Homework #4

ISCI 330 Game Theory

Due Tuesday April 10 (in class).
Note: If you turn this assignment in on time, you will get it back on Thursday (the final is the following Monday, April 16). If you use late days then we will try to grade your homework quickly, but we can only guarantee that you will get it back by Monday morning. Problem 3 should be answered on a separate piece of paper; for problems 1 and 2, you can use the back of the sheets if necessary.

1. **(30 Points)** As discussed in class, for a strategy to be an ESS (evolutionary stable strategy), it must satisfy either of the following two conditions:

   1) u(*S*, *S*) > u(*S′*, *S*) for any *S′* (strict Nash)
   2) u(*S*, *S*) = u(*S′*, *S*) <u>AND</u> u(*S*, *S′*) > u(*S′*, *S′*)

   where u is a function that gives the expected utility of the first strategy when playing the second strategy. Here we are thinking in terms of whether one strategy can invade another (i.e. does it have higher fitness when rare compared to the resident strategy). So the first argument of u can be thought of as an individual in a population full of players playing the strategy in the second argument of u. For example, TFT is an ESS with respect to ALLD in the iterated PD because it satisfies the first condition: given a sufficient number of iterations, u(TFT, TFT) > u(ALLD, TFT). We don't have to consider u(ALLD, ALLD) or u(TFT, ALLD) because ALLD is presumed to be a rare mutant, i.e. with very high probability it will play TFT, and TFT players with very high probability just play each other.

   Now consider the following Hawk-Dove game where fighting is very costly (note different payoffs than used in class):

Others

Focal Player		Dove	Hawk
	Dove	5, 5	0, 10
	Hawk	10, 0	-15, -15

   a) **(2 pts)** What is the expected utility (fitness) for a Hawk player in a population of Doves?

   b) **(2 pts)** What is the expected utility (fitness) for a Dove player in a population of Hawks?

c) **(4 pts)** How do these values compare to what Hawks get playing other Hawks and Doves get playing other Doves? Is pure Hawk or pure Dove an ESS? Justify your answer in terms of the two conditions above.

d) **(4 pts)** Find the mixed strategy Nash Equilibrium for the Hawk-Dove game above. Represent this equilibrium in terms of *p* where *p* equals proportion Hawk. Be sure to show your work. (You can think of this as a mixed strategy for individual players or the proportion of pure Hawk strategies in the population that makes the utility (fitness) of pure Hawks equal to pure Doves.

e) **(3 pts)** For a mixed Nash equilibrium, given that everybody else in the population keeps doing the same thing, the utilities are the same for pure Hawk, pure Dove, or any combination of Hawk and Dove. So now the first condition above for an ESS is not met, but the first part of the second condition is met. To decide if this mixed equilibrium is an ESS we need to decide whether the second part of the second condition is also satisfied:

$u(S, S') > u(S', S')$

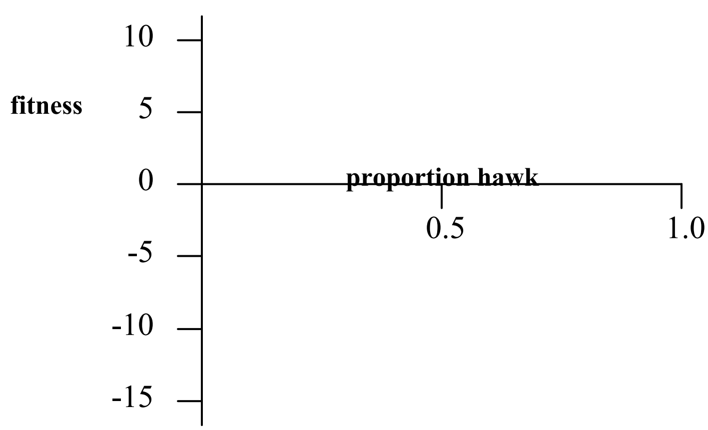
This condition is sometimes called the "stability condition". That is, consider S to be the mixed strategy you found in d). S' then represents a population playing some other mixture of Hawk and Dove. If we think in terms of individuals playing mixed strategies, this condition requires that our original mixed Nash equilibrium strategy does better than any alternative mixture. If it does do better (is more fit) then natural selection will increase our original mixed strategy in the population such that the population again becomes dominated by this strategy. If we think in terms of a mixed population of pure strategies, this condition requires that if the population is perturbed away from the equilibrium, then it tends to return to the equilibrium, i.e. it is a stable equilibrium.

So now let's think about the stability of the equilibrium you found in d). Calculate the utility of your mixed equilibrium strategy from d) playing a strategy with 0.01 more Hawk than your strategy. Here, your *p* from question d) is S and S' is *p* + 0.01. You are calculating $u(S, S')$.

f) **(3 pts)** Calculate $u(S', S')$

g) **(4 pts)** Now consider an alternative $S' = p - 0.01$. Calculate $u(S, S')$ and $u(S', S')$ for this S'.

h) **(8 pts)** In the graph below the x-axis is the proportion of Hawks (or Hawk behaviours) and the y-axis is fitness (utility). Draw two curves (lines) showing the fitness (payoff) for playing Hawk and for playing Dove. Clearly label each line. The end points for these lines can be found in your answers above. Do you think your answer in d) is an ESS? Use the graph and your answers above to justify your answer.



2. **(20 Points)** Now consider the Assurance game (also called Coordination or Stag Hunt) where the alternative pure strategies are Cooperate (C) and Defect (D):

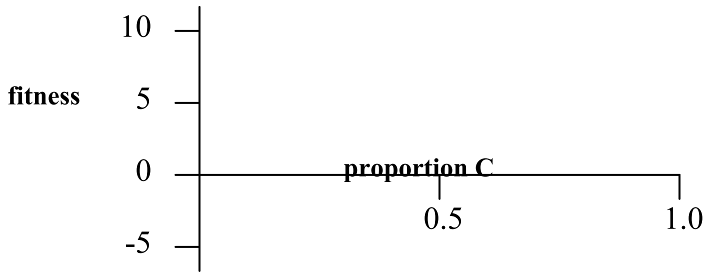
		Others	
		C	**D**
Focal Player	**C**	10, 10	-5, 5
	D	5, -5	0, 0

a) **(5 pts)** Find the mixed strategy Nash Equilibrium for this game. Represent this equilibrium in terms of p where p equals proportion C. Be sure to show your work.

b) **(3 pts)** Now consider the stability condition for this equilibrium:
$u(S, S') > u(S', S')$
Calculate the utility of your mixed equilibrium strategy from a) playing a strategy with 0.01 more C than your strategy. Again, your p from question a) is S and S' is $p + 0.01$ so you are calculating $u(S, S')$.

c) **(3 pts)** Now calculate $u(S', S')$

d) **(9 pts)** In the graph on the next page the x-axis is the proportion of C (or C behaviours) and the y-axis is fitness (utility). Draw two curves (lines) showing the fitness (payoff) for playing C and for playing D. Clearly label each line. The end points for these lines can be found in the payoff matrix above. Do you think your answer in a) is an ESS? Use the graph and your answers above to justify your answer.



3. **(30 Points)** Consider the following two games:

First game:

	C	D
C	6,4	1,2
D	3,2	4,5

Second game:

	C	D
C	6,6	1,2
D	2,4	4,1

Now consider a repeated game scenario in which the first game is repeated twice, and then the second game is repeated twice. Answer the following questions on a separate sheet of paper.

a) Find two *different* Nash equilibria of this repeated game, assuming that the agents' utilities are computed as the average of their payoffs in the stage games. Ensure that one or both of these equilibria involves randomization. Also ensure that one or both equilibria involves non-stationary strategies (e.g. the strategy depends in some way on the history of the game.) Give each agent's expected utility in each equilibrium.

b) Find two different Nash equilibria of this repeated game as before, now assuming that the agents' utilities are computed using **discounted rewards**, with a discount factor of 0.9. Again, ensure that one or both of these equilibria involves randomization. It's OK to use one or both of the equilibria you calculated for the previous part, but if you do you must explain why it is still an equilibrium under the new condition. Also ensure that one or both equilibria involves non-stationary strategies. Give each agent's expected utility in each equilibrium.

Academic Honesty Form

For this assignment, it is acceptable to collaborate with other students provided that you write up your solutions independently. Getting help from students or course materials from previous years is not acceptable.

List any people you collaborated with:

List any non-course materials you referred to:

Signature:

Fill in this page and include it with your assignment submission.