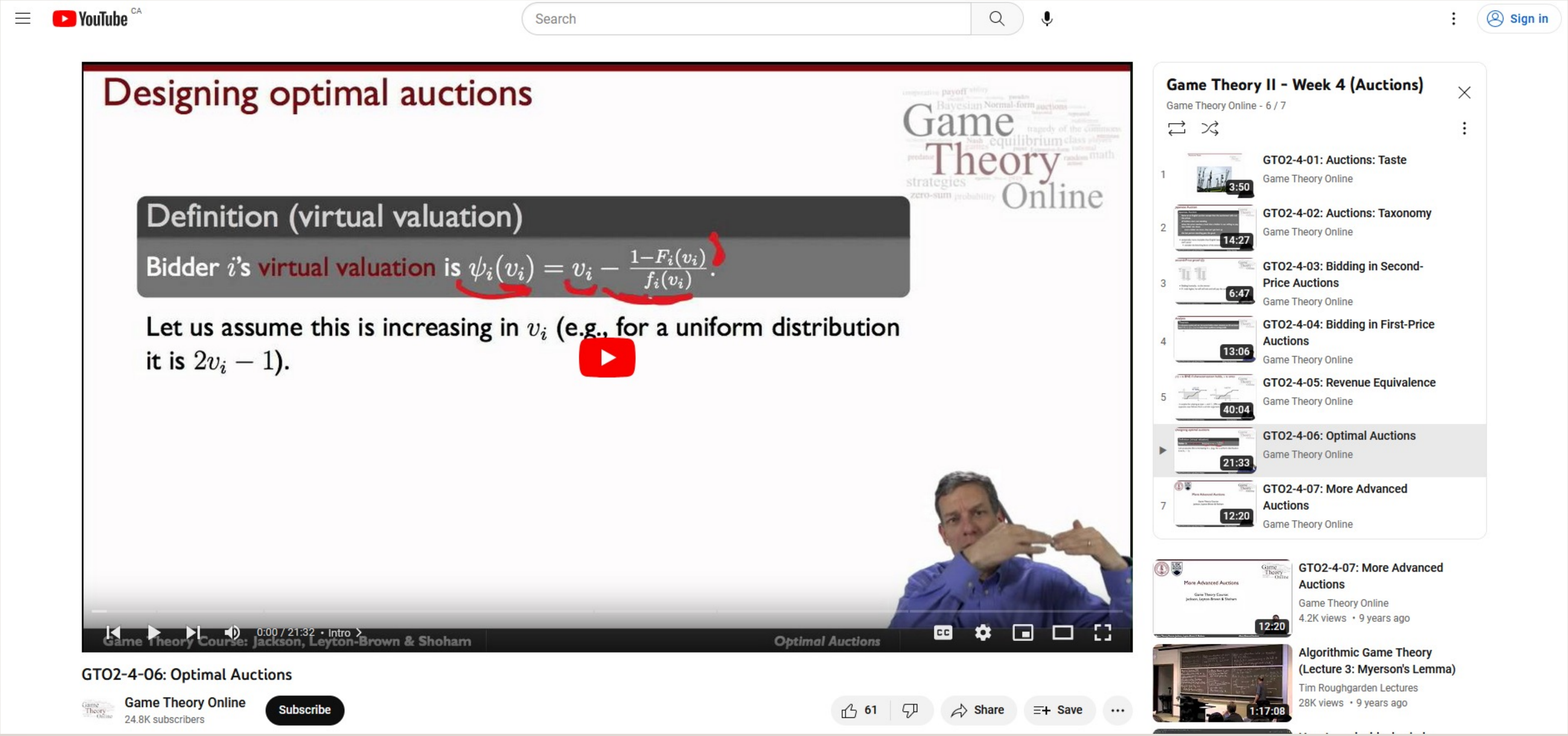
BEYOND OPTIMAL AUCTIONS

PRESENTED BY: MICHAEL LIU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Designing optimal auctions
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Definition (virtual valuation)
Bidder i's virtual valuation is $\psi_i(v_i) = v_i - \frac{1-F_i(v_i)}{f_i(v_i)}$.
Let us assume this is increasing in v_i (e.g., for a uniform distribution it is $2v_i - 1$).
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Can we do better than the optimal auction?

OUTLINE

- How to beat the optimal auction
  - The **Bulow-Klemperer** theorem
  - A geometric intuition
  - A boring proof

- An application of Bulow-Klemperer
  - Prior-free auctions
  - Other applications

BULOW-KLEMPERER THEOREM

The expected revenue of the second price auction on $n+1$ agents is at least the expected revenue of the optimal auction on n agents, provided valuations are drawn independently from a ***regular****, common distribution F*

A distribution is *regular* if it has a nondecreasing virtual value function

$$\phi_F(v) = v - \frac{1 - F(v)}{f(v)}$$

Examples: exponential, uniform, etc.

GEOMETRIC INTUITION

BULOW-KLEMPERER ($n = 1$)

The expected revenue of the second price auction on 2 *agents is at least the expected revenue of the optimal auction on 1 agent, provided valuations are drawn independently from a* ***regular****, common distribution* F

- What is the optimal auction on one agent?
- What is a second-price auction on two agents?

REVENUE FUNCTION

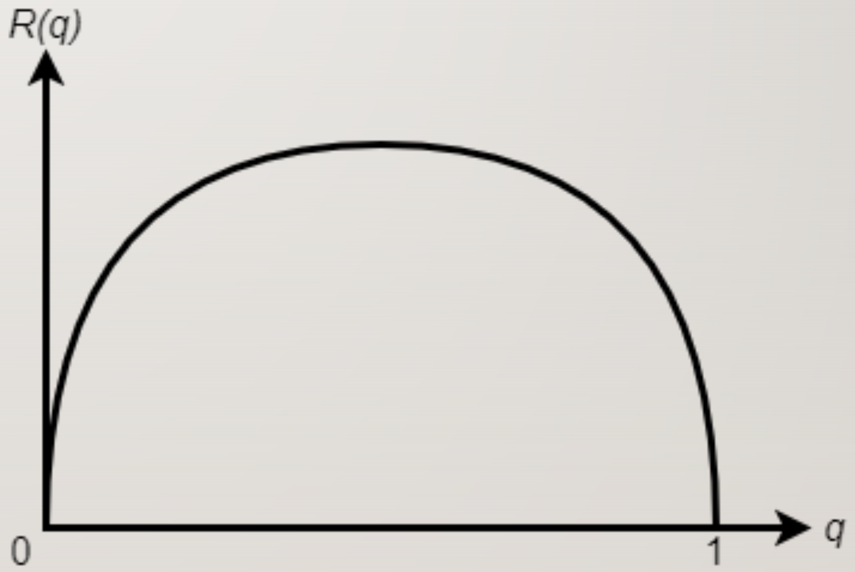
- Expected revenue given price p:

$$\hat{R}(p) = p \cdot \big(1 - F(p)\big)$$

- Expected revenue given probability of sale q:

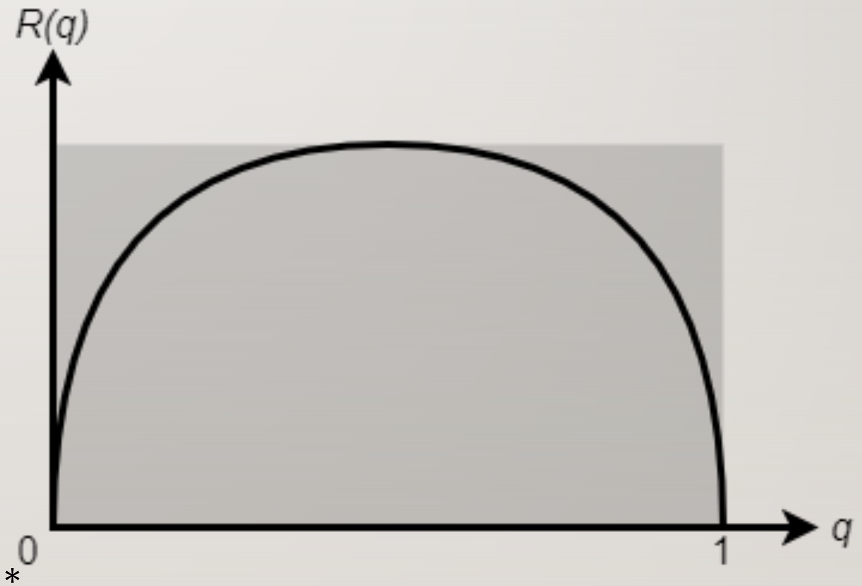
$$R(q) = q \cdot F^{-1}(1 - q)$$

- Regularity: revenue function is **concave**



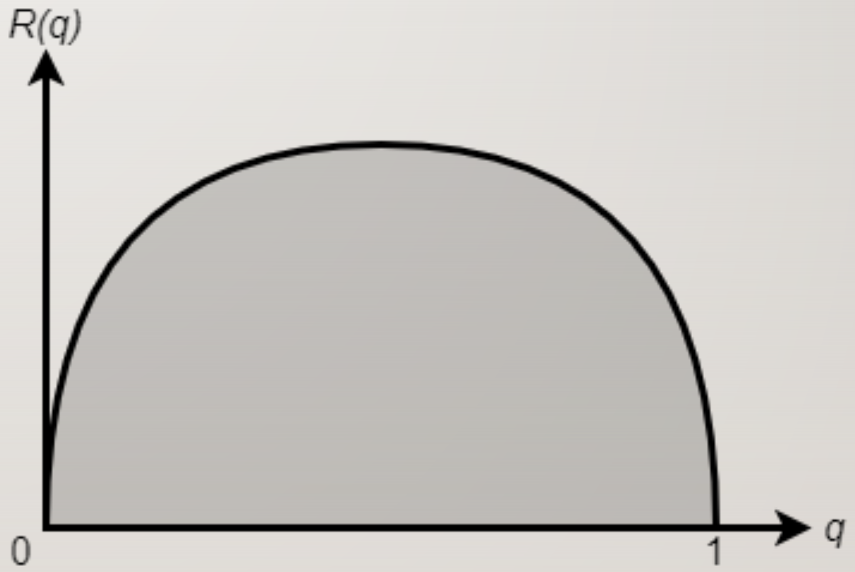
OPTIMAL AUCTIONS WITH ONE BUYER

- Recall:
  - Virtual value function: $\phi_F(v) = v - \frac{1-F(v)}{f(v)}$
- Set **reserve price** r^* where $\phi_F(r^*) = 0$
  - E.g., Uniform distribution on $[0, 1]$: $r^* = \frac{1}{2}$
  - E.g., Uniform distribution on $[0, a]$: $r^* = \frac{a}{2}$
- Reserve price corresponds to probability of sale q^*



SECOND PRICE AUCTIONS WITH TWO BUYERS

- Other agent's valuation is like a **reserve price**
- This reserve price is **randomly** drawn from F
- By **symmetry**, each agent contributes the **same amount** to expected revenue
- Expected revenue for one agent is the **area under** the revenue curve

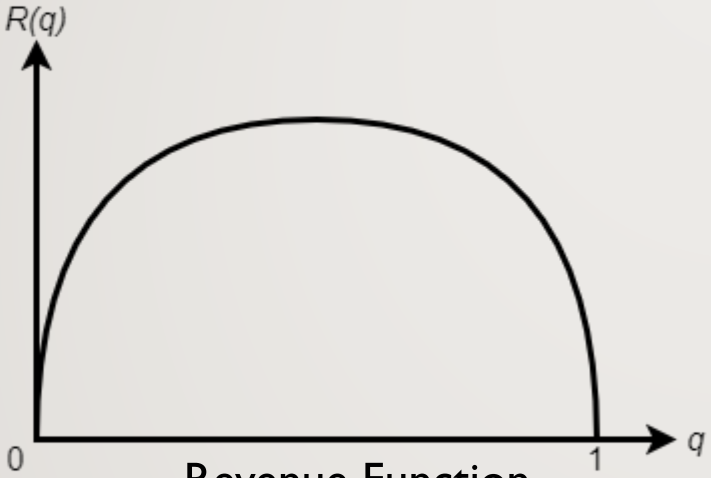


BULOW-KLEMPERER ($n = 1$)

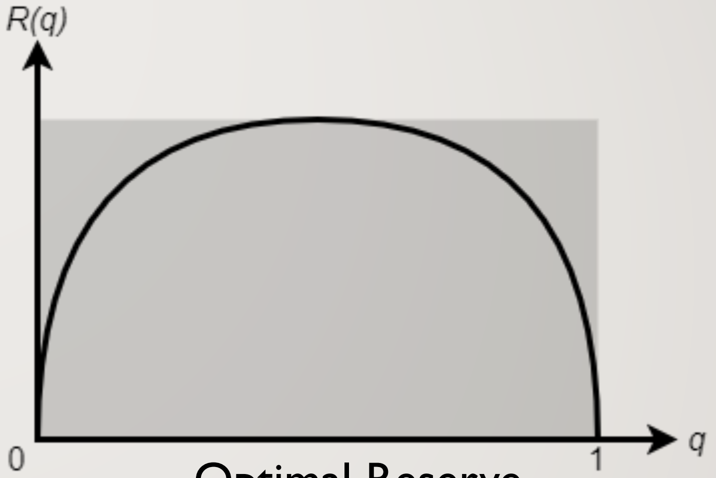
"For a bidder with a valuation drawn from a regular distribution F, the expected revenue from a ***random*** *posted price drawn from F is* ***at least half*** *that from an* ***optimal*** *posted price"*

– Dhangwatnotai, Roughgarden, and Yan

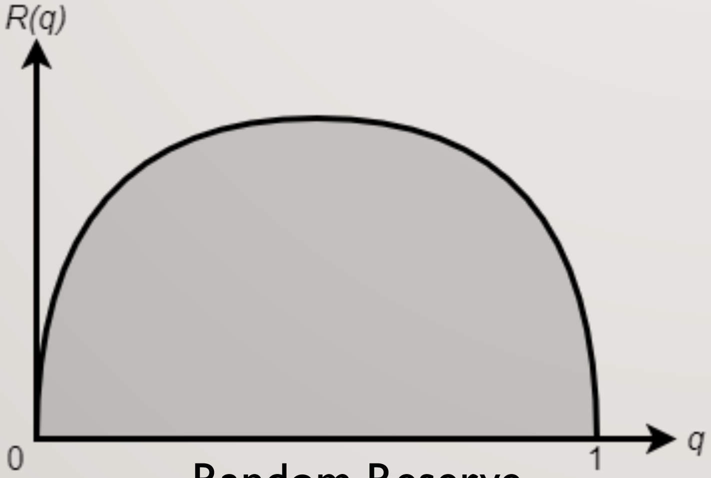
- **randomly** picking a reserve price is a good approximation to picking the **optimal** reserve price in the one-agent case



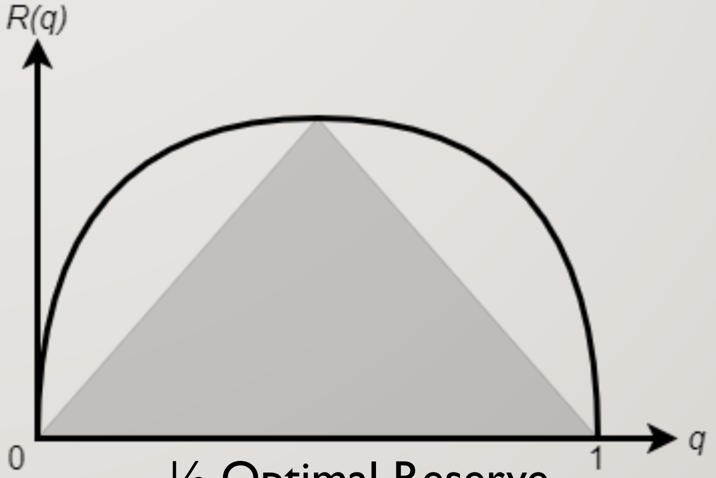
Revenue Function



Optimal Reserve



Random Reserve



½ Optimal Reserve

CHECKPOINT

- For 1 buyer, "random reserve revenue $\geq \frac{1}{2} \times$ Optimal revenue" in expectation
- For 2 buyers, "2nd price auction revenue $= 2 \times$ random reserve revenue"
- Conclusion: 2nd price auction with two buyers generates at least as much revenue as the optimal auction with one buyer.

PROOF OF BULOW-KLEMPERER

A SIMPLE LEMMA

The second price auction maximizes revenue provided the good is always allocated and valuations are drawn i.i.d. from a regular distribution

- The optimal auction allocates the good to the **highest virtual value**
- By assumptions on valuations, bidder with highest virtual value has **highest valuation**
  - The mechanism is **efficient**, **individually rational**
- VCG (2^{nd} price auction) is at least as **budget balanced** as this mechanism
  - Choice-set monotonicity, No negative externalities, No single-agent effect

A SIMPLE MECHANISM

- Mechanism:
  - Run the optimal auction on the first n buyers
  - If the good is not sold, give it to the $n+1^{\text{th}}$ buyer for free
- Observations:
  - Good is **always allocated**
  - Expected revenue is **equal to optimal auction** on n bidders
- By the previous lemma, the 2^{nd} price auction on $n+1$ buyers generates at least as much revenue as the optimal auction on n bidders.

BULOW-KLEMPLER APPLICATIONS

PRIOR-FREE MECHANISMS

- In the optimal auction, reserve price r^* is set so $r^* - \frac{1-F(r^*)}{f(r^*)} = 0$
- The seller needs to **know $\boldsymbol{F}$** in order to compute r^*
- We want a mechanism that optimizes for seller revenue **without knowing $\boldsymbol{F}$**

SINGLE-SAMPLE MECHANISM

- Mechanism:
  - Pick a **reserve bidder** $i \in N$ uniformly at random
  - Run a second price auction on the other agents $N \setminus \{i\}$ with reserve price v_i
- This mechanism is:
  - Prior-independent
  - Truthful in dominant strategies

SINGLE SAMPLE VS OPTIMAL

The Single-Sample mechanism generates at least $\frac{n-1}{2n}$ *of the revenue of the optimal auction*

- Removing an agent from the optimal auction loses at most $\frac{1}{n}$ revenue
- A random reserve price is a good approximation to the optimal reserve price
  - Fix reserve bidder j and non-reserve bidder i
  - i experiences a 2^{nd} price auction with reserve price $\max\{t, v_j\}$
  - Shown via extension of geometric argument

OTHER APPLICATIONS

- Optimal crowdsourcing contests
  - Each agent has skill v_i and can spend effort e_i to produce good with quality $p_i = v_i e_i$
  - A principal posts a monetary reward to buy the good from **one agent**
  - Goal is to maximize the **quality** of the chosen good
  - Bulow-Klemperer used to provide prior-independent mechanism
- Simple versus Optimal Mechanisms
  - 2^{nd} price auction with anonymous reserve generates at least $\frac{1}{4}$ of the optimal revenue
  - Other lower-bounds on revenue generated by “simple” mechanisms

CONCLUSION

- **Bulow-Klemperer**: Adding one bidder is better than running the optimal auction
- **Single-Sample Mechanism**: Prior-free auction that approximates the optimal auction
- **Other Applications**: Optimal crowdsourcing, simple vs optimal auctions, etc.

Can we do better than the optimal auction?

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