Evolutionary Game Theory

ISCI 330
Lecture 17
Outline

- A bit about historical origins of Evolutionary Game Theory
- Main (competing) theories about how cooperation evolves
- iPD and other social dilemma games
- Evolutionary Stable Strategy (ESS)
- N-player PD (and other games)
- Simpson’s paradox and the role of assortment
Evolution by Natural Selection

• Lewontin’s principles (from Darwin)
  – 1) Phenotypic variation
  – 2) Differential fitness
  – 3) Heritability

• In Evolutionary Game Theory
  – 1) Population of strategies
  – 2) Utility determines number of offspring (fitness)
  – 3) Strategies breed true

• Frequency-dependent selection
  – One of the first examples is Fisher’s sex ratio findings
  – Introduces idea of strategic phenotypes
## Ritualized Fighting

### Opponent’s Behaviour

<table>
<thead>
<tr>
<th></th>
<th>Dove</th>
<th>Hawk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dove</td>
<td>$V/2$</td>
<td>0</td>
</tr>
<tr>
<td>Hawk</td>
<td>10</td>
<td>$V/2 - c$</td>
</tr>
</tbody>
</table>

- $V = 10$; $c = 10$
- The rare strategy has an advantage (i.e. frequency dependent selection)
- Hawk-Dove, Chicken, Snowdrift, Brinkmanship
- If $c < V/2$, then game is PD instead
George Price’s Contributions

• Evolutionary Game Theory
  – Concept of an Evolutionary Stable Strategy (ESS)

• Formal description of Natural Selection and Fisher’s Fundamental Theorem

• Decomposition of selection at different hierarchical levels using covariance
  – Used to formalize Multilevel Selection Theory
What special circumstances or mechanisms thus favor cooperation? Currently, evolutionary biology offers a set of disparate explanations, and a general framework for this breadth of models has not emerged.

Main Theories: Evolution of Altruism

- **Multilevel Selection**
  \[ \Delta Q = \Delta Q_B + \Delta Q_W \] (Price Equation)

- **Inclusive Fitness/Kin Selection**
  \[ w_{incl.} = w_{direct} + w_{indirect} \]
  \[ \Delta Q > 0 \text{ if } rb > c \] (Hamilton’s rule)

- **Reciprocal Altruism**
  \[ \Delta Q > 0 \text{ if altruists are sufficiently compensated for their sacrifices via reciprocity (ESS)} \]
Prisoner’s Dilemma (PD)  
Actor's Fitness (Utility)

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

opponent’s behavior

actor’s behavior
Additive Prisoner’s Dilemma (PD)  
Actor's Fitness (Utility)

<table>
<thead>
<tr>
<th>actor's behaviour</th>
<th>opponent's behaviour</th>
</tr>
</thead>
<tbody>
<tr>
<td>C sacrificing 0</td>
<td>C contributes $b$</td>
</tr>
<tr>
<td>D sacrificing $c$</td>
<td>D contributes 0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>C contributing $b$</td>
<td>$w_0 + b - c$</td>
<td>$w_0 - c$</td>
</tr>
<tr>
<td>D contributing 0</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

$w_0 = 1; b = 4; c = 1$
### Non-Additive PD

**Actor's Fitness (Utility)**

<table>
<thead>
<tr>
<th>Actor’s Behavior</th>
<th>Opponent’s Behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C contributes ( b )</td>
</tr>
<tr>
<td><strong>C</strong> sacrifices ( c )</td>
<td>( w_0 + b - c )</td>
</tr>
<tr>
<td><strong>D</strong> sacrifices 0</td>
<td>((+d) 3)</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
w_0 & = 1; \quad b = 4; \quad c = 1; \quad d = -1
\end{align*}
\]
Main Theories: Evolution of Altruism

• Multilevel Selection
  – Predominate models are in terms of public good

• Inclusive Fitness/Kin Selection
  – Predominate models is in terms of individual contributions (b and c)

• Reciprocal Altruism
  – Predominate models in terms of iterated PD
Evolutionarily Social Dilemma Games

• What features do Hawk-Dove and the PD have in common?
  – Cs do better in CC pairs than Ds do in DD pairs
  – Ds do better than Cs in mixed pairs
• Given 4 utility levels (1^{st}, 2^{nd}, 3^{rd}, 4^{th}) how many 2-player, symmetric games are there that capture this idea of “social dilemma”?
• With a partner, find these other games. Can you name them?
6 evolutionarily interesting “social dilemmas”

• How do these games compare in terms of
  – Nash equilibria?
  – Pareto optimality?
  – Is it better to be rare or common?

• Consider populations of strategies rather than 2-players

• Relative vs. Absolute fitness
Common EGT Assumptions

• Population of strategies
• Replicator equations often assume
  – infinite populations
  – continuous (or discrete) time
  – complete mixing (random interactions)
  – strategies breed true (no sex)