

# Greedy Algorithms

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## Introduction

### Reading:

1. CLRS: “Greedy Algorithms” 16.1-16.2
2. GT: “The Greedy Method” 5.1

We have already discussed several algorithm design techniques:

- ▶ divide and conquer (sorting and integer multiplication)
- ▶ prune and search (select or randomized select)
- ▶ (now) greedy algorithms

### Definition

A greedy algorithm solves an optimization problem:

- ▶ makes a sequence of choices
- ▶ picks the choice that seems “best” without explicit consideration for past or future choices

## Greedy algorithm correctness

What property guarantees greedy solutions work?

### Greedy choice property

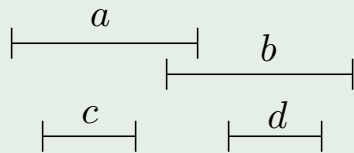
Some optimal solution can be obtained by combining

- ▶ a greedy choice with
- ▶ an optimal solution to remaining subproblem

### Problem: Activity selection

Find a largest subset of non-overlapping intervals. Variants are greatest weight subset or largest subset taking the least time.

### Example



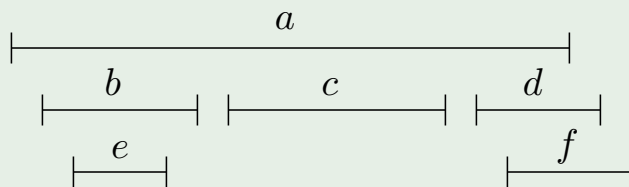
$\{a, d\}$  and  $\{b, c\}$  are the only valid solutions.

## Activity selection

Construct a solution by

- ▶ selecting intervals one at a time
- ▶ but never choosing an interval that overlaps a previous selection

### Example



Optimal is  $\{b, c, d\}$ ,  
 $\{b, c, f\}$ ,  $\{e, c, d\}$ , or  
 $\{e, c, f\}$

What order to consider the activities?

- ▶ Sorted alphabetically? No, selects  $\{a\}$ .
- ▶ Sorted by left endpoints? No, selects  $\{a\}$ .
- ▶ Sorted by right endpoints? Maybe, selects  $\{e, c, d\}$ .

## Sorting by right endpoints

- ▶ We choose the activity that ends the earliest.
- ▶ We leaves as much of the rest of the day available as possible.

```
Algorithm ActivitySelect(A)
  S ← ∅
  sort A by increasing right endpoints
  for j ← 0 to A.length-1
    if A[j].left ≥ maxRightEndPoint(S)
      S ← S ∪ A[j]
  return S
```

- ▶ The runtime is  $\Theta(n \log n)$  because we can make each comparison take  $\Theta(1)$  time.
- ▶ Why does it work?

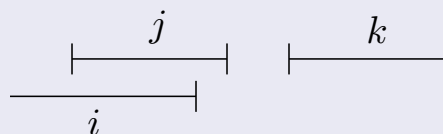
## Greedy choice property

### Lemma

*If activity  $i$  has the smallest right endpoint, some optimal activity selection includes  $i$ .*

### Proof.

Consider an optimal activity selection  $S$ . If  $i \in S$ , then  $S$  is the desired selection. Otherwise, let  $j \in S$  have the smallest right endpoint. Every  $k \in S$  other than  $j$  must lie to right of  $j$  because  $j$  and  $k$  do not overlap and  $j$ 's right endpoint is the smallest. So  $i$  and  $k$  do not overlap because  $i$ 's right endpoint is at least as small as  $j$ 's. Thus we can swap  $j$  for  $i$  to get an optimal selection.



## Activity selection correctness

### Theorem

*ActivitySelect returns an optimal activity selection.*

### Proof.

Let  $i$  be the activity of  $A$  with the earliest right endpoint. By the previous Lemma, some optimal selection  $T$  of  $A$  contains  $i$ .

Let  $S$  be the selection  $S$  chosen by our algorithm. To prove optimality, we must show that  $|S| \geq |T|$ . We do this by induction on  $|A|$ .

**Base case:**  $|A| \leq 1$ . Trivially,  $S = T = A$ .

## Activity selection correctness (cont'd)

### Proof. (cont'd)

**Induction step:** Suppose our selection algorithm works for all sets of activities with less than  $|A|$  activities (strong induction).

Let  $A'$  be the activities of  $A$  that do not conflict with  $i$ . Let  $S'$  be our algorithm's selection for  $A'$ .

By our greedy criteria,  $S = \{i\} \cup S'$ . By our inductive hypothesis,  $S'$  is an optimal selection of  $A'$ .

Let  $T' = T \setminus \{i\}$ . Then  $T' \subseteq A'$ . Therefore,  $|T'| \leq |S'|$  by optimality of  $S'$ . Hence,  $|T| \leq |S|$ . Thus,  $S$  is an optimal selection of  $A$ .