Lecture 6: Sampling (part 2)

Image Credit: https://en.wikibooks.org/wiki/Analog_and_Digital_Conversion/Nyquist_Sampling_Rate
Menu for Today (January 22, 2019)

Topics:

- **Sampling** (continued)
- **Aliasing**
- **Color Filter Arrays**
- **Bayer** patterns

Readings:

- **Today’s** Lecture: Forsyth & Ponce (2nd ed.) 4.5, 4.6
- **Next** Lecture: Forsyth & Ponce (2nd ed.) 4.6, 4.7

Reminders:

- **Assignment 1**: Image Filtering and Hybrid Images due **January 25th**
Today’s “fun” Example: Optical Illusions

Today’s “fun” Example: Nudging

Aerial view of the white stripes at the lake shore drive in Chicago.
Today’s “fun” Example: Anchoring and Ordering

<table>
<thead>
<tr>
<th>Champagne</th>
<th>Price</th>
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<tbody>
<tr>
<td>CH18 NV GREMILLET &quot;Brut Selection&quot; - Champagne</td>
<td>$65</td>
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<td>CH11 NV ERNEST RAPEAUX &quot;Selection Brut&quot; - Champagne</td>
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<tr>
<td>CH12 NV CHAMPAGNE ERNEST RAPEAUX - BRUT - Chardonnay/Pinot Noir/Pinot Meunier</td>
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<td>CH05 NV DRAPPER &quot;Cote d’O&quot; - Champagne</td>
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<td>CH12 NV ERNEST RAPEAUX &quot;Pavillon Cru Brut&quot; - Champagne</td>
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<tr>
<th>Sparkling Wines</th>
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<tr>
<td>CH06 NV IL CORTIGIANO - Prosecco Extra Dry - Veneto</td>
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<td>CH17 NV VALLEMMOSA &quot;Clasico&quot; Semli Seco - Cava</td>
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<td>CH24 NV VUrtU MOISANS &quot;Blanc de Blancs&quot; - Loire Valley</td>
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<td>CH25 NV VALDO - Prosecco Extra Dry - Treviso, Veneto</td>
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<td>CH33 NV VALDO &quot;Origine&quot; Rose - Veneto</td>
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<td>CH08 2012 CHATEAU MONTGIERET Samnur Sec Rose - Cabernet Franc - Loire Valley</td>
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<td>CH04 NV CAVA MAST RESERVA BRUT - Macibero/Xareto/Paella - Cava</td>
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<td>CH14 NV TRIVENTO &quot;Brut Nature&quot; - Mendoza</td>
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<td>CH21 2015 CAMASELLA - Gloria - Veneto</td>
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<td>CH01 NV VALDO &quot;ORO PURO&quot; Prosecco Superiore - Veneto</td>
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<td>CH40 NV MAISON DARRAGON - AOC Veuve Clicquot - Loire Valley</td>
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<td>CH09 NV LOU MIRANDA ESTATE LEONE - Sparkling Shira - Barossa Valley</td>
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<td>PO03 2014 CASAL MENDES Rose - Bagie - Portugal</td>
<td>$30</td>
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<tr>
<td>RH09 2014 LA VIE EN ROSE - Cissault - Langudoc</td>
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<td>RH08 2015 LES EMBRUNS &quot;La Croix des Silveres&quot; - Sable de Camargue</td>
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<td>RH06 2015 LES MATRICES VIGNOBLES DE ST TROPEZ - Cotes de Provence</td>
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<tr>
<td>RH15 2015 MAISON - COTES DE PROVENCE - Grenache/Chinon/Syrah - Provence</td>
<td>$34</td>
</tr>
<tr>
<td>RH04M 2015 LES MATRICES VIGNOBLES DE LA PRESAQUFIL DE SAINT TROPEZ - Grenache/Mourv</td>
<td>$36</td>
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<th>Sweet Wines</th>
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<tbody>
<tr>
<td>AR33 2015 TRIVENTO &quot;Birds &amp; Bees&quot; - White - Mendoza</td>
<td>$30</td>
</tr>
<tr>
<td>AR34 2015 TRIVENTO &quot;Birds &amp; Bees&quot; - Red - Mendoza</td>
<td>$30</td>
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<tr>
<td>AU05 2015 DEAKIN ESTATE - Moscato - Murray Darling</td>
<td>$30</td>
</tr>
<tr>
<td>AU12 2016 CHALK HIll - Moscato - McLaren Vale</td>
<td>$30</td>
</tr>
<tr>
<td>AU08 NV WESTEND ESTATE &quot;Richland&quot; - Moscato - New South Wales</td>
<td>$30</td>
</tr>
<tr>
<td>AU107 NV WESTEND ESTATE &quot;Richland&quot; - Pink Moscato - New South Wales</td>
<td>$30</td>
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In the **continuous** case, images are functions of two spatial variables, $x$ and $y$.

The **discrete** case is obtained from the continuous case via sampling (i.e. tessellation, quantization).
It is clear that *some* information may be lost when we work on a discrete pixel grid.
Lecture 5: Re-cap

**Question:** When is $I(X, Y)$ an exact characterization of $i(x, y)$?

**Question (modified):** When can we reconstruct $i(x, y)$ exactly from $I(X, Y)$?
Lecture 5: Re-cap

**Case 0:** Suppose $i(x, y) = k$ (with $k$ being one of our gray levels)

$I(X, Y) = k$. Any standard interpolation function would give $i(x, y) = k$ for non-integer $x$ and $y$ (irrespective of how coarse the sampling is)
Case 0: Suppose $i(x, y)$ has a discontinuity not falling precisely at integer $x, y$

We cannot reconstruct $i(x, y)$ exactly because we can never know exactly where the discontinuity lies.

This is impossible!
**Sampling Theory (informal)**

Exact reconstruction requires constraint on the rate at which \( i(x,y) \) can change between samples

- “rate of change” means derivative
- the formal concept is **bandlimited signal**
- “bandlimit” and “constraint on derivative” are linked

Think of music

- bandlimited if it has some maximum **temporal frequency**
- the upper limit of human hearing is about 20 kHz

Think of imaging systems. Resolving power is measured in

- “line pairs per mm” (for a bar test pattern)
- “cycles per mm” (for a sine wave test pattern)

An image is **bandlimited** if it has some maximum **spatial frequency**
Example: A Simple Sine Wave

How do we discretize the signal?
Example: A Simple Sine Wave

How do we discretize the signal?
Example: A Simple Sine Wave

How do we discretize the signal?

How many samples should I take?
Can I take as many samples as I want?
Example: A Simple Sine Wave

How do we discretize the signal?

How many samples should I take?
Can I take as few samples as I want?
**Example: A Simple Sine Wave**

How do we discretize the signal?

Signal can be confused with one at lower frequency
Example: A Simple Sine Wave

How do we discretize the signal?

Signal can be confused with one at lower frequency
Example: A Simple Sine Wave

How do we discretize the signal?

Signal can always be confused with one at higher frequency

Slide Credit: Ioannis (Yannis) Gkioulakeas (CMU)
Undersampling = **Aliasing**

Slide Credit: Ioannis (Yannis) Gkioulkas (CMU)
The challenge to intuition is the fact that music (in the 1D case) and images (in the 2D case) can be represented as linear combinations of individual sine waves of differing frequencies and phases (remember discussion on FFTs).

A fundamental result (Sampling Theorem) is:

For bandlimited signals, if you sample regularly at or above twice the maximum frequency (called the Nyquist rate), then you can reconstruct the original signal exactly.
**Question**: For a bandlimited signal, what if you *oversample* (i.e., sample at greater than the Nyquist rate)
Sampling Theory (informal)

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**Answer**: Nothing bad happens! Samples are redundant and there are wasted bits
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**Answer**: Nothing bad happens! Samples are redundant and there are wasted bits

**Question**: For a bandlimited signal, what if you **undersample** (i.e., sample at less than the Nyquist rate)

**Answer**: Two bad things happen! Things are missing (i.e., things that should be there aren’t). There are artifacts (i.e., things that shouldn’t be there are)
Sampling Theory (informal)

Forsyth & Ponce (2nd ed.) Figure 4.7
Sampling Theory (informal)

Forsyth & Ponce (2nd ed.) Figure 4.12
Reducing Aliasing Artifacts

1. **Oversampling** — sample more than you think you need and average (i.e., area sampling)
Aliasing

aliasing artifacts

anti-aliasing by oversampling

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)
Reducing Aliasing Artifacts

1. **Oversampling** — sample more than you think you need and average (i.e., area sampling)

2. **Smoothing** before sampling. Why?
Aliasing in Photographs

This is also known as “moire”
Imagine a spoked wheel moving to the right (rotating clockwise). Mark wheel with dot so we can see what’s happening.

If camera shutter is only open for a fraction of a frame time (frame time = 1/30 sec. for video, 1/24 sec. for film):

Without dot, wheel appears to be rotating slowly backwards! (counterclockwise)
Temporal Aliasing

Wagon wheel effect
Temporal Aliasing
Sampling Theory (informal)

Sometimes **undersampling** is unavoidable, and there is a trade-off between “things missing” and “artifacts.”

— **Medical imaging**: usually try to maximize information content, tolerate some artifacts

— **Computer graphics**: usually try to minimize artifacts, tolerate some information missing
Review: Continuous Case

— Images also can be considered a function of time. Then, we write $i(x, y, t)$ where $x$ and $y$ are spatial variable and $t$ is a **temporal variable**

— To make the dependence of brightness on wavelength explicit, we can instead write $i(x, y, t, \lambda)$ where $x$, $y$ and $t$ are as above and where $\lambda$ is a **spectral variable**

— More commonly, we think of “color” already as discrete and write

$$i_R(x, y)$$
$$i_G(x, y)$$
$$i_B(x, y)$$

for specific colour channels, R, G and B
Color is an Artifact of Human Perception

“Color” is not an objective physical property of light (electromagnetic radiation). Instead, light is characterized by its wavelength.

What we call “color” is how we subjectively perceive a very small range of these wavelengths.
Color Filter Arrays (CFA)
Color Filter Arrays (CFA)
Color Filters

Two design choices:
— What spectral sensitivity functions $f(\lambda)$ to use for each color filter?
— How to spatially arrange (“mosaic”) different color filters?
Color Filters

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— How to spatially arrange ("mosaic") different color filters?

Generally do not match human sensitivity

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)
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Color Filters

Two design choices:
— What spectral sensitivity functions $f(\lambda)$ to use for each color filter?
— How to spatially arrange ("mosaic") different color filters?

Why more green pixels?

Generally do not match human sensitivity

Canon 50D

$f(\lambda)$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)
Different Color Filter Arrays (CFAs)

Finding the “best” CFA mosaic is an active research area.

How would you go about designing your own CFA? What criteria would you consider?

CYGM
Canon IXUS, Powershot

RGBE
Sony Cyber-shot

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)
Many **Different Spectral Sensitivity** Functions

Each camera has its more or less unique, and most of the time secret, SSF

Same scene captured using 3 different cameras with identical settings

*Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)*
RAW Bayer Image

After all of this, what does an image look like?

- Kind of disappointing
- We call this the RAW image

lots of noise

mosaicking artifacts

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)
CFA Demosicing

Produce full RGB image from mosaiced sensor output

Any ideas on how to do this?
CFA Demosicing

Produce full RGB image from mosaiced sensor output

Interpolate from neighbors:
- Bilinear interpolation (needs 4 neighbors)
- Bicubic interpolation (needs more neighbors, may overblur)
- Edge-aware interpolation

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)
Demosaicing by Bilinear Interpolation

**Bilinear** interpolation: Simply average your 4 neighbors.

\[ G_? = \frac{G_1 + G_2 + G_3 + G_4}{4} \]

Neighborhood changes for different channels:

*Slide Credit:* Ioannis (Yannis) Gkioulekas (CMU)
The sequence of image processing operations applied by the camera’s image signal processor (ISP) to convert a RAW image into a “conventional” image.

- Analog front-end
- RAW image (mosaiced, linear, 12-bit)
- White balance
- CFA demosaicing
- Color transforms
- Tone reproduction
- Compression
- Final RGB image (non-linear, 8-bit)

**Slide Credit:** Ioannis (Yannis) Gkioulekas (CMU)
In the continuous case, images are functions of two spatial variables, $x$ and $y$.

The discrete case is obtained from the continuous case via sampling (i.e. tessellation, quantization).

If a signal is **bandlimited** then it is possible to design a sampling strategy such that the sampled signal captures the underlying continuous signal exactly.

Adequate sampling may not always be practical. In such cases there is a trade-off between “things missing” and “artifacts”.

— Different applications make the trade-off differently