THE UNIVERSITY OF BRITISH COLUMBIA

CPSC 425: Computer Vision

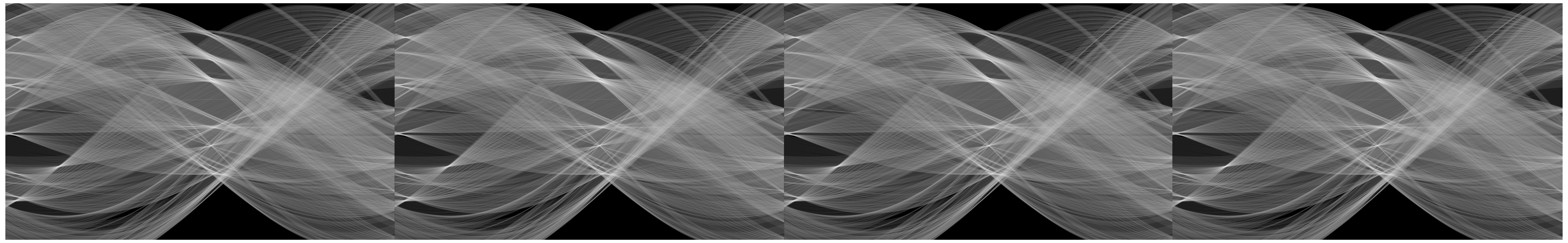


Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

Lecture 21: RANSAC cont., Hough Transform

Menu for Today (**October 26, 2018**)

Topics:

— RANSCA continued

— Hough Transform

— iClicker Quiz?

Redings:

— **Today's** Lecture: Forsyth & Ponce (2nd ed.) 10.1, 10.2

— **Next** Lecture: Forsyth & Ponce (2nd ed.) 7.1.1, 7.2.1, 7.4, 7.6

Reminders:

— **Assignment 3**: Texture Syntheis is **out,** due on **October 29th**

— Midterms **graded** (average is 64.5%)

Today’s “**fun**” Example: Everybody Dance Now

Lecture 20: Re-cap

RANSAC is a technique to fit data to a model
— divide data into inliers and outliers
— estimate model from minimal set of inliers
— improve model estimate using all inliers
— alternate fitting with re-classification as inlier/outlier

RANSAC is a general method suited for a wide range of model fitting problems
— easy to implement
— easy to estimate/control failure rate

RANSAC only handles a moderate percentage of outliers without cost blowing up

RANSAC: *k* Samples Chosen (p = 0.99)

Sample size	Proportion of outliers						
n	**5%**	**10%**	**20%**	**25%**	**30%**	**40%**	**50%**
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

Figure Credit: Hartley & Zisserman

After RANSAC

RANSAC divides data into inliers and outliers and yields estimate computed from minimal set of inliers

Improve this initial estimate with estimation over all inliers (e.g., with standard least-squares minimization)

But this may change inliers, so alternate fitting with re-classification as inlier/outlier

Example 2: Fitting a Line

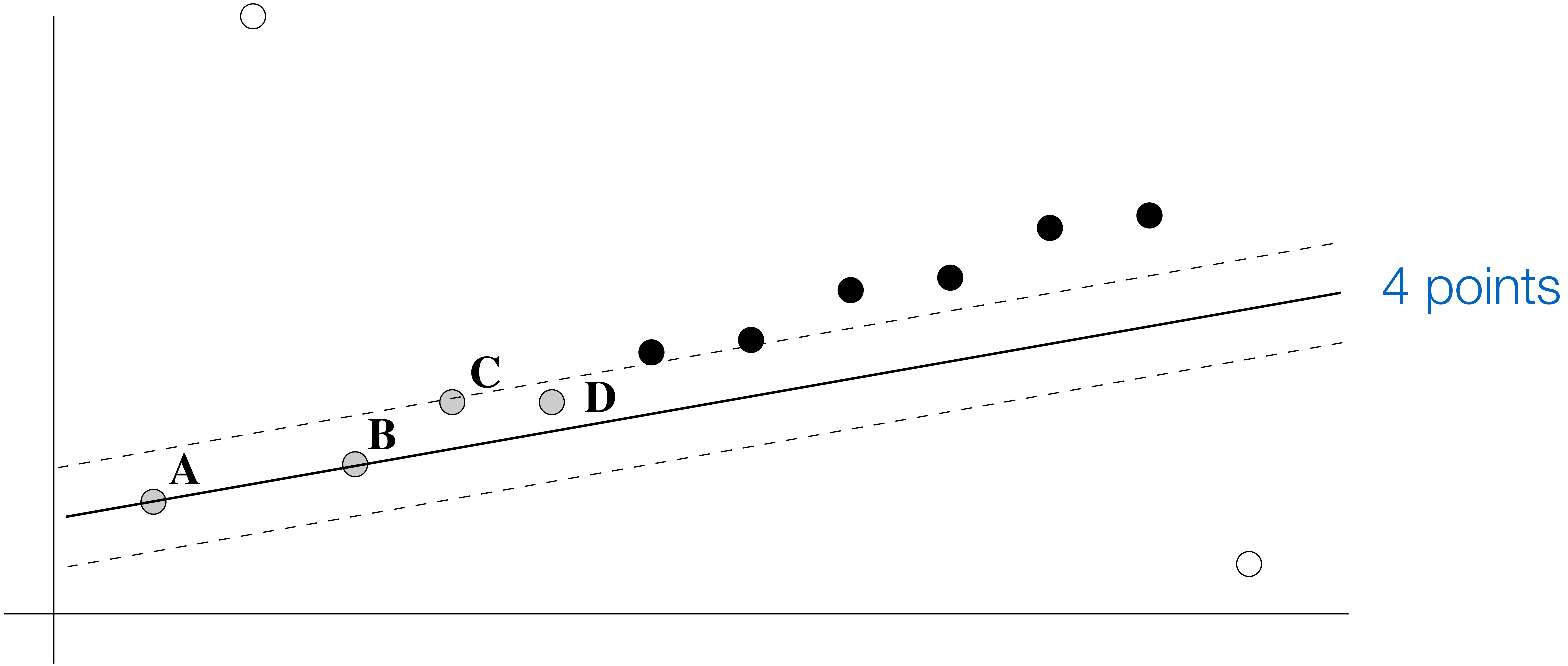


Figure Credit: Hartley & Zisserman

Example 2: Fitting a Line

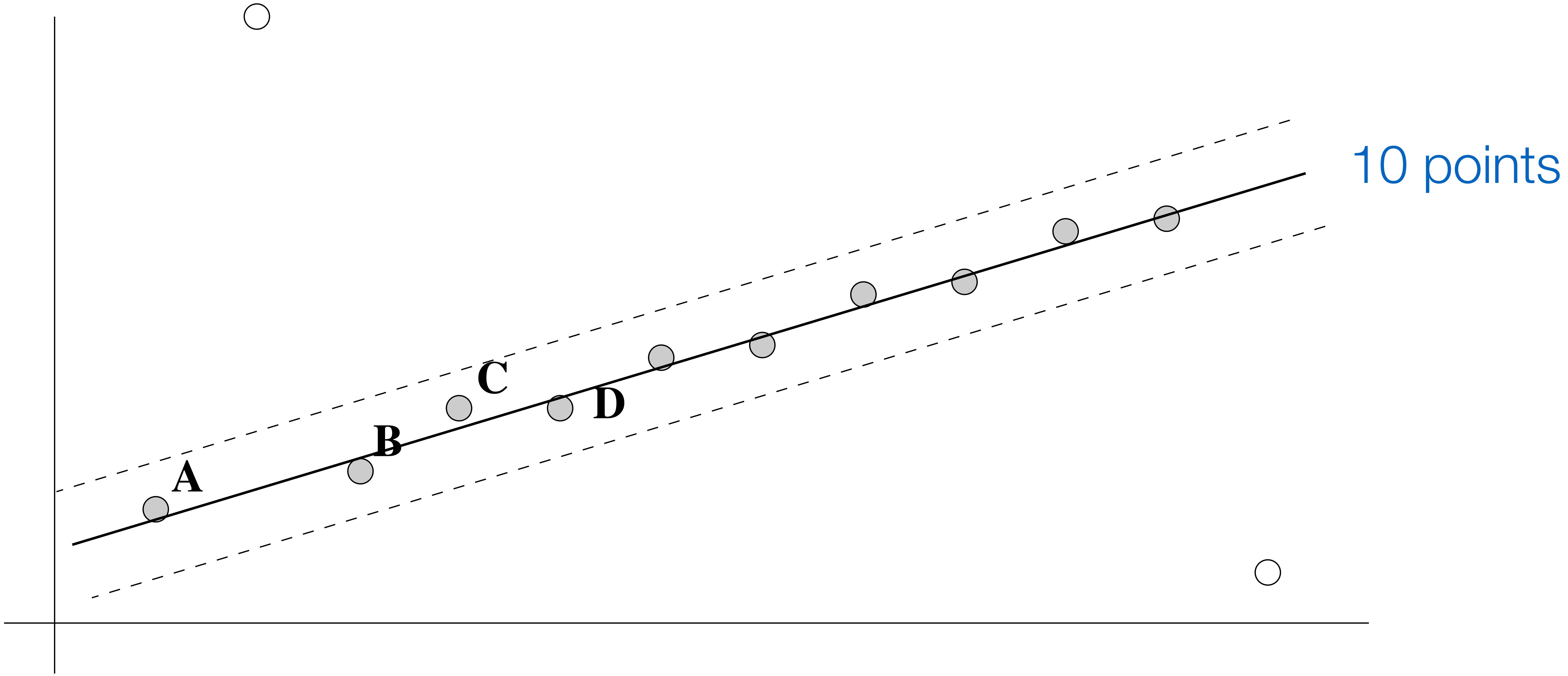


Figure Credit: Hartley & Zisserman

Example 3: Automatic Matching of Images

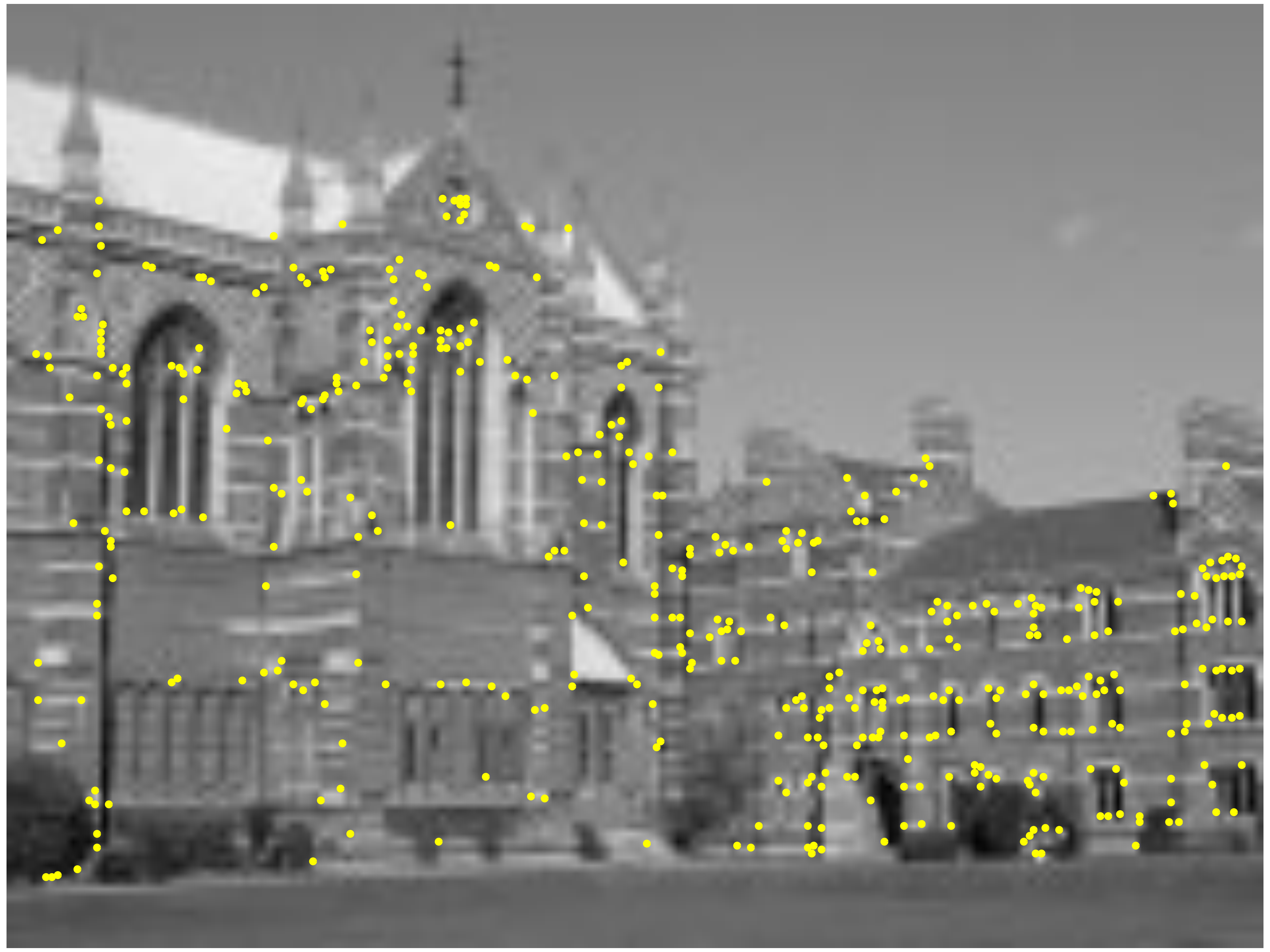
- How to get correct correspondences without human intervention?
- Can be used for image stitching or automatic determination of epipolar geometry

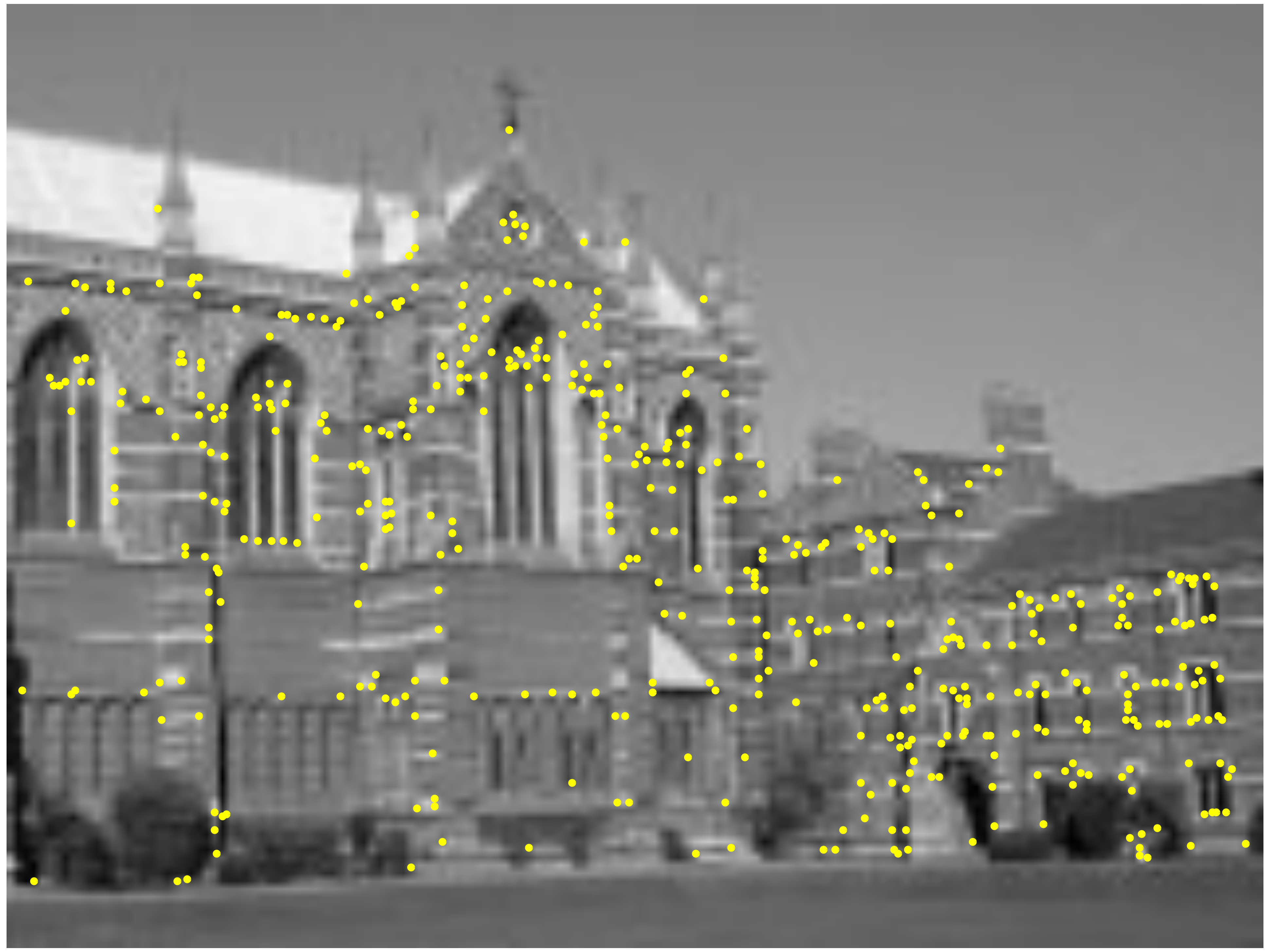
Figure Credit: Hartley & Zisserman

Example 3: Feature Extraction

— Find features in pair of images using Harris corner detector

— Assumes images are roughly the same scale



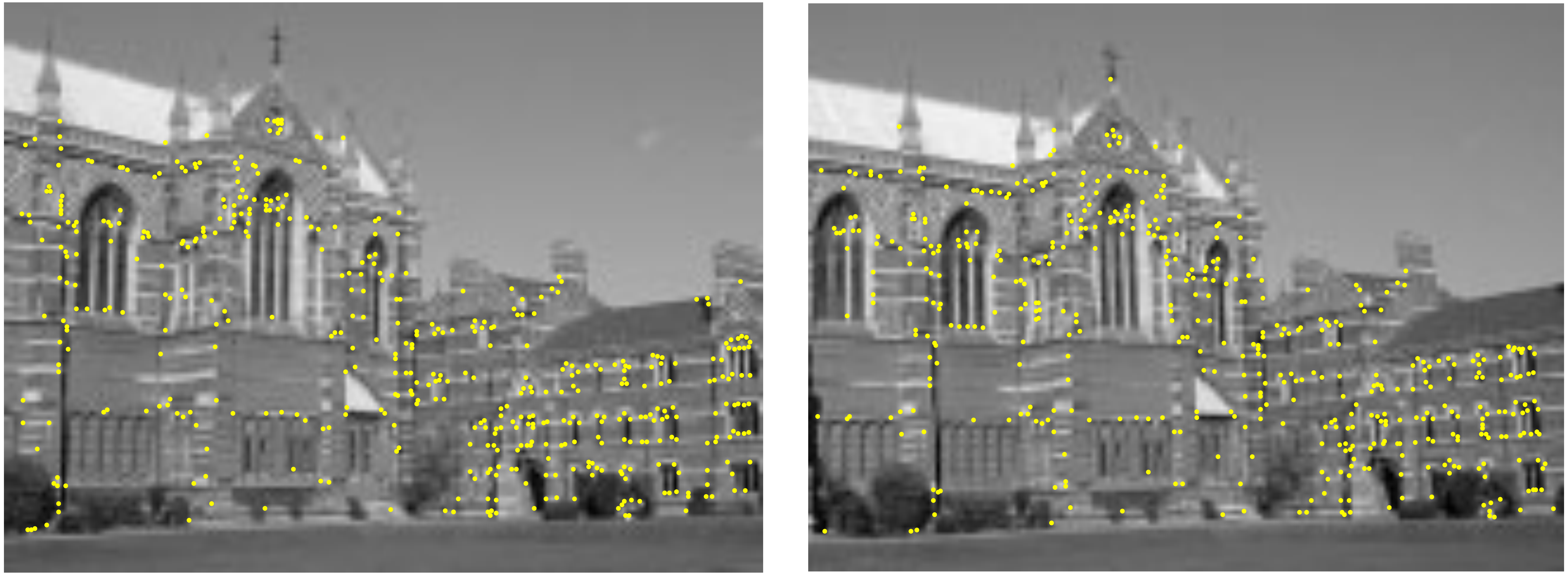


≈ 500 corner features found in each image

Figure Credit: Hartley & Zisserman

Example 3: Finding Feature Matches

Select best match over threshold within a square search window (here ±320 pixels) using SSD or (normalized) cross-correlation for small patch around the corner



≈ 500 corner features found in each image

Figure Credit: Hartley & Zisserman

Example 3: Initial Match Hypothesis

268 matched features (over SSD threshold) superimposed on left image (pointing to locations of corresponding feature in right image)

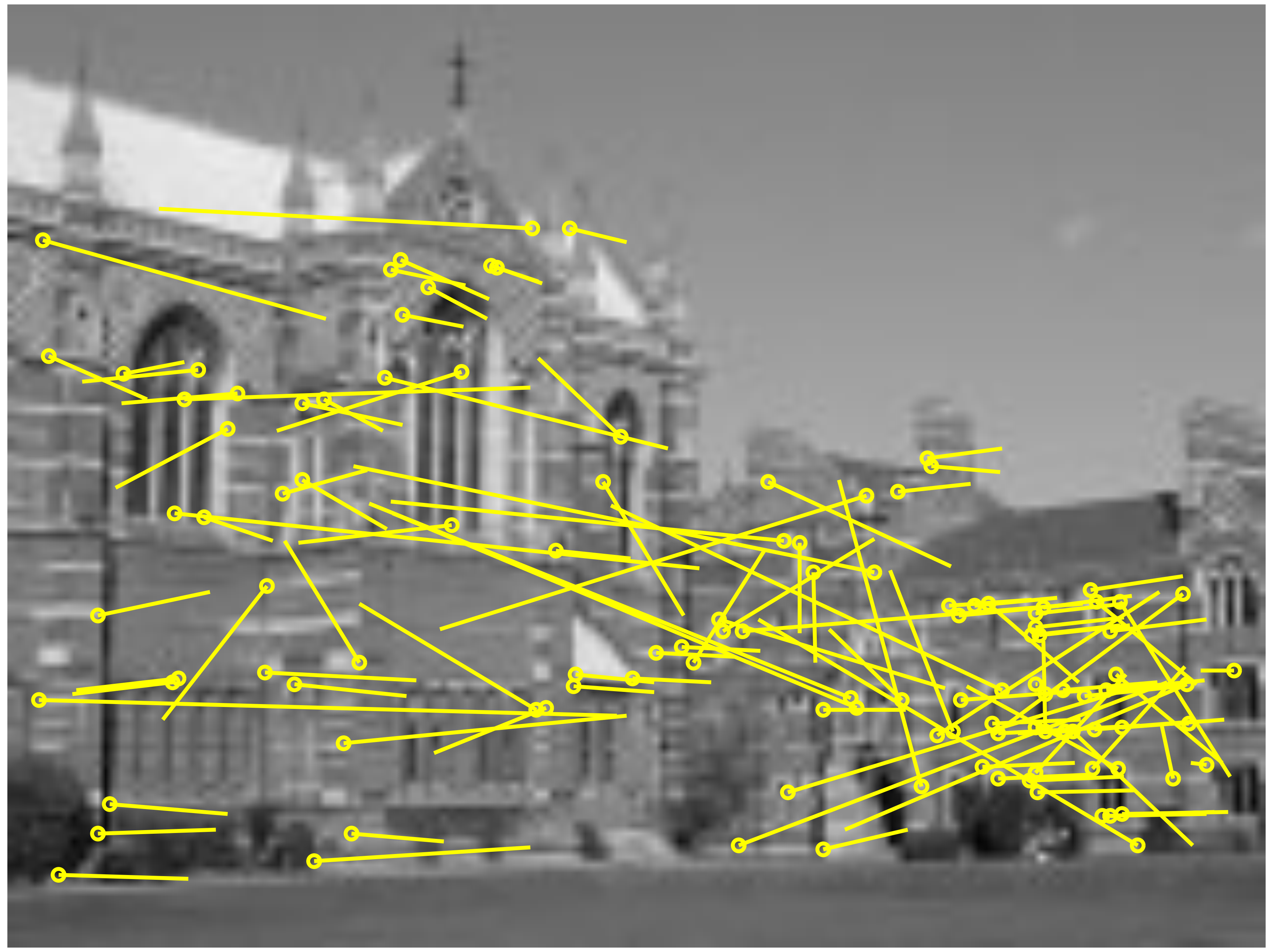
Figure Credit: Hartley & Zisserman

Example 3: Outliers & Inliers after RANSAC

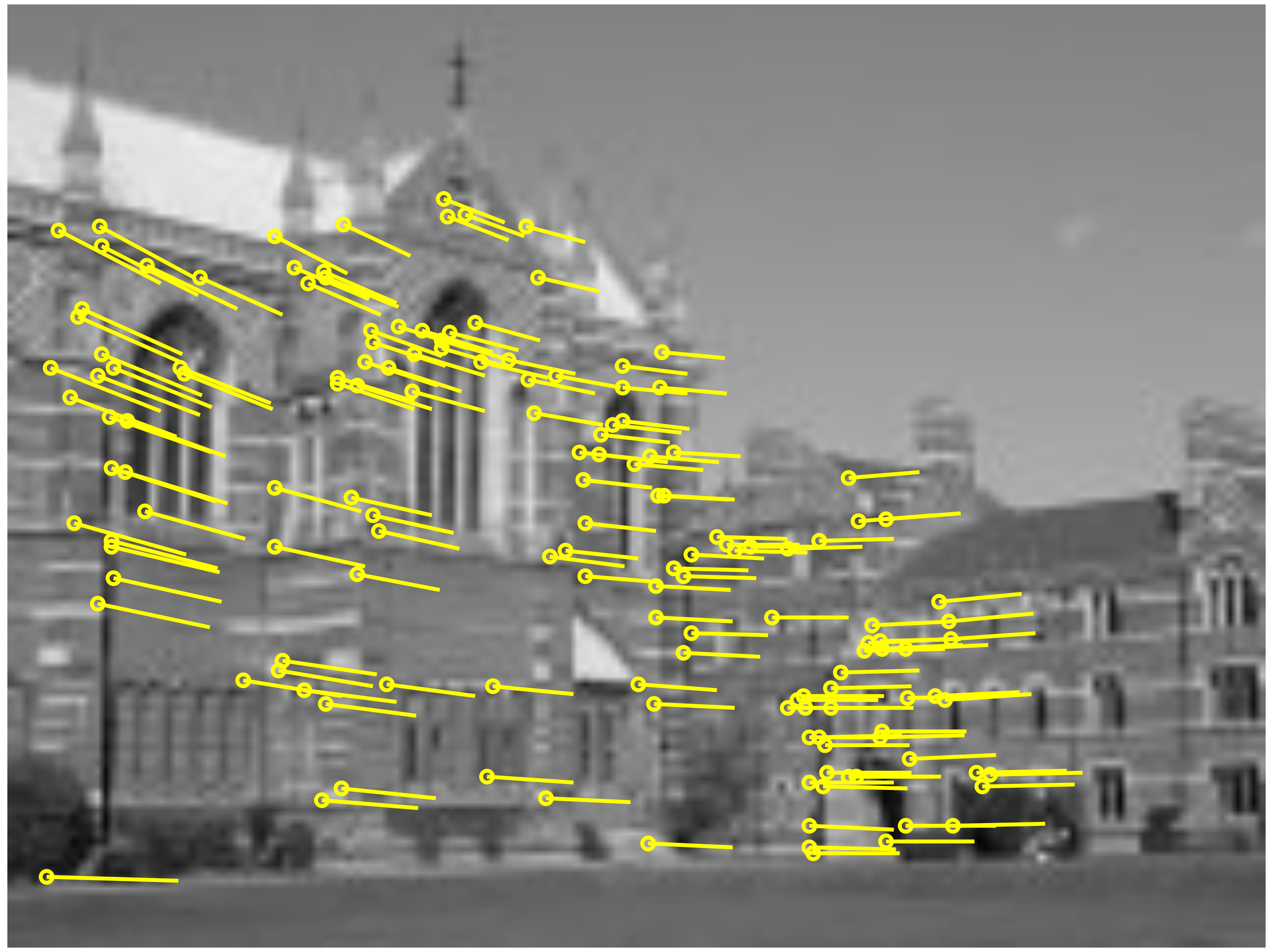
— *n* is 4 for this problem (a homography relating 2 images)

— Assume up to 50% outliers

— 43 samples used with t = 1.25 pixels



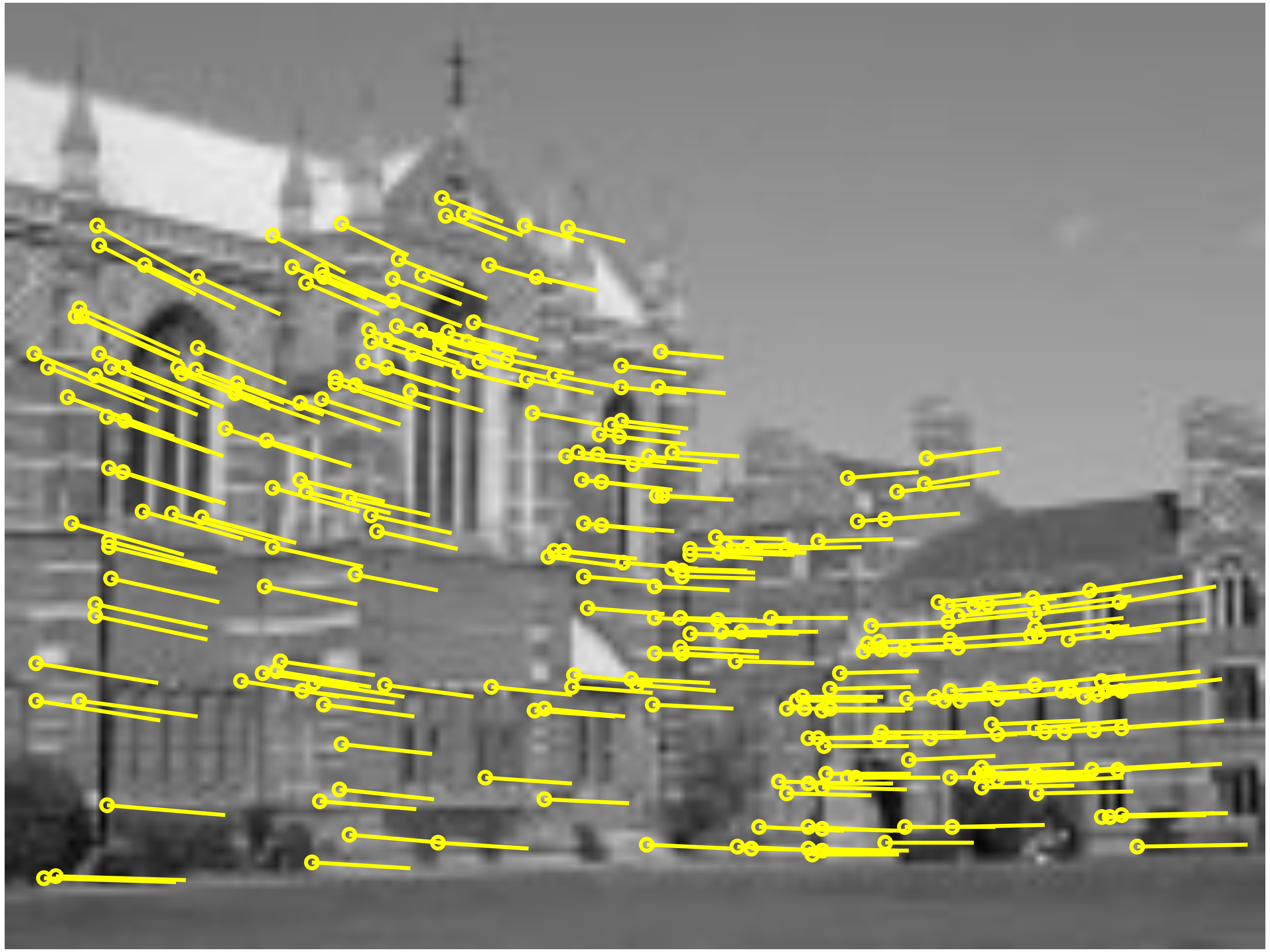
117 **outliers**



151 **inliers**

Figure Credit: Hartley & Zisserman

Example 3: Final Matches



final set of 262 matches

Figure Credit: Hartley & Zisserman

Discussion of RANSAC

Advantages:

— General method suited for a wide range of model fitting problems

— Easy to implement and easy to calculate its failure rate

Disadvantages:

— Only handles a moderate percentage of outliers without cost blowing up

— Many real problems have high rate of outliers (but sometimes selective choice of random subsets can help)

The Hough transform can handle high percentage of outliers

Example: Photo Tourism

Figure credit: Snavely et al. 2006

Takes as input unstructured collections of photographs and reconstructs each photo’s viewpoint and a sparse 3D model of the scene

Uses both SIFT and RANSAC

Fitting a Model

Suppose we want to fit a model to a set of tokens

— e.g. A line fits well to a set of points. This is unlikely to be due to chance, so we represent the points as a line.

— e.g. A 3D model can be scaled, rotated and translated to closely fit a set of points or line segments. If it fits well, the object is recognized.

Fitting a Model is Difficult

Difficulties arise owing to:

Extraneous data: clutter or multiple models

— We do not know what is part of the model

— Can we fit models with a few parts when there is significant background clutter?

Missing data: only some parts of model are present Noise

Computational cost:

— Not feasible to check all combinations of features by fitting a model to each possible subset

Hough Transform

Idea of **Hough transform**:

— For each token vote for all models to which the token could belong

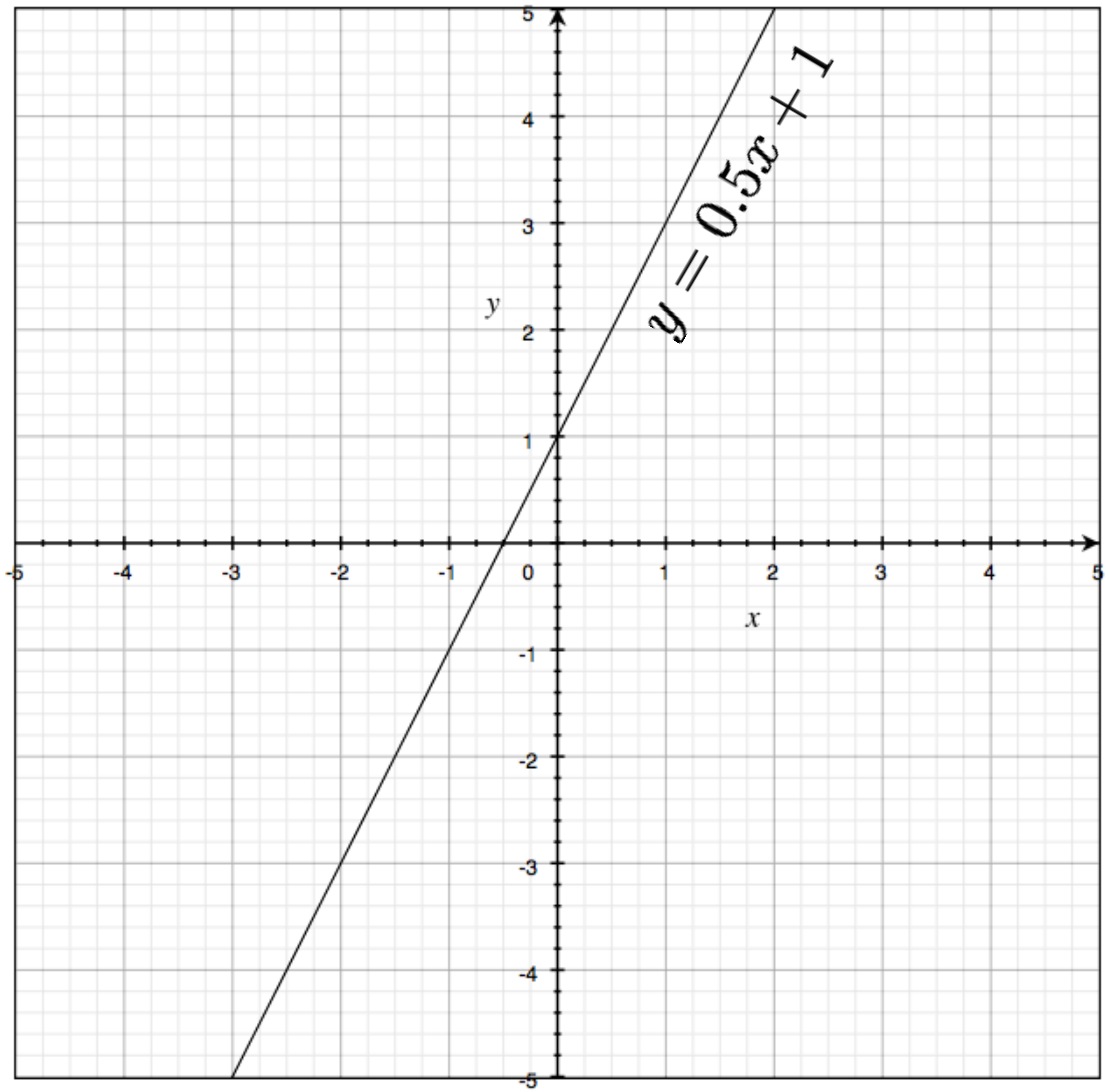
— Return models that get many votes

Example: For each point, vote for all lines that could pass through it; the true lines will pass through many points and so receive many votes

Lines: Slope intercept form

$$y = mx + b$$

slope y-intercept



Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Hough Transform: Image and Parameter Space

variables

$$y = mx + b$$

parameters

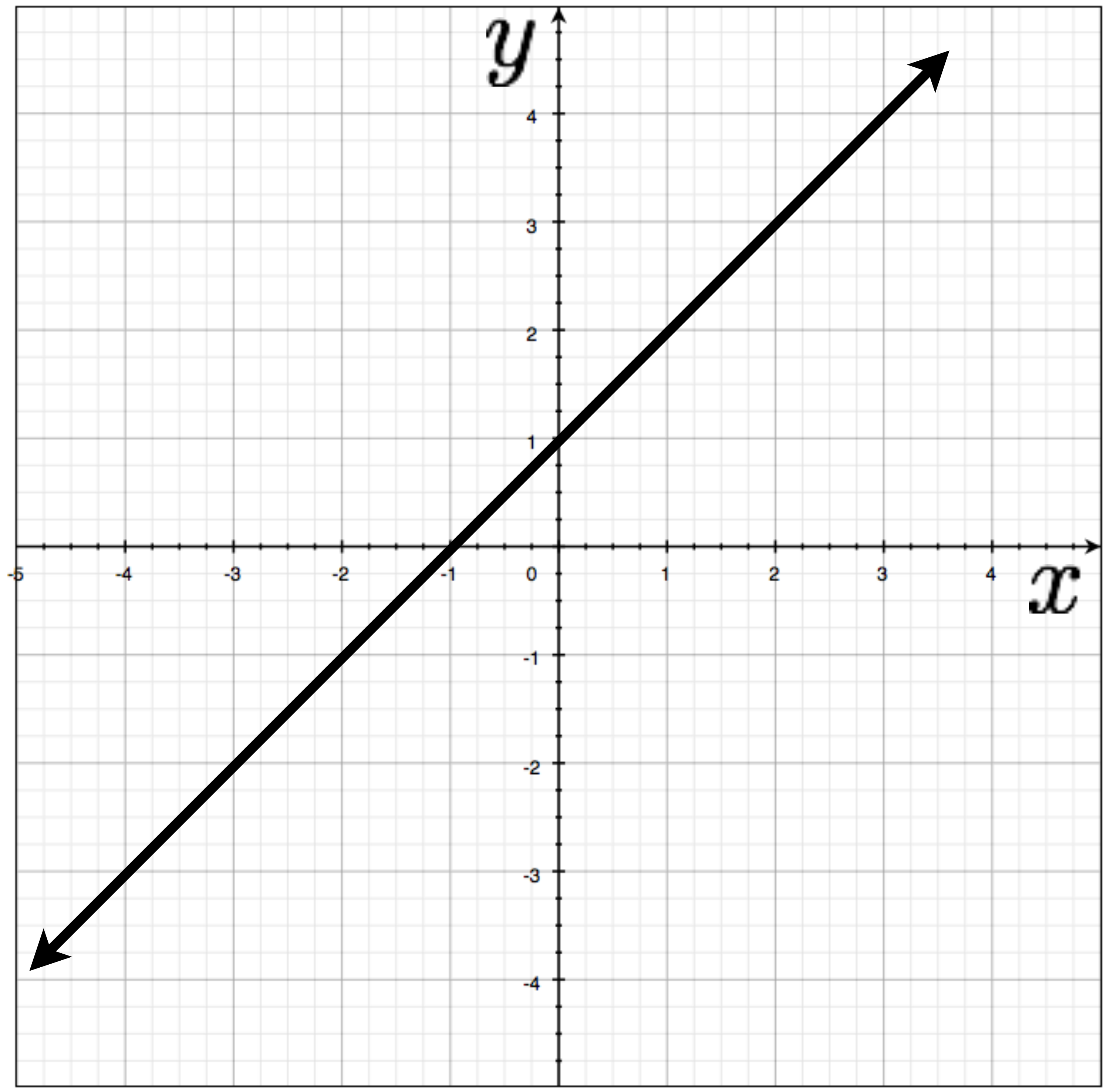


Image space

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Hough Transform: Image and Parameter Space

variables

$$y = mx + b$$

parameters

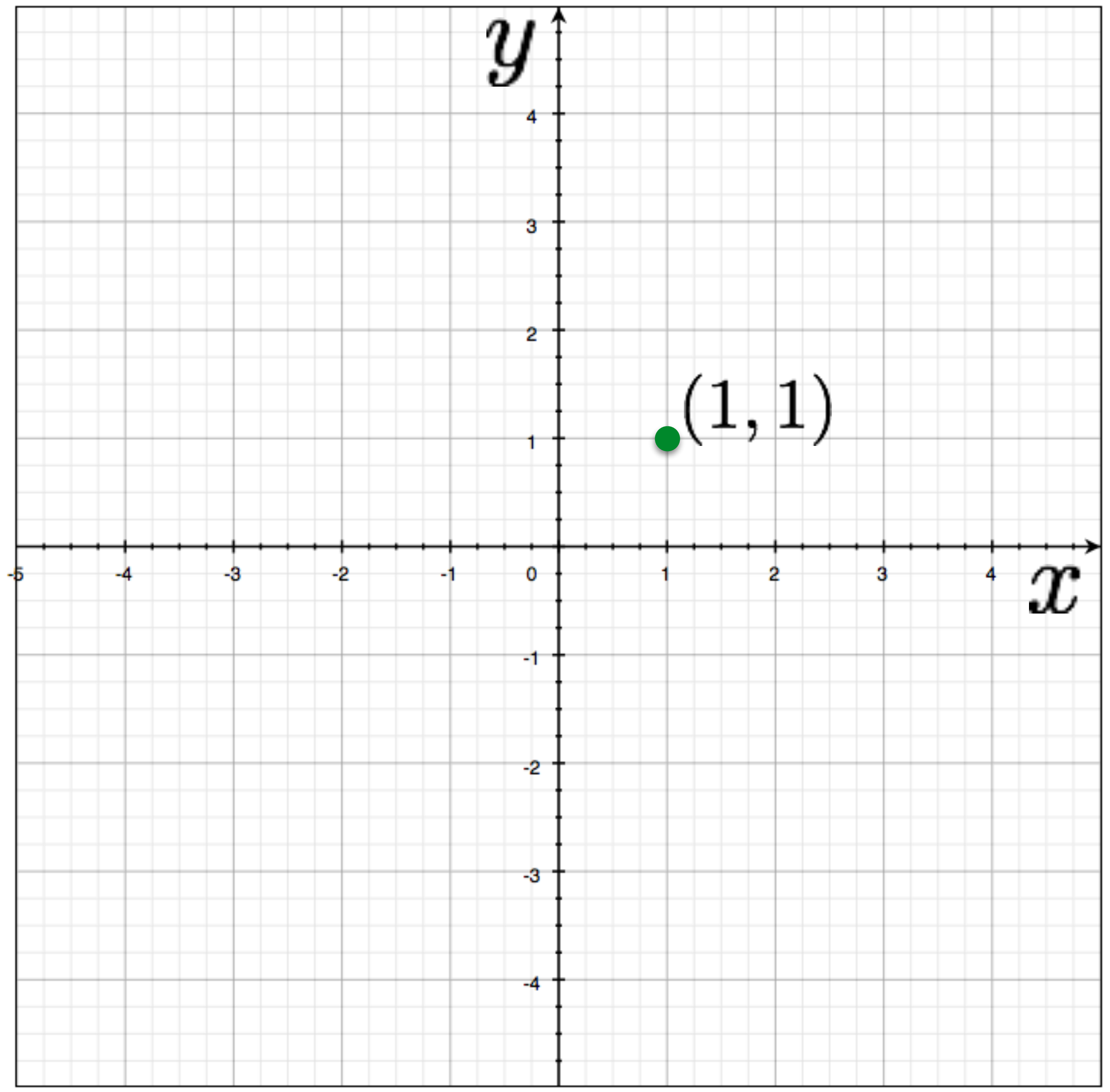


Image space

What would a **point** in image space become in **parameter space**?

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Hough Transform: Lines

variables

$$y = mx + b$$

parameters

variables

$$y - mx = b$$

parameters

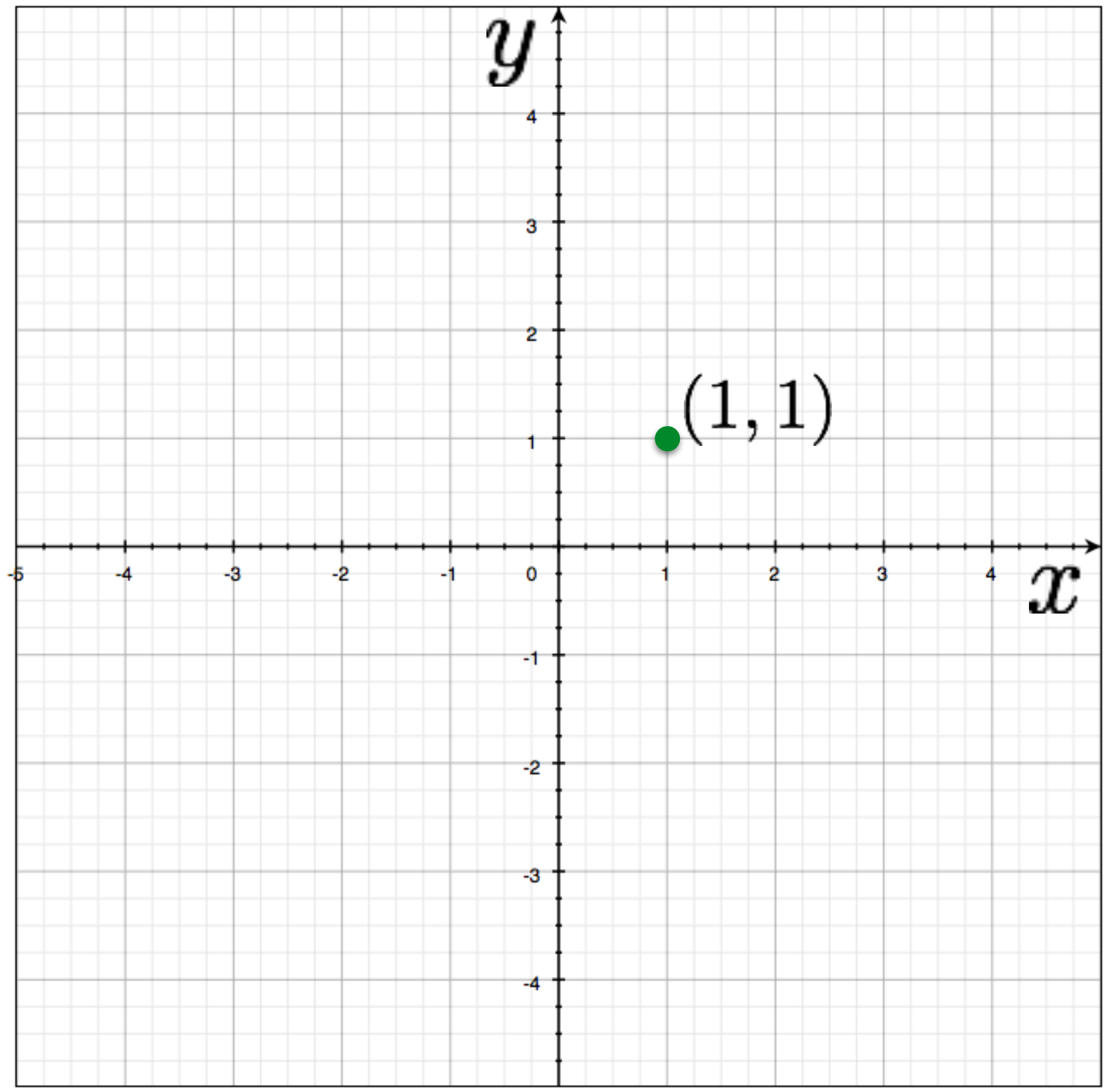
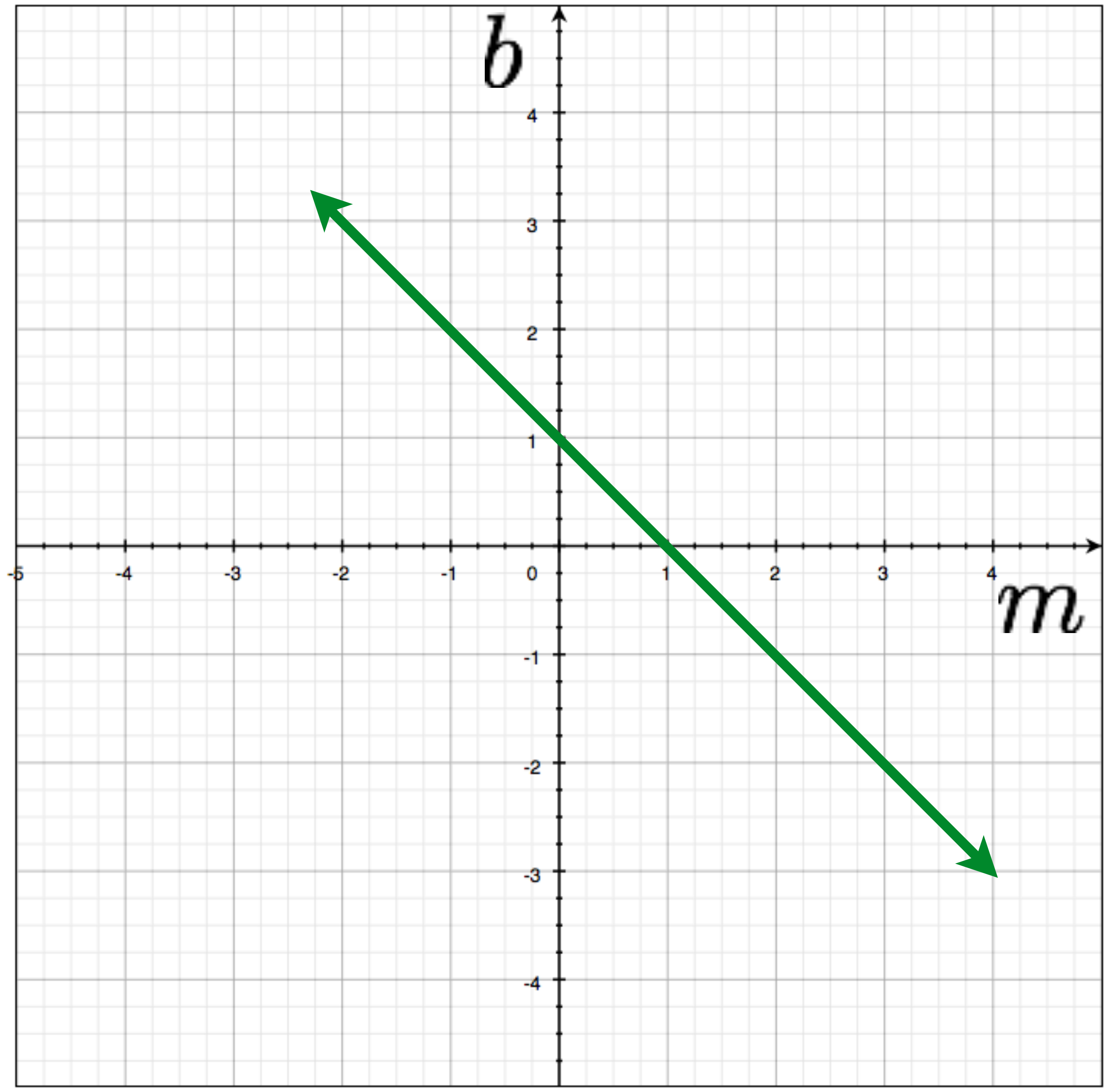


Image space

a point becomes a line



Parameter space

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Hough Transform: Lines

variables

$$y = mx + b$$

parameters

variables

$$y - mx = b$$

parameters

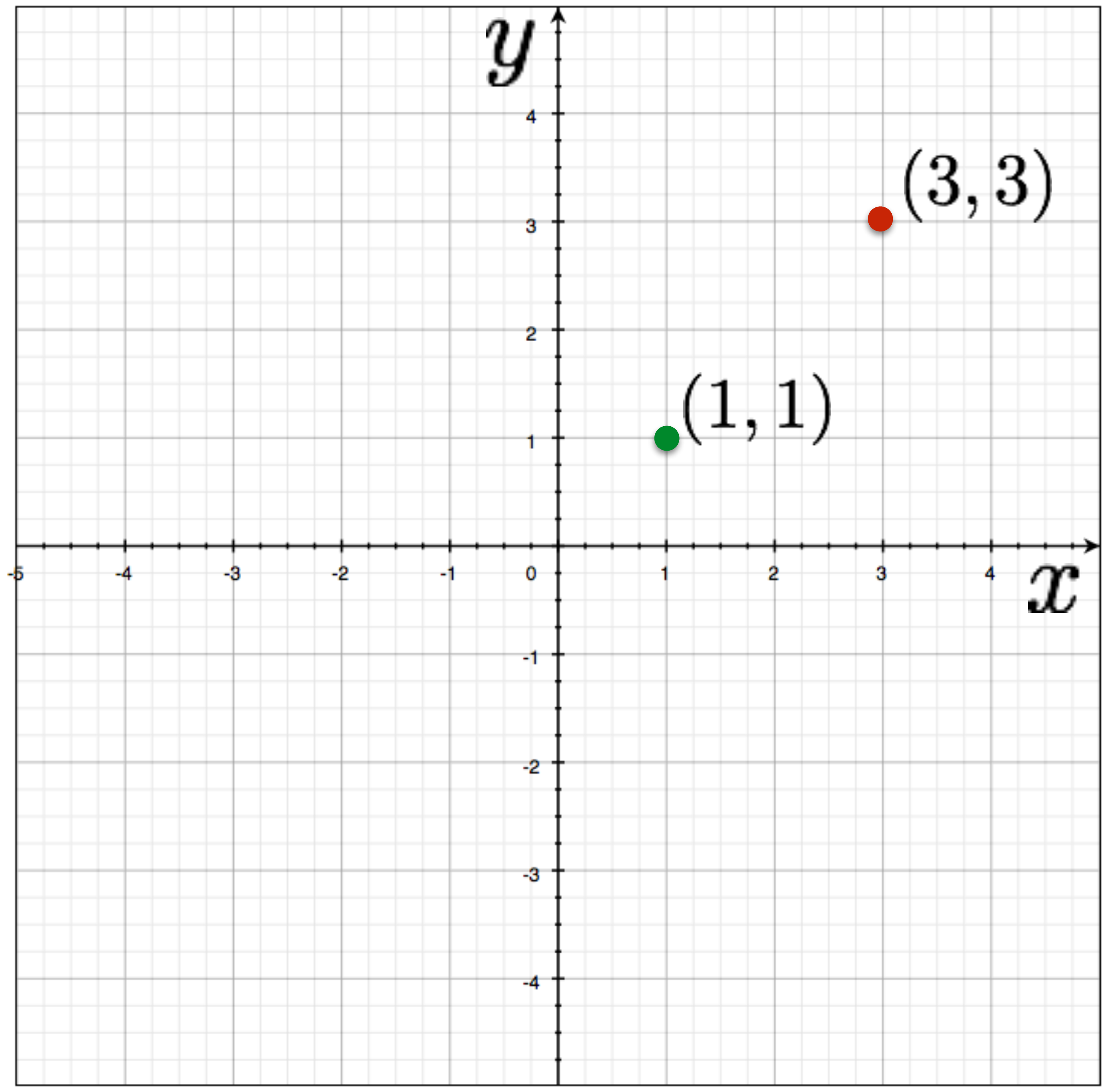
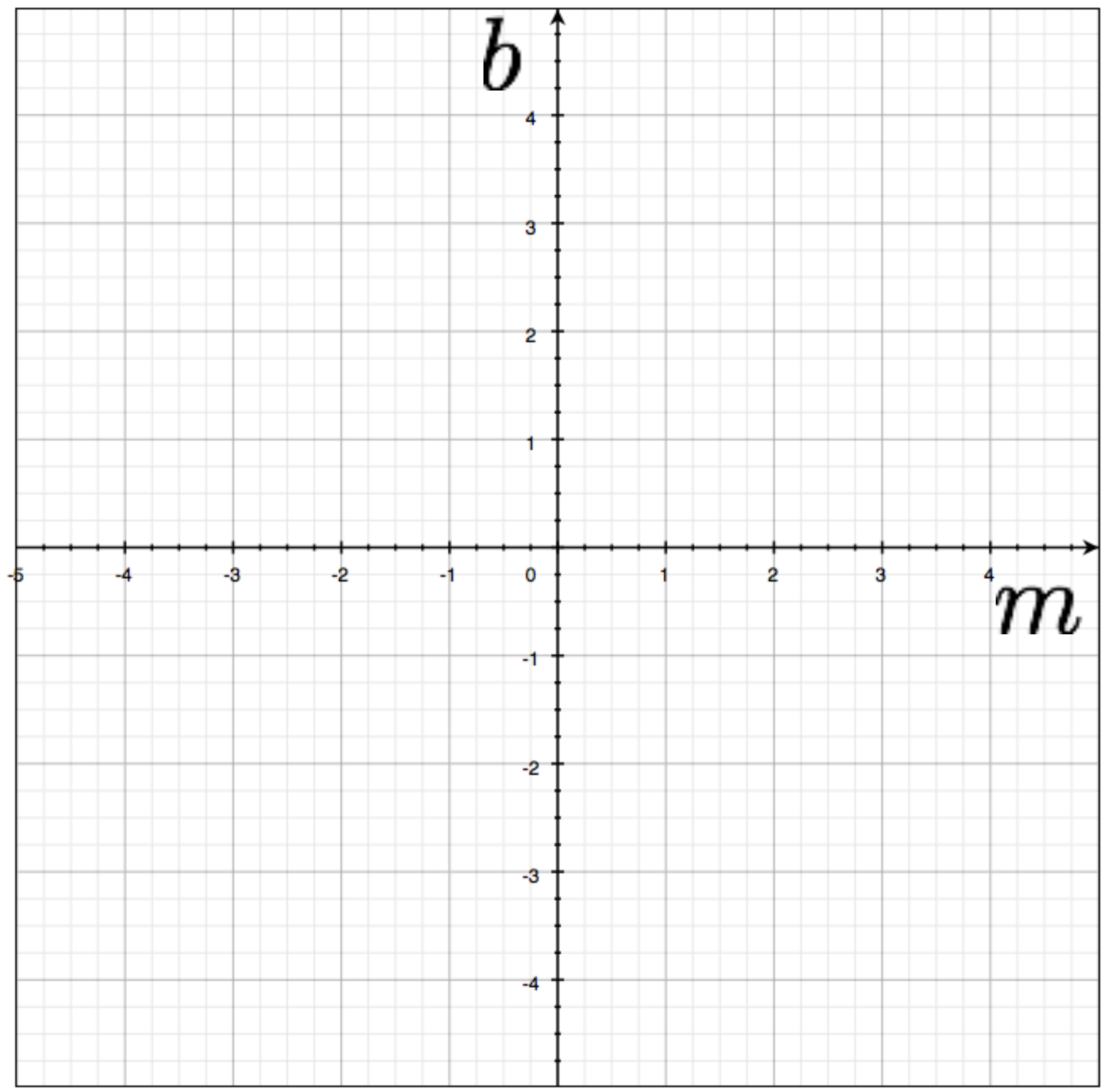


Image space

two points?



Parameter space

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Hough Transform: Lines

variables

$$y = mx + b$$

parameters

variables

$$y - mx = b$$

parameters

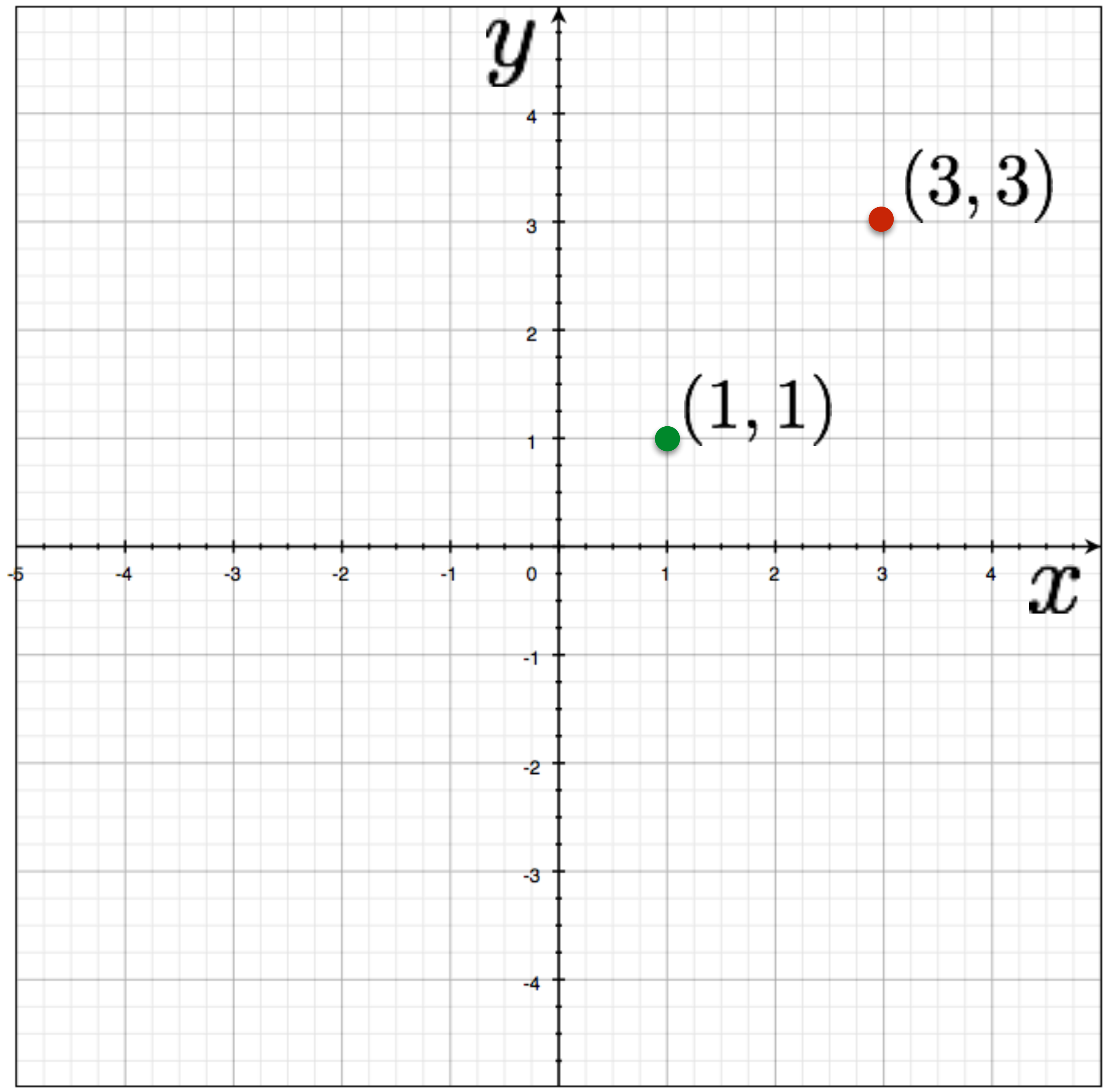
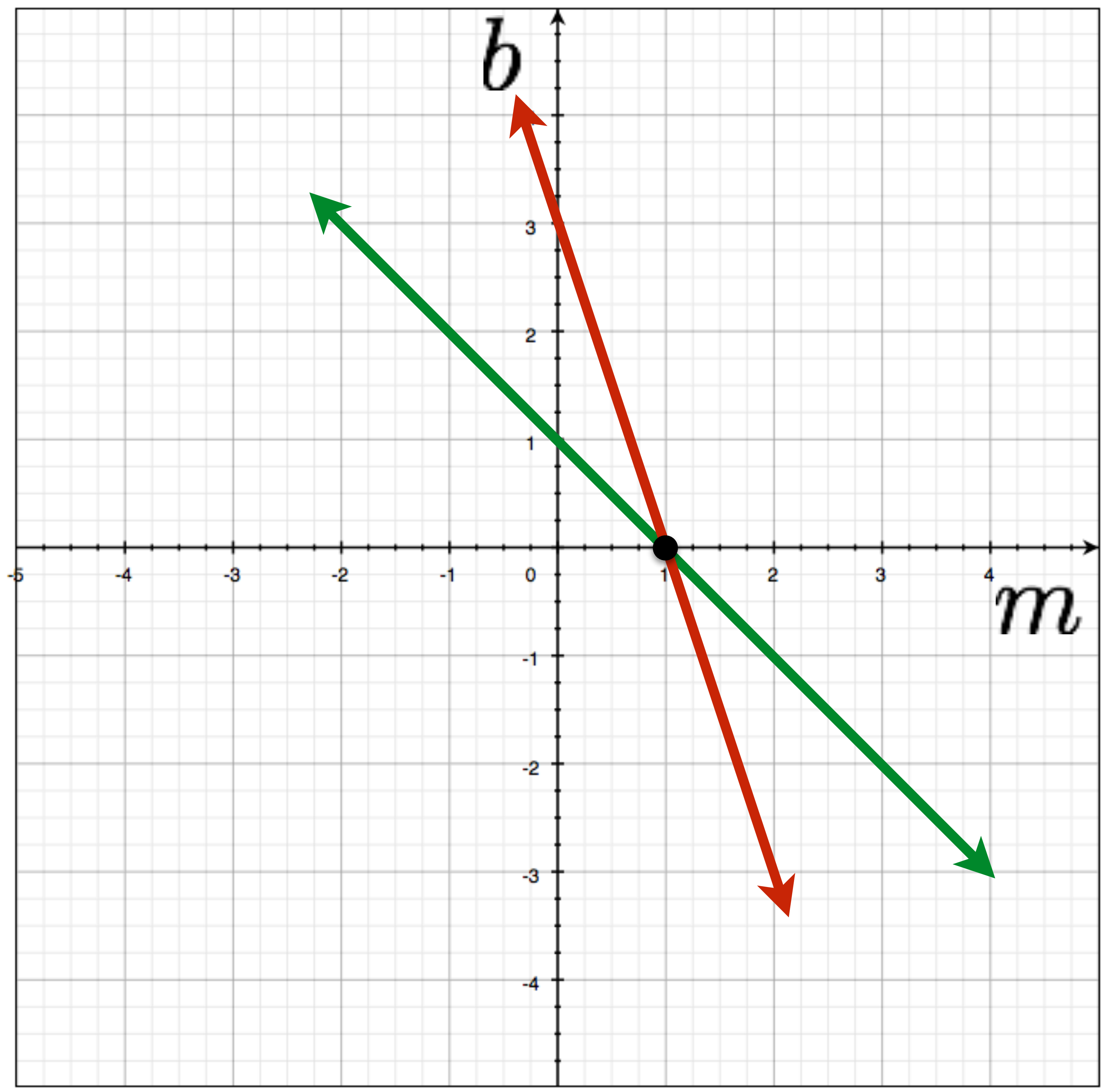


Image space

two
points?



Parameter space

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Hough Transform: Lines

variables

$$y = mx + b$$

parameters

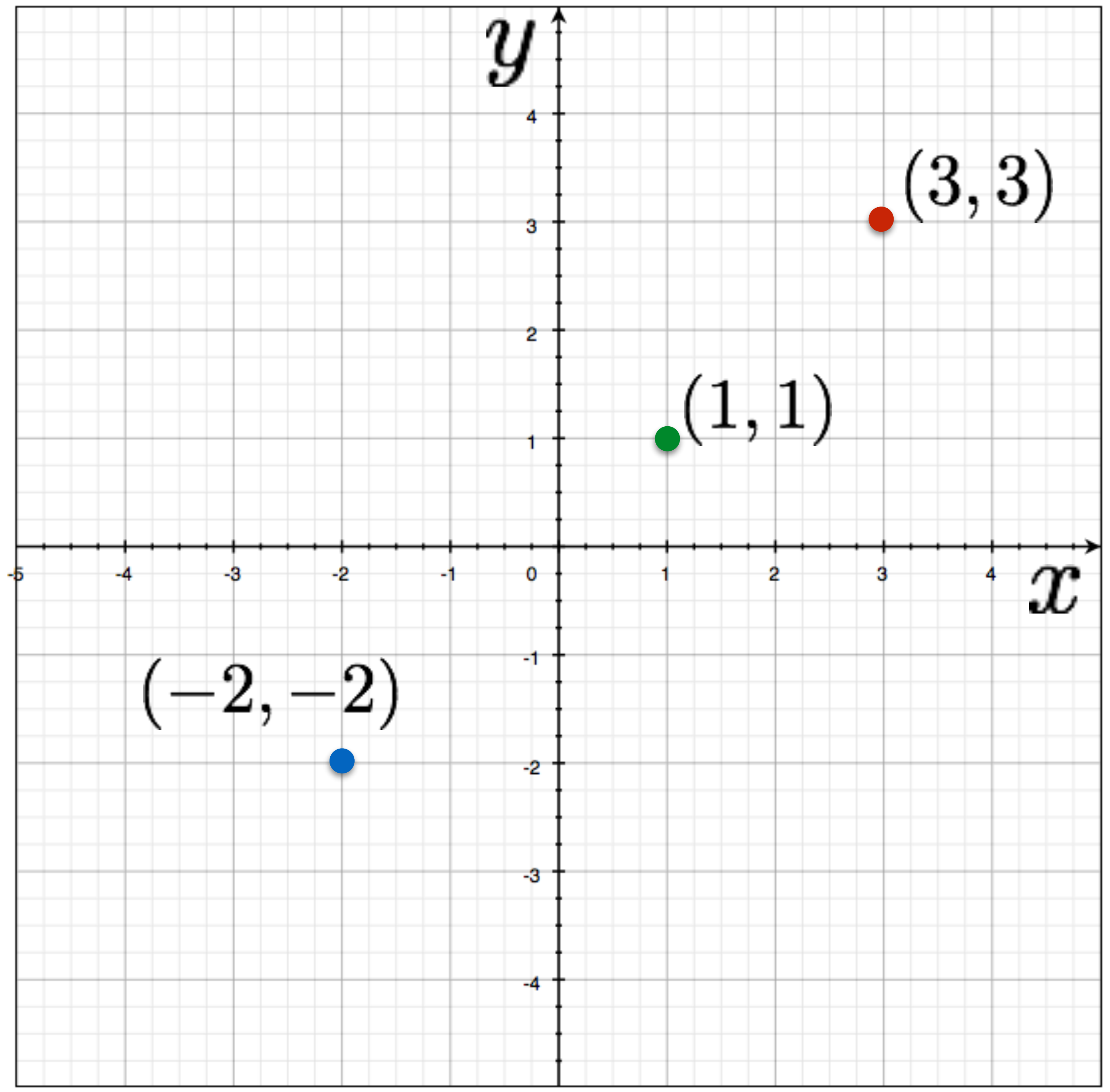


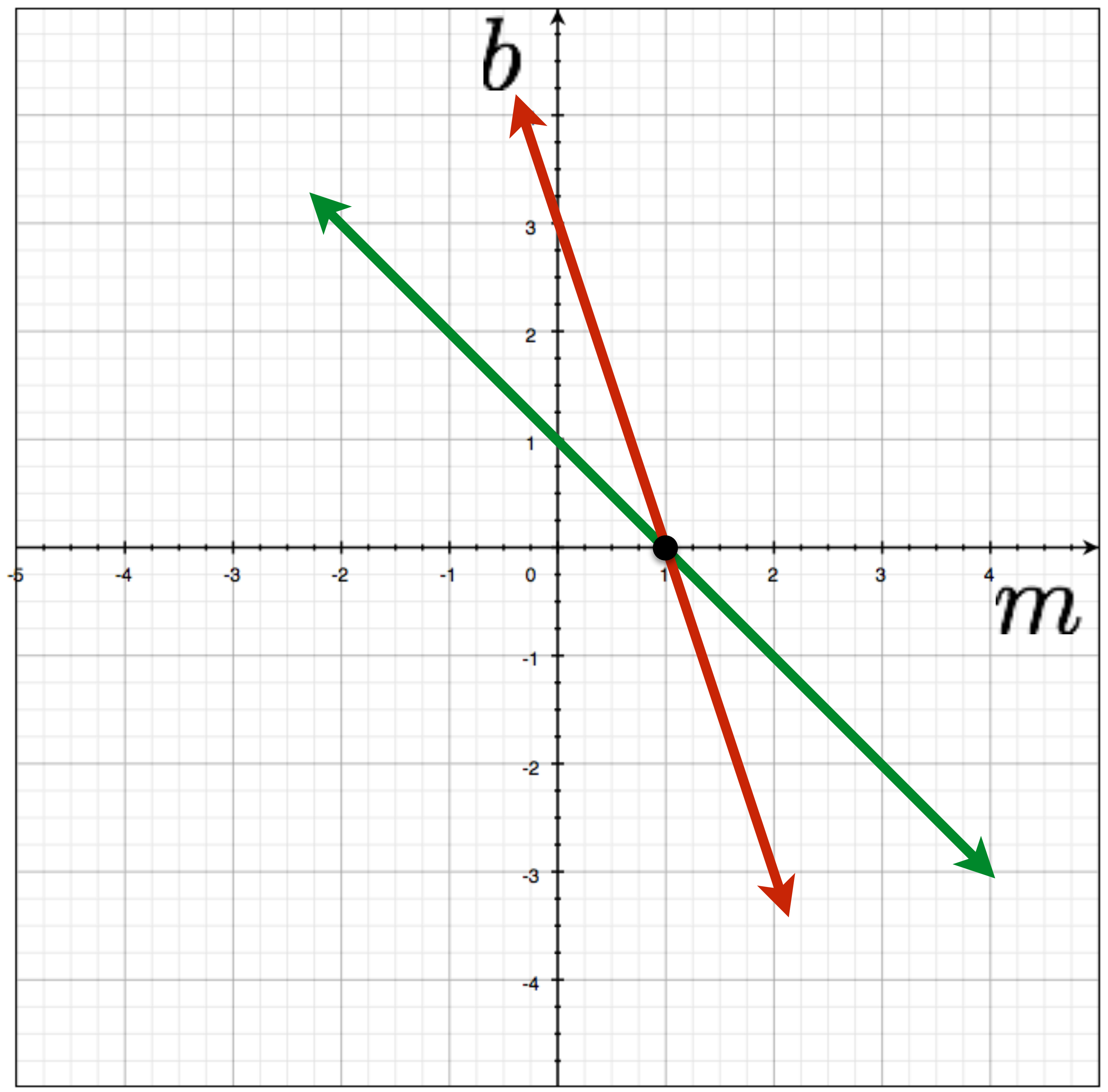
Image space

three
points?

variables

$$y - mx = b$$

parameters



Parameter space

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Hough Transform: Lines

variables

$$y = mx + b$$

parameters

variables

$$y - mx = b$$

parameters

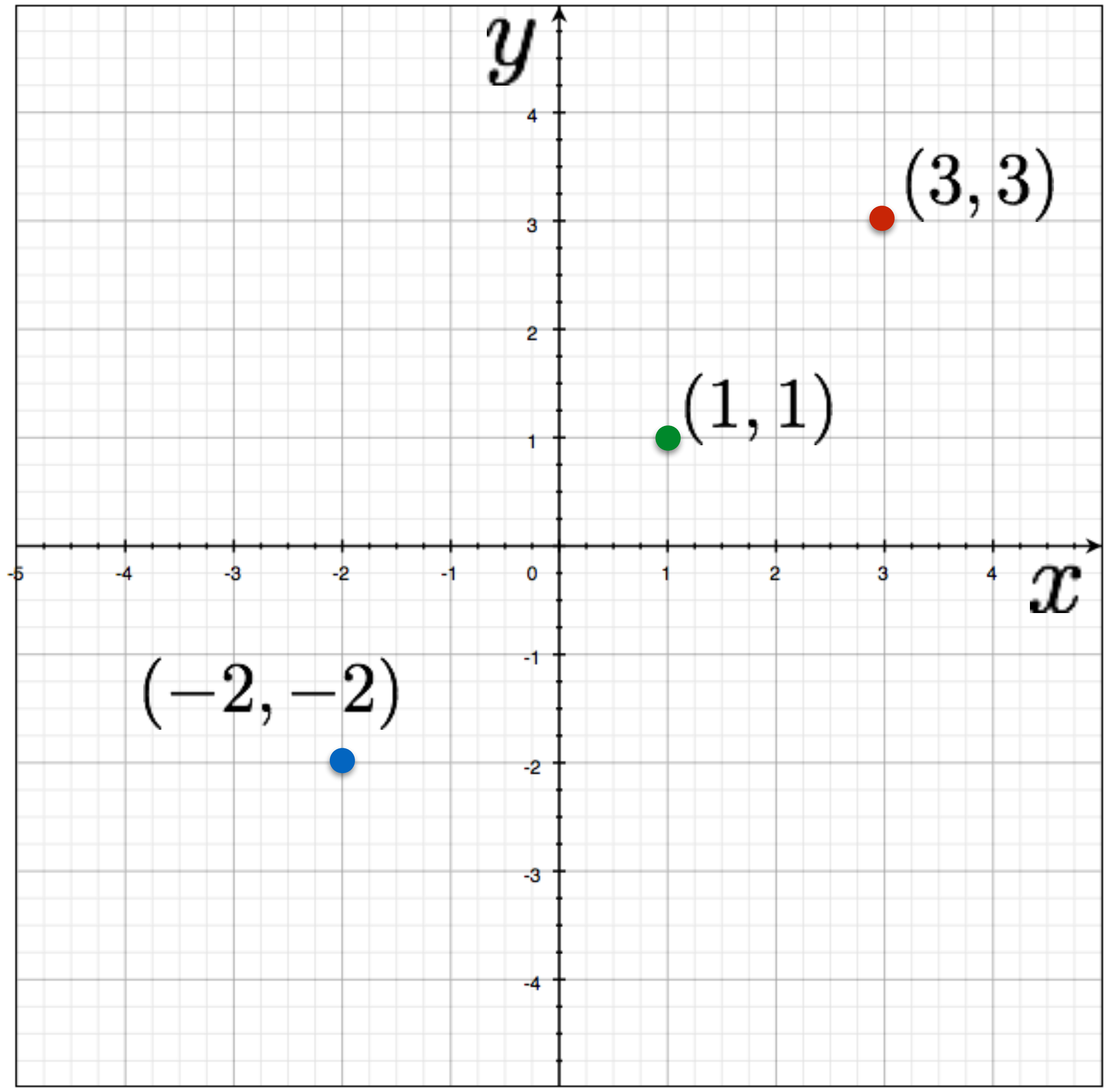
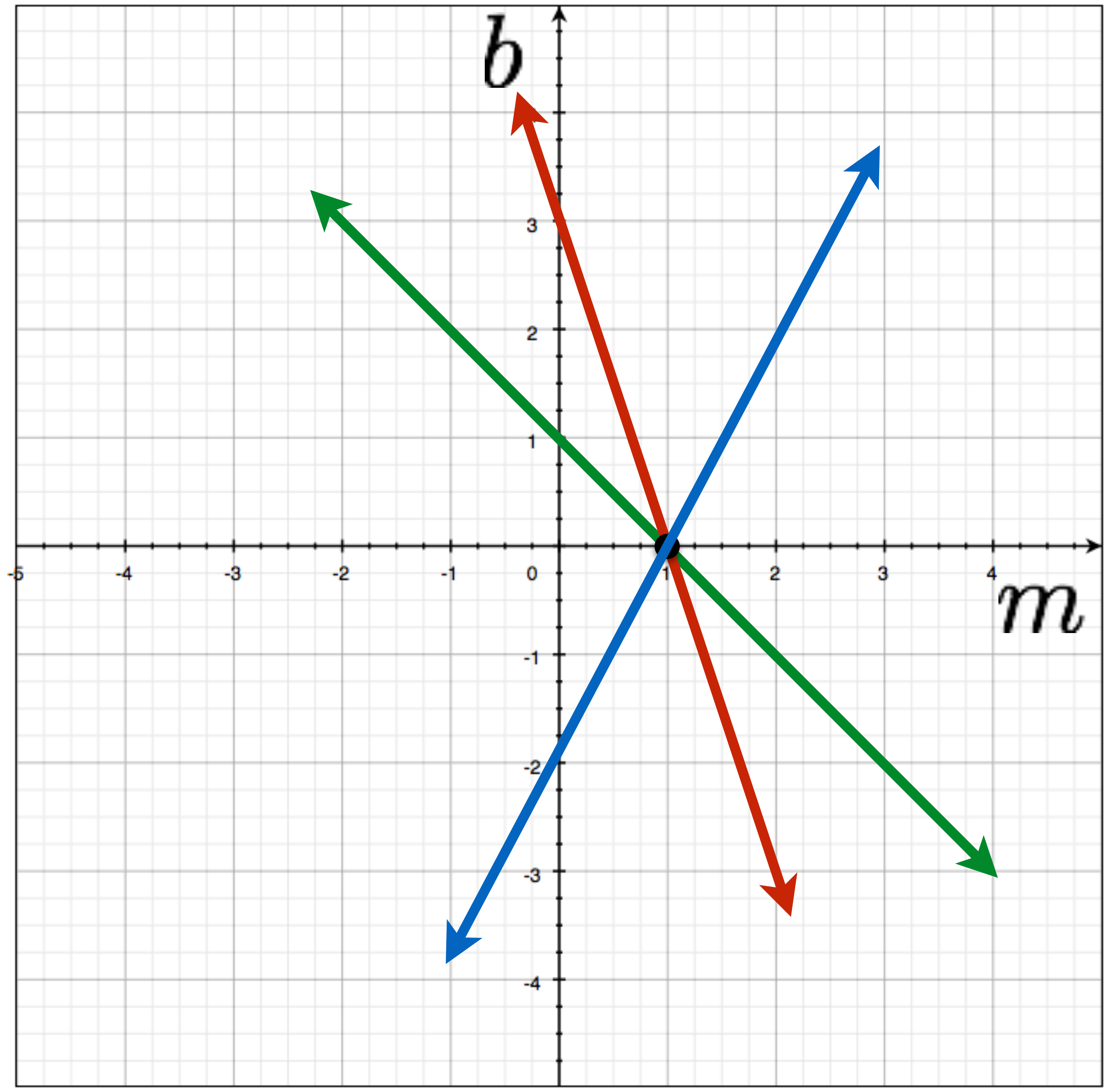


Image space

three
points?



Parameter space

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Hough Transform: Lines

variables

$$y = mx + b$$

parameters

variables

$$y - mx = b$$

parameters

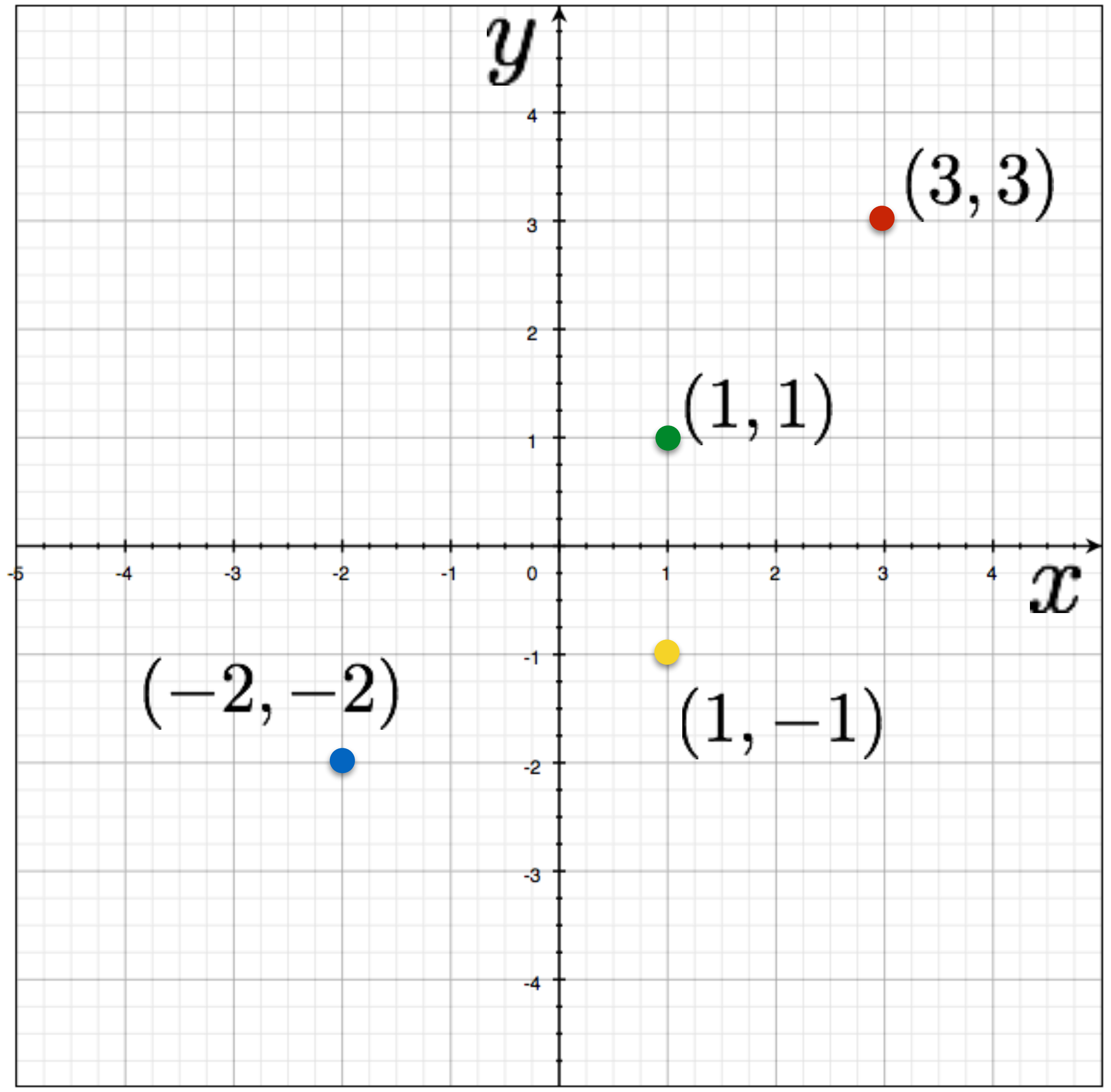
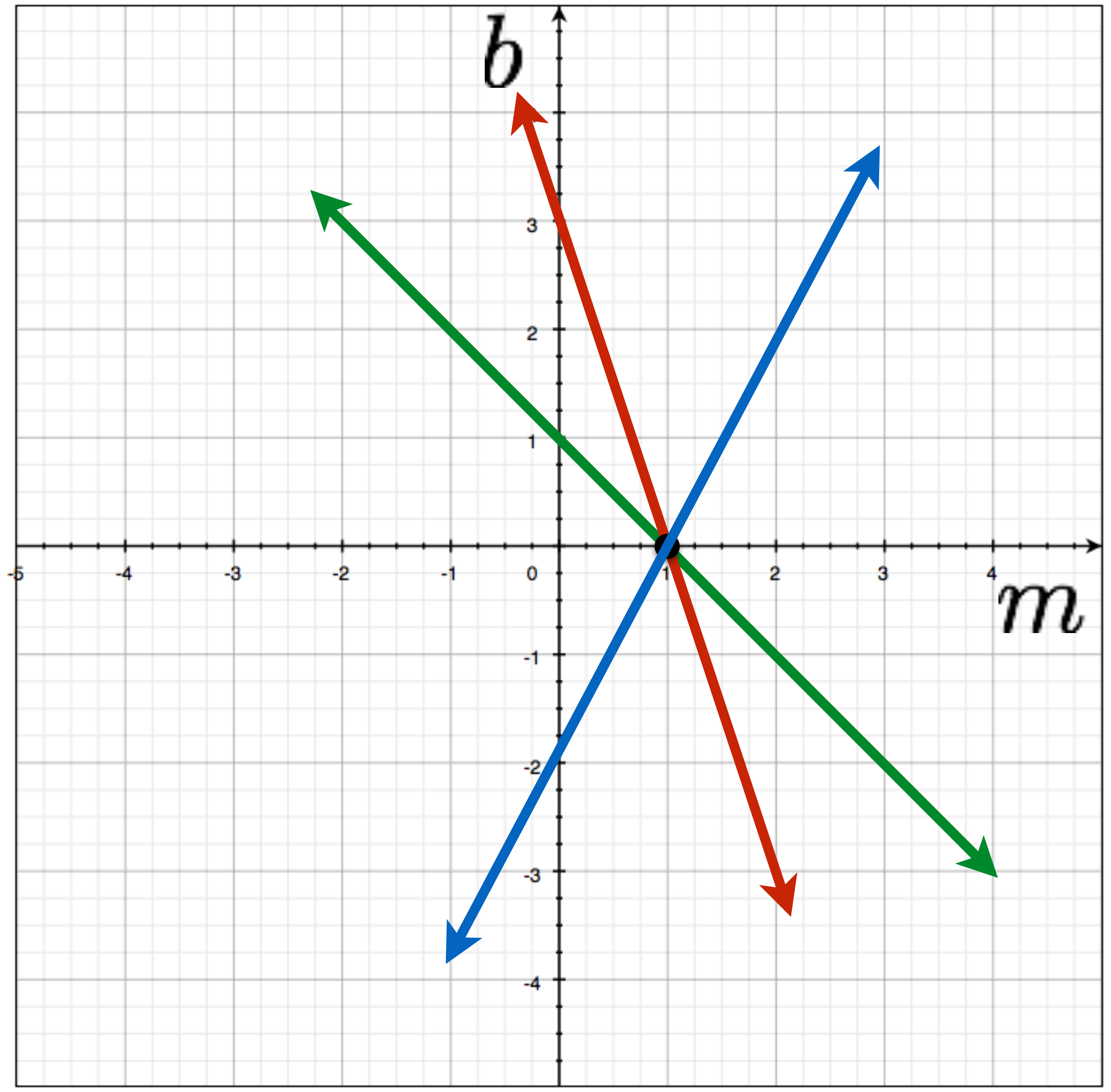


Image space



Parameter space

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Hough Transform: Lines

variables

$$y = mx + b$$

parameters

variables

$$y - mx = b$$

parameters

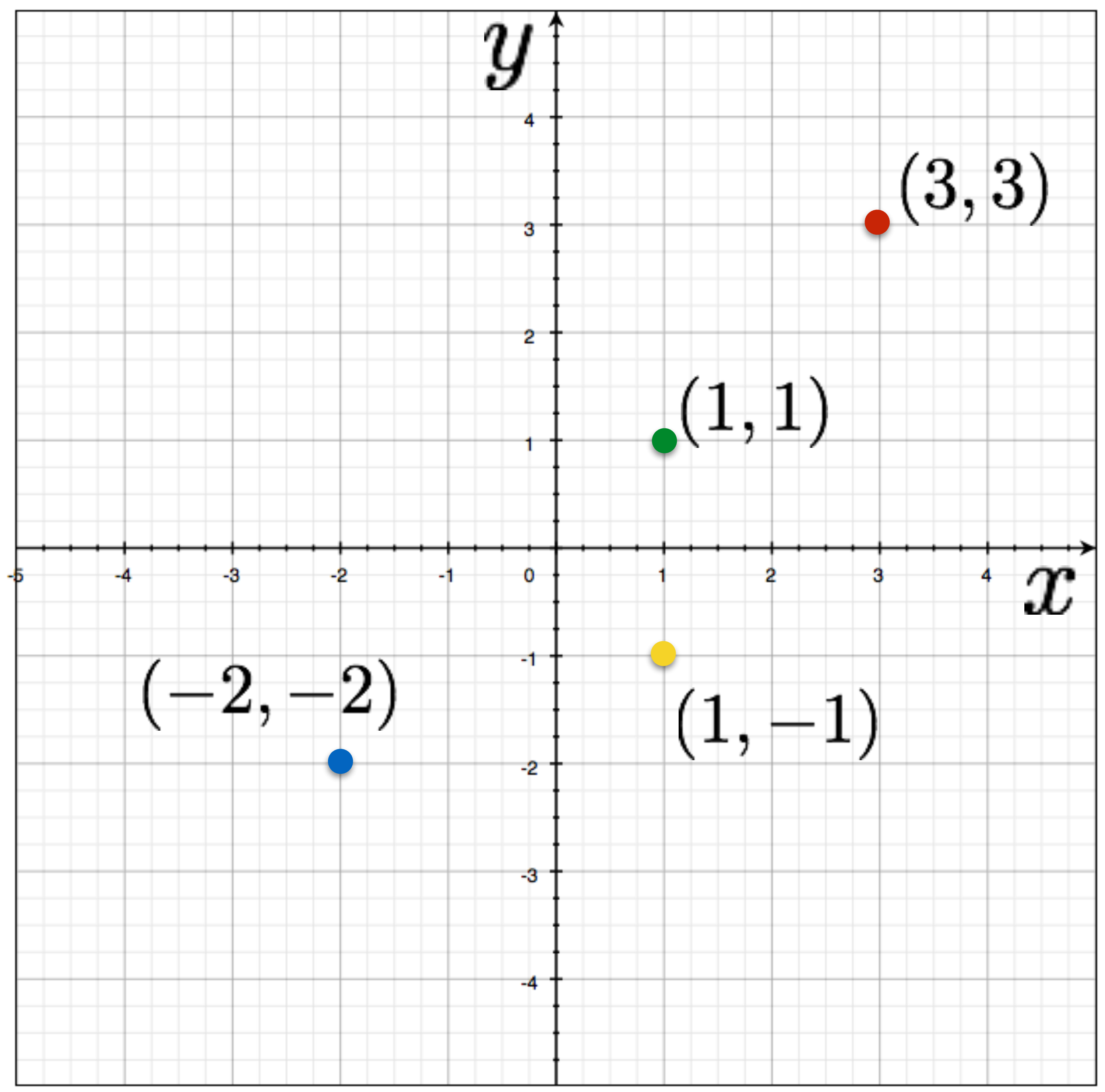
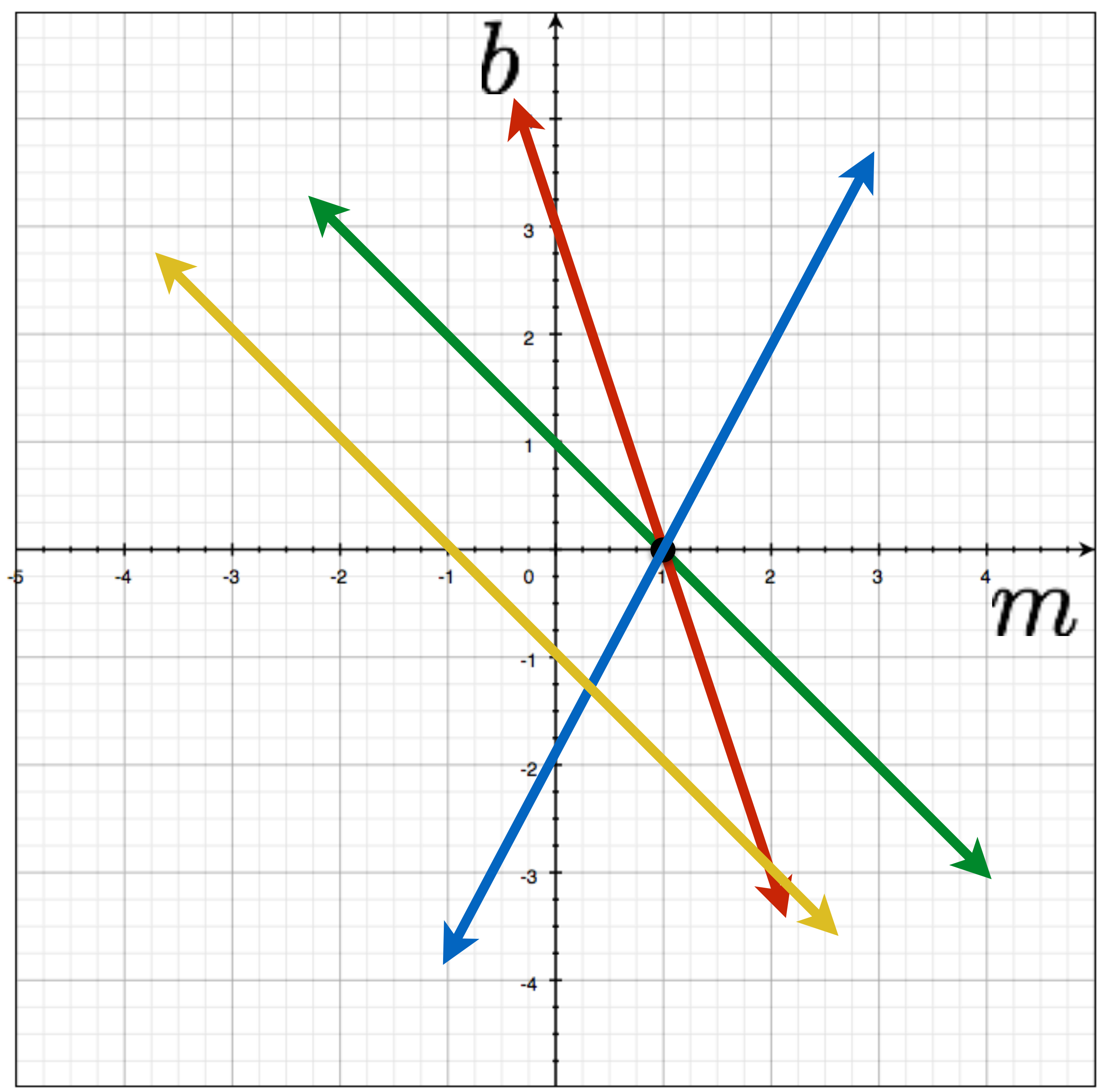


Image space

four
points?



Parameter space

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Hough Transform: Lines

How would you find the best fitting line?

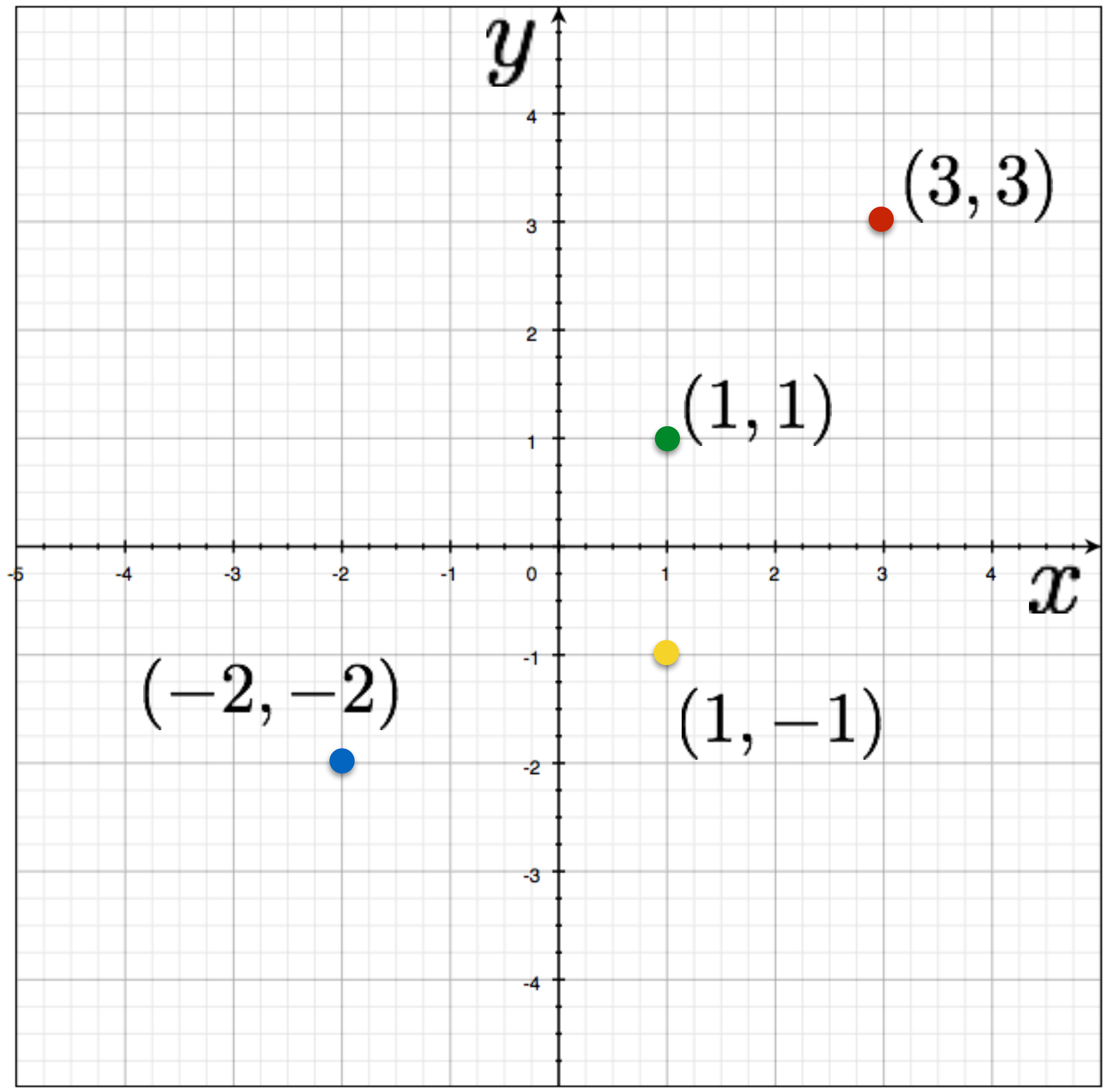
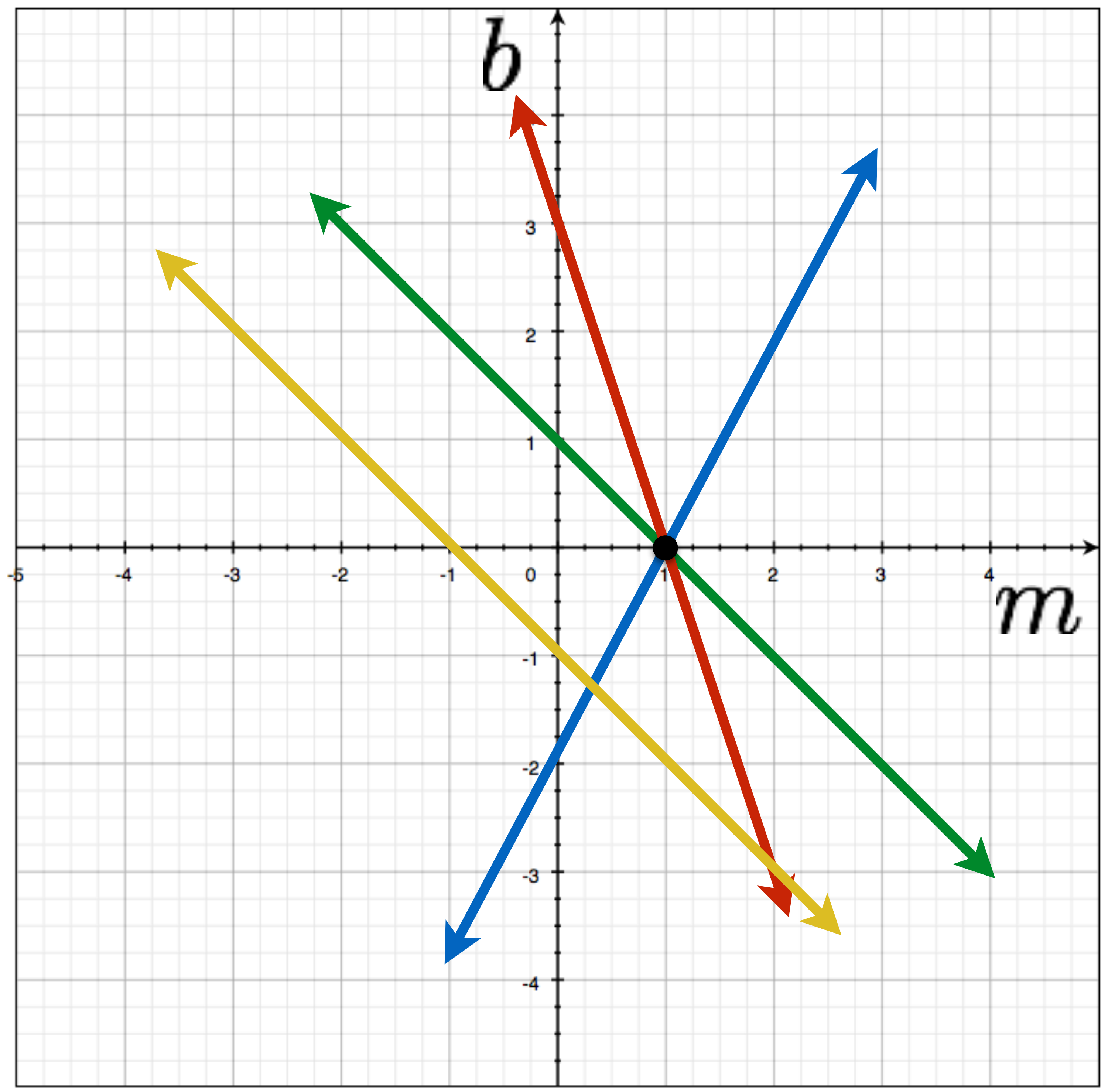


Image space



Parameter space

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Hough Transform: Lines

Is this method robust to measurement noise? clutter?

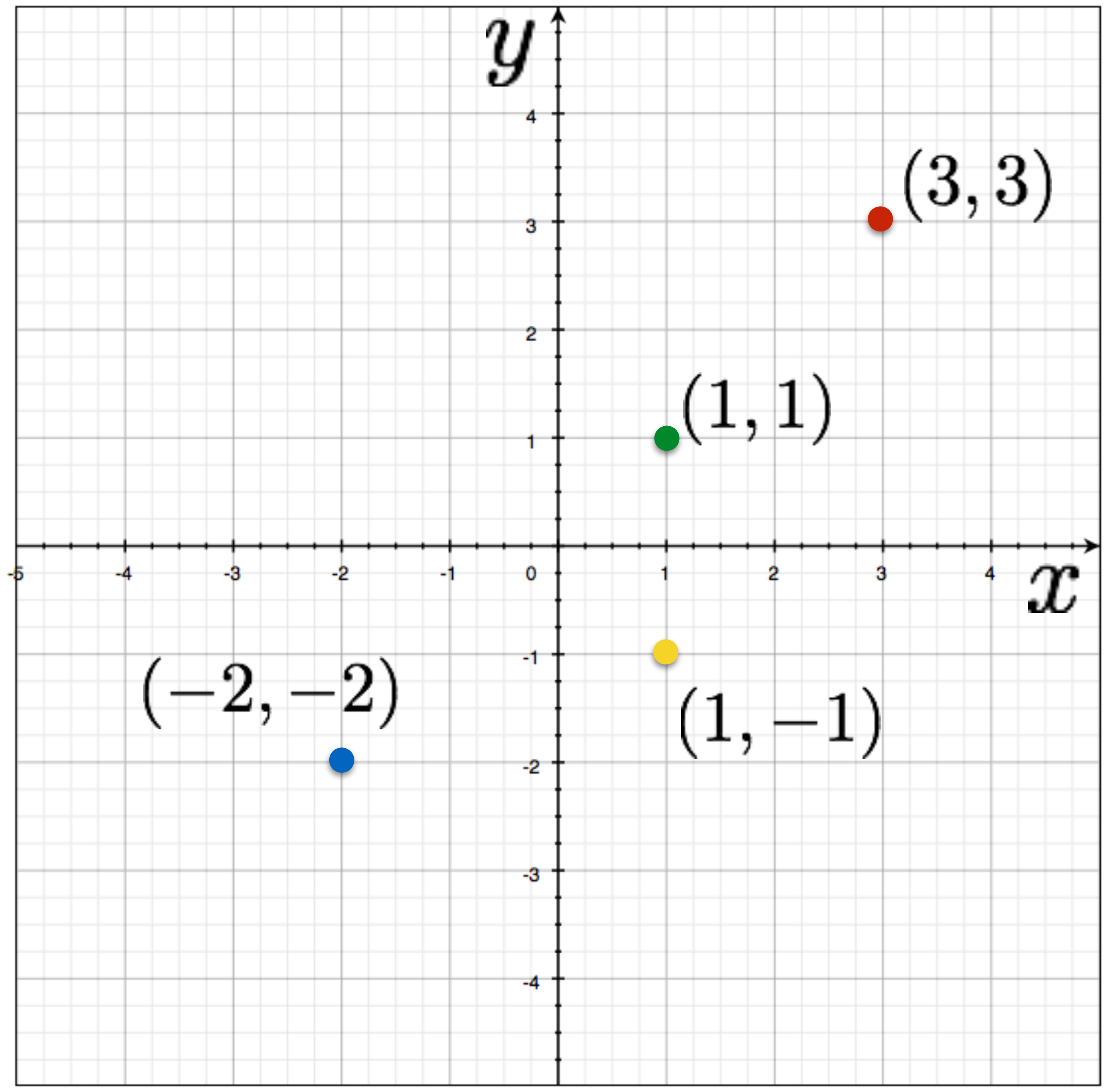
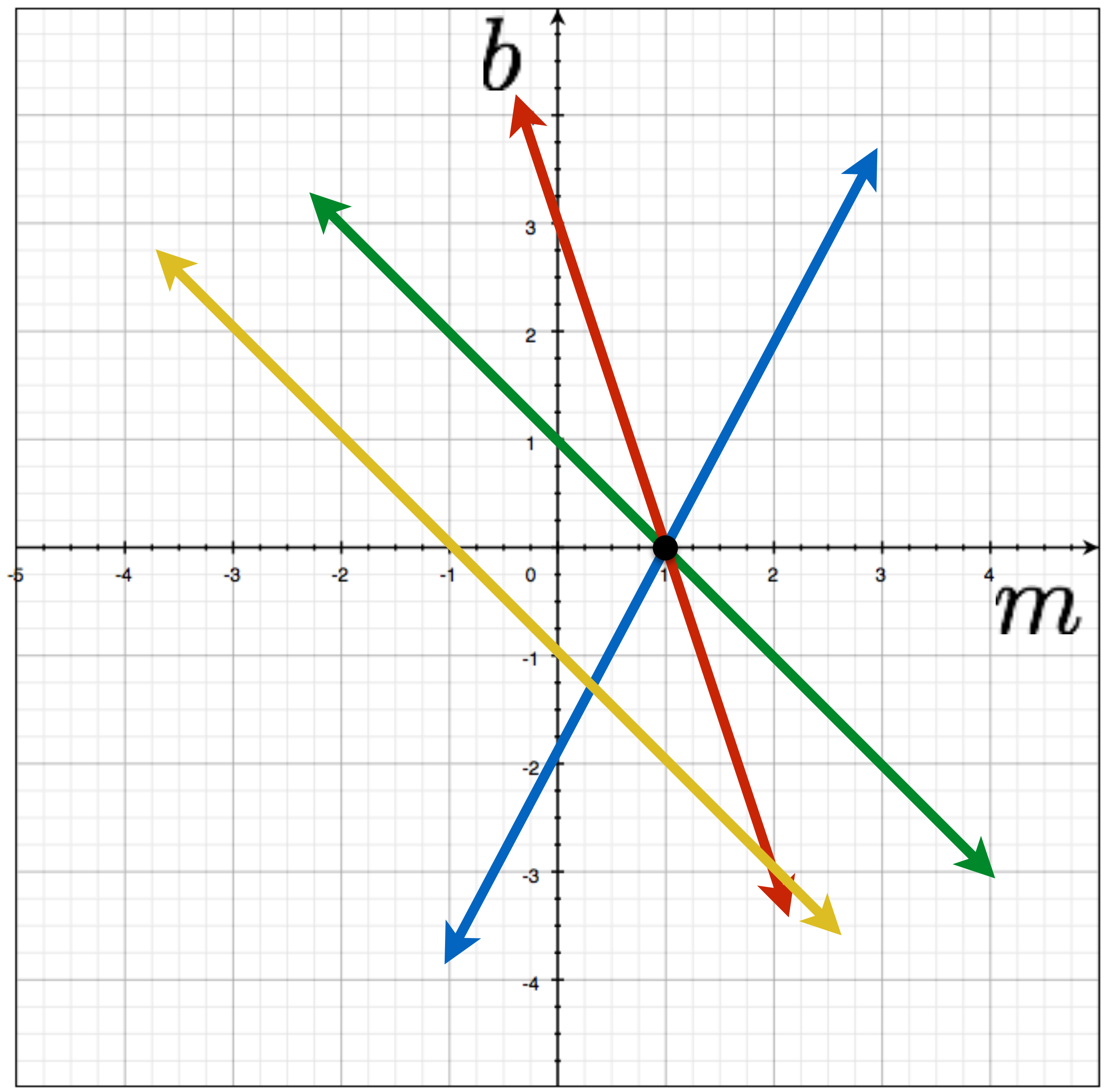


Image space



Parameter space

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Line Detection by Hough Transform

Algorithm:

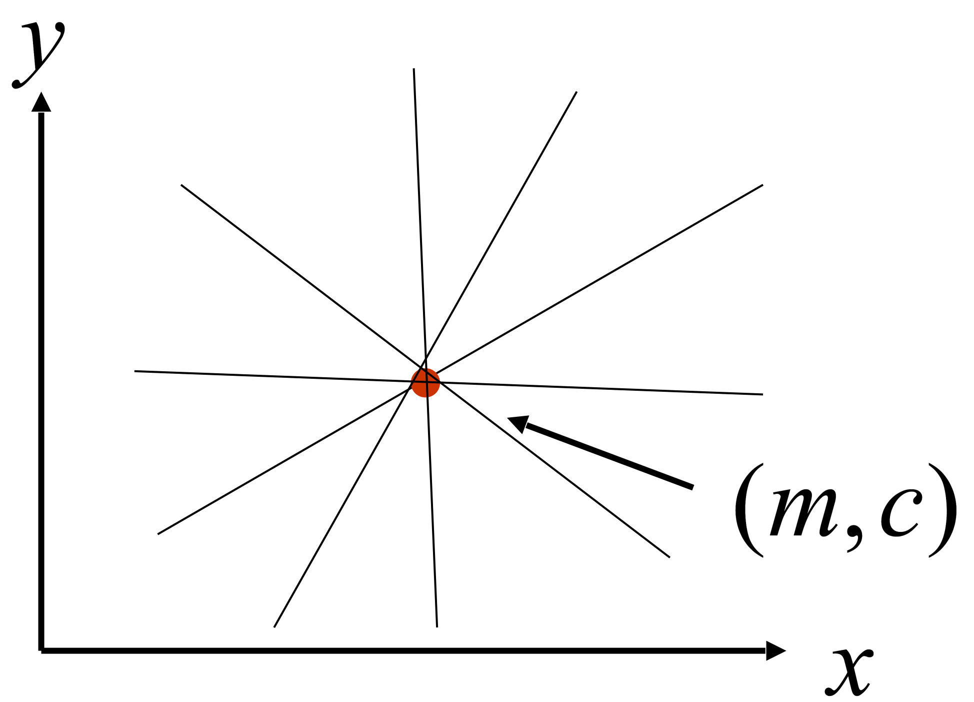
1. Quantize Parameter Space (m,c)

2. Create Accumulator Array $A(m,c)$

3. Set $A(m,c)=0 \quad \forall m,c$

4. For each image edge (x_i, y_i)
   For each element in $A(m,c)$
   If (m,c) lies on the line: $c=-x_i m + y_i$
   Increment $A(m,c)=A(m,c)+1$

5. Find local maxima in $A(m,c)$



Parameter Space

$A(m,c)$

	1						1		
		1				1			
			1		1				
				2					
			1		1				
		1				1			
	1						1		

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Problems with **Parametrization**

How big does the accumulator need to be for the parameterization (m, c)?

$A(m, c)$

	1						1		
		1				1			
			1		1				
				2					
			1		1				
		1				1			
	1						1		

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Problems with **Parametrization**

How big does the accumulator need to be for the parameterization (m, c)?

$A(m, c)$

	1						1		
		1				1			
			1		1				
				2					
			1		1				
		1				1			
	1						1		

The space of m is huge!

$$-\infty \leq m \leq \infty$$

The space of c is huge!

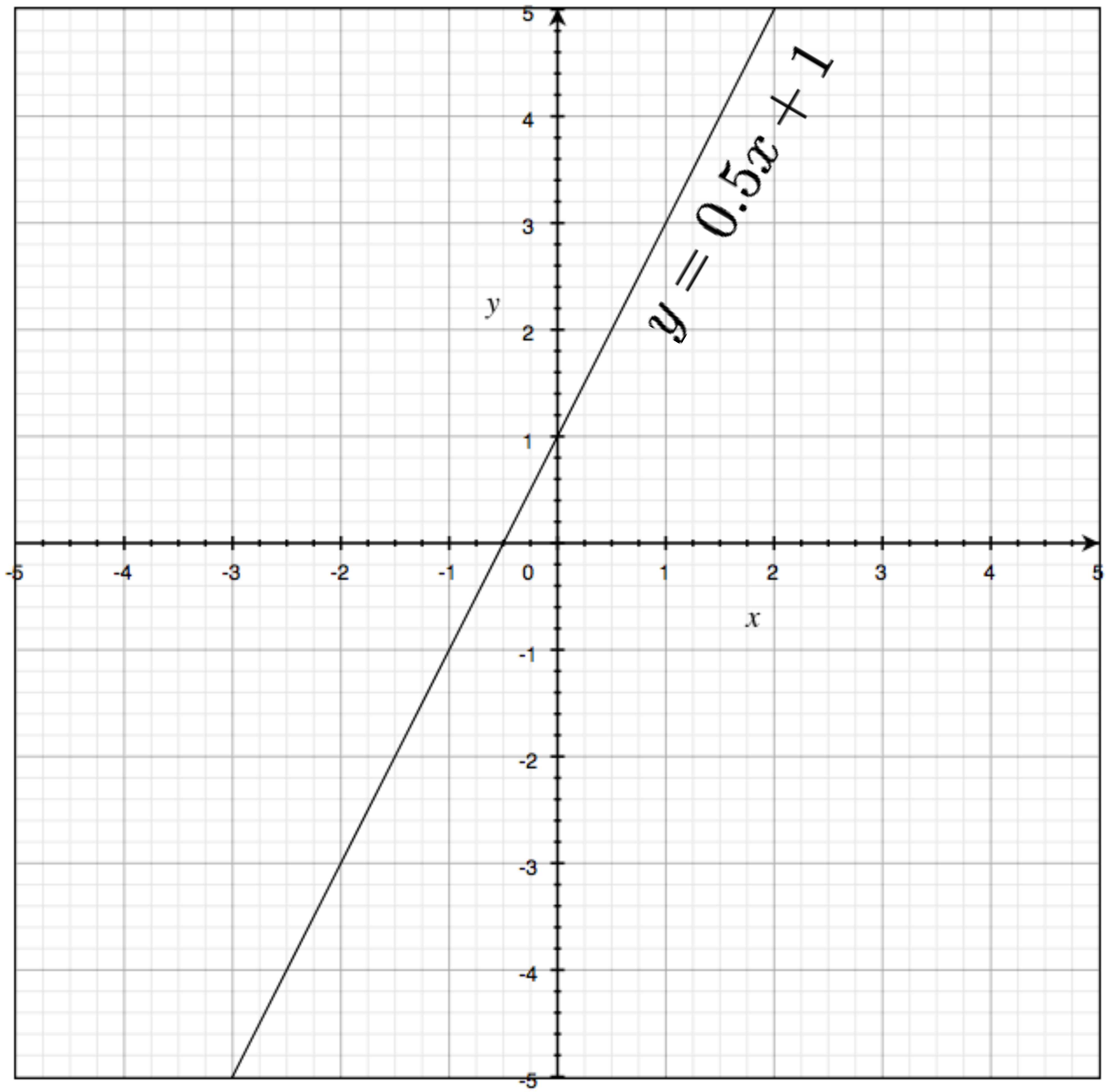
$$-\infty \leq c \leq \infty$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Lines: Slope intercept form

$$y = mx + b$$

slope y-intercept



Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Lines: Normal form

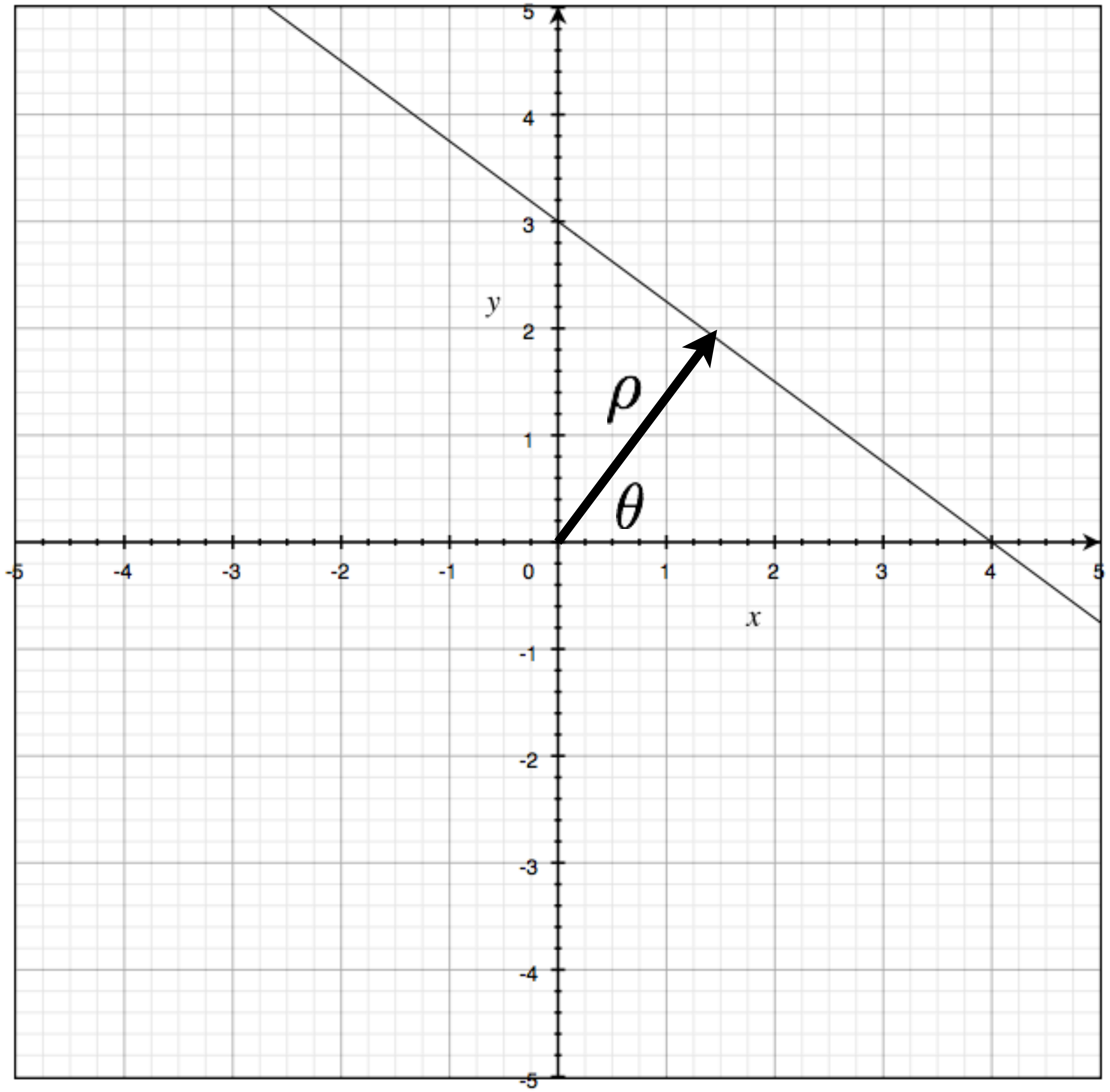
$$x \sin\theta + y \cos\theta = \rho$$

Book's convention

$$x \sin\theta + y \cos\theta + r = 0$$

$$r \geq 0$$

$$0 \leq \theta \leq 2\pi$$



Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Hough Transform: Lines

variables

$$y = mx + b$$

parameters

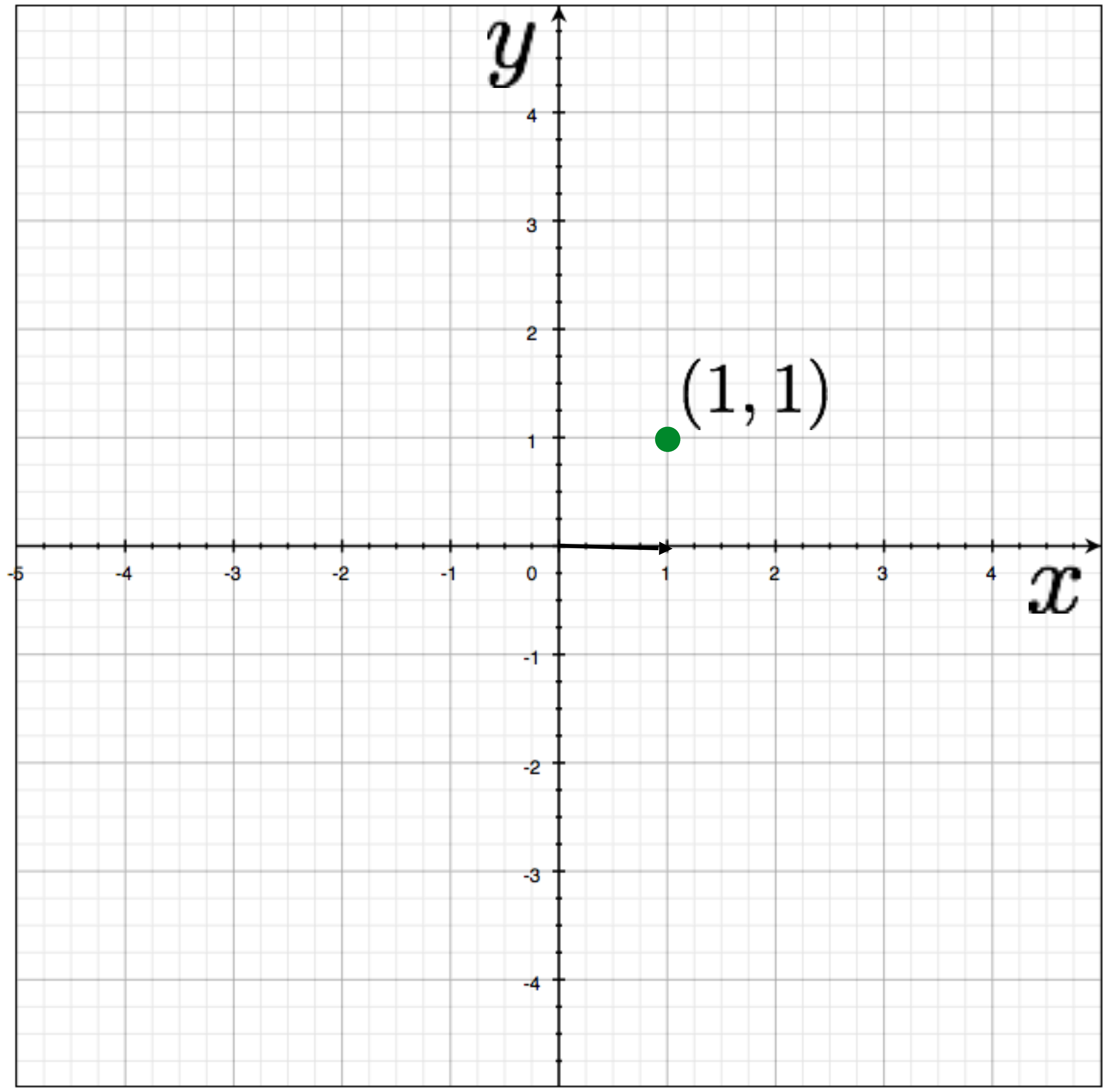


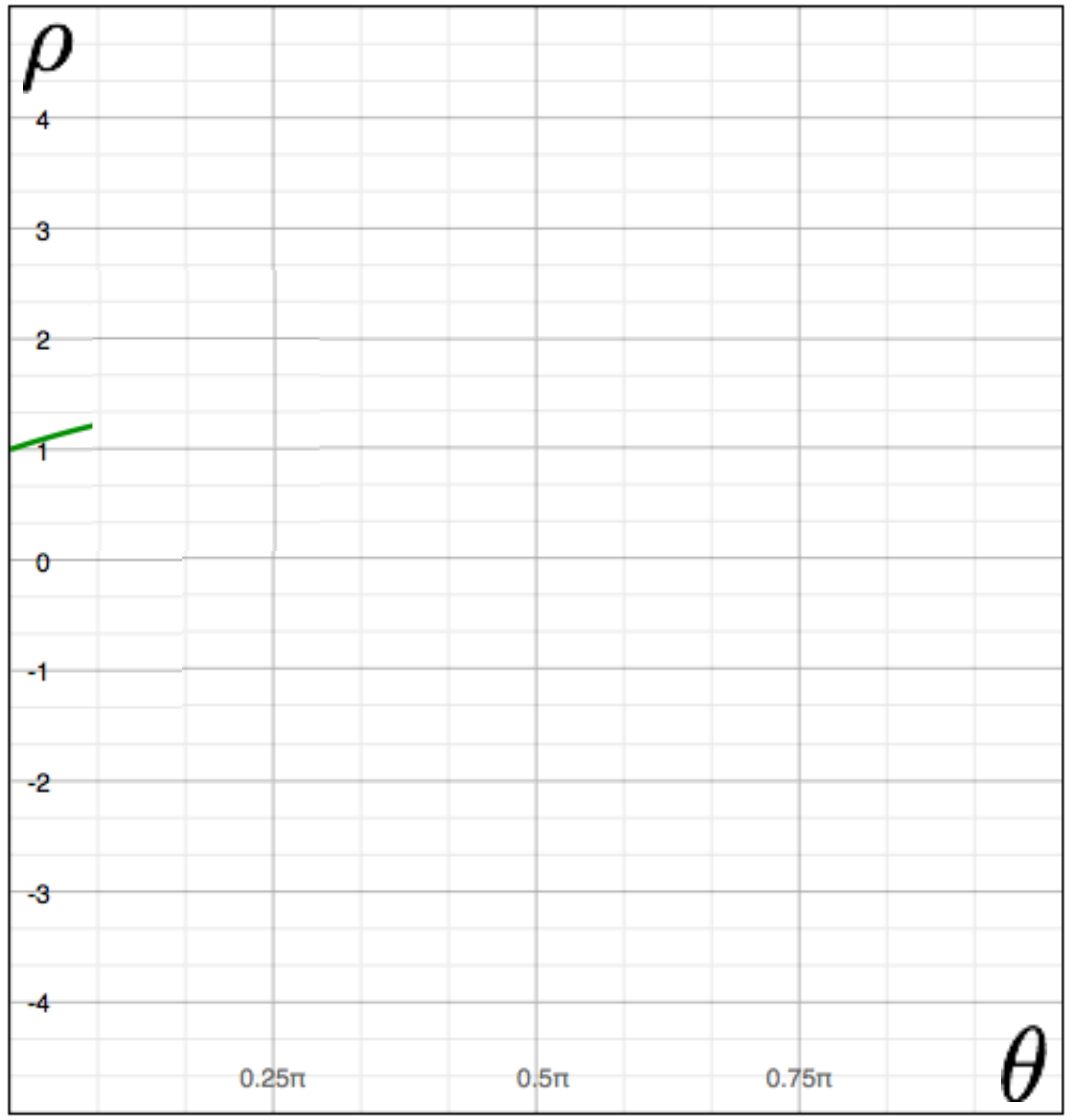
Image space

a point becomes?

parameters

$$x \sin\theta + y \cos\theta = \rho$$

variables



Parameter space

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Hough Transform: Lines

variables

$$y = mx + b$$

parameters

parameters

$$x \sin\theta + y \cos\theta = \rho$$

variables

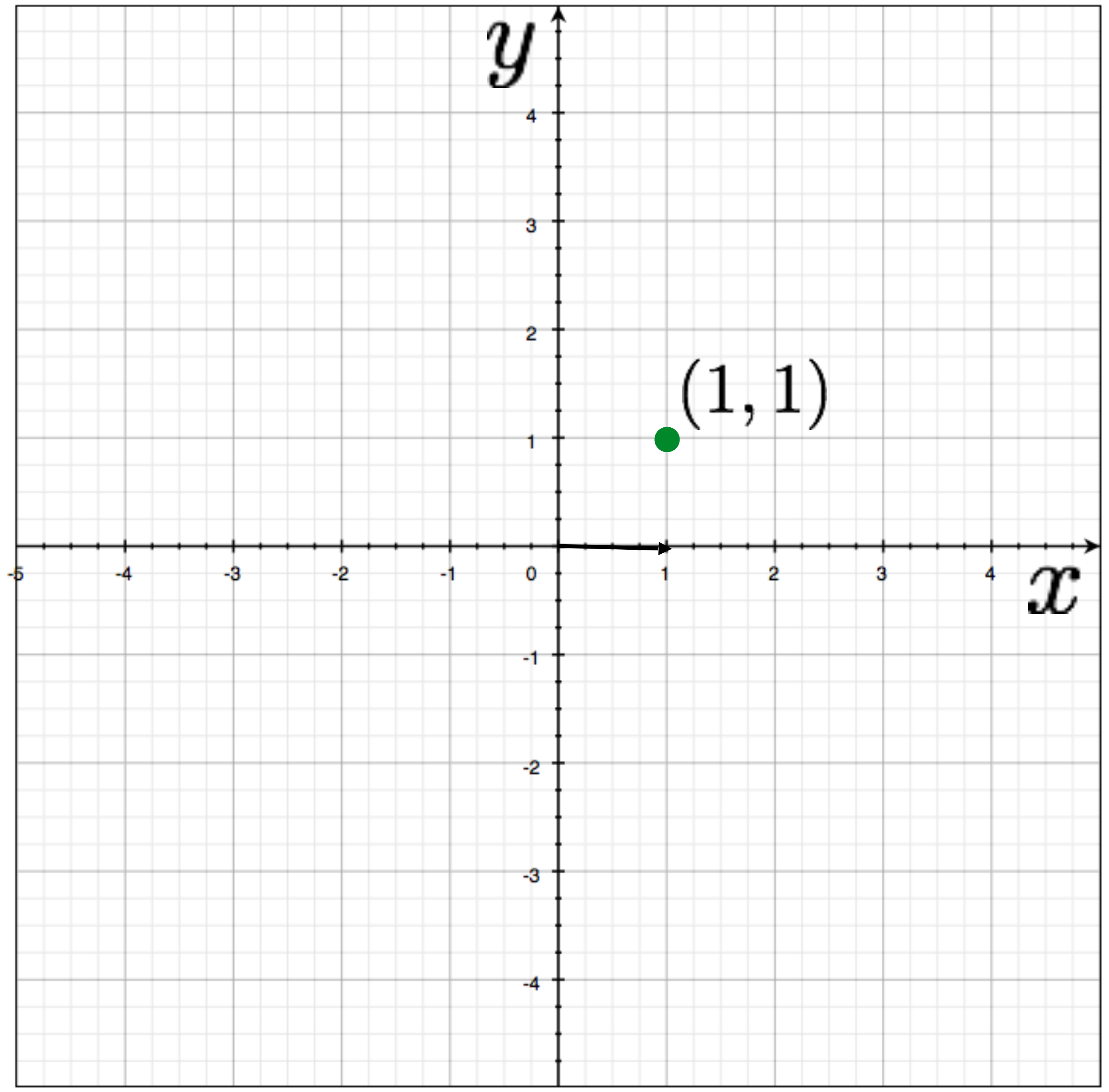
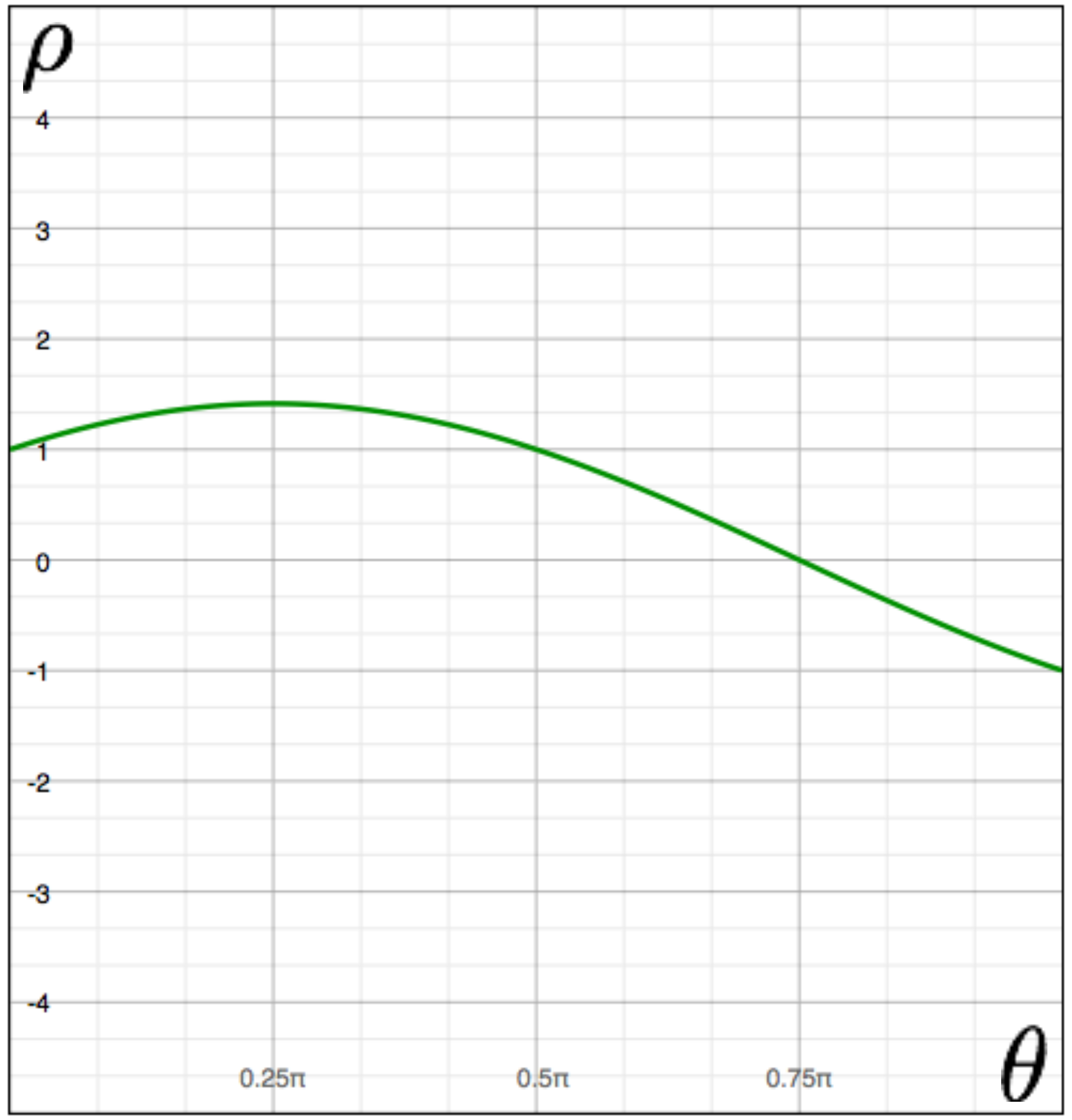


Image space

a point
becomes
a wave



Parameter space

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Hough Transform: Lines

variables

$$y = mx + b$$

parameters

parameters

$$x \sin\theta + y \cos\theta = \rho$$

variables

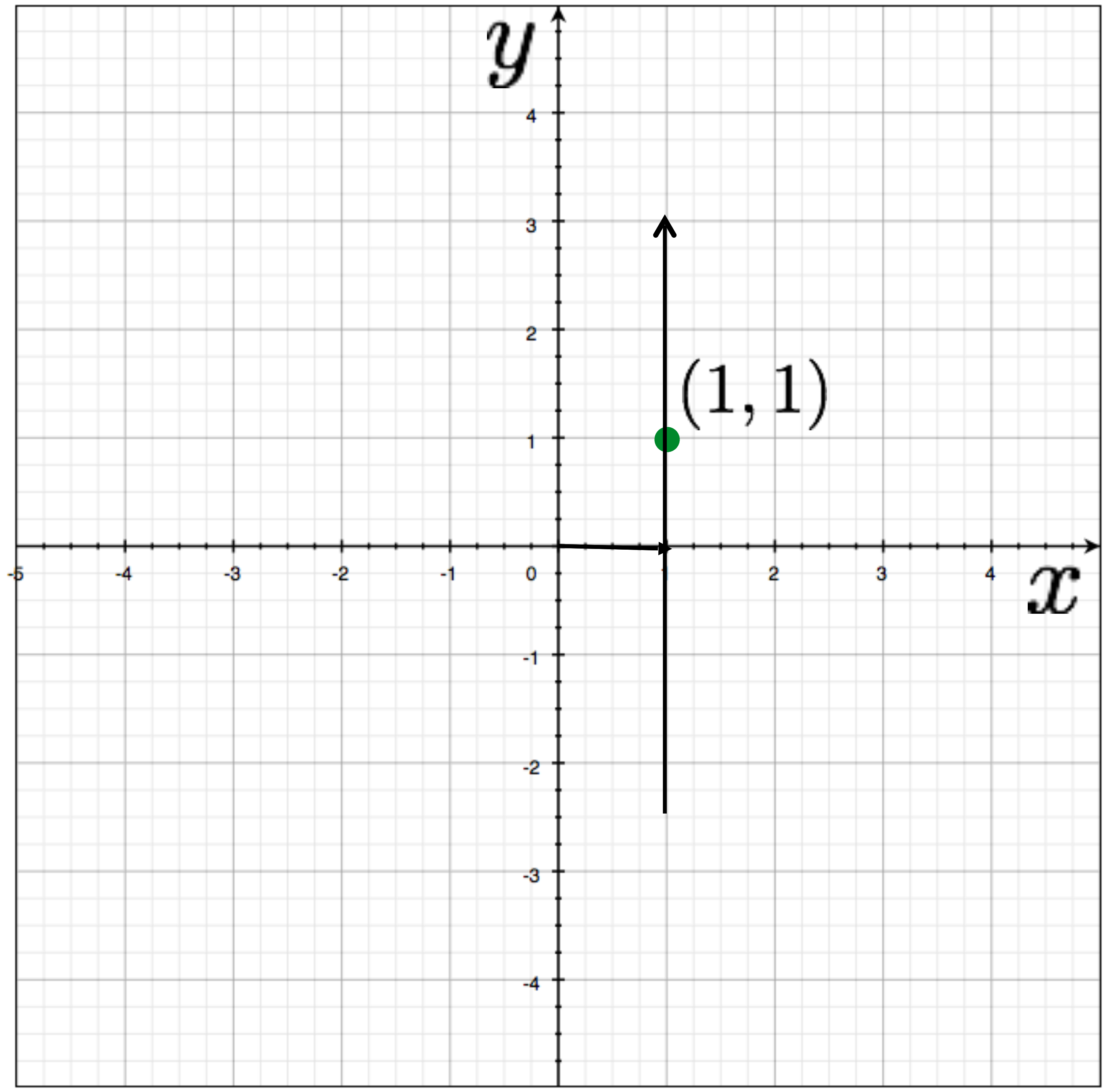
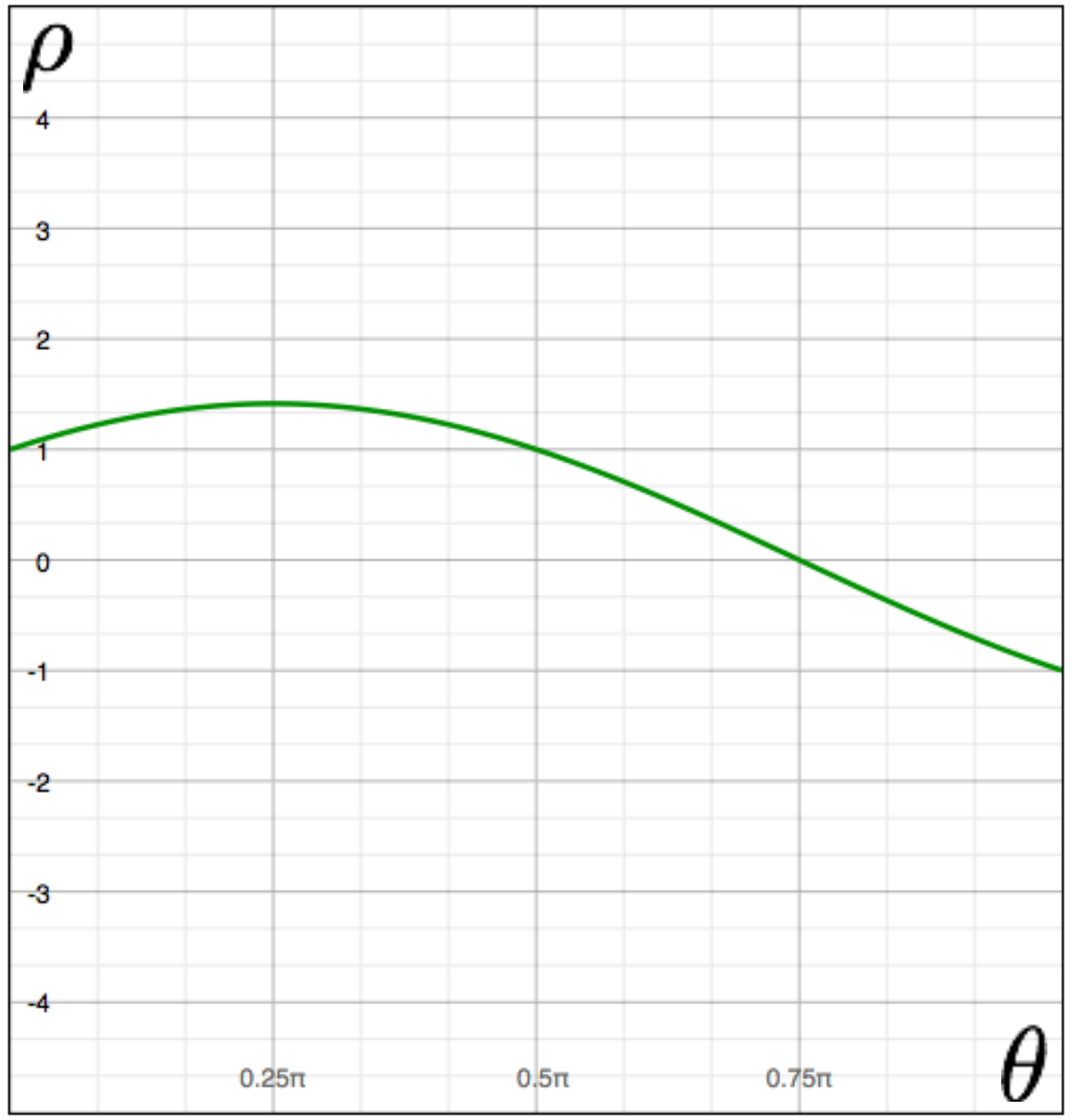


Image space

a line becomes?



Parameter space

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Hough Transform: Lines

variables

$$y = mx + b$$

parameters

parameters

$$x \sin\theta + y \cos\theta = \rho$$

variables

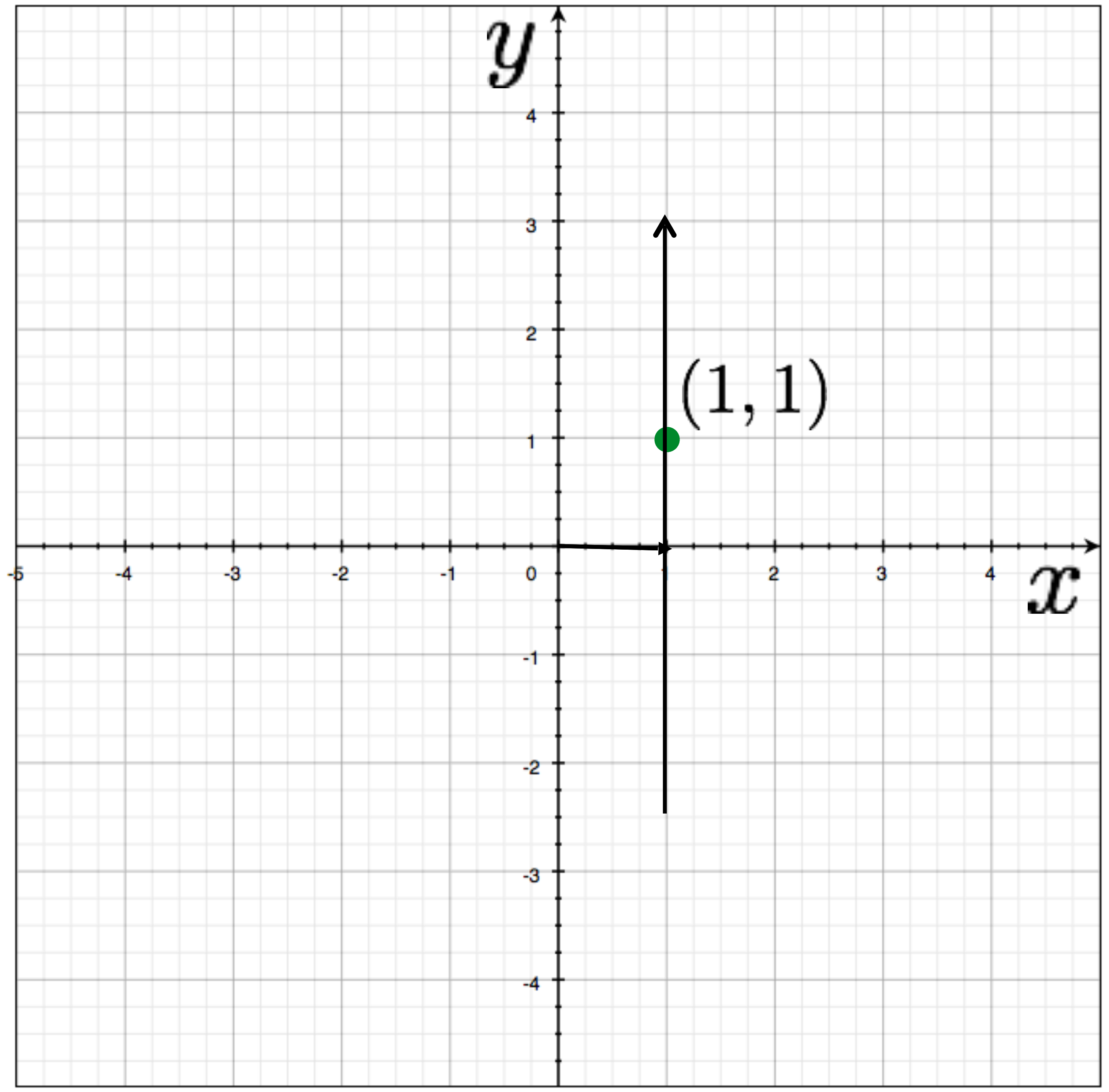
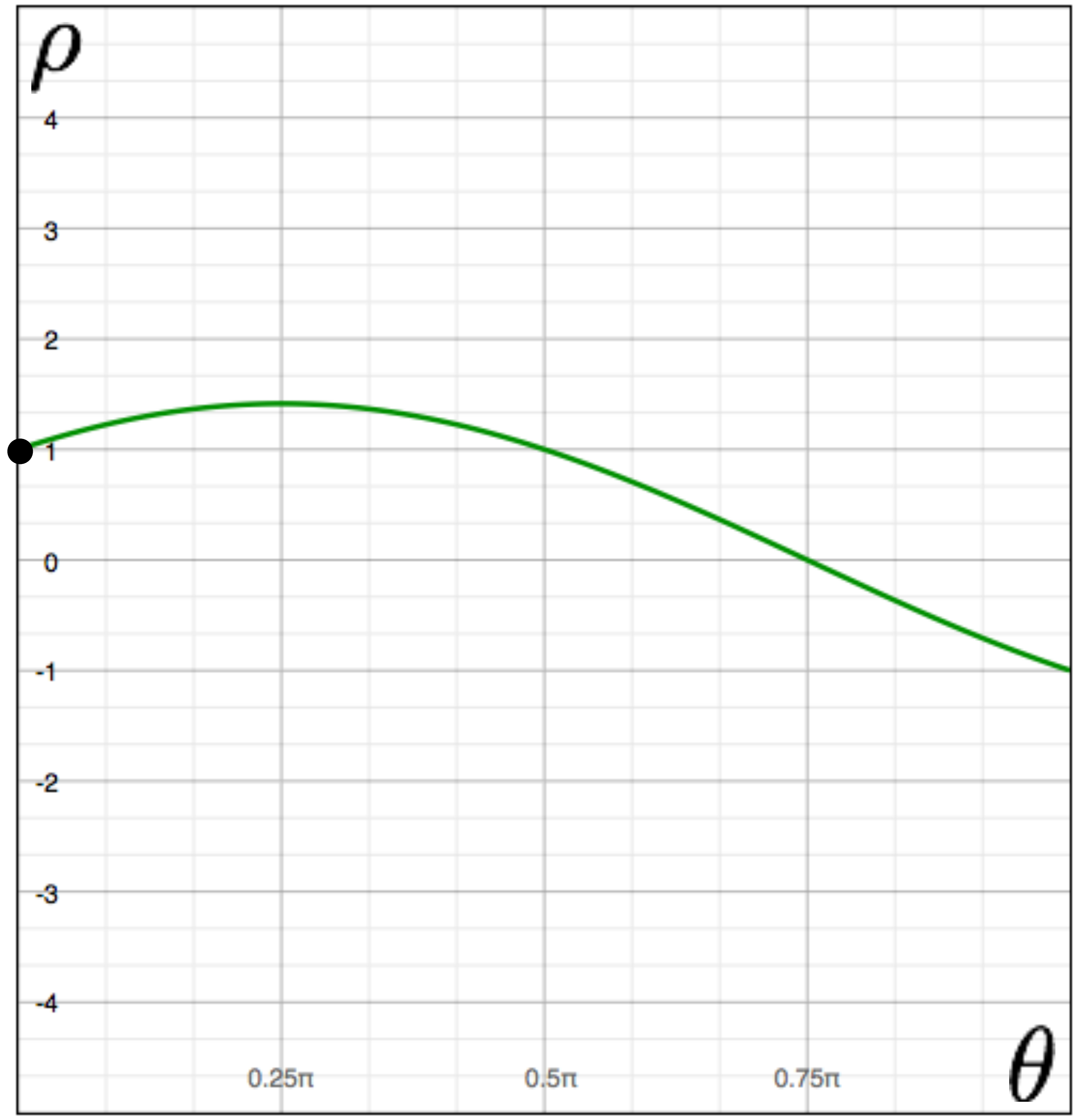


Image space

a line becomes a point



Parameter space

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Hough Transform: Lines

variables

$$y = mx + b$$

parameters

parameters

$$x \sin\theta + y\cos\theta = \rho$$

variables

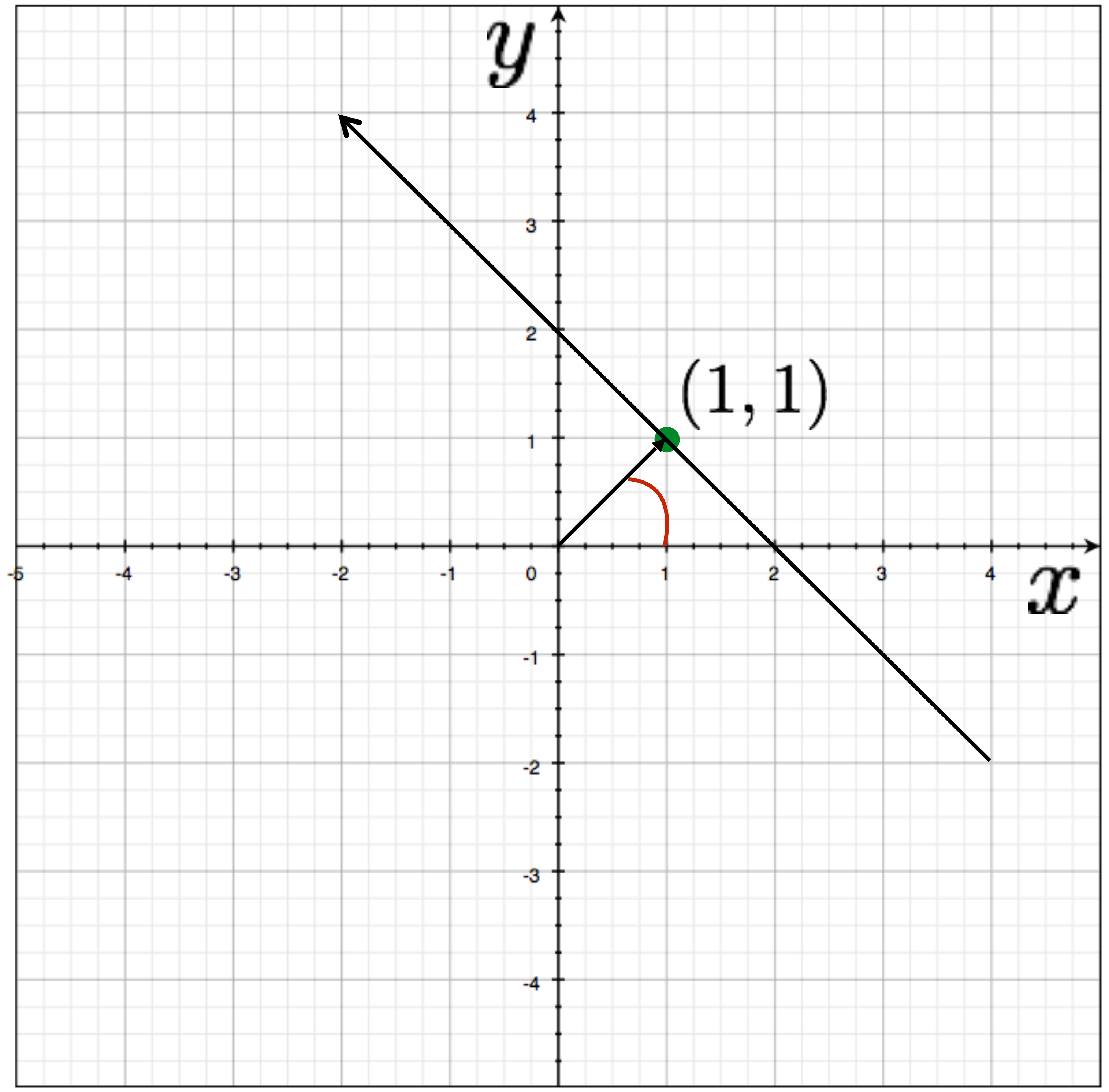
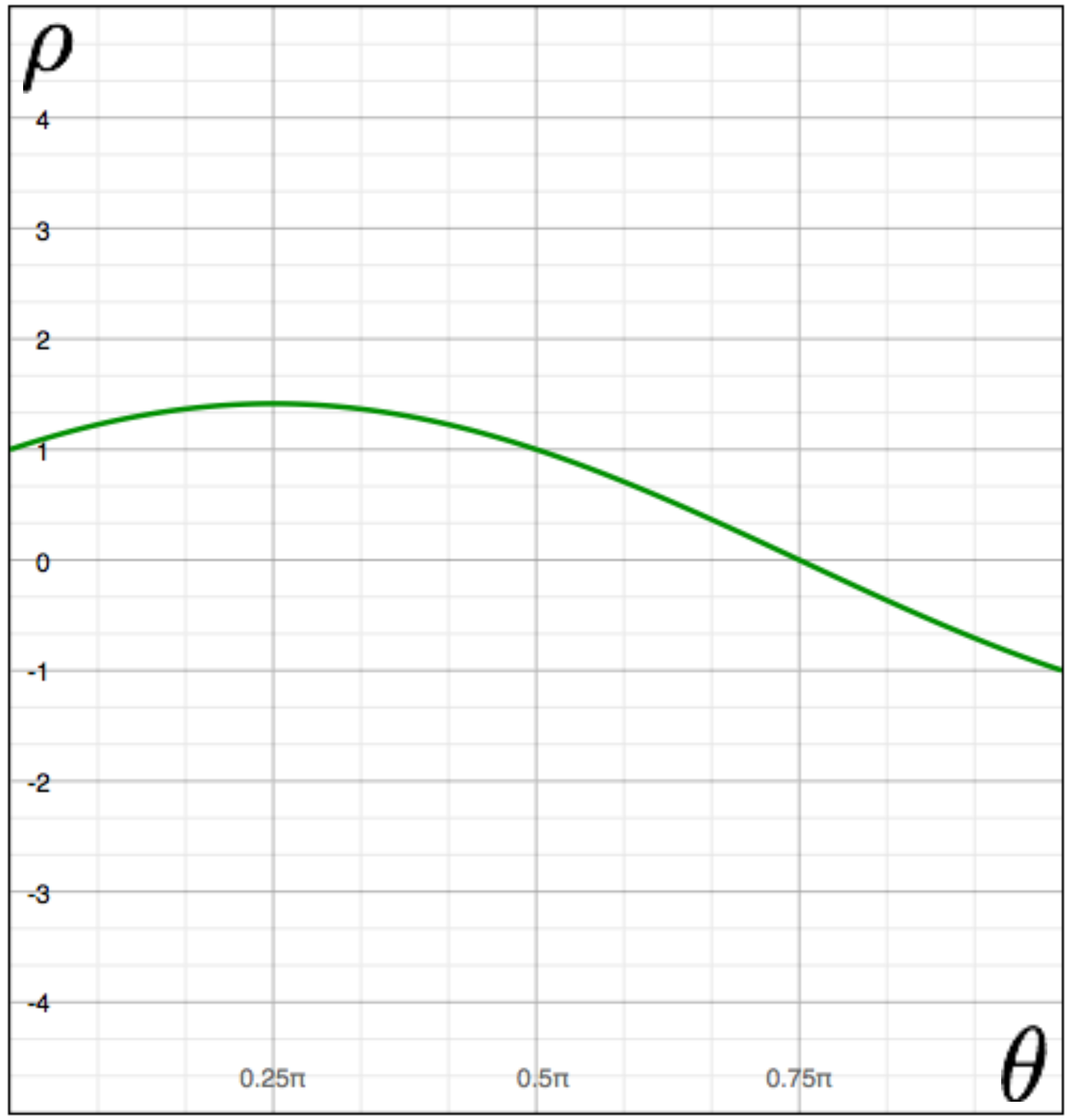


Image space

a line becomes?



Parameter space

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Hough Transform: Lines

variables

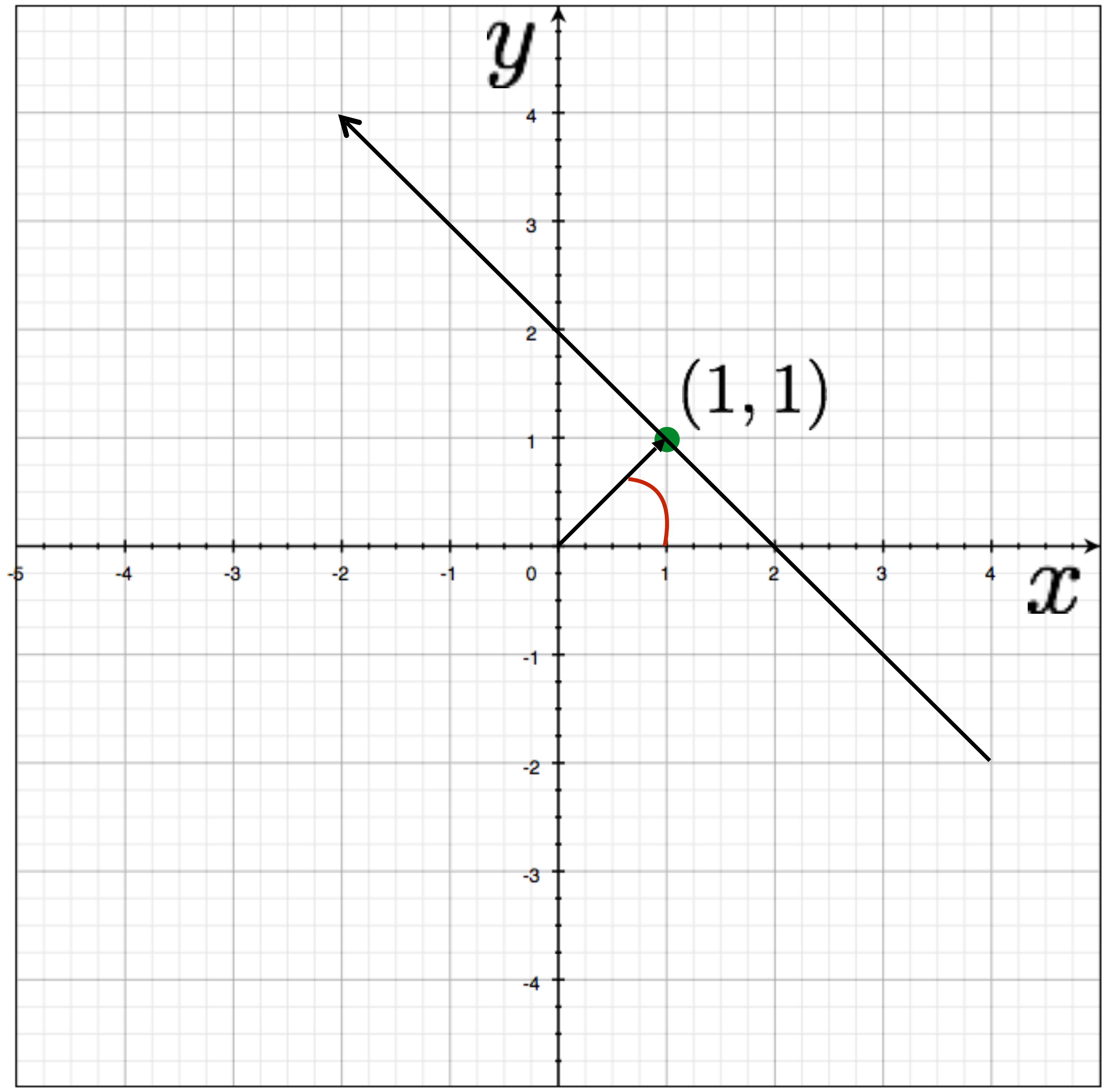
$$y = mx + b$$

parameters

parameters

$$x \sin\theta + y \cos\theta = \rho$$

variables



a line becomes a point

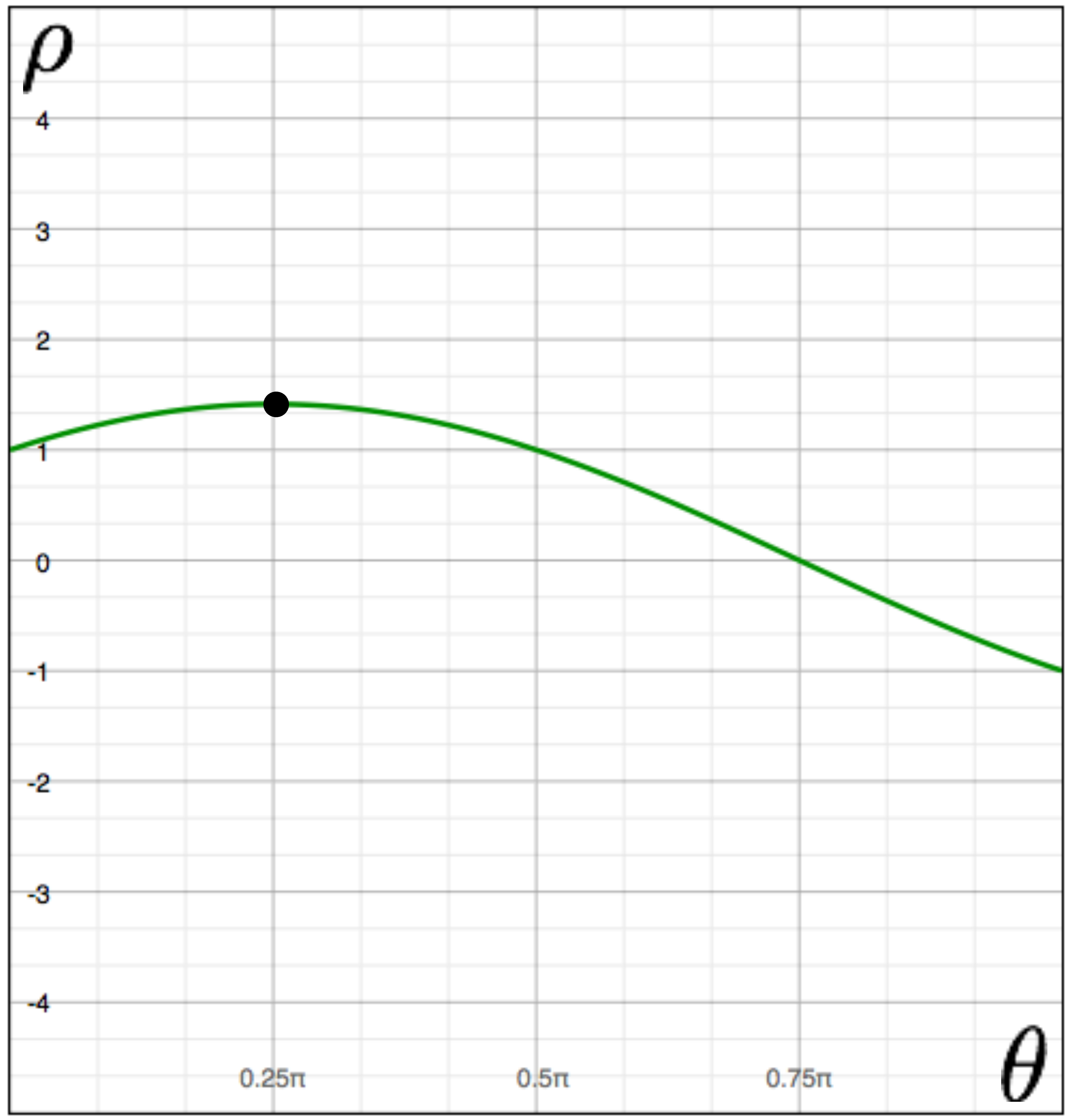


Image space

Parameter space

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Hough Transform: Lines

variables

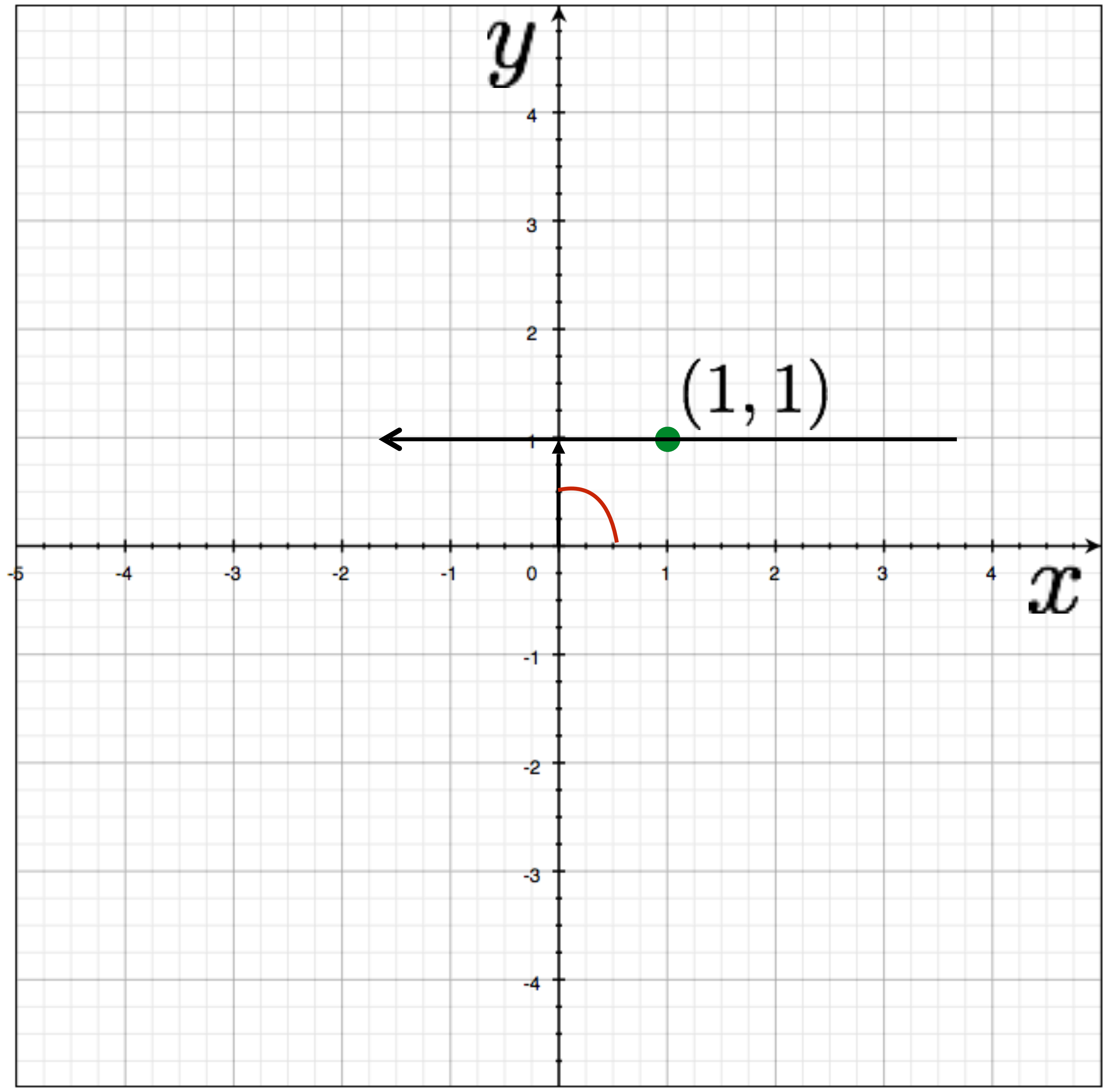
$$y = mx + b$$

parameters

parameters

$$x \sin\theta + y \cos\theta = \rho$$

variables



a line
becomes
a point

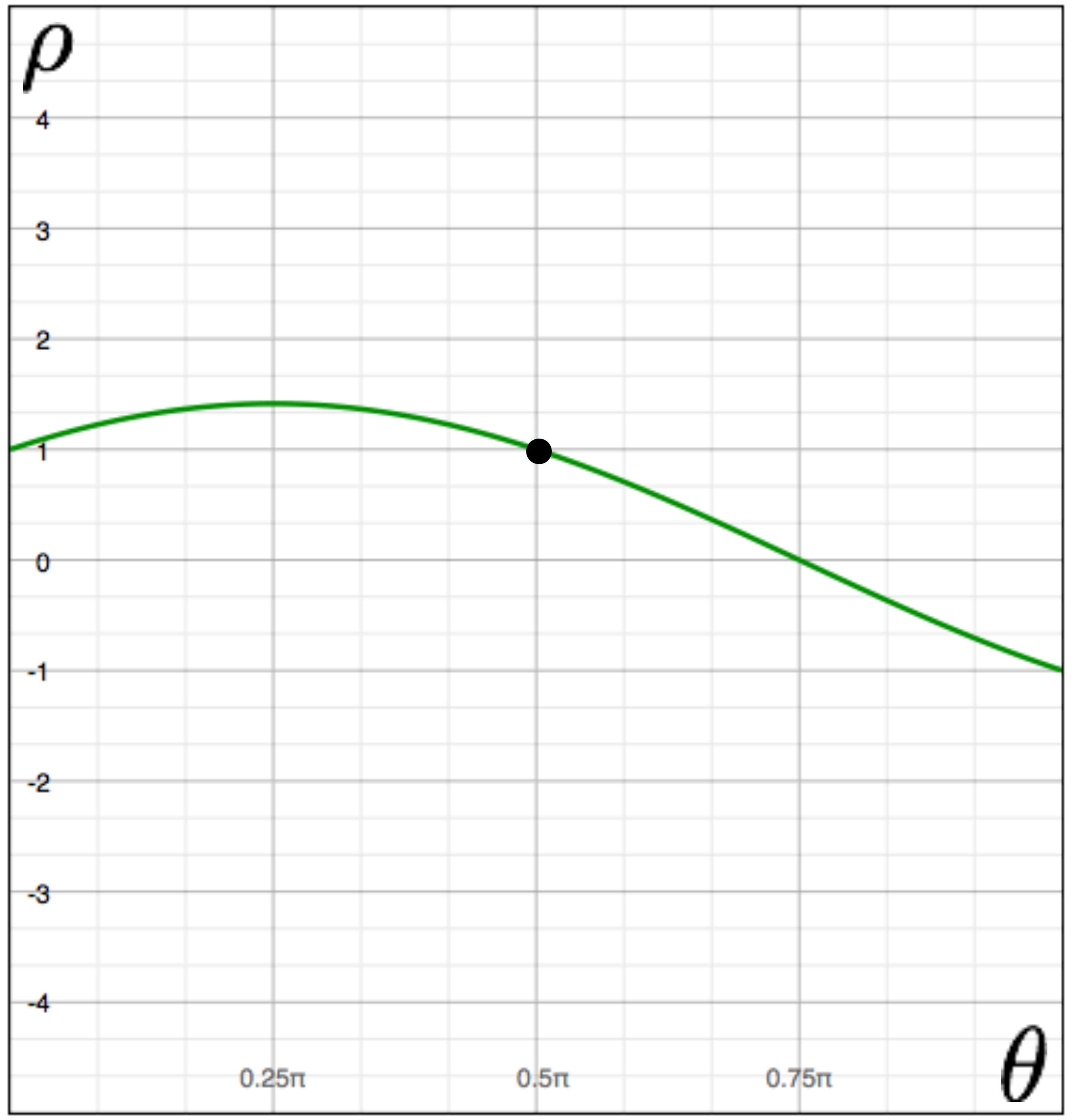


Image space

Parameter space

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Hough Transform: Lines

variables

$$y = mx + b$$

parameters

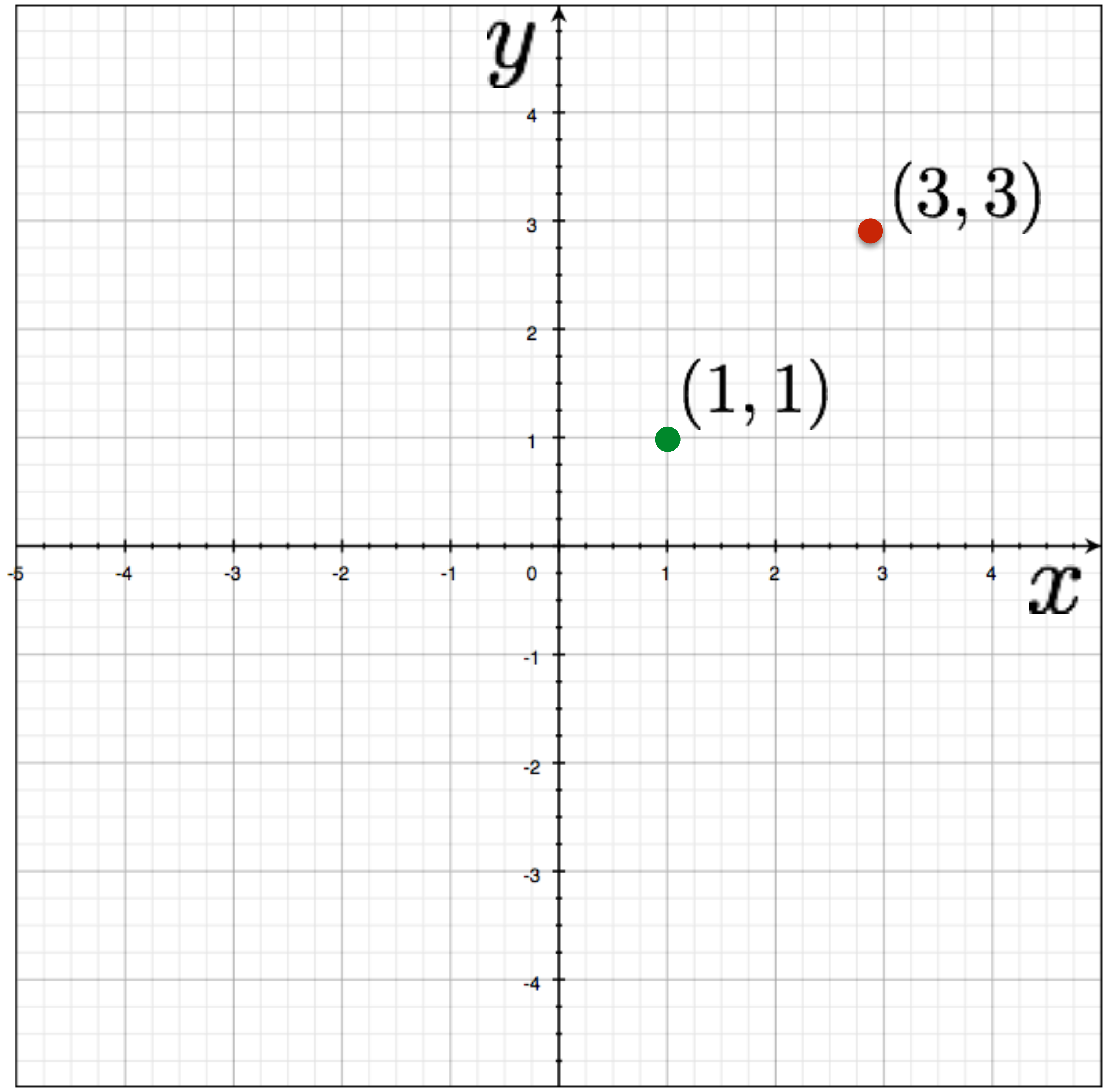


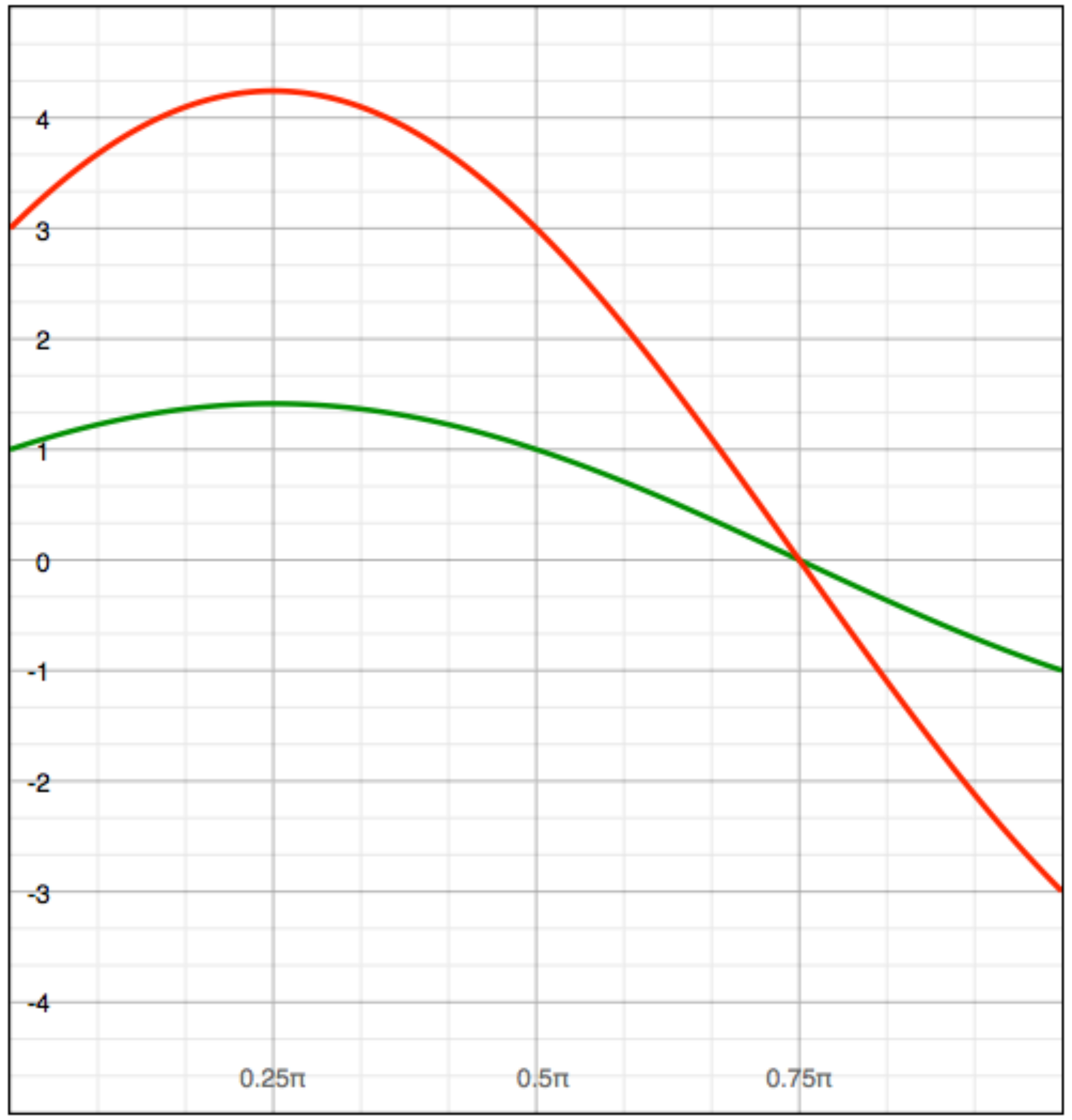
Image space

two points become?

parameters

$$x \sin\theta + y\cos\theta = \rho$$

variables



Parameter space

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Hough Transform: Lines

variables

$$y = mx + b$$

parameters

parameters

$$x \sin\theta + y \cos\theta = \rho$$

variables

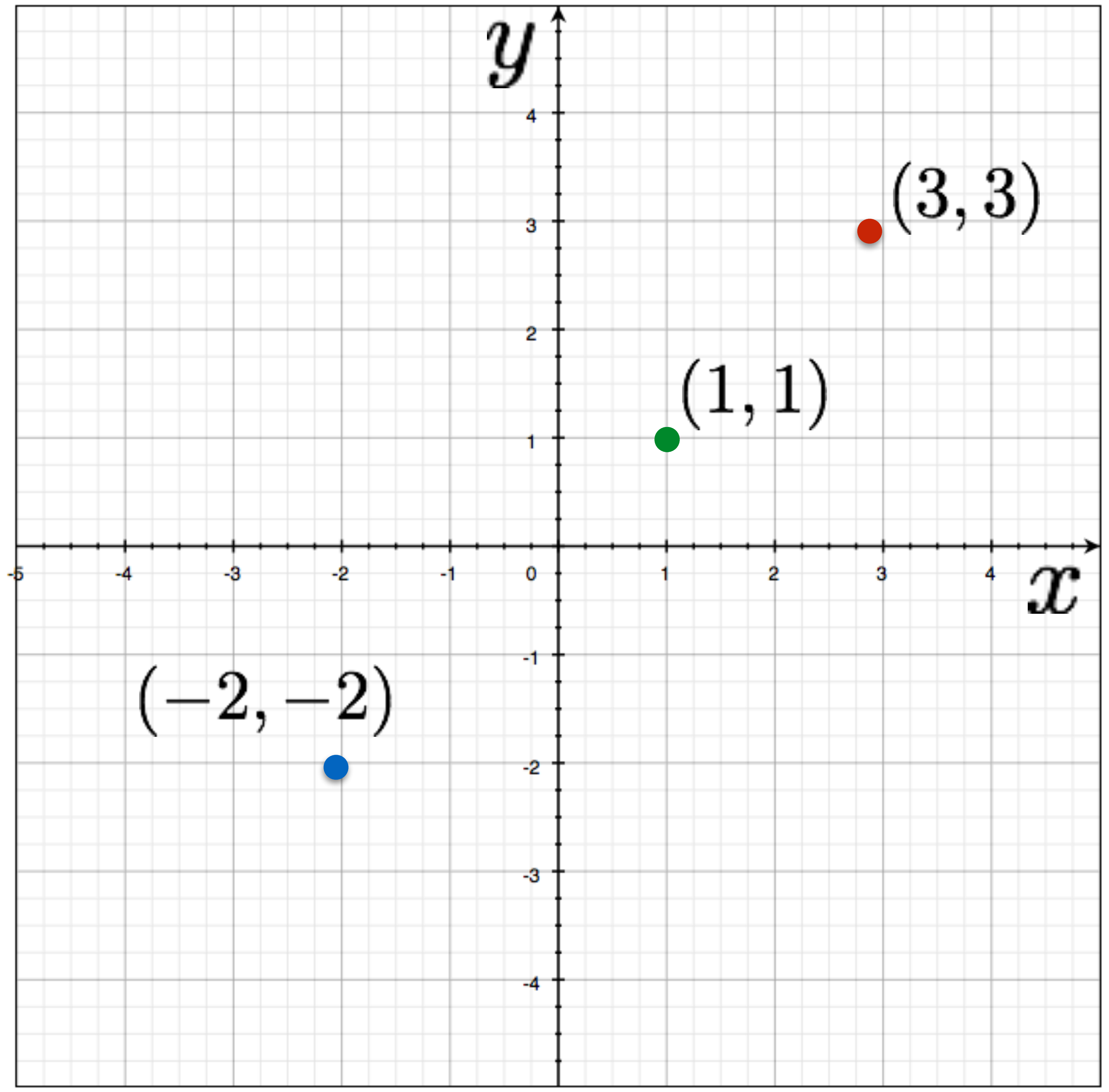
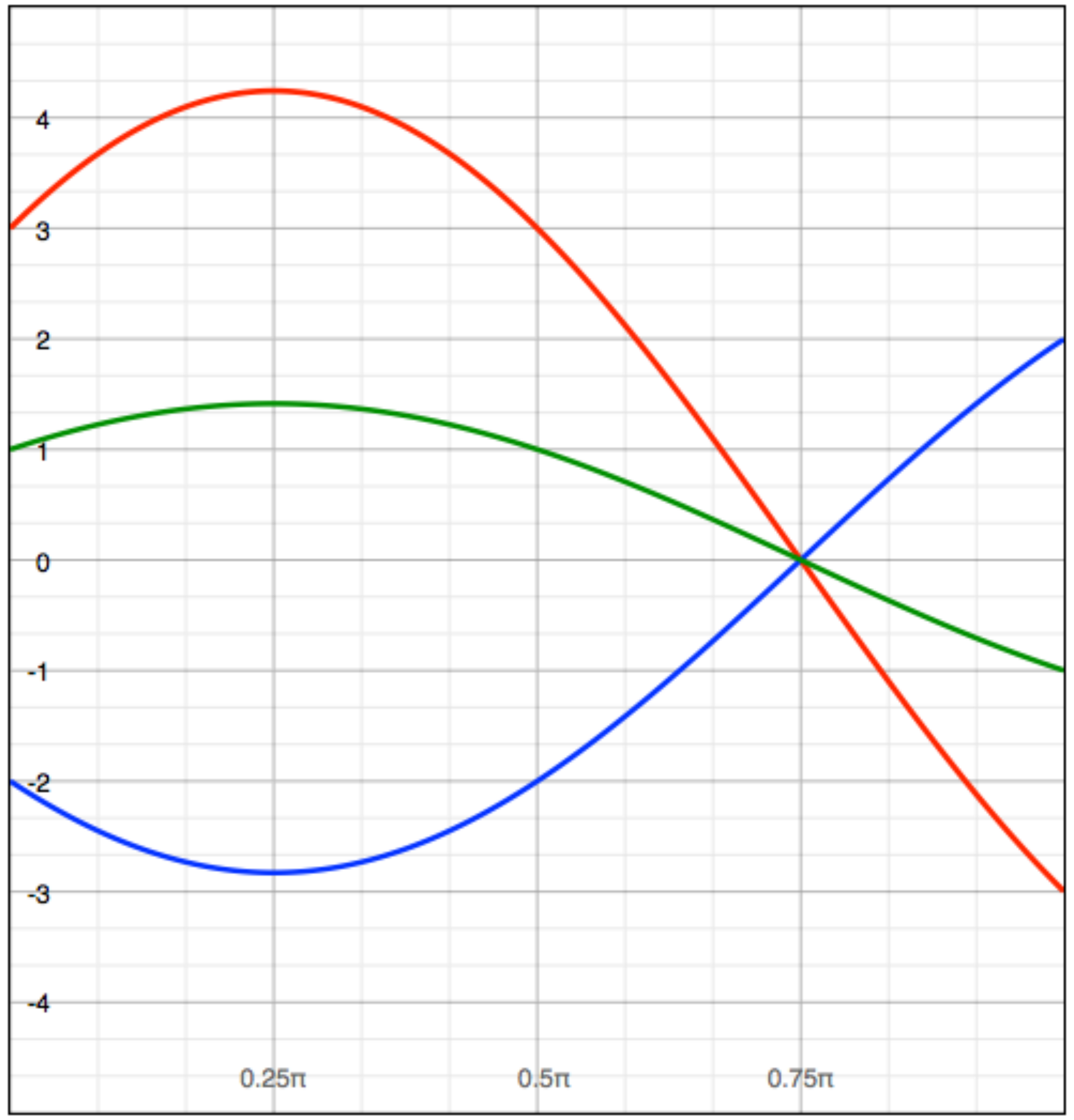


Image space

three points become?



Parameter space

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Hough Transform: Lines

variables

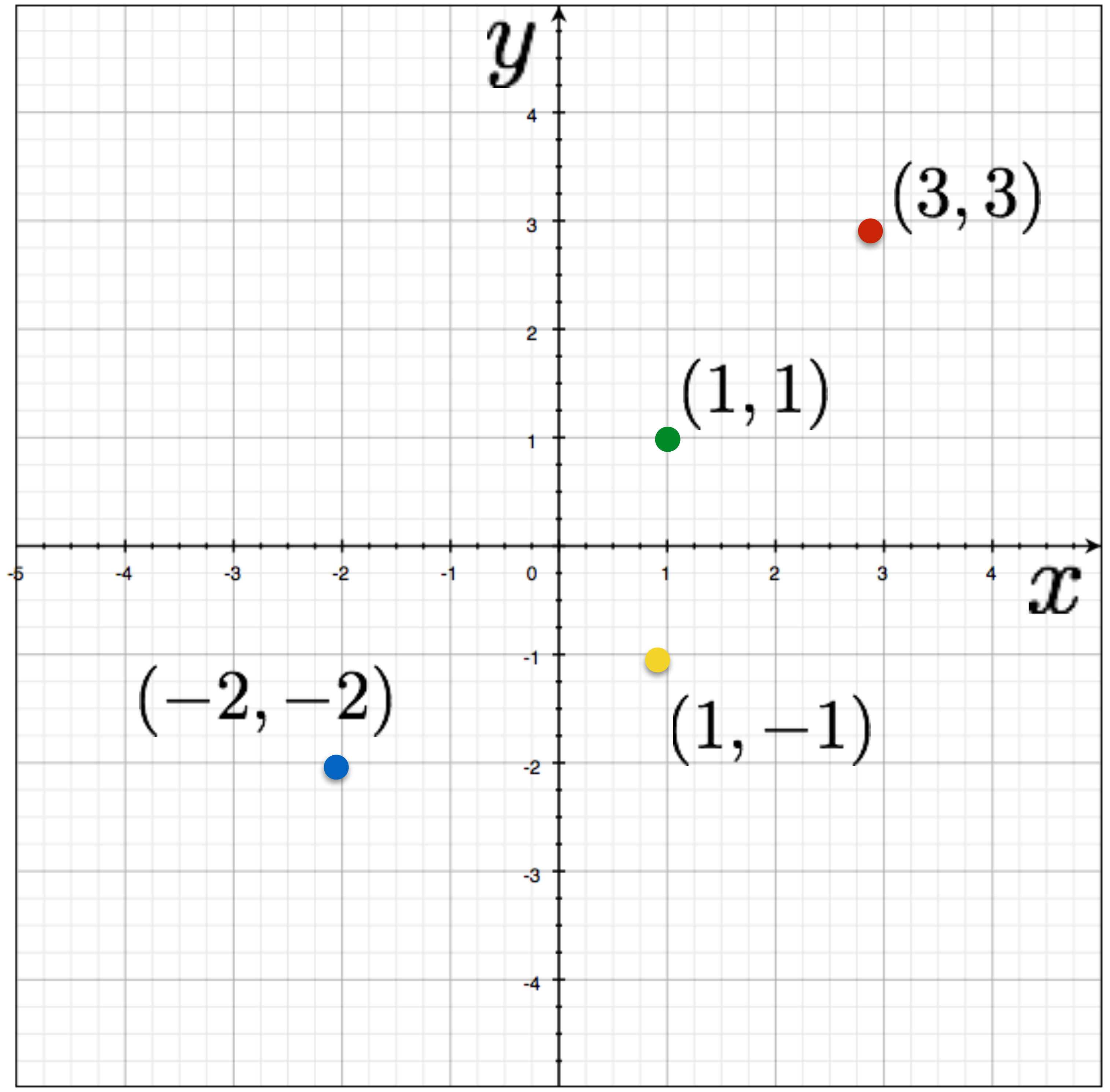
$$y = mx + b$$

parameters

parameters

$$x \sin\theta + y \cos\theta = \rho$$

variables



four points become?

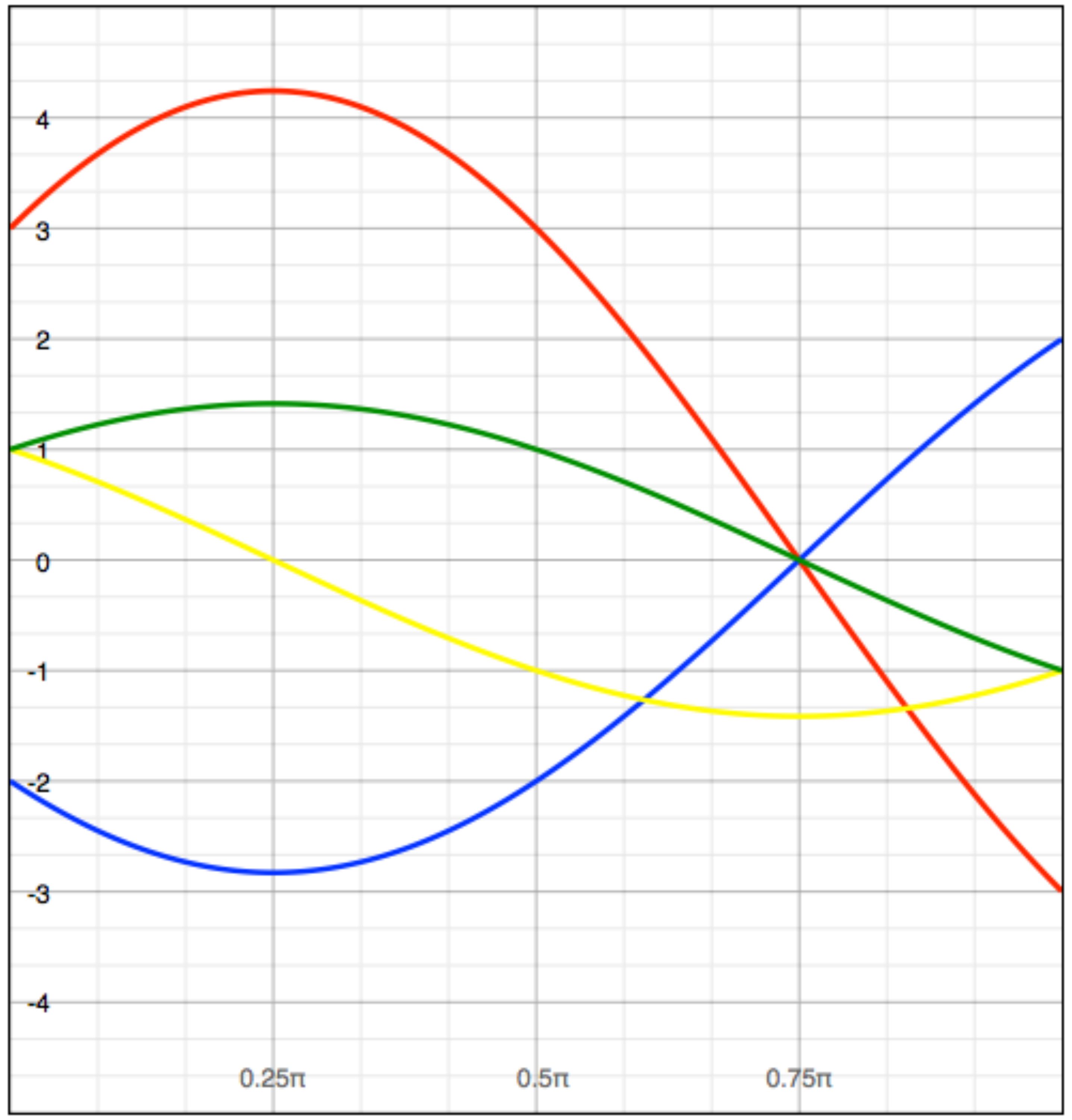


Image space

Parameter space

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