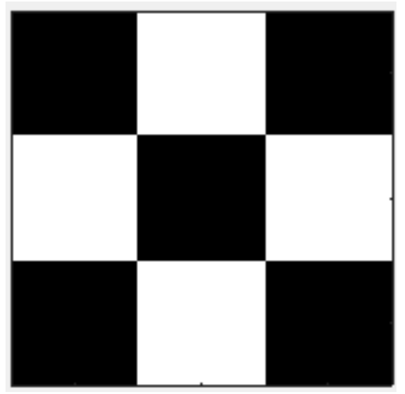
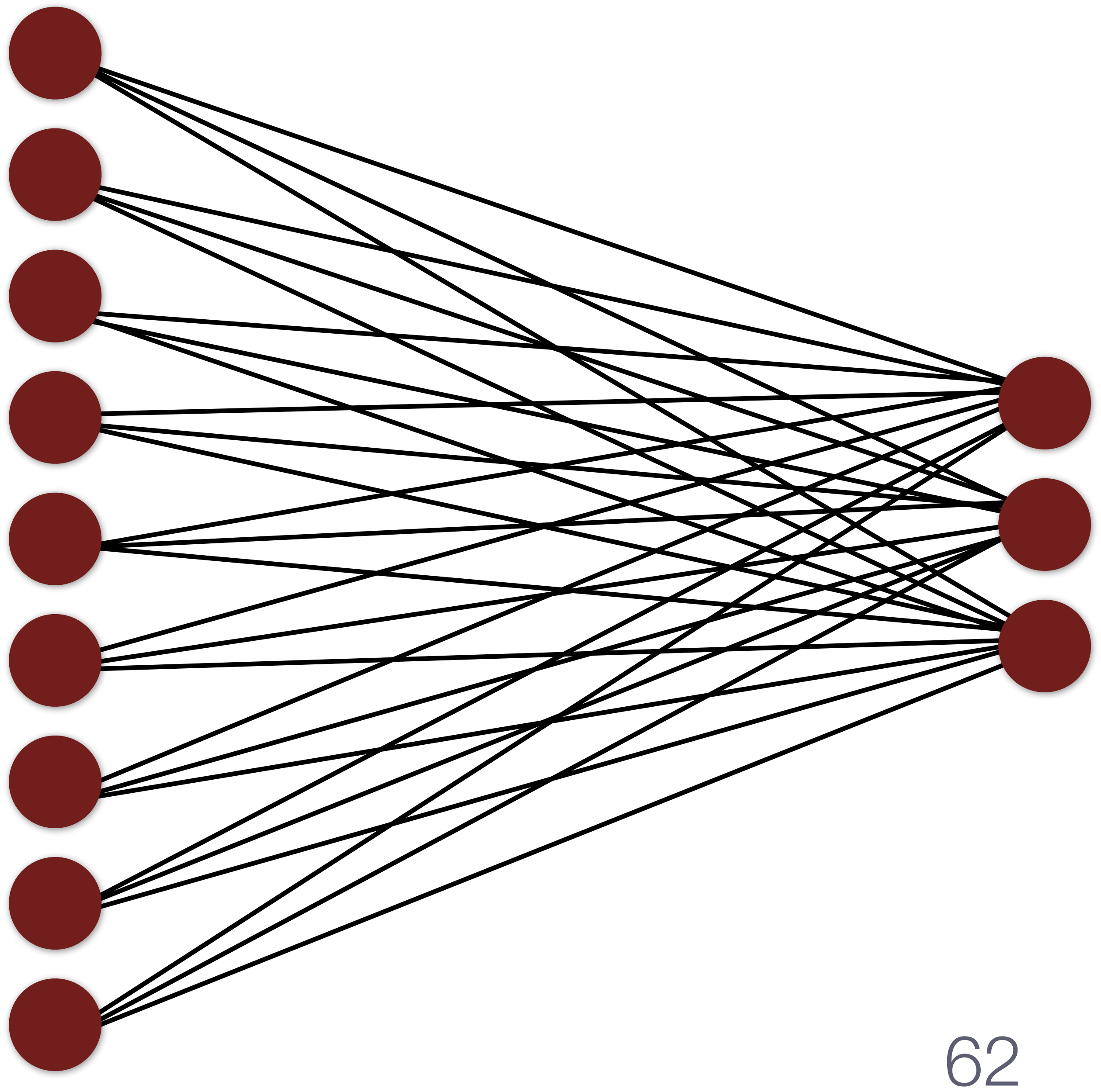






Input Layer **Output** Layer **Loss**



$$L_i = -\log\left(\frac{e^{f_{y_i}}}{\sum_j e^{f_{y_j}}}\right)$$

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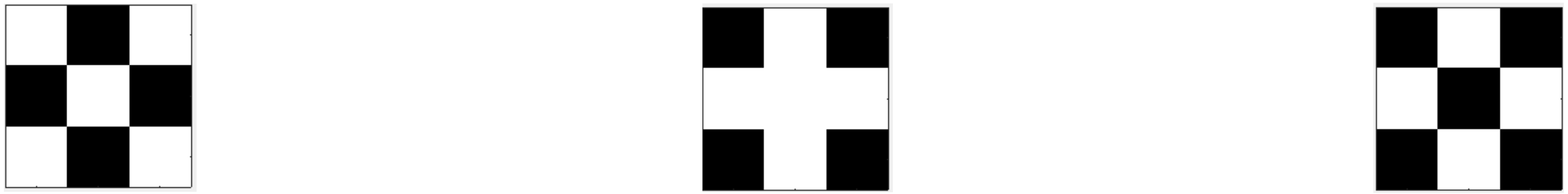

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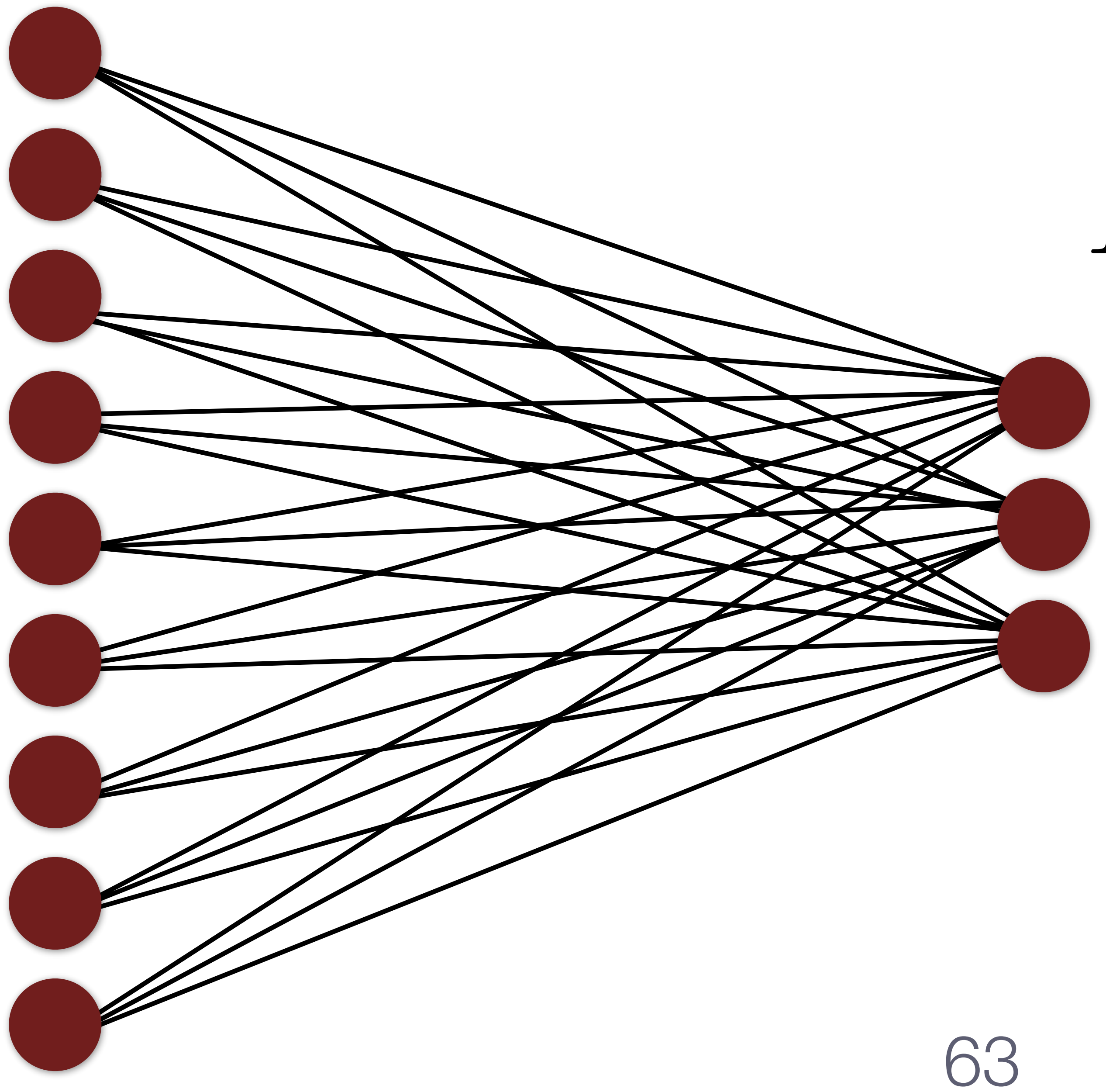
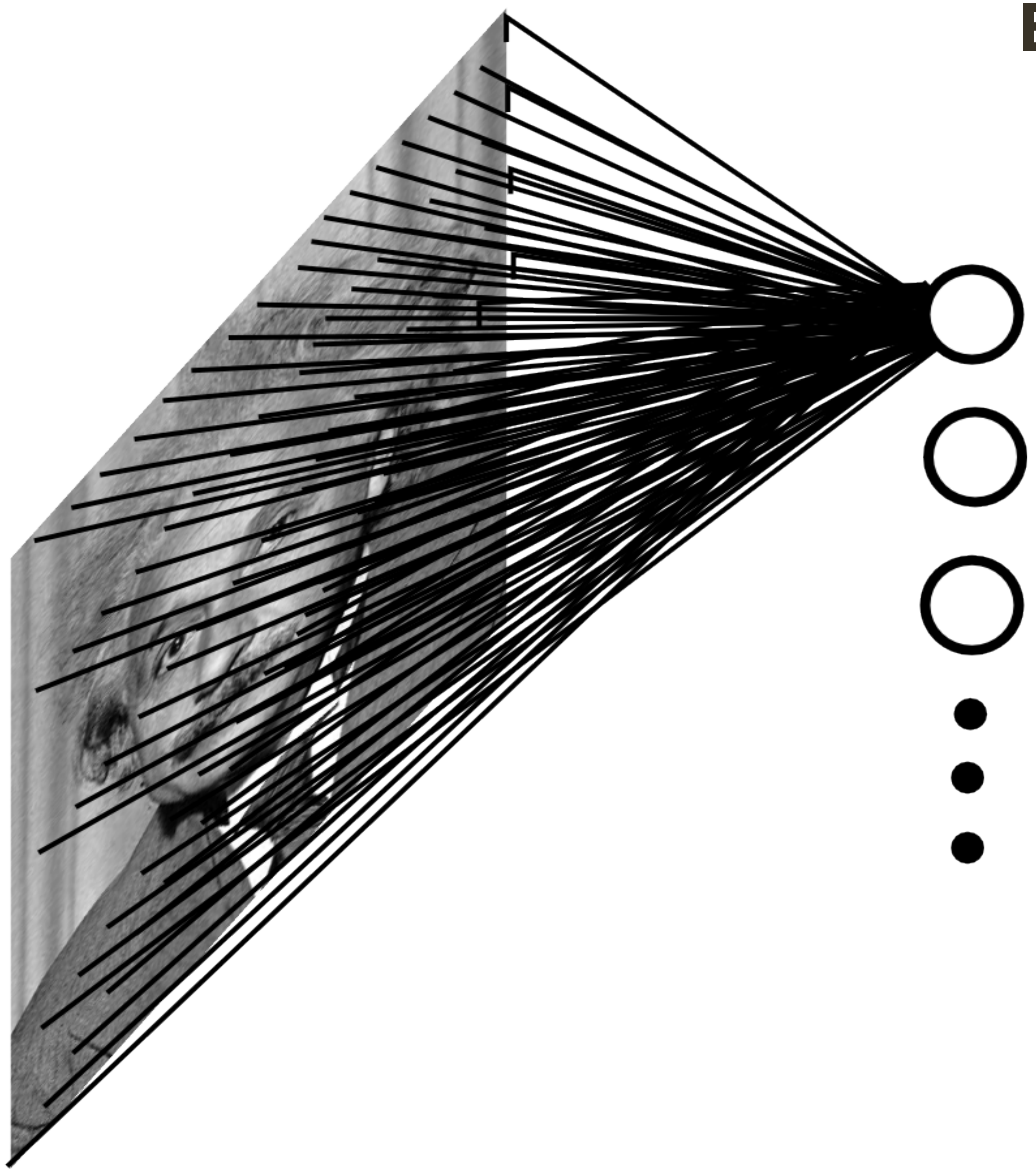


$$L_1 = -log\left(\frac{e^{\sum_{i=1}^{9}\sigma(w_{1,i}x_i+b_1)}}{\sum_{j=1}^{3}e^{\sum_{i=1}^{9}\sigma(w_{1,i}x_i+b_1)}}\right)$$

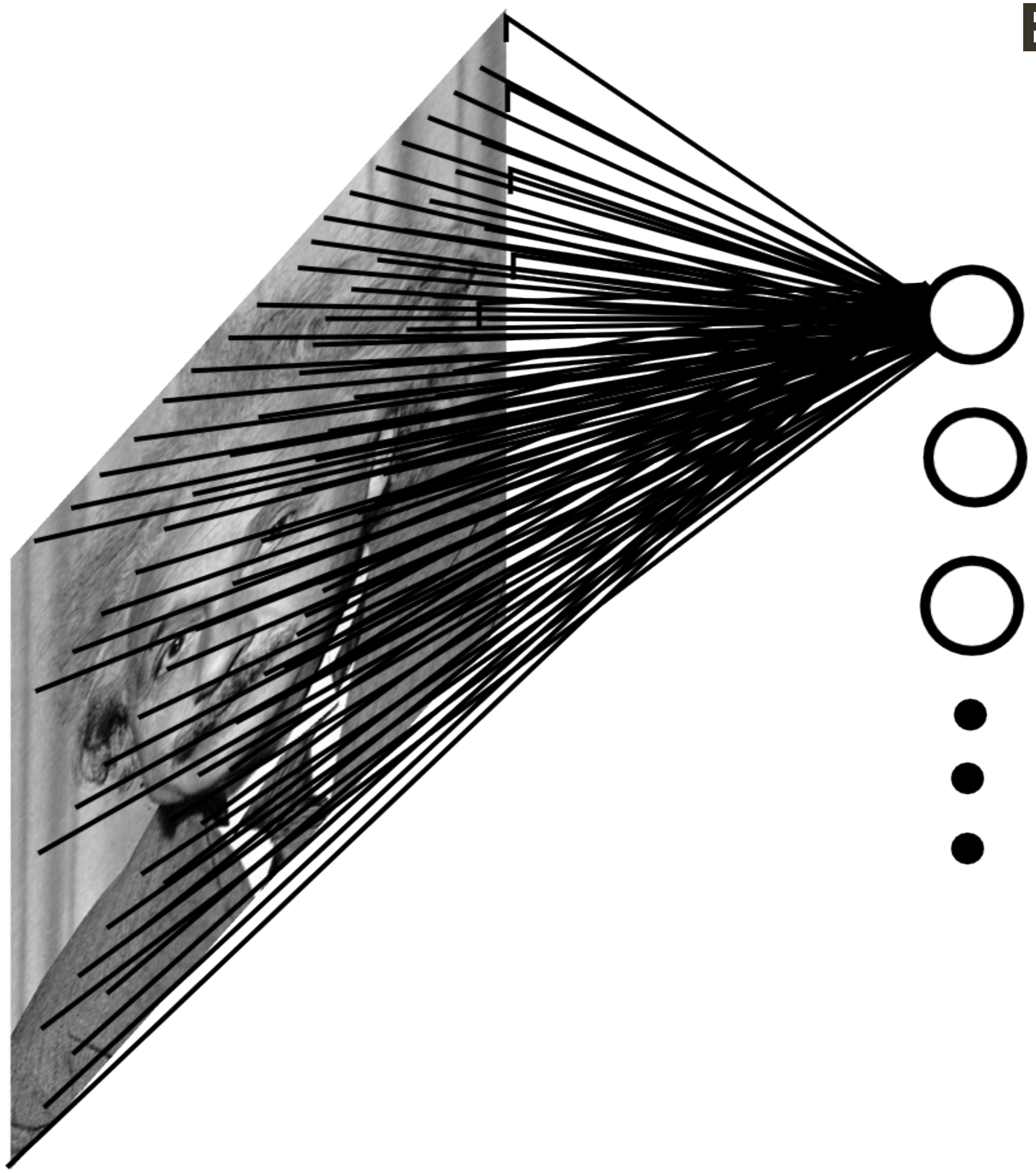
Fully Connected Layer



Example: 200 x 200 image (small)
x 40K hidden units

* slide from Marc'Aurelio Renzato

Fully Connected Layer



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= ~ **2 Billion** parameters (for one layer!)

* slide from Marc'Aurelio Renzato

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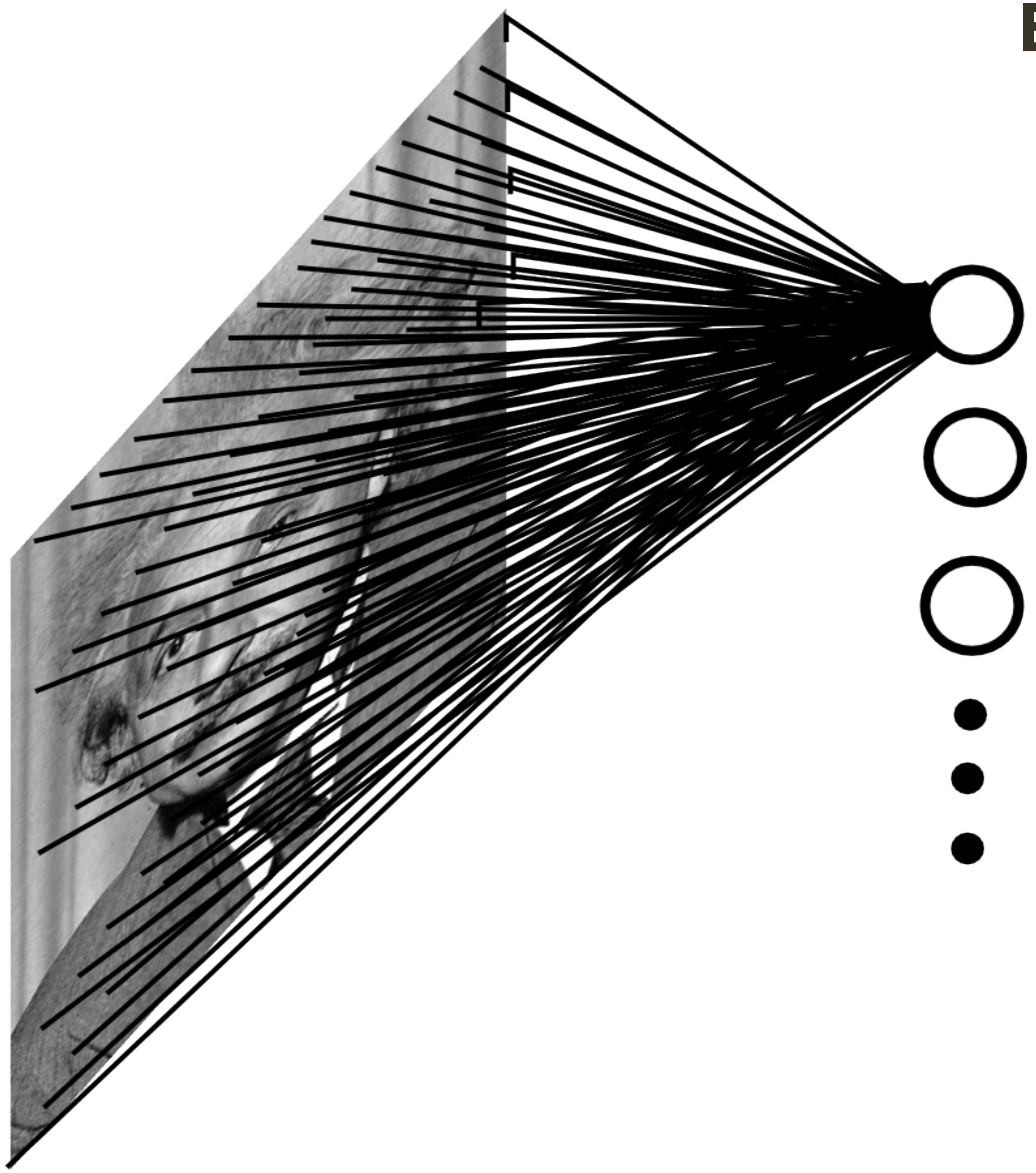

Example: 200 x 200 image (small)
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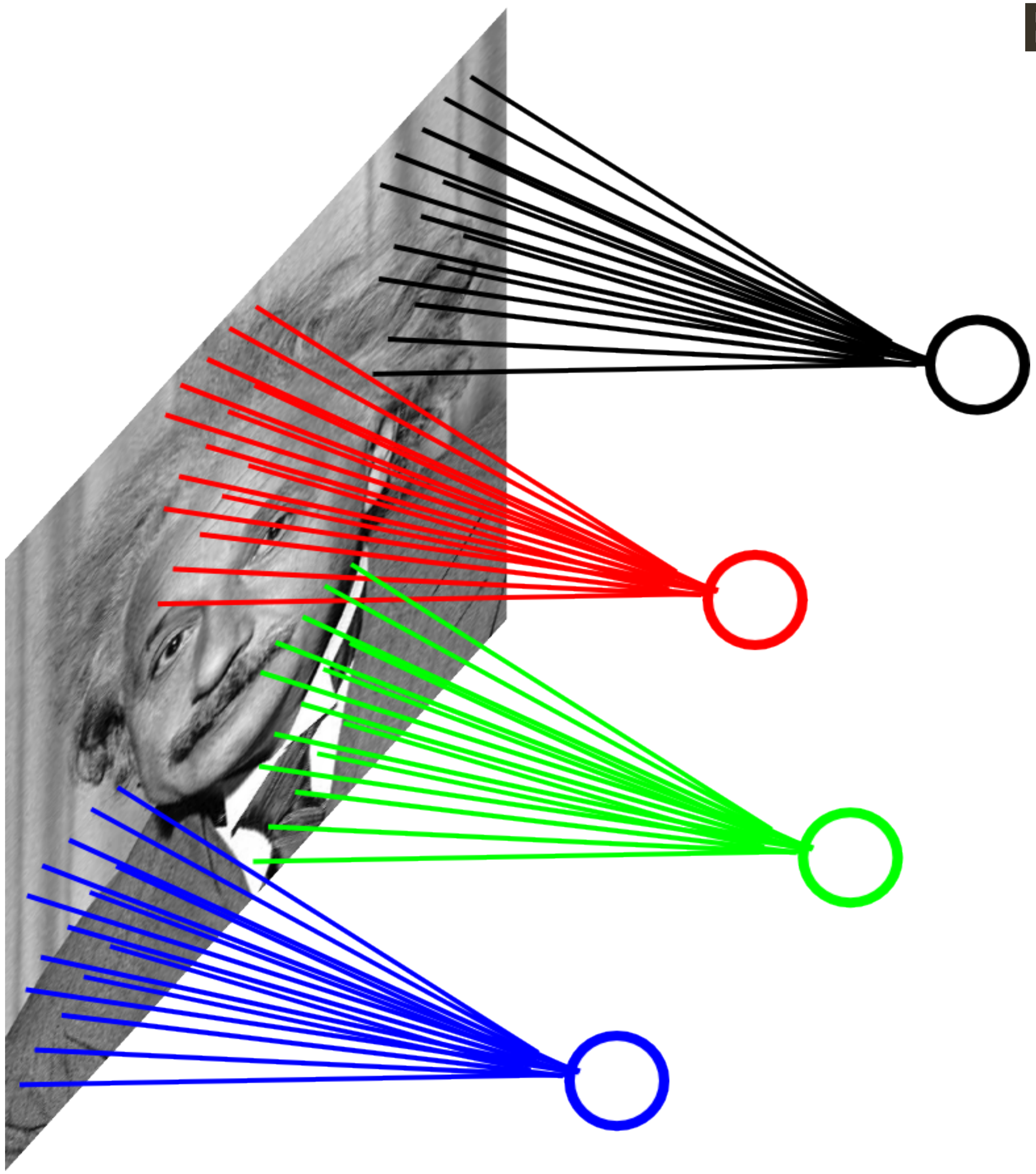
= ~ **2 Billion** parameters (for one layer!)

Spatial correlations are generally local

Waste of resources + we don't have enough data to train networks this large

* slide from Marc'Aurelio Renzato

Locally Connected Layer



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Filter size: 10 x 10

= ~ **4 Million** parameters

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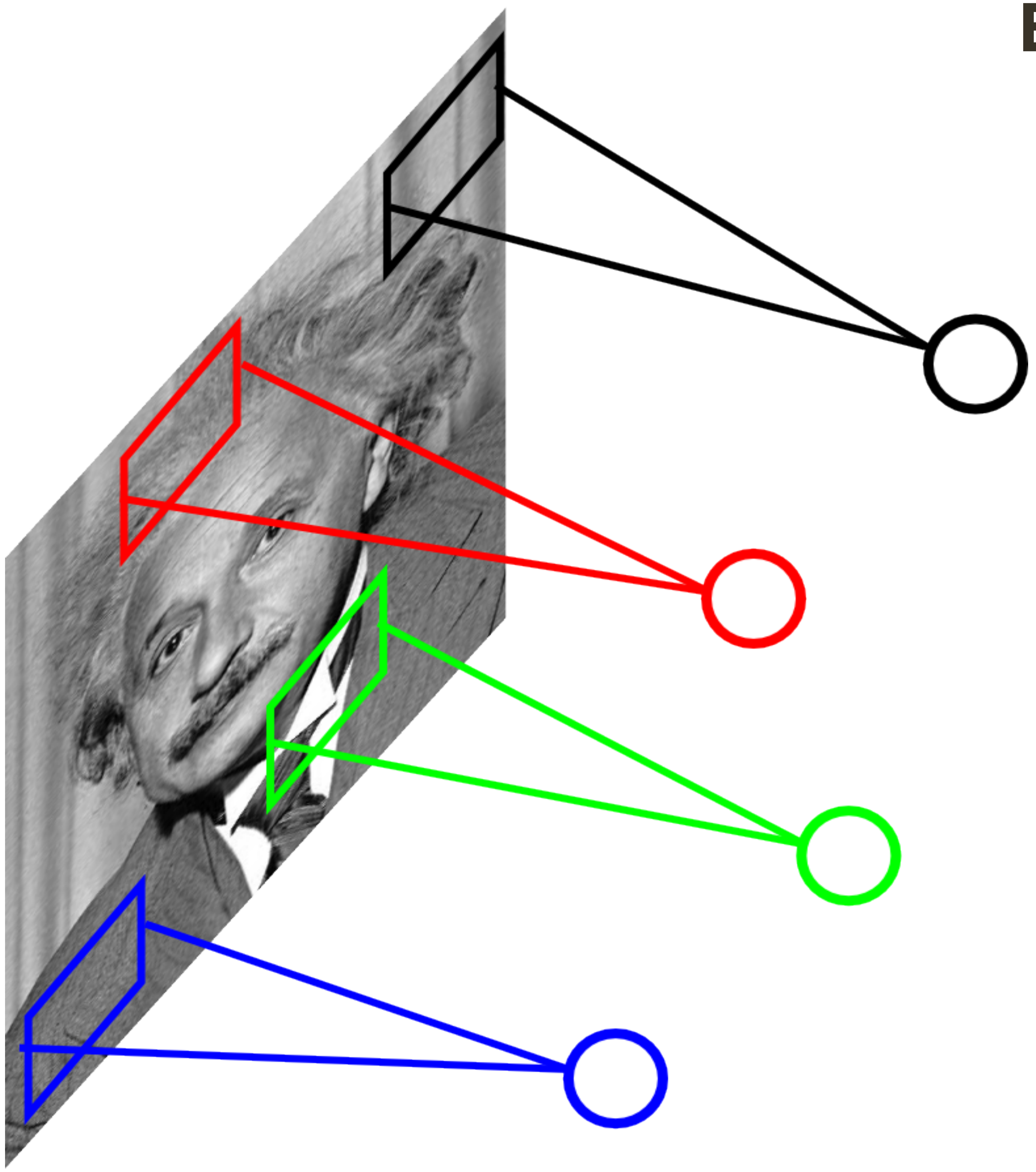

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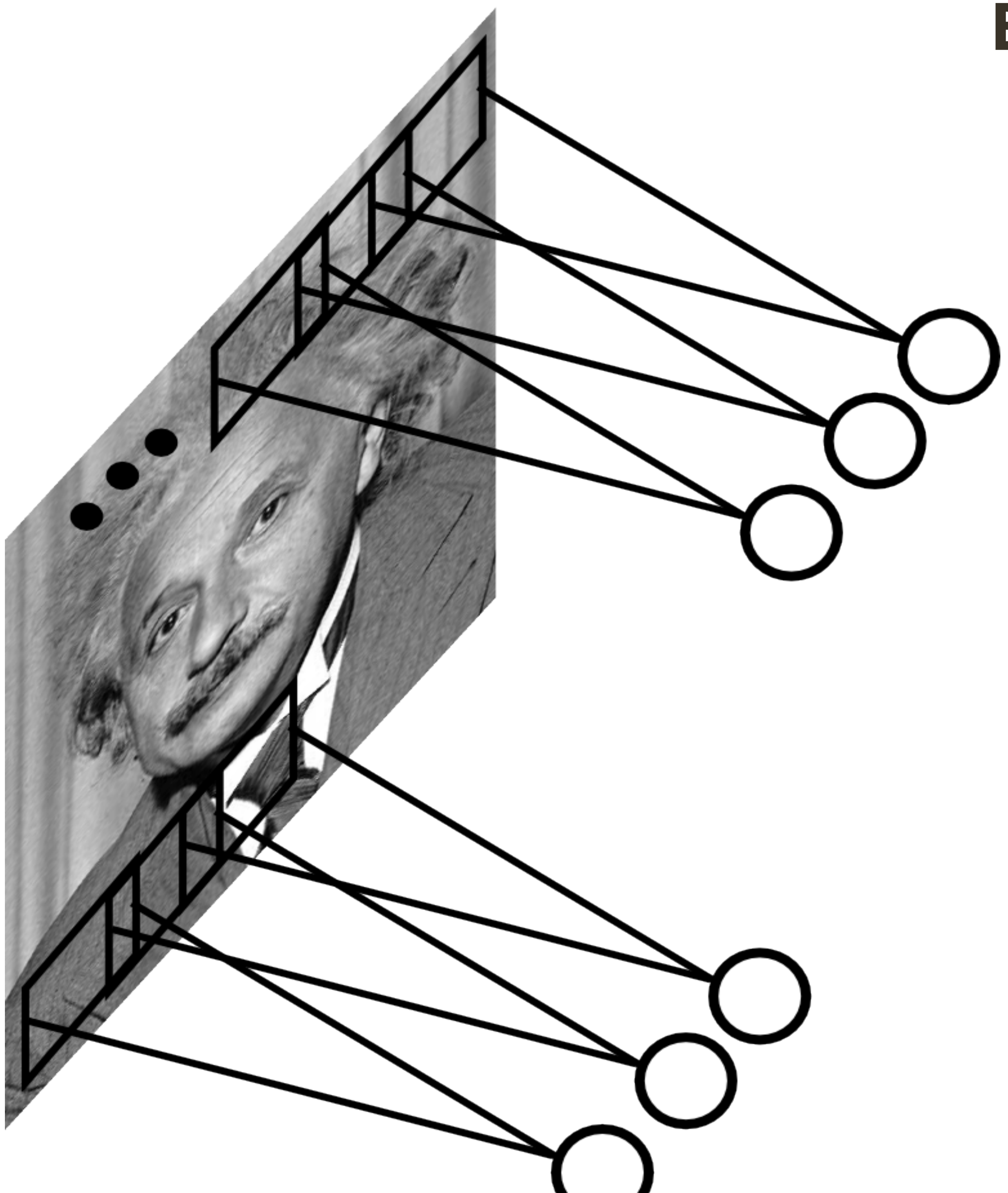
Stationarity — statistics is similar at different locations

* slide from Marc'Aurelio Renzato

Convolutional Layer



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Share the same parameters across the locations (assuming input is stationary)

* slide adopted from Marc'Aurelio Renzato

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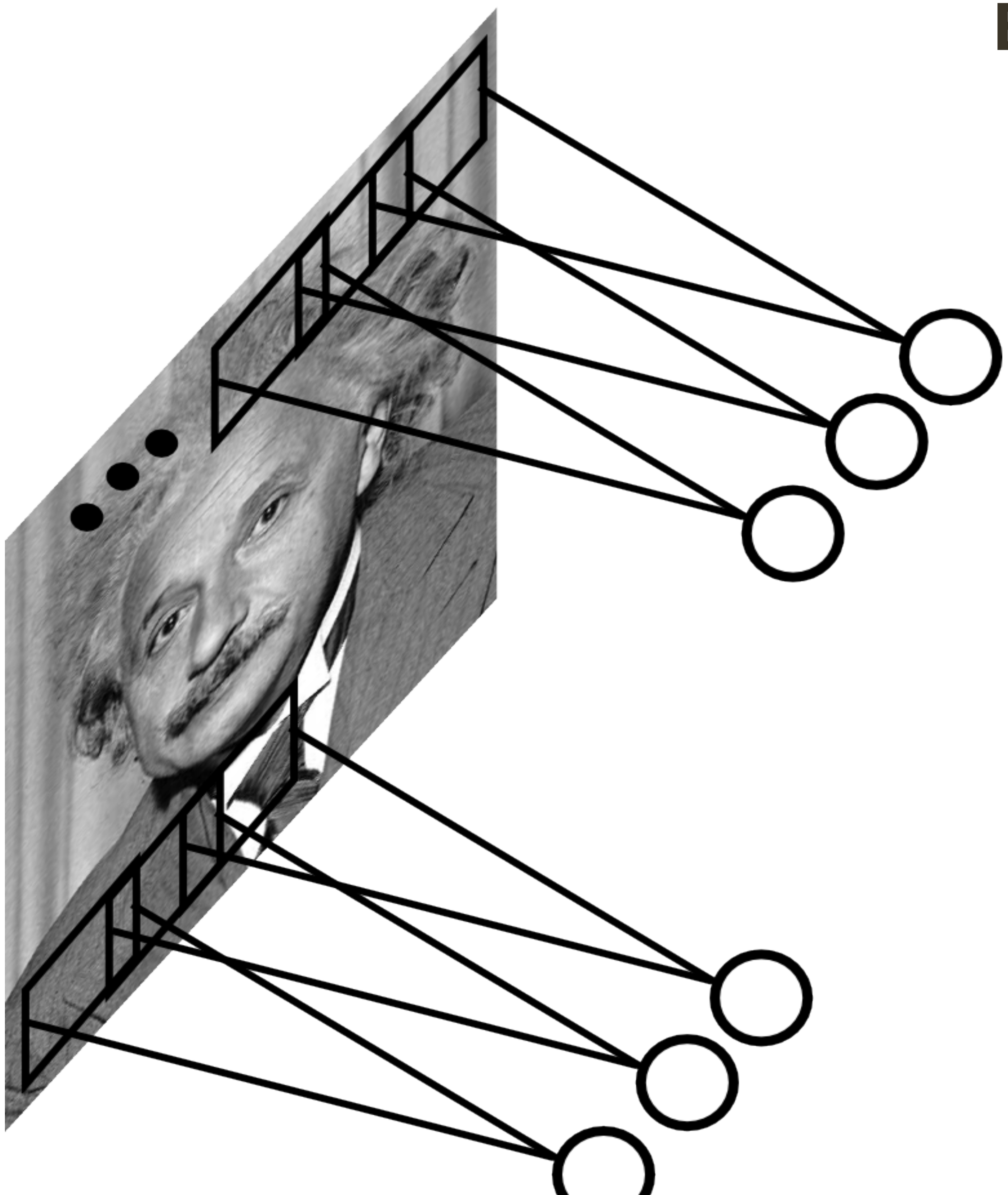

Example: 200 x 200 image (small)
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= ~ **~~4 Million~~** parameters

= 100 parameters

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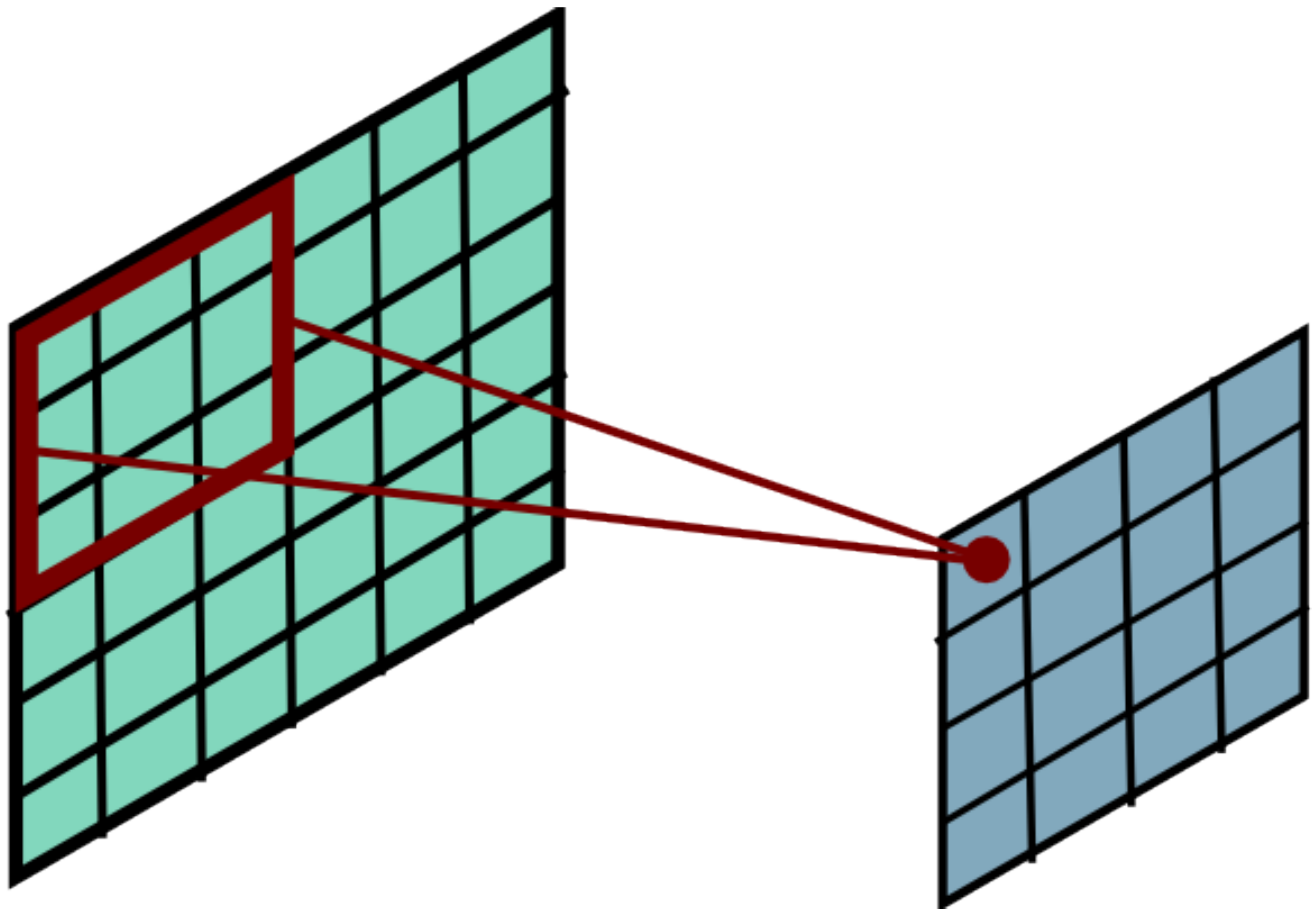

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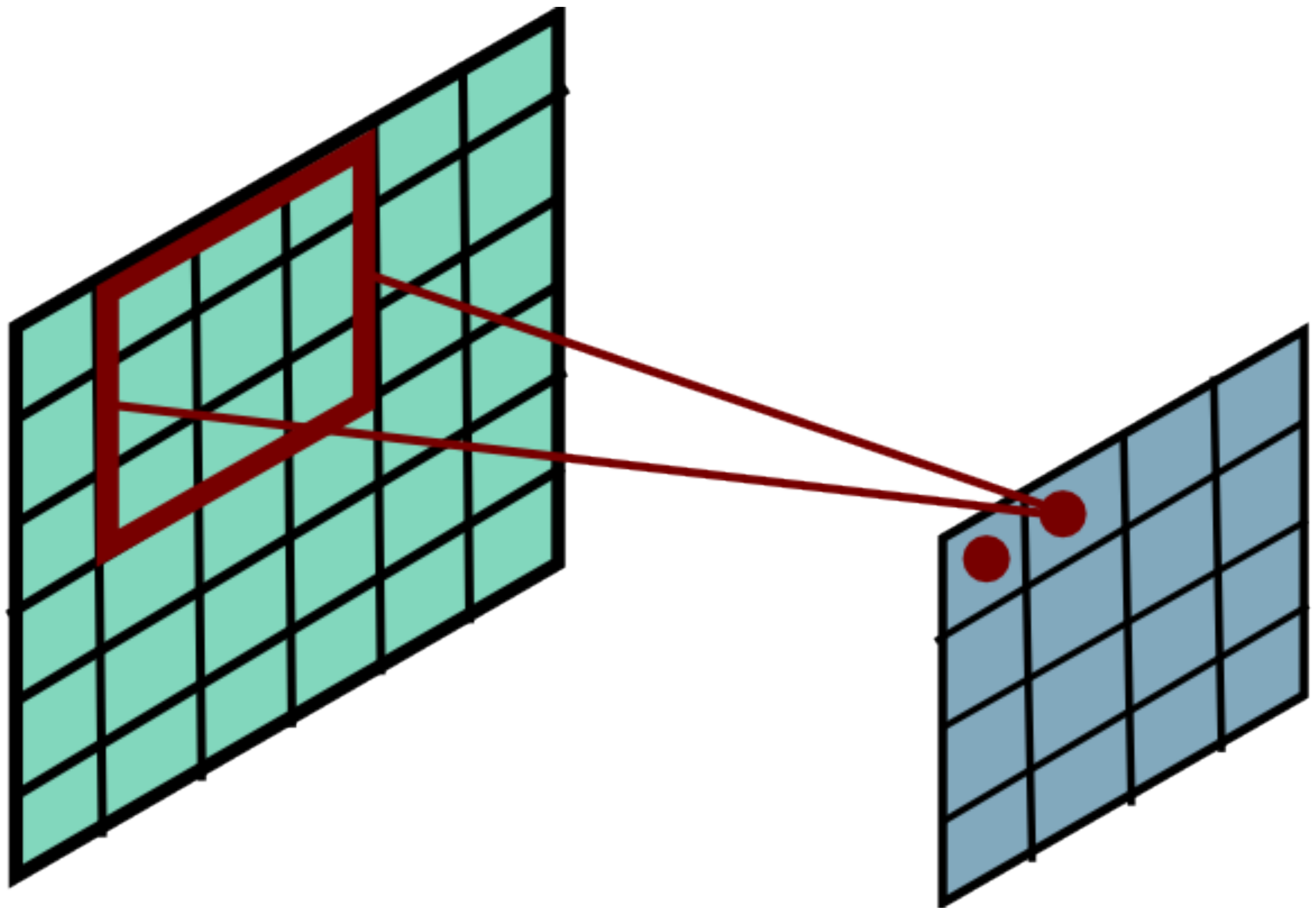

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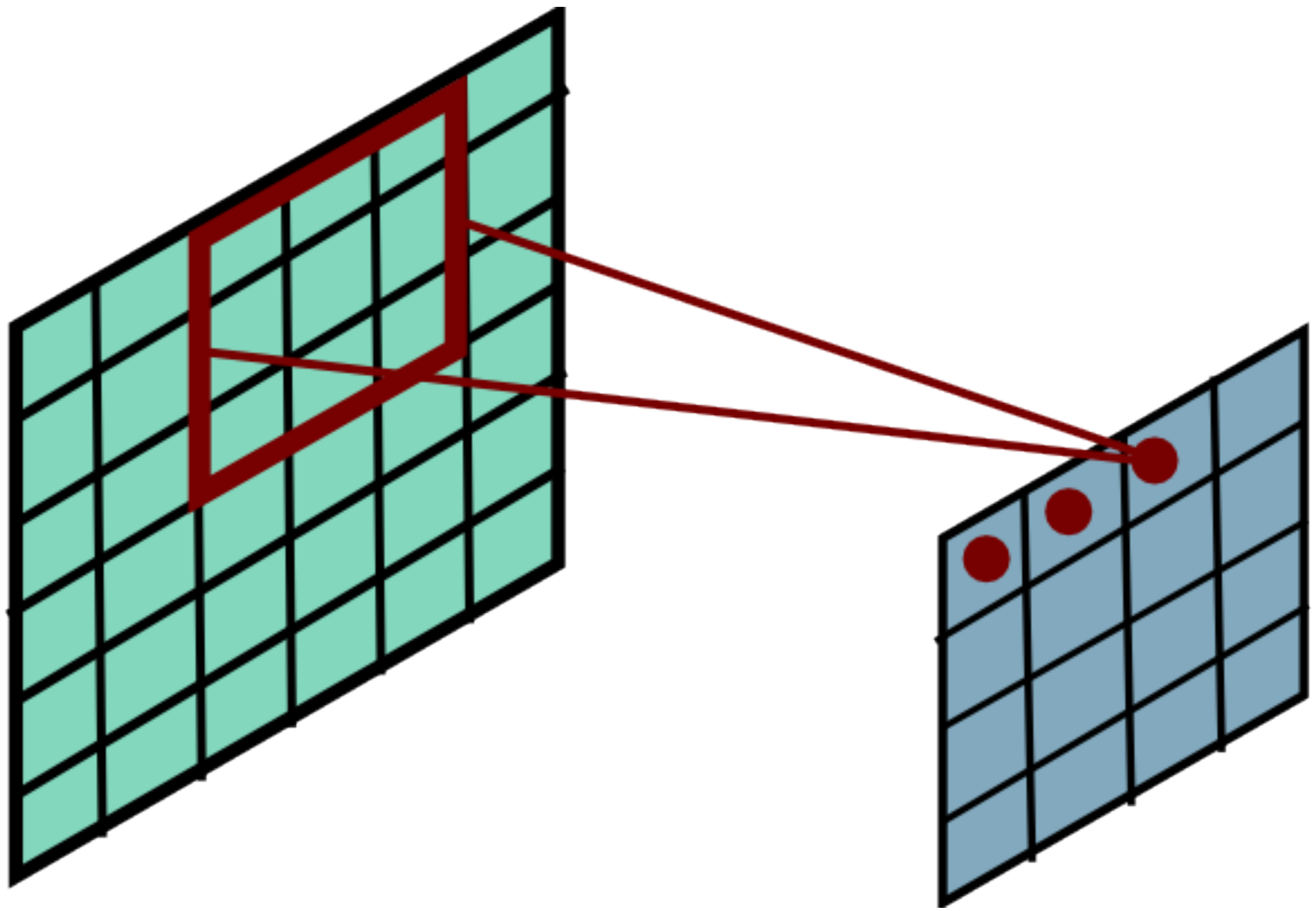

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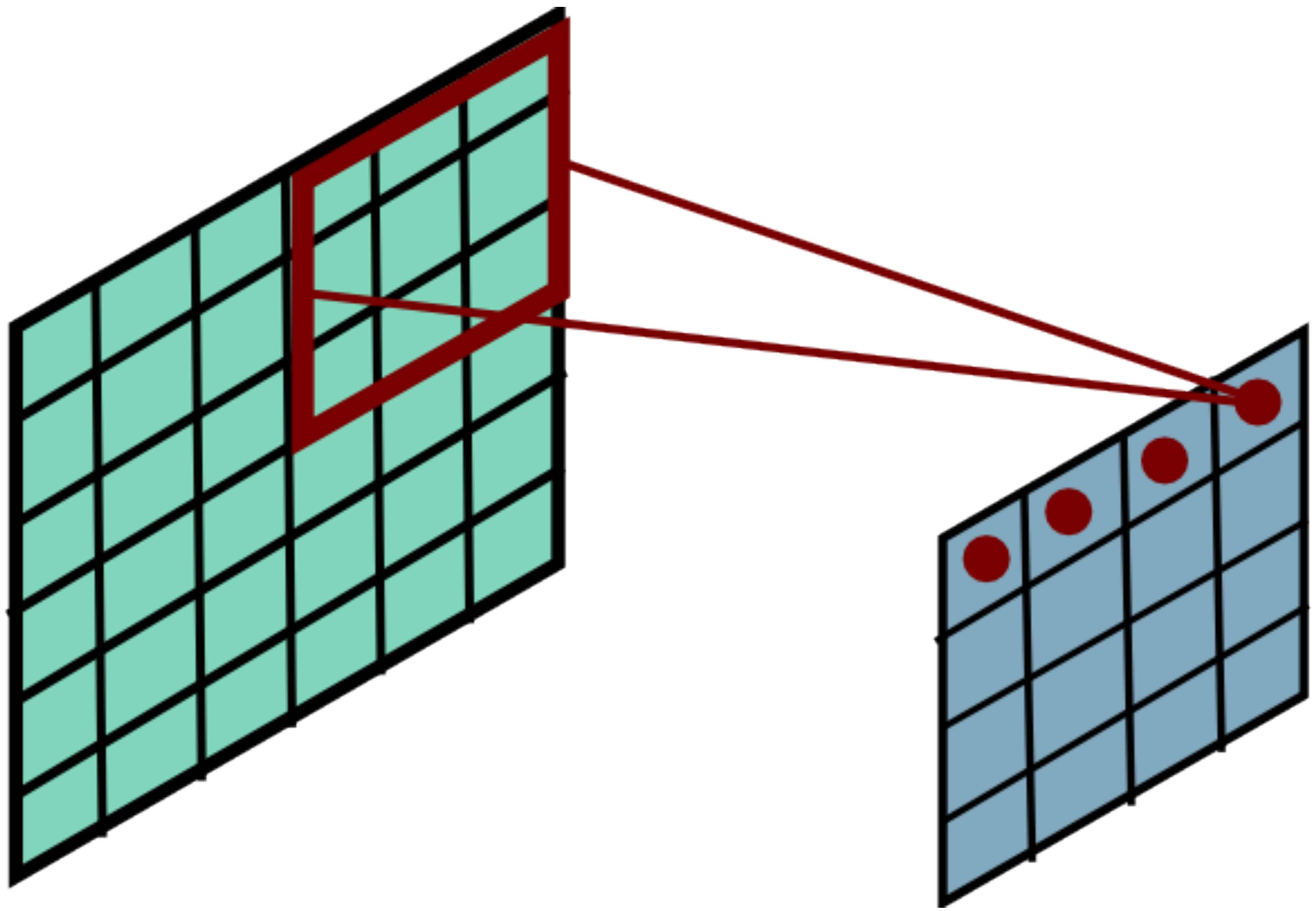

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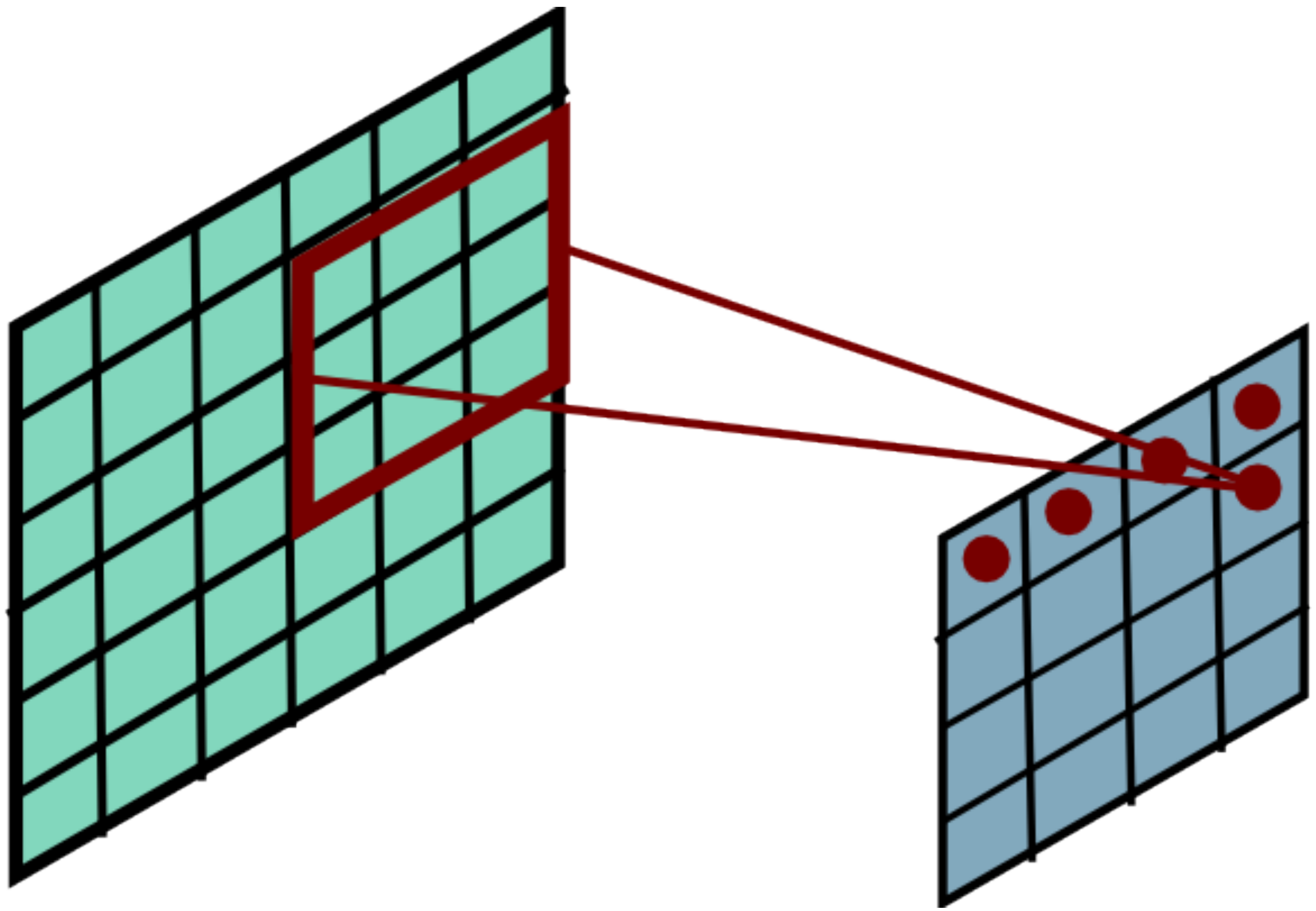
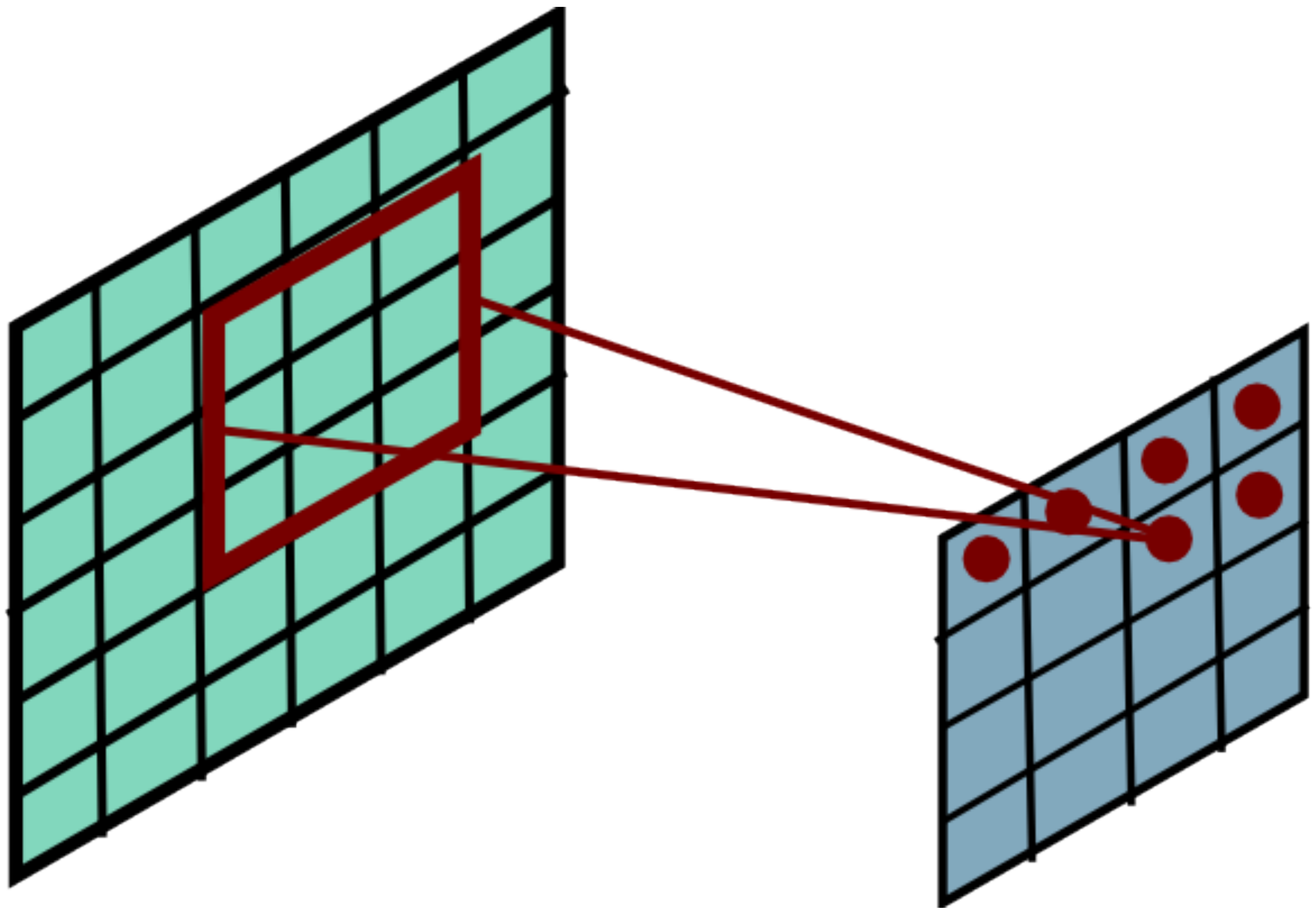


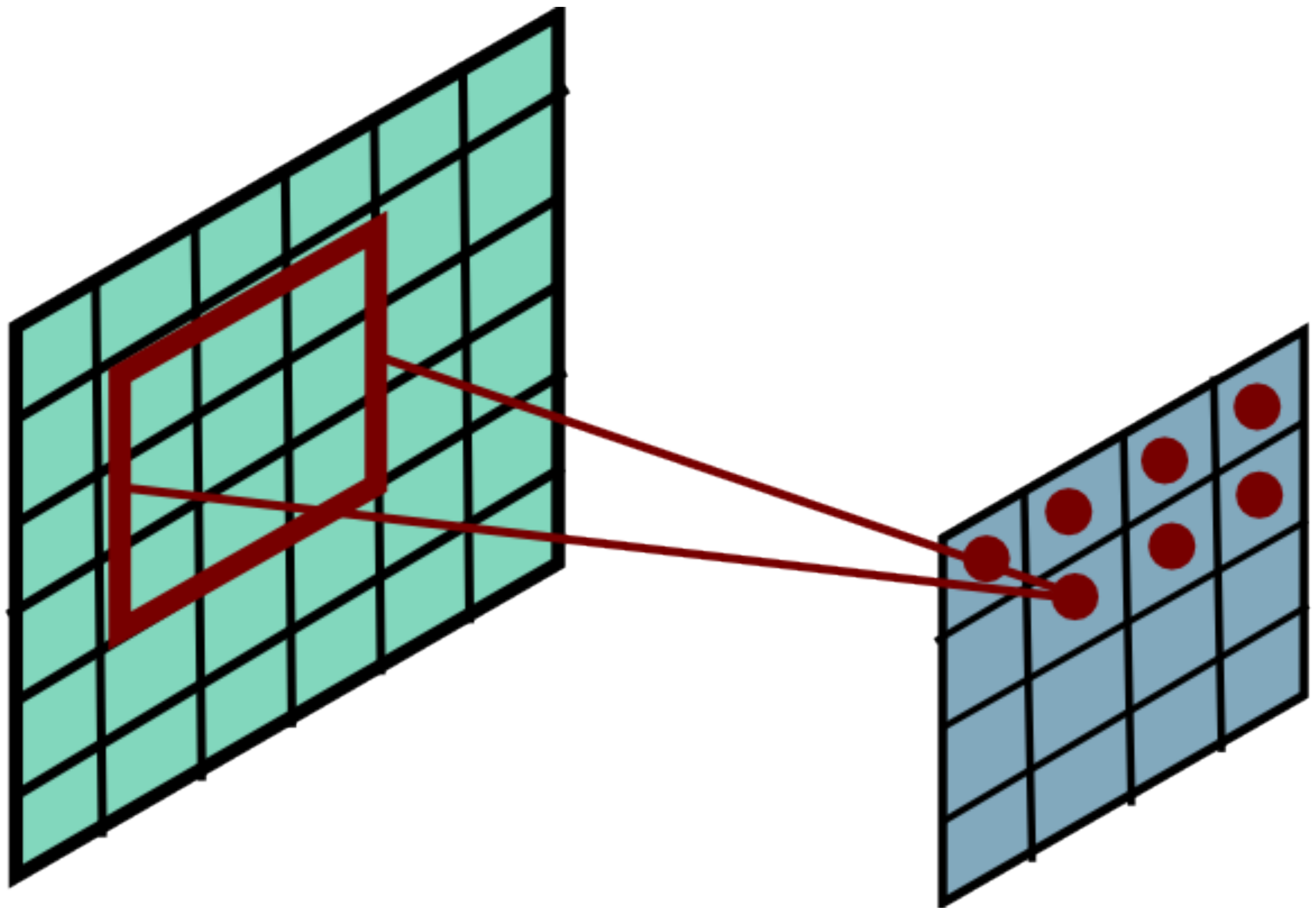
* slide from Marc'Aurelio Renzato

Convolutional Layer



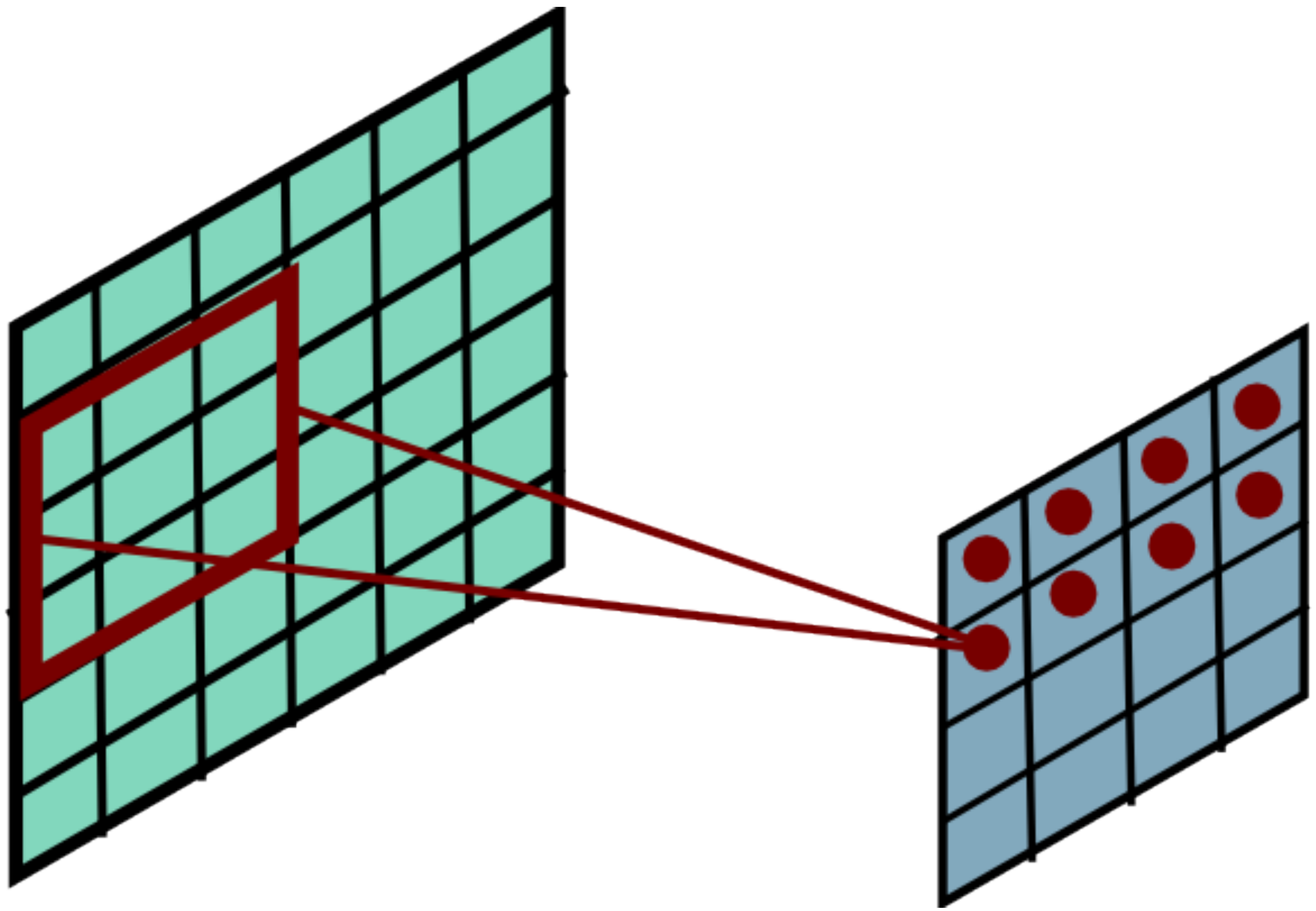
* slide from Marc'Aurelio Renzato

Convolutional Layer



* slide from Marc'Aurelio Renzato

Convolutional Layer



* slide from Marc'Aurelio Renzato

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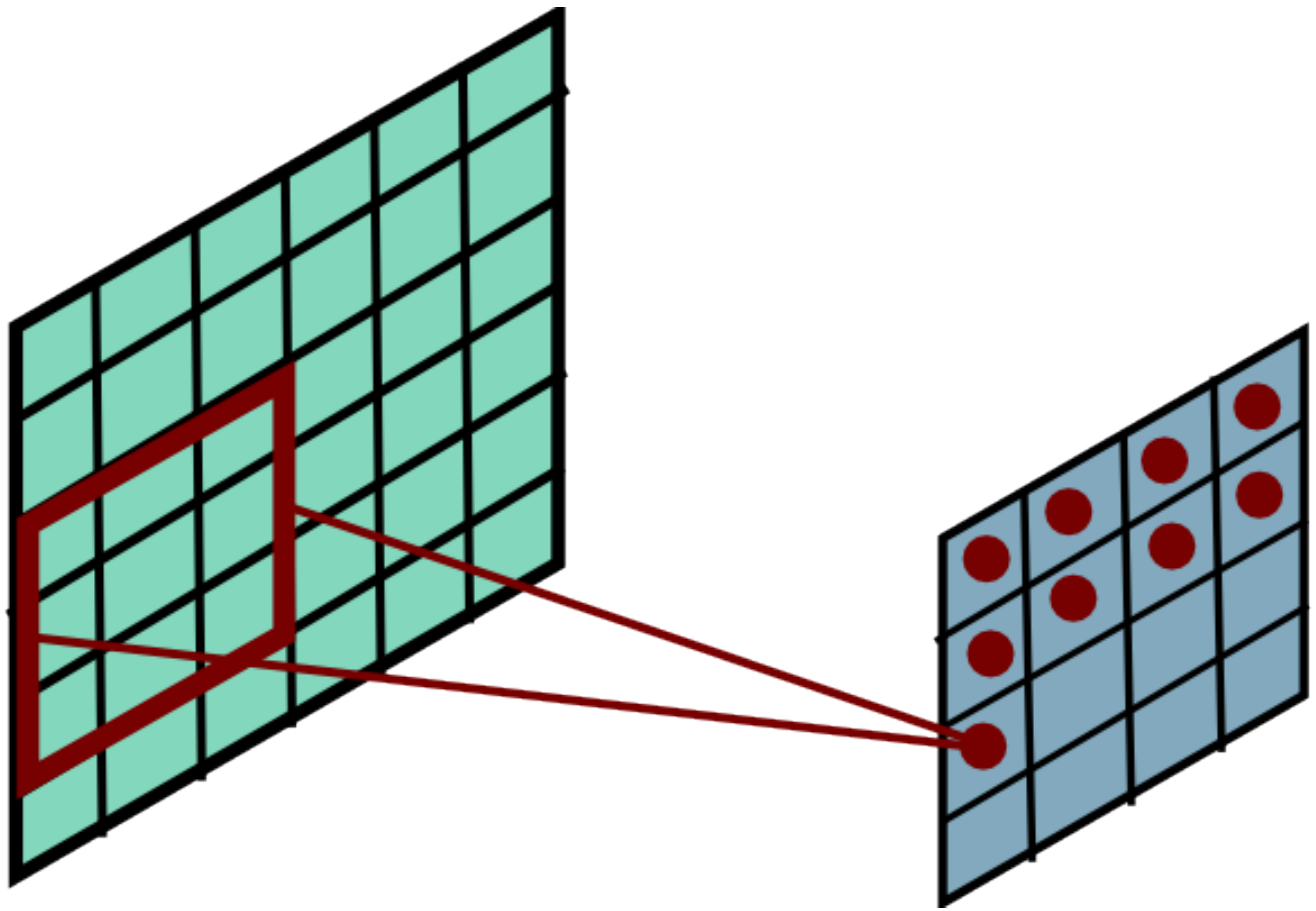

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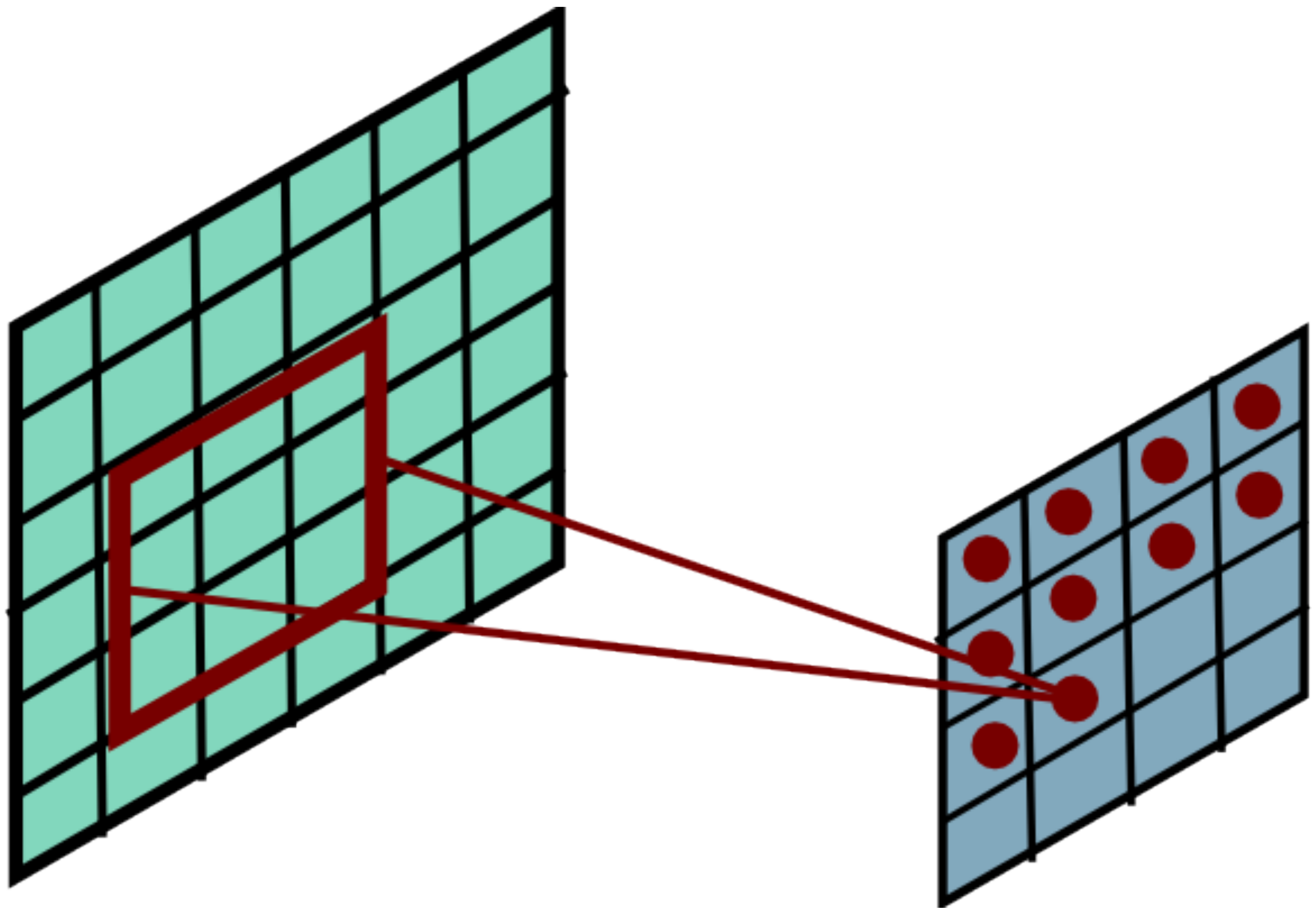

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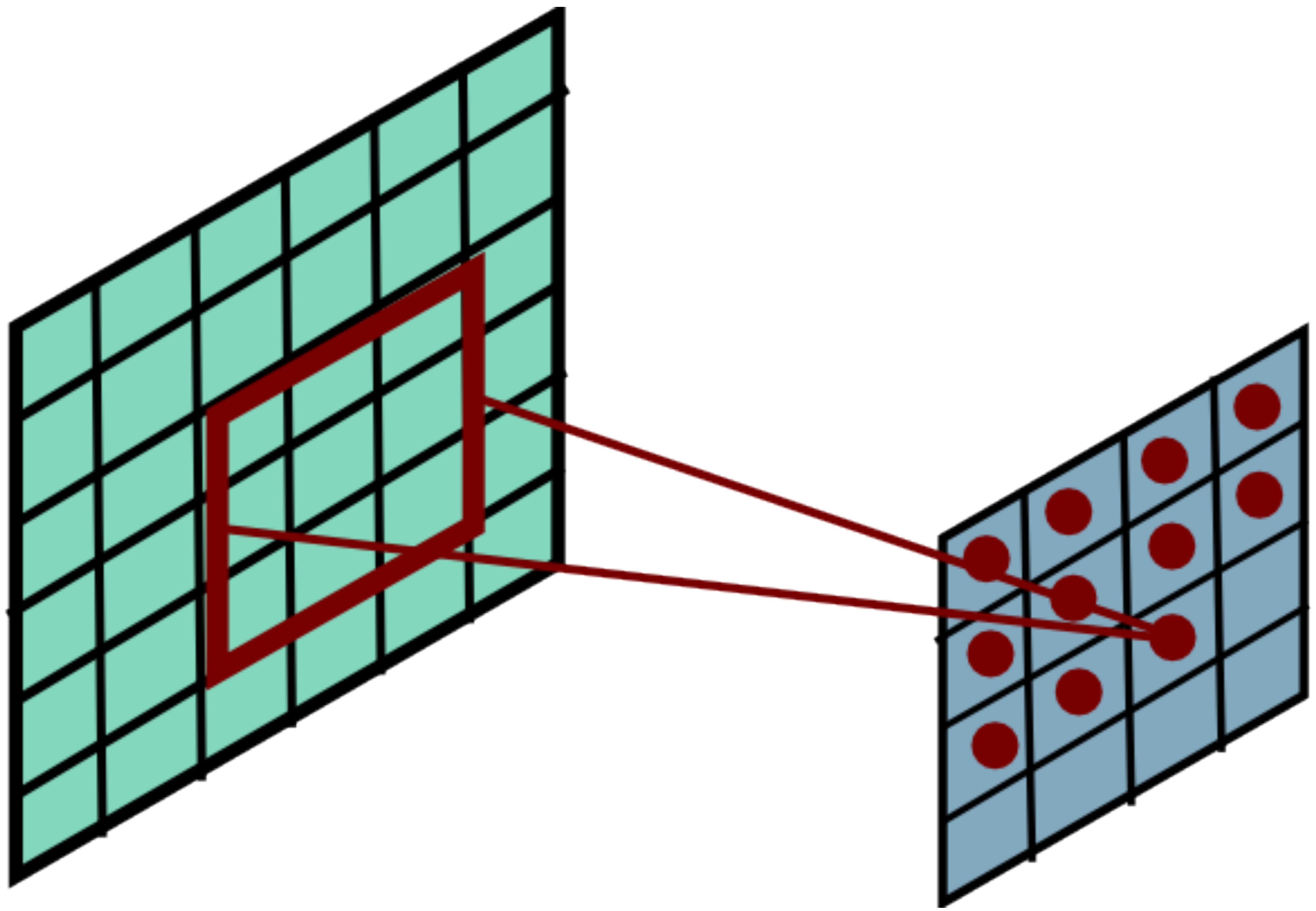
Convolutional Layer



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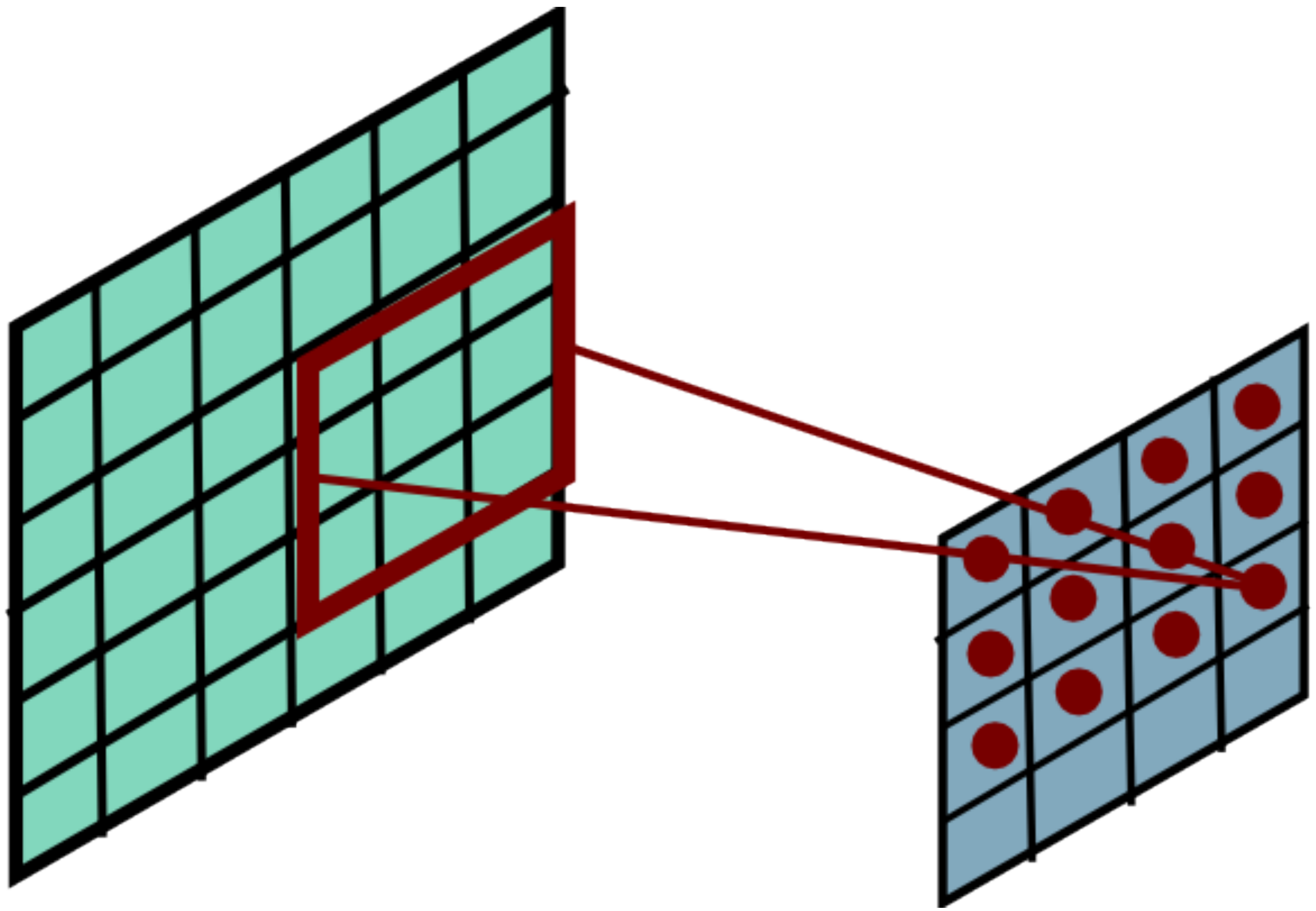
Convolutional Layer



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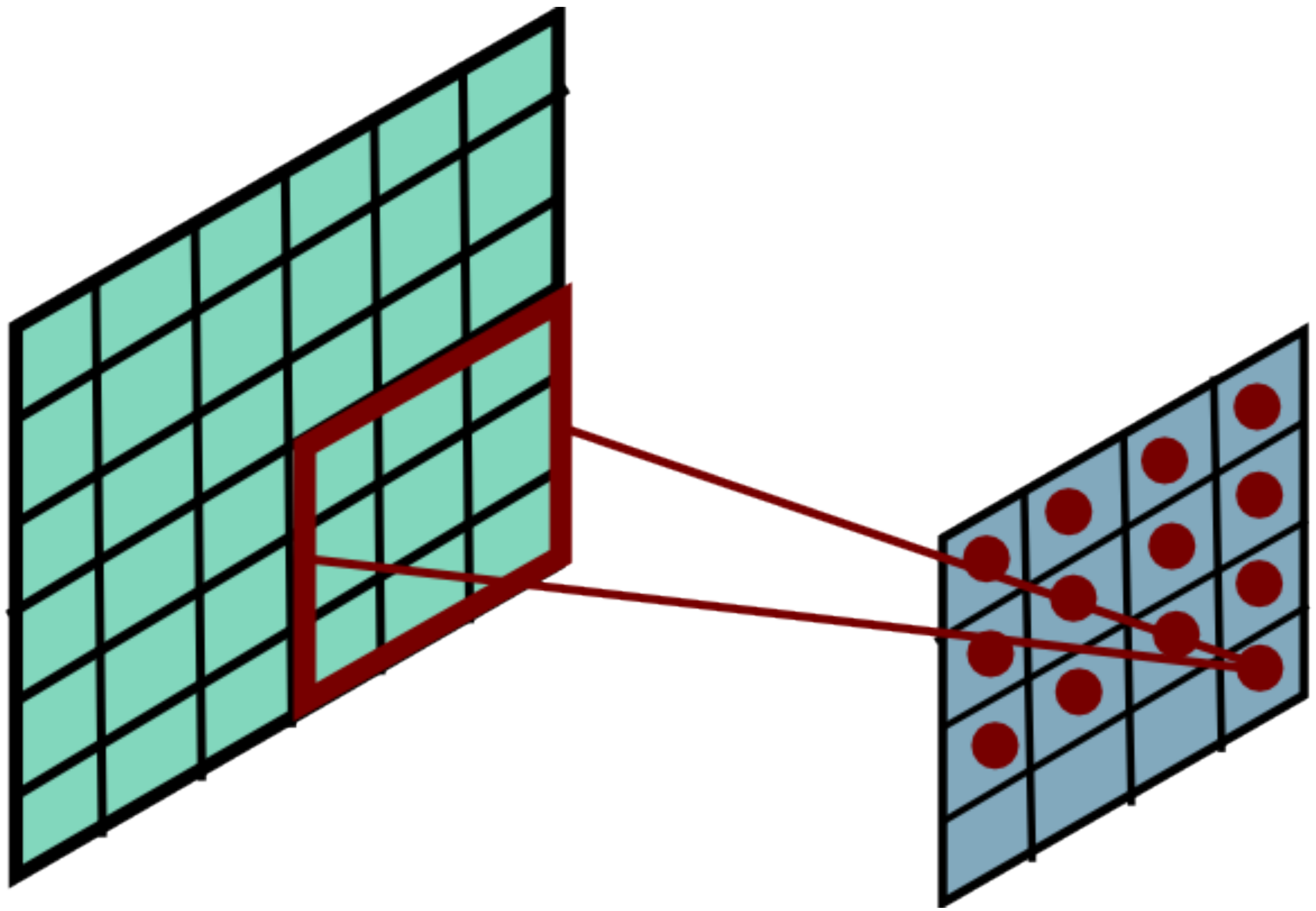
Convolutional Layer



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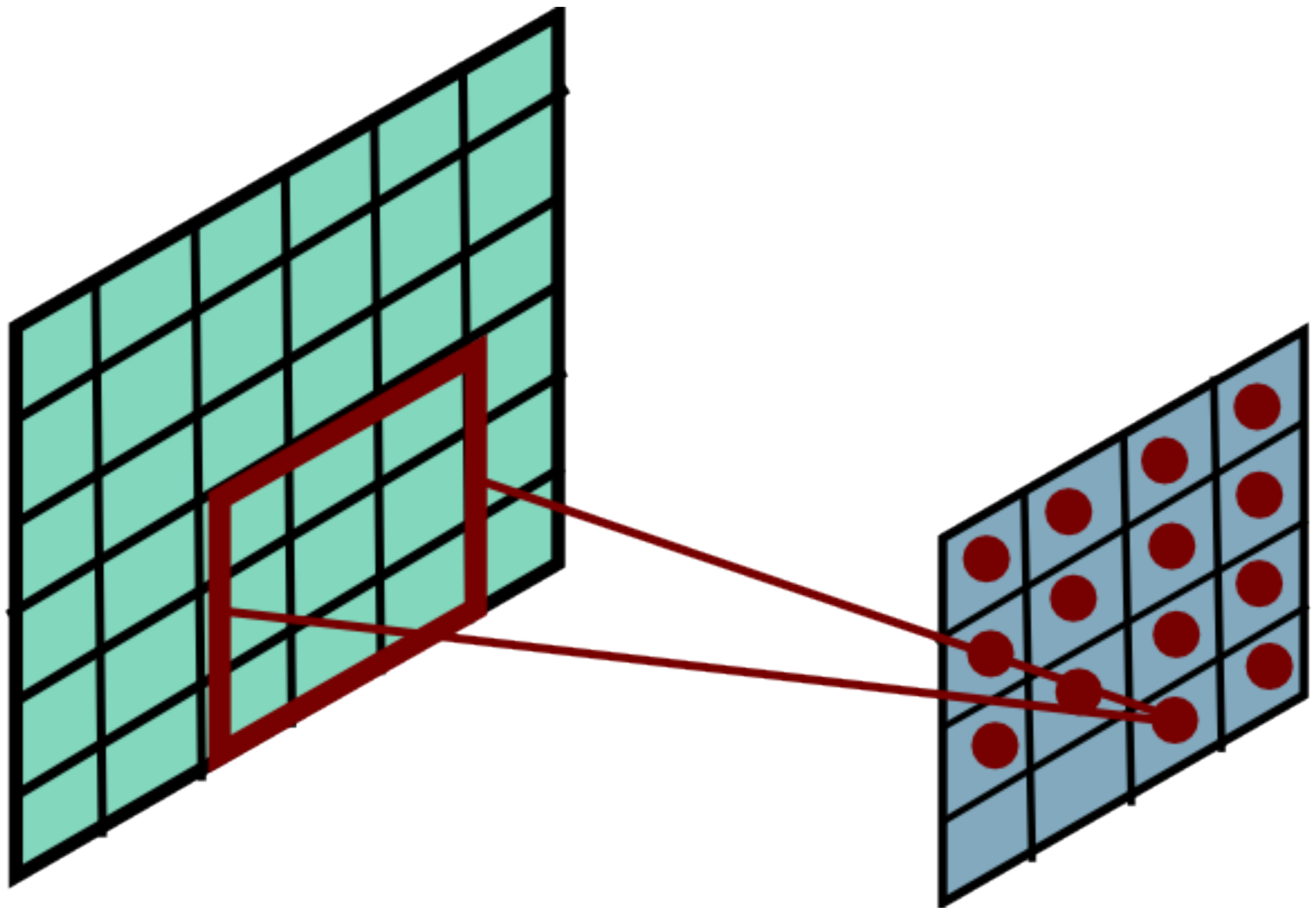

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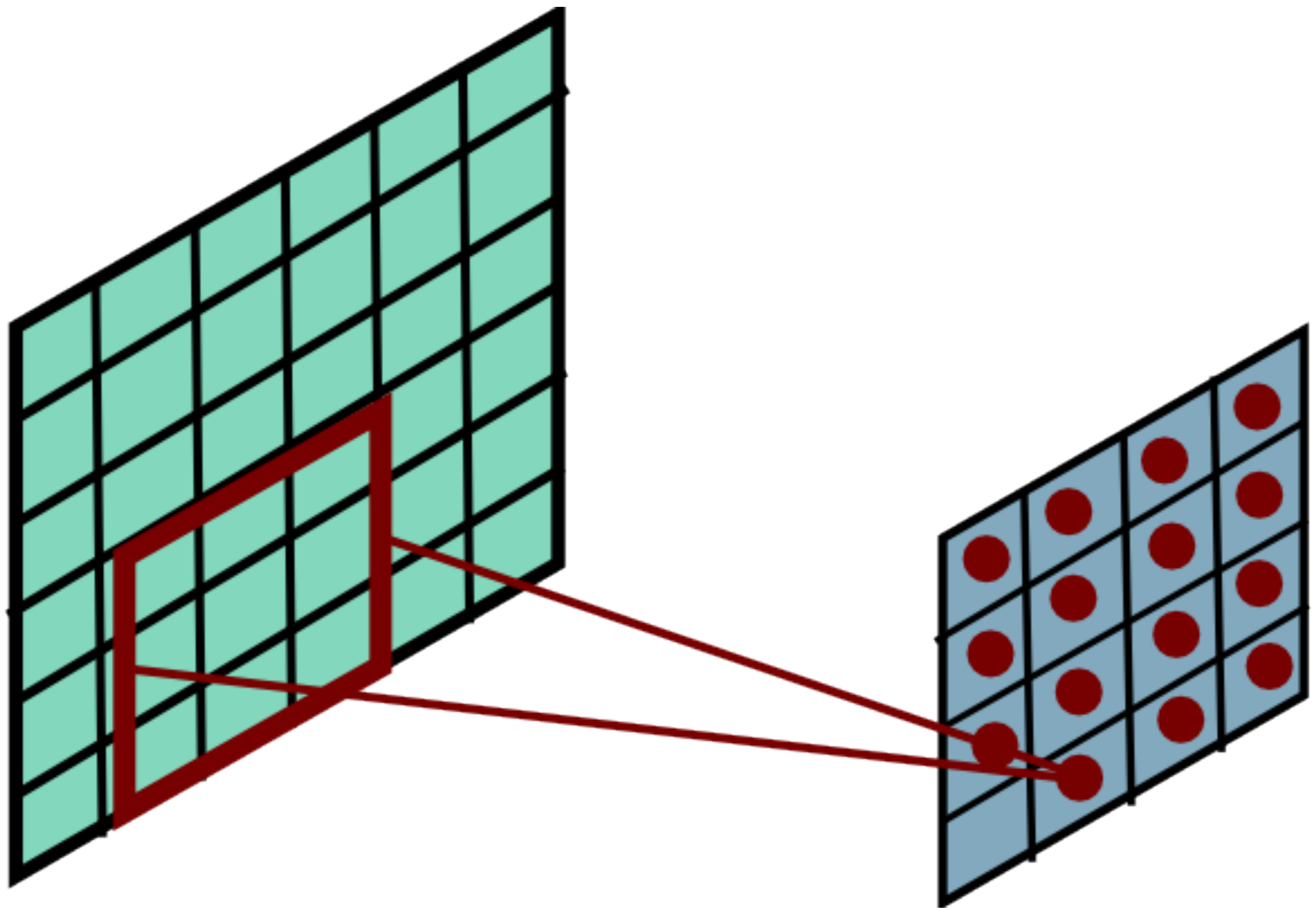

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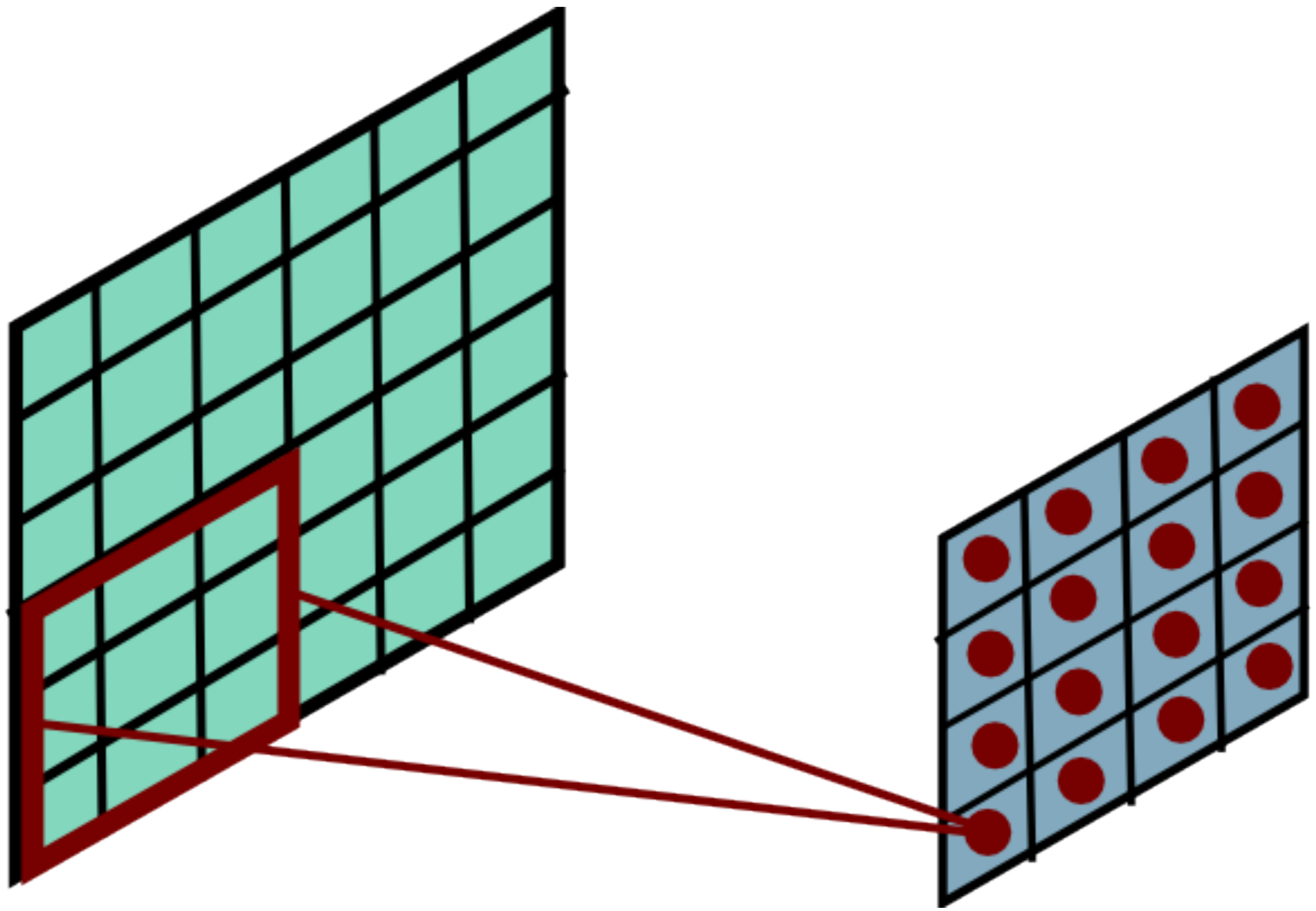
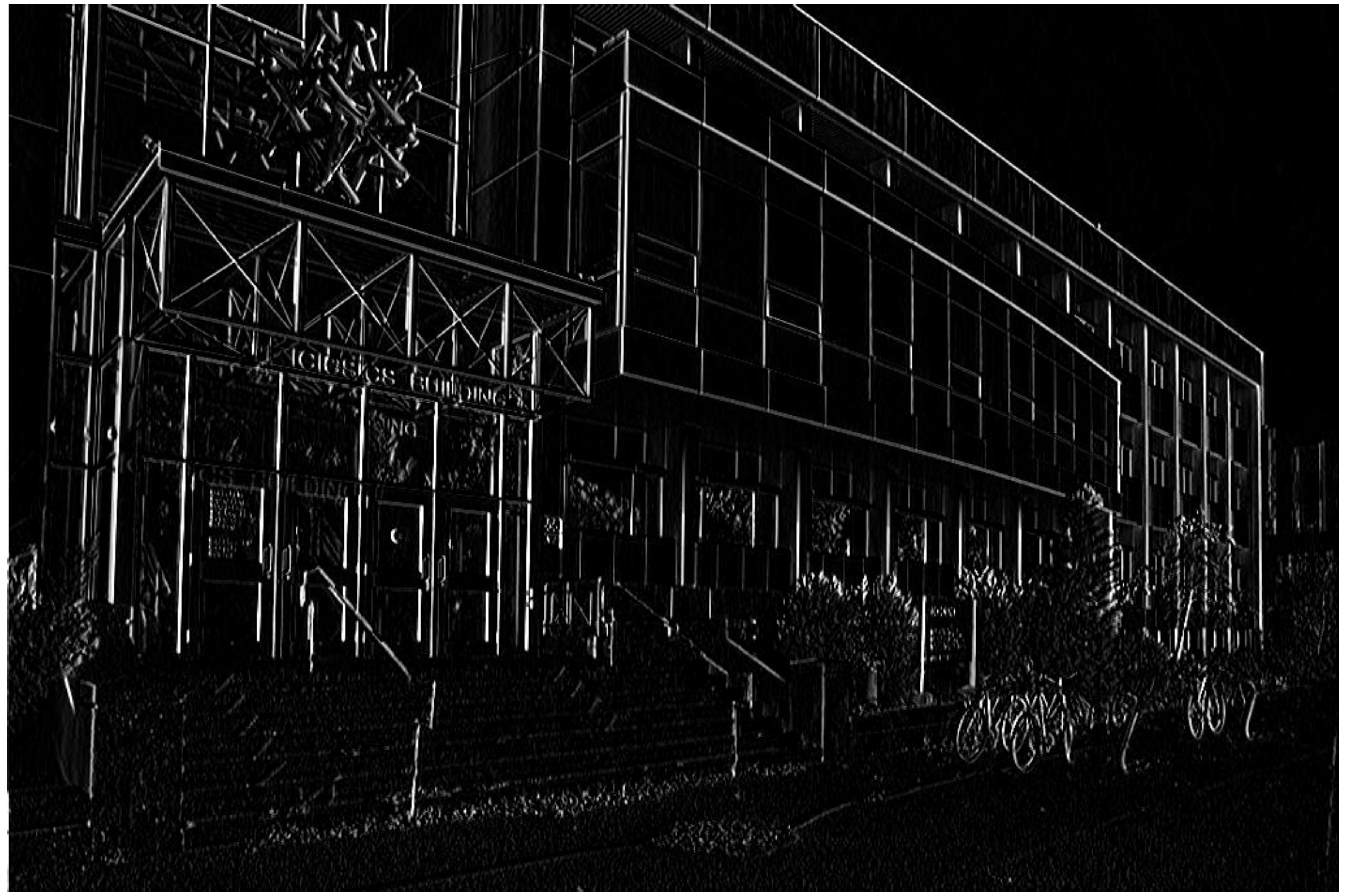
Convolutional Layer



* slide from Marc'Aurelio Renzato

Convolution Layer

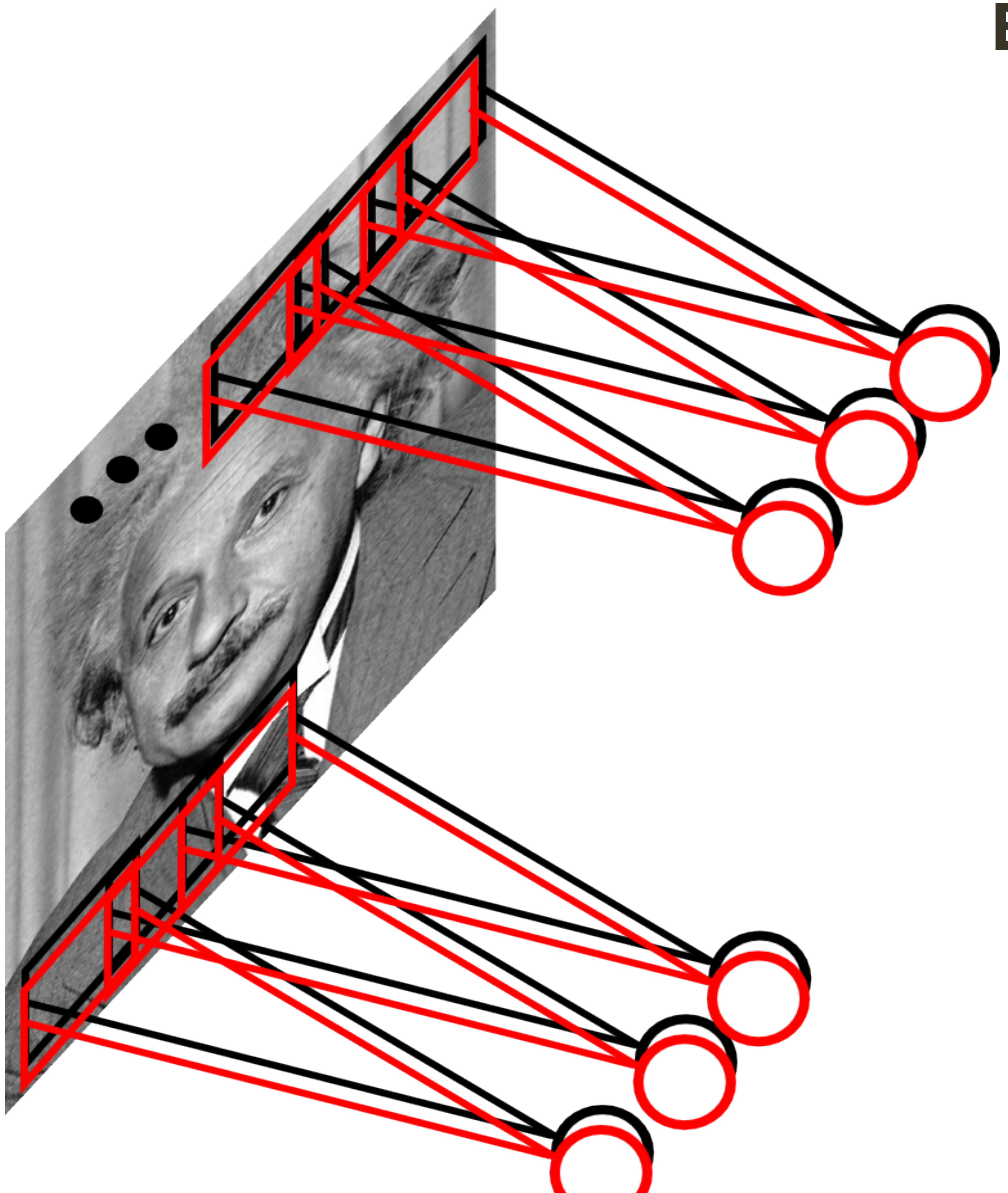
$$\star \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \longrightarrow$$



Convolution Layer

$$\star \begin{bmatrix} 0.11 & 0.11 & 0.11 \\ 0.11 & 0.11 & 0.11 \\ 0.11 & 0.11 & 0.11 \end{bmatrix} \rightarrow$$

Convolutional Layer



Example: 200 x 200 image (small)
x 40K hidden units

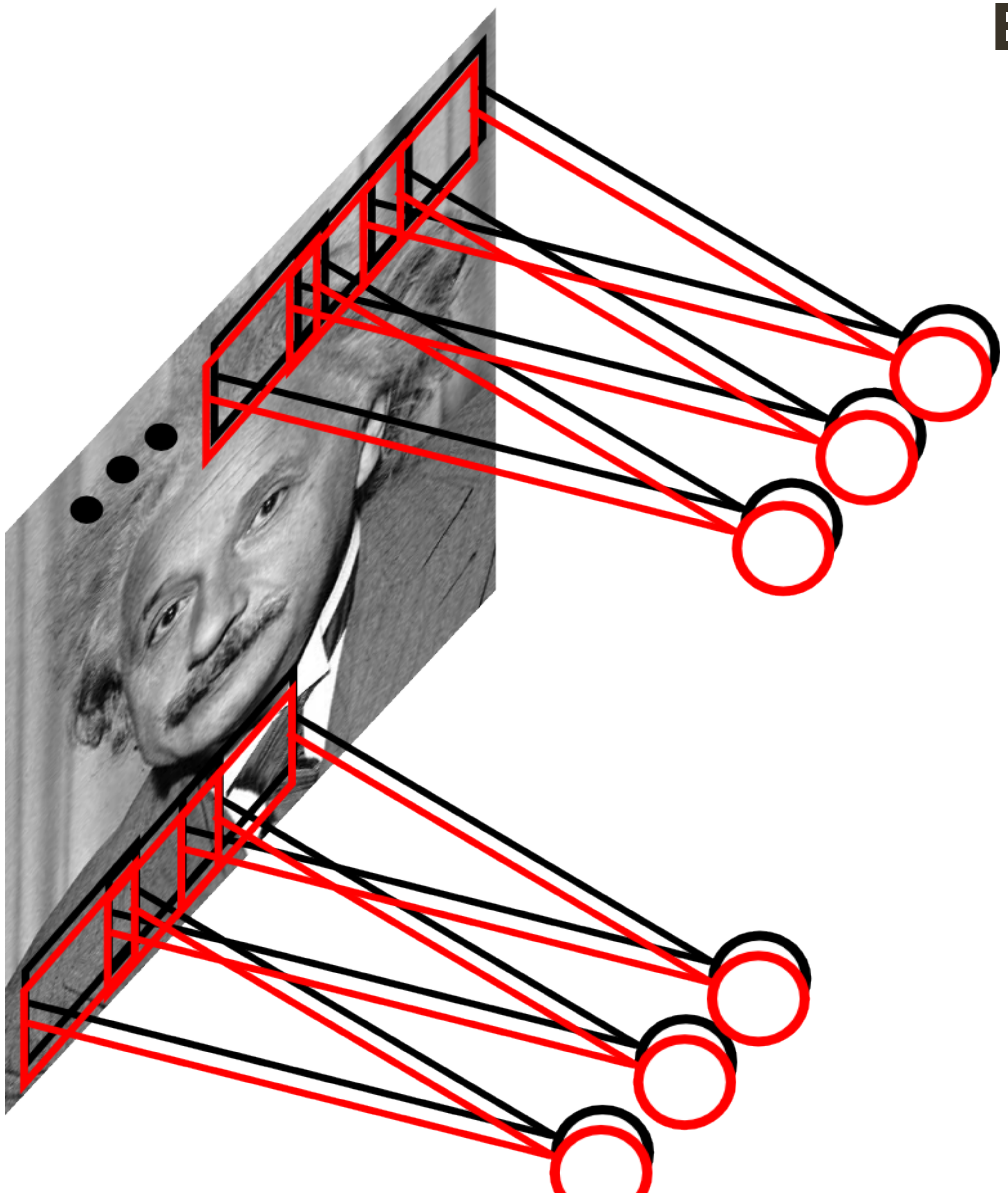
Filter size: 10 x 10

# of filters: 20

Learn **multiple filters**

* slide from Marc'Aurelio Renzato

Convolutional Layer



Example: 200 x 200 image (small)
x 40K hidden units

Filter size: 10 x 10

# of filters: 20

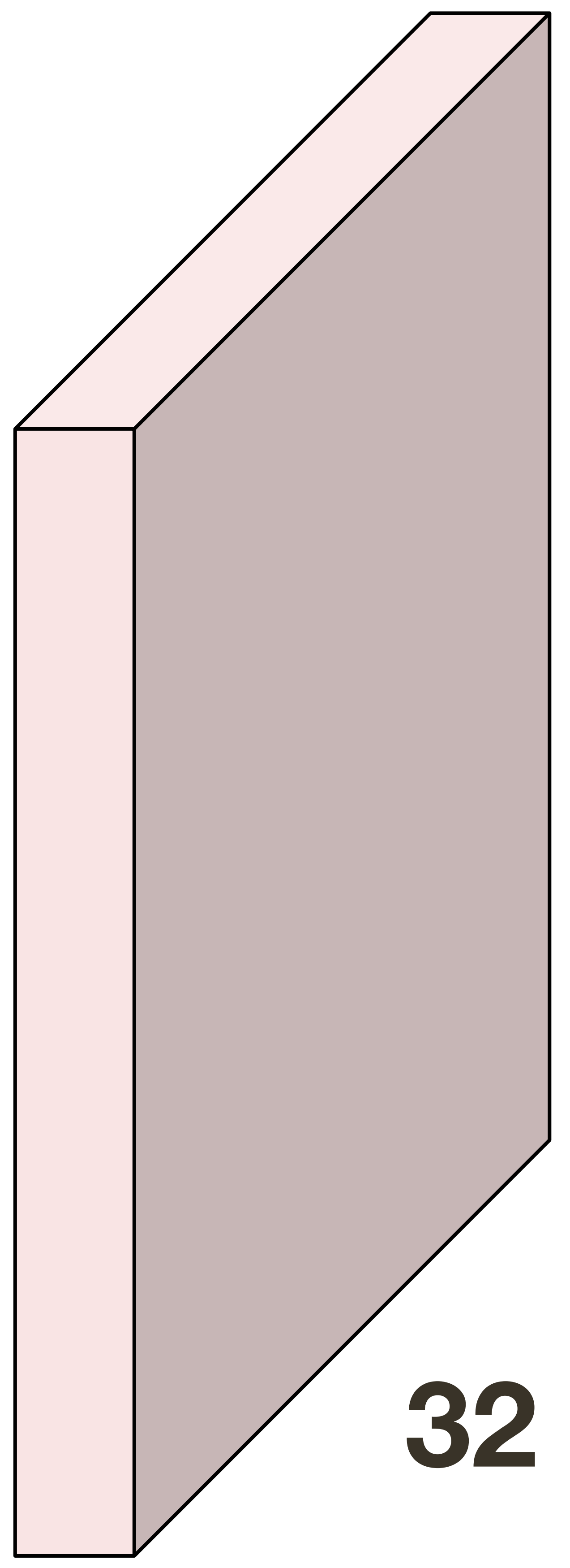
= 2000 parameters

Learn **multiple filters**

* slide from Marc'Aurelio Renzato

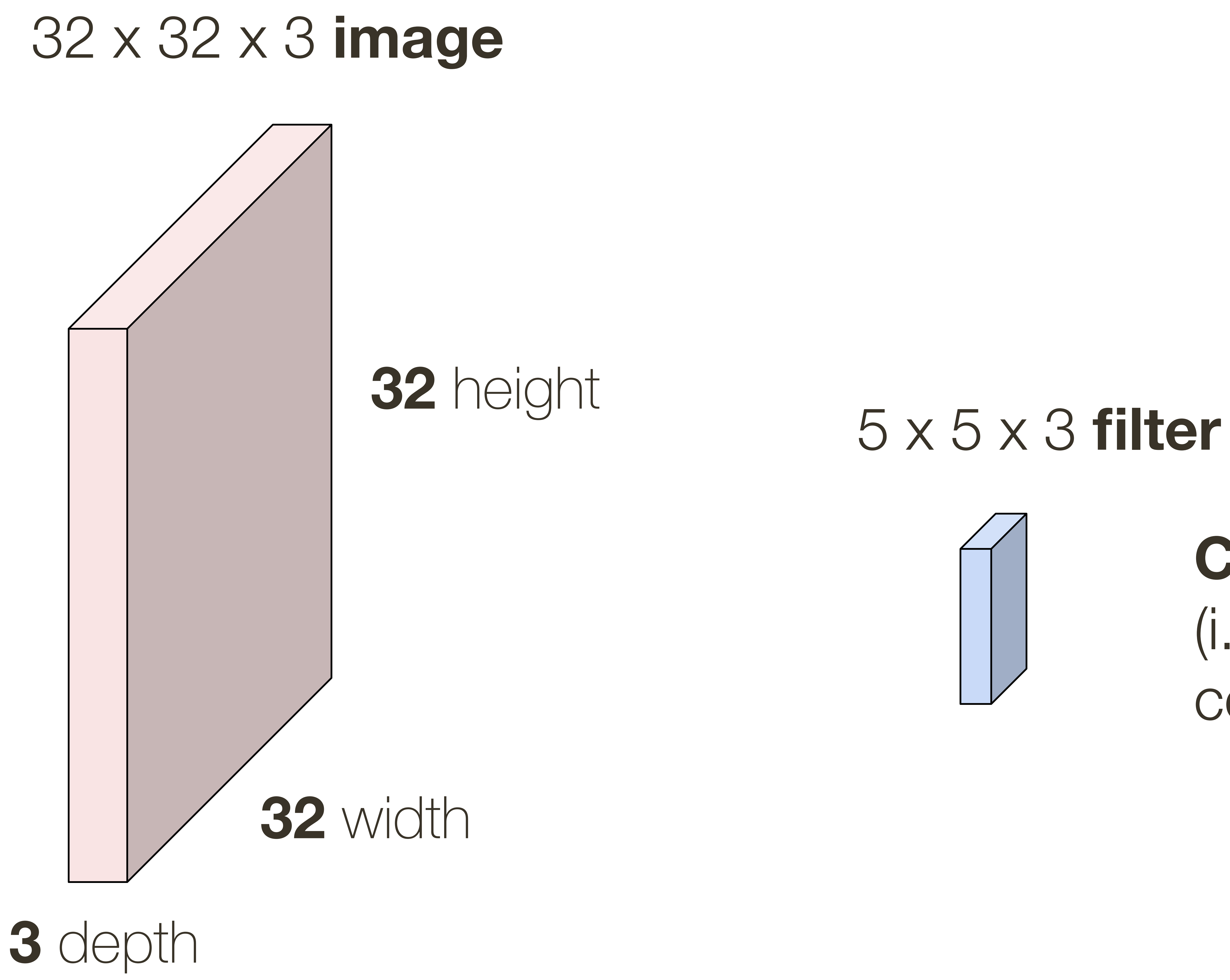
Convolutional Layer

32 x 32 x 3 **image** (note the image preserves spatial structure)



* slide from Fei-Dei Li, Justin Johnson, Serena Yeung, **cs231n Stanford**

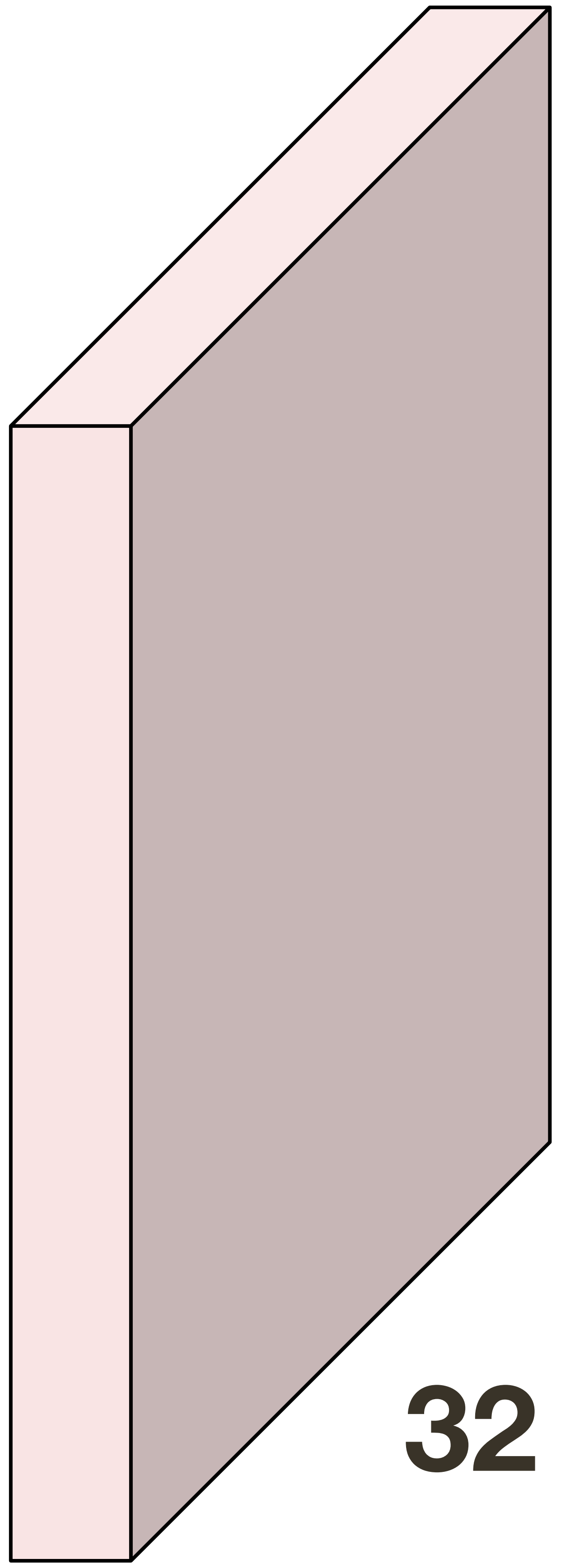
Convolutional Layer



Convolve the filter with the image
(i.e., "slide over the image spatially,
computing dot products")

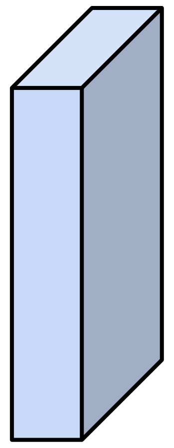
Convolutional Layer

32 x 32 x 3 **image**



Filters always extend the full depth of the input volume

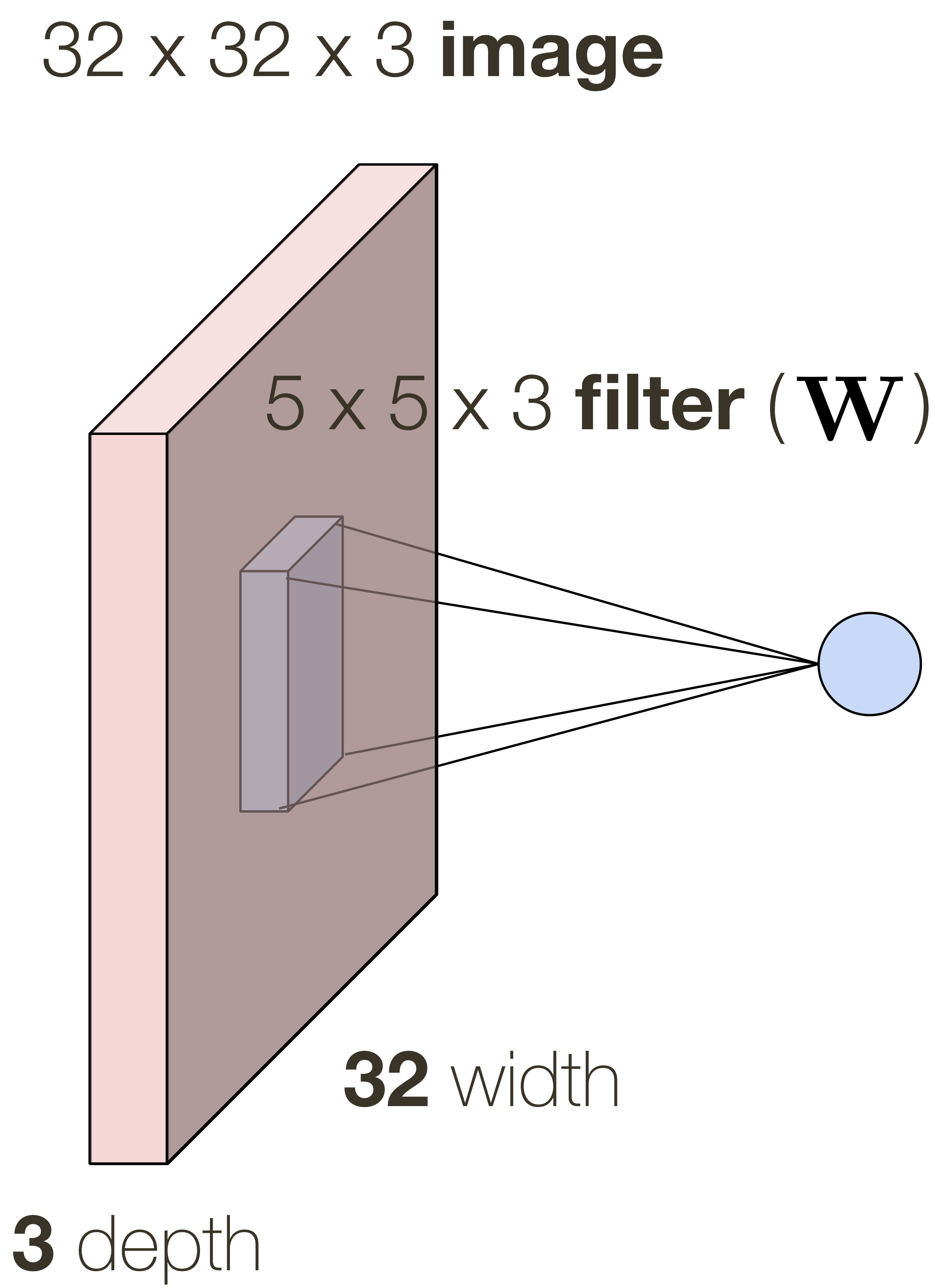
5 x 5 x 3 **filter**



Convolve the filter with the image (i.e., “slide over the image spatially, computing dot products”

* slide from Fei-Dei Li, Justin Johnson, Serena Yeung, **cs231n Stanford**

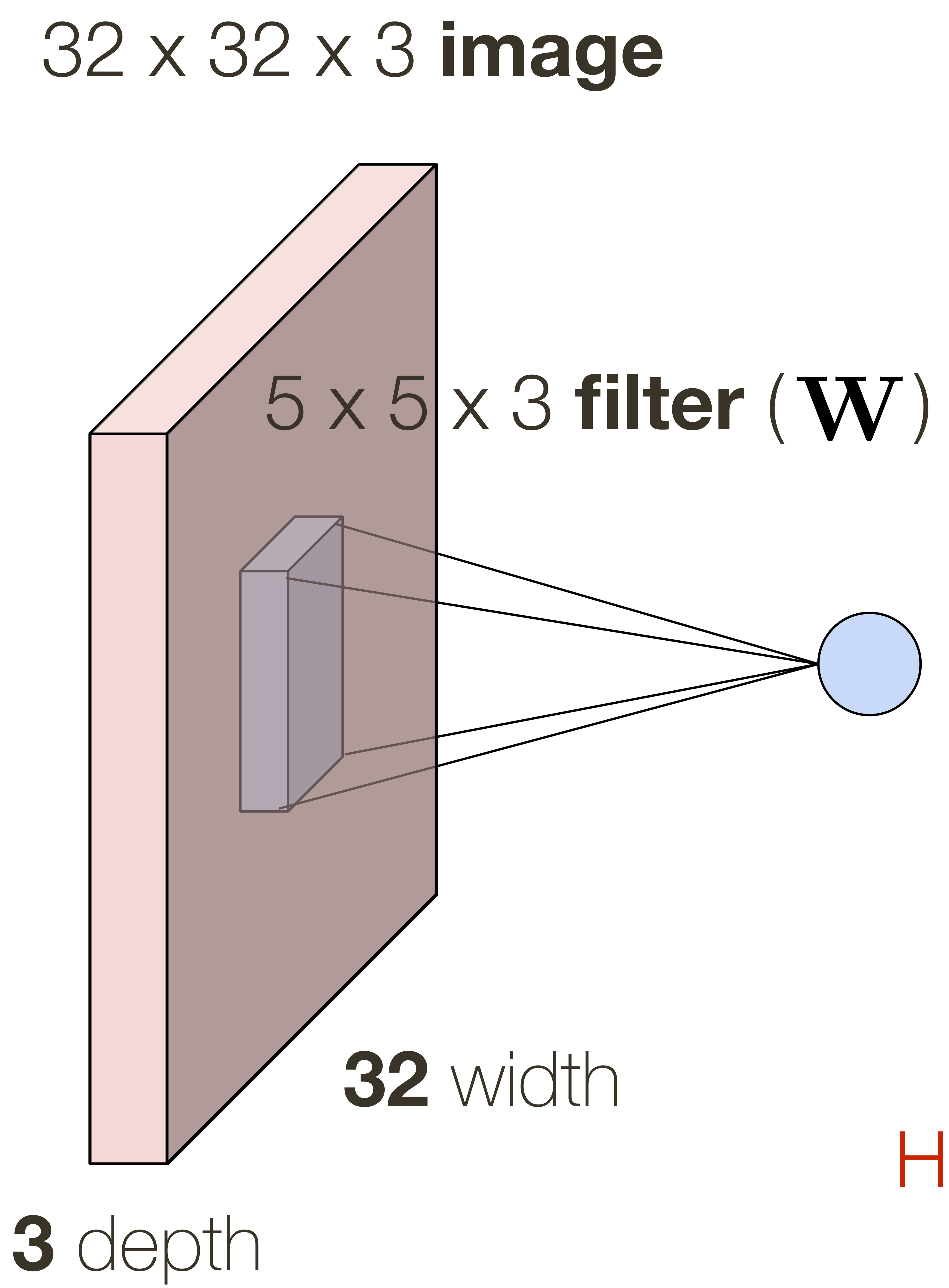
Convolutional Layer



1 number: the result of taking a dot product between the filter and a small 5 x 5 x 3 part of the image

$$\mathbf{W}^T\mathbf{x} + b, \text{where } \mathbf{W}, \mathbf{x} \in \mathbb{R}^{75}$$

Convolutional Layer



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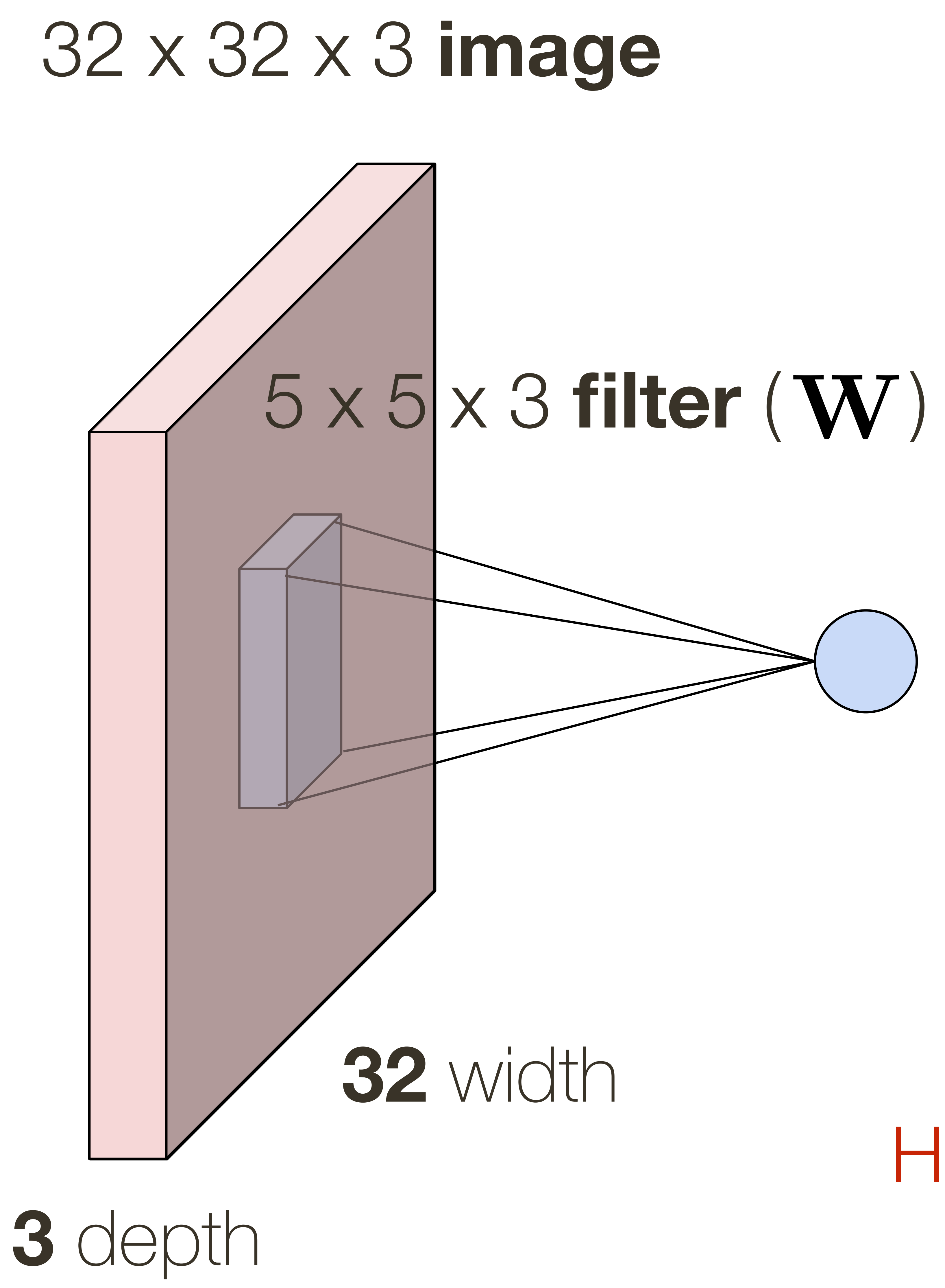
How many **parameters** does the layer have?

* slide from Fei-Dei Li, Justin Johnson, Serena Yeung, **cs231n Stanford**

Convolutional Layer



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$$\mathbf{W}^T\mathbf{x} + b, \text{where } \mathbf{W}, \mathbf{x} \in \mathbb{R}^{75}$$

How many **parameters** does the layer have? **76**

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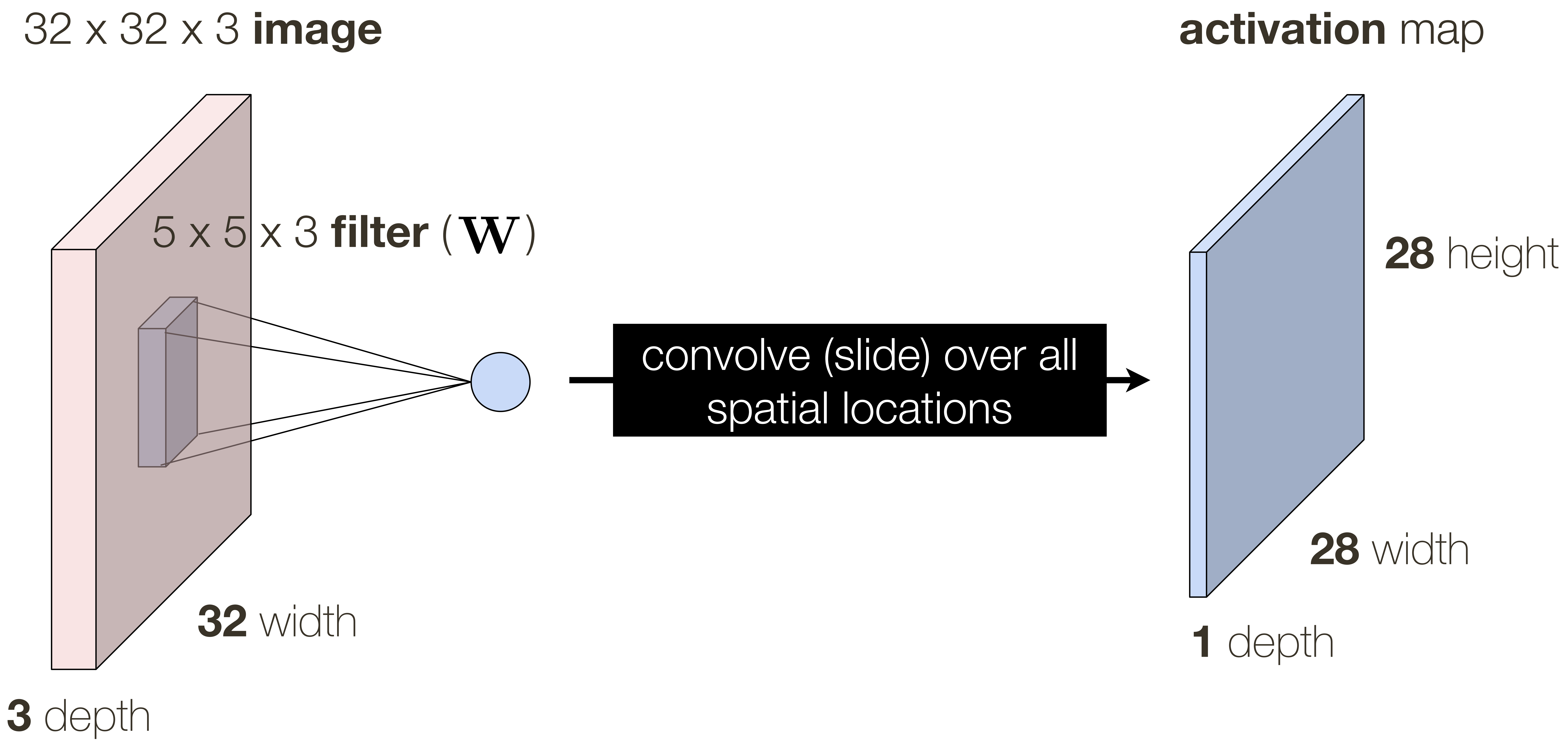
Convolutional Layer



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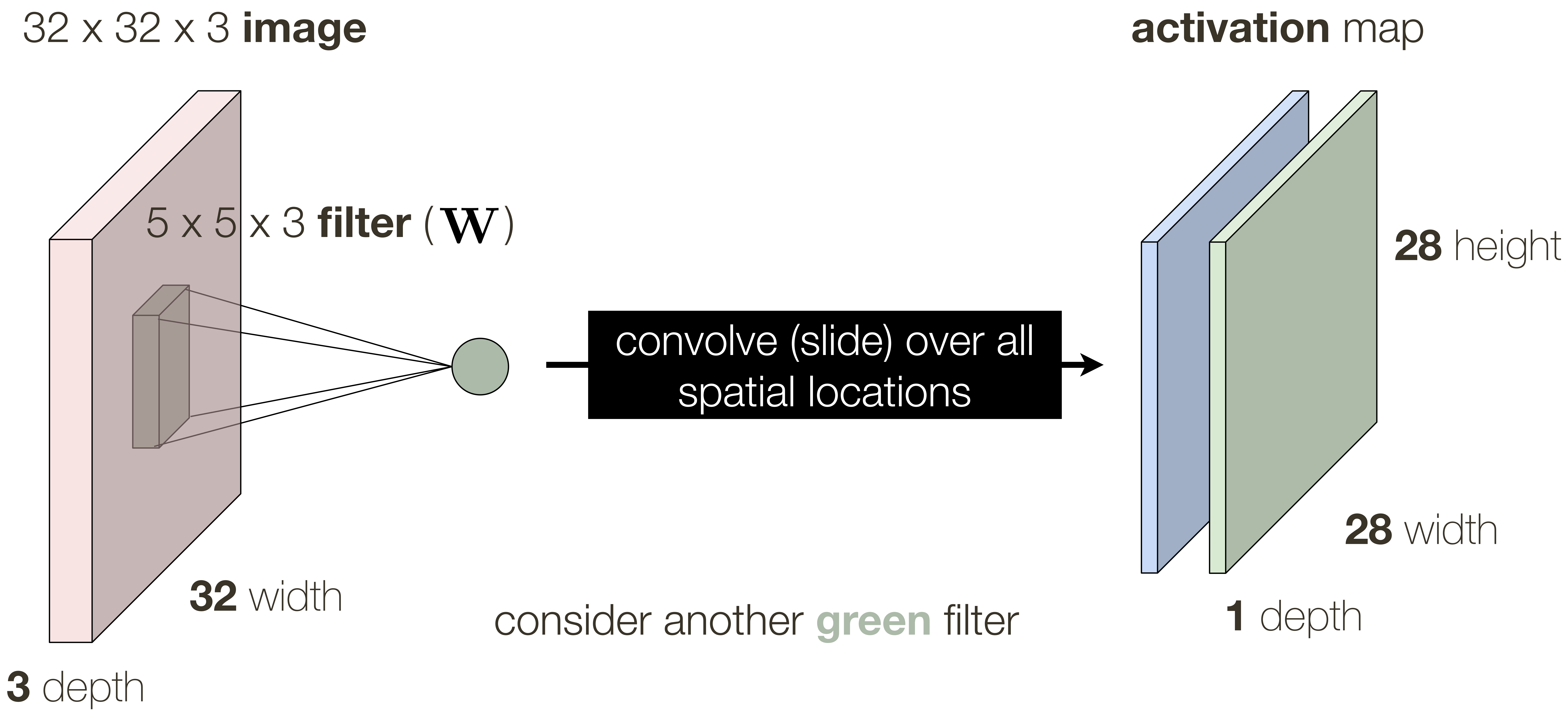
Convolutional Layer



consider another **green** filter

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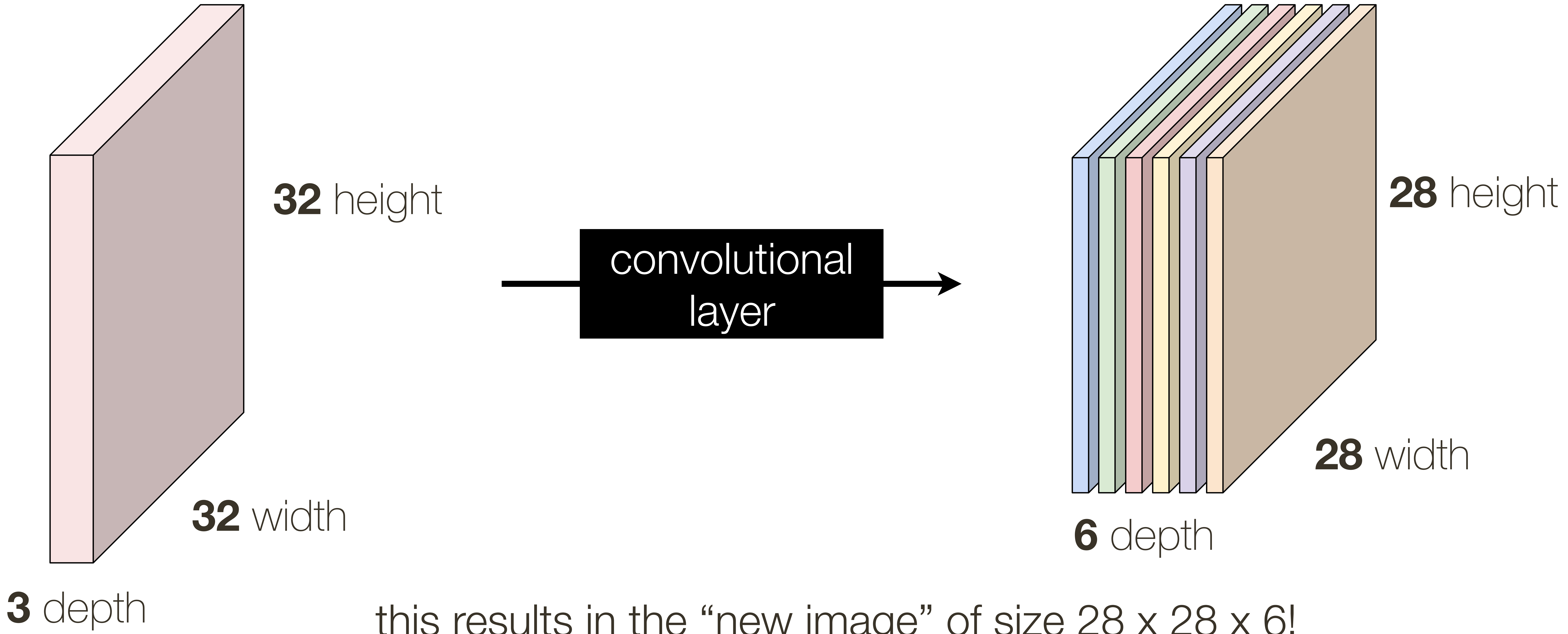
Convolutional Layer

If we have 6 5x5 filter, we'll get 6 separate activation maps: **activation** map



this results in the "new image" of size 28 x 28 x 6!

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Convolutional Layer

The number of neurons in a layer is determined by depth and stride parameter

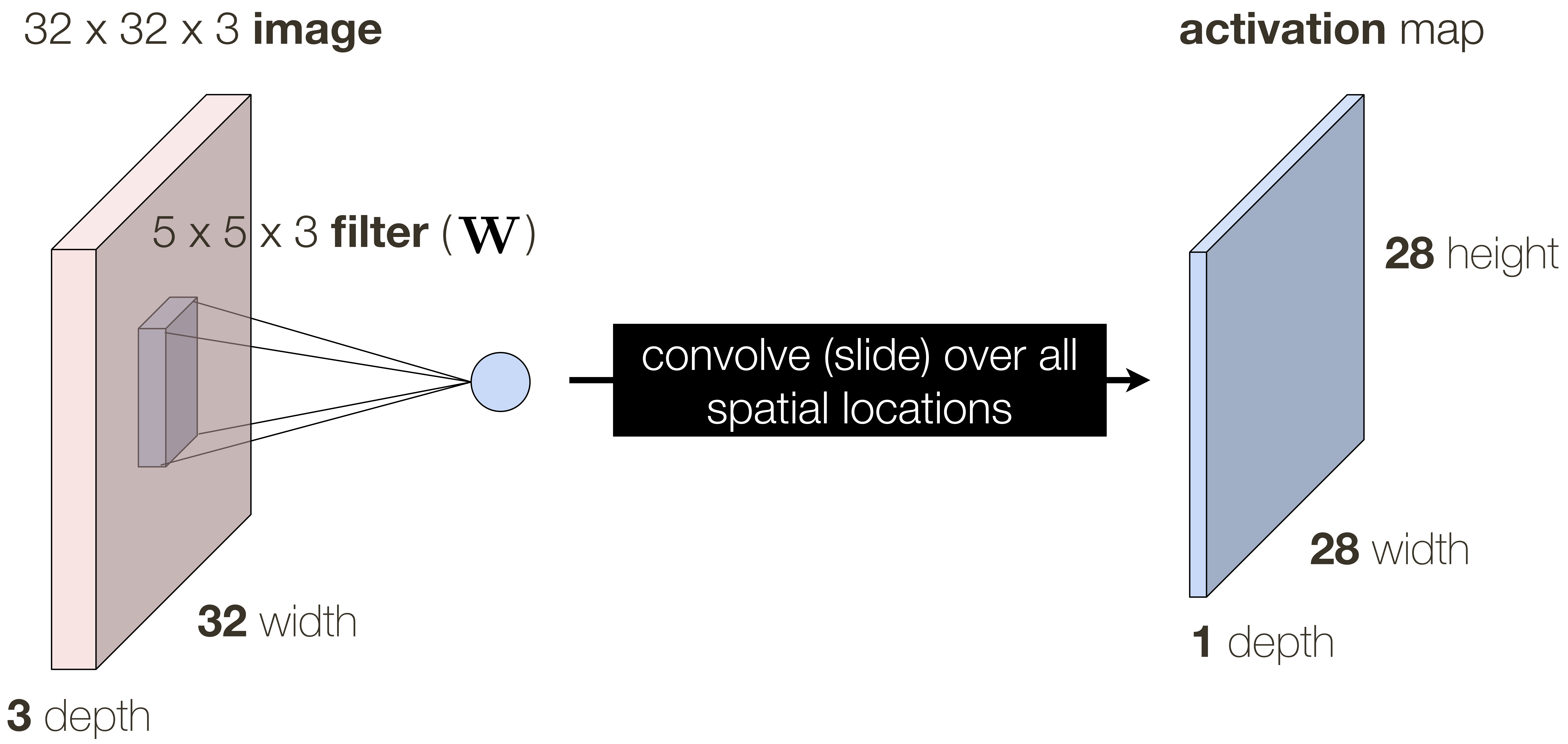
— also affected by zero-padding

Depth: Controls number of neurons that connect to the same region of the input layer

— a set of neurons connected to the same region is called a **depth column**

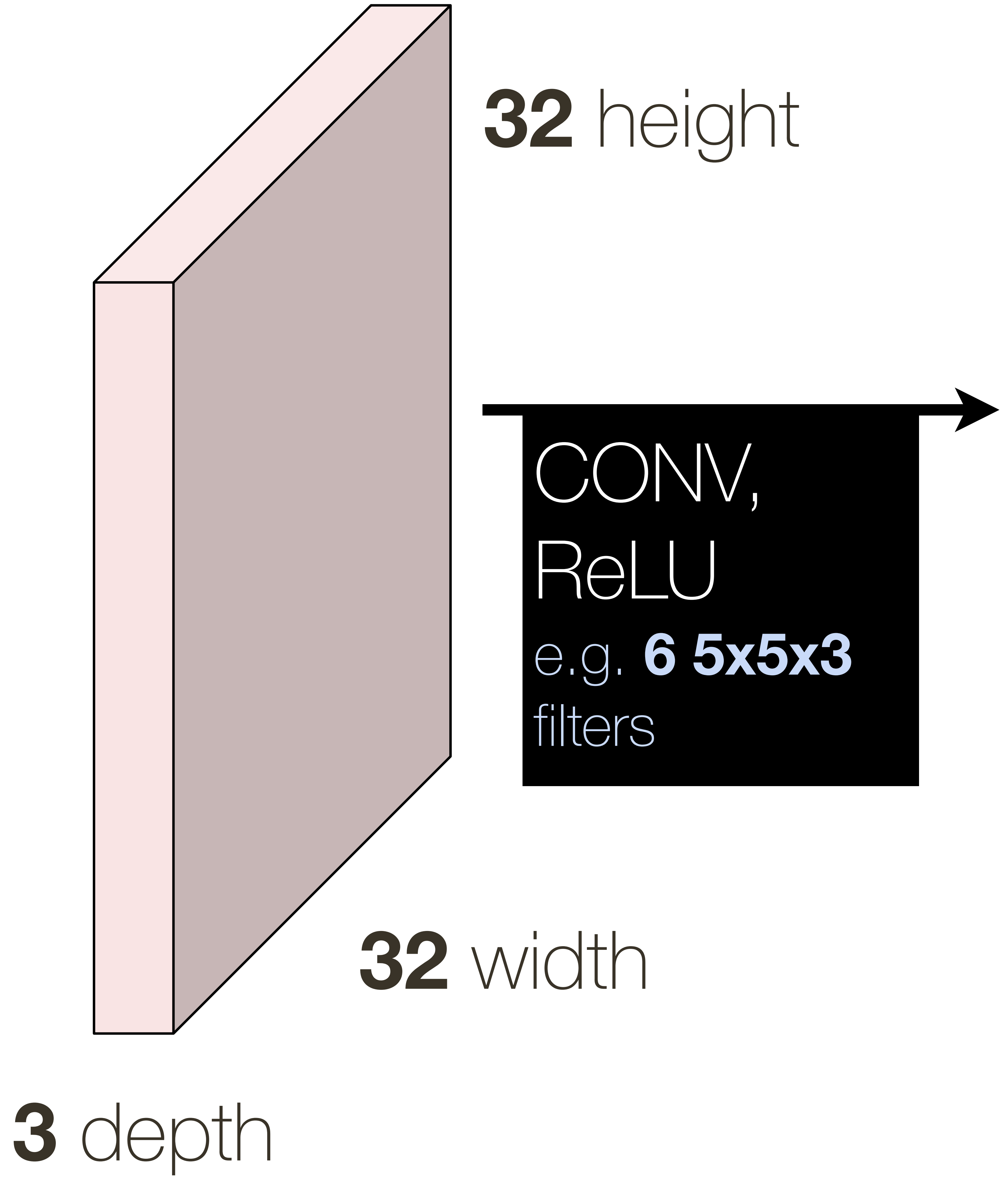
Stride: Controls spatial density. How far apart are depth columns?

Convolutional Layer: Closer Look at **Spatial Dimensions**



* slide from Fei-Dei Li, Justin Johnson, Serena Yeung, **cs231n Stanford**

Convolutional Neural Network (ConvNet)



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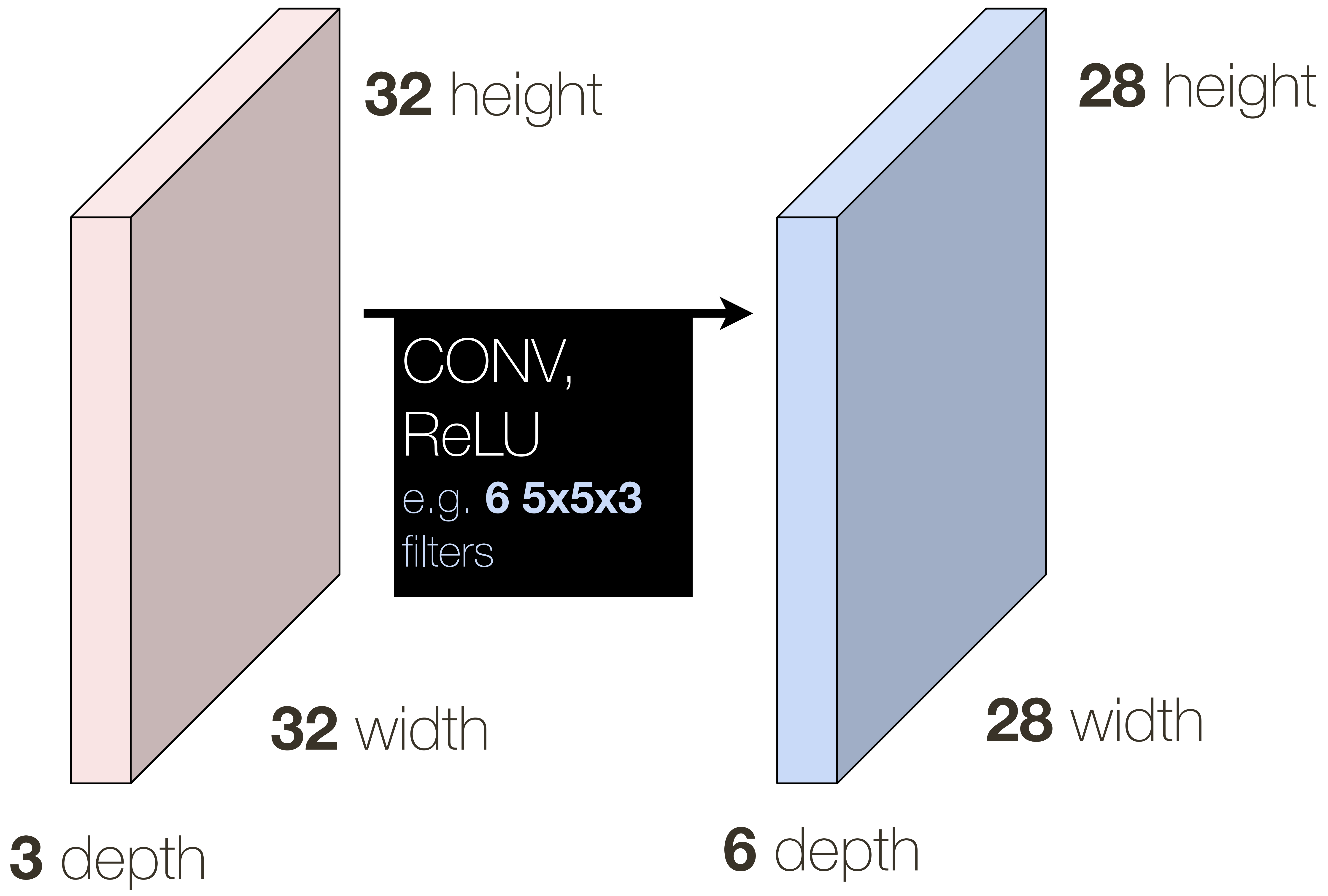
Convolutional Neural Network (ConvNet)



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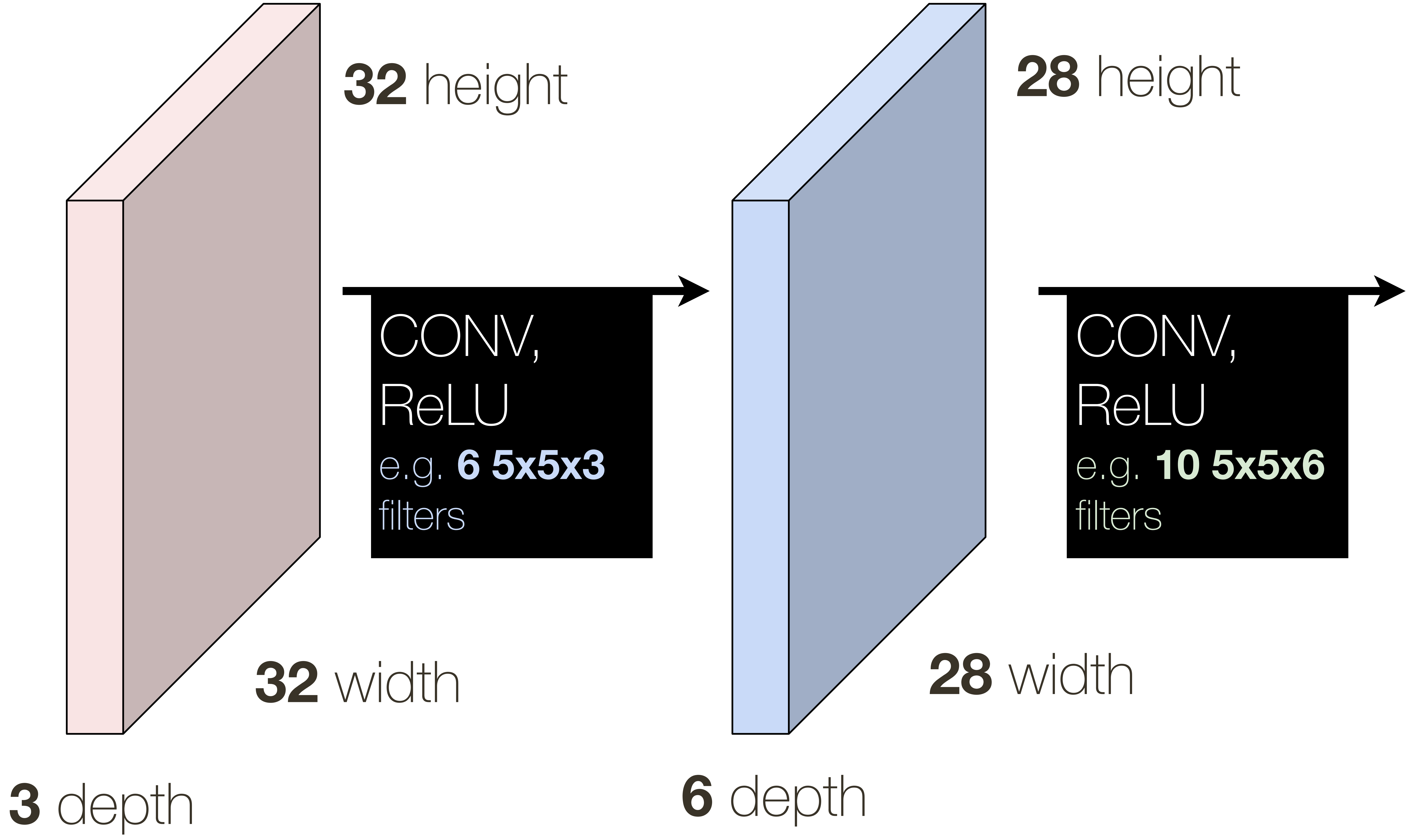
Convolutional Neural Network (ConvNet)



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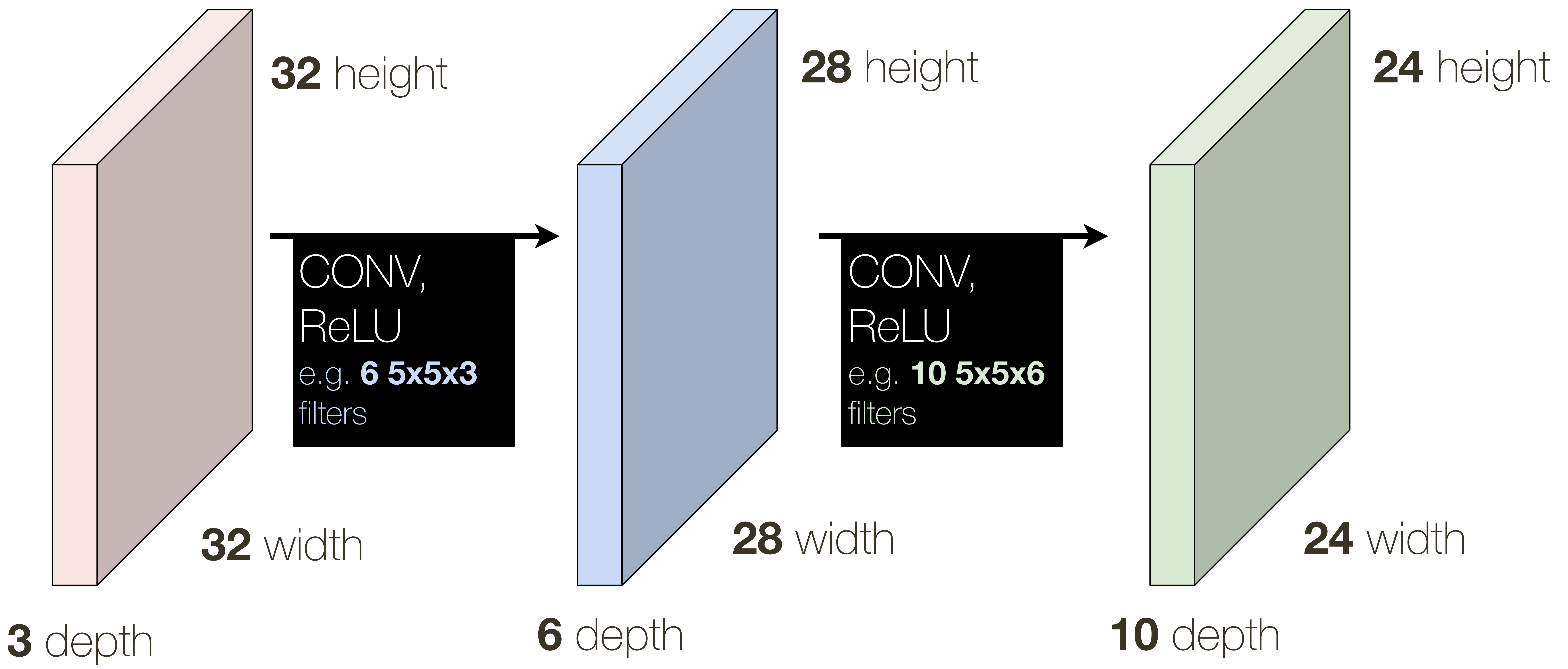
Convolutional Neural Network (ConvNet)



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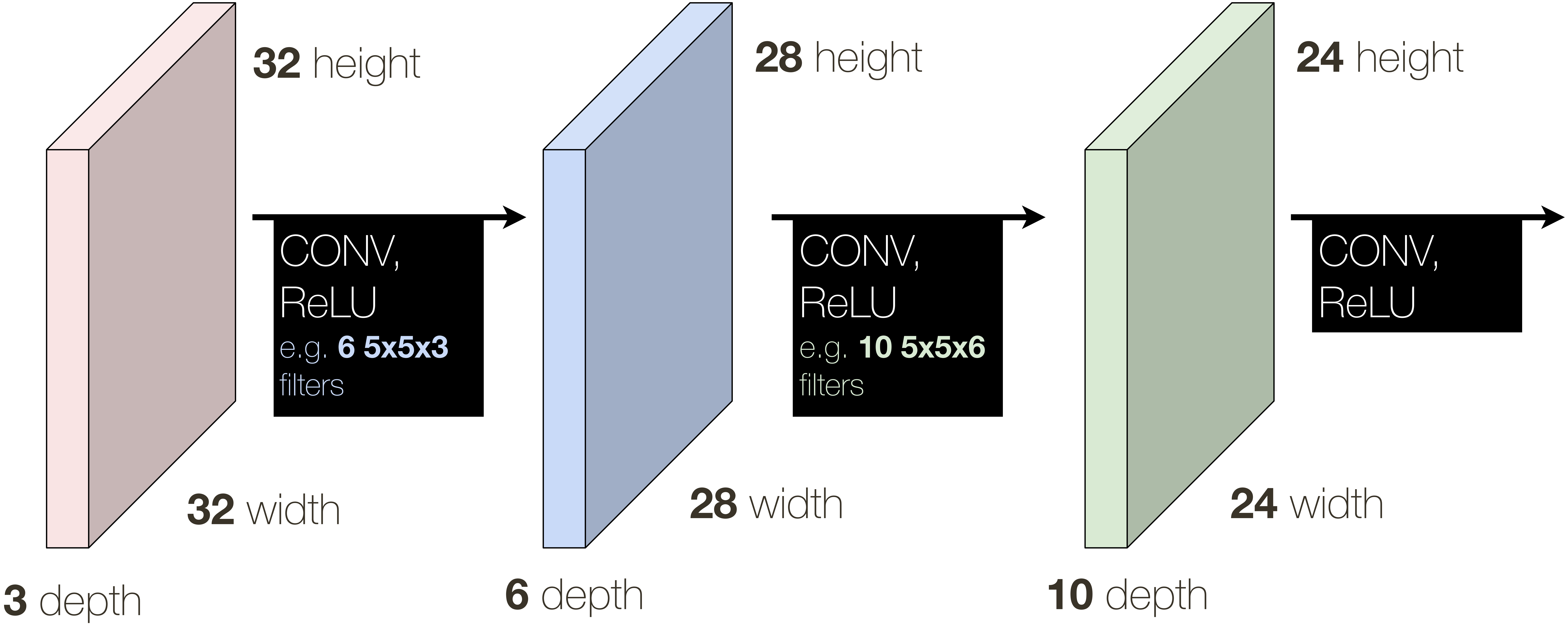
Convolutional Neural Network (ConvNet)



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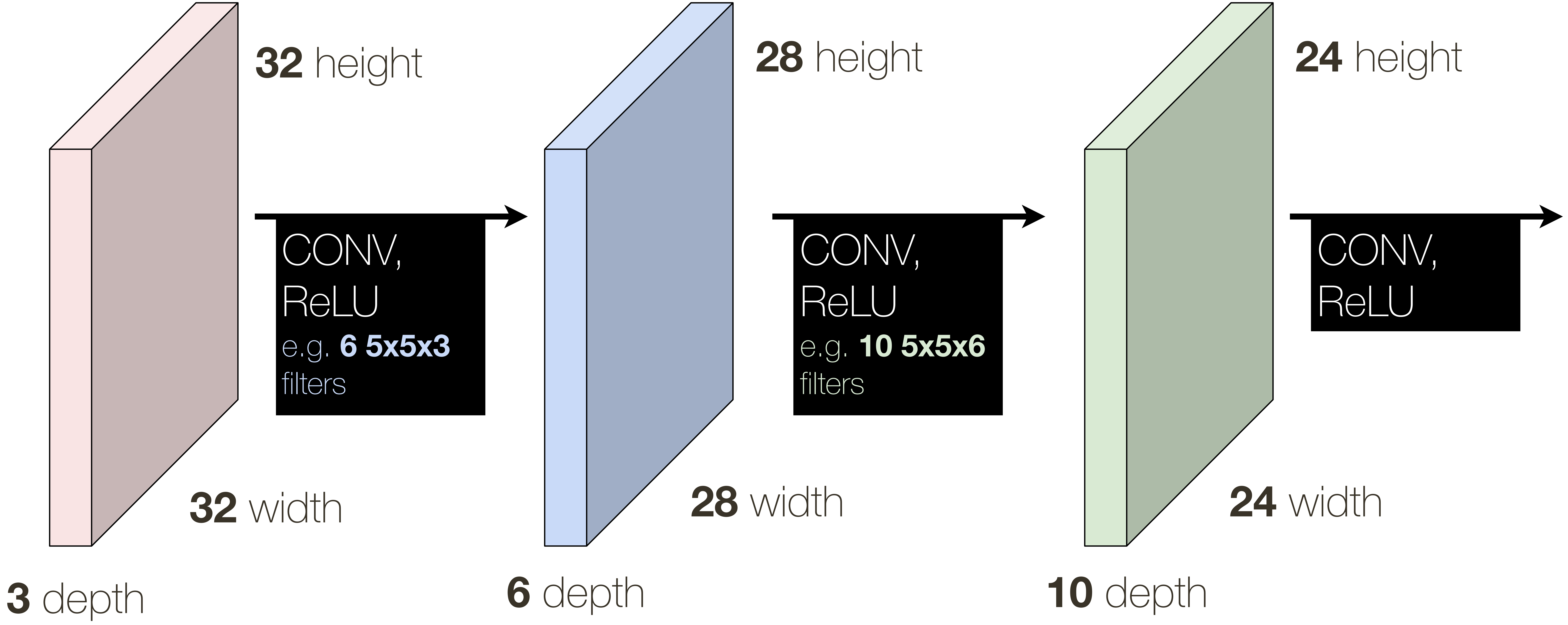
Convolutional Neural Network (ConvNet)

With padding we can achieve no shrinking (32 -> 28 -> 24); shrinking quickly (which happens with larger filters) doesn't work well in practice



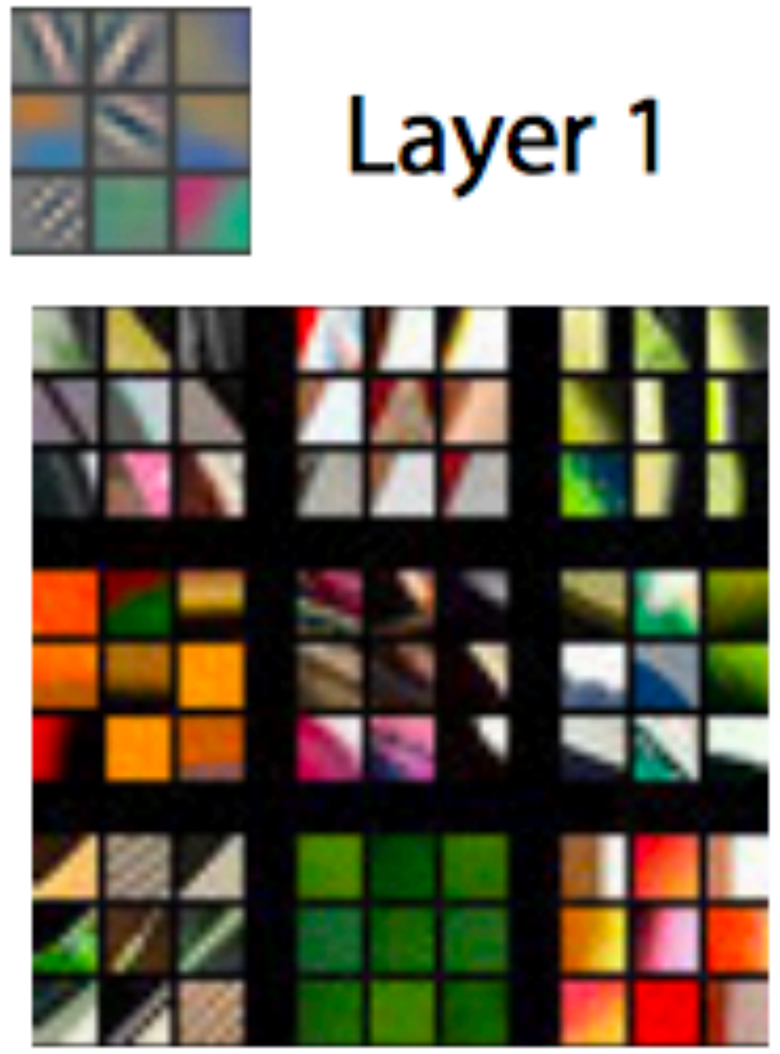
* slide from Fei-Dei Li, Justin Johnson, Serena Yeung, **cs231n Stanford**

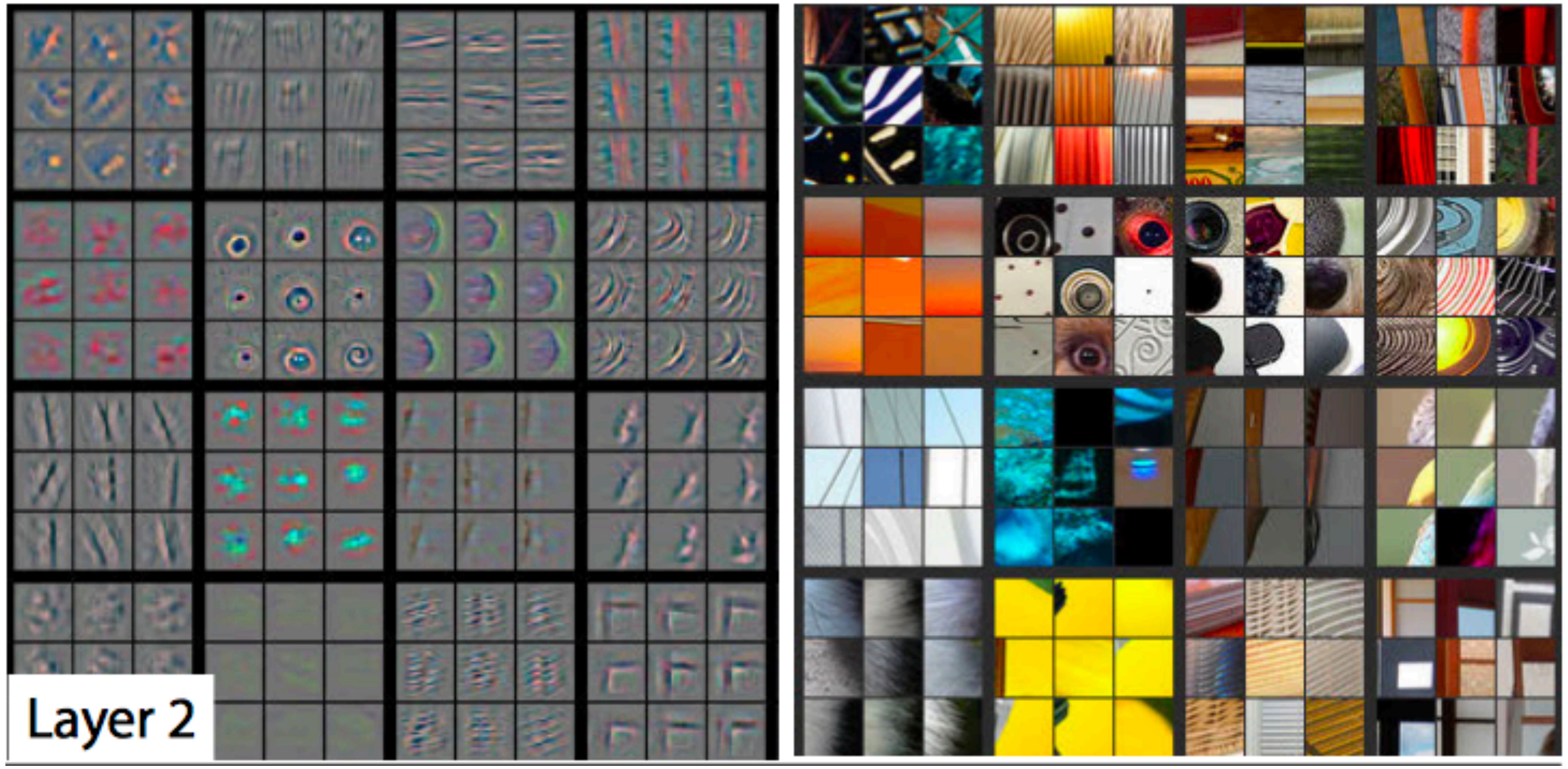
Convolutional Neural Network (ConvNet)

Convolutional neural networks can be seen as learning a hierarchy of filters.

As we go deeper in the network, filters learn and respond to increasingly specialized structures
— The first layers may contain simple orientation filters, middle layers may respond to common substructures, and final layers may respond to entire objects

What **filters** do networks learn?





[Zeiler and Fergus, 2013]

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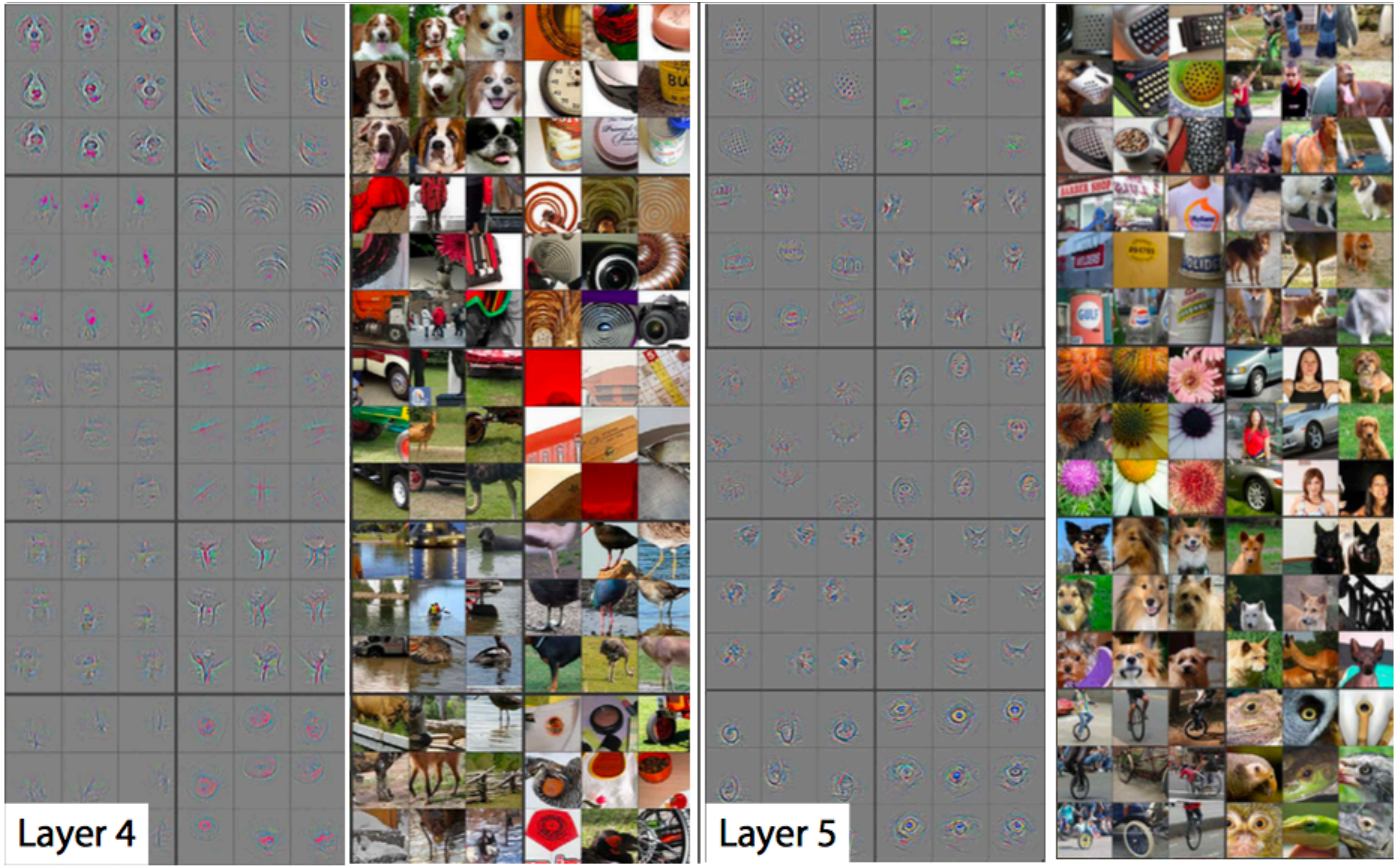
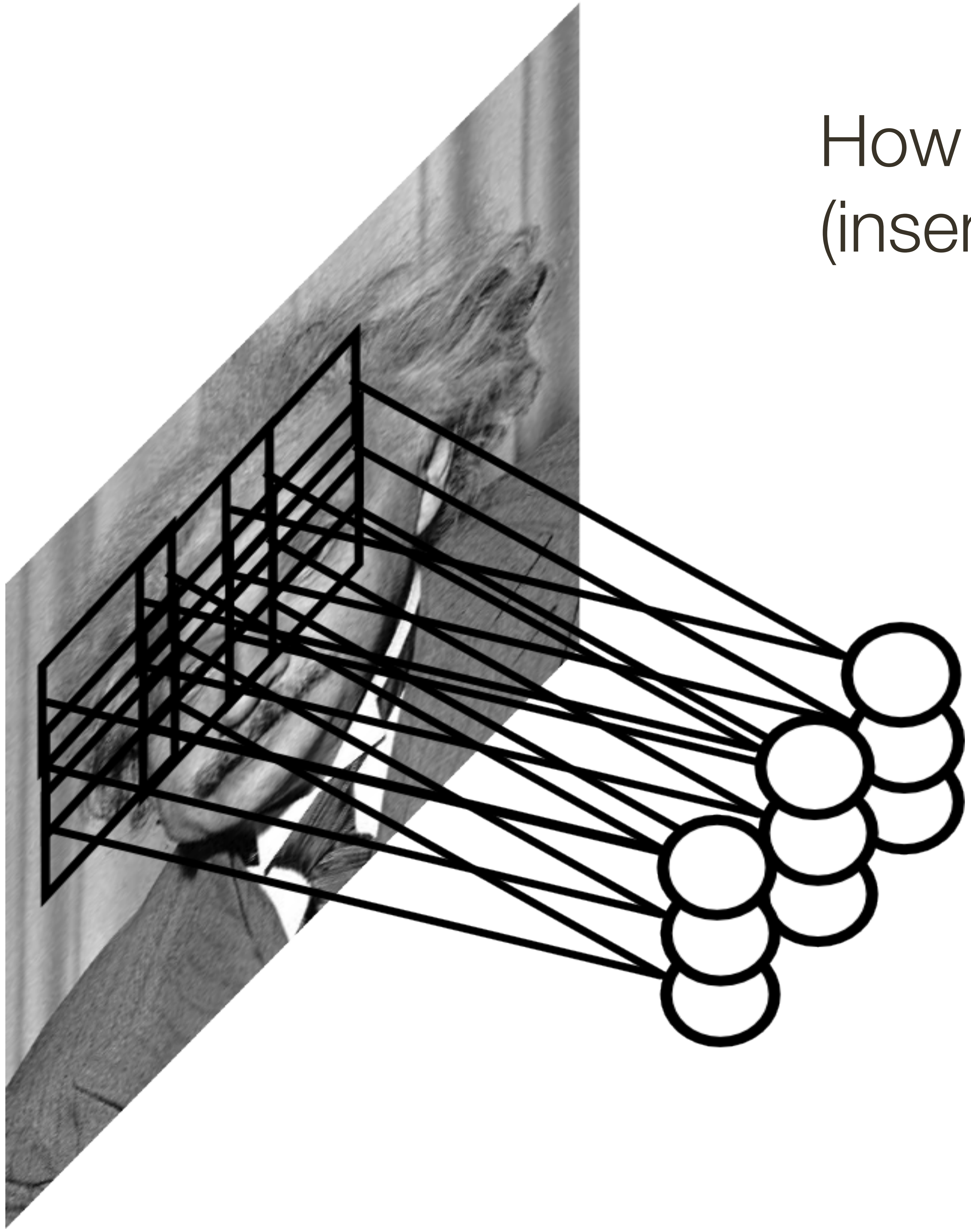


[Zeiler and Fergus, 2013]

Pooling Layer



Let us assume the filter is an “eye” detector

How can we make detection spatially invariant (insensitive to position of the eye in the image)

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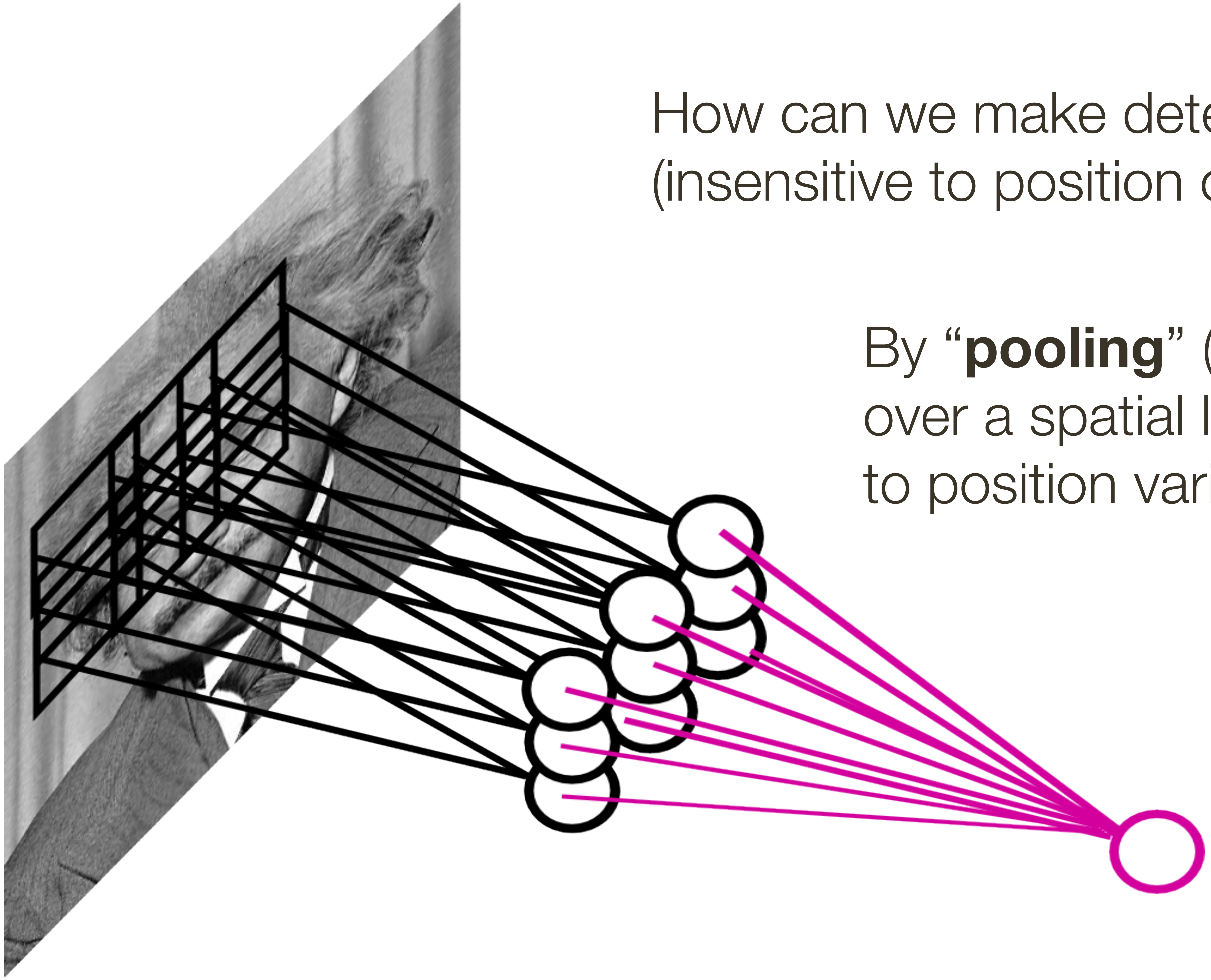
Pooling Layer



Let us assume the filter is an “eye” detector

How can we make detection spatially invariant (insensitive to position of the eye in the image)

By “**pooling**” (e.g., taking a max) response over a spatial locations we gain robustness to position variations

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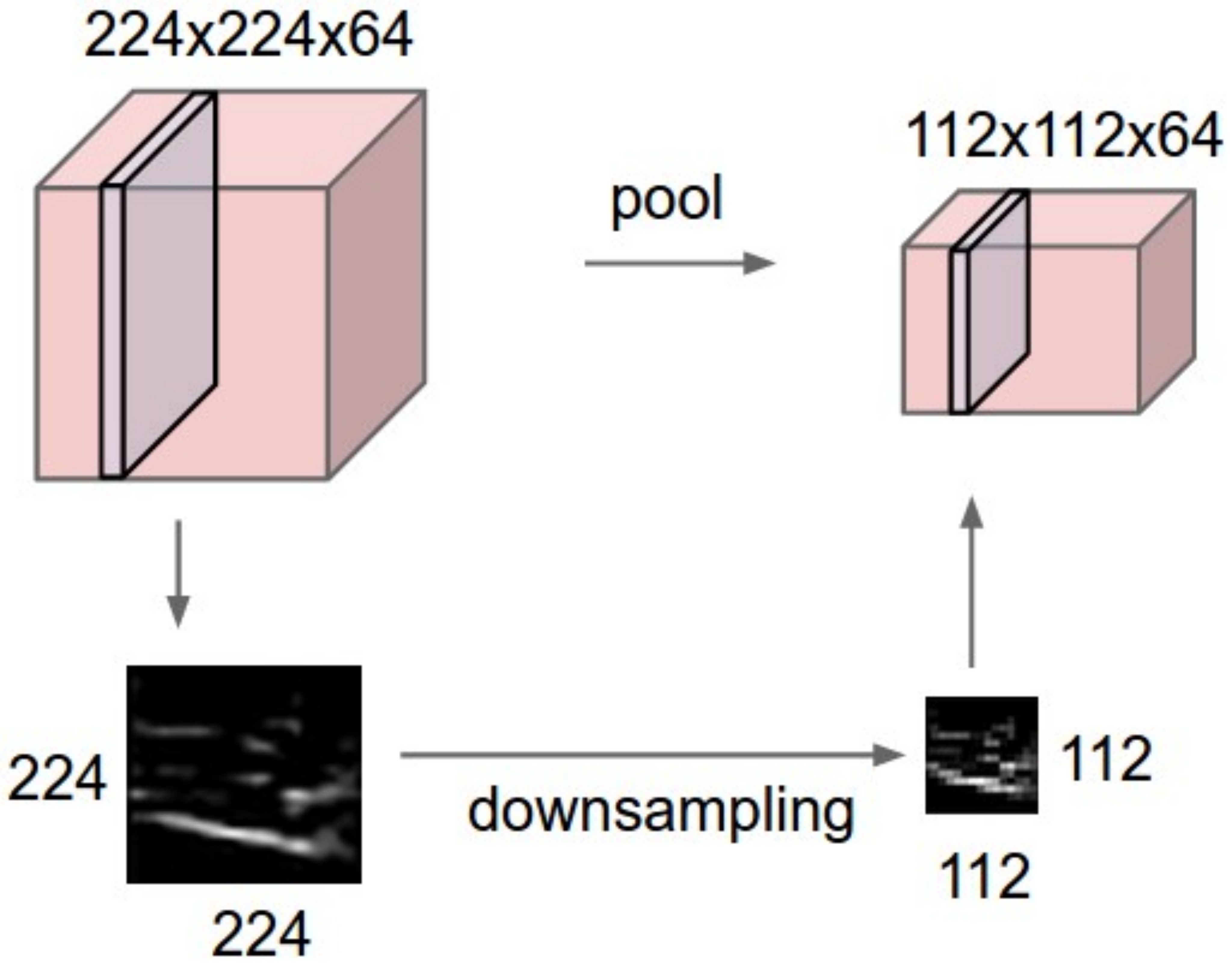
Pooling Layer

- Makes representation smaller, more manageable and spatially invariant
- Operates over each activation map independently



* slide from Fei-Dei Li, Justin Johnson, Serena Yeung, **cs231n Stanford**

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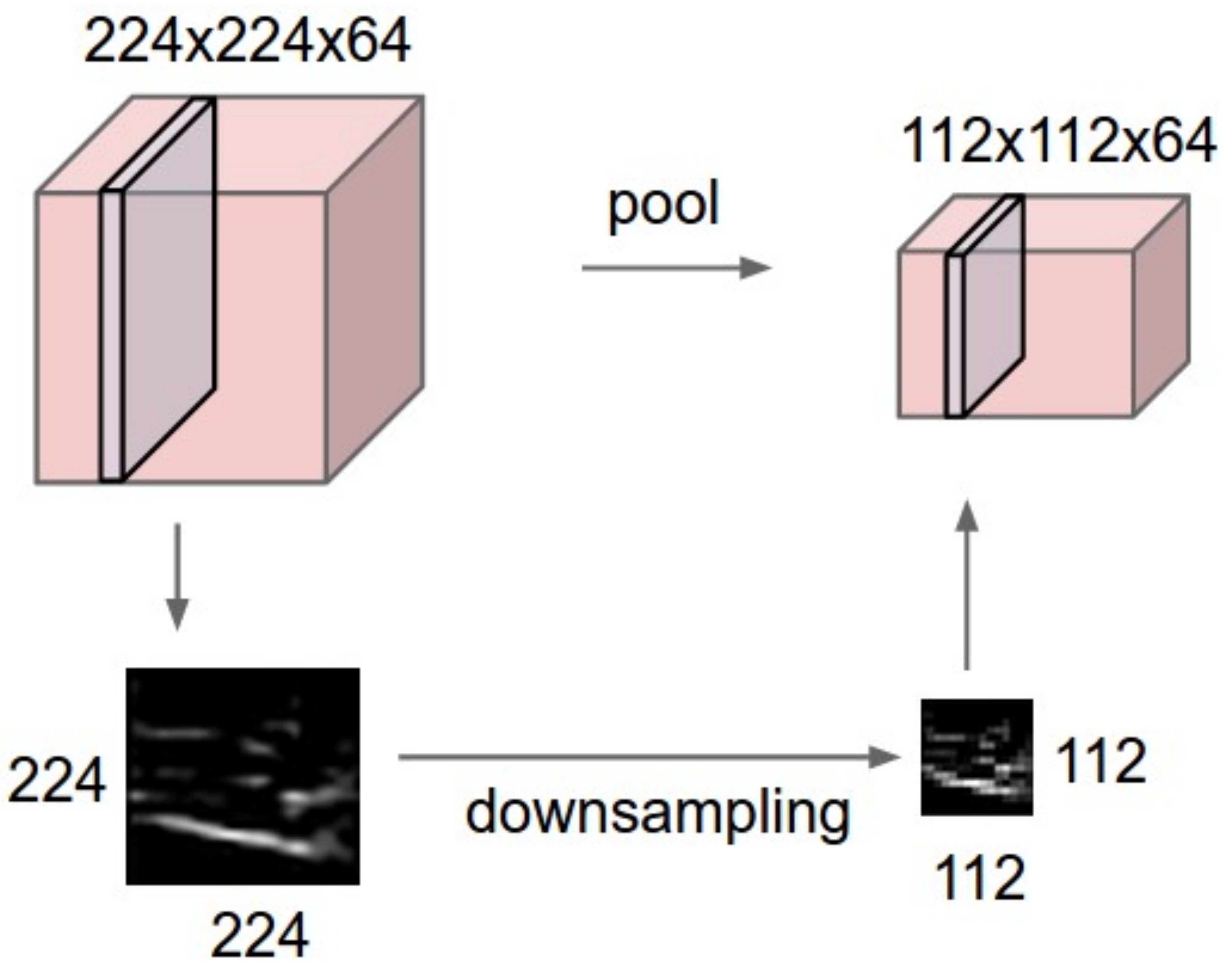

How many **parameters**?

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Pooling Layer

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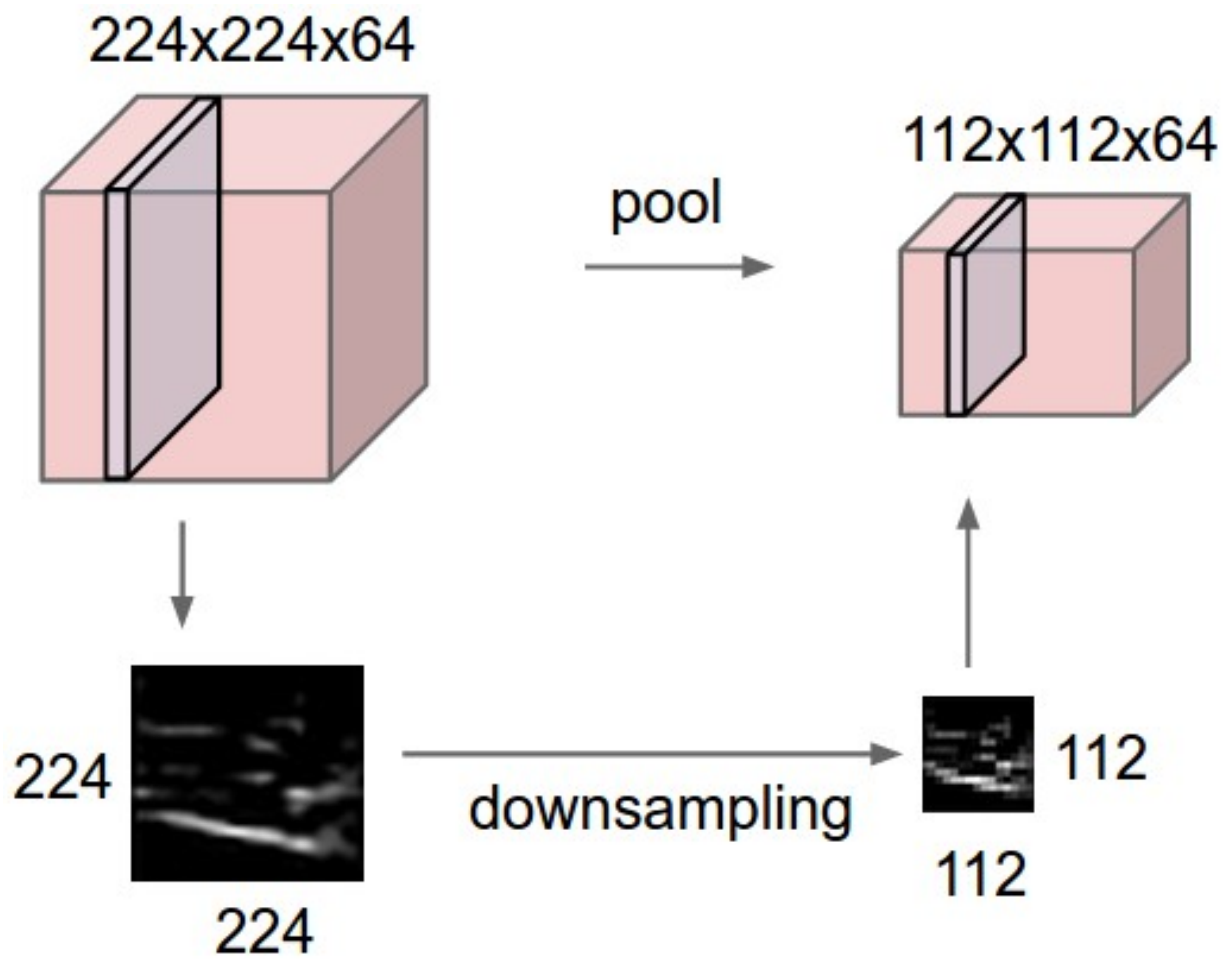
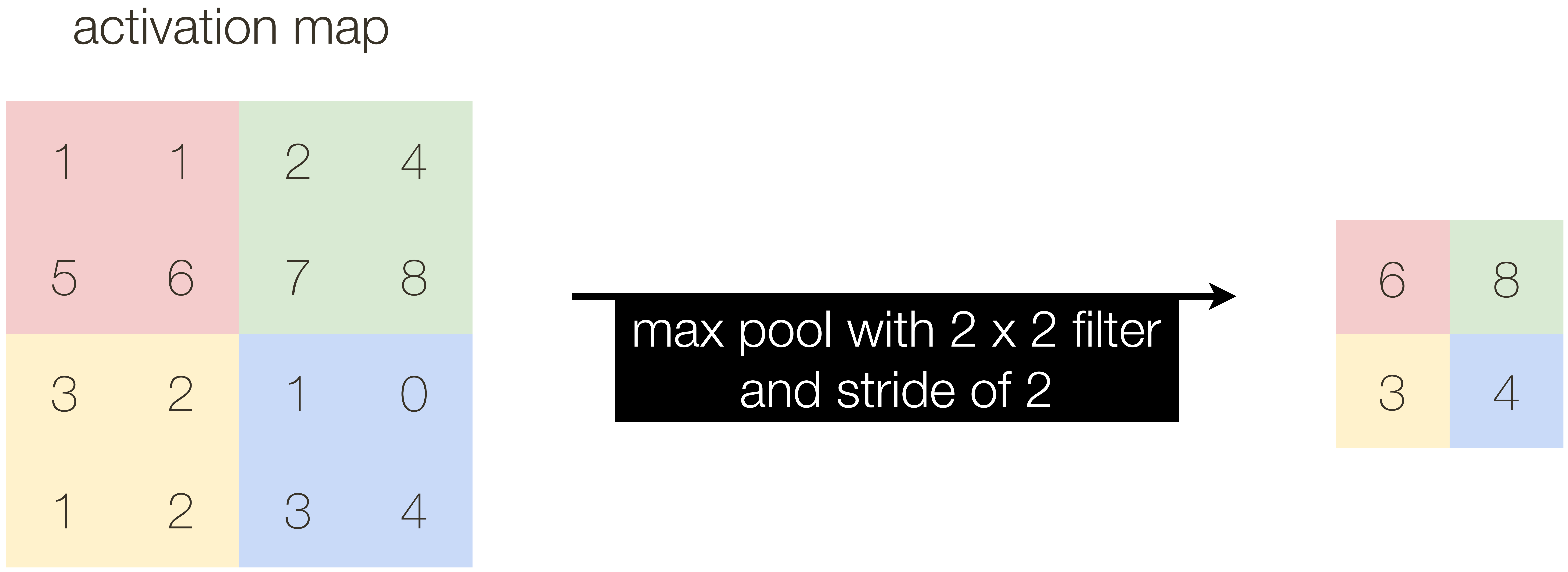


How many **parameters**?

None!

Max **Pooling**



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Average **Pooling**

activation map

1	1	2	4
5	6	7	8
3	2	1	0
1	2	3	4

avg pool with 2 x 2 filter
and stride of 2

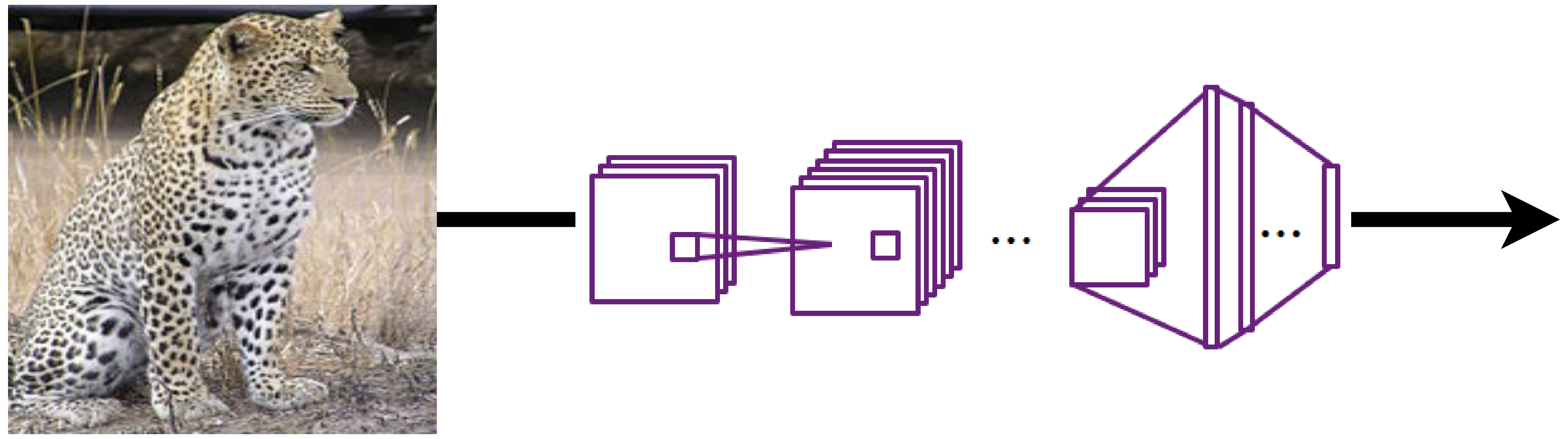
3.25	5.25
2	2

Object **Classification**

Category	Prediction
Dog	No
Cat	No
Couch	No
Flowers	No
Leopard	**Yes**
…	…

Problem: For each image predict which category it belongs to out of a fixed set

Object **Classification**



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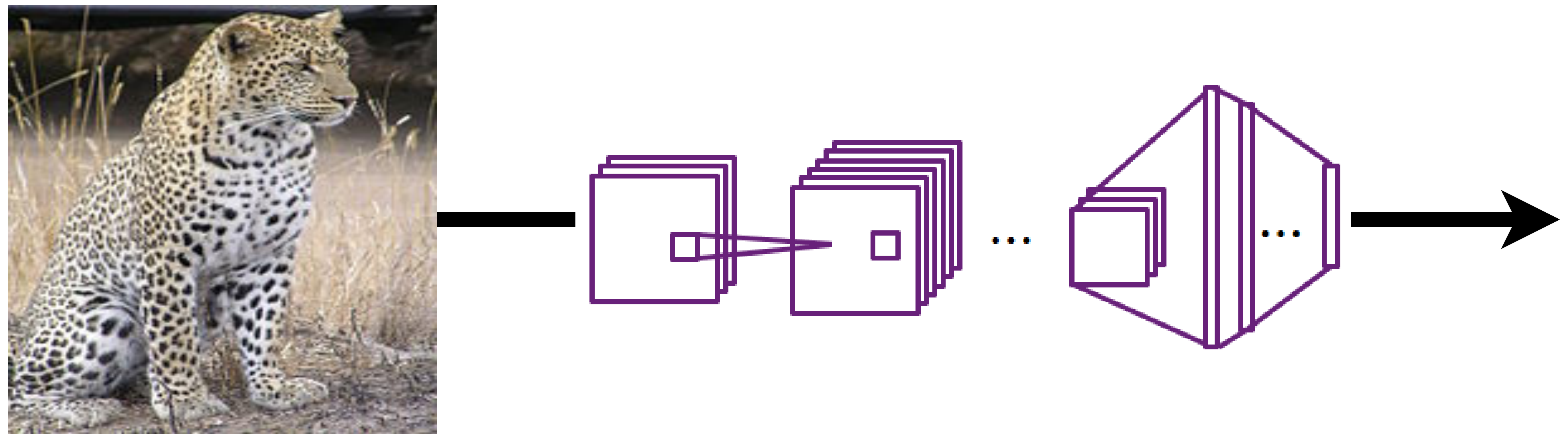
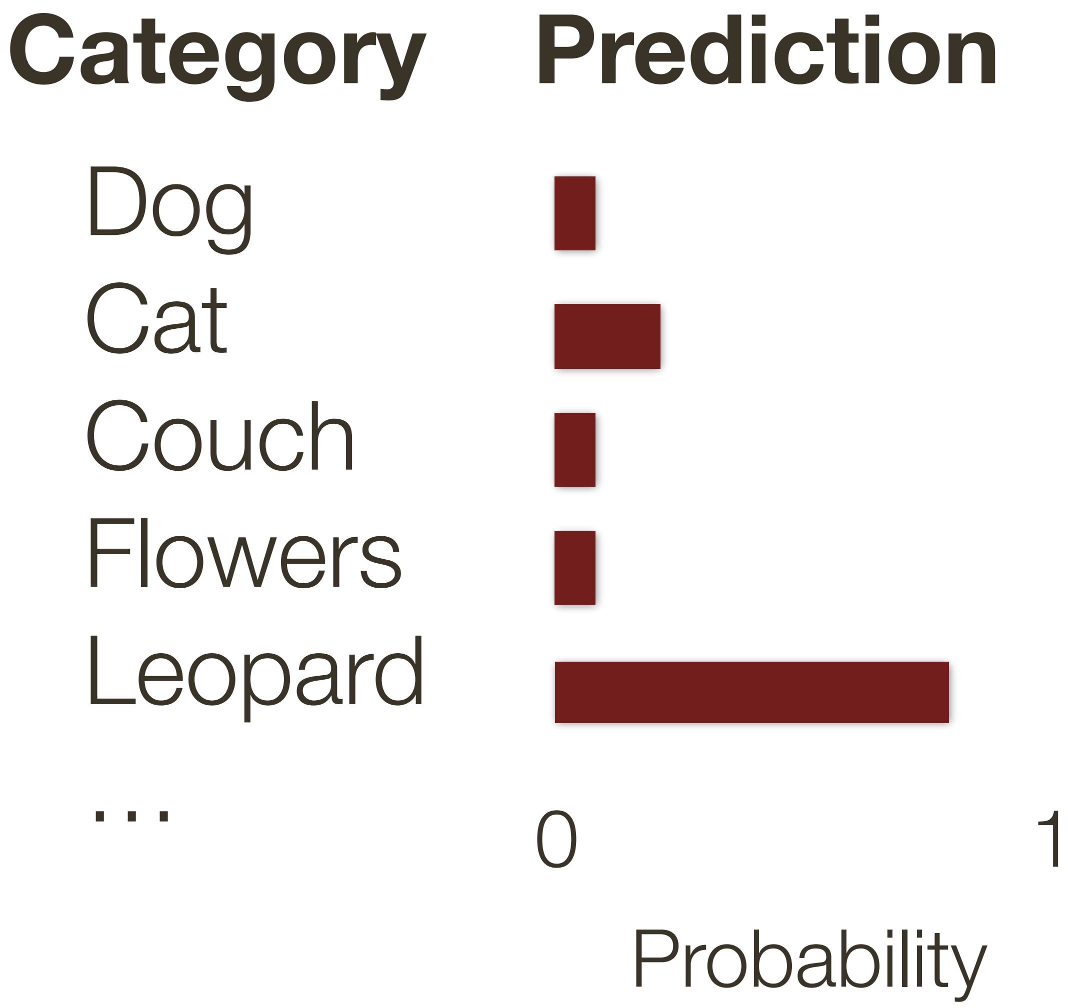




Problem: For each image predict which category it belongs to out of a fixed set

R-CNN

[Girshick et al, CVPR 2014]

Input **Image**

* image from Ross Girshick

R-CNN

[Girshick et al, CVPR 2014]

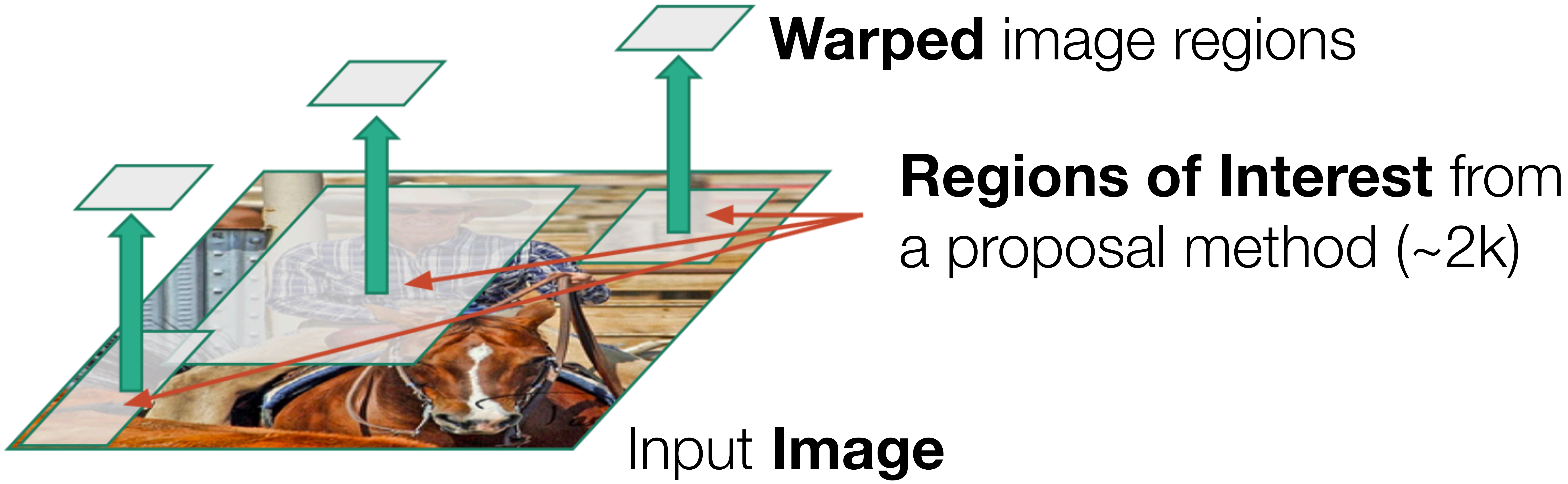
Regions of Interest from a proposal method (~2k)

Input **Image**

* image from Ross Girshick

R-CNN

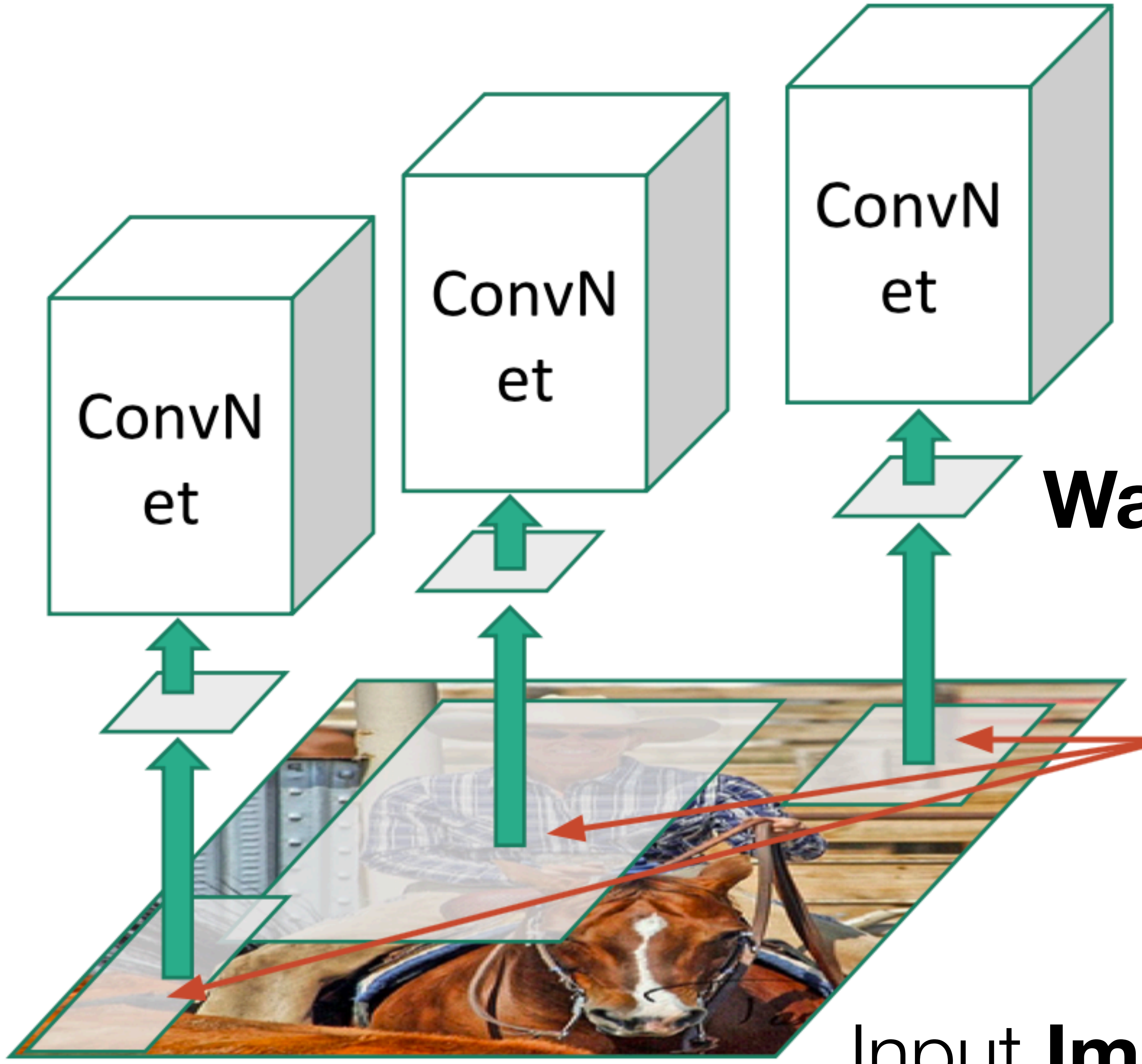
[Girshick et al, CVPR 2014]



* image from Ross Girshick

R-CNN

[Girshick et al, CVPR 2014]



Forward each region through a **CNN**

Warped image regions

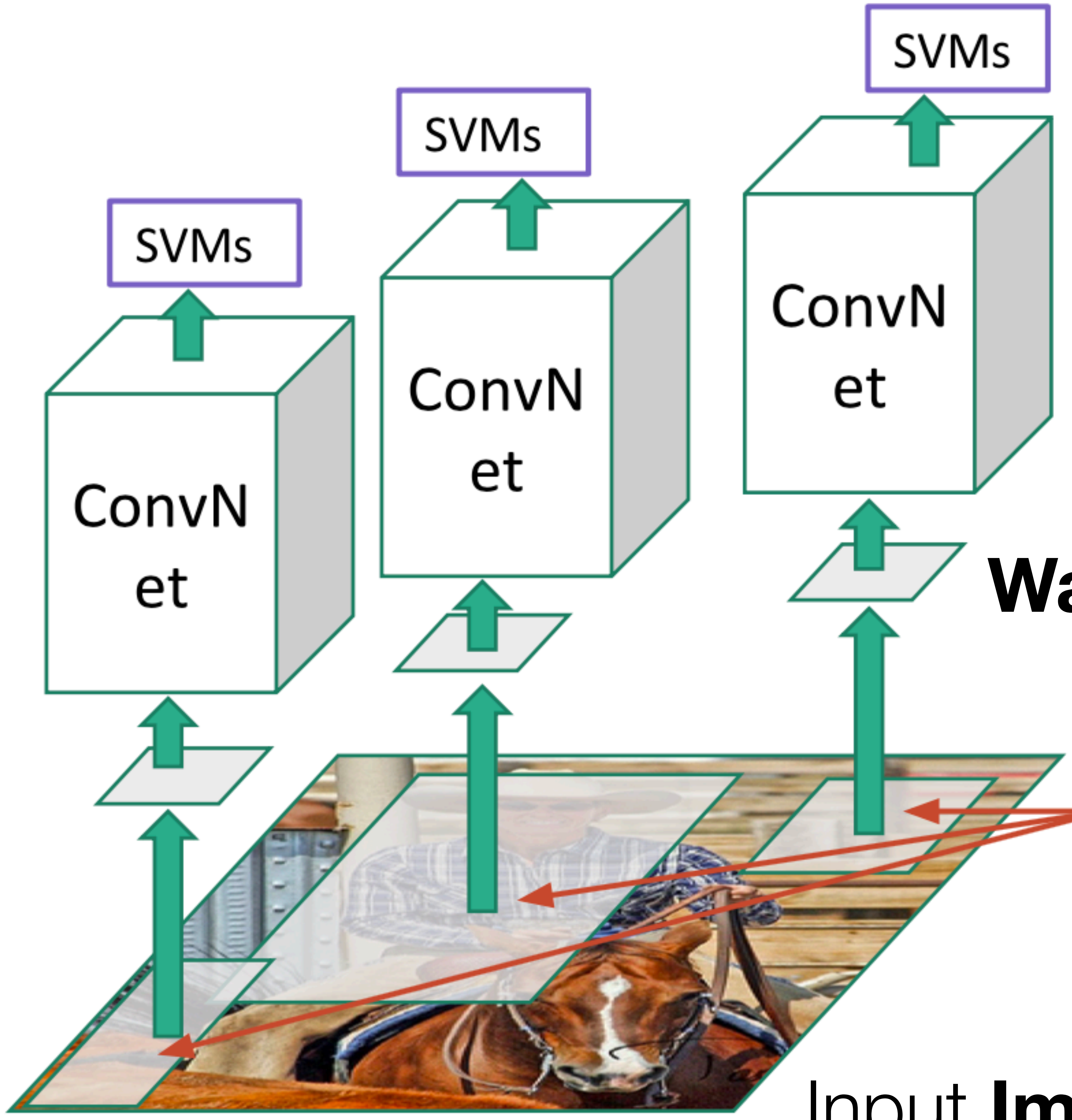
Regions of Interest from a proposal method (~2k)

Input **Image**

* image from Ross Girshick

R-CNN

[Girshick et al, CVPR 2014]



Classify regions with SVM

Forward each region through a **CNN**

Warped image regions

Regions of Interest from a proposal method (~2k)

Input **Image**

* image from Ross Girshick

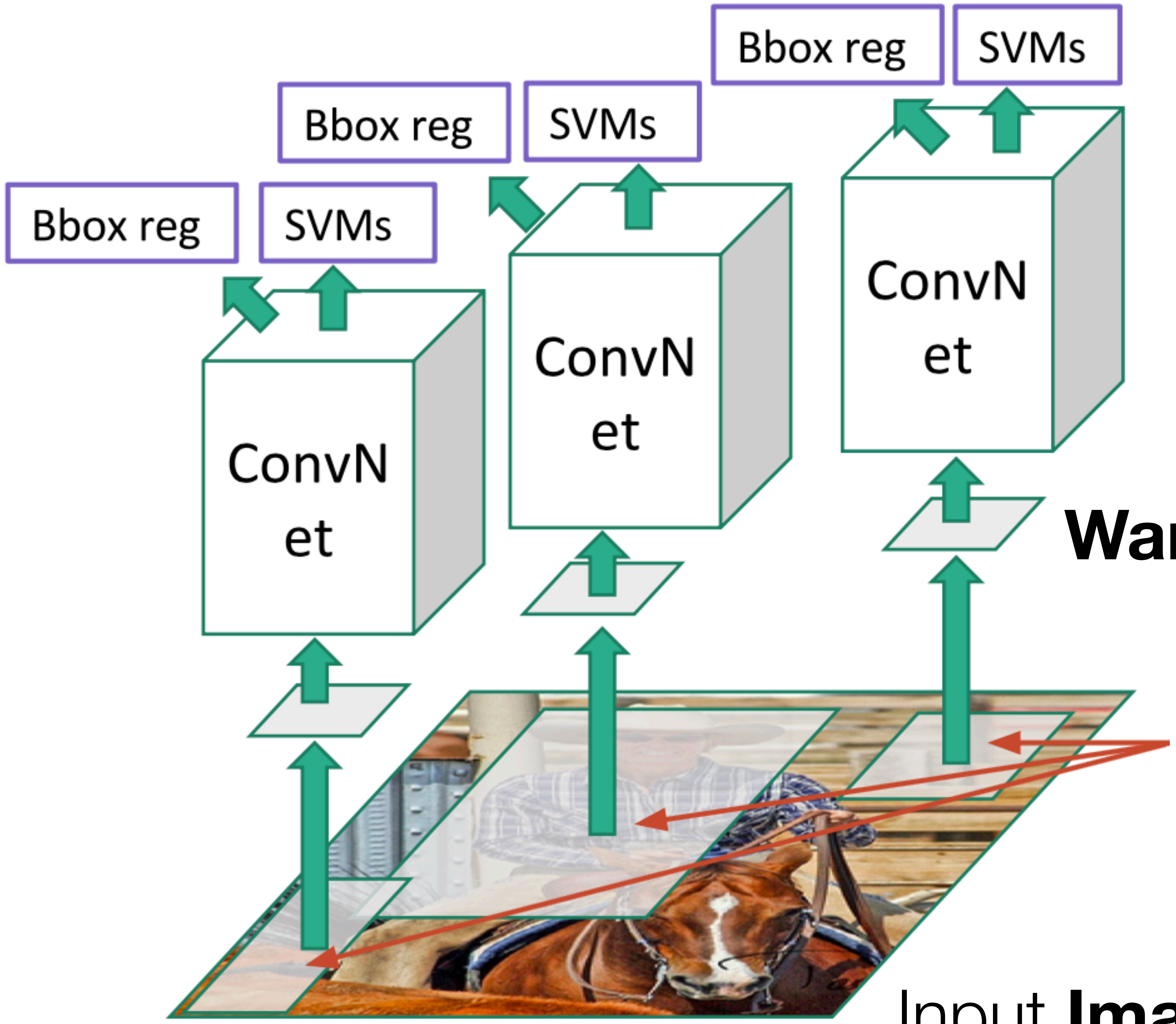
R-CNN

Linear Regression for bounding box offsets

[Girshick et al, CVPR 2014]

Classify regions with SVM

Forward each region through a **CNN**



Warped image regions

Regions of Interest from a proposal method (~2k)

Input Image

* image from Ross Girshick

R-CNN

R-CNN (Regions with CNN features) algorithm:

— Extract promising candidate regions using an object proposals algorithm

— Resize each proposal window to the size of the input layer of a trained convolutional neural network

— Input each resized image patch to the convolutional neural network

Implementation detail: Instead of using the classification scores of the network directly, the output of the final fully-connected layer can be used as an input feature to a trained support vector machine (SVM)

Summary

The parameters of a neural network are learned using **backpropagation**, which computes gradients via recursive application of the chain rule

A **convolutional neural network** assumes inputs are images, and constrains the network architecture to reduce the number of parameters

A **convolutional layer** applies a set of learnable filters

A **pooling layer** performs spatial downsampling

A **fully-connected** layer is the same as in a regular neural network

Convolutional neural networks can be seen as learning a hierarchy of filters

Please fill out

Student Evaluations

(on Canvas)