

Assignment #7

Due: March 21st, 2023, 1:00pm

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1 Ranking with a Distance Measure

Problem 1.1. [21 points] Let $d(\succ, \succ')$ denote the distance between two rankings \succ and \succ' , defined as the number of pairs o, o' for which $o \succ o'$ and $o' \succ' o$. For example, $d(A \succ B \succ C \succ D, C \succ B \succ A \succ D) = 3$ because the rankings disagree on (A, B) , (A, C) and (B, C) . Let \succ_i denote the preferences of agent i . Define the cost of a ranking $c(\succ \mid \succ_1, \succ_2, \dots)$ as the total distance between that ranking and the preferences of the agents:

$$c(\succ \mid \succ_1, \succ_2, \dots) = \sum_{i=1}^n d(\succ_i, \succ).$$

Now, we say that a social welfare function M is cost-minimizing if it chooses a ranking \succ that minimizes the cost. That is,

$$M(\succ_1, \succ_2, \dots) \in \arg \min_{\succ} c(\succ \mid \succ_1, \succ_2, \dots).$$

- (a) [5 points] Consider the following preferences:

$$\begin{aligned} D \succ_1 A \succ_1 B \succ_1 C \\ D \succ_2 A \succ_2 C \succ_2 B \\ C \succ_3 B \succ_3 D \succ_3 A \\ C \succ_4 D \succ_4 A \succ_4 B \\ C \succ_5 D \succ_5 A \succ_5 B \end{aligned}$$

What outcome will Borda select? What ranking will M select?

- (b) [8 points] Prove that if a Condorcet winner exists, it will be ranked first by M . Similarly, prove that if a Condorcet loser exists, it will be placed last by M . (For simplicity, you may assume that the number of agents is odd.)
- (c) [8 points] Consider the game that arises when a group of agents agree to use M to make a common decision. Each agent reports (simultaneously, by secret ballot) a ranking and the outcome ranked highest by the social welfare function is chosen. (When there are multiple cost-minimizing rankings, assume that M breaks ties lexicographically, e.g., alphabetically). Demonstrate (by providing a counterexample) that in this game it is not a (very weak) dominant strategy for the agents to report their true preferences, even for the case of three agents and three outcomes.

2 Strategic Voting

Problem 2.1. [4 points] Given the following Borda vote ordering, assume the same point allocation as in class (where the top choice get 3 points, 2nd get 2 points, 3rd gets 1 point and 4th gets no points).

Voter	1st	2nd	3rd	4th
Alice	D	A	C	B
Bob	A	B	C	D
Charlie	B	C	D	A

- (a) [2 points] How should Alice manipulate her vote to secure a more favorable outcome?
- (b) [2 points] In the Borda setting what conditions must hold for the voting scheme to be susceptible to the “Compromise” manipulation?

Problem 2.2. [6 points] Manipulable Voting Schemes

- (a) Recall from class in the “physics experiment” we saw that the least “fair” voting scheme was also the least manipulable. In this context you may treat fairness as the degree to which a voting scheme reflects the voters’ preferences. Do you believe there is a correlation between how fair a voting scheme is and how manipulable it is?
- (b) Suppose Sophie came up with a new and brilliant voting scheme and has proven it is NP-hard to manipulate. Should you trust Sophie’s new algorithm in your upcoming important election? Why, or why not?

Academic Honesty Form

For this assignment, it is acceptable to collaborate with other students provided that you write up your solutions independently. The only reference materials that you can use are the course notes and textbook, and the reference textbooks listed on the course web page. In particular, getting help from students or course materials from previous years is not acceptable.

List any people you collaborated with:

- 1.
- 2.
- 3.

List any non-course materials you referred to:

- 1.
- 2.
- 3.

Fill in this page and include it with your assignment submission.