Lecture 2: Image Formation

( unless otherwise stated slides are taken or adopted from Bob Woodham, Jim Little and Fred Tung )
Updates:

- **Canvas** should now be working now
- TA office hours are out
- **Assignment 0** is out (optional)
- Registration

Reminders:

- **WWW**: assignments, lecture notes, readings
- **Piazza**: discussion (lecture notes and assignment will also be posted here)
- **Canvas**: assignment hand in and grading
Menu for Today (January 8, 2019)

Topics:

— Image Formation
— Cameras and Lenses
— Projection
— Human eye (as camera)

Redings:

— Today’s Lecture: Forsyth & Ponce (2nd ed.) 1.1.1 — 1.1.3
— Next Lecture: Forsyth & Ponce (2nd ed.) 4.1, 4.5

Reminders:

— Complete Assignment 0 (ungraded) by Friday, January 11
— WWW: http://www.cs.ubc.ca/~lsigal/teaching.html
— Piazza: piazza.com/ubc.ca/winterterm12018/cpsc425
Today’s “fun” Example

Photo credit: reddit user Liammm
Today’s “fun” Example: **Eye Sink Illusion**

Photo credit: reddit user Liammm
Today’s “fun” Example: **Eye Sink Illusion**

“Tried taking a picture of a sink draining, wound up with a picture of an eye instead”

Photo credit: reddit user Liammm
Lecture 1: Re-cap

Types of computer vision problems:

- Computing properties of the 3D world from visual data (measurement)
- Recognition of objects and scenes (perception and interpretation)
- Search and interact with visual data (search and organization)
- Manipulation or creation of image or video content (visual imagination)

Computer vision challenges:

- Fundamentally ill-posed
- Enormous computation and scale
- Lack of fundamental understanding of how human perception works
Computer vision technologies have moved from research labs into commercial products and services. Examples cited include:

- broadcast television sports
- electronic games (Microsoft Kinect)
- biometrics
- image search
- visual special effects
- medical imaging
- robotics

... many others
Lecture 2: Goal

To understand how images are formed
The **image formation process** that produces a particular image depends on

- **Lightening** condition
- **Scene geometry**
- **Surface** properties
- **Camera optics**

Sensor (or eye) **captures amount of light** reflected from the object.
Graphics Review

- Source
- Normal
- Surface element
- Sensor
Surface reflection depends on both the viewing \((\theta_v, \phi_v)\) and illumination \((\theta_i, \phi_i)\) direction, with Bidirectional Reflection Distribution Function: \(\text{BRDF} (\theta_i, \phi_i, \theta_v, \phi_v)\)

\[
\text{BRDF} (\theta_i, \phi_i, \theta_v, \phi_v) = \frac{\rho d}{\pi}
\]

\textbf{Lambertian} surface:

\text{constant, called albedo}
Surface reflection depends on both the **viewing** \((\theta_v, \phi_v)\) and **illumination** \((\theta_i, \phi_i)\) direction, with Bidirectional Reflection Distribution Function: \(\text{BRDF}(\theta_i, \phi_i, \theta_v, \phi_v)\)

**Lambertian surface:**

\[
\text{BRDF}(\theta_i, \phi_i, \theta_v, \phi_v) = \frac{\rho d}{\pi}
\]

\[
L = \frac{\rho d}{\pi} I(\vec{i} \cdot \vec{n})
\]

**Slide adopted from:** Ioannis (Yannis) Gkioulekas (CMU)
Surface reflection depends on both the **viewing** $(\theta_v, \phi_v)$ and **illumination** $(\theta_i, \phi_i)$ direction, with Bidirectional Reflection Distribution Function: $\text{BRDF}(\theta_i, \phi_i, \theta_v, \phi_v)$

**Lambertian** surface:

$$\text{BRDF}(\theta_i, \phi_i, \theta_v, \phi_v) = \frac{\rho d}{\pi}$$

**Mirror** surface: all incident light reflected in one direction $(\theta_v, \phi_v) = (\theta_r, \phi_r)$

*Slide adopted from: Ioannis (Yannis) Gkioulekas (CMU)*
Let’s say we have a **sensor** ...
... and the **object** we would like to photograph.

What would an image taken like this look like?

Slide Credit: Ioannis (Yannis) Gkioulakes (CMU)
Bare-sensor imaging

real-world object

digital sensor (CCD or CMOS)

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Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)
Bare-sensor imaging

All scene points contribute to all sensor pixels

real-world object

digital sensor (CCD or CMOS)

Slide Credit: Ioannis (Yannis) Gkioulakas (CMU)
Bare-sensor imaging

All scene points contribute to all sensor pixels

**Slide Credit:** Ioannis (Yannis) Gkioulekas (CMU)
Pinhole Camera

real-world object

What would an image taken like this look like?

barrier (diaphragm)

pinhole (aperture)

digital sensor (CCD or CMOS)

Slide Credit: Ioannis (Yannis) Gkioulakas (CMU)
**Pinhole Camera**

real-world object

most rays are blocked

one makes it through

digital sensor (CCD or CMOS)

*Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)*
Pinhole Camera

Each scene point contributes to only one sensor pixel

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)
Camera Obscura (latin for “dark chamber”)

Reinerus Gemma-Frisius observed an eclipse of the sun at Louvain on January 24, 1544. He used this illustration in his book, “De Radio Astronomica et Geometrica,” 1545. It is thought to be the first published illustration of a camera obscura.

Credit: John H., Hammond, “Th Camera Obscure, A Chronicle”
First **Photograph** on Record

*La table servie*

*Credıt*: Nicéphore Niepce, 1822
Pinhole Camera

A pinhole camera is a box with a small hall (aperture) in it.

Forsyth & Ponce (2nd ed.) Figure 1.2
Image Formation

Forsyth & Ponce (2nd ed.) Figure 1.1

Credit: US Navy, Basic Optics and Optical Instruments. Dover, 1969
Accidental Pinhole Camera

Image Credit: Ioannis (Yannis) Gkioullekas (CMU)
Pinhole Camera (Simplified)

\[ f' \text{ is the } \textbf{focal length} \text{ of the camera} \]

![Diagram showing the relationship between the focal length (f'), object distance (z), and image plane size (x') in a pinhole camera.]

**Note:** In a pinhole camera we can adjust the focal length, all this will do is change the size of the resulting image.
It is convenient to think of the **image plane** which is in front of the pinhole.

What happens if object moves towards the camera? Away from the camera?
Perspective Effects

Far objects appear smaller than close ones

Size is inversely proportional to distance
Parallel lines meet at a point (vanishing point)
Vanishing Points

Draw a horizon line.
Vanishing Points

Draw a horizon line.

Make a vanishing point.
Vanishing Points

1. Draw a horizon line.
2. Make a vanishing point.
3. Draw a square or rectangle.
4. Draw orthogonals from shape corners to vanishing point.
5. Draw a horizontal line to end your form.
6. Draw a vertical line to make the form’s side.
7. Erase the orthogonals.
8. Draw another form!
9. Add windows and doors.

Slide Credit: David Jacobs
Vanishing Points

Each set of parallel lines meet at a different point
— the point is called \textit{vanishing point}
Vanishing Points

Each set of parallel lines meet at a different point
— the point is called **vanishing point**

Sets of parallel lines one the same plane lead to **collinear** vanishing points
— the line is called a **horizon** for that plane
Vanishing Points

Each set of parallel lines meet at a different point
— the point is called **vanishing point**

Sets of parallel lines on the same plane lead to **collinear** vanishing points
— the line is called a **horizon** for that plane

Good way to **spot fake images**
— scale and perspective do not work
— vanishing points behave badly
Vanishing Points

One point perspective

Add windows and doors.

Two point perspective

Vertical vanishing point (at infinity)

Vanishing point

Vanishing point

Slide Credit: Efros (Berkeley), photo from Criminisi
Perspective Aside

Image credit: http://www.martinacecilia.com/place-vanishing-points/
Properties of Projection

- Points project to points
- Lines project to lines
- Planes project to the whole or half image
- Angles are not preserved
Properties of Projection

- **Points** project to **points**
- **Lines** project to **lines**
- **Planes** project to the **whole** or **half** image
- Angles are **not** preserved

**Degenerate cases**

- Line through focal point projects to a point
- Plane through focal point projects to a line
Projection Illusion
Perspective Projection

3D object point

\[ P = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \]

projects to 2D image point

\[ P' = \begin{bmatrix} x' \\ y' \end{bmatrix} \]

where

\[ x' = f' \frac{x}{z} \]
\[ y' = f' \frac{y}{z} \]

Note: this assumes world coordinate frame at the optical center (pinhole) and aligned with the image plane, image coordinate frame aligned with the camera coordinate frame
Perspective Projection: Proof

Forsyth & Ponce (1st ed.) Figure 1.4

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Perspective Projection: Proof

Forsyth & Ponce (1st ed.) Figure 1.4

Note: this assumes world coordinate frame at the optical center (pinhole) and aligned with the image plane, image coordinate frame aligned with the camera coordinate frame
Weak Perspective

3D object point \( P = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \) in \( \Pi_0 \) projects to 2D image point \( P' = \begin{bmatrix} x' \\ y' \end{bmatrix} \)

where \( x' = mx \) and \( y' = my \) and \( m = \frac{f'}{z_0} \)

Forsyth & Ponce (1st ed.) Figure 1.5
**Orthographic Projection**

![Orthographic Projection Diagram](image)

Forsyth & Ponce (1st ed.) Figure 1.6

3D object point \( P = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \) projects to 2D image point \( P' = \begin{bmatrix} x' \\ y' \end{bmatrix} \)

where

\[
\begin{align*}
x' &= x \\
y' &= y
\end{align*}
\]
Summary of **Projection Equations**

3D object point \( P = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \) projects to 2D image point \( P' = \begin{bmatrix} x' \\ y' \end{bmatrix} \) where

<table>
<thead>
<tr>
<th>Type</th>
<th>Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perspective</td>
<td>( x' = f' \frac{x}{z} )  ( y' = f' \frac{y}{z} )</td>
</tr>
<tr>
<td>Weak Perspective</td>
<td>( x' = m x ) ( y' = m y ) ( m = \frac{f'}{z_0} )</td>
</tr>
<tr>
<td>Orthographic</td>
<td>( x' = x ) ( y' = y )</td>
</tr>
</tbody>
</table>
Projection Models: Pros and Cons

**Weak perspective** (including orthographic) has simpler mathematics

— accurate when object is small and/or distant

— useful for recognition

**Perspective** is more accurate for real scenes

When **maximum accuracy** is required, it is necessary to model additional details of a particular camera

— use perspective projection with additional parameters (e.g., lens distortion)
Why **Not** a Pinhole Camera?

— If pinhole is **too big** then many directions are averaged, blurring the image

— If pinhole is **too small** then diffraction becomes a factor, also blurring the image

— Generally, pinhole cameras are **dark**, because only a very small set of rays from a particular scene point hits the image plane

— Pinhole cameras are **slow**, because only a very small amount of light from a particular scene point hits the image plane per unit time

**Image Credit:** Credit: E. Hecht. “Optics,” Addison-Wesley, 1987
Snell’s Law

\[ n_1 \sin \alpha_1 = n_2 \sin \alpha_2 \]
Snell’s Law

\[ n_1 \sin \alpha_1 = n_2 \sin \alpha_2 \]
Reason for **Lenses**

The role of a lens is to **capture more light** while preserving, as much as possible, the abstraction of an ideal pinhole camera.
Pinhole Model (Simplified) with Lens
Thin Lens Equation

\[
\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}
\]

Forsyth & Ponce (1st ed.) Figure 1.9
Thin Lens Equation: Derivation

\[
\frac{y}{-z} = \frac{-y'}{z'}
\]
\[
\frac{y}{y'} = \frac{z}{z'}
\]

Forsyth & Ponce (1st ed.) Figure 1.9

\[
\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}
\]
Thin Lens Equation: Derivation

\[ \frac{y}{-z} = \frac{-y'}{z'} \]
\[ \frac{y}{y'} = \frac{z}{z'} \]

\[ \frac{-y'}{f} = \frac{y - y'}{-z} \]
\[ \frac{1}{f} = \frac{y - y'}{zy'} \]
\[ = \frac{y}{zy'} - \frac{y'}{zy'} = \frac{1}{z} \]

\[ \frac{1}{z'} - \frac{1}{z} = \frac{1}{f} \]
Thin Lens Equation: Derivation

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Forsyth & Ponce (1st ed.) Figure 1.9

\[
\frac{-y'}{f} = \frac{y - y'}{-z}
\]

\[
\frac{1}{f} = \frac{y - y'}{zy'}
\]

\[
= \frac{y}{zy'} - \frac{y'}{zy'}
\]

\[
= \frac{y}{zy'} - \frac{1}{z}
\]

Substitute: \[
\frac{1}{f} = \frac{1}{z} - \frac{1}{z'} - \frac{1}{z}
\]

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Focal Length

Another way of looking at the focal length of a lens. The incoming rays, parallel to the optical axis, converge to a single point a distance $f$ behind the lens. This is where we want to place the image plane.
Out-of-Focus

The image plane is in the wrong place, either slightly closer than the required focal length, f, or slightly further than the required focal length, f.
Spherical Aberration

Forsyth & Ponce (1st ed.) Figure 1.12a
A modern camera lens may contain multiple components, including aspherical elements.
Vignetting

Vignetting in a two-lens system

Forsyth & Ponce (2nd ed.) Figure 1.12

The shaded part of the beam *never reaches* the second lens
Vignetting
Chromatic Aberration

- Index of **refraction depends on wavelength**, \( \lambda \), of light
- Light of different colours follows different paths
- Therefore, not all colours can be in equal focus

Image Credit: Trevor Darrell
Other (Possibly Significant) **Lens Effects**

**Chromatic aberration**
- Index of refraction depends on wavelength, $\lambda$, of light
- Light of different colours follows different paths
- Therefore, not all colours can be in equal focus

**Scattering** at the lens surface
- Some light is reflected at each lens surface

There are other **geometric phenomena/distortions**
- pincushion distortion
- barrel distortion
- etc
Fish-eye Lens

Szeliski (1st ed.) Figure 2.13

Lines in the world are no longer lines on the image, they are curves!
Human Eye

- The eye has an iris (like a camera)
- Focusing is done by changing shape of lens
- When the eye is properly focused, light from an object outside the eye is imaged on the retina
- The retina contains light receptors called rods and cones

**pupil** = pinhole / aperture

**retina** = film / digital sensor

Slide adopted from: Steve Seitz
Human Eye

— The eye has an iris (like a camera)

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pupil = pinhole / aperture

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Slide adopted from: Steve Seitz
Two-types of **Light Sensitive Receptors**

**Rods**
- 75-150 million rod-shaped receptors
- *not* involved in color vision, gray-scale vision only
- operate at night
- highly sensitive, can responding to a single photon
- yield relatively poor spatial detail

**Cones**
- 6-7 million cone-shaped receptors
- color vision
- operate in high light
- less sensitive
- yield higher resolution

*Slide adopted from: James Hays*
Human Eye

Density of rods and cones

Density of rods and cones

Slide adopted from: James Hays
Lecture Summary

— We discussed a “physics-based” approach to image formation. Basic abstraction is the pinhole camera.

— **Lenses overcome limitations** of the pinhole model while trying to preserve it as a useful abstraction

— Projection equations: perspective, weak perspective, orthographic

— Thin lens equation

— Some “aberrations and distortions” persist (e.g. spherical aberration, vignetting)

— The **human eye** functions much like a camera
Reminders

**Readings:**

- *Today’s* Lecture: Forsyth & Ponce (2nd ed.) 1.1.1 — 1.1.3
- *Next* Lecture: Forsyth & Ponce (2nd ed.) 4.1, 4.5

**Reminders:**

- Complete **Assignment 0** (ungraded) by Wednesday, **September 12**
- **WWW:** [http://www.cs.ubc.ca/~lsigal/teaching.html](http://www.cs.ubc.ca/~lsigal/teaching.html)
- **Piazza:** [piazza.com/ubc.ca/winterterm12018/cpsc425](http://piazza.com/ubc.ca/winterterm12018/cpsc425)