15.1

patch patch

$$e = | I_1(x+\Delta u) - I_0(x) |^2$$

for a small flow Δu

$$I_1(x+\Delta u) \simeq I_1(x) + \frac{\partial I}{\partial x}^T \Delta u = I_1(x) + \begin{pmatrix} I_{1x} \\ I_{1y} \end{pmatrix} \cdot \begin{pmatrix} \Delta u_x \\ \Delta u_y \end{pmatrix}$$

$$e = | I_1(x) - I_0(x) + \frac{\partial I}{\partial x}^T \Delta u |^2$$

unknown

patches gradients least squares.

OF form $| J \Delta u - r |^2 \rightarrow \underline{\Delta u = (J^T J)^{-1} J^T r}$

15.2

let $e = 0$ for a single pixel

$$I_t(x) - I_{t+\Delta t}(x) = \frac{\partial I_1}{\partial x}^T \Delta x$$

$\div \Delta t$

$$- \frac{\partial I}{\partial t} = \frac{\partial I}{\partial x}^T \frac{\Delta x}{\Delta t}$$

velocity

temporal difference

$$I_t + \frac{\partial I}{\partial x}^T \frac{\partial x}{\partial t} = 0$$

$$I_t + \begin{pmatrix} I_x \\ I_y \end{pmatrix} \cdot \begin{pmatrix} v_x \\ v_y \end{pmatrix} = 0$$

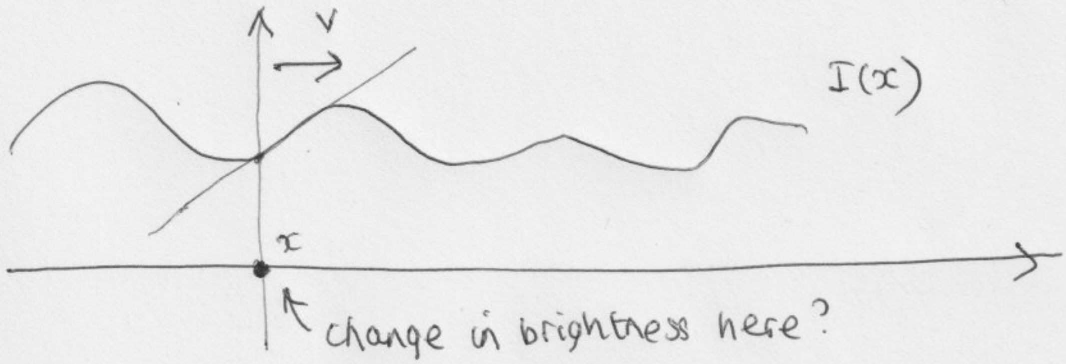
$$\underline{I_t + I_x v_x + I_y v_y = 0}$$

optical flow constraint eqn (single patch)

15.3

$$I_t + \begin{pmatrix} I_x \\ I_y \end{pmatrix} \cdot \begin{pmatrix} v_x \\ v_y \end{pmatrix} = 0 \qquad I_t + \nabla I \cdot \underline{v} = 0$$

Optical flow in 1D

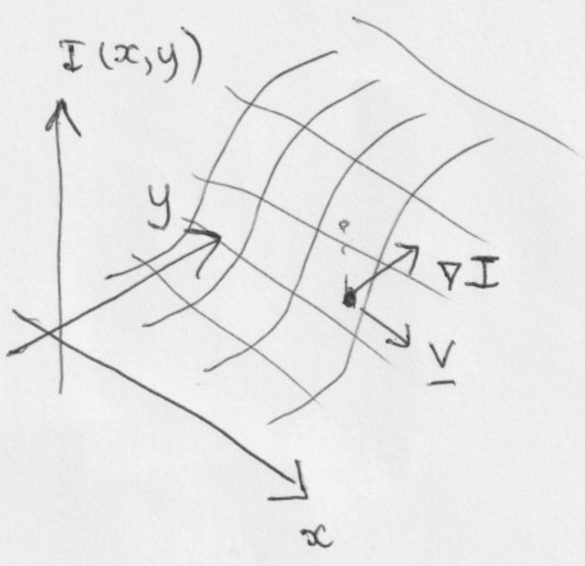


$$I(x, t+\Delta t) = I(x,t) - \Delta x \frac{\partial I}{\partial x}$$

Δt

$$\frac{\partial I}{\partial t} = -v \frac{\partial I}{\partial x}$$

Optical flow in 2D



If v is $\uparrow\uparrow$ to ∇I

$$I_t = -v\,|\nabla I|$$

If $\underline{v}$ is perpendicular to ∇I?

$$I_t = 0$$