Menu for Today (Nov 5, 2021)

Topics:

— Optical Flow

Readings:

— Today’s Lecture: Forsyth & Ponce (2nd ed.) 10.6, 6.2.2, 9.3.1, 9.3.3, 9.4.2
— Next Lecture: None

Reminders:

— Assignment 4: due Nov 8  Quiz 4: Tuesday Nov 9
Today’s “fun” Example: Visual Microphone

The Visual Microphone: Passive Recovery of Sound from Video

Abe Davis
Michael Rubinstein
Neal Wadhwa
Gautham J. Mysore
Fredo Durand
William T. Freeman

Follow-up work to previous lecture’s example of Eulerian video magnification
Optical Flow

**Problem:**
Determine how objects (and/or the camera itself) move in the 3D world

**Key Idea(s):**
Images acquired as a (continuous) function of time provide additional constraint.
Formulate motion analysis as finding (dense) point correspondences over time.
Optical Flow and 2D Motion

**Optical flow** is the apparent motion of brightness patterns in the image.

**Applications**
- image and video stabilization in digital cameras, camcorders
- motion-compensated video compression schemes such as MPEG
- image registration for medical imaging, remote sensing
- action recognition
- motion segmentation
Optical Flow and 2D Motion

Motion is geometric

Optical flow is radiometric

Usually we assume that optical flow and 2-D motion coincide ...

but this is not always the case!
Optical Flow and 2D Motion

Optical flow but no motion . . .
Optical Flow and 2D Motion

Optical flow but no motion . . .

. . . moving light source(s), lights going on/off, inter-reflection, shadows
Optical Flow and 2D Motion

Optical flow but no motion . . .
  . . . moving light source(s), lights going on/off, inter-reflection, shadows

Motion but no optical flow . . .
Optical Flow and 2D Motion

Optical flow but no motion . . .

. . . moving light source(s), lights going on/off, inter-reflection, shadows

Motion but no optical flow . . .

. . . spinning sphere.
Optical Flow and 2D Motion

Here’s a video example of a very skilled Japanese contact juggler working with a clear acrylic ball

Source: [http://youtu.be/CtztrcGkCBw?t=1m20s](http://youtu.be/CtztrcGkCBw?t=1m20s)

A key element to the illusion is motion without corresponding optical flow
Example 1: Rotating Ellipse
Example 1: Three “Percepts”

1. **Veridical**:  
   — a 2-D rigid, flat, rotating ellipse

2. **Amoeboid**:  
   — a 2-D, non-rigid “gelatinous” smoothly deforming shape

3. **Stereokinetic**:  
   — a circular, rigid disk rolling in 3-D
Example 1: Rotating Ellipse

A narrow ellipse oscillating rigidly about its center appears **rigid**
Example 1: Rotating Ellipse

However, a fat ellipse undergoing the same motion appears **nonrigid**

Video credits: Yair Weiss
Example 1: Rotating Ellipse

The apparent nonrigidity of a fat ellipse is not really a "visual illusion". A rotating ellipse or a nonrigid pulsating ellipse can cause the exact same stimulation on our retinas. In this sequence the ellipse contour is always doing the same thing, only the markers' motion changes.
Example 1: Rotating Ellipse

The ellipse's motion can be influenced by features not physically connected to the ellipse. In this sequence the ellipse is always doing the same thing, only the dots' motion changes.

Video credits: Yair Weiss
Bees have very limited stereo perception. How do they fly safely through narrow passages?
Bees have very limited stereo perception. How do they fly safely through narrow passages?

A simple strategy would be to balance the speeds of motion of the images of the two walls. If wall A is moving faster than wall B, what should you (as a bee) do?
Example: Flying Insects and Birds

Bee strategy: Balance the optical flow experienced by the two eyes

Figure credit: M. Srinivasan
**Example:** Flying Insects and Birds

How do bees land safely on surfaces?

During their approach, bees continually adjust their speed to hold constant the optical flow in the vicinity of the target

— approach speed decreases as the target is approached and reduces to zero at the point of touchdown

— no need to estimate the distance to the target at any time
Bees approach the surface more slowly if the spiral is rotated to augment the rate of expansion.  

**Example:** Flying Insects and Birds

**Figure credit:** M. Srinivasan
Example: Flying Insects and Birds

Figure credit: M. Srinivasan
In which direction is the line moving?

Image Credit: Ioannis (Yannis) Gkioulekas (CMU)
Aperture Problem

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Aperture Problem

- Without distinct features to track, the true visual motion is ambiguous

- Locally, one can compute only the component of the visual motion in the direction perpendicular to the contour
Aperture Problem

— Without distinct features to track, the true visual motion is ambiguous

— Locally, one can compute only the component of the visual motion in the direction perpendicular to the contour
Visual motion is determined when there are distinct features to track, provided:
- the features can be detected and localized accurately; and
- the features can be correctly matched over time
Motion as **Matching**

<table>
<thead>
<tr>
<th>Representation</th>
<th>Result is…</th>
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<tbody>
<tr>
<td>Point/feature based</td>
<td>(very) sparse</td>
</tr>
<tr>
<td>Contour based</td>
<td>(relatively) sparse</td>
</tr>
<tr>
<td>(Differential) gradient based</td>
<td>dense</td>
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</table>
Optical Flow \textbf{Constraint Equation}

Consider image intensity also to be a function of time, \( t \). We write \( I(x, y, t) \).
Consider image intensity also to be a function of time, \( t \). We write

\[ I(x, y, t) \]

Applying the **chain rule for differentiation**, we obtain

\[
\frac{dI(x, y, t)}{dt} = I_x \frac{dx}{dt} + I_y \frac{dy}{dt} + I_t
\]

where subscripts denote partial differentiation.
Consider image intensity also to be a function of time, $t$. We write
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$$ \frac{dI(x, y, t)}{dt} = I_x \frac{dx}{dt} + I_y \frac{dy}{dt} + I_t $$

where subscripts denote partial differentiation.

Define $u = \frac{dx}{dt}$ and $v = \frac{dy}{dt}$. Then $[u, v]$ is the 2-D motion and the space of all such $u$ and $v$ is the **2-D velocity space**.

$[u(x,y), v(x,y)]$ is the motion field – a vector $[u,v]$ at every point in the image.
Optical Flow **Constraint Equation**

Consider image intensity also to be a function of time, \( t \). We write

\[
I(x, y, t)
\]

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Define \( u = \frac{dx}{dt} \) and \( v = \frac{dy}{dt} \). Then \([u, v]\) is the 2-D motion and the space of all such \( u \) and \( v \) is the **2-D velocity space**.

Suppose \( \frac{dI(x, y, t)}{dt} = 0 \). Then we obtain the (classic) **optical flow constraint equation**

\[
I_x u + I_y v + I_t = 0
\]
Consider image intensity also to be a function of time, \( I(x, y, t) \). We write
\[
\frac{dI}{dt} = \nabla I \cdot \mathbf{v}
\]
where subscripts denote partial differentiation.

Define \( u(x, y, t) \) and \( v(x, y, t) \). Then \( u, v \) is the 2-D motion and the space of all

**Optical Flow Constraint Equation**

What does this mean, and why is it reasonable?
Consider image intensity also to be a function of time, $I(x, y, t)$. We write

$$I(x, y, t) = I_0(x, y) + \sum_{k=1}^{N} \Delta I_k(x, y, t)$$

Applying the chain rule for differentiation, we obtain

$$\frac{\partial I(x, y, t)}{\partial t} = \Delta I_1(x, y, t) + \sum_{k=2}^{N} \frac{\partial \Delta I_k(x, y, t)}{\partial t}$$

where subscripts denote partial differentiation.

Define $u(x, y, t) = \frac{\partial x}{\partial t}$ and $v(x, y, t) = \frac{\partial y}{\partial t}$. Then $\mathbf{u}(x, y, t)$ is the 2-D motion and the space of all $\mathbf{u}$ is the optical flow space.

**Optical Flow Constraint Equation**

Scene point moving through image sequence

- $(x(1), y(1))$
- $(x(2), y(2))$
- $\ldots$
- $(x(k), y(k))$

What does this mean, and why is it reasonable?

*Image Credit: Ioannis (Yannis) Gkioulekas (CMU)*
Consider image intensity also to be a function of time, \( t \). We write

\[
\frac{\partial I(x,y,t)}{\partial t} = \text{motion}.
\]

Applying the chain rule for differentiation, we obtain

\[
\frac{\partial I(x,y,t)}{\partial t} = \frac{\partial I(x,y,t)}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial I(x,y,t)}{\partial y} \frac{\partial y}{\partial t}.
\]

Define \( \mathbf{u} = (u(x,y), v(x,y)) \). Then \( \mathbf{u} \) is the 2-D motion and the space of all optical flow constraint equations.

What does this mean, and why is it reasonable?
Optical Flow **Constraint Equation**

**Brightness Constancy Assumption:** Brightness of the point remains the same

\[ I(x(t), y(t), t) = C \]

constant

What does this mean, and why is it reasonable?
Aside: Derivation of Optical Flow Constraint

\[ I(x + u\delta t, y + v\delta t, t + \delta t) = I(x, y, t) \]

For small space-time step, the brightness of a point in the scene remains the same.
Aside: Derivation of Optical Flow Constraint

\[ I(x + u\delta t, y + v\delta t, t + \delta t) = I(x, y, t) \]

For small space-time step, brightness of a point is the same

**Insight:**
If the time step is really small,
we can *linearize* the intensity function
(and motion is really-small ... think less than a pixel)

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*Slide Credit: Ioannis (Yannis) Gkioulkas (CMU)*
Aside: Derivation of Optical Flow Constraint

\[ I(x + u\delta t, y + v\delta t, t + \delta t) = I(x, y, t) \]

Multivariable Taylor Series Expansion
(First order approximation, two variables)

\[ f(x, y) \approx f(a, b) + f_x(a, b)(x - a) - f_y(a, b)(y - b) \]
**Aside: Derivation of Optical Flow Constraint**

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**Multivariable Taylor Series Expansion**
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\[ I(x, y, t) + \frac{\partial I}{\partial x} \delta x + \frac{\partial I}{\partial y} \delta y + \frac{\partial I}{\partial t} \delta t = I(x, y, t) \quad \text{assuming small motion} \]
Aside: Derivation of Optical Flow Constraint

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**Multivariable Taylor Series Expansion**
*(First order approximation, two variables)*

\[ f(x, y) \approx f(a, b) + f_x(a, b)(x - a) - f_y(a, b)(y - b) \]

Partial derivative

\[ I(x, y, t) + \frac{\partial I}{\partial x} \delta x + \frac{\partial I}{\partial y} \delta y + \frac{\partial I}{\partial t} \delta t = I(x, y, t) \]
*assuming small motion*

**Slide Credit:** Ioannis (Yannis) Gkioulekas (CMU)
Aside: Derivation of Optical Flow Constraint

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assuming small motion

\[ \frac{\partial I}{\partial x} \delta x + \frac{\partial I}{\partial y} \delta y + \frac{\partial I}{\partial t} \delta t = 0 \]
cancel terms
Aside: Derivation of Optical Flow Constraint

\[ I(x + u\delta t, y + v\delta t, t + \delta t) = I(x, y, t) \]

**Multivariable Taylor Series Expansion**
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\[ \frac{\partial I}{\partial x} \delta x + \frac{\partial I}{\partial y} \delta y + \frac{\partial I}{\partial t} \delta t = 0 \]
divide by \( \delta t \)
take limit \( \delta t \to 0 \)

*Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)*
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take limit \( \delta t \to 0 \)

\[ \frac{\partial I}{\partial x} \frac{dx}{dt} + \frac{\partial I}{\partial y} \frac{dy}{dt} + \frac{\partial I}{\partial t} = 0 \]
Brightness Constancy Equation

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)
How do we compute ...

\[ I_x u + I_y v + I_t = 0 \]
How do we **compute** ...

\[ I_x u + I_y v + I_t = 0 \]

\[ I_x = \frac{\partial I}{\partial x} \quad I_y = \frac{\partial I}{\partial y} \]

**spatial derivative**
How do we **compute** ... 

\[ I_x u + I_y v + I_t = 0 \]

\[ I_x = \frac{\partial I}{\partial x} \quad I_y = \frac{\partial I}{\partial y} \]

Spatial derivative

- Forward difference
- Sobel filter
- Scharr filter

\[ \cdots \]
How do we compute …

\[ I_x u + I_y v + I_t = 0 \]

\[
I_x = \frac{\partial I}{\partial x} \quad I_y = \frac{\partial I}{\partial y}
\]

spatial derivative

\[
I_t = \frac{\partial I}{\partial t}
\]

temporal derivative

Forward difference
  Sobel filter
  Scharr filter
  …

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)
How do we compute ...

\[ I_x u + I_y v + I_t = 0 \]

\[ I_x = \frac{\partial I}{\partial x}, \quad I_y = \frac{\partial I}{\partial y} \]

Spatial derivative

Forward difference
Sobel filter
Scharr filter

\[ I_t = \frac{\partial I}{\partial t} \]

Temporal derivative

Frame differencing

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)
### Frame Differencing: Example

\[ I_t = \frac{\partial I}{\partial t} \]

(Example of a forward temporal difference)

<table>
<thead>
<tr>
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**Slide Credit:** Ioannis (Yannis) Gkioulakas (CMU)
\[ I_x = \frac{\partial I}{\partial x} \]
\[ I_y = \frac{\partial I}{\partial y} \]
\[ I_t = \frac{\partial I}{\partial t} \]
How do we compute ... 

\[ I_x u + I_y v + I_t = 0 \]

- Spatial derivative:
  \[ I_x = \frac{\partial I}{\partial x} \quad I_y = \frac{\partial I}{\partial y} \]

- Optical flow:
  \[ u = \frac{dx}{dt} \quad v = \frac{dy}{dt} \]

- Temporal derivative:
  \[ I_t = \frac{\partial I}{\partial t} \]

- Forward difference
  - Sobel filter
  - Scharr filter
  ...

- How do you compute this?

- Frame differencing

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)
How do we compute ...

\[ I_x u + I_y v + I_t = 0 \]

\[ \begin{align*}
  I_x &= \frac{\partial I}{\partial x} \\
  I_y &= \frac{\partial I}{\partial y}
\end{align*} \]

Spatial derivative

\[ \begin{align*}
  u &= \frac{dx}{dt} \\
  v &= \frac{dy}{dt}
\end{align*} \]

Optical flow

\[ I_t = \frac{\partial I}{\partial t} \]

Temporal derivative

Forward difference
Sobel filter
Scharr filter
...

We need to solve for this!
(this is the unknown in the optical flow problem)

Frame differencing

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)
Optical Flow Constraint Equation

\[ I_x u + I_y v + I_t = 0 \]

many combinations of \( u \) and \( v \) will satisfy the equality

Equation determines a **straight line** in velocity space
Lucas-Kanade

Observations:

1. The 2-D motion, \([u, v]\) at a given point, \([x, y]\) has two degrees-of-freedom.
2. The partial derivatives, \(I_x, I_y, I_t\), provide one constraint.
3. The 2-D motion, \([u, v]\) cannot be determined locally from \(I_x, I_y, I_t\) alone.
Lucas-Kanade

Observations:
1. The 2-D motion $[u, v]$, at a given point $[x, y]$, has two degrees-of-freedom
2. The partial derivatives, $I_x, I_y, I_t$ provide one constraint
3. The 2-D motion, $[u, v]$ cannot be determined locally from $I_x, I_y, I_t$ alone

Lucas–Kanade Idea:
Obtain additional local constraint by computing the partial derivatives, $I_x, I_y, I_t$, in a window centred at the given $[x, y]$
Lucas-Kanade

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**Lucas–Kanade Idea:**
Obtain additional local constraint by computing the partial derivatives, $I_x, I_y, I_t$ in a window centered at the given $[x, y]$

**Constant Flow Assumption:** nearby pixels will likely have same optical flow
Lucas-Kanade

Optical Flow Constraint Equation: \( I_x u + I_y v + I_t = 0 \)

Suppose \([x_1, y_1] = [x, y]\) is the (original) centre point in the window. Let \([x_2, y_2]\) be any other point in the window. This gives us two equations that we can write

\[
\begin{align*}
I_{x_1} u + I_{y_1} v &= -I_{t_1} \\
I_{x_2} u + I_{y_2} v &= -I_{t_2}
\end{align*}
\]

and that can be solved locally for \(u\) and \(v\) as

\[
\begin{bmatrix}
u \\
v
\end{bmatrix} = - \begin{bmatrix}
I_{x_1} & I_{y_1} \\
I_{x_2} & I_{y_2}
\end{bmatrix}^{-1} \begin{bmatrix}
I_{t_1} \\
I_{t_2}
\end{bmatrix}
\]

provided that \(u\) and \(v\) are the same in both equations and provided that the required matrix inverse exists.
Considering all $n$ points in the window, one obtains

\[
\begin{align*}
I_{x1}u + I_{y1}v &= -I_{t1} \\
I_{x2}u + I_{y2}v &= -I_{t2} \\
& \vdots \\
I_{xn}u + I_{yn}v &= -I_{tn}
\end{align*}
\]

which can be written as the matrix equation $Av = b$

where $v = [u, v]^T$, $A = \begin{bmatrix} I_{x1} & I_{y1} \\
I_{x2} & I_{y2} \\
& \vdots & \vdots \\
I_{xn} & I_{yn} \end{bmatrix}$ and $b = -\begin{bmatrix} I_{t1} \\
I_{t2} \\
& \vdots \\
I_{tn} \end{bmatrix}$
The standard least squares solution, $\tilde{v}$, to this is

$$\tilde{v} = (A^T A)^{-1} A^T b$$

again provided that $u$ and $v$ are the same in all equations and provided that the rank of $A^T A$ is 2 (so that the required inverse exists)
Lucas-Kanade

Note that we can explicitly write down an expression for $\mathbf{A}^T \mathbf{A}$ as

$$
\mathbf{A}^T \mathbf{A} = \begin{bmatrix}
\sum I_x^2 & \sum I_x I_y \\
\sum I_x I_y & \sum I_y^2
\end{bmatrix}
$$

which is identical to the matrix $\mathbf{C}$ that we saw in the context of Harris corner detection.
Lucas-Kanade

Note that we can explicitly write down an expression for $A^T A$ as

$$A^T A = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}$$

which is identical to the matrix $C$ that we saw in the context of Harris corner detection.

What does that mean?
A dense method to compute motion $[u, v]$, at every location in an image

**Key Assumptions:**

1. Motion is slow enough and smooth enough that differential methods apply (i.e., that the partial derivatives $I_x, I_y, I_t$ are well-defined)

2. The optical flow constraint equation holds, i.e., $\frac{dI(x, y, t)}{dt} = 0$

3. A window size is chosen so that motion $[u, v]$, is constant in the window

4. A window size is chosen so that the rank of $A^T A$ is 2 for the window
Aside: Optical Flow Smoothness Constraint

Many methods trade off a ‘departure from the optical flow constraint’ cost with a ‘departure from smoothness’ cost. (Recall block matching and smoothness)

The optimization objective to minimize becomes

\[ E = \int \int (I_x u + I_y v + I_t)^2 + \lambda (\| \nabla u \|^2 + \| \nabla v \|^2) \]

where \( \lambda \) is a weighting parameter. That is, it trades off fit to the data (1\textsuperscript{st} term) with smoothness of the solution (the 2\textsuperscript{nd} term).

This is another “energy minimization” formulation (like stereo)
Horn-Schunck Optical Flow

\[
\min_{u,v} \sum_{i,j} \left\{ E_s(i, j) + \lambda E_d(i, j) \right\}
\]

Note that the lambda term can appear on the 1\textsuperscript{st} term or the 2\textsuperscript{nd} term – it doesn’t matter – only the ratio does.
Horn-Schunck Optical Flow

**Brightness constancy**

\[ E_d(i, j) = \left[ I_x u_{ij} + I_y v_{ij} + I_t \right]^2 \]

**Smoothness**

\[ E_s(i, j) = \frac{1}{4} \left[ (u_{ij} - u_{i+1,j})^2 + (u_{ij} - u_{i,j+1})^2 + (v_{ij} - v_{i+1,j})^2 + (v_{ij} - v_{i,j+1})^2 \right] \]
Summary

Motion, like binocular stereo, can be formulated as a matching problem. That is, given a scene point located at \((x_0, y_0)\) in an image acquired at time \(t_0\), what is its position \((x_1, y_1)\), in an image acquired at time \(t_1\)?

Assuming image intensity does not change as a consequence of motion, we obtain the (classic) **optical flow constraint equation**

\[
I_x u + I_y v + I_t = 0
\]

where \([u, v]\) is the 2-D motion at a given point, \([x, y]\) and \(I_x, I_y, I_t\) are the partial derivatives of intensity with respect to \(x, y,\) and \(t\).

**Lucas–Kanade** is a dense method to compute the motion, \([u, v]\) at every location in an image.