Computing Mixed Nash Equilibria

ISCI 330 Lecture 7

January 31, 2007

Lecture Overview

1. Recap
2. Computing Mixed Nash Equilibria
3. Fun Game

What are solution concepts?

- Solution concept: a subset of the outcomes in the game that are somehow interesting.
- There is an implicit computational problem of finding these outcomes given a particular game.
- Depending on the concept, existence can be an issue.

Solution concepts we've seen so far:

What are solution concepts?

- Solution concept: a subset of the outcomes in the game that are somehow interesting.
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Solution concepts we've seen so far:

- Pareto-optimal outcome
- Pure-strategy Nash equilibrium
- Mixed-strategy Nash equilibrium
- Other Nash variants:
  - weak Nash equilibrium
  - strict Nash equilibrium

Mixed Strategies

- It would be a pretty bad idea to play any deterministic strategy in matching pennies
- Idea: confuse the opponent by playing randomly
- Define a strategy s_i for agent i as any probability distribution over the actions A_i.
  - pure strategy: only one action is played with positive probability
  - mixed strategy: more than one action is played with positive probability
    - these actions are called the support of the mixed strategy
- Let the set of all strategies for i be S_i
- Let the set of all strategy profiles be $S = S_1 \times \ldots \times S_n$.

Utility under Mixed Strategies

- What is your payoff if all the players follow mixed strategy profile $s \in S$?
  - We can't just read this number from the game matrix anymore: we won't always end up in the same cell
- Instead, use the idea of expected utility from decision theory:

$$u_i(s) = \sum_{a \in A} u_i(a) Pr(a|s)$$

$$Pr(a|s) = \prod_{j \in N} s_j(a_j)$$

Best Response and Nash Equilibrium

Our definitions of best response and Nash equilibrium generalize from actions to strategies.

- Best response:
  - $s_i^* \in BR(s_{-i})$ iff $\forall s_i \in S_i$, $u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i})$
- Nash equilibrium:
  - $s = \langle s_1, \ldots, s_n \rangle$ is a Nash equilibrium iff $\forall i$, $s_i \in BR(s_{-i})$
- Every finite game has a Nash equilibrium! [Nash, 1950]
  - e.g., matching pennies: both players play heads/tails 50%/50%

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Computing Mixed Nash Equilibria: Battle of the Sexes

	B	F
B	2, 1	0, 0
F	0, 0	1, 2

- It's hard in general to compute Nash equilibria, but it's easy when you can guess the support
- For BoS, let's look for an equilibrium where all actions are part of the support

Computing Mixed Nash Equilibria: Battle of the Sexes

	B	F
B	$2, 1$	$0, 0$
F	$0, 0$	$1, 2$

- Let player 2 play B with p, F with $1 - p$.
- If player 1 best-responds with a mixed strategy, player 2 must make him indifferent between F and B (why?)

Computing Mixed Nash Equilibria: Battle of the Sexes

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$$u_1(B) = u_1(F)$$
$$2p + 0(1-p) = 0p + 1(1-p)$$
$$p = \frac{1}{3}$$

Computing Mixed Nash Equilibria: Battle of the Sexes

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- Likewise, player 1 must randomize to make player 2 indifferent.
  - Why is player 1 willing to randomize?

Computing Mixed Nash Equilibria: Battle of the Sexes

	B	F
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- Likewise, player 1 must randomize to make player 2 indifferent.
  - Why is player 1 willing to randomize?
- Let player 1 play B with q, F with $1 - q$.

$$u_2(B) = u_2(F)$$
$$q + 0(1-q) = 0q + 2(1-q)$$
$$q = \frac{2}{3}$$

- Thus the mixed strategies $(\frac{2}{3}, \frac{1}{3})$, $(\frac{1}{3}, \frac{2}{3})$ are a Nash equilibrium.

Interpreting Mixed Strategy Equilibria

What does it mean to play a mixed strategy? Different interpretations:

- Randomize to confuse your opponent
  - consider the matching pennies example
- Players randomize when they are uncertain about the other's action
  - consider battle of the sexes
- Mixed strategies are a concise description of what might happen in repeated play: count of pure strategies in the limit
- Mixed strategies describe population dynamics: 2 agents chosen from a population, all having deterministic strategies. MS is the probability of getting an agent who will play one PS or another.

Lecture Overview

Fun Game!

	L	R
T	$80, 40$	$40, 80$
B	$40, 80$	$80, 40$

- Play once as each player, recording the strategy you follow.

Fun Game!

	L	R
T	$320, 40$	$40, 80$
B	$40, 80$	$80, 40$

- Play once as each player, recording the strategy you follow.

Fun Game!

	L	R
T	$44, 40$	$40, 80$
B	$40, 80$	$80, 40$

- Play once as each player, recording the strategy you follow.

Fun Game!

	L	R
T	$80, 40$; $320, 40$; $44, 40$	$40, 80$
B	$40, 80$	$80, 40$

- Play once as each player, recording the strategy you follow.
- What does row player do in equilibrium of this game?

Fun Game!

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  - row player randomizes 50-50 all the time
  - that's what it takes to make column player indifferent

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- What happens when people play this game?

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- Play once as each player, recording the strategy you follow.
- What does row player do in equilibrium of this game?
  - row player randomizes 50-50 all the time
  - that's what it takes to make column player indifferent
- What happens when people play this game?
  - with payoff of 320, row player goes up essentially all the time
  - with payoff of 44, row player goes down essentially all the time