10.1

$$SSD = \sum_{R} | I(x+\Delta x) - I(x) |^2$$

R: region of patch

$$\nabla I = \begin{pmatrix} \frac{\partial I}{\partial x} \\ \frac{\partial I}{\partial y} \end{pmatrix} = \begin{pmatrix} I_x \\ I_y \end{pmatrix}$$

$$I(x+\Delta x) = I(x) + \nabla I^T \Delta x + \ldots$$ Taylor expansion

$$SSD = \sum_{R} | \nabla I^T \Delta x |^2 = \sum_{R} \Delta x^T \underbrace{\nabla I \, \nabla I^T}_{} \Delta x$$ $|x|^2 = x^T x$

$$SSD = \Delta x^T H \Delta x$$

where $\underline{H} = \sum_{R} \nabla I \nabla I^T = \sum_{R} \begin{pmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{pmatrix}$ = "Harris matrix"

10.2

$\underline{u}$ is an eigenvector of A if $A\underline{u} = \lambda \underline{u}$

where λ is a scalar, called the eigenvalue

$$(A - \lambda I)\underline{u} = 0$$

$$|A - \lambda I| = 0$$

e.g. $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$$\begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = (2-\lambda)^2 - 1 = 0, \Rightarrow \lambda = 1, 3$$

$$\boxed{\begin{matrix} \lambda_1 = 1 \\ \lambda_2 = 3 \end{matrix}}$$

$$(A - \lambda_1 I)\underline{u}_1 = 0$$

$$\begin{pmatrix} 2-1 & 1 \\ 1 & 2-1 \end{pmatrix} \underline{u}_1 = 0 \qquad \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \underline{u}_1 = 0 \qquad \underline{u}_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$(A - \lambda_2 I)\underline{u}_2 = 0 \qquad \Rightarrow \qquad \underline{u}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

10.3 Harris corners form matrix $\overset{C}{H} = \sum_R \nabla I \, \nabla I^T$ at each pixel

Find eigenvalues of $H = \lambda_1, \lambda_2$

corner if both λ_1, λ_2 large

it can be shown for a 2×2 matrix

$$Tr(H) = \lambda_1 + \lambda_2$$
$$Det(H) = \lambda_1 \lambda_2$$

so e.g. Harris $f_{strength} = det(H) - k \, Tr(H)^2$

$$= \lambda_1 \lambda_2 - k(\lambda_1 + \lambda_2)^2$$
$$= (1-k)\lambda_1\lambda_2 - k\lambda_1^2 - k\lambda_2^2$$

typically k is very small

$$H = \sum_R \begin{pmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{pmatrix}$$