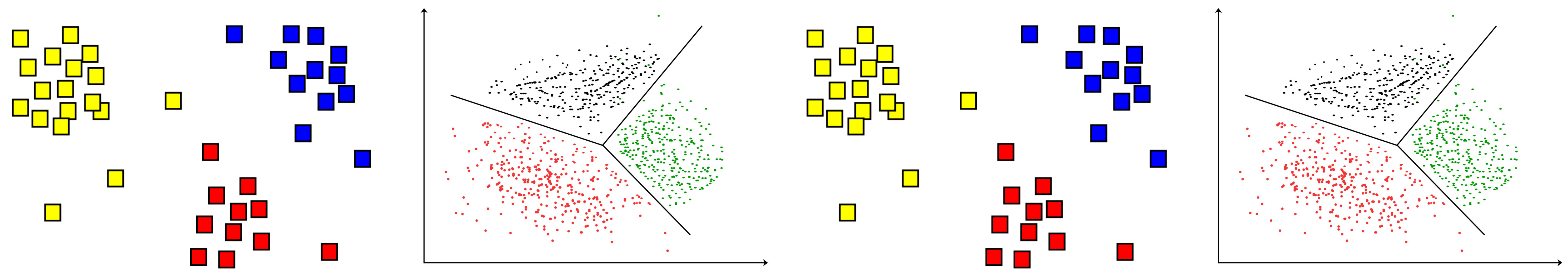
THE UNIVERSITY OF BRITISH COLUMBIA

CPSC 425: Computer Vision



Lecture 28 : Classification

Menu for Today (**November 14, 2018**)

Topics:

— Naive Bayes Classifier

— Bayes' Risk

— Error Measures, Cross Validation

— Nearest Neighbor Classifiers

Redings:

— **Today's** Lecture: Forsyth & Ponce (2nd ed.) 15.1, 15.2

— **Next** Lecture: Forsyth & Ponce (2nd ed.) 16.1.3, 16.1.4, 16.1.9

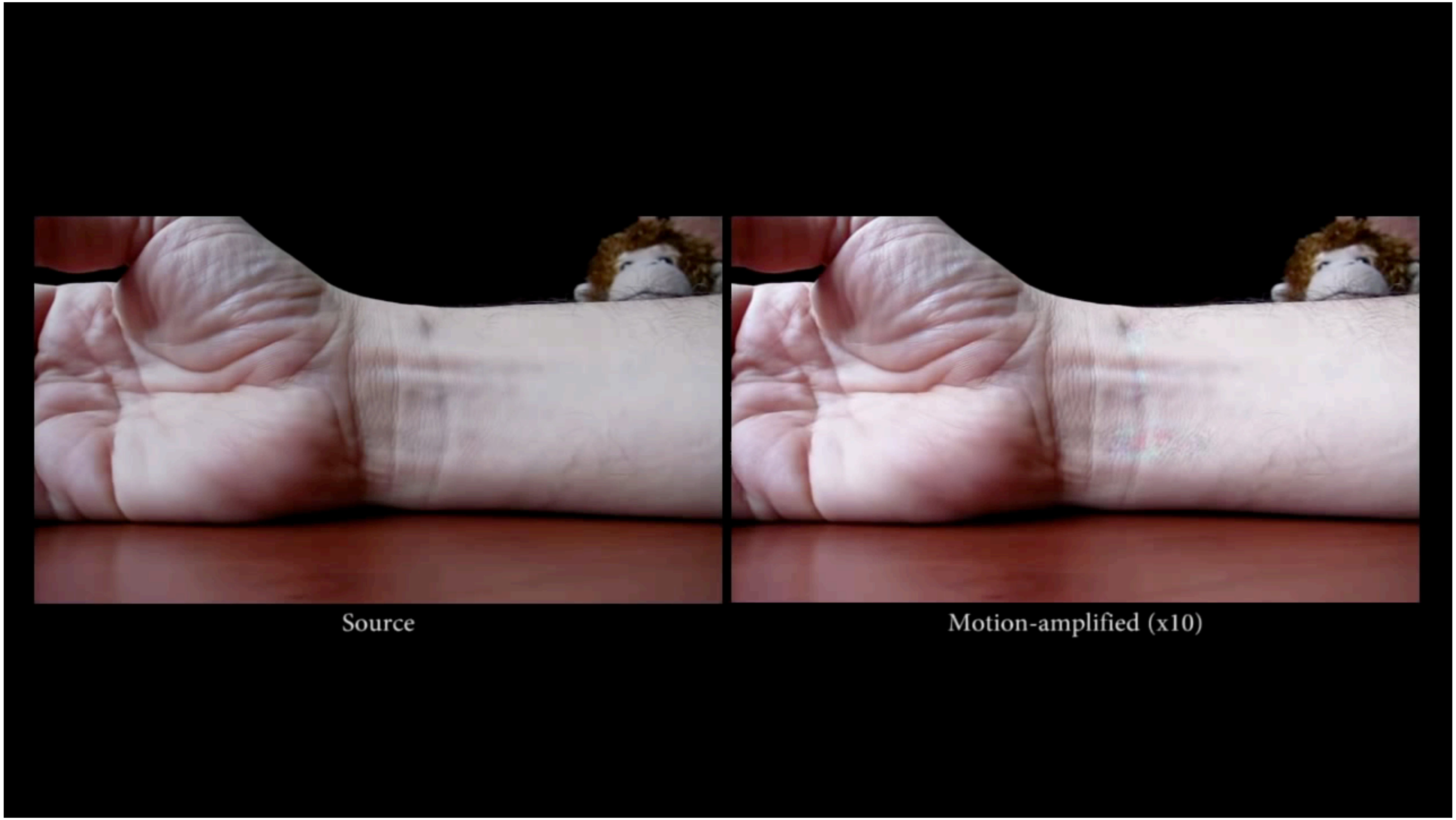
Reminders:

— **Assignment 4**: Local Invariant Features and RANSAC due **today**

— **Assignment 5**: Scene Recognition with Bag of Words due **last day of classes**

— Last week to pickup **Midterms**

Today's "**fun**" Example: Eulerian Video Magnification



Video From: Wu at al., Siggraph 2012

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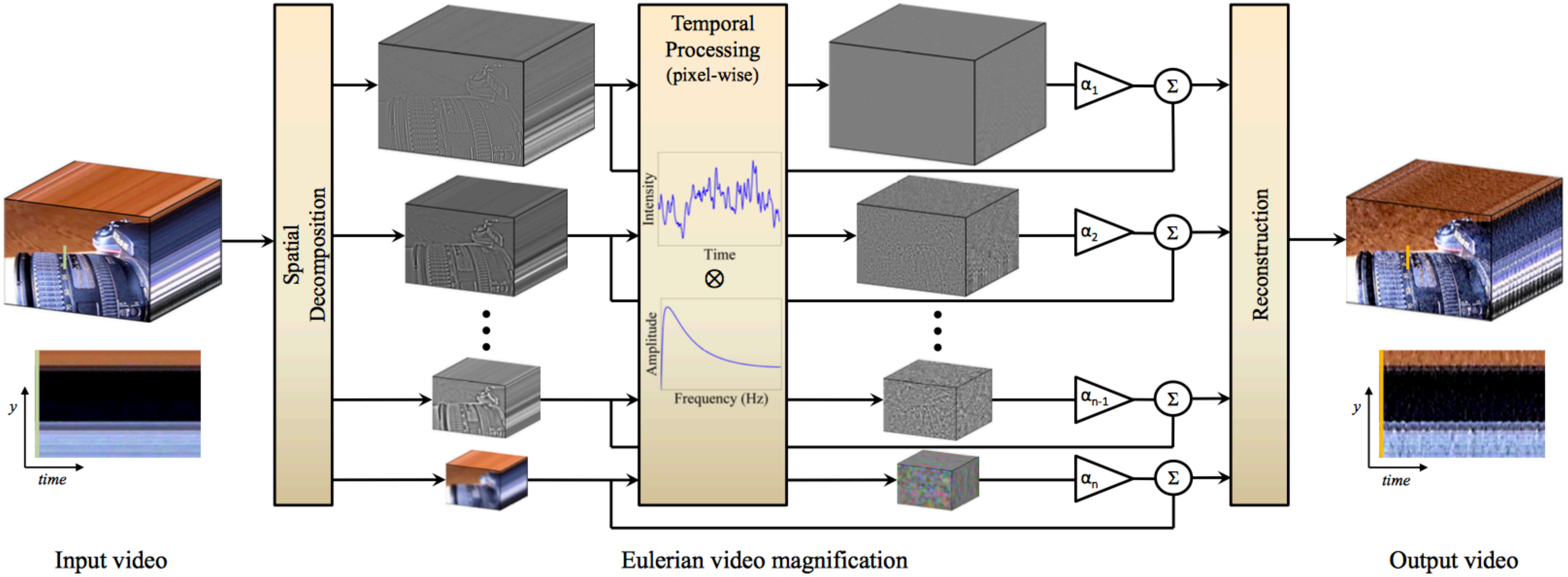

Figure From: Wu at al., Siggraph 2012

Lecture 27: Re-cap

— Collect a database of images with labels
— Use ML to train an image classifier
— Evaluate the classifier on test images

Label →

Feature vector computed from the image →

Lecture 27: Re-cap Bayes Rule

Let c be the class label and let x be the measurement (i.e., evidence)

class–conditional probability
(a.k.a. likelihood)

prior probability

$$P(c|x) = \frac{P(x|c)p(c)}{P(x)}$$

posterior probability

unconditional probability
(a.k.a. marginal likelihood)

Example: Discrete Bayes Classifier

Assume we have two classes: $c_1 = \textbf{male}$ $c_2 = \textbf{female}$

We have a person who's gender we don't know, who's name is *drew*

Example from: Eamonn Keogh

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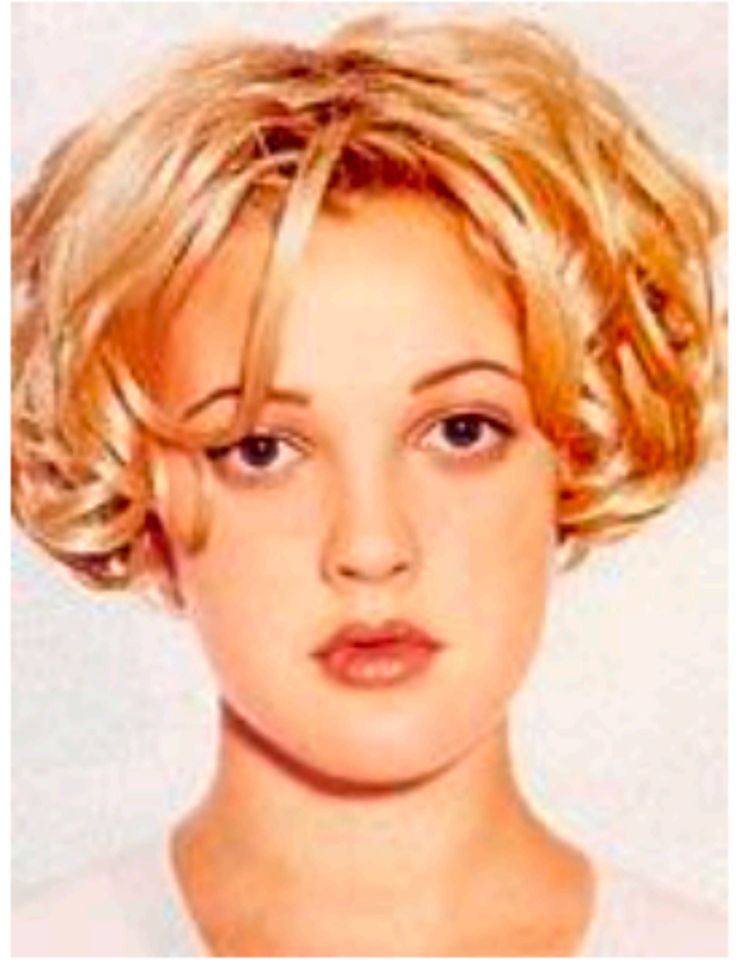
Drew Carey



Drew Barrymore

Example from: Eamonn Keogh

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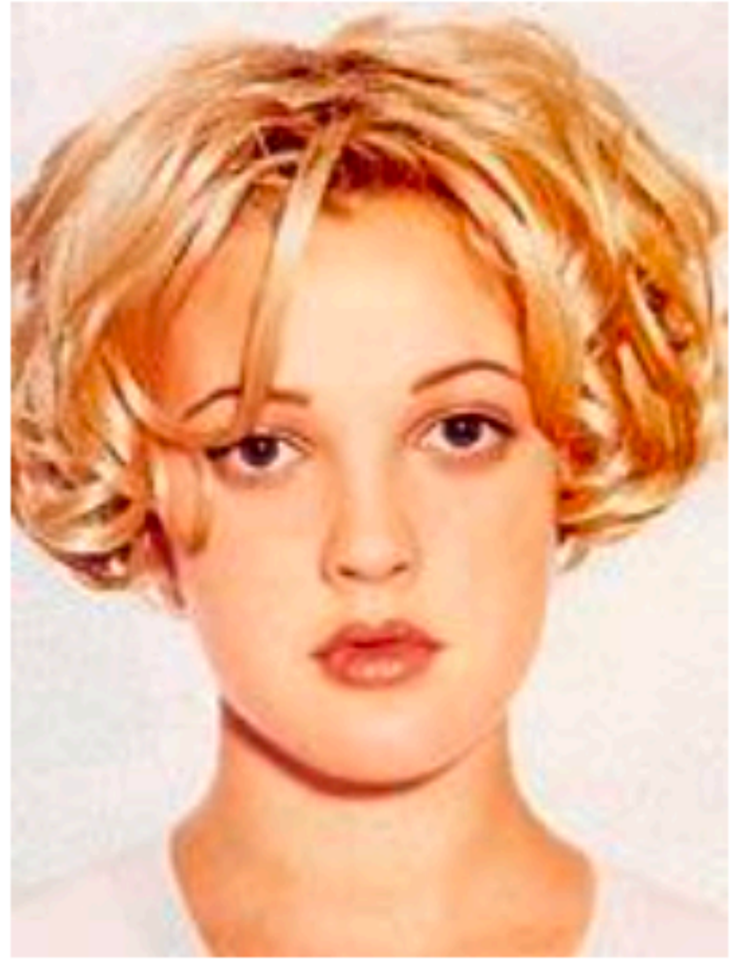
Assume we have two classes: $c_1 = \textbf{male}$ $c_2 = \textbf{female}$

We have a person who's gender we don't know, who's name is *drew*

Classifying drew as being male or female is equivalent to asking is it more probable that *drew* is male or female, i.e. which is greater $p(\textbf{male}|drew)$ $p(\textbf{female}|drew)$

Drew Carey



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We have a person who's gender we don't know, who's name is *drew*

Classifying drew as being male or female is equivalent to asking is it more probable that *drew* is male or female, i.e. which is greater $p(\textbf{male}|drew)$

$p(\textbf{female}|drew)$

$$p(\textbf{male}|drew) = \frac{p(drew|\textbf{male})p(\textbf{male})}{p(drew)}$$

Example from: Eamonn Keogh

Example: Discrete Bayes Classifier

Name	Gender
Drew	Male
Claudia	Female
Drew	Female
Drew	Female
Alberto	Male
Karin	Female
Nina	Female
Sergio	Male

$$p(\mathbf{male}|drew) = \frac{p(drew|\mathbf{male})p(\mathbf{male})}{p(drew)}$$

Example from: Eamonn Keogh

Example: Discrete Bayes Classifier

Name	Gender
Drew	Male
Claudia	Female
Drew	Female
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Sergio	Male

$$p(\mathbf{male}) =$$

$$p(drew|\mathbf{male}) =$$

$$p(drew) =$$

$$p(\mathbf{male}|drew) = \frac{p(drew|\mathbf{male})p(\mathbf{male})}{p(drew)}$$

Example from: Eamonn Keogh

Example: Discrete Bayes Classifier

Name	Gender
Drew	Male
Claudia	Female
Drew	Female
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Sergio	Male

$$p(\mathbf{male}) = \frac{3}{8}$$

$$p(drew|\mathbf{male}) =$$

$$p(drew) =$$

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$$p(drew|\mathbf{male}) = \frac{1}{3}$$

$$\cancel{p(drew) = \frac{3}{8}}$$

$$p(\mathbf{male}|drew) = \frac{p(drew|\mathbf{male})p(\mathbf{male})}{\cancel{p(drew)}} = 0.125$$

Example from: Eamonn Keogh

Example: Discrete Bayes Classifier

Name	Gender
Drew	Male
Claudia	Female
Drew	Female
Drew	Female
Alberto	Male
Karin	Female
Nina	Female
Sergio	Male

$$p(\textbf{male}) = \frac{3}{8} \qquad p(\textbf{female}) = \frac{5}{8}$$

$$p(drew|\textbf{male}) = \frac{1}{3} \qquad p(drew|\textbf{female}) = \frac{2}{5}$$

$$\cancel{p(drew) = \frac{3}{8}}$$

$$p(\textbf{male}|drew) = \frac{p(drew|\textbf{male})p(\textbf{male})}{\cancel{p(drew)}} = 0.125$$

$$p(\textbf{female}|drew) = \frac{p(drew|\textbf{female})p(\textbf{female})}{\cancel{p(drew)}} = 0.25$$

Example from: Eamonn Keogh

Bayes Rule (Review and Definitions)

Let c be the **class label** and let x be the **measurement** (i.e., evidence)

Simple case:

— binary classification; i.e., $c \in \{1, 2\}$

— features are 1D; i.e., $x \in \mathbb{R}$

$$P(c|x) = \frac{P(x|c)p(c)}{P(x)}$$

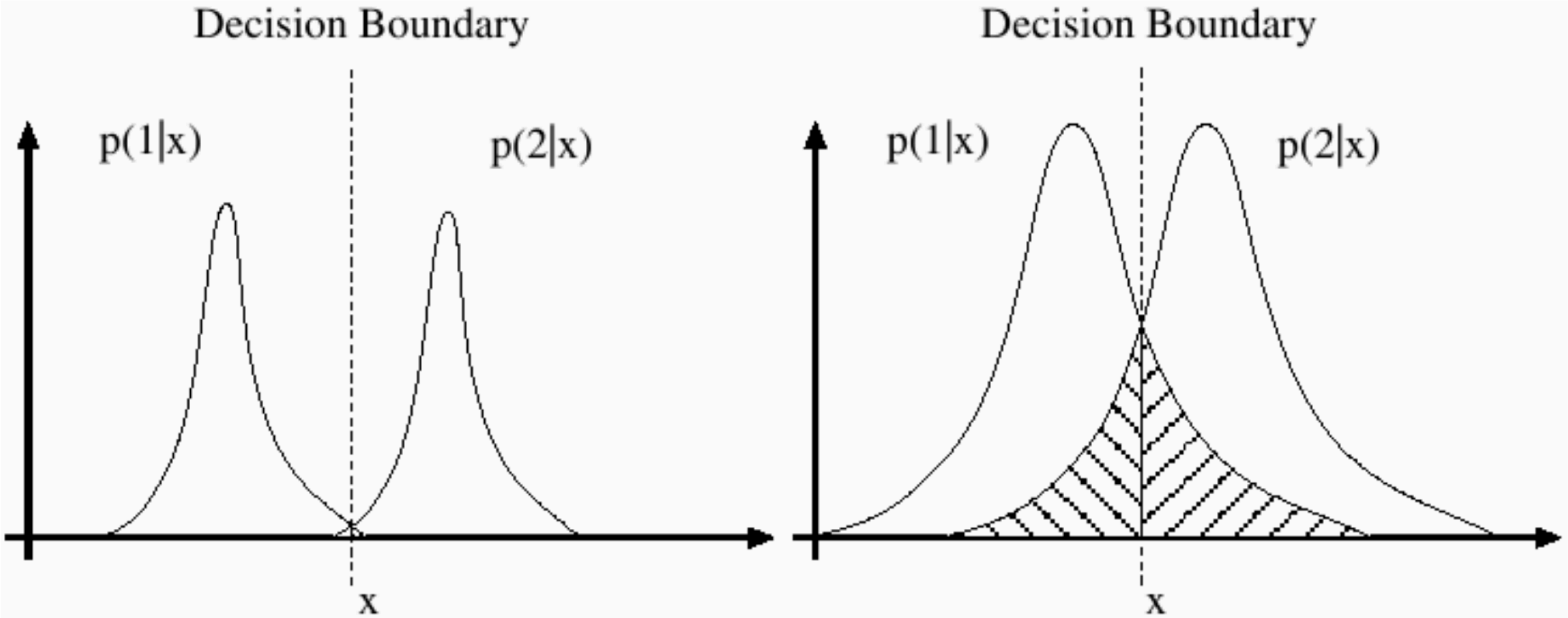
General case:

— multi-class; i.e., $c \in \{1, ..., 1000\}$

— features are high-dimensional; i.e., $x \in \mathbb{R}^{2,000+}$

Bayes' Risk

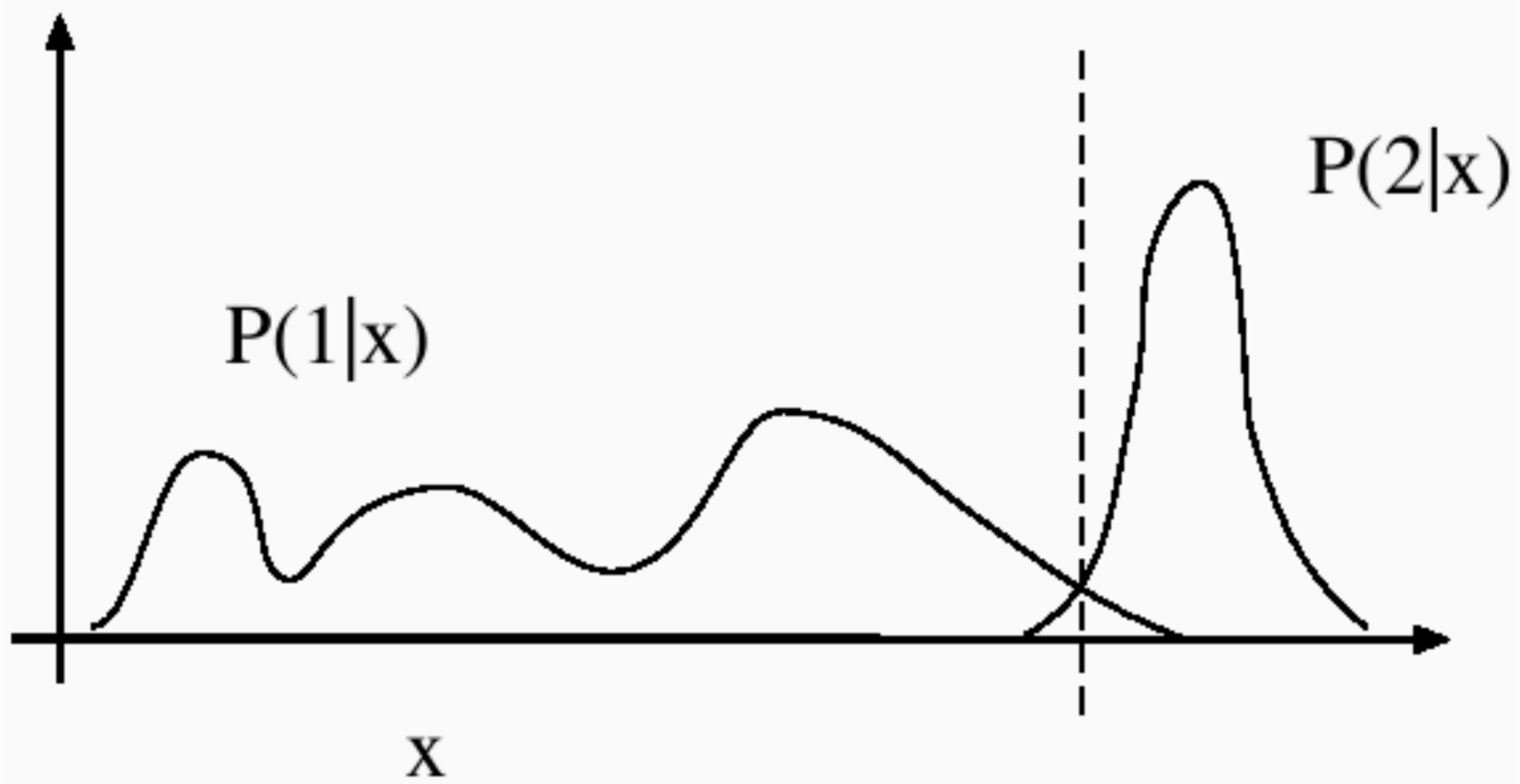
Some errors may be inevitable: the minimum risk (shaded area) is called the **Bayes' risk**



Forsyth & Ponce (2nd ed.) Figure 15.1

Discriminative vs. **Generative**

Finding a **decision boundary** is not the same as modeling a **conditional density** — while a normal density here is a poor fit to P(1|x), the quality of the classifier depends only on how well the boundary is positioned



Forsyth & Ponce (2nd ed.) Figure 15.5

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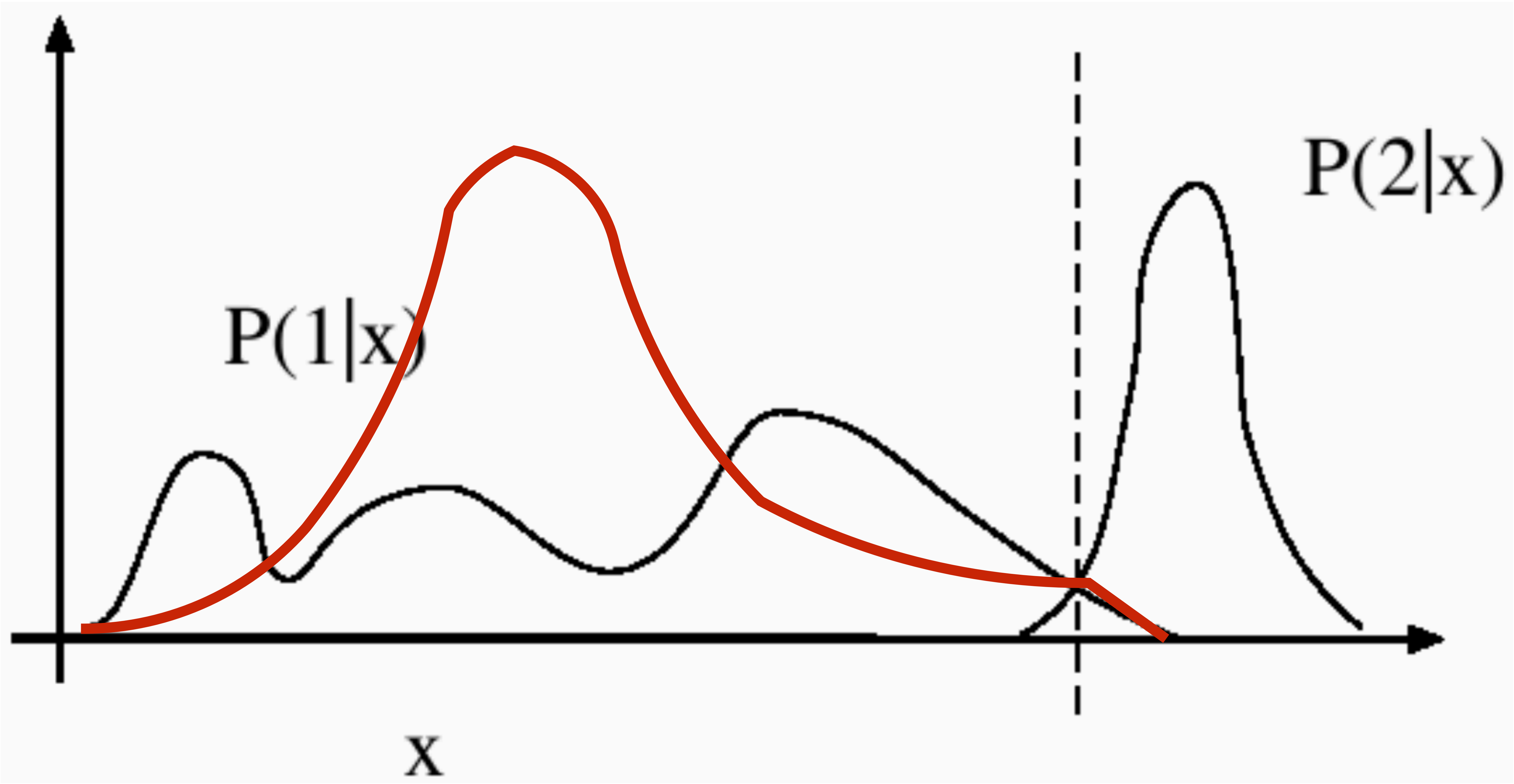


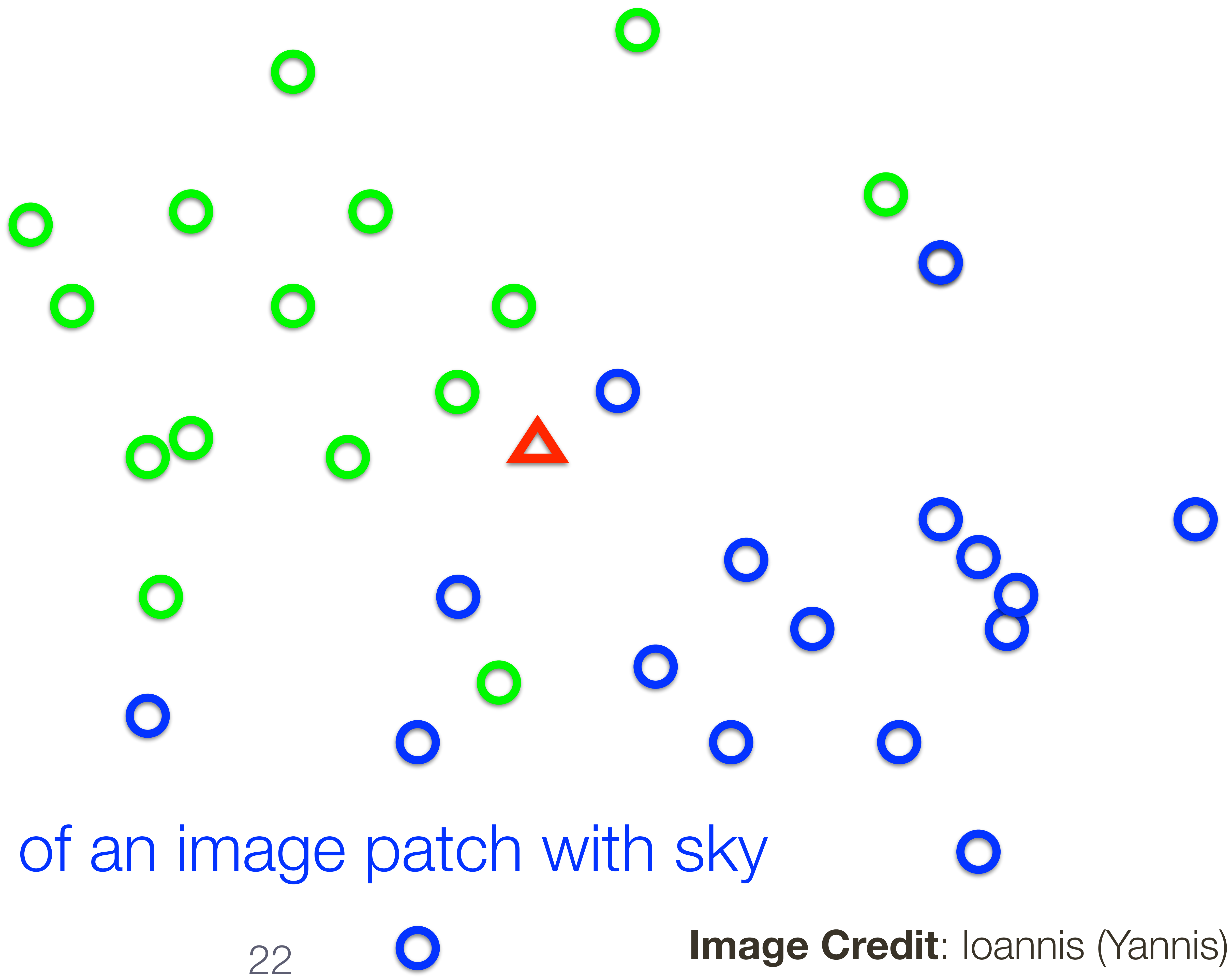
Forsyth & Ponce (2nd ed.) Figure 15.5

Example: 2D Bayes Classifier

- 17 samples (blue)
- 15 samples (green)

These could be (g,b) pixel value of an image patch with grass

Given a (g,b) pixel value from a new patch is it more likely to be be grass or sky?



These could be (g,b) pixel value of an image patch with sky

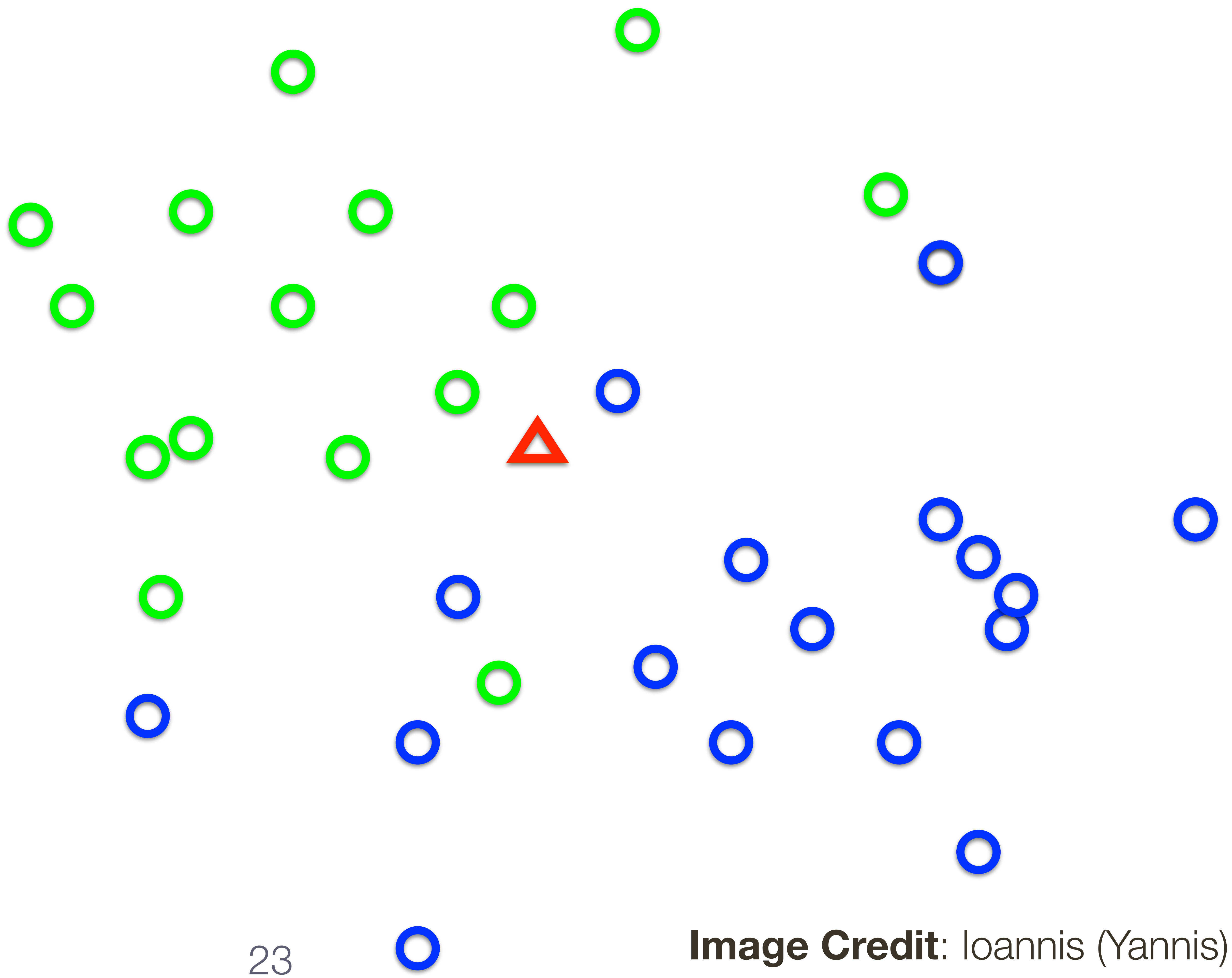
Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

Example: 2D Bayes Classifier

- 17 samples
- 15 samples

$$p(blue) = \frac{17}{17+15}$$

$$p(green) = \frac{15}{17+15}$$



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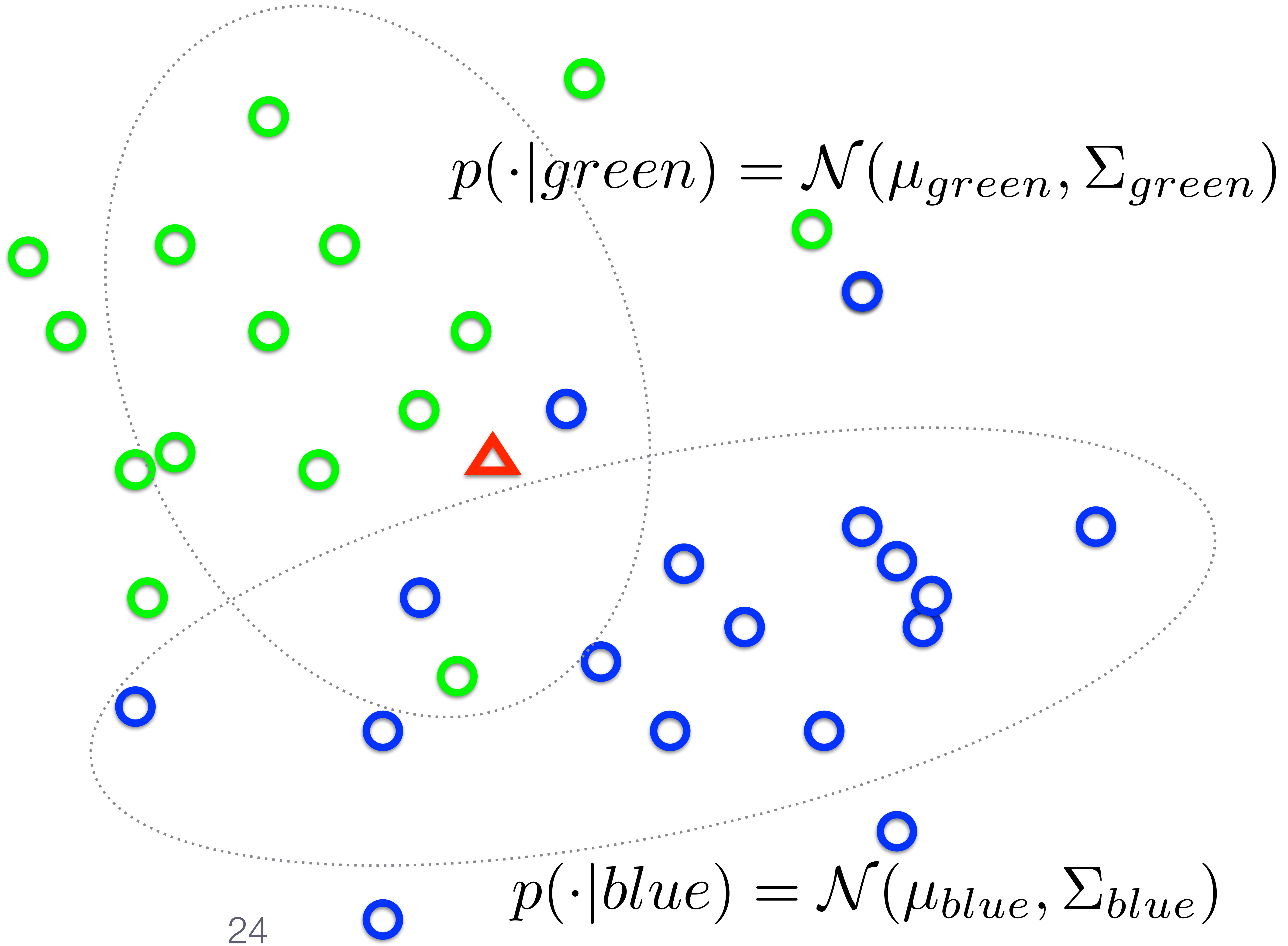


Example: 2D Bayes Classifier

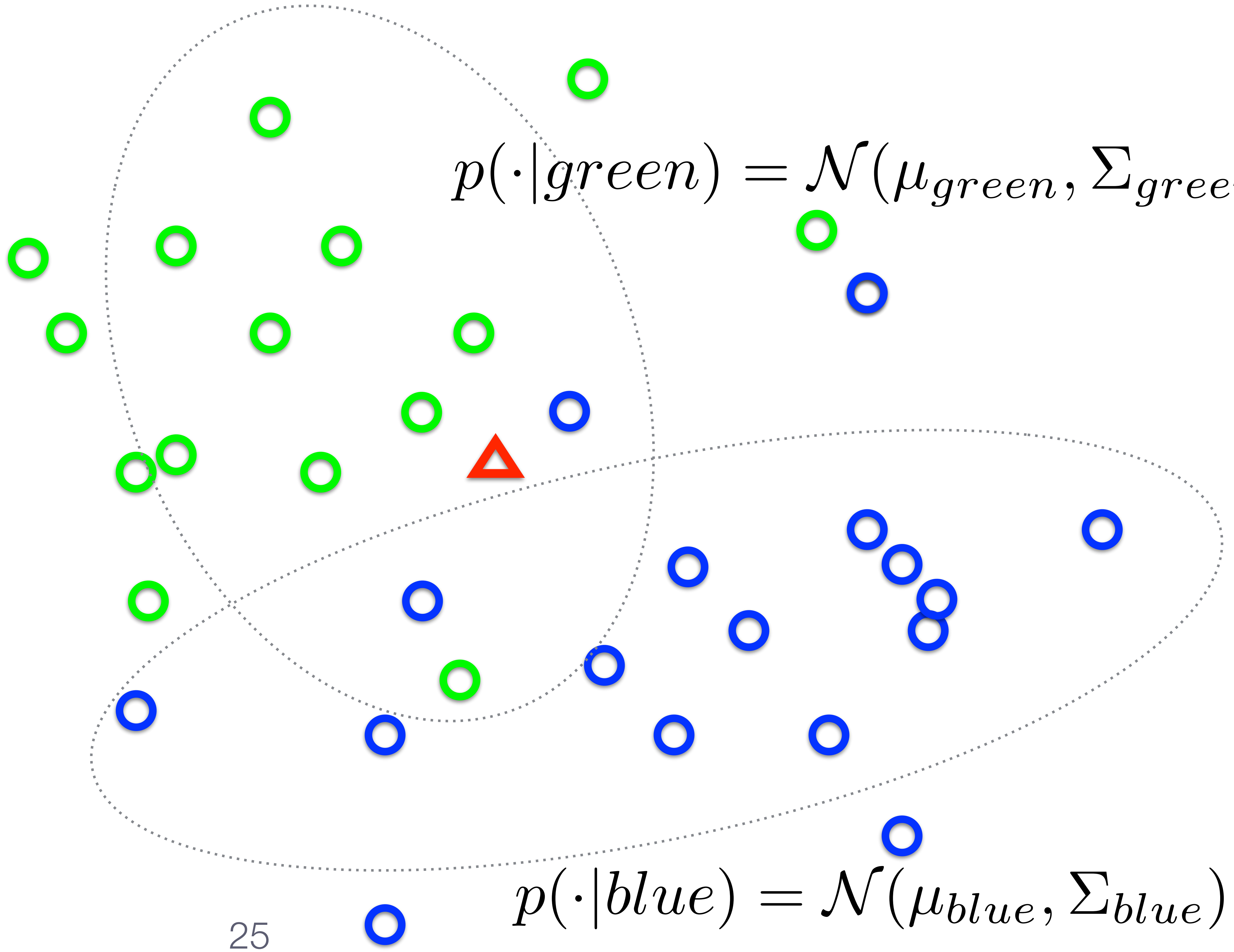
$$p(green|\triangle) \propto \mathcal{N}(\triangle; \mu_{green}, \Sigma_{green})p(green)$$

$$p(blue|\triangle) \propto \mathcal{N}(\triangle; \mu_{blue}, \Sigma_{blue})p(blue)$$

- 17 samples
- 15 samples

$$p(blue) = \frac{17}{17+15}$$

$$p(green) = \frac{15}{17+15}$$



$$p(\cdot|green) = \mathcal{N}(\mu_{green}, \Sigma_{green})$$

$$p(\cdot|blue) = \mathcal{N}(\mu_{blue}, \Sigma_{blue})$$

Loss Functions and Classifiers

Loss

- Some errors may be more expensive than others

  Example: A fatal disease that is easily cured by a cheap medicine with no side-effects. Here, false positives in diagnosis are better than false negatives

- We discuss two class classification:
  $L(1 \rightarrow 2)$ is the loss caused by calling 1 a 2

Total risk of using classifier ***s*** is

$$R(s) = \Pr\{1 \rightarrow 2 \mid \text{using } \boldsymbol{s}\}\, L(1 \rightarrow 2) + \Pr\{2 \rightarrow 1 \mid \text{using } \boldsymbol{s}\}\, L(2 \rightarrow 1)$$

Two Class Classification

Generally, we should classify as 1 if the expected loss of classifying as 1 is less than for 2

Classify $\boldsymbol{x}$ as

$$1 \quad \text{if } p(1|\boldsymbol{x})\, L(1 \rightarrow 2) > p(2|\boldsymbol{x})\, L(2 \rightarrow 1)$$

$$2 \quad \text{if } p(1|\boldsymbol{x})\, L(1 \rightarrow 2) < p(2|\boldsymbol{x})\, L(2 \rightarrow 1)$$

Decision boundary: points where the loss is the same for either class.

Training Error, **Testing** Error, and Overfitting

Training error is the error a classifier makes on the training set

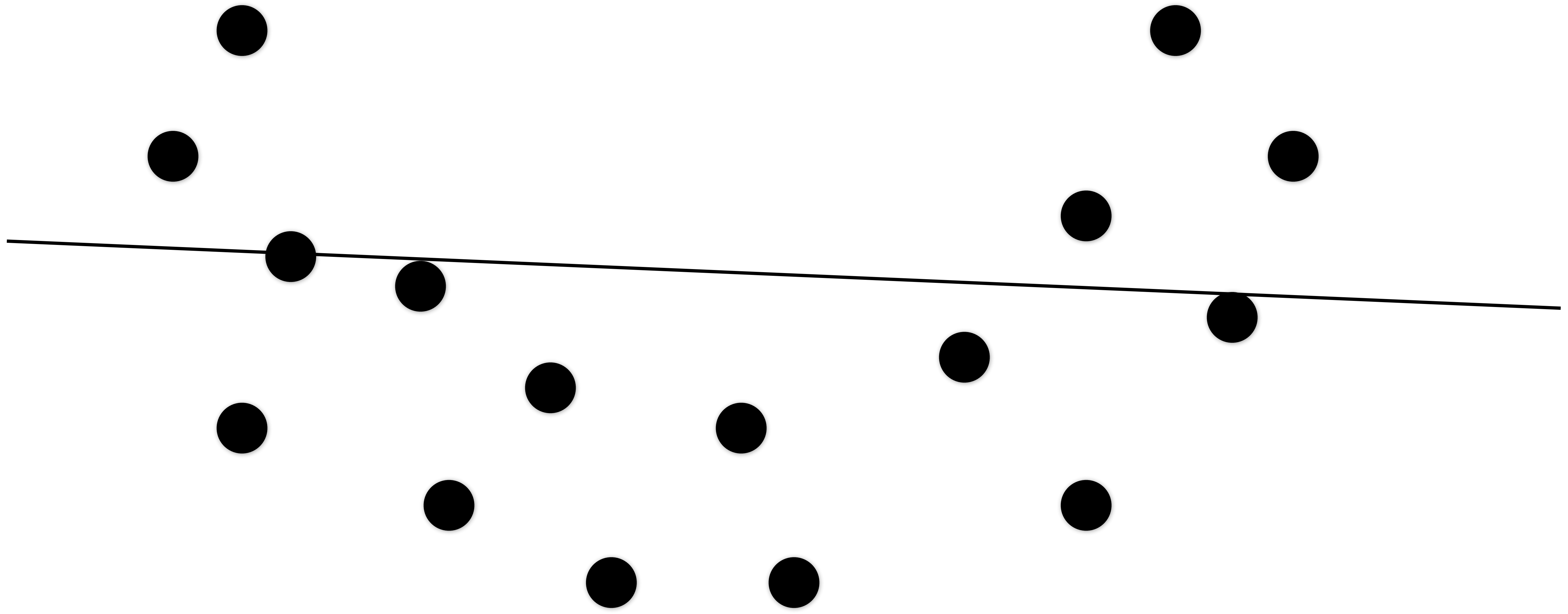
We want to minimize the **testing error** – the error the classifier makes on an unseen testing set

Classifiers that have small training error may not necessarily have small testing error

The phenomenon that causes testing error to be worse than training error is called **overfitting**

Training Error, Testing Error, and **Overfitting**

Underfitting: model is too simple to represent all the relevant class characteristics



Training Error, Testing Error, and **Overfitting**

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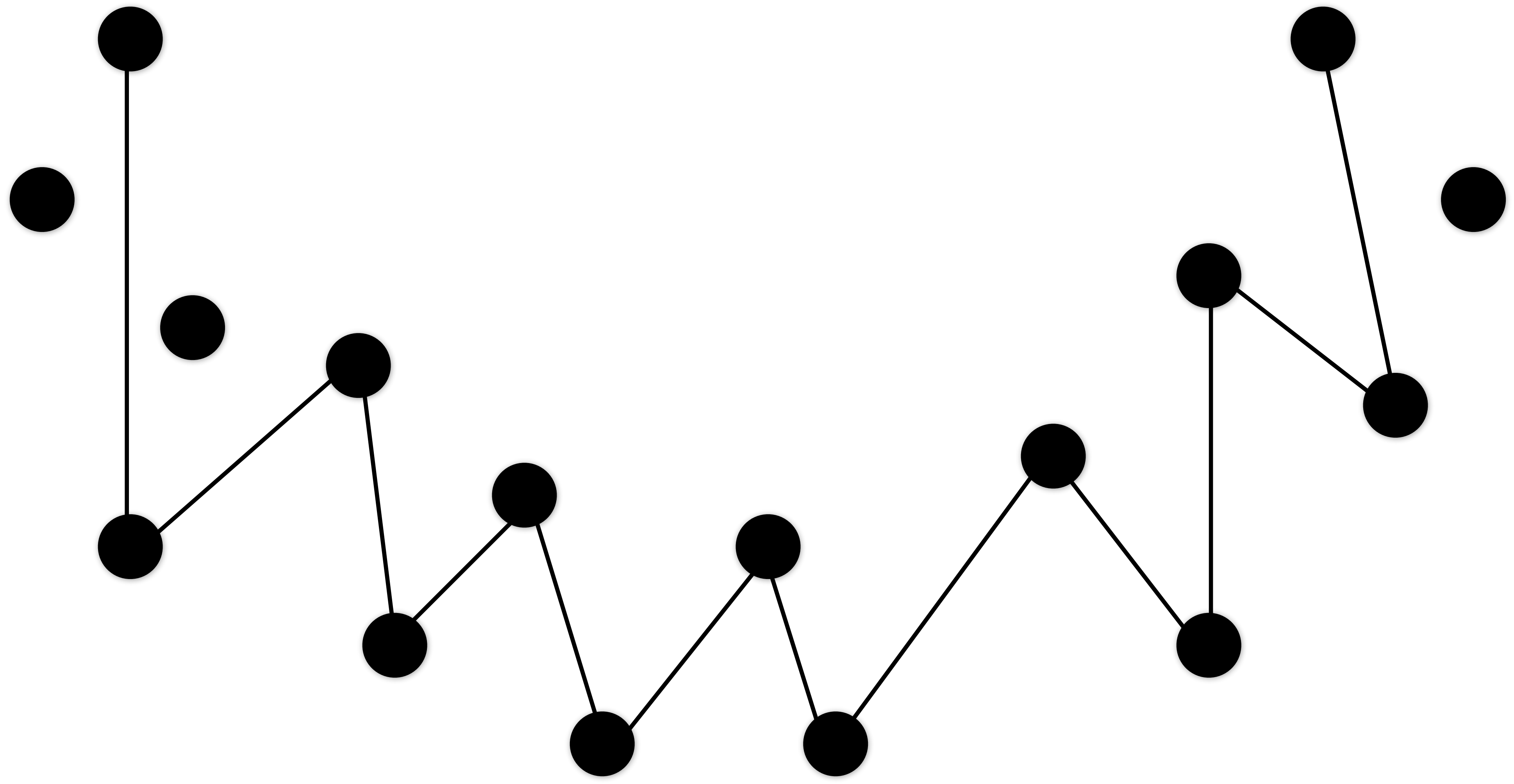
Overfitting: model is too complex and fits irrelevant characteristics (noise) in the data



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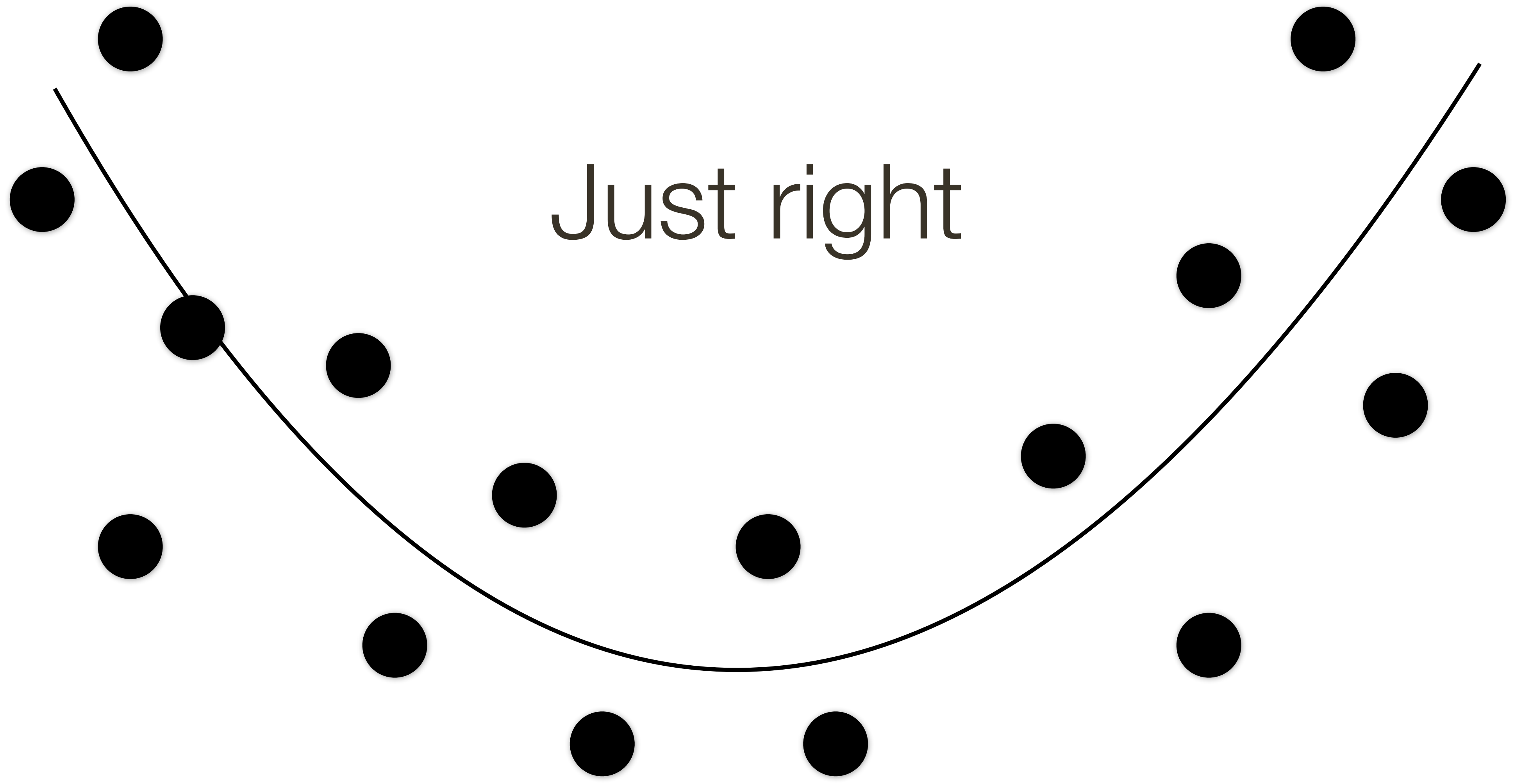
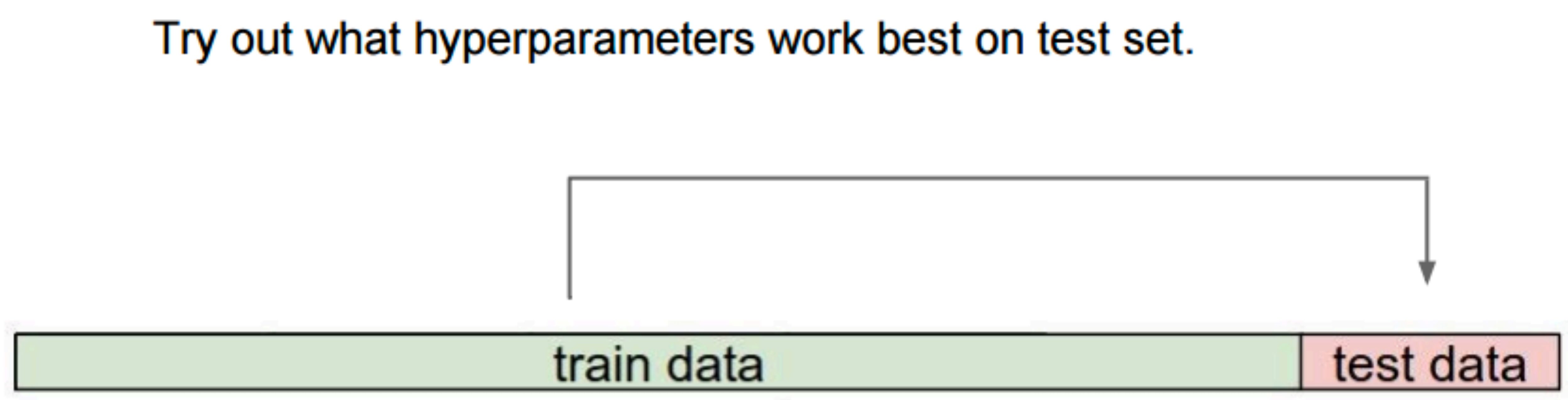


Cross-Validation

We cannot reliably estimate the error rate of the classifier using the training set

An alternative is to split some training data to form a **validation** set, then train the classifier on the rest of the data and evaluate on the validation set



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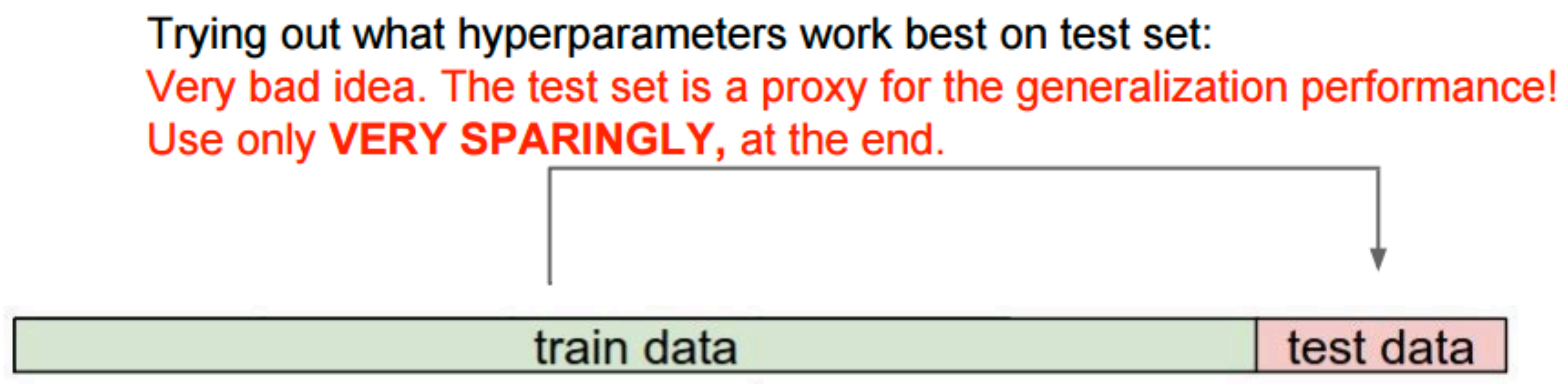

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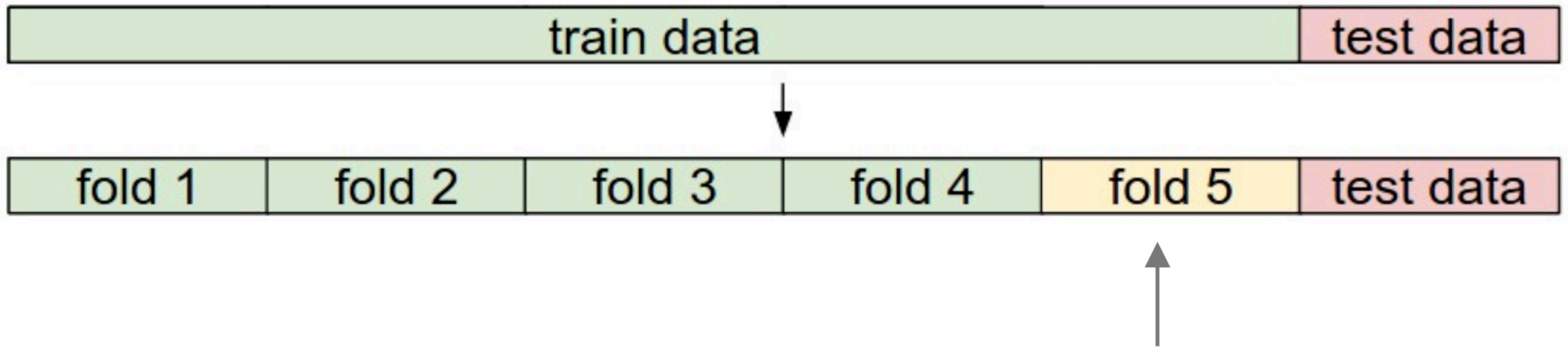
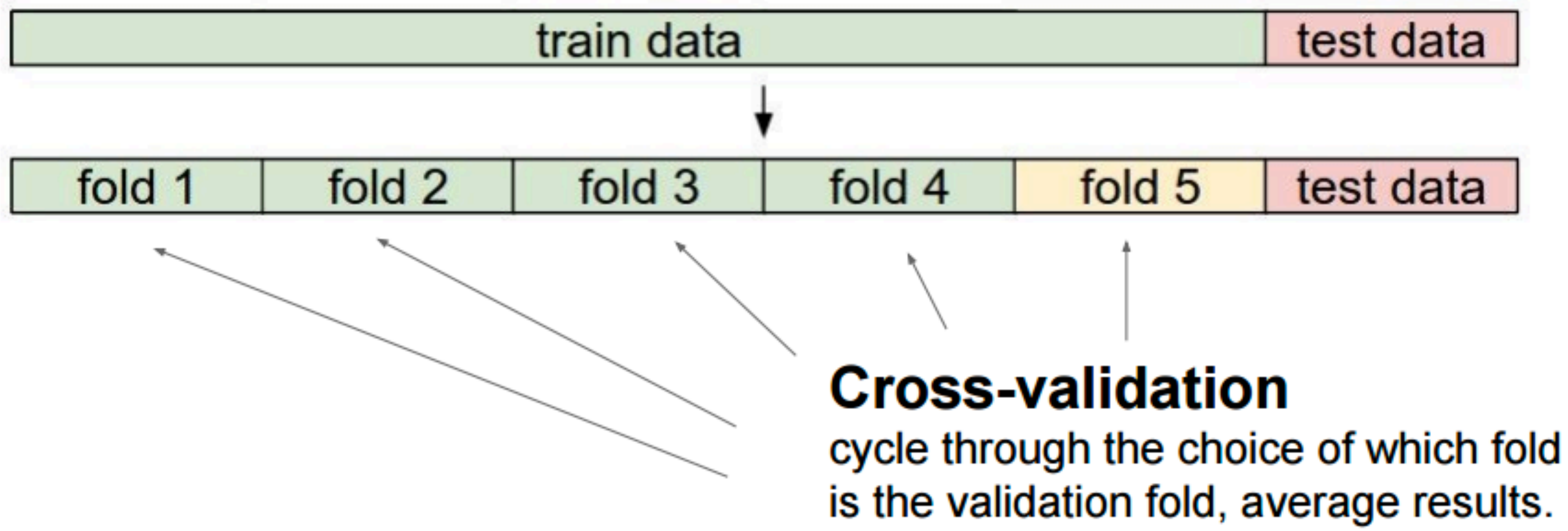


Validation data
use to tune hyperparameters
evaluate on test set ONCE at the end

Cross-Validation

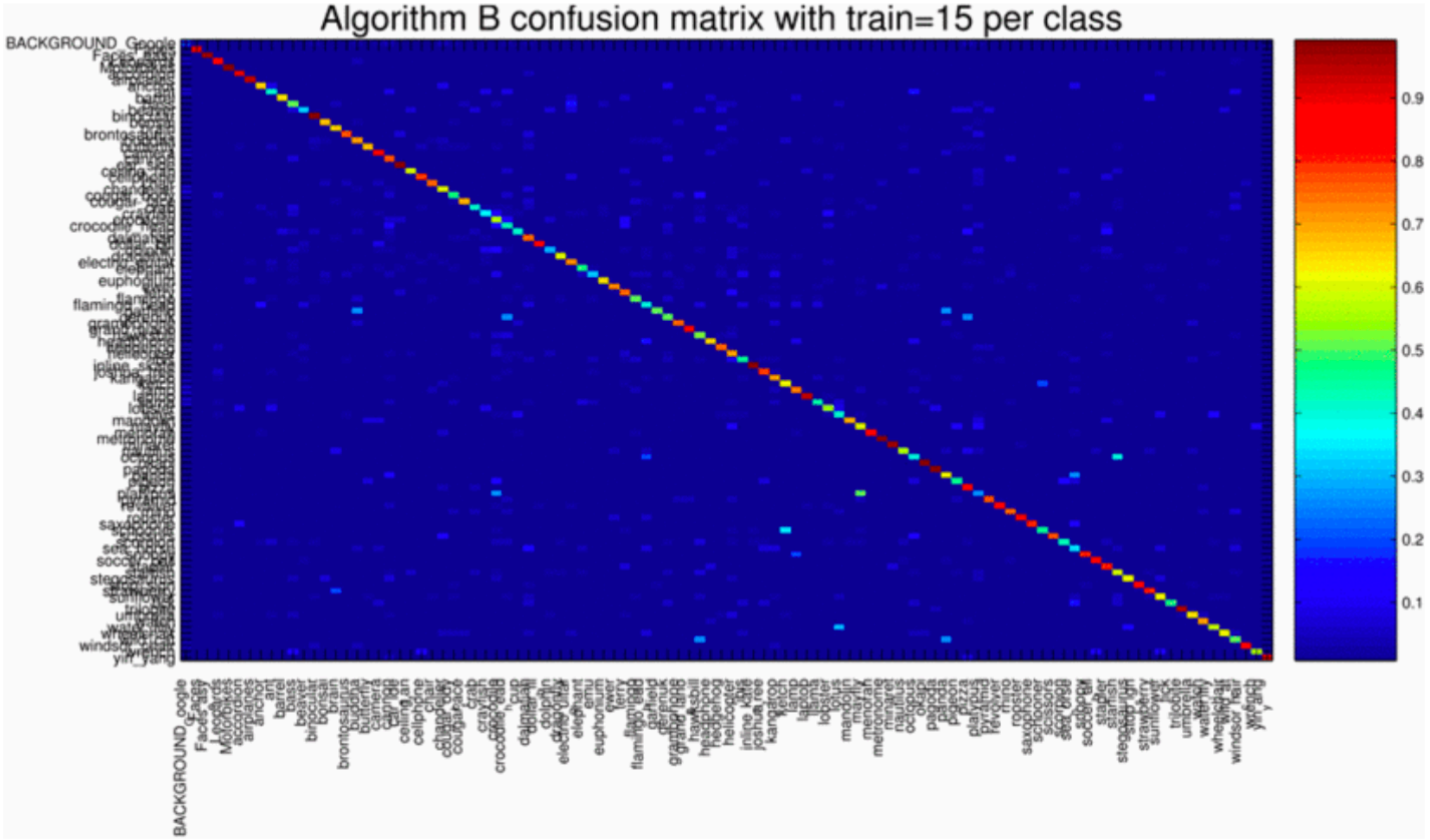
Cross-validation involves performing multiple splits and averaging the error over all splits



Confusion Matrix

When evaluating a multi-class classifier, it may be useful to know how often certain classes are often misclassified as others.

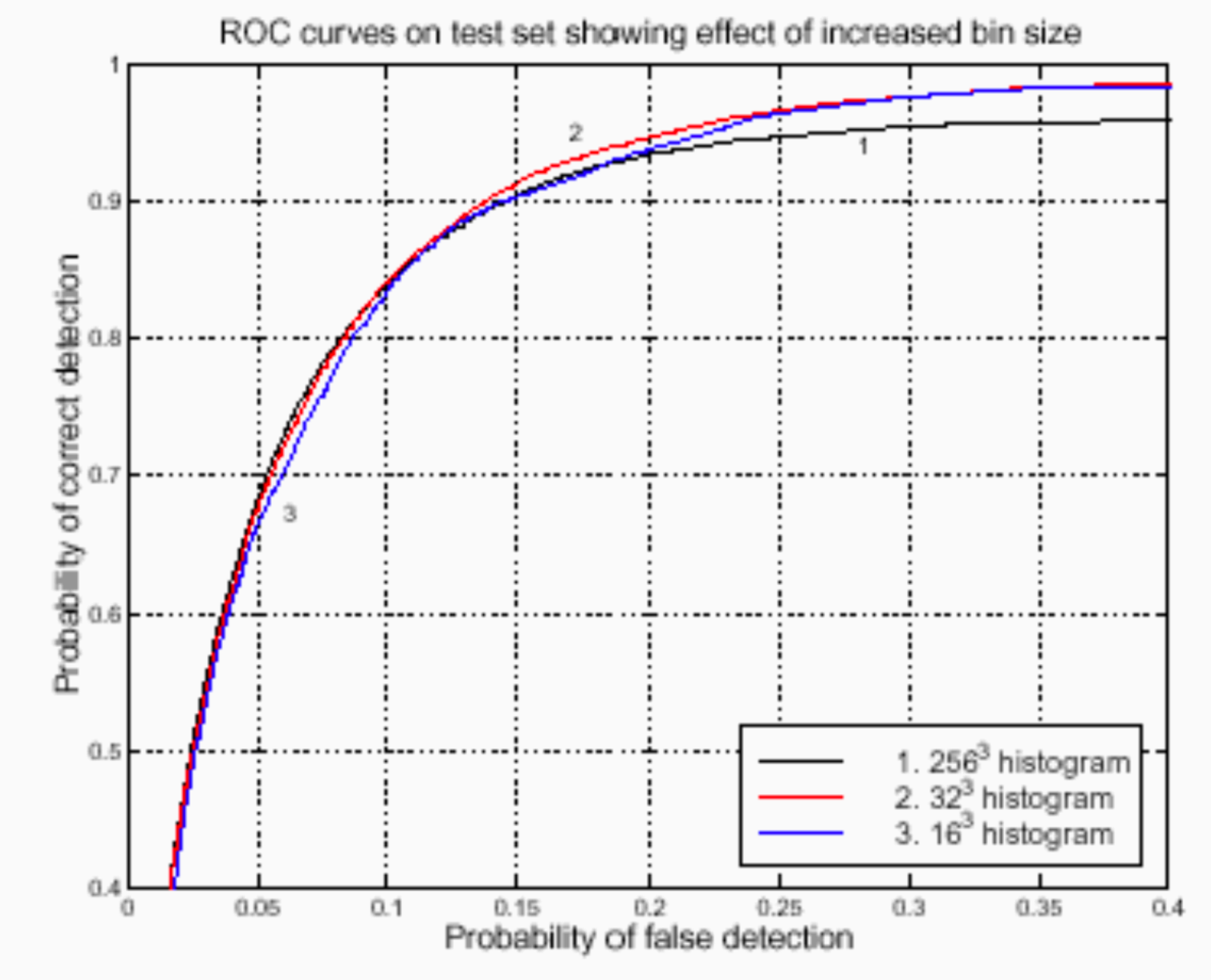
A **confusion matrix** is a table whose (i,j)th entry is the frequency (or proportion) an item of true class i was labelled as j by the classifier.



Forsyth & Ponce (2nd ed.) Figure 15.3. Original credit: H. Zhang et al., 2006.

Receiver Operating Characteristics (ROC)

ROC curves plot trade-off between false positives and false negatives



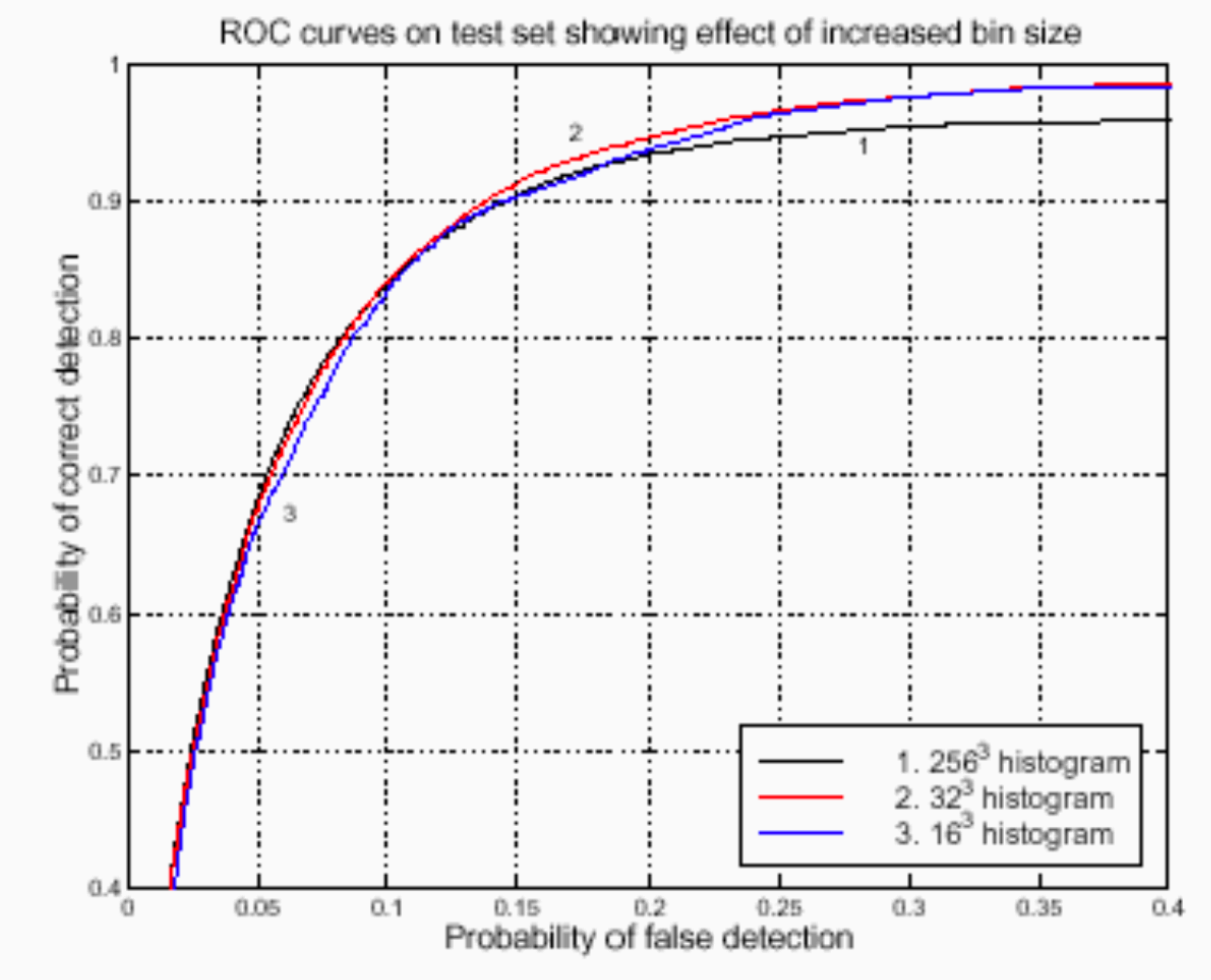
Forsyth & Ponce (2nd ed.) Figure 15.4

Figure from M. J. Jones and J. Rehg, “Statistical color models with application to skin detection,” Proc. CVPR, 1999, IEEE

Receiver Operating Characteristics (ROC)

What is a ROC curve for a perfect classifier?

ROC curves plot trade-off between false positives and false negatives



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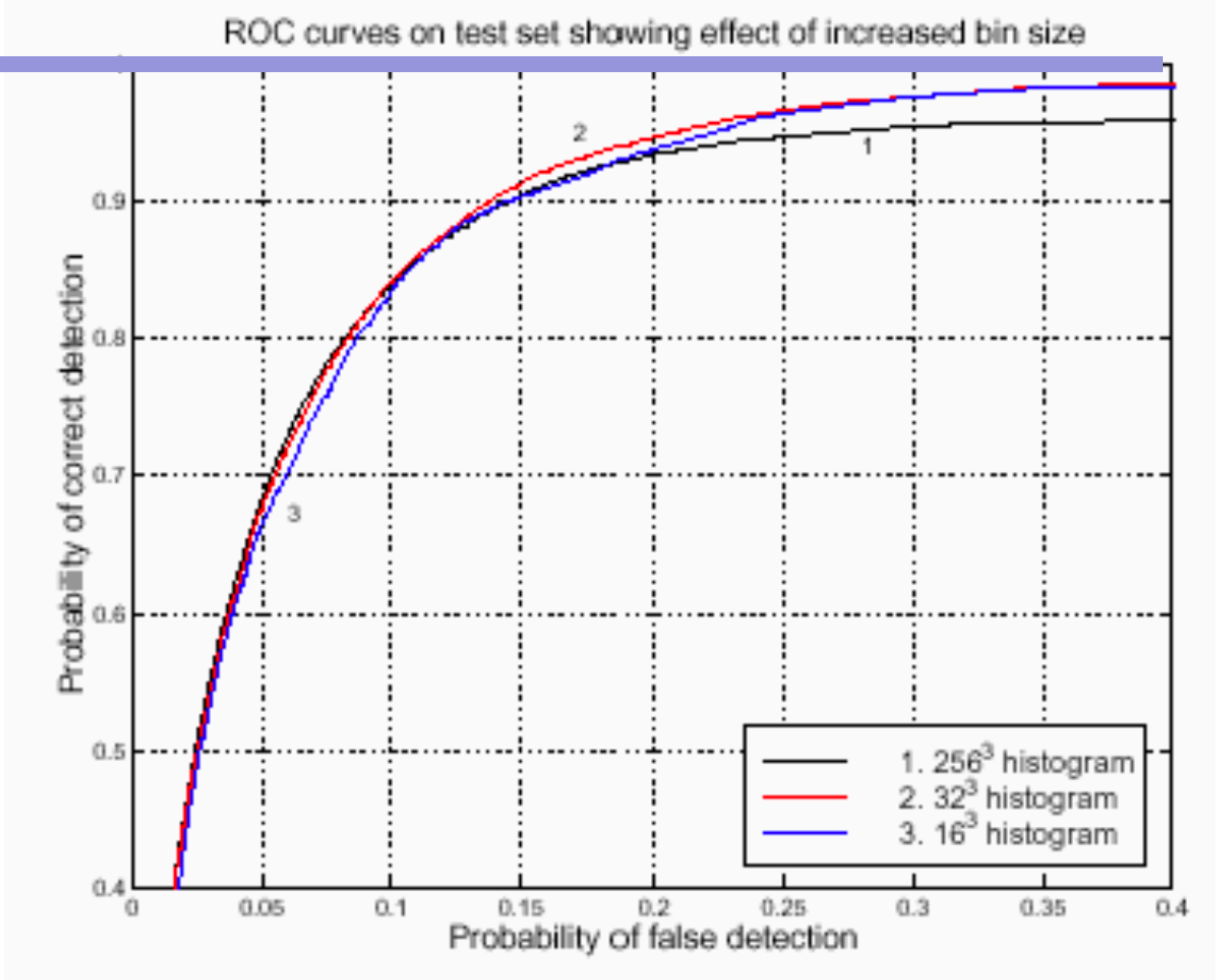

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Classifier Strategies

Classification strategies fall under two broad types: **parametric** and **non-parametric**.

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Parametric classifiers are **model driven**. The parameters of the model are learned from training examples. New data points are classified by the learned model.

— fast, compact

— flexibility and accuracy depend on model assumptions

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Classification strategies fall under two broad types: **parametric** and **non-parametric**.

Parametric classifiers are **model driven**. The parameters of the model are learned from training examples. New data points are classified by the learned model.
— fast, compact
— flexibility and accuracy depend on model assumptions

Non-parametric classifiers are **data driven**. New data points are classified by comparing to the training examples directly. "The data is the model".
— slow
— highly flexible decision boundaries

Nearest Neighbor Classifier

Given a new data point, assign the label of nearest training example in feature space.

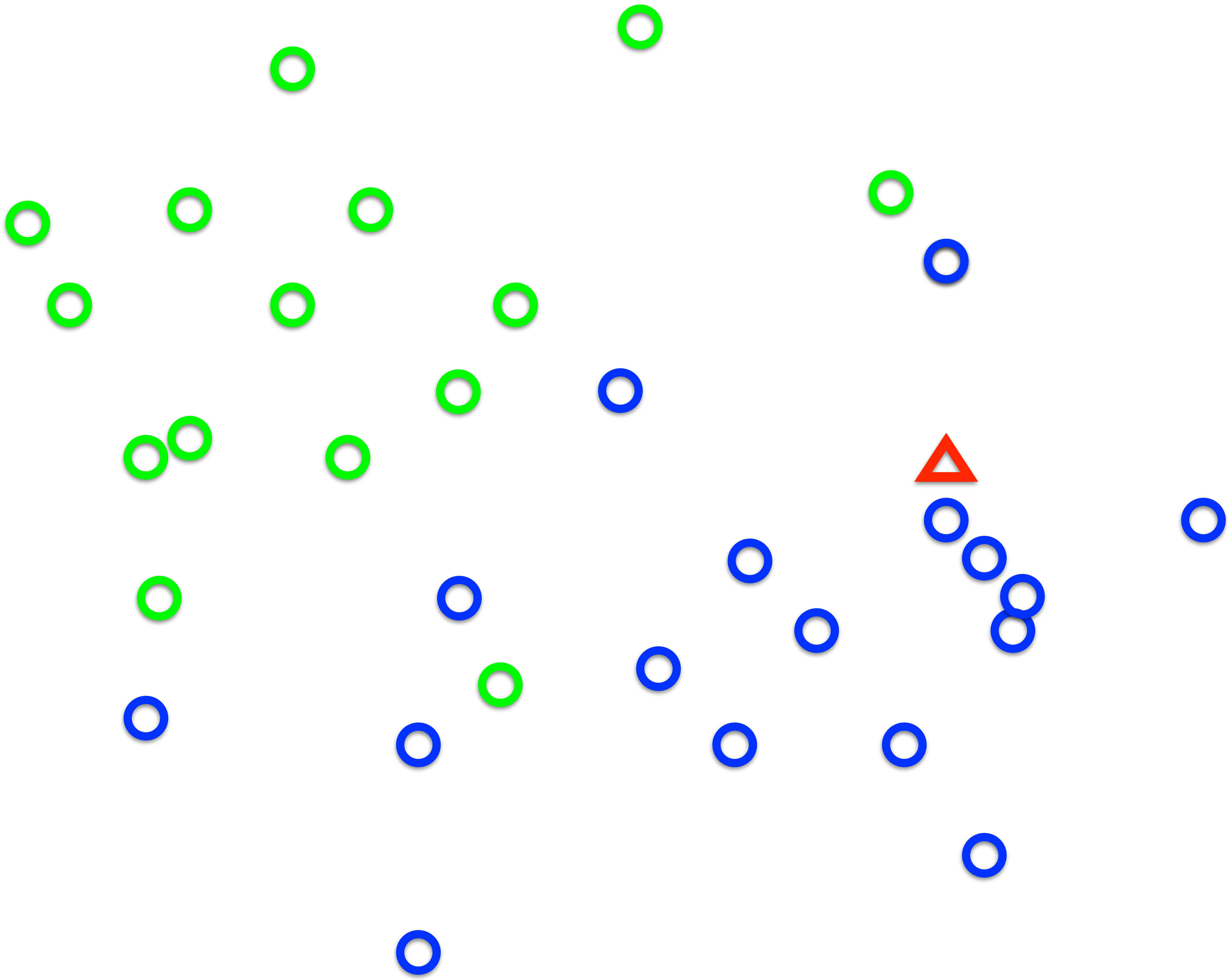

Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

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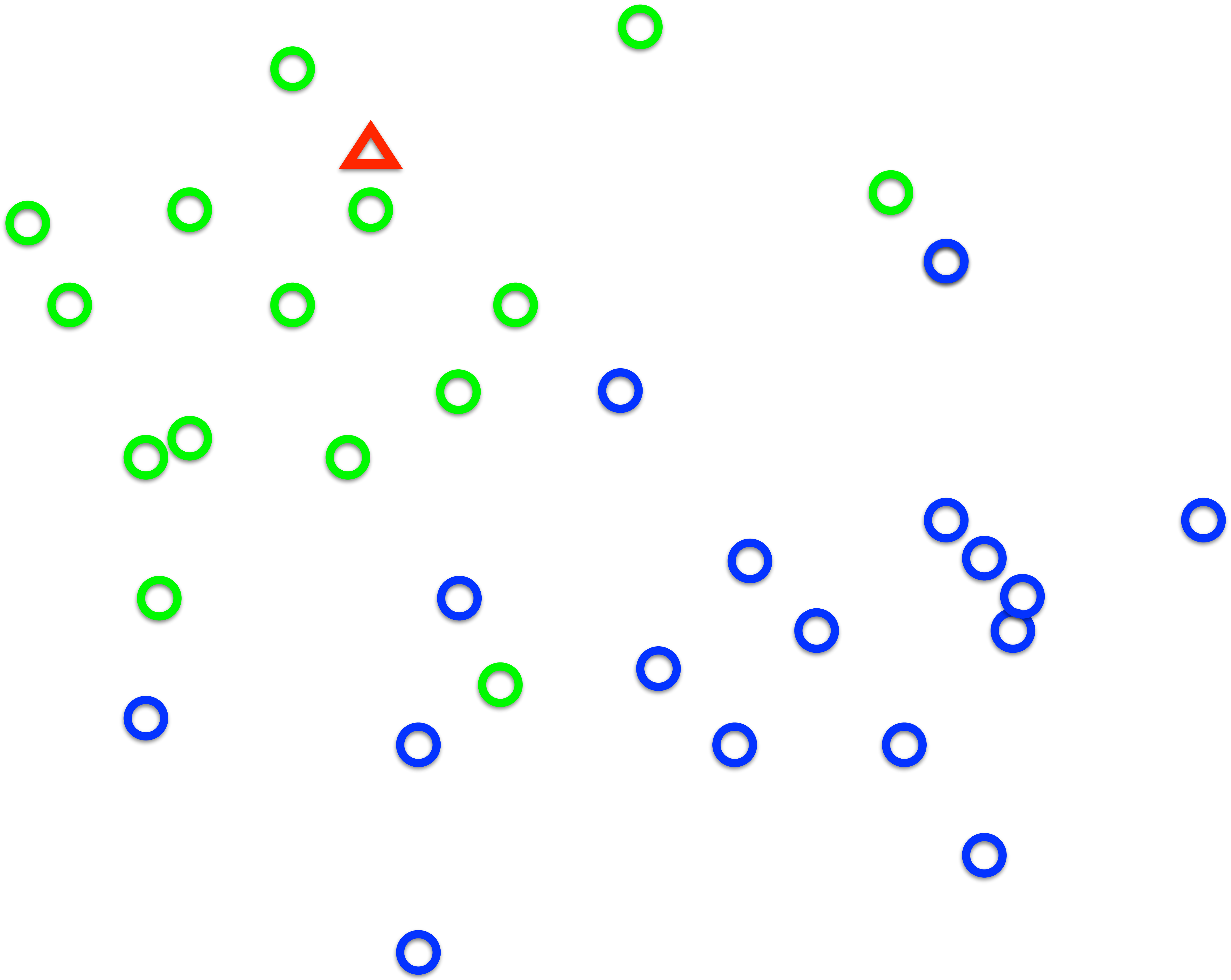


Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

k-**Nearest Neighbor** (kNN) Classifier

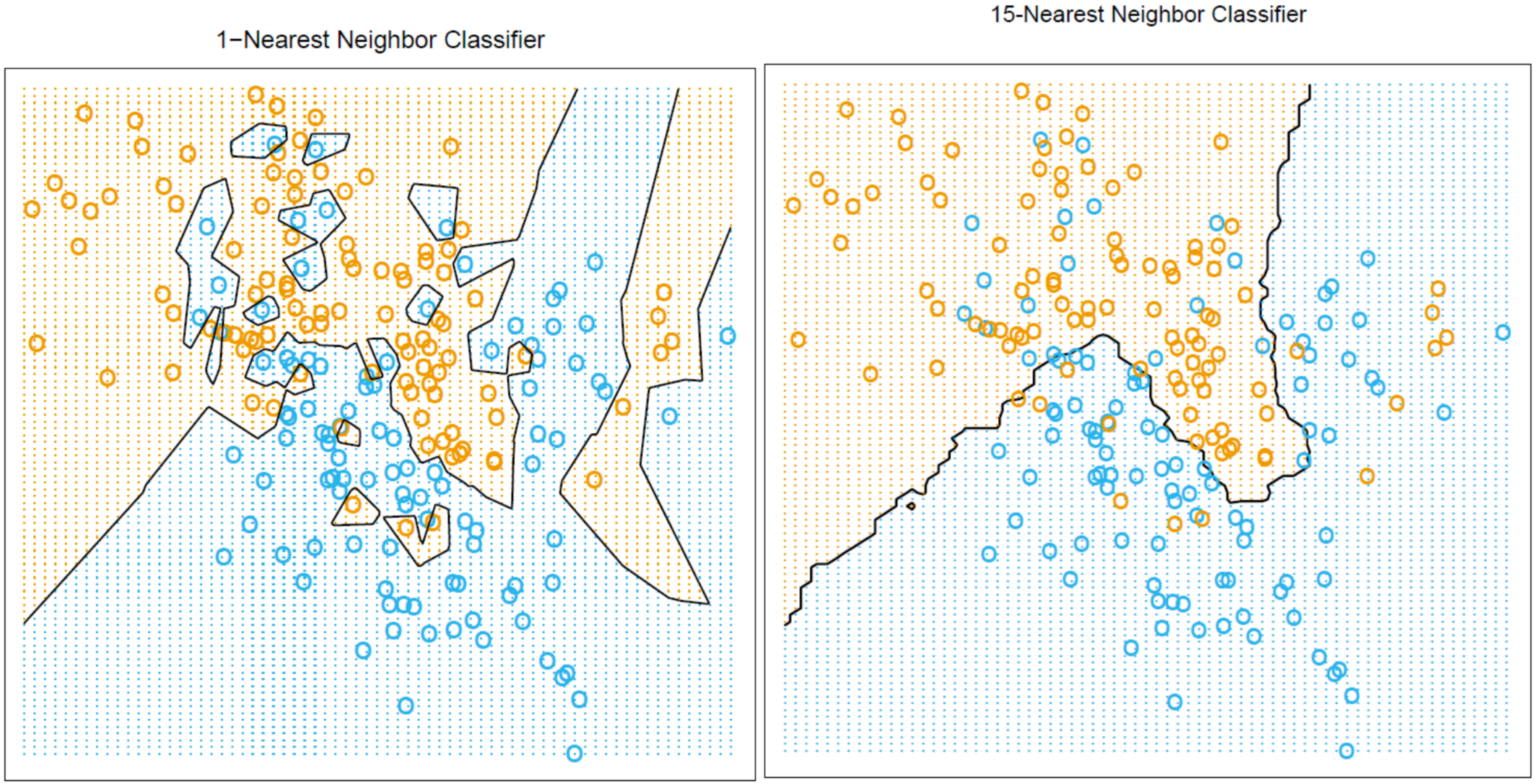
We can gain some robustness to noise by voting over multiple neighbours.

Given a new data point, find the k nearest training examples. Assign the label by majority vote.

Simple method that works well if the distance measure correctly weights the various dimensions

For large data sets, as k increases kNN approaches optimality in terms of minimizing probability of error

k-**Nearest Neighbor** (kNN) Classifier



kNN decision boundaries respond to local clusters where one class dominates

Figure credit: Hastie, Tibshirani & Friedman (2nd ed.)

Support Vector Machines (SVM)

Idea: Try to obtain the decision boundary directly

The decision boundary is parameterized as a **separating hyperplane** in feature space.
— e.g. a separating line in 2D

We choose the hyperplane that is as far as possible from each class - that maximizes the distance to the closest point from either class.

Linear Classifier

Defines a score function:

$$f(\mathbf{x}_i, \mathbf{W}, \mathbf{b}) = \mathbf{W}\mathbf{x}_i + \mathbf{b}$$

image features

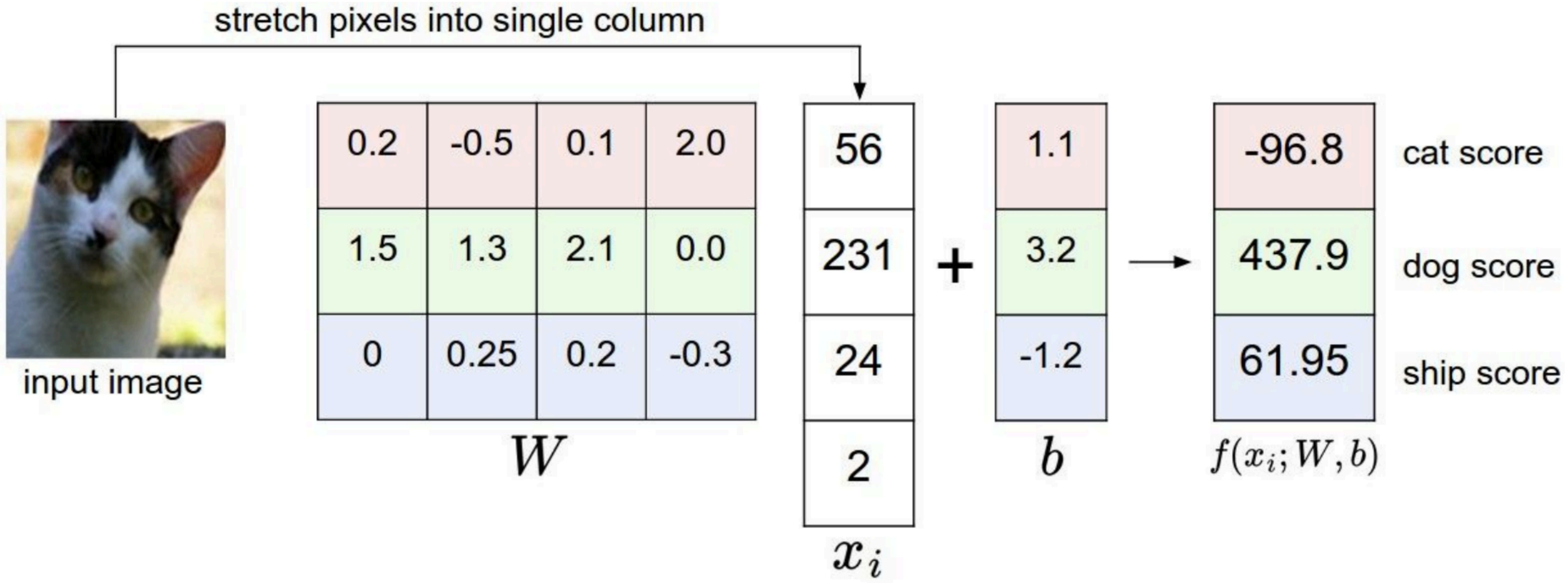
weights (parameters)

bias vector

Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

Linear Classifier

Example with an image with 4 pixels, and 3 classes (cat/dog/ship)



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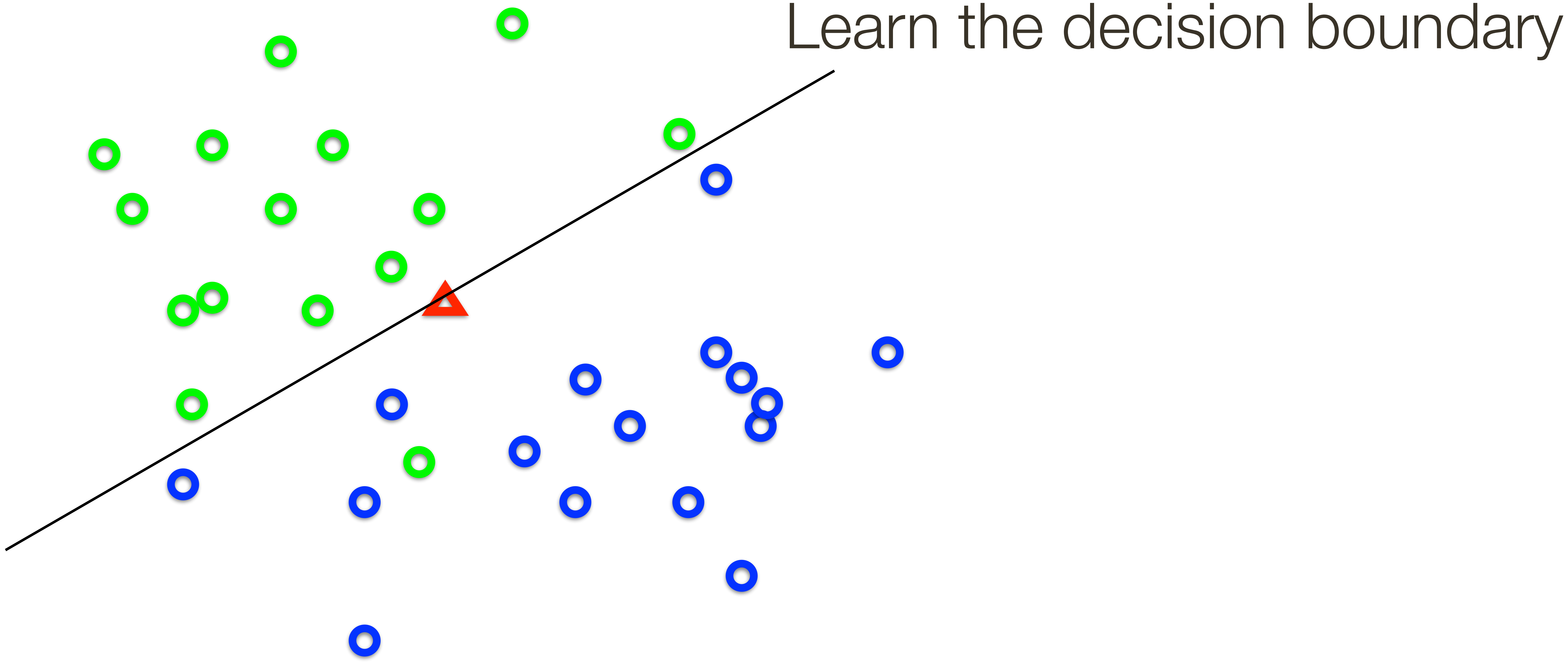
Support Vector Machines (SVM)



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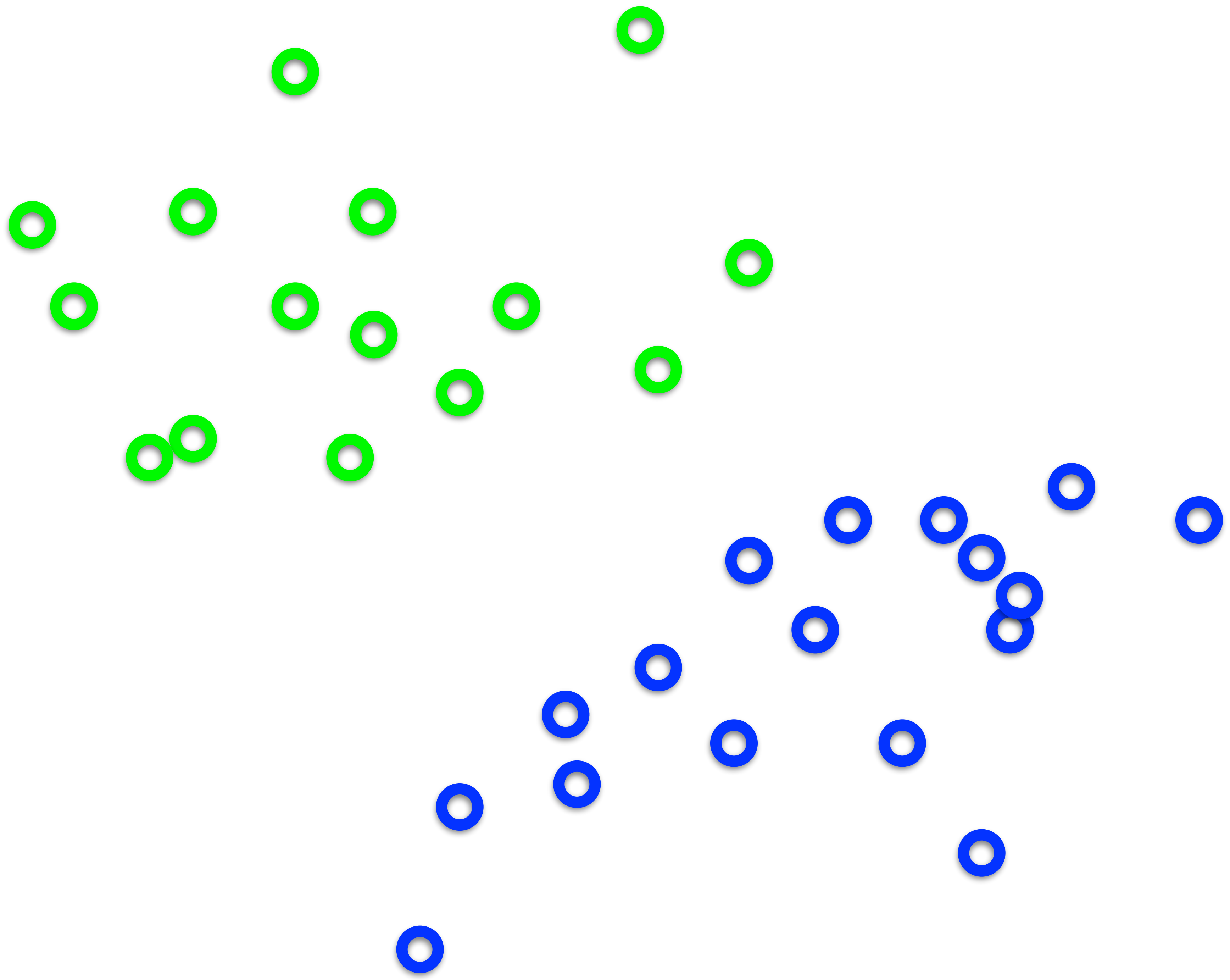
Support Vector Machines (SVM)

What's the best **w** ?



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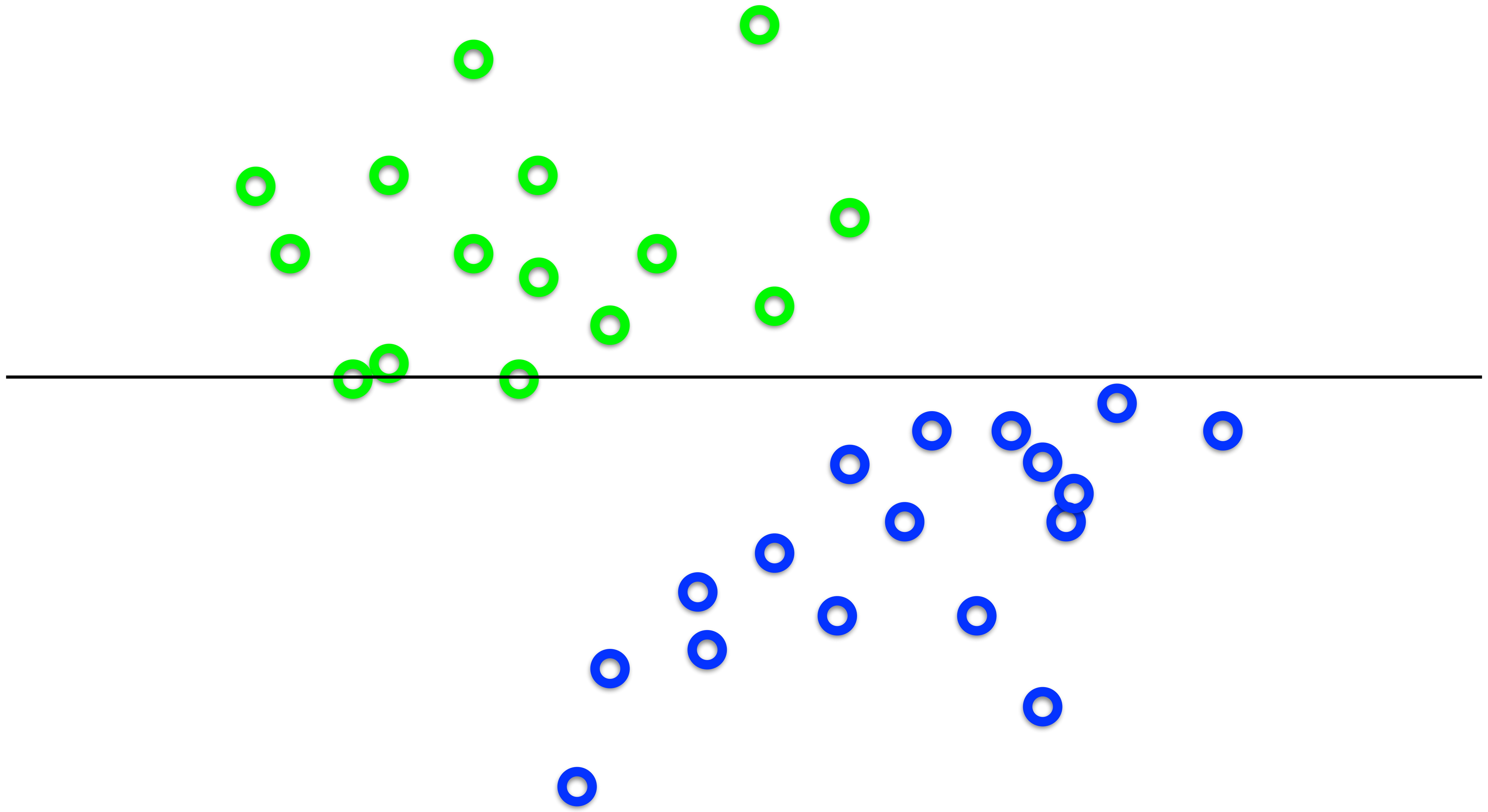

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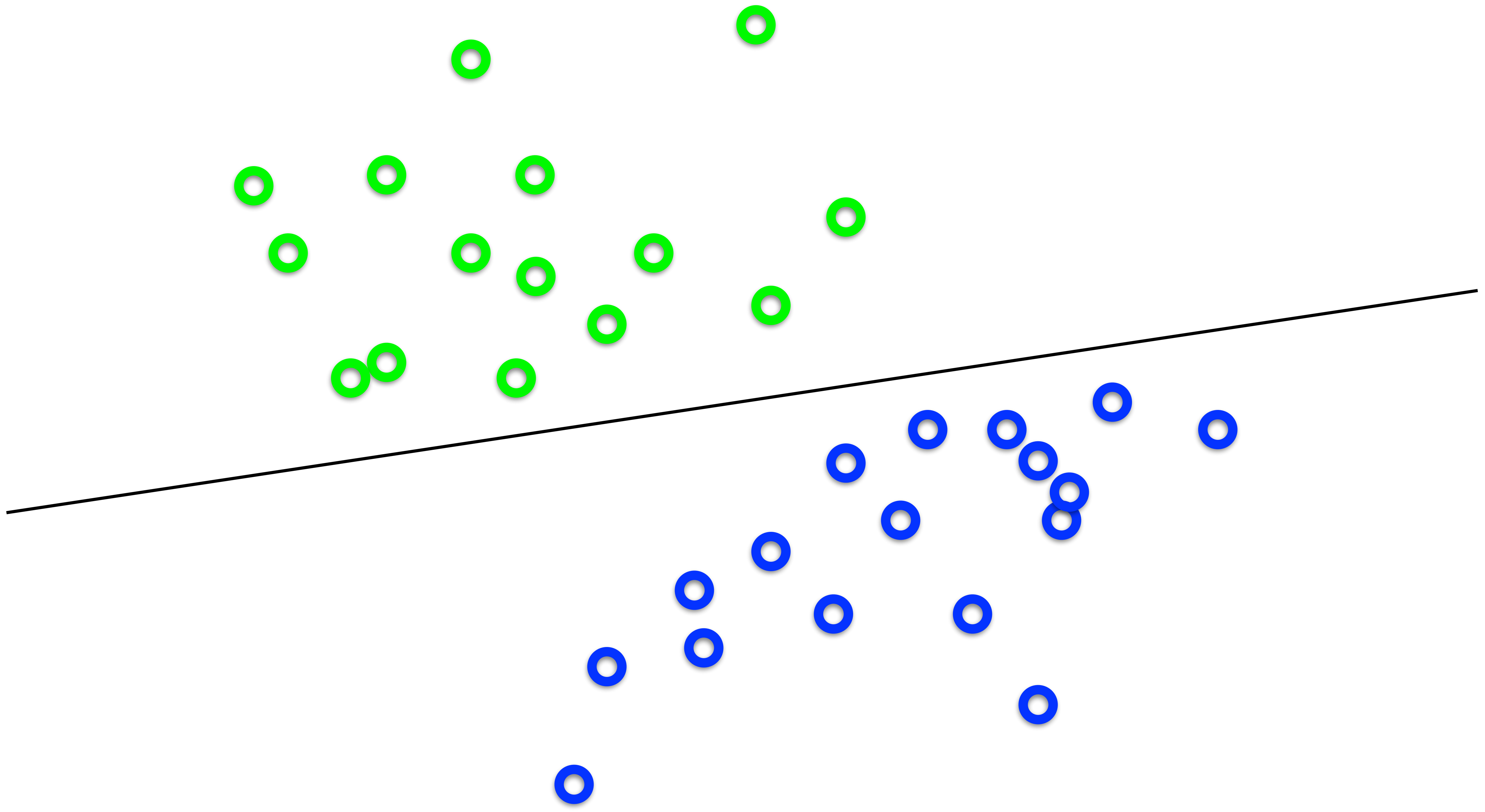

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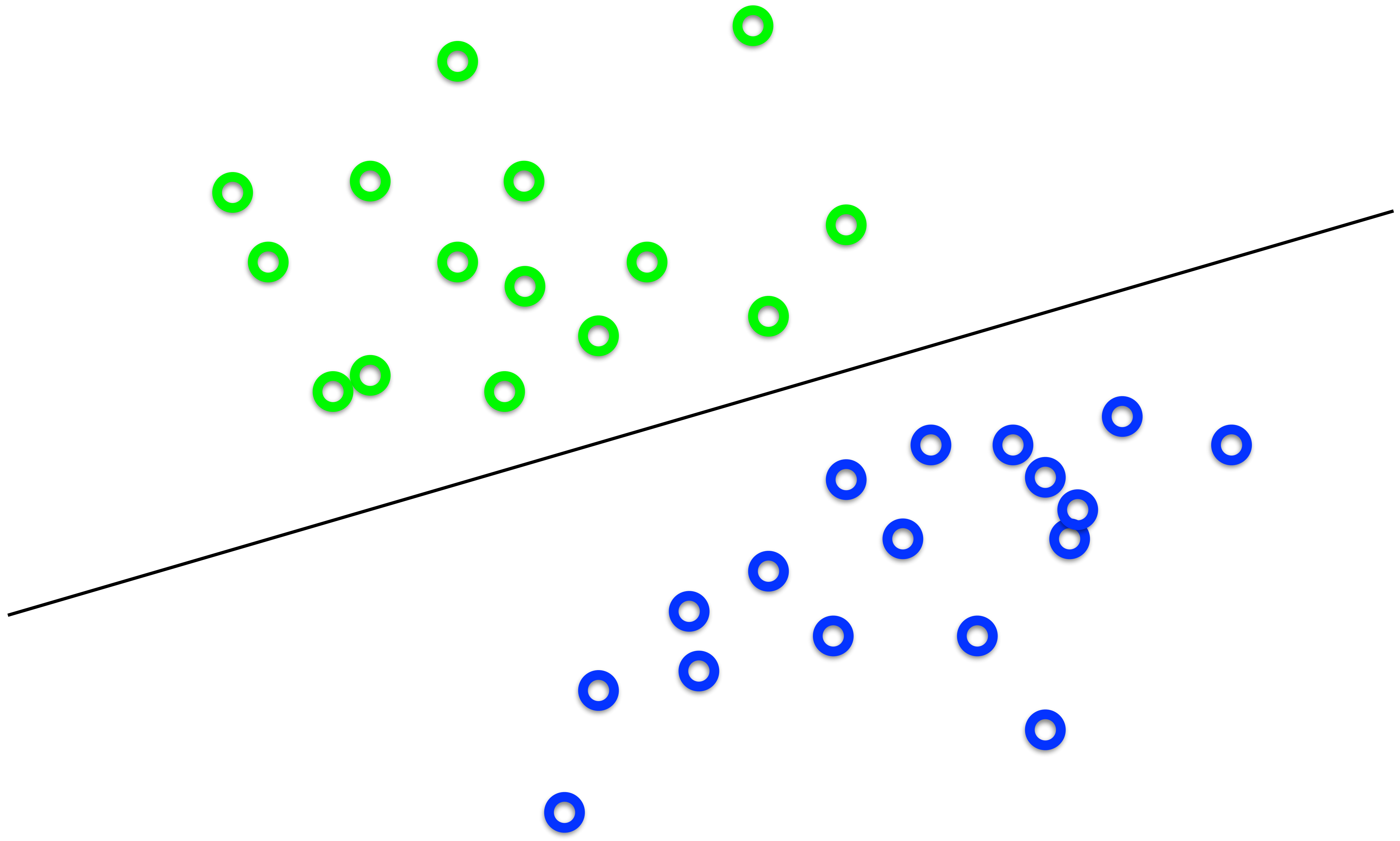

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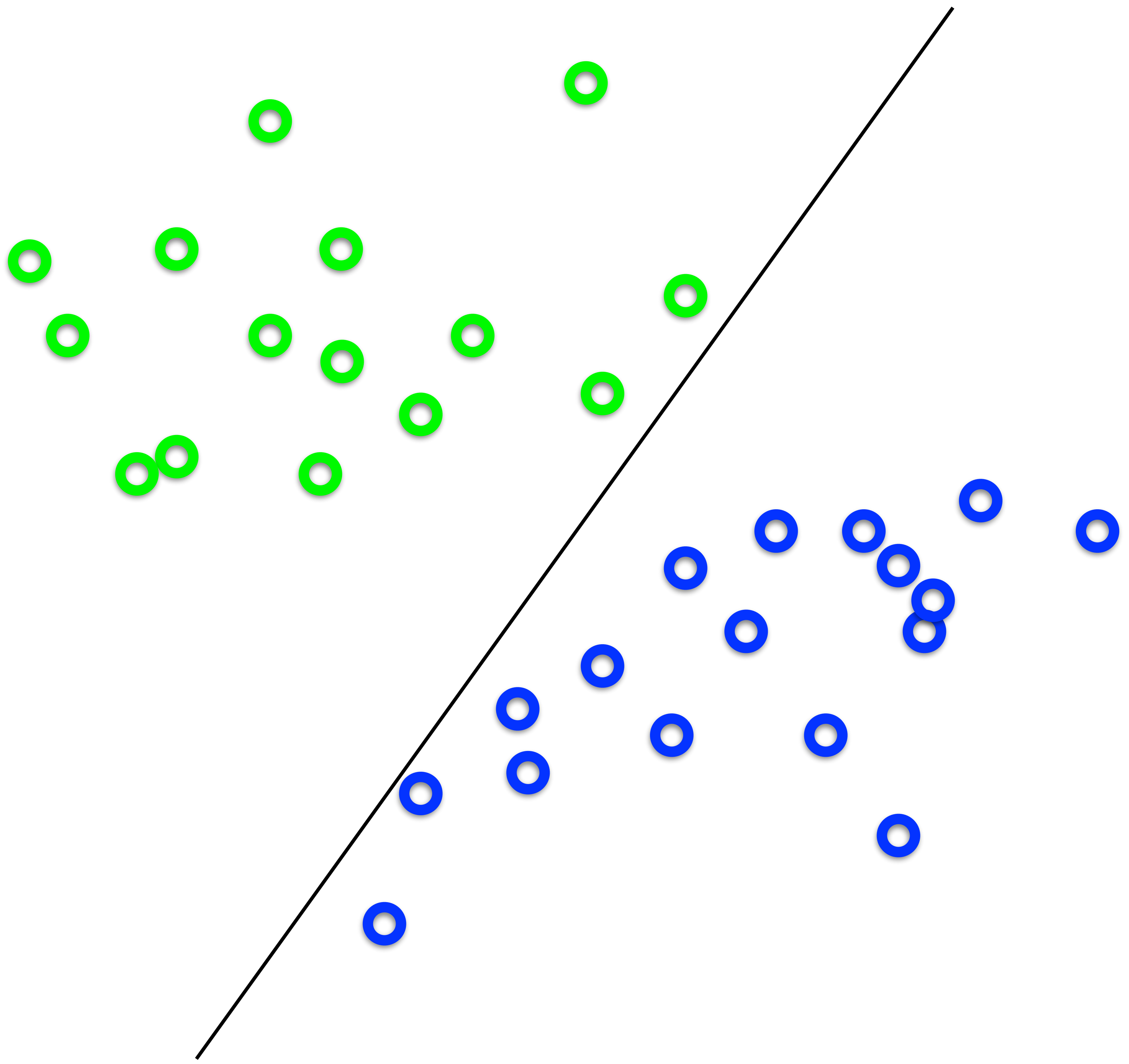
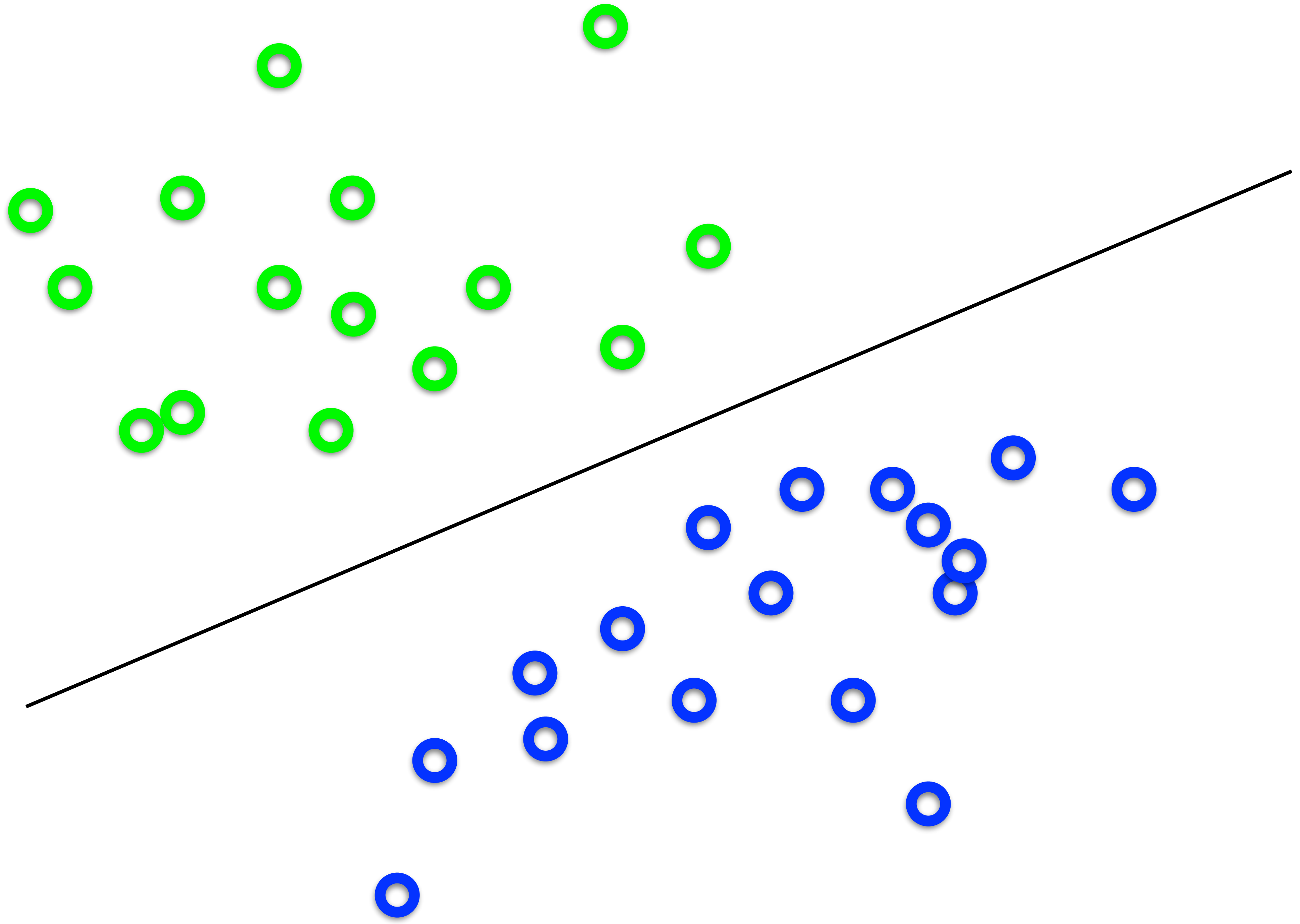


Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

Support Vector Machines (SVM)

What's the best **w** ?

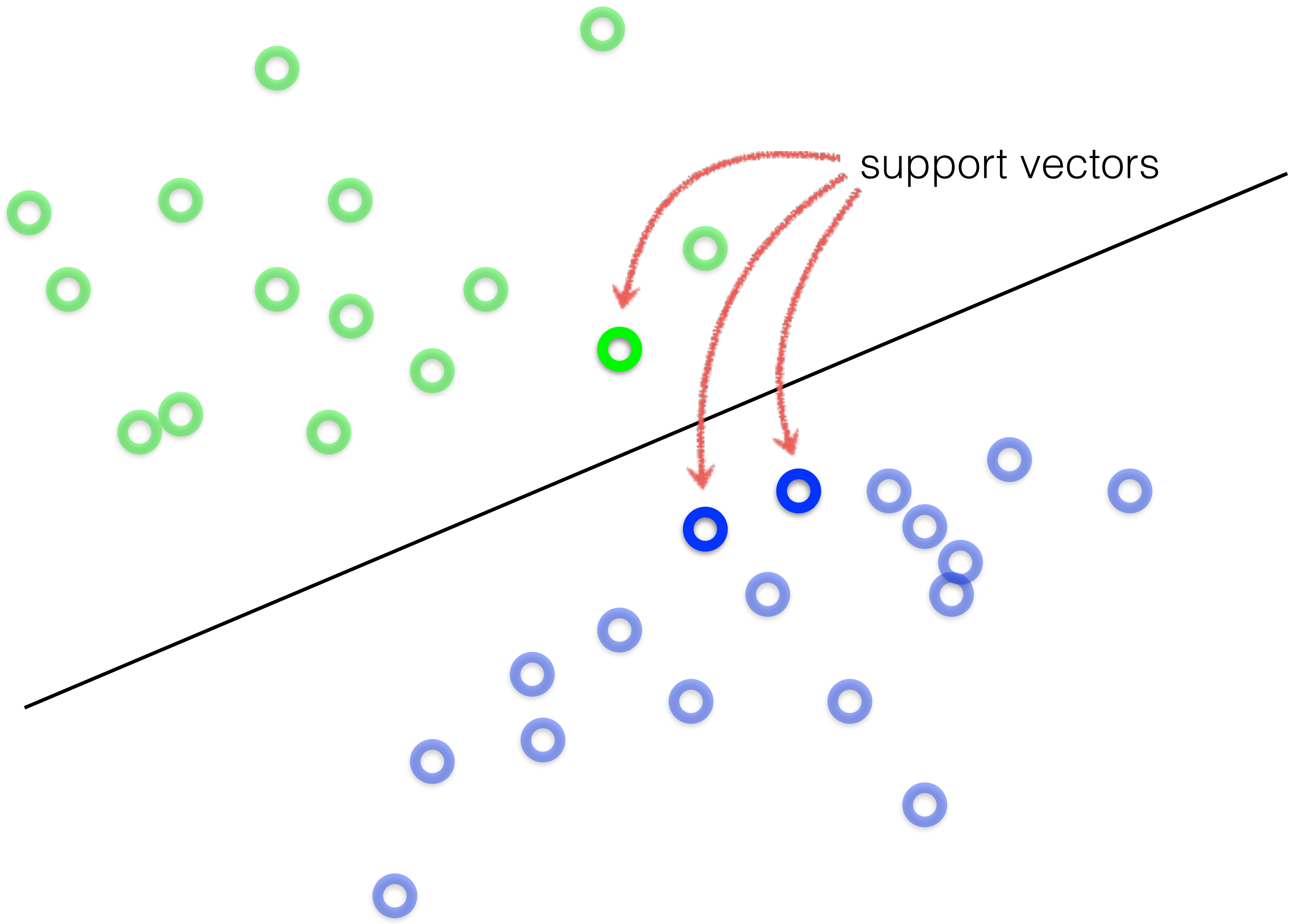


Intuitively, the line that is the farthest
from all interior points

Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

Support Vector Machines (SVM)

What's the best **w** ?



Want a hyperplane that is far away from 'inner points'

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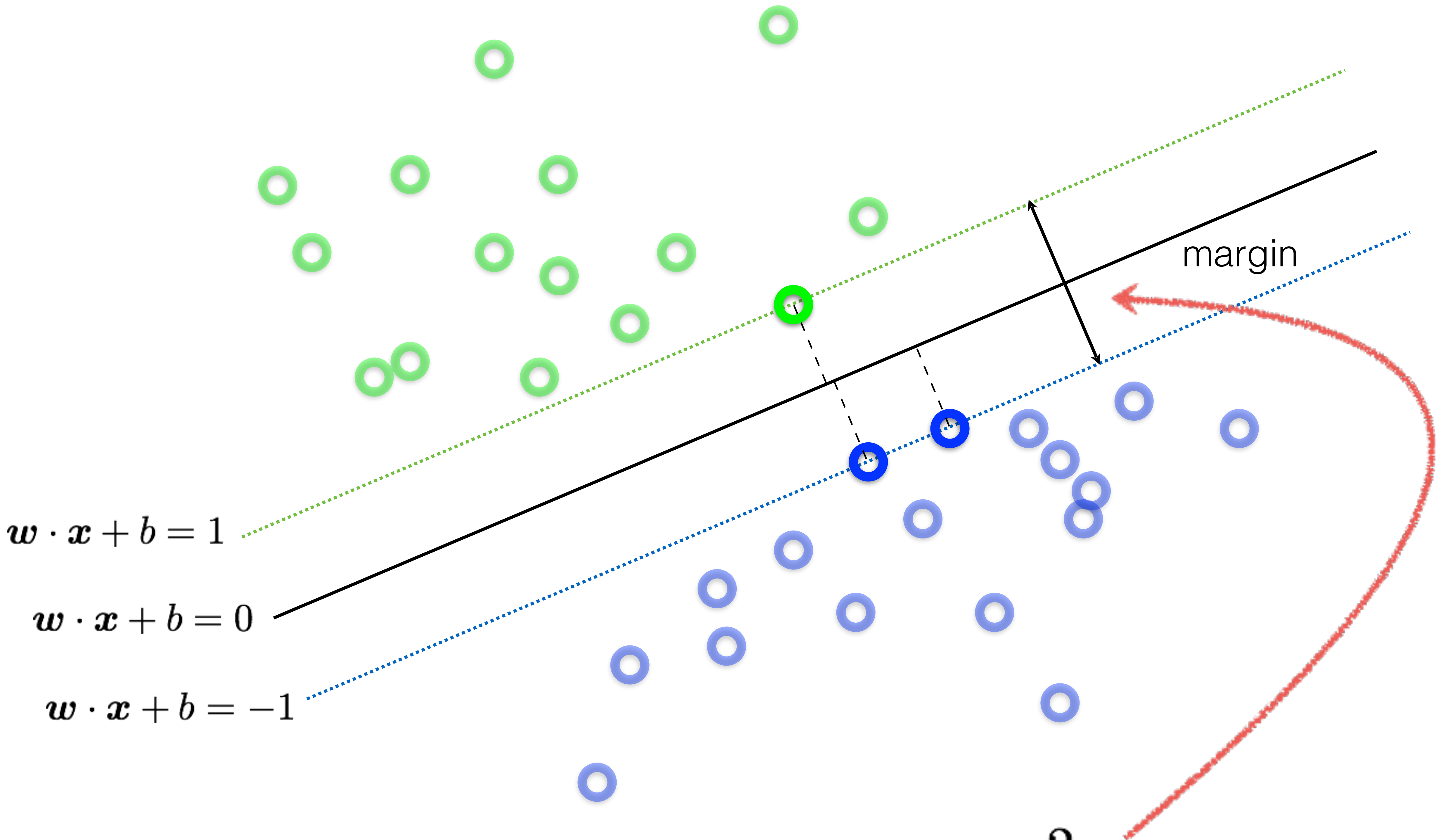
Support Vector Machines (SVM)

Find hyperplane **w** such that …



the gap between parallel hyperplanes $\frac{2}{\|\boldsymbol{w}\|}$ is maximized

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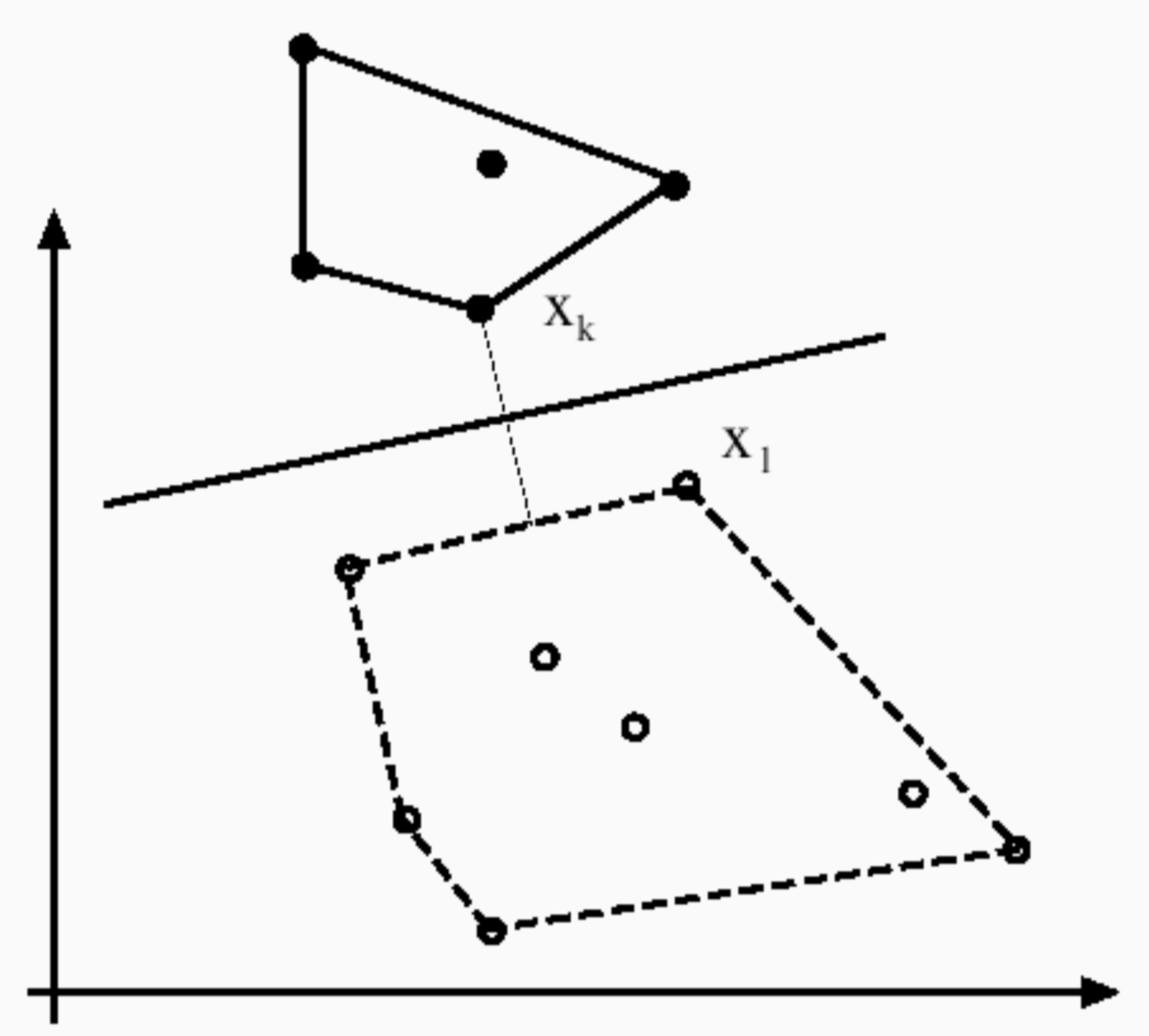
Support Vector Machines (SVM)



Forsyth & Ponce (2nd ed.) Figure 15.6

Example: Pedestrian Detection with SVM

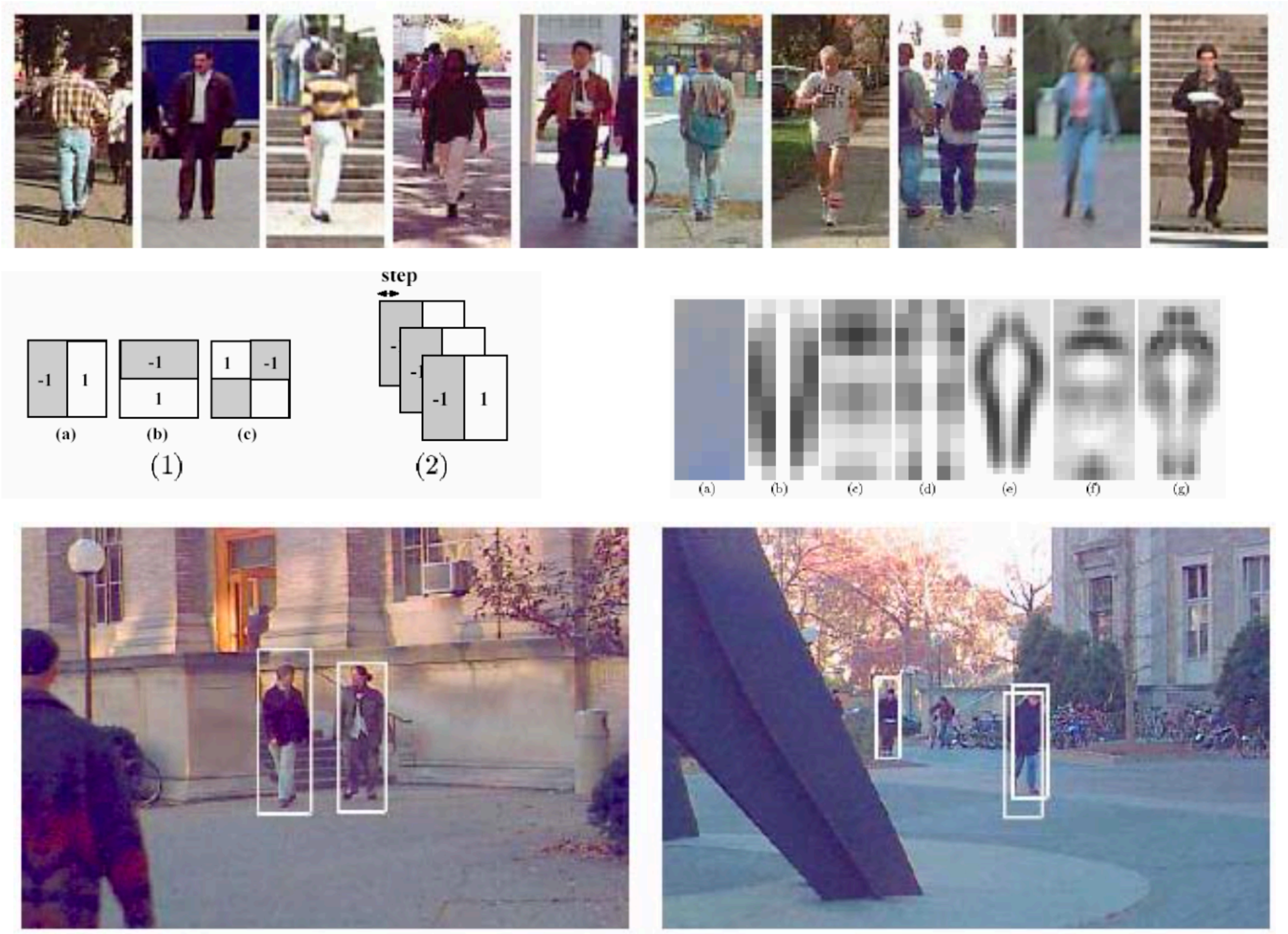


Figure credit: Papageorgiou, Oren, and Poggio, 1998

Summary

A **classifier** accepts as input a set of features and outputs (predicts) a class label

Classifiers need to take into account "loss" associated with each kind of classification error

A Receiver Operating Characteristic (ROC) curve plots the trade-off between false negatives and false positives

Parametric classifiers are model driven. The parameters of the model are learned from training examples
— e.g. support vector machine, decision tree

Non-parametric classifiers are data driven. New data points are classified by comparing to the training examples directly
— e.g. k-nearest neighbour