CPSC 532L: Models of Strategic Behaviour **Spring 2023**

Assignment #8

Due: March 28th, 2023, 1:00 pm

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TA: Narun Raman

1 Bayesian Games

Student 1	Student 2	Probability
b_1	b_2	0.1
b_1	$\neg b_2$	0.6
$\neg b_1$	b_2	0.1
$\neg b_1$	$\neg b_2$	0.2

Table 1: The common prior joint distribution on student preferences. b_i means that student i prefers to go to the bar, $\neg b_i$ means she prefers to work in the lab.

Problem 1.1. [20 points] Consider a problem where two students must simultaneously decide between working on their research in their (separate) offices and going to Koerner's. Each student has a preference for one of the two choices. The students don't know each other's preferences, but know that they are drawn from a commonly known joint distribution. This distribution is described in Table 1. Starting from a baseline utility of zero, a student gains 2 units of utility if she goes to the place that she prefers. However, the students are working on a course project together, and so both students lose 3 units of utility if they both attend the bar and reveal to each other that they were slacking off (independent of whether they gained 2 units of utility based on their preference). Thus, for example, if they both prefer bar, and they both go to the bar, they each get a utility of $0 + 2 - 3 = -1$.

(a) [**5 points**] Model the setting as a Bayesian game. Recall that you need a set of agents N, a set of actions A, a set of types for each agent Θ_i, a probability function mapping from one agent's type to a distribution over the types of the other agent(s) $p_i : \Theta_i \to \Delta(\Theta_{-i})$, and a payoff function for each agent mapping from the agents' joint actions and types to a real number $u_i : A \times \Theta \to \mathbb{R}$. Denote by B and L the actions of going to the bar and staying in the lab, respectively. Let $N = \{1, 2\}$ be the set of agents, G the set of games, $\Theta = \Theta_1 \times \Theta_2$ the set of joint agent types, and $I = \{I_1, I_2\}$ the partitions over games for the two agents. Your entire answer can be a figure similar to Figure 6.7 in the textbook, which shows the games, the common prior, and the partitions of the agents.

(b) [**5 points**] Find all Bayes-Nash equilibria of this game.

(c) [**5 points**] Draw the payoff matrix of the induced normal form of the game and justify why your equilibrium/equilibria hold(s). Explicitly state the meaning of an action in the induced normal form game; please write the actions in alphabetical order.

(d) [**1 point**] What is the *ex-ante* expected utility to player 1 of the strategy profile (LB, BL)? ("not enough information" is a potential answer)

(e) [**1 point**] What is the *ex-interim* utility to player 1 of the strategy profile (LB, BL) if player 1 has type b_1? ("not enough information" is a potential answer)

(f) [**3 points**] What is the *ex-post* utility to player 1 of the strategy profile (LB, BL) if player 1 has type b_1? ("not enough information" is a potential answer)

2 Bayesian Persuasion

Problem 2.1. [10 points] A recruiter heard that many of Kevin's past students were excellent. However, the recruiter doesn't know if Kevin's current students, e.g. Narun, are excellent. The recruiter asks Kevin for his recommendations. Assume the students can either be $\Omega = \{(\mathbf{l})lazy, (\mathbf{a})average, (\mathbf{e})excellent\}$ probabilities $\mu_0 = (\frac{1}{2}, \frac{1}{4}, \frac{1}{4})$. The recruiter will hire a student if she believes they are at least average with probability $\geq \frac{3}{5}$. If the recruiter hires a student, she will have to choose a salary $x \in [\underline{x}, \bar{x}]$. The salary is linearly increasing with the conditional probability of the student being excellent, up to a cap of $\bar{x}$ reached at probability $\frac{1}{2}$. Kevin obviously wants the best for his students, wanting to maximize their expected salaries:

$$V(\mu) = \mathbf{1}_{\{\mu(a)+\mu(e)\geq\frac{3}{5}\}} min\{\bar{x}, \underline{x} + 2\frac{\mu(e)}{\mu(e)+\mu(a)}(\bar{x} - \underline{x})\}$$

(a) [**1 point**] Who is the receiver and who is the sender in this scenario?

(b) [**2 points**] What is the average expected salary if Kevin provides the recruiter with no information?

(c) [**2 points**] What is the average expected salary if Kevin provides the recruiter with perfect information?

(d) [**5 points**] What is a signalling strategy that Kevin can give such that the average expected salary is higher than that of perfect information? What is the average expected salary in this case?

Academic Honesty Form

For this assignment, it is acceptable to collaborate with other students provided that you write up your solutions independently. The only reference materials that you can use are the course notes and textbook, and the reference textbooks listed on the course web page. In particular, getting help from students or course materials from previous years is not acceptable.

List any people you collaborated with:

1.
2.
3.

List any non-course materials you refered to:

1.
2.
3.

Fill in this page and include it with your assignment submission.