Lecture 7: Edge Detection

( unless otherwise stated slides are taken or adopted from Bob Woodham, Jim Little and Fred Tung )
**Edge Detection**

**Goal**: Identify sudden changes in image intensity

This is where most shape information is encoded

**Example**: artist’s line drawing (but artist also is using object-level knowledge)
What Causes **Edges**?

- Depth discontinuity
- Surface orientation discontinuity
- Reflectance discontinuity (i.e., change in surface material properties)
- Illumination discontinuity (e.g., shadow)

*Slide Credit: Christopher Rasmussen*
**Smoothing and Differentiation**

**Edge**: a location with high gradient (derivative)

Need smoothing to reduce noise prior to taking derivative

Need two derivatives, in x and y direction

We can use **derivative of Gaussian** filters
  — because differentiation is convolution, and
  — convolution is associative

Let \( \otimes \) denote convolution

\[
D \otimes (G \otimes I(X,Y)) = (D \otimes G) \otimes I(X,Y)
\]
1D Example

Let's consider a row of pixels in an image:

$I(X, 245)$

Where is the edge?
1D Example: Derivative

Let's consider a row of pixels in an image:

\[ I(X, 245) \]

\[ \frac{\partial I(X, 245)}{\partial x} \]

Where is the edge?
1D Example: Smoothing + Derivative

Let's consider a row of pixels in an image:

\[ I(X, 245) \]

\[ G \]

\[ G \otimes I(X, Y) \]
1D Example: Smoothing + Derivative

Let's consider a row of pixels in an image:

$I(X, 245)$

$G$

$G \otimes I(X, Y)$

$\frac{\partial G \otimes I(X, Y)}{\partial x}$
1D Example: Smoothing + Derivative (efficient)

Let's consider a row of pixels in an image:

\[ I(X, 245) \]

\[ \frac{\partial G}{\partial x} \]

\[ \frac{\partial G}{\partial x} \otimes I(X, Y) \]
Partial Derivatives of Gaussian

\[
\frac{\partial}{\partial x} G_\sigma
\]

\[
\frac{\partial}{\partial y} G_\sigma
\]

Slide Credit: Christopher Rasmussen
Let $I(X, Y)$ be a (digital) image

Let $I_x(X, Y)$ and $I_y(X, Y)$ be estimates of the partial derivatives in the $x$ and $y$ directions, respectively.

Call these estimates $I_x$ and $I_y$ (for short) The vector $[I_x, I_y]$ is the gradient.

The scalar $\sqrt{I_x^2 + I_y^2}$ is the gradient magnitude.
The gradient of an image: \[ \nabla f = \begin{bmatrix} \frac{\partial f}{\partial x}, & \frac{\partial f}{\partial y} \end{bmatrix} \]

\[ \nabla f = \left[ \frac{\partial f}{\partial x}, 0 \right] \]

\[ \nabla f = [0, \frac{\partial f}{\partial y}] \]
The gradient of an image: \[ \nabla f = \left[ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right] \]

\[ \nabla f = \left[ \frac{\partial f}{\partial x}, 0 \right] \]

\[ \nabla f = \left[ 0, \frac{\partial f}{\partial y} \right] \]

The gradient points in the direction of most rapid increase of intensity:
The gradient of an image: \( \nabla f = \left[ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right] \)

The gradient points in the direction of most rapid increase of intensity:

The gradient direction is given by: \( \theta = \tan^{-1} \left( \frac{\partial f}{\partial y} / \frac{\partial f}{\partial x} \right) \)

(how is this related to the direction of the edge?)
The edge strength is given by the gradient magnitude:

\[ \| \nabla f \| = \sqrt{\left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2} \]

The gradient direction is given by:

\[ \theta = \tan^{-1} \left( \frac{\partial f}{\partial y} / \frac{\partial f}{\partial x} \right) \]

The gradient of an image:

\[ \nabla f = \left[ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right] \]

The gradient points in the direction of most rapid increase of intensity:

Image **Gradient**

The gradient points in the direction of most rapid increase of intensity:

The **gradient direction** is given by:

\[ \theta = \tan^{-1} \left( \frac{\partial f}{\partial y} / \frac{\partial f}{\partial x} \right) \]

(how is this related to the direction of the edge?)

The edge strength is given by the **gradient magnitude**:

\[ \| \nabla f \| = \sqrt{\left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2} \]
Increased smoothing:

- eliminates noise edges
- makes edges smoother and thicker
- removes fine detail
Sobel Edge Detector

1. Use central differencing to compute gradient image (instead of first forward differencing). This is more accurate.

2. **Threshold** to obtain edges

Original Image | Sobel Gradient | Sobel Edges

Thresholds are brittle, we can do better!