CS 516 -1
Computational Geometry & Graph Drawing
(Spring 2013)

Today…

- go over admin details

  www.cs.ubc.ca/~kirk/cs516

- high-level discussion/illustration of topics
- start a (motivating) example: finding near neighbours (within fixed distance)

What *is* computational geometry?

- The study of geometric problems from a computational (algorithmic) perspective

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- The study of (familiar) computational problems under geometric constraints
- Note: it means other things to other people

What *is* computational geometry?

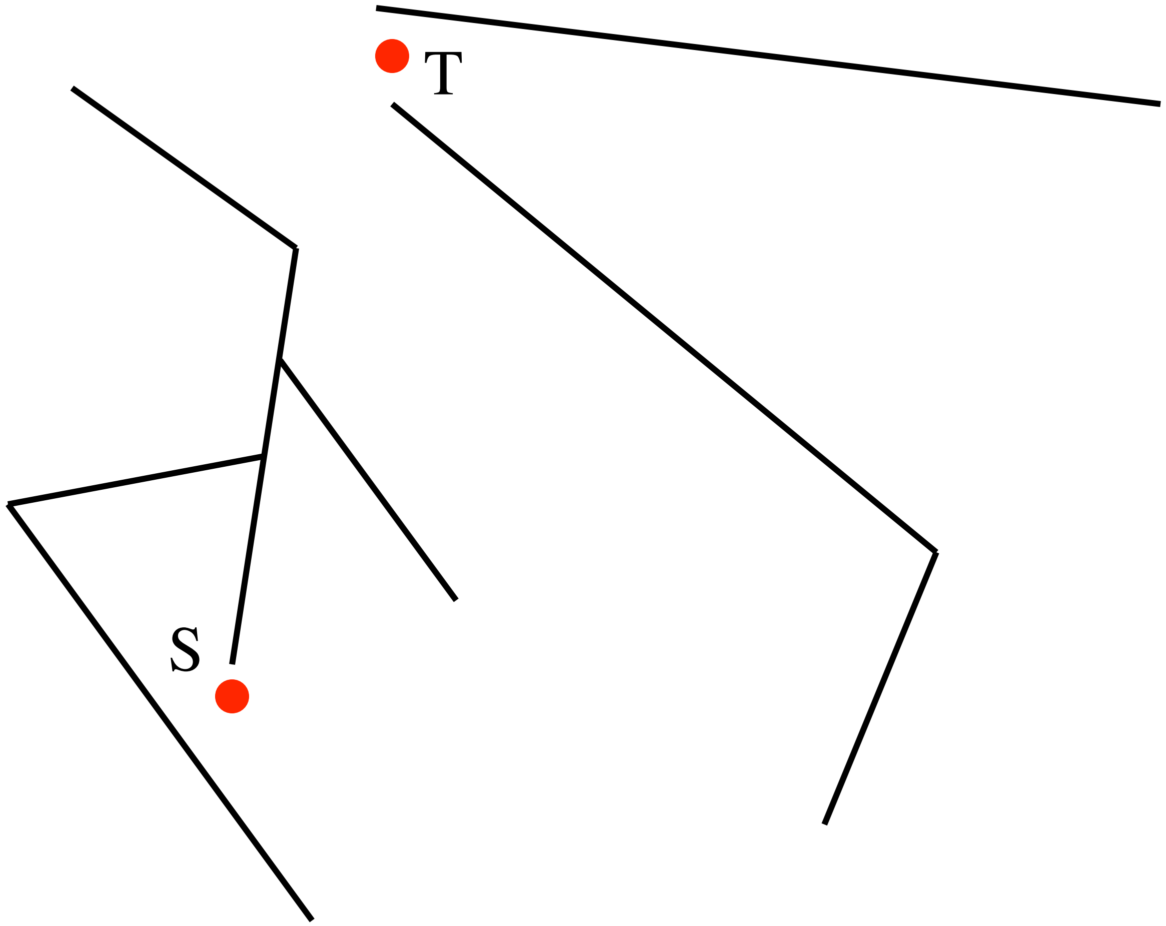
Topics in surface modeling: b-splines, non-uniform rational b-splines, physically based deformable surfaces, sweeps and generalized cylinders, offsets, blending and filleting surfaces. Non-linear solvers and intersection problems. Solid modeling: constructive solid geometry, boundary representation, non-manifold and mixed-dimension boundary representation models, octrees. Robustness of geometric computations. Interval methods. Finite and boundary element discretization methods for continuum mechanics problems. Scientific visualization. Variational geometry. Tolerances. Inspection methods. Feature representation and recognition. Shape interrogation for design, analysis, and manufacturing. Involves analytical and programming assignments.

How does it relate to graph drawing?

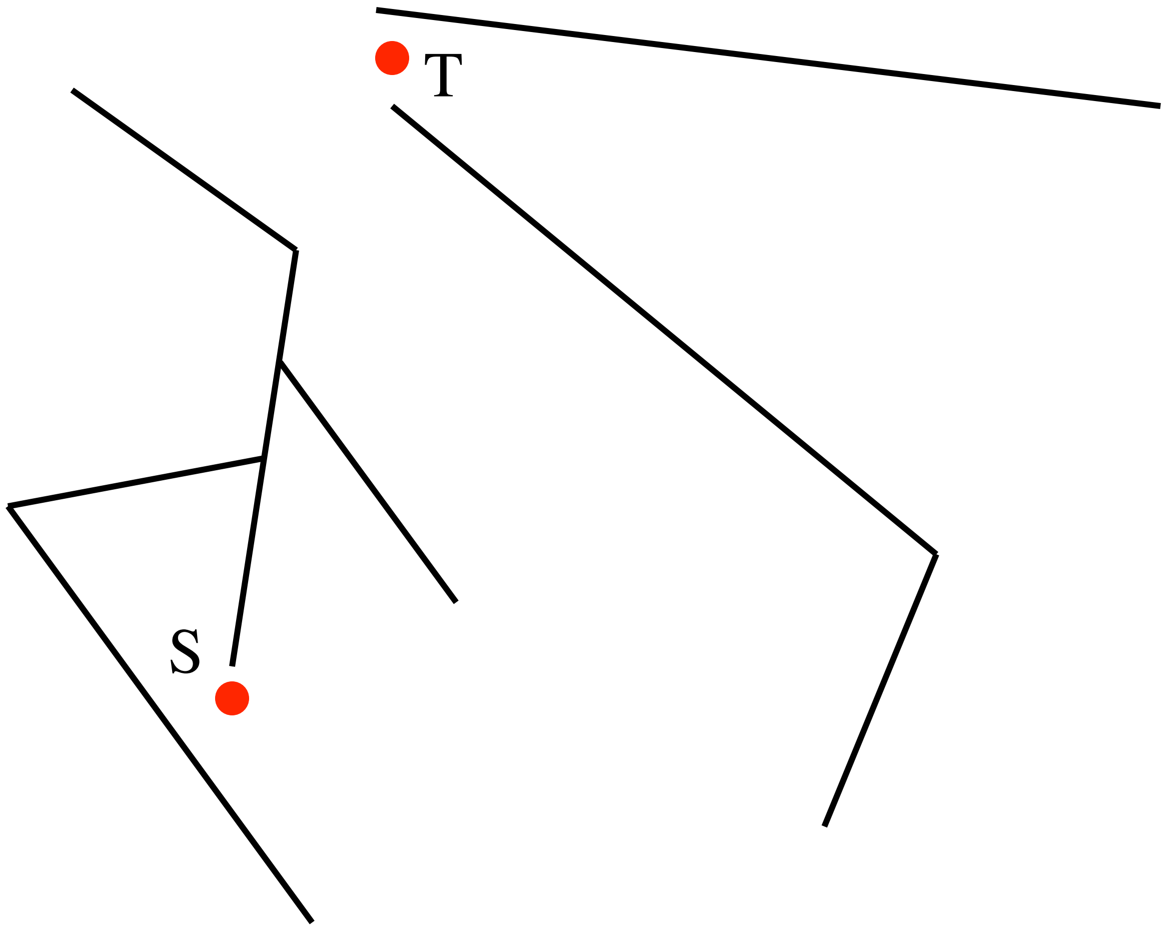
- Graph drawing uses methods from computational geometry, as well as geometric graph theory and information visualization, to construct pleasing and informative renderings of graphs arising from areas such as social networks, cartography, and bioinformatics.
- Computational geometry looks for the structure implicit in configurations of geometric objects. Graph drawing seeks geometric configurations that realize/embody given abstract structures.

An illustrative example

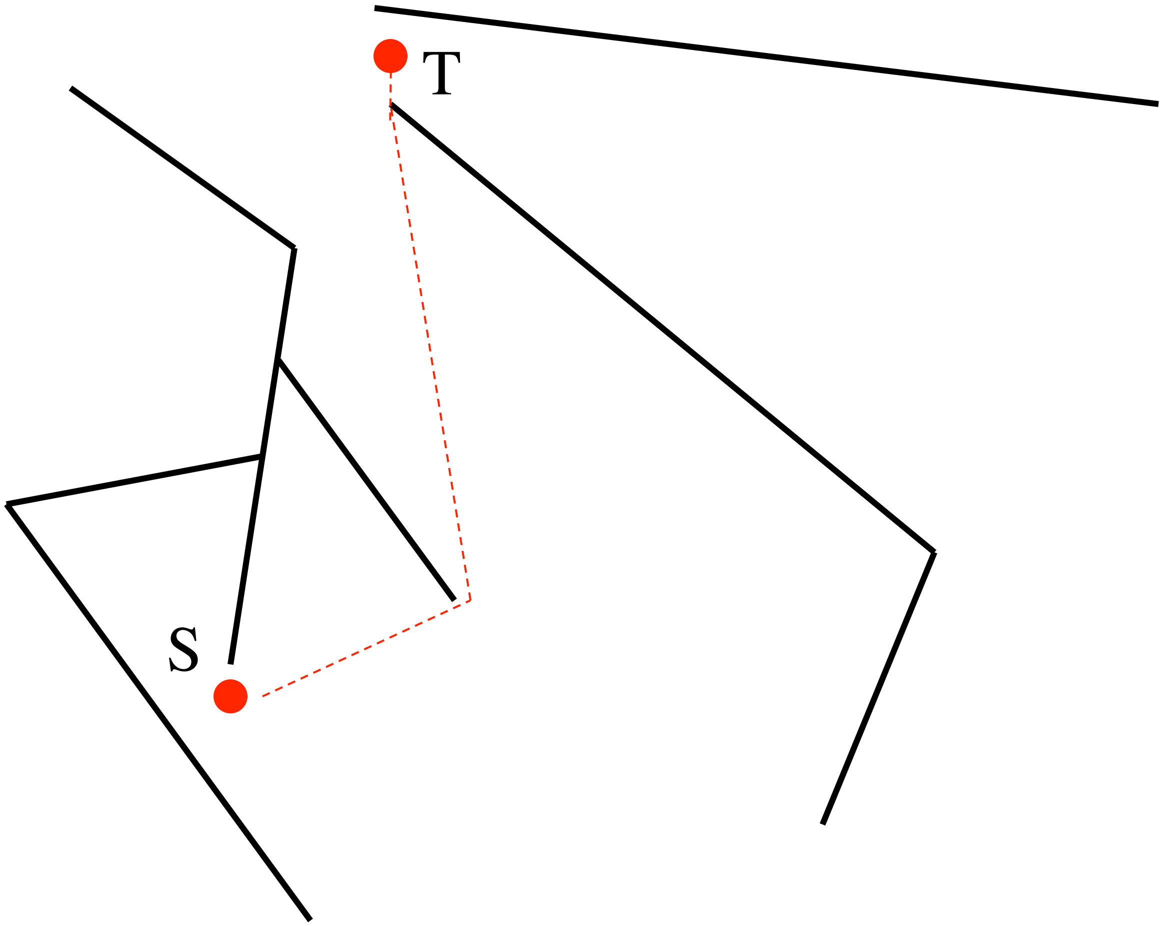
- Finding an *optimal* path from here to there…



planning optimal motion in the plane
is “easy”



shortest motion



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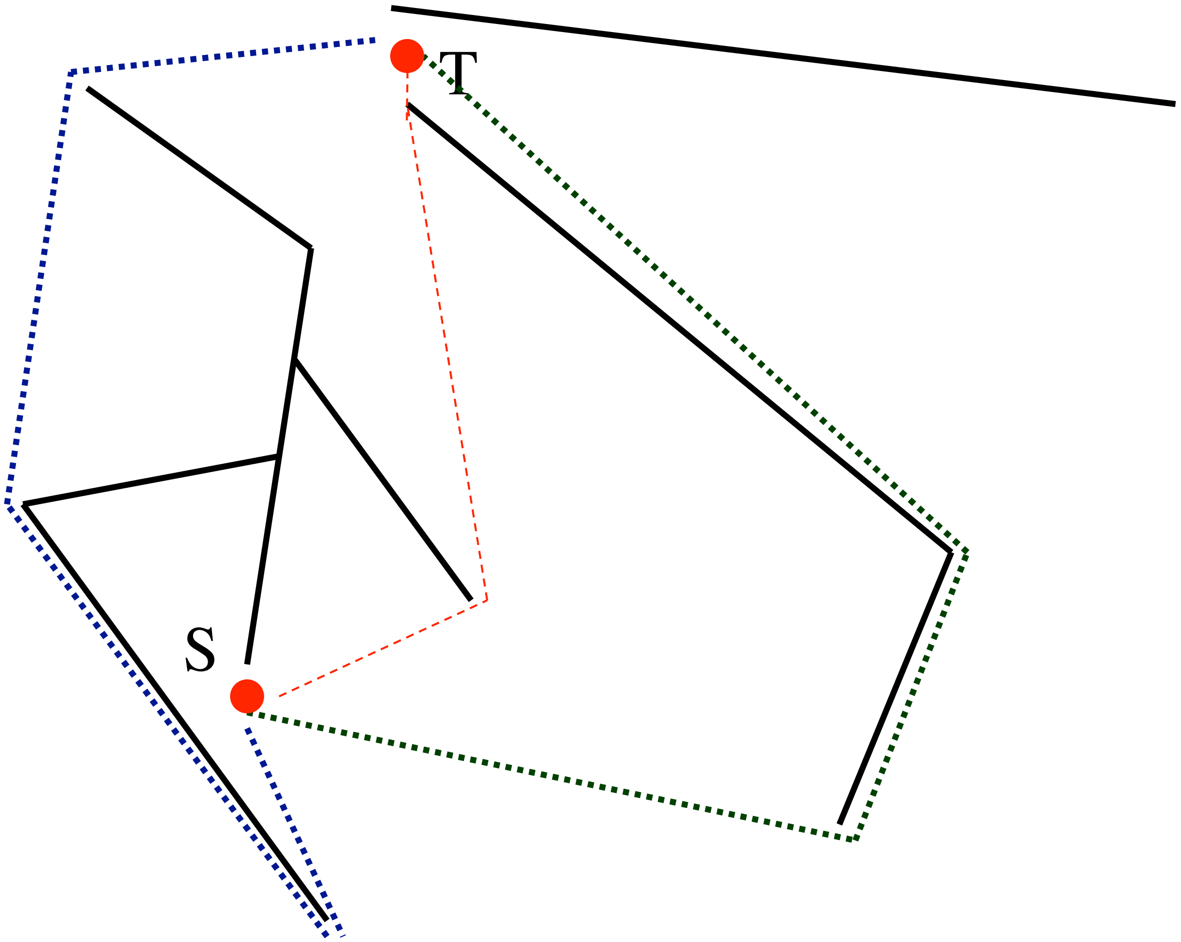
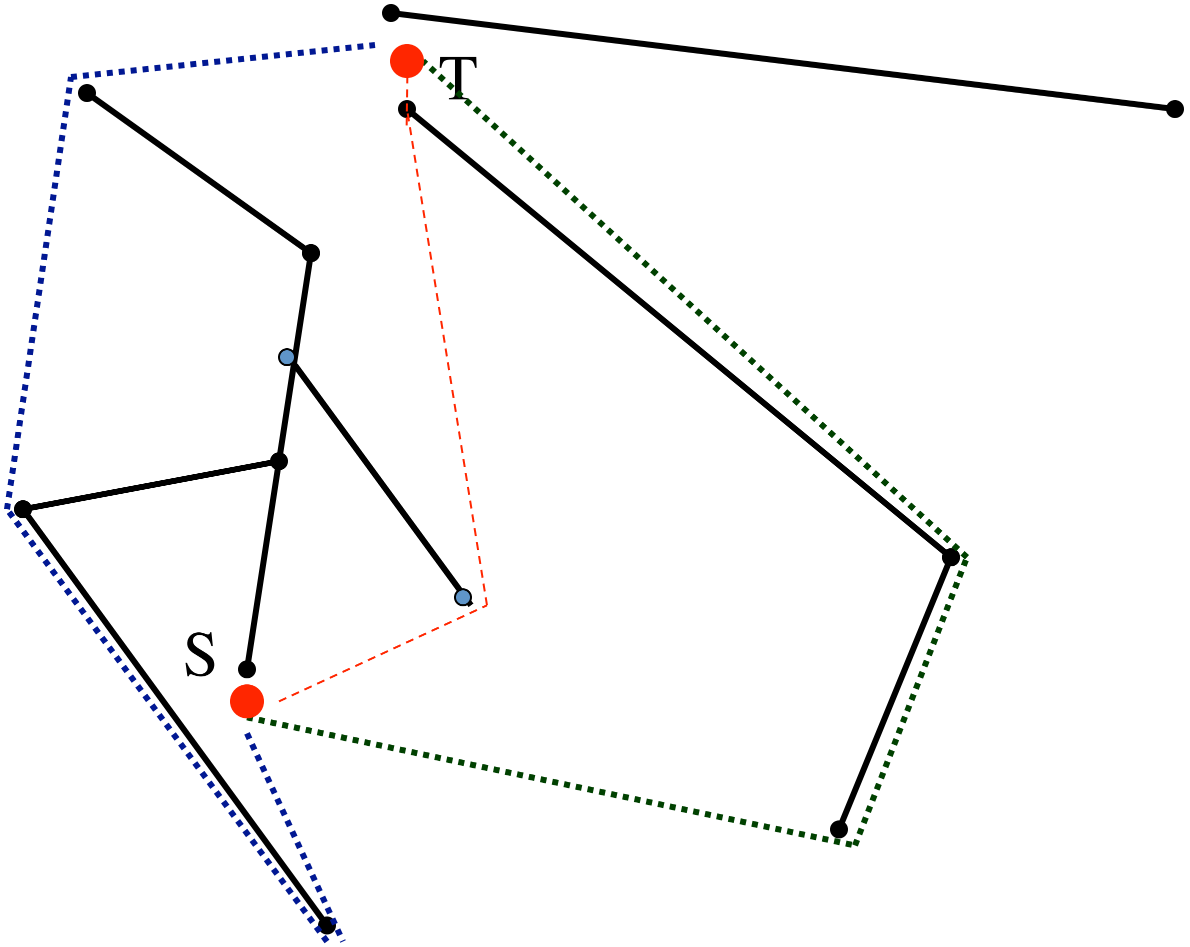
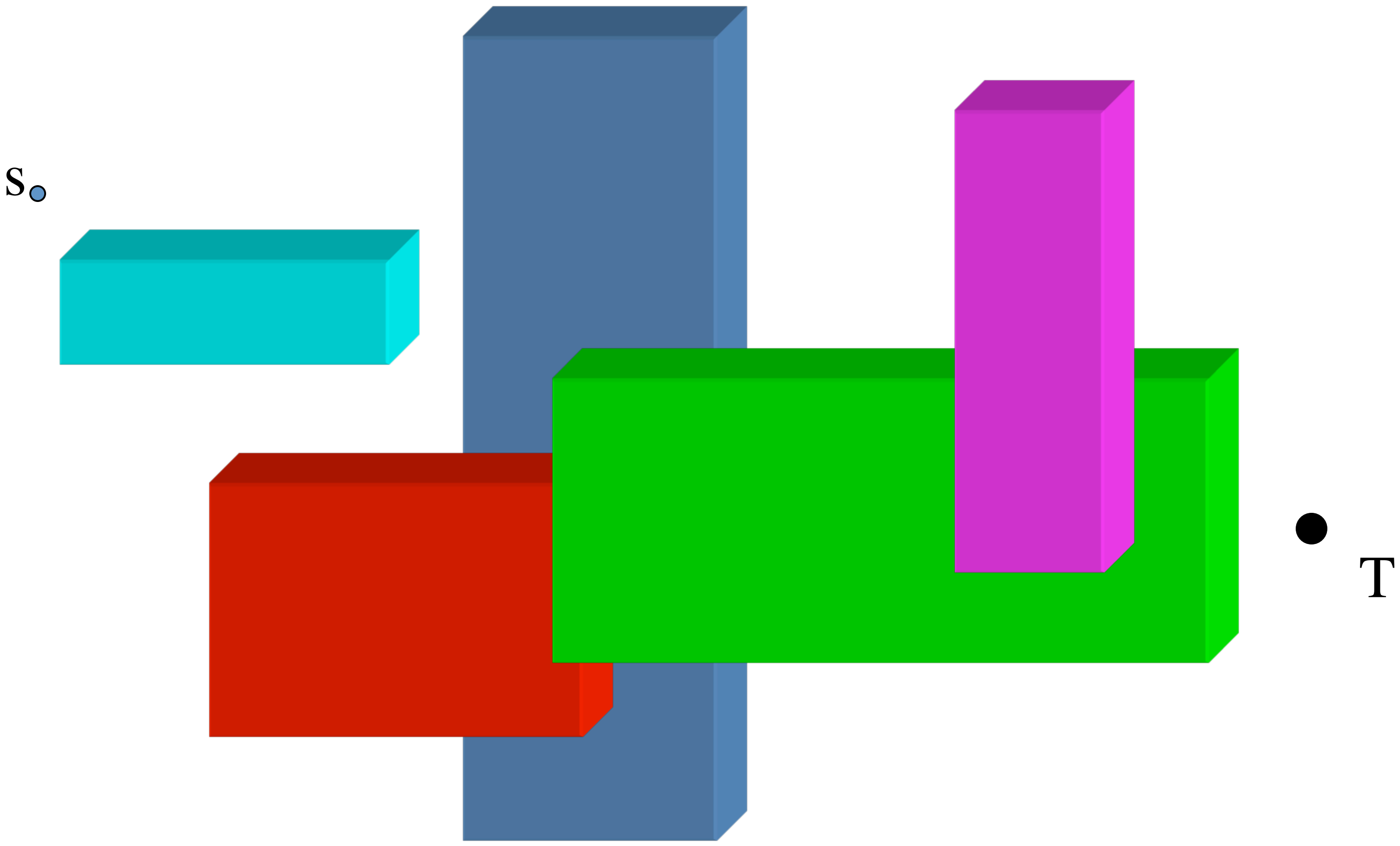


other locally shortest motions

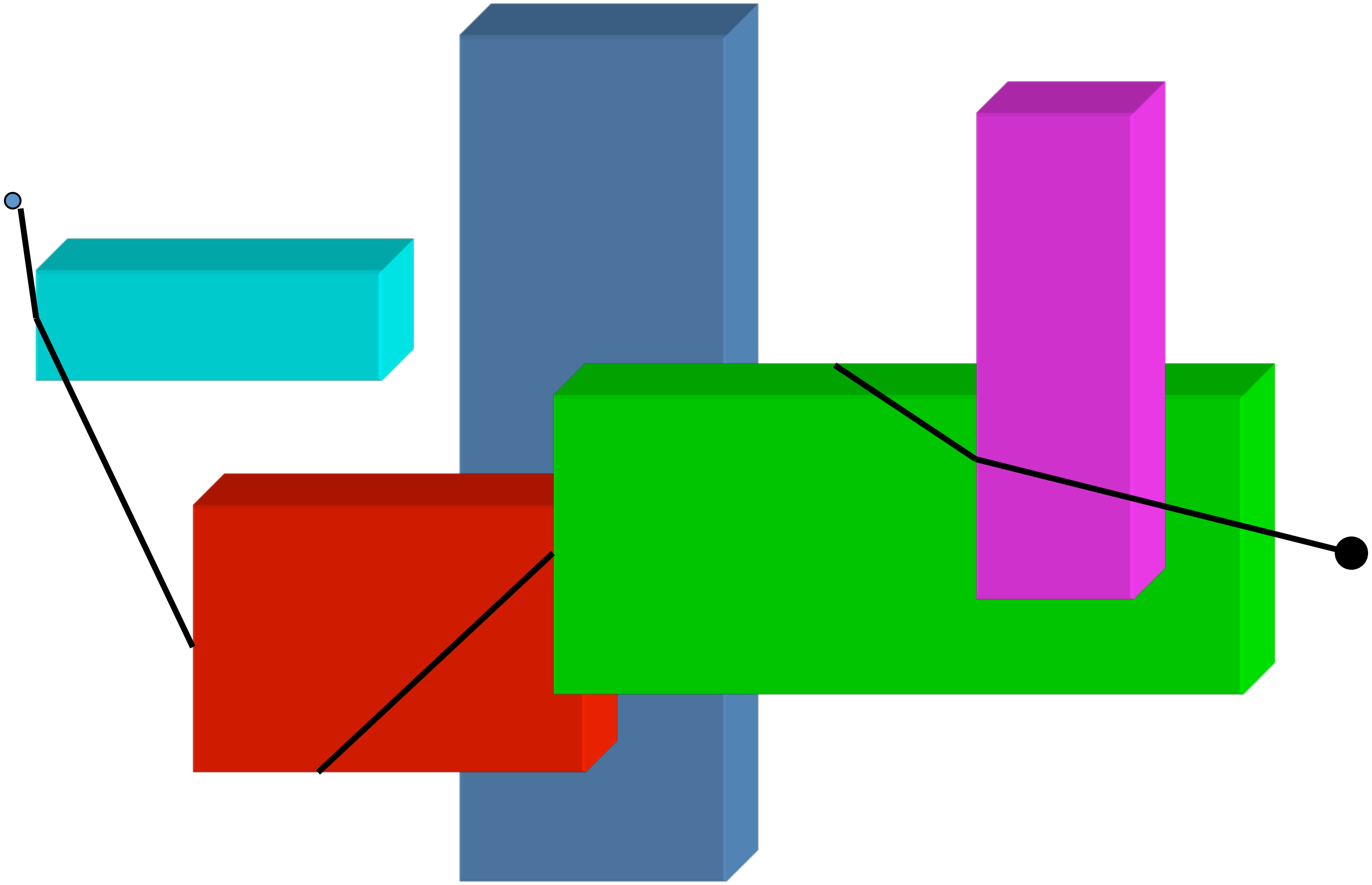


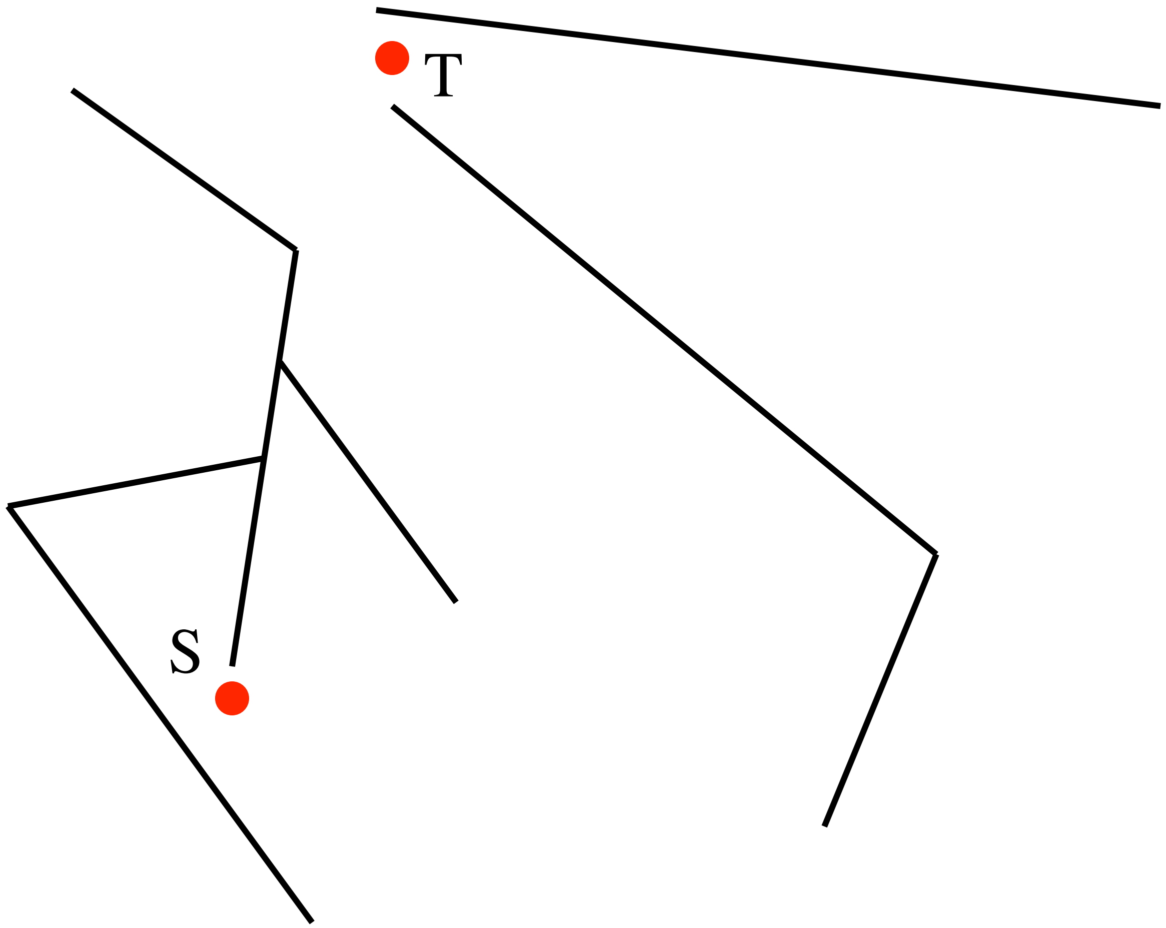
reduction to shortest path
in a (visibility) graph

three-dimensional obstacles

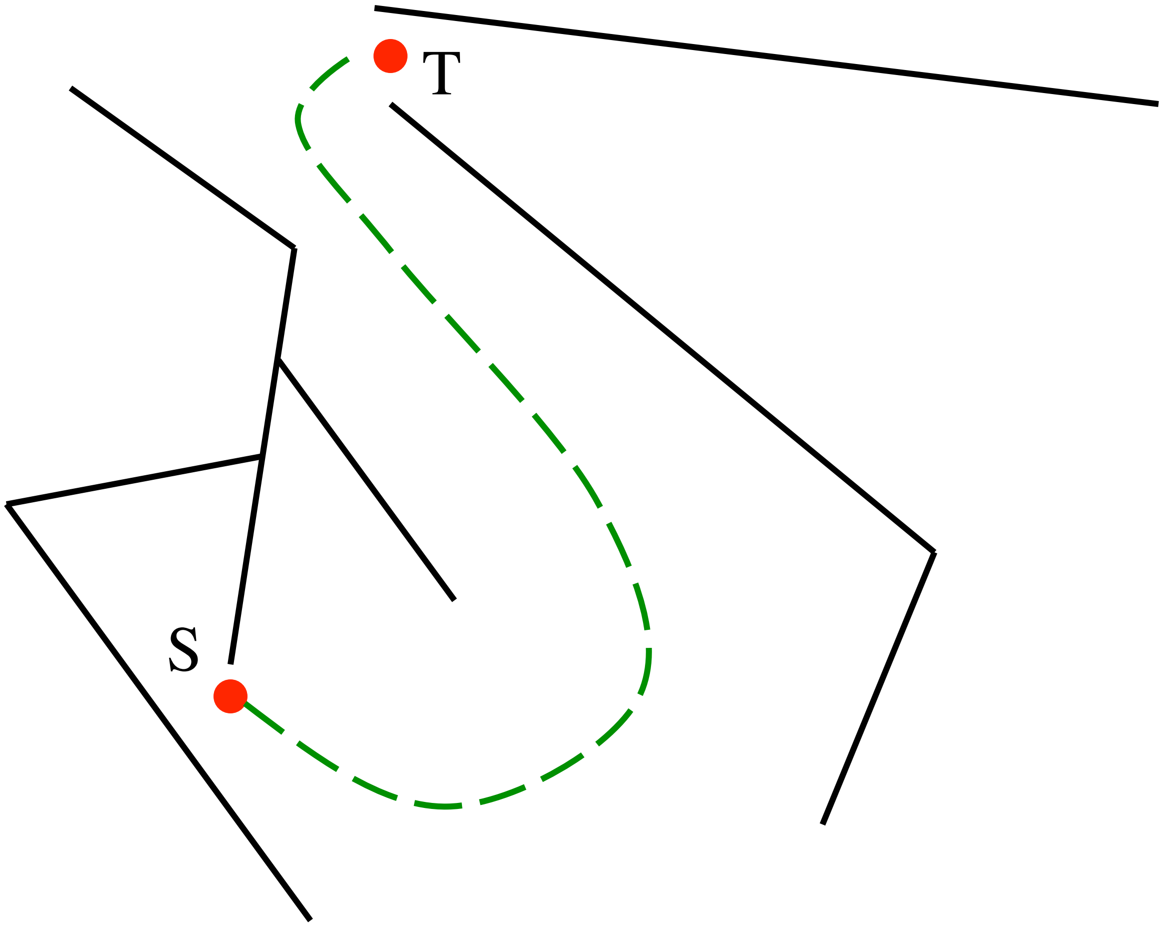


bends could occur anywhere along edges

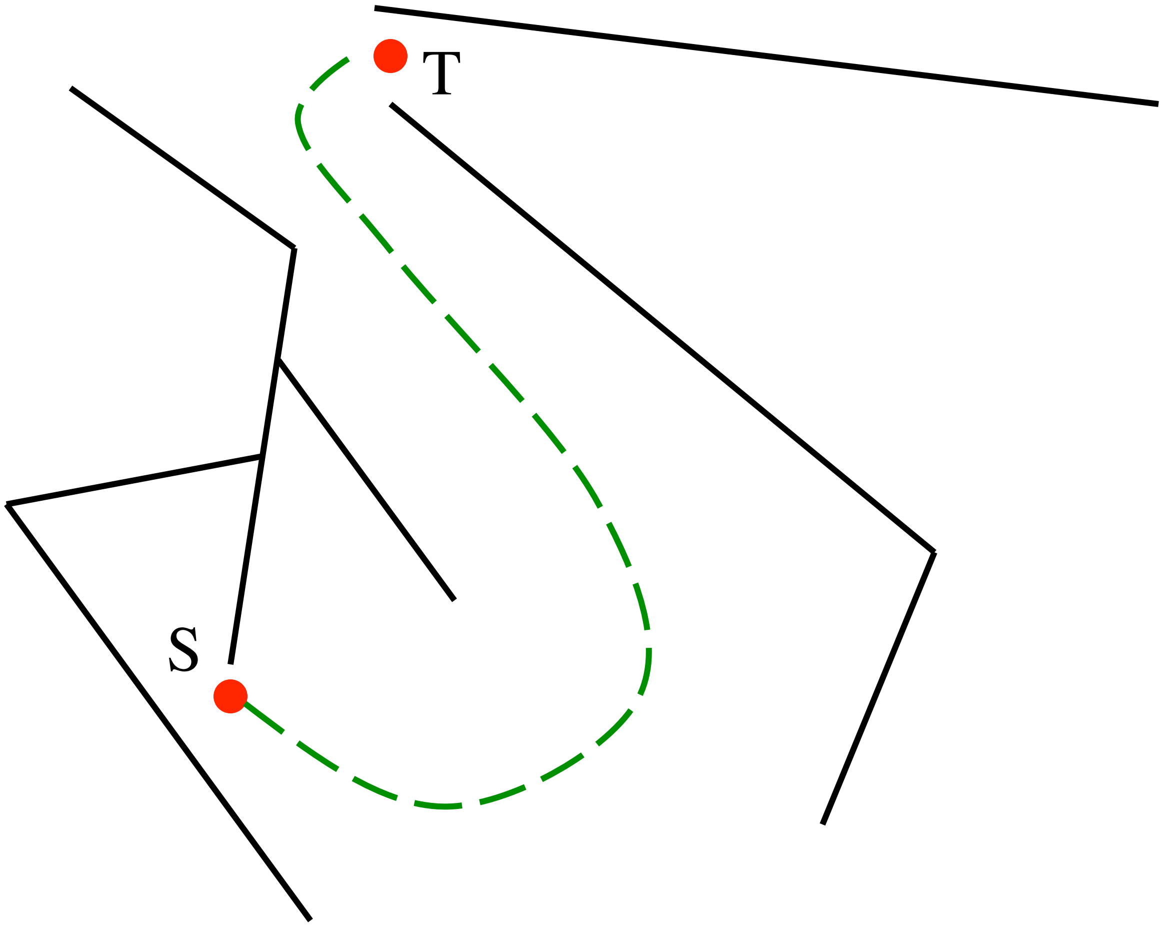




other notions of optimality:



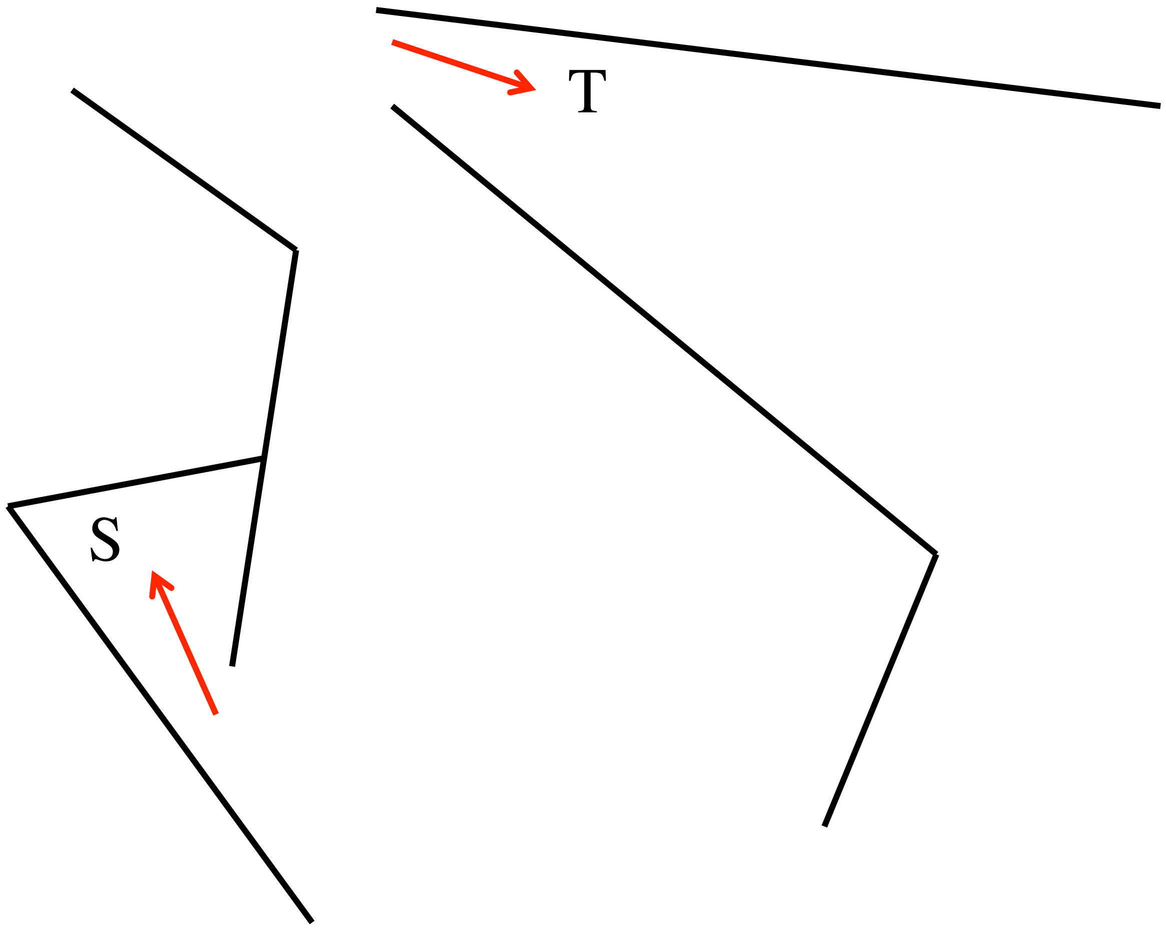
other notions of optimality:
maximum clearance



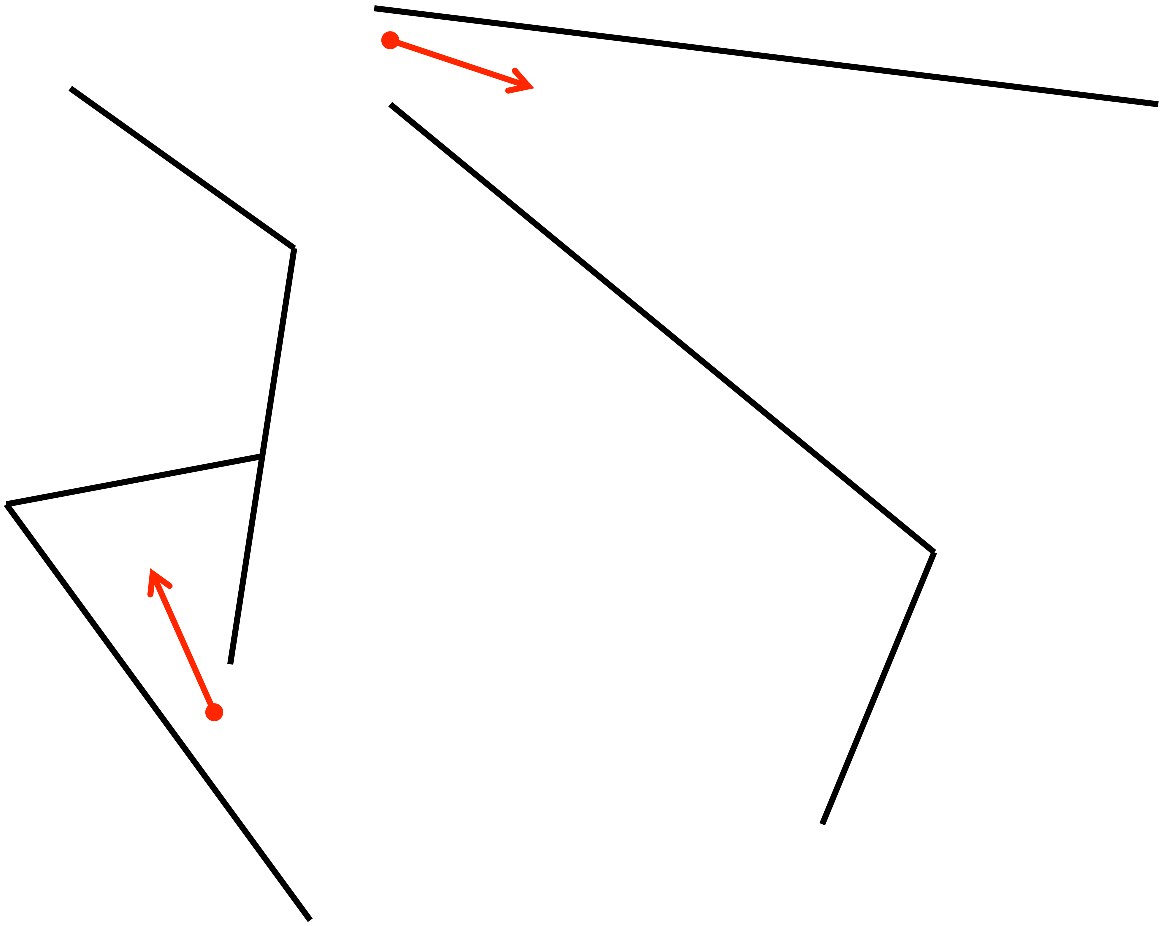
other notions of optimality:

maximum clearance

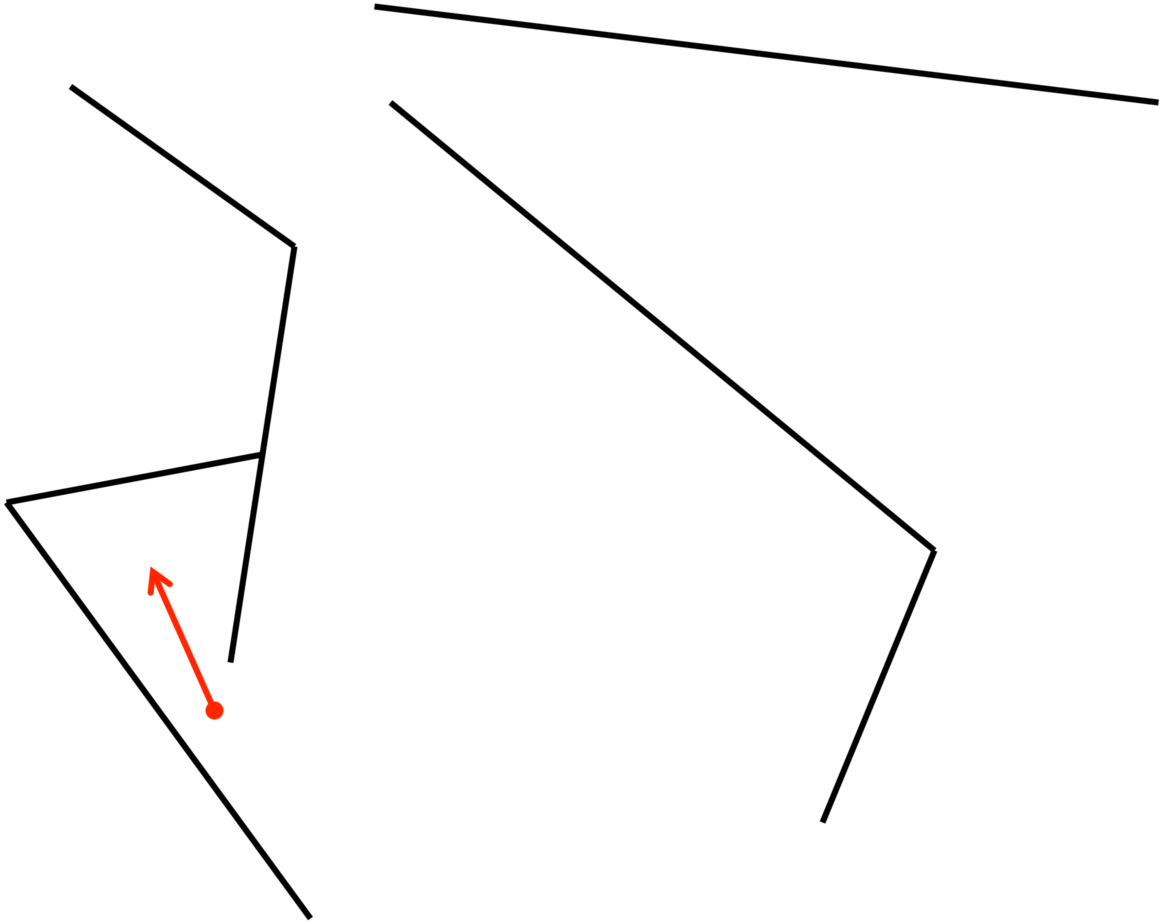
minimum curvature

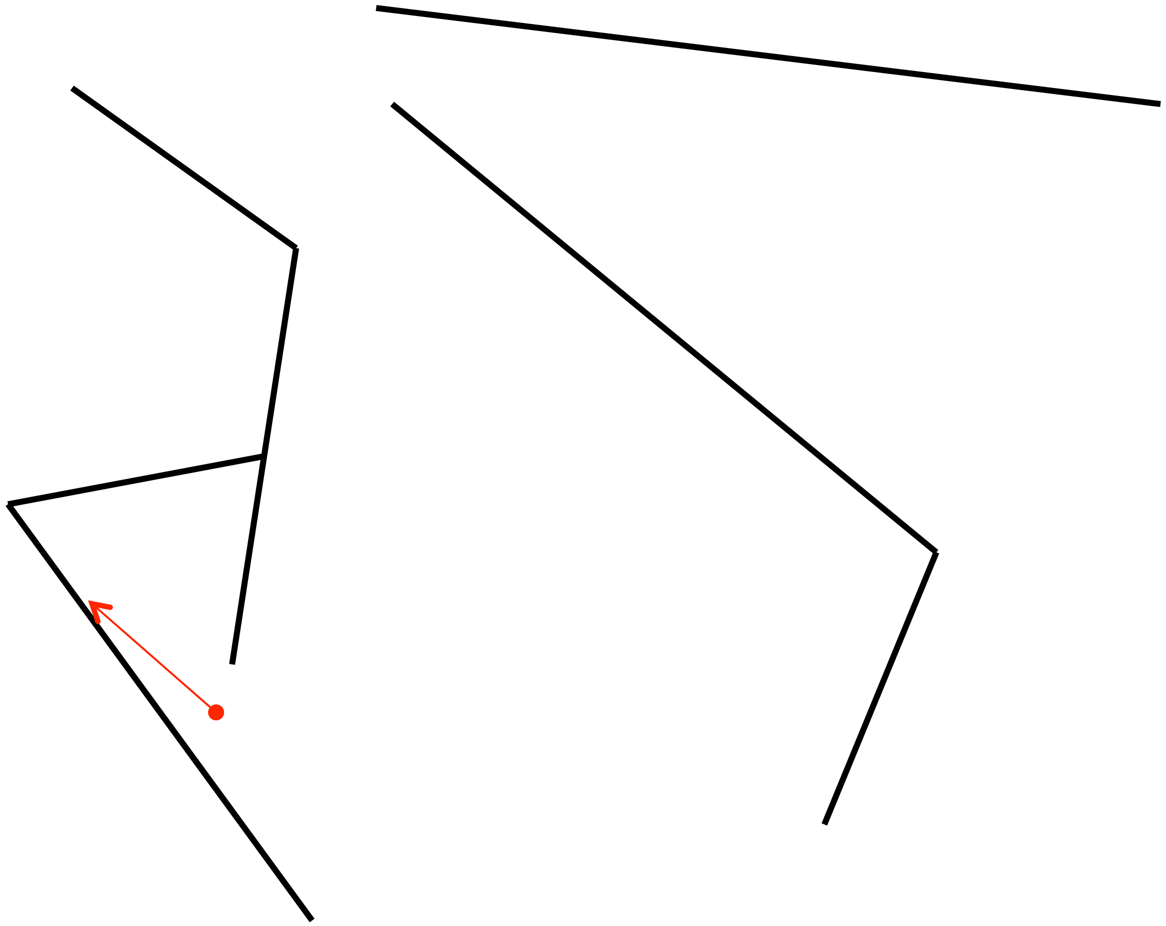


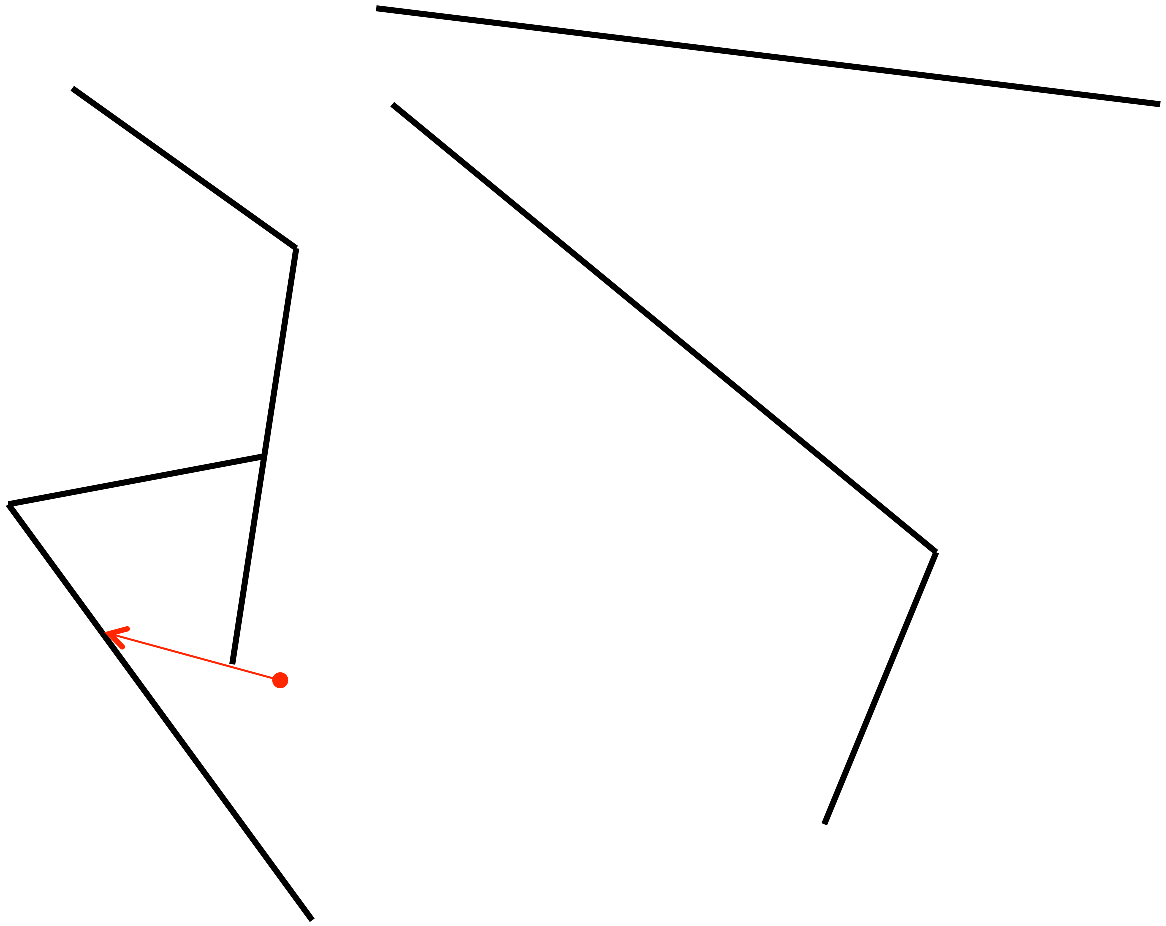
what is the shortest motion?

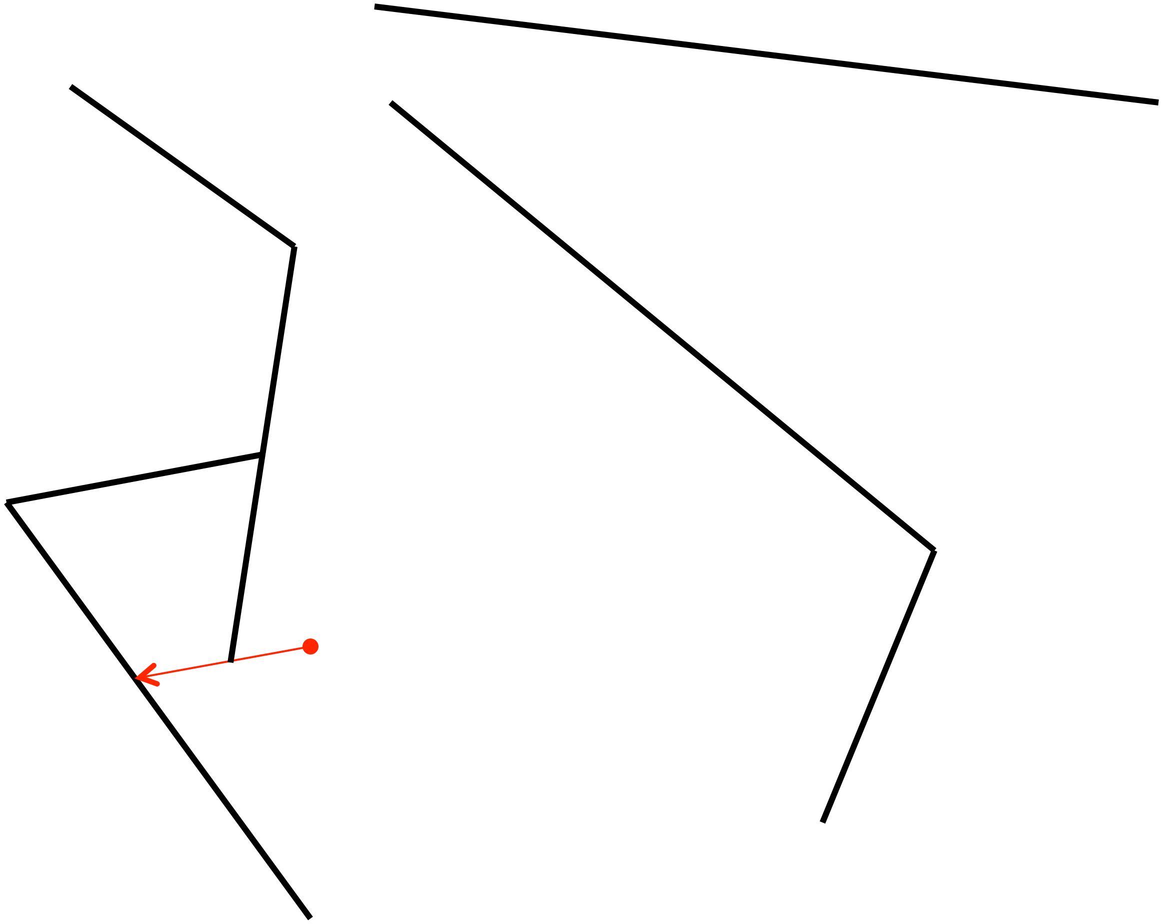


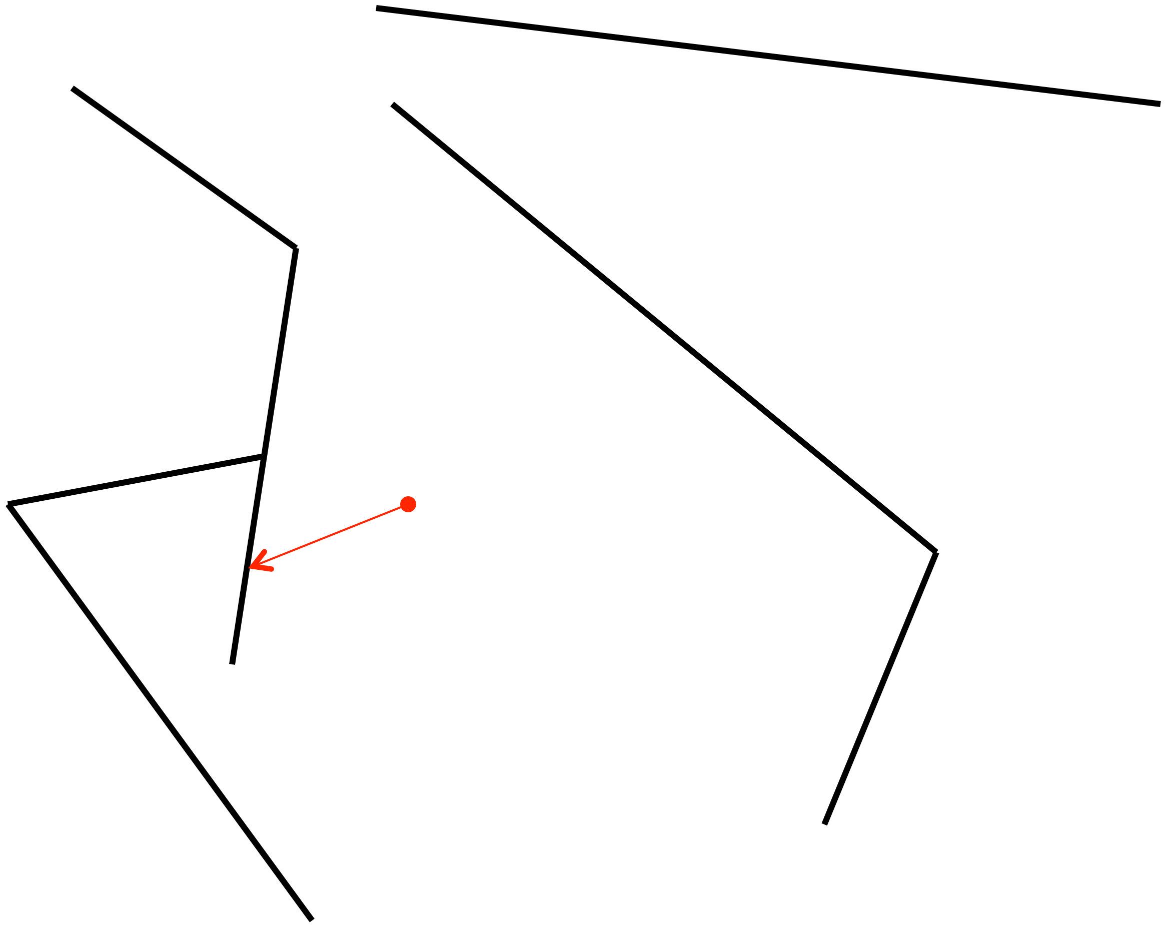
fix a point and minimize its trace

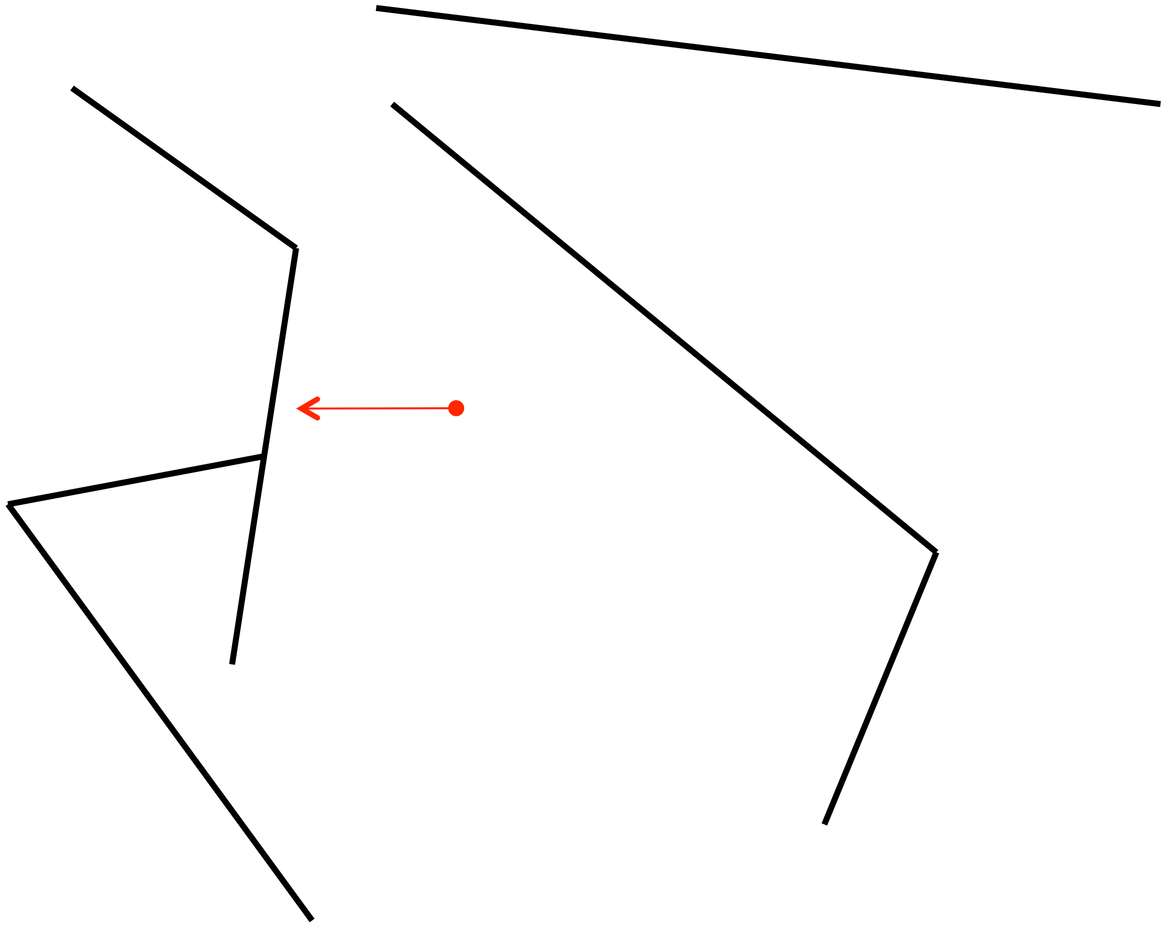


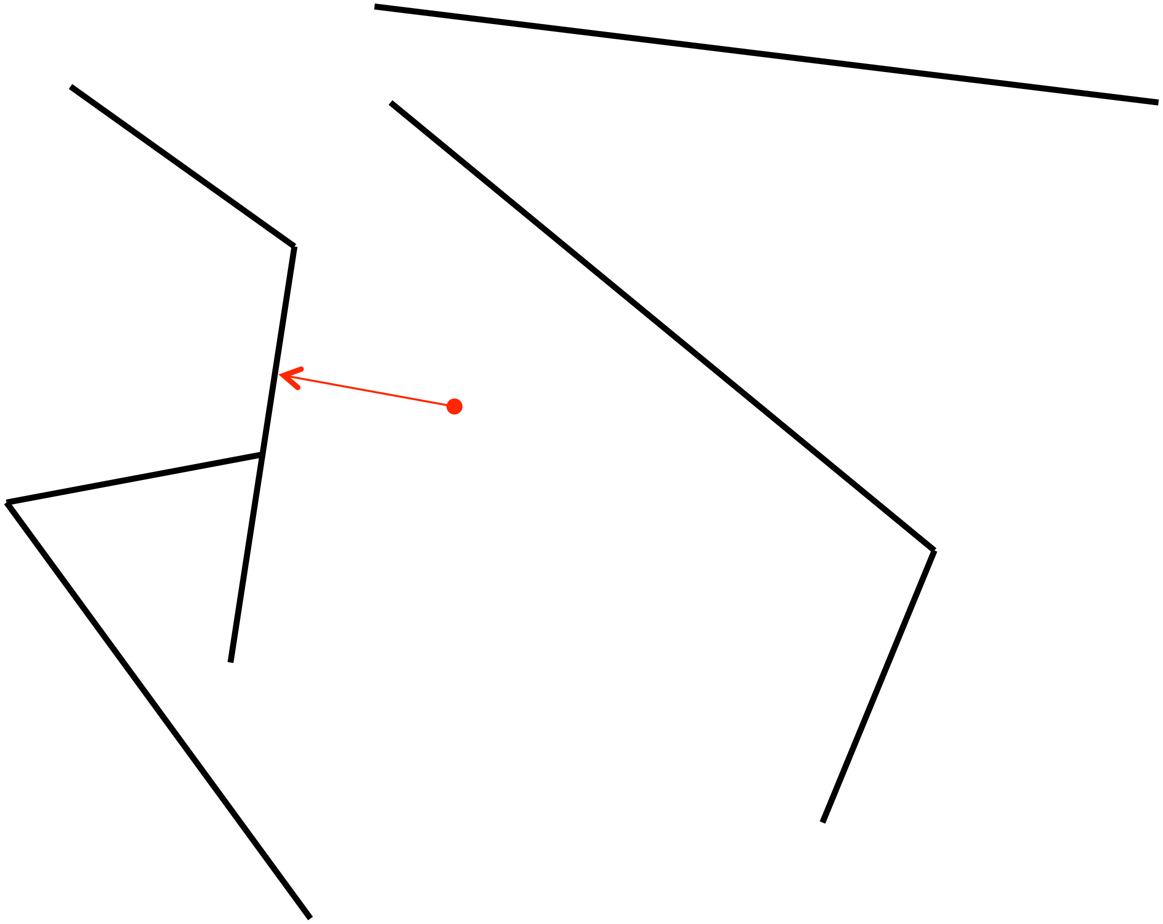


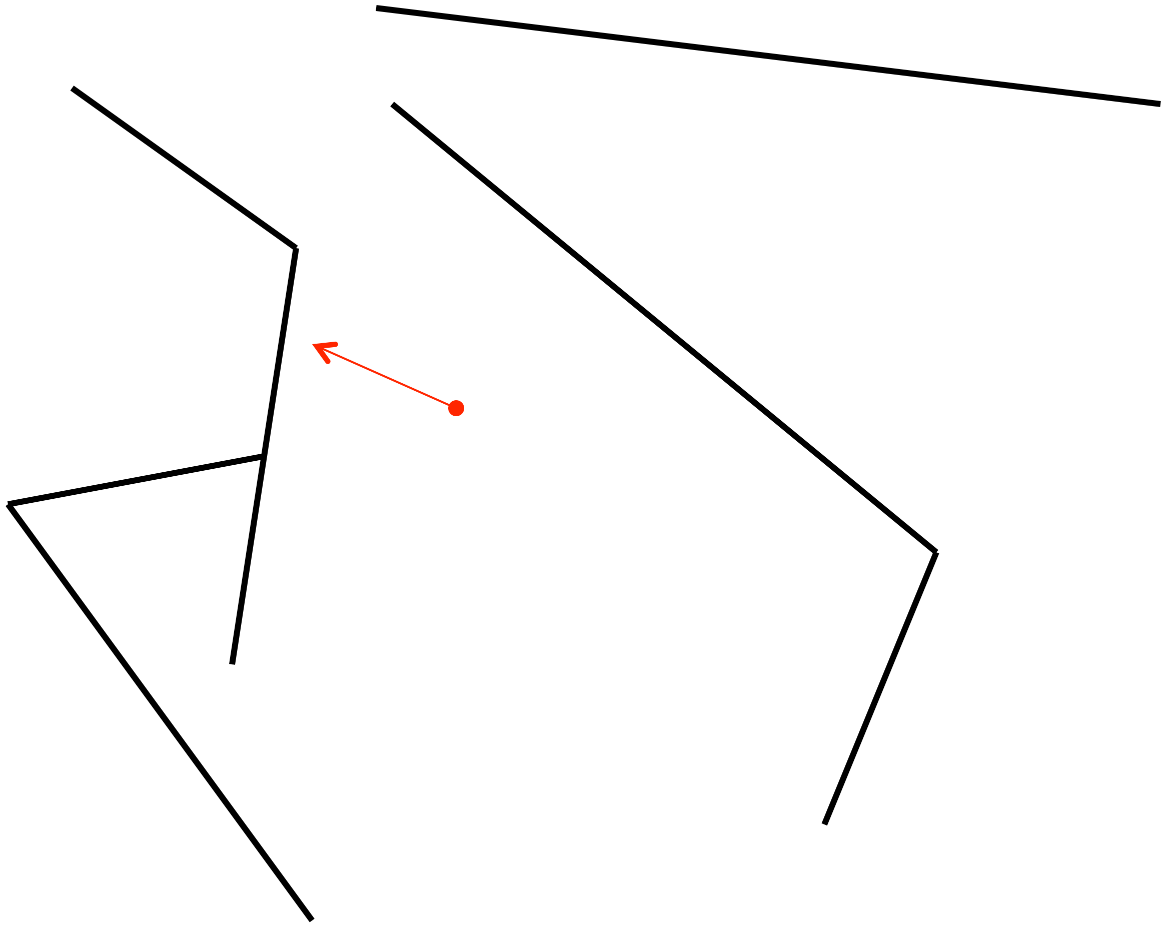


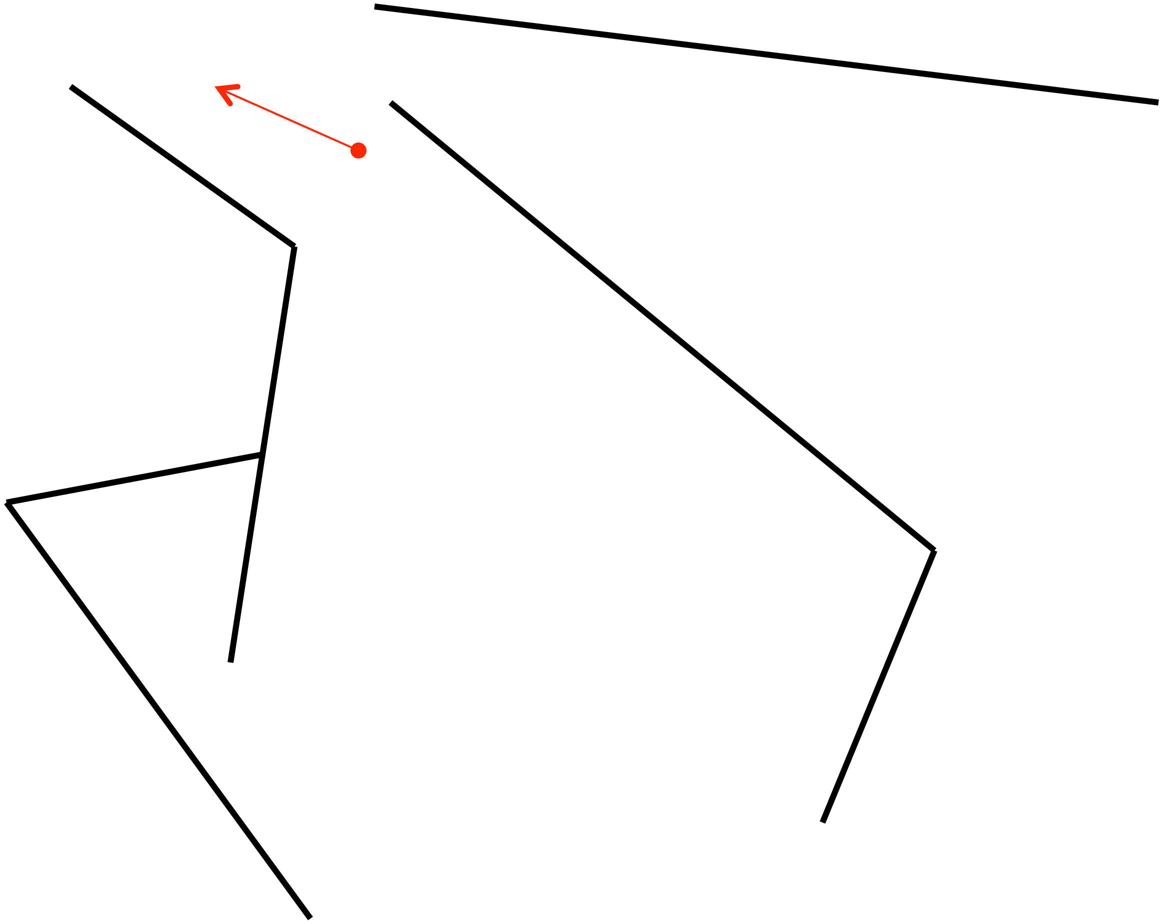


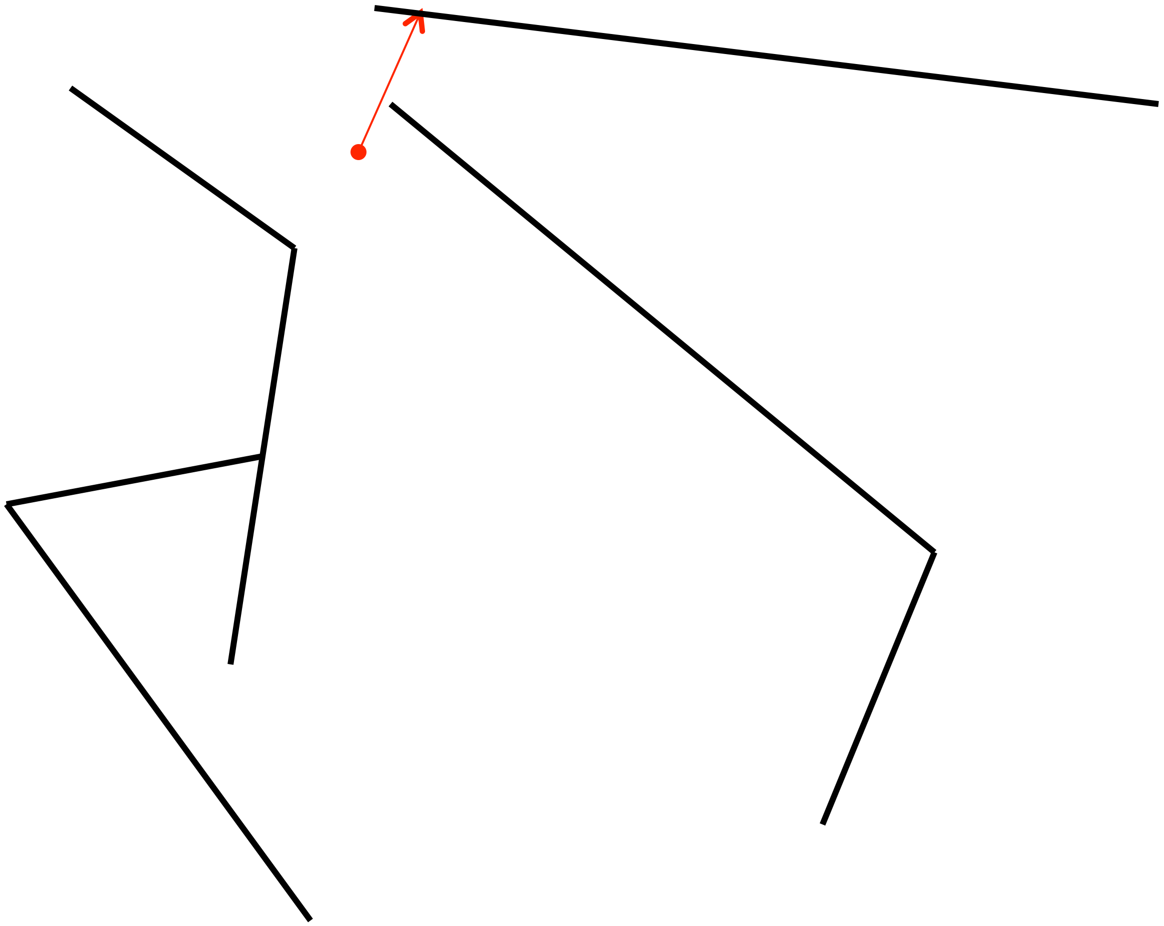


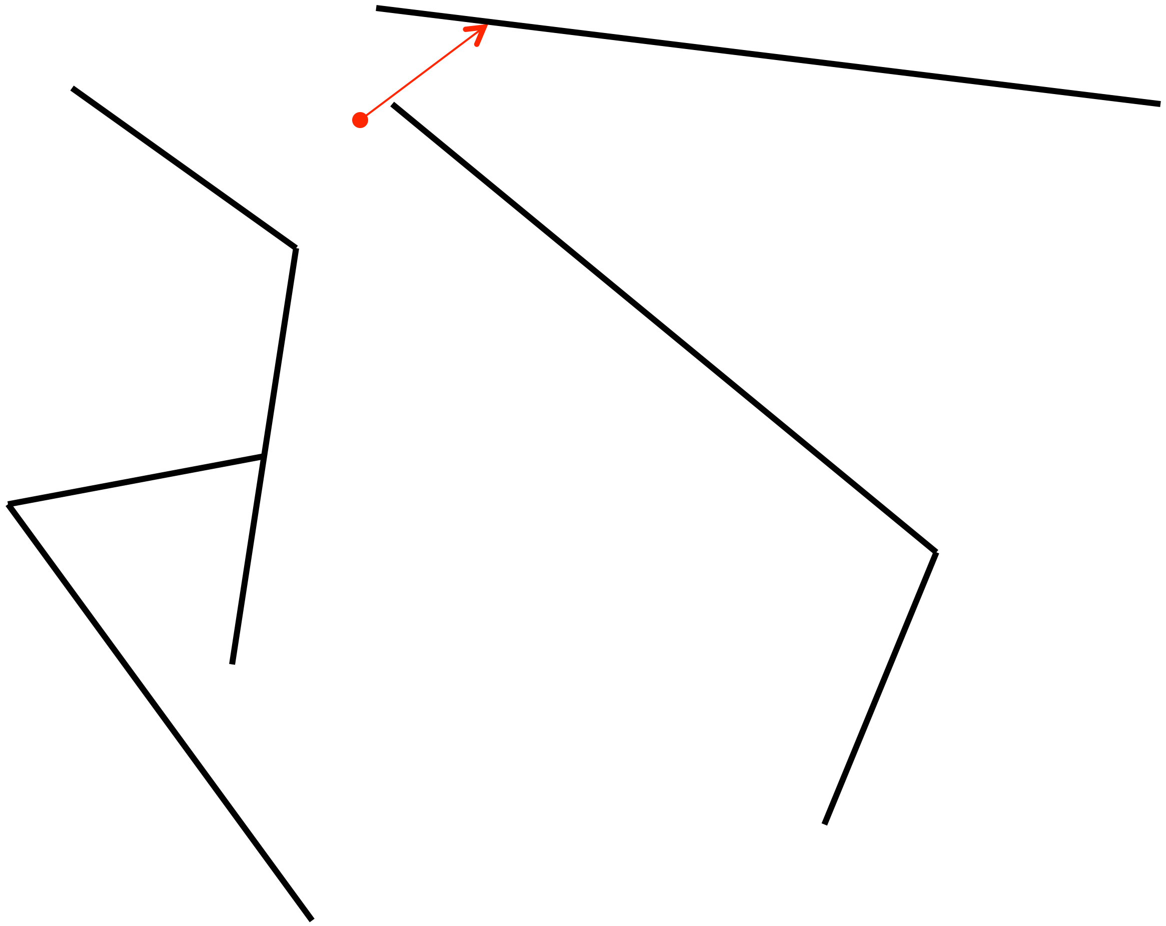


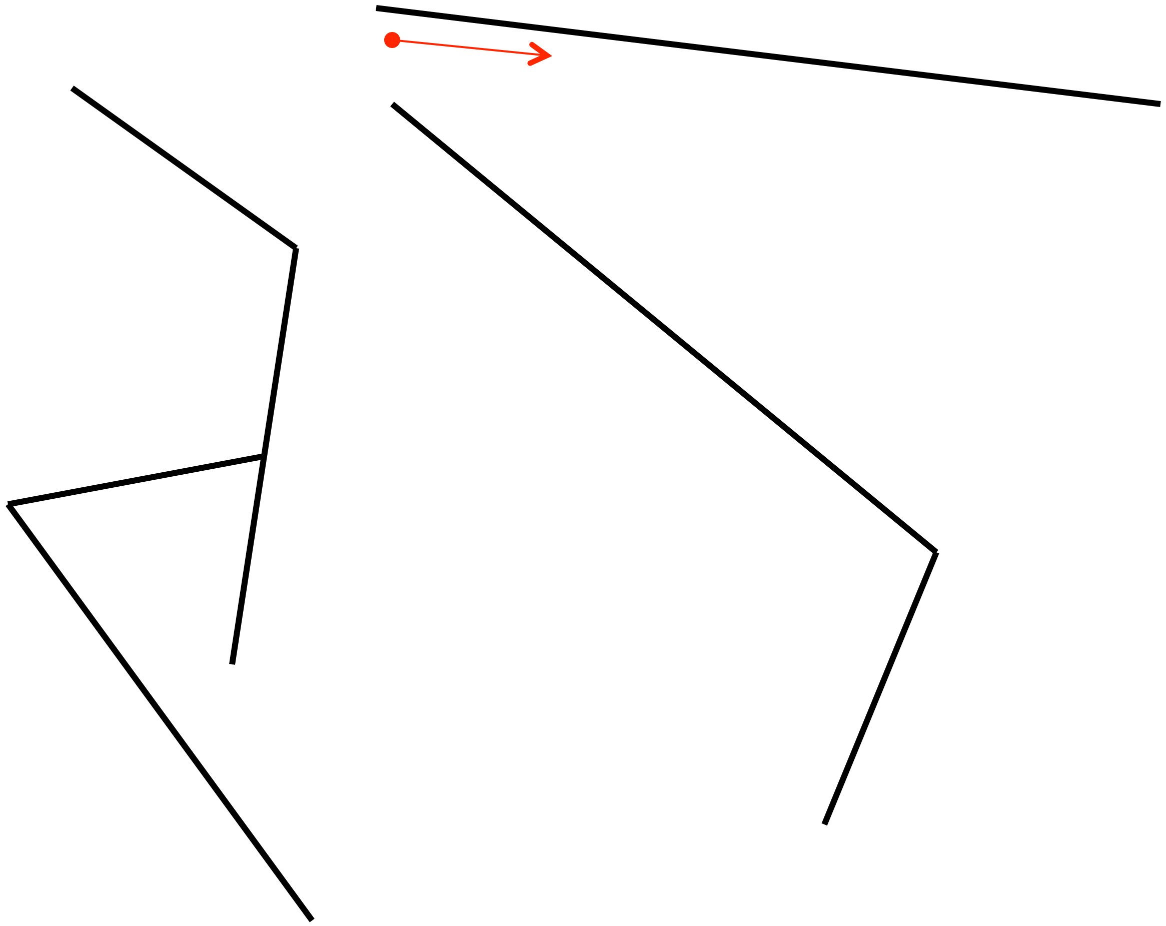


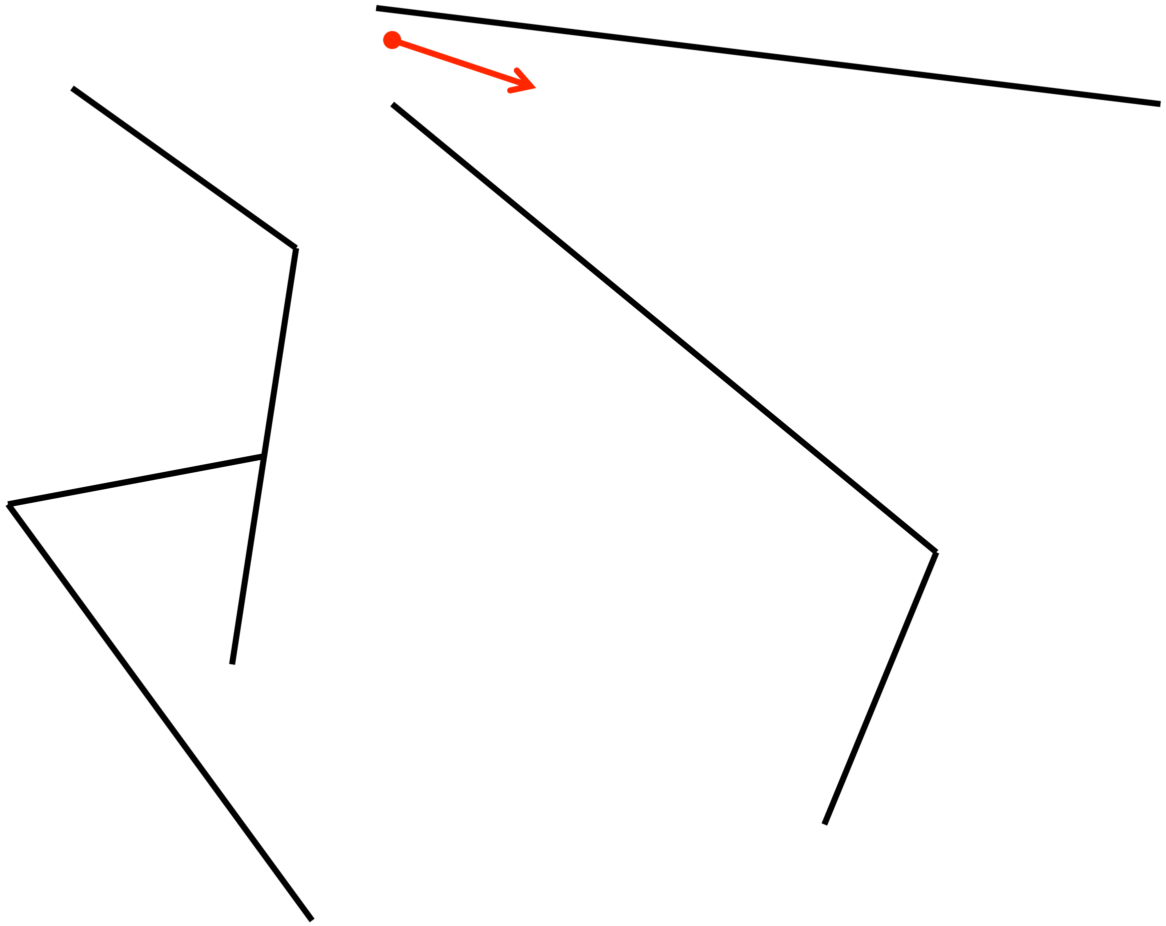


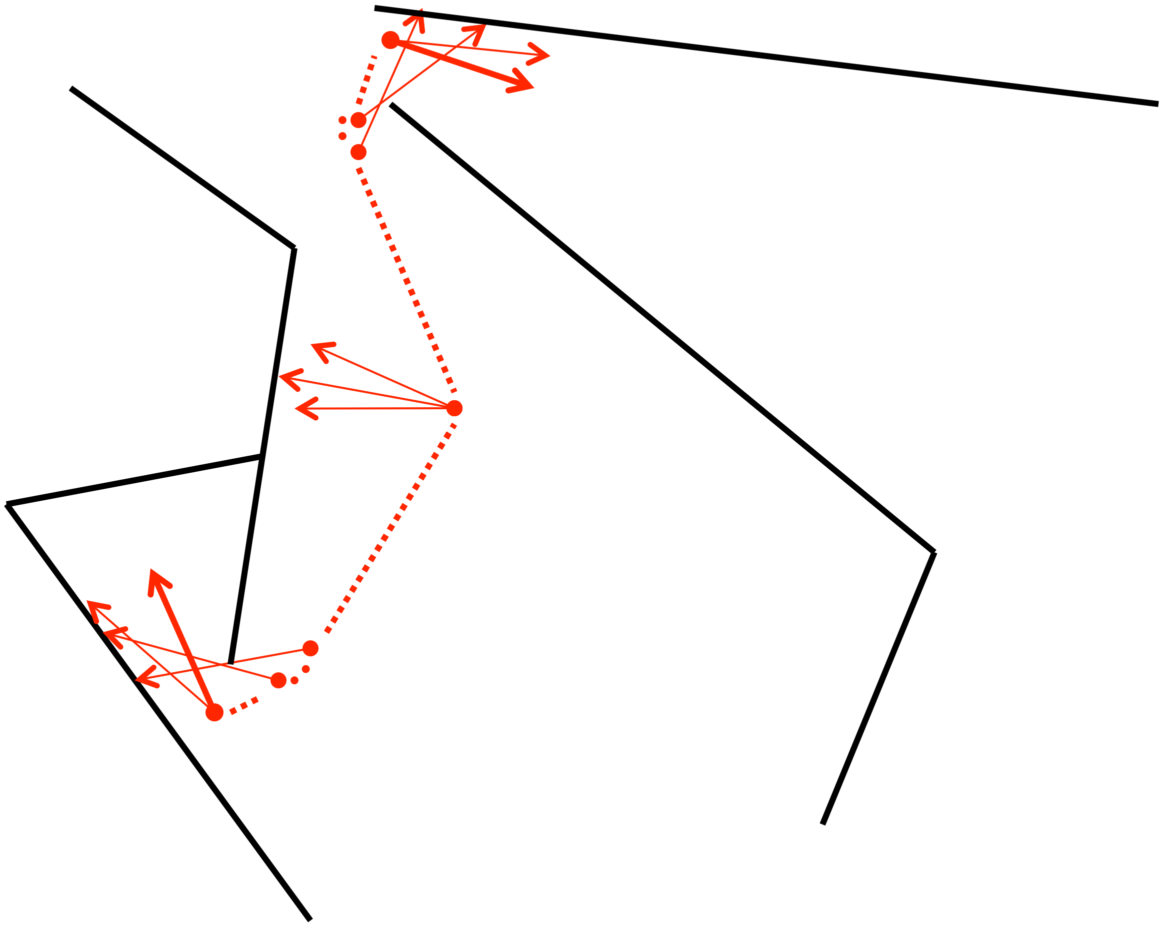




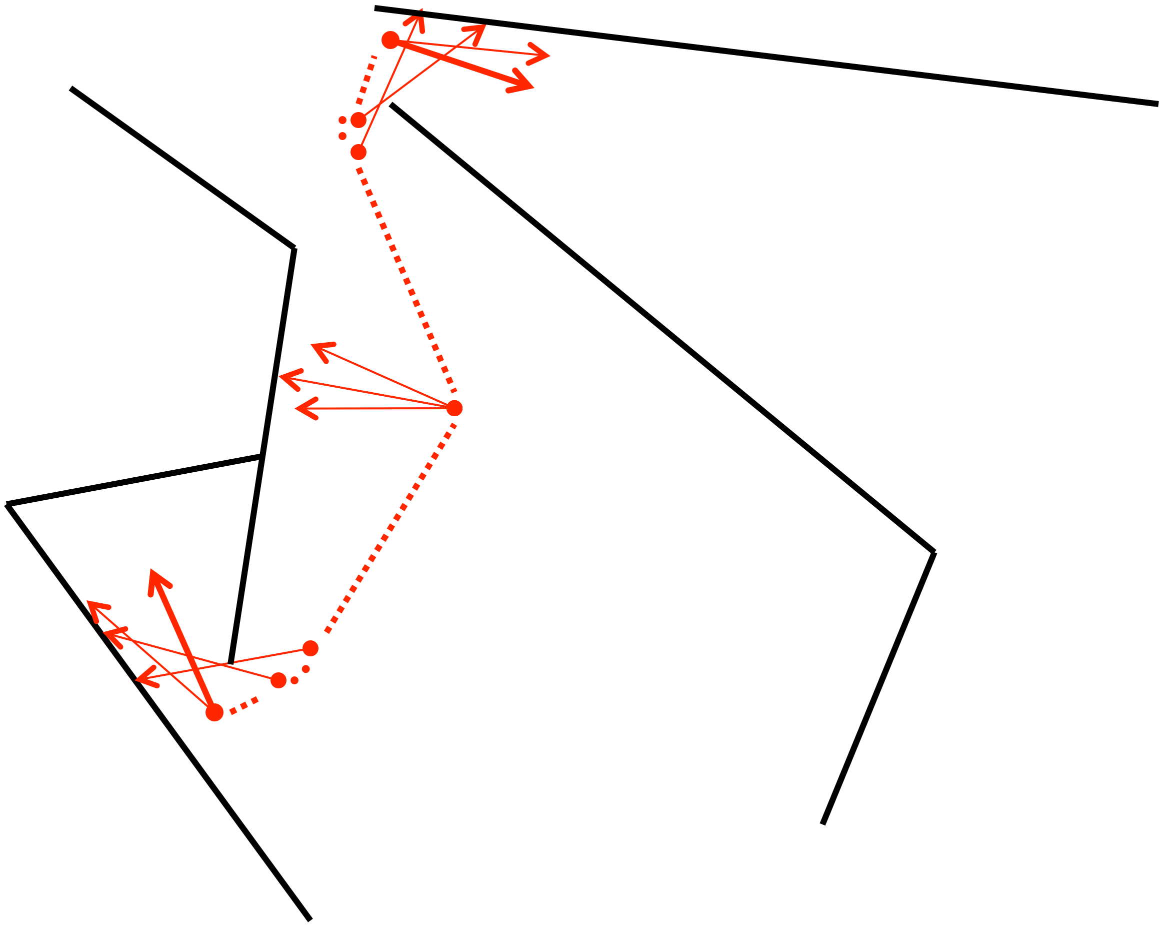








shortest trace of endpoint



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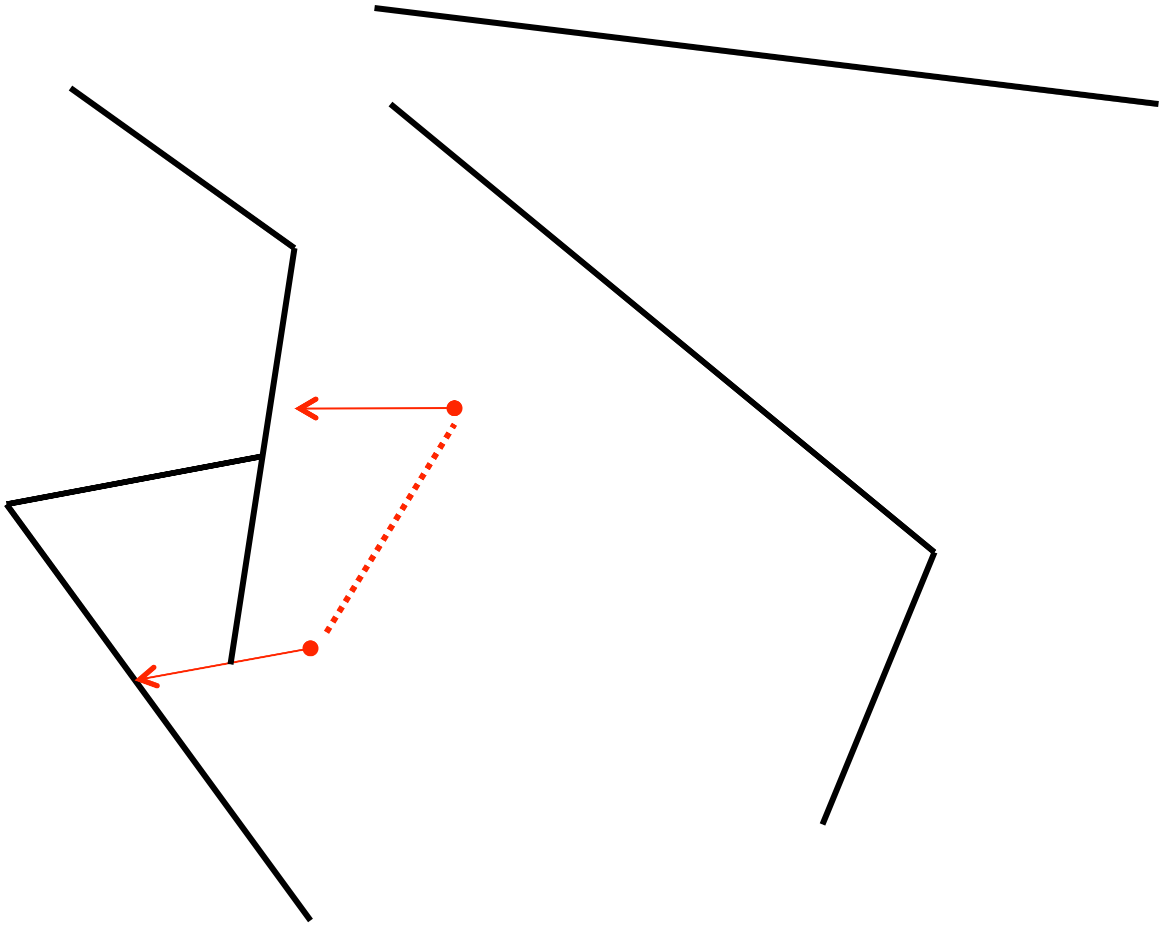
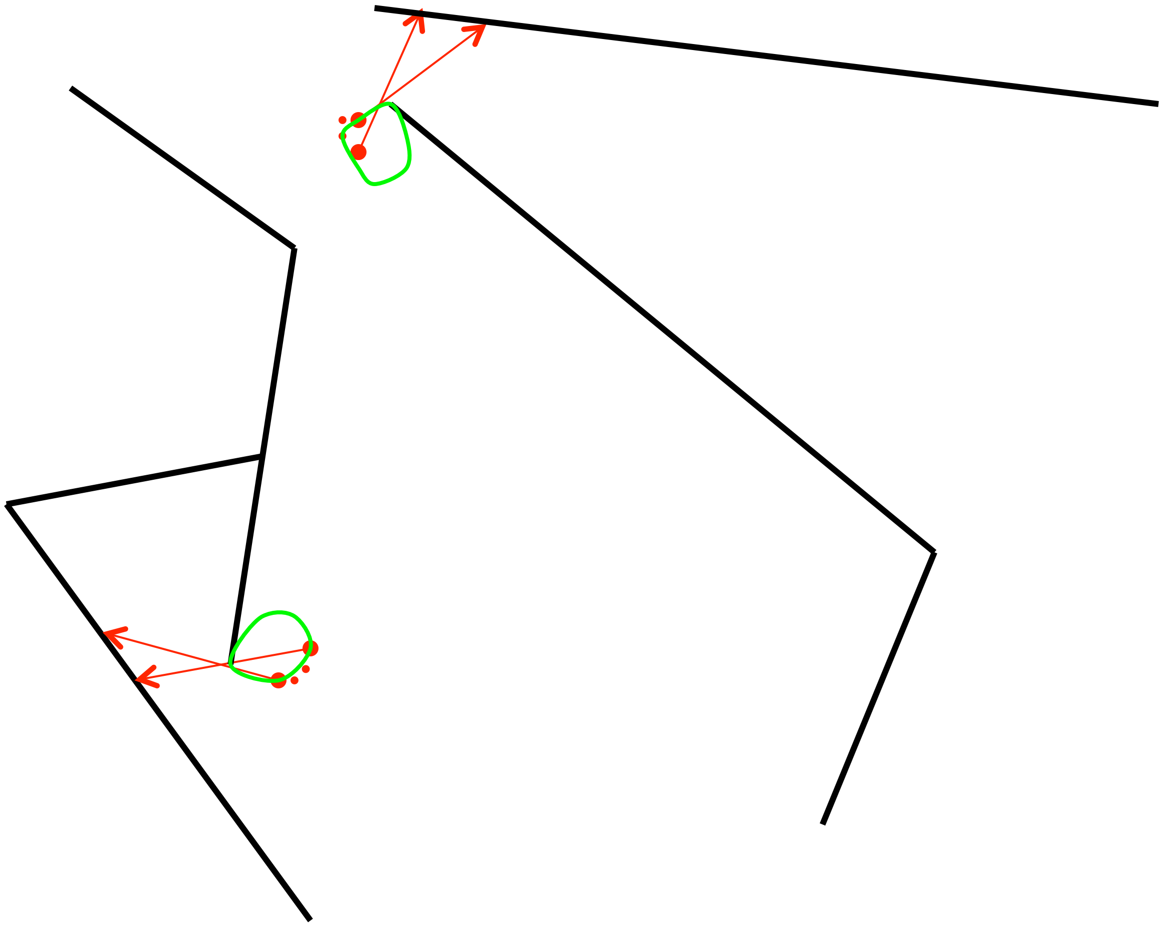
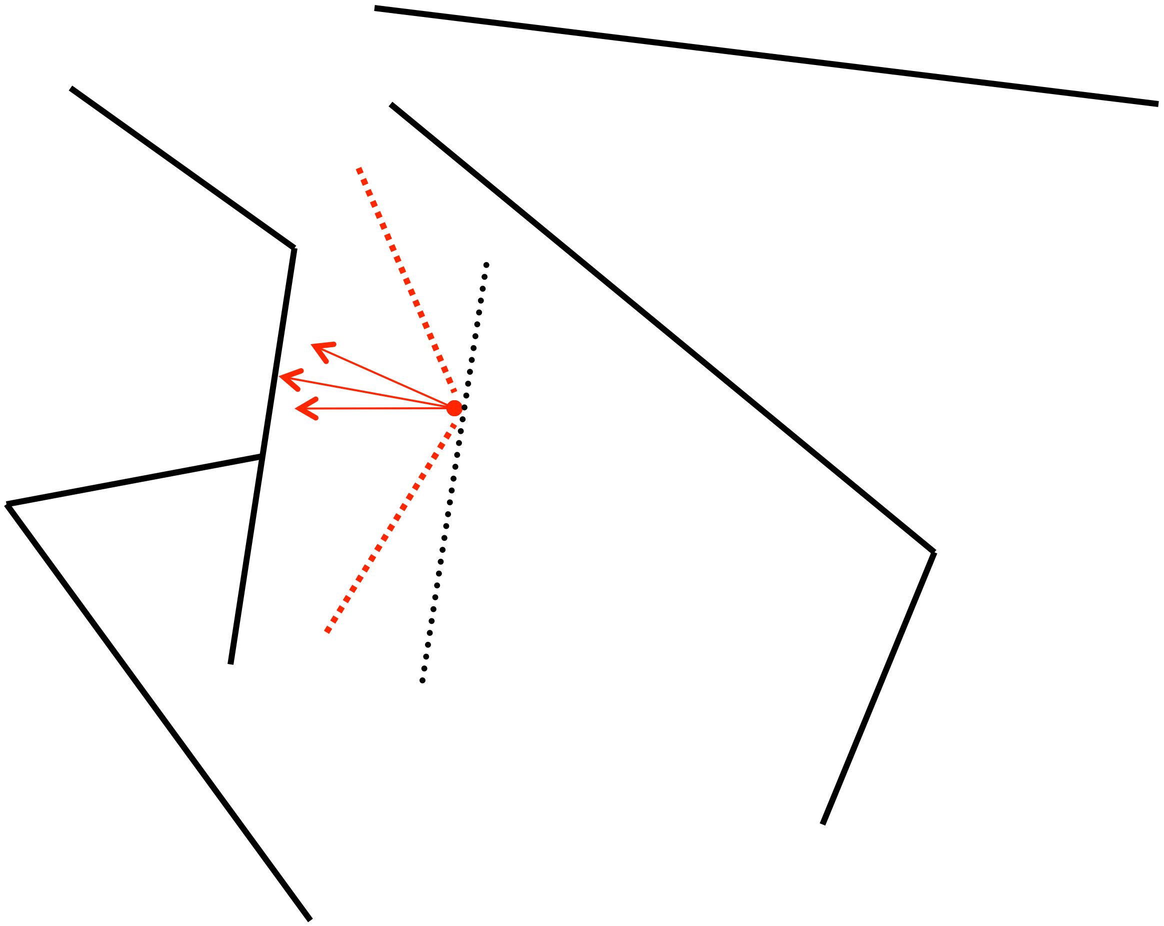


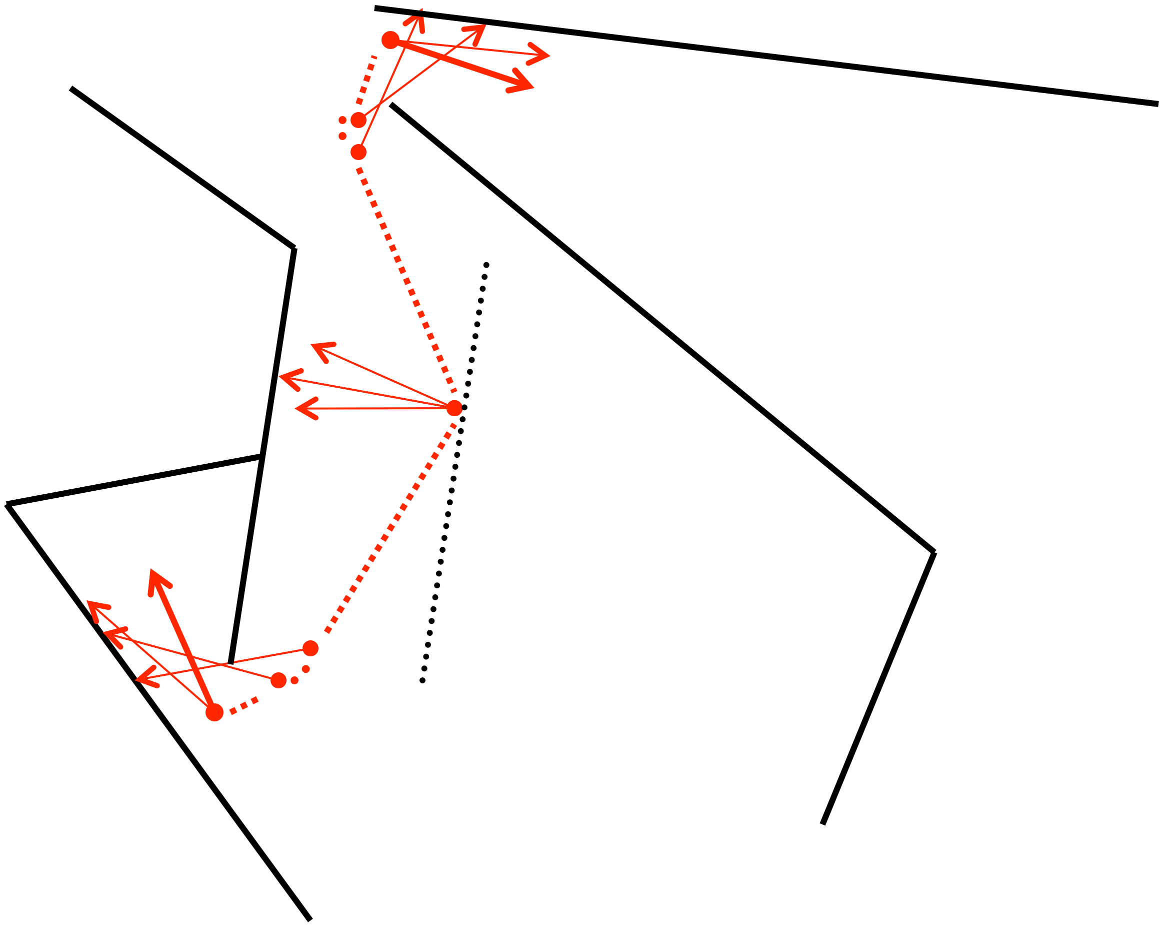
translation



sliding (conchoid curve)



reflection (rotation)



can’t predetermine all
“stopover” points

Course Topics

- Geometric optimization
  - finding extreme points
  - low dimensional linear programming
  - facility location
- Geometric search
  - Planar point location
  - Data structures that facilitate geometric search
- Intersection detection
  - Segments, polygons, polyhedra…

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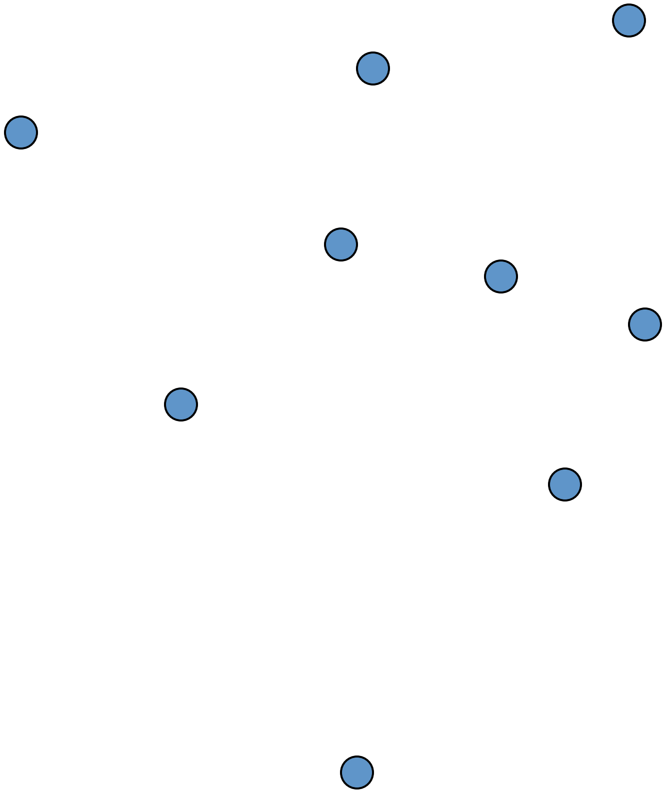

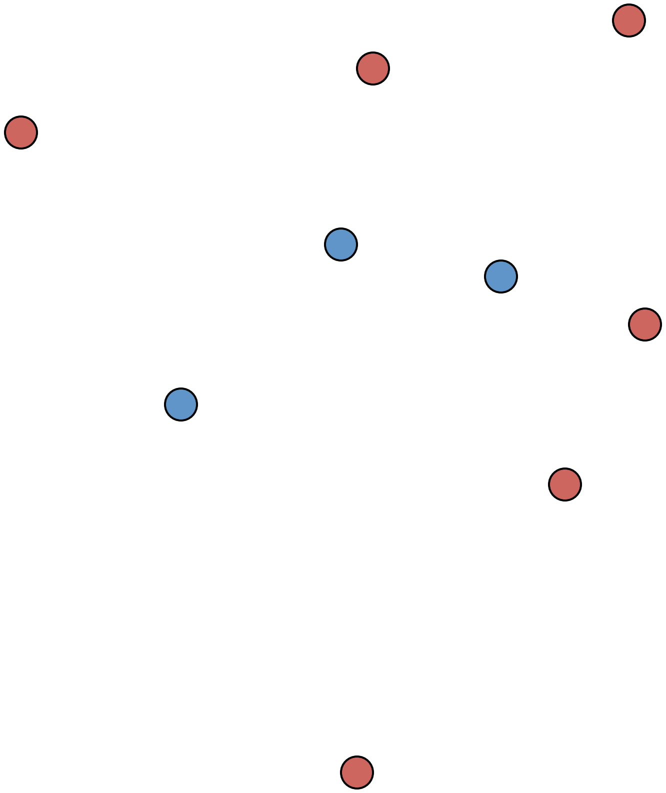

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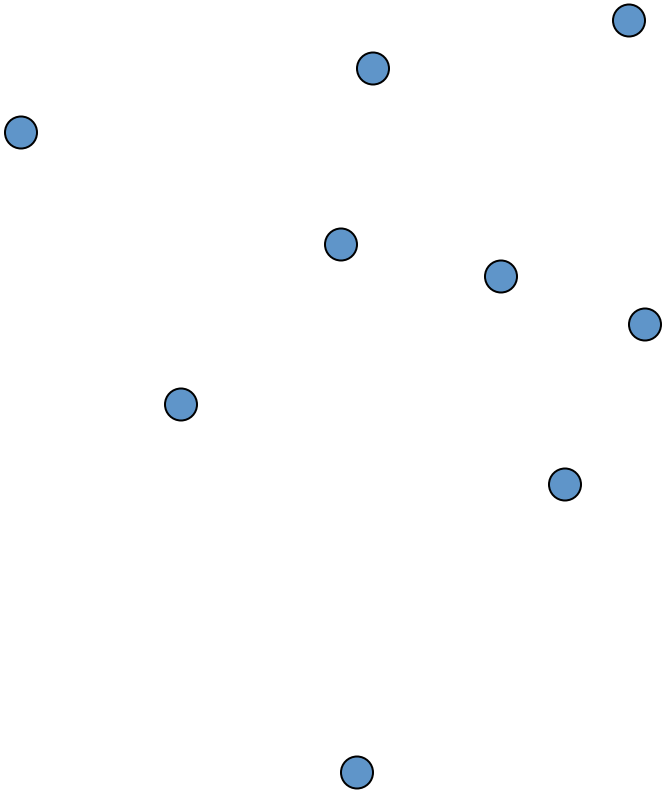
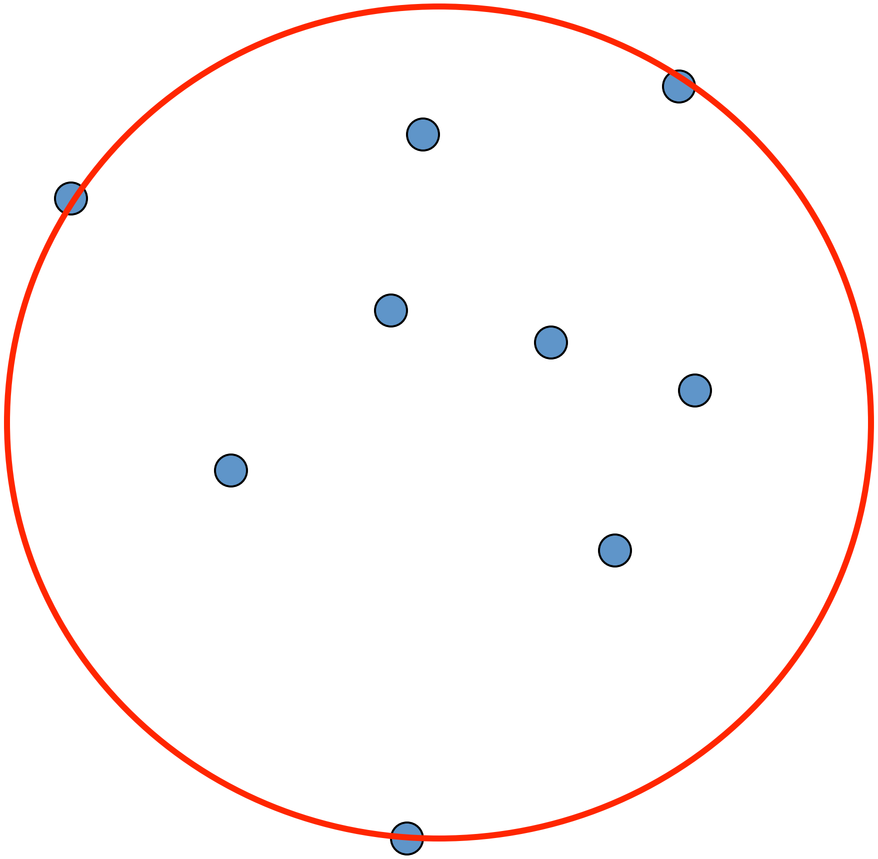
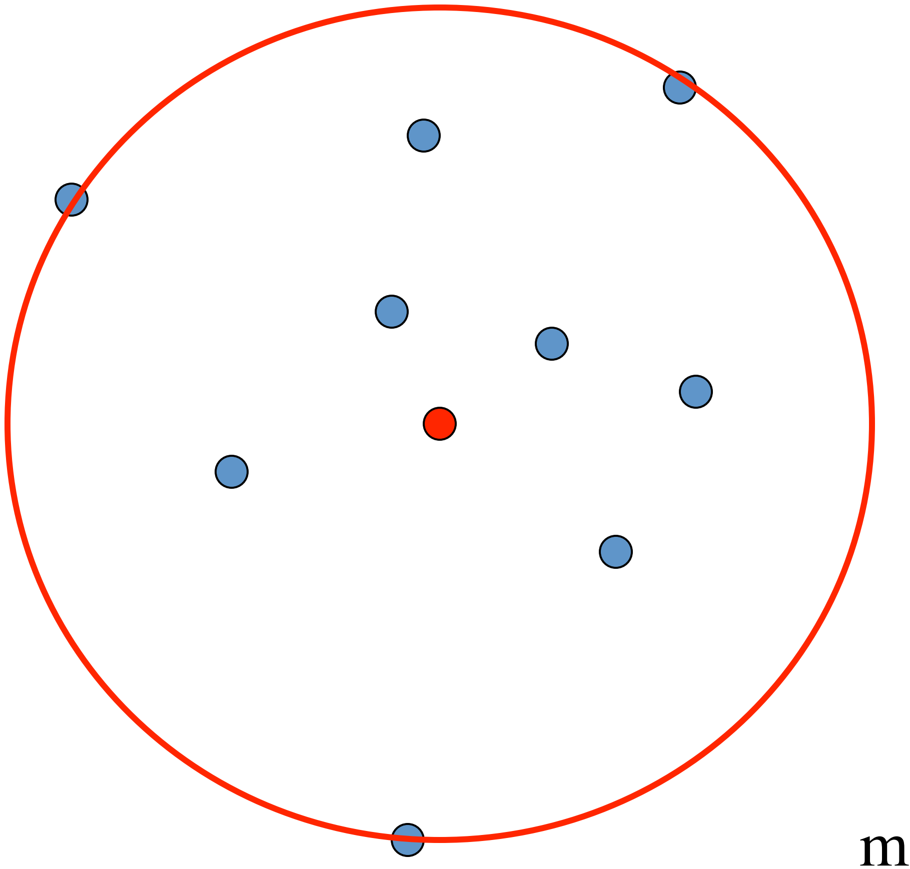
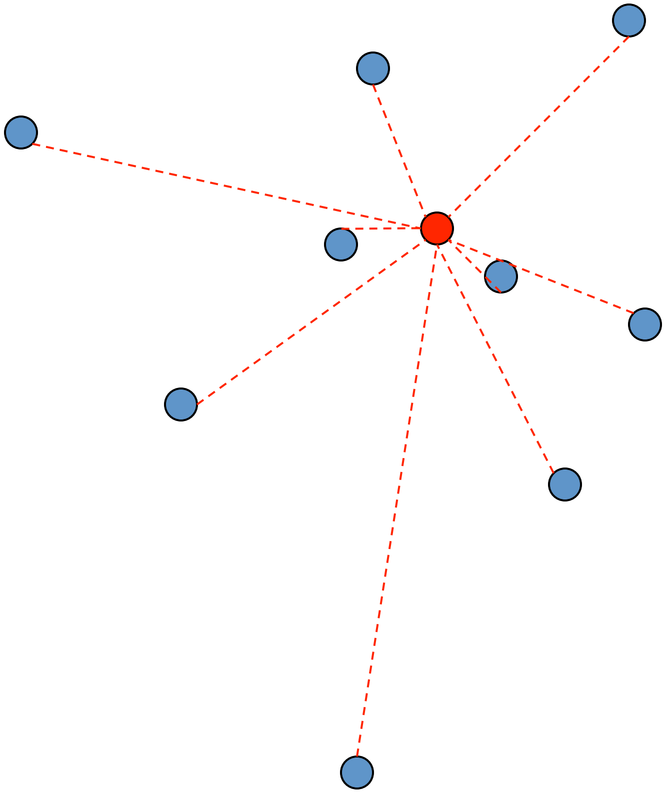






minimize max
distance



minimize sum of distances

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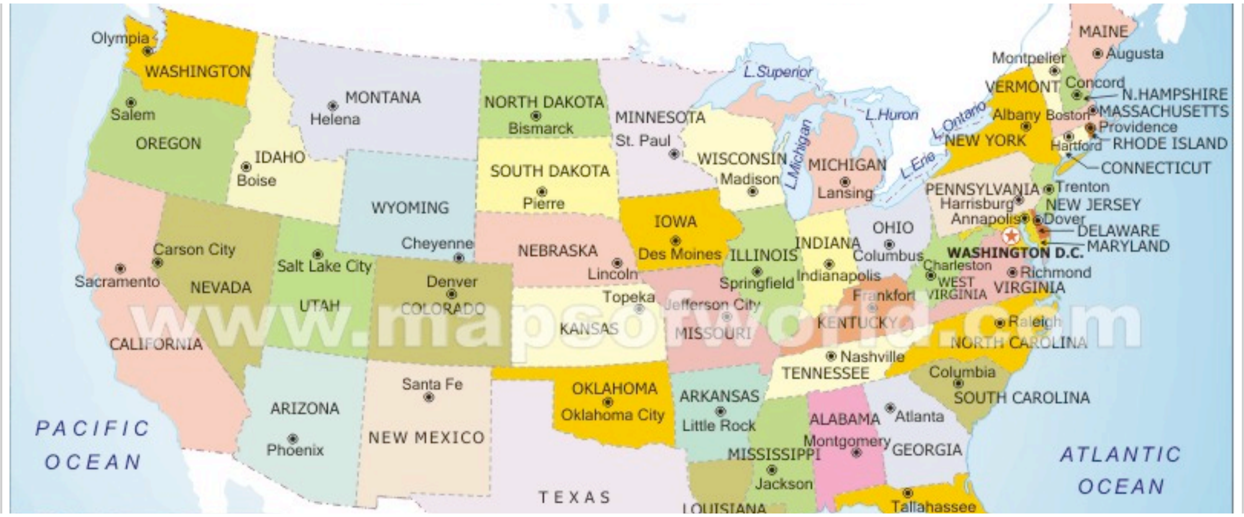
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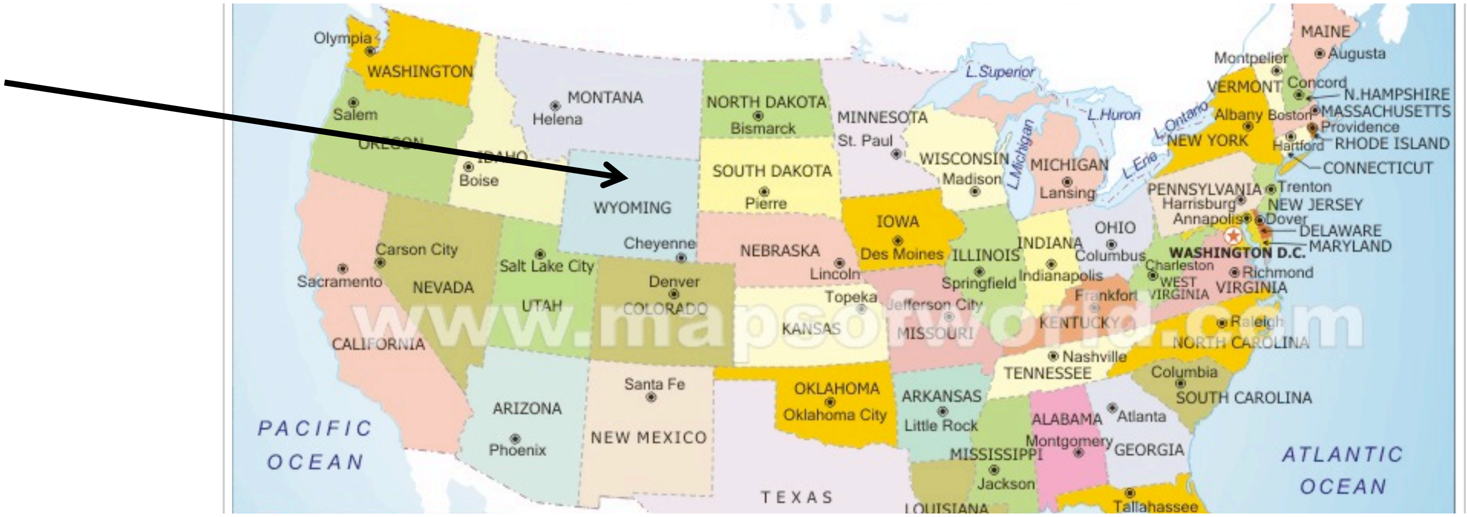
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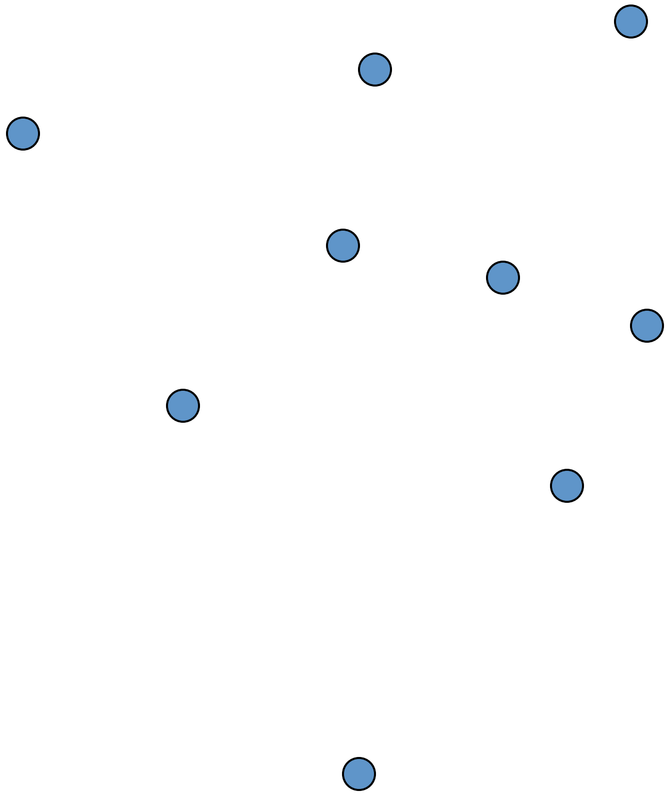
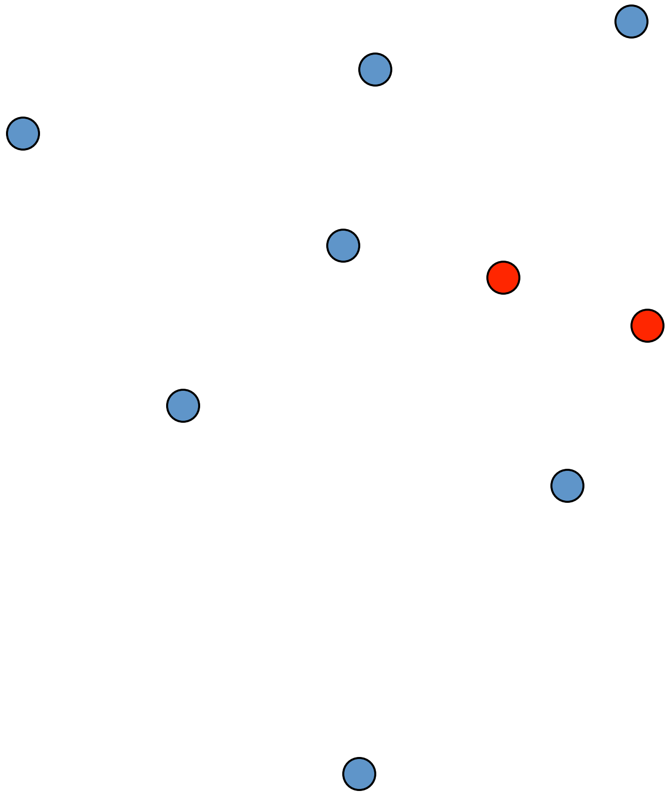
Topics (cont.)

- Proximity problems
    - Nearest neighbours
    - Voronoi diagrams
- Spatial decomposition
    - Triangulations, mesh generation
- Combinatorial geometry
    - Line arrangements
    - Ham sandwiches





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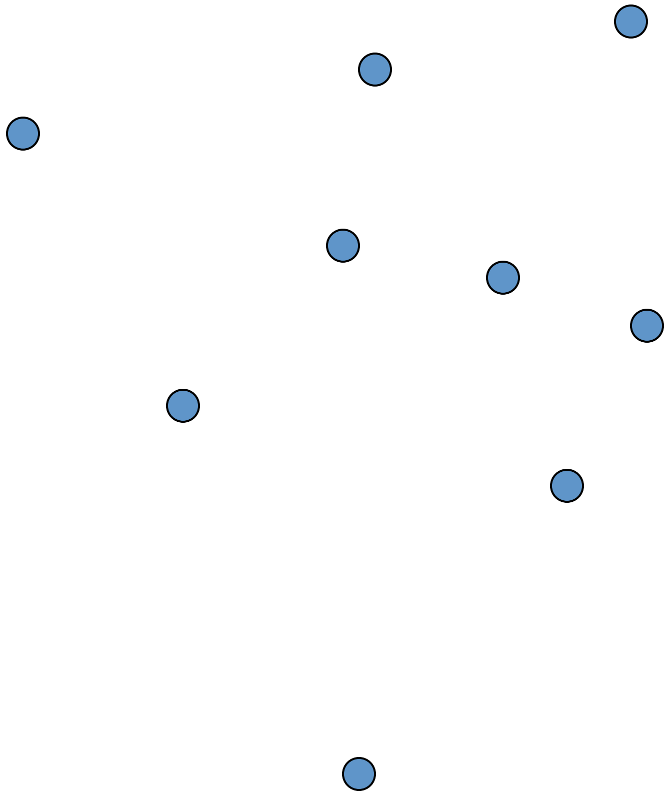

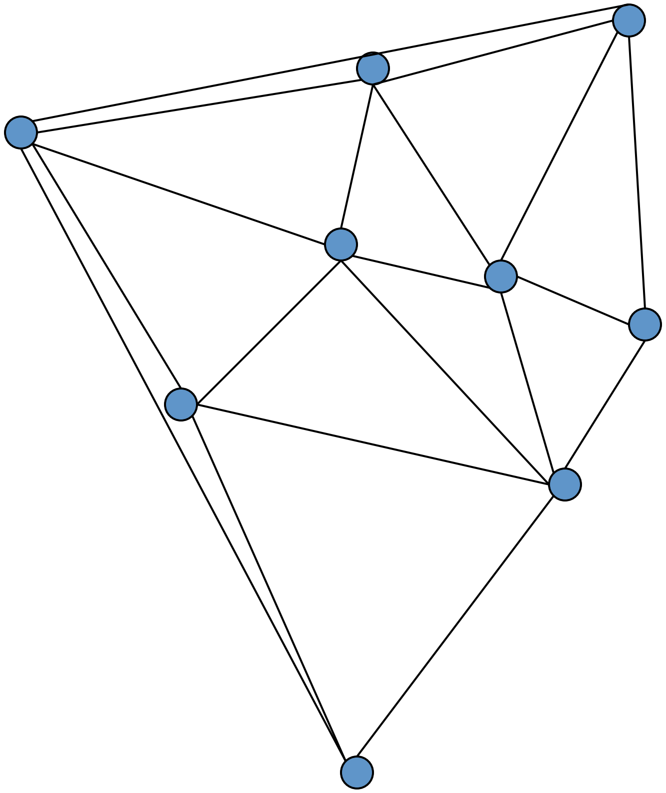

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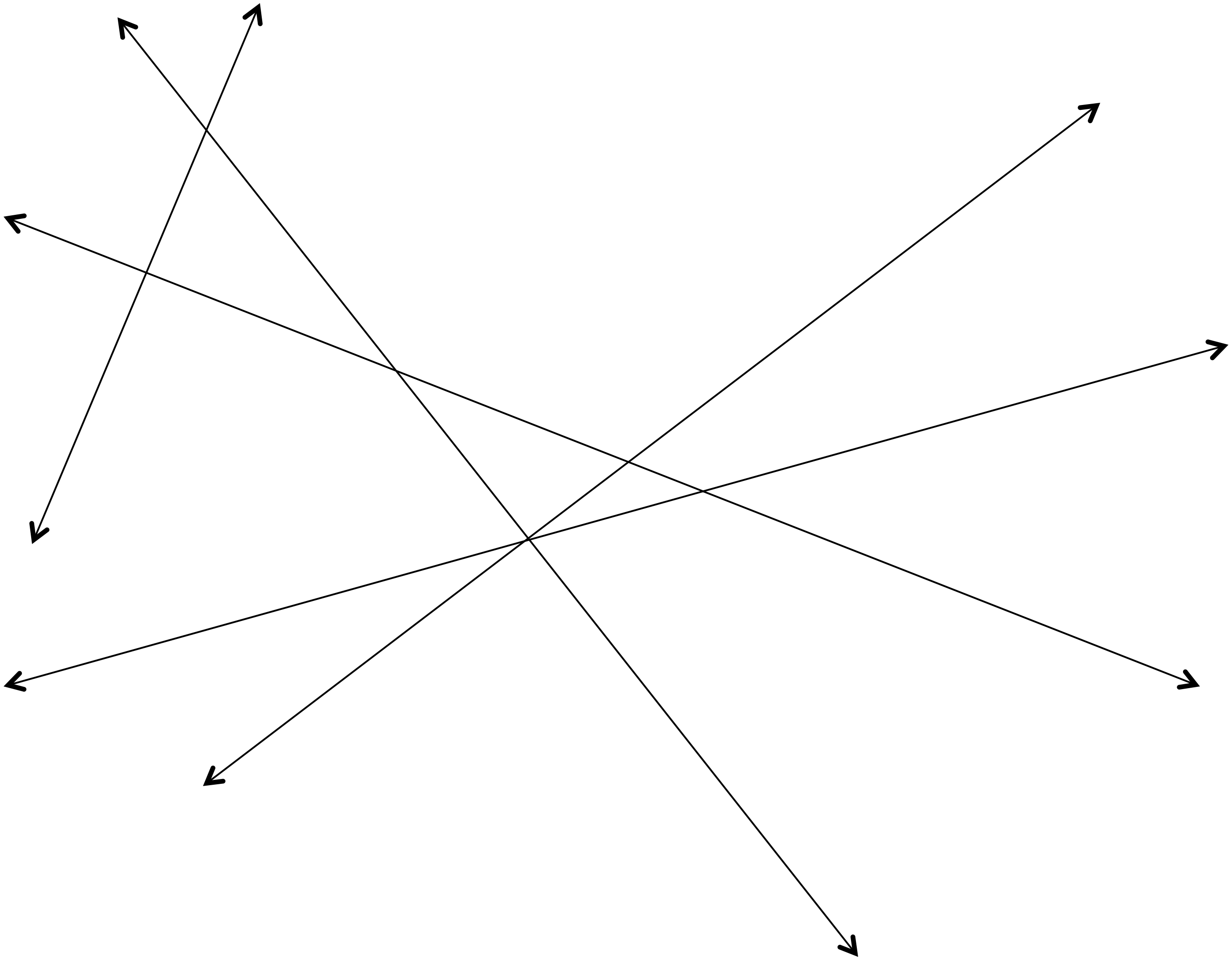
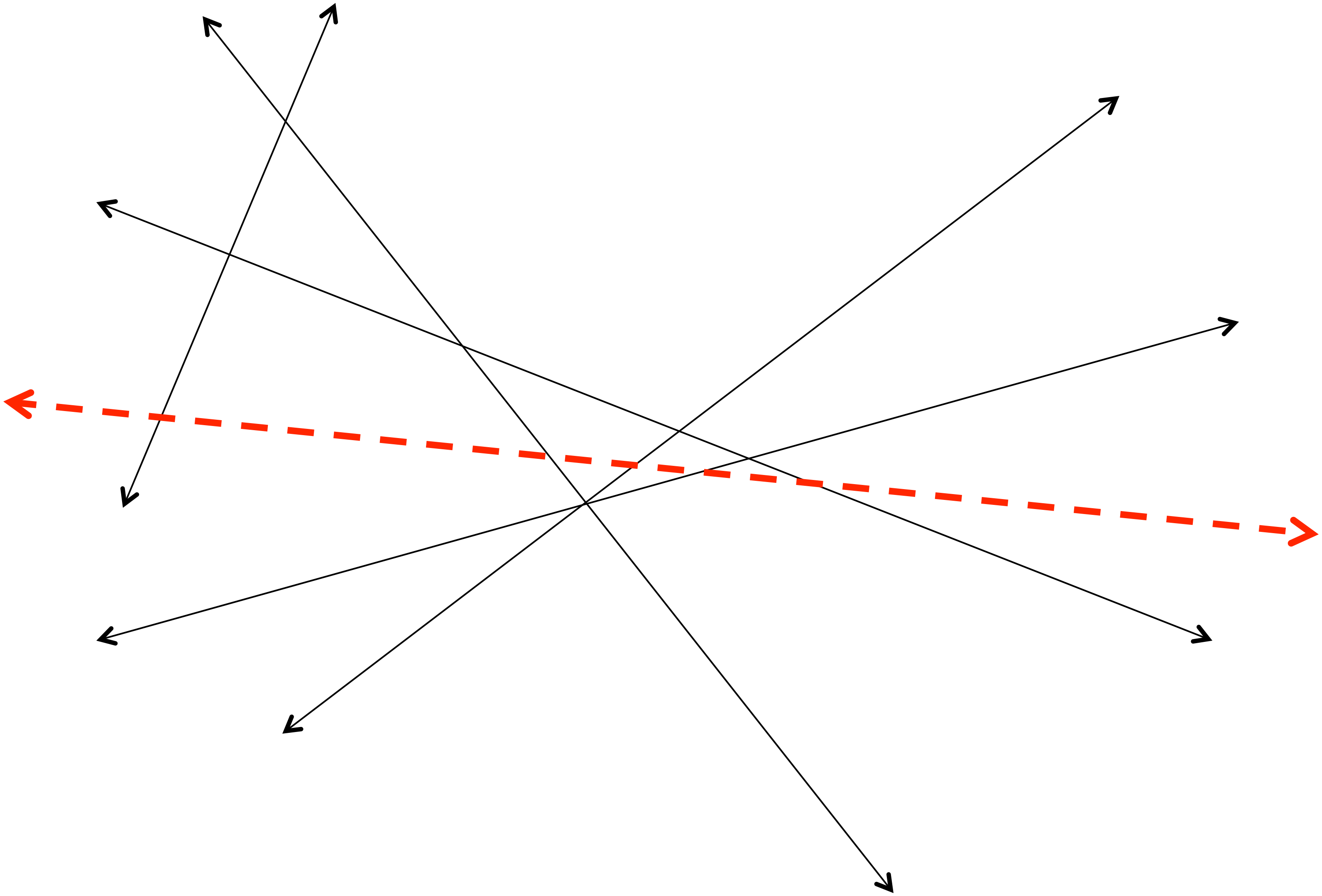




Topics (cont.)

- Complexity and approximation techniques
    - Output-size sensitivity
    - Lower bounds
    - Random sampling
- Other models
    - Parallel, kinetic & streaming algorithms
    - Robustness and uncertainty

More Topics (cont.)

- “Nice” layouts of trees, wires and planar graphs
- Contact & intersection representation of graphs
- Proximity graphs
- Visibility graphs
- Geographic graphs
- Force-directed graph drawing

Graph drawing

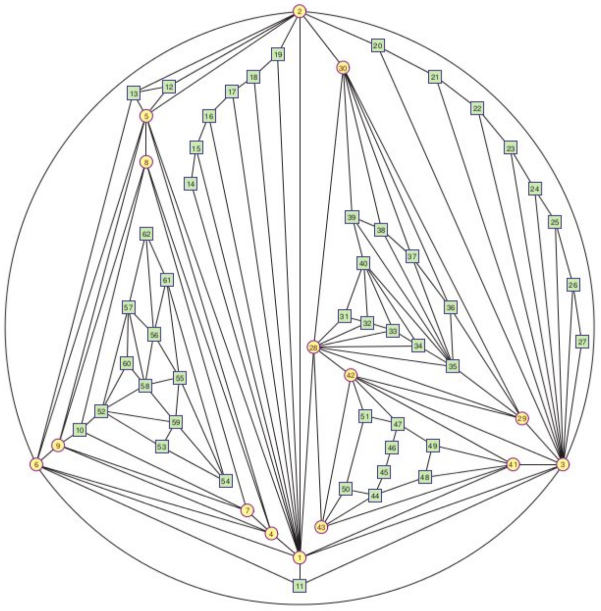
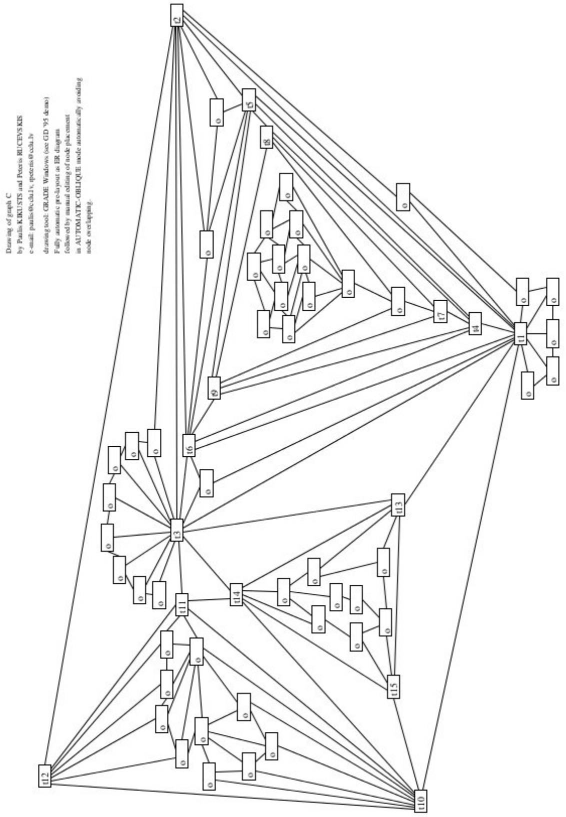
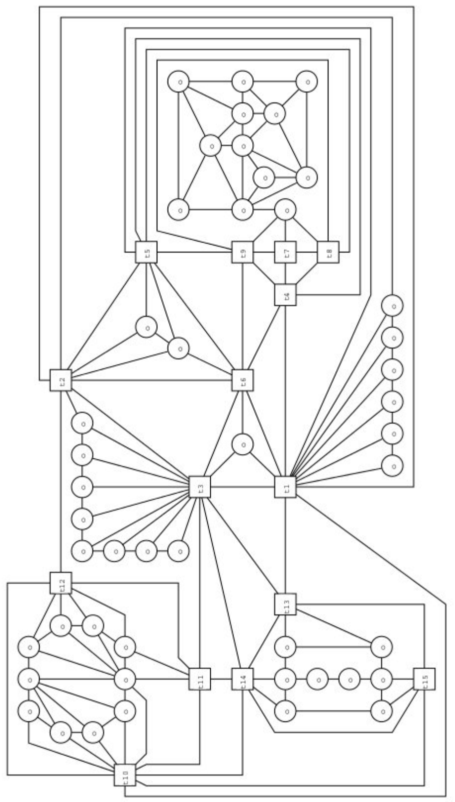


Figure 8: Joint winner, Graph C.


Drawing of graph C
by Paulis KIKUSTS and Peteris RUCEVSKIS
e-mail: paulis@cclu.lv, rpeteris@cclu.lv
drawing tool: GRADE Windows (see GD '95 demo)
Fully automatic pre-layout as ER diagram
followed by manual editing of node placement
in AUTOMATIC-OBLIQUE mode automatically avoiding
node overlapping.



Planar graph drawing

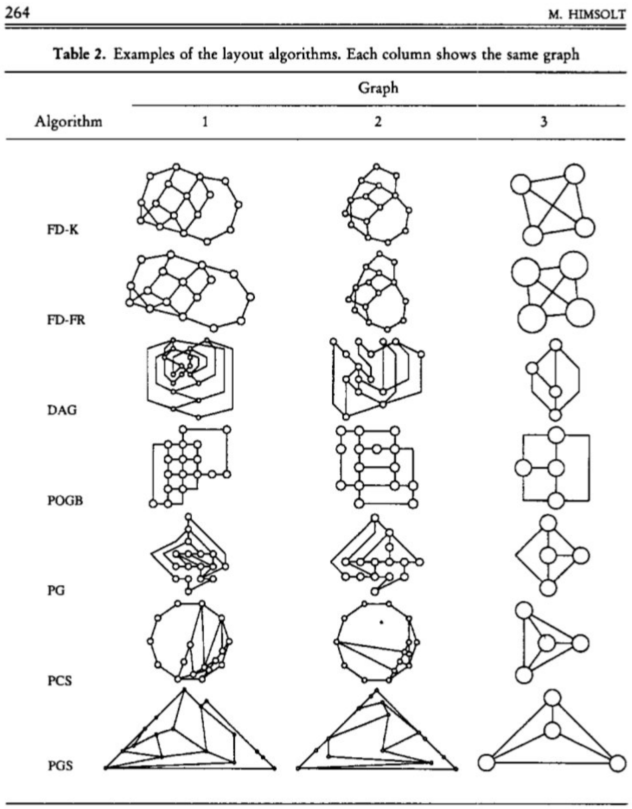


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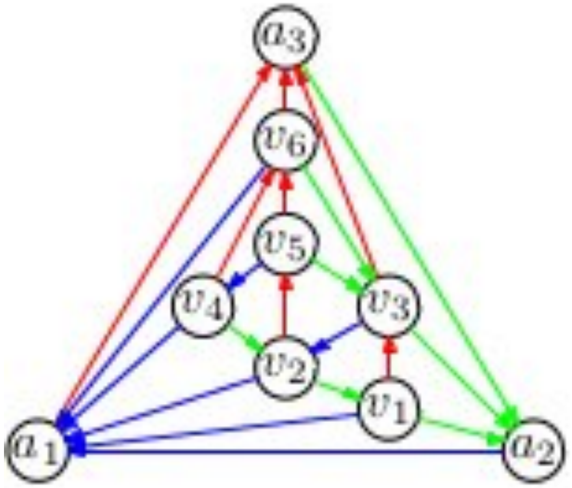
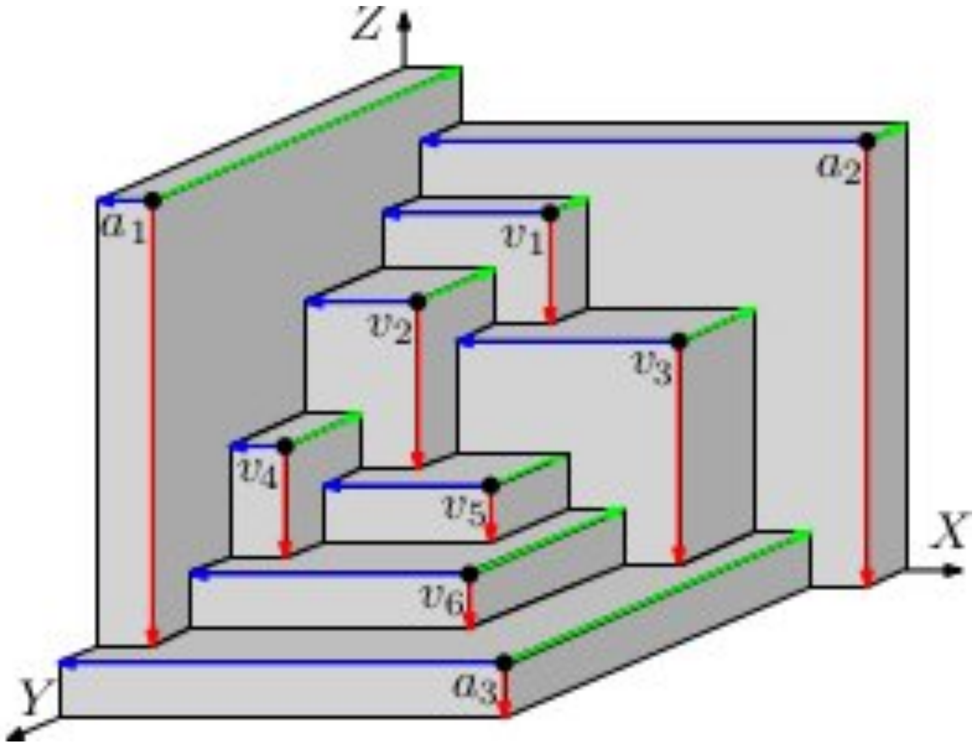
Table 2. Examples of the layout algorithms. Each column shows the same graph

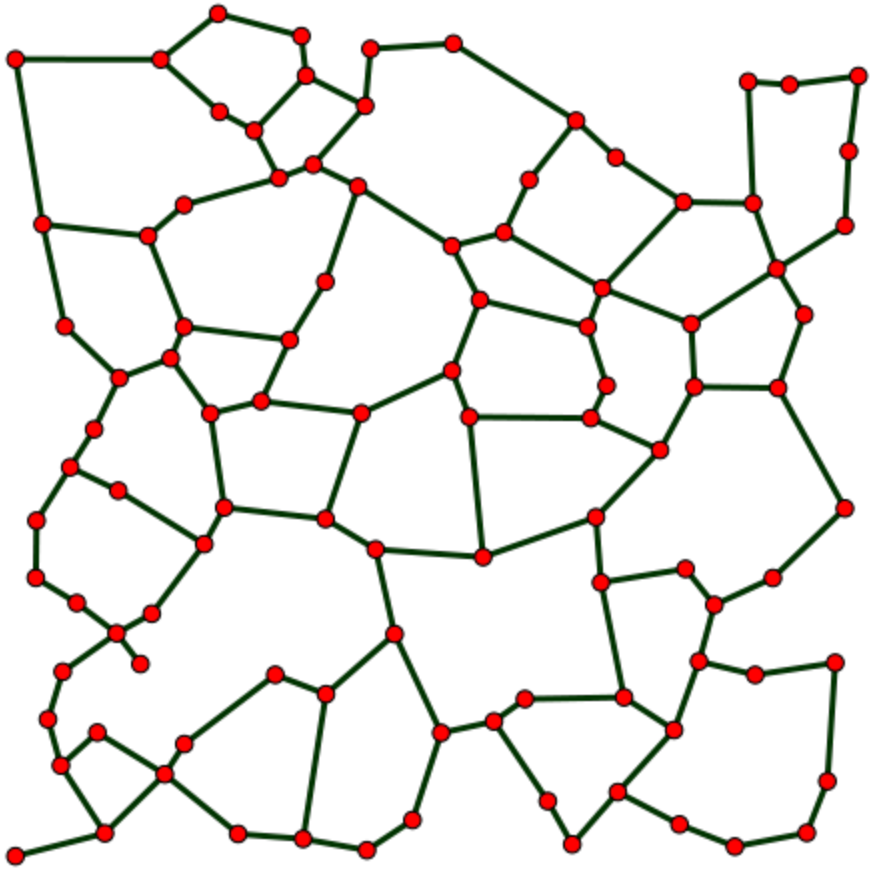
	Graph		
Algorithm	1	2	3
FD-K			
FD-FR			
DAG			
POGB			
PG			
PCS			
PGS			

Contact representation

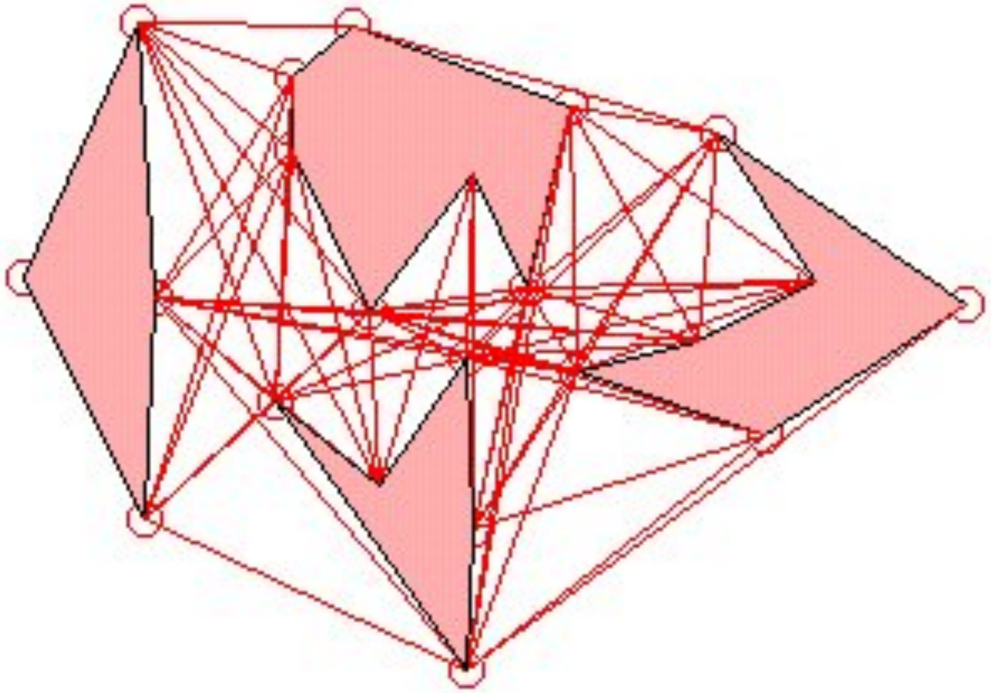




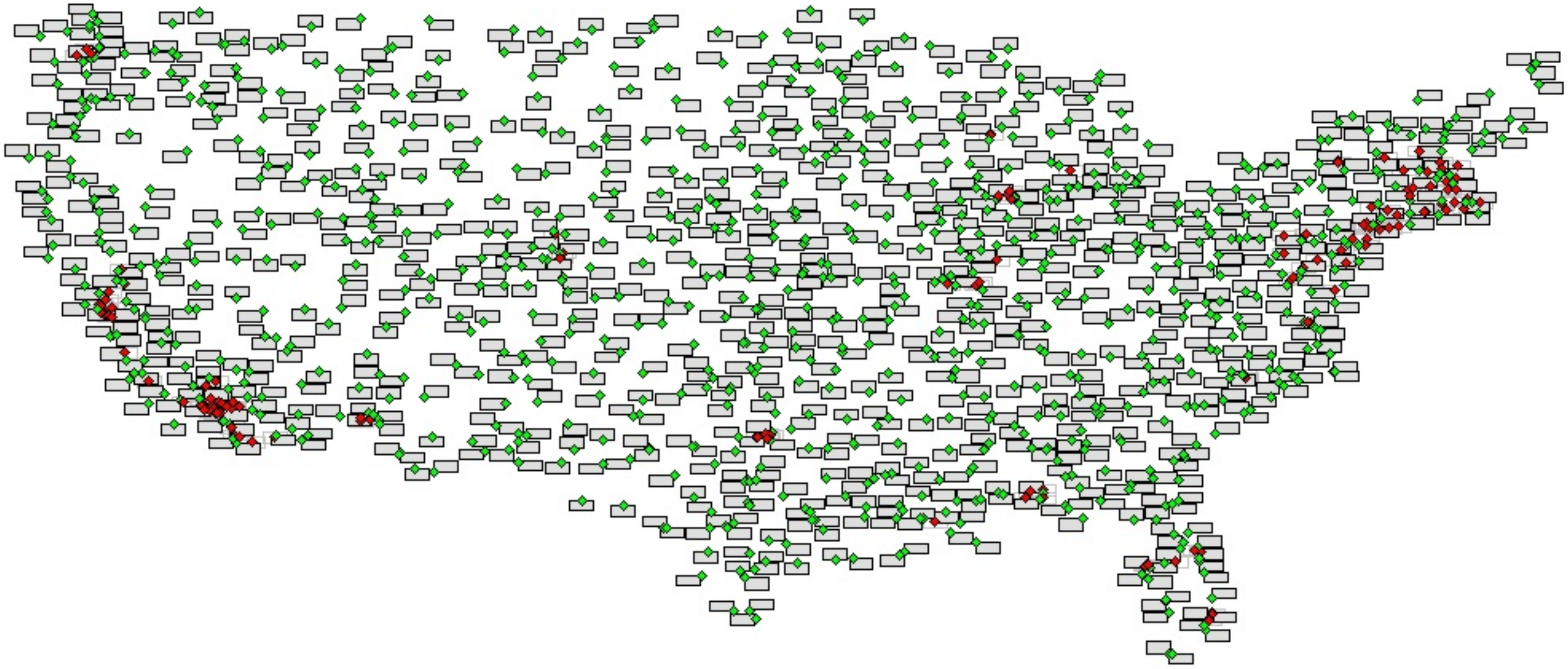
Relative Neighborhood Graph



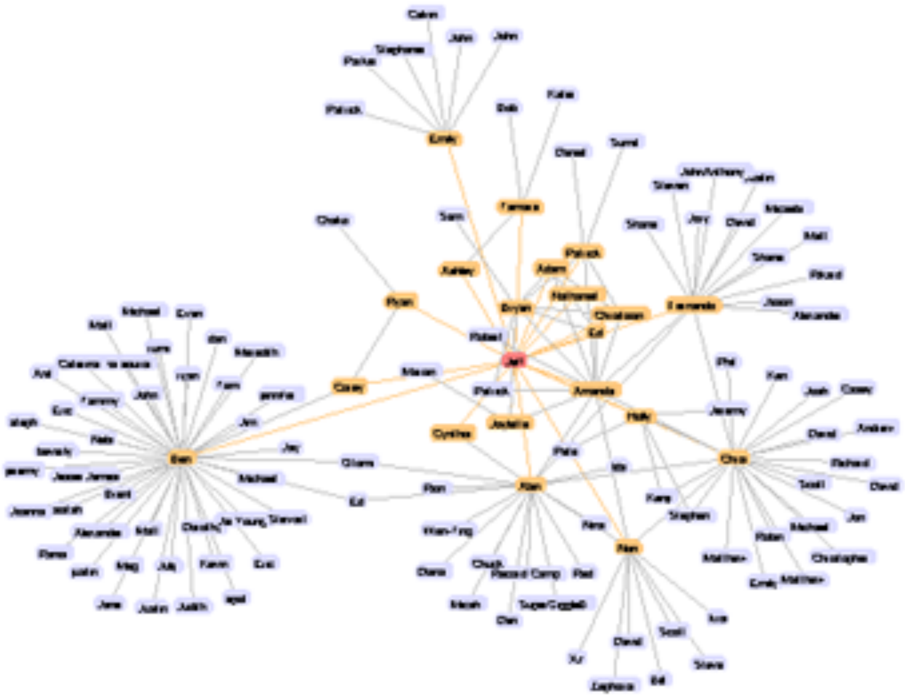
Visibility Graph



Map labelling



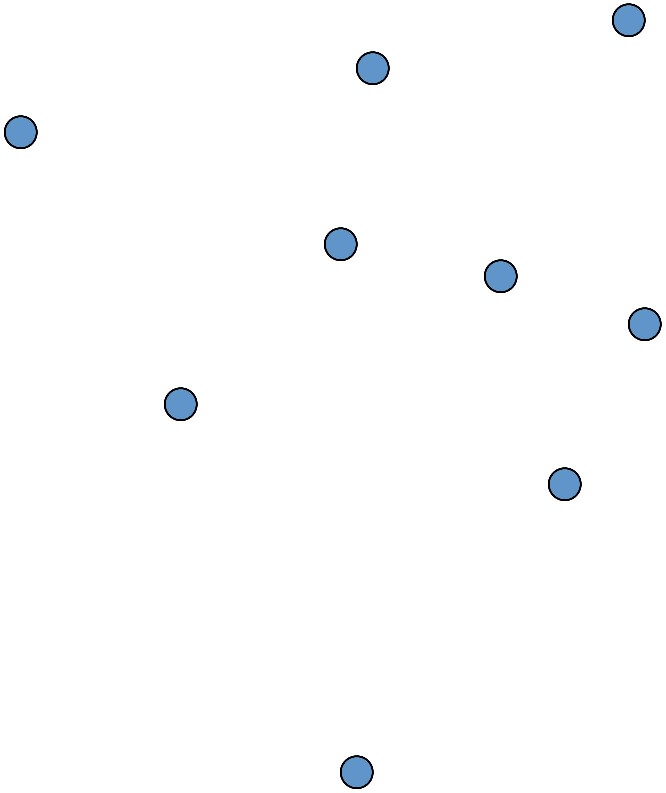
Force-directed layout



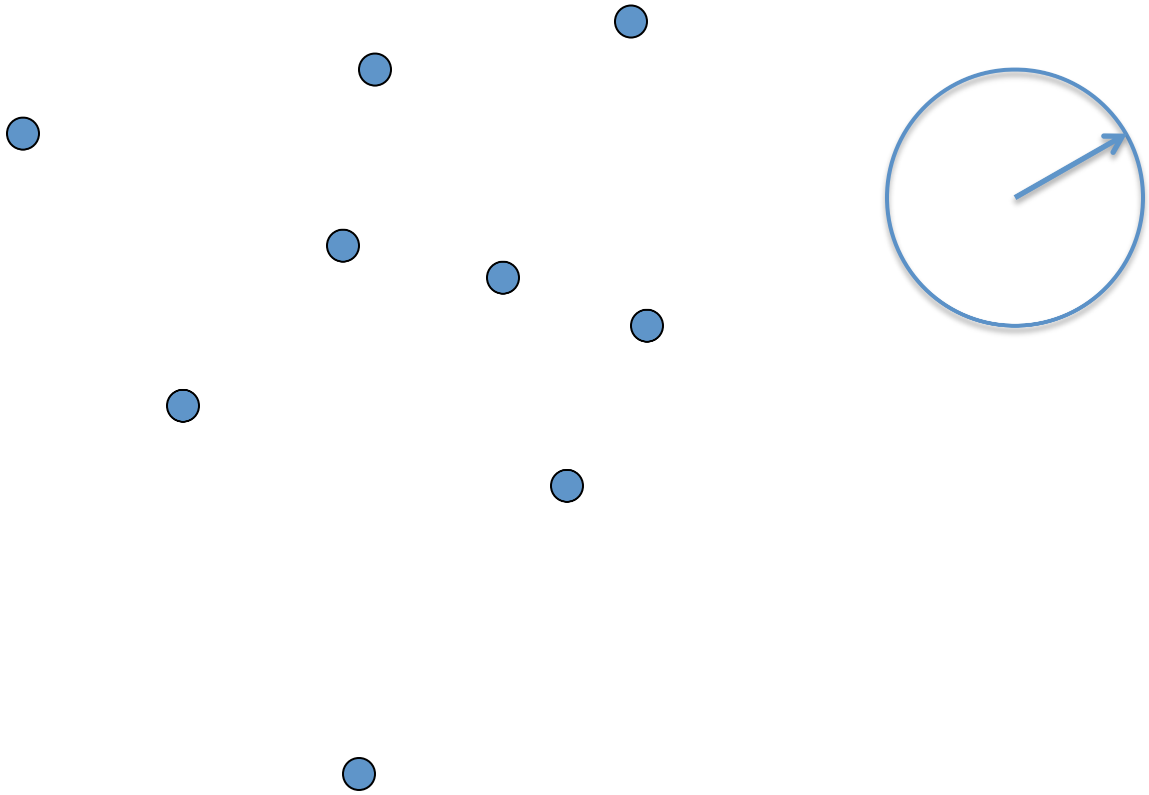
Topics (cont.)

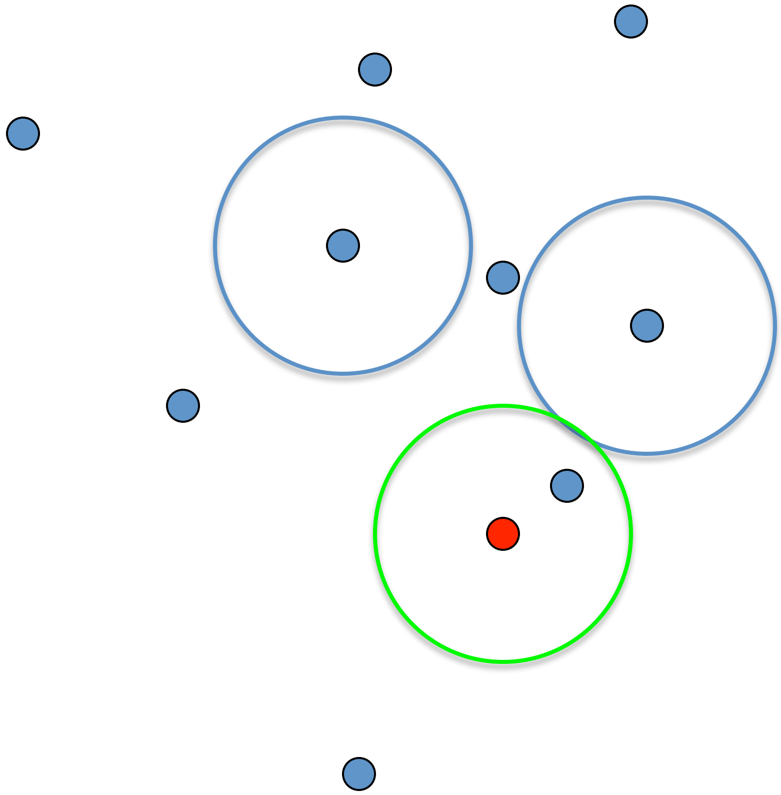
- Applications
  - Geographic information systems
  - Guarding & sensor networks
  - Motion planning
  - Manufacturing
  - …

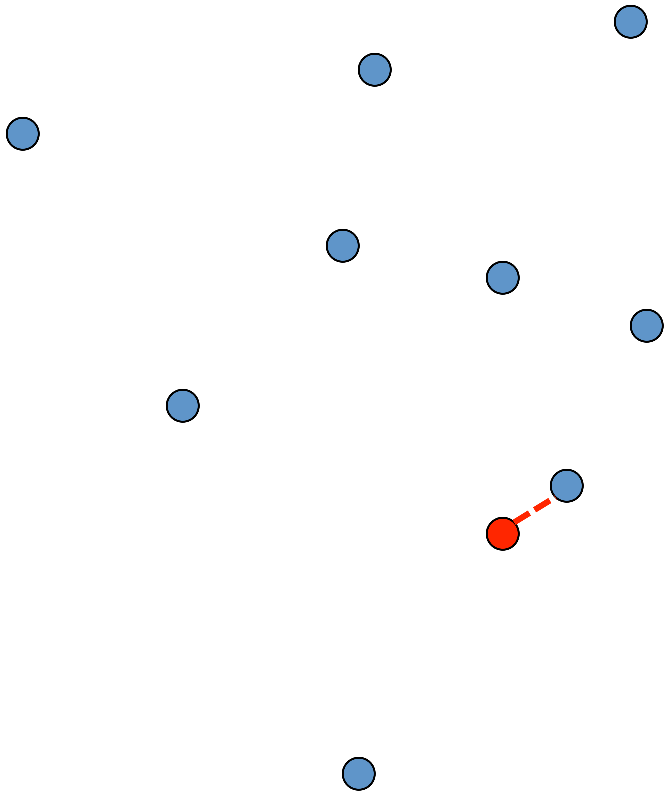
Case study: finding all near neighbours



Case study: finding all near neighbours







Reading

MountNotes Chapters 1-4,7,8