Domination

ISCI 330 Lecture 9

February 6, 2007

Lecture Overview

Max-Min Strategies

- Player i's maxmin strategy is a strategy that maximizes i's worst-case payoff, in the situation where all the other players (whom we denote $-i$) happen to play the strategies which cause the greatest harm to i.
- The maxmin value (or safety level) of the game for player i is that minimum amount of payoff guaranteed by a maxmin strategy.
- Why would i want to play a maxmin strategy?
  - a conservative agent maximizing worst-case payoff
  - a paranoid agent who believes everyone is out to get him

Definition

The maxmin strategy for player i is $\arg\max_{s_i} \min_{s_{-i}} u_i(s_1, s_2)$, and the maxmin value for player i is $\max_{s_i} \min_{s_{-i}} u_i(s_1, s_2)$.

Min-Max Strategies

- Player i's minmax strategy in a 2-player game is a strategy that minimizes the other player $-i$'s best-case payoff.
- The minmax value of the 2-player game for player i is that maximum amount of payoff that $-i$ could achieve under i's minmax strategy.
- Why would i want to play a minmax strategy?
  - to punish the other agent as much as possible

Definition

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Definition

In a two-player game, the minmax strategy for player i is $\arg\min_{s_i} \max_{s_{-i}} u_{-i}(s_1, s_2)$, and the minmax value for player i is $\min_{s_i} \max_{s_{-i}} u_{-i}(s_1, s_2)$.

Minmax Theorem

Theorem (Minmax theorem (von Neumann, 1928))

In any finite, two-player, zero-sum game, in any Nash equilibrium each player receives a payoff that is equal to both his maxmin value and his minmax value.

- The maxmin value for one player is equal to the minmax value for the other player. By convention, the maxmin value for player 1 is called the value of the game.
- For both players, the set of maxmin strategies coincides with the set of minmax strategies.
- Any maxmin strategy profile (or, equivalently, minmax strategy profile) is a Nash equilibrium. Furthermore, these are all the Nash equilibria. Consequently, all Nash equilibria have the same payoff vector (namely, those in which player 1 gets the value of the game).

How to find maxmin and minmax strategies

Consider maxmin strategies for player i in a 2-player game.

- Notice that i's maxmin strategy depends only on i's utilities
  - thus changes to $-i$'s utilities do not change i's maxmin strategy
- Consider the game where player i has the same utilities as before, but player $-i$'s utilities are replaced with the negatives of i's utilities
  - this is now a zero-sum game
- Because of the minmax theorem, we know that any Nash equilibrium strategy in this game is also a maxmin strategy
  - Thus, find player i's equilibrium strategy in the new game and we have i's maxmin strategy in the original game
- We can use a similar approach for minmax.

Lecture Overview

Traveler's Dilemma

Two travelers purchase identical African masks while on a tropical vacation. Their luggage is lost on the return trip, and the airline asks them to make independent claims for compensation. In anticipation of excessive claims, the airline representative announces: "We know that the bags have identical contents, and we will entertain any claim between $180 and $300, but you will each be reimbursed at an amount that equals the minimum of the two claims submitted. If the two claims differ, we will also pay a reward R *to the person making the smaller claim and we will deduct a penalty* R *from the reimbursement to the person making the larger claim."*

Traveler's Dilemma

- Action: choose an integer between 180 and 300
- If both players pick the same number, they both get that amount as payoff
- If players pick a different number:
  - the low player gets his number (L) plus some constant R
  - the high player gets $L - R$.
- Play this game *once* with a partner; play with as many different partners as you like.
  - $R = 5$.

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  - $R = 5$.
  - $R = 180$.

Traveler's Dilemma

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Traveler's Dilemma

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- What happens?

Traveler's Dilemma

- What is the equilibrium?
  - $(180, 180)$ is the only equilibrium, for all $R \geq 2$.
- What happens?
  - with $R = 5$ most people choose 295–300
  - with $R = 180$ most people choose 180

Lecture Overview

Domination

- Let s_i and s_i' be two strategies for player i, and let S_{-i} be is the set of all possible strategy profiles for the other players

Definition

s_i strictly dominates s_i' if $\forall s_{-i} \in S_{-i}$, $u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$

Definition

s_i weakly dominates s_i' if $\forall s_{-i} \in S_{-i}$, $u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$ and $\exists s_{-i} \in S_{-i}$, $u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$

Definition

s_i very weakly dominates s_i' if $\forall s_{-i} \in S_{-i}$, $u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$

Equilibria and dominance

- If one strategy dominates all others, we say it is dominant.
- A strategy profile consisting of dominant strategies for every player must be a Nash equilibrium.
  - An equilibrium in strictly dominant strategies must be unique.

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- If one strategy dominates all others, we say it is dominant.
- A strategy profile consisting of dominant strategies for every player must be a Nash equilibrium.
    - An equilibrium in strictly dominant strategies must be unique.
- Consider Prisoner's Dilemma again
    - not only is the only equilibrium the only non-Pareto-optimal outcome, but it's also an equilibrium in strictly dominant strategies!