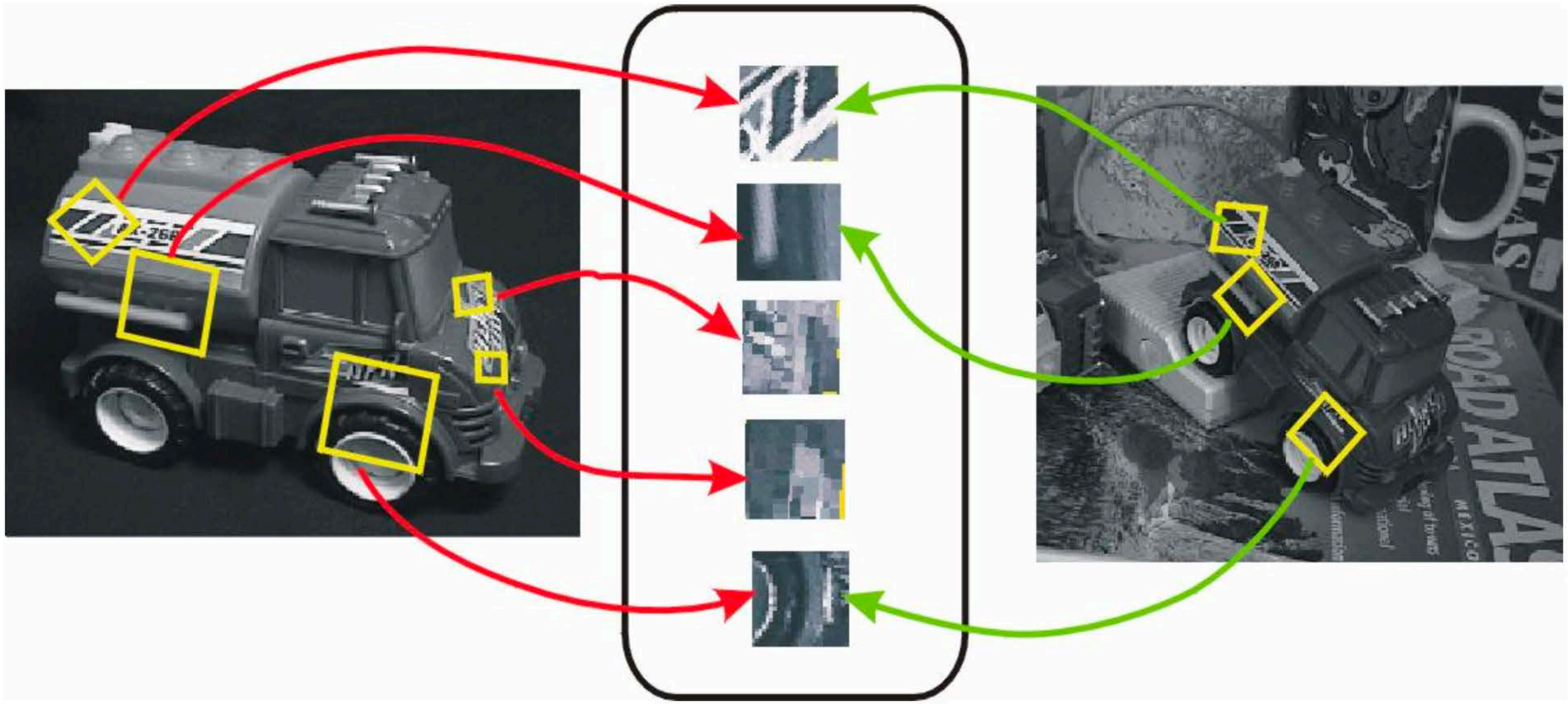
Review: Learning **Goals**

1. The design philosophy behind SIFT

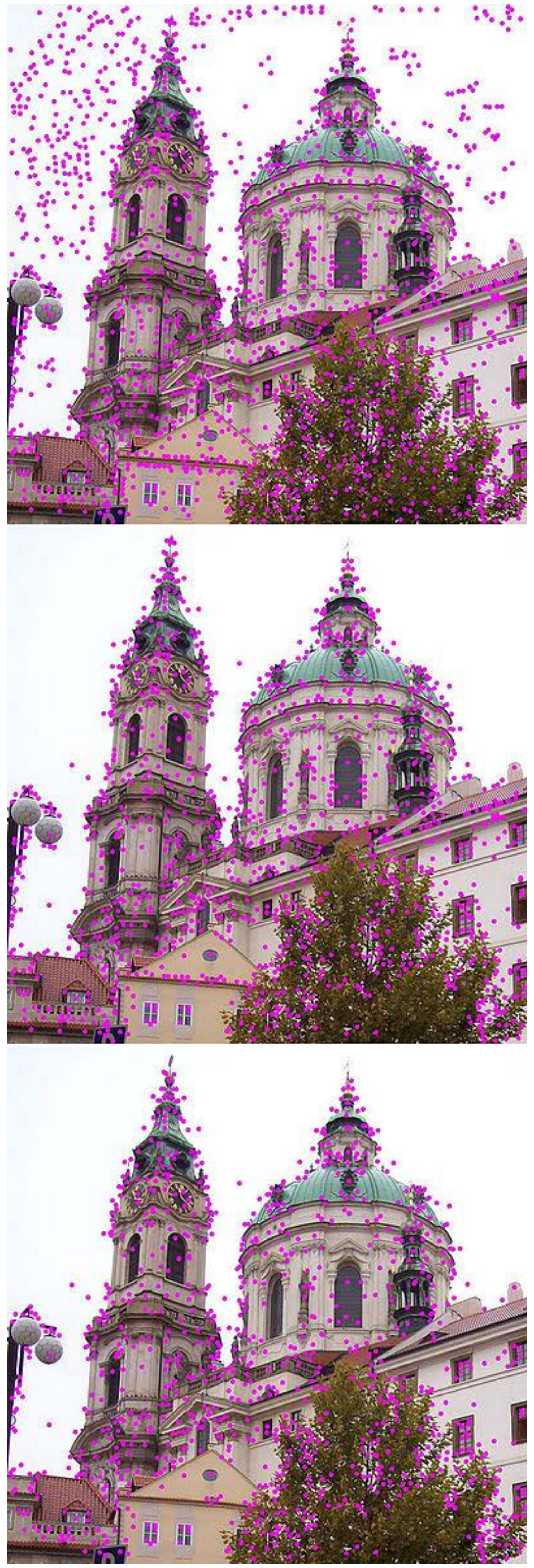
David Lowe's Invariant Local Features

Image content is transformed into local feature coordinates that are invariant to translation, rotation, scale, and other imaging parameters



SIFT Features

Scale Invariant Feature Transform (**SIFT**)

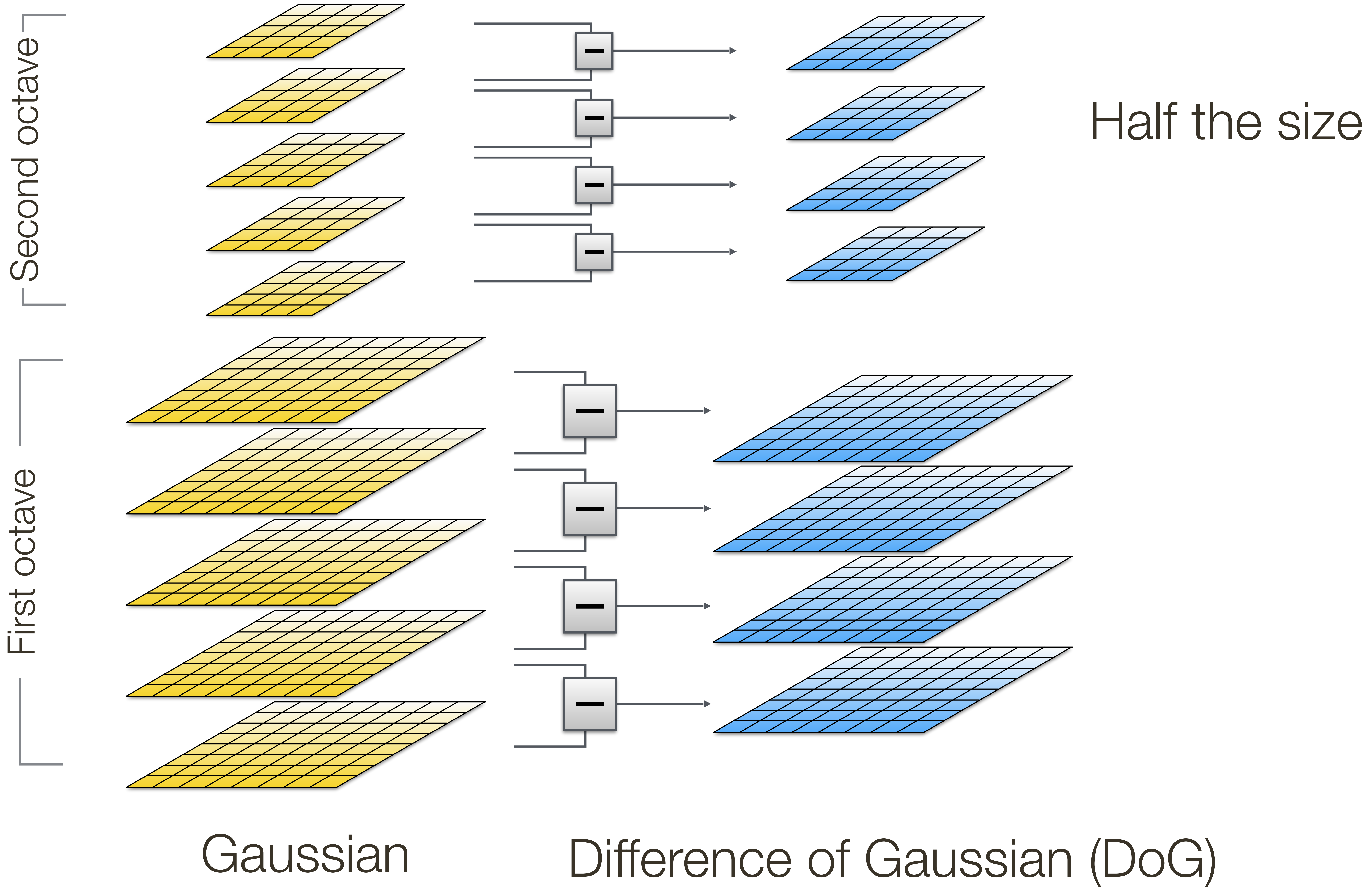


SIFT describes both a **detector** and **descriptor**

1. Multi-scale extrema detection
2. Keypoint localization
3. Orientation assignment
4. Keypoint descriptor

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

1. Multi-scale Extrema Detection

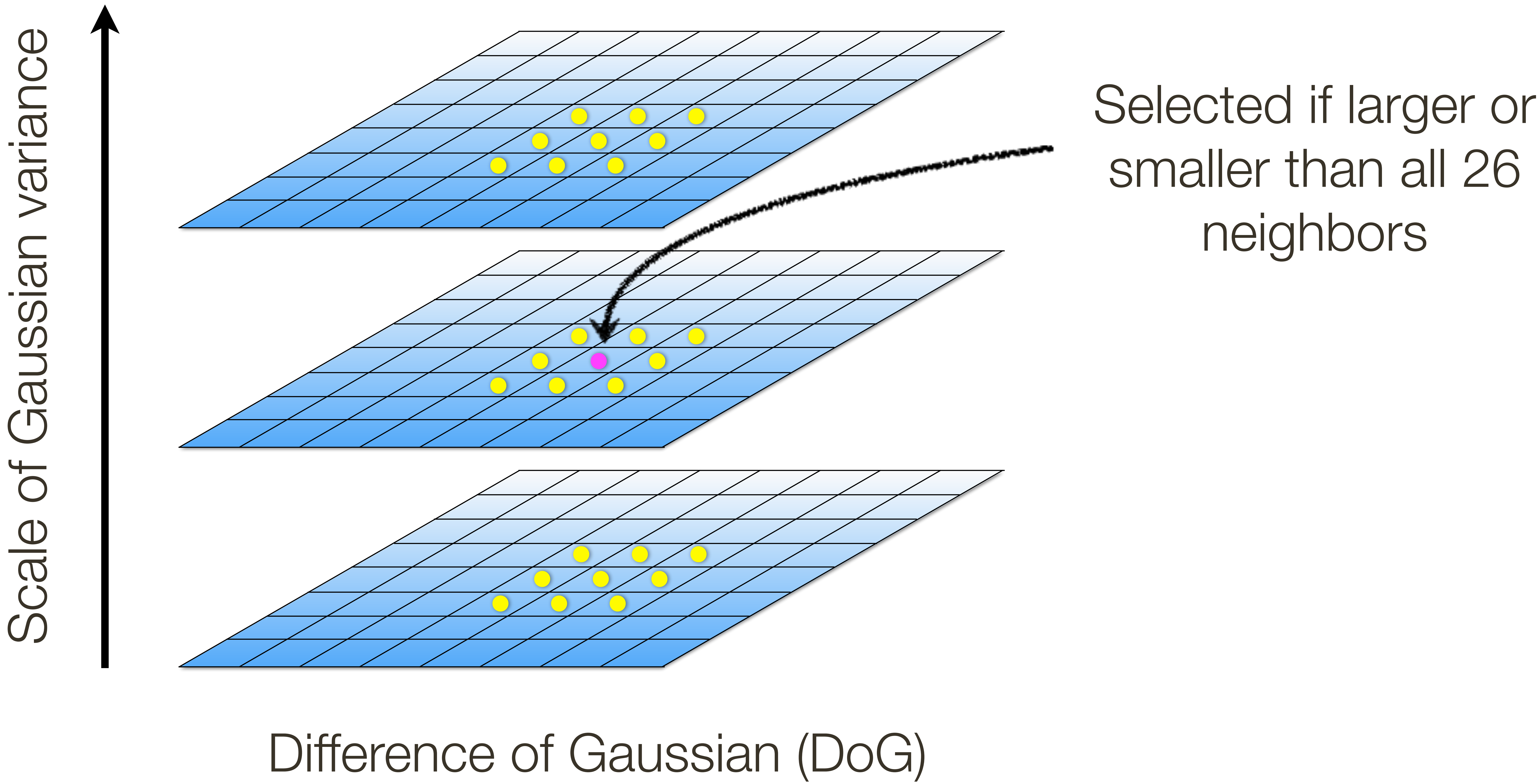


Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

1. Multi-scale Extrema Detection

Detect maxima and minima of Difference of Gaussian in scale space

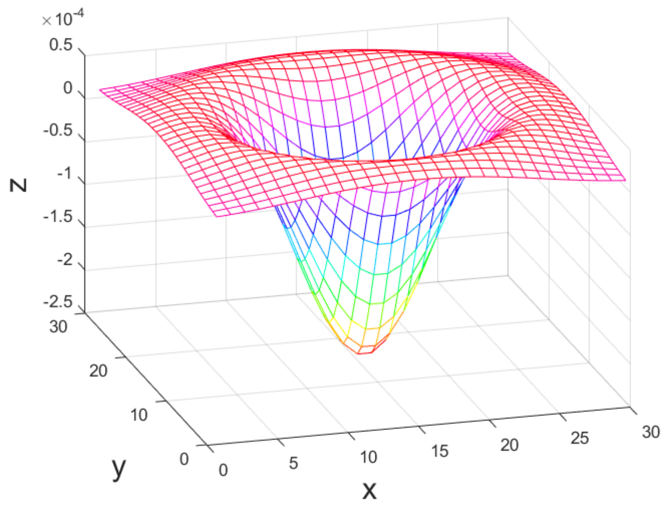
$$\sigma = 2^{1/s}$$



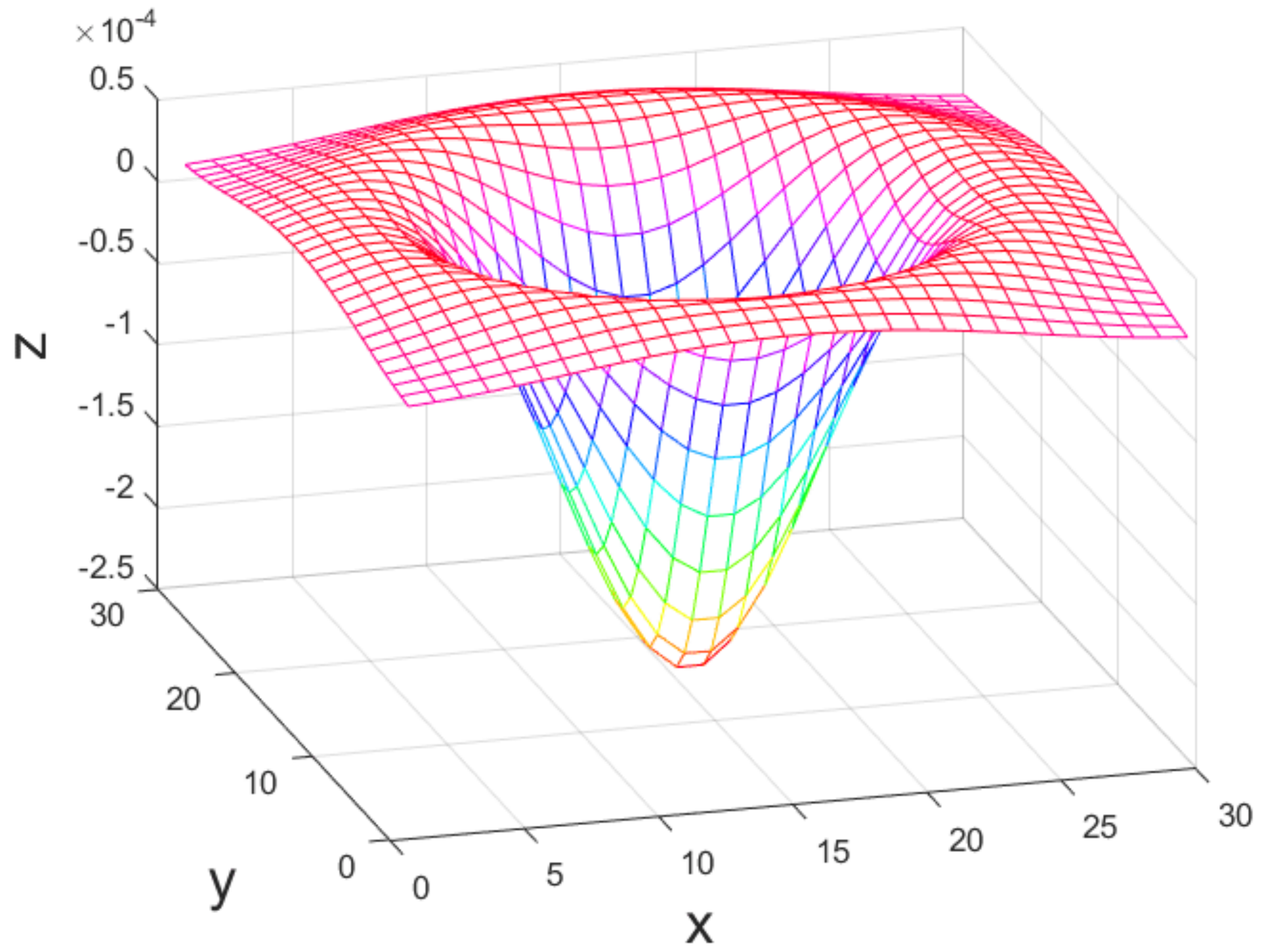
Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Searching over **Scale**-space

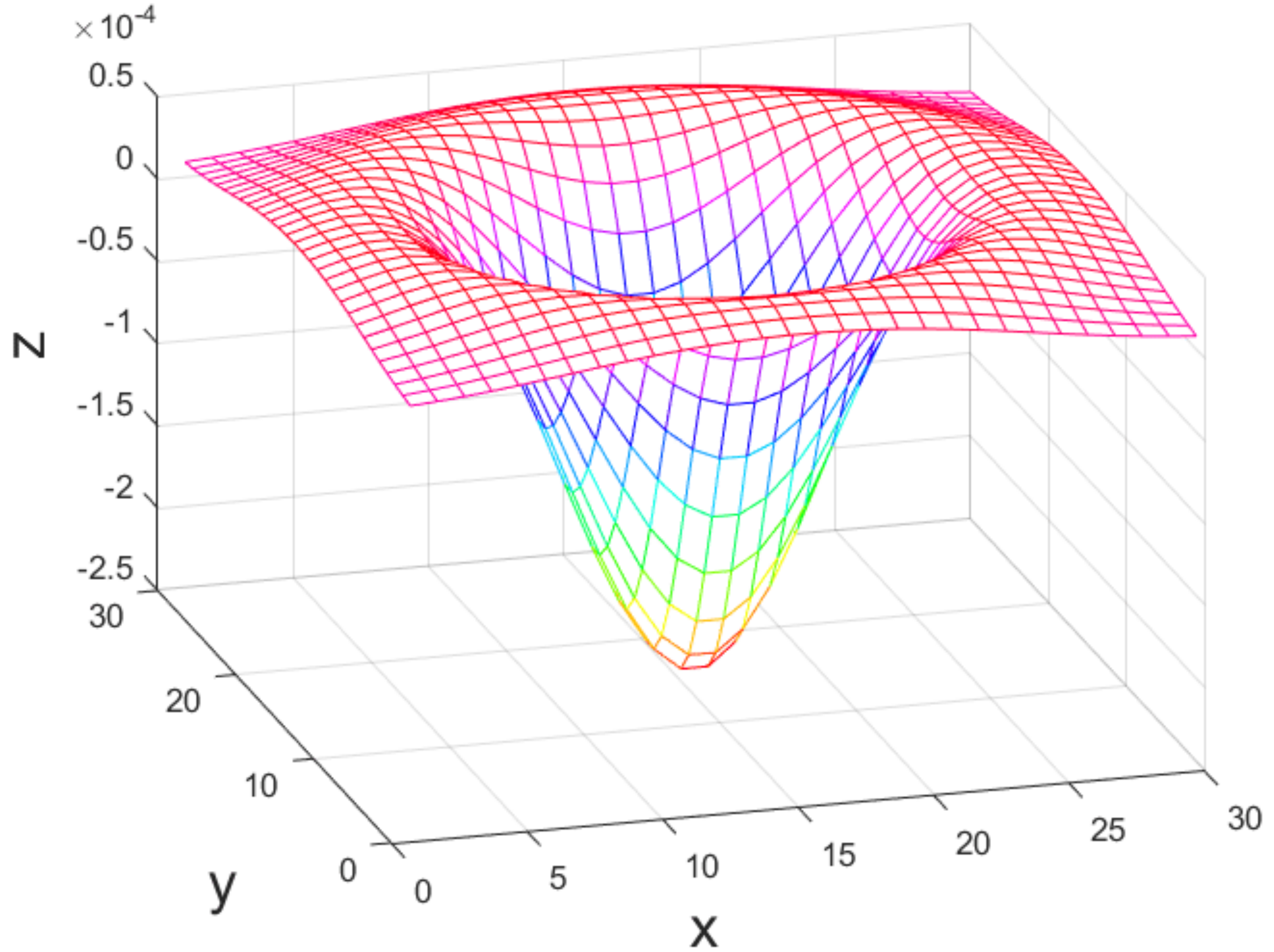
σ

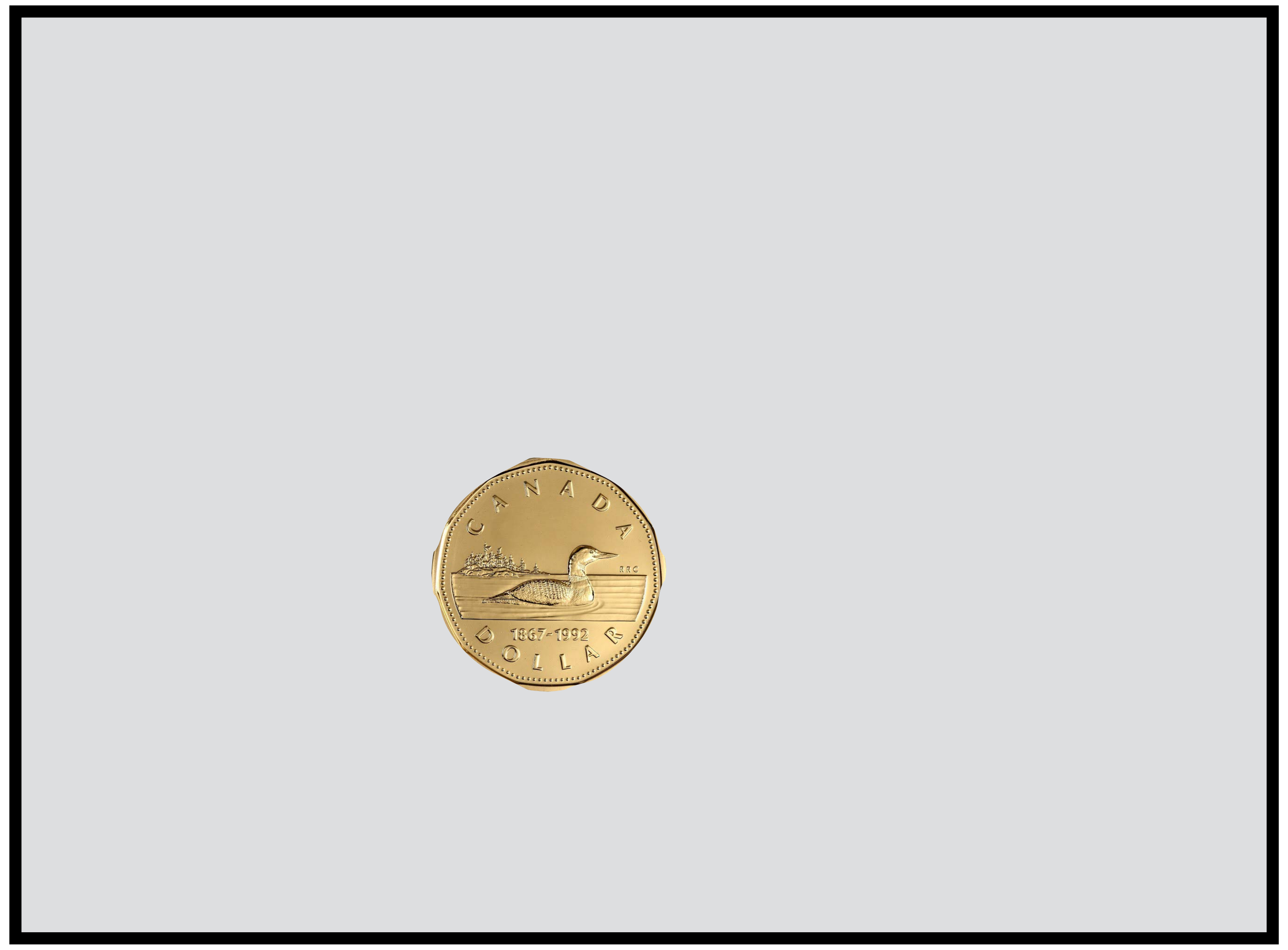


$\sigma' = 2\sigma$



$\sigma' = 3\sigma$





2. Keypoint Localization

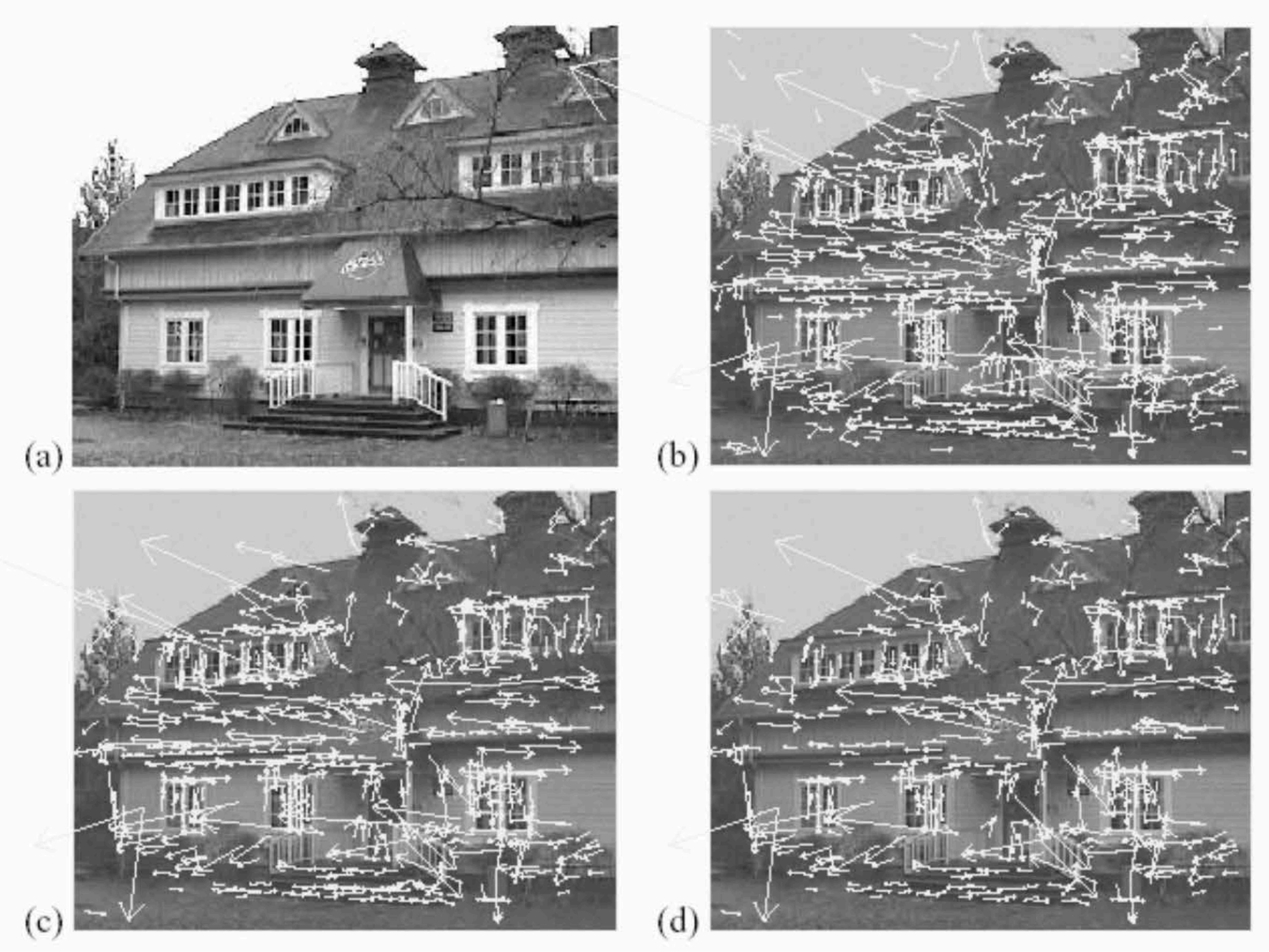
— After keypoints are detected, we remove those that have **low contrast** or are **poorly localized** along an edge

How do we decide whether a keypoint is poorly localized, say along an edge, vs. well-localized?

$$C = \begin{bmatrix} \sum\limits_{p \in P} I_x I_x & \sum\limits_{p \in P} I_x I_y \\ \sum\limits_{p \in P} I_y I_x & \sum\limits_{p \in P} I_y I_y \end{bmatrix}$$

2. Keypoint Localization

Example:



(a) 233 × 189 image

(b) 832 DOG extrema

(c) 729 left after peak value threshold

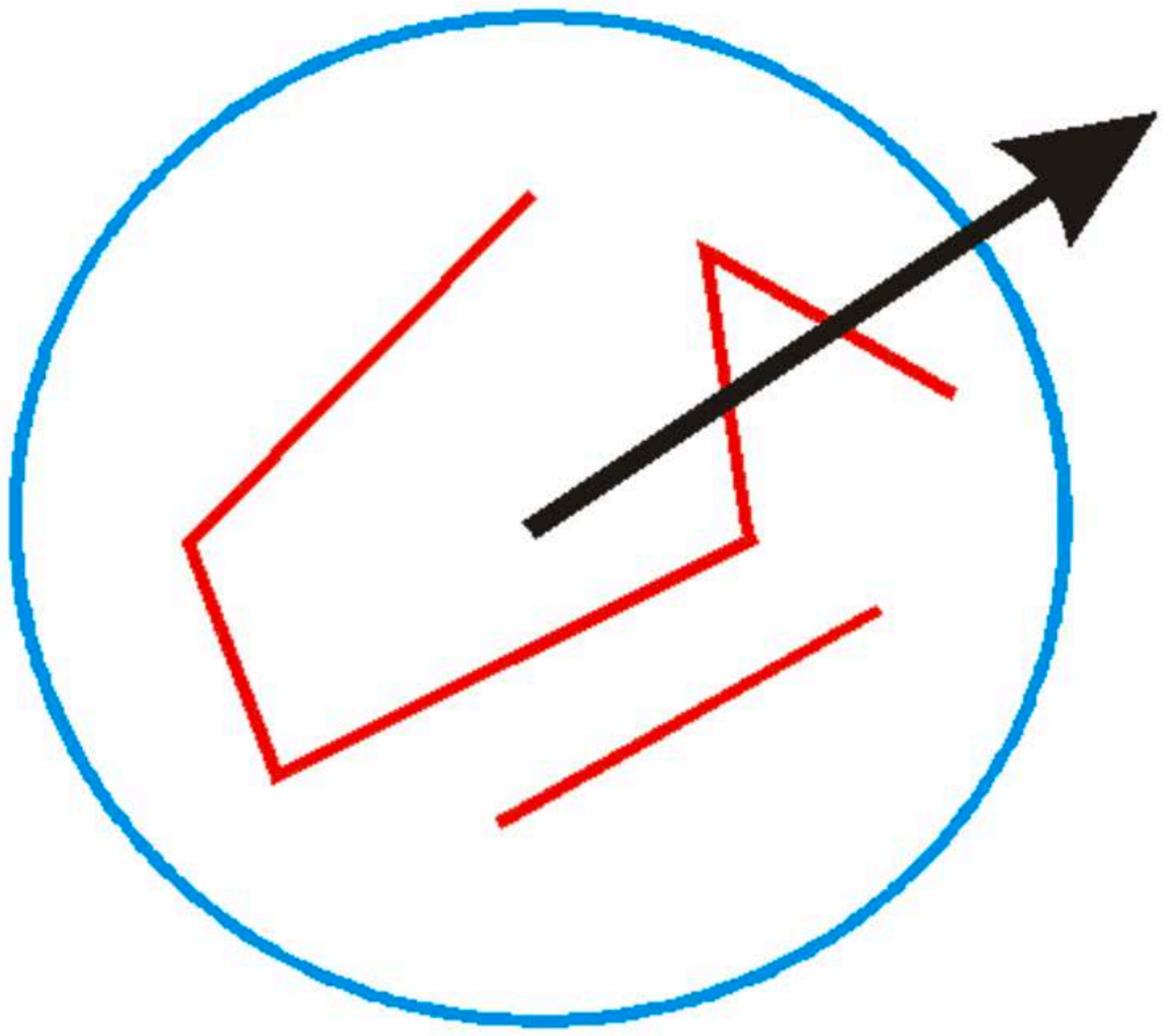
(d) 536 left after testing ratio of principal curvatures

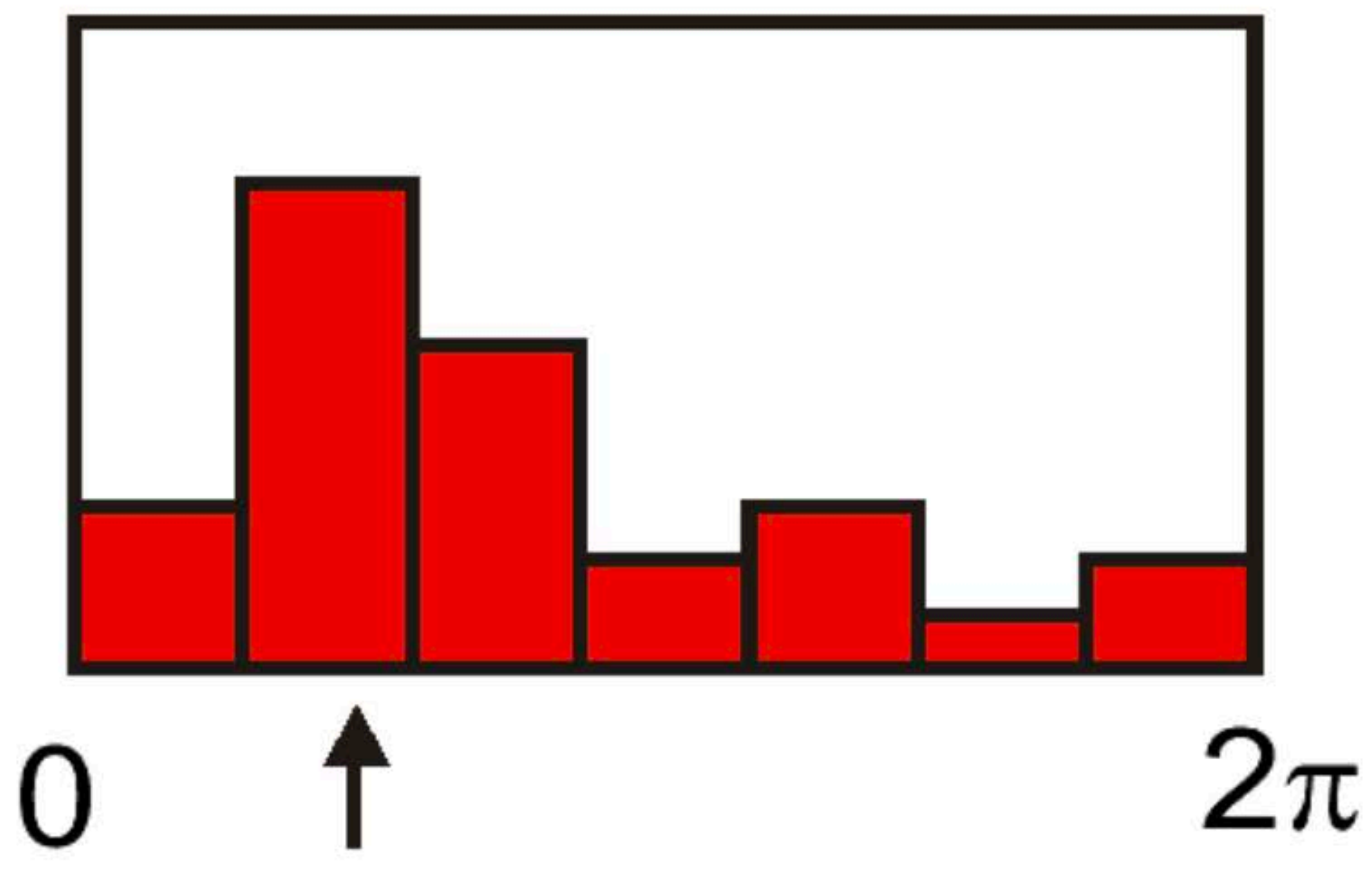
3. Orientation Assignment

— Create **histogram** of local gradient directions computed at selected scale

— Assign **canonical orientation** at peak of smoothed histogram

— Each key specifies stable 2D coordinates (x , y , scale, orientation)



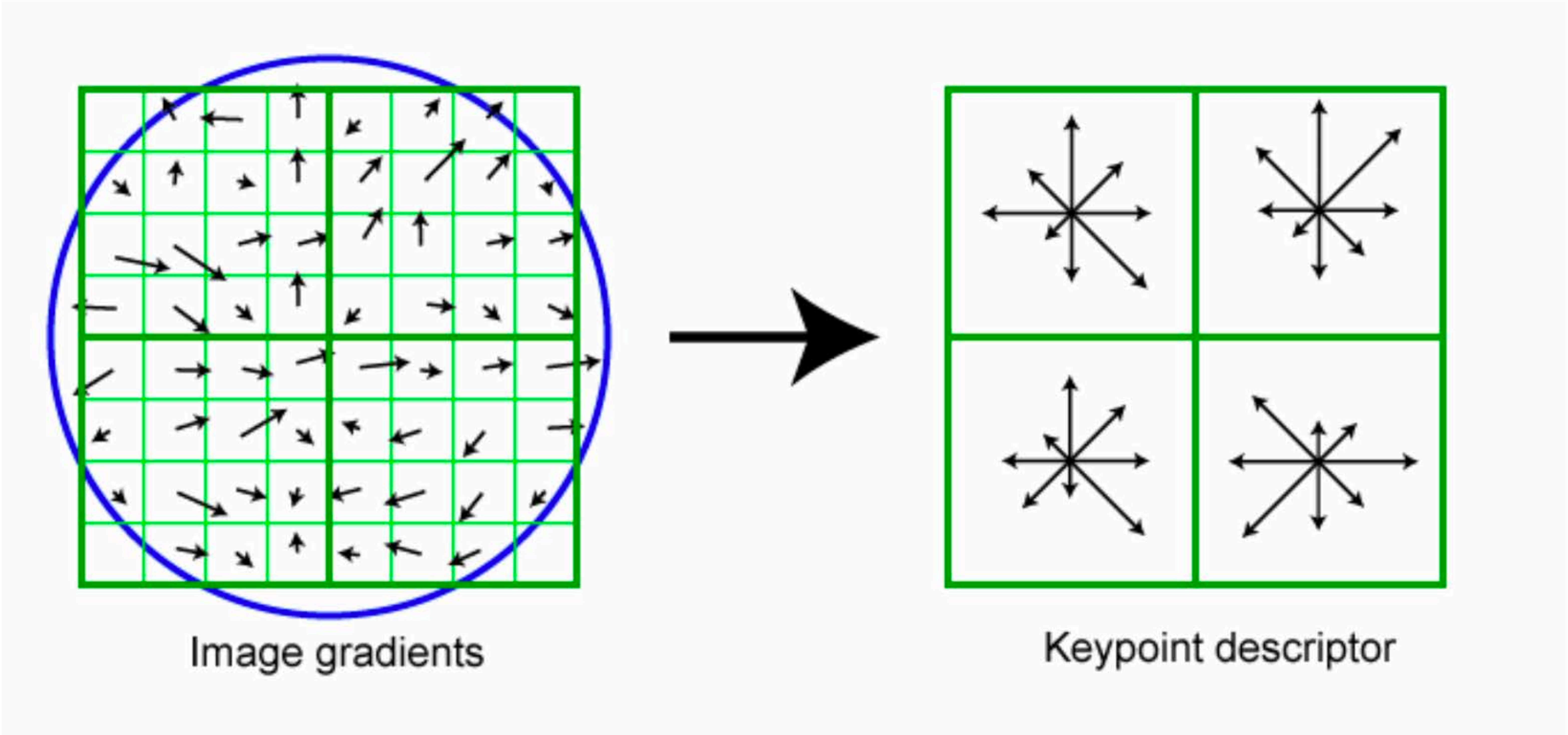


4. SIFT Descriptor

— Image gradients are sampled over 16 × 16 array of locations in scale space (weighted by a Gaussian with sigma half the size of the window)

— Create array of orientation histograms

— 8 orientations × 4 × 4 histogram array



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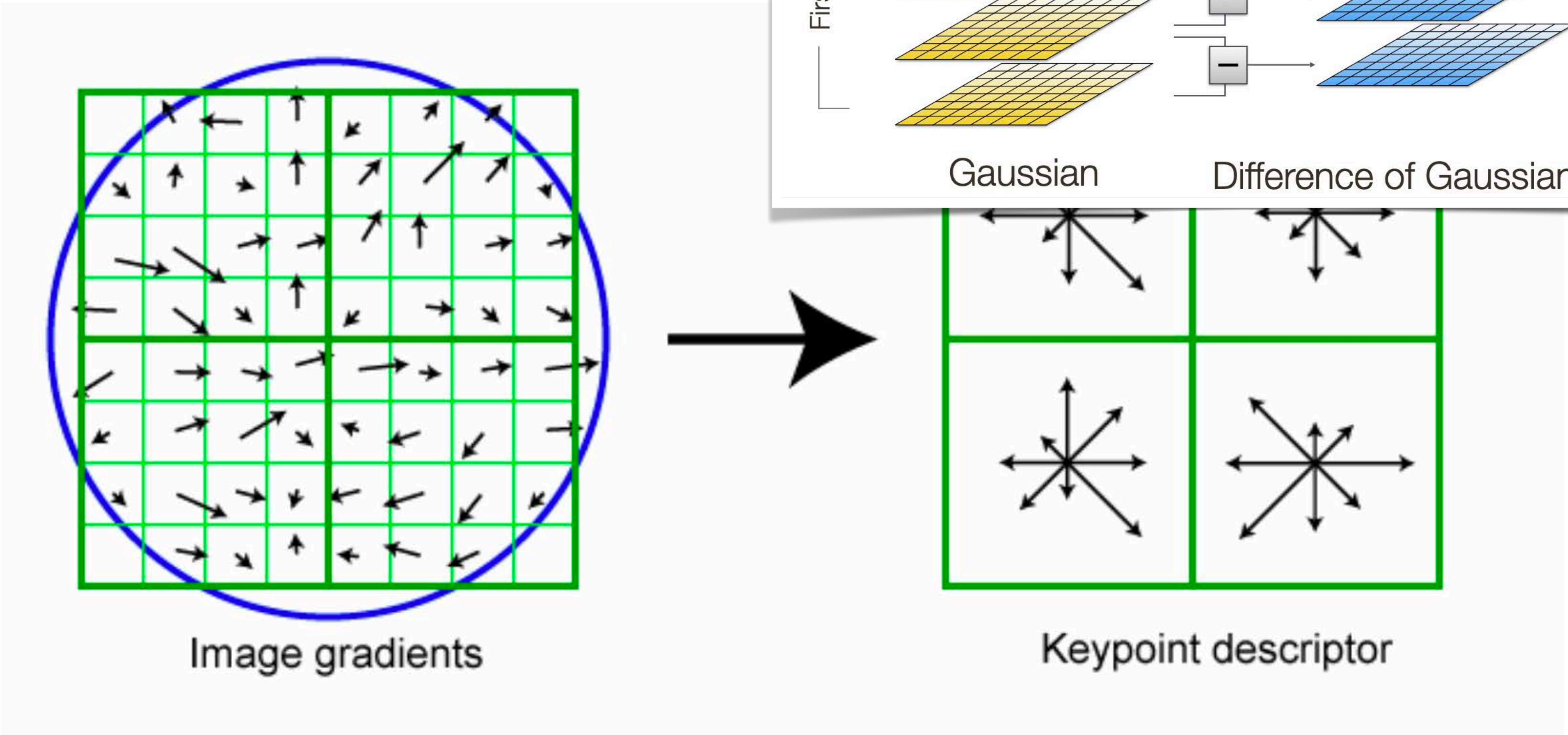

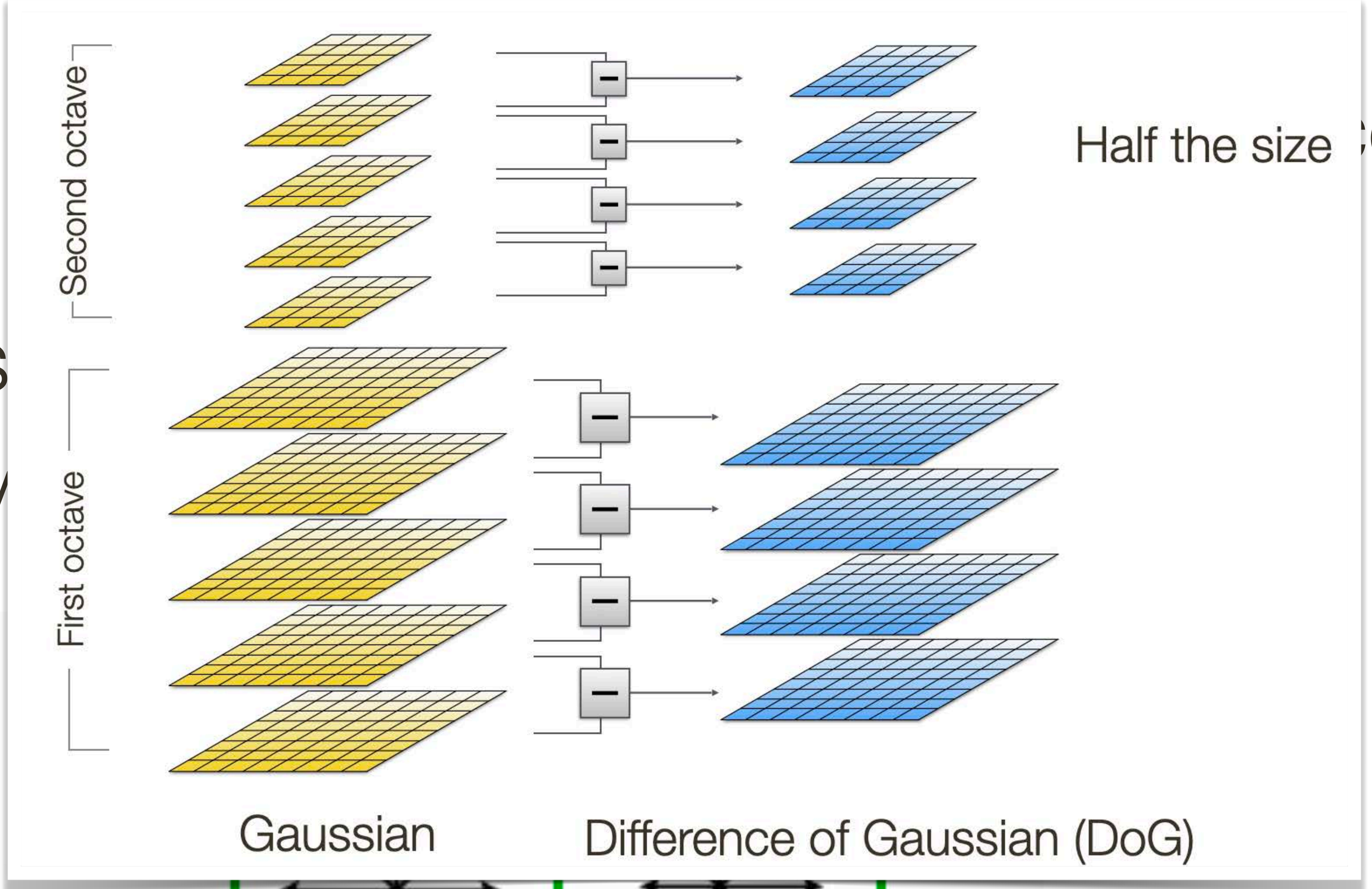

SIFT Matching

Extract features from the image …

Each image might generate 100’s or 1000’s of SIFT descriptors

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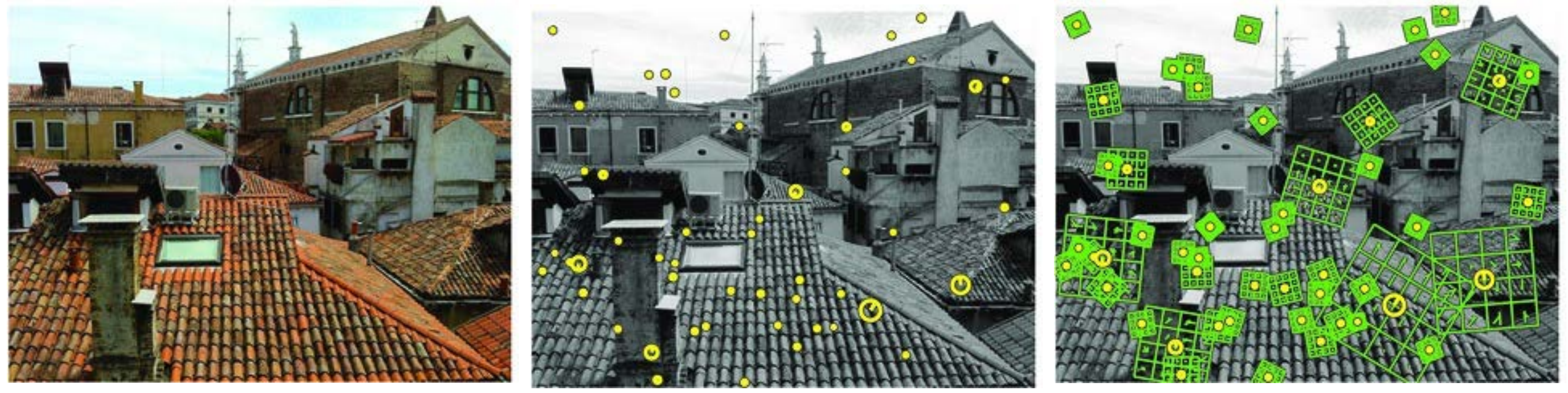
CPSC 425: Computer Vision



Lecture 13: Planar Geometry and RANSAC

Menu for Today

Topics:

- **Planar** Geometry
- **Image Alignment**, Object Recognition
- **RANSAC**

Readings:

- **Today's** Lecture: Szeliski 2.1, 8.1, Forsyth & Ponce 10.4.2

Reminders:

- **Assignment 3:** due **March 6th!**

Learning **Goals**

1. Linear (Projective) Transformations
2. Good results don't happen by chance (or do they?)
3. Good == more support

Image **Alignment**

Aim: warp our images together using a 2D transformation

Image **Alignment**

Aim: warp our images together using a 2D transformation

Image **Alignment**

Find corresponding (matching) points between the images

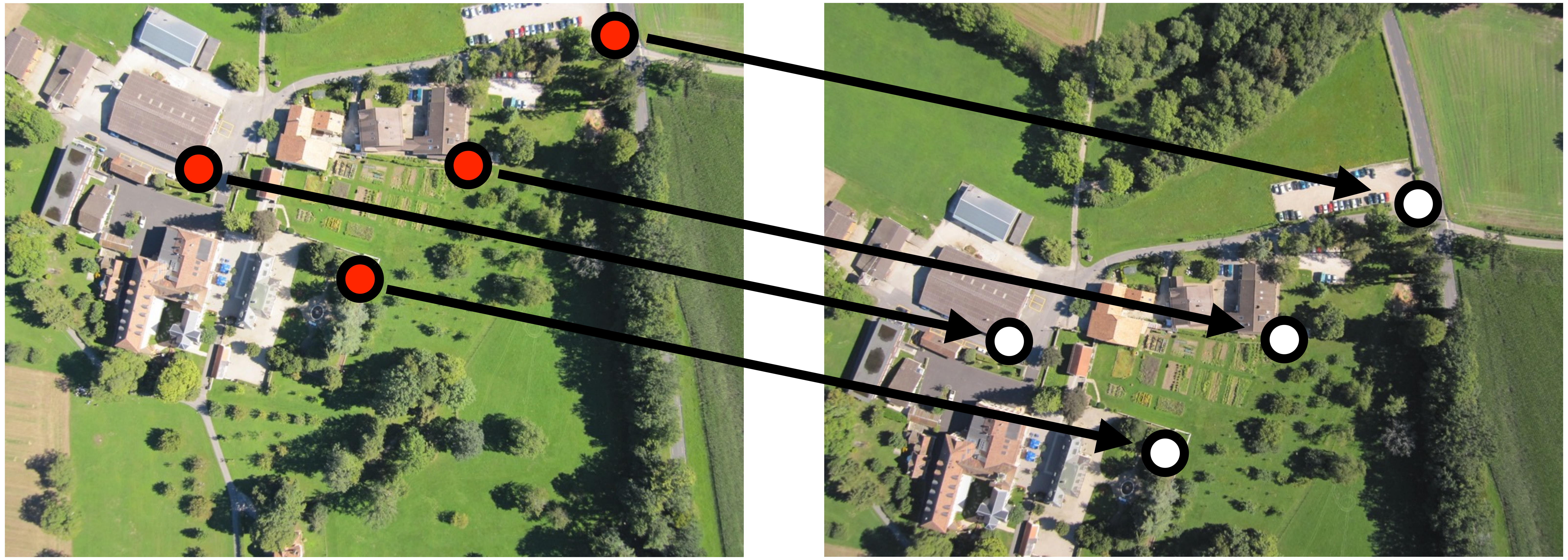


Image **Alignment**

Compute the transformation to align the points

Image **Alignment**

We can also use this transformation to reject outliers

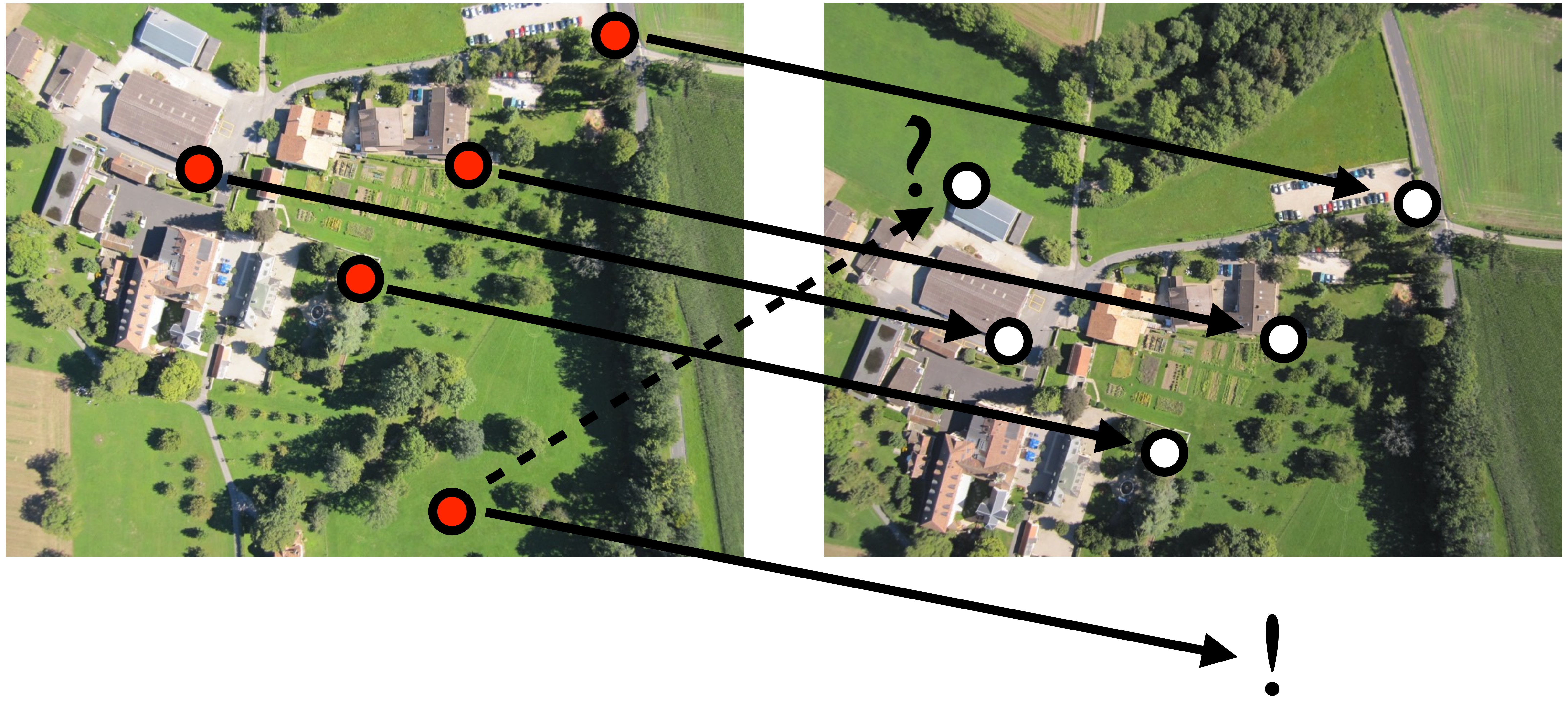


Image **Alignment**

We can also use this transformation to reject outliers

Planar Geometry

— 2D Linear + **Projective** transformations

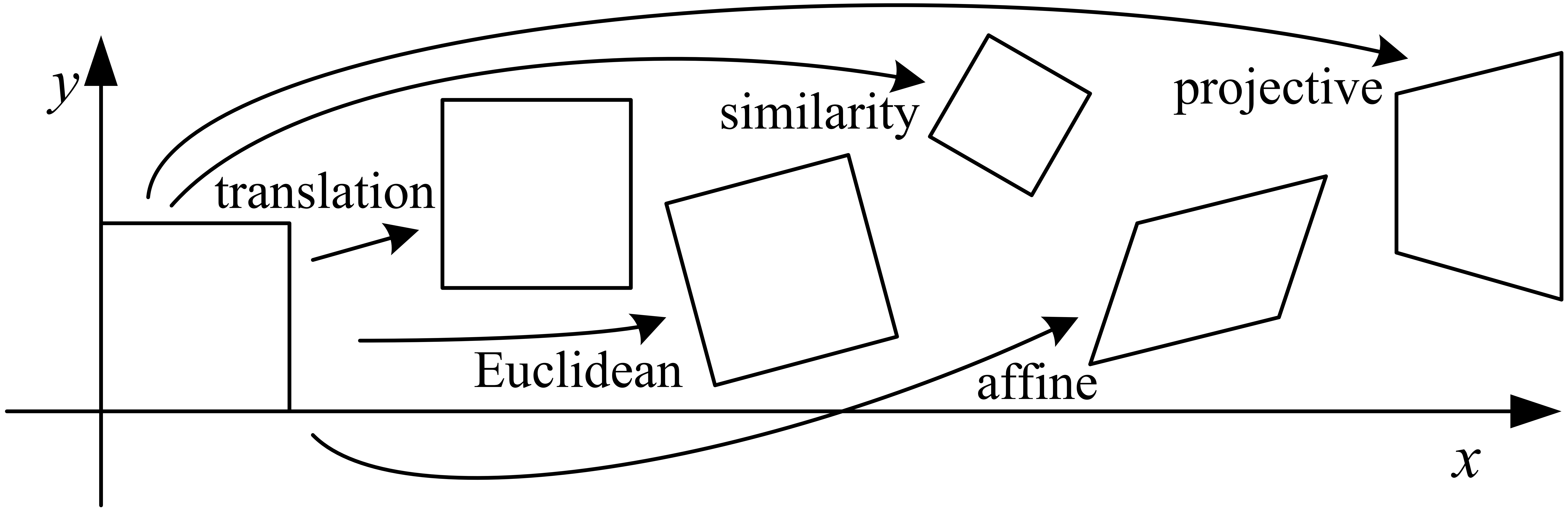
Euclidean, Similarity, Affine, Homography

— Robust Estimation and **RANSAC**

Estimating 2D transforms with noisy correspondences

2D **Transformations**

— We will look at a family that can be represented by 3x3 matrices



— This group represents perspective projections of **planar surfaces**

Affine Transformation

– Transformed points are a **linear function** of the input points

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} a_{13} \\ a_{23} \end{bmatrix}$$

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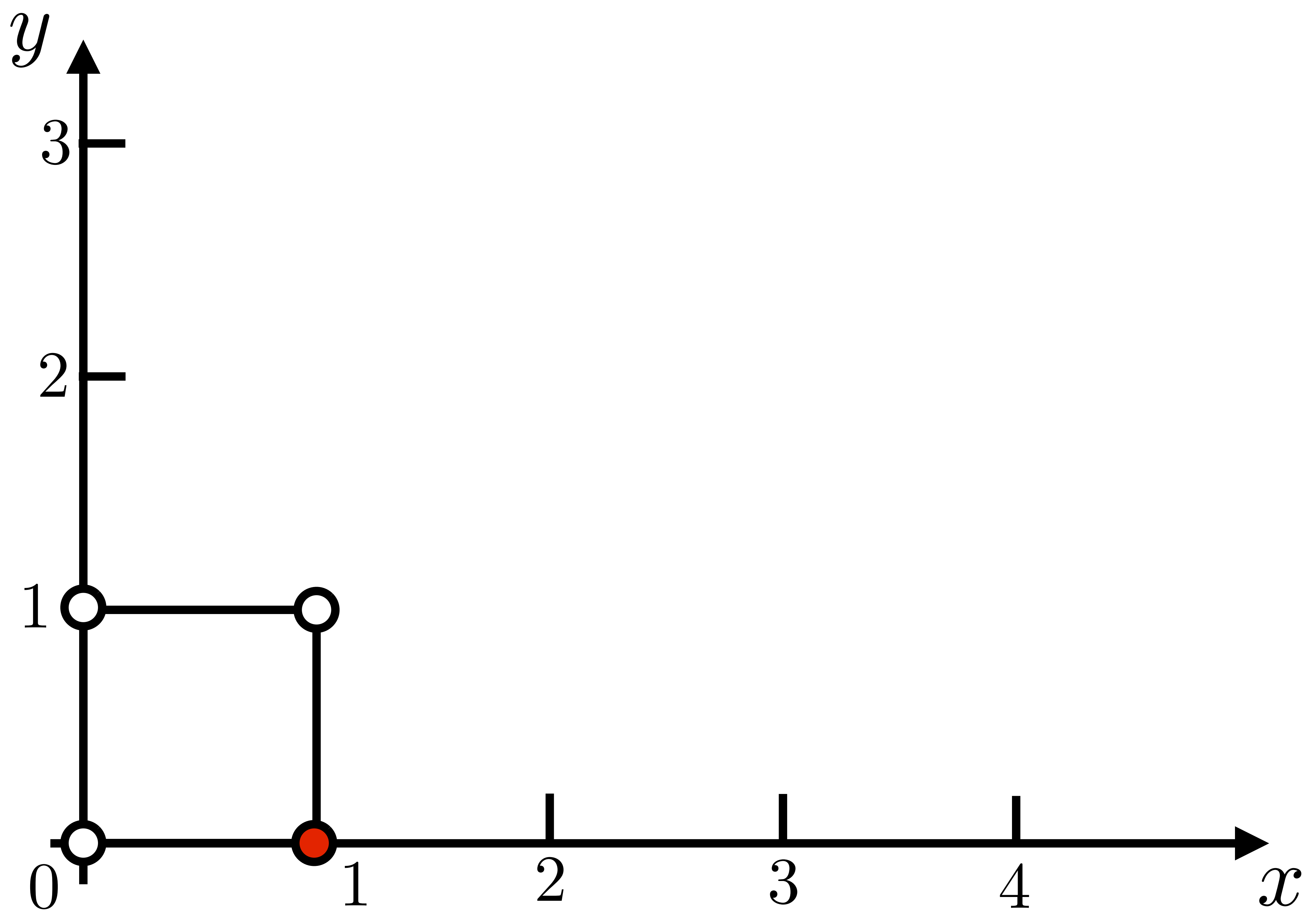
$$\begin{bmatrix} x'_1 \\ y'_1 \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

Linear Transformation

– Consider the action of the unit square under, sample transform $\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$



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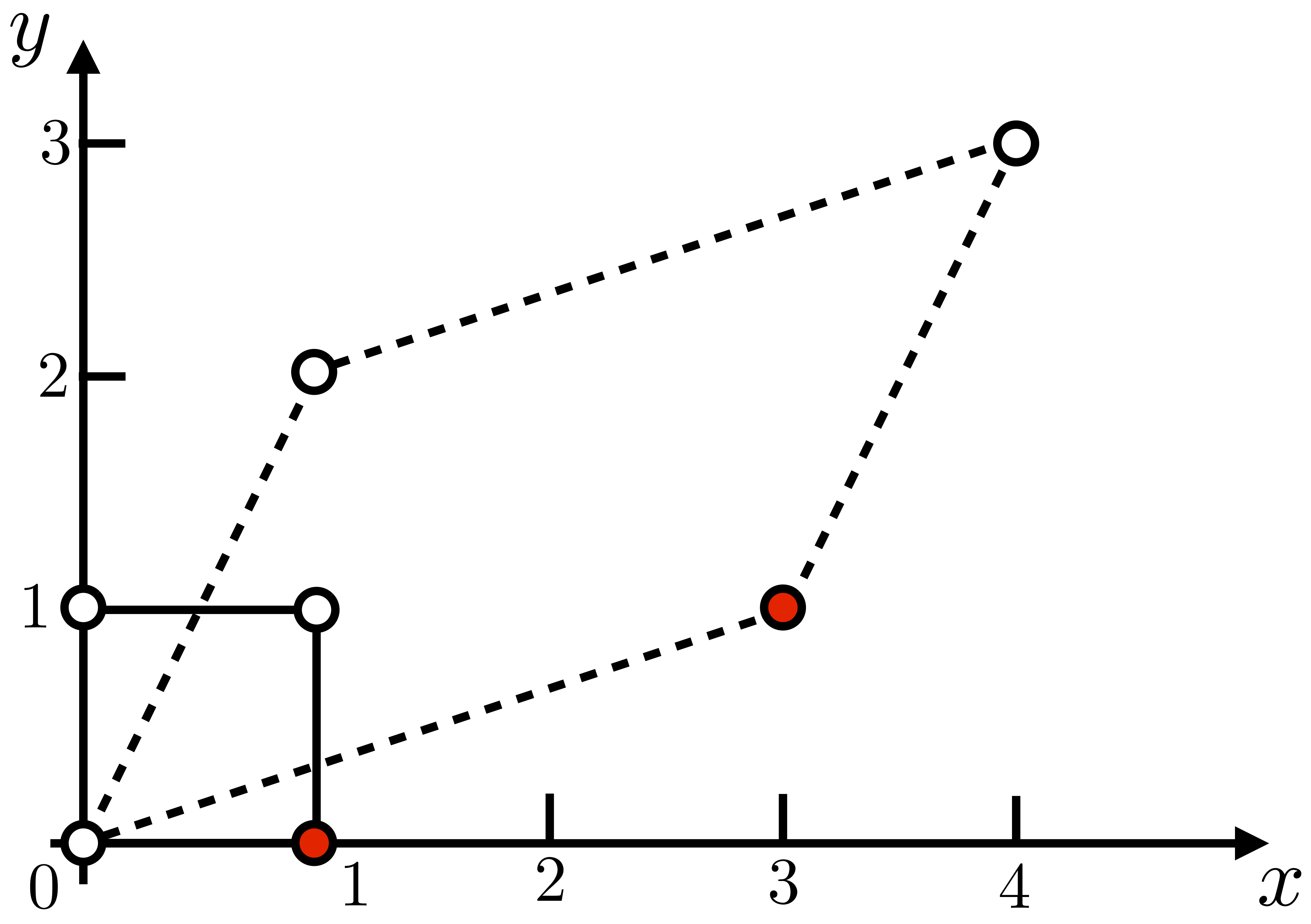
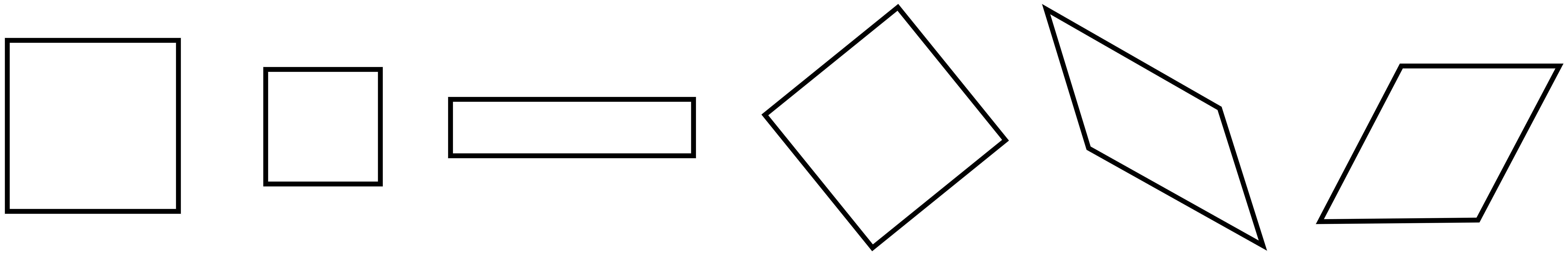


$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 3 & 4 \\ 0 & 2 & 1 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

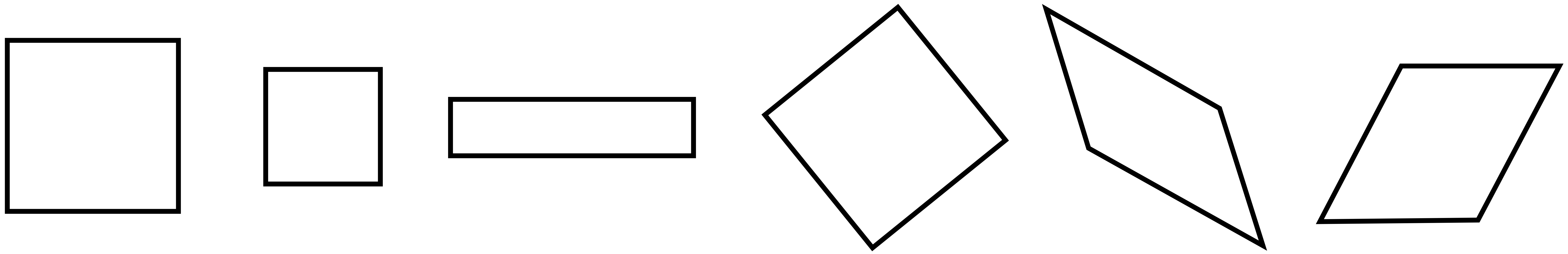
Transformation　　Points　　Transformed Points

Linear (or Affine) Transformations

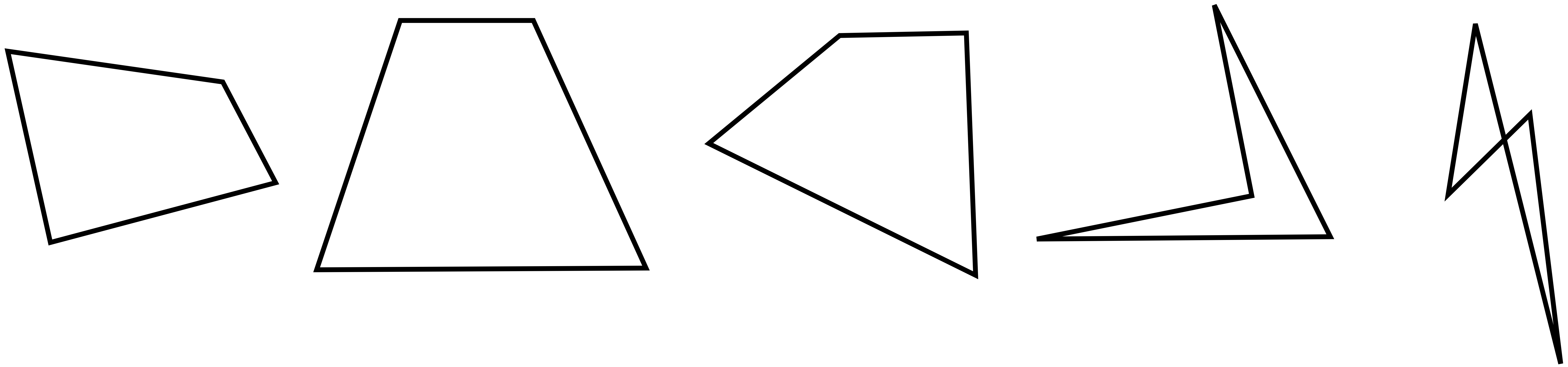


Translation, rotation, scale, shear (parallel lines preserved)

Linear (or Affine) Transformations



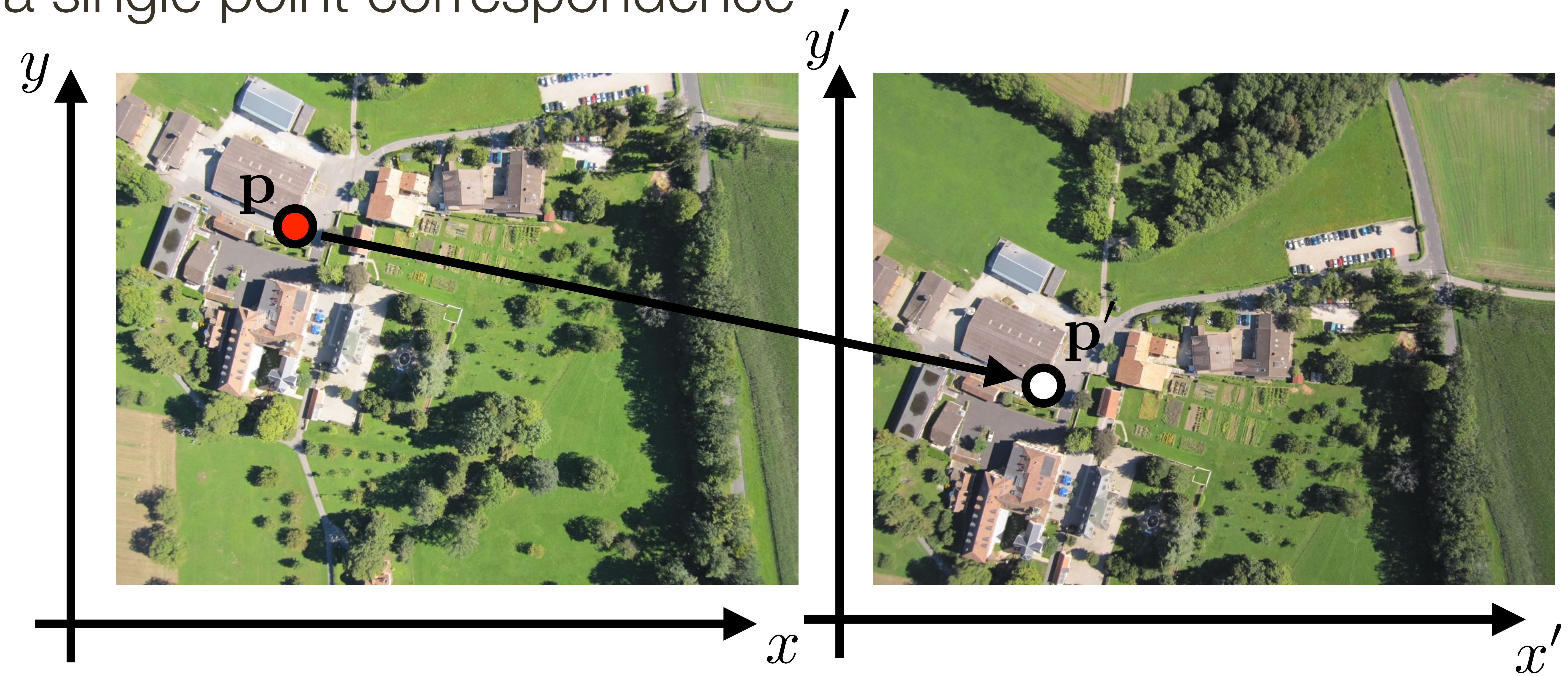
Translation, rotation, scale, shear (parallel lines preserved)



These transforms are not affine (parallel lines not preserved)

Linear (or Affine) Transformations

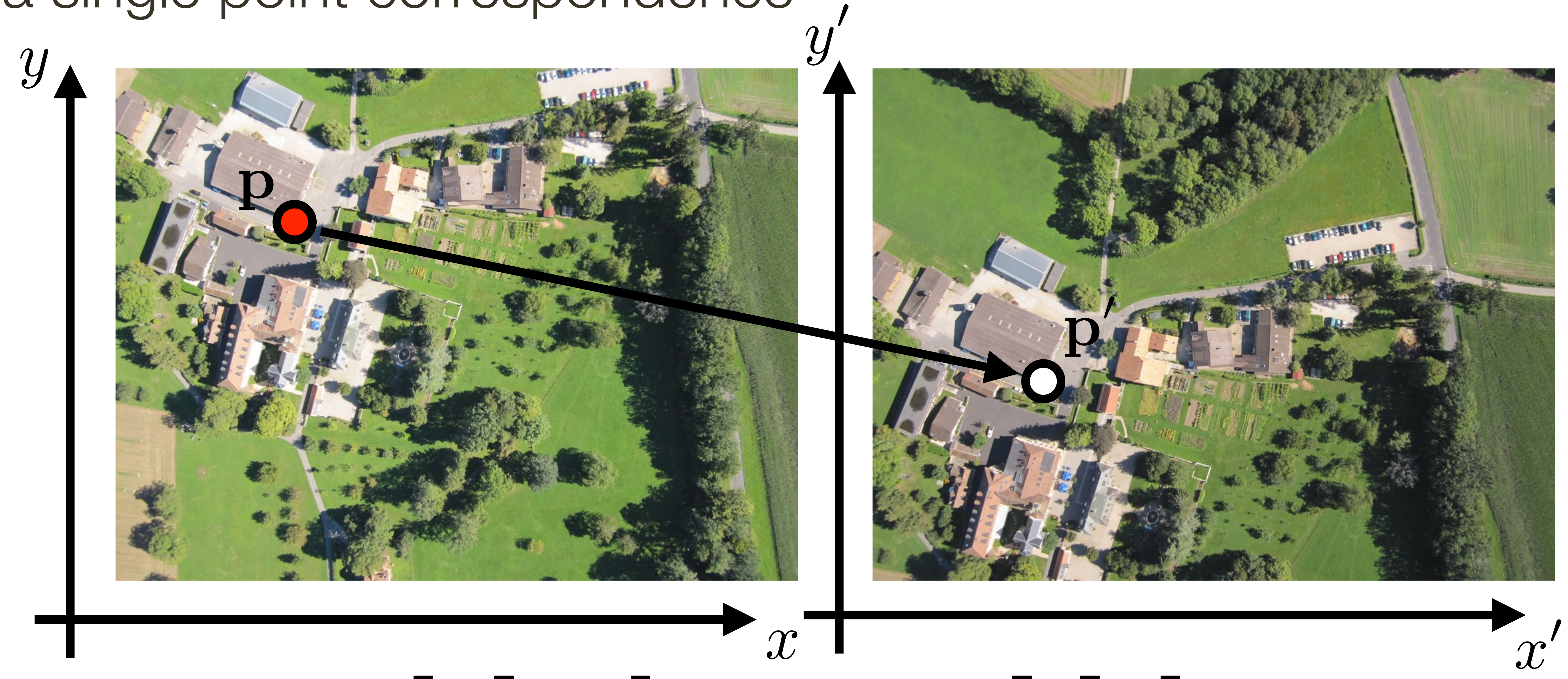
Consider a single point correspondence



$$\begin{bmatrix} x'_1 \\ y'_1 \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

Linear (or Affine) Transformations

Consider a single point correspondence



$$\begin{bmatrix} x_1' \\ y_1' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

How many points are needed to solve for **a**?

Computing Affine Transform

Lets compute an affine transform from correspondences:

$$\begin{bmatrix} x_1' \\ y_1' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

Re-arrange unknowns into a vector

Computing Affine Transform

Lets compute an affine transform from correspondences:

$$\begin{bmatrix} x'_1 \\ y'_1 \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

Re-arrange unknowns into a vector

$$\begin{bmatrix} x'_1 \\ y'_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{21} & a_{22} & a_{23} \end{bmatrix} \begin{bmatrix} 0 & x_1 \\ 0 & y_1 \\ 0 & 1 \\ x_1 & 0 \\ y_1 & 0 \\ 1 & 0 \end{bmatrix}$$

Computing Affine Transform

Linear system in the unknown parameters **a**

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ x_3 & y_3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_3 & y_3 & 1 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{21} \\ a_{22} \\ a_{23} \end{bmatrix} = \begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ x'_3 \\ y'_3 \end{bmatrix}$$

Of the form

$$\mathbf{Ma} = \mathbf{y}$$

Computing Affine Transform

Linear system in the unknown parameters **a**

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ x_3 & y_3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_3 & y_3 & 1 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{21} \\ a_{22} \\ a_{23} \end{bmatrix} = \begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ x'_3 \\ y'_3 \end{bmatrix}$$

Of the form

$$\mathbf{Ma} = \mathbf{y}$$

Solve for **a** using Gaussian Elimination

Computing Affine Transform

Once we solve for a transform, we can now map any <u>other points</u> between the two images … or resample one image in the coordinate system of the other

$$\begin{bmatrix} x_1' \\ y_1' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

Computing Affine Transform

Once we solve for a transform, we can now map any <u>other points</u> between the two images … or resample one image in the coordinate system of the other

This allows us to "stitch" the two images

Linear Transformations

Other linear transforms are special cases of **affine** transform:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

Linear Transformations

Other linear transforms are special cases of **affine** transform:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

e.g.,

$$\begin{bmatrix} 1 & 0 & a_{13} \\ 0 & 1 & a_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

translation transform

Linear Transformations

Other linear transforms are special cases of **affine** transform:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

e.g., $\begin{bmatrix} \cos\theta & \sin\theta & a_{13} \\ -\sin\theta & \cos\theta & a_{23} \\ 0 & 0 & 1 \end{bmatrix}$ **euclidian** transform

Linear Transformations

Other linear transforms are special cases of **affine** transform:

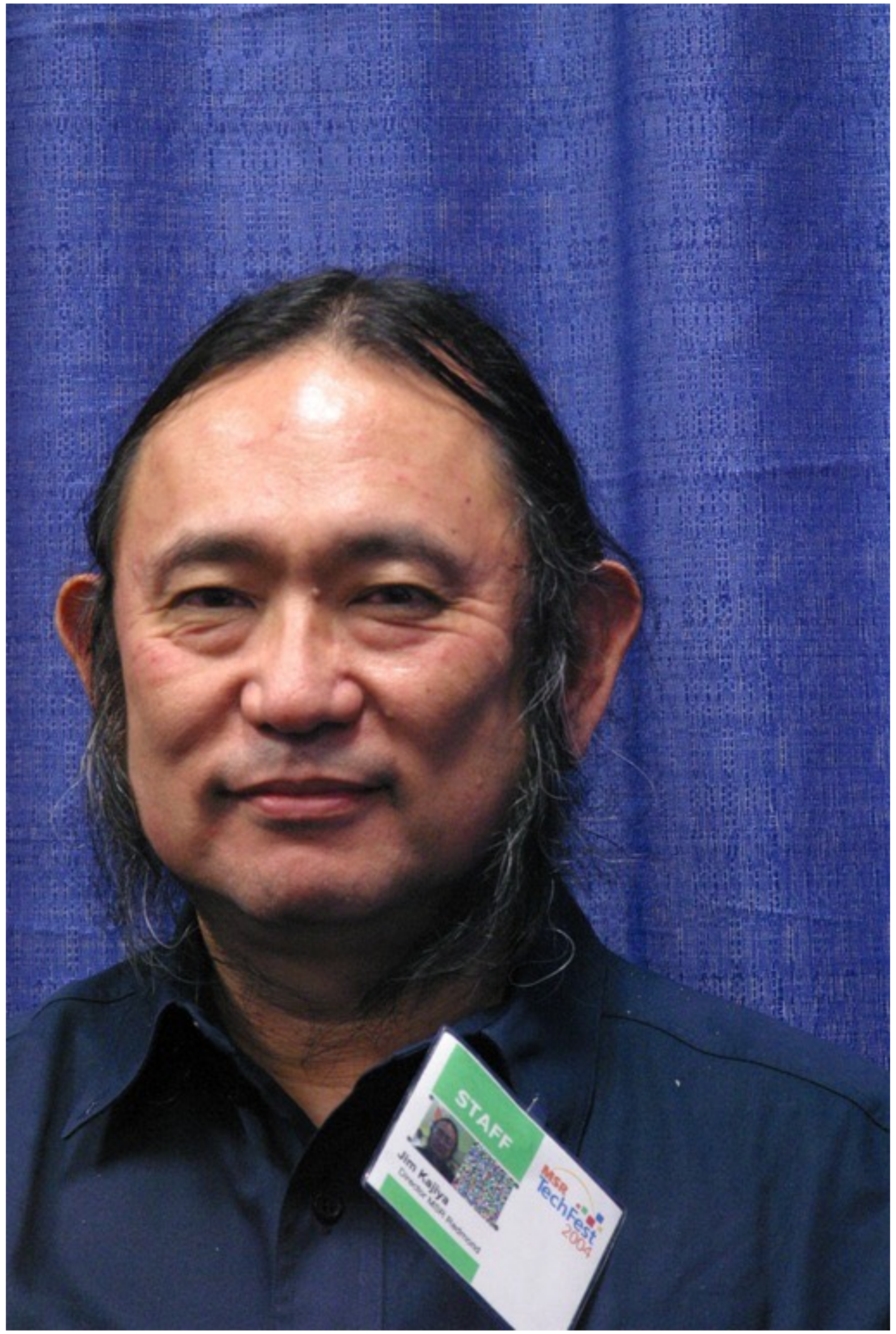
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

e.g.,

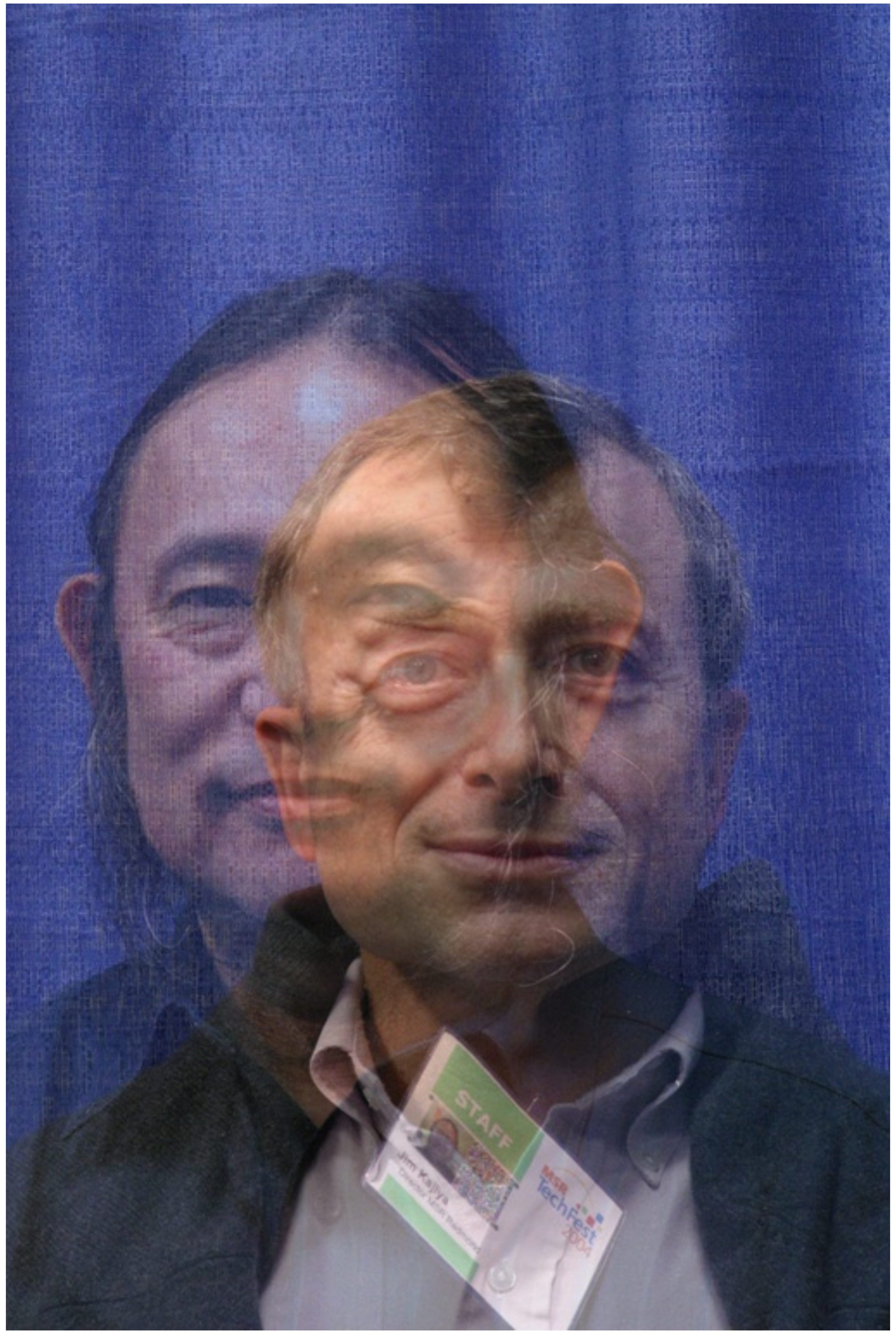
$$\begin{bmatrix} s\cos\theta & s\sin\theta & a_{13} \\ -s\sin\theta & s\cos\theta & a_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

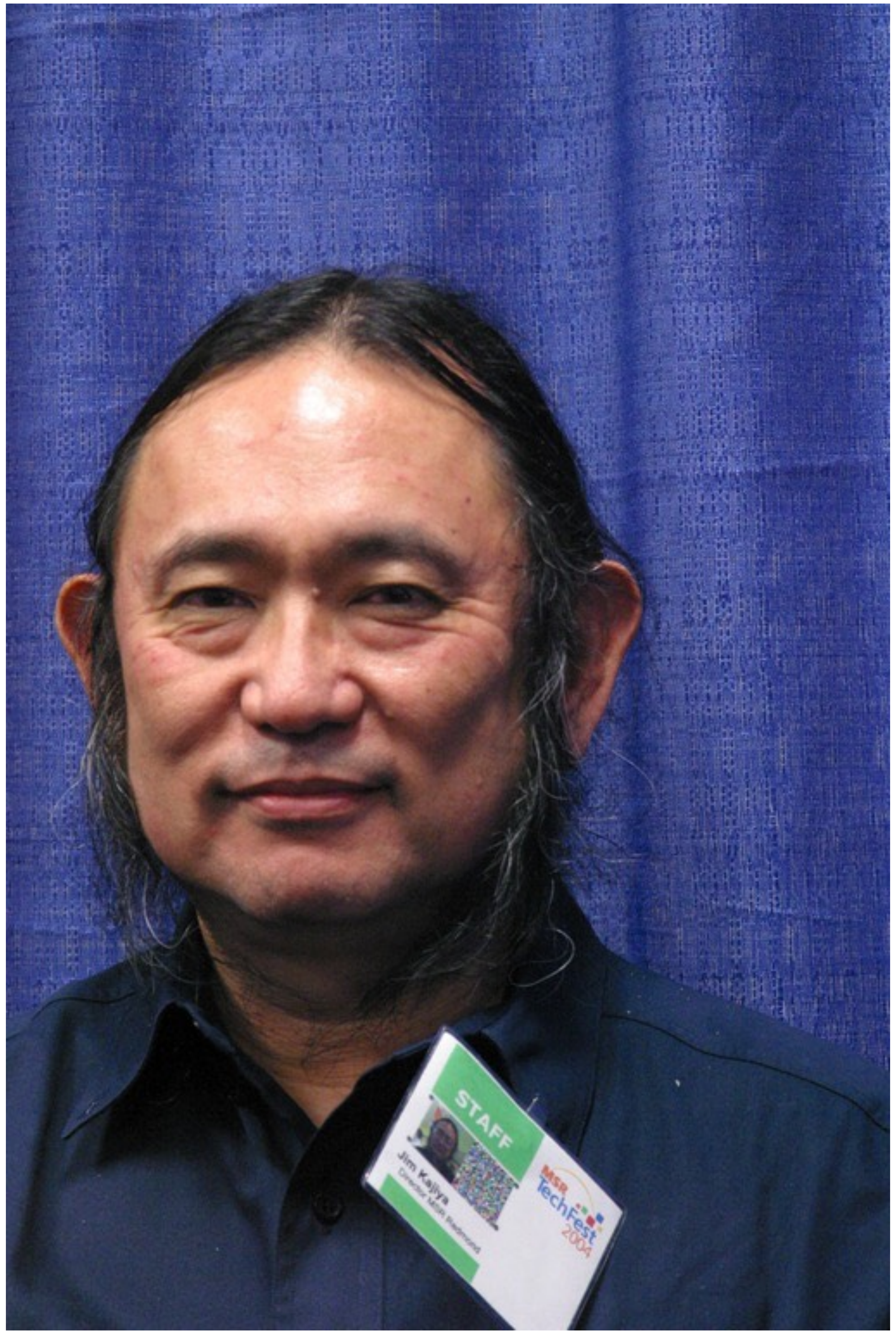
similarity transform

Face Alignment



Face Alignment





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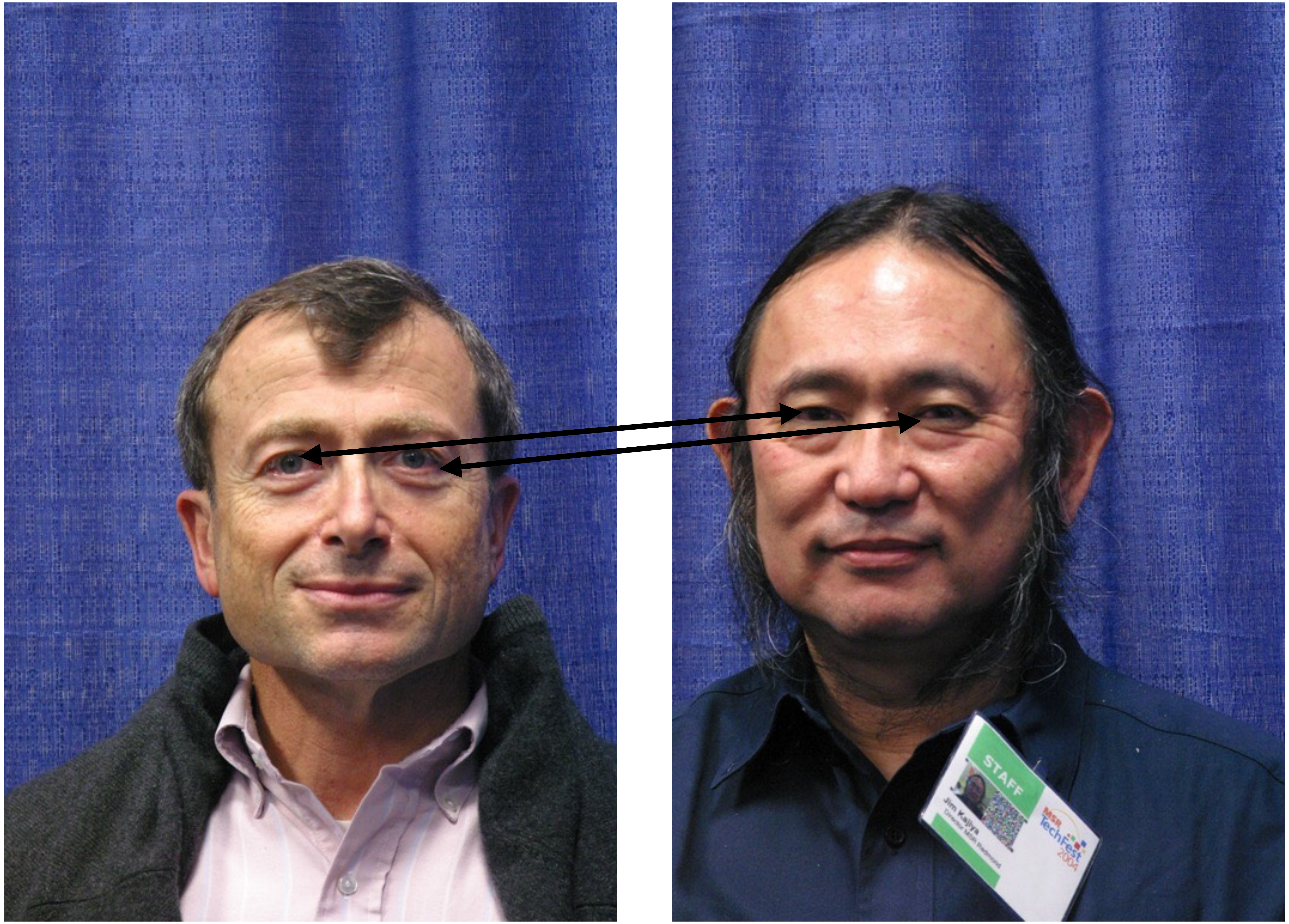

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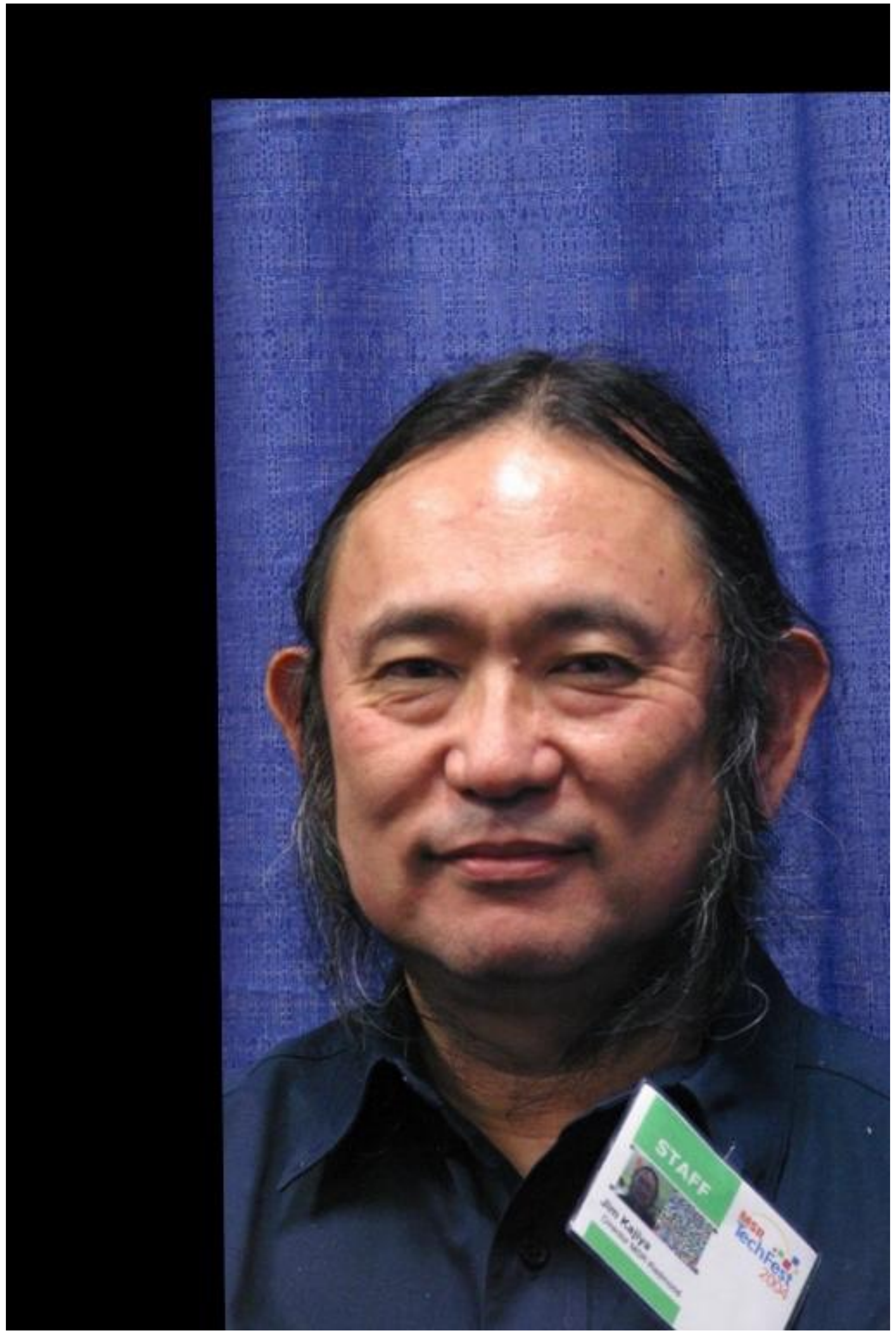

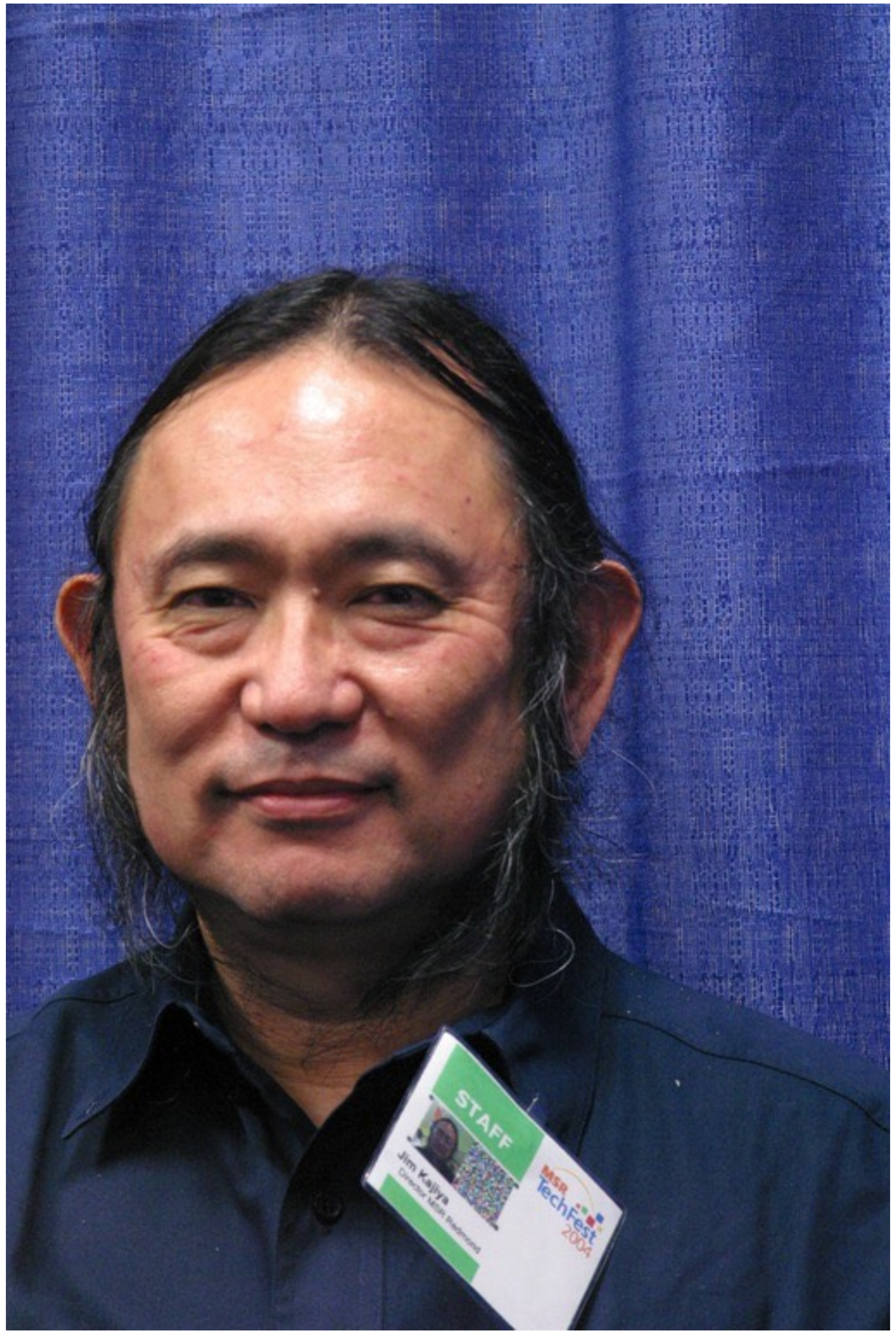

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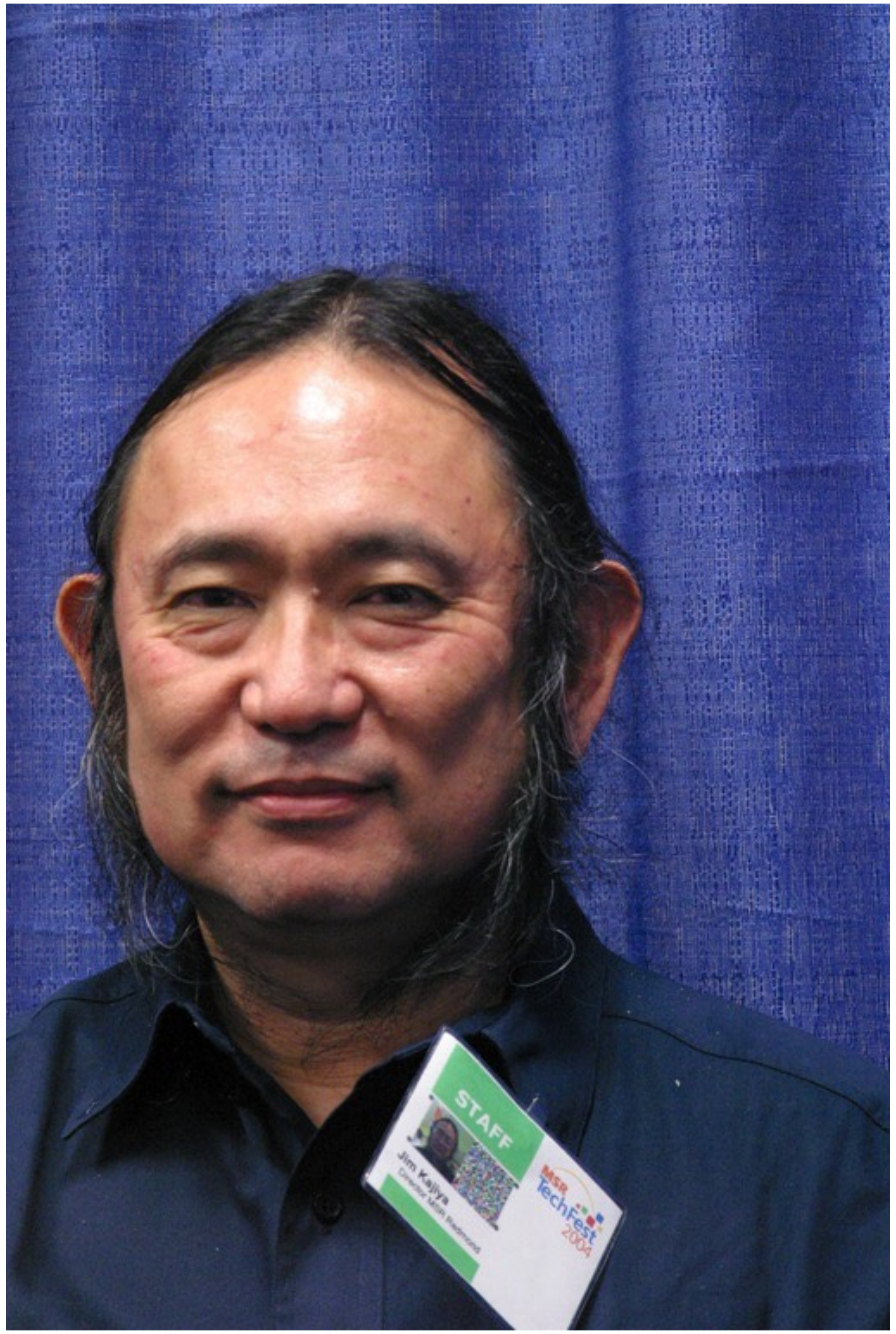

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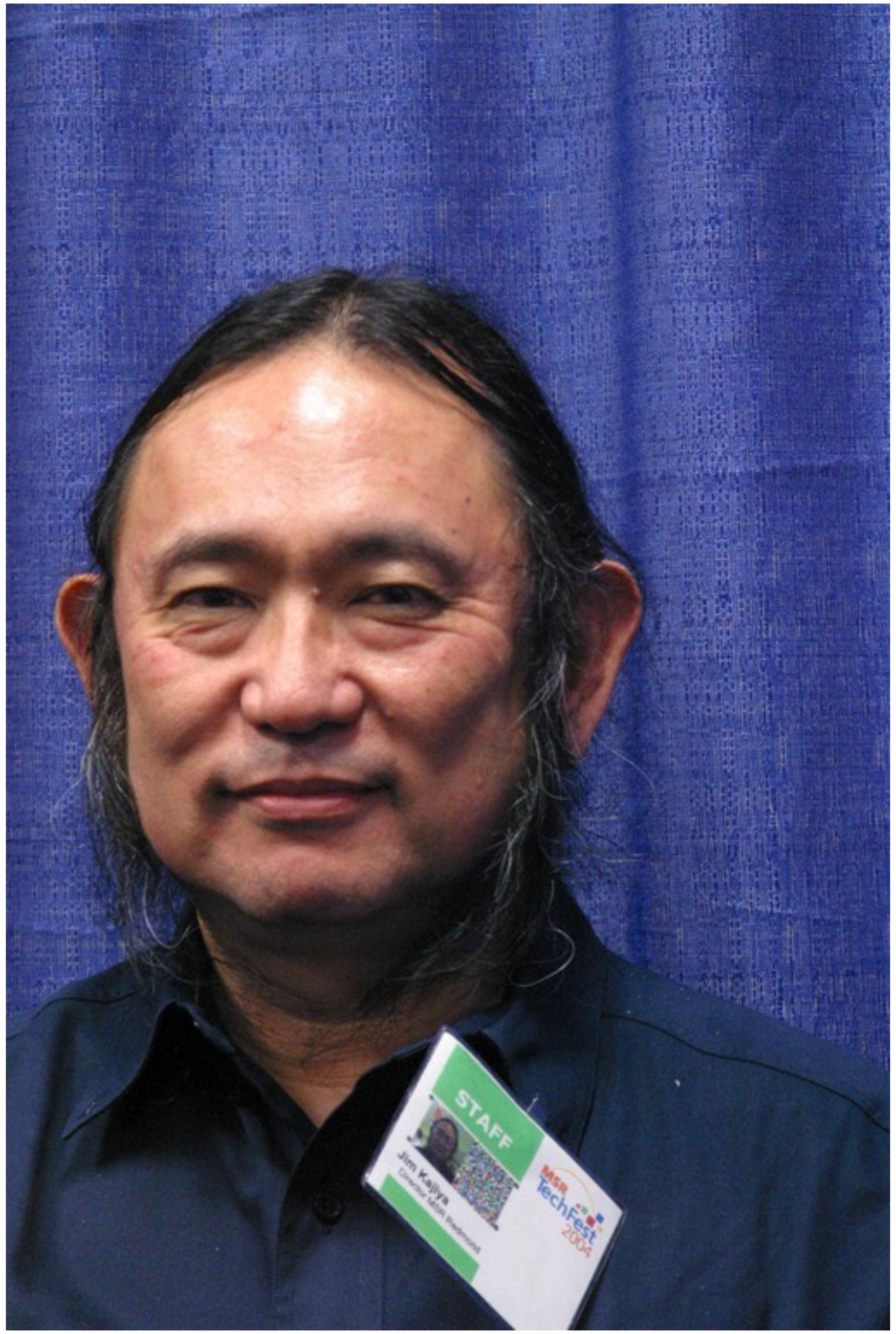
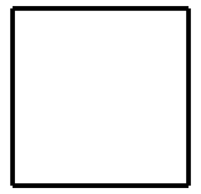
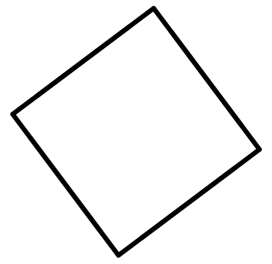
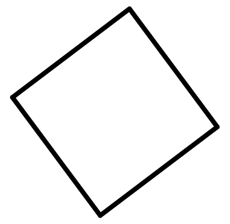
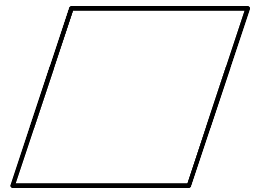
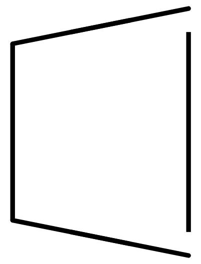


2D **Transformations**

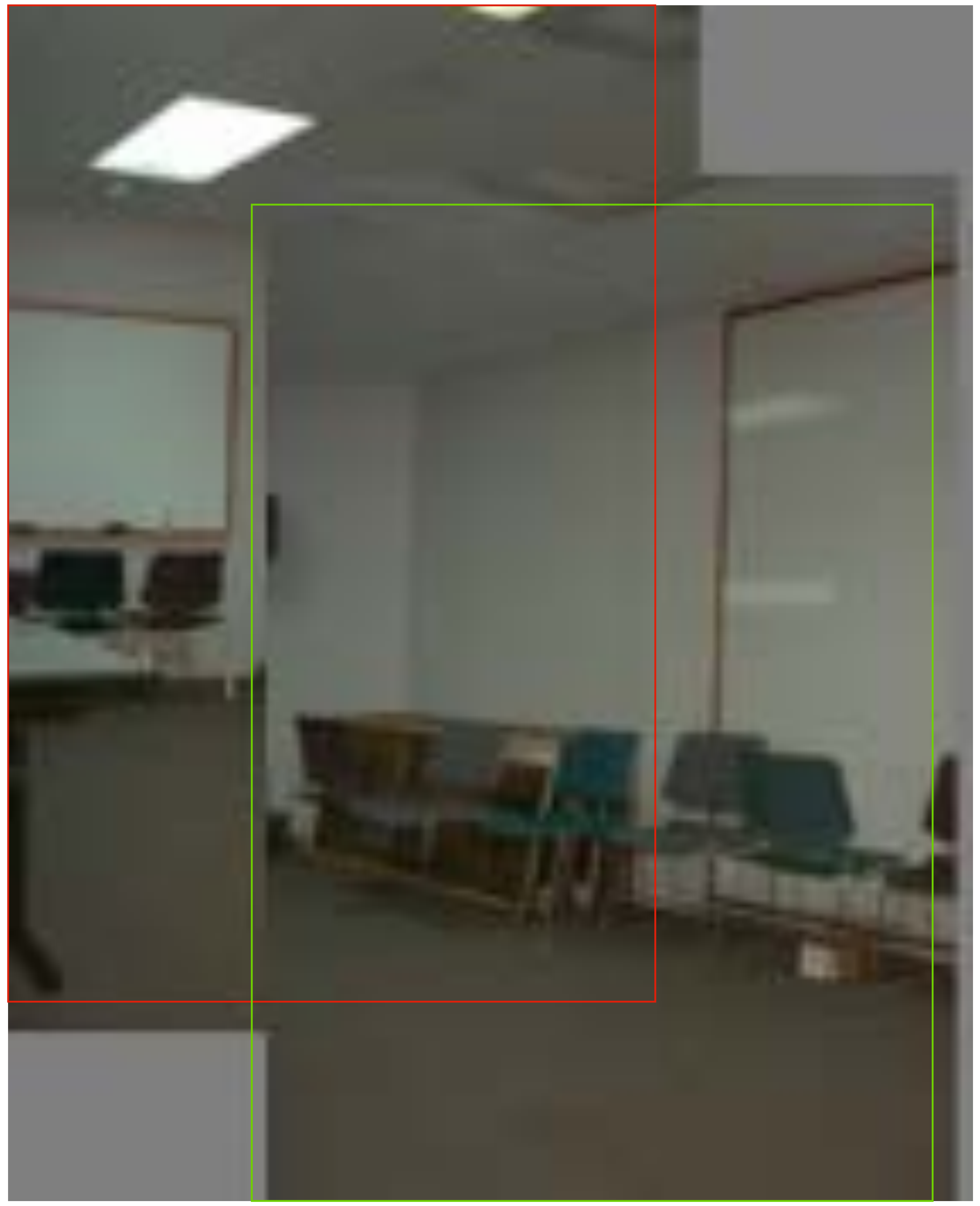
Transformation	Matrix	# DoF	Preserves	Icon
translation	$\left[\begin{array}{c\|c} \boldsymbol{I} & \boldsymbol{t} \end{array}\right]_{2\times 3}$	2	orientation	
rigid (Euclidean)	$\left[\begin{array}{c\|c} \boldsymbol{R} & \boldsymbol{t} \end{array}\right]_{2\times 3}$	3	lengths	
similarity	$\left[\begin{array}{c\|c} s\boldsymbol{R} & \boldsymbol{t} \end{array}\right]_{2\times 3}$	4	angles	
affine	$\left[\begin{array}{c} \boldsymbol{A} \end{array}\right]_{2\times 3}$	6	parallelism	
projective	$\left[\begin{array}{c} \tilde{\boldsymbol{H}} \end{array}\right]_{3\times 3}$	8	straight lines	

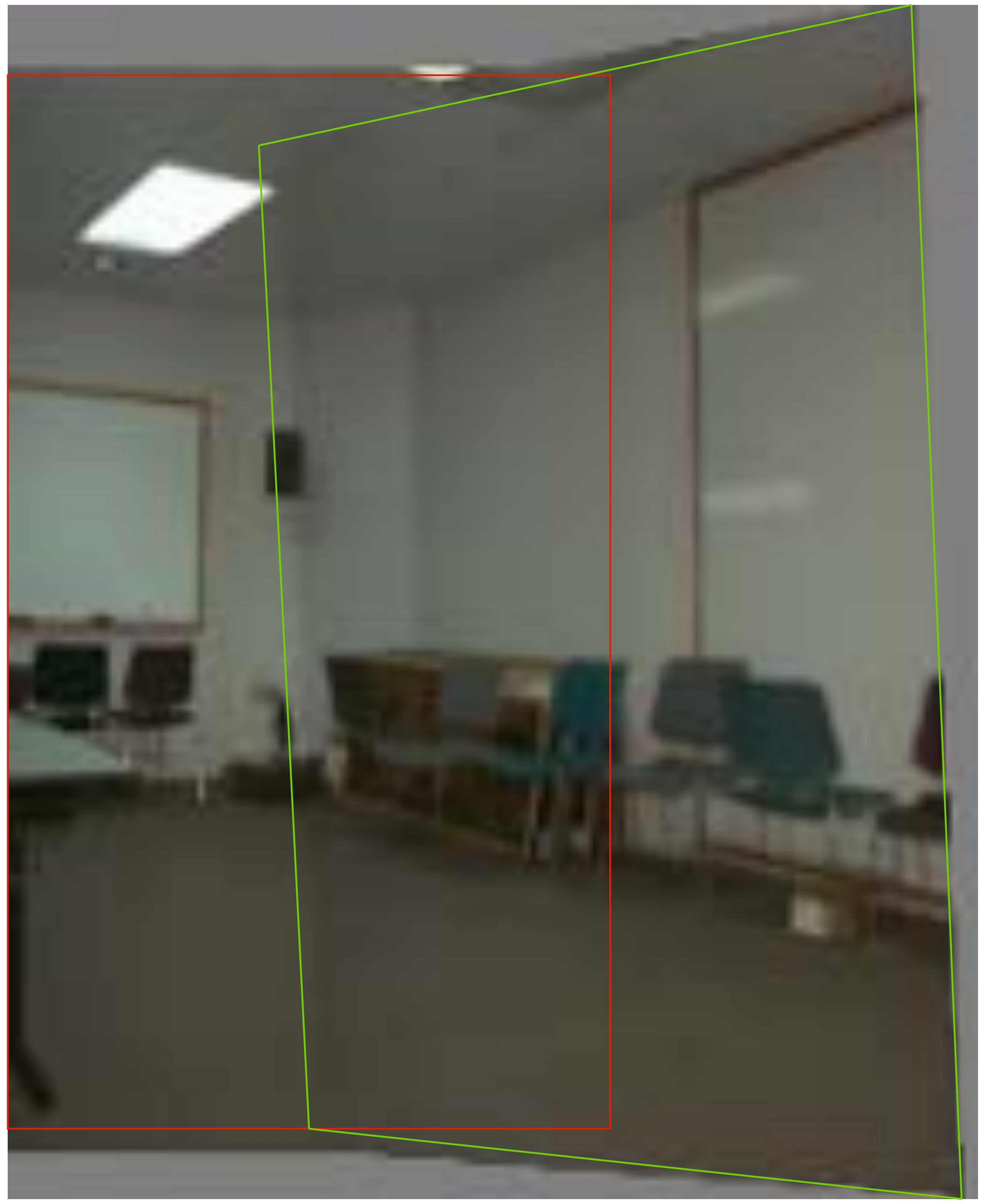
Example: Warping with Different Transformations

Translation

Affine

Projective (homography)





Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Aside: We can use homographies when …

1.… the scene is planar; or

2.… the scene is very far or has small (relative) depth variation → scene is approximately planar

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Aside: We can use homographies when …

3.… the scene is captured under camera rotation only (no translation or pose change)

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Projective Transformation

General 3x3 matrix transformation

$$\begin{bmatrix} x_1' \\ y_1' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

Projective Transformation

General 3x3 matrix transformation

$$\begin{bmatrix} x'_1 \\ y'_1 \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

Lets try an example:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \mathbf{H} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}}_{\text{Transformation}} \underbrace{\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}}_{\text{Points}} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 2 & 1 & 2 \end{bmatrix}}_{\text{Transformed Points}}$$

Projective Transformation

General 3x3 matrix transformation

$$\begin{bmatrix} x'_1 \\ y'_1 \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

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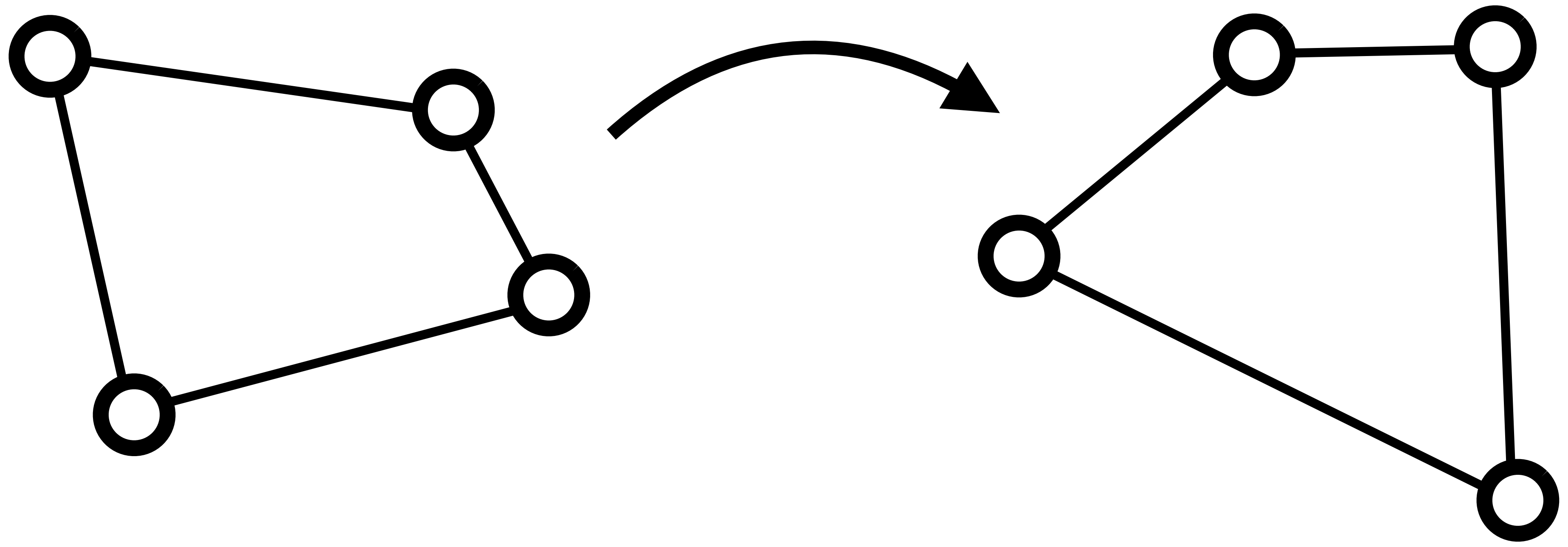
Divide by the last row: $\begin{bmatrix} 0 & 0 & 1 & 0.5 \\ 0 & 0.5 & 0 & 0.5 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

Compute **H** from Correspondences

Each match gives 2 equations to solve for **8** parameters

$$\begin{bmatrix} x_1' \\ y_1' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$



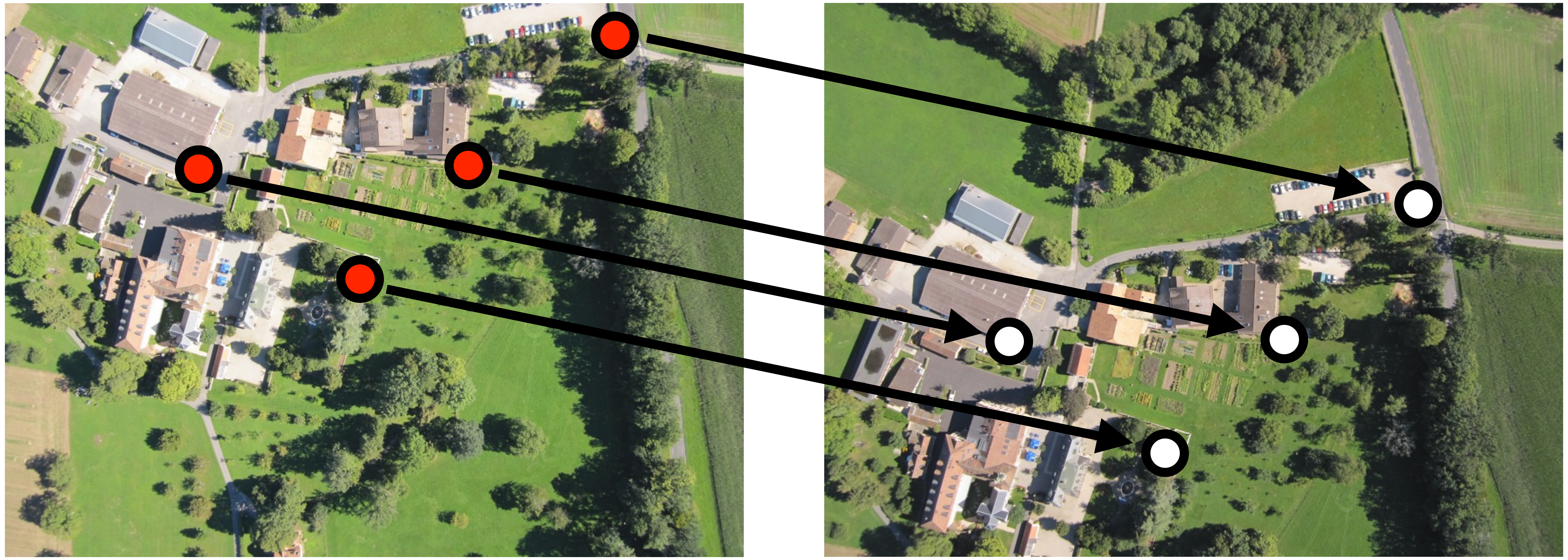
→ 4 correspondences to solve for **H** matrix

Solution uses **Singular Value Decomposition** (SVD)

In **Assignment 4** you can compute this using `cv2.findHomography`

Image **Alignment**

Find **corresponding** (matching) points between the image



$$\mathbf{u} = \mathbf{H}\mathbf{x}$$

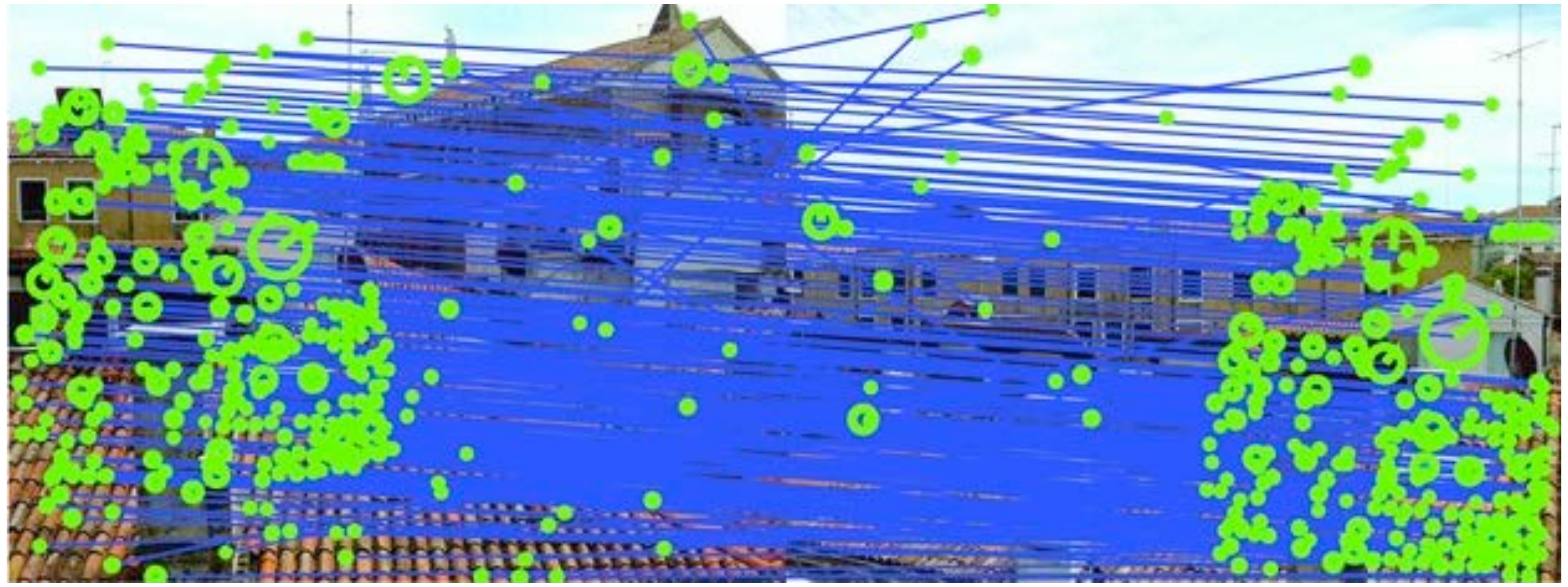
2 points for Similarity

3 for Affine

4 for Homography

Image **Alignment**

In practice we have many noisy correspondences + **outliers**



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e.g., for an affine transform we have a linear system in the parameters **a**:

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ x_3 & y_3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_3 & y_3 & 1 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{21} \\ a_{22} \\ a_{23} \end{bmatrix} = \begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ x'_3 \\ y'_3 \end{bmatrix}$$

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It is **overconstrained** (more equations than unknowns) and subject to **outliers** (some rows are completely wrong)

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Let's deal with these problems in a simpler context …

Fitting a Model to Noisy Data

If we draw pairs of points uniformly at random, what fraction of pairs will consist entirely of ‘good’ data points (inliers)?

Fitting a Model to Noisy Data

Suppose we are **fitting a line** to a dataset that consists of 50% outliers

We can fit a line using two points

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— A better estimate of the line can be obtained by refitting the line to the points that lie close to the line

RANSAC (**RAN**dom **SA**mple **C**onsensus)

1. Randomly choose minimal subset of data points necessary to fit model (a **sample**)

2. Points within some distance threshold, t, of model are a **consensus set**. Size of consensus set is model's **support**

3. Repeat for N samples; model with biggest support is most robust fit
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Slide Credit: Christopher Rasmussen

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RANSAC is very useful for variety of applications

Slide Credit: Christopher Rasmussen

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   Fitting a Line: 2 points

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Example 1: Fitting a Line

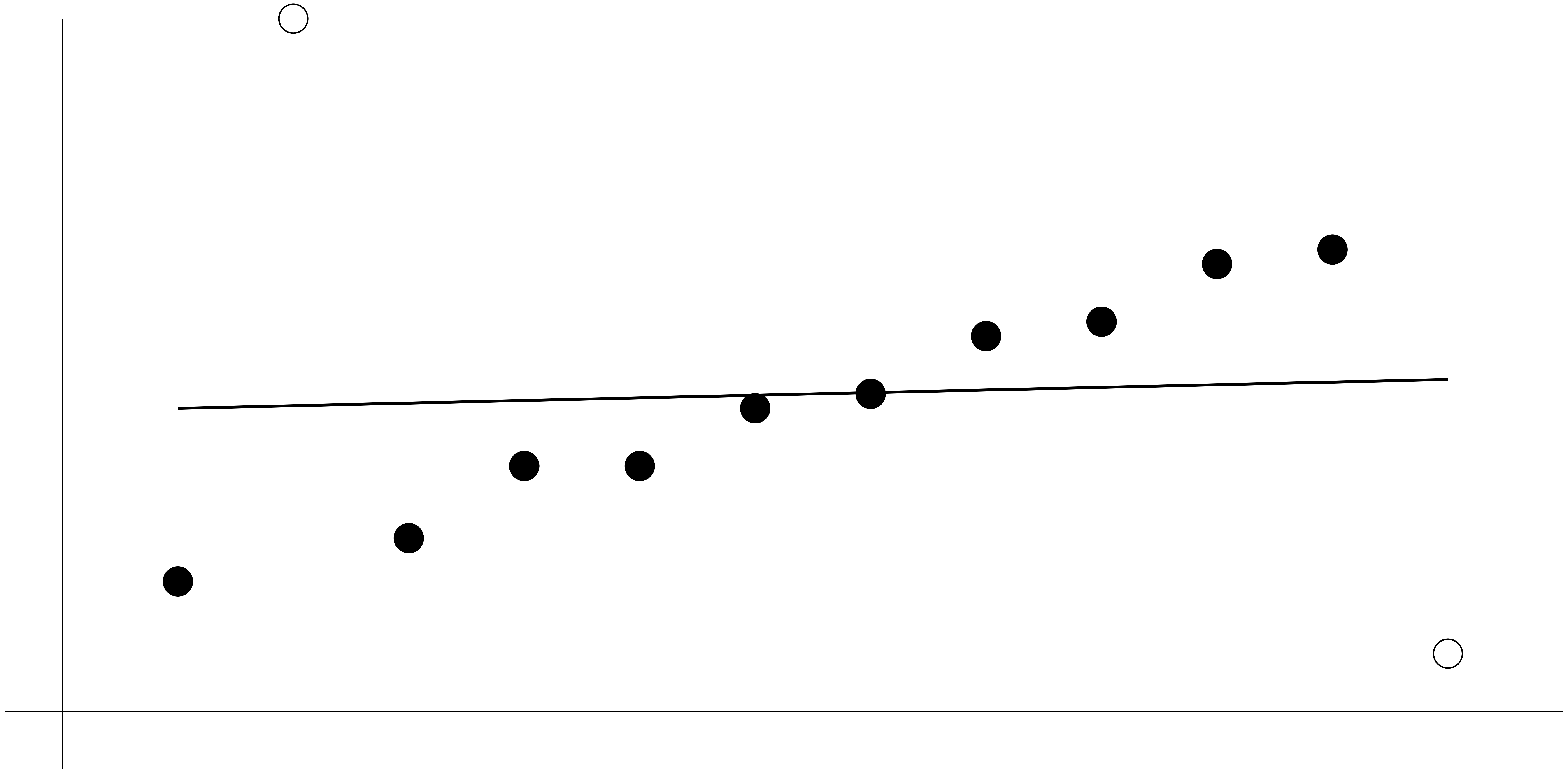

Figure Credit: Hartley & Zisserman

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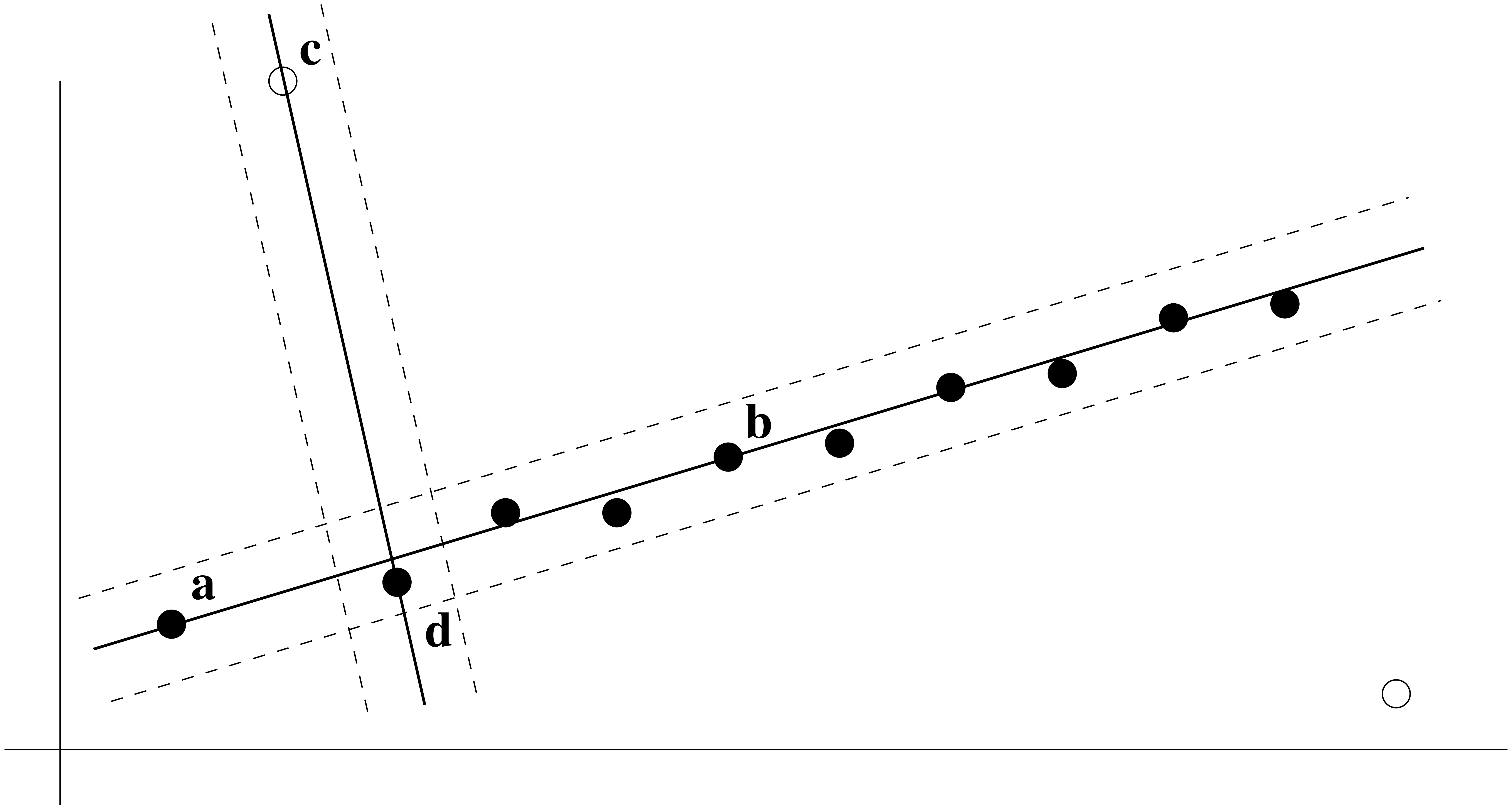

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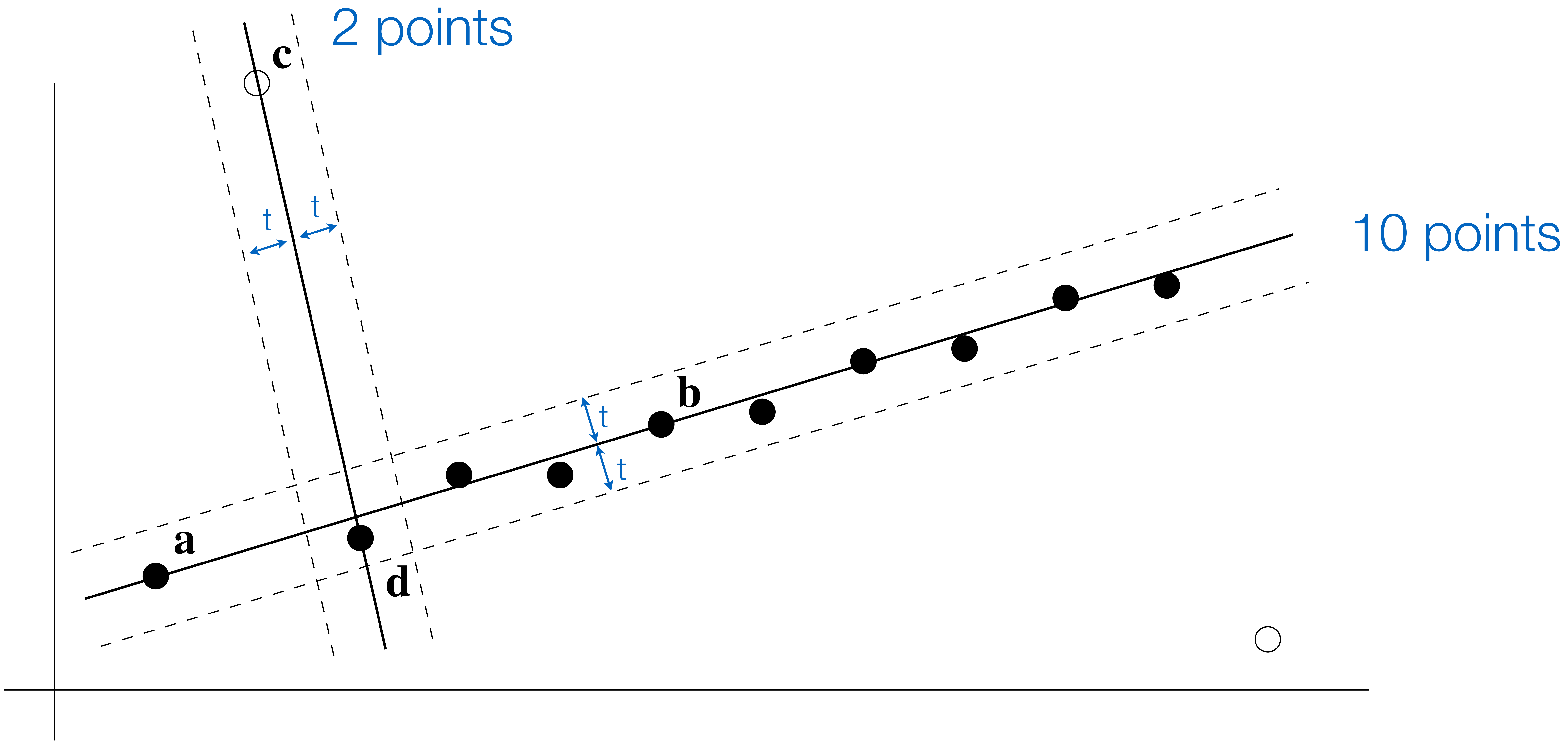


Figure Credit: Hartley & Zisserman

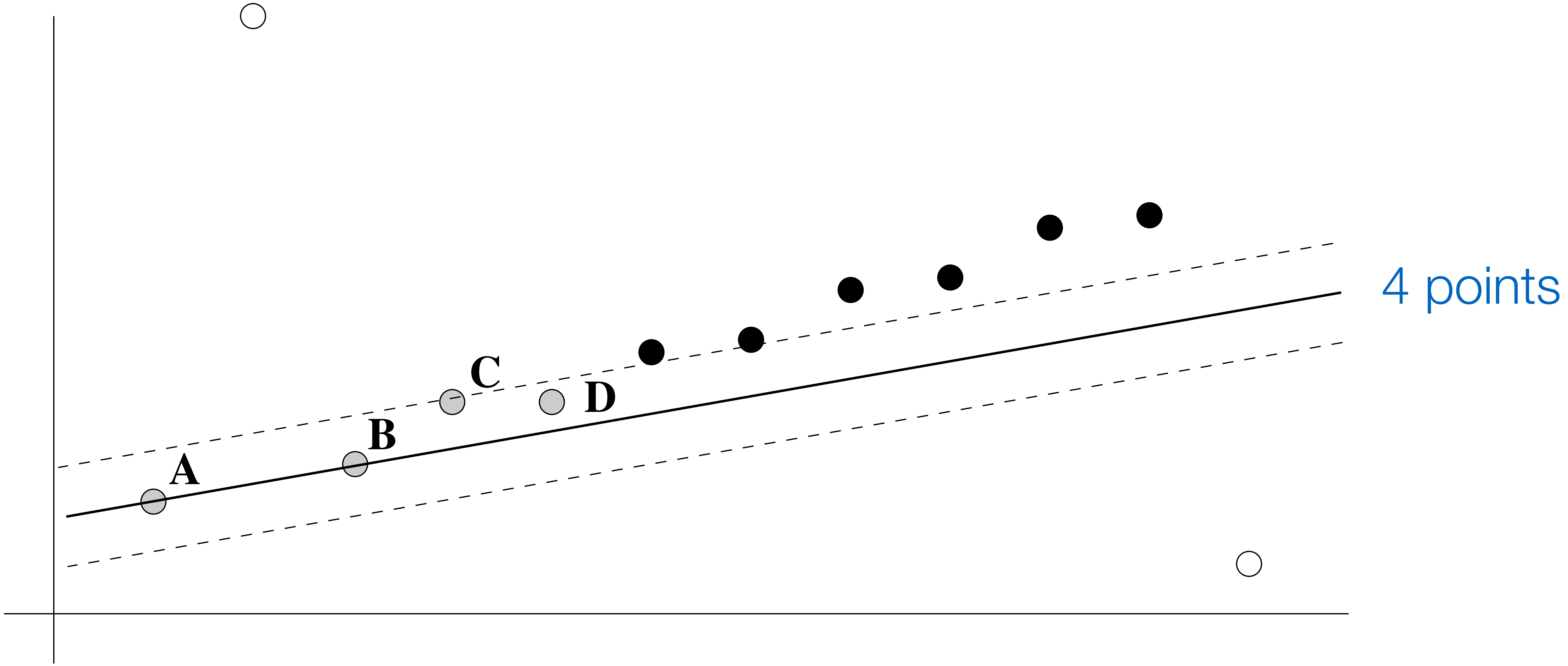
After RANSAC

RANSAC divides data into inliers and outliers and yields estimate computed from minimal set of inliers

Improve this initial estimate with estimation over all inliers (e.g., with standard least-squares minimization)

But this may change inliers, so alternate fitting with re-classification as inlier/outlier

Example 2: Fitting a Line



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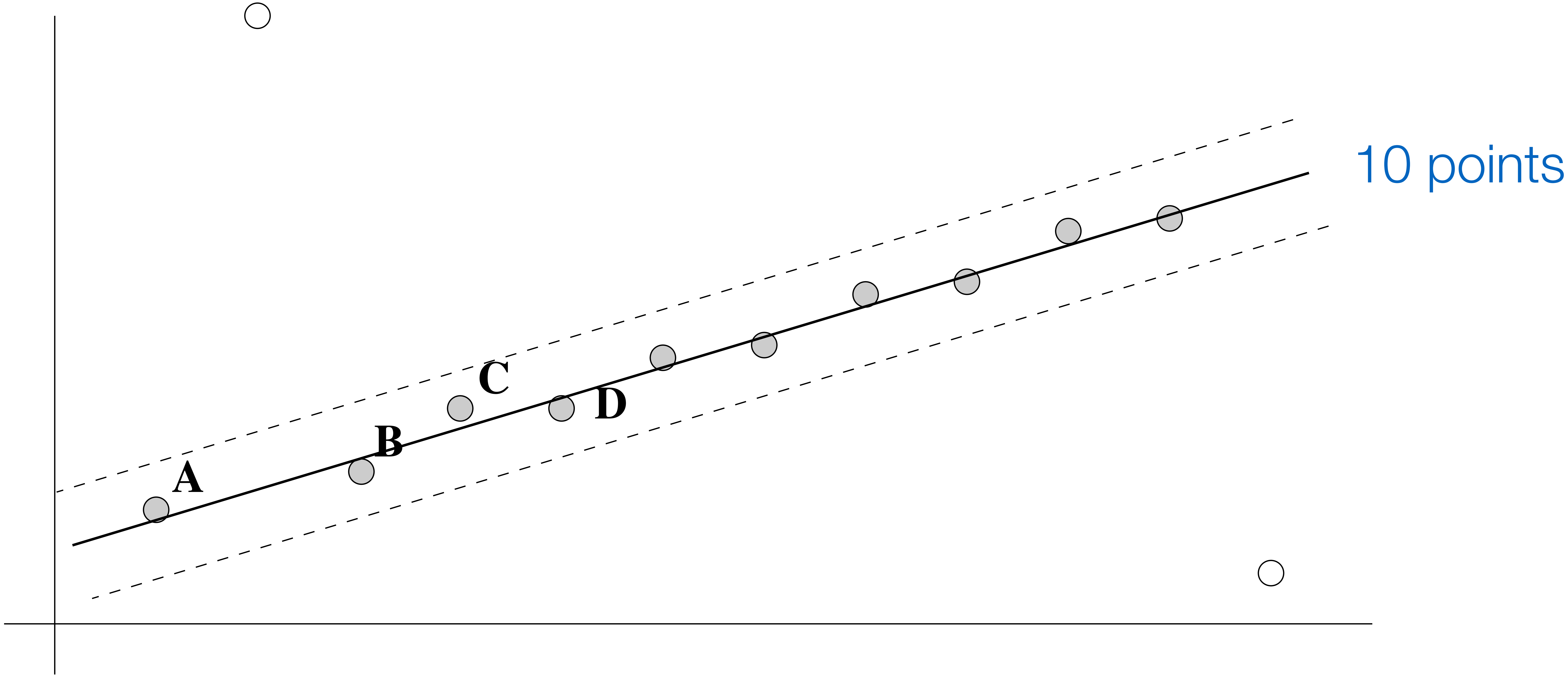

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Image **Alignment + RANSAC**

In practice we have many noisy correspondences + **outliers**

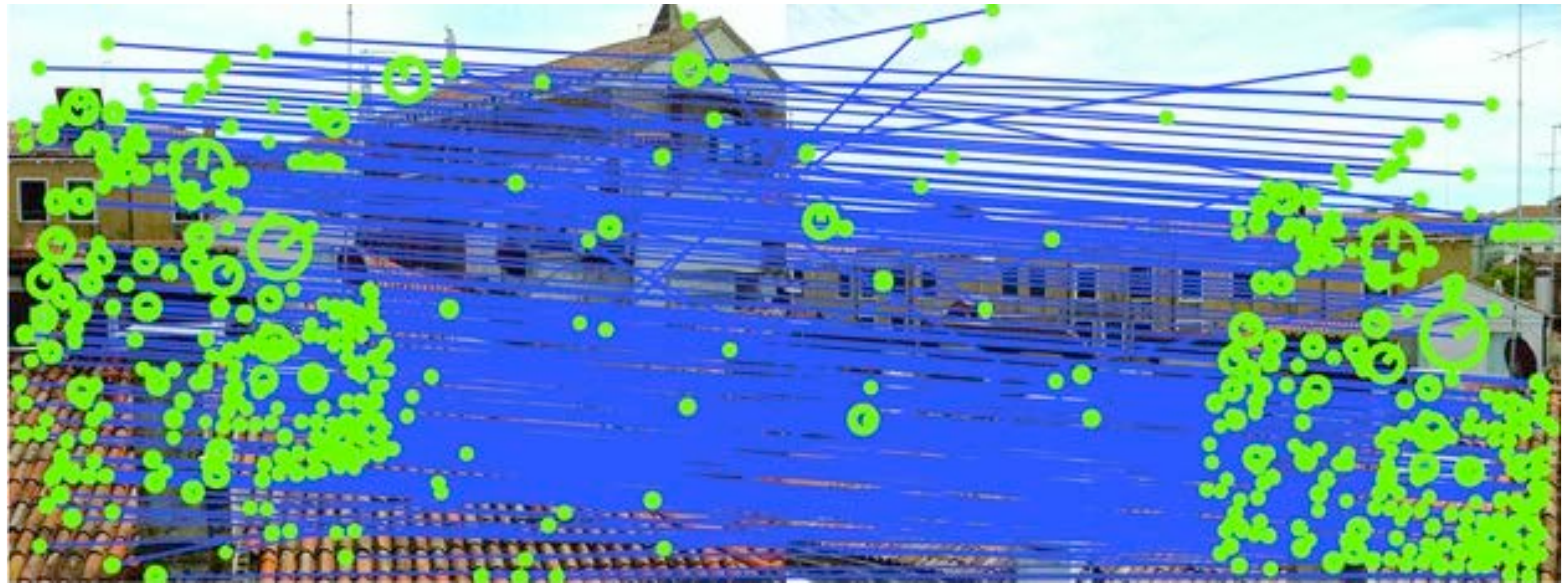

Image **Alignment + RANSAC**

RANSAC solution for Similarity Transform (2 points)

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4 inliers (red, yellow, orange, brown),

Image **Alignment + RANSAC**

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4 outliers (blue, light blue, purple, pink)

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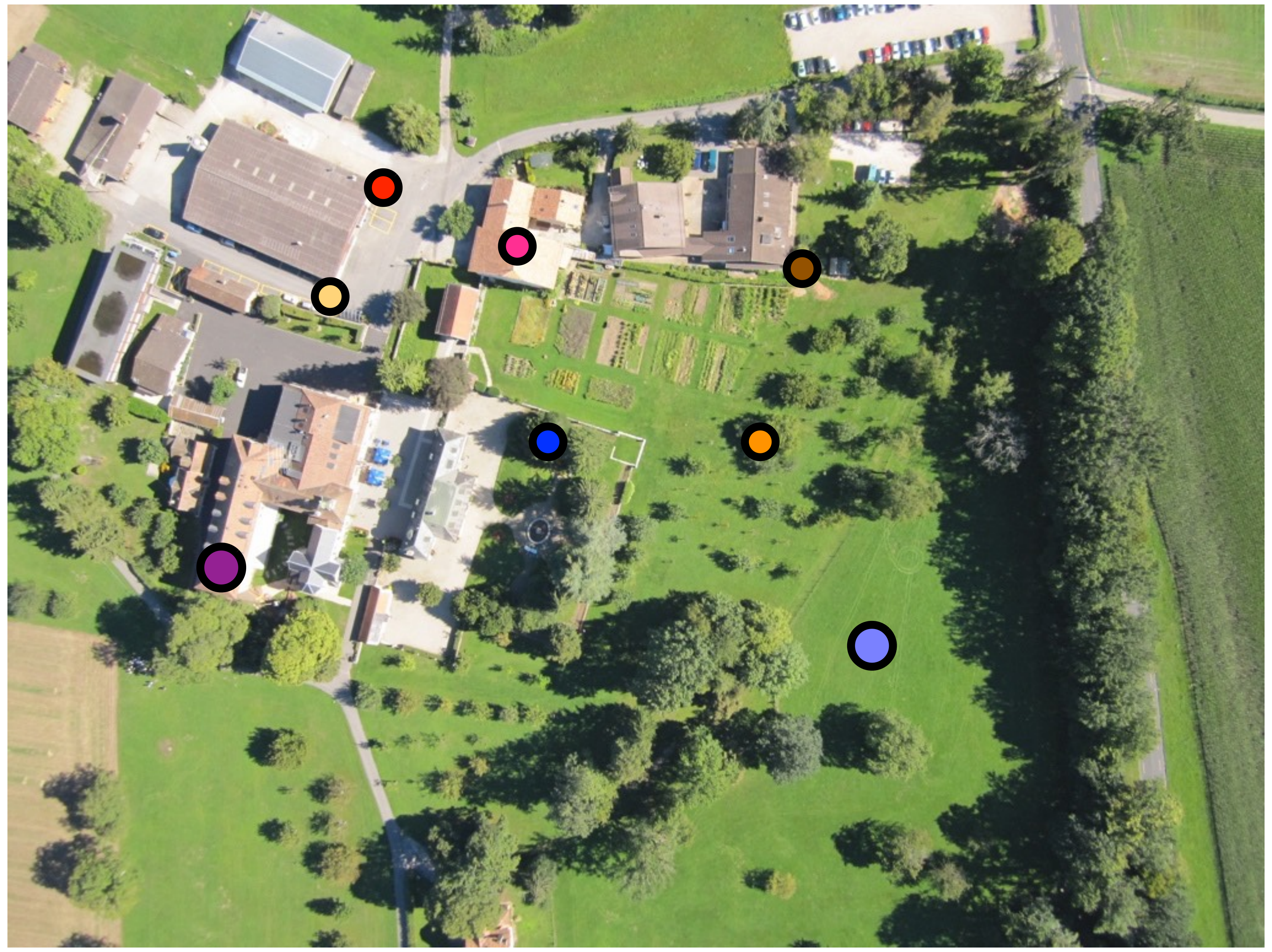

choose light blue, purple

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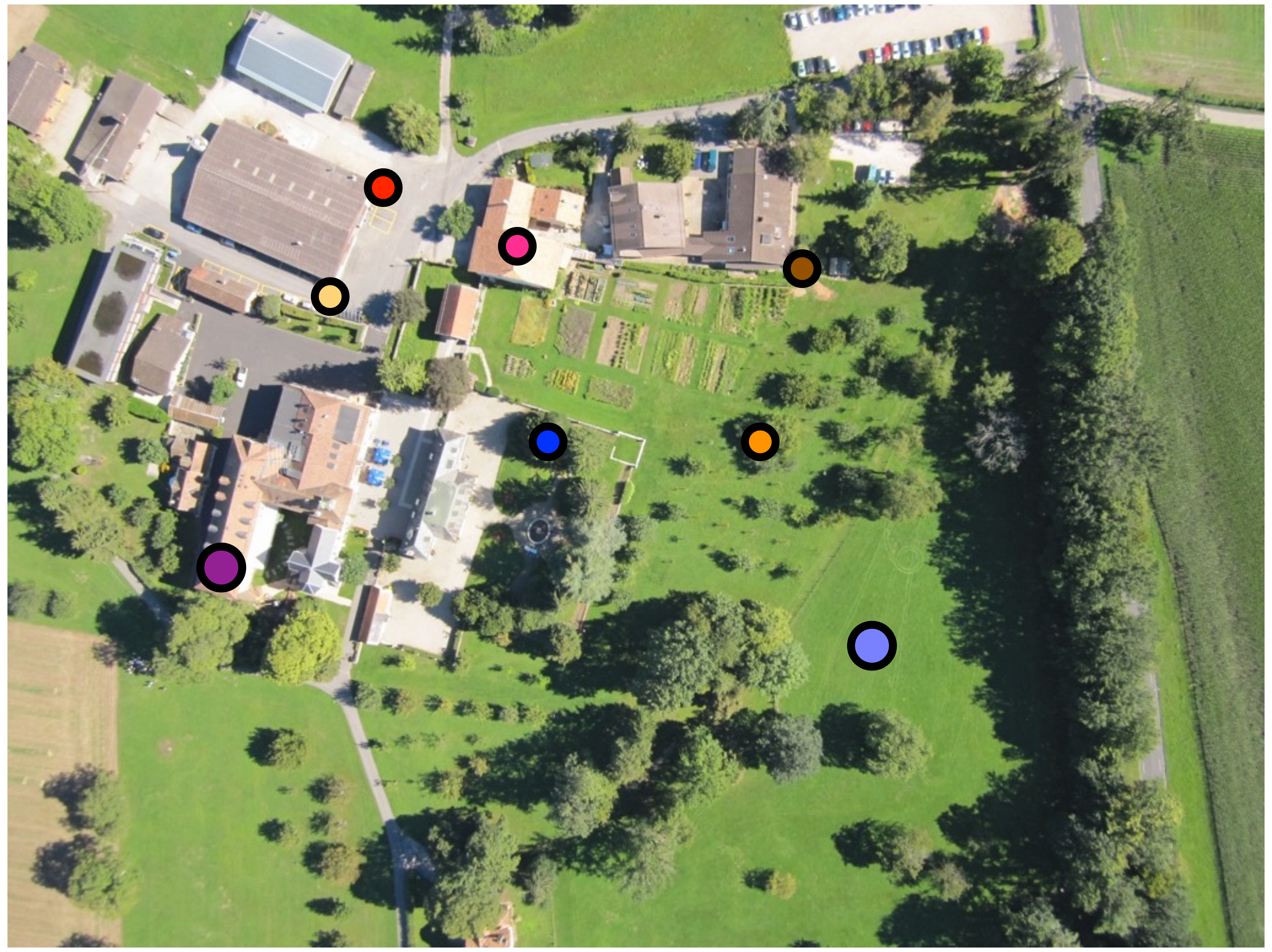

warp image

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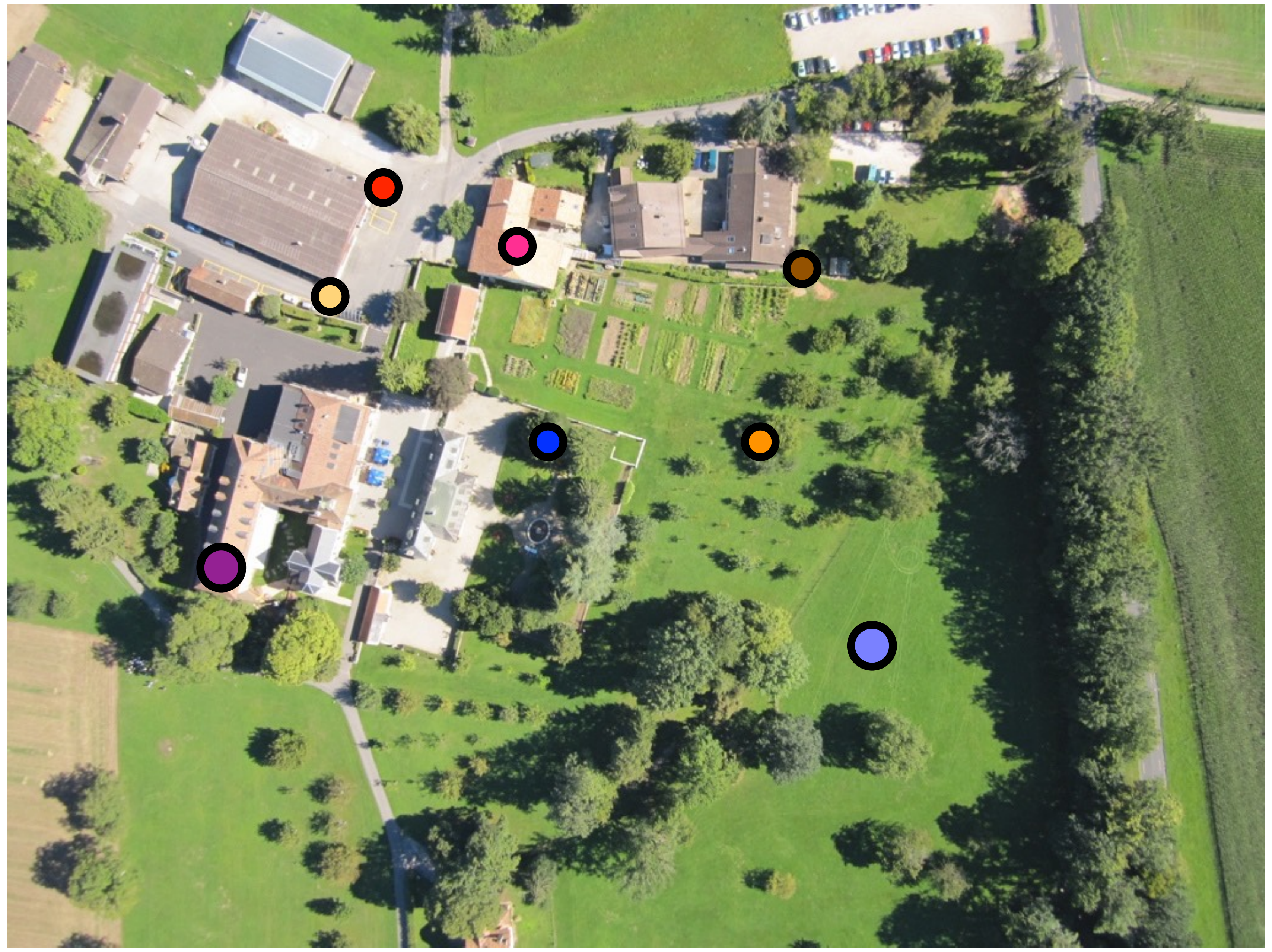

check match distances

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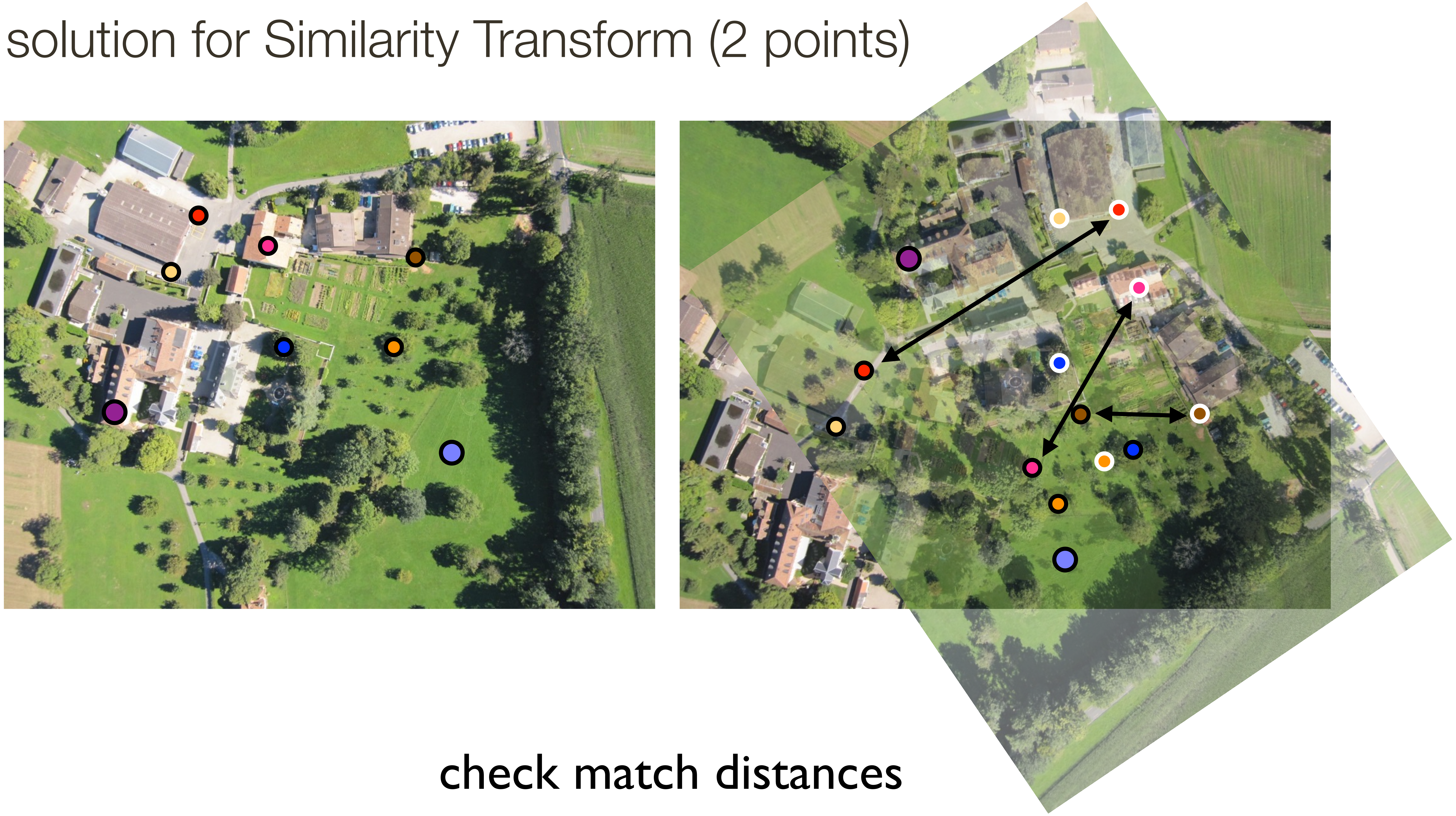

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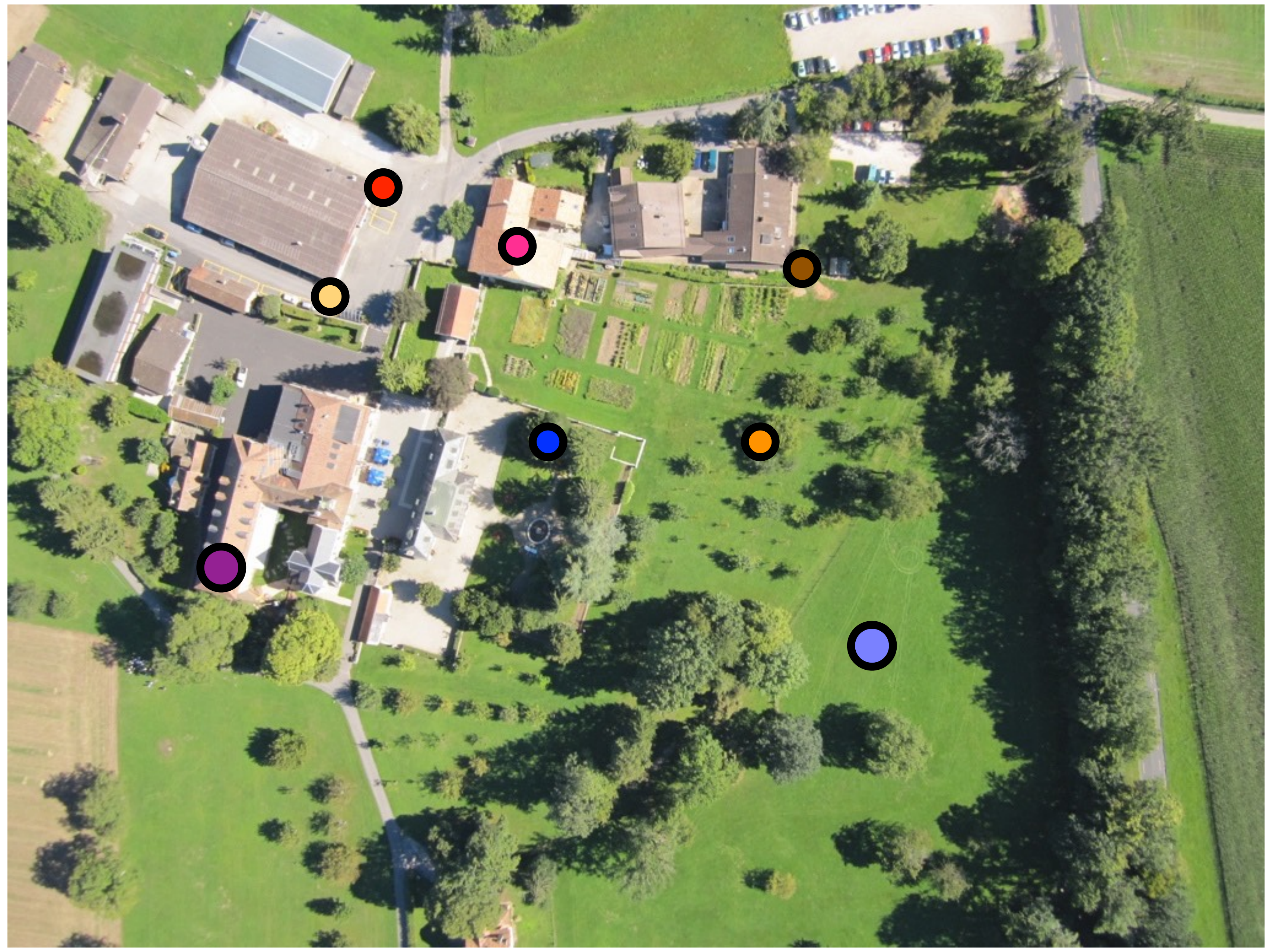

check match distances

#inliers = 2

Image **Alignment + RANSAC**

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Image **Alignment + RANSAC**

RANSAC solution for Similarity Transform (2 points)

choose pink, blue

Image **Alignment + RANSAC**

RANSAC solution for Similarity Transform (2 points)

warp image

Image **Alignment + RANSAC**

RANSAC solution for Similarity Transform (2 points)

check match distances

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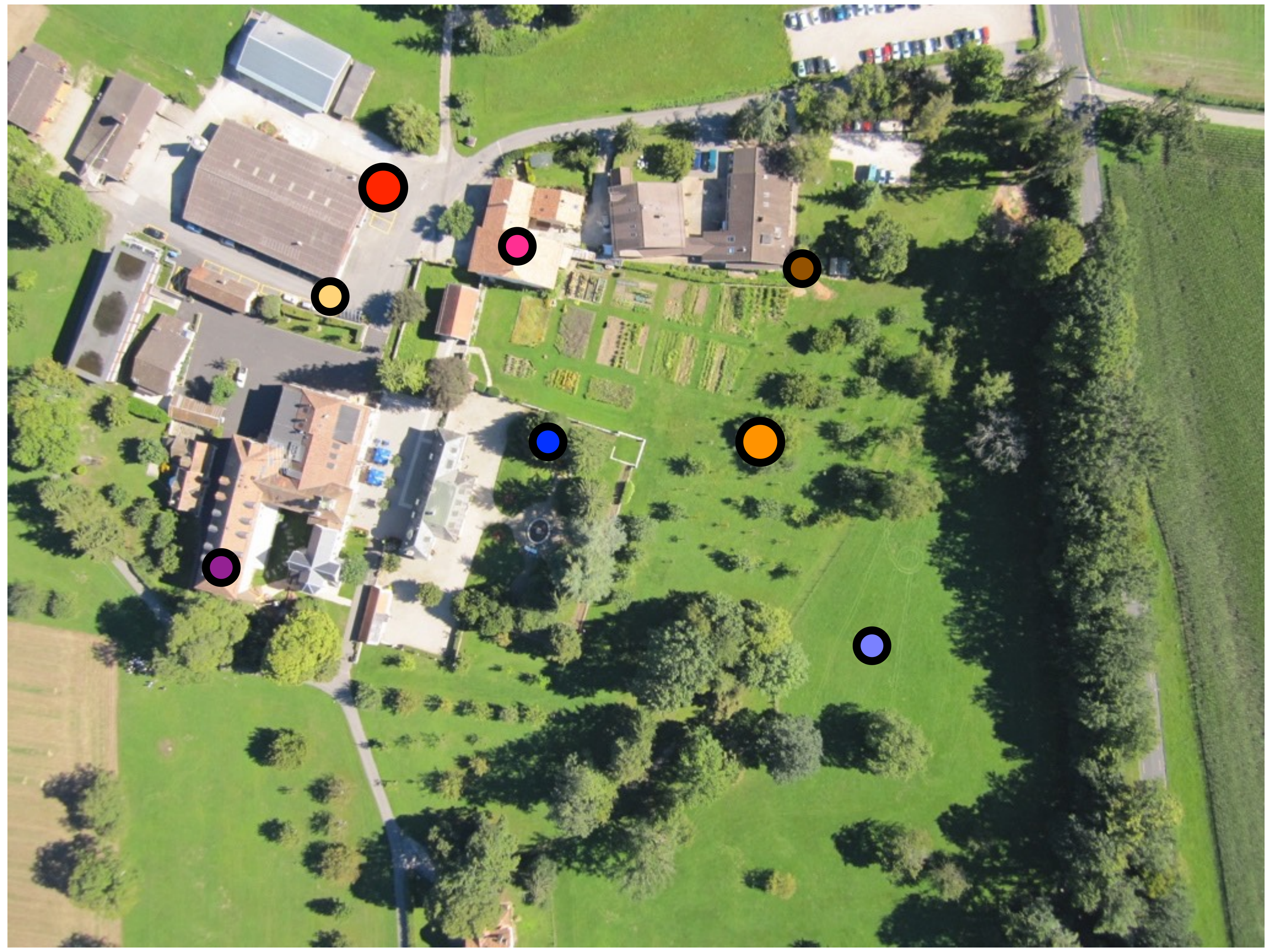
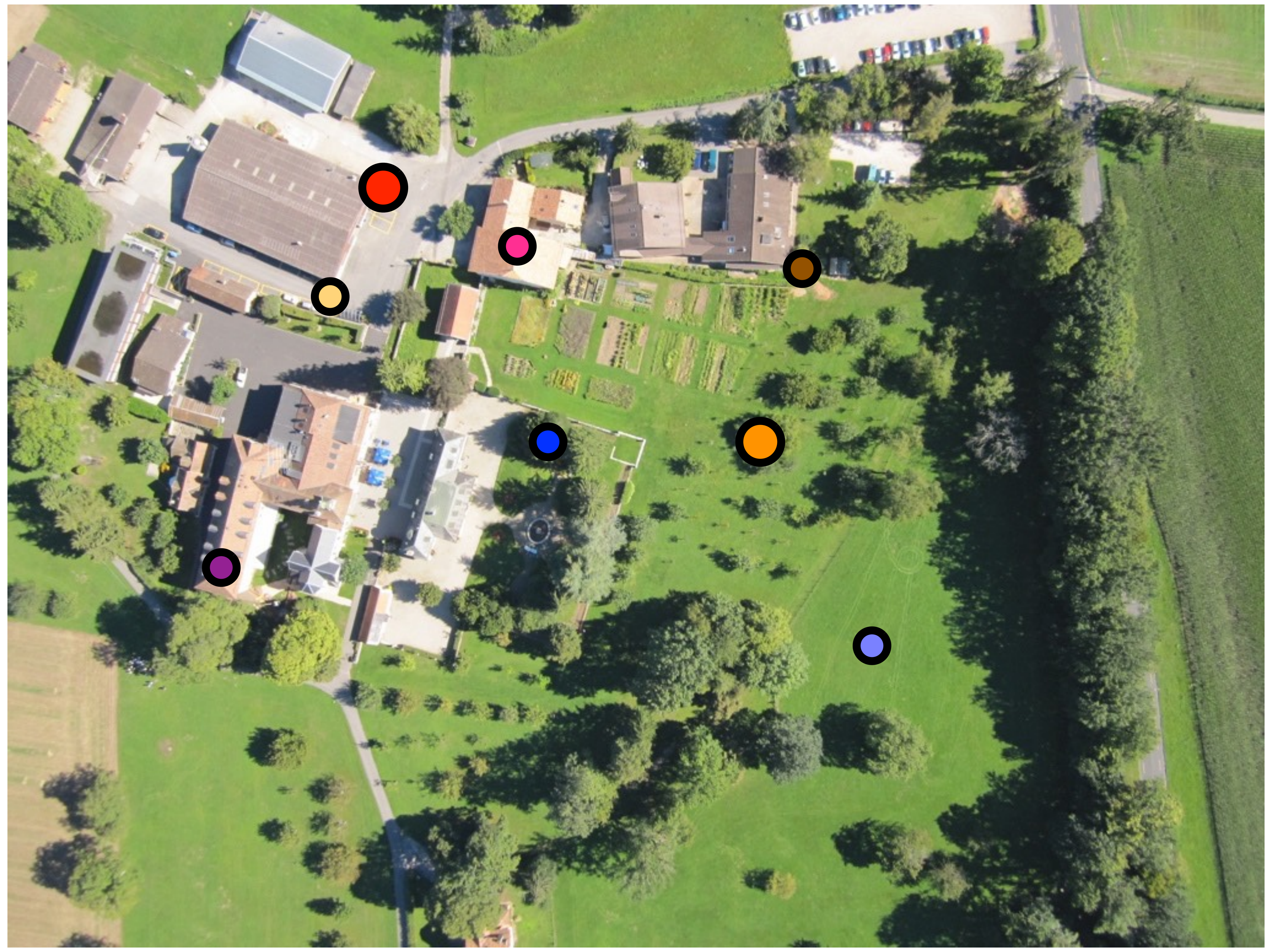
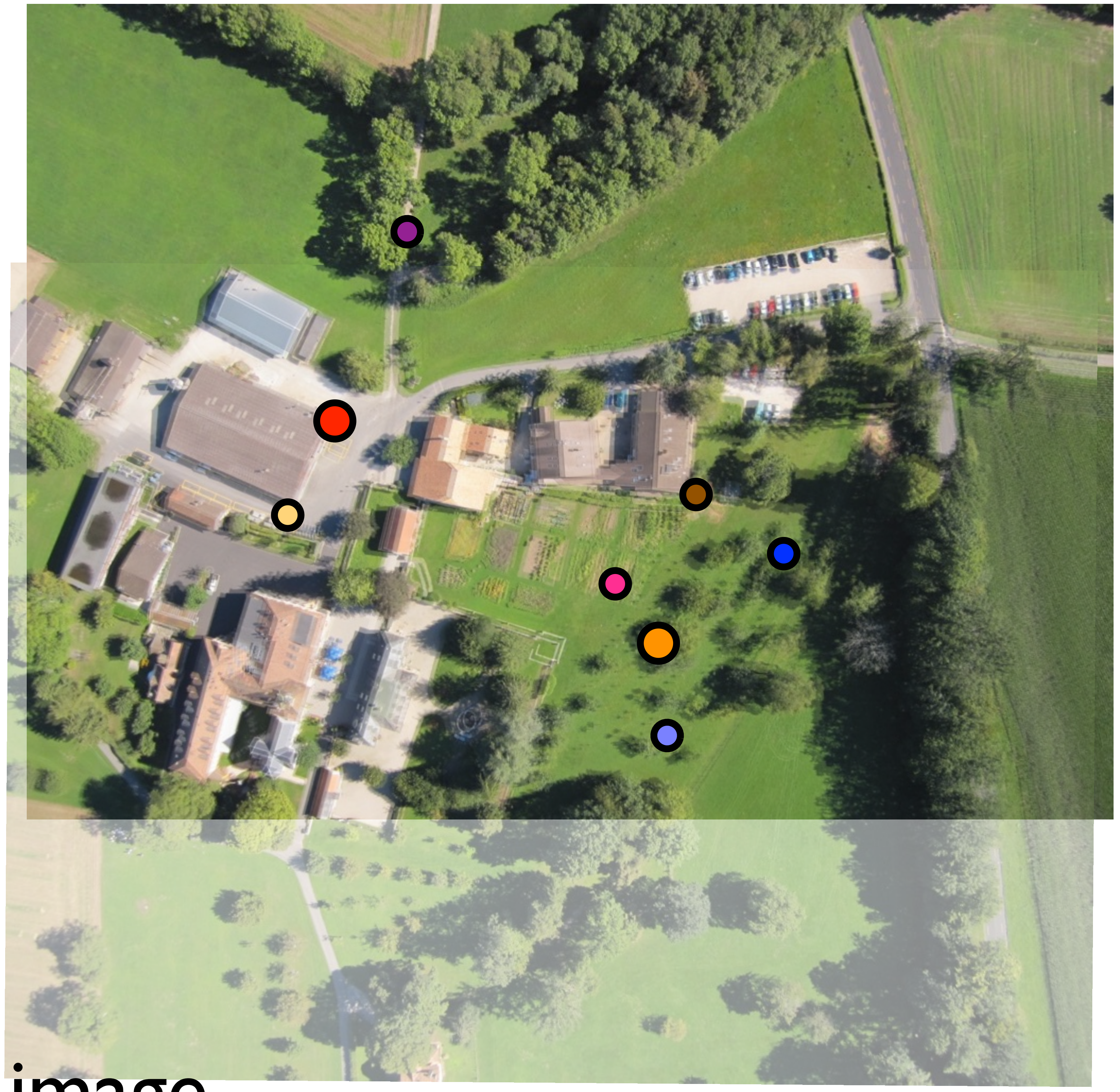


choose red, orange

Image **Alignment + RANSAC**

RANSAC solution for Similarity Transform (2 points)





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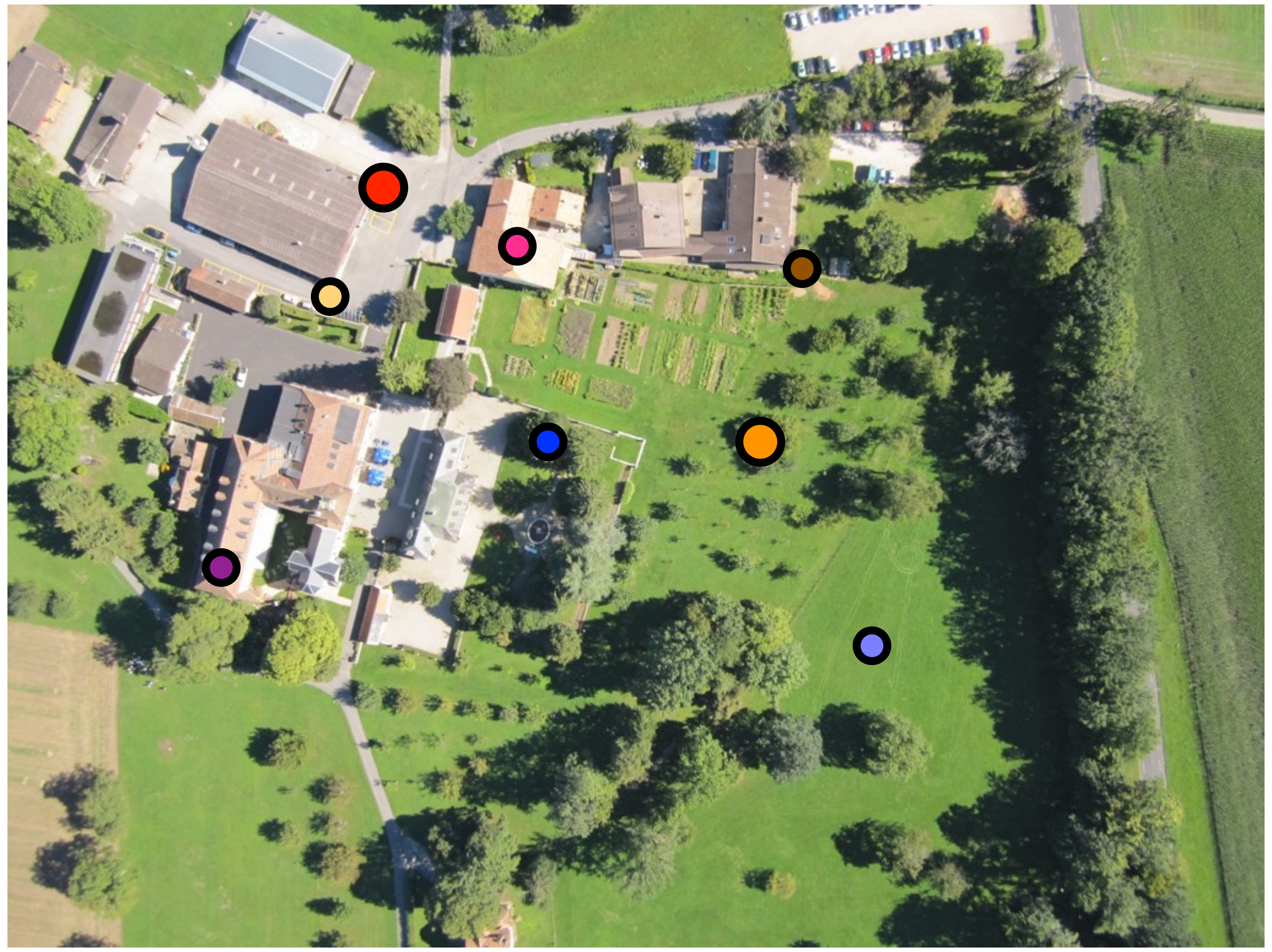

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RANSAC solution for Similarity Transform (2 points)

check match distances

Image **Alignment + RANSAC**

RANSAC solution for Similarity Transform (2 points)

check match distances

#inliers = 4

Image **Alignment + RANSAC**

RANSAC solution for Similarity Transform (2 points)

Image **Alignment + RANSAC**

Assignment 4

1. Match feature points between 2 views

2. Select minimal subset of matches*

3. Compute transformation T using minimal subset

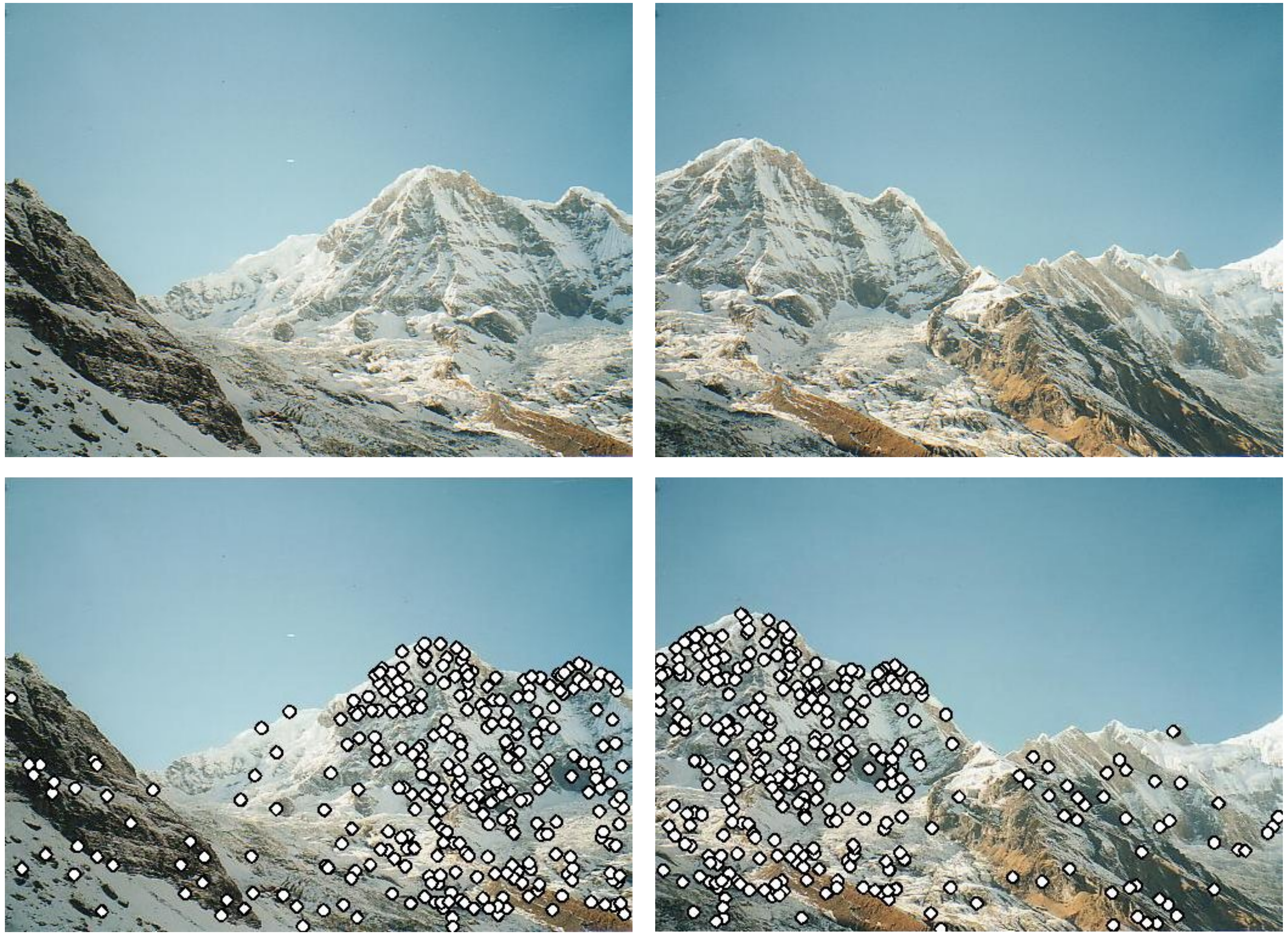
4. Check consistency of all points with T — compute projected position and count #inliers with distance < threshold

5. Repeat steps 2-4 to maximize #inliers

* Similarity transform = 2 points, Affine = 3, Homography = 4

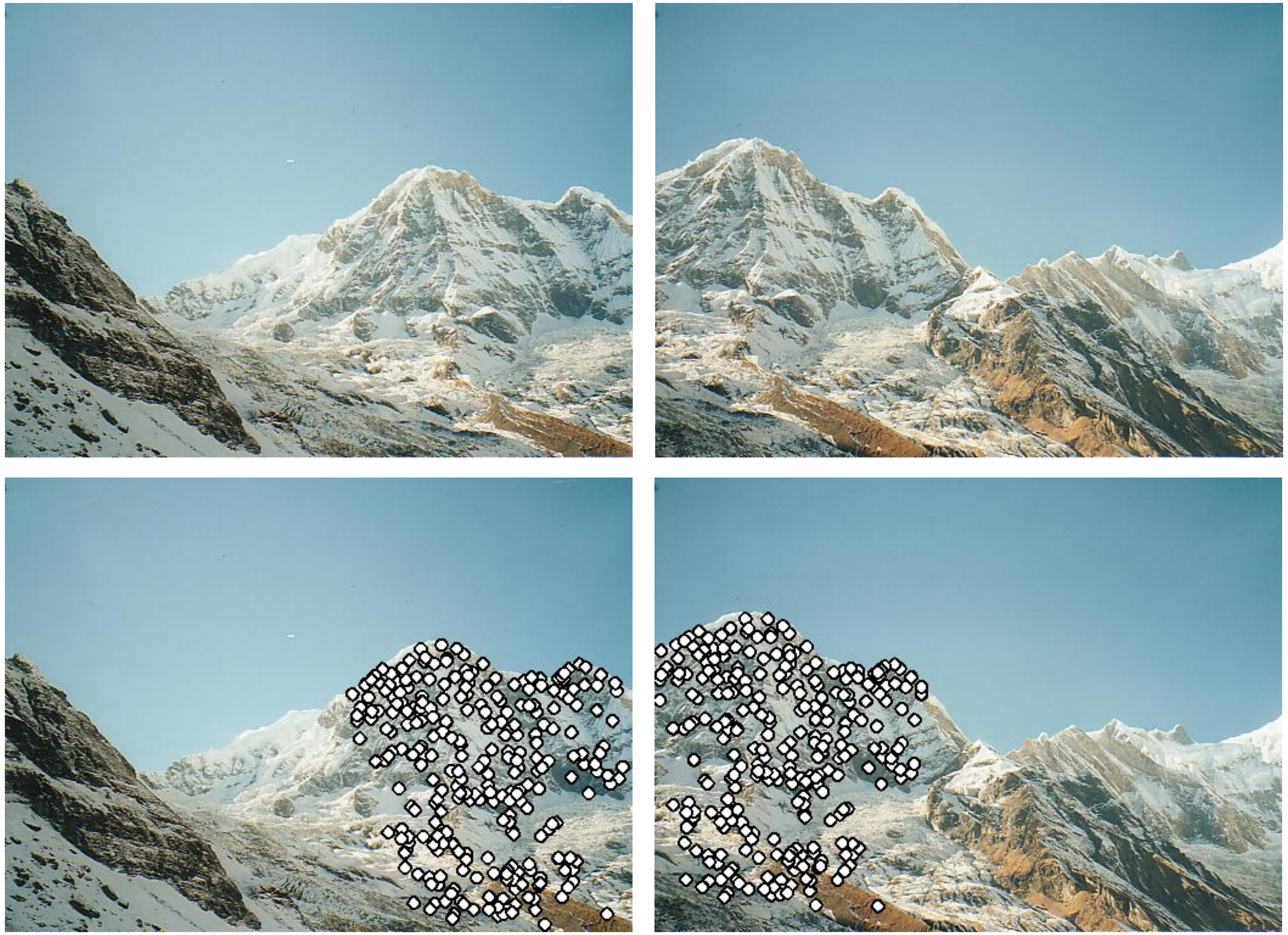
2-view **Rotation** Estimation

Find features + raw matches, use RANSAC to find Similarity



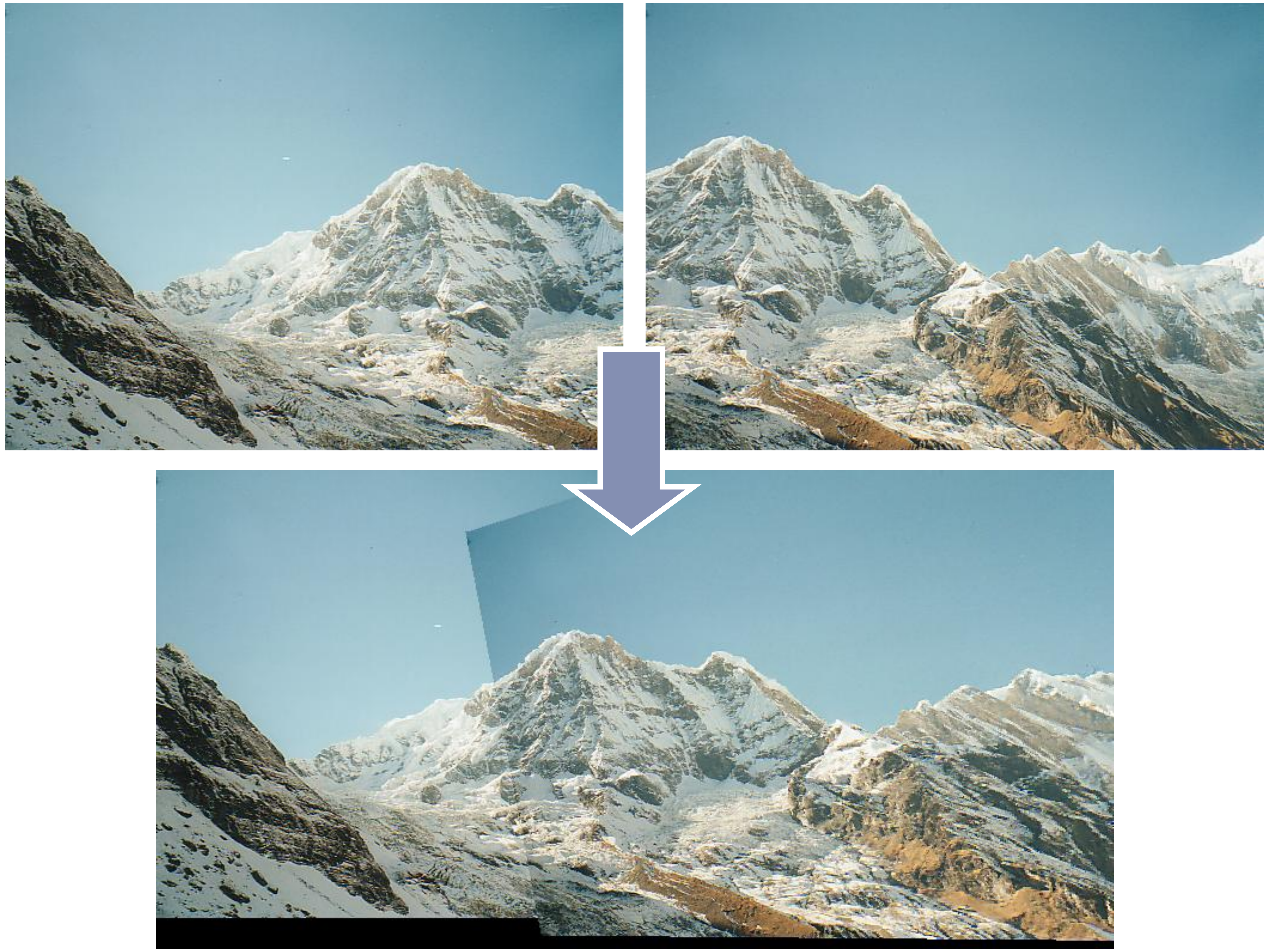
2-view **Rotation** Estimation

Remove outliers, can now solve for R using least squares



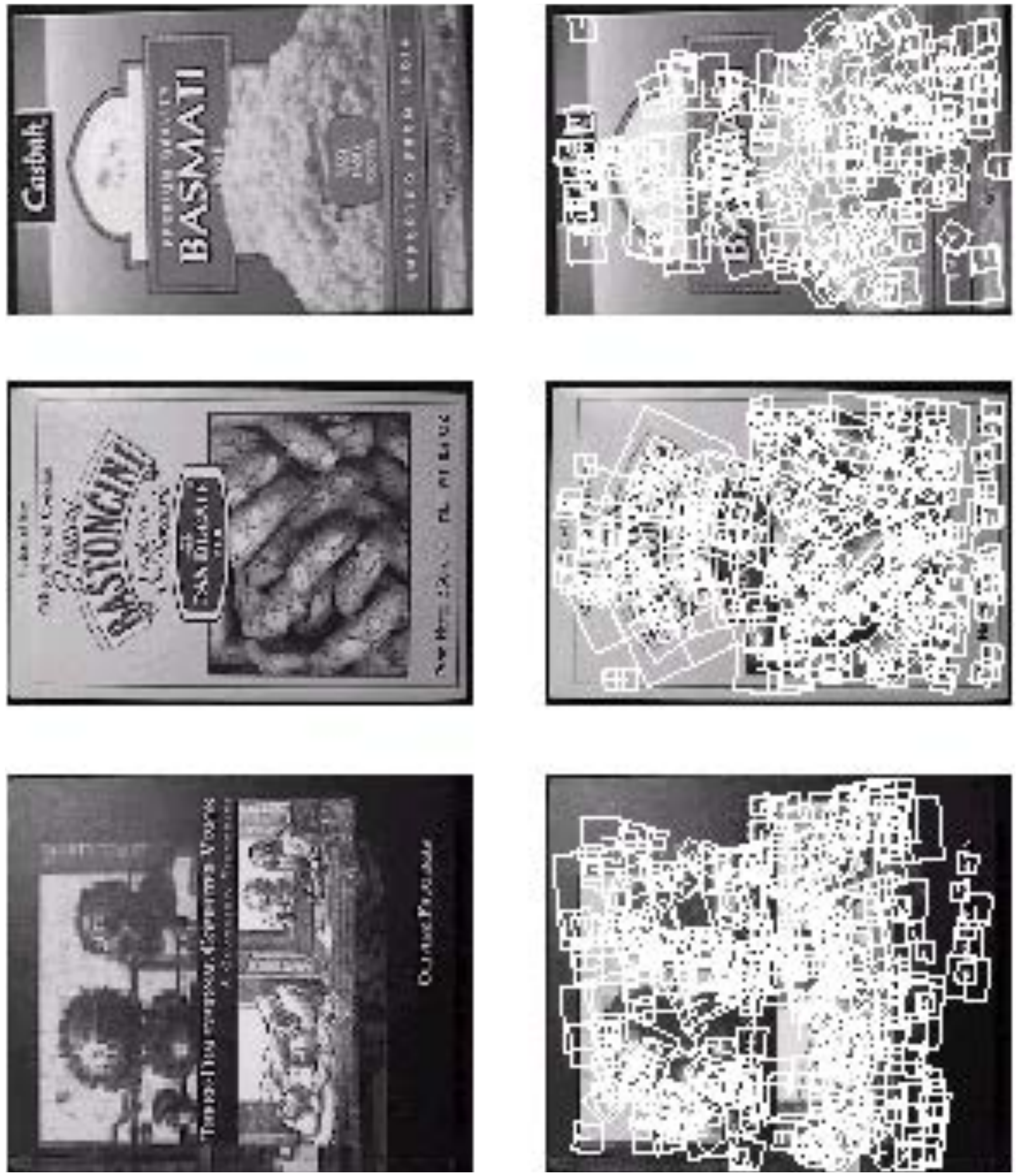
2-view **Rotation** Estimation

Final rotation estimation



Object **Instance Recognition**

Database of planar objects



Instance recognition

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Object **Instance Recognition** with SIFT

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Match SIFT descriptors between **query image** and a database of known keypoints extracted from **training examples**

— use fast (approximate) nearest neighbour matching

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Suppose we are **fitting a line** to a dataset that consists of 50% outliers

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If we draw pairs of points uniformly at random, what fraction of pairs will consist entirely of 'good' data points (inliers)?

RANSAC: How many samples?

Let p_0 be the fraction of outliers (i.e., points on line)

Let n be the number of points needed to define hypothesis
($n = 2$ for a line in the plane)

Suppose k samples are chosen

How many samples do we need to find a good solution?

13.7

RANSAC: How many samples? (p = 0.99)

Sample size	Proportion of outliers						
n	**5%**	**10%**	**20%**	**25%**	**30%**	**40%**	**50%**
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

Figure Credit: Hartley & Zisserman

In practice…

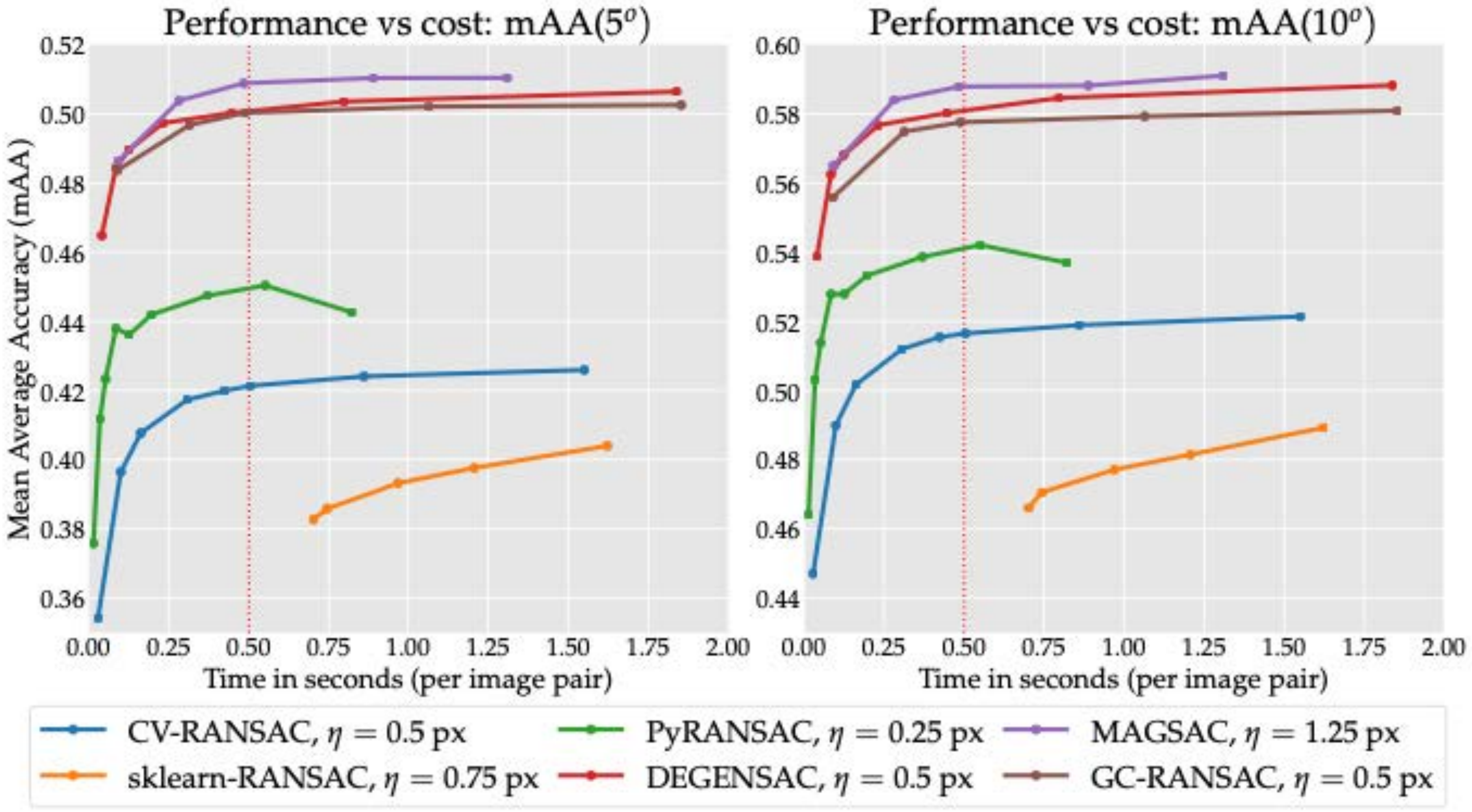

Fig. 9 Validation – Performance vs. cost for RANSAC. We evaluate six RANSAC variants, using 8k SIFT features with "both" matching and a ratio test threshold of r=0.8. The inlier threshold η and iterations limit Γ are variables – we plot only the best η for each method, for clarity, and set a budget of 0.5 seconds per image pair (dotted red line). For each RANSAC variant, we pick the largest Γ under this time "limit" and use it for all validation experiments. Computed on 'n1-standard-2' VMs on Google Compute (2 vCPUs, 7.5 GB).

[Jin et al., 2021]

Re-cap: RANSAC

RANSAC is a technique to fit data to a model

— divide data into inliers and outliers

— estimate model from minimal set of inliers

— improve model estimate using all inliers

— alternate fitting with re-classification as inlier/outlier

RANSAC is a general method suited for a wide range of model fitting problems

— easy to implement

— easy to estimate/control failure rate

RANSAC only handles a moderate percentage of outliers without cost blowing up