Lecture 2: Image Formation

( unless otherwise stated slides are taken or adopted from Bob Woodham, Jim Little and Fred Tung )
Menu for Today (January 9, 2020)

Updates:

- Assignment 0 is updated (optional)
- All assignments release and due dates are posted
- Midterm is scheduled for (right after the break)

Reminders:

- WWW: assignments, lecture notes, readings
- Piazza: discussion (lecture notes and assignment will also be posted here)
- Canvas: assignment hand in and grading
Menu for Today (January 9, 2020)

Topics:

- Image Formation
- Cameras and Lenses
- Projection
- Human eye (as camera)

Readings:

- **Today's** Lecture: Forsyth & Ponce (2nd ed.) 1.1.1 — 1.1.3
- **Next** Lecture: Forsyth & Ponce (2nd ed.) 4.1, 4.5

Reminders:

- Complete **Assignment 0** (ungraded) by Tuesday, **January 14**
- **WWW**: [http://www.cs.ubc.ca/~lsigal/teaching.html](http://www.cs.ubc.ca/~lsigal/teaching.html)
- **Piazza**: [piazza.com/ubc.ca/winterterm22020/cpsc425201/home](piazza.com/ubc.ca/winterterm22020/cpsc425201/home)
Today’s “fun” Example

Photo credit: reddit user Liammm
Today’s “fun” Example: Eye Sink Illusion

Photo credit: reddit user Liammm
Today’s “fun” Example: Eye Sink Illusion

“Tried taking a picture of a sink draining, wound up with a picture of an eye instead”

Photo credit: reddit user Liammm
Types of computer vision problems:

- Computing properties of the 3D world from visual data (measurement)
- Recognition of objects and scenes (perception and interpretation)
- Search and interact with visual data (search and organization)
- Manipulation or creation of image or video content (visual imagination)

Computer vision challenges:

- Fundamentally ill-posed
- Enormous computation and scale
- Lack of fundamental understanding of how human perception works
Computer vision technologies have moved **from research labs into commercial products and services**. Examples cited include:

- broadcast television sports
- electronic games (Microsoft Kinect)
- biometrics
- image search
- visual special effects
- medical imaging
- robotics

... many others
Lecture 2: Goal

To understand how images are formed

(and develop relevant mathematical concepts and abstractions)
What is **Computer Vision**?

Compute vision, broadly speaking, is a research field aimed to enable computers to **process and interpret visual data**, as sighted humans can.


blue sky, trees, fountains, UBC, …
What is **Computer Vision**?

Compute vision, broadly speaking, is a research field aimed to enable computers to **process and interpret visual data**, as sighted humans can.

[Diagram showing the process of computer vision from sensing to interpretation]
The **image formation process** that produces a particular image depends on:

- **Lightening condition**
- **Scene geometry**
- **Surface properties**
- **Camera optics and viewpoint**

Sensor (or eye) **captures amount of light** reflected from the object.
source

normal

surface element

sensor
source

normal

direction

sensor
Surface reflection depends on both the **viewing** \((\theta_v, \phi_v)\) and **illumination** \((\theta_i, \phi_i)\) direction, with Bidirectional Reflection Distribution Function: \(\text{BRDF}(\theta_i, \phi_i, \theta_v, \phi_v)\)

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**Slide adopted from:** Ioannis (Yannis) Gkioulekas (CMU)
Surface reflection depends on both the viewing \((\theta_v, \phi_v)\) and illumination \((\theta_i, \phi_i)\) direction, with Bidirectional Reflection Distribution Function: \(\text{BRDF}(\theta_i, \phi_i, \theta_v, \phi_v)\)

**Lambertian surface:**

\[
\frac{\rho d}{\pi}
\]

constant, called **albedo**

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**Lambertian** surface:

\[
\text{BRDF}(\theta_i, \phi_i, \theta_v, \phi_v) = \frac{\rho d}{\pi}
\]

\[
L = \frac{\rho d}{\pi} I(\vec{i} \cdot \vec{n})
\]

Slide adopted from: Ioannis (Yannis) Gkioulekas (CMU)
Surface reflection depends on both the **viewing** \((\theta_v, \phi_v)\) and **illumination** \((\theta_i, \phi_i)\) direction, with Bidirectional Reflection Distribution Function: \(\text{BRDF}(\theta_i, \phi_i, \theta_v, \phi_v)\)

**Lambertian** surface:

\[
\text{BRDF}(\theta_i, \phi_i, \theta_v, \phi_v) = \frac{\rho d}{\pi}
\]

**Mirror** surface: all incident light reflected in one directions \((\theta_v, \phi_v) = (\theta_r, \phi_r)\)

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*Slide adopted from: Ioannis (Yannis) Gkioulekas (CMU)*
Cameras

Old school film camera

Digital CCD/CMOS camera
Let’s say we have a sensor …

**Digital** CCD/CMOS camera

*Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)*
Let’s say we have a sensor ...  

**Digital** CCD/CMOS camera
Let’s say we have a **sensor** …

**Digital** CCD/CMOS camera

*Slide Credit:* Ioannis (Yannis) Gkioulekas (CMU)
... and the **object** we would like to photograph

What would an image taken like this look like?

real-world object

digital sensor (CCD or CMOS)

*Slide Credit:* Ioannis (Yannis) Gkioulekas (CMU)
Bare-sensor imaging

real-world object

digital sensor (CCD or CMOS)

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Bare-sensor imaging

real-world object

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Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)
Bare-sensor imaging

All scene points contribute to all sensor pixels

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)
Bare-sensor imaging

All scene points contribute to all sensor pixels

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)
Pinhole Camera

What would an image taken like this look like?

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)
Pinhole Camera

real-world object

most rays are blocked

digital sensor (CCD or CMOS)

one makes it through

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)
Pinhole Camera

Each scene point contributes to only one sensor pixel

real-world object

digital sensor (CCD or CMOS)

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)
Reinerus Gemma-Frisius observed an eclipse of the sun at Louvain on January 24, 1544. He used this illustration in his book, “De Radio Astronomica et Geometrica,” 1545. It is thought to be the first published illustration of a camera obscura.

Credit: John H., Hammond, “Th Camera Obscure, A Chronicle”
Camera Obscura (Latin for "dark chamber")

Reinerus Gemma-Frisius observed an eclipse of the sun at Louvain on January 24, 1544. He used this illustration in his book, “De Radio Astronomica et Geometrica,” 1545. It is thought to be the first published illustration of a camera obscura.

Credit: John H., Hammond, “The Camera Obscure, A Chronicle”
First **Photograph** on Record

*La table servie*

Credit: Nicéphore Niepce, 1822
A pinhole camera is a box with a small hall (aperture) in it.
Image Formation

Forsyth & Ponce (2nd ed.) Figure 1.1

Credit: US Navy, Basic Optics and Optical Instruments. Dover, 1969
Accidental Pinhole Camera

Image Credit: Ioannis (Yannis) Gkioulekas (CMU)
$f'$ is the **focal length** of the camera

Note: In a pinhole camera we can adjust the focal length, all this will do is change the **size** of the resulting image.
Pinhole Camera (Simplified)

It is convenient to think of the image plane which is in from of the pinhole.
It is convenient to think of the **image plane** which is in front of the pinhole.

What happens if the object moves towards the camera? Away from the camera?
Perspective Effects

Forsyth & Ponce (1st ed.) Figure 1.3a
Perspective Effects

Far objects appear smaller than close ones

Forsyth & Ponce (1st ed.) Figure 1.3a
Perspective Effects

Far objects appear smaller than close ones.

Size is inversely proportional to distance.
Perspective Effects

Forsyth & Ponce (1st ed.) Figure 1.3b
Parallel lines meet at a point (vanishing point)
Vanishing Points

Each set of parallel lines meet at a different point
— the point is called **vanishing point**
Vanishing Points

Each set of parallel lines meet at a different point
— the point is called **vanishing point**

Sets of parallel lines one the same plane lead to **collinear** vanishing points
— the line is called a **horizon** for that plane
Vanishing Points

Draw a horizon line.

Slide Credit: David Jacobs
vanishing points

<table>
<thead>
<tr>
<th>Draw a horizon line.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make a vanishing point.</td>
</tr>
</tbody>
</table>
Vanishing Points

1. Draw a horizon line.
2. Make a vanishing point.
3. Draw a square or rectangle.

Slide Credit: David Jacobs
Vanishing Points

1. Draw a horizon line.
2. Make a vanishing point.
3. Draw a square or rectangle.
4. Draw orthogonals from shape corners to vanishing point.

Slide Credit: David Jacobs
Vanishing Points

Draw a horizon line.

Make a vanishing point.

Draw a square or rectangle.

Draw orthogonals from shape corners to vanishing point.

Draw a horizontal line to end your form.

Slide Credit: David Jacobs
Vanishing Points

1. Draw a horizon line.
2. Make a vanishing point.
3. Draw a square or rectangle.
4. Draw orthogonal lines from shape corners to vanishing point.
5. Draw a horizontal line to end your form.
6. Draw a vertical line to make the form’s side.

Slide Credit: David Jacobs
Vanishing Points

1. Draw a horizon line.
2. Make a vanishing point.
3. Draw a square or rectangle.
4. Draw orthogonals from shape corners to vanishing point.
5. Draw a horizontal line to end your form.
6. Draw a vertical line to make the form’s side.
7. Erase the orthogonals.

Slide Credit: David Jacobs
Vanishing Points

1. Draw a horizon line.
2. Make a vanishing point.
3. Draw a square or rectangle.
4. Draw orthogonals from shape corners to vanishing point.
5. Draw a horizontal line to end your form.
6. Draw a vertical line to make the form's side.
7. Erase the orthogonals.
8. Draw another form!

Slide Credit: David Jacobs
Vanishing Points

1. Draw a horizon line.
2. Make a vanishing point.
3. Draw a square or rectangle.
   - Draw orthogonal lines from shape corners to vanishing point.
4. Draw a horizontal line to end your form.
5. Draw a vertical line to make the form’s side.
6. Erase the orthogonal lines.
7. Draw another form!
8. Add windows and doors.

Slide Credit: David Jacobs
Vanishing Points

Each set of parallel lines meet at a different point
— the point is called *vanishing point*

Sets of parallel lines on the same plane lead to *collinear* vanishing points
— the line is called a *horizon* for that plane

Good way to *spot fake images*
— scale and perspective do not work
— vanishing points behave badly
Vanishing Points

Slide Credit: Efros (Berkeley), photo from Criminisi
Vanishing Points

Slide Credit: Efros (Berkeley), photo from Criminisi
Vanishing Points

Slide Credit: Efros (Berkeley), photo from Criminisi
Vanishing Points

Vertical vanishing point (at infinity)

Vanishing point

Vanishing point

Slide Credit: Efros (Berkeley), photo from Criminisi
Vanishing Points

Two/three point perspective

Vertical vanishing point (at infinity)

Vanishing point

Slide Credit: Efros (Berkeley), photo from Criminisi
Vanishing Points

One point perspective

Two/three point perspective

Vertical vanishing point (at infinity)

Vanishing point

Slide Credit: Efros (Berkeley), photo from Criminisi
Perspective Aside

Image credit: http://www.martinacecilia.com/place-vanishing-points/
Perspective Aside

Image credit: http://www.martinacecilia.com/place-vanishing-points/
Properties of Projection

- **Points** project to **points**
- **Lines** project to **lines**
- **Planes** project to the **whole** or **half** image
- Angles are **not** preserved
Properties of Projection

- **Points** project to **points**
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- **Planes** project to the **whole** or **half** image
- Angles are **not** preserved

**Degenerate** cases

- Line through focal point projects to a point
- Plane through focal point projects to a line
Projection Illusion
Perspective Projection

3D object point

\[ P = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \]

projects to 2D image point

\[ P' = \begin{bmatrix} x' \\ y' \end{bmatrix} \]

where

\[ x' = f' \frac{x}{z} \]
\[ y' = f' \frac{y}{z} \]

Note: this assumes world coordinate frame at the optical center (pinhole) and aligned with the image plane, image coordinate frame aligned with the camera coordinate frame.
Perspective Projection: Proof

3D object point

\[ P = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \]

projects to 2D image point

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\[ y' = f' \frac{y}{z} \]

Note: this assumes world coordinate frame at the optical center (pinhole) and aligned with the image plane, image coordinate frame aligned with the camera coordinate frame.
**Perspective Projection: Proof**

Forsyth & Ponce (1st ed.) Figure 1.4

3D object point

\[ P = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \]

projects to 2D image point

\[ P' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \]

where

\[ P' = CP \]

**Camera Matrix**

\[
C = \begin{bmatrix}
f' & 0 & 0 & 0 \\
0 & f' & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

**Note:** This assumes world coordinate frame at the optical center (pinhole) and aligned with the image plane, image coordinate frame aligned with the camera coordinate frame.
Weak Perspective

3D object point $P = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ in $\Pi_0$ projects to 2D image point $P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$

where $x' = mx$  
$y' = my$  
and $m = \frac{f'}{z_0}$
Orthographic Projection

3D object point \( P = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \) projects to 2D image point \( P' = \begin{bmatrix} x' \\ y' \end{bmatrix} \)

where
\[
\begin{align*}
x' &= x \\
y' &= y
\end{align*}
\]

Forsyth & Ponce (1st ed.) Figure 1.6
## Summary of Projection Equations

A 3D object point \( P = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \) projects to a 2D image point \( P' = \begin{bmatrix} x' \\ y' \end{bmatrix} \) where the equations depend on the type of projection:

### Perspective
\[
\begin{align*}
x' &= f' \frac{x}{z} \\
y' &= f' \frac{y}{z}
\end{align*}
\]

### Weak Perspective
\[
\begin{align*}
x' &= m x \\
y' &= m y
\end{align*}
\]
where \( m = \frac{f'}{z_0} \)

### Orthographic
\[
\begin{align*}
x' &= x \\
y' &= y
\end{align*}
\]
Projection Models: Pros and Cons

**Weak perspective** (including orthographic) has simpler mathematics

- accurate when object is small and/or distant
- useful for recognition

**Perspective** is more accurate for real scenes

When **maximum accuracy** is required, it is necessary to model additional details of a particular camera

- use perspective projection with additional parameters (e.g., lens distortion)
Why Not a Pinhole Camera?

— If pinhole is too big then many directions are averaged, blurring the image

— If pinhole is too small then diffraction becomes a factor, also blurring the image

— Generally, pinhole cameras are dark, because only a very small set of rays from a particular scene point hits the image plane

— Pinhole cameras are slow, because only a very small amount of light from a particular scene point hits the image plane per unit time

Image Credit: Credit: E. Hecht. “Optics,” Addison-Wesley, 1987
Snell’s Law

\[ n_1 \sin \alpha_1 = n_2 \sin \alpha_2 \]
Snell’s Law

Index of refraction

\[ n_1 \sin \alpha_1 = n_2 \sin \alpha_2 \]
Reason for **Lenses**

The role of a lens is to **capture more light** while preserving, as much as possible, the abstraction of an ideal pinhole camera.
Pinhole Model (Simplified) \textbf{with Lens}
Thin Lens Equation

Forsyth & Ponce (1st ed.) Figure 1.9

\[
\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}
\]
Thin Lens Equation: Derivation

\[ \frac{y}{-z} = \frac{-y'}{z'} \]

\[ \frac{y}{y'} = \frac{z}{z'} \]

\[ \frac{1}{z'} - \frac{1}{z} = \frac{1}{f} \]

Forsyth & Ponce (1st ed.) Figure 1.9
Thin Lens Equation: Derivation

\[
\frac{y}{-z} = -\frac{y'}{z'}
\]

\[
\frac{y}{y'} = \frac{z}{z'}
\]

Forsyth & Ponce (1st ed.) Figure 1.9

\[
\frac{-y'}{f} = \frac{y - y'}{-z}
\]

\[
\frac{1}{f} = \frac{y - y'}{z y'}
\]

\[
= \frac{y}{z y'} - \frac{y'}{z y'}
\]

\[
= \frac{y}{z y'} - \frac{1}{z}
\]

\[
\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}
\]
Thin Lens Equation: Derivation

\[ \frac{y}{-z} = -\frac{y'}{z'} \]
\[ \frac{y}{y'} = \frac{z}{z'} \]

Forsyth & Ponce (1st ed.) Figure 1.9

\[ \frac{-y'}{f} = \frac{y - y'}{-z} \]
\[ \frac{1}{f} = \frac{y - y'}{zy'} \]
\[ = \frac{y}{zy'} - \frac{y'}{zy'} \]
\[ = \frac{y}{zy'} - \frac{1}{z} \]

\[ \frac{1}{z'} - \frac{1}{z} = \frac{1}{f} \]

Substitute: \[ \frac{1}{f} = \frac{1}{\frac{z'}{z}} - \frac{1}{z} \]
Possible Uses of Thin Lens Abstraction

\[ \frac{1}{z'} - \frac{1}{z} = \frac{1}{f} \]
Another way of looking at the focal length of a lens. The incoming rays, parallel to the optical axis, converge to a single point a distance $f$ behind the lens. This is where we want to place the image plane.
Out-of-Focus

The image plane is in the wrong place, either slightly closer than the required focal length, $f$, or slightly further than the required focal length, $f$. 
Spherical Aberration

Forsyth & Ponce (1st ed.) Figure 1.12a
Spherical Aberration

Un-aberrated image

Image from lens with Spherical Aberration
A modern camera lens may contain multiple components, including aspherical elements.
Vignetting

Vignetting in a two-lens system

The shaded part of the beam **never reaches** the second lens
Vignetting

Image Credit: Cambridge in Colour
Chromatic Aberration

– Index of **refraction depends on wavelength**, \( \lambda \), of light
– Light of different colours follows different paths
– Therefore, not all colours can be in equal focus

*Image Credit: Trevor Darrell*
Other (Possibly Significant) **Lens Effects**

**Chromatic aberration**
- Index of refraction depends on wavelength, $\lambda$, of light
- Light of different colours follows different paths
- Therefore, not all colours can be in equal focus

**Scattering** at the lens surface
- Some light is reflected at each lens surface

There are other **geometric phenomena/distortions**
- pincushion distortion
- barrel distortion
- etc
Lens Distortion

Szeliski (1st ed.) Figure 2.13

Lines in the world are no longer lines on the image, they are curves!
Human Eye

— The eye has an iris (like a camera)

— Focusing is done by changing shape of lens

— When the eye is properly focused, light from an object outside the eye is imaged on the retina

— The retina contains light receptors called rods and cones

\[ \text{pupil} = \text{pinhole / aperture} \]

\[ \text{retina} = \text{film / digital sensor} \]

Slide adopted from: Steve Seitz
Fun Aside

https://io9.gizmodo.com/does-your-brain-really-have-the-power-to-see-the-world-5905180

George M. Stratton
Human Eye

— The eye has an iris (like a camera)
— Focusing is done by changing shape of lens
— When the eye is properly focused, light from an object outside the eye is imaged on the retina
— The retina contains light receptors called rods and cones

**pupil** = pinhole / aperture

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Slide adopted from: Steve Seitz
Human Eye

- The eye has an iris (like a camera)
- Focusing is done by changing shape of lens
- When the eye is properly focused, light from an object outside the eye is imaged on the retina.
- The retina contains light receptors called rods and cones

\[
pupil = \text{pinhole / aperture}
\]

\[
\text{retina} = \text{film / digital sensor}
\]

Slide adopted from: Steve Seitz
Two types of **Light Sensitive Receptors**

**Rods**
- 75-150 million rod-shaped receptors
- **not** involved in color vision, gray-scale vision only
- operate at night
- highly sensitive, can responding to a single photon
- yield relatively poor spatial detail

**Cones**
- 6-7 million cone-shaped receptors
- color vision
- operate in high light
- less sensitive
- yield higher resolution

*Slide adopted from: James Hays*
Density of rods and cones

Slide adopted from: James Hays
Lecture Summary

— We discussed a “physics-based” approach to image formation. Basic abstraction is the **pinhole camera**.

— **Lenses overcome limitations** of the pinhole model while trying to preserve it as a useful abstraction

— Projection equations: **perspective**, weak perspective, orthographic

— Thin lens equation

— Some “aberrations and **distortions**” persist (e.g. spherical aberration, vignetting)

— The **human eye** functions much like a camera
Reminders

Readings:

— **Today’s** Lecture: Forsyth & Ponce (2nd ed.) 1.1.1 — 1.1.3
— **Next** Lecture: Forsyth & Ponce (2nd ed.) 4.1, 4.5

Reminders:

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