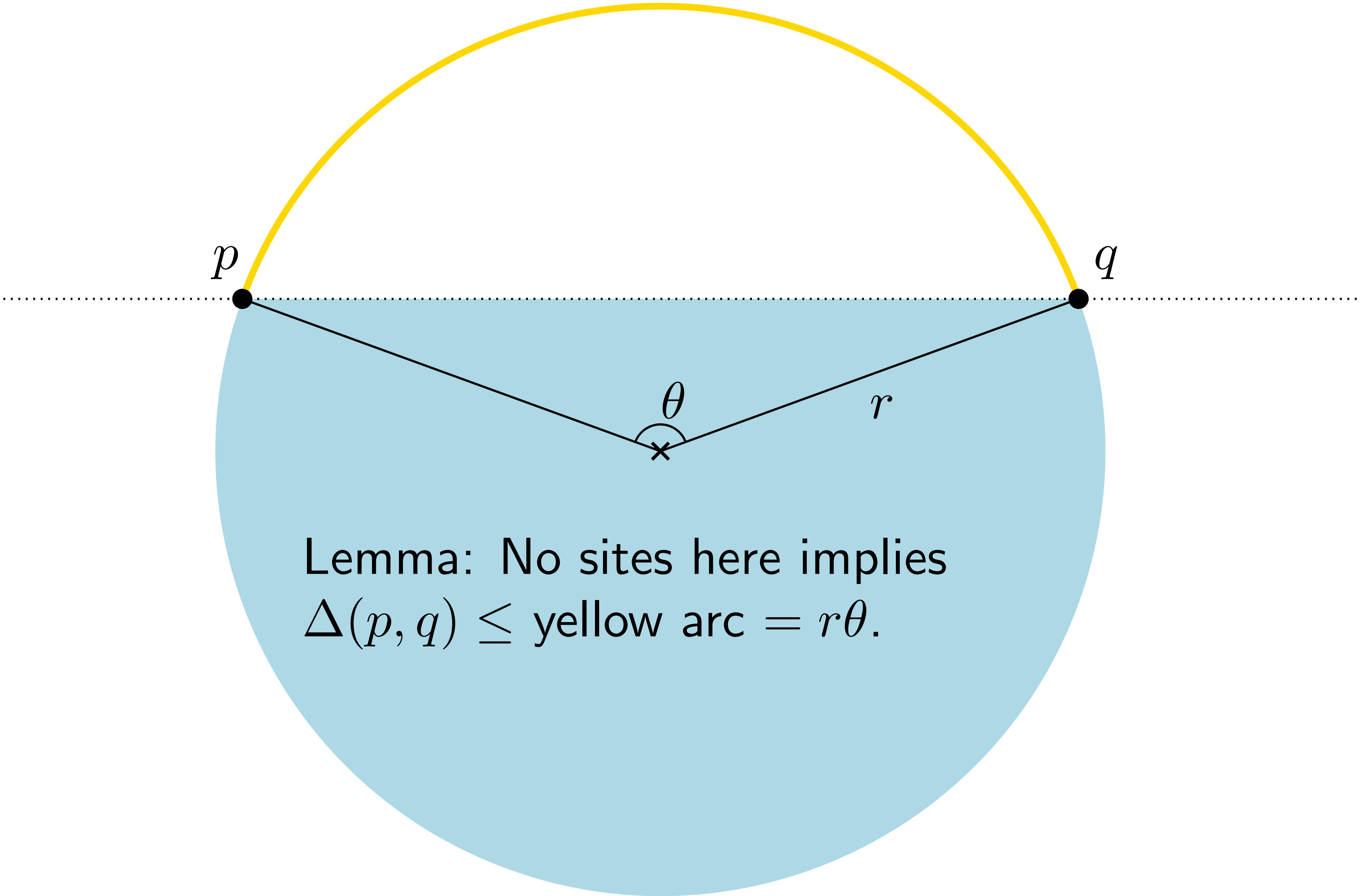
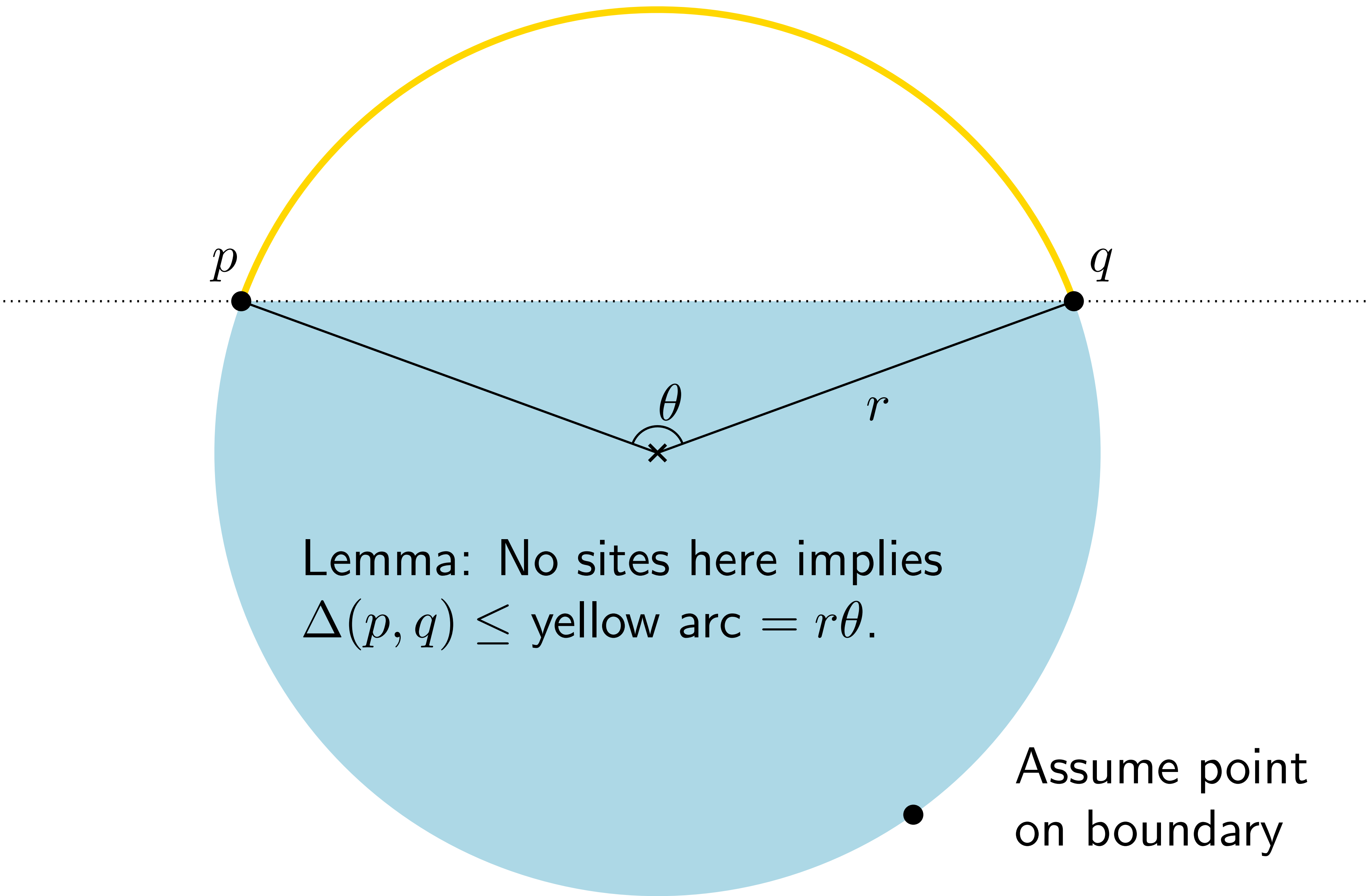
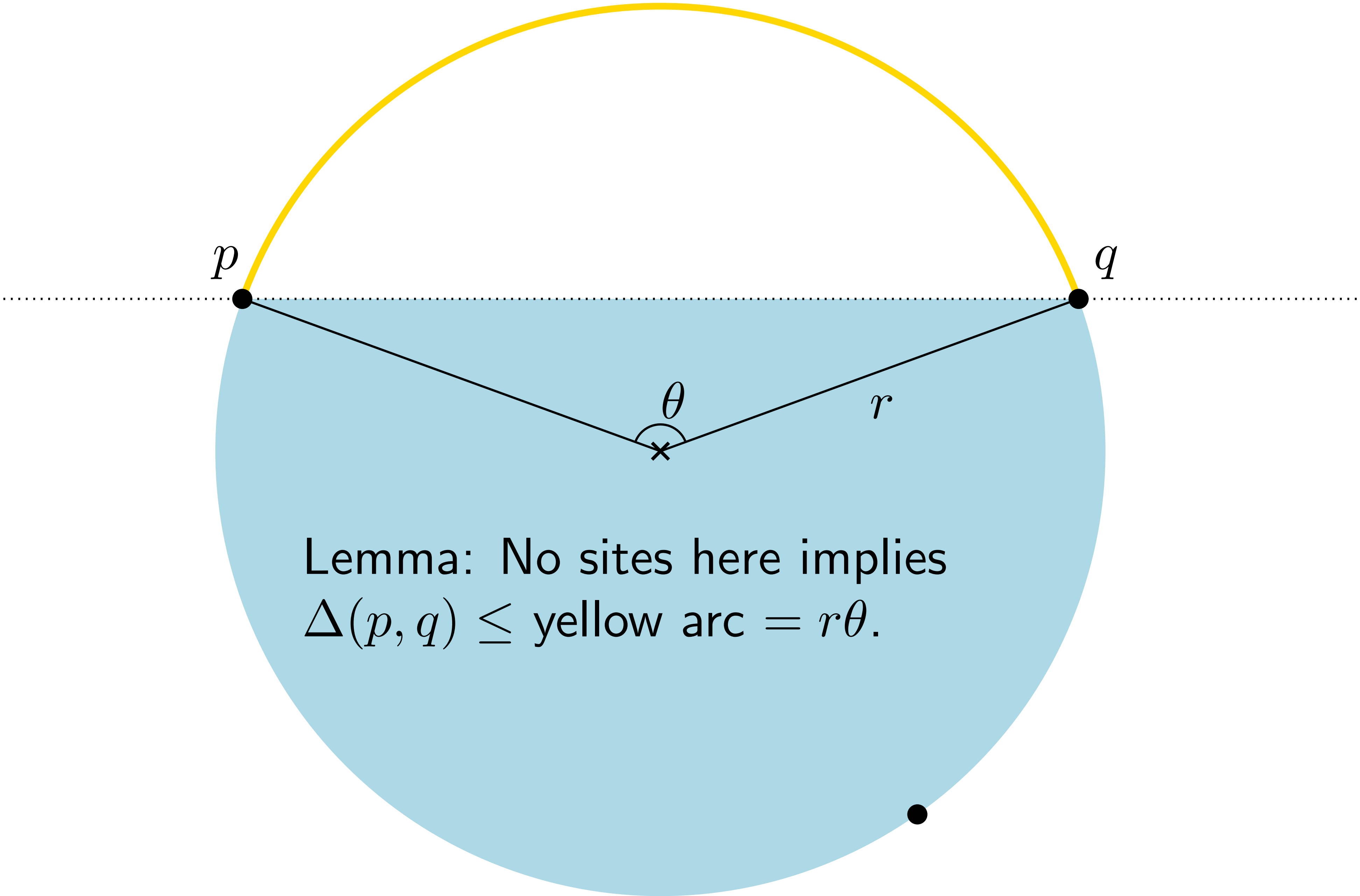

p
q
θ
r
Lemma: No sites here implies
$\Delta(p, q) \leq$ yellow arc $= r\theta$.

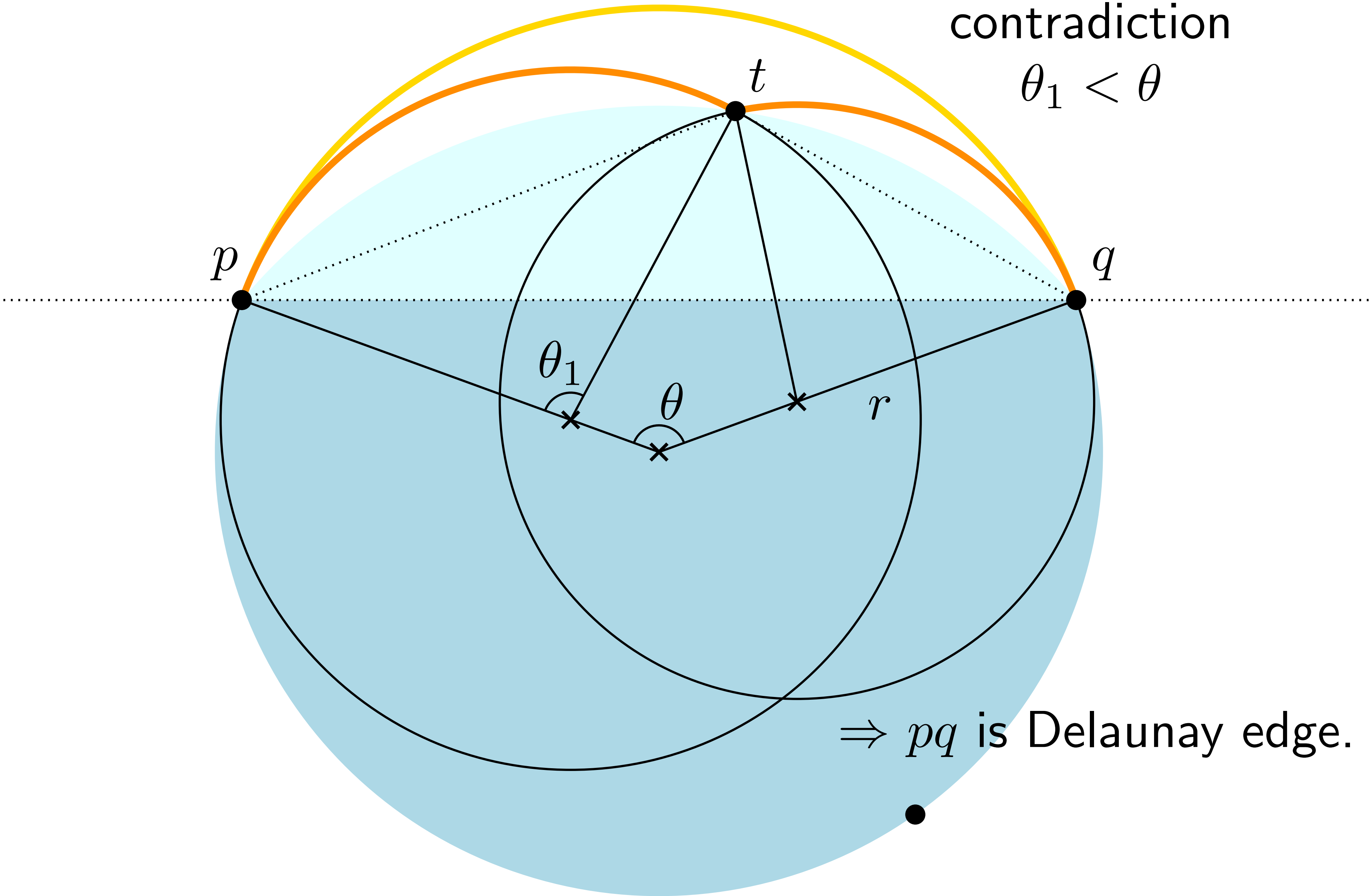

p
q
θ
r
Lemma: No sites here implies
$\Delta(p, q) \leq$ yellow arc $= r\theta$.
Assume point
on boundary

Proof by induction on the rank of θ



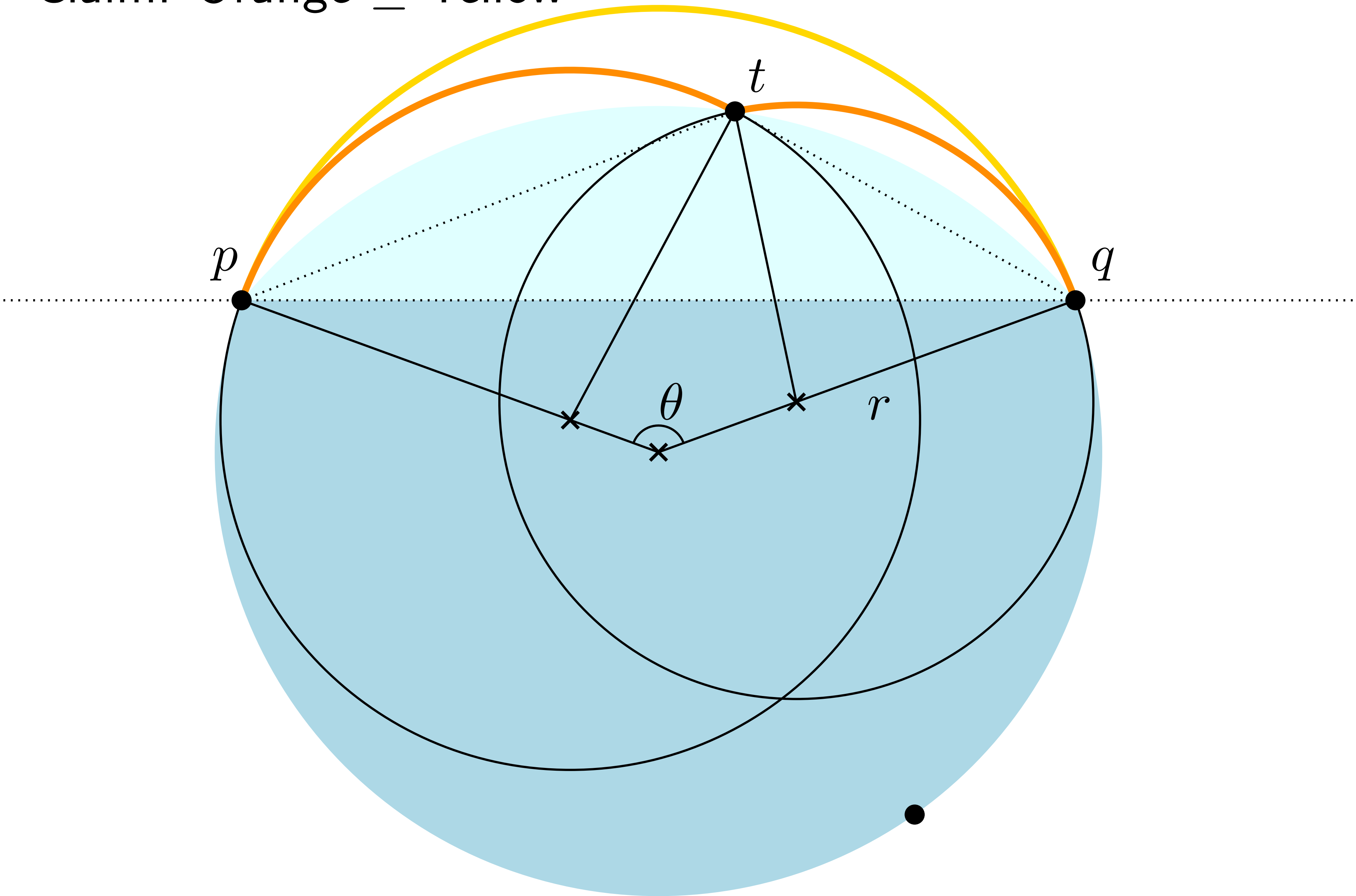
Base case: Min $\theta \Rightarrow$ no points in disk, otherwise contradiction

$\theta_1 < \theta$

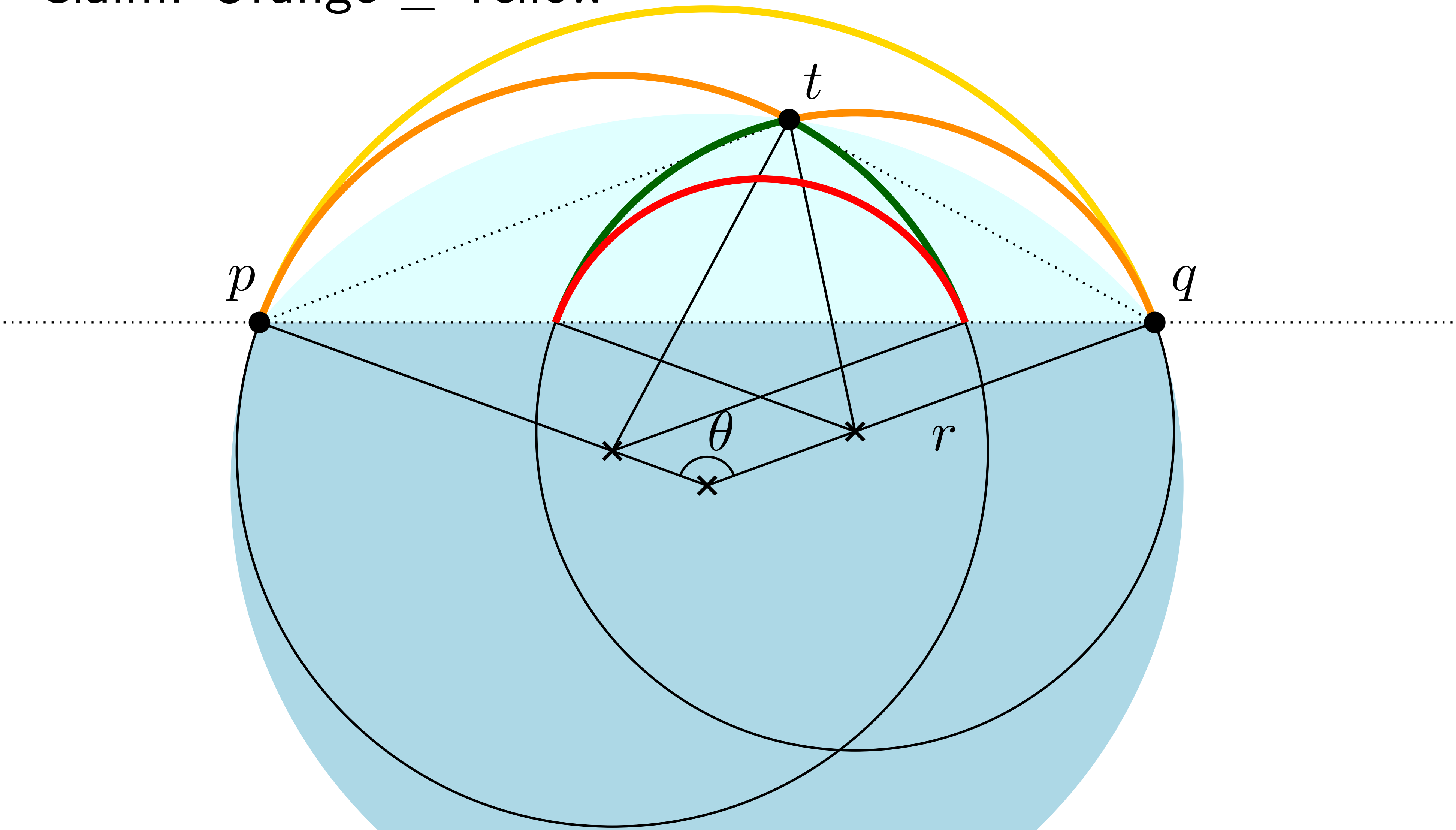


$\Rightarrow pq$ is Delaunay edge.

Claim: Orange $\leq$ Yellow



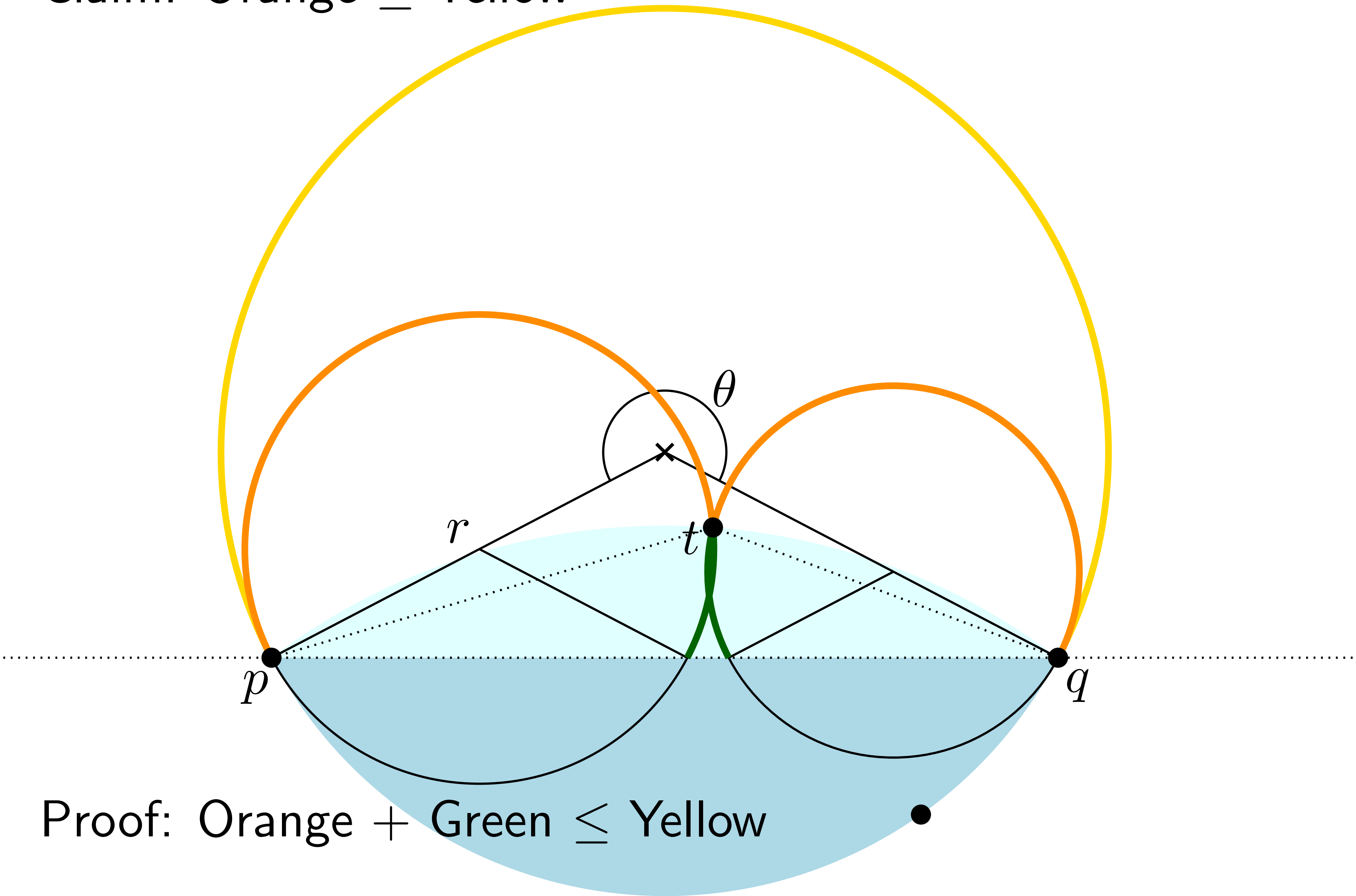
Claim: Orange $\leq$ Yellow



Proof: Orange + Green - Red = Yellow ●

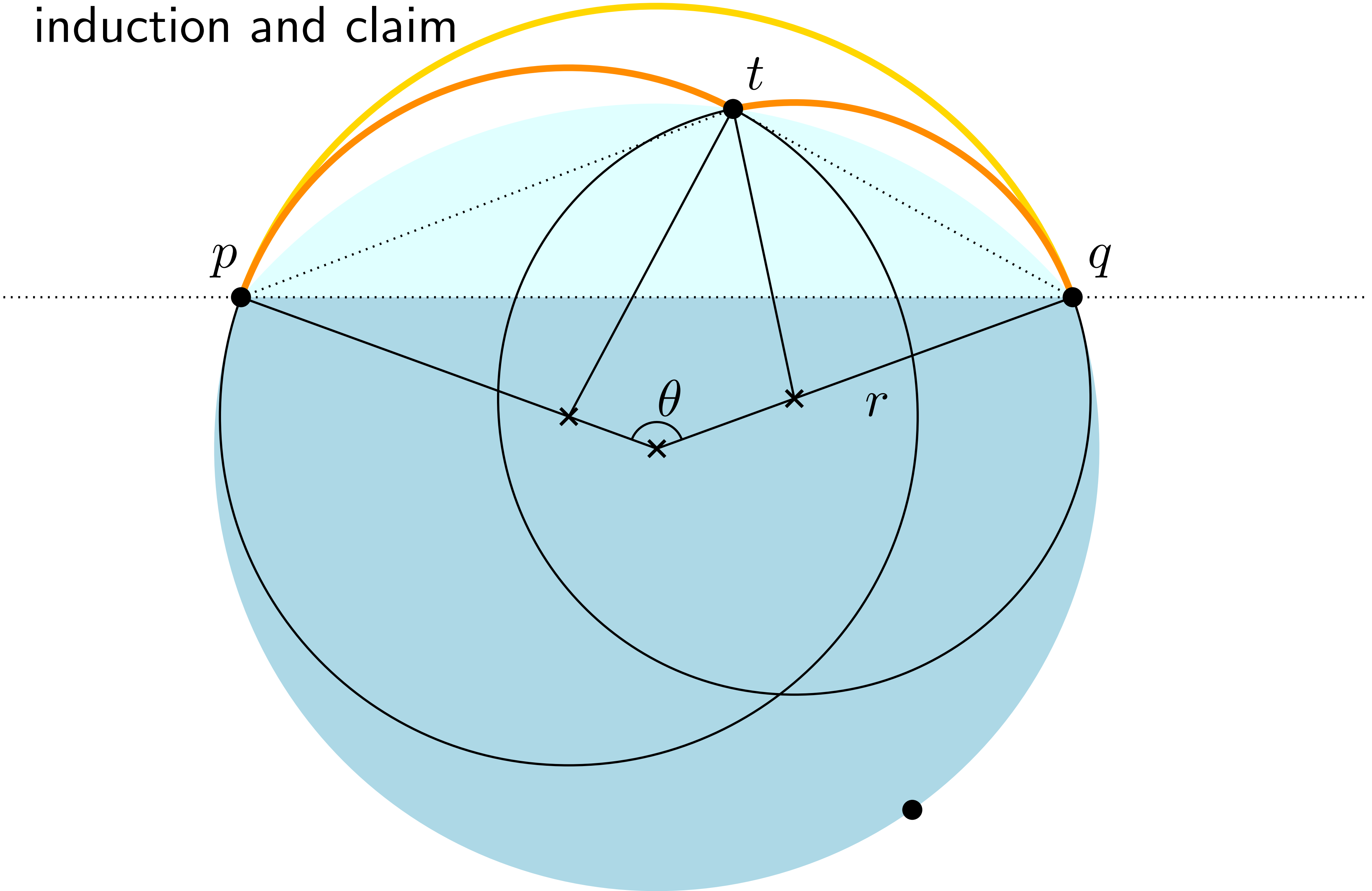
Green > Red

Claim: Orange $\leq$ Yellow



Proof: Orange + Green $\leq$ Yellow

Proof (of Lemma): Use $\Delta(p,q) \leq \Delta(p,t) + \Delta(t,q)$ and induction and claim



Theorem: For any two sites p, q in a Delaunay triangulation,

$$\frac{\Delta(p,q)}{d(p,q)} \leq \frac{2\pi}{3\cos(\pi/6)} \approx 2.42 \ .$$

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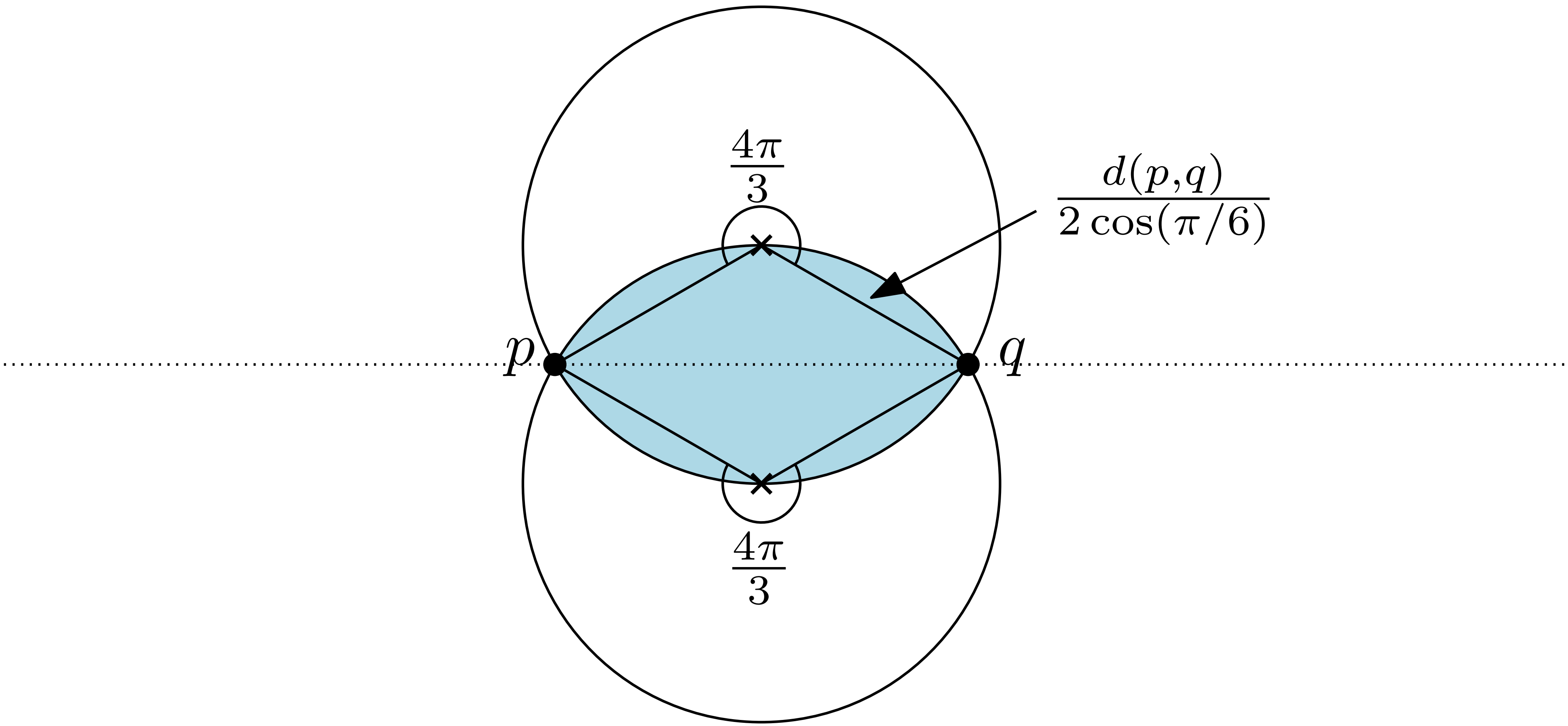
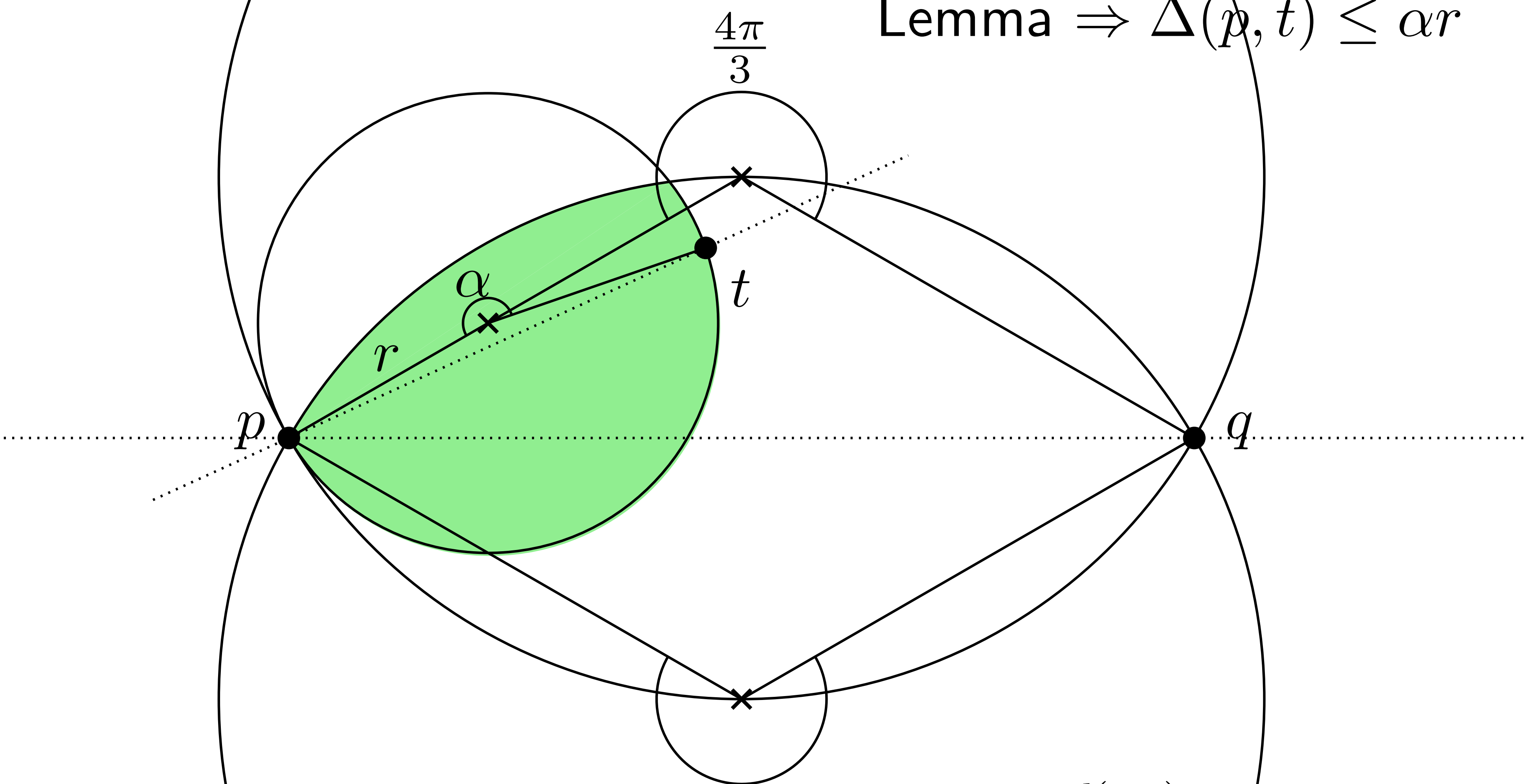


Blue empty $\Rightarrow$ done (by Lemma).

Otherwise, pick t so that Green is empty.

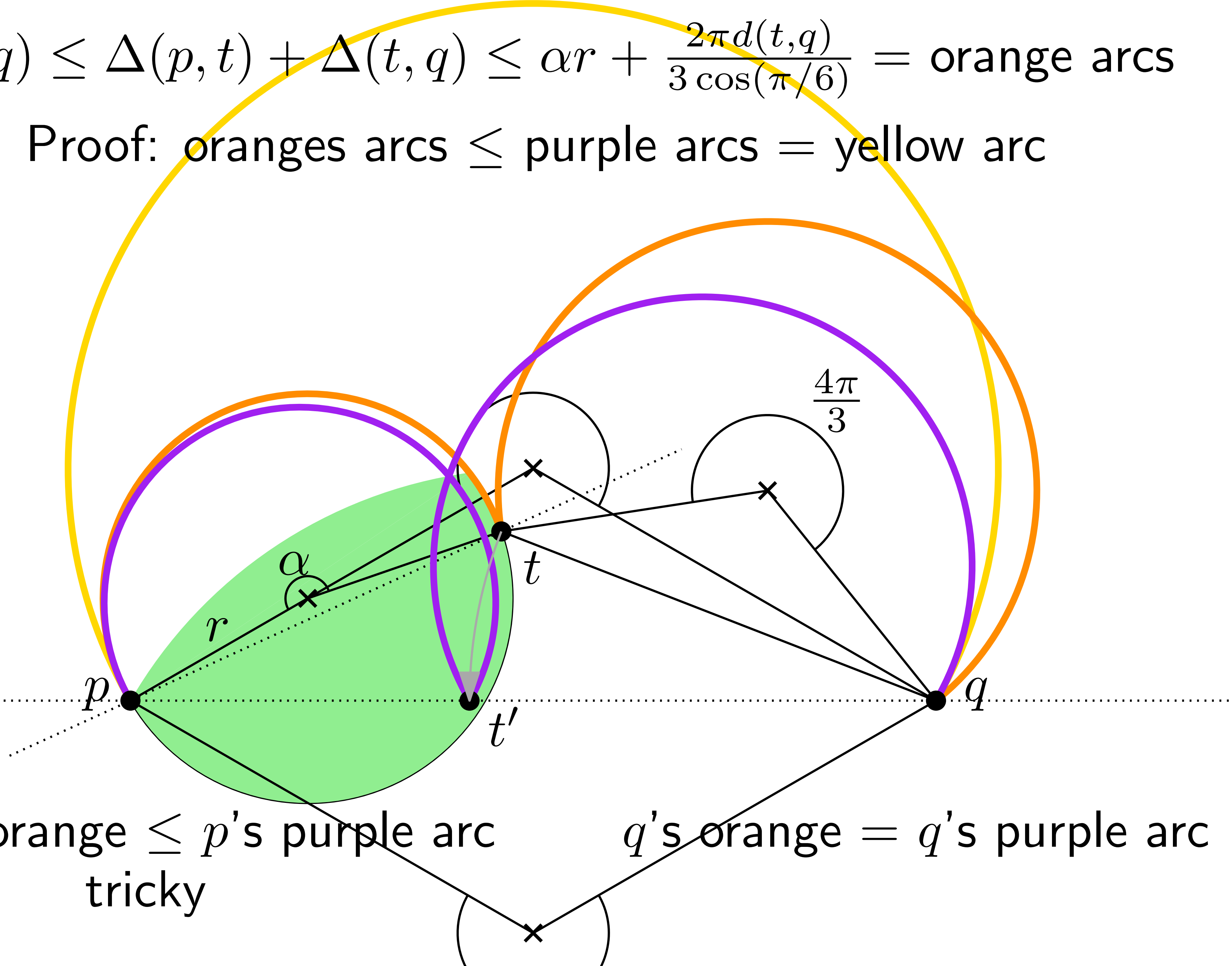
Lemma $\Rightarrow \Delta(p,t) \leq \alpha r$



Induction on rank $d(p,q) \Rightarrow \Delta(t,q) \leq \frac{2\pi d(t,q)}{3\cos(\pi/6)}$

$$\Delta(p,q) \le \Delta(p,t) + \Delta(t,q) \le \alpha r + \frac{2\pi d(t,q)}{3\cos(\pi/6)} = \text{orange arcs}$$

Proof: oranges arcs $\le$ purple arcs $=$ yellow arc

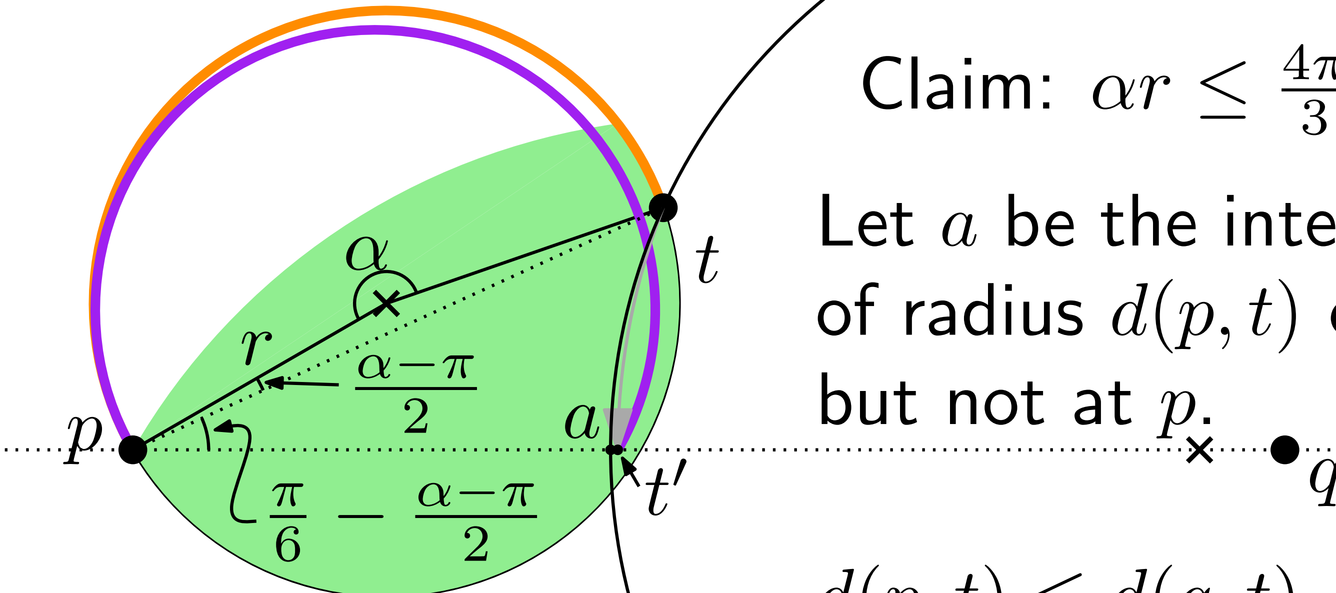


p's orange $\le$ p's purple arc
tricky

q's orange $=$ q's purple arc

Claim: $\alpha r \leq \frac{4\pi}{3} \frac{d(p,t')}{2\cos(\pi/6)}$

Let a be the intersection of the circle of radius $d(p,t)$ centered on line pq but not at p.



$$d(p,t) \leq d(q,t) \Rightarrow d(p,a) \leq d(p,t').$$

$$r = \frac{d(p,t)}{2\cos((\alpha-\pi)/2)}$$

$$d(p,a) = 2d(p,t)\cos(\pi/6 - (\alpha-\pi)/2) - d(p,t)$$

Claim $\Leftarrow \frac{\alpha}{2\cos\left(\frac{\alpha-\pi}{2}\right)} \leq \frac{4\pi}{3} \frac{2\cos(\pi/6-(\alpha-\pi)/2)-1}{2\cos(\pi/6)}$ for $\pi \leq \alpha \leq \frac{4\pi}{3}$

calculus...