Analyzing Games: Nash Equilibrium Recap

ISCI 330 Lecture 6

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Lecture Overview

1 Recap

2 Interpreting Mixed Strategies
What are solution concepts?

- **Solution concept**: a subset of the outcomes in the game that are somehow interesting.
- There is an implicit computational problem of finding these outcomes given a particular game.
- Depending on the concept, existence can be an issue.

Solution concepts we’ve seen so far:
What are solution concepts?

- **Solution concept**: a subset of the outcomes in the game that are somehow interesting.
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Solution concepts we’ve seen so far:

- Pareto-optimal outcome
- Pure-strategy Nash equilibrium
- Mixed-strategy Nash equilibrium
- Other Nash variants:
  - **weak** Nash equilibrium
  - **strict** Nash equilibrium
Mixed Strategies

- It would be a pretty bad idea to play any deterministic strategy in matching pennies
- Idea: confuse the opponent by playing randomly
- Define a strategy $s_i$ for agent $i$ as any probability distribution over the actions $A_i$.
  - pure strategy: only one action is played with positive probability
  - mixed strategy: more than one action is played with positive probability
    - these actions are called the support of the mixed strategy
- Let the set of all strategies for $i$ be $S_i$
- Let the set of all strategy profiles be $S = S_1 \times \ldots \times S_n$. 
Utility under Mixed Strategies

What is your payoff if all the players follow mixed strategy profile \( s \in S \)?

- We can’t just read this number from the game matrix anymore: we won’t always end up in the same cell.

- Instead, use the idea of expected utility from decision theory:

\[
u_i(s) = \sum_{a \in A} u_i(a) Pr(a|s)
\]

\[
Pr(a|s) = \prod_{j \in N} s_j(a_j)
\]
Best Response and Nash Equilibrium

Our definitions of best response and Nash equilibrium generalize from actions to strategies.

- **Best response:**
  \[ s^*_i \in BR(s_{-i}) \text{ iff } \forall s_i \in S_i, \ u_i(s^*_i, s_{-i}) \geq u_i(s_i, s_{-i}) \]

- **Nash equilibrium:**
  \[ s = \langle s_1, \ldots, s_n \rangle \text{ is a Nash equilibrium iff } \forall i, \ s_i \in BR(s_{-i}) \]

- **Every finite game has a Nash equilibrium!** [Nash, 1950]
  - e.g., matching pennies: both players play heads/tails 50%/50%
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Interpreting Mixed Strategy Equilibria

What does it mean to play a mixed strategy? Different interpretations:

- Randomize to **confuse** your opponent
  - consider the matching pennies example
- Players randomize when they are **uncertain** about the other’s action
  - consider battle of the sexes
- Mixed strategies are a concise description of what might happen in **repeated play**: count of pure strategies in the limit
- Mixed strategies describe **population dynamics**: 2 agents chosen from a population, all having deterministic strategies. MS is the probability of getting an agent who will play one PS or another.