CPSC 532L: Models of Strategic Behaviour **Spring 2023**

Assignment #6

Due: March 14th, 2023, 1:00pm

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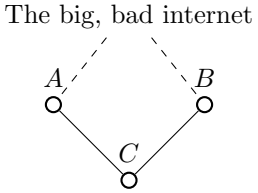
TA: Narun Raman

1 Network Security Game

Problem 1.1. **[10 points]** Consider the following network graph:



Each node (except for "the big, bad internet") represents one agent's computer in the network. Each agent can pay cost 2 to "protect" his computer from attackers. If a computer is protected, then it cannot be attacked, and other computers cannot be attacked through it. Any node will be attacked if it is not protected and is reachable (i.e., there is a path from the internet to that node, which does not pass through any protected nodes). Each agent i gets a payoff of v_i if his computer is not attacked, and zero if it is; $v_A = 3$, $v_B = 1$, and $v_C = 4$.

The three agents in this game want to decide which of their computers to protect and how to distribute the costs. (Do not treat any would-be attackers as part of the cooperative game.)

(a) **[3 points]** Write a characteristic-function representation of this game. Assume that agents who are not part of the coalition cannot be depended on to provide security.

(b) **[3 points]** Calculate the Shapley values for this game.

(c) **[4 points]** Characterize the core for this game.

2 Core-Selecting Auction

Problem 2.1. **[8 points]** A single seller has two pieces of home-theater equipment that he does not value, a TV and a sound system. He has three prospective buyers with the following valuations:

Agent	TV	Sound System	Both
S	0	0	0
B_1	2	5	6
B_2	4	2	4
B_3	1	3	8

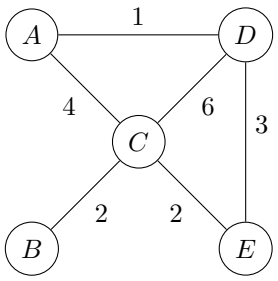
In the interest of fairness (and profit) the seller decides to use the core to determine how to distribute the goods and how much to charge for them. (Note, for this question we will assume that money and utility are identical.)

(a) **[5 points]** Write a characteristic-function representation of this game. (Note that the seller is an agent in the game.)

(b) **[3 points]** Identify an outcome (i.e., a way of distributing the goods and transferring money) that is in the core. Justify that your answer is consistent with the definition of core.

3 Compact Representations

Problem 3.1. **[6 points]** Consider the coalitional game represented by the following weighted graph:



(a) [**1 point**] What is the payoff for the coalition $\{B, C, D\}$?

(b) [**2 points**] Using the Shapley value, is the following payoff assignment a valid solution concept to the grand coalition of this game? Justify your answer.

$$\{A : 2.5, B : 1, C : 7, D : 6, E : 1.5\}$$

(c) [**3 points**] Show that the weighted graph representation is not complete by showing that it cannot represent the following majority voting game:

$$G = (N, v) \text{ where } N = \{1, 2, 3, 4\}, \quad v(S) = \begin{cases} 0, & \text{if } |S| \leq 1 \\ 1, & \text{if } |S| = 2 \\ 3, & \text{if } |S| = 3 \\ 10, & \text{if } |S| = 4 \end{cases}$$

Problem 3.2. **[4 points]** Consider the coalitional game represented by the following marginal-contribution net:

$$\begin{aligned} A &\mapsto 3 \\ B &\mapsto 4 \\ B \wedge C &\mapsto 5 \\ A \wedge B \wedge D &\mapsto 6 \end{aligned}$$

(a) [**1 point**] What is the Shapley value of player B ?

(b) [**1 point**] Consider we add a new rule $(\neg A \wedge C \mapsto -2)$, where $\neg A$ is a negative literal indicating what would happen in the absence of A. With this new rule, what is the payoff of the coalition $\{B, C\}$?

(c) [**2 points**] How does adding a rule with more complex boolean conditions to an otherwise basic form of the MC-net affect the trade-off between representational power, compactness, and ease of computation?

4 The Shapley Value in Machine Learning

Problem 4.1. [7 points] Applying the Shapley value

(a) [**3 points**] Suppose you have a set of machine learning classifiers. You use them to form an ensemble model, that is, a machine learning model that makes a prediction by aggregating the predictions of the individual classifiers. You want to know which classifiers in the ensemble are the most useful for prediction. How would you represent this problem as a coalitional game? Explain qualitatively what the "dummy player" Shapley axiom would mean in this context.

(b) [**4 points**] Consider the data valuation setting, and suppose you have three data points, x_1, x_2, and x_3. Let the performance of a model trained on some subset of $\{x_i\}$ be defined by:

$$\begin{aligned} P(\emptyset) &= 0 & P(\{x_1, x_3\}) &= 3 \\ P(\{x_i\}) &= 1, i = 1, 2, 3 & P(\{x_1, x_2\}) &= 0 \\ P(\{x_1, x_2, x_3\}) &= 2 & P(\{x_2, x_3\}) &= 2 \end{aligned}$$

- [**3 points**] What are Shapley values for each data point in the corresponding coalitional game? Which point is the least useful?
- [**1 point**] Is the least useful point a dummy player? Why or why not? *(Hint: think carefully about the definition of a dummy player).*

Problem 4.2. [3 points] Efficient Computation

This is the (weighted, constrained) linear regression definition of the Shapley value:

$$\begin{aligned} \phi_1^*, ..., \phi_n^* = \arg \min_{\phi_0, \phi_1, ..., \phi_n} & \sum_{S \subseteq N} w_S \left(\left[\phi_0 + \sum_{i \in S} \phi_i \right] - v(S) \right) \\ \text{subject to} \quad & \phi_0 = v(\emptyset) \\ & \phi_0 + \sum_{i \in N} \phi_i = v(N) \end{aligned}$$

where $w_S = \frac{|N|-1}{\binom{|N|}{|S|}|S|(|N|-|S|)}$.

(a) [**1 point**] How many different subsets S must the objective sum over?

(b) [**2 points**] In practice, the SHAP algorithm subsamples subsets S to approximate the objective.

- Will the estimated values still form an allocation of the overall performance measure $v(N) - v(\emptyset)$? Why or why not?
- We saw in the presentation that the Shapley axioms are not guaranteed to hold here. Is it possible to find a method that efficiently approximates the Shapley value but still guarantees that solutions will satisfy the axioms? Why or why not?

Academic Honesty Form

For this assignment, it is acceptable to collaborate with other students provided that you write up your solutions independently. The only reference materials that you can use are the course notes and textbook, and the reference textbooks listed on the course web page. In particular, getting help from students or course materials from previous years is not acceptable.

List any people you collaborated with:

1.

2.

3.

List any non-course materials you refered to:

1.

2.

3.

Fill in this page and include it with your assignment submission.