Game Theory intro

ISCI 330 Lecture 3

January 16, 2007
Lecture Overview

1. Recap: Utility Theory
2. Game Theory
3. Example Matrix Games
Self-interested agents

What does it mean to say that an agent is self-interested?
- not that they want to harm other agents
- not that they only care about things that benefit them
- that the agent has its own description of states of the world that it likes, and that its actions are motivated by this description

Utility theory:
- quantifies degree of preference across alternatives
- understand the impact of uncertainty on these preferences
- utility function: a mapping from states of the world to real numbers, indicating the agent’s level of happiness with that state of the world
- Decision-theoretic rationality: take actions to maximize expected utility.
Preferences and utility functions

Theorem (von Neumann and Morgenstern, 1944; informal version)

If an agent’s preference relation satisfies the axioms Completeness, Transitivity, Substitutability, Decomposability, Monotonicity and Continuity then there exists a function $u : O \rightarrow [0, 1]$ with the properties that:

1. $u(o_1) \geq u(o_2)$ iff the agent prefers $o_1$ to $o_2$; and
2. when faced about uncertainty about which outcomes he will receive, the agent always prefers outcomes with larger expected values of $u$ to outcomes with smaller expected values of $u$. 
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1. Recap: Utility Theory
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Non-Cooperative Game Theory

- What is it?
Non-Cooperative Game Theory

- **What is it?**
  - mathematical study of interaction between *rational*, *self-interested* agents
Non-Cooperative Game Theory

- What is it?
  - mathematical study of interaction between rational, self-interested agents

- Why is it called non-cooperative?
Non-Cooperative Game Theory

- What is it?
  - mathematical study of interaction between rational, self-interested agents

- Why is it called non-cooperative?
  - while it’s most interested in situations where agents’ interests conflict, it’s not restricted to these settings
  - the key is that the individual is the basic modeling unit, and that individuals pursue their own interests
    - cooperative/coalitional game theory has teams as the central unit, rather than agents
TCP Backoff Game

Consider this situation as a two-player game:
- Both use a correct implementation: both get 1 ms delay.
- One correct, one defective: 4 ms delay for correct, 0 ms for defective.
- Both defective: both get a 3 ms delay.

Should you send your packets using correctly-implemented TCP (which has a "backoff" mechanism) or using a defective implementation (which doesn't)?
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- Play this game with someone near you, repeating five times.
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Questions:

- What action should a player of the game take?
- Would all users behave the same in this scenario?
- What global patterns of behaviour should the system designer expect?
- Under what changes to the delay numbers would behavior be the same?
- What effect would communication have?
- Repetitions? (finite? infinite?)
- Does it matter if I believe that my opponent is rational?
Defining Games

- Finite, $n$-person game: $\langle N, A, u \rangle$:
  - $N$ is a finite set of $n$ players, indexed by $i$
  - $A = \langle A_1, \ldots, A_n \rangle$ is a tuple of action sets for each player $i$
    - $a \in A$ is an action profile
  - $u = \langle u_1, \ldots, u_n \rangle$, a utility function for each player, where $u_i : A \mapsto \mathbb{R}$

- Writing a 2-player game as a matrix:
  - row player is player 1, column player is player 2
  - rows are actions $a \in A_1$, columns are $a' \in A_2$
  - cells are outcomes, written as a tuple of utility values for each player
1 Recap: Utility Theory

2 Game Theory

3 Example Matrix Games
Games in Matrix Form

Here’s the **TCP Backoff Game** written as a matrix (“normal form”).

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>C</strong></td>
<td>-1, -1</td>
<td>-4, 0</td>
</tr>
<tr>
<td><strong>D</strong></td>
<td>0, -4</td>
<td>-3, -3</td>
</tr>
</tbody>
</table>
Prisoner’s dilemma is any game

\[
\begin{array}{cc}
C & D \\
C & a, a & b, c \\
D & c, b & d, d
\end{array}
\]

with \( c > a > d > b \).
Players have exactly opposed interests

- There must be precisely two players (otherwise they can’t have exactly opposed interests)
- For all action profiles \( a \in A \), \( u_1(a) + u_2(a) = c \) for some constant \( c \)
  - Special case: zero sum
- Thus, we only need to store a utility function for one player
  - in a sense, it’s a one-player game
Matching Pennies

One player wants to match; the other wants to mismatch.

<table>
<thead>
<tr>
<th></th>
<th>Heads</th>
<th>Tails</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heads</td>
<td>1</td>
<td>-1</td>
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Play this game with someone near you, repeating five times.
Rock-Paper-Scissors

Generalized matching pennies.

<table>
<thead>
<tr>
<th></th>
<th>Rock</th>
<th>Paper</th>
<th>Scissors</th>
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</thead>
<tbody>
<tr>
<td>Rock</td>
<td>0</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
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<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
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...Believe it or not, there’s an annual international competition for this game!