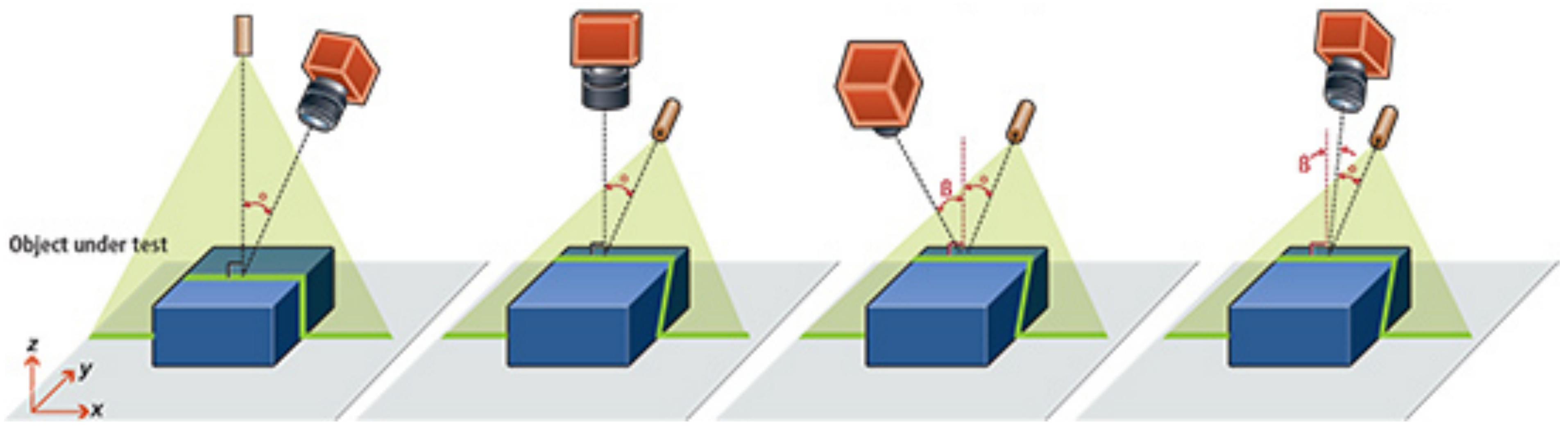
THE UNIVERSITY OF BRITISH COLUMBIA

CPSC 425: Computer Vision



Lecture 3: Image Formation (continued)

(unless otherwise stated slides are taken or adopted from **Bob Woodham, Jim Little** and **Fred Tung**)

Menu for Today (**January 10, 2019**)

Topics: (continue) Image Formation

— Human **eye** (as camera)

Redings:

— **Today's** Lecture: Forsyth & Ponce (2nd ed.) 4.1, 4.5

— **Next** Lecture: none

Reminders:

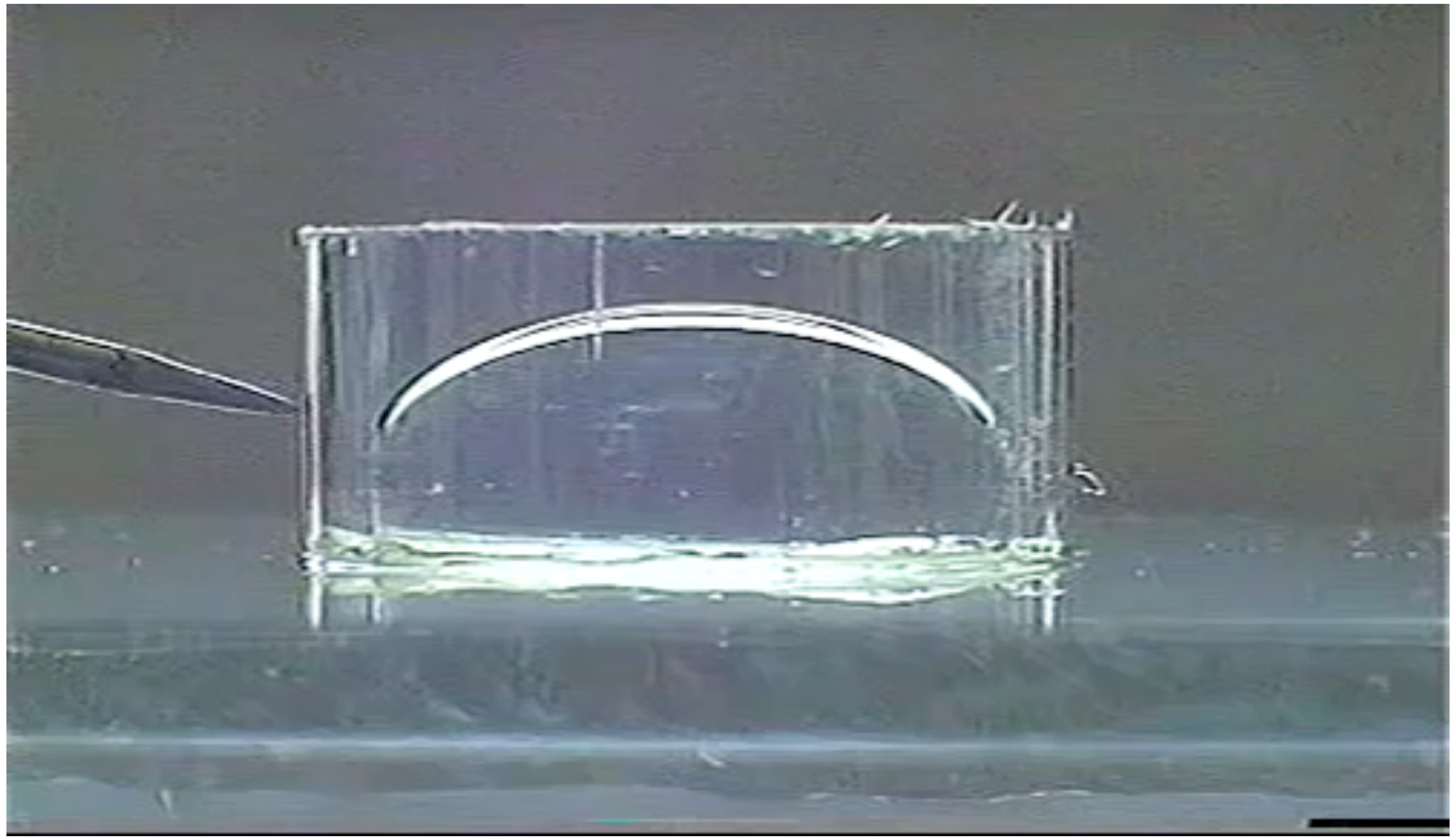
— Complete **Assignment 0** (ungraded) by Friday, **January 11**

— **Assignment 1:** Image Filtering and Hybrid Images (will be out **January 11**)

Today’s “**fun**” Example:

Developed by the French company **Varioptic**, the lenses consist of an oil-based and a water-based fluid sandwiched between glass discs. Electric charge causes the boundary between oil and water to change shape, altering the lens geometry and therefore the lens focal length

The intended applications are: **auto-focus** and **image stabilization**. No moving parts. Fast response. Minimal power consumption.



Video Source: https://www.youtube.com/watch?v=2c6lCdDFOY8

Today's "**fun**" Example:

Electrostatic field between the column of water and the electron (other side of power supply attached to the pipe) — see full video for complete explanation

Video Source: https://www.youtube.com/watch?v=NjLJ77IuBdM

Today's "**fun**" Example:

As one example, in 2010, **Cognex** signed a licence agreement with Varioptic to add auto-focus capability to it DataMan line of industrial ID readers (press release May 29, 2012)

Video Source: https://www.youtube.com/watch?v=EU8LXxip1NM

Lecture 2: Re-capWe take a “physics-based” approach to image formation
— Treat camera as an instrument that takes measurements of the 3D world

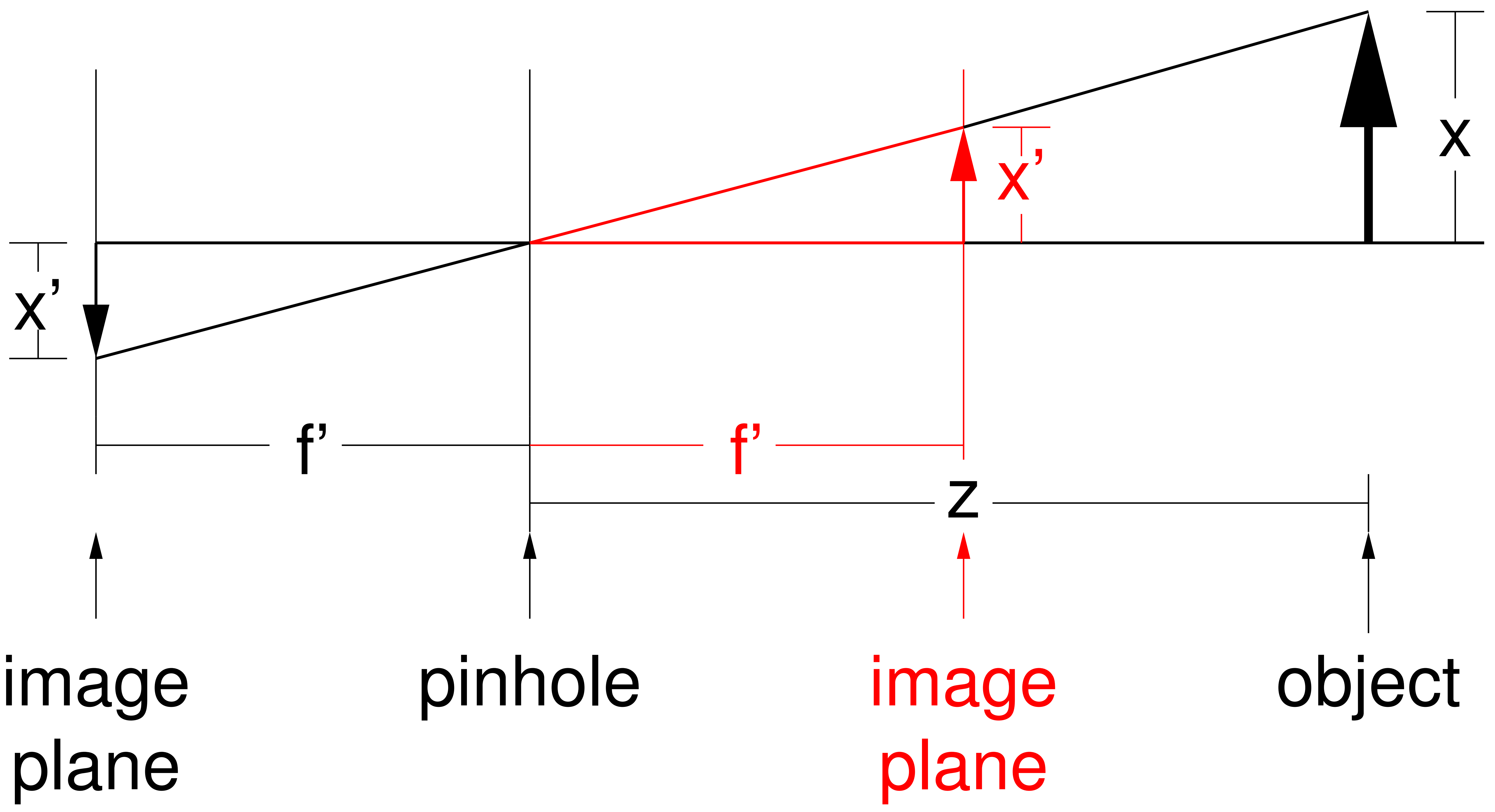
Basic abstraction is the **pinhole camera**

Lenses overcome limitations of the pinhole model while trying to preserve it as a useful abstraction

When **maximum accuracy** required, it is necessary to model additional details of each particular camera (and camera setting)
— Aside: This is called camera calibration

Lecture 2: Re-cap Pinhole Camera Abstraction

Pinhole Camera Abstraction



Lecture 2: Re-cap Projection

3D object point $P = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ projects to 2D image point $P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$ where

Perspective	$x' = f' \frac{x}{z}$ $y' = f' \frac{y}{z}$	
Weak Perspective	$x' = m\,x$ $y' = m\,y$	$m = \frac{f'}{z_0}$
Orthographic	$x' = x$ $y' = y$	

Lecture 2: Re-cap

— If pinhole is **too big** then many directions are averaged, blurring the image

— If pinhole is **too small** then diffraction becomes a factor, also blurring the image

— Generally, pinhole cameras are **dark**, because only a very small set of rays from a particular scene point hits the image plane

— Pinhole cameras are **slow**, because only a very small amount of light from a particular scene point hits the image plane per unit time

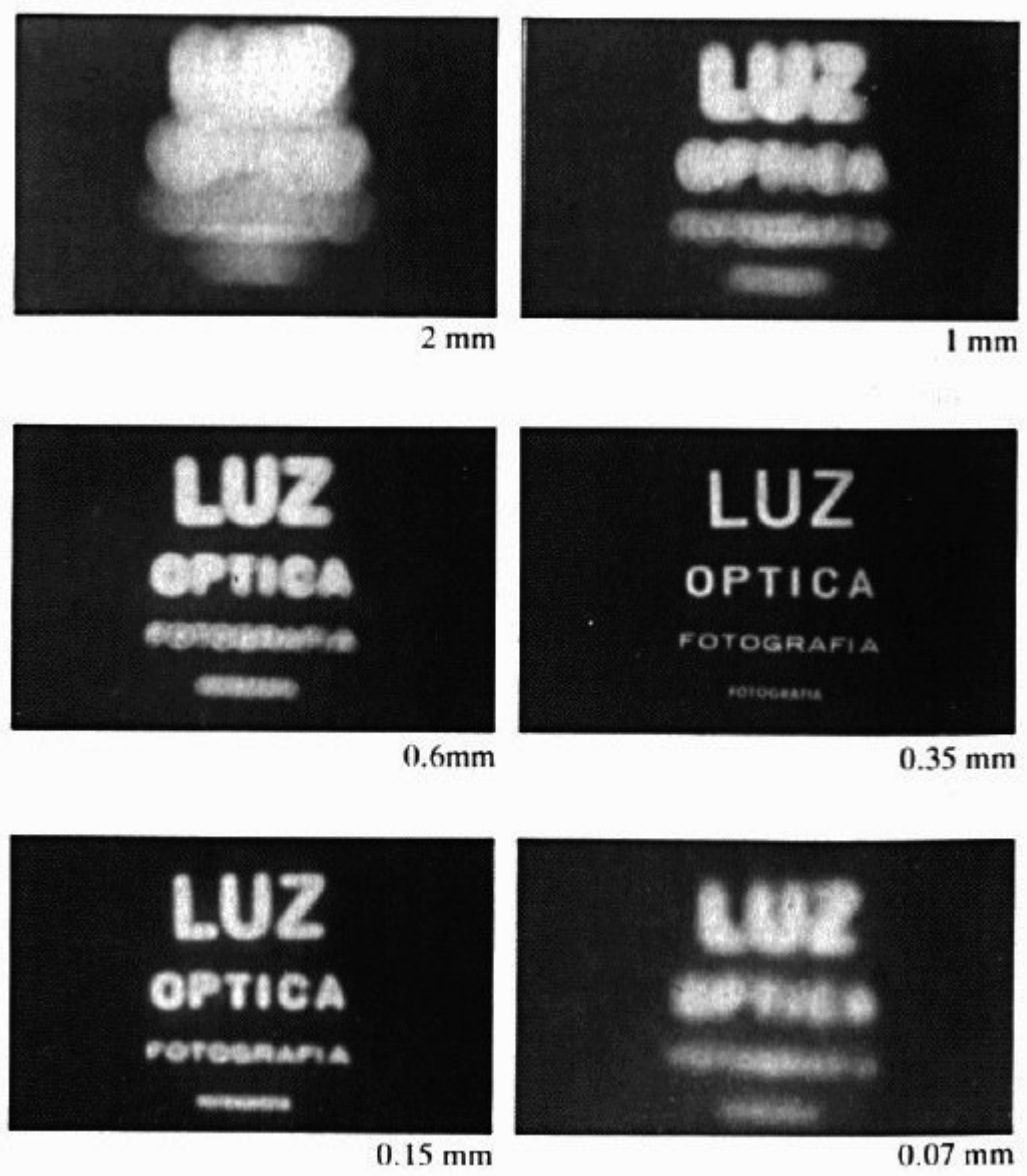
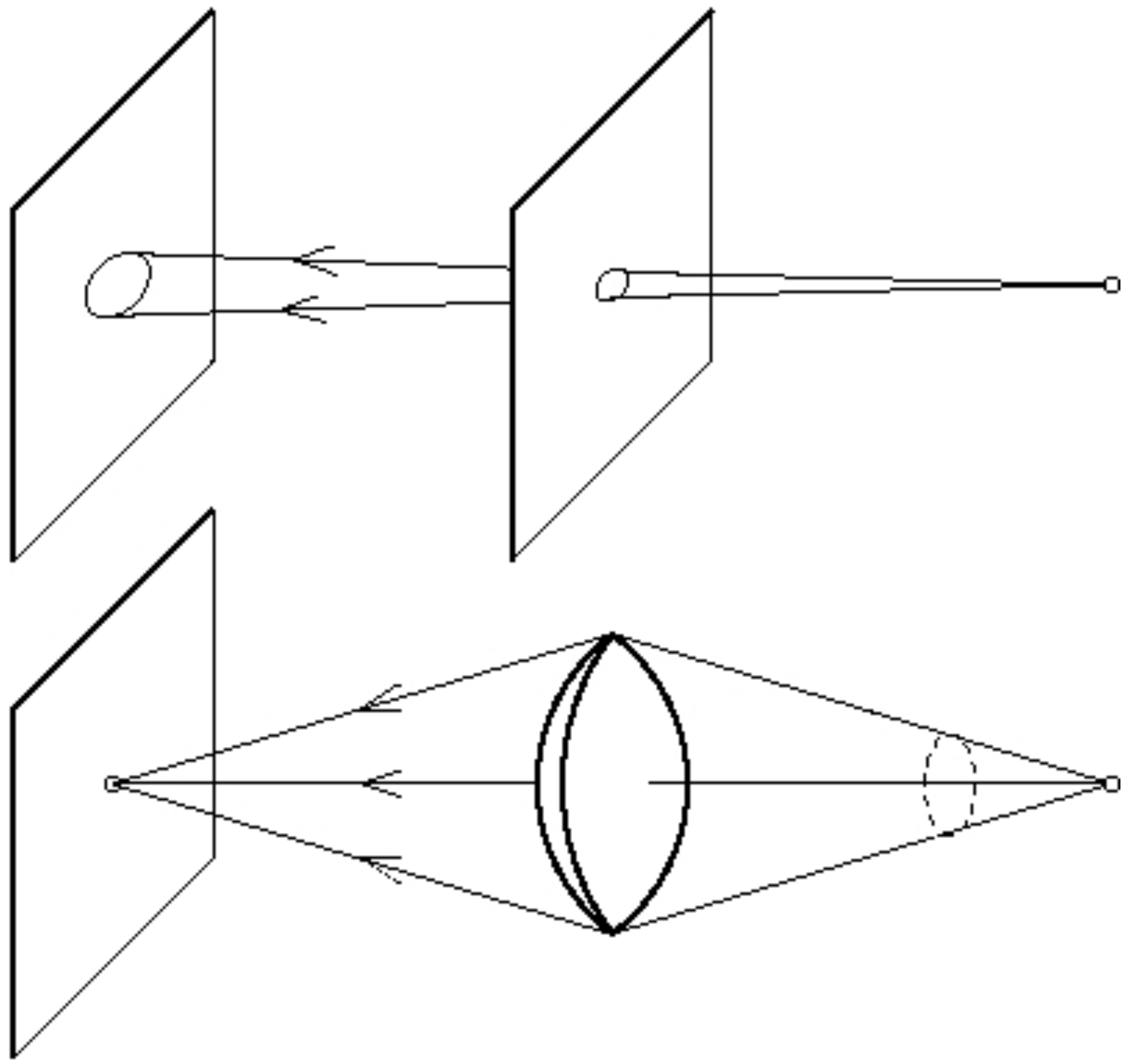


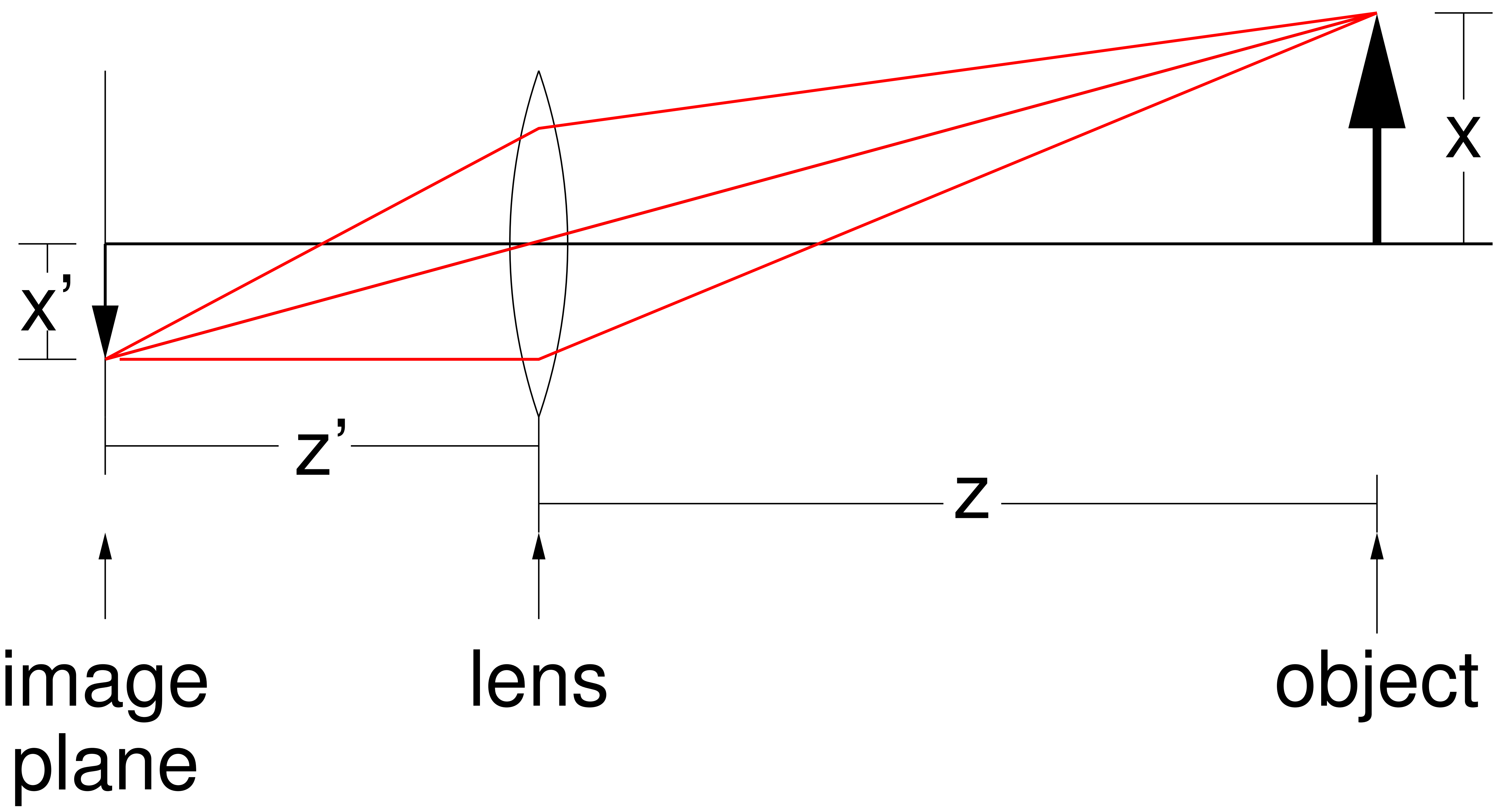
Image Credit: Credit: E. Hecht. “Optics,” Addison-Wesley, 1987

Lecture 2: Re-cap Lenses

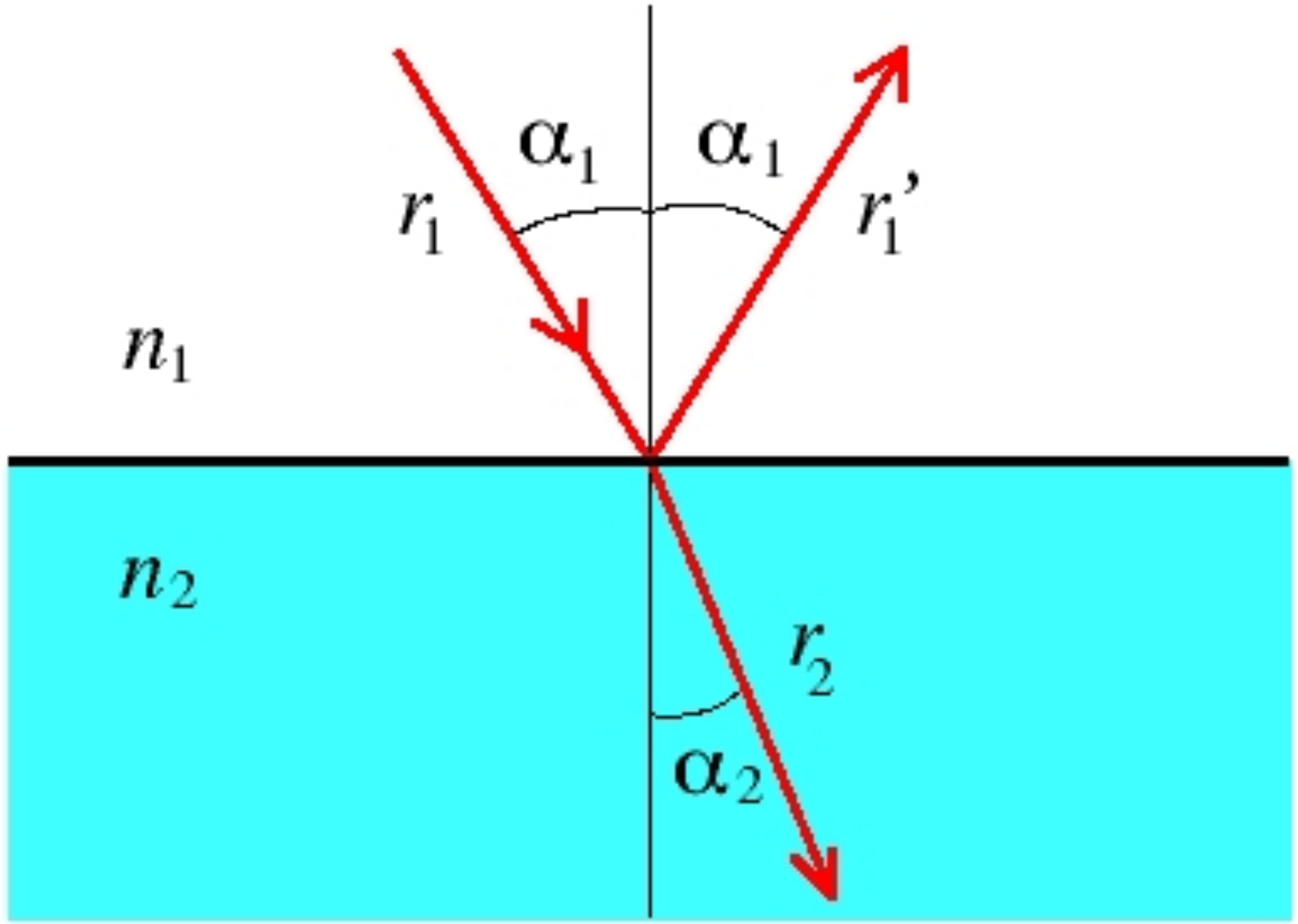
The role of a lens is to **capture more light** while preserving, as much as possible, the abstraction of an ideal pinhole camera.



Lecture 2: Re-cap Lenses

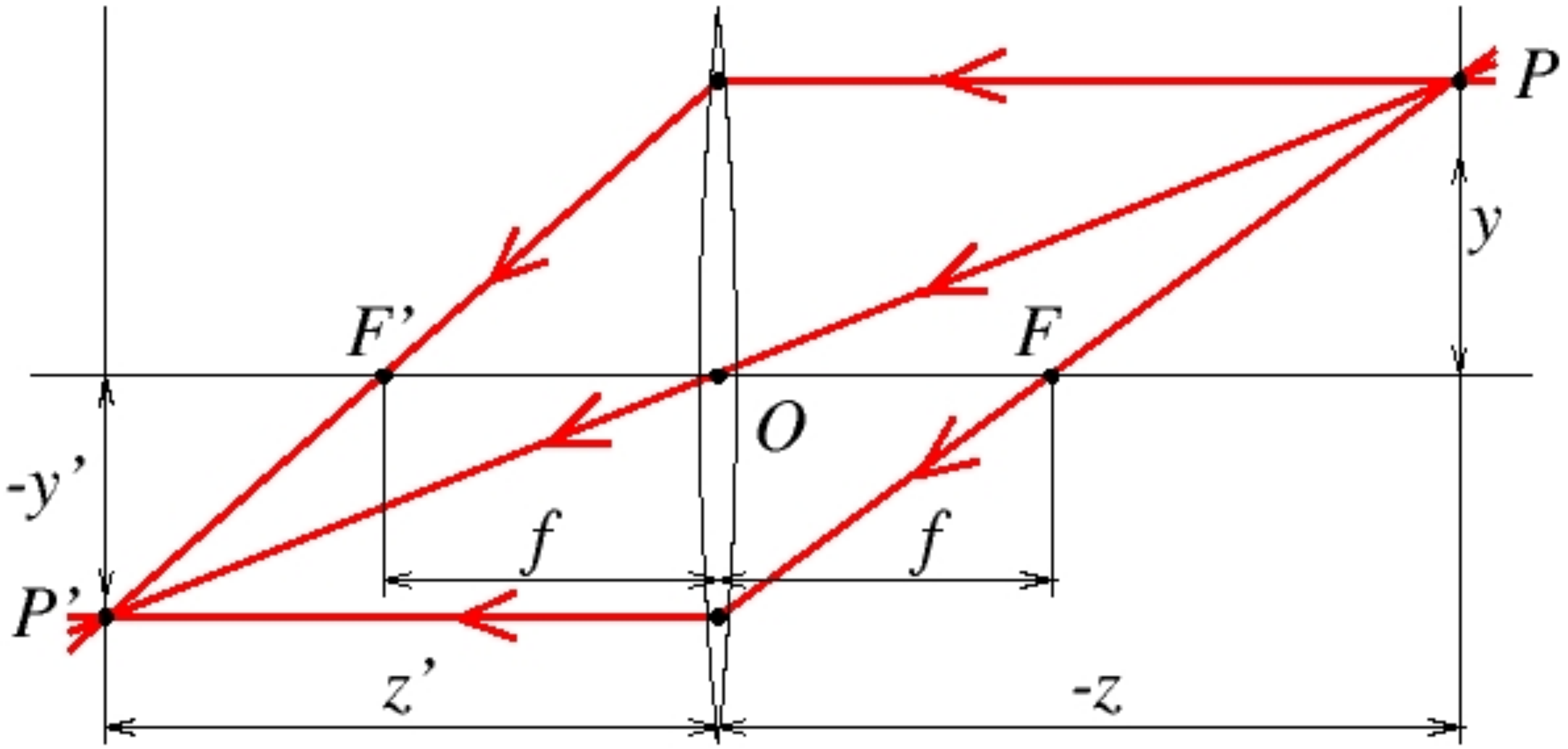


Lecture 2: Re-cap Snell's Law



$$\boxed{n_1 \sin \alpha_1 = n_2 \sin \alpha_2}$$

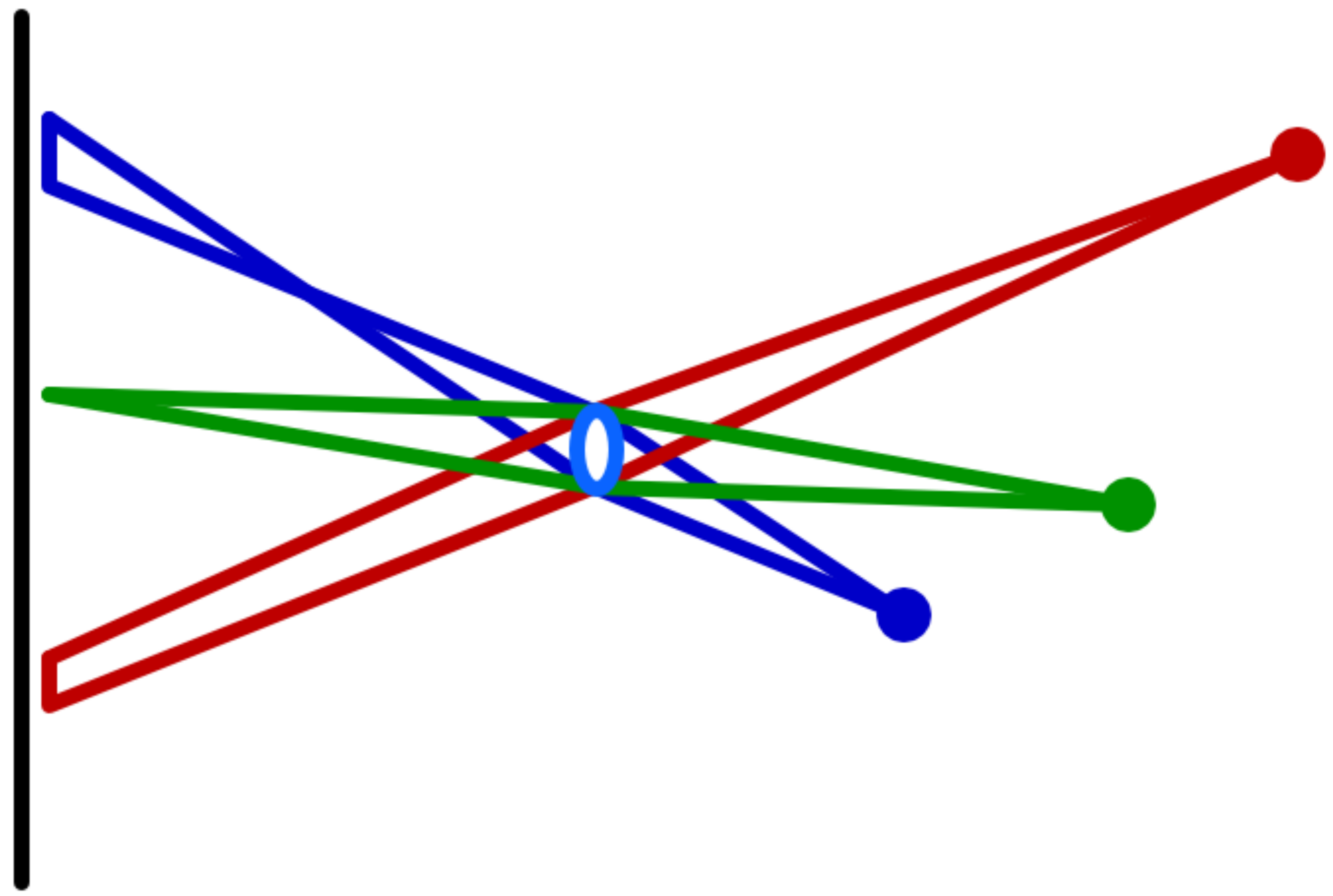
Lecture 2: Re-cap Thin Lens Equation



Forsyth & Ponce (1st ed.) Figure 1.9

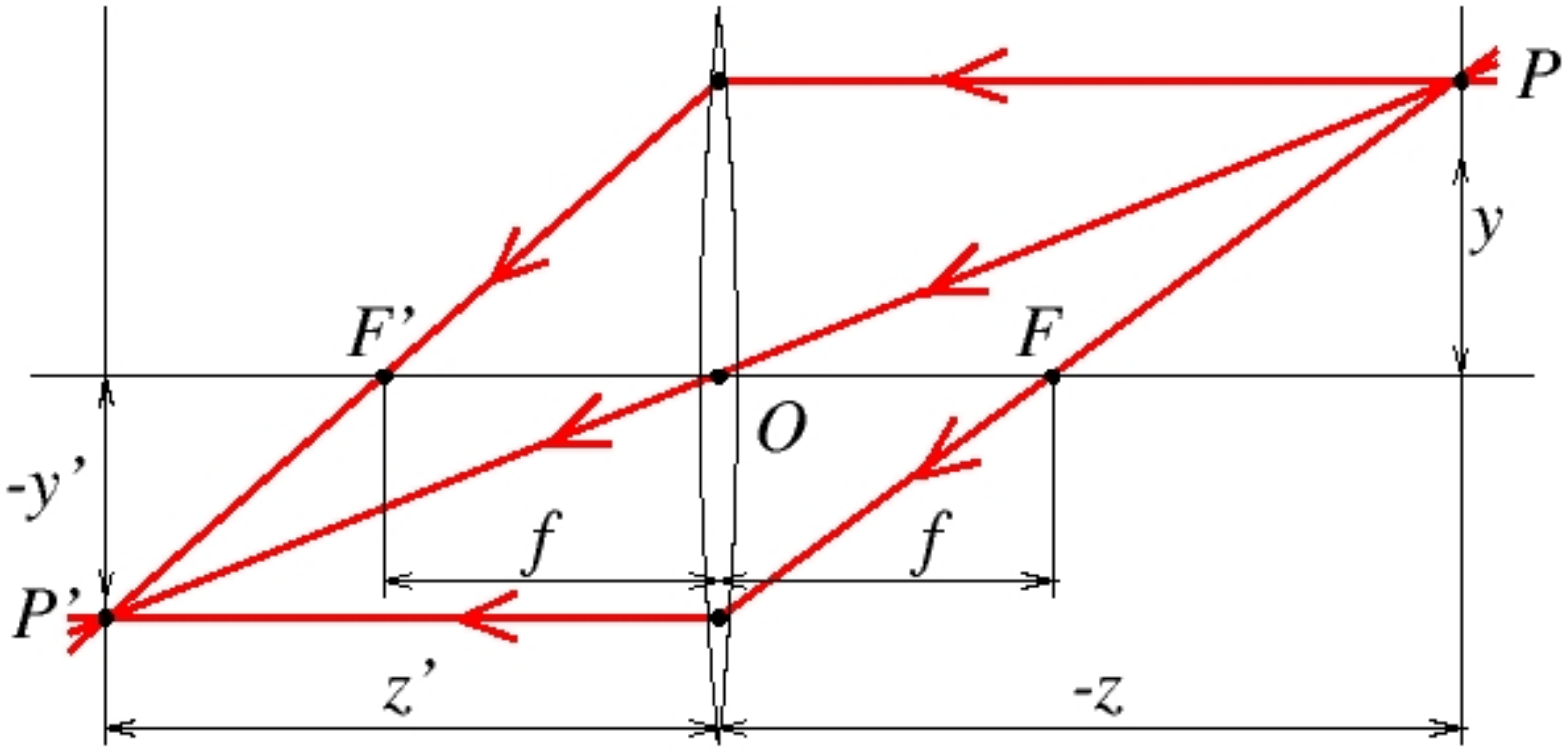
$$\boxed{\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}}$$

Lecture 2: Re-cap



* image credit: https://catlikecoding.com/unity/tutorials/advanced-rendering/depth-of-field/circle-of-confusion/lens-camera.png

Lecture 2: Re-cap Thin Lens Equation

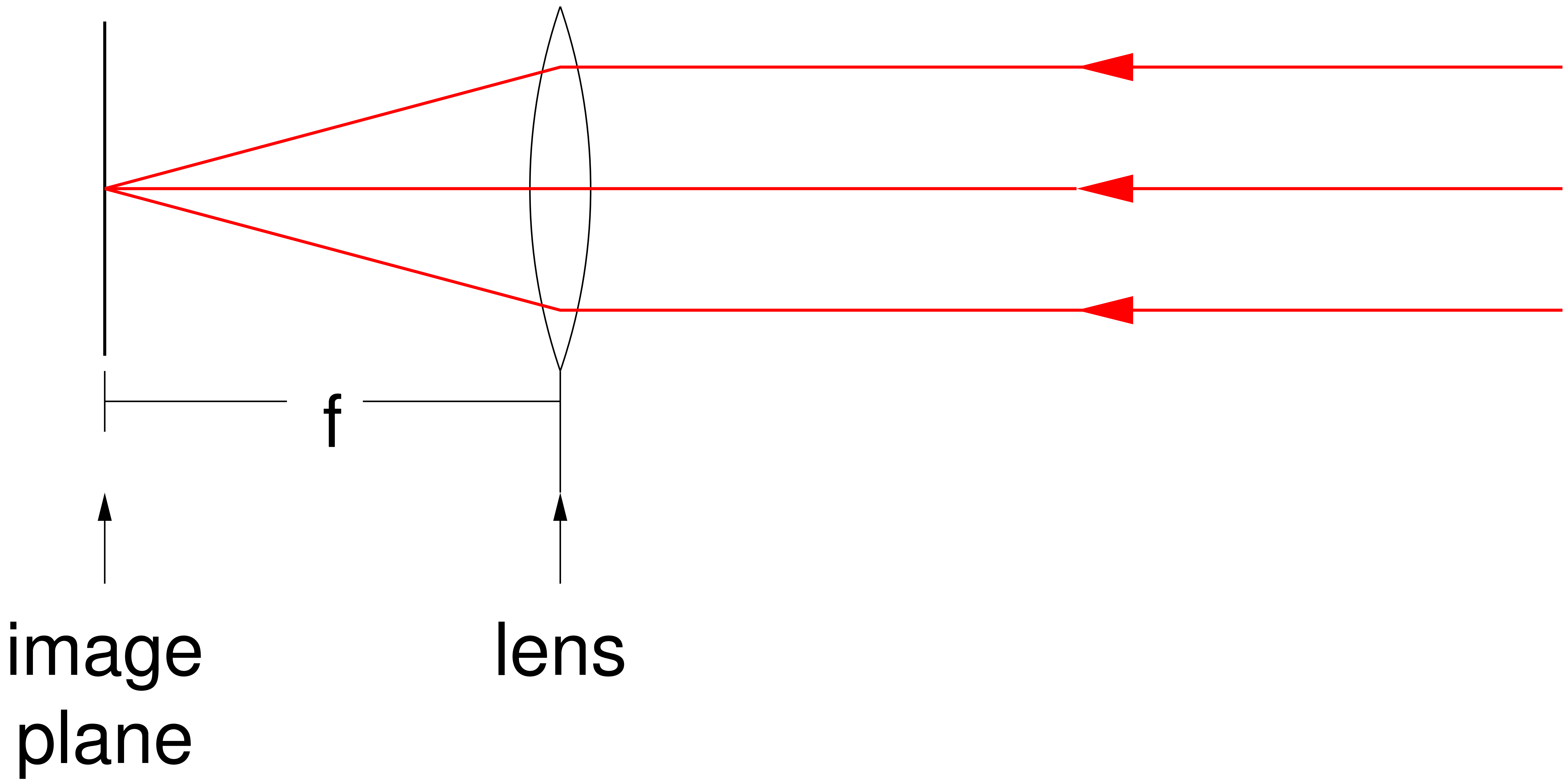


Forsyth & Ponce (1st ed.) Figure 1.9

$$\boxed{\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}}$$

Lecture 2: Re-cap

Another way of looking at the **focal length** of a lens. The incoming rays, parallel to the optical axis, **converge to a single point a distance f behind the lens**. This is where we want to place the image plane.



Lecture 2: Re-capChromatic **aberration**

— Index of refraction depends on wavelength, λ, of light

— Light of different colours follows different paths

— Therefore, not all colours can be in equal focus

Scattering at the lens surface

— Some light is reflected at each lens surface

There are other **geometric phenomena/distortions**

— pincushion distortion

— barrel distortion

— etc

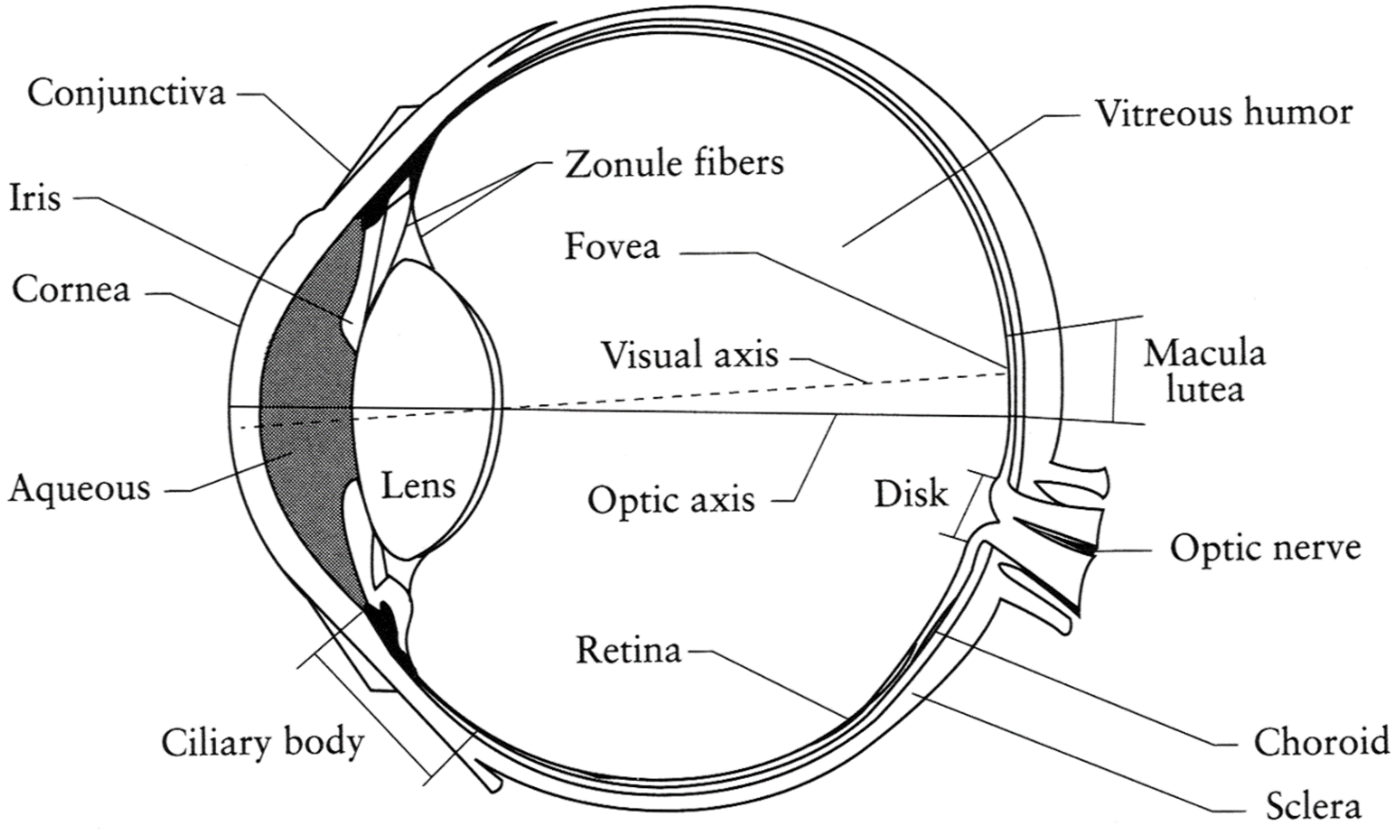
Human Eye

— The eye has an **iris** (like a camera)

— **Focusing** is done by changing shape of lens

— When the eye is properly focused, light from an object outside the eye is imaged on the **retina**

— The retina contains light receptors called **rods** and **cones**



pupil = pinhole / aperture

retina = film / digital sensor

Slide adopted from: Steve Seitz

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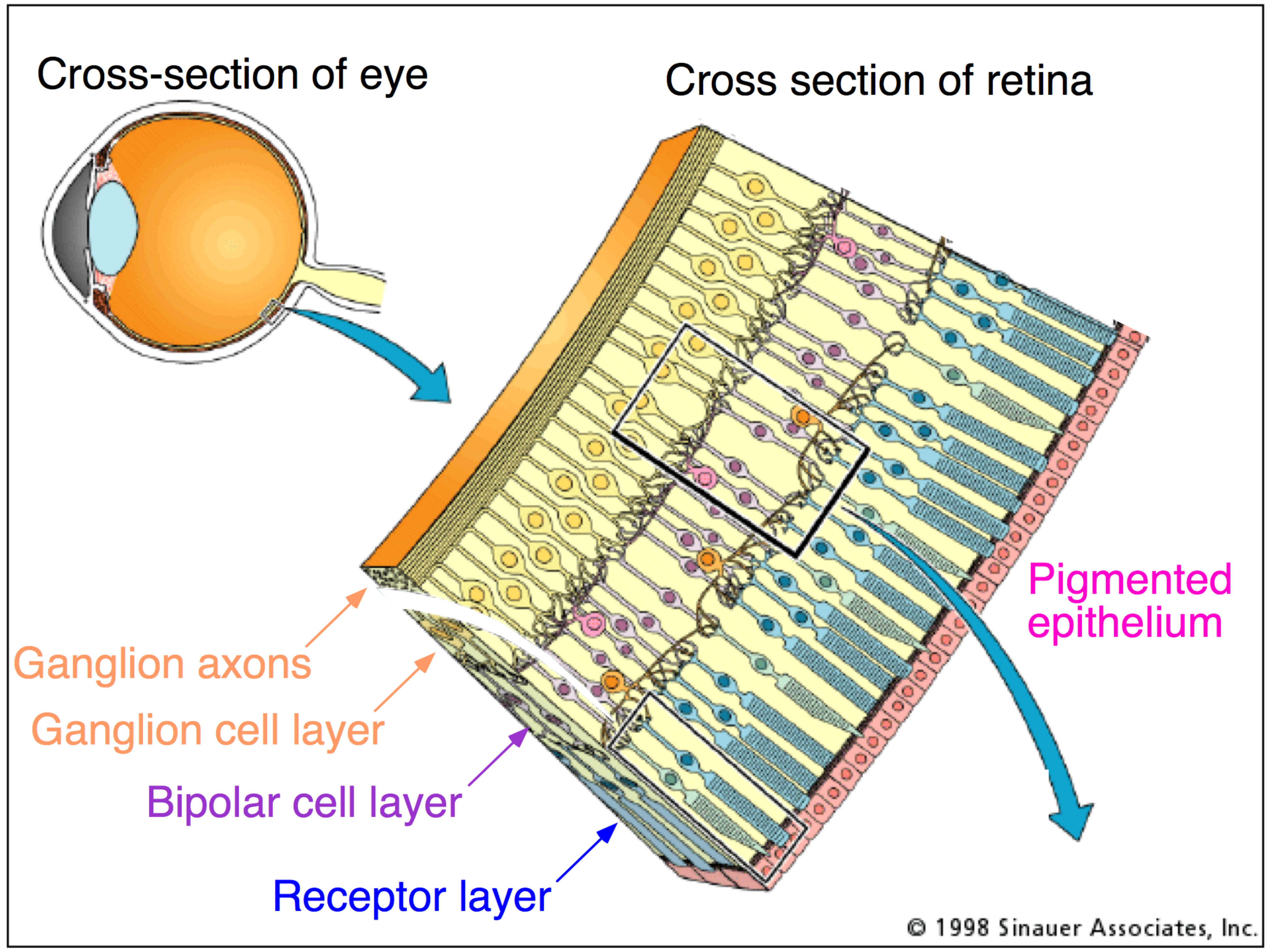


pupil = pinhole / aperture

retina = film / digital sensor

Slide adopted from: Steve Seitz

Two-types of **Light Sensitive Receptors**

Rods

75-150 million rod-shaped receptors

not involved in color vision, gray-scale vision only

operate at night

highly sensitive, can responding to a single photon

yield relatively poor spatial detail

Cones

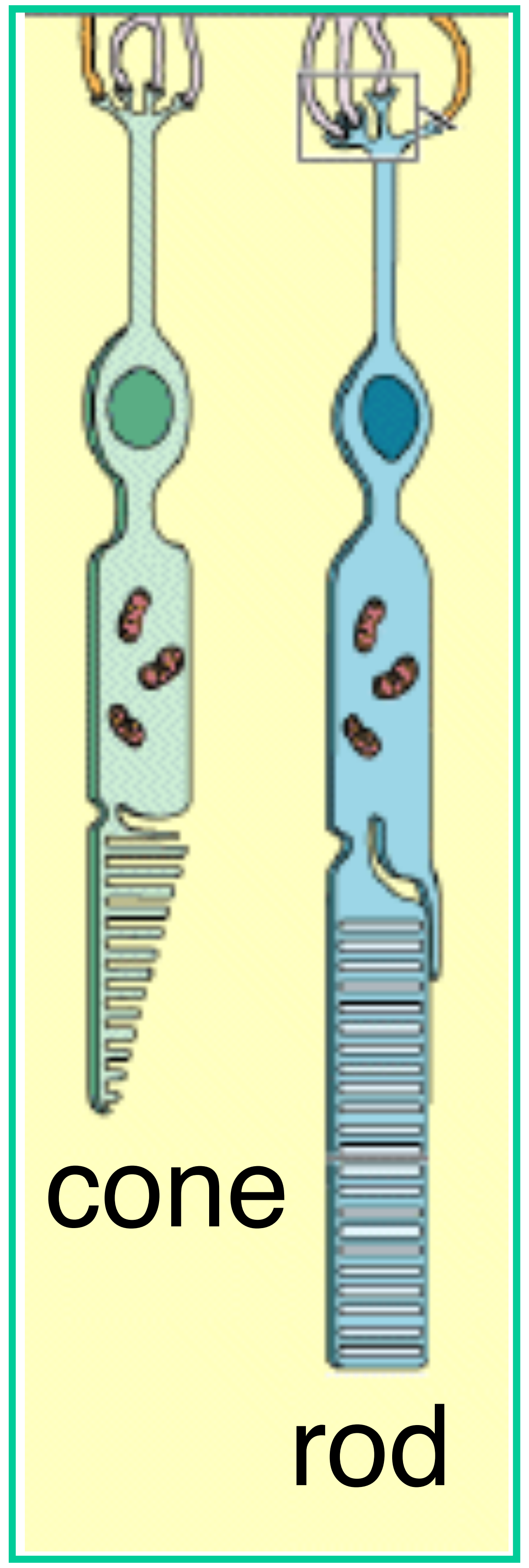
6-7 million cone-shaped receptors

color vision

operate in high light

less sensitive

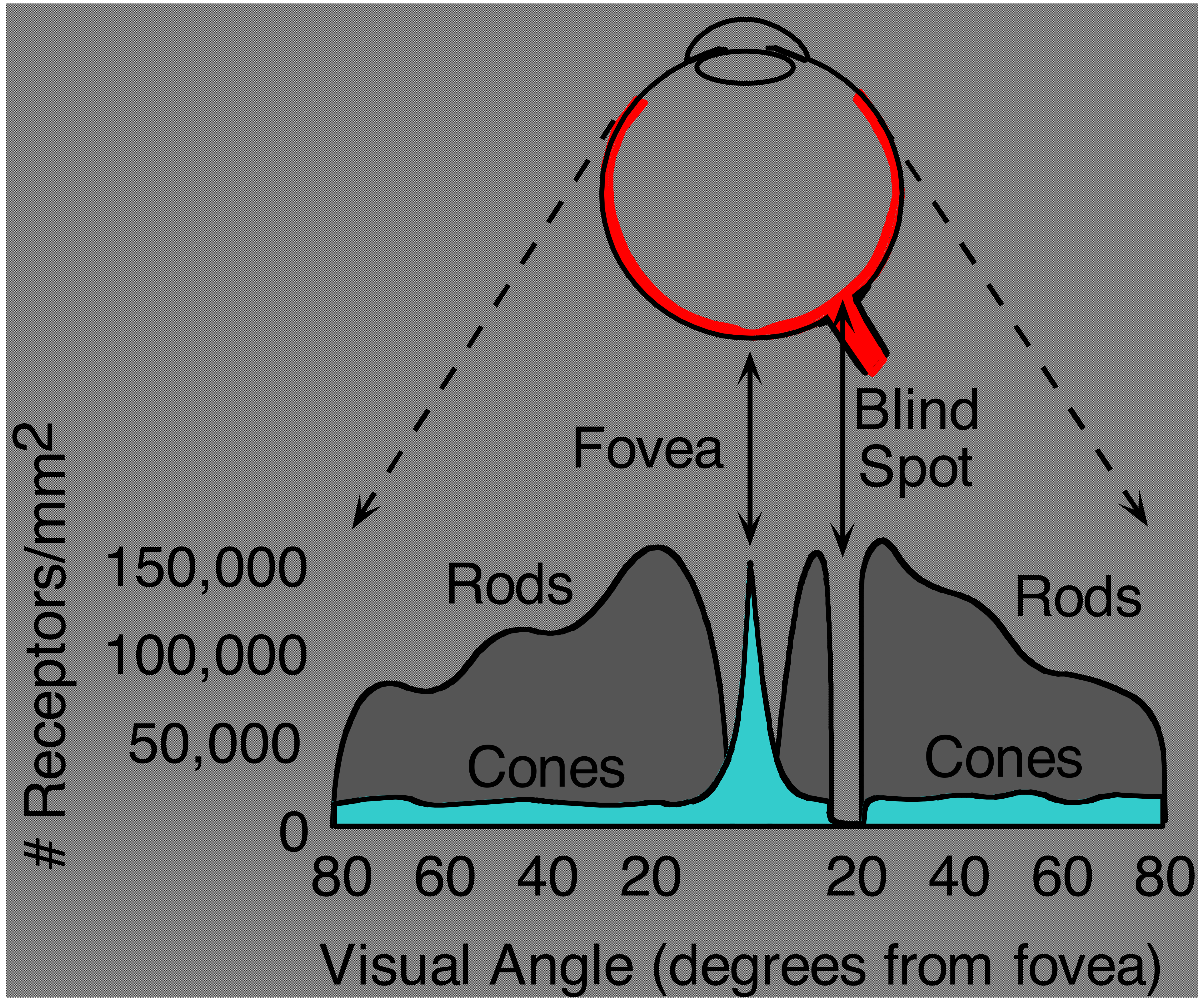
yield higher resolution



Slide adopted from: James Hays

Human Eye

Density of rods and cones



Slide adopted from: James Hays

Lecture **Summary**

— We discussed a “physics-based” approach to image formation. Basic abstraction is the **pinhole camera**.

— **Lenses overcome limitations** of the pinhole model while trying to preserve it as a useful abstraction

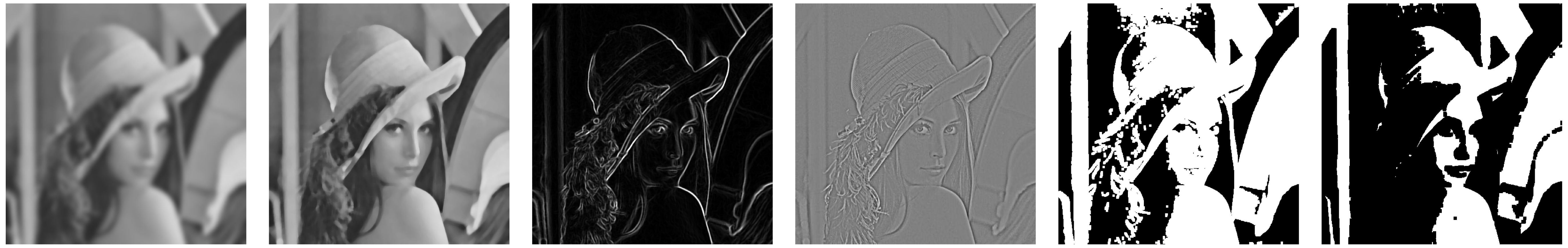
— Projection equations: **perspective**, weak perspective, orthographic

— Thin lens equation

— Some “aberrations and **distortions**” persist (e.g. spherical aberration, vignetting)

— The **human eye** functions much like a camera

CPSC 425: Computer Vision



Lecture 3: Image Filtering

(unless otherwise stated slides are taken or adopted from **Bob Woodham, Jim Little** and **Fred Tung**)

Menu for Today (**January 10, 2019**)

Topics: Image Filtering (also topic for next week)

— **Image** as a **function**
— **Linear** filters
— **Correlation** / **Convolution**
— Filter **examples**: Box, Gaussian

Redings:

— **Today's** Lecture: Forsyth & Ponce (2nd ed.) 4.1, 4.5
— **Next** Lecture: none

Reminders:

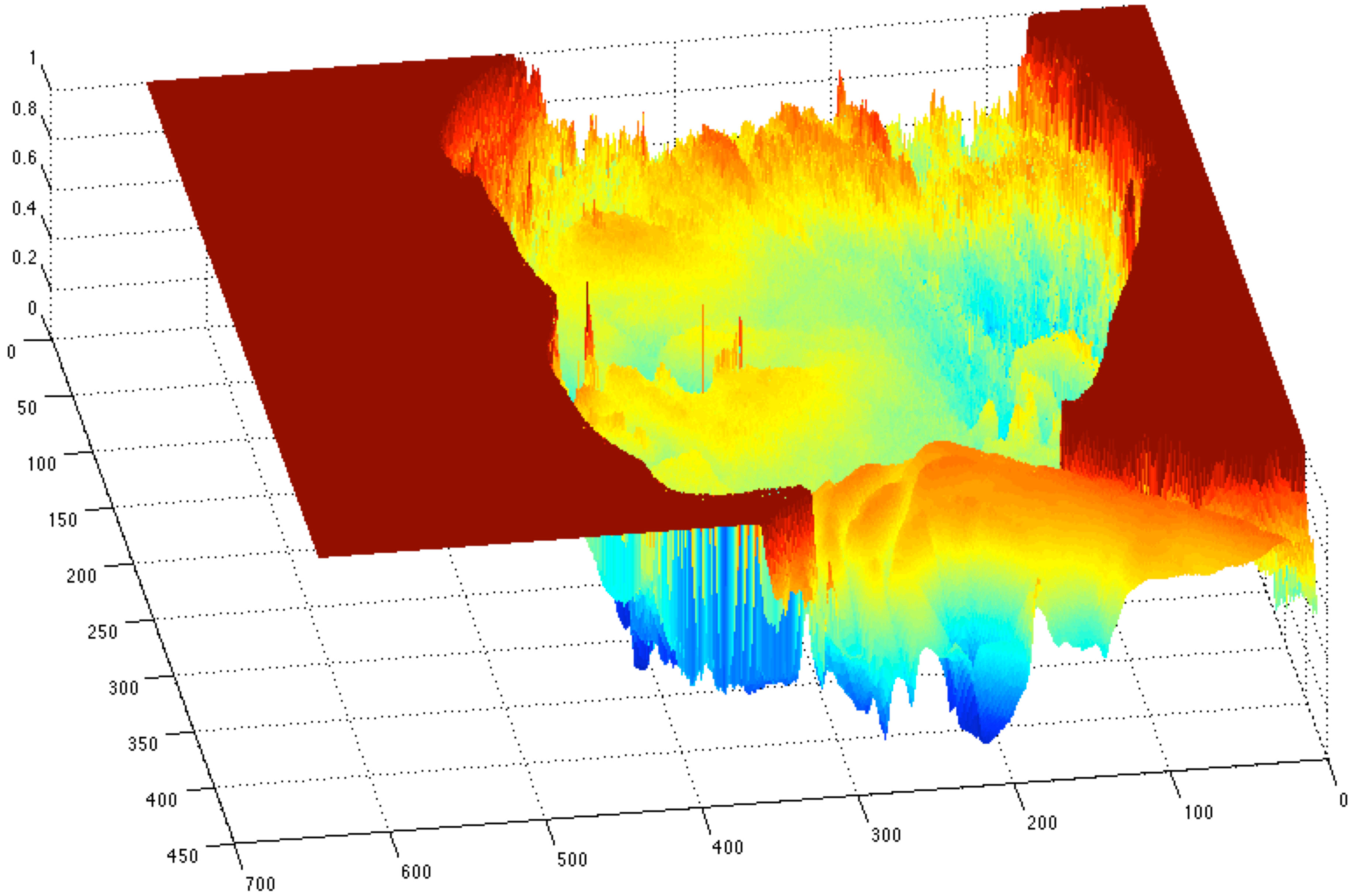
— Complete **Assignment 0** (ungraded) by Friday, **January 11**
— **Assignment 1:** Image Filtering and Hybrid Images (will be out **January 11**)

Image as a **2D Function**

A (grayscale) image is a 2D function

grayscale image

$I(X, Y)$



What is the **range** of the image function?

$$I(X, Y) \in [0, 255] \in \mathbb{Z}$$

domain: $(X, Y) \in ([1, width], [1, hight])$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Adding two Images

Since images are functions, we can perform operations on them, e.g., **average**

$I(X,Y)$

$G(X,Y)$

$$\frac{I(X,Y)}{2} + \frac{G(X,Y)}{2}$$

Adding two Images

$$a = \frac{I(X,Y)}{2} + \frac{G(X,Y)}{2}$$

$$b = \frac{I(X,Y) + G(X,Y)}{2}$$

Question:

$a = b$

$a > b$

$a < b$

Adding two Images

Red pixel in camera man image = 98

Red pixel in moon image = 200

$$\frac{98}{2} + \frac{200}{2} = 49 + 100 = 149$$

$$\frac{98 + 200}{2} = \frac{\lfloor 298 \rfloor}{2} = \frac{255}{2} = 127$$

Question:

$a = b$

$\boxed{a > b}$

$a < b$

Adding two Images

$$a = \frac{I(X,Y)}{2} + \frac{G(X,Y)}{2}$$

$$b = \frac{I(X,Y) + G(X,Y)}{2}$$

Question:

$$a = b$$

$$a > b$$

$$b < a$$

Adding two Images

It is often convenient to convert images to **doubles** when doing processing

In Python

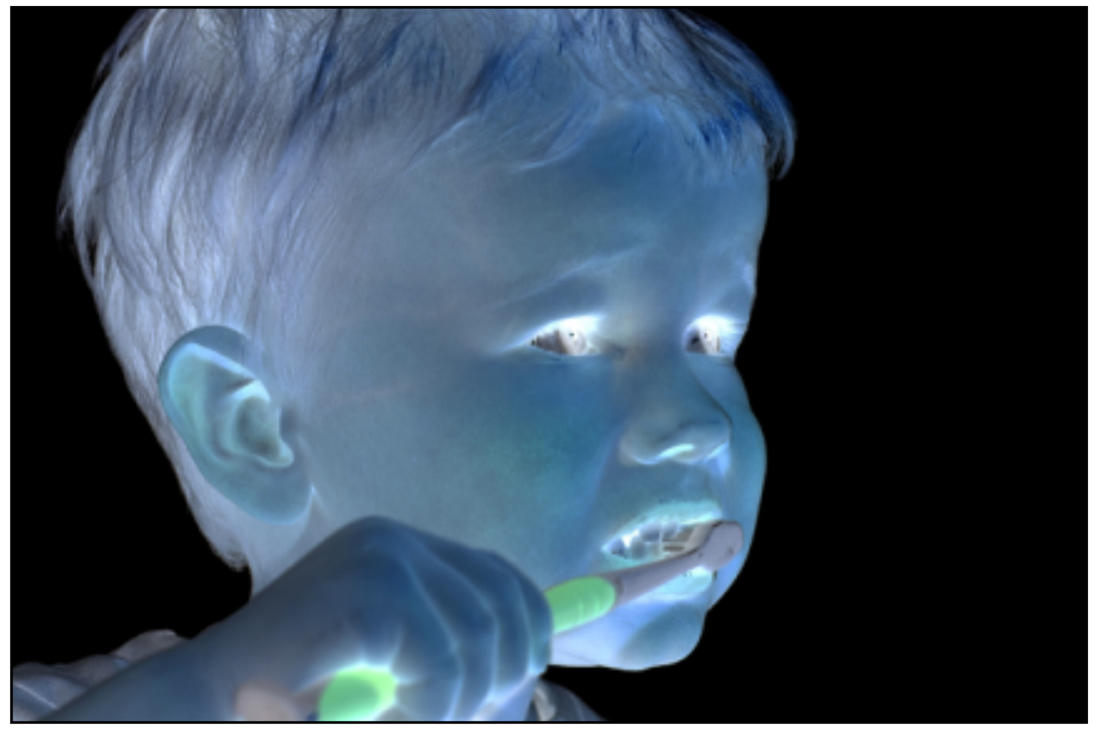
```
from PIL import Image
img = Image.open('cameraman.png')
import numpy as np
imgArr = np.asfarray(img)

# Or do this
import matplotlib.pyplot as plt
camera = plt.imread('cameraman.png');
```

What types of **transformations** can we do?

$I(X, Y)$

Filtering

$I'(X, Y)$

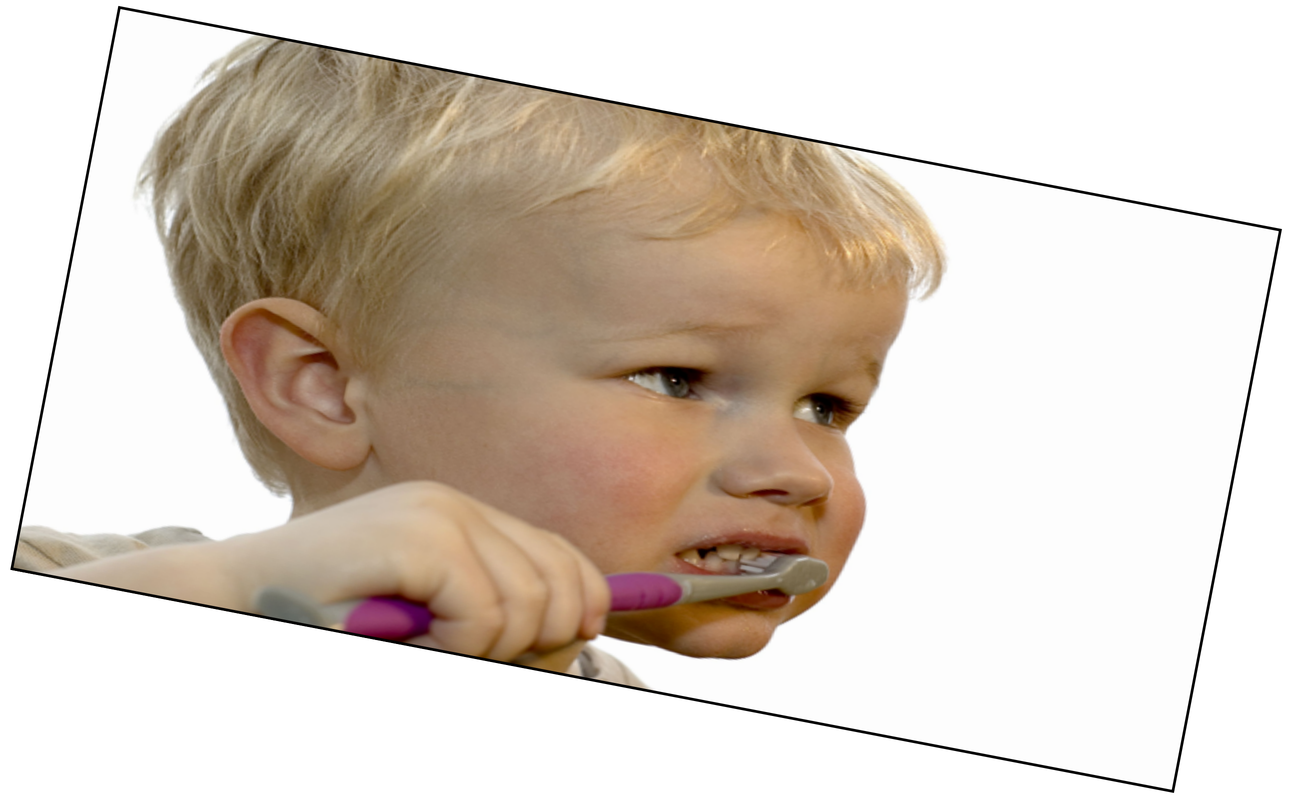


changes range of image function

$I(X, Y)$

Warping

$I'(X, Y)$

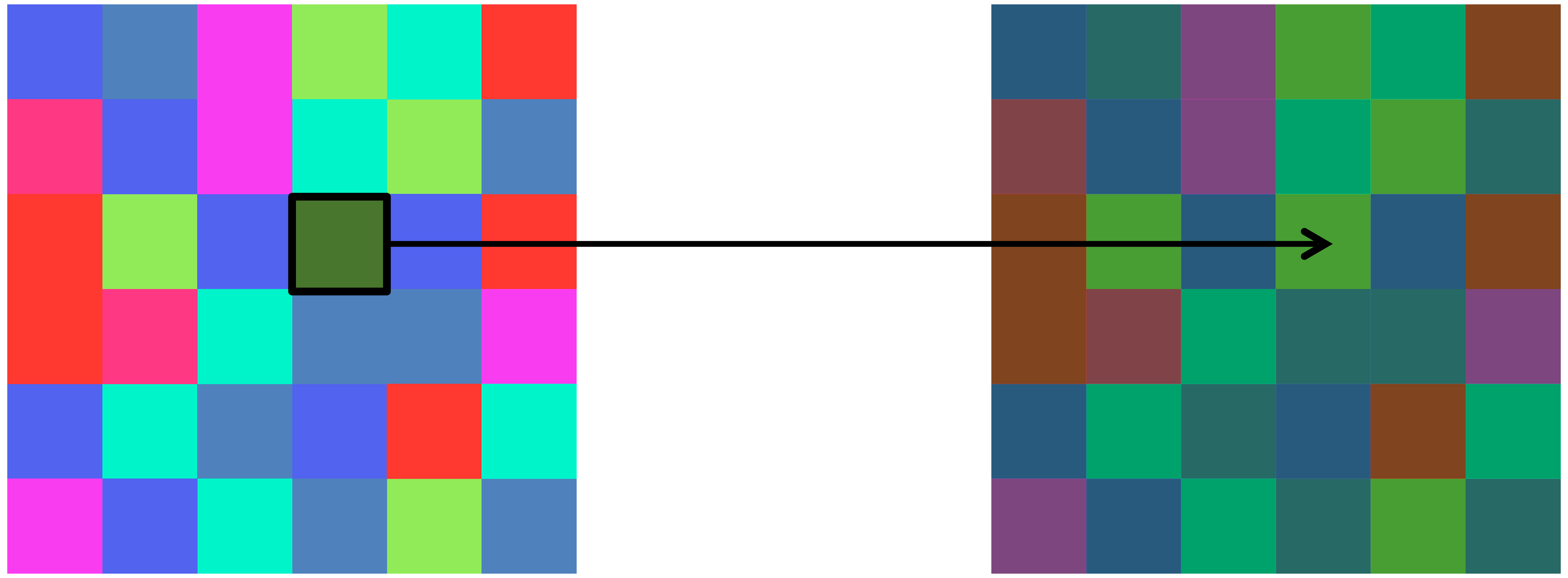


changes domain of image function

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

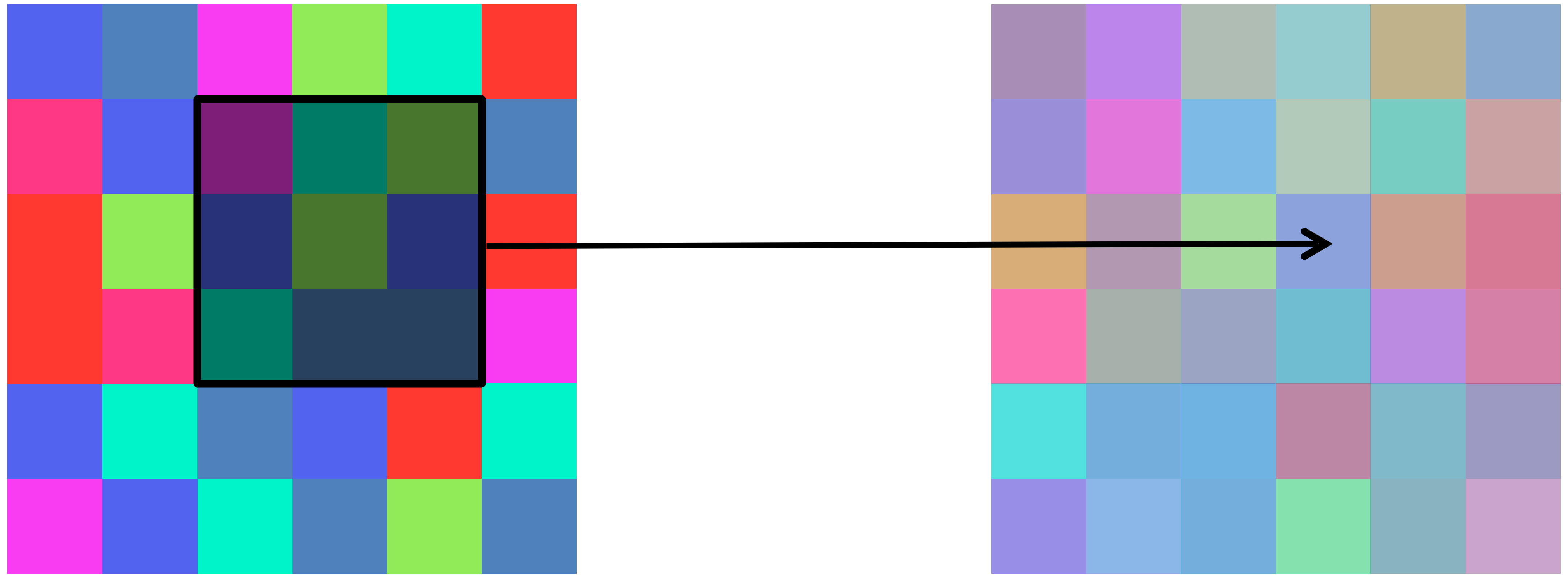
What types of **filtering** can we do?

Point Operation



point processing

Neighborhood Operation



“filtering”

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Examples of **Point Processing**

original

$$I(X,Y)$$

darken

$$I(X,Y) - 128$$

lower contrast

$$\frac{I(X,Y)}{2}$$

non-linear lower contrast

$$\left(\frac{I(X,Y)}{255}\right)^{1/3} \times 255$$

invert

$$255 - I(X,Y)$$

lighten

$$I(X,Y) + 128$$

raise contrast

$$I(X,Y) \times 2$$

non-linear raise contrast

$$\left(\frac{I(X,Y)}{255}\right)^{2} \times 255$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Examples of **Point Processing**

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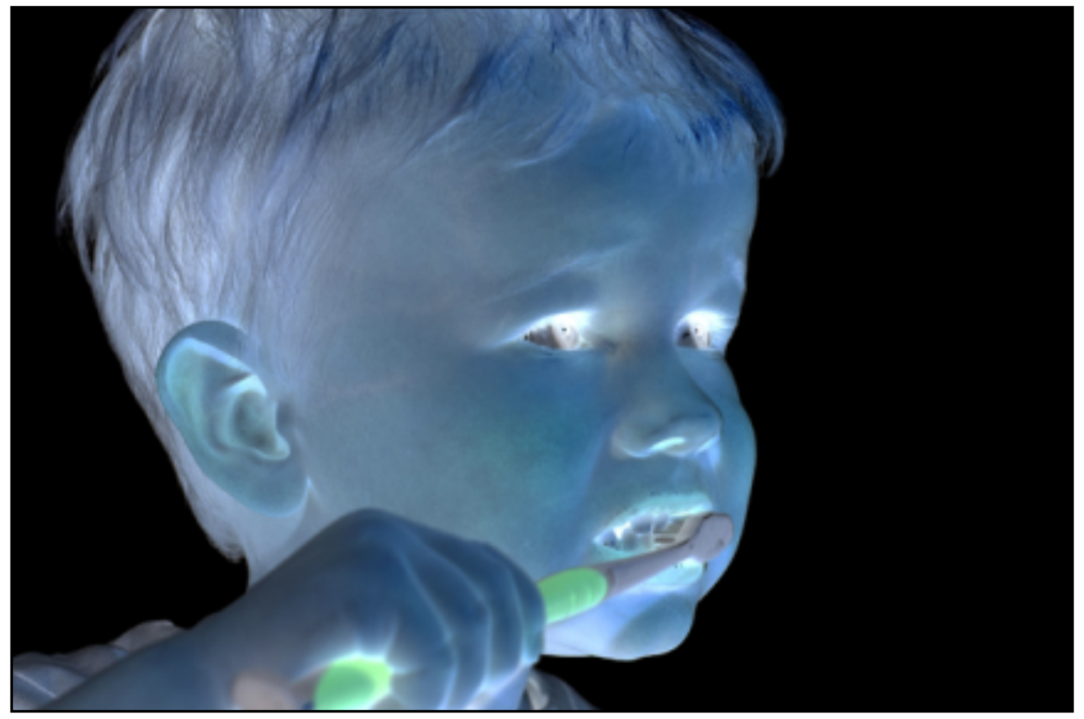
invert



$$255 - I(X,Y)$$

lighten

$$I(X,Y) + 128$$

raise contrast

$$I(X,Y) \times 2$$

non-linear raise contrast

$$\left(\frac{I(X,Y)}{255}\right)^{2} \times 255$$

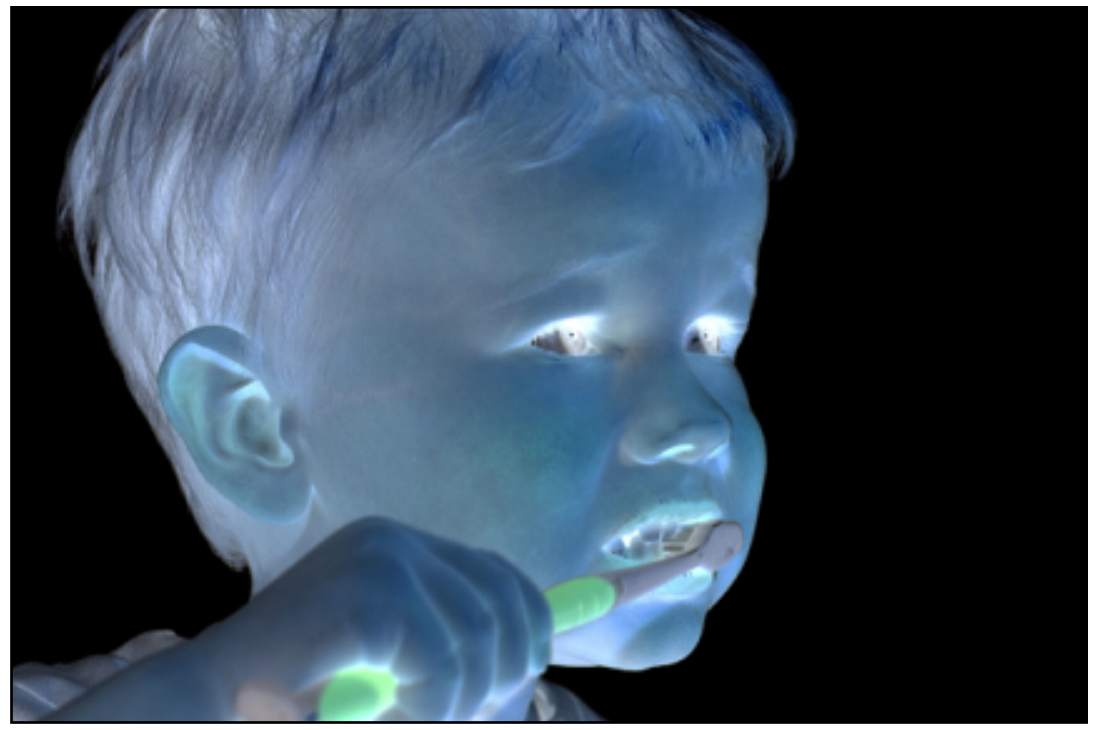
Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

What types of **transformations** can we do?

$I(X,Y)$

Filtering

$I'(X,Y)$

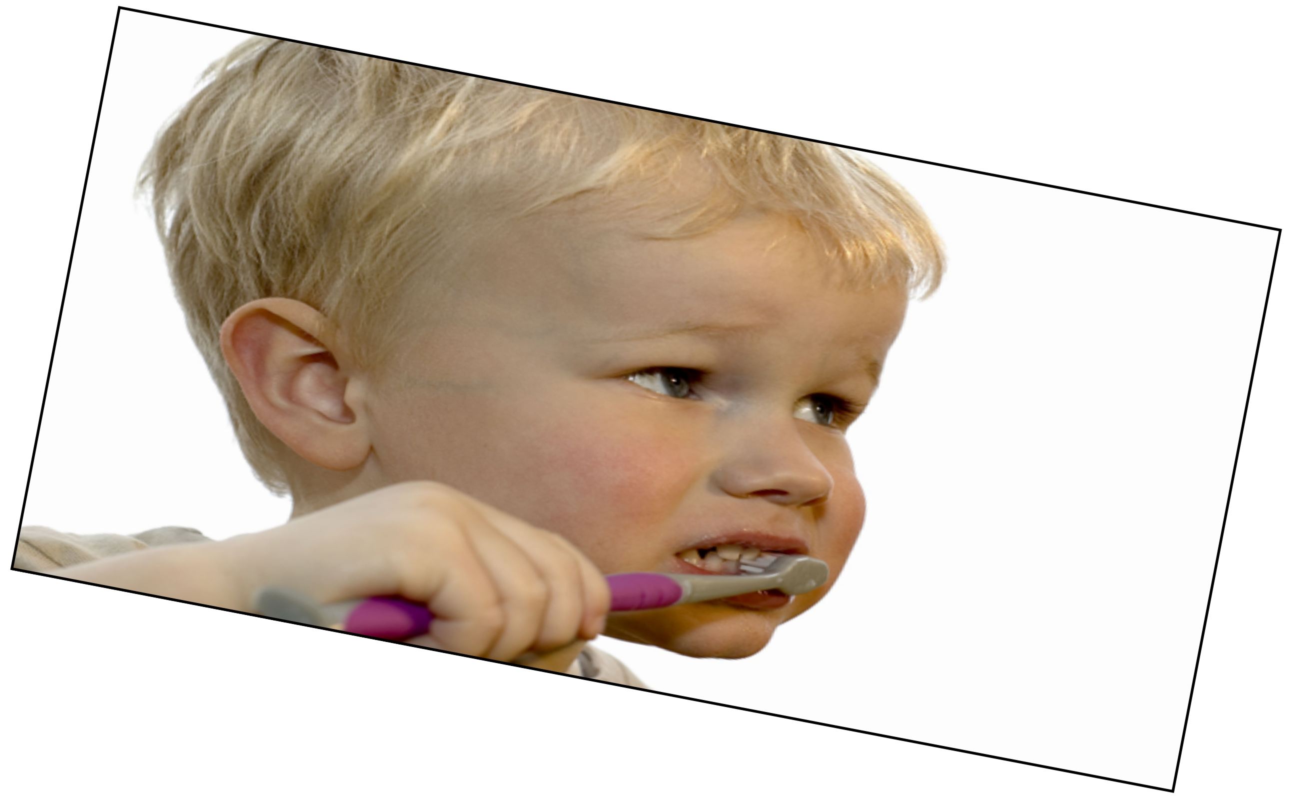


changes range of image function

$I(X,Y)$

Warping

$I'(X,Y)$



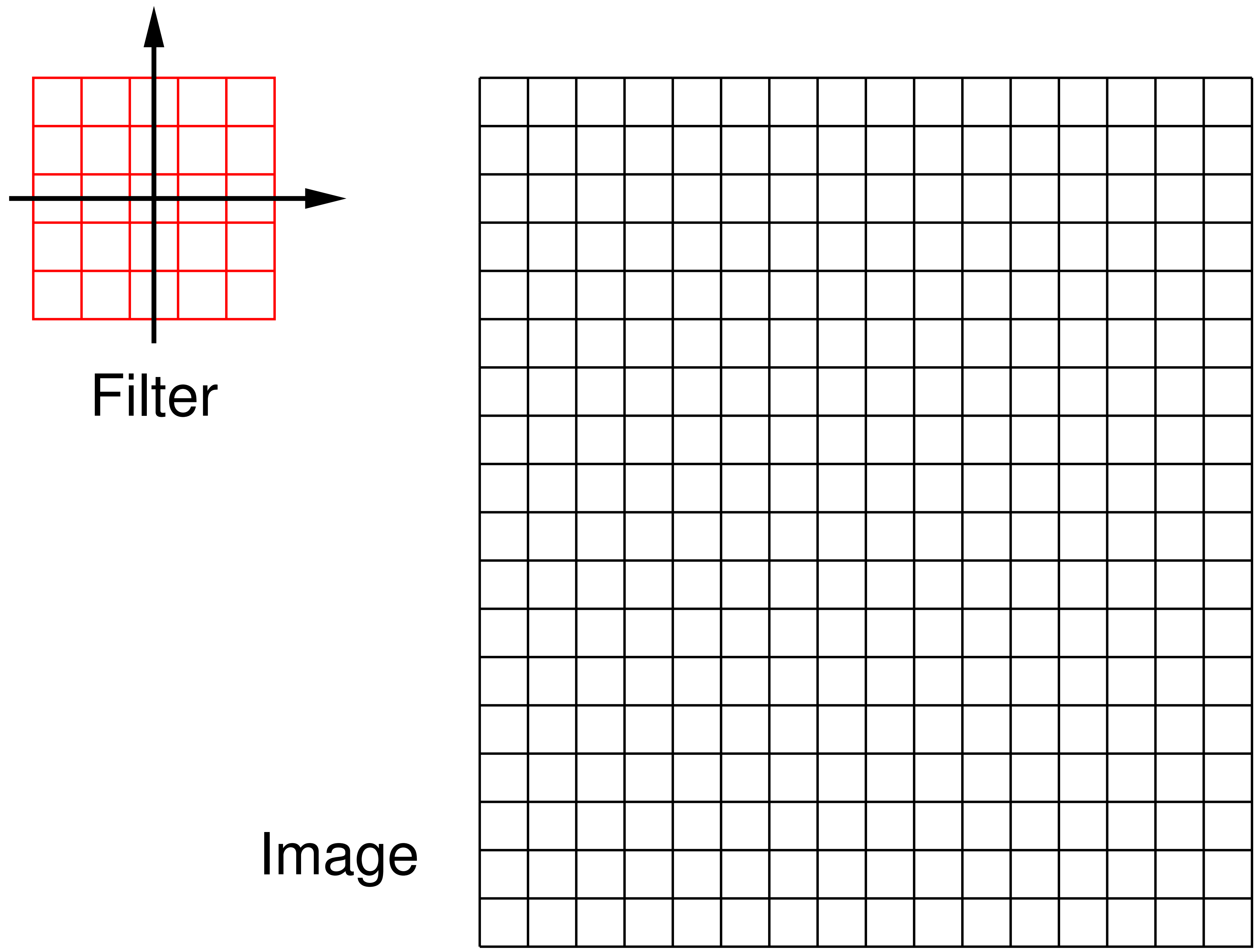
changes domain of image function

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear **Filters**

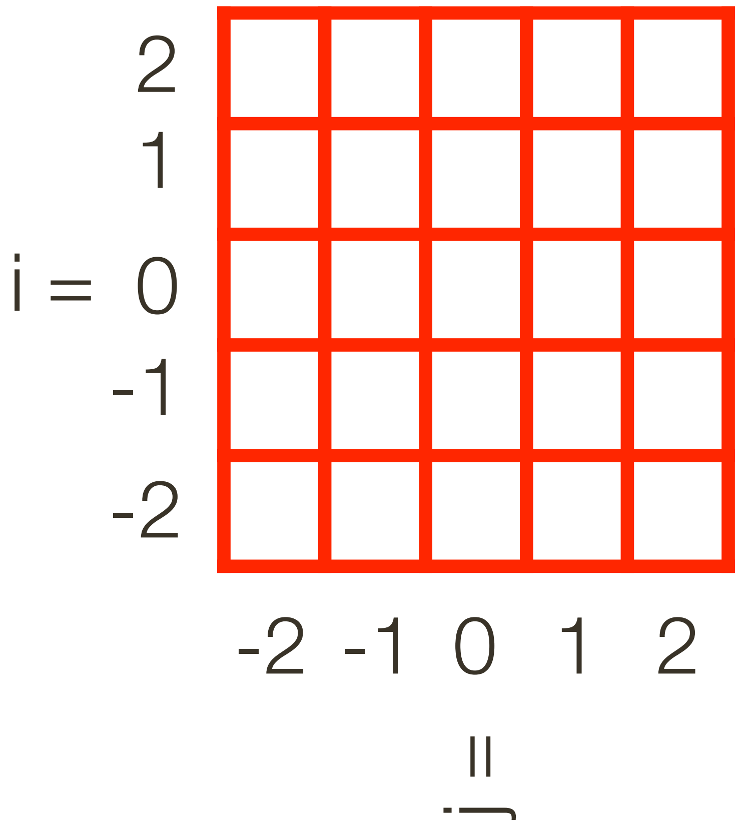
Let $I(X, Y)$ be an $n \times n$ digital image (for convenience we let width = height)

Let $F(X, Y)$ be another $m \times m$ digital image (our "**filter**" or "**kernel**")



For convenience we will assume m is odd. (Here, $m = 5$)

Linear **Filters**



Let $k = \left\lfloor \frac{m}{2} \right\rfloor$

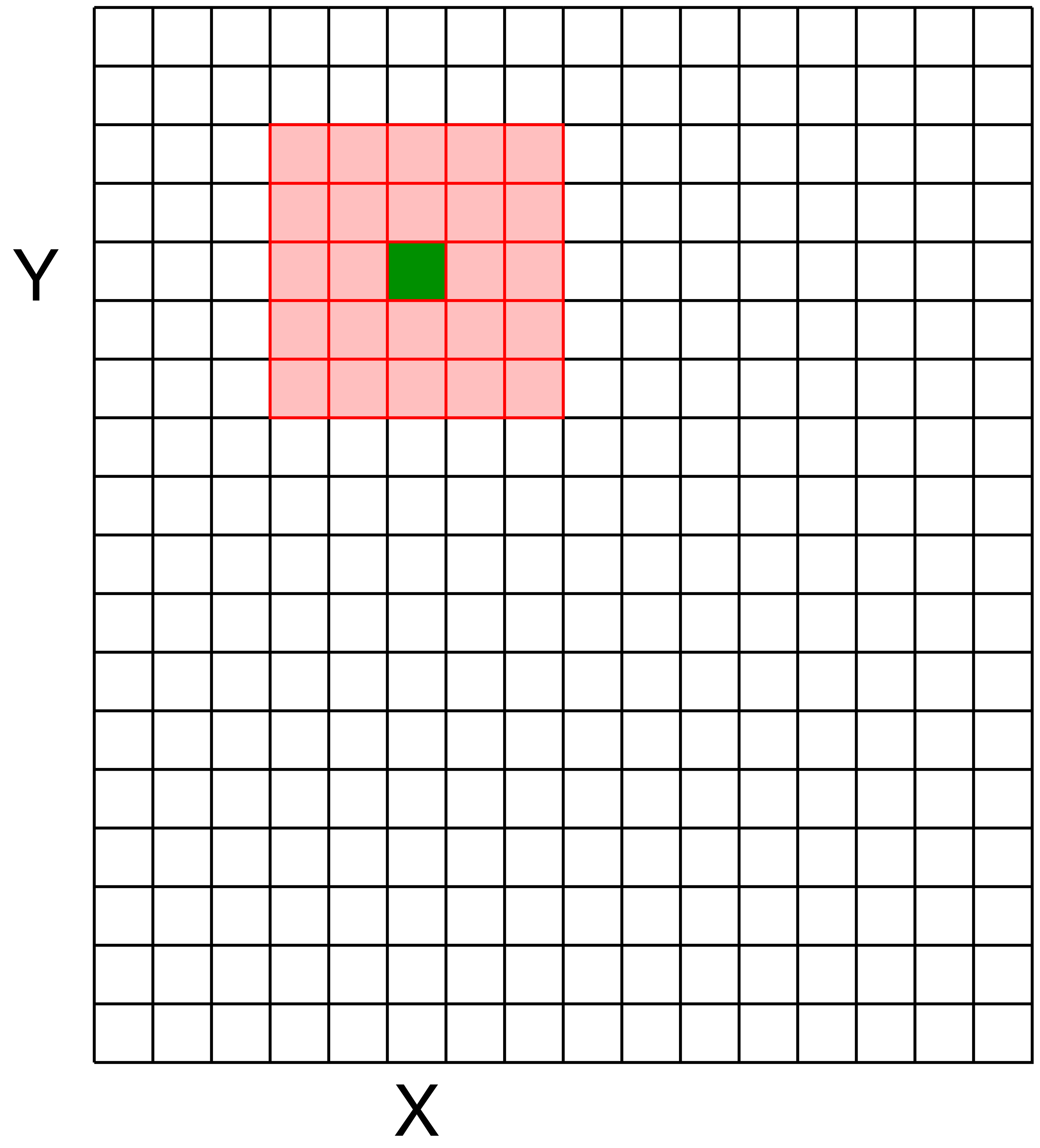
Compute a new image, $I'(X,Y)$, as follows

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k} \sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}} \underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

Intuition: each pixel in the output image is a linear combination of the same pixel and its neighboring pixels in the original image

Linear **Filters**

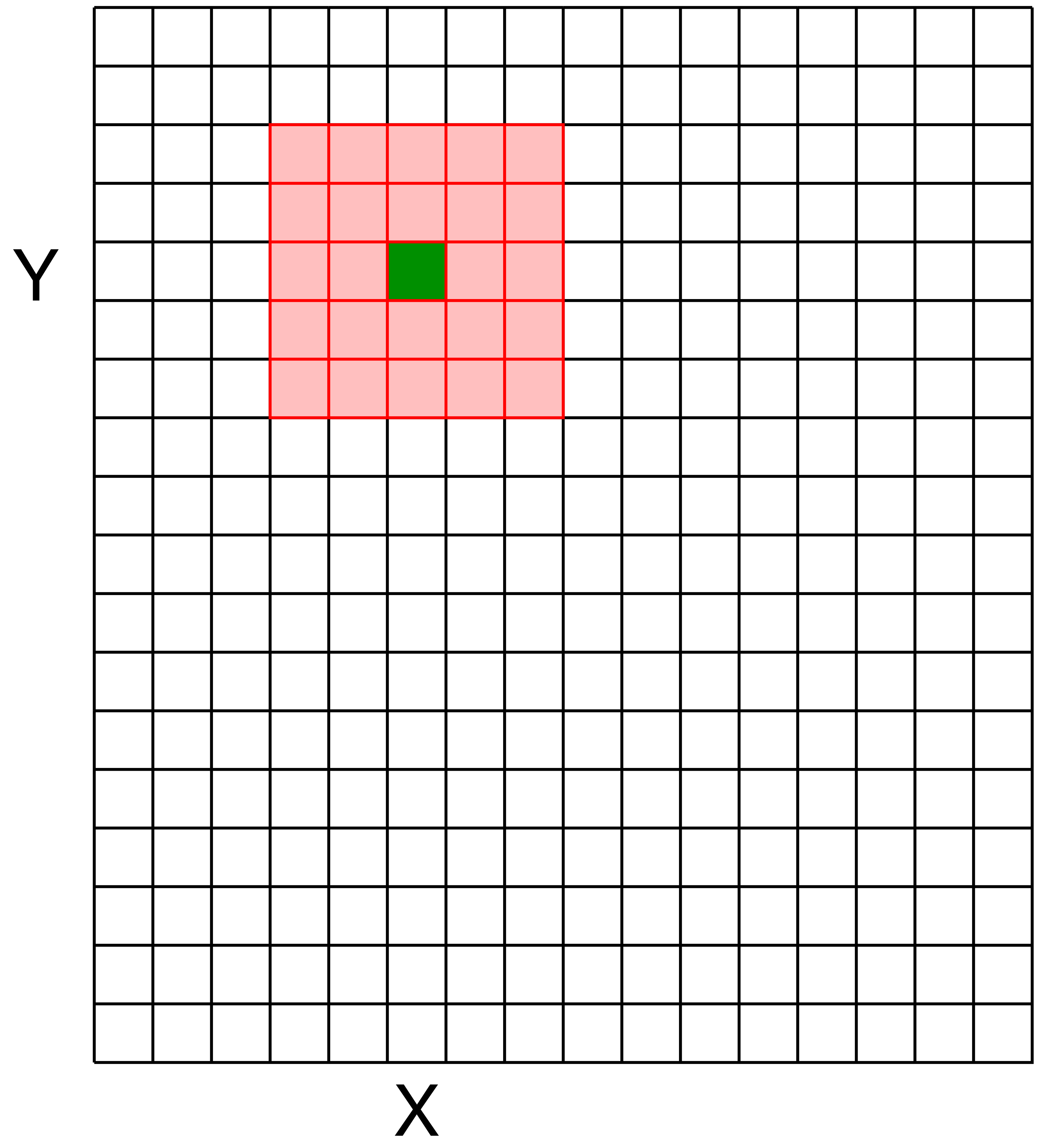
For a give X and Y, superimpose the filter on the image centered at (X, Y)



Linear **Filters**

For a give X and Y, superimpose the filter on the image centered at (X, Y)

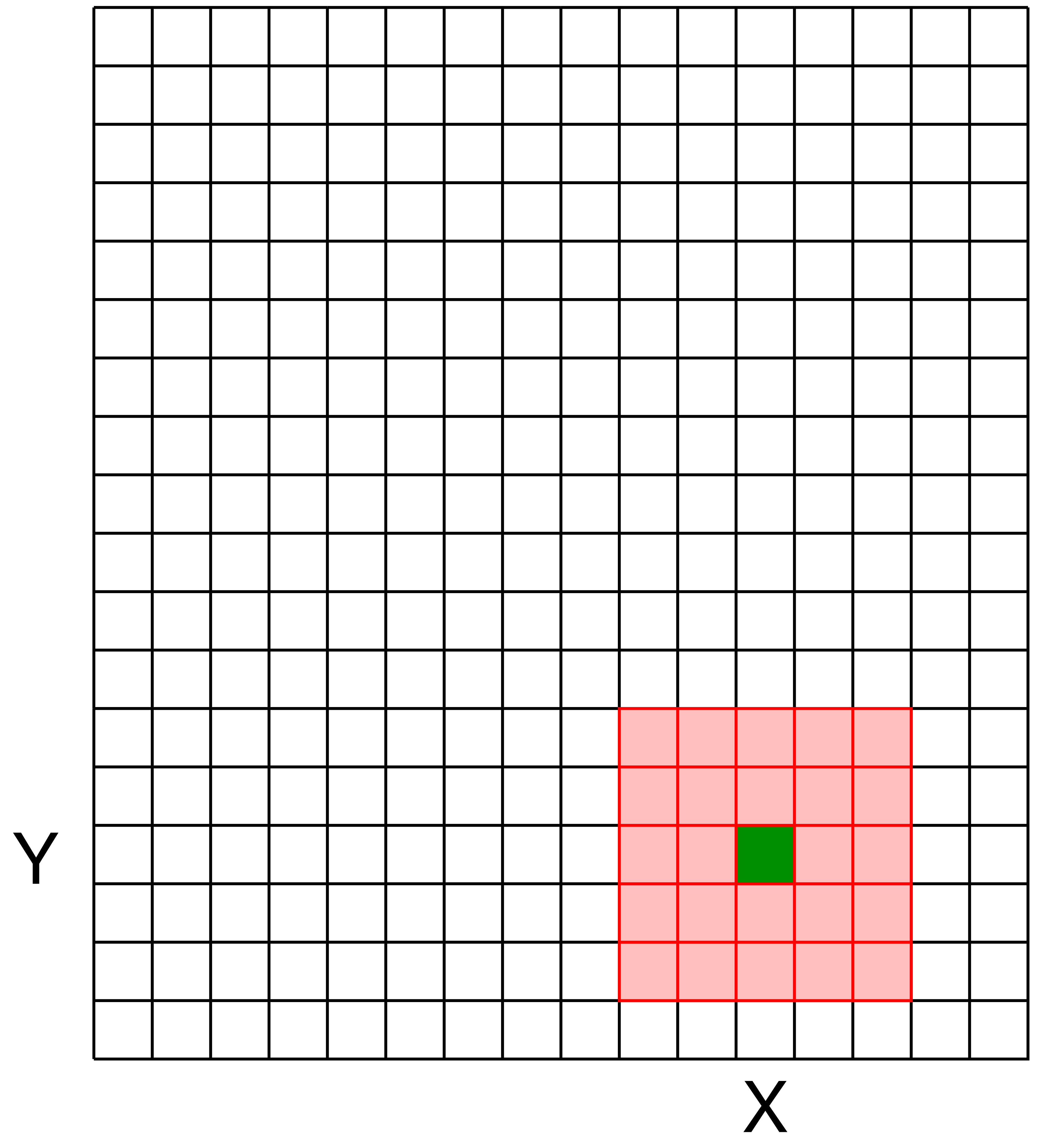
Compute the new pixel value, $I'(X, Y)$, as the sum of $m \times m$ values, where each value is the product of the original pixel value in $I(X, Y)$ and the corresponding values in the filter



Linear **Filters**

The computation is repeated for each

$$(X, Y)$$



Linear Filter **Example**

filter $F(X,Y)$

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image $I(X,Y)$

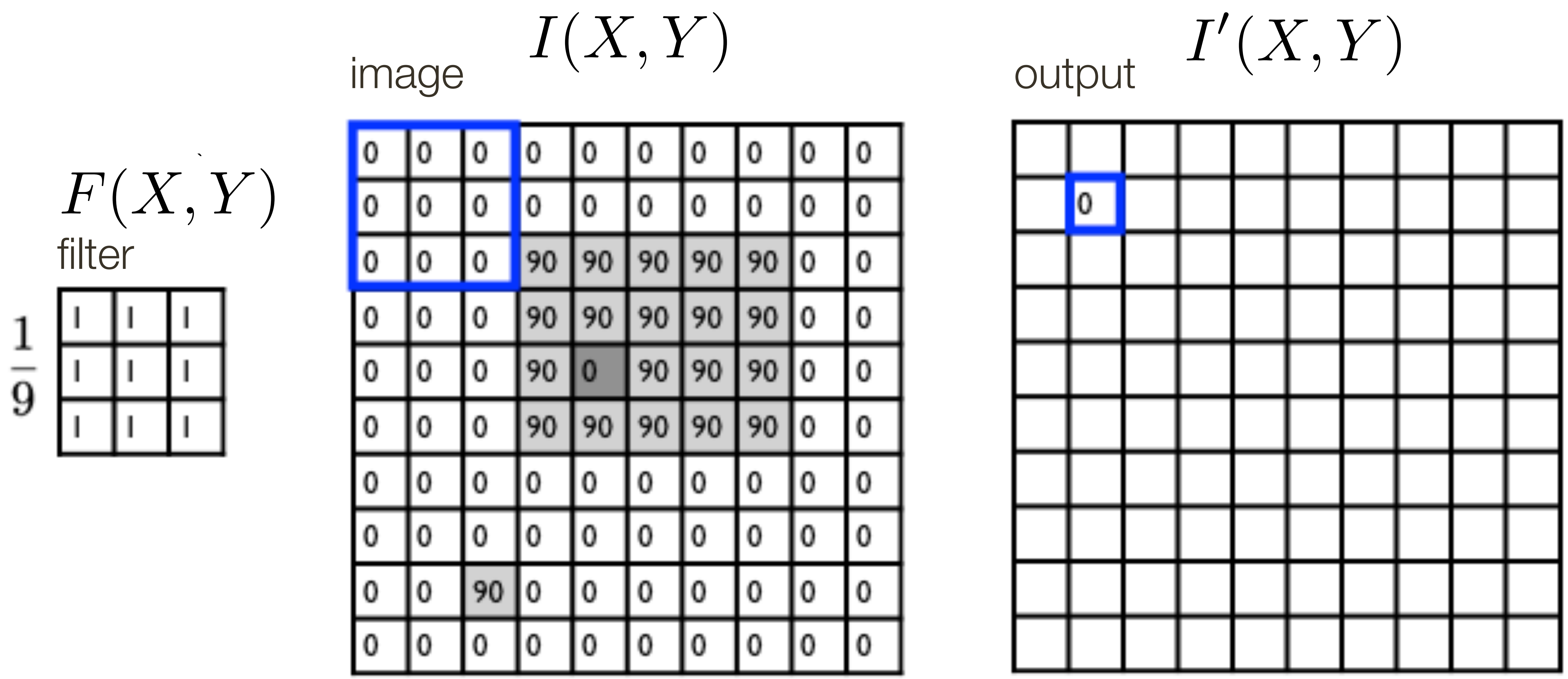
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k} \sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}} \underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filter **Example**



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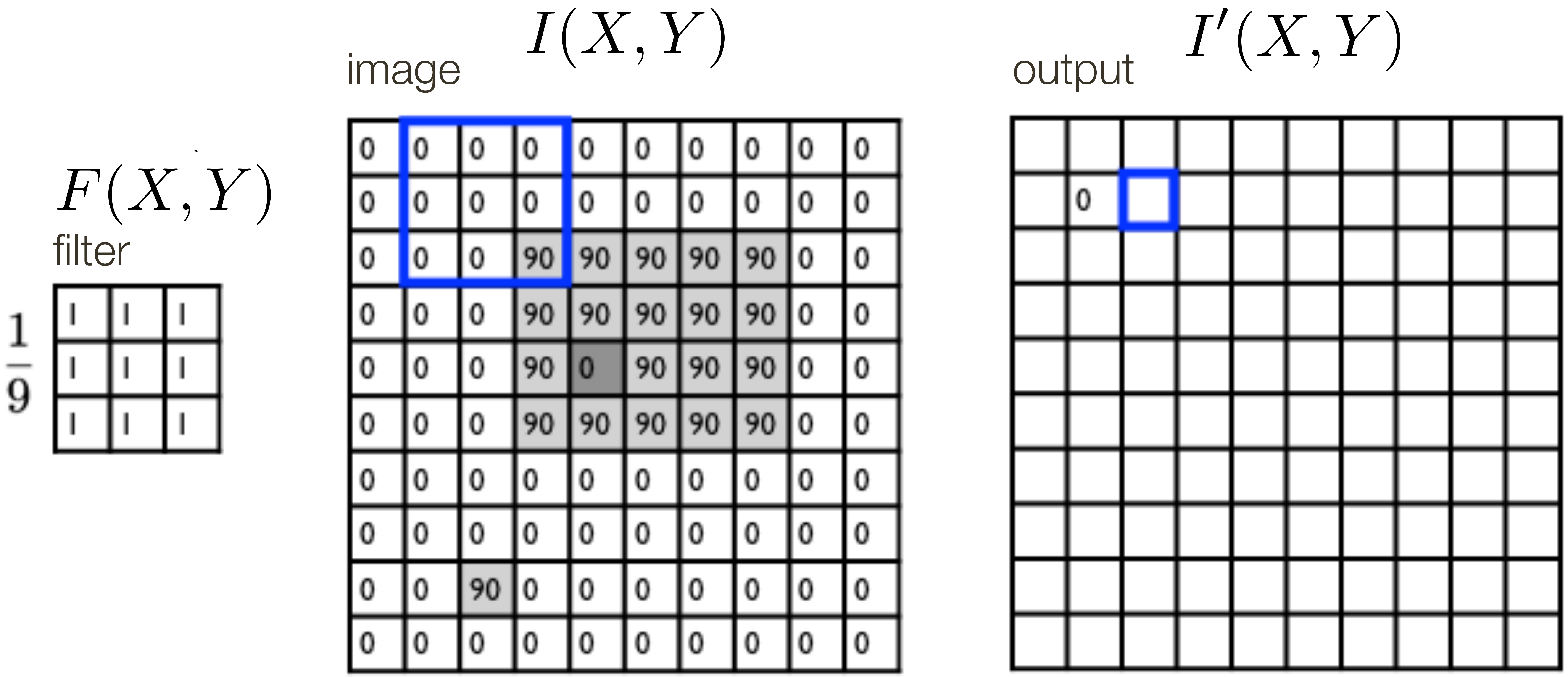
Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filter **Example**



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Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filter **Example**

$F(X,Y)$

filter

$$\frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10							

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k}\sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}}\underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filter **Example**

$F(X,Y)$

filter

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10							

$$\boxed{I'(X,Y)} = \sum_{j=-k}^{k} \sum_{i=-k}^{k} \boxed{F(I,J)}\boxed{I(X+i,Y+j)}$$

output

filter

image (signal)

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filter **Example**

filter $F(X,Y)$

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10	20						

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k}\sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}}\,\underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filter **Example**

filter $F(X,Y)$

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

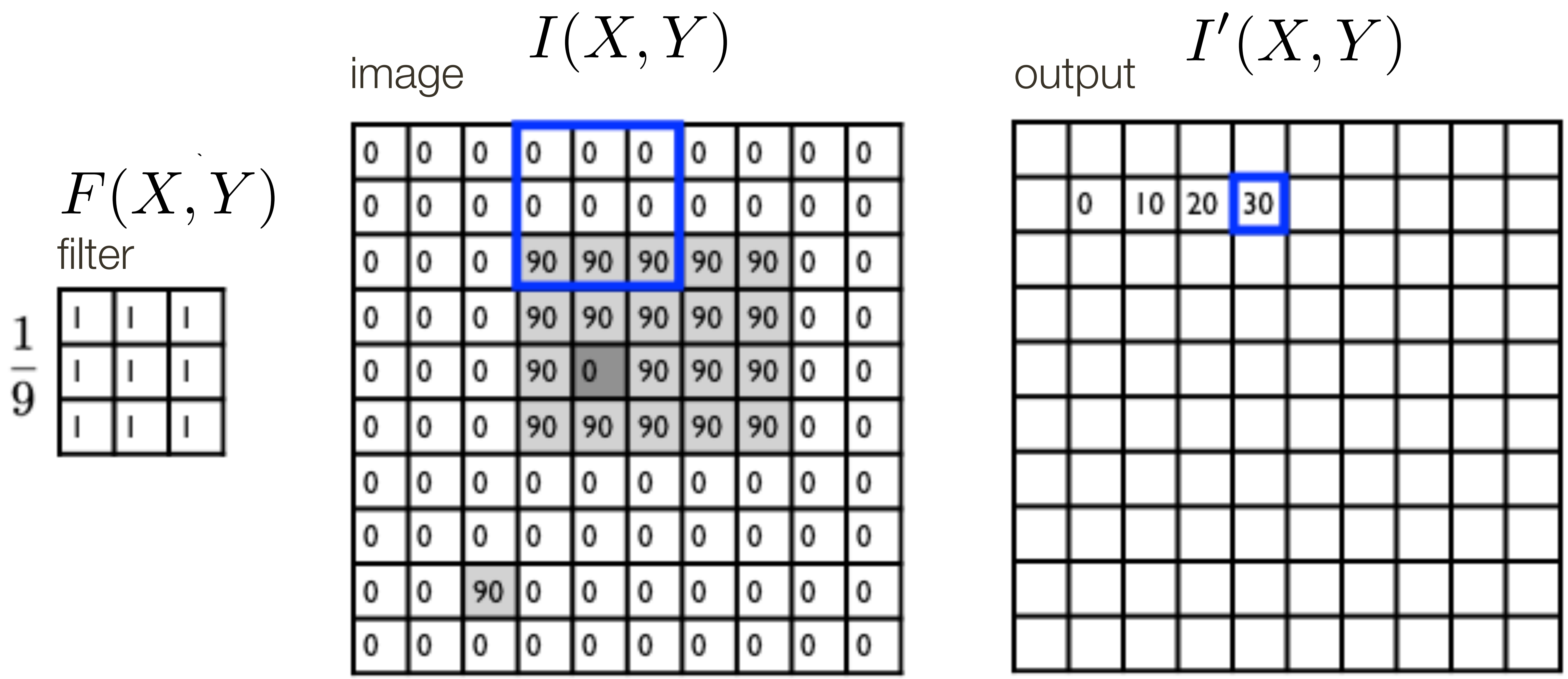
output $I'(X,Y)$

	0	10	20						

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k} \sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}} \underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

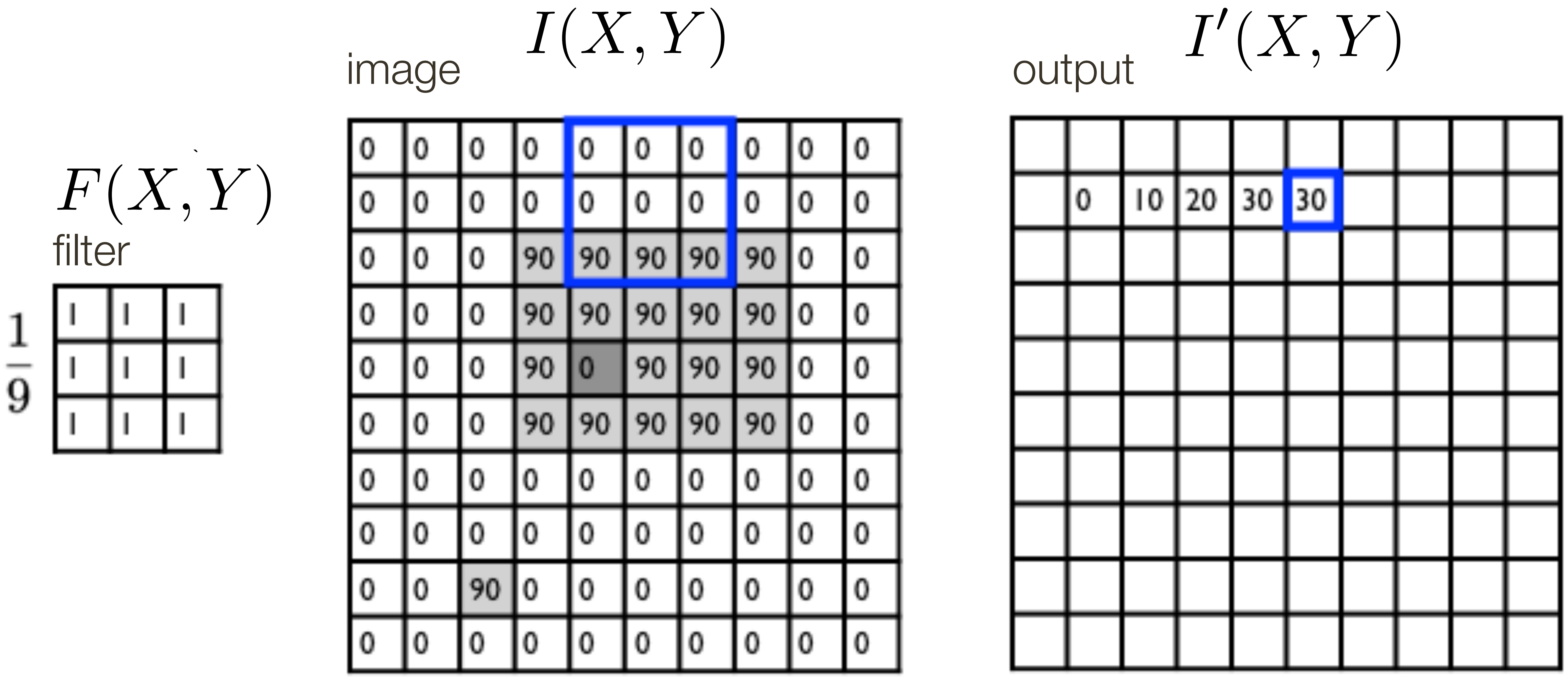
Linear Filter **Example**



$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k} \sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}} \underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filter **Example**



$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k} \sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}} \underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filter **Example**

filter $F(X,Y)$

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10	20	30	30	30			

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k}\sum_{i=-k}^{k}\underbrace{F(I,J)}_{\text{filter}}\underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filter **Example**

$F(X,Y)$

filter

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10	20	30	30	30	20		

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k} \sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}} \underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filter **Example**

$F(X,Y)$

filter

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10	20	30	30	30	20	10	

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k} \sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}} \underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filter **Example**

$F(X,Y)$

filter

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10	20	30	30	30	20	10	
	0								

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k}\sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}}\,\underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filter **Example**

filter $F(X,Y)$

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10	20	30	30	30	20	10	
	0	20							

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k}\sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}}\,\underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filter **Example**

filter $F(X,Y)$

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10	20	30	30	30	20	10	
	0	20	40						

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k}\sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}}\underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filter **Example**

$F(X,Y)$

filter

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0								

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k} \sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}} \underbrace{I(X+i, Y+j)}_{\text{image (signal)}}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filter **Example**

$F(X,Y)$

filter

$$\frac{1}{9}\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30							

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k}\sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}}\underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filter **Example**

$F(X,Y)$

filter

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	0	10	20	30	30	30	20	10	
	10	10	10	10	0	0	0	0	
	10								

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k} \sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}} \underbrace{I(X+i, Y+j)}_{\text{image (signal)}}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filter **Example**

$F(X,Y)$

filter

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	0	10	20	30	30	30	20	10	
	10	10	10	10	0	0	0	0	
	10	10	10	10	0	0	0	0	

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k} \sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}} \underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filter **Example**

$F(X,Y)$

filter

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image $I(X,Y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output $I'(X,Y)$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	0	10	20	30	30	30	20	10	
	10	10	10	10	0	0	0	0	
	10	10	10	10	0	0	0	0	

$$\underbrace{I'(X,Y)}_{\text{output}} = \sum_{j=-k}^{k} \sum_{i=-k}^{k} \underbrace{F(I,J)}_{\text{filter}} \underbrace{I(X+i,Y+j)}_{\text{image (signal)}}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear **Filters**

$$\underbrace{\boxed{I'(X,Y)}}_{\text{output}} = \sum_{j=-k}^{k} \sum_{i=-k}^{k} \underbrace{\boxed{F(I,J)}}_{\text{filter}} \underbrace{\boxed{I(X+i,Y+j)}}_{\text{image (signal)}}$$

For a give X and Y, superimpose the filter on the image centered at (X,Y)

Compute the new pixel value, $I'(X,Y)$, as the sum of $m \times m$ values, where each value is the product of the original pixel value in $I(X,Y)$ and the corresponding values in the filter

Linear **Filters**

Let's do some accounting …

$$\underbrace{\boxed{I'(X,Y)}}_{\text{output}} = \sum_{j=-k}^{k} \sum_{i=-k}^{k} \underbrace{\boxed{F(I,J)}}_{\text{filter}} \underbrace{\boxed{I(X+i,Y+j)}}_{\text{image (signal)}}$$

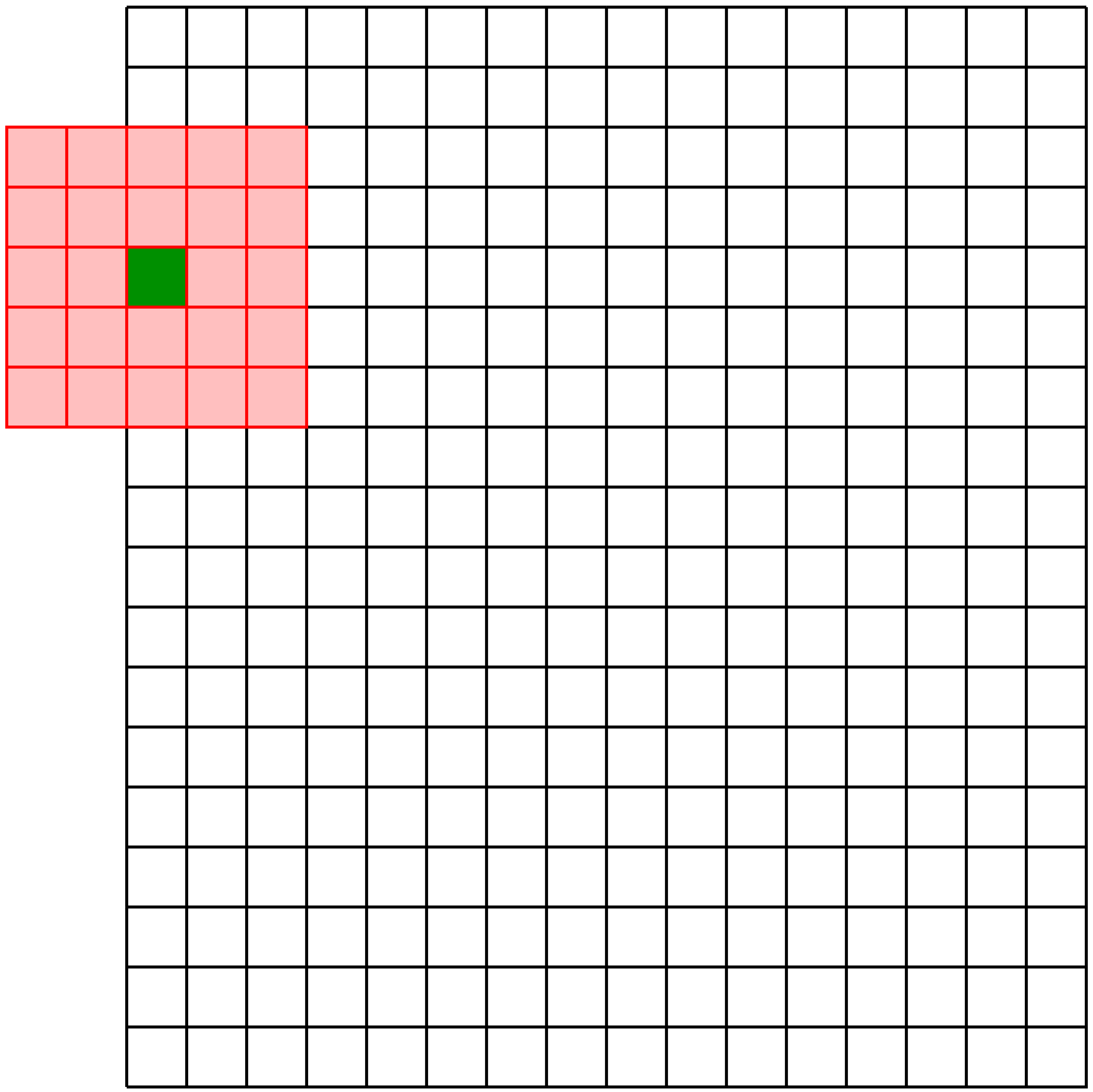
At each pixel, (X,Y), there are $m \times m$ multiplications

There are $n \times n$ pixels in (X,Y)

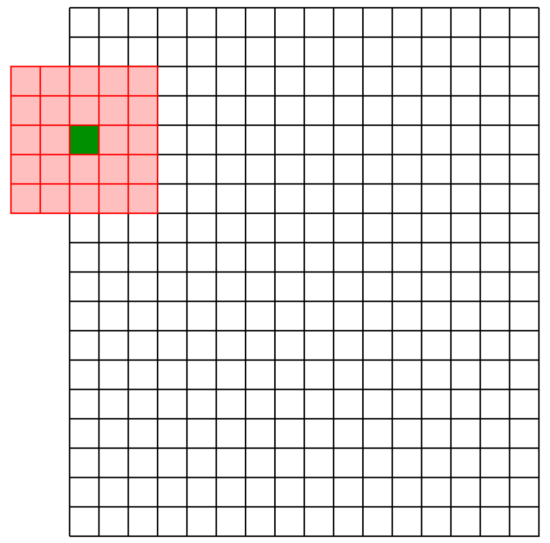
Total: $m^2 \times n^2$ multiplications

When m is fixed, small constant, this is $\mathcal{O}(n^2)$. But when $m \approx n$ this is $\mathcal{O}(m^4)$.

Linear Filters: **Boundary** Effects



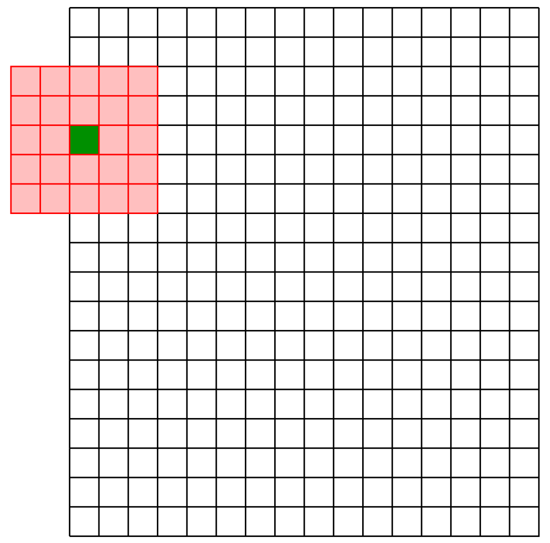
Linear Filters: **Boundary** Effects



Three standard ways to deal with boundaries:

1. **Ignore these locations:** Make the computation undefined for the top and bottom k rows and the leftmost and rightmost k columns

Linear Filters: **Boundary** Effects

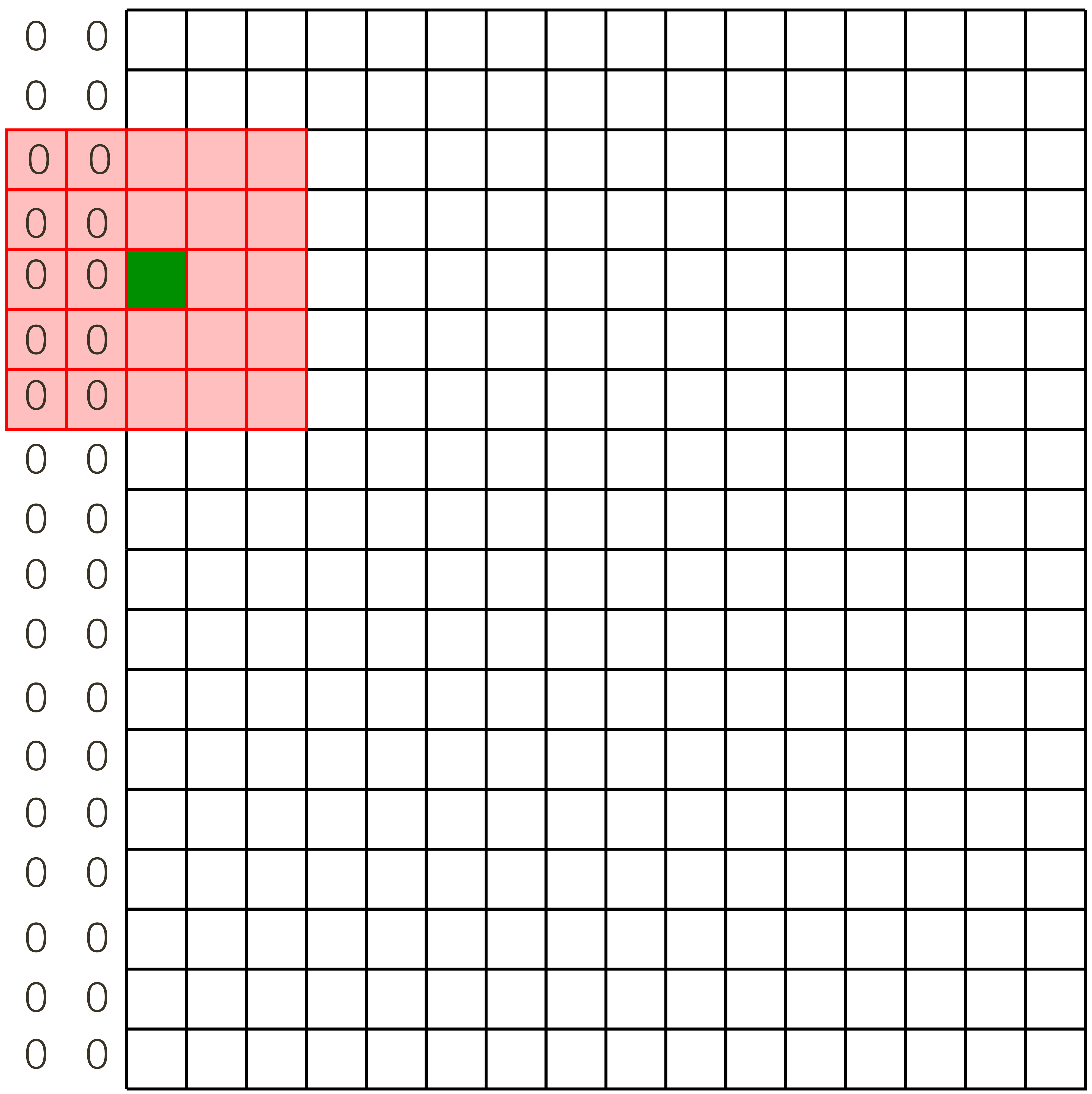


Three standard ways to deal with boundaries:

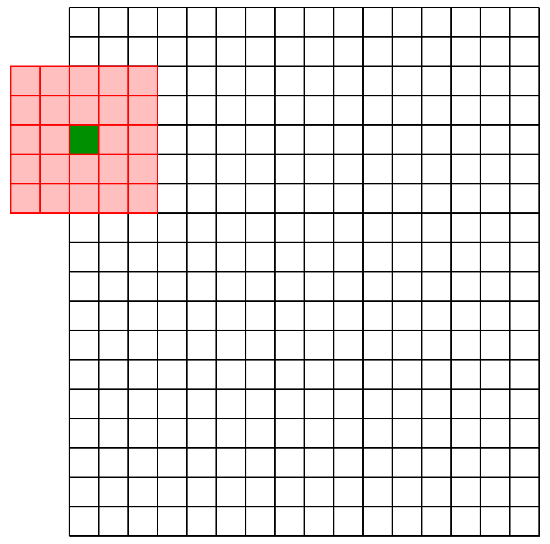
1. **Ignore these locations:** Make the computation undefined for the top and bottom k rows and the leftmost and rightmost k columns

2. **Pad the image with zeros**: Return zero whenever a value of I is required at some position outside the defined limits of X and Y

Linear Filters: **Boundary** Effects



Linear Filters: **Boundary** Effects



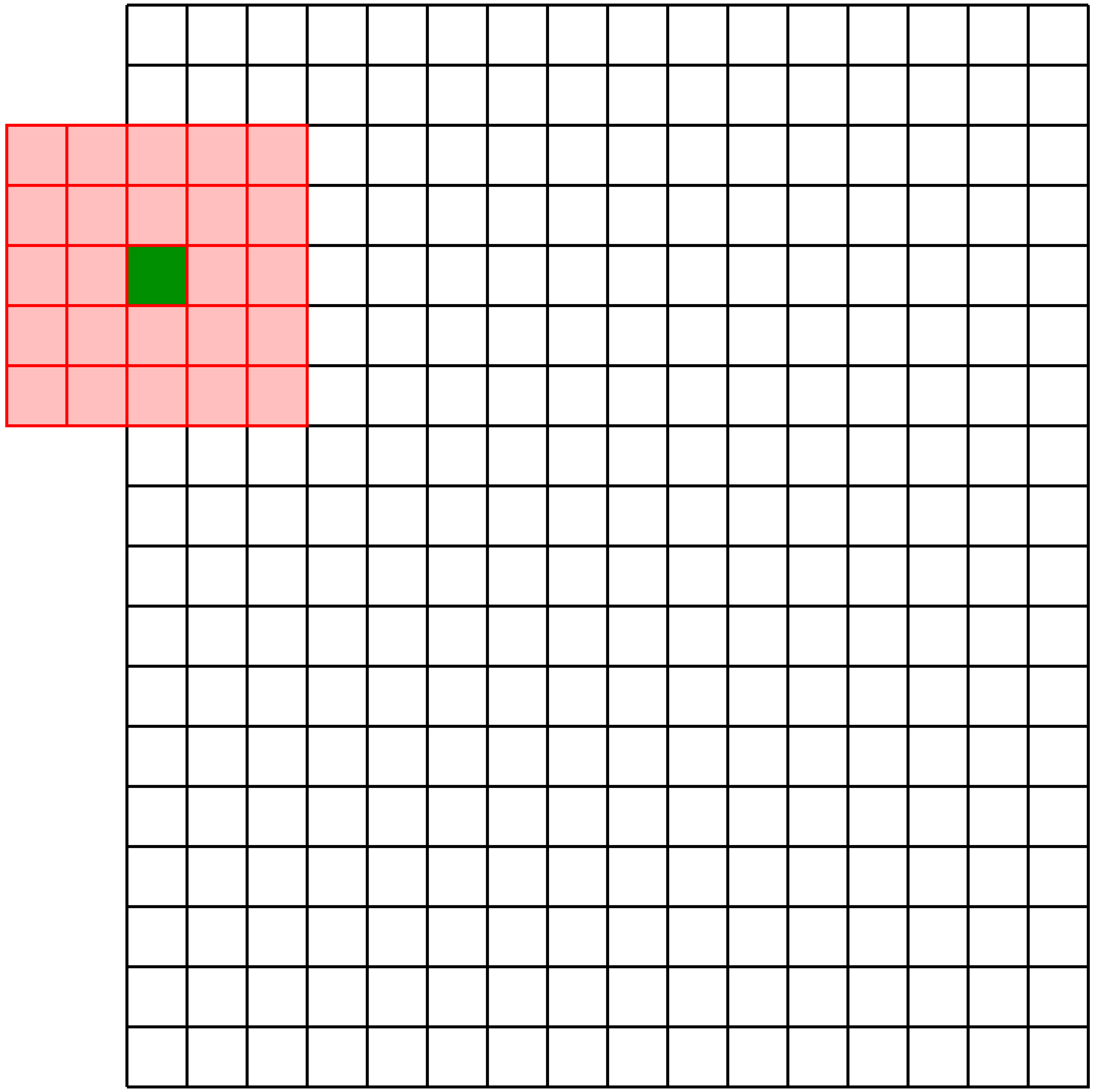
Three standard ways to deal with boundaries:

1. **Ignore these locations:** Make the computation undefined for the top and bottom k rows and the leftmost and rightmost k columns

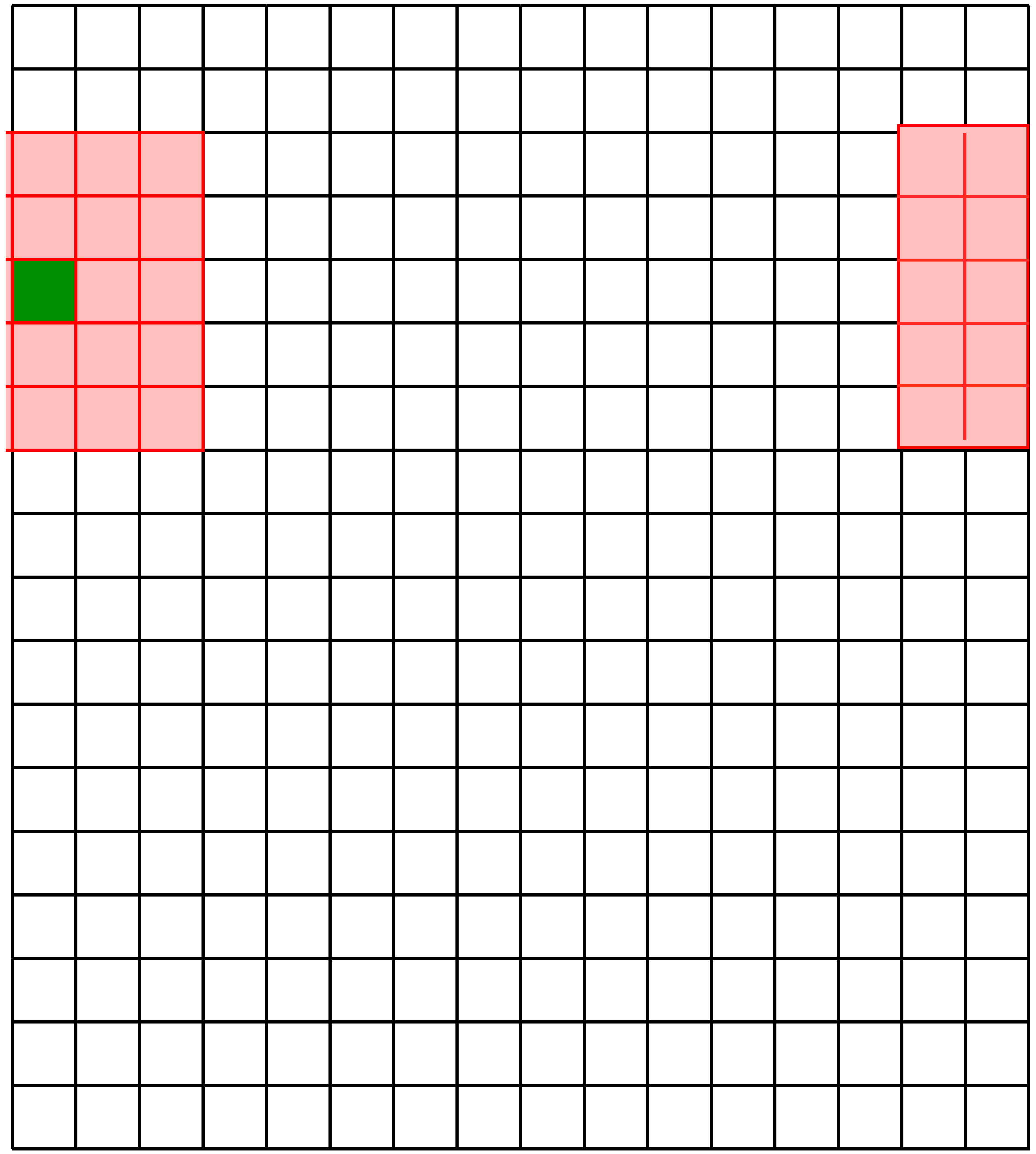
2. **Pad the image with zeros**: Return zero whenever a value of I is required at some position outside the defined limits of X and Y

3. **Assume periodicity**: The top row wraps around to the bottom row; the leftmost column wraps around to the rightmost column

Linear Filters: **Boundary** Effects

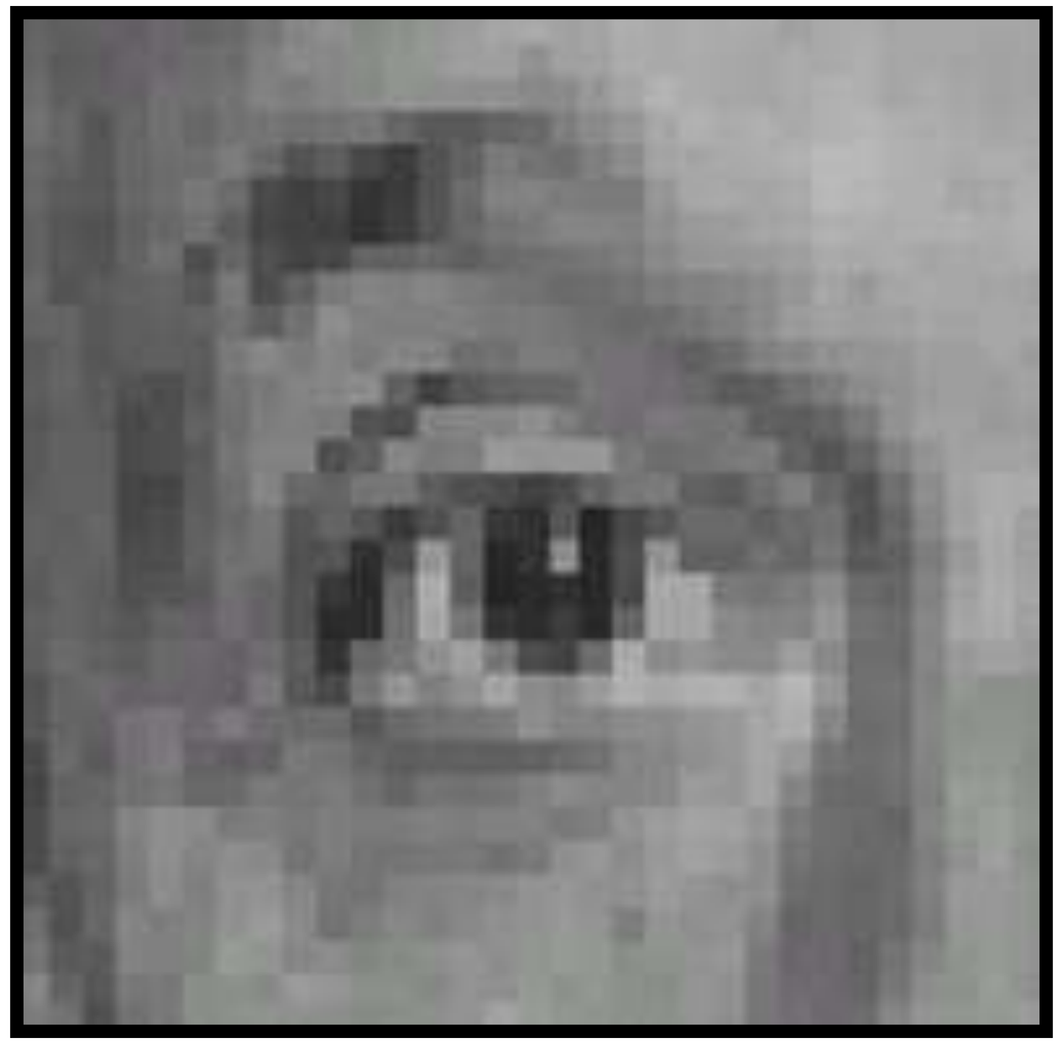


Linear Filters: **Boundary** Effects



A short exercise …

Example 1: Warm up



Original

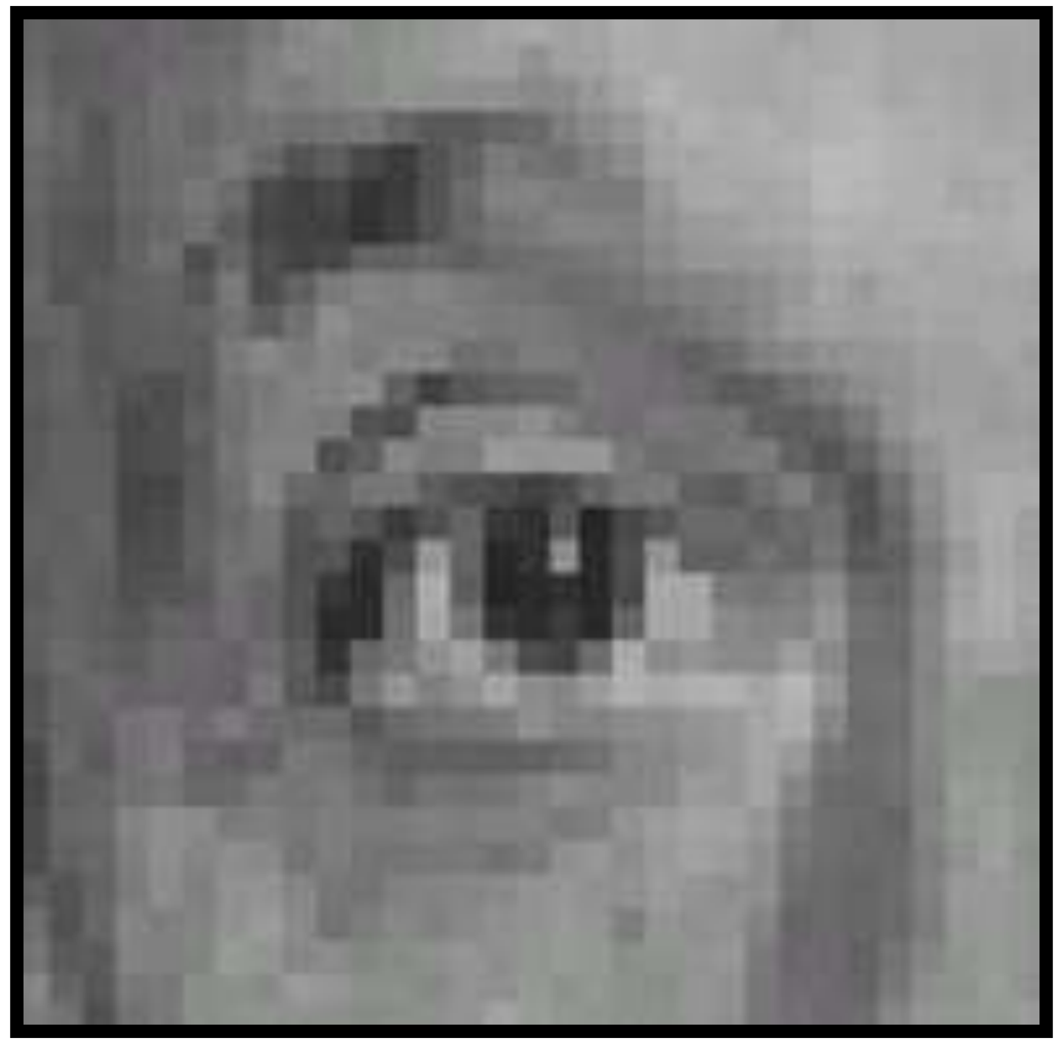
0	0	0
0	1	0
0	0	0

Filter

?

Result

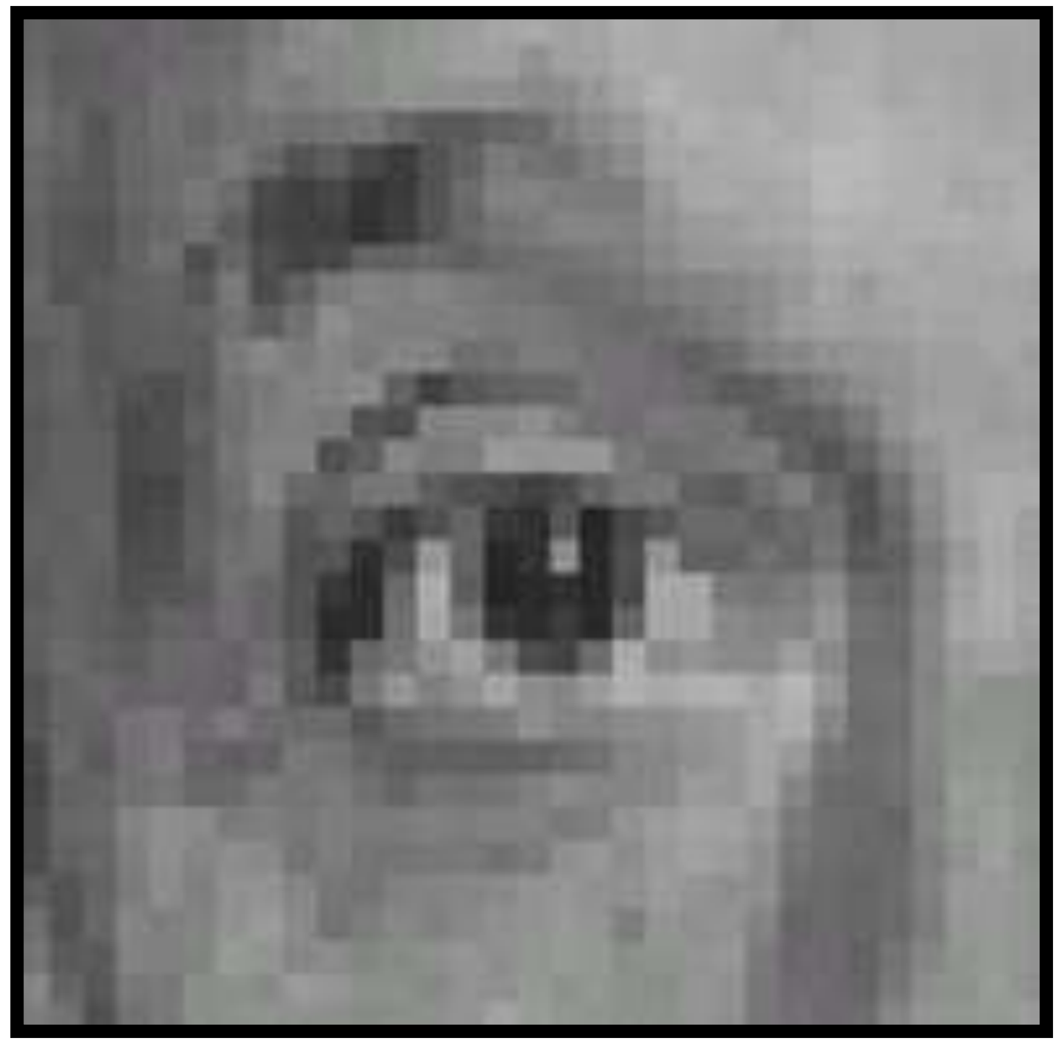
Example 1: Warm up



Original

0	0	0
0	1	0
0	0	0

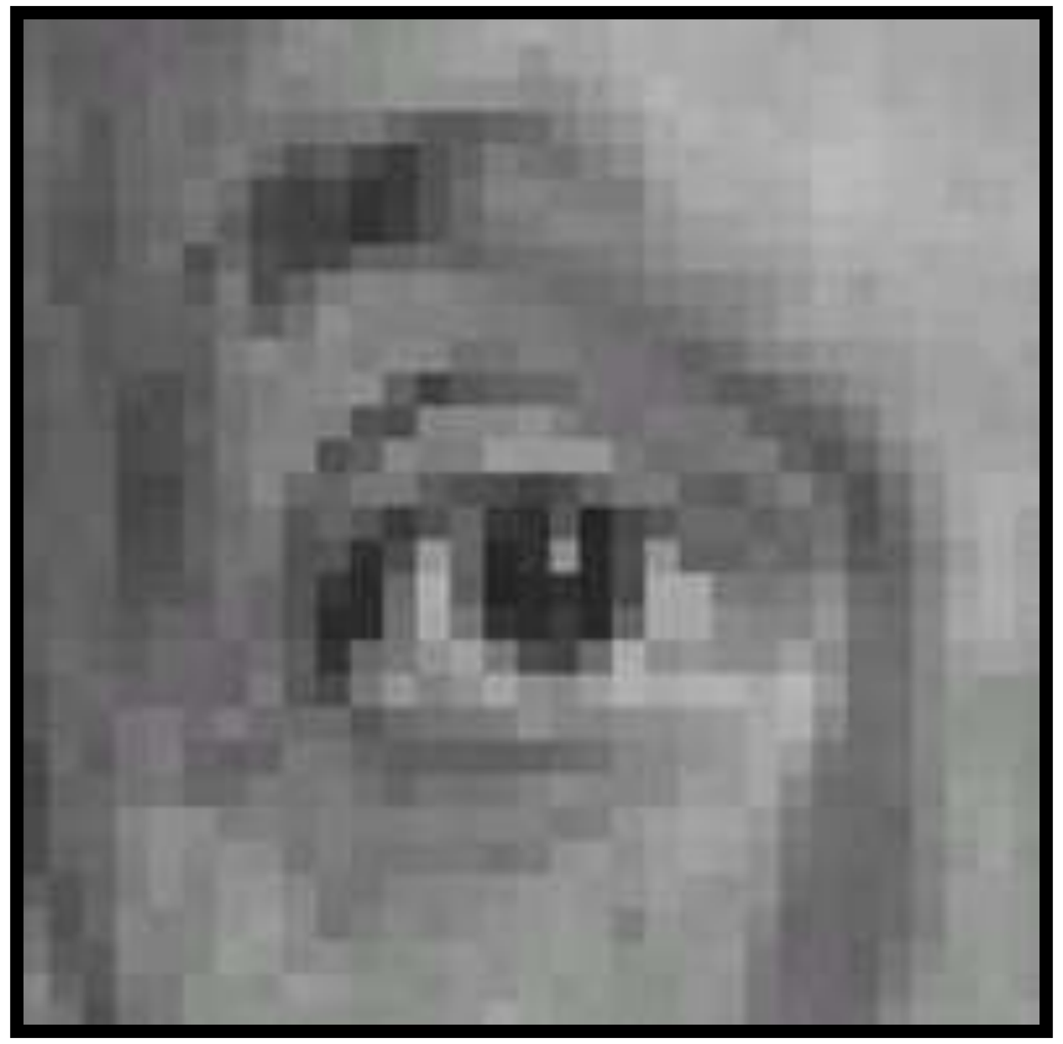
Filter



Result

(no change)

Example 2:



Original

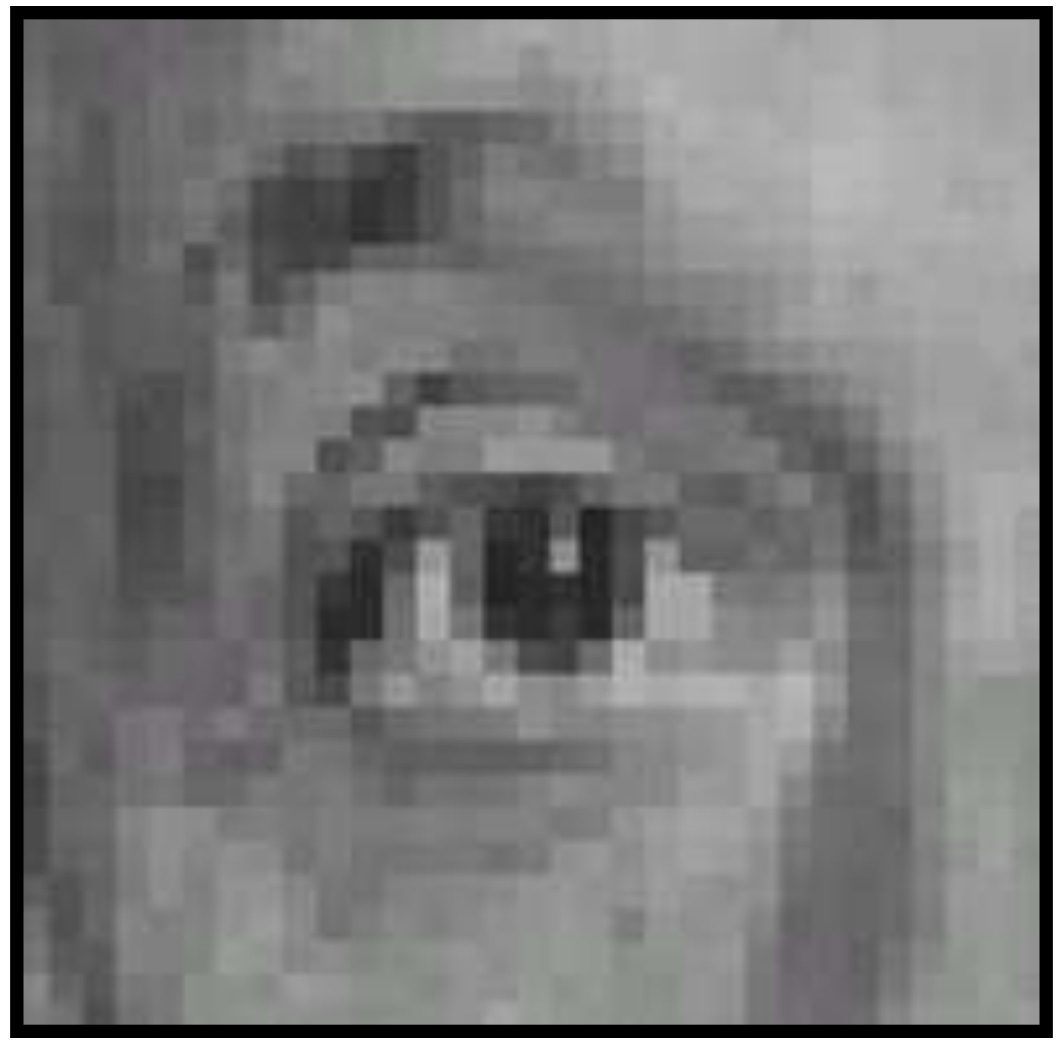
0	0	0
0	0	1
0	0	0

Filter

?

Result

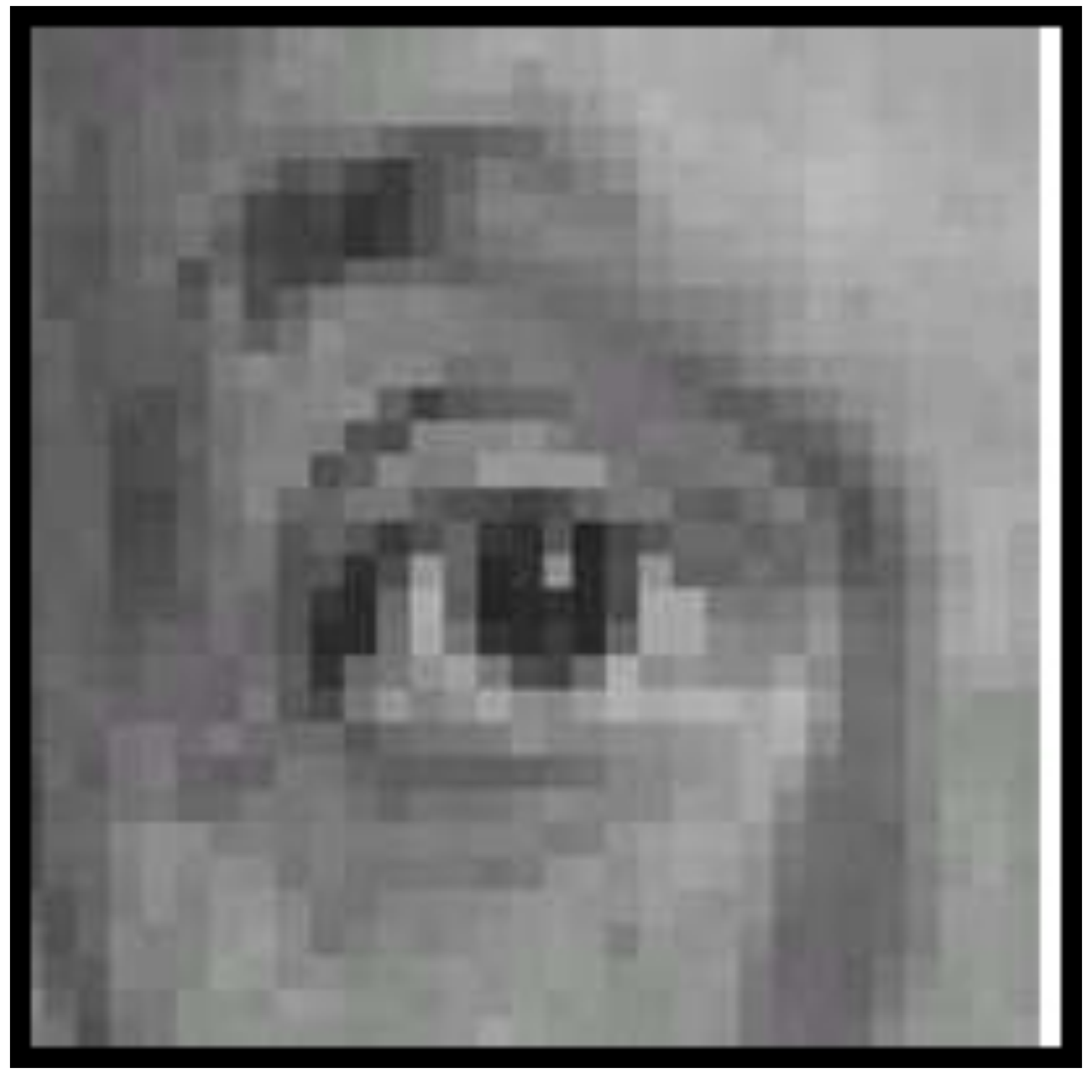
Example 2:



Original

0	0	0
0	0	1
0	0	0

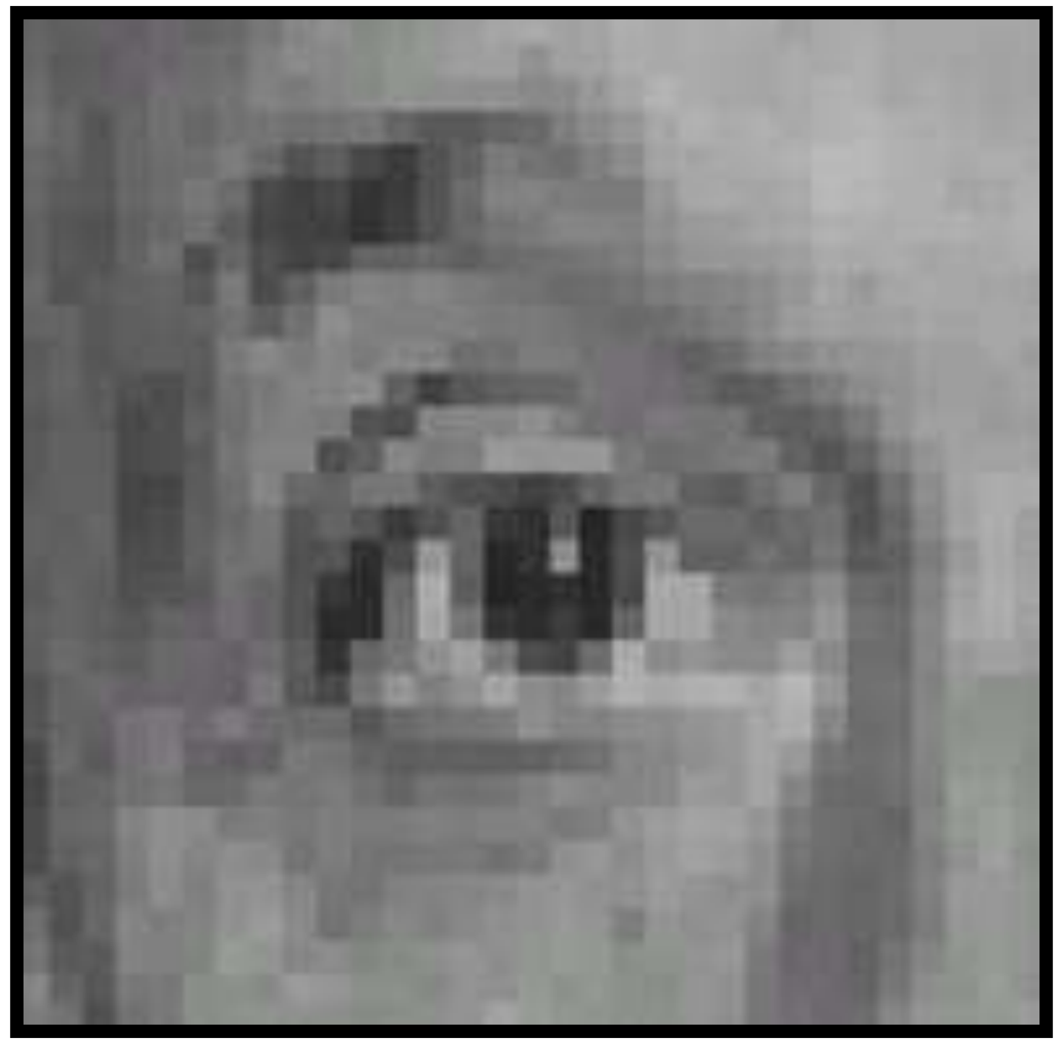
Filter



Result

(sift left by 1 pixel)

Example 3:



Original

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\1 & 1 & 1\\1 & 1 & 1\end{bmatrix}$$

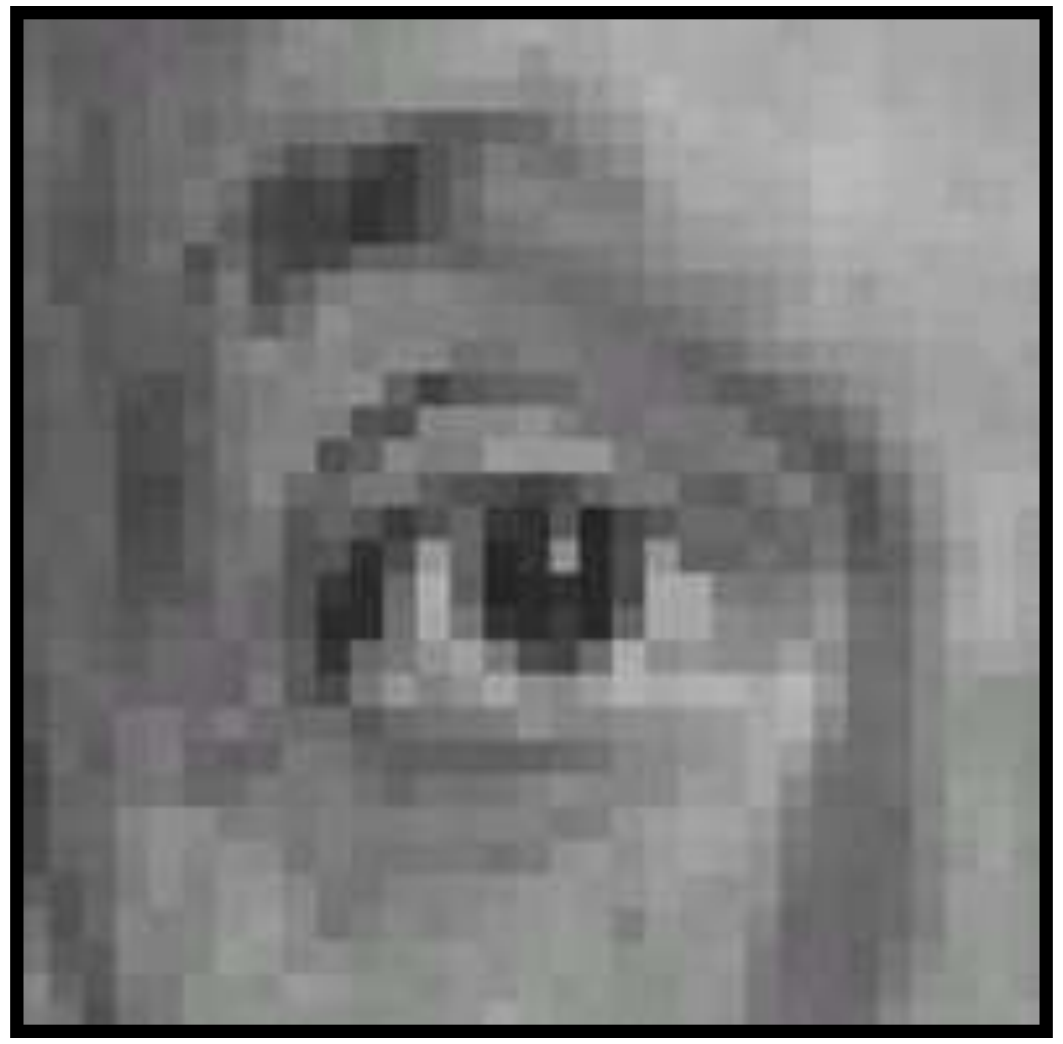
Filter

(filter sums to 1)

?

Result

Example 3:

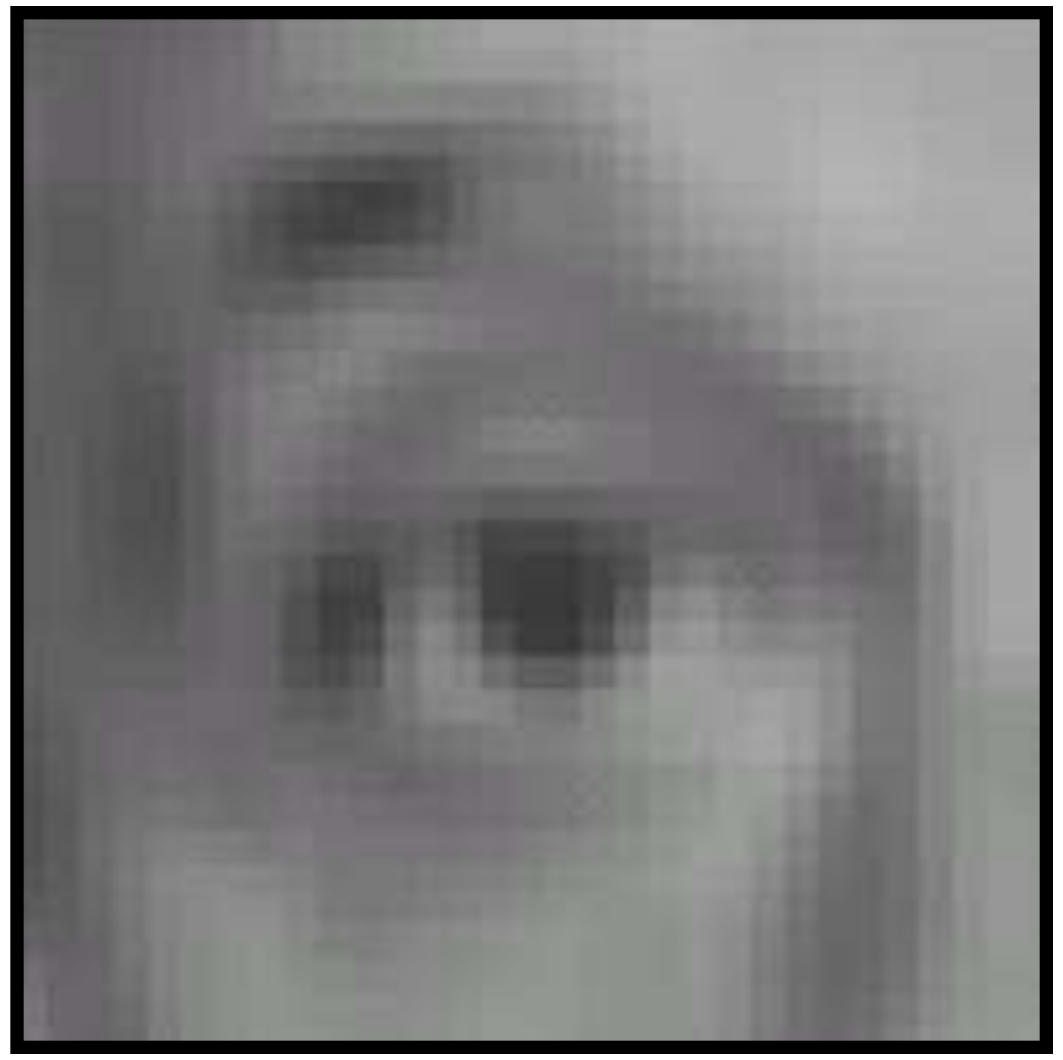


Original

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Filter

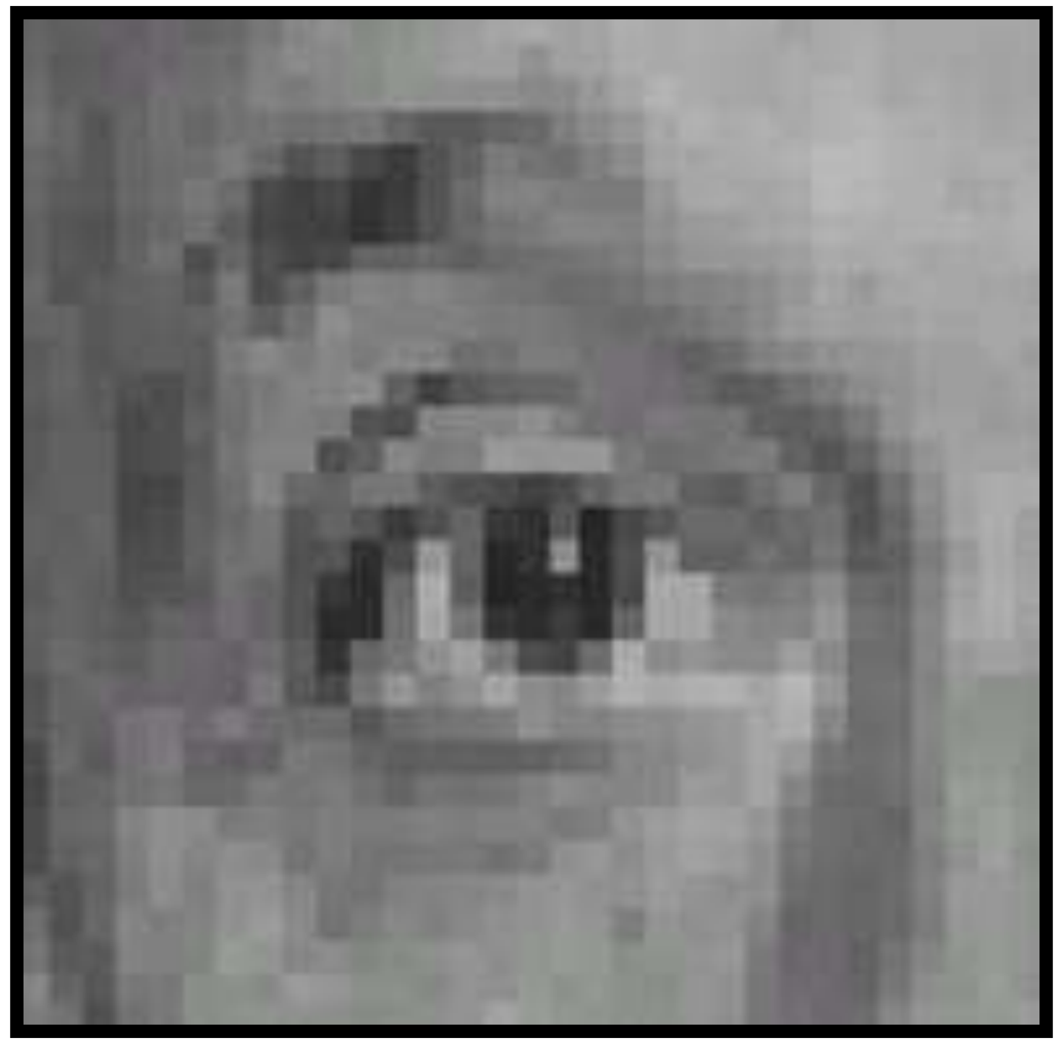
(filter sums to 1)



Result

(blur with a box filter)

Example 4:



$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

?

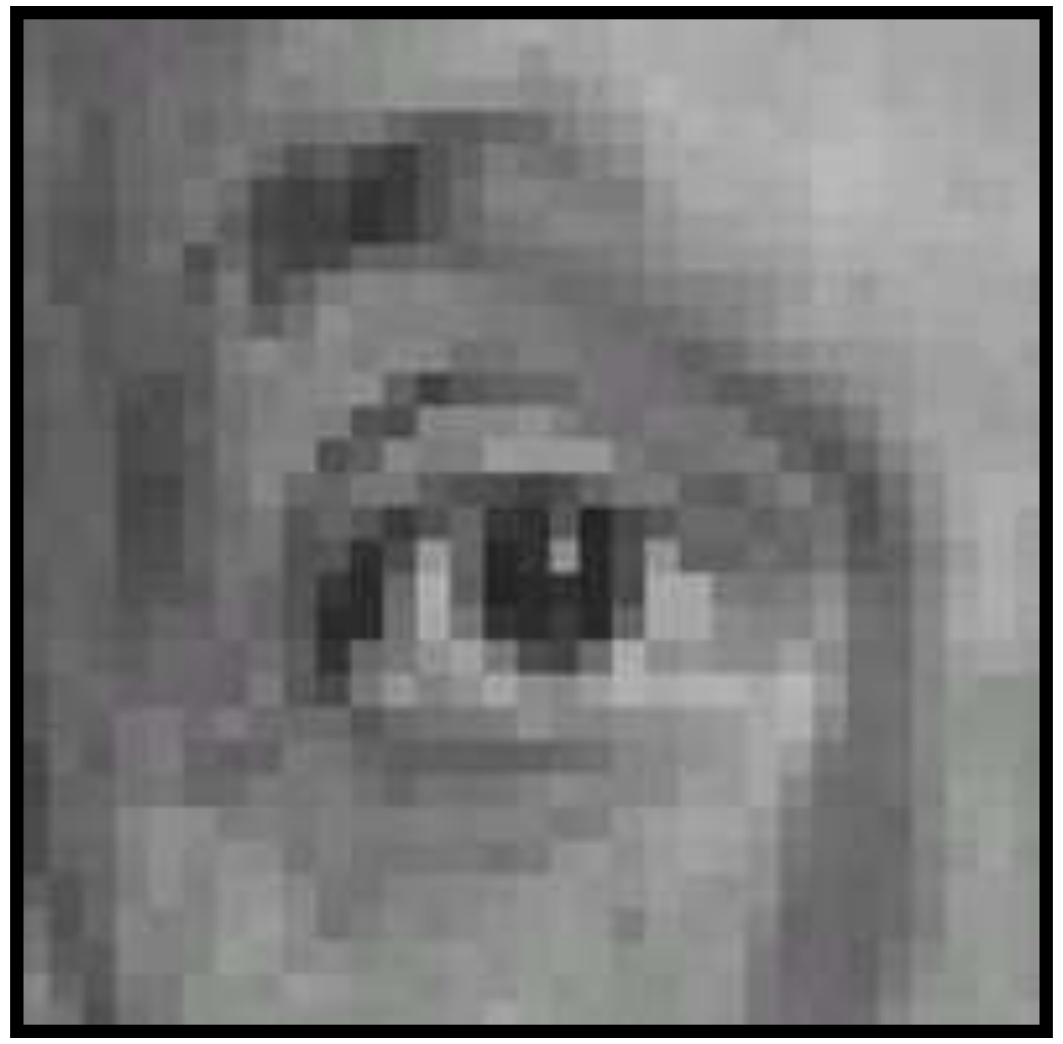
Original

Filter

(filter sums to 1)

Result

Example 4:



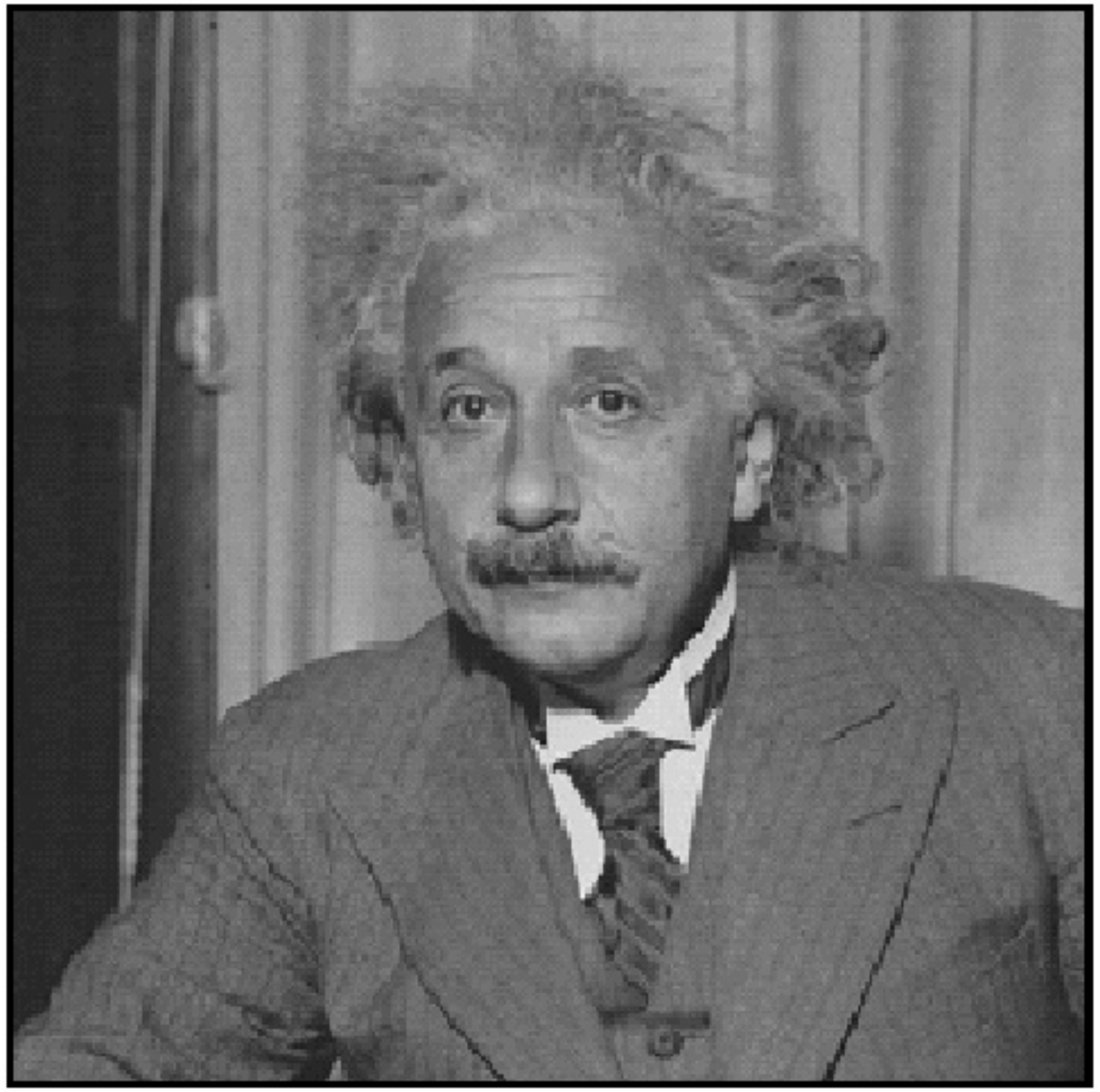
Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

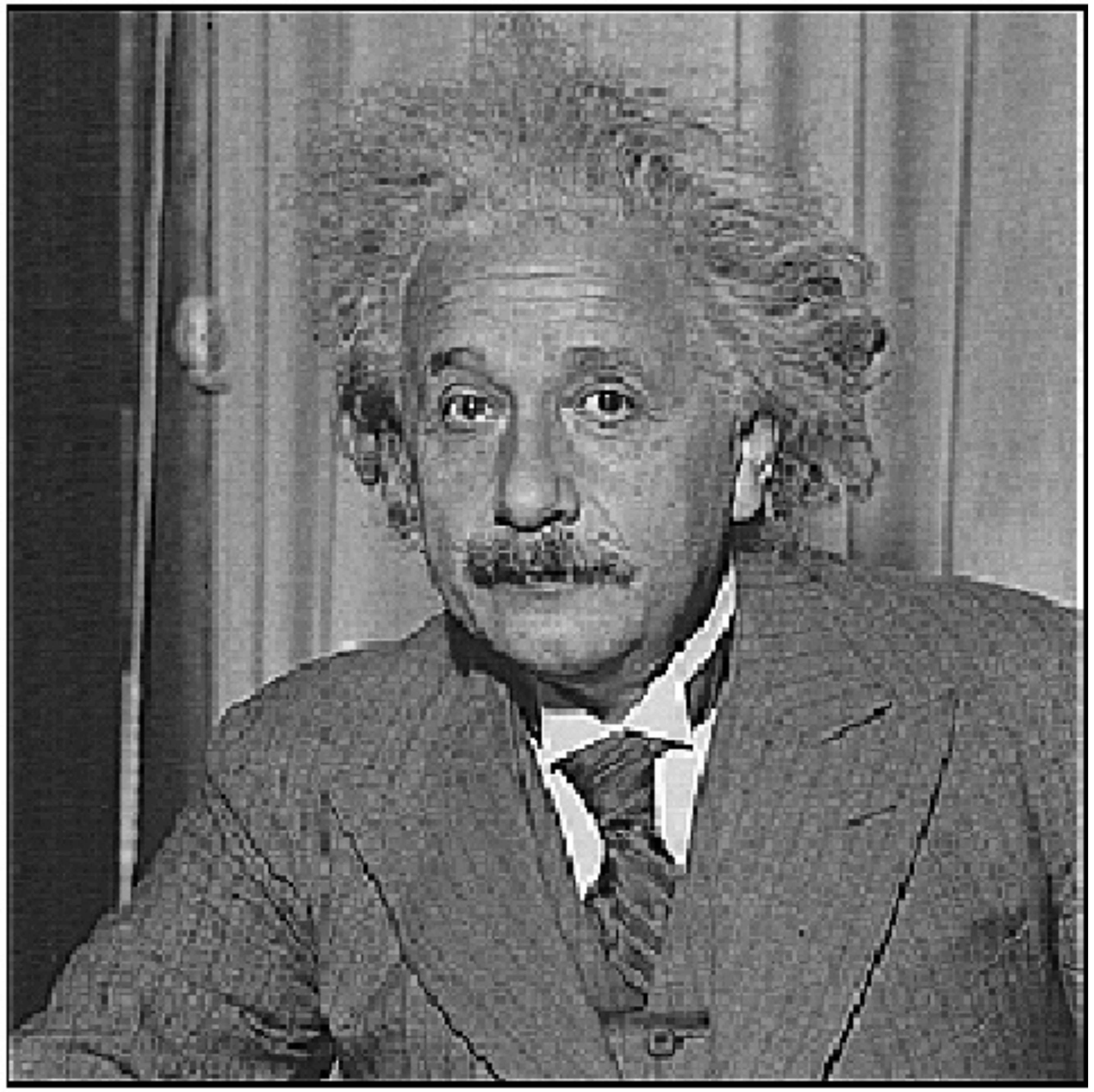
Filter
(filter sums to 1)

Result
(sharpening)

Example 4: Sharpening



Before



After

Example 4: Sharpening

Before

After

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Filters: Correlation vs. Convolution

Definition: **Correlation**

$$I'(X,Y) = \sum_{j=-k}^{k} \sum_{i=-k}^{k} F(i,j) I(X+i, Y+j)$$

Linear Filters: Correlation vs. Convolution

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Definition: **Convolution**

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Linear Filters: Correlation vs. Convolution

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a	b	c
d	e	f
g	h	i

Filter

1	2	3
4	5	6
7	8	9

Image

	$\boxed{}$	

Output

= 1a + 2b + 3c
+ 4d + 5e + 6f
+ 7g + 8h + 9i

Linear Filters: Correlation vs. Convolution

Definition: **Correlation**

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a	b	c
d	e	f
g	h	i

Filter

1	2	3
4	5	6
7	8	9

Image

Output

= 9a + 8b + 7c
+ 6d + 5e + 4f
+ 3g + 2h + 1i

Linear Filters: Correlation vs. Convolution

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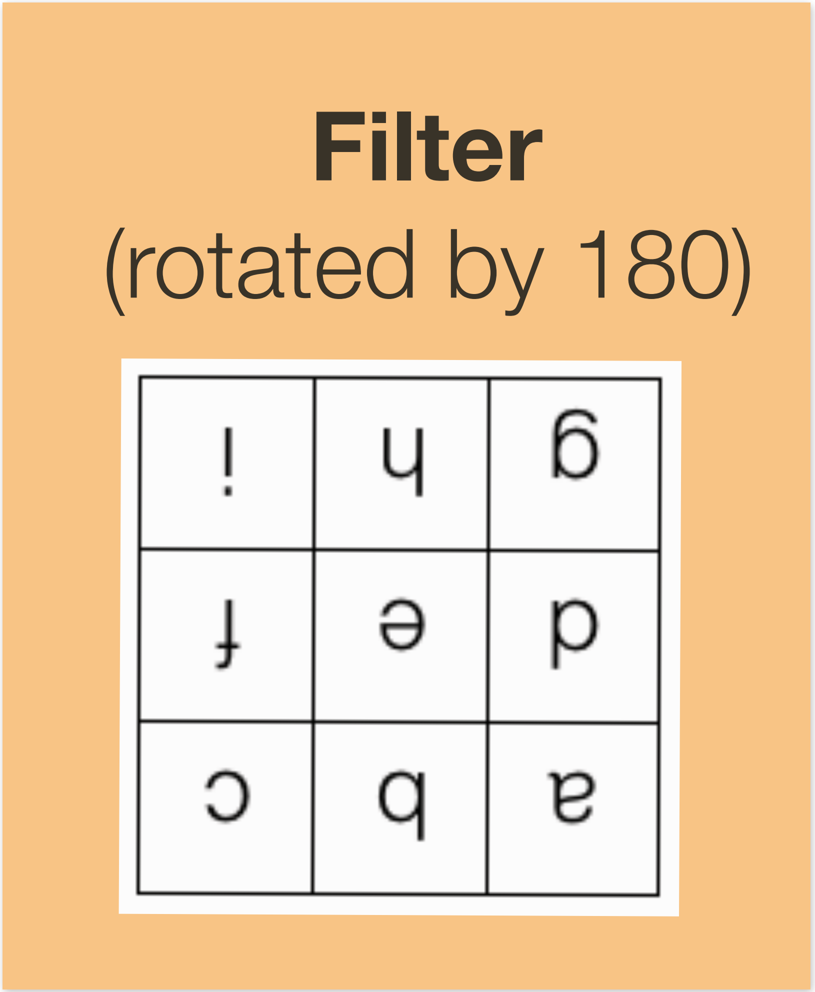
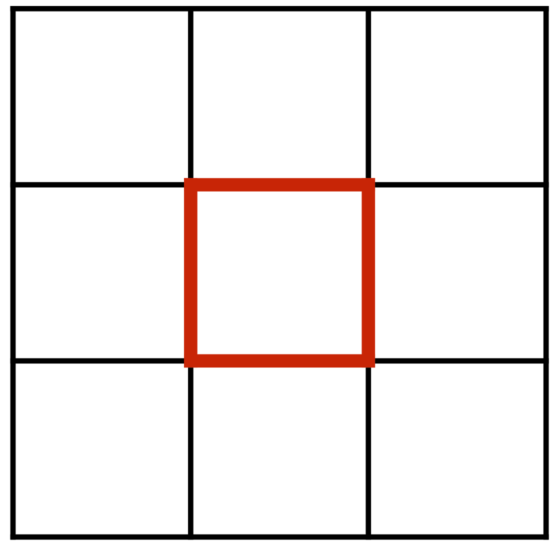


a	b	c
d	e	f
g	h	i

Filter

1	2	3
4	5	6
7	8	9

Image



Output

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Linear Filters: Correlation vs. Convolution

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Definition: **Convolution**

$$I'(X,Y) = \sum_{j=-k}^{k} \sum_{i=-k}^{k} F(i,j) I(X-i, Y-j)$$

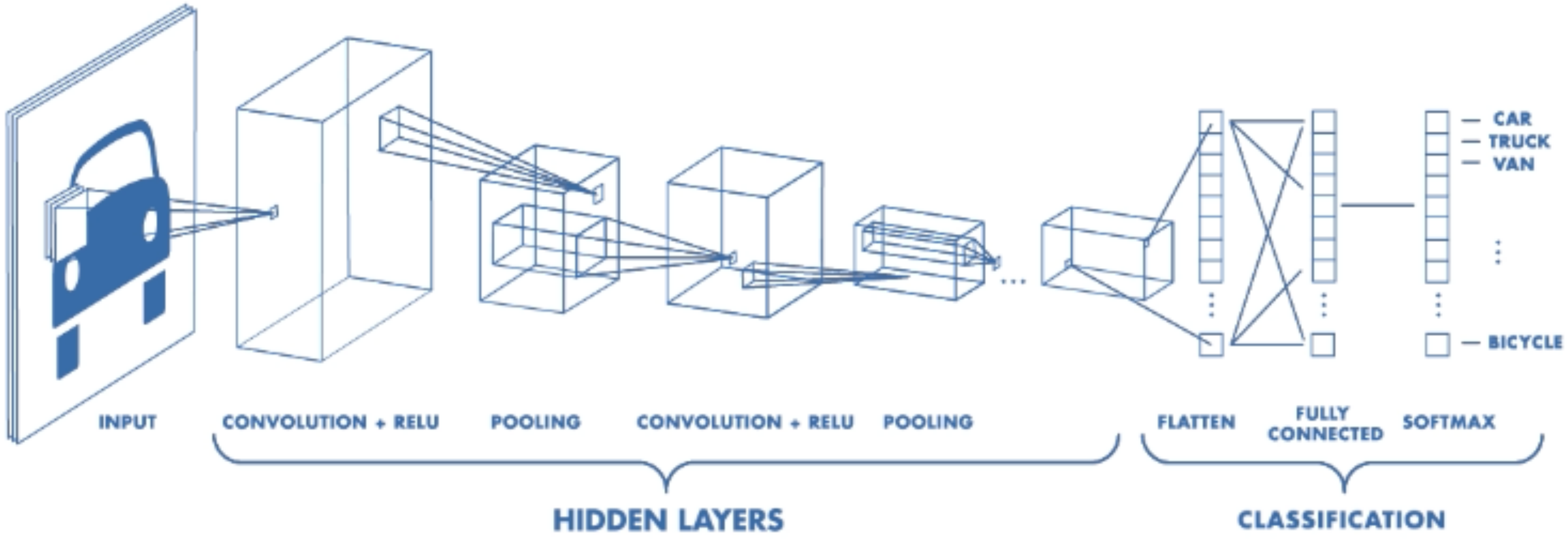
$$= \sum_{j=-k}^{k} \sum_{i=-k}^{k} F(-i,-j) I(X+i, Y+j)$$

Note: if $F(X,Y) = F(-X,-Y)$ then correlation = convolution.

Preview: Why convolutions are important?

Who has heard of **Convolutional Neural Networks** (CNNs)?

What about **Deep Learning**?



Basic operations in CNNs are convolutions (with learned linear filters) followed by non-linear functions.

Note: This results in non-linear filters.

Linear Filters: **Properties**

Let $\otimes$ denote convolution. Let $I(X,Y)$ be a digital image

Superposition: Let F_1 and F_2 be digital filters

$$(F_1 + F_2) \otimes I(X,Y) = F_1 \otimes I(X,Y) + F_2 \otimes I(X,Y)$$

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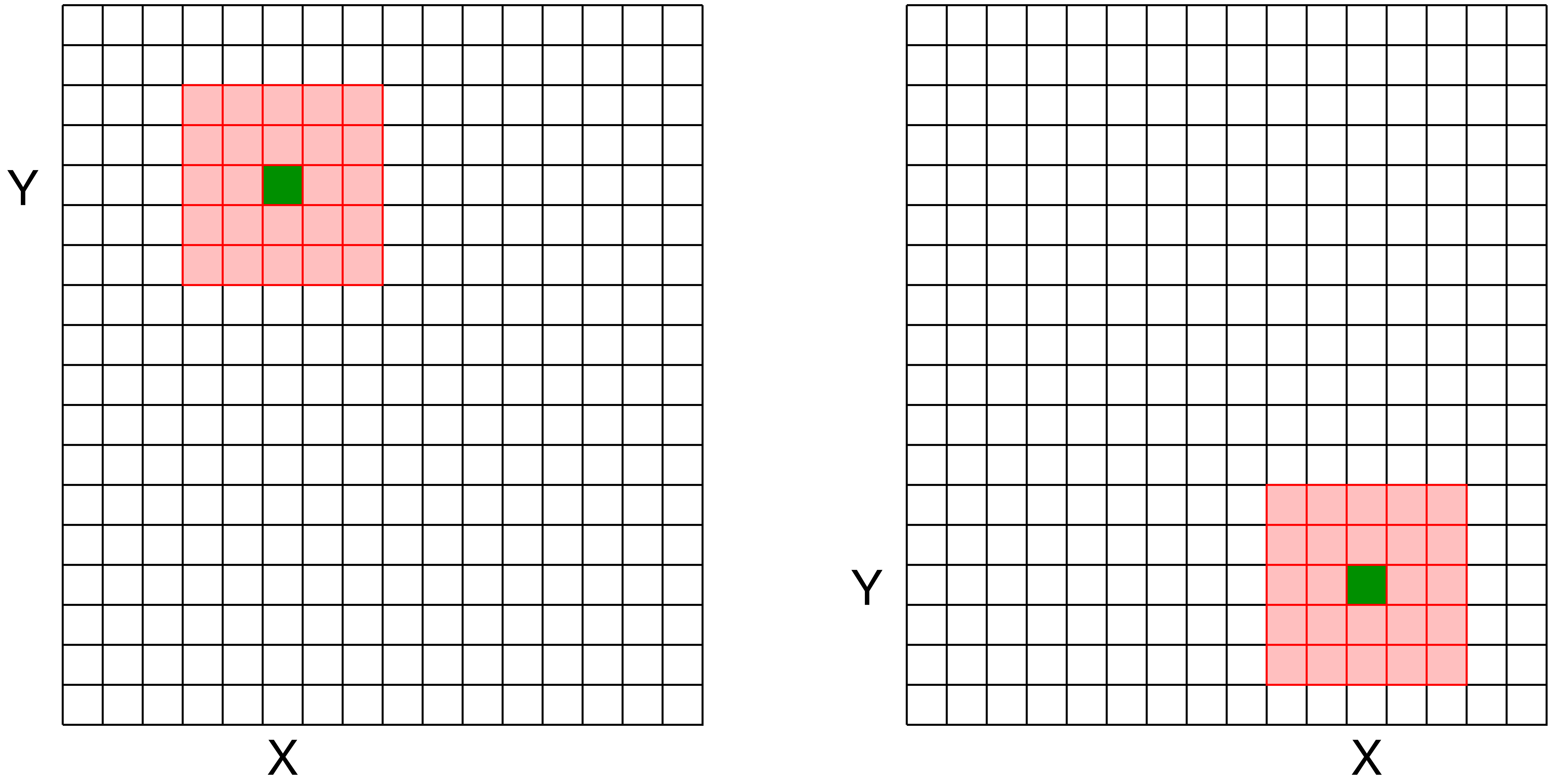
Shift Invariance: Output is local (i.e., no dependence on absolute position)

Linear Filters: Shift Invariance

Output does **not** depend on absolute position



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Shift Invariance: Output is local (i.e., no dependence on absolute position)

An operation is **linear** if it satisfies both **superposition** and **scaling**

Linear Systems: Characterization Theorem

Any linear, shift invariant operation can be expressed as convolution

Example 5: Smoothing with a Box Filter

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

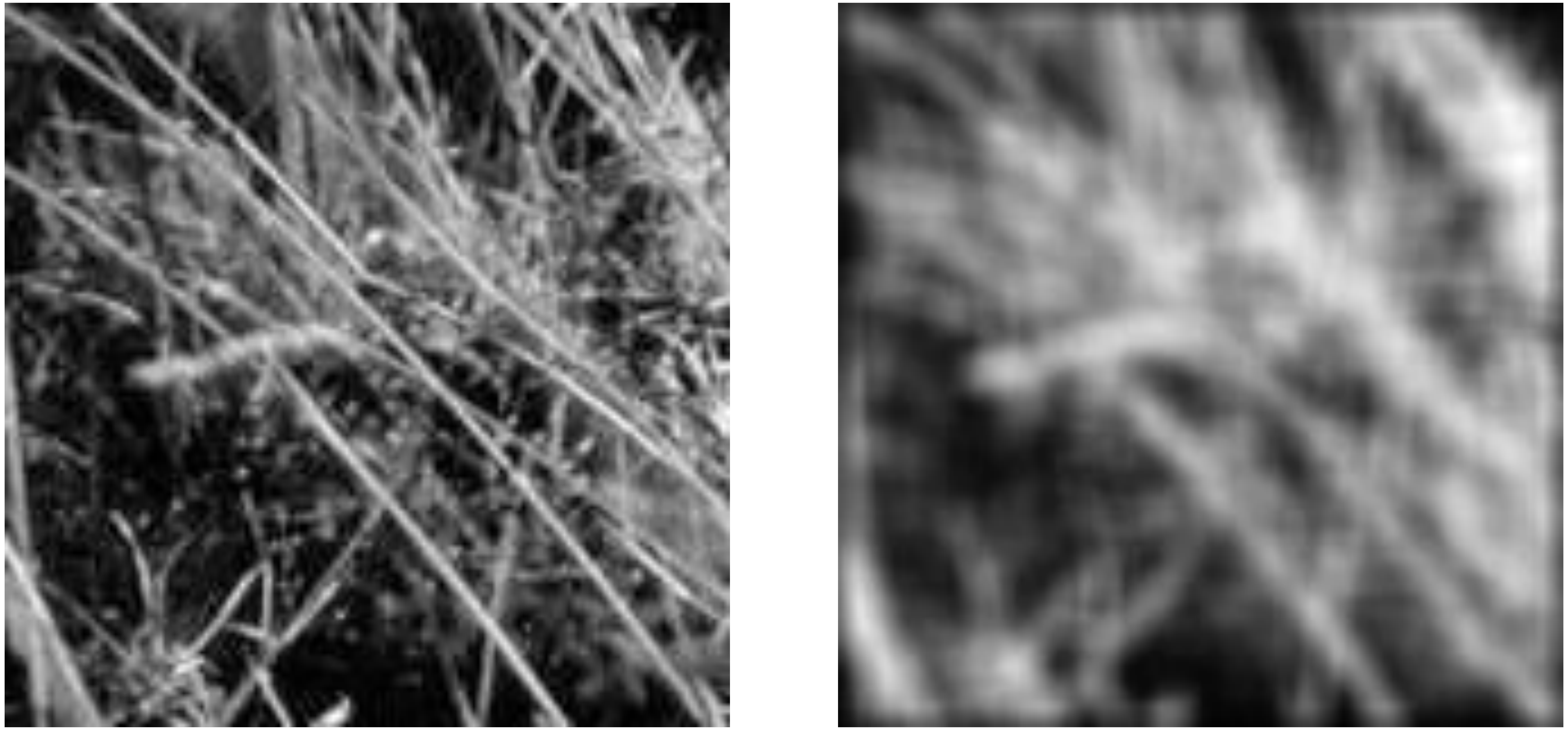
Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

Filter has equal positive values that some up to 1

Replaces each pixel with the average of itself and its local neighborhood

— Box filter is also referred to as **average filter** or **mean filter**

Example 5: Smoothing with a Box Filter



Forsyth & Ponce (2nd ed.) Figure 4.1 (left and middle)

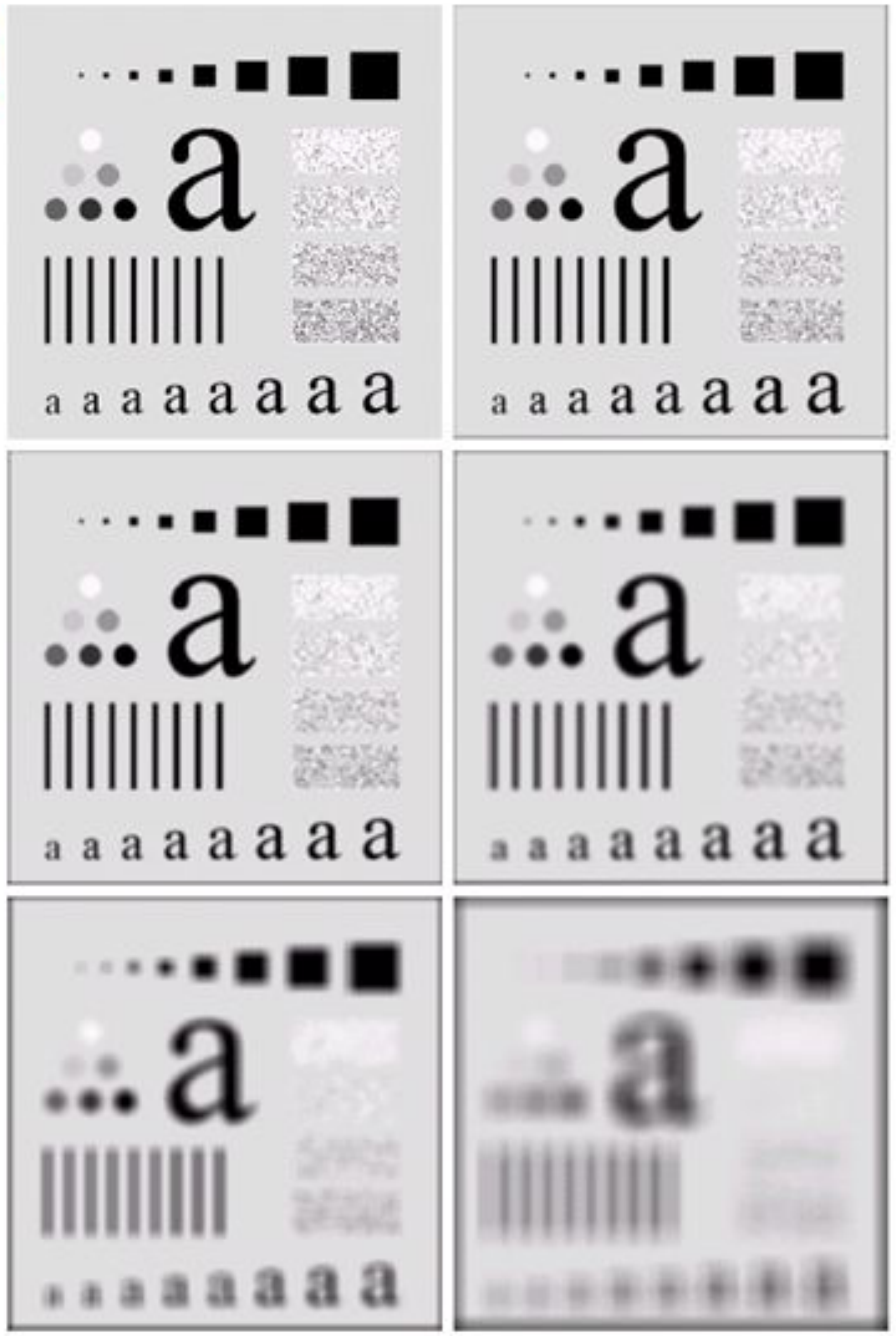
Example 5: Smoothing with a Box Filter

What happens if we increase the width (size) of the box filter?

Example 5: Smoothing with a Box Filter

Original

3x3



5x5

9x9

15x15

35x35

Gonzales & Woods (3rd ed.) Figure 3.3

Example 6: Smoothing with a Gaussian

Smoothing with a box **doesn’t model lens defocus** well

— Smoothing with a box filter depends on direction

— Image in which the center point is 1 and every other point is 0

Example 6: Smoothing with a Gaussian

Smoothing with a box **doesn't model lens defocus** well

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— Image in which the center point is 1 and every other point is 0

$$\frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

Filter

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

Image

Example 6: Smoothing with a Gaussian

Smoothing with a box **doesn't model lens defocus** well

— Smoothing with a box filter depends on direction

— Image in which the center point is 1 and every other point is 0

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

Filter

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

Image

0	0	0	0	0
0	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	0
0	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	0
0	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	0
0	0	0	0	0

Result

Example 6: Smoothing with a Gaussian

Smoothing with a box **doesn't model lens defocus** well

— Smoothing with a box filter depends on direction

— Image in which the center point is 1 and every other point is 0

Smoothing with a (circular) **pillbox** is a better model for defocus (in geometric optics)

The **Gaussian** is a good general smoothing model

— for phenomena (that are the sum of other small effects)

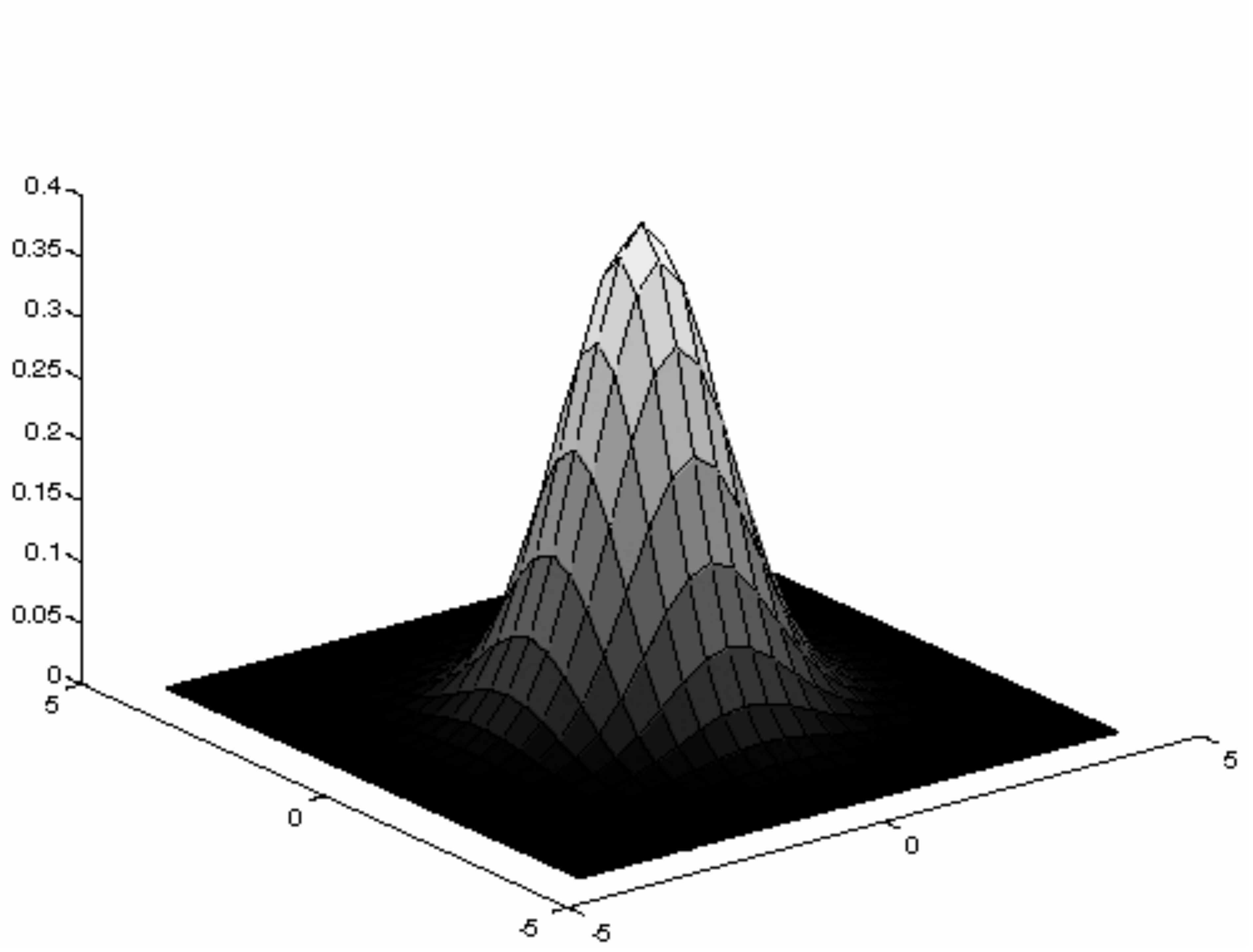
— whenever the Central Limit Theorem applies

Example 6: Smoothing with a Gaussian

Idea: Weight contributions of pixels by spatial proximity (nearness)

2D **Gaussian** (continuous case):

$$G_\sigma(x, y) = \frac{1}{2\pi\sigma^2} \exp^{-\frac{x^2+y^2}{2\sigma^2}}$$



Forsyth & Ponce (2nd ed.)
Figure 4.2

Summary

— The **correlation** of $F(X,Y)$ and $I(X,Y)$ is:

$$I'(X,Y) = \sum_{j=-k}^{k} \sum_{i=-k}^{k} F(i,j) I(X+i, Y+j)$$

— **Visual interpretation**: Superimpose the filter F on the image I at (X,Y), perform an element-wise multiply, and sum up the values

— **Convolution** is like **correlation** except filter "flipped"

if $F(X,Y) = F(-X,-Y)$ then correlation = convolution.

— **Characterization Theorem**: Any linear, spatially invariant operation can be expressed as a convolution