

# Game Theory ISCI 330 Class Notes (2007)

## ***Class 2: Jan. 11 Thursday – Utility Theory***

### **Follow-up (10 min)**

- 1) Their thoughts on game results from Tuesday
  - a) Winning strategy? (depends on how others behave)
  - b) What if everyone rational?
    - i) (point 3a from last time) GT not just descriptive, but also normative
    - ii) Individual rationality vs. collective rationality
  - c) Example of Public Goods game (e.g. fisheries)
    - i) Tragedy of the Commons or n-player Prisoner's Dilemma
- 2) Other questions comments?

### **Logistics (5 min)**

- 1) Registered students, auditors, Gillian?
- 2) Quick roll
- 3) Readings are broken down in schedule
- 4) Course website again: <http://www.cs.ubc.ca/~kevinlb/teaching/isci330/index.html>

### **Agency (15 min)**

- 1) We mentioned GT has abstract notion of agents, but what are essential characteristics of agents? (get their ideas)
- 2) Self-interested decision making or action (not in the sense of representing others)
  - a) Doesn't mean selfish or wants to harm others
  - b) Agents perceive different states of the world and have preferences for certain states over others
  - c) Agents' decisions/actions are determined by these preferences
  - d) Does not mean agents are always conscious beings
    - i) Bacteria – sense their environment, “prefer” some states of the environment to others, and move accordingly (e.g. towards light or food or moisture)
    - ii) Nations – assess their economic health (e.g. trade deficits/surpluses), prefer some states of the world to others, and may act accordingly
  - e) Note that in the bacteria example we are thinking in terms of only agent and environment, but in national trade example we are thinking in terms of other agents (other nations)
    - i) DT vs. GT -- both have agents
    - ii) In multi-agent systems (MAS) we are interested in interactions among agents

### **Utility Theory (40 min)**

- 1) Before GT and interactions between self-interested actors, need to be clearer on what we mean by self interest

- 2) Game theory rests on the foundation of Utility Theory (which helps define self-interest)
- 3) First let's see why it might be useful not only to know what players prefer, but also how these preferences interact with uncertainty about what others will do
  - a) Later today we'll address this more formally
  - b) We began to see this in our neighbor/nature example with the umbrella last time
  - c) Build on this briefly
- 4) Friends and enemies example from text 3.1.1 (not allowed to look up in text)
  - a) Set up problem together
    - i) First do ordinal preferences, then with numbers
  - b) Form groups of 3 or 4 (different from last time)
    - i) Introductions
    - ii) Formally figure out what Alice should prefer to do
    - iii) After they do this, have them discuss whether Alice should always do this preferred behavior (e.g. every night)?
- 5) Formalizing the notion of utility—utility function
  - a) Under what circumstances can we compare different outcomes on one single-dimensional scale? E.g.:
    - i) Comparing a 0.1 chance of having to endure Bob at a club alone vs. a 0.45 chance of watching a movie with just Carol?
    - ii) Get there ideas about what is necessary?
- 6) von Neumann and Morgenstern's axioms (see Appendix C)
  - a) Briefly cover each, especially decomposability, Lemma C.1.6
    - i) probably won't do proofs
  - b) How well does Alice's situation conform to these axioms (assumptions)?
  - c) Given these assumptions about preferences, (von Neumann & Morgenstern 1944) showed we can be assured that a utility function exists that:
    - i) Gives  $\geq$  numerical values for outcomes that are preferred or indifferent
    - ii) Given a probability distribution of outcomes, utility function gives the expectation of utility across individual outcomes and probabilities
  - d) Why not just use money?
    - i)  $[0.5, 2M; 0.5, 0] \sim 1M$ , but most people would prefer sure thing (maximin?)
    - ii) Court systems do, i.e. value of accidental death, pain and suffering
    - iii) Is this reasonable? Is there a better way? Briefly discuss