THE UNIVERSITY OF BRITISH COLUMBIA

CPSC 425: Computer Vision

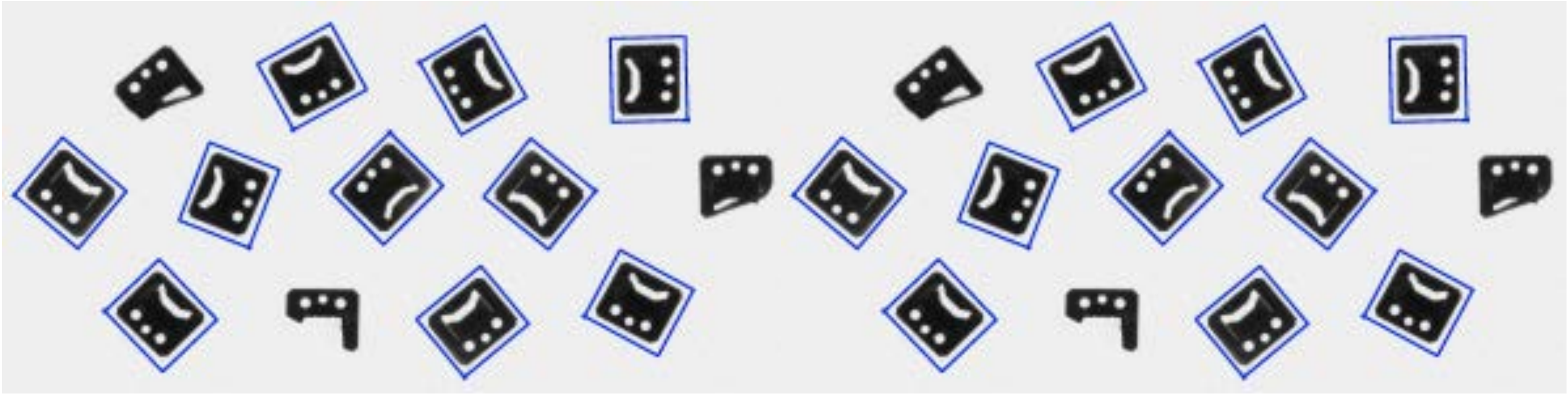


Image Credit: https://docs.adaptive-vision.com/4.7/studio/machine_vision_guide/TemplateMatching.html

Lecture 8: Scaled Representations

(unless otherwise stated slides are taken or adopted from **Bob Woodham, Jim Little** and **Fred Tung**)

Menu for Today

Topics:

— **Scaled** Representations

— Image **Pyramid**

— Multi-scale Template **Matching**

Readings:

— **Today's** Lecture: Szeliski 2.3, 3.5, Forsyth & Ponce (2nd ed.) 4.5 - 4.7

Reminders:

— **Assignment 2**: Scaled Representations, Face Detection and Image Blending available now

Next: Please get your **iClickers** —

Quiz 2: **6** questions

Bilateral Filter

image $I(X,Y)$

25	0	25	255	255	255
0	0	0	230	255	255
0	25	25	255	230	255
0	0	25	255	255	255

→

Normalised image $I(X,Y)$

0.1	0	0.1	1	1	1
0	0	0	0.9	1	1
0	0.1	0.1	1	0.9	1
0	0	0.1	1	1	1

Domain Kernel

$\sigma_d = 1$

0.08	0.12	0.08
0.12	0.20	0.12
0.08	0.12	0.08

domain kernel

$$\exp^{-\frac{x^2+y^2}{2\sigma_d^2}}$$

Bilateral Filter

image $I(X,Y)$

25	0	25	255	255	255
0	0	0	230	255	255
0	25	25	255	230	255
0	0	25	255	255	255

→

Normalised image $I(X,Y)$

0.1	0	0.1	1	1	1
0	0	0	0.9	1	1
0	0.1	0.1	1	0.9	1
0	0	0.1	1	1	1

Domain Kernel

$\sigma_d = 1$

0.08	0.12	0.08
0.12	0.20	0.12
0.08	0.12	0.08

Range Kernel

$\sigma_r = 0.45$

0.98	0.98	0.2
1	1	0.1
0.98	1	0.1

(differences based on **centre pixel**)

$$\exp^{-\frac{(I(X+x,Y+y)-I(X,Y))^2}{2\sigma_r^2}}$$

range kernel

Bilateral Filter

image $I(X,Y)$

25	0	25	255	255	255
0	0	0	230	255	255
0	25	25	255	230	255
0	0	25	255	255	255

→

Normalised image $I(X,Y)$

0.1	0	0.1	1	1	1
0	0	0	0.9	1	1
0	0.1	0.1	1	0.9	1
0	0	0.1	1	1	1

Domain Kernel

$\sigma_d = 1$

0.08	0.12	0.08
0.12	0.20	0.12
0.08	0.12	0.08

Range Kernel

$\sigma_r = 0.45$

0.98	0.98	0.2
1	1	0.1
0.98	1	0.1

(differences based on **centre pixel**)

multiply →

Range * **Domain** Kernel

0.08	0.12	0.02
0.12	0.20	0.01
0.08	0.12	0.01

Bilateral Filter

image $I(X,Y)$

25	0	25	255	255	255
0	0	0	230	255	255
0	25	25	255	230	255
0	0	25	255	255	255

→

Normalised image $I(X,Y)$

0.1	0	0.1	1	1	1
0	0	0	0.9	1	1
0	0.1	0.1	1	0.9	1
0	0	0.1	1	1	1

Domain Kernel

$\sigma_d = 1$

0.08	0.12	0.08
0.12	0.20	0.12
0.08	0.12	0.08

Range Kernel

$\sigma_r = 0.45$

0.98	0.98	0.2
1	1	0.1
0.98	1	0.1

(differences based on **centre pixel**)

multiply →

Range * **Domain** Kernel

0.08	0.12	0.02
0.12	0.20	0.01
0.08	0.12	0.01

sum to 1 →

0.11	0.16	0.03
0.16	0.26	0.01
0.11	0.16	0.01

Bilateral Filter

image $I(X,Y)$

25	0	25	255	255	255
0	0	0	230	255	255
0	25	25	255	230	255
0	0	25	255	255	255

→

Normalised image $I(X,Y)$

0.1	0	0.1	1	1	1
0	0	0	0.9	1	1
0	0.1	0.1	1	0.9	1
0	0	0.1	1	1	1

Domain Kernel

$\sigma_d = 1$

0.08	0.12	0.08
0.12	0.20	0.12
0.08	0.12	0.08

Range Kernel

$\sigma_r = 0.45$

0.98	0.98	0.2
1	1	0.1
0.98	1	0.1

(differences based on **centre pixel**)

multiply →

Range * **Domain** Kernel

0.08	0.12	0.02
0.12	0.20	0.01
0.08	0.12	0.01

$$\sum \begin{bmatrix} 0.11 & 0.16 & 0.03 \\ 0.16 & 0.26 & 0.01 \\ 0.11 & 0.16 & 0.01 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0.9 \\ 0.1 & 0.1 & 1 \\ 0 & 0.1 & 1 \end{bmatrix} = 0.1$$

Bilateral Filter

Bilateral Filter

image $I(X,Y)$

25	0	25	255	255	255
0	0	0	230	255	255
0	25	25	255	230	255
0	0	25	255	255	255

→

Normalised image $I(X,Y)$

0.1	0	0.1	1	1	1
0	0	0	0.9	1	1
0	0.1	0.1	1	0.9	1
0	0	0.1	1	1	1

Domain Kernel

$\sigma_d = 1$

$$\sum \begin{bmatrix} 0.08 & 0.12 & 0.08 \\ 0.12 & 0.20 & 0.12 \\ 0.08 & 0.12 & 0.08 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0.9 \\ 0.1 & 0.1 & 1 \\ 0 & 0.1 & 1 \end{bmatrix} = 0.3$$

Gaussian Filter (only)

Range Kernel

$\sigma_r = 0.45$

0.98	0.98	0.2
1	1	0.1
0.98	1	0.1

(differences based on **centre pixel**)

multiply →

Range * **Domain** Kernel

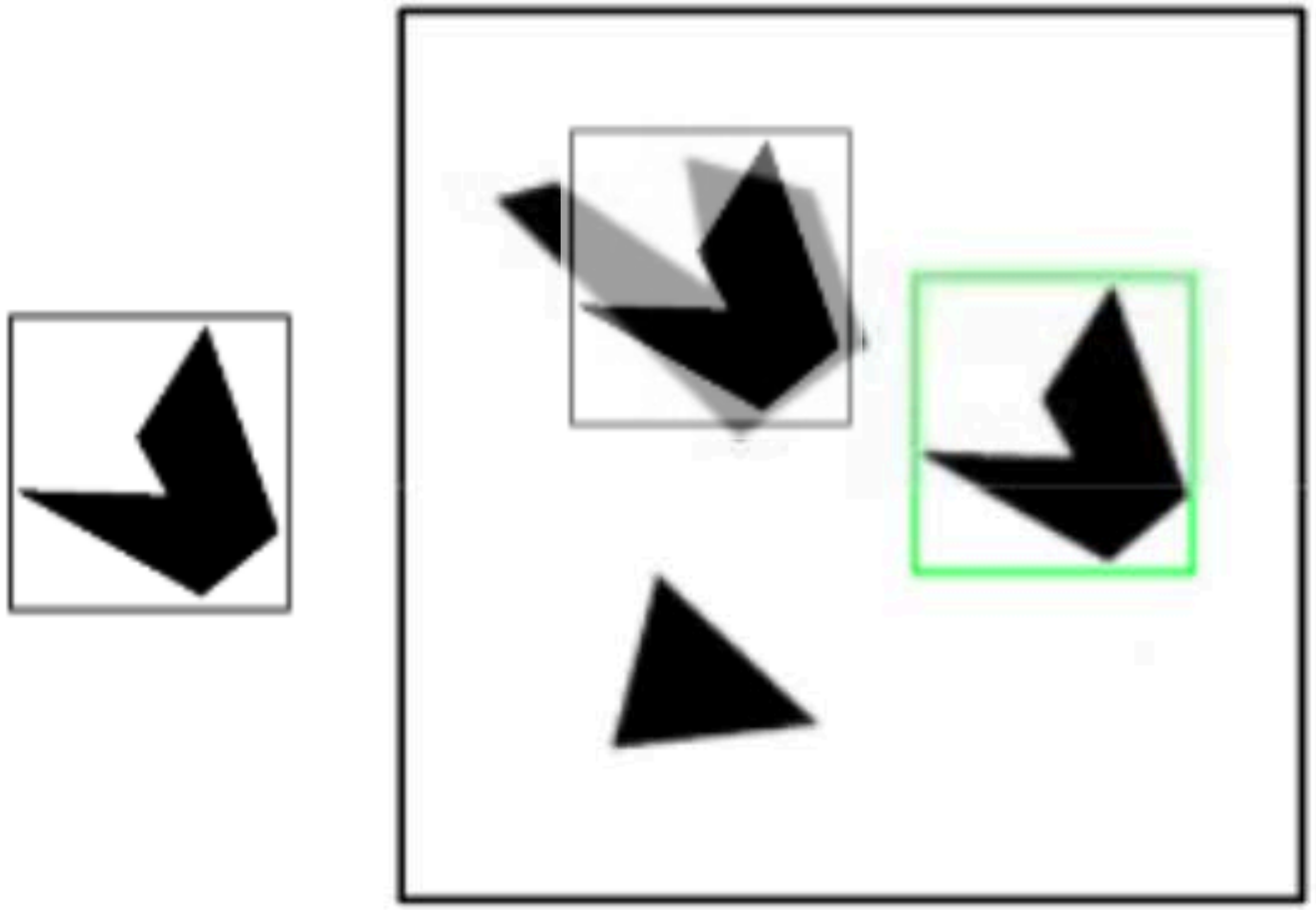
0.08	0.12	0.02
0.12	0.20	0.01
0.08	0.12	0.01

$$\sum \begin{bmatrix} 0.11 & 0.16 & 0.03 \\ 0.16 & 0.26 & 0.01 \\ 0.11 & 0.16 & 0.01 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0.9 \\ 0.1 & 0.1 & 1 \\ 0 & 0.1 & 1 \end{bmatrix} = 0.1$$

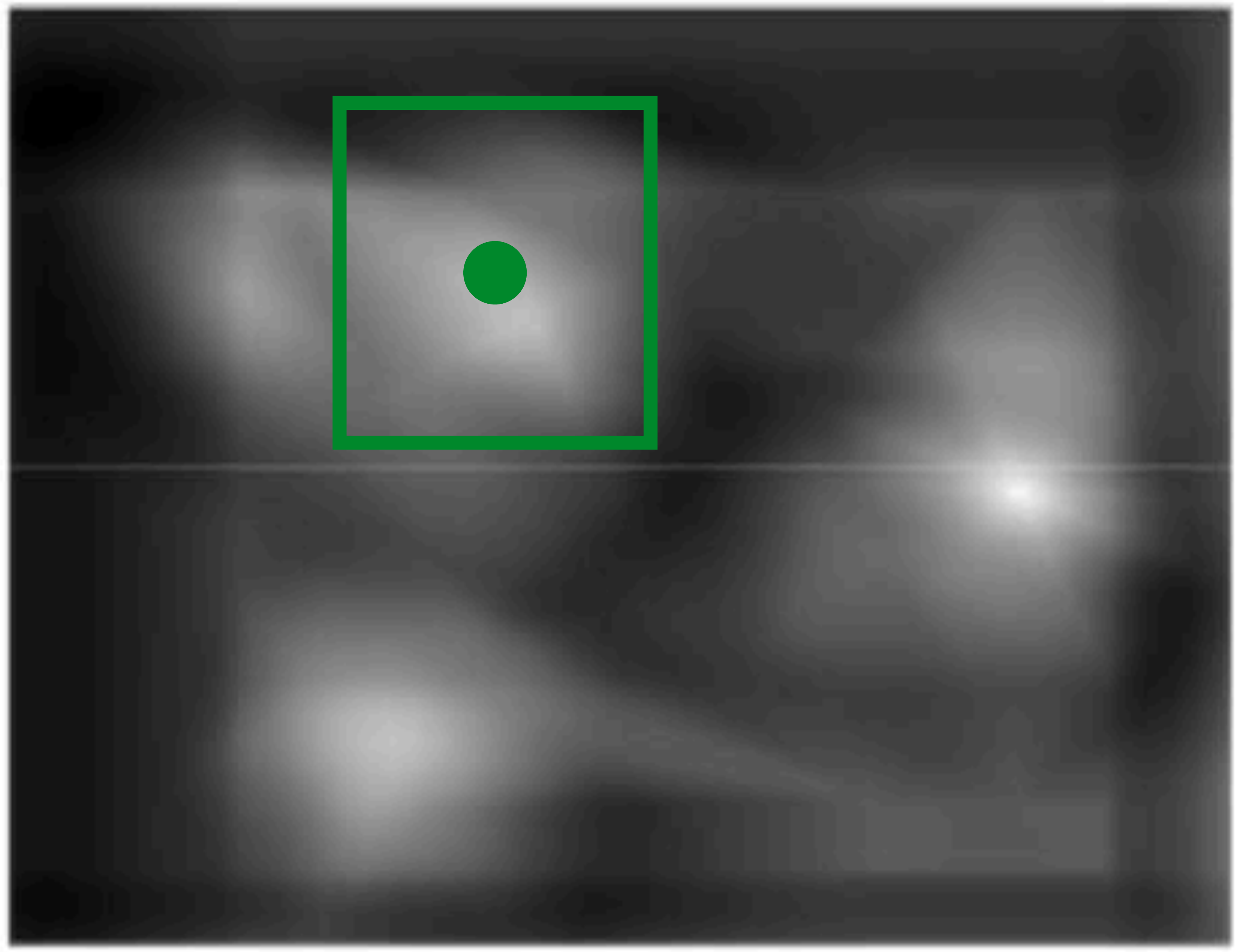
Bilateral Filter

Template Matching

Assuming template is all positive, what does this tell us about correlation map?



Detected template



Correlation map

$$\frac{a}{|a|}\frac{b}{|b|} = ?$$

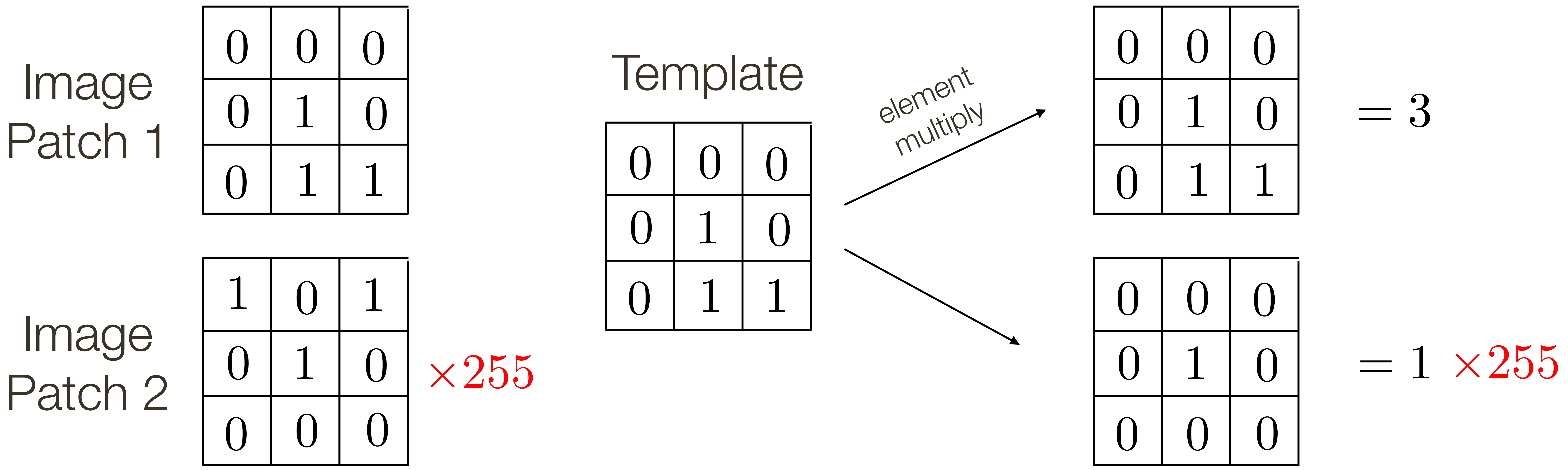
Slide Credit: Kristen Grauman

Template Matching

We can think of convolution/**correlation** as comparing a template (the filter) with each local image patch.

— Consider the filter and image patch as vectors.

— Applying a filter at an image location can be interpreted as computing the dot product between the filter and the local image patch.



Template Matching

Let a and b be vectors. Let θ be the angle between them. We know

$$\cos\theta = \frac{a \cdot b}{|a||b|} = \frac{a \cdot b}{\sqrt{(a \cdot a)(b \cdot b)}} = \frac{a}{|a|}\frac{b}{|b|}$$

where · is dot product and | | is vector magnitude

1. Normalize the template / filter (b) in the beginning
2. Compute norm of $|a|$ by convolving squared image with a filter of all 1's of equal size to the the template and square-rooting the response
3. We can compute the dot product by correlation of image (a) with normalized filter (b)
4. We can finally compute the normalized correlation by dividing element-wise result in Step 3 by result in Step 2

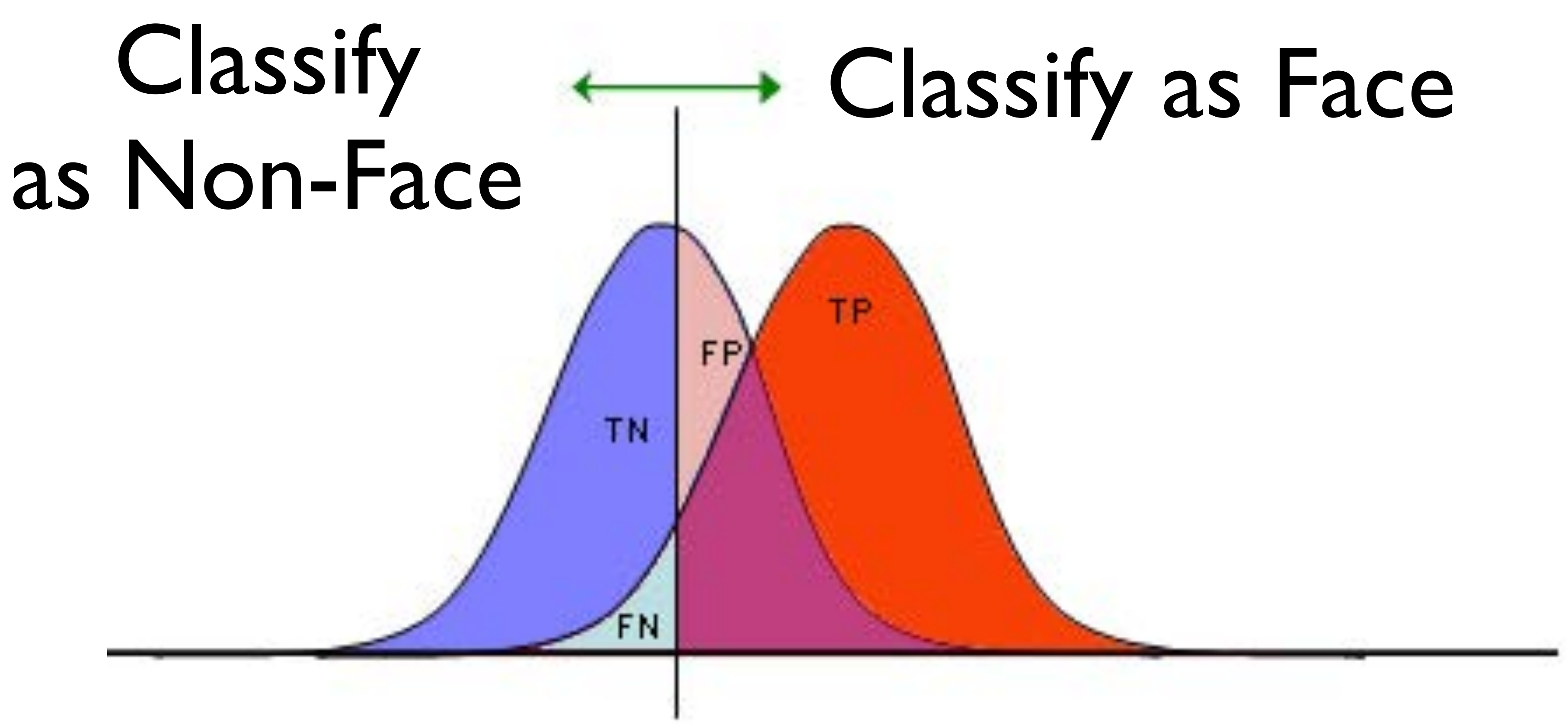
ROC Curves

Note that we can easily get 100% true positives (if we are prepared to get 100% false positives as well!)

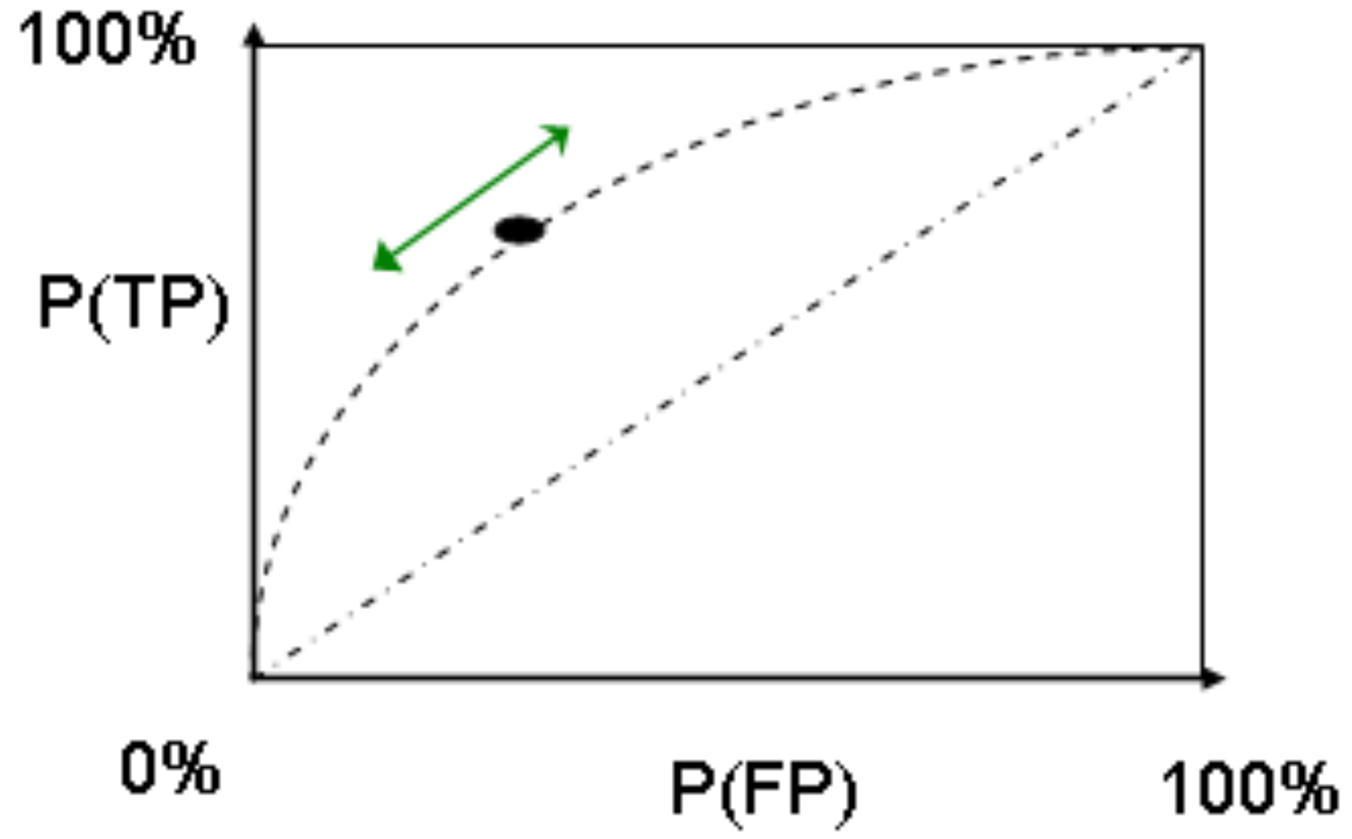
It is a tradeoff between **true positive rate (TP)** and **false positive rate (FP)**

We can plot a curve of all TP rates vs FP rates by varying the classifier threshold

This is a **Receiver Operating Characteristic (ROC)** curve



red = actual faces, blue = actual non-faces



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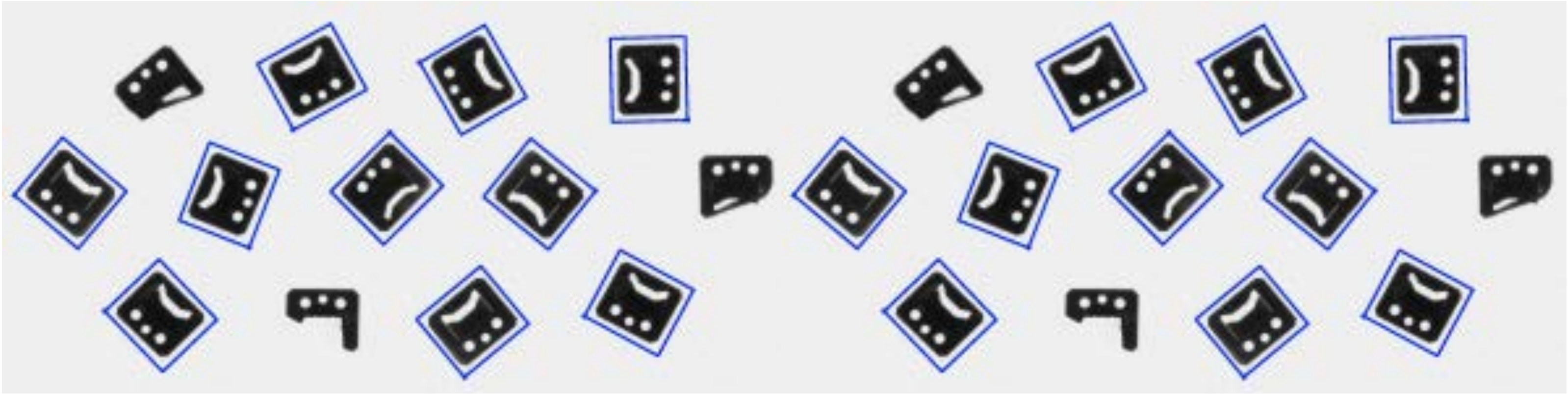

Image Credit: https://docs.adaptive-vision.com/4.7/studio/machine_vision_guide/TemplateMatching.html

Lecture 8: Scaled Representations

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— Multi-scale Template **Matching**

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Goal

1. Understand the idea behind image pyramids

2. Understand laplacian pyramids

Multi-Scale Template Matching

Problem: Make template matching robust to changes in 2D (spatial) scale.

Key Idea(s): Build a scaled representation: the Gaussian image pyramid

Alternatives:

— use multiple sizes for each given template

— ignore the issue of 2D (spatial) scale

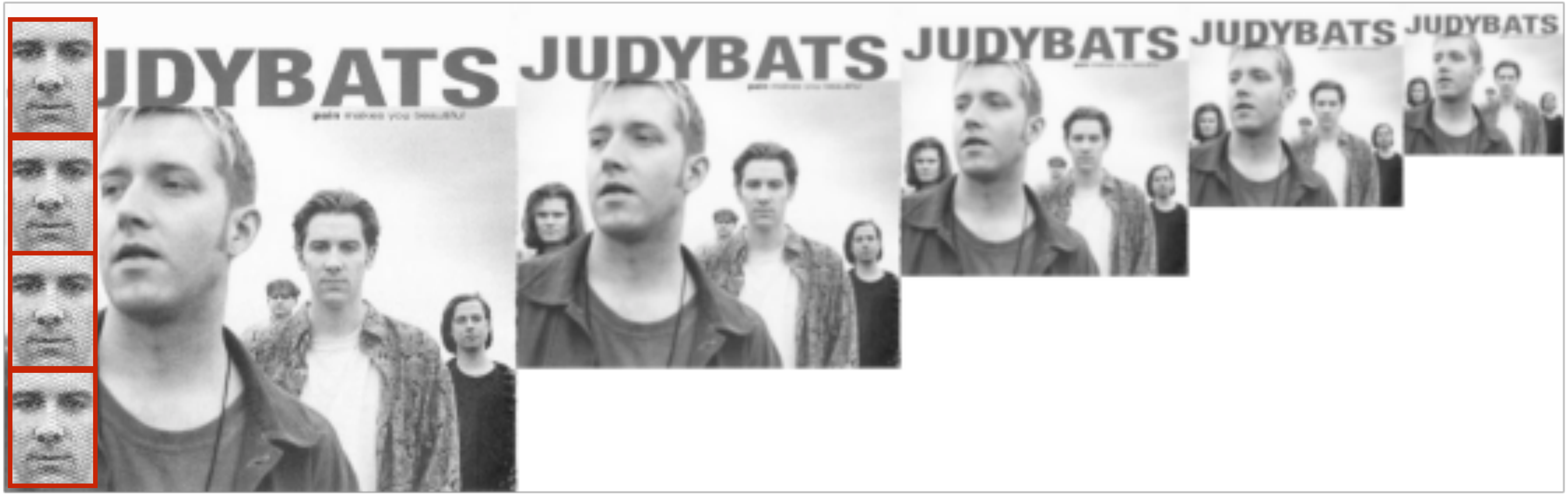
Theory: Sampling theory allows us to build image pyramids in a principled way

“**Gotchas**:”

— template matching remains sensitive to 2D orientation, 3D pose and illumination

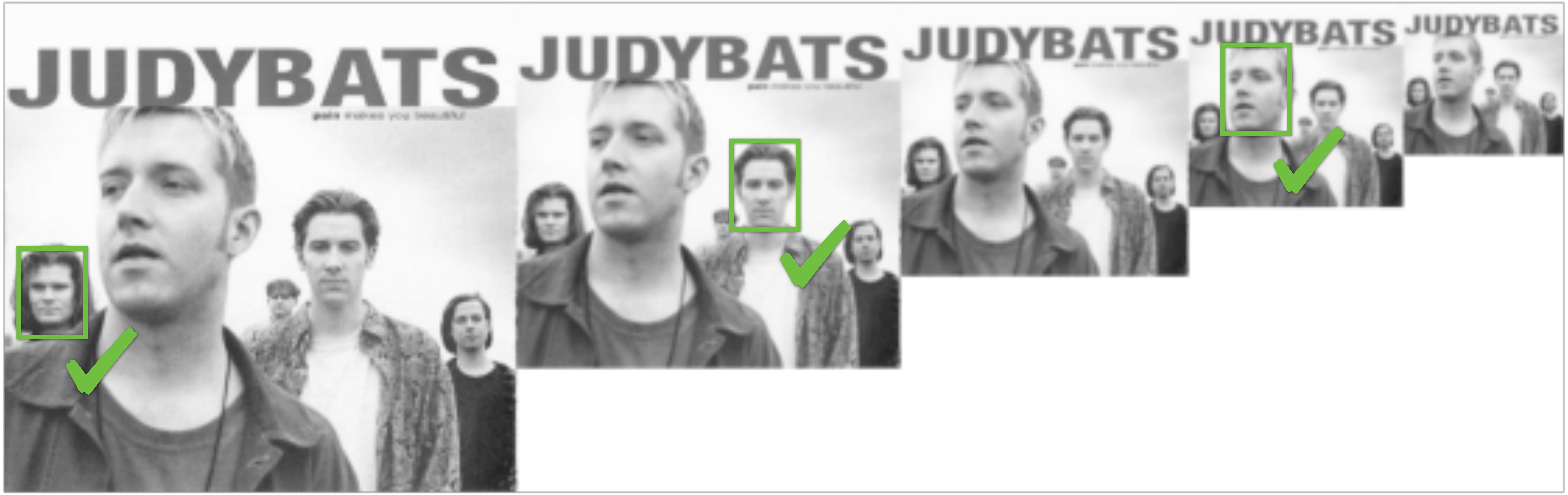
Multi-Scale Template Matching

Correlation with a **fixed-sized template** only detects faces at **specific scales**



Multi-Scale Template Matching

Solution: form a Gaussian Pyramid and convolve with the template at each scale

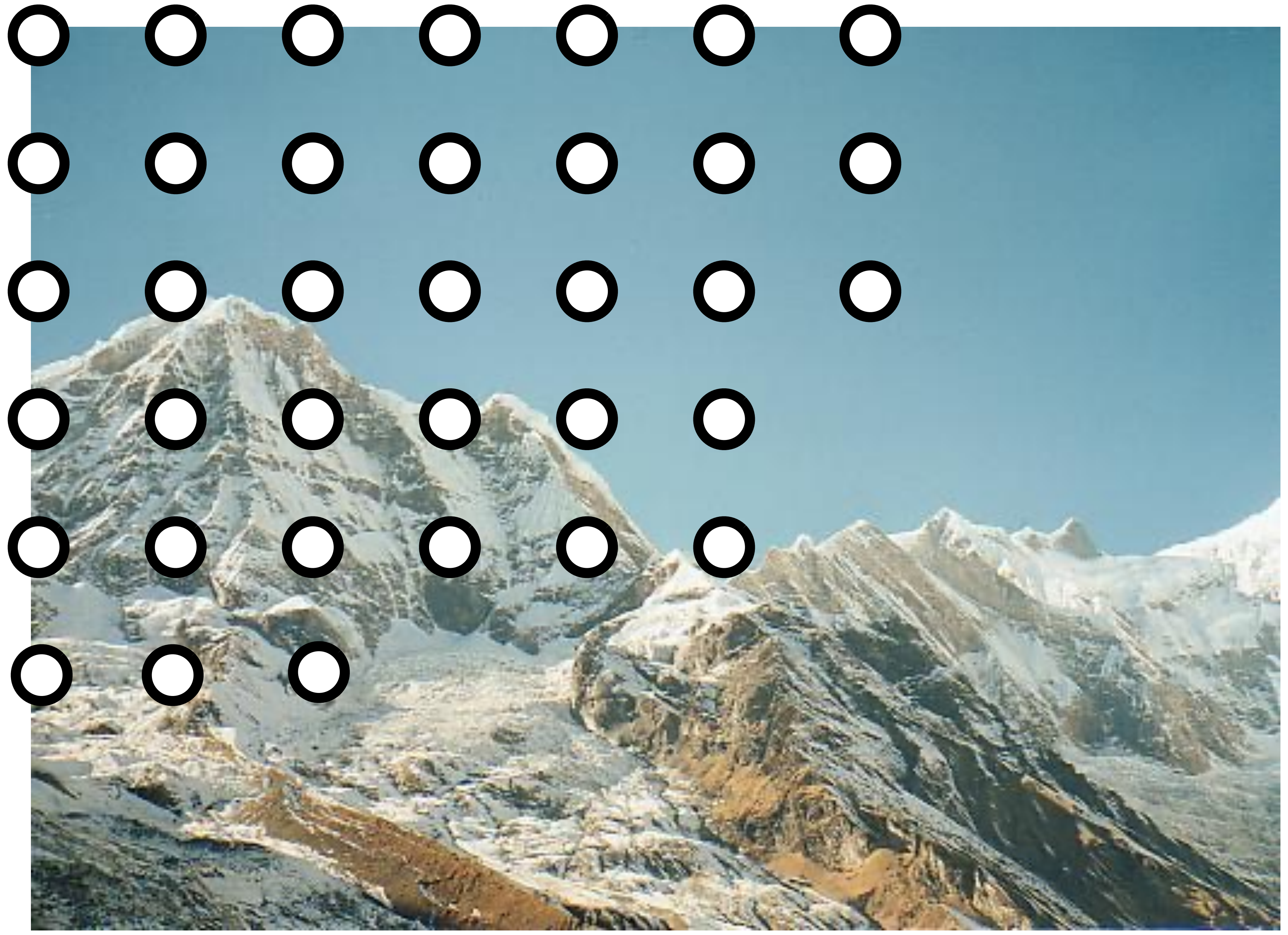


Shrinking the Image

We can't shrink an image simply by taking every second pixel

 Why?

Aliasing Example

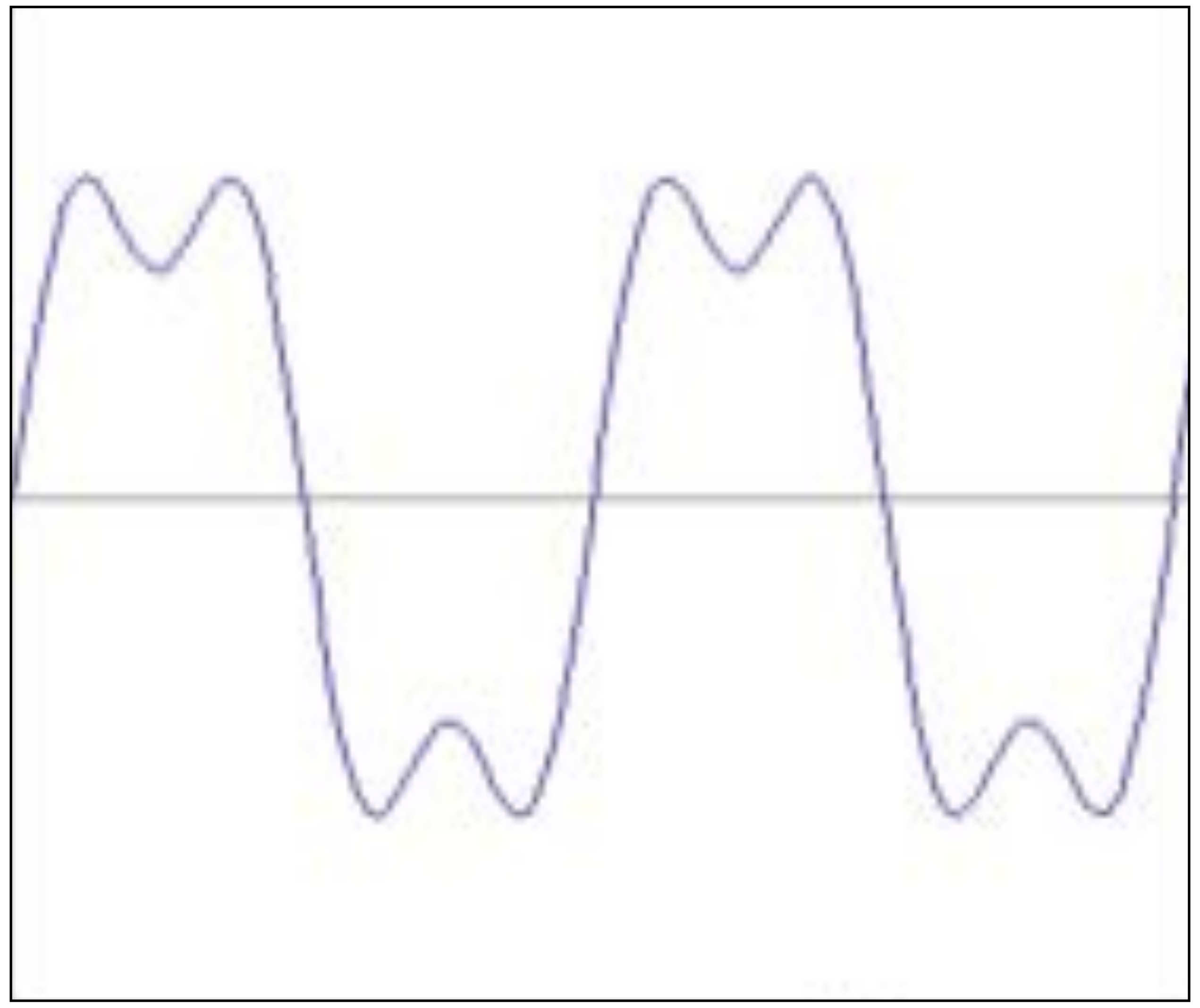


No filtering

Gaussian Blur $\sigma = 3.0$

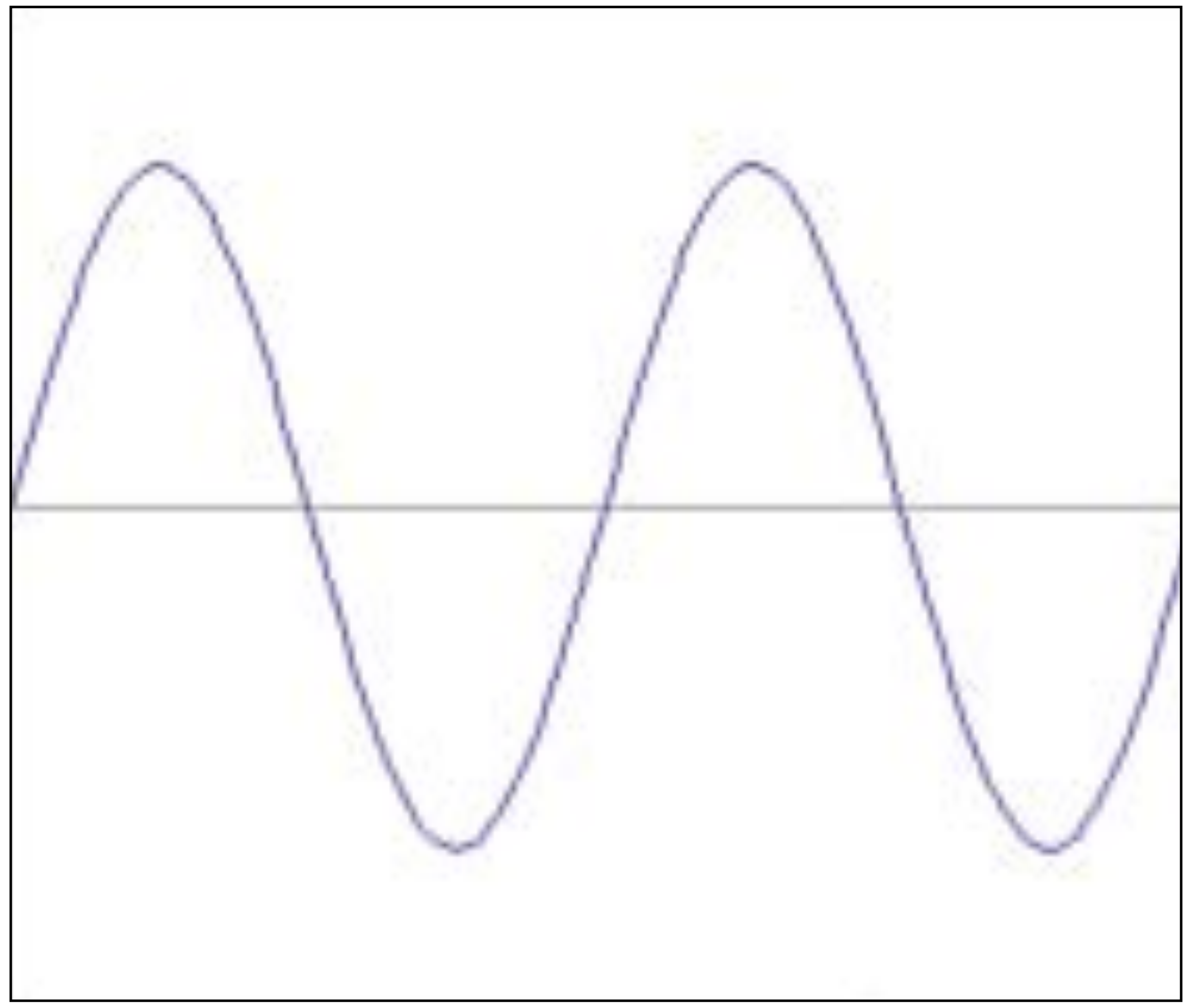
Recall: **Fourier** Representation

Any signal can be written as a sum of sinusoidal functions



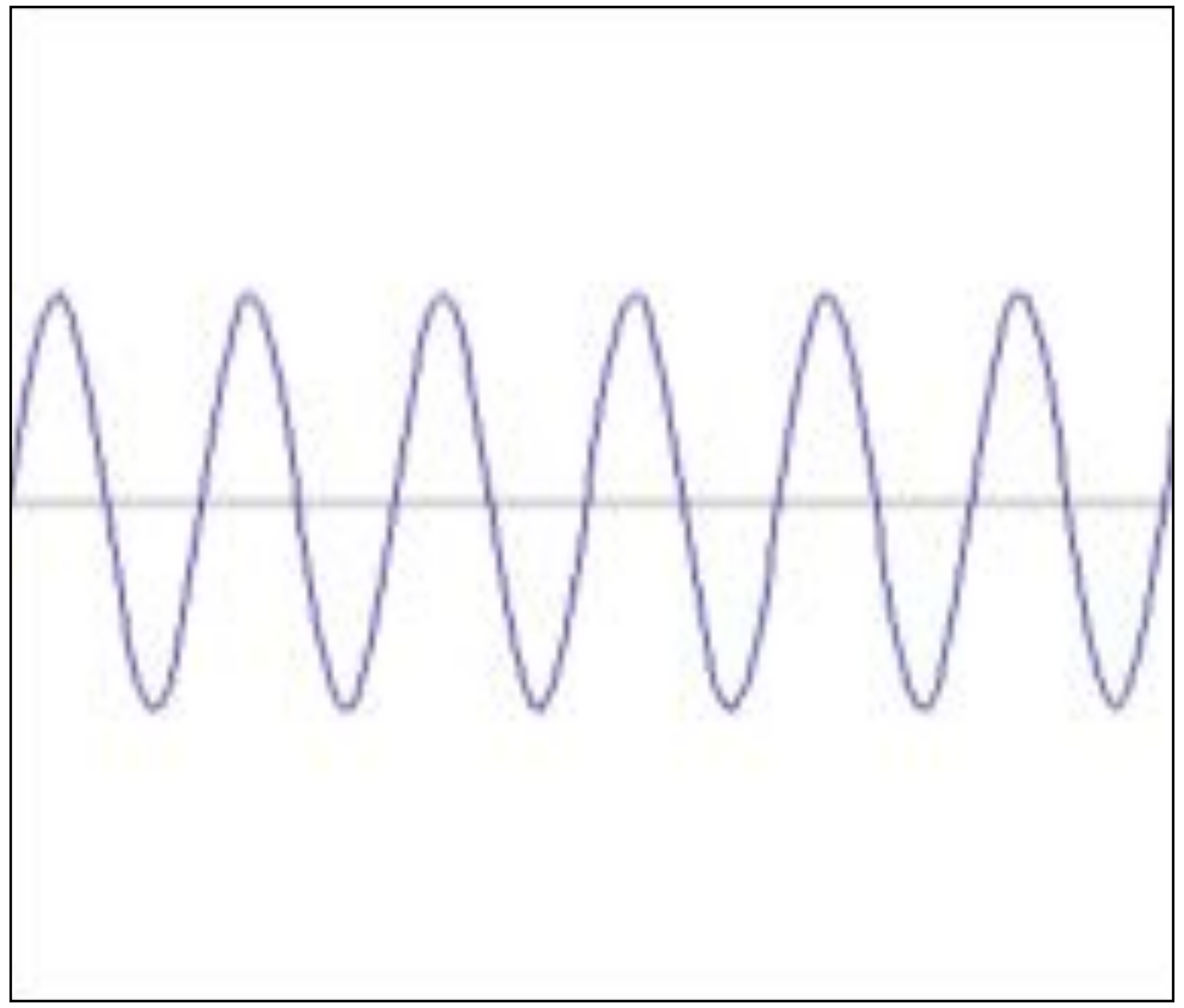
$$f(x) = \sin(2\pi x) + \frac{1}{3}\sin(2\pi 3x)$$

=



$$\sin(2\pi x)$$

+

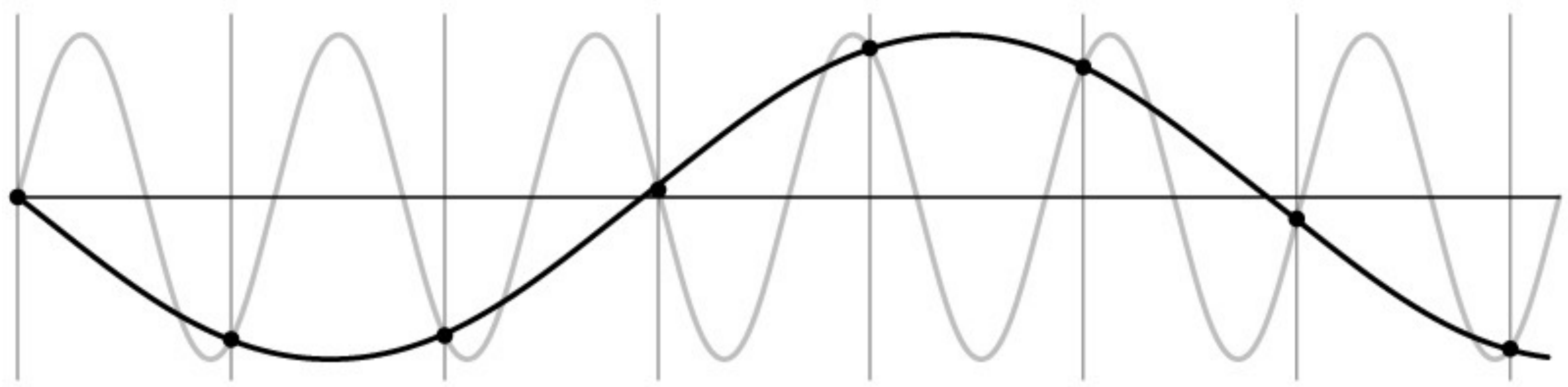


$$\frac{1}{3}\sin(2\pi 3x)$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Recall: Aliasing

Signal has been sampled too infrequently — result = **Aliasing**



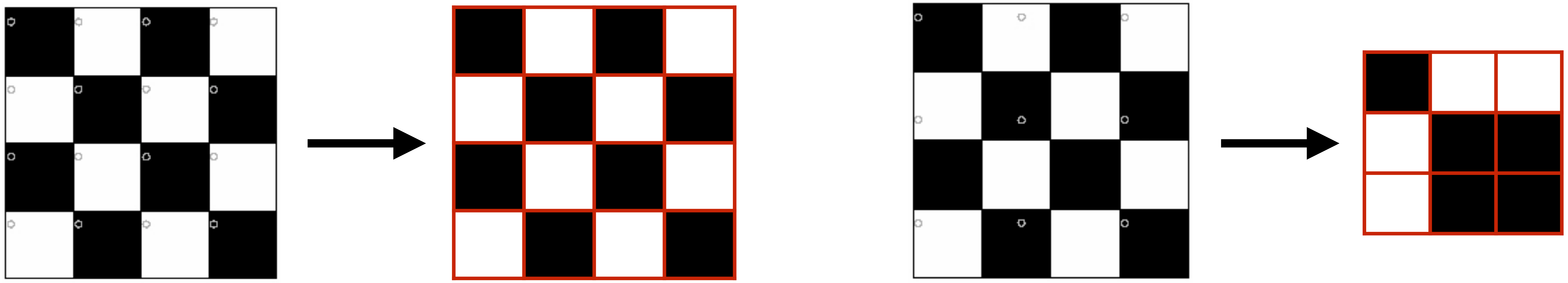
Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Nyquist Sampling

To avoid aliasing a signal must be sampled at twice the maximum frequency:

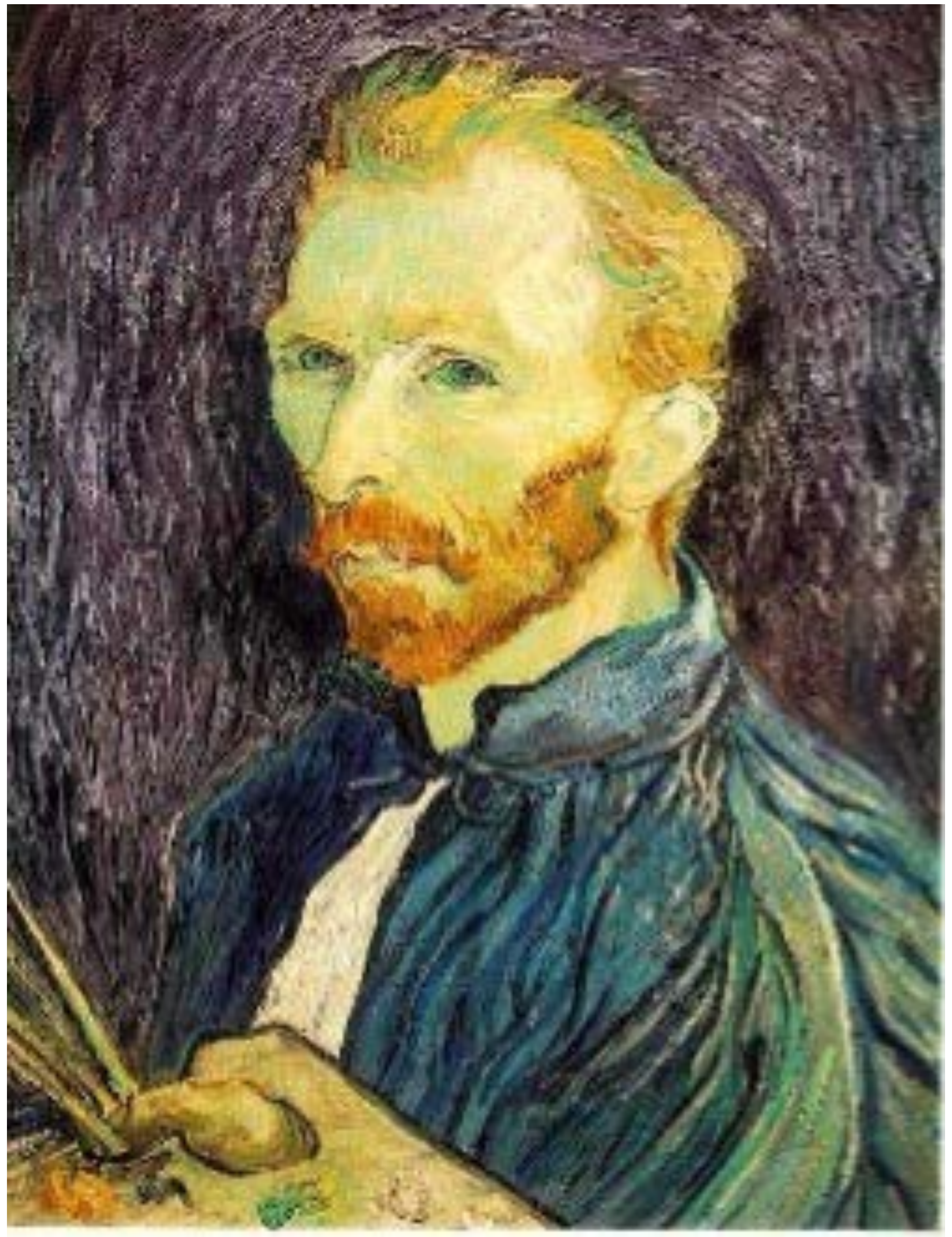
$$f_s > 2 \times f_{max}$$

For Images: We need to sample the underlying continuous signal **at least once per pixel** to avoid aliasing (assuming a correctly sampled image)



undersampling = aliasing

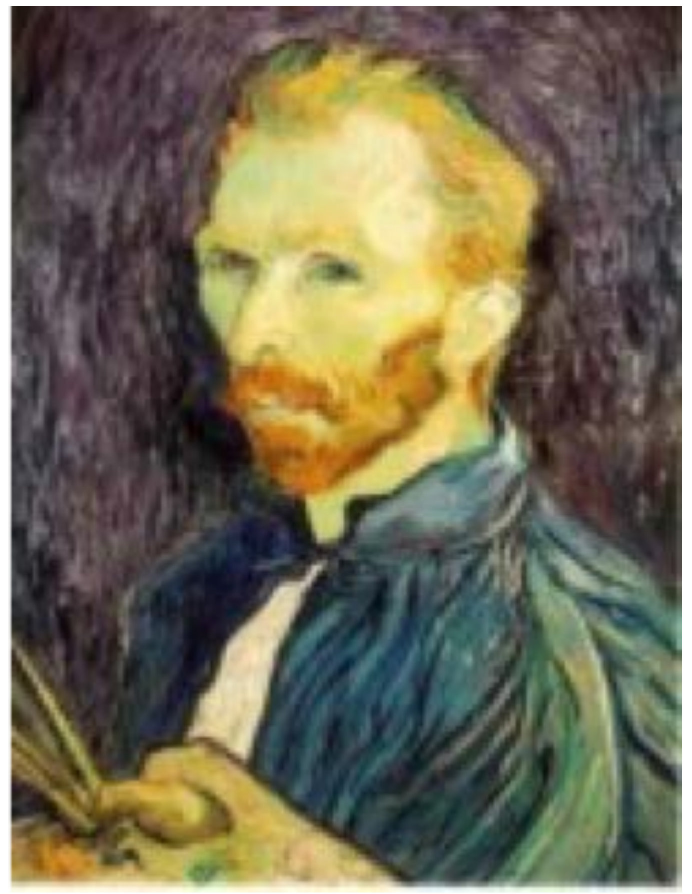
Template Matching: Sub-sample with Gaussian Pre-filtering



1/2

Apply a smoothing filter first, then throw away half the rows and columns

Gaussian filter
delete even rows
delete even columns



1/4

Gaussian filter
delete even rows
delete even columns

1/8

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

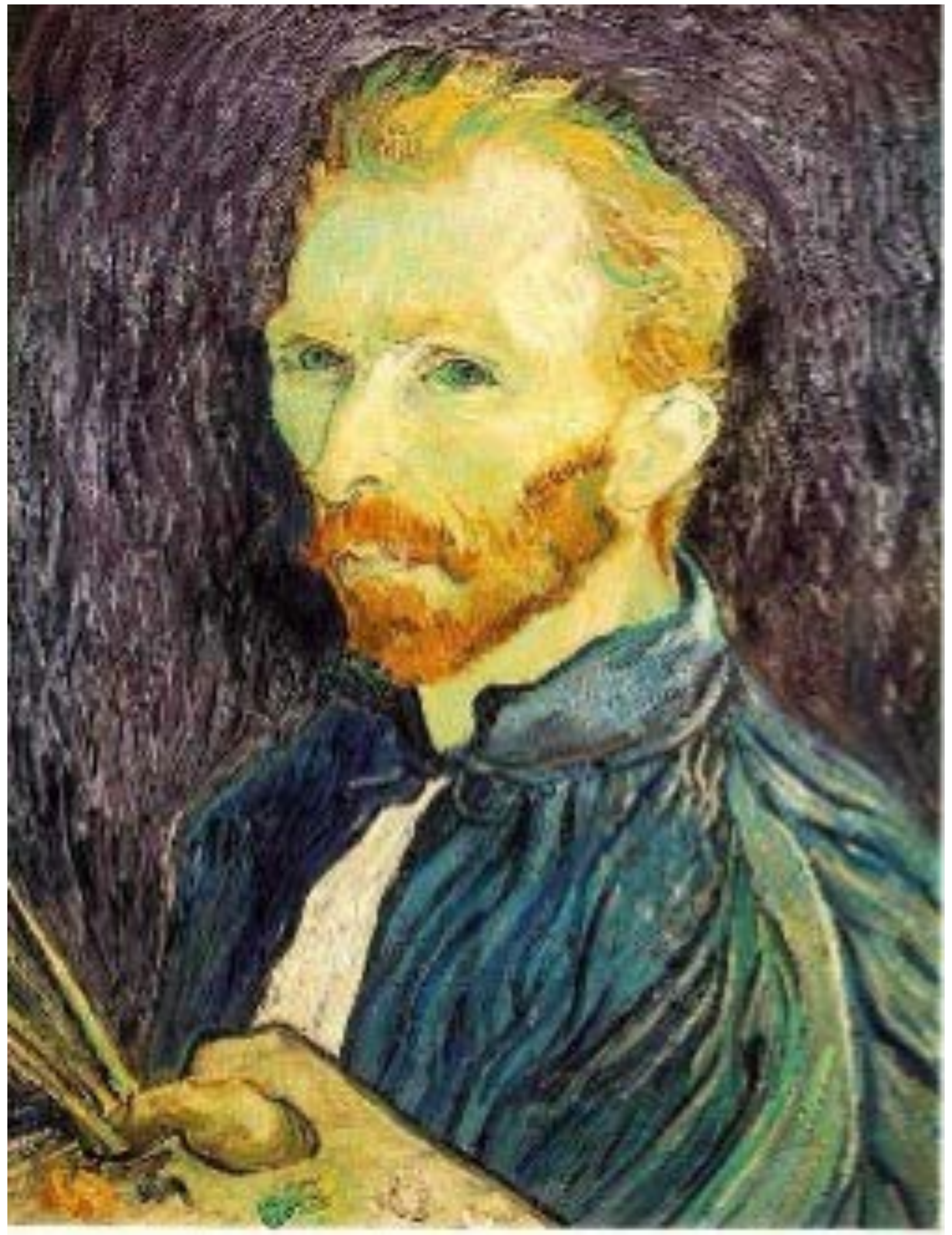
Gaussian Pre-filtering

Question: How much smoothing is needed to avoid aliasing?

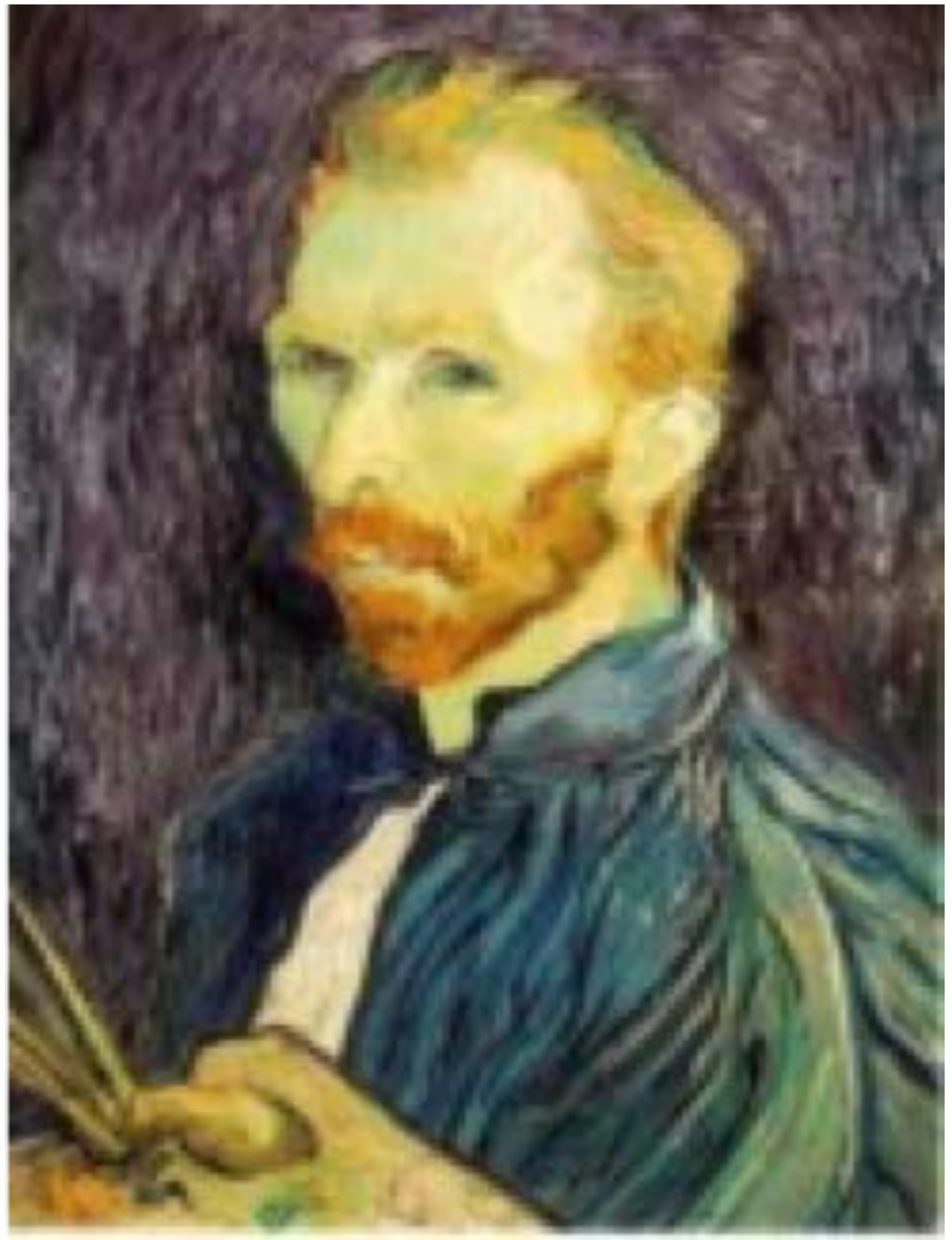
Answer: Smoothing should be sufficient to ensure that the resulting image is band limited "enough" to ensure we can sample every other pixel.

Practically: For every image reduction of 0.5, smooth by $\sigma = 1$

Template Matching: Sub-sample with Gaussian Pre-filtering



1/2

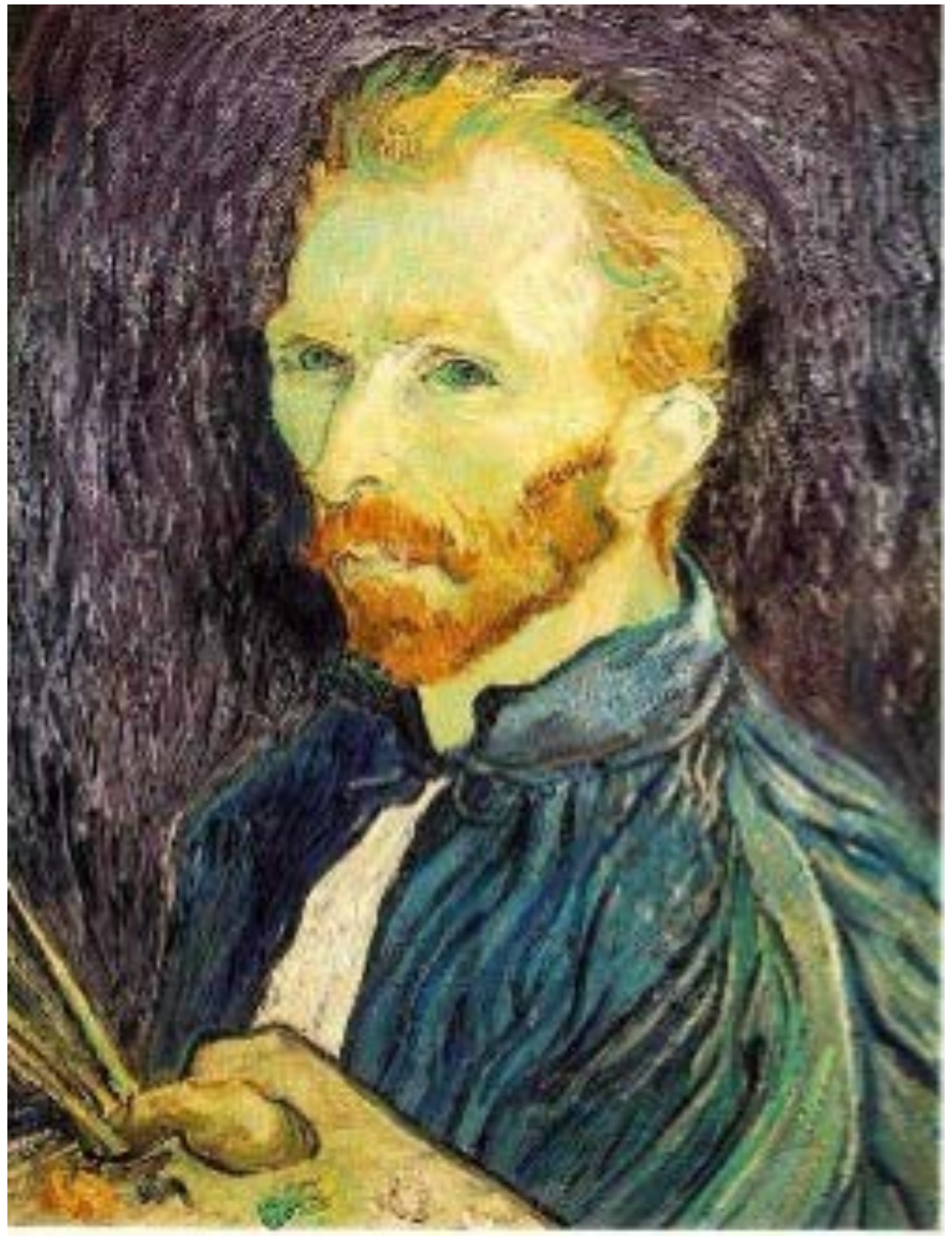


1/4 (2x zoom)

1/8 (4x zoom)

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Template Matching: Sub-sample with NO Pre-filtering



1/2

1/4 (2x zoom)

1/8 (4x zoom)

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Image **Pyramid**

An **image pyramid** is an efficient way to represent an image at multiple scales

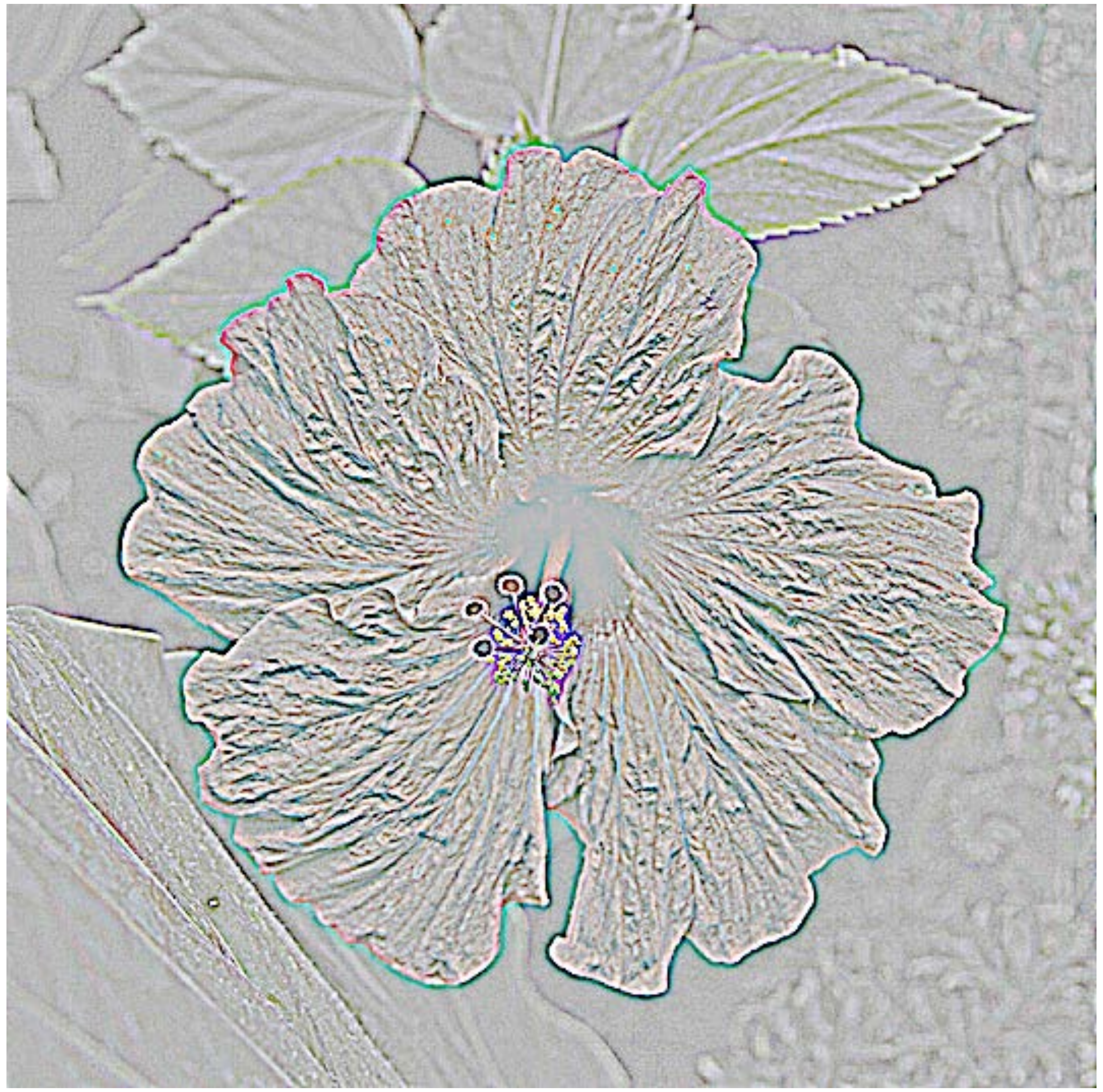
In a **Gaussian pyramid**, each layer is smoothed by a Gaussian filter and resampled to get the next layer, taking advantage of the fact that

$$G_{\sigma_1}(x) \otimes G_{\sigma_2}(x) = G_{\sqrt{\sigma_1^2+\sigma_2^2}}(x)$$

Gaussian vs **Laplacian** Pyramid

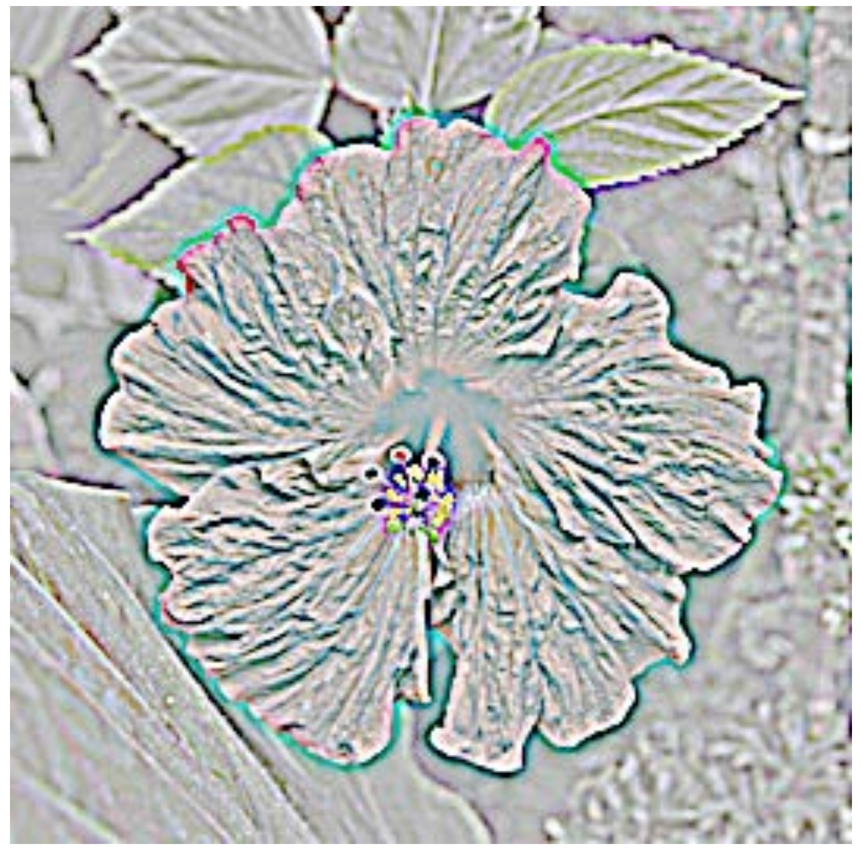



Shown in opposite order for space

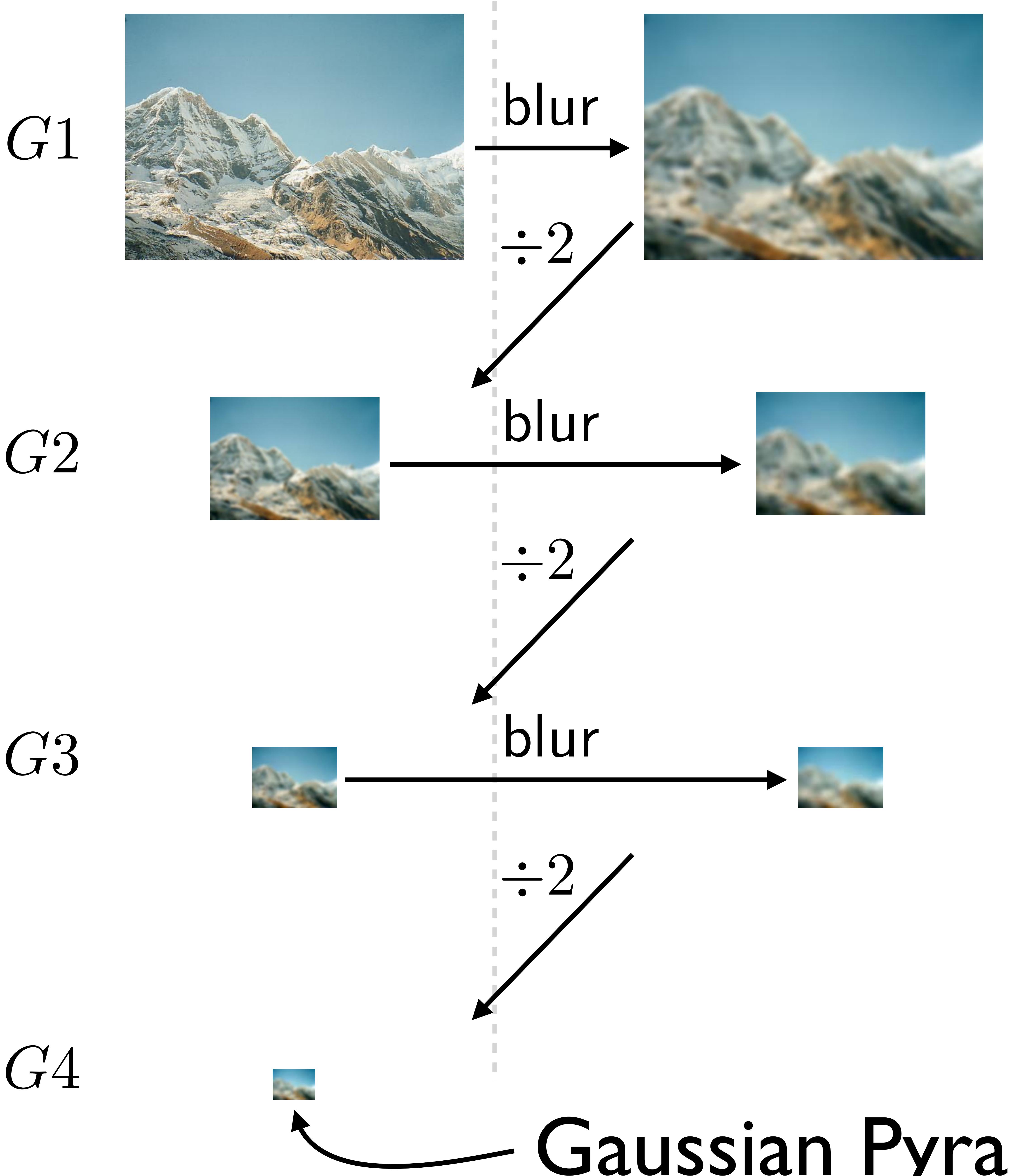


Which one takes more space to store?





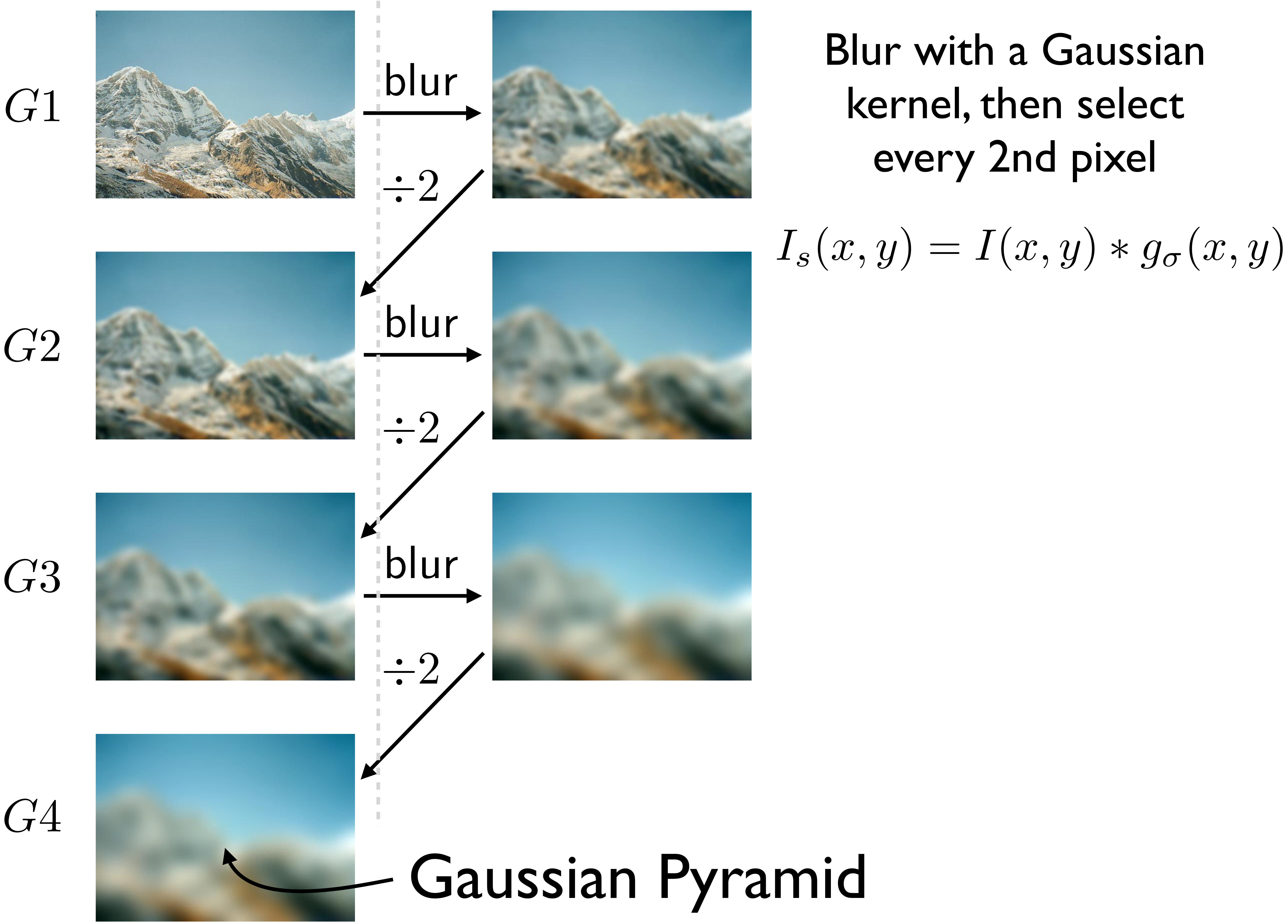
Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)



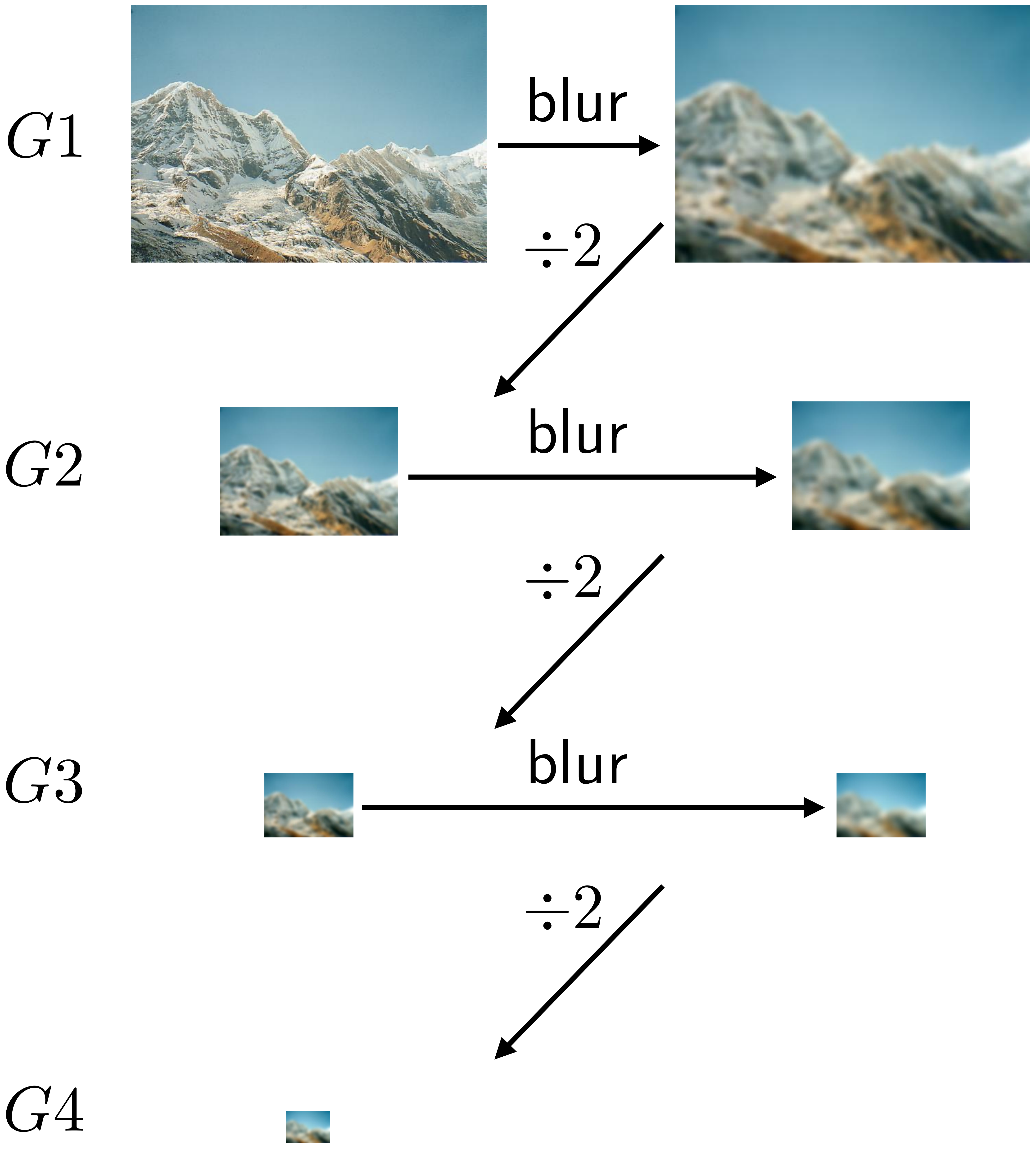
Gaussian Pyramid

Blur with a Gaussian kernel, then select every 2nd pixel

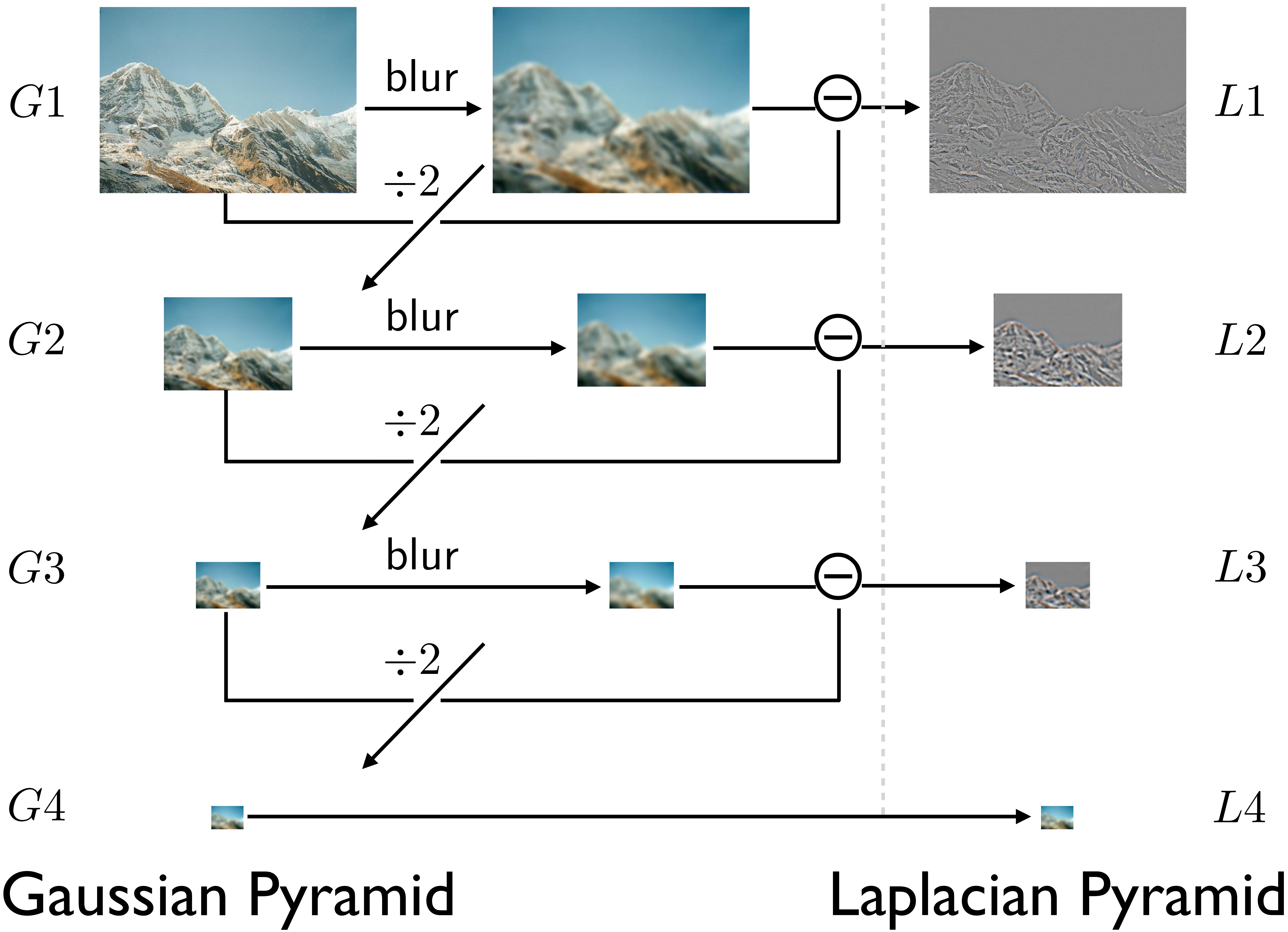
$$I_s(x, y) = I(x, y) * g_\sigma(x, y)$$

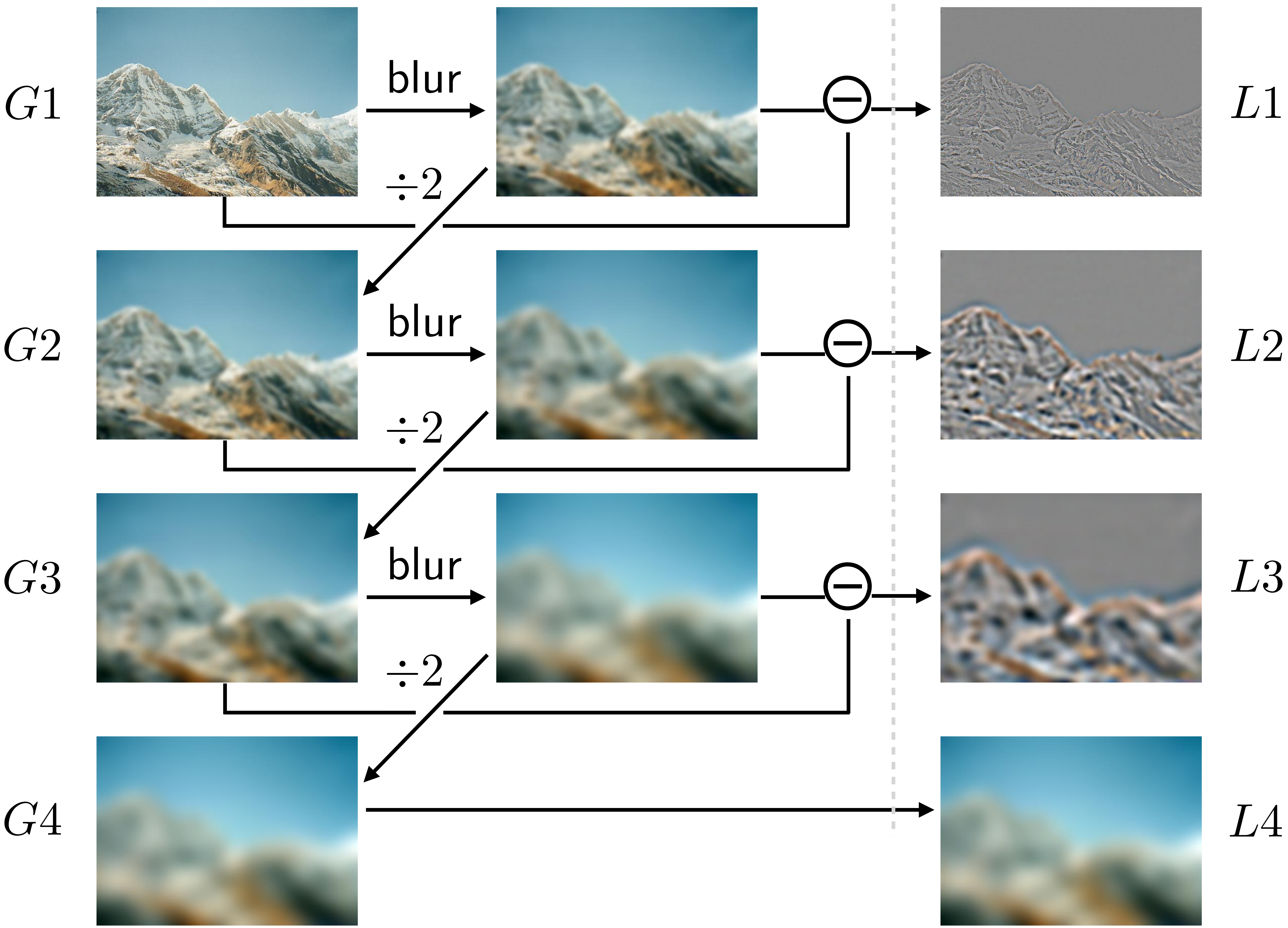


G1
G2
G3
G4
blur
blur
blur
÷2
÷2
÷2
Blur with a Gaussian kernel, then select every 2nd pixel
$I_s(x,y) = I(x,y) * g_\sigma(x,y)$
Gaussian Pyramid

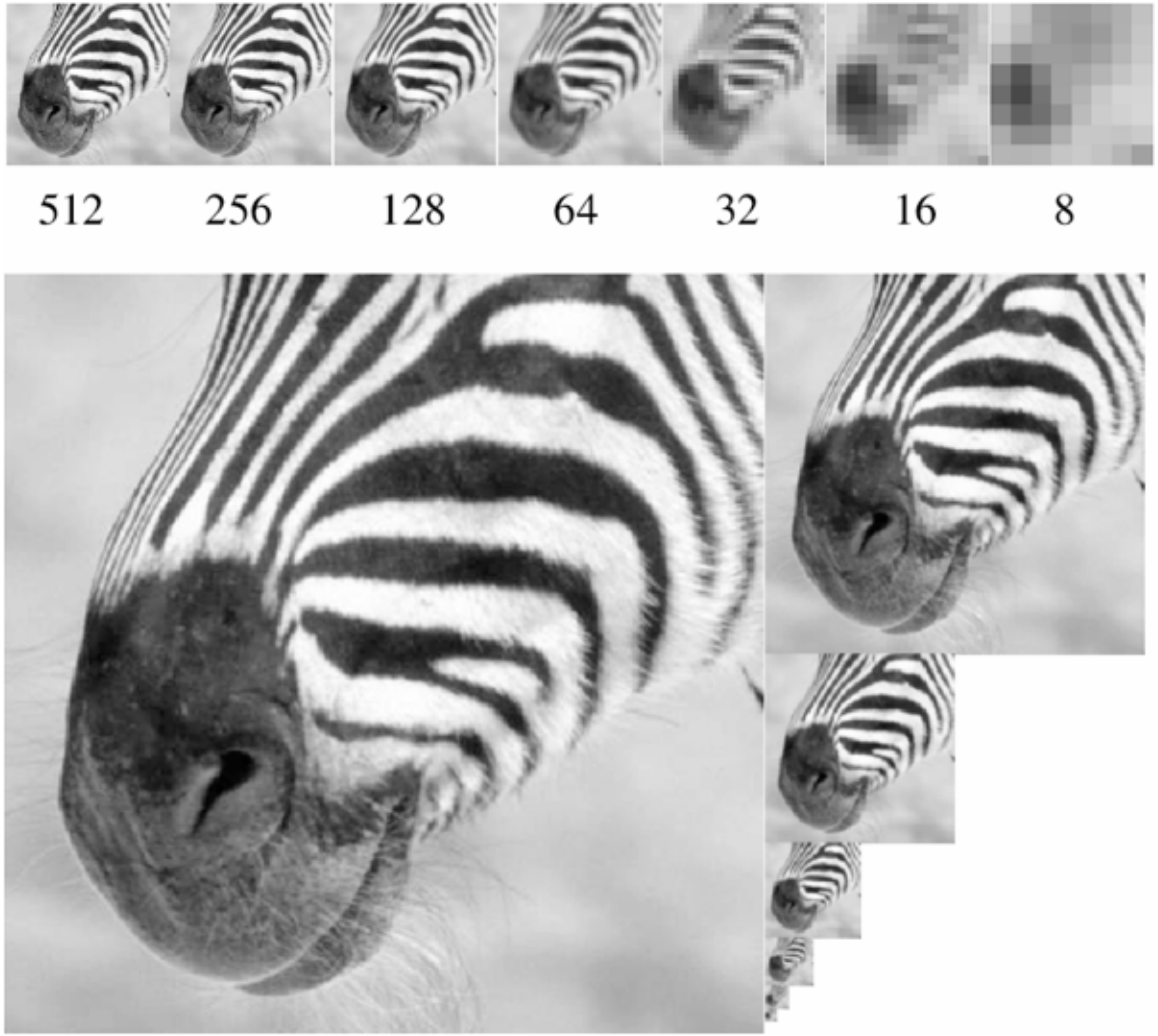


Gaussian Pyramid


G1
blur
÷2
L1
G2
blur
÷2
L2
G3
blur
÷2
L3
G4
L4
Gaussian Pyramid
Laplacian Pyramid


G1
blur
÷2
L1
G2
blur
÷2
L2
G3
blur
÷2
L3
G4
L4

Gaussian Pyramid



Forsyth & Ponce (2nd ed.) Figure 4.17

What happens to the details?

– They get smoothed out as we move to higher levels

What is preserved at the higher levels?

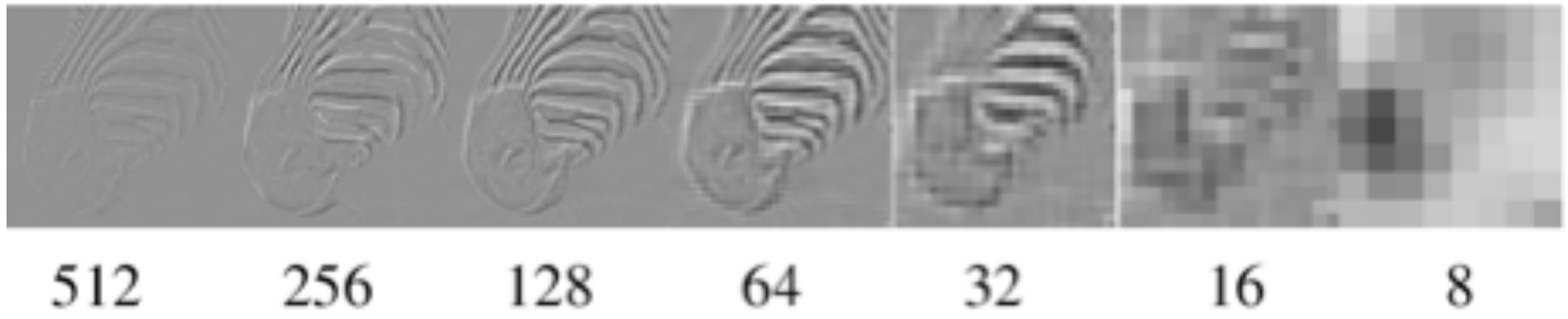
– Mostly large uniform regions in the original image

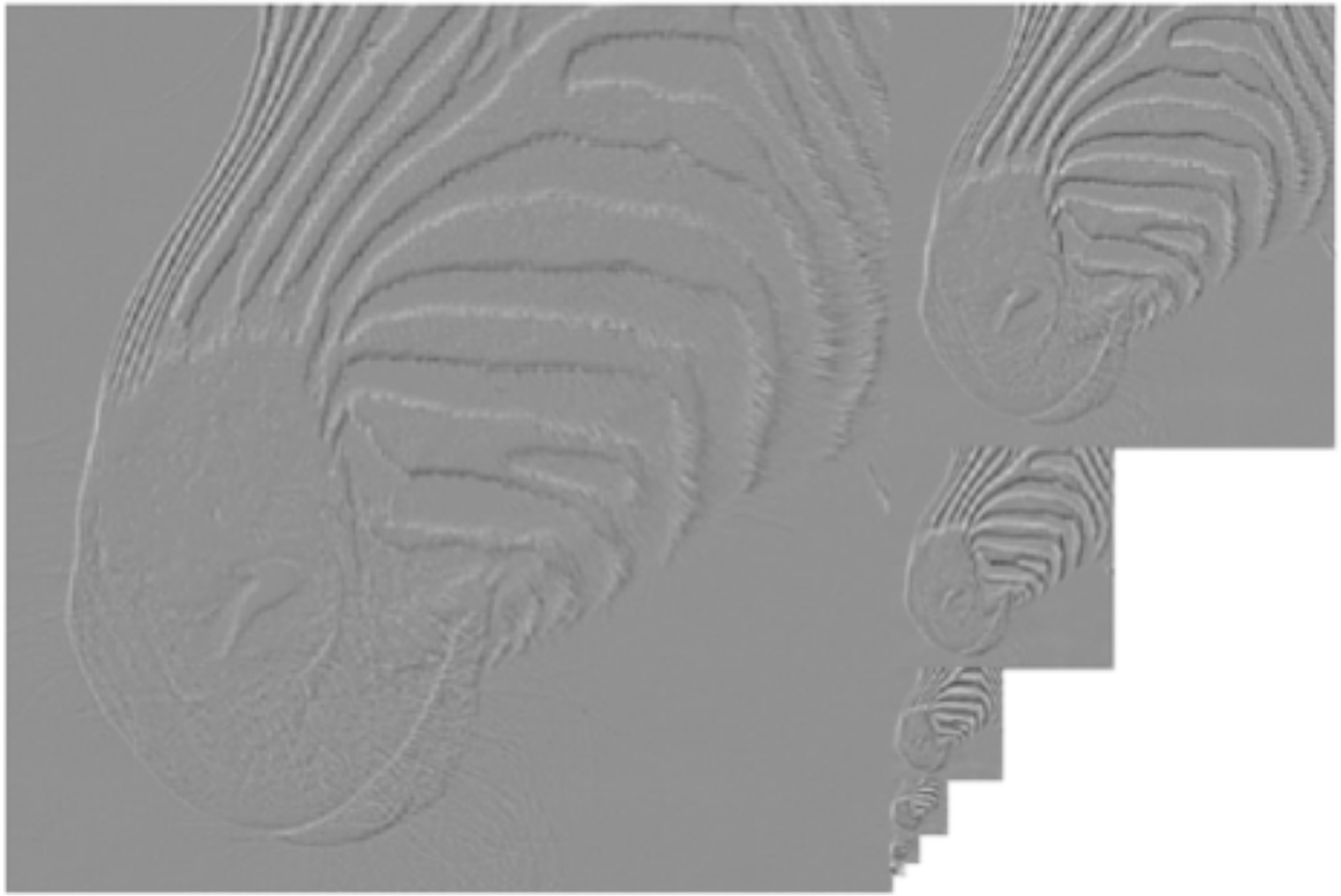
How would you reconstruct the original image from the image at the upper level?

– That's not possible

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Laplacian Pyramid





At each level, retain the residuals instead of the blurred images themselves.

Can we reconstruct the original image using the pyramid?
— Yes we can!

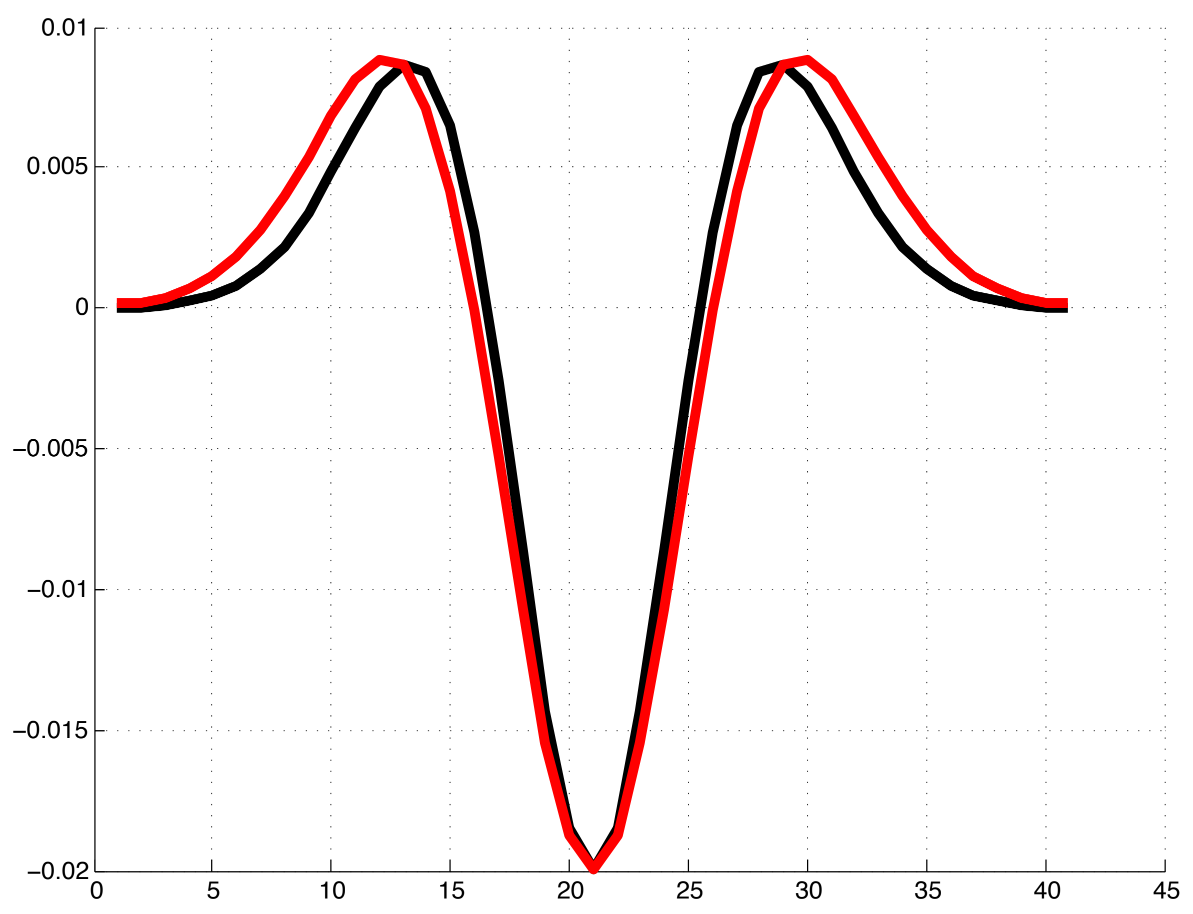
What do we need to store to be able to reconstruct the original image?

Why is it called Laplacian Pyramid?

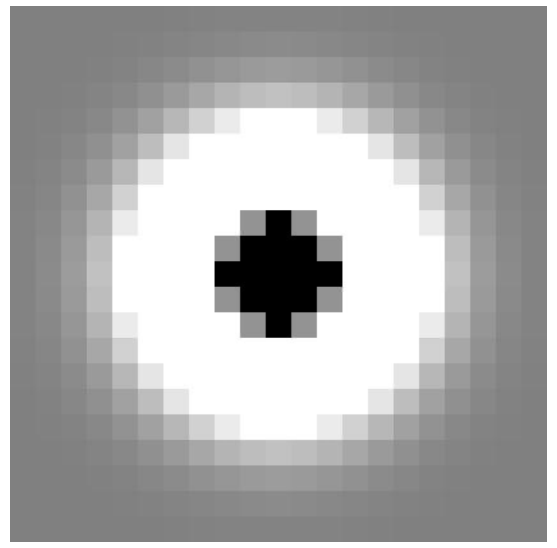
Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Why **Laplacian** Pyramid?

$$\text{red} = [1\ \ -2\ 1] * g(x;\ 5.0)$$
$$\text{black} = g(x;\ 5.0) - g(x;\ 4.0)$$



- Laplacian/DoG = centre-surround filter

 $*$  $=$ 

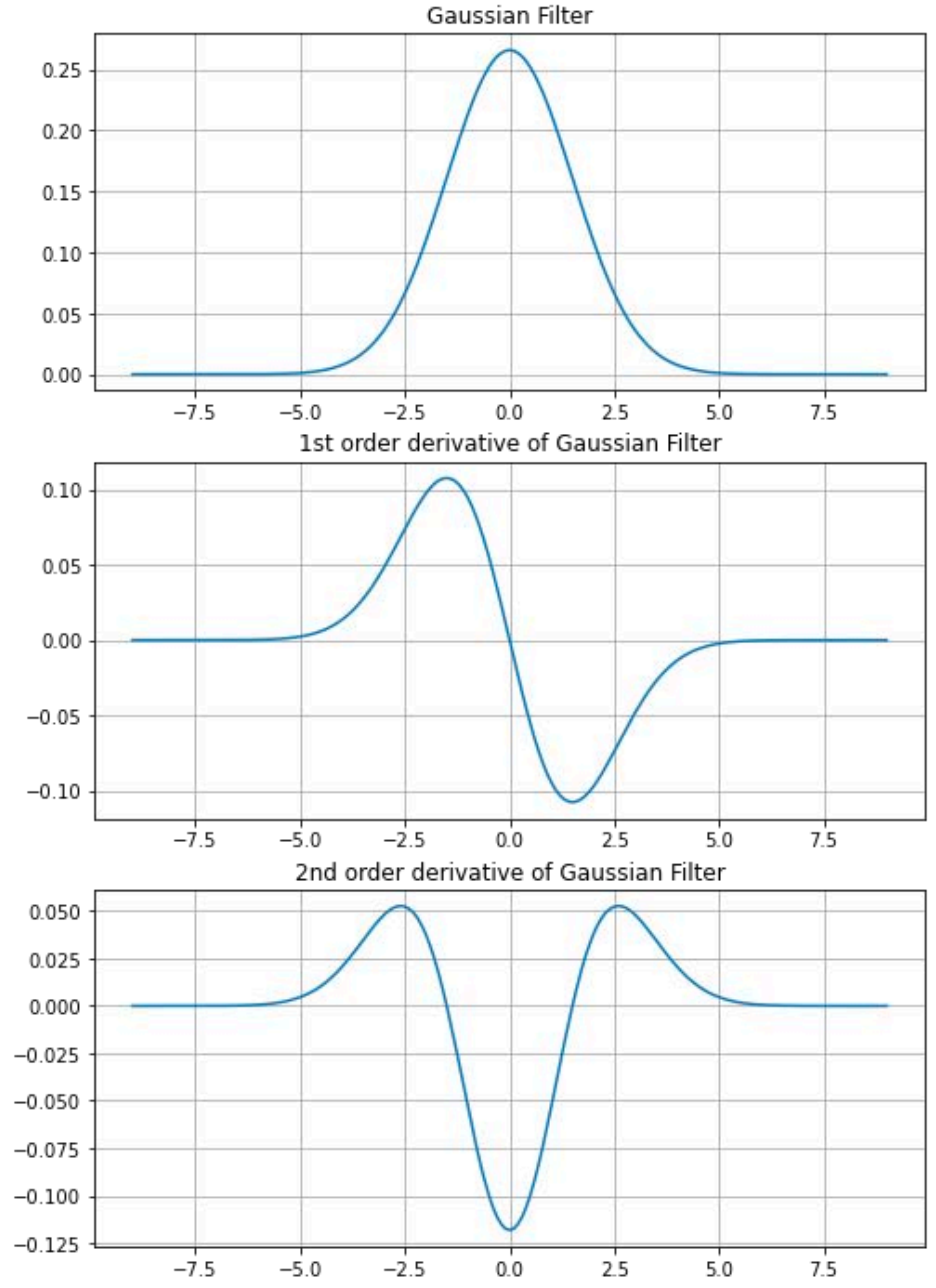
Why **Laplacian** Pyramid?

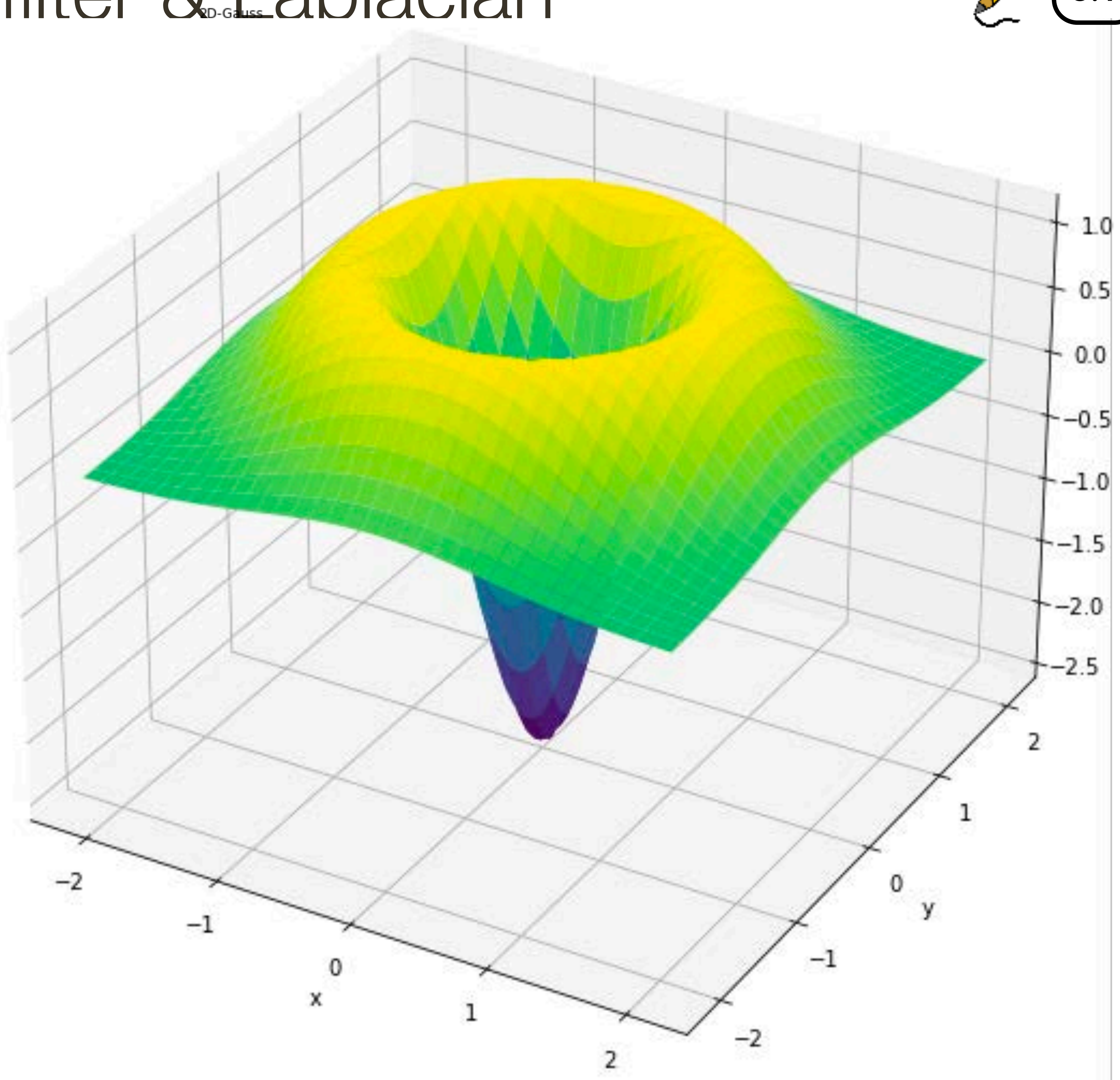
8.1

https://www.desmos.com/calculator/a9snuia3la

Derivatives of a Gaussian filter & Laplacian

8.1





Images from https://hannibunny.github.io/orbook/preprocessing/04gaussianDerivatives.html

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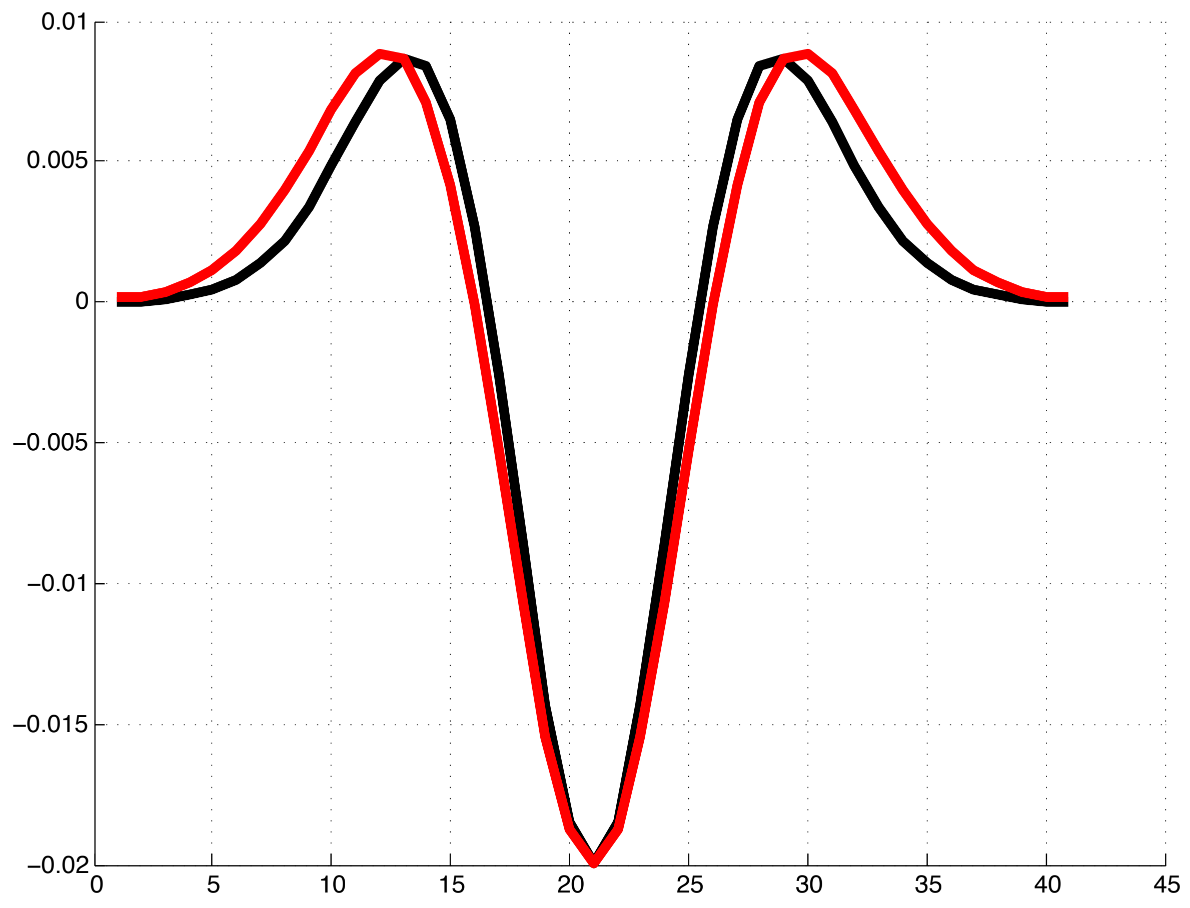
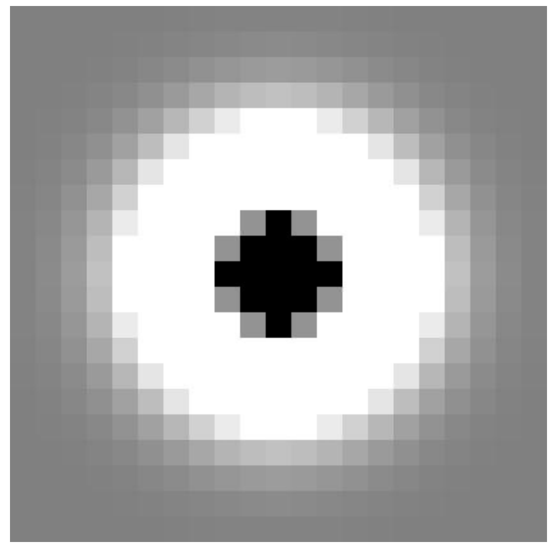
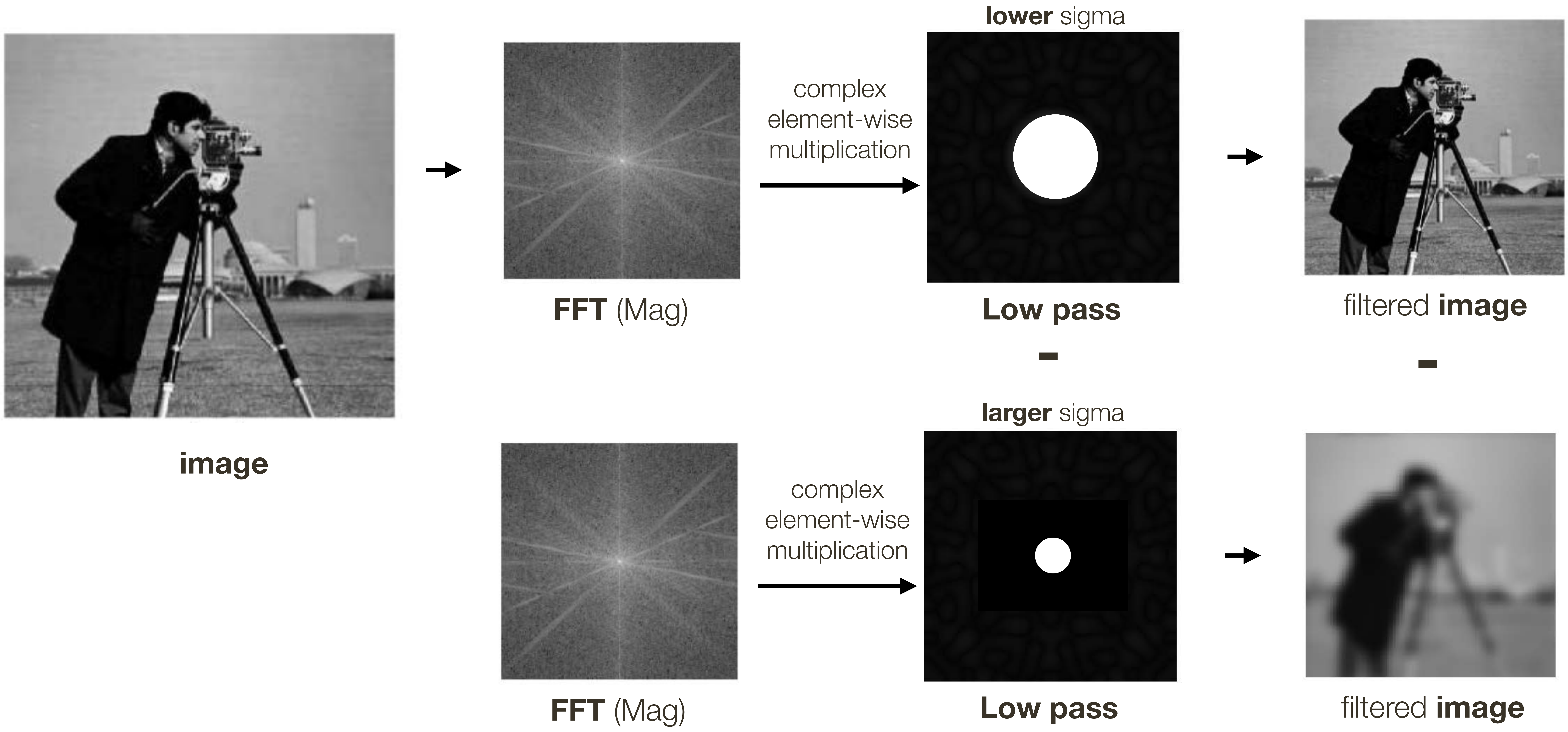
$$\text{red} = [1\ \ -2\ 1] * g(x;\ 5.0)$$
$$\text{black} = g(x;\ 5.0) - g(x;\ 4.0)$$



- Laplacian/DoG = centre-surround filter

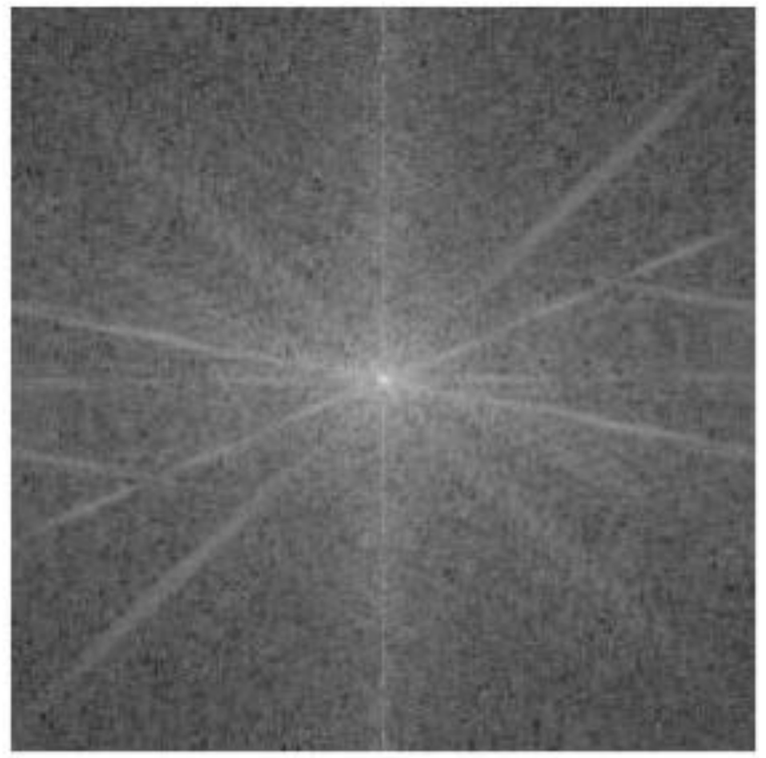

*

=


Laplacian is a Bandpass Filter



Laplacian is a Bandpass Filter

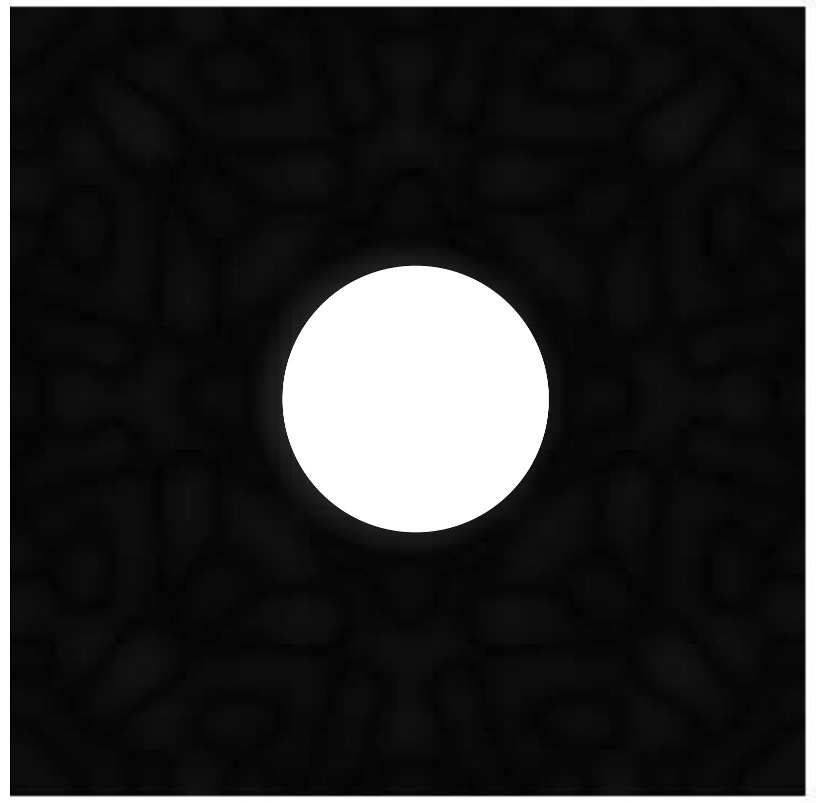
image



FFT (Mag)

complex
element-wise
multiplication

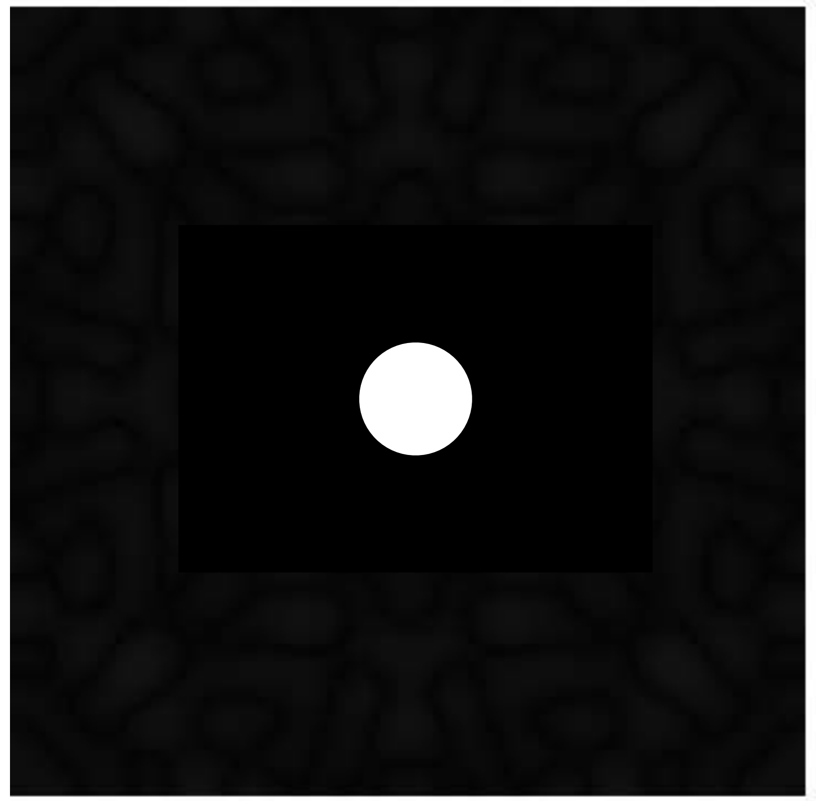
lower sigma



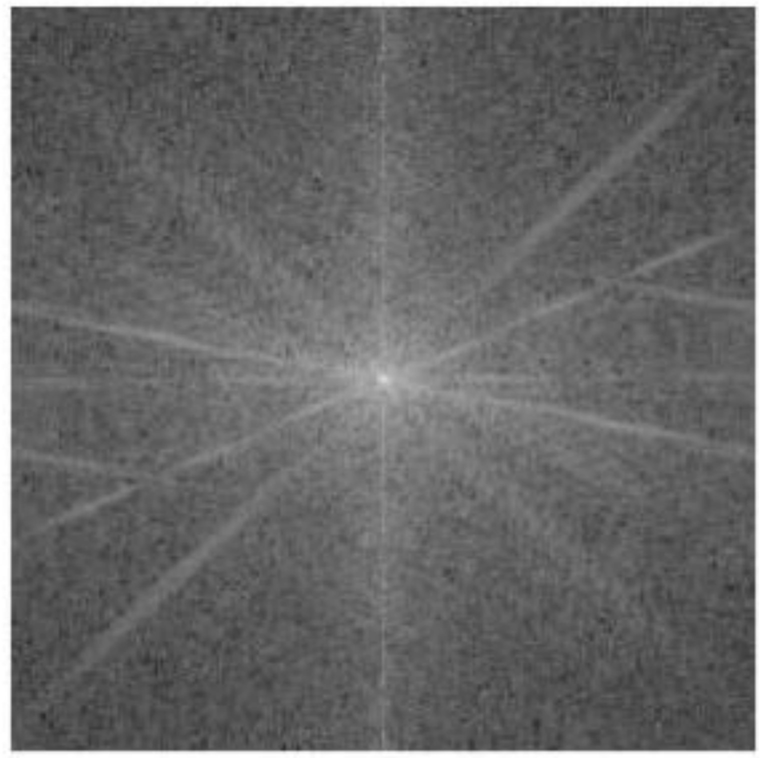
Low pass

-

larger sigma

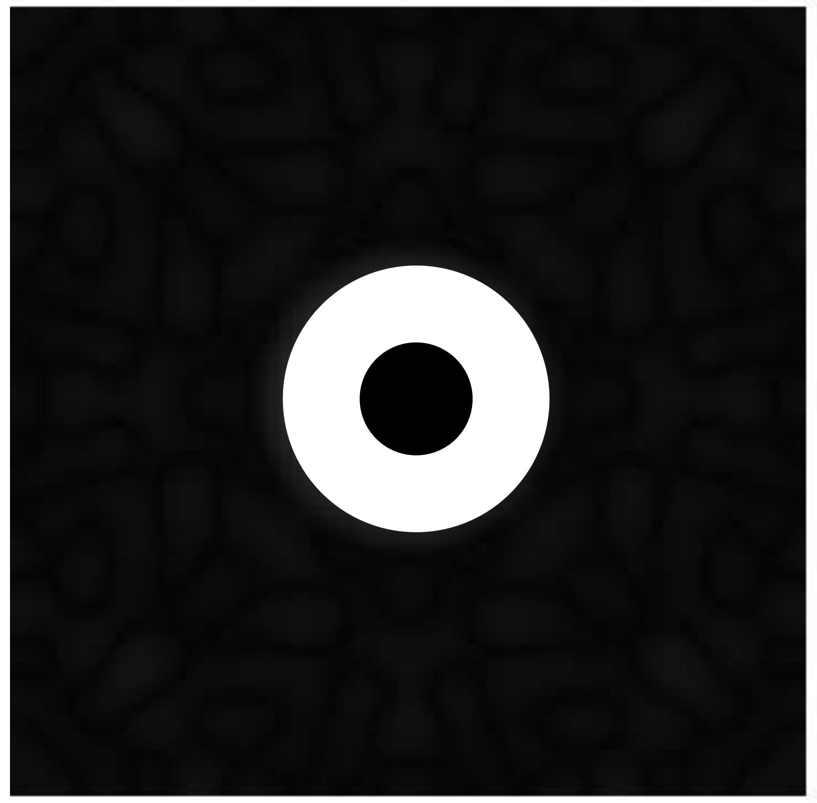


Low pass



FFT (Mag)

complex
element-wise
multiplication



Laplacian Pyramid

Building a **Laplacian** pyramid:

— Create a Gaussian pyramid

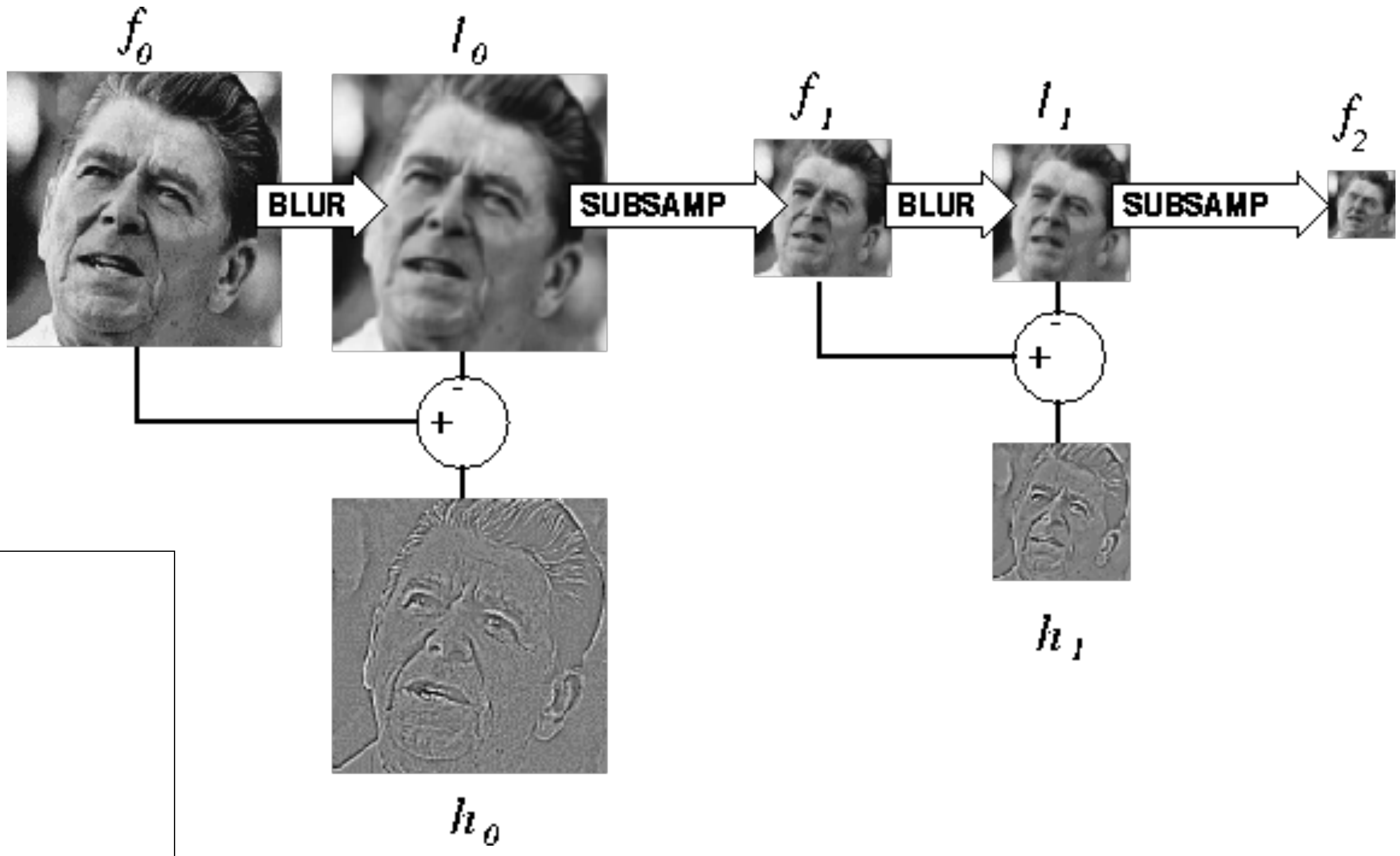
— Take the difference between one Gaussian pyramid level and the next

Properties

— Computes a Laplacian / Difference-of-Gaussian (DoG) function of the image at multiple scales

— It is a band pass filter – each level represents a different band of spatial frequencies

Constructing a **Laplacian** Pyramid — Implementation

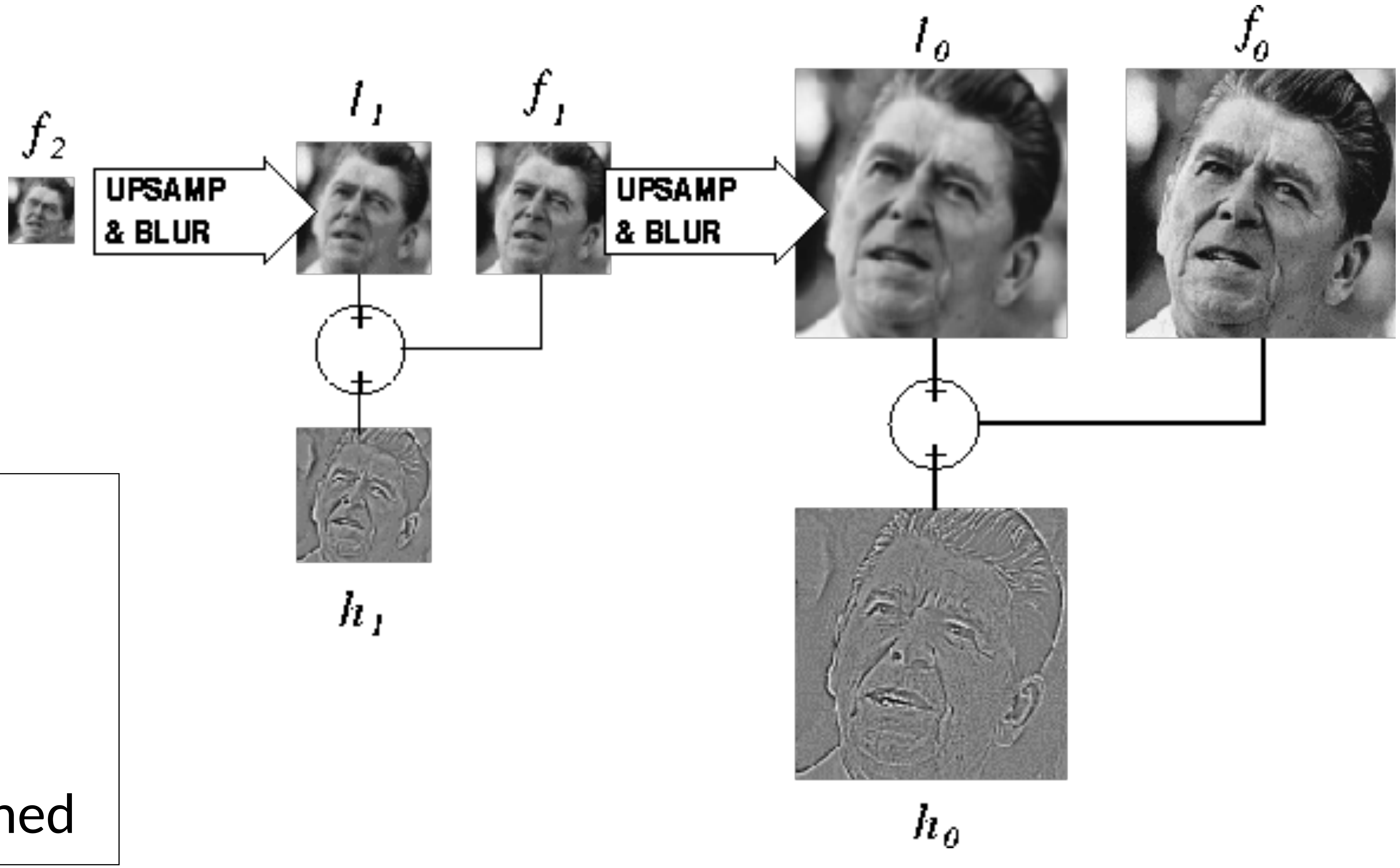


Algorithm

repeat:
  filter
  compute residual
  subsample
until min resolution reached

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Reconstructing the Original Image — Implementation



Algorithm

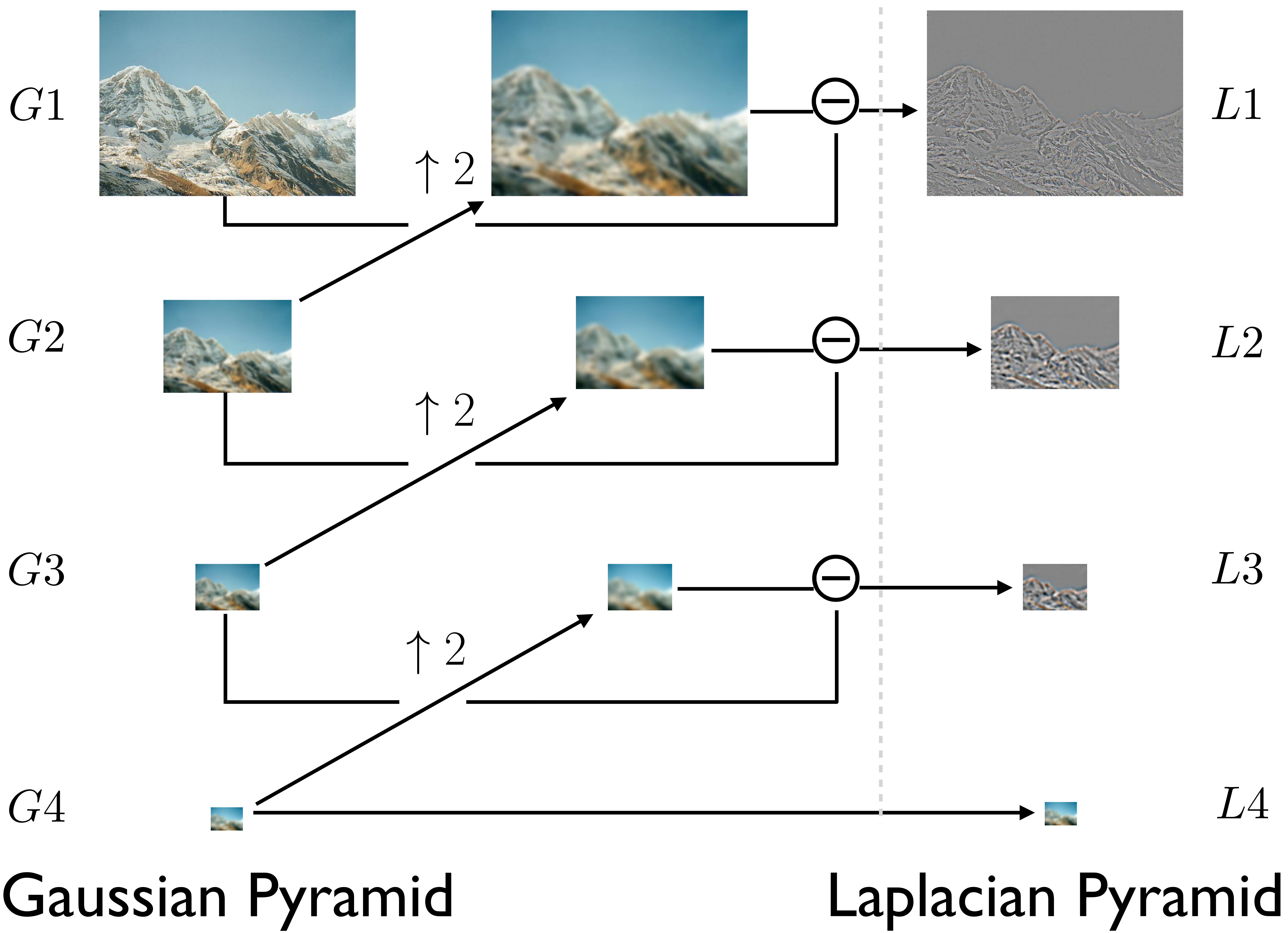
repeat:

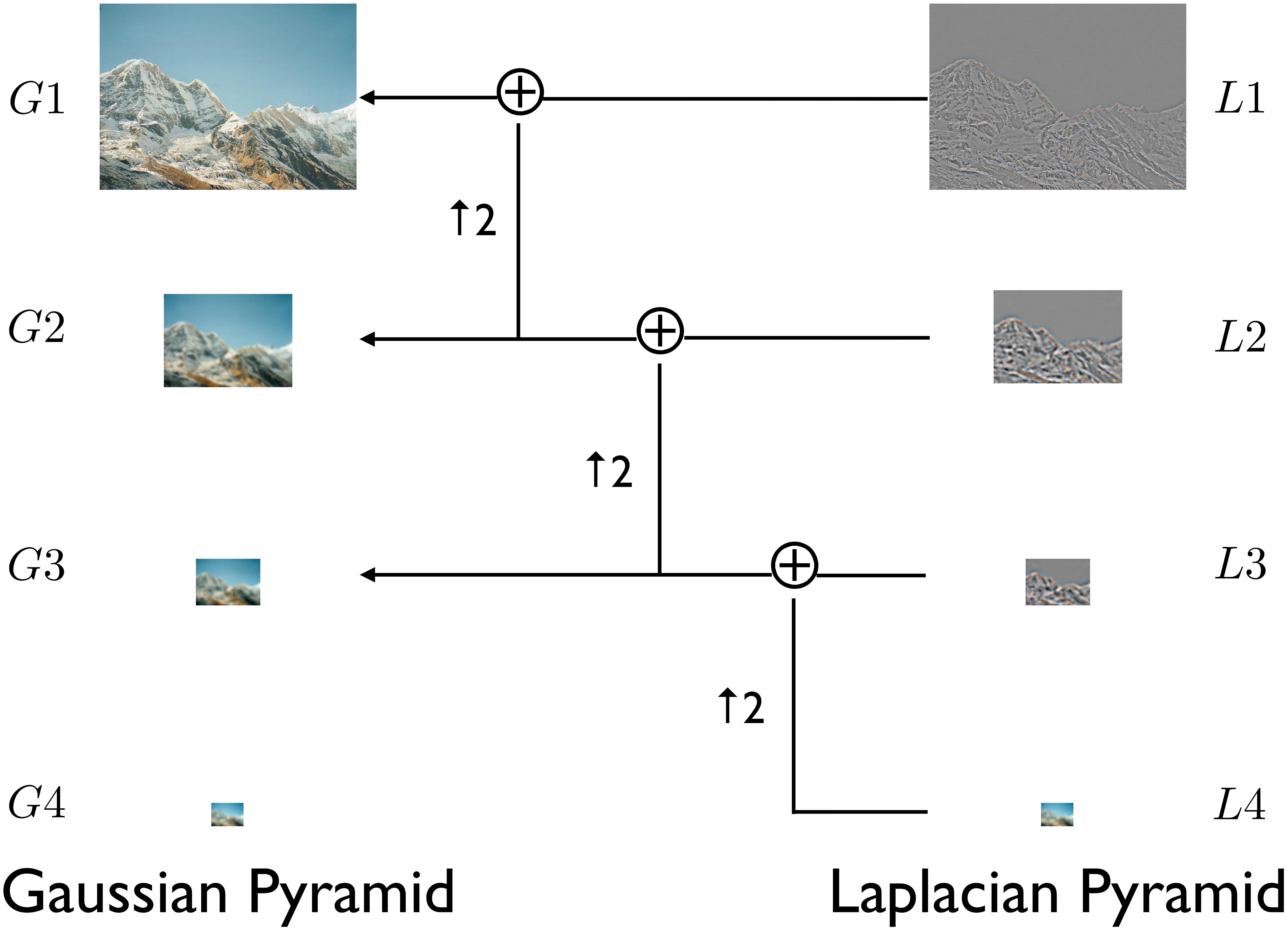
upsample

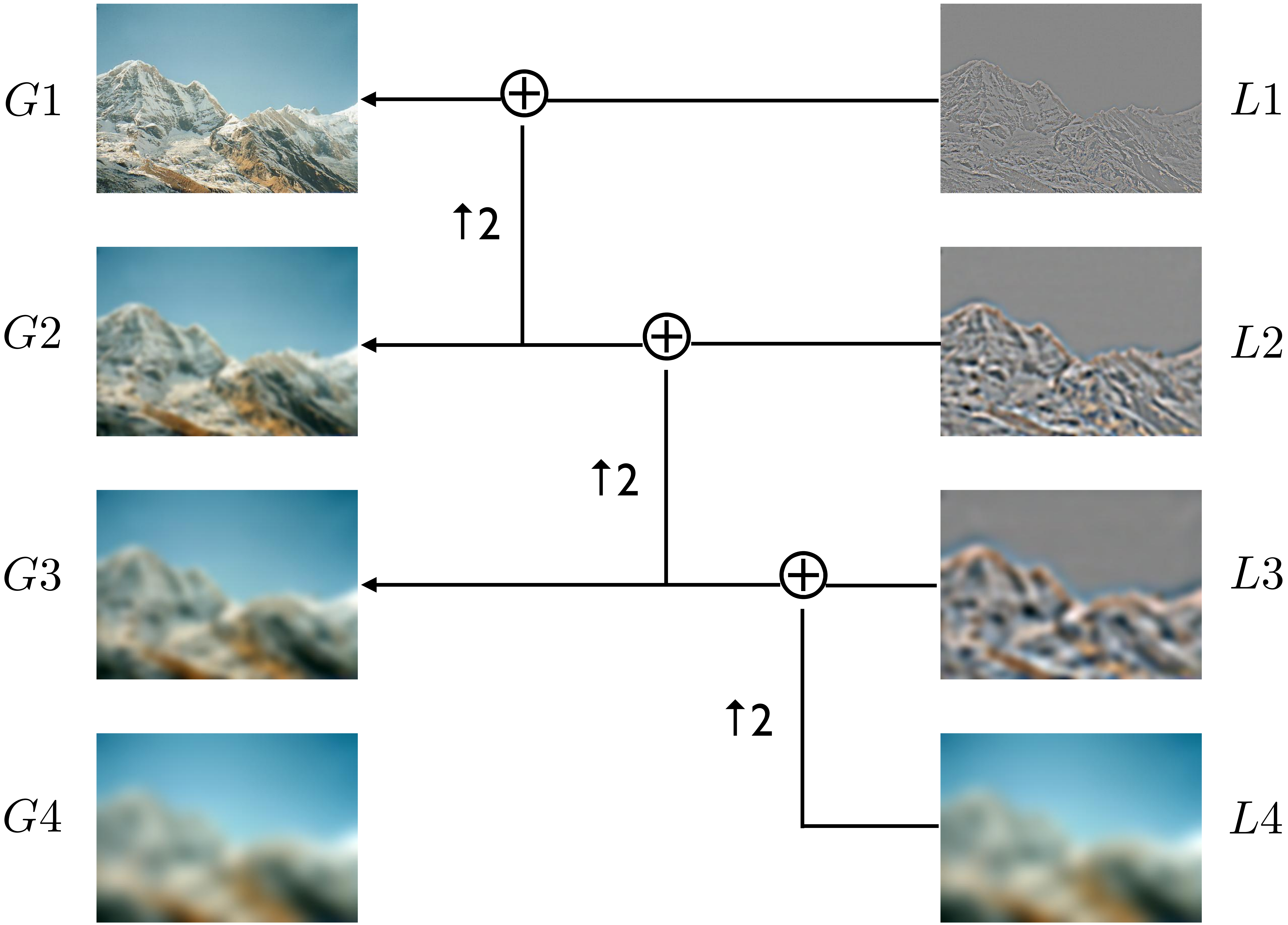
sum with residual

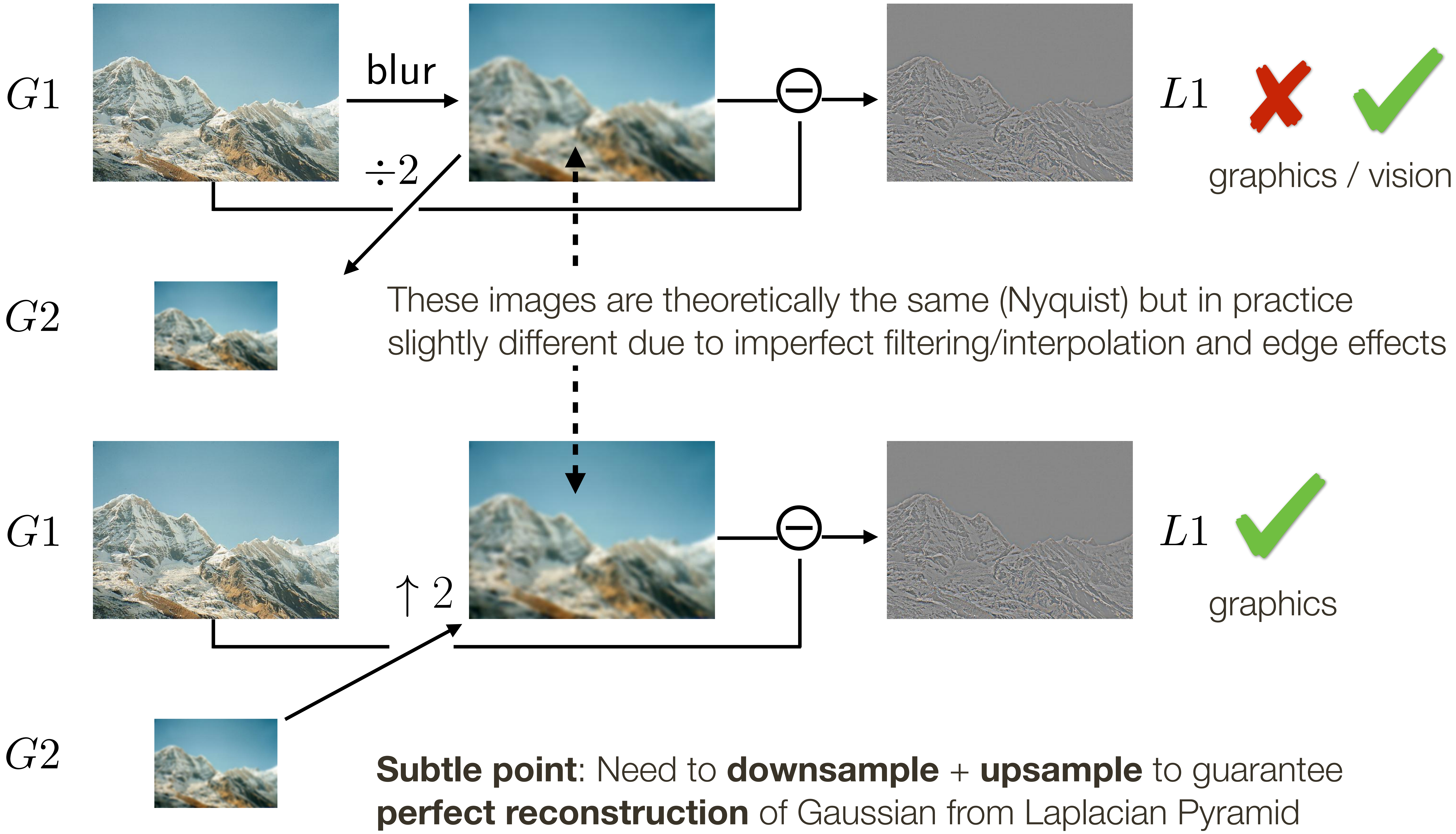
until orig resolution reached

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)


G1
G2
G3
G4
↑ 2
↑ 2
↑ 2
L1
L2
L3
L4
Gaussian Pyramid
Laplacian Pyramid

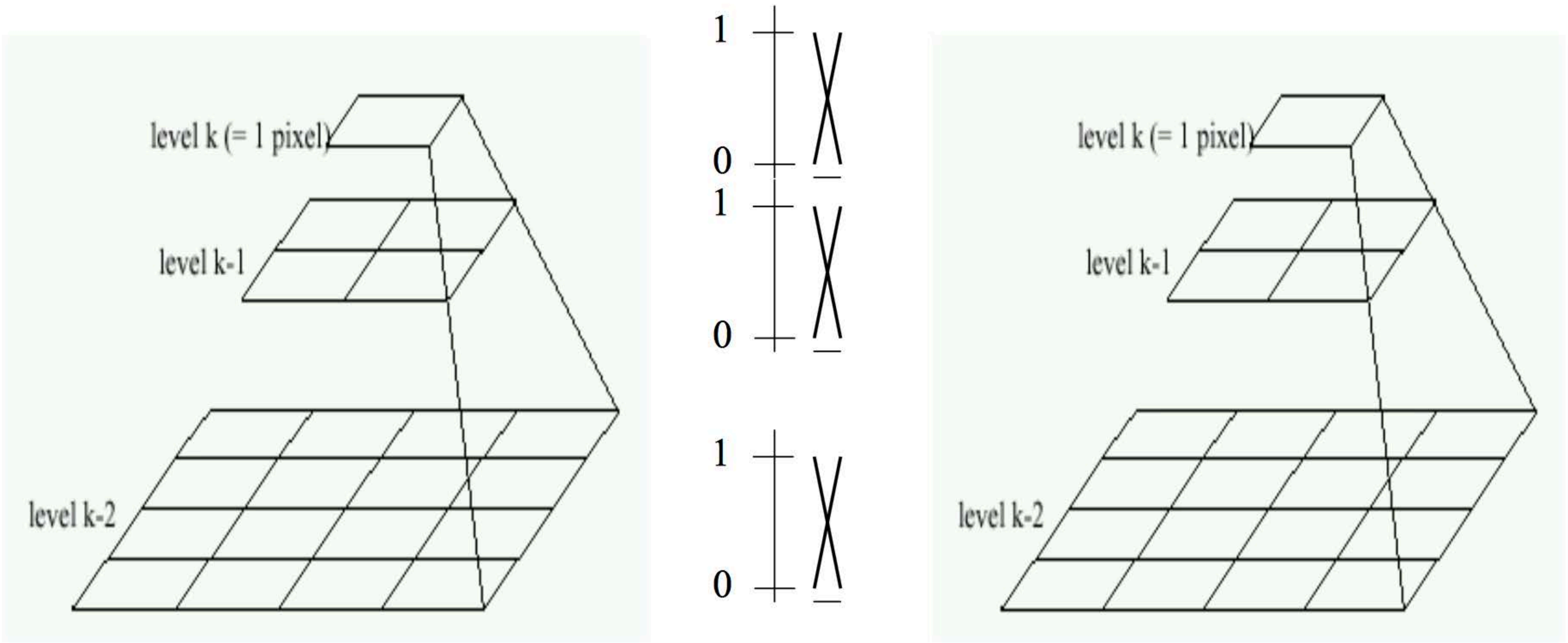

$G1$
$G2$
$G3$
$G4$
↑2
↑2
↑2
$L1$
$L2$
$L3$
$L4$
Gaussian Pyramid
Laplacian Pyramid


$G1$
$G2$
$G3$
$G4$
$L1$
$L2$
$L3$
$L4$
↑2
↑2
↑2



$G1$
blur
$\div 2$
$L1$
graphics / vision
$G2$
These images are theoretically the same (Nyquist) but in practice slightly different due to imperfect filtering/interpolation and edge effects
$G1$
$\uparrow 2$
$L1$
graphics
$G2$
Subtle point: Need to **downsample** + **upsample** to guarantee **perfect reconstruction** of Gaussian from Laplacian Pyramid

Application: Image Blending

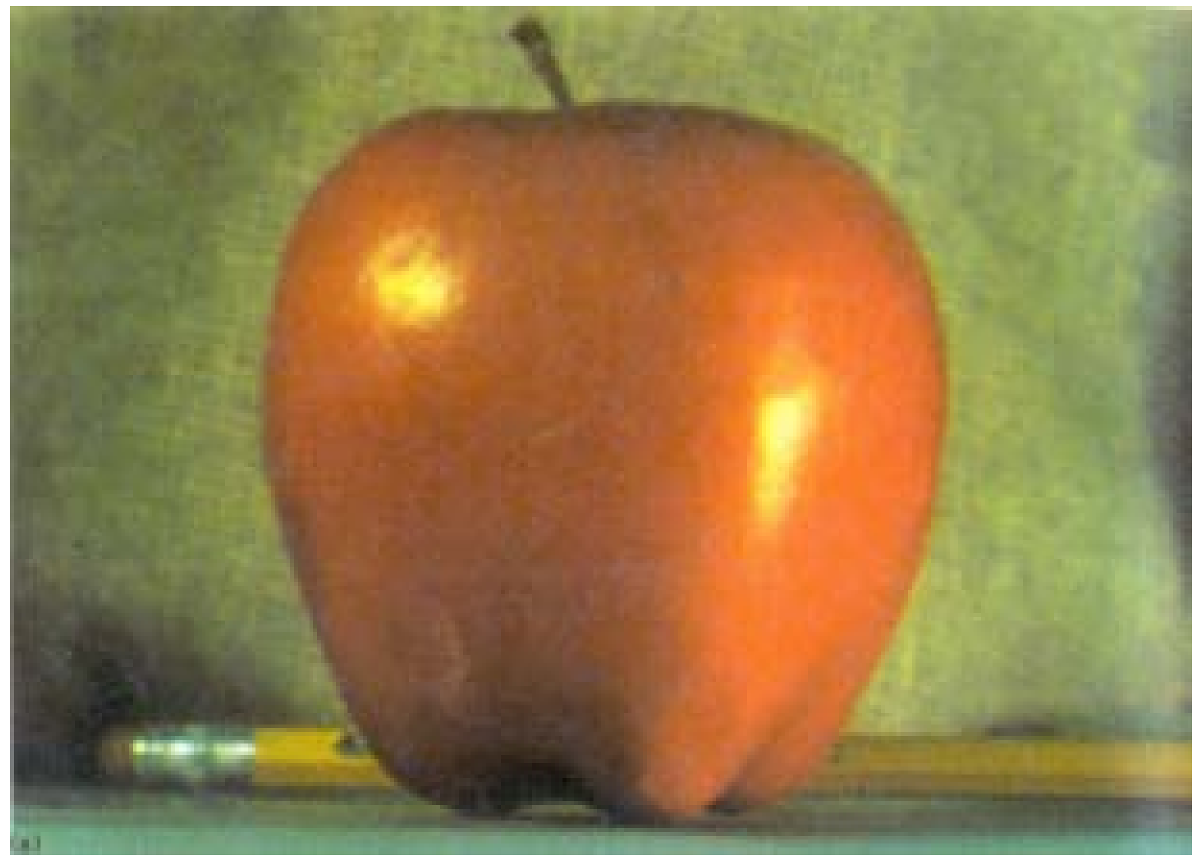


Left pyramid blend Right pyramid

Burt and Adelson, "A multiresolution spline with application to image mosaics,"ACM Transactions on Graphics, 1983, Vol.2, pp.217-236.

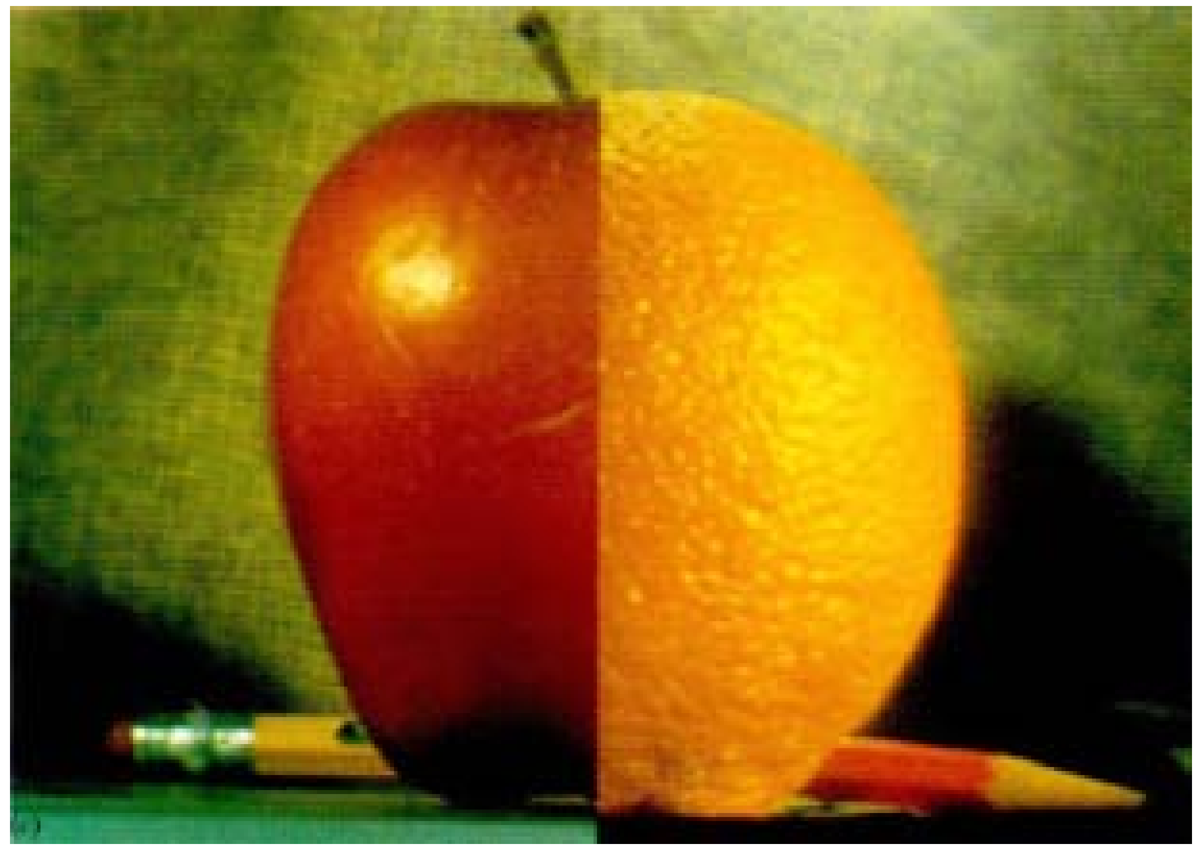
Pyramid Blending

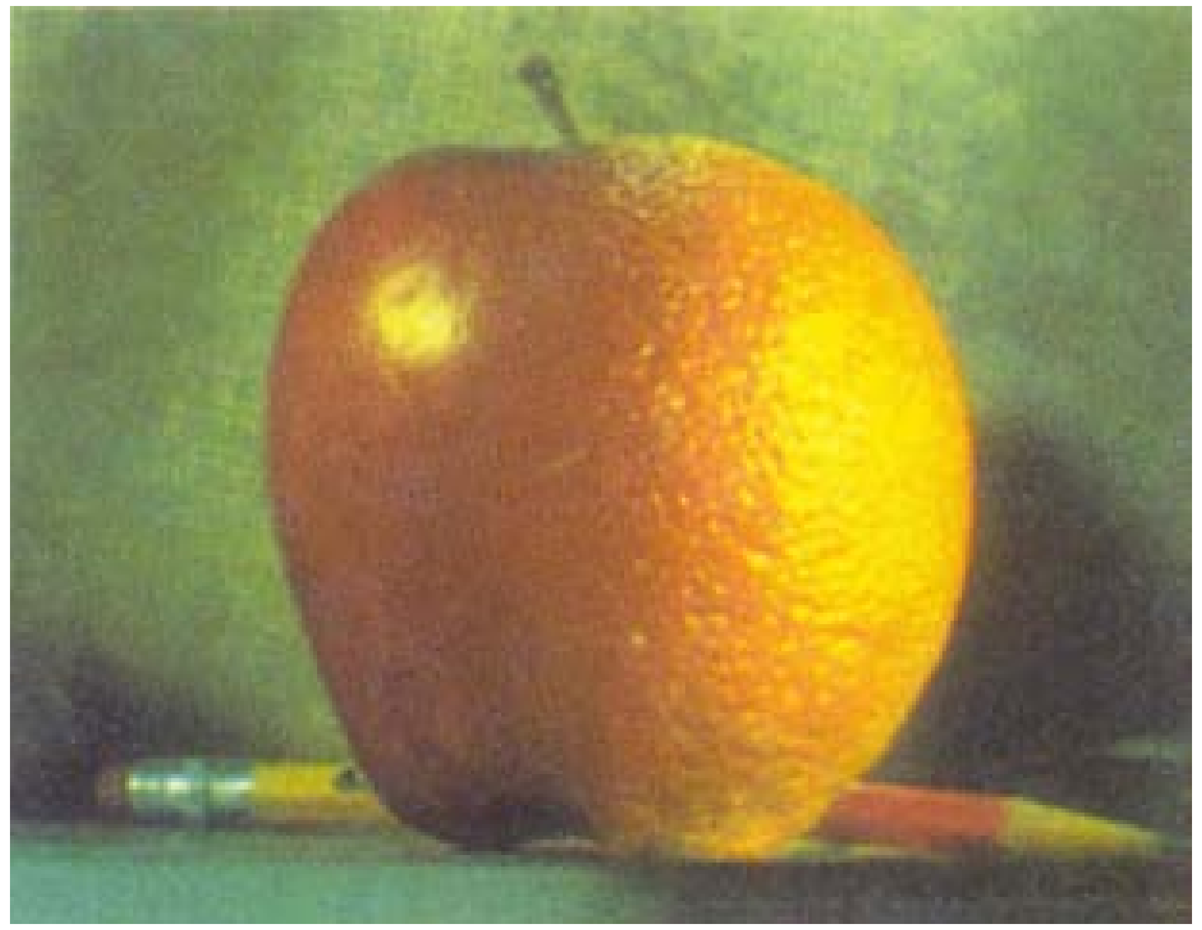
Smooth low frequencies, whilst preserving high frequency detail



(a)

(b)

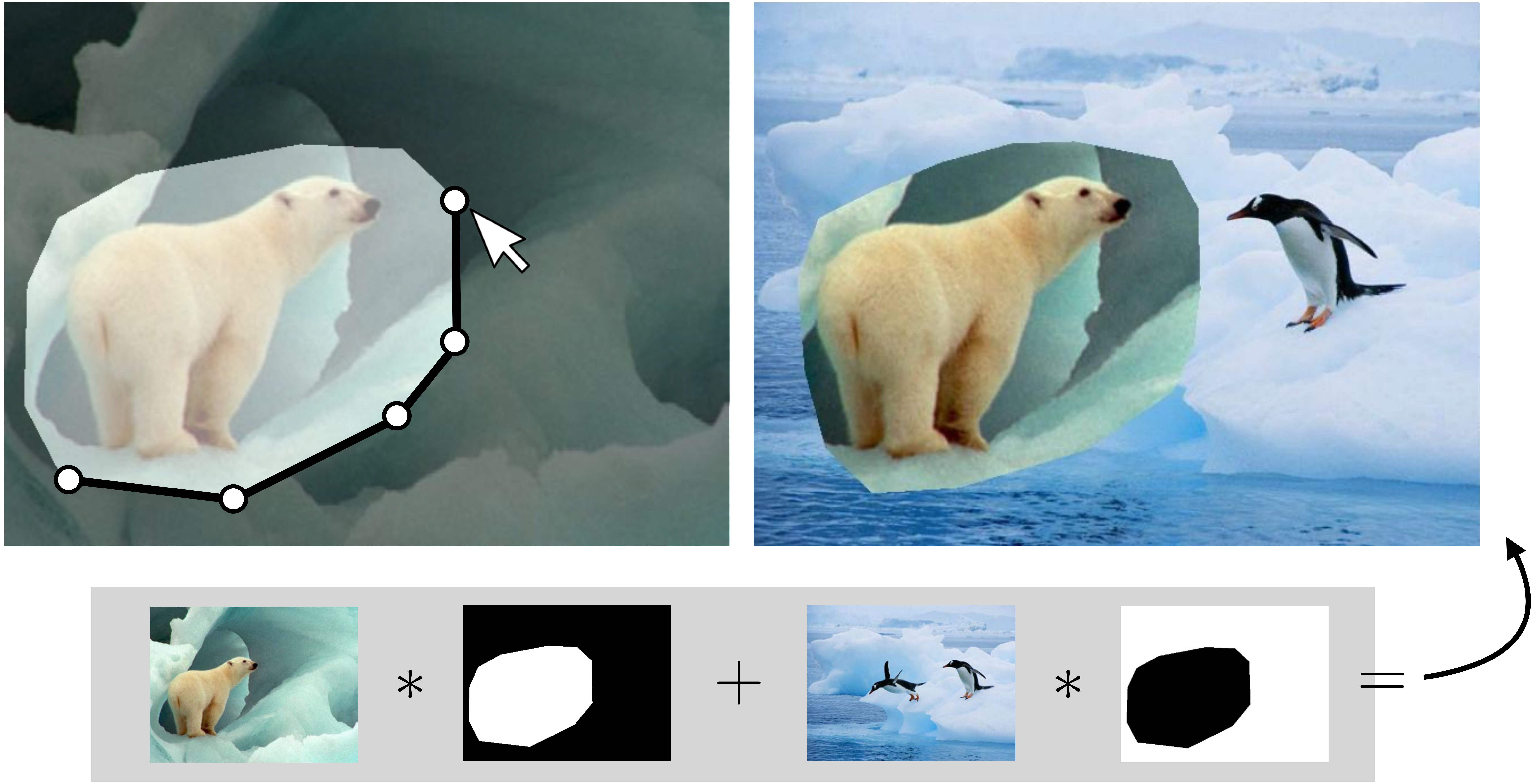




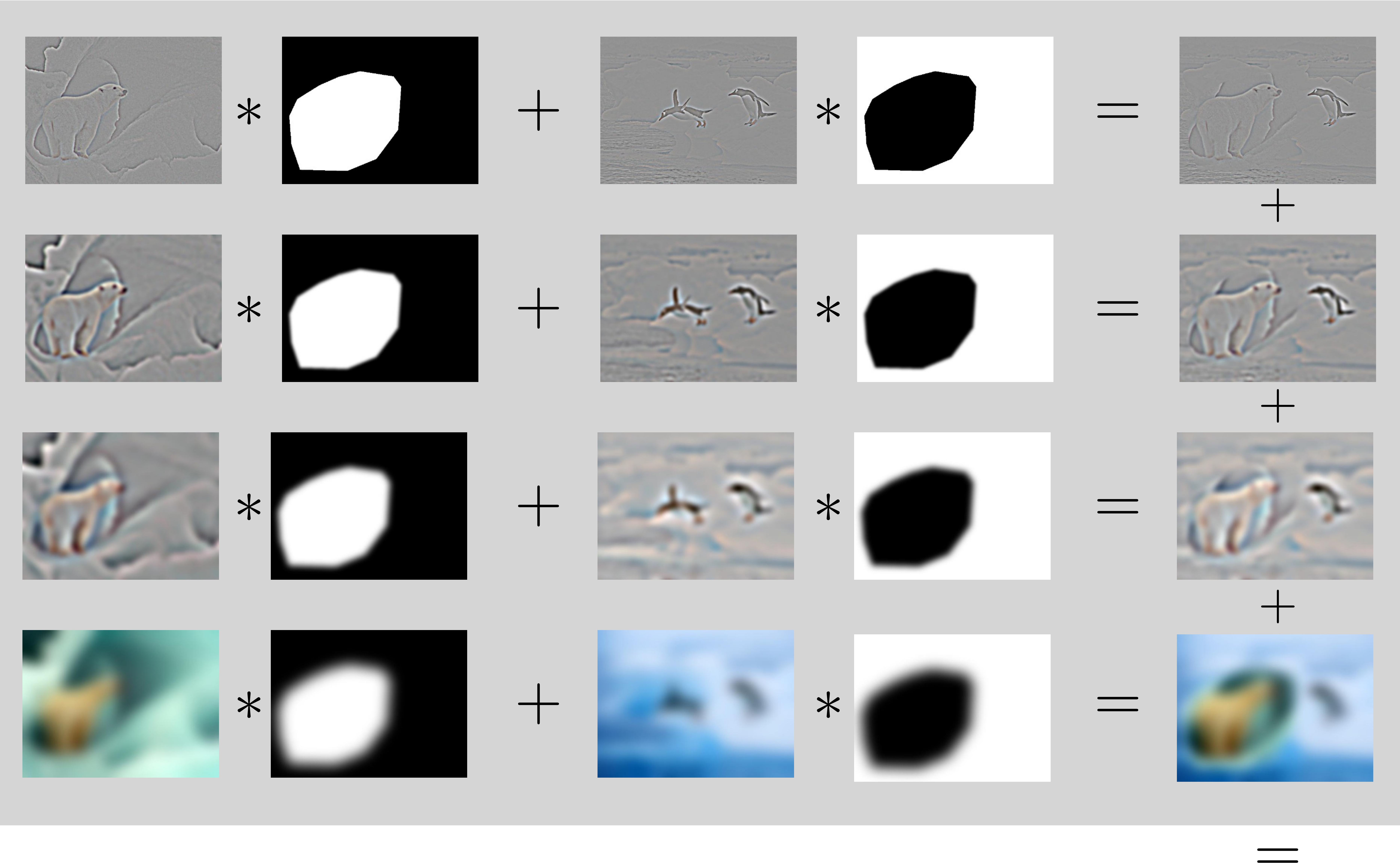
[Burt Adelson 1983]

Pyramid Blending

Pyramid Blending



Step 1: Specify an Image Mask



Step 2: blend lower frequency bands over larger spatial ranges, high frequency bands over small spatial ranges

Application: Image Blending

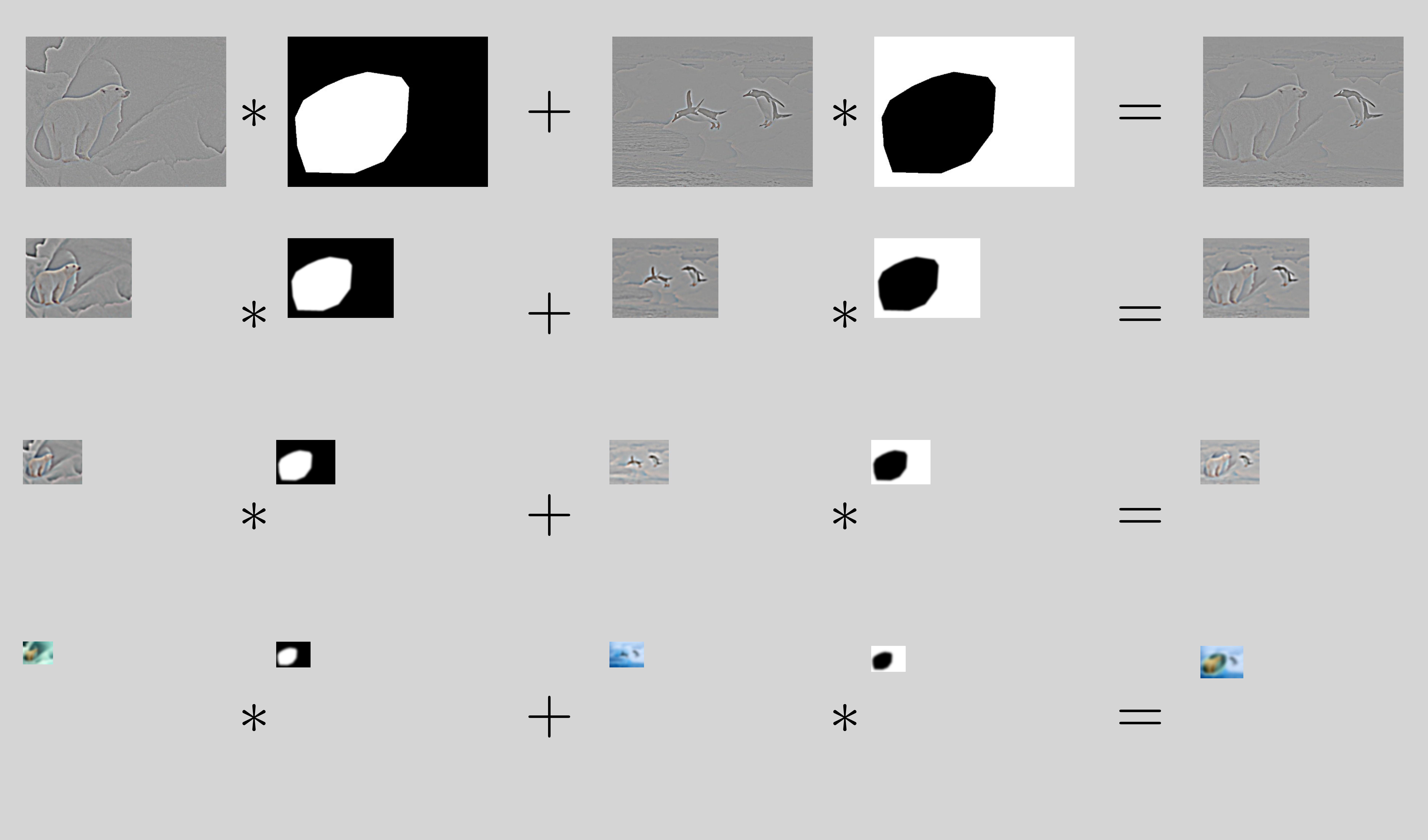
Algorithm:

1. Build Laplacian pyramid LA and LB from images A and B

2. Build a Gaussian pyramid GR from mask image R (the mask defines which image pixels should be coming from A or B)

3. From a combined (blended) Laplacian pyramid LS, using nodes of GR as weights: $LS(i,j) = GR(i,j) * LA(i,j) + (1-GR(i,j)) * LB(i,j)$

4. Reconstruct the final blended image from LS



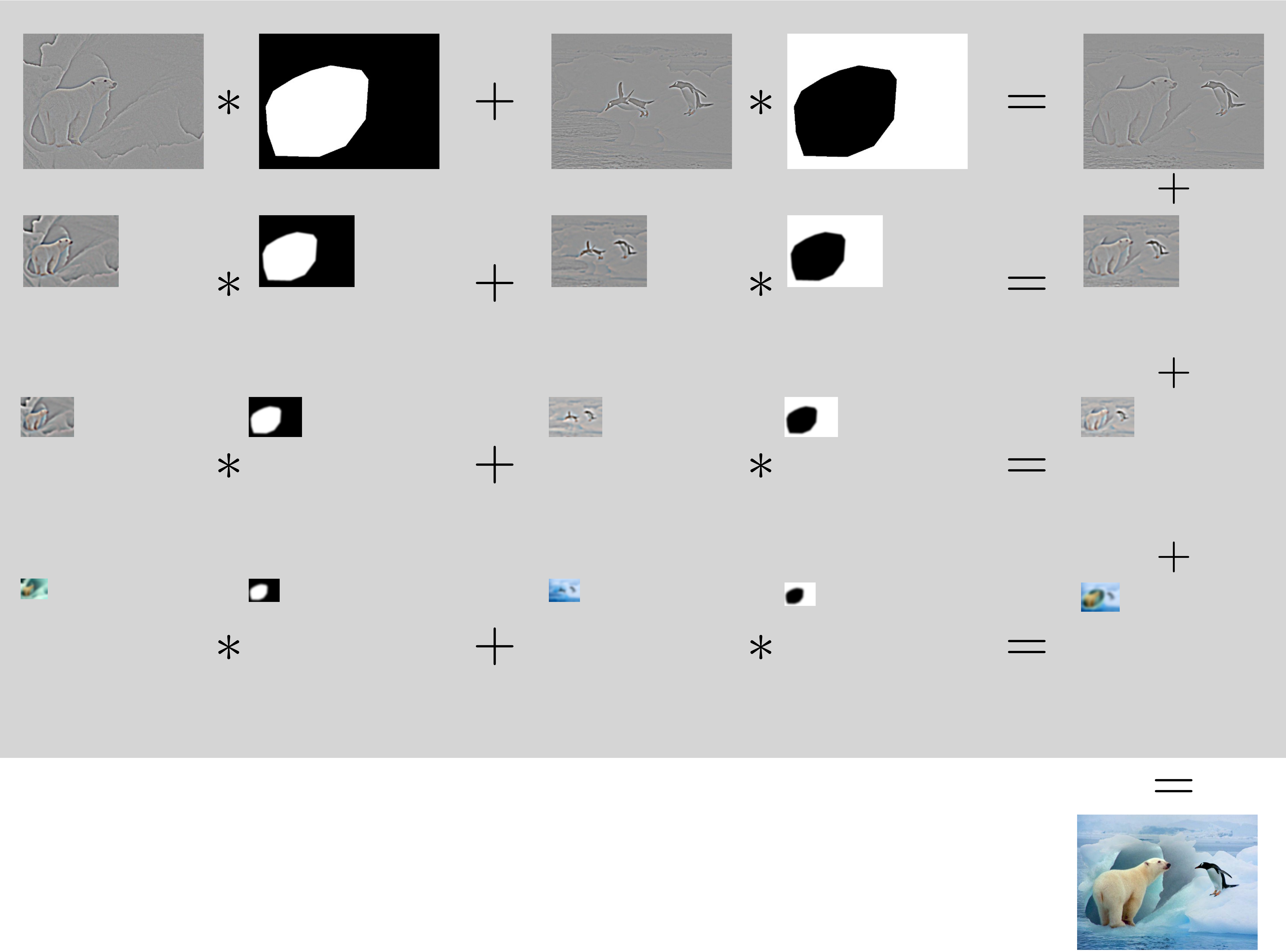
Polar Bear Laplacian Pyramid

Mask Gaussian Pyramid

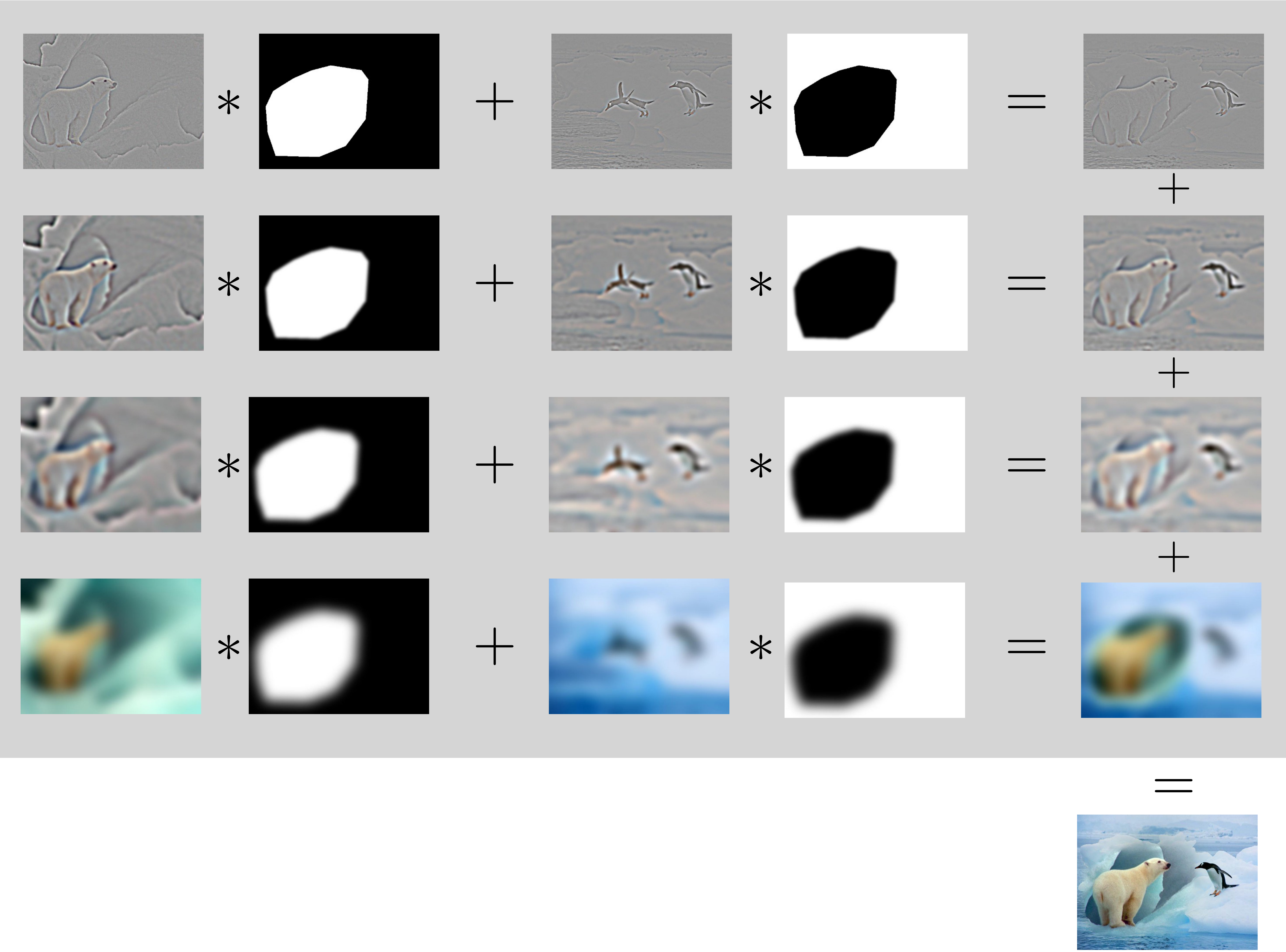
Penguin Laplacian Pyramid

1 - Mask Gaussian Pyramid

Result Pyramid



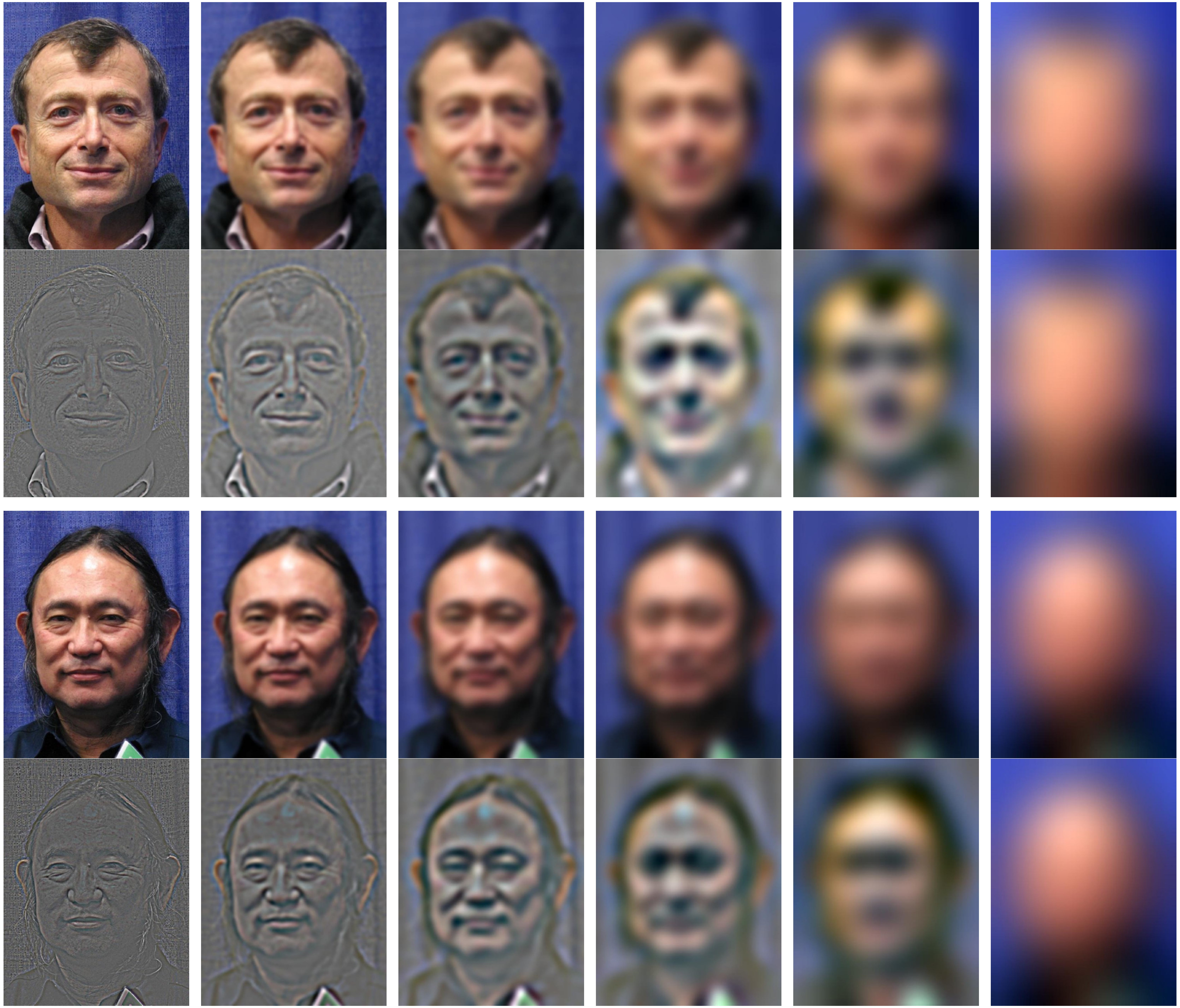
Reconstruct
Result



Reconstruct Result


[Jim Kajiya, Andries van Dam]

[Jim Kajiya, Andries van Dam]



Alpha blend with sharp fall-off

Alpha blend with gradual fall-off

Pyramid Blend

Summary: **Scaled Representations**

Gaussian Pyramid

— Each level represents a **low-pass** filtered image at a different scale

— Generated by successive Gaussian blurring and downsampling

— Useful for image resizing, sampling

Laplacian Pyramid

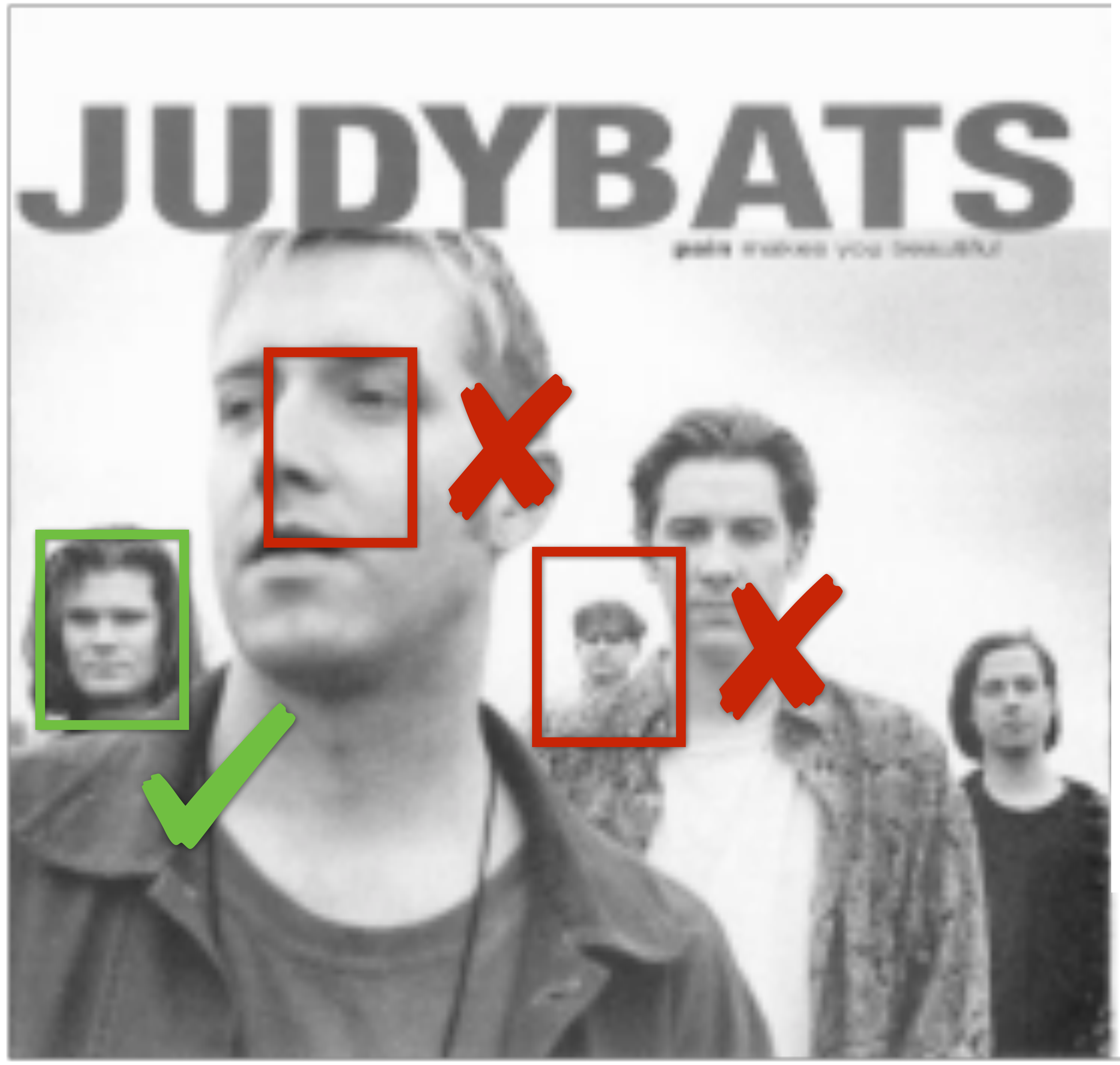
— Each level is a **band-pass** image at a different scale

— Generated by differences between successive levels of a Gaussian Pyramid

— Used for pyramid blending, feature extraction etc.

Recap: **Multi-Scale** Template Matching

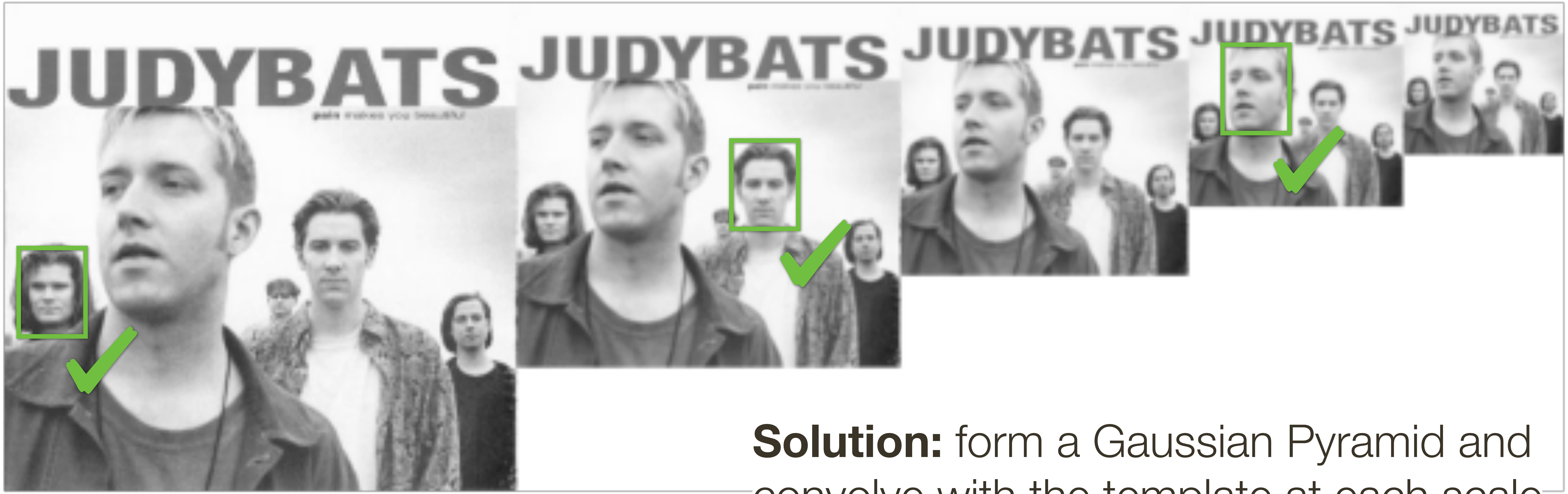
Correlation with a **fixed-sized image** only detects faces at **specific scales**



= Template

Recap: **Multi-Scale** Template Matching

Correlation with a **fixed-sized image** only detects faces at **specific scales**



Solution: form a Gaussian Pyramid and convolve with the template at each scale

Q. **Why scale** the **image** and not the **template**?

= Template

Improving Template Matching

Consider the problem of finding images of an elephant using a template

An elephant looks different from different viewpoints

— from above (as in an aerial photograph or satellite image)

— head on

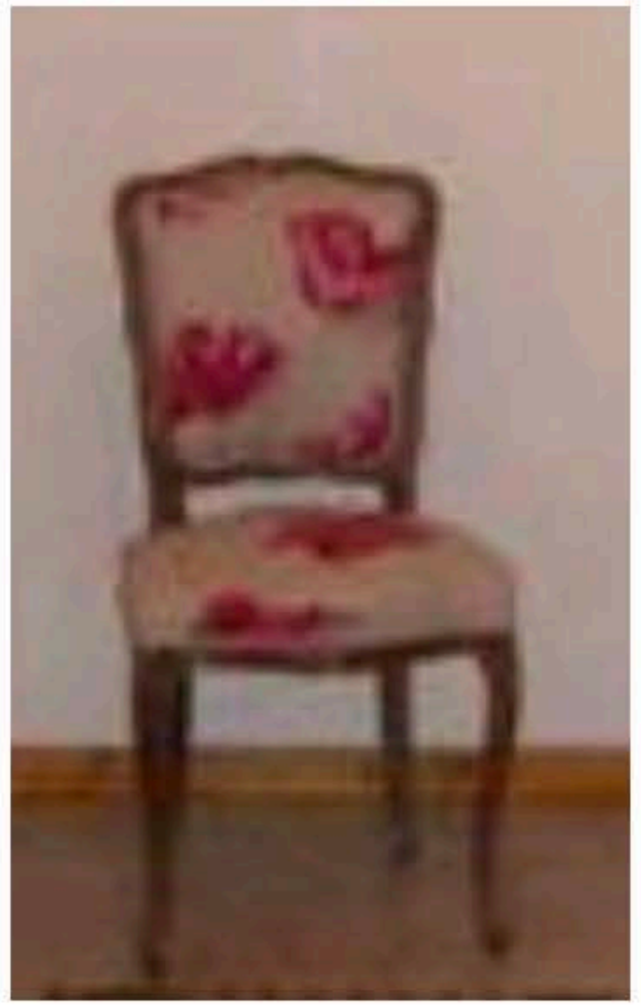
— sideways (i.e., in profile)

— rear on

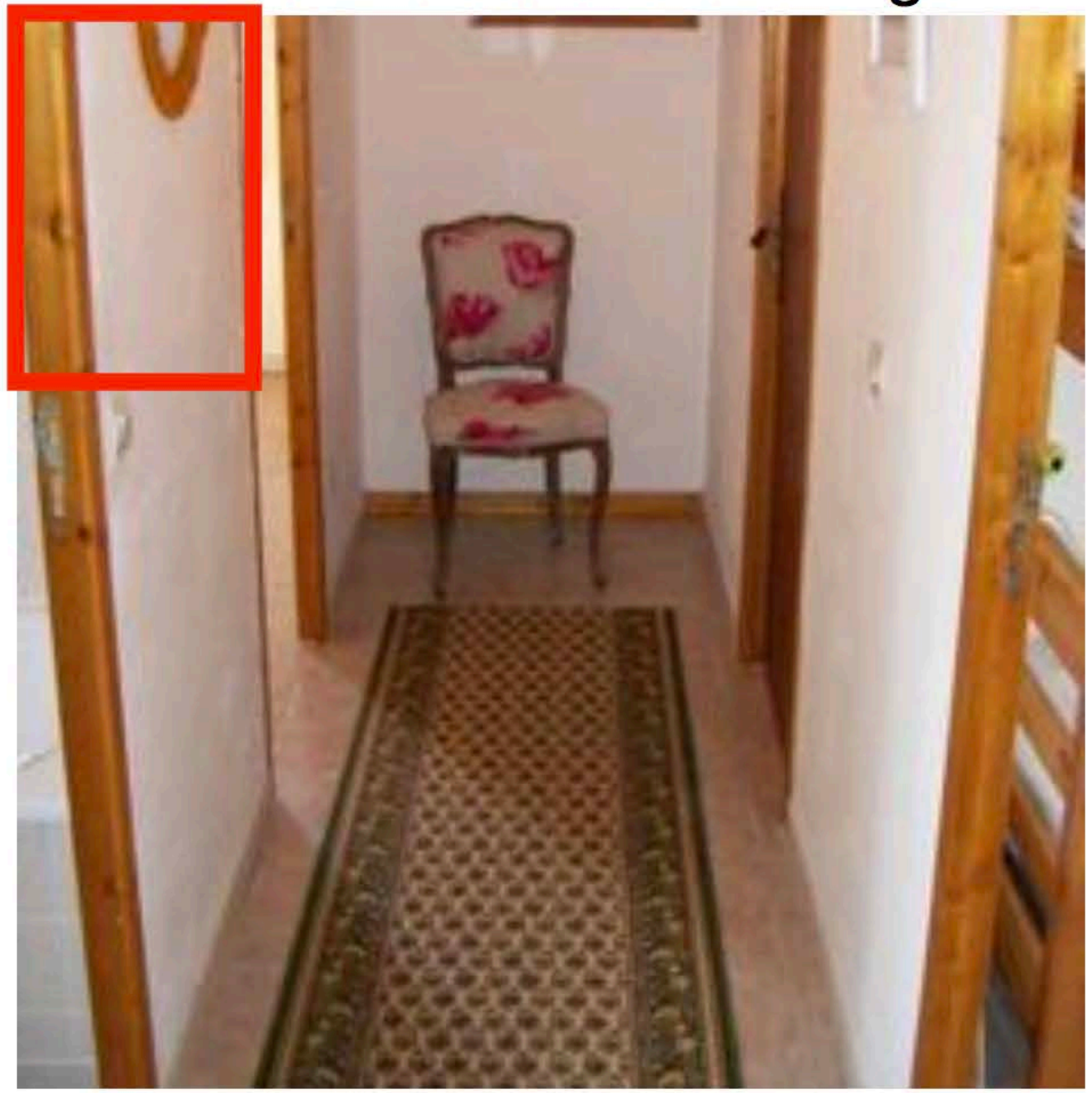
What happens if parts of an elephant are obscured from view by trees, rocks, other elephants?

Improving Template Matching

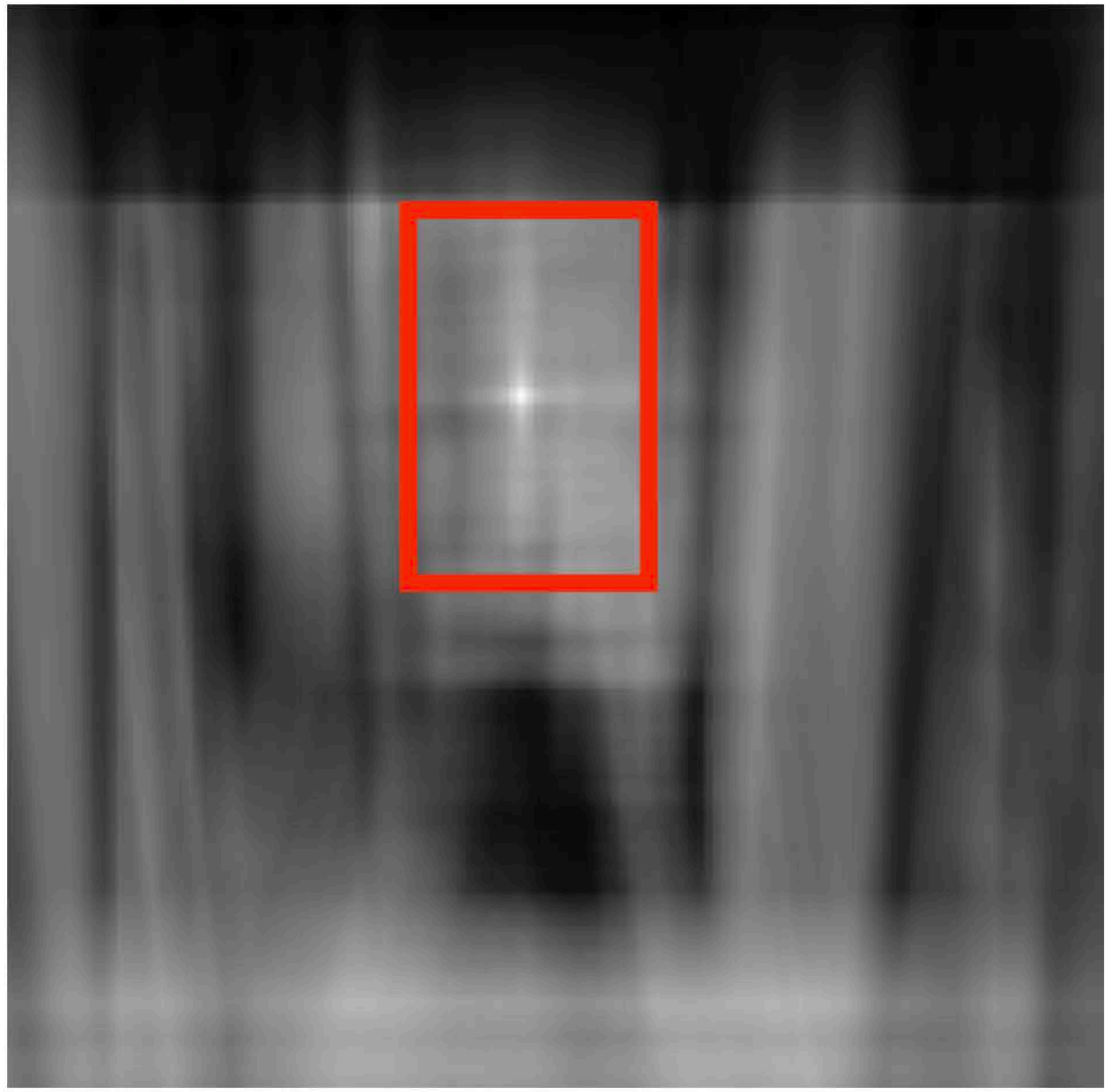
This is a chair



Find the chair in this image

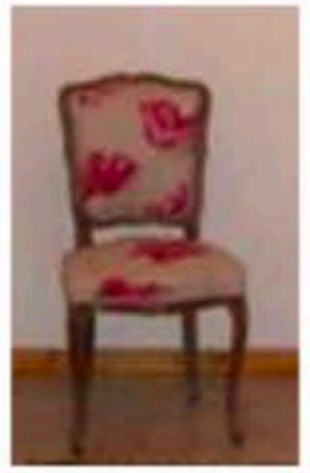


Output of normalized correlation

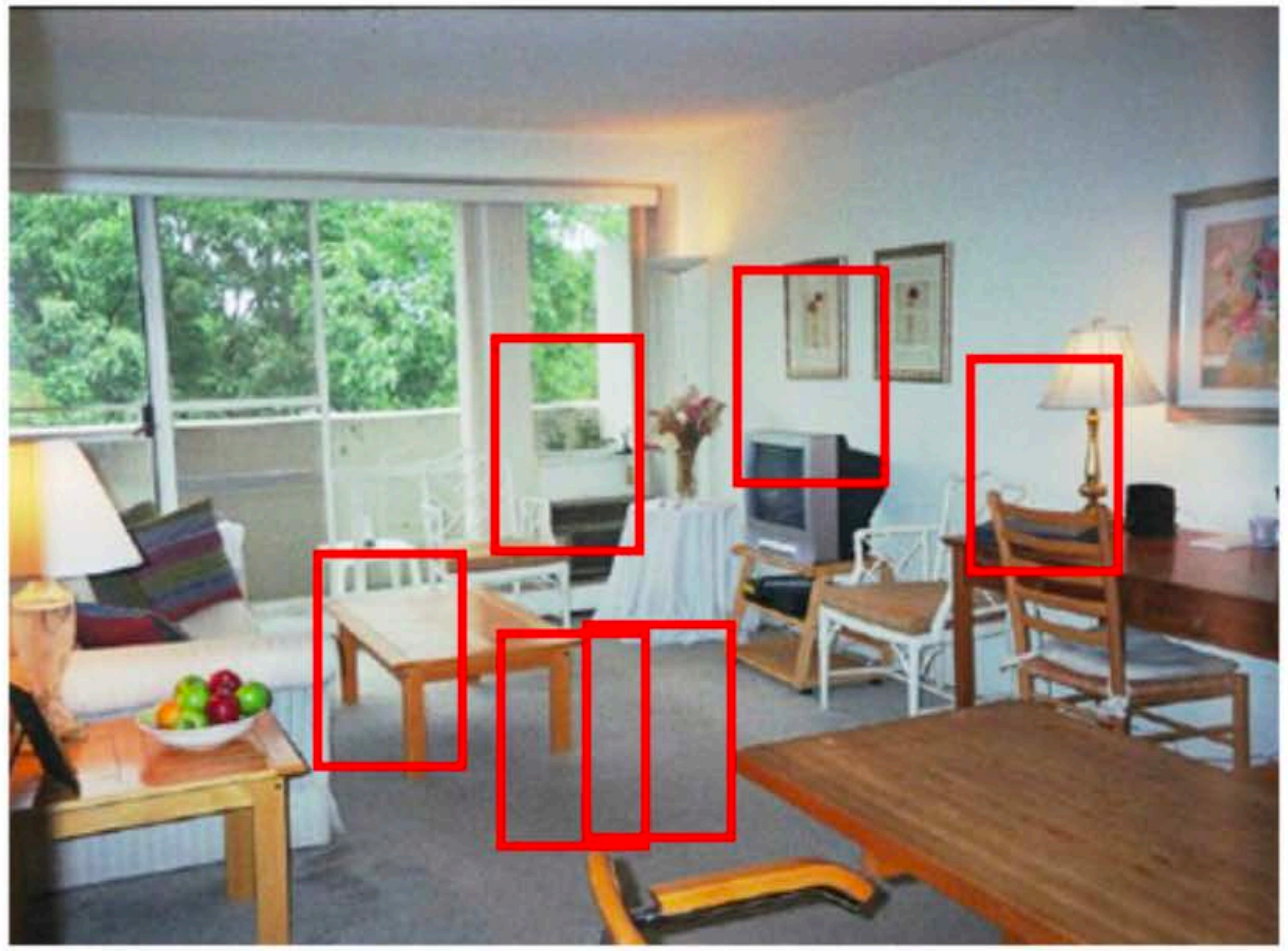


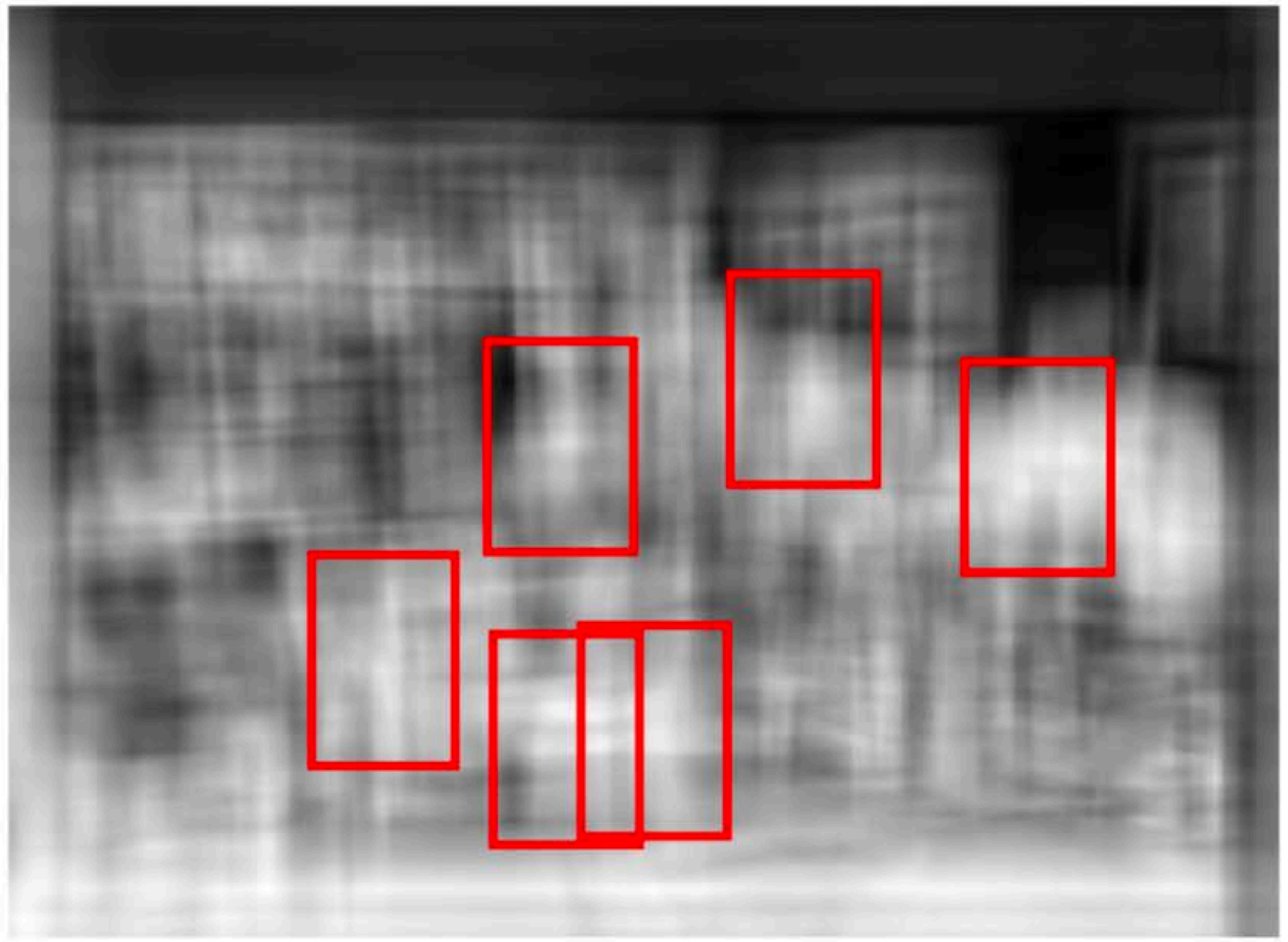
Slide Credit: Li Fei-Fei, Rob Fergus, and Antonio Torralba

Improving Template Matching



Find the chair in this image





Pretty much garbage
Simple template matching is not going to make it

Slide Credit: Li Fei-Fei, Rob Fergus, and Antonio Torralba

Improving Template Matching

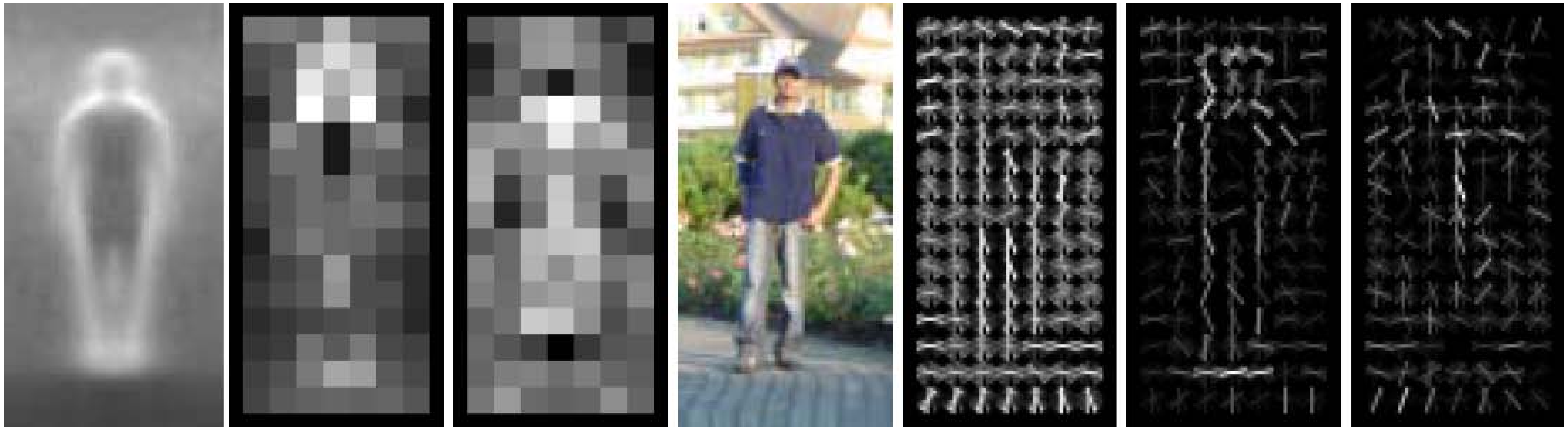
Improved detection algorithms make use of **image features**

These can be **hand coded** or **learned**

Template Matching with HOG

Template matching can be improved by using better features, e.g., Histograms of Gradients (HOG) [Dalal Triggs 2005]

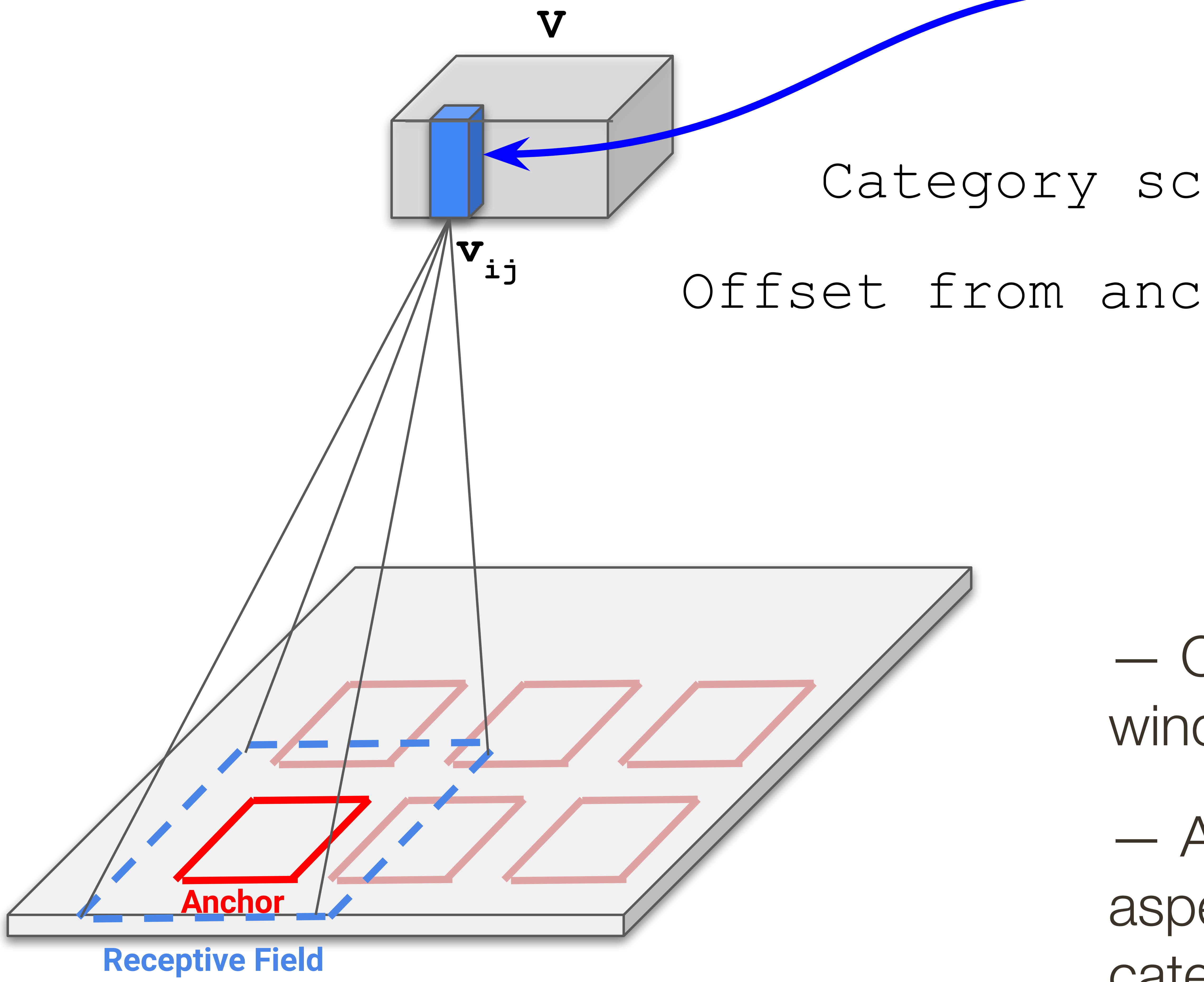
The authors use a Learning-based approach (Support Vector Machine) to find an optimally weighted template



avg grad   SVM weights   HOG   weighted HOG

+   −

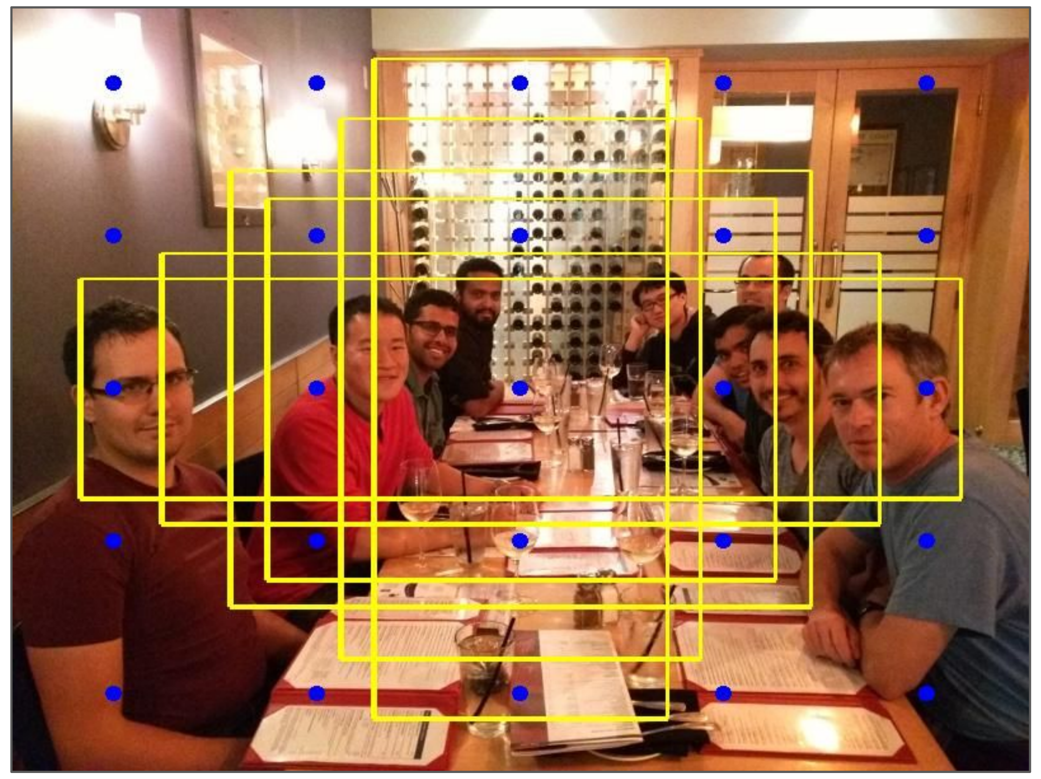
Convnet Object Detection



Think of each feature vector $\mathbf{v}_{ij}$ as corresponding to a sliding window (anchor).

$$\text{Category score} = \text{SoftMax}(W^{cls} \cdot \mathbf{v}_{ij})$$

$$\text{Offset from anchor} = W^{loc} \cdot \mathbf{v}_{ij}$$



— Convnet based object detectors resemble sliding window template matching in feature space

— Architectures typically involve multiple scales and aspect ratios, and regress to a 2D offset in addition to category scores

[Images: Jonathan Huang]

Summary

Template matching as (normalized) correlation. Template matching is not robust to changes in:

— 2D spatial scale and 2D orientation

— 3D pose and viewing direction

— illumination

Scaled representations facilitate

— template matching at multiple scales

— efficient search for image–to–image correspondences

— image analysis at multiple levels of detail

A **Gaussian pyramid** reduces artifacts introduced when sub-sampling to coarser scales