15.1

patch patch

$$e = |I_1(x+\Delta u) - I_0(x)|^2$$

for a small flow Δu

$$I_1(x+\Delta u) \simeq I_1(x) + \frac{\partial I}{\partial x}^T \Delta u = I_1(x) + \begin{pmatrix} I_{1x} \\ I_{1y} \end{pmatrix} \cdot \begin{pmatrix} \Delta u_x \\ \Delta u_y \end{pmatrix}$$

$$e = |\underbrace{I_1(x) - I_0(x)}_{\text{patches}} + \underbrace{\frac{\partial I}{\partial x}^T}_{\text{gradients}} \underbrace{\Delta u}_{\text{unknown}}|^2$$

least squares.

OF form $|J\Delta u - r|^2 \rightarrow \Delta u = (J^T J)^{-1} J^T r$

15.2

let $e = 0$ for a single pixel

$$I_t(x) - I_{t+\Delta t}(x) = \frac{\partial I_1}{\partial x}^T \Delta x$$

$\div \Delta t$

$$-\frac{\partial I}{\partial t} = \frac{\partial I}{\partial x}^T \frac{\Delta x}{\Delta t}$$

temporal difference velocity

$$I_t + \frac{\partial I}{\partial x}^T \frac{\partial x}{\partial t} = 0$$

$$I_t + \begin{pmatrix} I_x \\ I_y \end{pmatrix} \cdot \begin{pmatrix} v_x \\ v_y \end{pmatrix} = 0$$

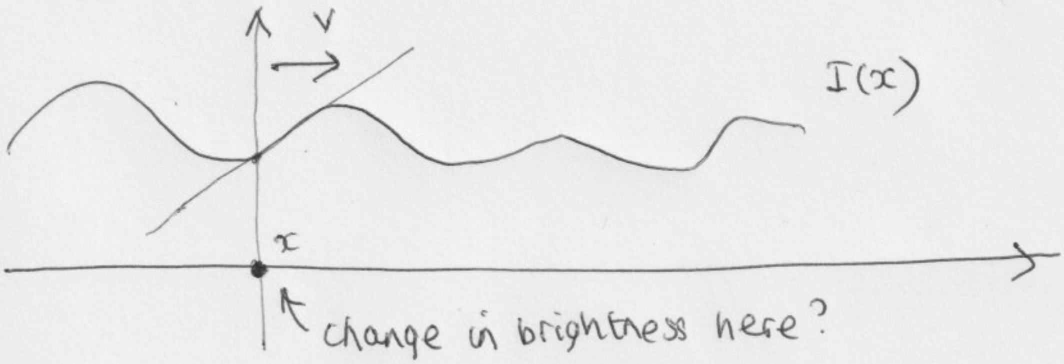
$$I_t + I_x v_x + I_y v_y = 0$$

optical flow constraint eqn (single patch)

15.3

$$I_t + \begin{pmatrix} I_x \\ I_y \end{pmatrix} \cdot \begin{pmatrix} v_x \\ v_y \end{pmatrix} = 0 \qquad I_t + \nabla I \cdot \underline{v} = 0$$

Optical flow in 1D

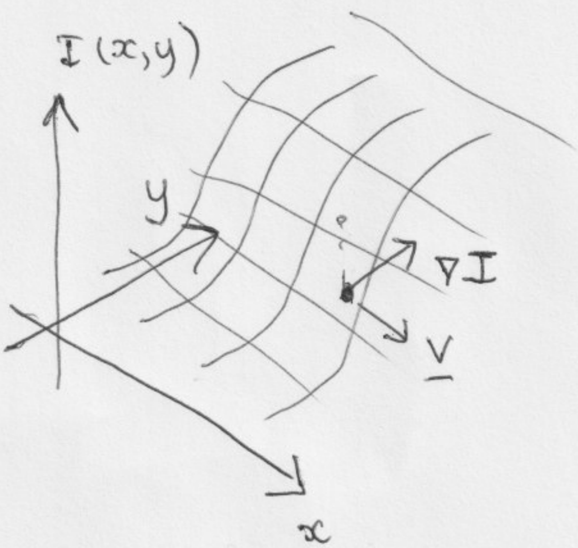


$$I(x, t+\Delta t) = I(x,t) - \Delta x \frac{\partial I}{\partial x}$$

Δt

$$\frac{\partial I}{\partial t} = -v \frac{\partial I}{\partial x}$$

Optical flow in 2D



If v is $\uparrow\uparrow$ to ∇I

$$I_t = -v\,|\nabla I|$$

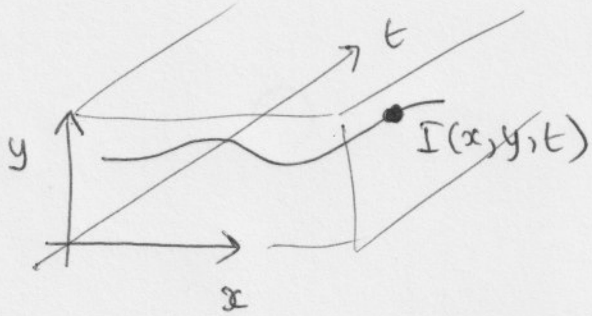
If $\underline{v}$ is perpendicular to ∇I?

$$I_t = 0$$

15.4

Brightness constancy

$$I(x, y, t) = \text{constant}$$



$$\frac{dI}{dt}(x, y, t) = \frac{\partial I}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial I}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial I}{\partial t} = 0$$

$$\nabla I . V + I_t = 0$$