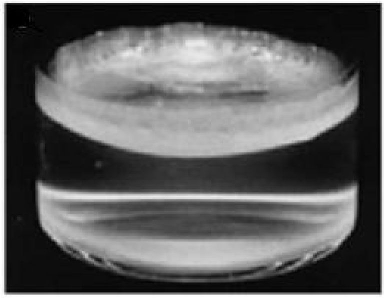
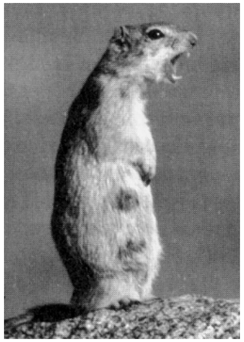
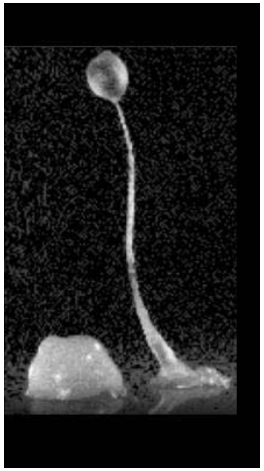
Evolutionary Game Theory

ISCI 330
Lecture 18

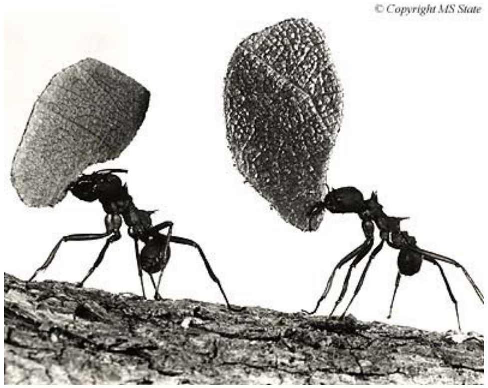






Outline

- A bit about historical origins of Evolutionary Game Theory
- Main (competing) theories about how cooperation evolves
- PD and other social dilemma games
- Iterated PD, TFT, etc.
- Evolutionary Stable Strategy (ESS)
- N-player PD (and other games)
- Simpson's paradox and the role of assortment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Evolution by Natural Selection

- Lewontin’s principles (from Darwin)
  - 1) Phenotypic variation
  - 2) Differential fitness
  - 3) Heritability
- In Evolutionary Game Theory
  - 1) Population of strategies
  - 2) Utility determines number of offspring (fitness)
  - 3) Strategies breed true
- Frequency-dependent selection
  - One of the first examples is Fisher’s sex ratio findings
  - Introduces idea of strategic phenotypes

Ritualized Fighting

		opponent's behaviour	
		Dove	**Hawk**
actor's behaviour	**Dove**	$V / 2$ 5	0
	Hawk	V 10	$(V - c) / 2$ -5

- $V = 10$; $c = 20$
- The rare strategy has an advantage (i.e. frequency dependent selection)
- Hawk-Dove, Chicken, Snowdrift, Brinkmanship
- If $0 < c < V$, then game is PD instead

Main Theories: Evolution of Altruism

- Multilevel Selection

  $\Delta Q = \Delta Q_B + \Delta Q_W$ (Price Equation)

- Inclusive Fitness/Kin Selection
  - $w_{incl.} = w_{direct} + w_{indirect}$

  $\Delta Q > 0$ if $rb > c$ (Hamilton's rule)

- Reciprocal Altruism

  $\Delta Q > 0$ if altruists are sufficiently compensated for their sacrifices via reciprocity (ESS)

Additive Prisoner's Dilemma (PD)

Actor's Fitness (Utility)

		opponent's behaviour	
		C contributes b	**D** contributes 0
actor's behaviour	**C** sacrifices c	$w_0 + b - c$ 4	$w_0 - c$ 0
	D sacrifices 0	$w_0 + b$ 5	w_0 1

- $w_0 = 1$; $b = 4$; $c = 1$

Non-Additive PD

Actor's Fitness (Utility)

		opponent's behavior	
		C contributes b	**D** contributes 0
actor's behavior	**C** sacrifices c	$w_0 + b - c$ $(+d)$ 3	$w_0 - c$ 0
	D sacrifices 0	$w_0 + b$ 5	w_0 1

- $w_0 = 1$; $b = 4$; $c = 1$; $d = -1$

Main Theories: Evolution of Altruism

- Multilevel Selection
  - Predominate models are in terms of public good
- Inclusive Fitness/Kin Selection
  - Predominate models is in terms of individual contributions (b and c)
- Reciprocal Altruism
  - Predominate models in terms of iterated PD (iPD)

Evolutionarily Social Dilemma Games

- What features do Hawk-Dove and the PD have in common?
  - Cs do better in CC pairs than Ds do in DD pairs
  - Ds do better than Cs in mixed pairs
- Given 4 utility levels (1st, 2nd, 3rd, 4th) how many 2-player, symmetric games are there that capture this idea of "social dilemma"?
- With a partner, find these other games. Can you name them?

6 evolutionarily interesting “social dilemmas”

- How do these games compare in terms of
  - Nash equilibria?
  - Pareto optimality?
  - Is it better to be rare or common?
- Consider populations of strategies rather than 2-players
- Relative vs. Absolute fitness

Common EGT Assumptions

- Population of strategies
- Replicator equations
  - Number of individuals of a certain strategy in next generation depends on:
    - Average fitness (utility) of individuals with that strategy
    - Which depends on frequency distribution of strategies
- Often assume
  - infinite populations where replicator equations give proportion of strategy (scaled by average fitness)
  - continuous (or discrete) time
  - complete mixing (random interactions)
  - strategies breed true (no sex or mutation)

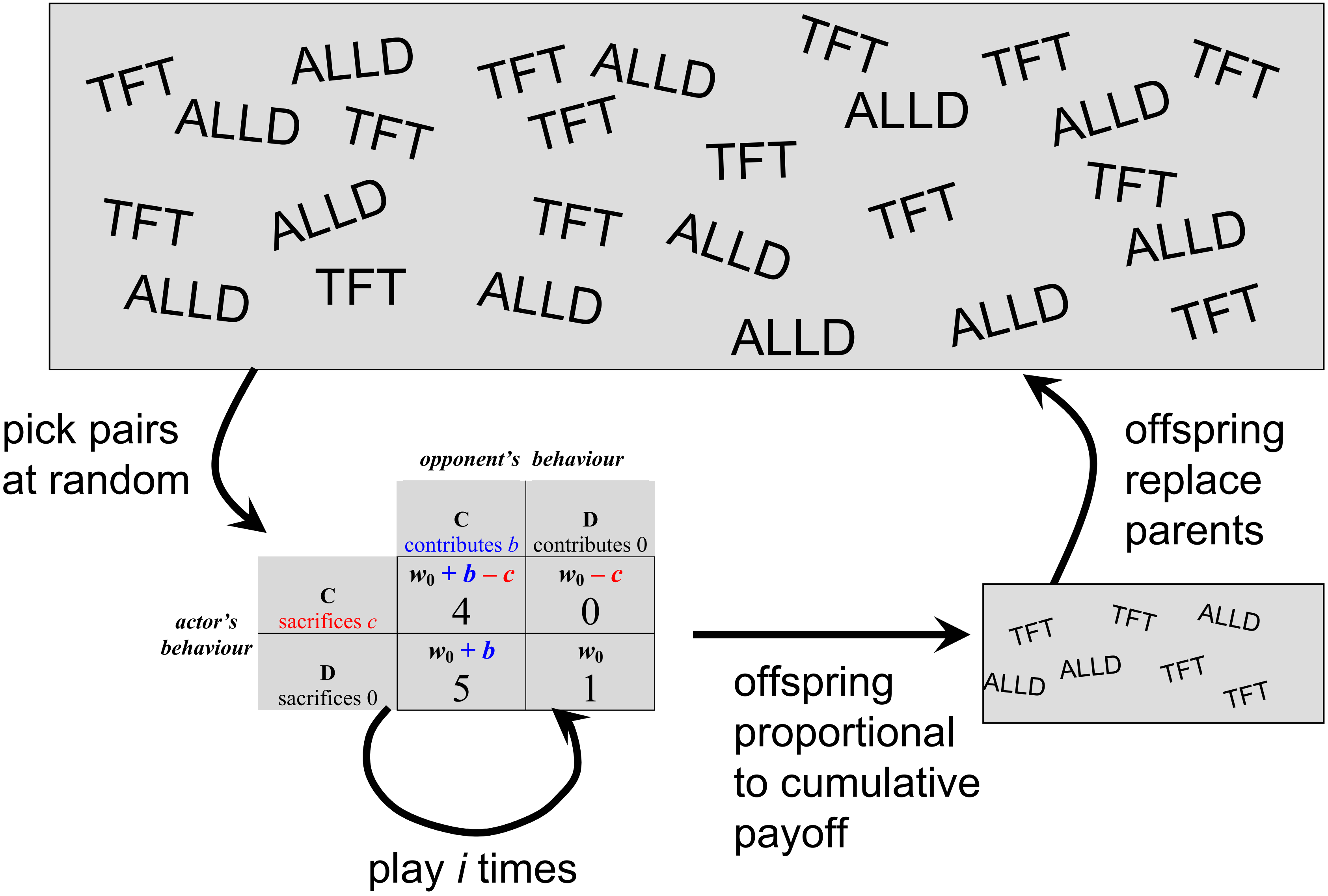
Iterated Prisoner's Dilemma

- Robert Triver's concept of reciprocal altruism
- Robert Axelrod's tournaments
  - Every strategy plays every other one
    - Or at random for evolutionary experiments
  - On average 200 rounds
  - Final score is cumulative payoffs from all rounds
  - Can condition current behaviour on any amount of history
    - opponent's and actor's

Conditional Strategies

- Anatol Rappoport's Tit-For-Tat strategy
  - Unless provoked, the agent will always cooperate
  - If provoked, the agent will retaliate swiftly
  - The agent is quick to forgive
  - Susceptible to noise
  - Backwards induction issue
    - Imagine gene for when to start defecting
- Alternatives
  - Forgiving TFT, TF2T, Pavlov, walk-away
- Under what conditions does TFT beat it's opponent?

Simple Iterated PD Model



pick pairs at random

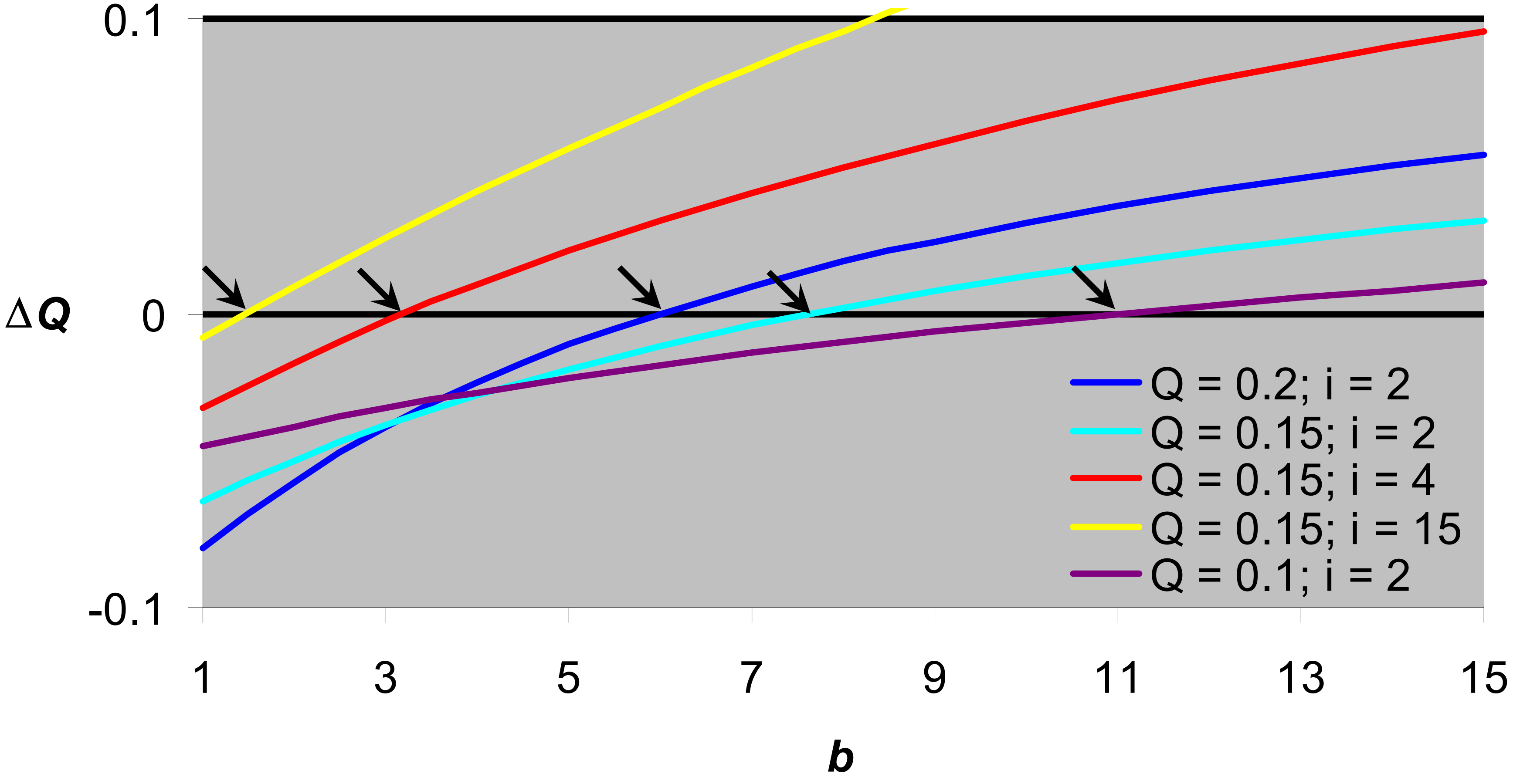
actor's behaviour	*opponent's behaviour* **C** contributes b	 **D** contributes 0
C sacrifices c	$w_0 + b - c$ 4	$w_0 - c$ 0
D sacrifices 0	$w_0 + b$ 5	w_0 1

play *i* times

offspring proportional to cumulative payoff

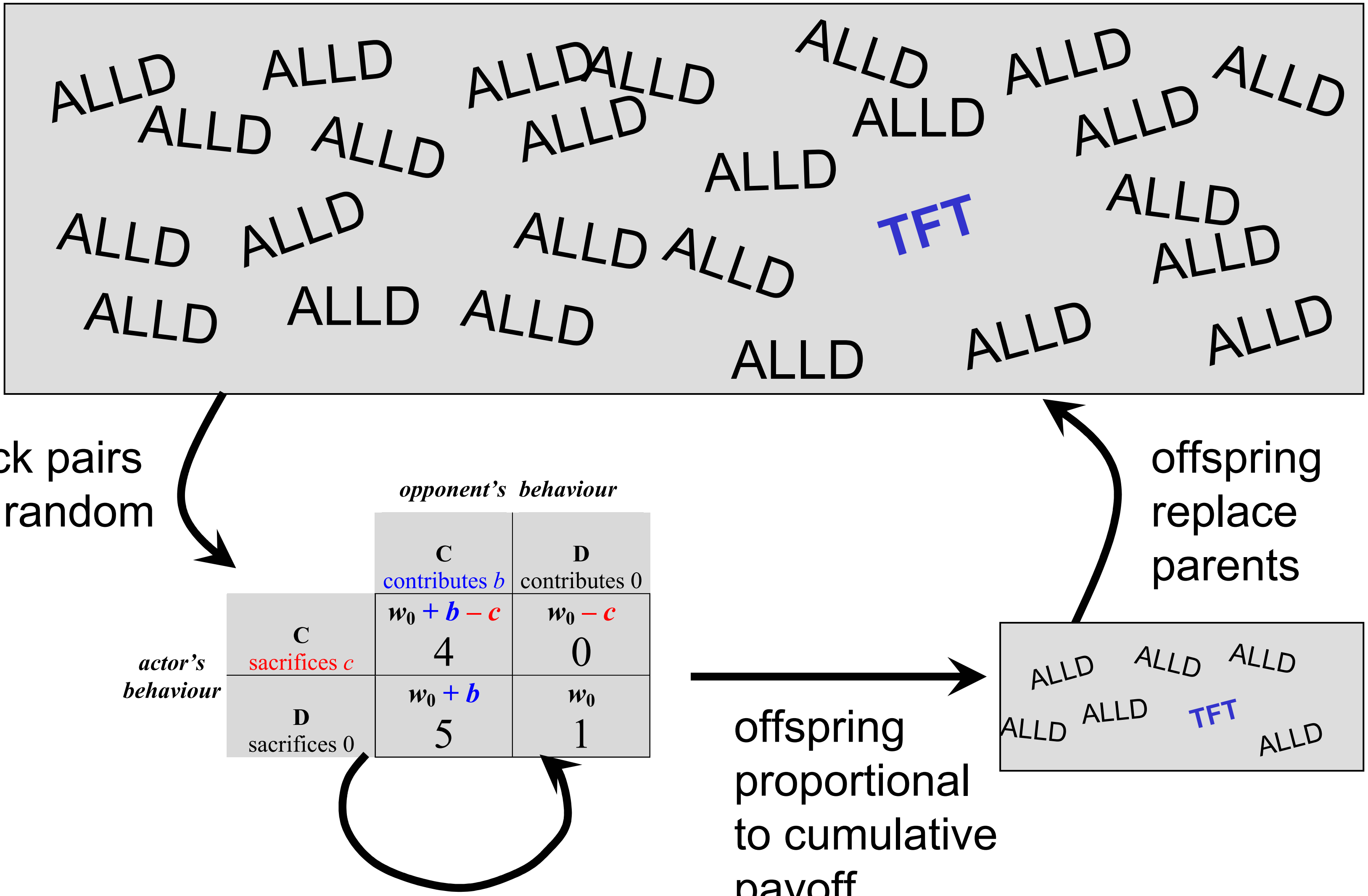
offspring replace parents

Numerical Simulations of Iterated PD varying *Q*, *i*, and *b* (*c* = 1)



– Fletcher & Zwick, 2006. *The American Naturalist*

Simple Iterated PD Model



pick pairs
at random

opponent's behaviour

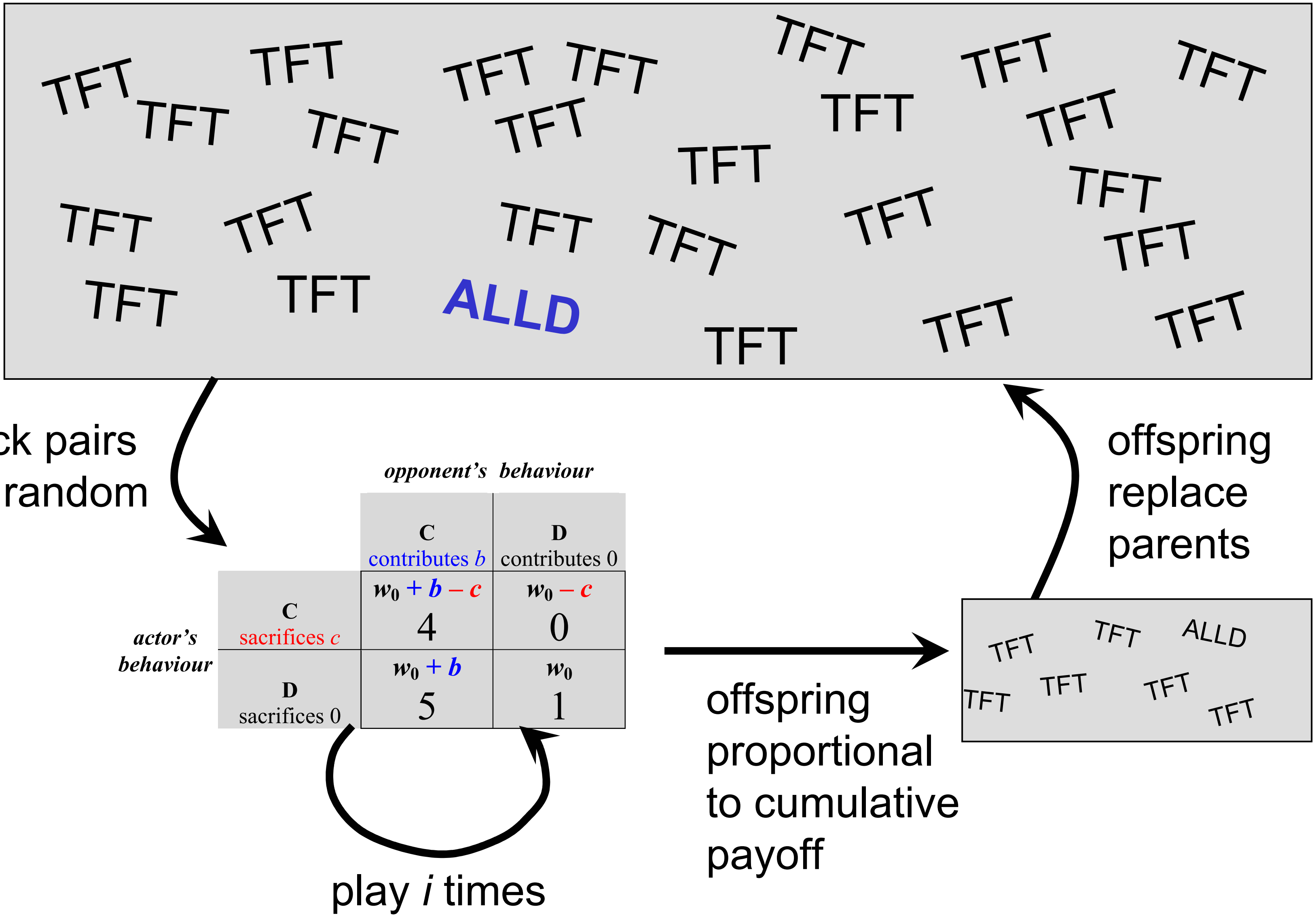
actor's behaviour		**C** contributes *b*	**D** contributes 0
	C sacrifices *c*	$w_0 + b - c$ 4	$w_0 - c$ 0
	D sacrifices 0	$w_0 + b$ 5	w_0 1

play *i* times

offspring
proportional
to cumulative
payoff

offspring
replace
parents

Simple Iterated PD Model



pick pairs
at random

		opponent's behaviour	
		C contributes b	**D** contributes 0
actor's behaviour	**C** sacrifices c	$w_0 + b - c$ 4	$w_0 - c$ 0
	D sacrifices 0	$w_0 + b$ 5	w_0 1

play *i* times

offspring
proportional
to cumulative
payoff

offspring
replace
parents

Evolutionary Stable Strategy (ESS)

- A strategy which if adopted by a population cannot be invaded by any competing alternative strategy
- How does this compare to a Nash Equilibrium?
  - Here assume *almost* all players play the same strategy (call it S)
  - S is a Nash Eq. if $u(S, S) \geq u(S', S)$ for any S'
  - S is an ESS if
    - $u(S, S) > u(S', S)$ for any S' (strict Nash)
    - Or $u(S, S) = u(S', S)$ AND $u(S, S') > u(S', S')$
  - If S is an ESS, then it is a Nash Eq.
  - If S is a strict Nash Eq. (given a population of S), then it is an ESS

ESS and Hawk-Dove (Chicken)

- Is Hawk an ESS? Is Dove and ESS?
- Is there a mixture of playing Hawk and Dove that is an ESS?
  - Find it assuming original game
- Note that we can think of this in 2 ways
  - Agents playing a mixed strategy
    - Monomorphic solution where an allele for this mixture has fixed in the population
  - A mixed population of strategies at this ratio
    - Polymorphic stable equilibrium
- Learning vs. Evolving

		opponent's behaviour	
		Dove	**Hawk**
actor's behaviour	**Dove**	5	0
	Hawk	10	-5

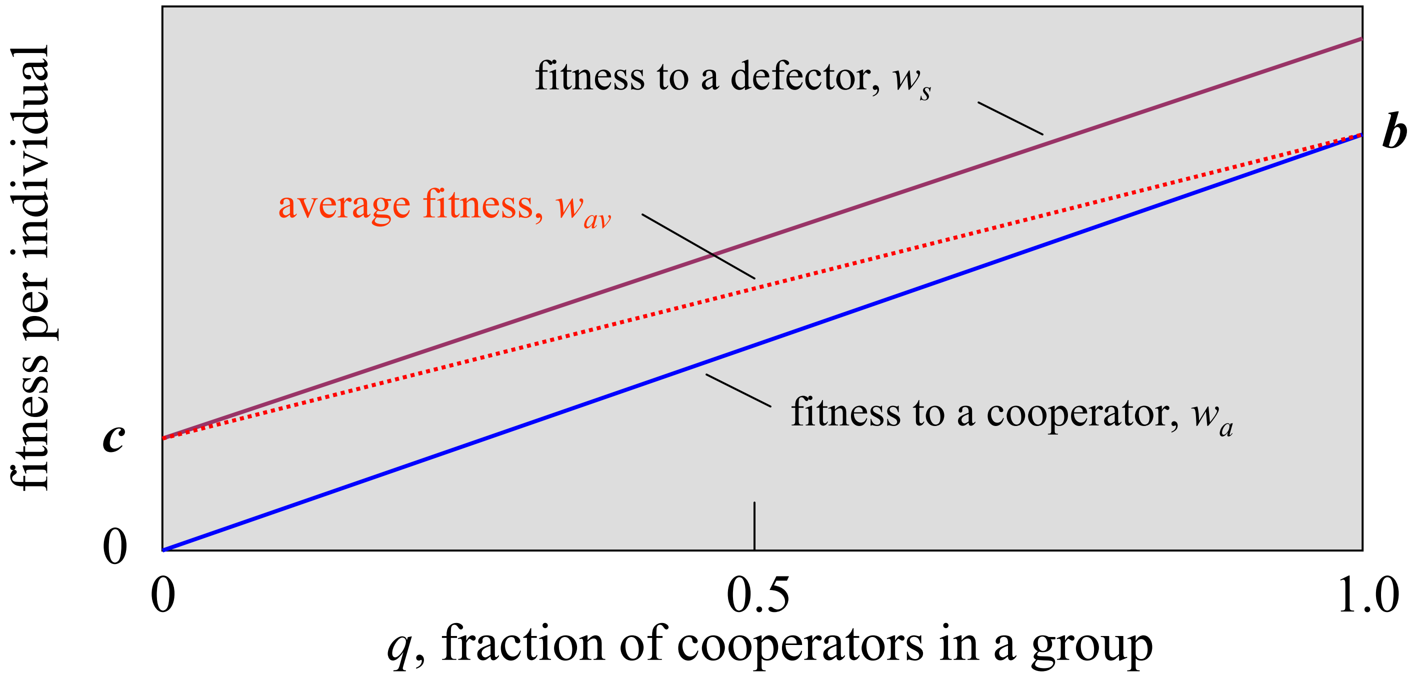
ESS and the PD

- Is C an ESS? Is D and ESS?
- Is there a mixture of playing C and D that is an ESS?
- What if we break the assumption of random interactions?
- Is TFT an ESS? Is ALLD and ESS?
  - Does it depend on iterations in iPD?
  - Find game (payoff matrix) for TFT vs. ALLD if i = 3
  - What do we call this game?
- Can think of TFT in an iPD in two ways:
  - Conditional behaviour causes C behaviours of others to be more assorted with TFT than with ALLD
  - In iPD, TFT and ALLD change the game to Assurance

		opponent's behaviour	
		C contributes b	**D** contributes 0
actor's behaviour	**C** sacrifices c	$w_0 + b - c$ 4	$w_0 - c$ 0
	D sacrifices 0	$w_0 + b$ 5	w_0 1

N-Player Prisoner's Dilemma

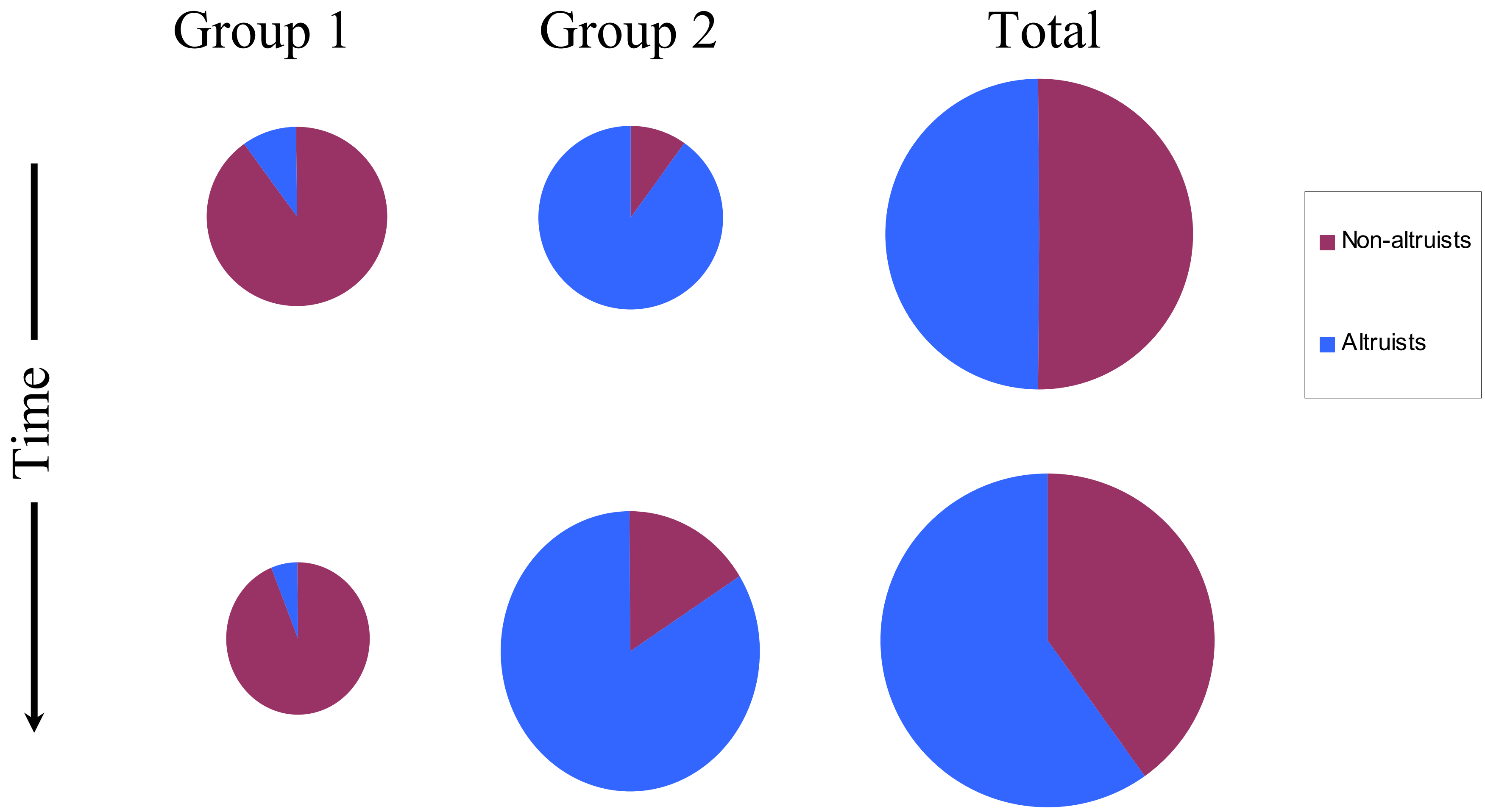
(Tragedy of the Commons)



- $a_i' = a_i\,[1 + w_a(q_i)]$ $\qquad s_i' = s_i\,[1 + w_s(q_i)]$
- $w_a(q_i) = bq_i - c + w_0$ $\qquad w_s(q_i) = bq_i + w_0$

– Fletcher & Zwick, 2007. *Journal of Theoretical Biology*

Simpson's Paradox



- Altruists become a smaller portion of each group
- But altruists become a larger portion of the whole